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A back-of-the-envelope analysis of house prices: Czech Republic, 2013-2021*

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Abstract

A simple practical method for quantitative analysis of house prices is proposed. Similar to consumer theory, housing demand is decomposed into changes in income and a relative price. The latter includes implicit costs of mortgage finance, determined by monetary policy and future disposable income growth and inflation expectations. The method is applied to the 63% increase in real house prices in Czechia, 2013-2021. The income effect accounts for 32% of the increase, implicit mortgage costs for another 20%. Most of the latter hinges on income growth expectations, reflecting the robust 2013-2020 economic recovery. Going forward, the paper explores hypothetical scenarios in light of the recent increase in mortgage rates to 5.33%. As an example, at long-term inflation expectations of 6%, a dire scenario of zero expected future growth in real disposable income leads to a decline in real house prices of 13%. However, if real income growth expectations remained unchanged from the boom period of 2013-2020, the increase in mortgage rates, at inflation expectations of 6%, would lead to only a modest drop in house prices. Across the various scenarios, the risks for house prices are nonetheless skewed downwards.

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1 Introduction

A day does not pass by in the Czech Republic that would not bring media reports on the housing market. And rightly so. Since 2013, the Czech housing market experienced one of the highest growth rates of house prices among the 38 OECD countries, surpassed only by Luxembourg and Hungary. In nominal terms, house prices grew by 78\% between 2013.Q1—the turning point after the Global Financial Crisis—and 2021.Q1; see Figure 1. This corresponds to 9.75\% per year on average. Adjusted for CPI inflation, the increase in real house prices between 2013.Q1 and 2021.Q1 was 63\%. To explain the fast growth, market analysts and commentators point to the usual suspects: low interest rates, slow and bureaucratic process for construction permits, abundant savings, inflation fears, etc. Real estate professionals and business economists seem to also almost universally agree that, unless construction significantly picks up, the only way for house prices is up. What is lacking in this debate, however, is a systematic quantitative analysis, evaluating the various factors that have contributed to the increase in house prices so far, and assessing their likely effects going forward.¹

The contribution of the paper is twofold. First, to offer a quantitative account of the 2013-2021 boom in the Czech Republic house prices and to evaluate future risks. Second, to propose a practical method for assessing movements in house prices that would be of use to market analysts. The method has theoretical underpinnings in the general equilibrium model of Garriga, Kydland, and Šustek (2017). Abstracting from the technical (and some substantive) dimensions of the dynamic stochastic problem describing a house purchase, as well as from many aspects of the equilibrium, the method provides a simple back-of-the-envelope calculation for the contribution of various factors to house price movements. The advantage to practitioners is that, while grounded in theory, the method does not require any

¹To the best of my knowledge, the only other study offering some quantitative assessment of the recent trends is the work done by the Czech National Bank as a part of the Financial Stability Report 2020-21. The approach followed by this paper in relation to the work done by the Czech National Bank is discussed below. For a quantitative analysis preceding the current boom, see, for instance, Cadil (2009) and Hlaváček and Komárek (2011).
knowledge of the techniques used in the state-of-the-art academic research and can be easily implemented in a spreadsheet. The limitation is that due to the simplifications involved, the method gives only a first-pass, back-of-the-envelope, account of the movements in house prices.

The method centers around an optimality condition for a house purchase of a hypothetical household, assumed to approximate reasonably well the marginal home buyer. Effectively, it compares the economic conditions of the marginal home buyer in two different periods. Critically, the household finances the home purchase with a mortgage and faces financial markets that are sufficiently incomplete. As in consumer theory, the optimality condition relates the relative price of the house to the marginal rate of substitution between consumption and future housing services. Here, the relative price consists of the actual house price and the cost of financing the house purchase. Under some additional assumptions, the optimality condition can be used to decompose the change in house prices between two periods into two effects, an income effect and an implicit mortgage cost effect, and their deeper economic determinants. The total change in real house prices accounted for by the method is the sum of the two effects (adding the CPI inflation rate on top of the two effects then gives the change in house prices in nominal terms).

The income effect is straightforward and relates a change in house prices between two periods to a change in household income. Specifically, the income effect is given as the percentage change in income of the marginal home buyer less the percentage change in the aggregate housing stock per household.

The implicit mortgage cost effect is more involved conceptually, but also easy to implement in practice. It requires calculating the present value of mortgage payments expected to be made by the marginal home buyer over the term of the loan. However, due to an assumed limited access of the home buyer to financial markets, the discount rates are not based on market interest rates but given by internal values the household attaches to those payments. A payment that is made in a period with relatively scarce consumption is assigned a higher
value, as such a payment is more burdensome in terms of foregone consumption, than a payment in a period with abundant consumption.\(^2\) To compute the internal discount factors, the analyst only needs to construct a sequence of the household’s future consumption (and take a stand on the functional form of the household’s utility function). This involves specifying expected future income growth, various mandatory expenses, and inflation, and calculating the mortgage payments on the basis of given mortgage rates and the term of the loan. The present value can be expressed in a form that looks like an implicit distortionary tax (a wedge) in the optimality condition for the house purchase. Consequently, changes in this wedge work like changes in a tax on the house purchase, similar to a stamp duty tax. The loan-to-value (LTV) ratio, the mortgage rate, the statutory tax code, and expectations of future income growth and inflation, among other factors, affect this implicit tax, making the house purchase more or less expensive. The \textit{implicit mortgage cost effect} is a percentage change in house prices between two periods equal to the change in the implicit tax, with a minus sign.\(^3\) The qualifier “implicit” is used to stress that the mortgage cost is not determined purely by market interest rates but is specific to the marginal home buyer.

When applied to the Czech Republic 2013.Q1-2021.Q1, under a baseline calibration, the income effect produces a 32% increase and the implicit mortgage cost a 20% increase in real house prices between the two periods. Thus, together, the two effects generate a 52% increase in real house prices between 2013.Q1 and 2021.Q1, leaving 11% of the 63% increase in the data to additional factors (for instance, the abolishment of the 4% stamp duty tax, demand for more space following the outbreak of COVID-19, a decline in risk premia, or search and matching frictions in the housing market that have led to a tighter market for buyers). When CPI inflation between the two periods is taken into account, the joint effect is 67% (vs. 78% in the data). The implicit mortgage cost effect is responsible for the often-cited detachment

\(^2\)It seems reasonable to assume that the typical home buyer does not have the same access to financial markets as the institutional investors holding mortgage debt (eg, banks, pension funds), who price mortgages on the basis of arbitrage with other instruments in financial markets. Consequently, the discount factors of the home buyer and mortgage investors differ, with the latter being based on market interest rates, as in standard present value calculations, but the former differing from market rates.

\(^3\)A percentage point decline in the tax translates to a percentage increase in real house prices.
of house prices from people’s income.\footnote{The frequently-cited supply constraints, such as the slow and bureaucratic planning and permits process in the Czech Republic, are implicitly taken into account in the above figures—the calculation is produced under the assumption that the housing stock per household remained constant during the period under investigation, which was roughly the case in the data.}

What economic forces are behind these figures? The income effect is due to the observed change in household income, which has grown by 32\% in real terms in the eight-year period under investigation, when measured by the average salary. The housing stock per household stayed roughly constant. The 20\% implicit mortgage cost effect during this period had two major drivers: a decline in mortgage rates and an increase in income growth expectations, with a bulk of the effect being due to the latter factor. Whereas mortgage rates are observed, income growth expectations are not. The baseline findings are produced under the assumption that during 2013.Q1-2021.Q1, household expectations for the annual income growth were 3.4\%, the average real wage growth during 2014-2020, an increase from 0.3\% for the period 2009-2013. Essentially, expectations of higher future income led to a substantial reduction in the implicit mortgage cost for the \textit{marginal} buyer.\footnote{As the focus of the paper is on house prices, the discussion centers around the marginal home buyer. Issues related to inequality in the housing market (eg, decreased affordability for those to the left from the marginal buyer in the distribution of income and wealth) are not addressed here.} Experimenting with alternative plausible values for income growth expectations produces the increase in real house prices due to the implicit mortgage cost effect from 17\% to about 23\%. The above estimates are fairly robust to alternative calibrations of the other inputs entering the calculation, including the dwelling size and household characteristics.\footnote{The importance of expectations about fundamentals echoes Garriga, Manuelli, and Peralta-Alva (2019), although the variables about which expectations are formed are different—income growth here, whereas credit conditions in their paper. Piazzesi and Schneider (2009) use survey evidence directly on house price expectations.}

More construction could not have prevented the dramatic increase in house prices so far.\footnote{That does not mean that more construction of social or municipal housing could not ease housing constraints of those far to the left of the marginal buyer in the income and wealth distribution.} Based on the back-of-the-envelope estimates in this paper, just to undo the implicit mortgage cost effect, and thus let house prices grow in line with income, construction would have had to increase to unrealistic levels during 2013-21. To undo half of the effect, annual construction would have had to be about 75\% higher than the average annual completions
seen during that period.

Re-calibrating the inputs to 2021.Q1 values, the method can be used to get a sense of the range of possible changes in real house prices going forward, which again consist of the income and implicit mortgage cost effects (and CPI inflation to convert real house price growth to nominal terms). The income effect will depend on the actual growth rate of household income and the housing stock per person. The size and direction of the implicit mortgage cost effect is somewhat more involved. The paper provides scenario analysis for the implicit mortgage cost effect under various combinations of long-term inflation expectations and real income growth expectations, taking as given the recent increase in mortgage rates to 5.33%. Assigning probabilities to the different scenarios, and thus providing forecasts for future house prices, is however beyond the scope of the paper. As an example, at long-term inflation expectations of 6%, a dire scenario of zero expected future real income growth leads to a decline in real house prices of 13%. However, if income growth expectations remained unchanged from the boom period of 2013-2020, the recent increase in mortgage rates, at the long-term inflation expectations of 6%, would lead to only a modest, 1.5%, drop in real house prices. The most severe fall in house prices would result from the combination of low inflation and real income growth expectations. In contrast, a combination of inflation expectations sufficiently exceeding the mortgage rate and optimistic outlook for future real income growth would even push house prices above the 2021 levels. Nonetheless, taking all the scenarios considered together, the risks for house prices are skewed to the downside.\(^8\)

One possible reason for the absence of any quantitative analysis in the current debate on the Czech housing market is that such investigation is demanding, as the past 15 years or so of academic research into the determinants of house prices demonstrates. The quantitative techniques used to study house prices and the macroeconomy in academic research are technically involved and unaccessible to most practitioners (and even for those who have mastered them, the state-of-the-art approach is highly time intensive).\(^9\) This leaves market

\(^8\)An implicit assumption throughout the paper is that house prices are determined by fundamentals and expectations about fundamentals. That is, bubbles, while a possibility, are not considered here.

\(^9\)Davis and Van Nieuwerburgh (2015), Guerrieri and Uhlig (2016) and Piazzesi and Schneider (2016) pro-
economists and real estate analysts with essentially three options: reduced-form regressions, present-value discounting of future rents (a dynamic Gordon model), and static affordability ratios, such as debt-to-income (DTI) and debt-servicing-to-income (DSTI). The methodology employed by the Czech National Bank as a part of its macro-prudential assessment follows the second and third approaches (see Plašil and Andrle, 2019). While each method has its merits, they are not without important limitations. Regression analysis requires a long enough sample in a stationary environment. The dynamic Gordon model has been shown to critically depend on a “housing premium”, a time-varying residual by which observed interest rates have to be adjusted for the present-value calculations to fit the time series data on house prices and rents (Campbell, Davis, Gallin, and Martin, 2009). And affordability ratios ignore the fact that a house purchase decision is inherently dynamic, with contractual mortgage payments made over many years, during which aggregate and household-specific economic conditions may differ from the current conditions. The method proposed here can be viewed as combining elements of both the dynamic Gordon model and affordability ratios by making the affordability considerations dynamic.\footnote{As an example, while a given house purchase may result in a high current DTI or DSTI ratio, if the household knows it will receive a significant and permanent pay rise in a one year’s time, dynamically the purchase is not as financially burdensome as it may appear on the basis of the static ratios. Income growth expectations are thus an important determinant of affordability in a dynamic setting.} Alternatively, the method can be viewed as a dynamic Gordon model of an agent whose access to financial markets is limited.

The paper proceeds as follows. The next section explains the method. Section 3 calibrates the inputs required for the calculation to the Czech data and the 2013.Q1 conditions specifically. Section 4 reports the findings. Section 5 concludes.

# The method

This section provides a theoretical framework for the method. A practitioner not interested in the theoretical background may skip this section and move on to Section 3 without loosing the ability to use the method in practice.
2.1 Overview

The method can be explained using a three-period example, which I borrow (and appropriately modify) from Garriga et al. (2017). An extension to longer time horizons is straightforward and is used in the actual application of the method.

There are four implicit assumptions. First, house prices are determined by fundamentals. That is, bubbles are excluded, meaning that the method excludes the possibility that houses are valued even if they never pay any dividends either in the form of rent or utility of owner-occupiers. Second, the marginal home buyer can be described as a household that finances the house purchase with a mortgage but has otherwise limited access to financial markets.\textsuperscript{11} Third, the household is assumed to make a once-and-for-all house purchase. This is at the opposite extreme of the more typical assumption, according to which the household trades in the housing market every period. In reality, household decisions to enter the housing market are infrequent and both state and time dependent (an example of the former is divorce, an example of the latter is a person’s life-cycle). The implicit assumption is thus that the once-and-for-all decision is a reasonable approximation to the otherwise complex dynamic stochastic optimisation, which takes into account the various future eventualities. The less likely the future eventualities involving a sale of the property are, or the further out in the life-cycle they are expected to occur, the more appropriate this assumption is.

The same applies to the mortgage, where state- and time-dependent default, pre-payment, equity withdrawal, or refinancing are abstracted from. An explicit treatment of the above complexities would require the use of dynamic programming and numerical methods, making the method unaccessible to a large spectrum of practitioners. The trade-off is thus between accessibility and thoroughness of the analysis.\textsuperscript{12} And finally, housing is a homogenous and

\textsuperscript{11}Available empirical research covering a few OECD countries (though, unfortunately, not the Czech Republic) suggests that the typical homeowner is financially constrained, exhibiting hand-to-mouth behavior; see, for instance, Kaplan and Violante (2014), Kaplan, Violante, and Weidner (2014), Cava, Hughson, and Kaplan (2016), Di Magio, Kermani, Keys, Piskorski, Ramcharan, Seru, and Yao (2017), Flodén, Kilström, Sigurdsson, and Vestman (2018), Cloyne, Ferreira, and Surico (2020), and Slačálek, Tristani, and Violante (2020). Greenwald (2018) offers evidence that home purchases are made close to a threshold LTV ratio, after which the mortgage terms become substantially more expensive.

\textsuperscript{12}For an explicit treatment of some of the complexities ignored here see, among others,
one-dimensional commodity, whose quantity can be summarized by an index \( h \). The housing market is integrated, meaning that any quantity of the existing housing stock can be used either for rental purposes or owner occupancy. In principle thus, the household can buy for owner occupancy the housing it is renting. Housing supply can therefore be summarized by the aggregate housing stock divided by the number of households.

In describing the method, I skip some subtle theoretical underpinnings discussed in detail in Garriga et al. (2017), which have to do with financial market incompleteness and mortgage pricing in general equilibrium, where both borrowers and lenders are taken into account; see also Garriga, Kydland, and Šustek (2021). Mild technical assumptions are laid out along the way.

2.2 Details

Consider three periods in the life of a household, denoted by \( j = 0, 1, 2 \), in a deterministic setting. Calendar time is denoted by \( t \). In each of the three periods, the household gets real income \( \tilde{w}_{t+j} \), which is net of various taxes/subsidies and mandatory expenses, such as utilities and maintenance. The elements making up \( \tilde{w}_{t+j} \) are treated explicitly in the next section describing calibration, and can be as detailed as desired by the analyst, but are inessential here. In period \( j = 0 \), the household is assumed to be a renter, have no outstanding debt, and have sufficient funds set aside for a downpayment on a house purchase. In \( j = 0 \), the household makes a once-and-for-all house purchase, financing a fraction \( \theta_t \) of the purchase with a loan and a fraction \( 1 - \theta_t \) with the downpayment. The house lasts for periods \( j = 1, 2 \), during which the household enjoys its services, then it fully depreciates (this reflects the notion of a once-and-for-all purchase, as there is no sale of the property and a realisation of capital gains/losses). During the periods \( j = 1, 2 \), the household also repays the mortgage, with interest. As is the case with standard mortgage loans, the mortgage payments are

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set in nominal terms, making it is necessary below to distinguish between real and nominal variables.\textsuperscript{13}

The lifetime utility function of the household is $U_t = \sum_{j=0}^{2} \beta^j \log(c_{t+j}) + \log(\xi h_t^R) + \log(h_t) \sum_{j=0}^{2} \beta^j$, where $\beta$ is a discount factor, $c_{t+j}$ is non-housing consumption (henceforth referred to simply as “consumption”), $h_t^R$ is the amount of housing the households is renting in period $j = 0$, $\xi \in (0, 1]$ controls the extent to which housing when rented provides fewer services than the same housing when owned (eg, due to the inability to decorate the place to one’s own taste), and $h_t$ is the amount of housing purchased for owner-occupancy in periods $j = 1, 2$.\textsuperscript{14} The log function for per-period utility is considered for convenience. Nonetheless, the arguments of Gomme and Rupert (2007) ensure that the results below apply also to any per-period utility function in consumption and housing, $u(a(c, h))$, in which the aggregator $a$ is homogenous of degree one.

For the ease of the exposition, the household is taken to be a complete hand-to-mouth household, with no access to financial markets other than the mortgage market.\textsuperscript{15} The household’s three per-period budget constraints are $c_t + r_t h_t^R = \tilde{w}_t + \omega_t - (1 - \theta_t) q_t h_t$, $c_{t+1} = \tilde{w}_{t+1} - m_{t+1}/P_{t+1}$, and $c_{t+2} = \tilde{w}_{t+2} - m_{t+2}/P_{t+2}$, where $r_t$ is the real rental rate, $\omega_t$ denotes the funds set aside for the downpayment, $q_t$ is the real house price, $m_{t+1}$ and $m_{t+2}$ are nominal mortgage payments, and $P_t$ is the general price level in period $t$. Let $l_t = \theta_t p_t h_t$ denote the nominal loan size, where $p_t = P_t q_t$ is the nominal house price. The nominal mortgage payments are determined as $m_{t+1} = (i_t + \phi) l_t$ and $m_{t+2} = (i_t + 1)(1 - \phi) l_t$, where $i_t$ is the market nominal mortgage interest rate at the time of origination and $\phi$ is the amortisation rate; that is, a fraction $\phi$ of the loan is repaid in period $j = 1$, with the remaining fraction, $(1 - \phi)$, in period $j = 2$. Mortgage contracts have such an amortisation

\textsuperscript{13}The case of a buy-to-let investor is similar. The difference is that the owner-occupier’s housing services are replaced by rent. In a rental market equilibrium, rent is in turn related to the housing services enjoyed by the tenant.

\textsuperscript{14}The parameter $\xi$ is only incorporated to provide a theoretical justification for why the household may want to own housing even when it is more expensive then renting. Otherwise the parameter is unimportant.

\textsuperscript{15}The arguments go qualitatively through also when the household can borrow/save, as long as the financial markets accessible to the household are sufficiently incomplete (see Garriga et al., 2017, 2021). The case of a pure hand-to-mouth household, however, simplifies the exposition. The next section discusses how the effect of some borrowing and saving can be taken into account in the quantitative application of the method.
schedule implicitly built in. It is possible to let the interest rate on the loan change over its term, thus specifying different $i$'s for $j = 1, 2$. To keep the exposition simple, I set $i_{t,j=1} = i_{t,j=2} = i_t$.

The method is based on the optimality condition for $h_t$ to the problem of maximising $U_t$ subject to the three per-period budget constraints. The optimality (first-order) condition takes the form

$$U_{c,t} \left[ (1 - \theta_t) + \theta_t v_t \right] q_t = \beta (1 + \beta) U_{h,t}, \quad (1)$$

where $U_{c,t} = 1/c_t$ denotes the marginal utility of $c_t$ and $U_{h,t} = 1/h_t$ denotes the marginal utility of $h_t$, received in the two periods $j = 1, 2$. In (1), the expression $q_t^*$ is the relative price of the house, consisting of $q_t$ and the expression in the square brackets, which summarizes the financing cost. The financing cost is a weighted average of the cost of the downpayment and the cost of the mortgage. The cost of the downpayment is equal to one (as one unit of wealth given up today has a value equal to one), while the cost of the mortgage, $v_t$, is equal to the present value of future mortgage payments (on one unit of the loan taken out today)

$$v_t = \mu_{t,t+1} \frac{i_t + \phi}{1 + \pi_{t+1}} + \mu_{t+1,t+2} \frac{(i_t + 1)(1 - \phi)}{(1 + \pi_{t+1})(1 + \pi_{t+2})}.$$  

Here, $\mu_{t,t+1} \equiv \beta(U_{c,t+1}/U_{c,t}) = \beta(c_t/c_{t+1})$ is the marginal rate of intertemporal substitution between periods $t$ and $t + 1$ (similarly for $\mu_{t+1,t+2}$). It is the implicit price the homeowner attaches to the cash outflows in period $t + 1$, as of period $t$. If $U_{c,t+1} = U_{c,t}$, the price would be simply the discount factor $\beta$, which captures impatience. The marginal utilities adjust the discount factor for the relative amount of consumption enjoyed in different periods. The lower is consumption in a given period, the more valuable are the cash outflows in that period to the household, as instead of servicing the mortgage, the resources could be used to increase the relatively scarce consumption. In other words, the lower is consumption, the more burdensome are the mortgage cash outflows in that period. The present value $v_t$ is

\[\text{Strictly speaking, } \phi \text{ depends on the mortgage rate, and is such that } m_{t+1} = m_{t+2}, \text{ but this detail is ignored here. See Garriga et al. (2017).}\]
thus determined the same way as in standard present value calculations, except that the cash flows are evaluated with the household’s internal discount factors, rather than discount factors based on market interest rates.

The optimality condition (1) can be expressed in a slightly different way as

\[
(1 + \tau_{H,t}) q_t = \beta (1 + \beta) \frac{U_{h,t}}{U_{c,t}}
\]

where

\[
\tau_{H,t} = -\theta_t \left\{ 1 - \left[ \mu_{t,t+1} \frac{i_t + \phi}{1 + \pi_{t+1}} + \mu_{t+1,t+2} (i_t + 1)(1 - \phi) \right] \right\}.
\]

An advantage of writing the optimality condition this way is that \(\tau_{H,t}\), which depends on the present value \(v_t\), appears in the optimality condition (2) like a tax on the house purchase, similar to a stamp duty tax (except that it can be negative). We can thus view the optimality condition for the house purchase like any other optimality condition in consumer theory, where the marginal rate of substitution between two goods, \(\beta (1 + \beta) \frac{U_{h,t}}{U_{c,t}}\), is equated to the relative price, \(q^*_t\). The relative price consists of the actual price, \(q_t\), and a distortionary tax, \(\tau_{H,t}\). Observe that \(\tau_{H,t}\) depends on a number of other variables. Specifically, on the LTV ratio, \(\theta_t\), and thus on macroprudential policy, and on nominal variables \(i_t\), \(\pi_{t+1}\), and \(\pi_{t+2}\), and thus on monetary policy (\(i_t\) can be also affected by the competition among mortgage lenders). Furthermore, through the discount factors, \(\mu_{t,t+1}\) and \(\mu_{t+1,t+2}\), the tax depends on future consumption growth, and thus (through the budget constraints) on future income growth.

### 2.3 Income and implicit mortgage cost effects

The optimality condition (2) is used to carry out a decomposition of a change in house prices between periods \(t\) and \(t + T\) into two effects and their deeper economic determinants. Effectively, it compares the economic conditions of the marginal home buyer in the two
periods. At the aggregate level, all variables entering equation (2) are endogenous. A certain structure is therefore imposed on the dynamics of the variables to facilitate the decomposition in a technically undemanding way. Because of the simplifying assumptions, the decomposition is only a back-of-the-envelope calculation. To prepare the ground, I start with two simple propositions.

**Proposition 1:** $\tau_{H,t}$ is scale invariant. It depends on expected consumption growth from period $t$ on, but not on the level of consumption in period $t$.

To show this result, let $c_{t+1} = (1 + \lambda_{1,t})c_t$ and $c_{t+2} = (1 + \lambda_{1,t})(1 + \lambda_{2,t})c_t$, where $\lambda_{1,t}$ and $\lambda_{2,t}$ are the growth rates for periods $j = 1$ and $j = 2$, starting with period $t$. Substituting into $\mu_{t,t+1}$ and $\mu_{t+1,t+2}$ shows that $\tau_{H,t}$ depends on the growth rates but not on $c_t$.

Next, consider a balanced growth path defined as a situation in which the LTV ratio, the inflation rate and the nominal interest rate are constant and all other variables grow at some constant growth rates.

**Proposition 2:** The optimality condition (2) satisfies a balanced growth path along which a percentage change in real house prices is proportional to the percentage change in income.

To show this result, start with Proposition 1, which implies that on the balanced growth path, $\tau_{H,t}$ is constant. Next, consider two periods $t$ and $t+T$. Consider growth rates between the two periods, $\eta$ and $\kappa$, such that $c_{t+T} = (1 + \eta)c_t$, $\tilde{w}_{t+T} = (1 + \eta)\tilde{w}_t$, $\omega_{t+T} = (1 + \eta)\omega_t$, and $h_{t+T} = (1 + \kappa)h_t$. Further, let $q_{t+T} = (1 + \eta)/(1 + \kappa)q_t$ and $r_{t+T} = (1 + \eta)/(1 + \kappa)r_t$. Substituting $c_{t+T}$, $w_{t+T}$, $\omega_{t+T}$, $h_{t+T}$, $q_{t+T}$, and $r_{t+T}$ into the budget constraint in period $j = 0$ and equation (2) shows that if these equations hold in period $t$ they also hold in period $t+T$. The growth rate of house prices is $q_{t+T}/q_t = (1 + \eta)/(1 + \kappa)$, which is proportional to the growth rate of income as claimed.

The two propositions are used to decompose a change in $q_t$ between two periods into a change due to income (and the aggregate housing stock per household) and a change due to the household’s internal cost of mortgage finance. I will refer to the first effect as the
income effect and to the second effect as the implicit mortgage cost effect. I use the qualifier “implicit” to stress that \( \tau_{H,t} \) depends on the household’s future consumption (income) growth and future inflation, not just the variables characterizing the mortgage contract, \( i_t \) and \( \theta_t \). I will use \( \Delta q_{w,t+T} \) to denote the part of the percentage change in house prices between \( t \) and \( t + T \) determined by the income effect and \( \Delta q_{\tau_{H,t+T}} \) to denote the part determined by the implicit mortgage cost effect.

The starting point of the decomposition is to view equation (2) as an equilibrium condition. In this interpretation, we can think of \( \kappa \) as the growth rate in the housing stock per household, determined by factors not modelled here, such as the growth rate in permits for land development and construction costs (for the numerator) and socio-demographic factors (for the denominator).

The simplifying structure imposed on the dynamics of the variables entering equation (2) is that all changes in income, wealth, consumption, and the housing stock per household occur on a balance growth path. This assumption makes the decomposition easily operational and provides the most fundamental sense in which house prices, income, the housing stock, and the population are related to each other. The implicit mortgage cost effect is then assumed to account for the deviations of house prices from the balanced growth path.

The income effect is related to the balanced growth path and given by \((1 + \eta)/(1 + \kappa)\). Taking logs, \( \Delta q_{w,t+T} \approx \eta - \kappa \), where \( \eta \) is the observed growth rate of income between \( t \) and \( t + T \) and \( \kappa \) is the observed growth rate of the housing stock per household.

The implicit mortgage cost effect is given by the change in \( \tau_{H,t} \), with a negative sign. For \( c_t \) and \( h_t \) given by the balanced growth path, Proposition 1 and equation (2) imply that, after taking logs, \( \Delta q_{\tau_{H,t+T}} \approx -\Delta \tau_{H} \), where \( \Delta \tau_{H} = \tau_{H,t+T} - \tau_{H,t} \) and \( \Delta q_{\tau_{H,t+T}} \) is orthogonal to \( \Delta q_{w,t+T} \). In the implementation of the method, \( \tau_{H,t} \) is calculated exactly as in equation (2), except that the number of periods in the present value calculation is an arbitrary \( J \) and \( \tilde{w}_{t+j} \) includes as many details determining after-tax income and mandatory expenses as desired by the analyst. The calculation of \( \tau_{H,t} \) can be done in a spreadsheet. It is then
It is straightforward to quantitatively evaluate how $\tau_{H,t}$, and thus house prices, change as one of the inputs entering $\tau_{H,t}$ changes. One can thus decompose $\Delta q_{t+T}$ further into the contribution of the different economic factors embedded in equation (3).\footnote{Strictly speaking, as house prices enter the budget constraints through both the downpayment and mortgage payments, the percentage change in house prices due to a factor $x$ affecting $\tau_{H,t}$ is: $\Delta q_{t+T} \approx -\Delta \tau_{H} - (\partial \tau_{H}/\partial q) dq$. The implicit mortgage cost effect for simplicity abstracts from the feedback effect of $q$ to $\tau_{H,t}$, reflected in the partial derivative $(\partial \tau_{H}/\partial q)$. As long as the effect of a change in $q$ is approximately proportional across the budget constraints, the partial derivative is close to zero.}

The total effect is the sum of the two effects: $\Delta q_{t+T} = \Delta q_{w,t+T} + \Delta q_{\tau_{H,t+T}}$. For nominal house prices: $\Delta p_{t+T} = \Delta q_{t+T} + \pi_{t+T}$, where $\pi_{t+T} = \log P_{t+T} - \log P_t$ is the CPI inflation rate between $t$ and $t + T$.

### 2.4 Relationship to the dynamic Gordon model

The dynamic Gordon model is a simple, widely-used, method to analyse house prices, taught in almost every textbook on real estate finance. It postulates that the house price in period $t$ is determined by the discounted flow of future rents.\footnote{It is closely related to the Poterba (1984) model and results by a recursive substitution of the asset pricing equation in Poterba (1984), which relates the house price in period $t$ to a sum of the rental price (net of expenses, taxes and subsidies) and the house price in period $t + 1$, divided by a gross interest rate. The “static” Gordon model is a special case of the dynamic version under the assumption that rates of return and the growth rate of the rental price are constant.}

Continuing with our three-period deterministic example, the dynamic Gordon model would state that

$$ q_t = \frac{d_{t+1}}{1 + r_{t+1}} + \frac{d_{t+2}}{(1 + r_{t+1})(1 + r_{t+2})}, $$

where $1 + r_{t+1}$ and $1 + r_{t+2}$ are the real interest rates in the two respective periods. For the owner occupier, the rental price is implicit and equals the marginal rate of substitution between consumption and housing. Specifically, in the case of the once-and-for-all purchase in period $j = 0$

$$ q_t = \frac{1}{1 + r_{t+1}} \left( \frac{U_{h,t}}{U_{c,t+1}} \right) + \frac{1}{(1 + r_{t+1})(1 + r_{t+2})} \left( \frac{U_{h,t}}{U_{c,t+2}} \right). $$
If the home buyer has an unrestricted access to the instruments paying $1 + r_{t+1}$ and $1 + r_{t+2}$, then

$$\frac{1}{1 + r_{t+1}} = \beta \frac{U_{c,t+1}}{U_{c,t}} \quad \text{and} \quad \frac{1}{1 + r_{t+2}} = \beta \frac{U_{c,t+2}}{U_{c,t+1}}$$

and consequently, after substituting into (4),

$$q_t = \beta (1 + \beta) \frac{U_{h,t}}{U_{c,t}}$$

Comparing equation (5) with equation (2), the two equations differ only by the presence of $\tau_{H,t}$ in equation (2). As demonstrated by Campbell et al. (2009), to match the volatility of house prices over time, given that rents are relatively stable, observed interest rates have to be adjusted by a sufficiently volatile premium. Another way to see this property of the Gordon model is from equation (5). The housing stock and consumption in the data are much smoother than house prices. Thus, equation (5) would have a hard time matching the data. The wedge $\tau_{H,t}$ in equation (2) introduces an additional source of volatility.\(^{19}\)

3 Calibration

A reader interested purely in the method, rather than its application to the Czech housing market, may find some of the details in this section tedious. In such a case, the reader may just skim through this section to get a general idea of how the framework can be calibrated.

The baseline calculation is based on the average household, deriving income from labour, and the average-size dwelling. However, a sensitivity analysis is carried out to evaluate the effect of alternative plausible calibrations. I make one period in the model a quarter, but report various values as either on per month or per annum basis, depending on the convention (they are then converted to quarterly values in the actual calculations). As in

\(^{19}\)In the model of Garriga et al. (2019), a similar wedge in the optimality condition for housing, and also originating from credit market imperfections, helps account for the boom-bust period in the US housing market in the 2000s, which the Gordon model without the “housing premium” is unable to. In Kydland, Rupert, and Šustek (2016) the wedge considered here is critical in reproducing the observed dynamics of housing construction over the business cycle in a number of countries.
the conceptual framework above, the calculation is deterministic; i.e., it abstracts from any probabilistic distributions over future states of the world (both aggregate and individual).

I use the CRRA utility function for consumption, \( u(c) = c^{1-\gamma}/(1-\gamma) \), of which the log function \((\gamma = 1)\) is a special case. The annual discount factor \( \beta \) is set equal to 0.98. This is a fairly common value, implying an annual real interest rate of 2\%, a compromise between the long-run average of the real interest rate on short-term bonds in OECD countries, which is closer to 1\%, and the estimates of after-tax net rates of return on aggregate business capital, which vary between 4\% and 6\%, and aggregate housing capital, which is around 2.5\% (Gomme, Ravikumar, and Rupert, 2011). The parameter controlling the curvature of the utility function, \( \gamma \), is set equal to 0.5, which implies the elasticity of intertemporal substitution equal to 2. This corresponds to more willingness to substitute intertemporally than the log case used in the previous section and proxies the effect on marginal utilities of some ability of the household to smooth out consumption intertemporally through borrowing and saving, the explicit treatment of which is abstracted from the back-of-the-envelope calculation.\(^{20}\)

I work with nominal values, which are then converted into real consumption \((c_{t+j})\) by deflating by CPI inflation. The calibration is based on the initial conditions in the base period 2013.Q1 (the period in which \( j = 0 \) is thus \( t = 2013.Q1 \)). The average nominal house price in 2021.Q1 was CZK 77,800 per square meter for a dwelling in an apartment building. The price in 2013.Q1 \( (p_{2013Q1}) \) is set equal to CZK 43,708, implying an increase between the two periods of 78\% (about 9.75\% per year).\(^{21}\) The house size \((h)\) is kept constant across \( t \) and set equal to 68.5 square meters, the average floor area for a dwelling in an apartment building based on the 2011 Census. This corresponds to the average size of an apartment building.

\(^{20}\)The quantitative results would be stronger if the log case was used. Thus, assuming \( \gamma = 0.5 \) provides more conservative estimates, approximating the ability of the household to smooth out consumption to some extent (both borrowing/saving and increasing the elasticity of intertemporal substitution result in a smoother sequence of marginal utilities).

\(^{21}\)The 2021 price is taken from the Deloitte Real Index Q1 2021. It refers to the average realised apartment price as recorded in the Cadastre of Real Estate \((\text{Katastr nemovitostí})\) in the given quarter. The Deloitte Real Index tracks apartment prices in the 13 regional capitals plus Prague (an apartment is a dwelling in a residential building with four or more independent units). The Deloitte data are not publicly available for 2013. The price in 2013.Q1 is thus obtained by dividing the price in 2021.Q1 by a factor of 1.78, which is the increase between the two periods in the index of nominal Czech house prices reported by OECD (unlike the Deloitte data, the OECD data are just an index and do not contain actual prices).
with a kitchen (incl. open-plan) and two rooms.\footnote{See Vybrané údaje o bydlení 2018 (červen 2019), published by the Ministry of Regional Development (Ministerstvo pro místní rozvoj) and based on the latest available Census, carried out in 2011. The alternative common measurement of dwelling sizes—the living area—excludes kitchenettes, bathrooms, halls, storage rooms, etc. For the average dwelling in an apartment building, the living area is 52.6 square meters. The Deloitte Real Index prices are as per floor area, which is the area recorded in the Cadastre of Real Estate. Both measurements exclude balconies, terraces, etc. Data on new construction, contained in the above publication by the Ministry of Regional Development (Table 4.12), show that the average size of the typical newly constructed dwelling in an apartment building is somewhat lower than for the stock in the 2011 Census (e.g., for 2013, the values are 64.9 and 48.9 square meters for the floor and living areas, respectively).} These considerations imply the value of the typical apartment in 2013.Q1 ($p_{2013Q1}$) of CZK 2,993,998. According to the Financial Stability Report for 2015-16, published by the Czech National Bank, LTV ratios on new loans in 2014 (the first year for which the Report contains this indicator) were concentrated in the 80-90% bracket.\footnote{See Chart IV.16 in Zpráva o finanční stabilitě 2015/2016. About a third of new loans were in the 80-90% bracket.} I therefore take 85% as the typical LTV ratio in 2013.Q1 and set $\theta_{2013Q1}$ equal to 0.85. This yields a loan size ($l_{2013Q1}$) of CZK 2,544,898.

According to the Czech Statistical Office, the average gross nominal salary in 2013.Q1 ($W_{2013Q1}$) was CZK 24,061 per month. Applying the effective income tax rate ($\tau^*$) of 31.1% yields net salary ($W_{2013Q1}^*$) of CZK 16,585 per month. The effective tax rate is worked out as follows: according to the Czech tax code, from 2008 up until the end of 2020, the tax base for income from labour was the so-called super gross salary, derived as the gross salary plus the employer’s health insurance and social security contributions. These were, respectively, 9% and 24.8% of the gross salary. The tax base in our example is thus the super gross salary of CZK 24,061 \times 1.338 = CZK 32,194. Applying the statutory tax rate ($\tau$) of 15% to the super gross salary and the employee’s health insurance (4.5%) and social security contributions (6.5%) to the gross salary gives total deductions of CZK 7,476, or 31.1% of the gross salary.\footnote{The Czech tax code has effectively a flat income tax rate. A higher tax rate of 23%, introduced in 2013 in the aftermath of the financial crisis under the name “Solidarity tax”, applies only to monthly salaries well above the average. In 2013, for instance, the threshold was CZK 103,536 per month. In 2021, the higher rate became a part of the regular tax code.} I consider a childless household consisting of two adults, each earning the average salary, thus setting $W_{H,2013Q1}^* = 2W_{2013Q1}^* = 33,170$.

Let $J$ be the term of the loan and recall that $j = 1$ denotes the first period in which the households makes mortgage payments. For periods $j = 1, \ldots, J$, nominal household
consumption \((C_{t+j})\), is derived as after-tax labor income \((W_{H,t+j}^*)\) less a property tax \((X)\), utilities & maintenance \((D_{t+j})\), subsistence \((S_{t+j})\), and a mortgage payment \((m_t = m_{I,t+j} + m_{A,t+j})\), which includes both interest \((m_{I,t+j})\) and amortisation \((m_{A,t+j})\) and is determined in the base period \(t\). A tax deductability of mortgage interest payments under the Czech tax code is taken into account by adding to the after-tax income 15\% \times the interest payment on the mortgage (ie, \(\tau m_{I,t+j}\)). In 2013.Q1, the annual property tax for the dwelling of the size considered here would be CZK 784, which is a negligible amount and has essentially no effect on the calculations. Utilities & maintenance for the two-person household are set equal to CZK 1,743 per month. Subsistence for the household is set equal to CZK 348 per day (CZK 174 per person). Mortgage payments are determined below.

What is important to the household when evaluating the cost of future mortgage payments on a new loan is the stream of real consumption in all future periods over the life of the loan. First, I construct a stream of nominal consumption by considering a nominal wage growth rate \((g)\) and a CPI inflation rate \((\pi)\) and then deflate the nominal stream back to 2013.Q1 prices. This is done as follows. The nominal after-tax household income \(W_{H,t+j}^*\) is adjusted between two periods after the base period by \(1 + g\) and utilities & maintenance and subsistence by \(1 + \pi\). The base period here is 2013.Q1. Mortgage payments, of course, stay constant in nominal terms throughout the life of the loan (assuming no foreseeable changes in the mortgage rate, prepayment, etc.). The negligible property taxes are also assumed to remain constant in nominal terms, as in reality they have not changed for many years; the tax code is also assumed to remain constant. The resulting nominal consumption in period \(t + j\) is then converted to real consumption, in prices of the base period, by deflating by

\(^{25}\)An inherent property of mortgage payments, resulting from amortisation, is that over the life of the loan the share of interest payments declines to zero, whereas the share of amortisation payments increases to one. Thus, the two components of \(m_t\) are indexed by \(j\), even if \(m_t\) stays constant through out its term (as would be the case under a fixed mortgage rate, no prepayment, etc.), as considered here.

\(^{26}\)Household utilities and maintenance expenses are published at a per-person basis by the Czech Statistical Office. Subsistence is based on an educated guess of CZK 200 per person per day in 2021.Q1 prices, deflated to 2013.Q1 by CPI inflation. Property taxes partially depend on municipality. The tax in my example is calculated for Prague.

\(^{27}\) Implicitly, thus, the household in 2013.Q1 does not foresee the change in the tax code in 2021 that abolished the concept of super gross salary, thereby reducing the effective tax rate from 31.1% to 26%.
To summarise, real household consumption in period \( t + j \) \((j = 1, \ldots, J)\) is given by

\[
c_{t+j} = [(1+g)^j W_{H,t}^* + \tau m_{t,t+j} - X - (1+\pi)^j (D_t + S_t) - m_t]/(1+\pi)^j.
\]

I discuss the choices for \( g \) and \( \pi \) after describing mortgage payments.

Mortgage payments are computed using the standard annuity formula under the maintained assumption that the same mortgage rate applies throughout the life of the loan

\[
m_t = \frac{i_t}{1 - (1+i_t)^{-J\ell_t}},
\]

where \( J \) is the term of the loan, \( i_t \) is the market nominal mortgage interest rate at origination, and \( \ell_t \) is the loan taken out in the base period \( t \).\(^{28}\) I consider a 25-year loan, the typical term in the Czech mortgage market, as reported by the European Mortgage Federation. As one period in the model corresponds to a quarter, \( J = 100 \).\(^{29}\) The mortgage rate is set equal to 3.21\%, the average mortgage rate in 2013.Q1, according to the Fincentrum Hypoindex. For the size of the loan \( l_{2013Q1} \) worked out above, the implied monthly payments are CZK 12,286, of which (in the first period) 55\% is interest and 45\% is amortisation. As a share of the household after-tax income in 2013.Q1, the mortgage payments make up 37\% (the DSTI ratio).\(^{30}\)

Recall that in period \( j = 0 \), the household is a renter. Its consumption is given by

\[
C_t = W_{H,t}^* - D_t^R - S_t - r_t h^R,
\]

where \( D_t^R \) is utilities (not including maintenance), \( h^R \) is the size of the rental property, and \( r_t \) is a rental price per square meter (as \( t = 2013.Q1 \) is the base period, there is no distinction between nominal and real variables and \( c_t = C_t \)). The above equation implicitly assumes that all existing savings are used for the downpayment.

\(^{28}\)It is straightforward to replace the constant mortgage rate with a sequence of future mortgage rates, for instance, following the end of a fixation period or in each future period under an adjustable rate contract (the interest rate sequence can be based, eg, on the yield curve in period \( t \)). The mortgage payments in such a case would be re-calculated in each \( j \) when the interest rate changes, using the new rate, the remaining term, and the outstanding mortgage balance. The implicit assumption would be that the household can anticipate such changes at the time of origination.

\(^{29}\)To calculate a monthly \( m \), I convert the annual mortgage rate to a monthly rate and set \( J = 300 \). The monthly payments are then converted to quarterly by multiplying them by three.

\(^{30}\)In 2018, the Czech National Bank recommended that the DSTI ratio should not exceed 45\%, subsequently increasing the threshold to 50\% in April 2020 and abolishing it altogether in July 2020.
(in our example, the downpayment is CZK 449,100, which corresponds to the household’s 14 monthly after-tax joint salaries). I am going to further assume that the household rents a dwelling of the same size as the one it wants to purchase (ie, \( h^R = h \)). A stamp duty tax and various legal and transaction fees related to the home purchase are abstracted from, or are assumed to be covered by the savings. The values of \( W_{H,t} \) and \( S_t \) are the aforementioned values for 2013.Q1. The value of \( D_t^R \) is adjusted for the absence of maintenance expenses and set equal to CZK 1,163 per month. The rental price \( (r_t) \) is set equal to CZK 200 per square meter, implying a monthly rent of CZK 13,700.31,32

To set the growth rate of nominal wages \( (g) \) and the inflation rate \( (\pi) \) that are used to compute the future stream of consumption, I make the following heuristic assumptions. Figure 2 plots the annual growth rate of nominal wages and the CPI inflation rate from 2009 to 2020 (the data are from the Czech Statistical Office). 2009 is the year the Czech Republic entered recession, following the 2008 global financial crisis. Observing the time path of the nominal wage growth, it appears that the economy operated during the 2009-2020 period in two modes, a recession mode (2009-2013) and a recovery/expansion mode (2014-2020).33 The average growth rate of nominal wages in 2009-2013 was 2.2%, whereas in 2014-2020 it was 5.2%. The average inflation rate in the two subperiods, however, was almost the same, 1.9% and 1.8%, respectively. Real wages thus stayed almost constant in the first subperiod, but achieved a robust consistent growth of 3.4% on average in the second subperiod. I will therefore proceed under the assumption that in 2013.Q1 household expectations were still based on the recession mode and set \( g \) equal to 2.2% and \( \pi \) to 1.9%. The inputs described

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31 The rental price is worked out as the price in 2021.Q1, published by the Deloitte Rent Index Q1 2021, divided by a factor of 1.19, which is the increase between 2021.Q1 and 2013.Q1 in the index of nominal Czech rental prices reported by OECD. The Deloitte data are based on apartment rental prices in the 13 regional capitals plus Prague, collected from real estate portals. As a side note, the rental yield in 2013.Q1 based on the above rental price was 5.5%.

32 Comparing the monthly rent of CZK 13,700 with the monthly mortgage payment of CZK 12,286, the standard conclusion offered by real estate professionals would be that owning is cheaper than renting (abstracting from property taxes and maintenance). The method proposed here does not attempt to solve the dynamic tenure choice problem, which (at the minimum) would require to take a stand on a sequence of future rental prices and rates of return on the alternative uses of the downpayment.

33 The COVID-19 induced recession in 2020 has not so far changed this basic split for wages, with the real wage growth rate in 2020 still exceeding all but one year in the period 2009-2013.
above are summarized in Table 1, column 2013.Q1.\footnote{As in the case of the nominal interest rate, it is possible to create a sequence of future inflation and nominal income growth rates and carry out the exercise conditional on such sequences, implicitly assuming these are the sequences the household expects as of the base period. I abstract from such complication here.}

4 Findings

4.1 Accounting for the 2013.Q1-2021.Q1 increase in house prices

I start with a back-of-the-envelope calculation for the contribution of different factors to house prices during the boom period 2013.Q1-2021.Q1. Recall that during this period nominal house prices increased by 78%. The CPI inflation rate during the same period was 15%. Thus, 15% of the 78% increase in nominal house prices is simply due general inflation. This leaves 63% of the increase in real house prices to be accounted for by the income and implicit mortgage cost effects.

Recall that the income effect is \( \Delta q_{w,t+T} \approx \eta - \kappa \), where \( \eta \) is the growth rate of real income per household and \( \kappa \) is the growth rate of the housing stock per household. Based on the average salary, as reported by the Czech Statistical Office, nominal income growth during 2013.Q1-2021.Q1 was 47%. Adjusting this figure for CPI inflation, I set \( \eta = 32\% \).

The data on the number of households are unfortunately not available at the frequency corresponding to the period under investigation. I therefore use the population 20+ as a basis for a proxy. The data are available at annual frequency from the United Nation’s World Population Prospects. During 2013-2020, the population 20+ stayed roughly constant, with an annual increase of only 0.05\%.\footnote{An additional difficulty of using the number of households is that, at least partially, household formation is endogenous to house prices; see, eg, Peter, Piazzesi, and Schneider (2020). Exogenous socio-demographic factors driving household formation can be thought of as demand shifts for housing per person 20+.}

The housing stock data are available for 2011 from the 2011 Census and are reported in Vybrané údaje o bydlení 2018 (červen 2019), published by the Ministry of Regional Development (Ministerstvo pro místní rozvoj). According to the Census, there were 4,104,635 occupied dwellings in 2011 (most of unoccupied dwellings were family recreational cabins). Adding to this figure completions in 2012 (and assuming...}
no demolishions or additional occupation of existing unoccupied dwellings) yields 4,134,102 dwellings at the start of 2013. Completions during 2013-2020 totalled 234,846 units, which is a 5.7% increase on the stock at the start of 2013, assuming no demolishions and additional occupation of the existing stock. This corresponds to an annual growth rate of the housing stock of 0.72% (29,356 units per year on average).\textsuperscript{36} In light of this figure and the roughly constant population 20+, and taking into account that the number of adults per dwelling has been steadily declining over the past 50 years (from 2.34 in 1961 to 2.09 in 2011), I take the housing stock per household as being approximately constant during 2013.Q1-2021.Q1 and set \( \kappa = 0\% \). The income effect is thus \( \Delta q_{w,t+T} = 32\% \).

To compute the implicit mortgage cost effect for 2013.Q1-2021.Q1, I start by computing \( \tau_{H,t} \) on the basis of the 2013.Q1 inputs discussed above (Table 1, column 2013.Q1). This gives the implicit mortgage cost for the marginal buyer in 2013.Q1. I then change the four factors in equation (3), \( \theta, g, i, \pi \), to their values in 2021.Q1 and compute the implicit tax again to see how the implicit cost has changed for the marginal buyer over time.\textsuperscript{37} Specifically, the four factors are changed as follows: \( \theta : 85\% \mapsto 80\% \), \( g : 0.3\% \mapsto 3.4\% \), \( i : 3.21\% \mapsto 2.13\% \), and \( \pi : 1.9\% \mapsto 1.8\% \). The new LTV ratio is based on the Financial Stability Report 2020-21, which shows that after 2017, when the Czech National Bank tightened the recommendations for the LTV ratio, most new loans became concentrated in the 70 – 80% bracket, with close to 40% of the loans in this range. The mortgage rate is again based on the Fincentrum Hypoindex. As discussed above, the real income growth rate, \( g \), and the inflation rate, \( \pi \), are based on their respective averages in the expansion period 2014 – 2020. As a result of the changes in these inputs, \( \Delta q_{\tau_{H,t+T}} = -\Delta \tau_{H} = 20\% \).

Taken together, my back-of-the-envelope calculation suggests that, out of the 78% in-
crease in nominal house prices between 2013.Q1 and 2021.Q1, 15% was due to general inflation, 32% due to an increase in real salaries, and 20% due to a reduction in the implicit cost to the marginal buyer of financing the house purchase. Taking into account the abolishment of the stamp duty tax would add another 4%, leaving 7% for factors not taken into account. These may include demand for more space following the outbreak of COVID-19, a decline in risk premia, tighter search and matching frictions in the housing market, and potentially also speculative purchases.

The implicit mortgage cost effect can be further decomposed into the contribution of the individual four factors. The results of this decomposition are summarised in Table 2. The decomposition is carried out by keeping the factor of interest at the 2013.Q1 level, while changing the other three factors as above. The table shows that income growth expectations were the single most important factor behind the 20% decline in the implicit cost of mortgage finance. If expectations remained at the 2009-2013 recession level, the implicit mortgage cost would decline by only 8.1%, mainly due to a decline in the mortgage rate. The contribution of the small change in the LTV ratio and even smaller change in the inflation rate was minuscule.

In sum, much of the growth in real house prices between 2013.Q1 and 2021.Q1 can be accounted for by economic forces: predominantly, fast growth in household real income and expectations of a similarly strong growth continuing into the future, with such expectations reducing the cost of servicing a mortgage over time.

4.2 Sensitivity analysis

Table 3 reports sensitivity of the above findings for the change in the implicit mortgage cost to alternative calibrations of the inputs entering the calculation. Changing income growth expectations from the assumed 3.4% by one percentage point in either direction does not dramatically change the findings, leading to a range of $\Delta q_{tH,t+T}$ from 16.9% to 22.5%. A more material change occurs if income growth expectations were changed to 1.4%, only one
percentage point higher than in the recession phase. Then the resulting $\Delta q_{\tau H,t+T}$ would be only 13.1%. Similarly, marginally changing the parameter of intertemporal substitution from the baseline value of 1/2 to either 1/3 (more willingness/ability to substitute intertemporally) to 2/3 (less willingness/ability) leads to a similar range, from 17.2% to 21.7%. Changing the discount factor $\beta$ to correspond to an annual real interest rate of 1%, instead of the baseline 2%, gives also a similar range of values for the implicit mortgage cost effect. Finally, changing the characteristics of the household and the dwelling, either to match the average loan size in 2013.Q1, as reported by the Czech National Bank, or assuming that the marginal home buyer lies to the right of the average in the income distribution, does not materially change the conclusion either.\textsuperscript{38}

### 4.3 The role of construction

A slow and bureaucratic planning and permits process in the Czech Republic is frequently cited by market commentators, business economists, and politicians alike as the main culprit for the sharp rise in house prices since 2013.

As discussed above, the aggregate housing stock grew over the period under investigation roughly in line with the number of households. In fact, in relation to the population 20+, the growth rate was above the historical average. According to the decennial Census starting in 1961, the average growth rate of the housing stock between 1961 and 2011 was 0.81% per annum, while the average growth rate of the population 20+ was 0.57% per annum. During 2013-2020, these figures were 0.72% and 0.05%, respectively, as noted above. In fact, the same as in 1980-1991. In relation to total population, the growth rates were roughly at par with the historical averages: 0.81% and 0.19% for the housing stock and total population, respectively, in 1961-2011 vs. 0.72% and 0.15% in 2013-20.\textsuperscript{39}

\textsuperscript{38}The baseline calibration implies a loan size that is greater than the average loan size reported by the Czech National Bank for 2013.Q1 in the Financial Stability Report. Cases D.1 and D.2 in the table address this issue (see the caption to the table for details).

\textsuperscript{39}The aggregate housing stock may not be a sufficient statistic for housing supply if construction and household formation are, for instance, mismatched in terms of location or the required size of the dwelling. Also, the number of dwellings (floor size) used here to measure the housing stock does not take into account
Nevertheless, could have higher construction activity prevented the increase in real house prices above the 35% increase justifiable by the increase in real salaries? In other words, could have higher construction wiped out the 20% increase due to the implicit mortgage cost effect? A back-of-the-envelope answer to this question can be obtained from equation (2). In the decomposition for 2013.Q1-2021.Q1, I used $\kappa = 0\%$. The question is then, how large $\kappa$ would have to be to undo the 20% increase due to the implicit mortgage cost effect? For the log utility function in housing, the answer is 20%. That is, a 20% increase in the housing stock on the level at the start of 2013 in just eight years. This corresponds to 2.5% annually, which is 103,353 units a year (or the equivalent in square meters)—a number exceeding even the peak of construction activity in 1975 (97,104 units).\footnote{See Table 4.2 in \textit{Vybrané údaje o bydlení 2018 (červen 2019)}, published by the Ministry of Regional Development (Ministerstvo pro místní rozvoj).} This is unrealistic in the current conditions and probably not even desirable. To undo a half of the implicit mortgage cost effect, the number of units completed would have to be 51,676 a year, which was last seen in 1989 (55,073 units), but is not too far from the 41,649 units completed in 2007 (vs. the average of 29,356 units per annum in 2013-2020). This does not mean that realistically higher, sustained, construction levels cannot have material downward pressure on house prices, but such effects will only be felt in the long run. Unless increased construction activity shifts people’s expectations about future house prices, the short- to medium-run effects of more construction on house prices are likely to be limited and dominated by other factors.\footnote{Similar conclusions for the United Kingdom have been reached by Oxford Economics (2016) using different methodology.}

### 4.4 Scenario analysis

The method can be used to evaluate the effect on house prices of different future scenarios. On the basis of the back-of-the-envelope calculation proposed in this paper, the percentage change in house prices going forward will be the sum of the income effect, the implicit depreciation and thus required renovations.
mortgage cost effect, and the CPI inflation rate, for a change in nominal house prices. The
future scenarios are only defined by the change in the values of the inputs, not a time
dimension. The period over which house prices adjust thus corresponds to the period over
which the change in inputs is considered to take place. As an example, suppose the analyst
considers the following future values of the inputs: the growth rate of real income per
household is 1% per annum, the housing stock per household is constant, the CPI inflation
rate is 2% per annum, and the implicit mortgage cost effect is −10% (e.g., due to a decline in
income growth expectations). If this change in expectations is considered to take place over a
five-year period, then the total change in house prices according to the back-of-the-envelope
calculation is: \[ 5 \times (1\% + 2\%) - 10\% = 5\% \] in nominal terms and −5% in real terms. If,
however, the change is considered to happen over two years, then the change in house prices
is \[ 2 \times (1\% + 2\%) - 10\% = -4\% \] in nominal terms and −8% in real terms. The income effect
is straightforward to calculate and trivially depends on future forecasts for the two variables
that define it. I therefore focus only on the implicit mortgage cost effect.

To carry out the scenario analysis, the relevant inputs are recalibrated to the 2021.Q1
conditions, with the values summarised in Table 1, panel 2021.Q1. The scenarios are
defined by the change in the values of the inputs investigated, holding all other inputs at
the 2021.Q1 values reported in Table 1.

Figures 3-5 contain the results. Figure 3 considers the increase in mortgage rates to
5.33%, the level reached in 2022.Q2. The figure plots the change in the implicit mortgage
cost under three different expected future growth rates of real disposable income and a range
of expected future inflation rates. Figure 4 plots the change in the implicit mortgage cost
under three different expected inflation rates and a range of expected future real income
growth rates. As an example, at long-term inflation expectations of 6%, a dire scenario of
zero expected future real income growth leads to a decline in real house prices of 13% (holding
the income effect constant). However, if income growth expectations remained unchanged
from the boom period of 2013-2020, the recent increase in mortgage rates, at long-term

\[ \text{The change in the effective tax results from the abolition of the super gross salary as a tax base.} \]
inflation expectations of 6%, would lead to only a modest, 1.5%, drop in house prices. The most severe drop in house prices would result from the combination of low inflation and real income growth expectations. In contrast, a combination of inflation expectations sufficiently exceeding the mortgage rate and optimistic outlook for future real income growth would even push house prices above the 2021 levels. While assigning probabilities to the different scenarios, and thus providing forecasts for future house prices, is beyond the scope of the paper, taking all the scenarios together, the risks for house prices are skewed to the downside. Observe that the shape of the graphs is convex. A given negative change in income growth expectations, for instance, results in a larger increase in the implicit mortgage cost than is the decline in the implicit mortgage cost resulting from an increase in income growth expectations of the same magnitude.

Finally, Figure 5 considers a more modest increase in the mortgage rate than the increase in the data, accompanied by relatively modest inflation expectations of 4%. This scenario may be thought of as a state to which the economy converges after the current turbulent period ends but inflation expectations remain somewhat de-anchored. In this case, zero expected real income growth would result in a decline in real house prices of about 15% (holding the income effect constant), while no change in real income growth expectations from the boom period would lead to only a 2% drop in real house prices. In contrast, an increase in real house prices by 5% would require real income growth expectations of 6%, almost double the growth rate observed during the boom period.

5 Conclusions

The paper aims to propose a new, easy-to-use, method that would allow practitioners to carry out a back-of-the-envelope analysis of past developments in house prices and to assess future risks. The method is applied to the rapid increase in the Czech Republic house prices between 2013.Q1 and 2021.Q1, the third fastest growth among OECD countries.

Much of the growth in real house prices between 2013.Q1 and 2021.Q1 can be accounted
for by economic forces. Predominantly, fast growth in household real income and expectations of a similarly strong growth continuing into the future. Such expectations have significantly reduced the expected burden of mortgage debt over the life of the loan for the marginal home buyer (the implicit mortgage cost).

In more detail, out of the 78% increase in nominal house prices between 2013.Q1 and 2021.Q1, the back-of-the-envelope calculation suggests that 15% was due to general inflation, 32% due to an increase in real salaries, and 20% due to a reduction in the implicit mortgage cost. Most of the last effect was due to expectations of strong future income growth, with a decline in mortgage rates playing a secondary role. Taking into account the abolishment of the stamp duty tax would add another 4%, leaving 7% to other factors. These may include demand for more space following the outbreak of COVID-19, a decline in risk premia, tighter search and matching frictions in the housing market, and potentially also speculative purchases.

The housing stock grew during 2013-2020 at a slightly higher rate that the adult population. Higher construction activity could not have prevented the dramatic increase in house prices so far. Just to undo the implicit mortgage cost effect, and thus let house prices grow in line with income, construction would have had to increase to unrealistic levels during 2013-20. To undo about half of the implicit mortgage cost effect, annual completions would have had to be sustained during the eight-year period at least at the levels of the peak observed so far in the post-1989 era.

Going forward, contrary to the popular opinion, house prices can decline even if construction activity remains at the current, relatively low, levels. A combination of mortgage rates sufficiently high in relation to inflation expectations, coinciding with an economic slowdown that leads to a downward revision in future income growth expectations, constitute one of the possible scenarios that would lead to a fall in real house prices. The paper quantitatively evaluates a number of possible scenarios. While a number of scenarios suggest continued growth, it is clear that the generally-believed “only-way-up” is not the only possible out-
come for house prices, even at the current subdued construction levels. In fact, the risks for house prices are skewed downwards.

State-of-the-art techniques used in academic research are time-demanding and increasingly inaccessible to practitioners. The method proposed here, while grounded in theory, can be implemented in spreadsheet. Due to various simplifications to ensure practicability, it offers only a first-pass, back-of-the-envelope, account of house price movements. The textbook dynamic Gordon model, a popular way to assess house price movements, requires a reduced-form time-varying “housing premium” to generate the volatility of house prices typically observed in the data. Affordability ratios, another popular method used in practice, ignores the fact that a house purchase is inherently a dynamic decision shaped by future, not just current, affordability considerations. The method proposed here makes the affordability considerations dynamic by constructing a marginal home buyer who is constrained in financial markets. A resulting wedge in the household’s optimality condition, working like a distortionary tax/subsidy on the house purchase, summarises the implicit cost of the mortgage to the household and introduces a source of volatility above and beyond that in the dynamic Gordon model. Unlike in the Gordon model, future cash flows are not discounted by market interest rates, but by internal discount factors specific to the marginal buyer and can be easily computed in spreadsheet. With additional simplifying assumptions, house prices can be easily decomposed into two effects: an income effect, capturing household income and housing supply per household, and an implicit mortgage cost effect, capturing the burden of mortgage payments to the marginal buyer over the life of the loan. CPI inflation then converts the growth in real house prices to nominal values.
References


Table 1: Baseline calibration

<table>
<thead>
<tr>
<th></th>
<th>2013.Q1</th>
<th>2021.Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Utility function</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount factor ($\beta$, annual)</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>CRRA parameter ($\gamma$)</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td><strong>House purchase</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price (CZK, m$^2$)</td>
<td>43,708</td>
<td>77,800</td>
</tr>
<tr>
<td>Dwelling size (m$^2$)</td>
<td>68.5</td>
<td>68.5</td>
</tr>
<tr>
<td><strong>Income</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of wage earners</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Household size</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Gross salary (CZK, monthly, person)</td>
<td>24,061</td>
<td>35,285</td>
</tr>
<tr>
<td>Effective tax rate</td>
<td>31.1%</td>
<td>26%</td>
</tr>
<tr>
<td><strong>Mandatory expenses</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Util. &amp; maintenance (CZK, monthly, household)</td>
<td>1,743</td>
<td>2,004</td>
</tr>
<tr>
<td>Property tax (CZK, annual, dwelling)</td>
<td>784</td>
<td>784</td>
</tr>
<tr>
<td>Subsistence (CZK, daily, person)</td>
<td>174</td>
<td>200</td>
</tr>
<tr>
<td><strong>Mortgage</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LTV</td>
<td>85%</td>
<td>80%</td>
</tr>
<tr>
<td>Term (years)</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>3.21%</td>
<td>2.13%</td>
</tr>
<tr>
<td>Mortgage payment (CZK, monthly)</td>
<td>12,286</td>
<td>18,299</td>
</tr>
<tr>
<td>Resulting DTI</td>
<td>6.4</td>
<td>6.8</td>
</tr>
<tr>
<td>Resulting DSTI</td>
<td>37%</td>
<td>35%</td>
</tr>
<tr>
<td><strong>Expectations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real wage growth (annual)</td>
<td>0.3%</td>
<td>3.4%</td>
</tr>
<tr>
<td>CPI inflation rate (annual)</td>
<td>1.9%</td>
<td>1.8%</td>
</tr>
<tr>
<td><strong>Renter</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rental price (CZK, m$^2$, monthly)</td>
<td>200</td>
<td>237</td>
</tr>
<tr>
<td>Dwelling size (m$^2$)</td>
<td>68.5</td>
<td>68.5</td>
</tr>
<tr>
<td>Rent (CZK, dwelling, monthly)</td>
<td>13,700</td>
<td>16,235</td>
</tr>
<tr>
<td>Utilities (CZK, household, monthly)</td>
<td>1,163</td>
<td>1,337</td>
</tr>
<tr>
<td>Subsistence (CZK, daily, person)</td>
<td>174</td>
<td>200</td>
</tr>
</tbody>
</table>

Notes: DTI and DSTI are based on after-tax household income, in accordance with how these indicators are calculated by the Czech National Bank. All CZK values are nominal.
Table 2: Change in $\tau_H$ (the implicit mortgage cost): 2013.Q1-2021.Q1

<table>
<thead>
<tr>
<th>Change in inputs</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$ :</td>
<td>85% $\mapsto$ 80%</td>
<td>$\theta$ const.</td>
<td>$\theta$ const.</td>
<td>$\theta$ const.</td>
<td>$\theta$ const.</td>
</tr>
<tr>
<td>$g$ :</td>
<td>0.3% $\mapsto$ 3.4%</td>
<td>$g$ const.</td>
<td>$g$ const.</td>
<td>$g$ const.</td>
<td>$g$ const.</td>
</tr>
<tr>
<td>$i$ :</td>
<td>3.21% $\mapsto$ 2.13%</td>
<td>$i$ const.</td>
<td>$i$ const.</td>
<td>$i$ const.</td>
<td>$i$ const.</td>
</tr>
<tr>
<td>$\pi$ :</td>
<td>1.9% $\mapsto$ 1.8%</td>
<td>$\pi$ const.</td>
<td>$\pi$ const.</td>
<td>$\pi$ const.</td>
<td>$\pi$ const.</td>
</tr>
<tr>
<td>$\Delta \tau_H$ :</td>
<td>-20.0%</td>
<td>-22.1%</td>
<td>-8.1%</td>
<td>-14.3%</td>
<td>-20.1%</td>
</tr>
</tbody>
</table>

Notes: The change in the real house price due to the implicit mortgage cost effect = $-\Delta \tau_H$. Based on the baseline calibration in Table 1, column 2013.Q1; “const.” refers to the respective factor being held constant at the 2013.Q1 value, while the other factors are changing as described in column (a).
Table 3: Sensitivity of the change in $\tau_H$ (the implicit mortgage cost): 2013.Q1-2021.Q1

<table>
<thead>
<tr>
<th>A. Expected real income growth (expansion)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g$: 4.4% 3.4% 2.4% 1.4%</td>
</tr>
<tr>
<td>$\Delta \tau_H$: -22.5% -20.0% -16.9% -13.1%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Parameter of intertemporal subst.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$: 2/3 1/2 1/3 1/5</td>
</tr>
<tr>
<td>$\Delta \tau_H$: -21.7% -20.0% -17.2% -14.1%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C. Discount factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$: 0.98 0.99</td>
</tr>
<tr>
<td>$\Delta \tau_H$: -20.0% -23.1%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D. Household and dwelling characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) 44m$^2$, 1.4 wage earners, 2 h/h members</td>
</tr>
<tr>
<td>$\Delta \tau_H$: -20.9%</td>
</tr>
<tr>
<td>(2) 44m$^2$, 1.4×average salary, 1 h/h member</td>
</tr>
<tr>
<td>$\Delta \tau_H$: -19.3%</td>
</tr>
<tr>
<td>(3) 68.5m$^2$, 1.5×average salary, 2 h/h members</td>
</tr>
<tr>
<td>$\Delta \tau_H$: -19.1%</td>
</tr>
<tr>
<td>(4) 68.5m$^2$, 2×average salary, 2 h/h members</td>
</tr>
<tr>
<td>$\Delta \tau_H$: -18.3%</td>
</tr>
</tbody>
</table>

Notes: The change in the real house price due to the implicit mortgage cost effect = $-\Delta \tau_H$. The change in $(\theta, g, i, \pi)$ is the same as in Table 2, column (a), except panel A, where the new value of $g$ is as specified. In case (D.1), the home size and salary are chosen to match the average loan size in 2013.Q1 (CZK 1,630,000) as reported by the Czech National Bank, while preserving the most typical DTI = 6 (one household member in this case gets the average salary, the other only 0.4×the average salary; the implied DSTI=34%). Case (D.2) is the same as (D.1), but for a single-person household, earning 1.4×the average salary; DTI and DSTI are the same as in (D.1). Case (D.3) is the baseline dwelling size but with both household members earning 1.5× the average salary (implied DTI=4, DSTI=0.25%). Case (D.4) is the baseline dwelling size but with both household members earning 2× the average salary (implied DTI=3, DSTI=0.19%).
Figure 1: Czech Republic house prices. Source: OECD.
Figure 2: Nominal wage growth and inflation, 2013-2020. Source: Czech Statistical Office.
Increase in the mortgage rate from 2.13% to 5.33%

Figure 3: Change in $\tau_h$ (the implicit cost of mortgage finance) under various scenarios. Combinations of future expected inflation and real income growth rates ($g$), assuming the mortgage rate increases from 2.13% to 5.33%. As a point of reference, during the boom period 2013-2020, the average inflation and real income growth rates were 1.8% and 3.4%, respectively, and in 2021.Q1 the mortgage rate was 2.13%. At these values going forward, the change in $\tau_h$ would be zero.
Increase in the mortgage rate from 2.13% to 5.33%

Figure 4: Change in $\tau_h$ (the implicit cost of mortgage finance) under various scenarios. Combinations of future expected inflation ($\pi$) and real income growth rates, assuming the mortgage rate increases from 2.13% to 5.33%. Other variables are set to their 2021.Q1 values. As a point of reference, during the boom period 2013-2020, the average inflation and real income growth rates were 1.8% and 3.4%, respectively, and in 2021.Q1 the mortgage rate was 2.13%. At these values going forward, the change in $\tau_h$ would be zero.
Increase in the mortgage rate from 2.13% to 4%

Figure 5: Change in $\tau_h$ (the implicit cost of mortgage finance) under various scenarios. An increase in the mortgage rate from 2.13% to 4%, assuming expected future inflation rate ($\pi$) of 4%, under various expected future real income growth rates. As a point of reference, during the boom period 2013-2020, the average inflation and real income growth rates were 1.8% and 3.4%, respectively, and in 2021.Q1 the mortgage rate was 2.13%. At these values going forward, the change in $\tau_h$ would be zero.
Abstrakt
