

# Firm Leverage and Wealth Inequality\*

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## Abstract

This paper studies the effects of a change in firm leverage on wealth inequality and macroeconomic aggregates. The question is studied in a general equilibrium model with a continuum of heterogeneous agents, life-cycle, incomplete markets, and idiosyncratic and aggregate risk. The analysis focuses on the particular change in firm leverage that occurred in the U.S. during the 1980s, when firm leverage increased significantly, and subsequently has been dropping since the early 1990s. In the benchmark model, an increase in firm leverage of the size that occurred during the 1980s increases capital accumulation by 5.38%, decreases wealth inequality by 1.07 Gini points and decreases government revenues by 0.11% of output. An increase in firm leverage increases average after-tax returns on savings, as firm debt has beneficial tax treatment. This increases the saving rates of all households, and disproportionately increases the saving rates of relatively poorer households. Consequently, the model implies that the increase in firm leverage did not contribute to rising inequality in the U.S. in the 1980s, but rather the opposite; that the reduction in leverage from the early 1990s to 2008 has contributed to rising wealth inequality. Furthermore, I show that if the model abstracts from beneficial tax treatment of corporate debt, the change in leverage has only minor effects on macro aggregates and inequality, despite having significant implications for asset prices. This is consistent with the previous result in the literature showing that the Modigliani-Miller theorem approximately holds in the heterogeneous agents model with imperfect markets.

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# 1 Introduction

The data shows that households with different amounts of wealth hold starkly different portfolios (Survey of Consumer Finance, 1998, 2016). Considering only financial assets, poorer households invest mostly in safe financial assets, with 46% of US households not holding any risky financial assets (equity) (Chang et al., 2018). At the same time, the richest households invest most of their financial savings in equity.<sup>1</sup> These facts motivate the study in this paper, which analyses how the change in the corporate structure affects inequality and macroeconomic aggregates. In particular, Graham et al. (2015) documents that firm leverage has significantly risen in the US since the end of the Second World War, and one of the sharp increases occurred during the late 1980s.<sup>2</sup> Given the patterns observed in the data, one could speculate that the change in corporate structure (i.e. leverage) will have heterogeneous effects on the population, as it changes the supply and the riskiness of financial assets, which are disproportionately distributed among households.

The Modigliani-Miller theorem (see Modigliani and Miller (1958) and Modigliani and Miller (1963)) on the irrelevance of firms' financial (leverage) policy has been shown to be valid in a wide range of environments. Algan et al. (2009) have shown that, although the theorem does not hold exactly in an environment with a borrowing constraint, the effects of a change in the representative firms' financial policy is sufficiently small that the theorem holds approximately. This is because, similar to the original models of Krusell and Smith (1997) and Krusell and Smith (1998), most agents are well insured and can offset the effects of the firm's financing decision by adjusting their portfolio. The households that are not sufficiently insured are very poor households, close to the borrowing constraint. However, there are not many of such households, and moreover, they hold a very small amount of wealth, so a change in their behavior does not influence the aggregate dynamics. Furthermore, in Algan et al. (2009), the equity risk premium is not sufficiently high, as it is common in standard macroeconomic models (Mehra and Prescott, 1985), making the two types of assets similar from the household's point of view.

This paper studies the effects of a change in firm leverage on macroaggregates and wealth

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<sup>1</sup>Chang et al. (2018) and Carroll (2000)

<sup>2</sup>See the graph in the Appendix B.

inequality in a model with a continuum of households, imperfect markets, borrowing and portfolio choice constraints, and idiosyncratic and aggregate risk. The model augments the Krusell and Smith (1997, 1998) and Algan et al. (2009) models by adding the parsimonious life cycle in the style of Krueger et al. (2016), capital depreciation shocks, capital taxation, and accounting for the beneficial tax treatment of debt. These additional features help generate moments from the data in the model, which are essential to match in order to examine the question at hand. In particular, life cycle helps the model to generate a more realistic mass of households close to the borrowing constraint,<sup>3</sup> depreciation shocks help to generate a realistic equity premium,<sup>4</sup> and the beneficial tax treatment of debt (debt tax shield) is another feature of the model that breaks the Modigliani-Miller theorem on the neutrality of debt.<sup>5</sup>

This paper studies how firm leverage influences inequality and macroaggregates in a dynamic general equilibrium model. In particular, it performs an exercise that compares the long-run equilibrium in a model with the level of leverage from the early 1980s (which is very similar to the value in 2008), with the one with the level of leverage in the early 1990s. This is a particularly interesting time period because, at the same time that the changes in leverage were happening, the wealth inequality has been steadily increasing (Saez and Zucman, 2016). Firm leverage in the US economy (measured as total debt over total capital) during the 1980s increased from roughly 35 to 48 percent in the early 1990s (see Graham et al. (2015)), who use Moody's Industrial Manual to compute it), and subsequently fell back to approximately 35 – 37% percent in 2010 (see Graham et al. (2015), who use Moody's Industrial Manual data).<sup>6</sup> The study of the causes of the increase of firm leverage is beyond the scope of this paper. Instead, the analysis is limited to the consequences of such an increase. The potential reasons for the change in leverage in the mentioned period is briefly discussed in the Appendix G. The results of the model suggest that the sudden increase in firm leverage that occurred in the US during the 1980s did not contribute to the increase in wealth inequality, but on the contrary, it is the steady reduction in firm leverage since the early 1990s that has contributed

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<sup>3</sup>This is because, in the infinite horizon models, majority of households live “long enough” to eventually accumulate a relatively large capital buffer to insure themselves against a series of bad shocks that would move them close to the borrowing constraints. However, if the households die periodically, many of them will not be very far from the borrowing constraints.

<sup>4</sup>Depreciation shocks increase the equity premium, as they increase the stock returns volatility.

<sup>5</sup>Modigliani and Miller (1963)

<sup>6</sup>To see the historical changes, see Figure 5. in Appendix B.

to the subsequent increases in wealth inequality.

The benchmark model finds that the increase of leverage of the magnitude observed in the late 80s leads to an increase in capital accumulation, and a decrease in both wealth inequality and government revenue. In the version of the model without capital taxation and beneficial tax treatment of debt, the changes in leverage lead to significant changes in asset prices but do not have quantitatively significant effects on inequality and macroaggregates. This result is consistent with the results of Algan et al. (2009), who find that in the Krusell-Smith (1997,1998) setting, Modigliani-Miller theorem holds approximately.

In terms of the model complexity, this paper includes idiosyncratic risk, aggregate risk, portfolio choice with discrete participation decision, and parsimonious life cycle. In a general context, the model is built on the tradition of the models of Bewley (1977), Huggett (1993) and Aiyagari (1994) with incomplete markets. Huggett (1993) studies a model where infinitely lived agents face idiosyncratic risk and borrowing constraint, and can only trade bonds which are in zero supply. Aiyagari (1994) adds an endogenous supply of bonds (capital) from firms, and constructs a general equilibrium model. Krusell and Smith (1998) add aggregate risk to the Aiyagari (1994) model, which means that the aggregate capital and prices in the economy are not constant, as in Aiyagari (1994). Furthermore, Krusell and Smith (1997) add portfolio choice to the model with both idiosyncratic and aggregate risk and allow the households to save in both capital and zero supply bonds. Papers such as Harenberg and Ludwig (2018), Krueger and Kubler (2006) and Gomes et al. (2013) add an overlapping-generations (OLG) structure on top of the complexity of Krusell and Smith (1997). One of the more similar models is in Algan et al. (2009), which does not have OLG structure, but adds a positive supply of both bonds and capital, which are issued by a leveraged firm. The model in this dissertation adds a parsimonious OLG structure, risk in capital depreciation, and a discrete choice of households whether to participate in the stock market or not. This means that the agents do not only choose how much they went to invest in stocks and bonds, but also whether to pay stock market participation and participate in this market in the first place. Therefore, my model differentiates from Algan et al. (2009) in that it contains OLG structure and an additional discrete decision. Furthermore, it is different from Harenberg and Ludwig (2018), Gomes et al. (2013),

and Krueger and Kubler (2006), because it has a more parsimonious OLG structure, and at the same time it has additional discrete decision and positive supply of bonds from a leveraged firm.

The remainder of the paper describes the benchmark model and calibration, presents and discusses the results, studies several decompositions and extensions, and finally concludes.

## 2 Model

I construct a model based on Algan et al. (2009), and in the tradition of Krusell and Smith (1997). The model consists of a continuum of heterogeneous agents facing aggregate risk, uninsurable idiosyncratic labor risk and a borrowing constraint, and who save in two assets: risky equity and safe bonds. Unlike Algan et al. (2009), the model parsimoniously captures the life cycle dynamics of the households in the fashion of Krueger et al. (2016), where working-age agents face the retirement shock and retired households face the risk of dying. Furthermore, this model captures the beneficial tax treatment of debt.

### 2.1 Production technology

In each period  $t$ , the representative firm uses aggregate capital  $K_t$ , and aggregate labor  $L_t$ , to produce  $y$  units of a final good with the aggregate technology  $y_t = f(z_t, K_t, L_t)$ , where  $z_t$  is an aggregate total factor productivity (TFP) shock. I assume that  $z_t$  follows a stationary Markov process with transition function  $\Pi_t(z, z') = Pr(z_{t+1} = z' | z_t = z)$ . The production function is continuously differentiable, strictly increasing, strictly concave and homogeneous of degree one in  $K$  and  $L$ . Capital depreciates at the stochastic rate  $\delta_t \in (0, 1)$  and it accumulates according to the standard law of motion:

$$K_{t+1} = I_t + (1 - \delta_t)K_t$$

where  $I_t$  is aggregate investment. Particular aggregate production technology is:

$$Y_t = z_t A K_t^\Delta L_t^{1-\Delta}$$

## 2.2 Preferences

Households are indexed by  $i$ , and they have Epstein-Zin preferences (Epstein and Zin, 1989).

They are maximizing their lifetime utility, expressed recursively for the retired agents:

$$V_{R,i,t} = \{c_t^{1-\rho} + v\beta[E_t V_{R,i,t+1}^{(1-\alpha)}]^{\frac{1-\rho}{1-\alpha}}\}^{\frac{1}{1-\rho}}$$

where  $V_{R,i,t}$  is the recursively defined value function of a retired household  $i$ , at time period  $t$ .

Working-age agents maximize:

$$V_{W,i,t} = \{c_t^{1-\rho} + \beta[(1-\theta)E_t V_{W,i,t+1}^{1-\alpha} + \theta E_t V_{R,i,t+1}^{1-\alpha}]\}^{\frac{1-\rho}{1-\alpha}}$$

where  $V_{W,i,t}$  is the recursively defined value function of a working-age household  $i$ , at time period  $t$ . Furthermore,  $\beta$  denotes the subjective discount factor,  $E_t$  denotes expectations conditional on information at time  $t$ ,  $\alpha$  is the risk aversion,  $\frac{1}{\rho}$  is the intertemporal elasticity of substitution,  $\theta$  is the probability of retiring, and  $v$  is the probability of retired agents dying.

## 2.3 Life cycle structure

In each period, working-age households have a chance of retiring  $1 - \theta$ , and retired households have a chance of dying  $1 - v$ , as in Castaneda et al. (2003) and Krueger et al. (2016). Therefore, the share of working age households in the total population is:

$$\Pi_W = \frac{1 - v}{(1 - \theta) + (1 - v)}$$

and the share of the retired households in the total population is:

$$\Pi_R = \frac{1 - \theta}{(1 - \theta) + (1 - v)}$$

The retired households who die in period  $t$  are replaced by new-born agents who start at a working age without any assets. For simplicity, the retired households have perfect annuity markets, which make their returns larger by a fraction of  $\frac{1}{v}$ , as in Krueger et al. (2016).

It is common that models with idiosyncratic risk, aggregate risk, and imperfect markets

generate fewer households that hold zero or almost zero wealth (or are near the borrowing constraint) than is observed in the data. Consequently, these models often do not generate aggregate dynamics, which are very different from the representative agent model. However, Krueger et al. (2016) show that it is possible to generate a realistic mass of households with zero wealth, or near the borrowing constraint, by adding parsimonious life-cycle to the model. More importantly, they also demonstrate that once the model generates a realistic lower tail of wealth distribution, the heterogeneity matters for the aggregate dynamics. Simultaneously, the approximate aggregation result from Krusell and Smith (1997, 1998) still holds if the change in wealth distribution is captured well enough by the changes of aggregate capital and aggregate productivity shock (or any other state variable that directly enters the aggregate capital perceived laws of motion).

Furthermore, Chang et al. (2018) study the Survey of Consumer Finance and show that the financial portfolio composition changes with the age of the households. They find that the share of savings invested in risky assets initially increases in the age of the household (controlling for the wealth of the household), but as the households retire, the households begin to reduce their risky share as they become older. As the model has only two life cycle stages; working-age and retired, it is not possible to replicate this inverse U-shaped curve, but the model will replicate the fact that the retired households save disproportionately more in the risky asset compared to the working-age households.<sup>7</sup>

## 2.4 Idiosyncratic uncertainty

In each period, working-age households are subject to an idiosyncratic labor income risk that can be decomposed into two parts. The first part is the employment probability that depends on aggregate risk and is denoted by  $e_t \in (0, 1)$ .  $e = 1$  denotes that the agent is employed, and  $e = 0$  that the agent is unemployed. Conditional on  $z_t$  and  $z_{t+1}$ , I assume that the period  $t + 1$  realization of the employment shock follows the Markov process.

$$\Pi_e(z, z', e, e') = Pr(e_{t+1} = e' | e_t = e, z_t = z, z_{t+1} = z')$$

This labor risk structure allows idiosyncratic shocks to be correlated with the aggregate

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<sup>7</sup>See also Fagereng et al. (2017)

productivity shocks, which is consistent with the data and generates the portfolio choice profile such that the share of wealth invested in risky assets is increasing in wealth. The condition imposed on the transition matrix and the law of large numbers implies that aggregate employment is only a function of the aggregate productivity shock.

If  $e = 1$  and the agent is employed, one can assume that the agent is endowed with  $l_t \in L \equiv \{l_1, l_2, l_3, \dots, l_m\}$  efficiency labor units, which she can supply to the firm. Labor efficiency is independent of the aggregate productivity shock, and it is governed by a stationary Markov process with transition function  $\Pi_l(l, l') = Pr(l_{t+1} = l' | l_t = l)$ . If the agent is unemployed, (s)he receives unemployment benefits  $g_{u,t}$ , which are financed by the government.

Comparing the calibration to the cases in Davila et al. (2012), it is not a priori clear whether the model will generate excessive or insufficient aggregate capital accumulation (in a constrained efficiency sense). This is because the Markov process capturing labor income uncertainty is not made to replicate wealth inequality (as in the insufficient capital accumulation case), nor the unemployment economy (excessive capital accumulation case). Instead, the income process is supposed to capture the actual labor income risks that workers face.

## 2.5 The representative firm

As in Algan et al. (2009), firm leverage in this model is given exogenously. The leverage of the firm is determined exogenously, by the parameter  $\lambda$ . The Modigliani-Miller theorem (1958, 1963) does not hold, as some of the agents are borrowing-constrained, and some are portfolio-constrained. Therefore, theoretically, the leverage of the firm has some macroeconomic relevance. Additionally, debt is taxed differently than equity returns, and this additionally breaks the Modigliani-Miller theorem.

In the economy, the representative firm can finance its investment with two types of contracts. The first is a one-period risk-free bond that promises to pay a fixed return to the owner. The second is a risky equity that entitles the owner to claim the residual profits of the firm after the firm pays out wages and debt from the previous period. Both of these assets are freely



traded in competitive financial markets. By construction, there is no default in the equilibrium.

The return on the bond  $r_{t+1}^b$  is determined by the clearing of the bond market:

$$\int_S g^{b,j,e} d\mu = \lambda K'$$

where  $g^{b,j,e}$  are the individual policy functions for bonds.

In each period  $t$ , the firm redistributes all the residual value of the firm, after production and depreciation have taken place, and wages and debt have been paid. Therefore, the return on the risky equity depends on the realizations of the aggregate shocks and is given by the equation:

$$(1 + r_{t+1}^s) = \frac{f(z_{t+1}, K_{t+1}, L_{t+1}) - f_L(z_{t+1}, K_{t+1}, L_{t+1})L_{t+1} - \lambda K_{t+1}(1 + r_{t+1}^b) + (1 - \delta_{t+1})K_{t+1}}{(1 - \lambda)K_{t+1}}$$

Since the return to stocks subject to corporate income tax, the post-tax gross return to equity  $1 + r_{t+1}^{s,p}$  is:

$$1 + r_{t+1}^{s,p} = 1 + r_{t+1}^s(1 - \tau_s)$$

An important caveat in having heterogeneous households who own the firm is that they do not necessarily have the same stochastic discount factor  $m_{t+1}^j$ , and therefore the definition of the objective function of the firm is not straightforward. I follow Algan et al. (2009), who assume that the firm is maximizing the welfare of the agents who have an interior portfolio choice, and consequently, the firm has the same stochastic discount factor  $m_{t+1}$  as the agents with the interior portfolio choice.

As in Algan et al. (2009), it is possible to use the fact that, for a given stochastic discount factor,  $V_t = K_{t+1}$ , which enables the elimination of the capital Euler equation from the equilibrium conditions.

## 2.6 Financial markets

As stated earlier, households can save in two assets: risky equity and safe bonds (firm debt). There are borrowing constraints for both assets, and thus the lowest amounts of equity and debt that households can hold in period  $t$  are:  $\kappa^s$  and  $\kappa^b$ , respectively. Markets are assumed to be incomplete, in the sense that there are no markets for the assets contingent on the realization of individual idiosyncratic shocks. Furthermore, if the household wants to save a positive amount of resources in equity in period  $t$ , it has to pay  $\phi$  as a per-period cost of participating in the stock market.

## 2.7 Government

The government runs two social programs: social security (retirement benefits), and unemployment insurance, and are modeled as in Krueger et al. (2016). Both are financed by separate labor taxes. Social security is financed with a constant labor tax rate:  $\tau^{lss}$ , and the revenues  $T_t^{ss} = \frac{L_t}{\Pi_R} w_t L_t \tau^{lss}$  are equally distributed in period  $t$  to all retired households, irrespective of their past contributions. Unemployment benefits are financed with a labor tax rate  $\tau_t^u$ . The amount of unemployment benefits  $g_{u,t}$  is determined by a constant  $\eta$ , which represents the fraction of the average wage in each period.

To satisfy the budget constraint, the government has to tax labor with the tax rate:

$$\tau_t^u = \frac{1}{1 + \frac{1 - \Pi_u(z)}{\Pi_u(z)\eta}}$$

where  $\Pi_u$  is the share of unemployed people in the total working-age population.

Furthermore, the government taxes the net profit of the firm by a corporate income tax with the rate  $\tau_s$ . Therefore, the net return to the investment in stock is  $r_{t+1}^s(1 - \tau_s)$ . The revenue collected by corporate income tax is spent on wasteful government consumption. This is a simplifying assumption, since studying government expenditure is not a central question for this paper.

## 2.8 Household problem

Retired household  $i$  maximizes its lifetime utility subject to the following constraints:

$$\begin{aligned}
c_{i,t} + s_{i,t+1} + b_{i,t+1} + \phi \mathbb{I}_{\{s_{i,t+1} \neq 0\}} &\leq \omega_{i,t} \\
\omega_{i,t+1} &= T_{ss,t+1} + [(1 + r_{t+1}^s) s_{i,t+1} + (1 + r_{t+1}^b) b_{i,t+1}] \frac{1}{v} \\
(c_{i,t}, b_{i,t+1}, s_{i,t+1}) &\geq (0, \kappa^b, \kappa^s)
\end{aligned}$$

Working age household  $i$  maximizes its expected lifetime utility subject to the constraints below:

$$\begin{aligned}
c_{i,t} + s_{i,t+1} + b_{i,t+1} + \phi \mathbb{I}_{\{s_{i,t+1} \neq 0\}} &\leq \omega_{i,t} \\
\omega_{i,t+1} &= \begin{cases} w_{t+1} l_{i,t+1} (1 - \tau_{t+1}^l) + (1 + r_{t+1}^s (1 - \tau_s)) s_{i,t+1} + (1 + r_{t+1}^b) b_{i,t+1} & \text{if } e = 1 \\ g_{u,t+1} (1 - \tau_{t+1}^l) + (1 + r_{t+1}^s (1 - \tau_s)) s_{i,t+1} + (1 + r_{t+1}^b) b_{i,t+1} & \text{if } e = 0 \end{cases} \\
(c_{i,t}, b_{i,t+1}, s_{i,t+1}) &\geq (0, \kappa^b, \kappa^s)
\end{aligned}$$

## 2.9 Recursive household problem

The idiosyncratic state variables of the household problem are: current wealth  $\omega$ , households age, and if the household is not retired: current employment and productivity state  $e, l$ . By  $\Theta$ , I denote the vector of all discrete individual states (all except the current wealth).<sup>8</sup>

The aggregate state variables of the household problem are: state of the TFP shock  $z$ , state of the capital depreciation shock  $\delta$ , and distribution captured by the probability measure  $\mu$ .  $\mu$  is a probability measure on  $(S, \beta_s)$ , where  $S = [\underline{\omega}, \bar{\omega}] \times \Theta$ , and  $\beta_s$  is the Borel  $\sigma$ -algebra.  $\underline{\omega}$  and  $\bar{\omega}$  denote the minimal and maximal allowed amount of wealth the household can hold.<sup>9</sup> Therefore, for  $B \in \beta_s$ ,  $\mu(B)$  indicates the mass of households whose individual states fall in  $B$ . Intuitively, one can think of  $\mu$  as a distribution variable that measures the mass of agents in a certain interval of wealth, for each possible combination of other idiosyncratic variables.

<sup>8</sup>In the benchmark model, there will be five elements of  $\Theta$ : retired, unemployed, and three levels of productivity for the employed households.

<sup>9</sup> $\underline{\omega}$  is determined by the borrowing constraint, and  $\bar{\omega}$  is chosen such that there are always no agents with that amount of wealth in equilibrium.

The recursive household problem for the retired households:

$$v_R(\omega; z, \mu, \delta) = \max_{c, b', s'} \left\{ u(c - \gamma)^{1-\rho} + v \beta E_{z', \mu', \delta' | z, \mu, \delta} [v_R(\omega'; z', \mu', \delta')^{1-\alpha}]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}$$

subject to:

$$c + s' + b' + \phi \mathbb{I}_{\{s' \neq 0\}} = \omega$$

$$\omega' = T'_{ss} + [s'(1 + r^s) + b'(1 + r^b)] \frac{1}{v}$$

$$\mu' = \Gamma(\mu, z, z', d, d')$$

$$(c, b', s') \geq (0, \kappa^b, \kappa^s)$$

Working-age households:

$$v_W(\omega, e, l; z, \mu, \delta) =$$

$$\max_{c, b', s'} \left\{ u(c - \gamma)^{1-\rho} + \beta E_{e', l', z', \mu', \delta' | e, l, z, \mu, \delta} [(1 - \theta)v_W(\omega', e', l'; z', \mu', \delta')^{1-\alpha} + \theta v_R(\omega', e', l'; z', \mu', \delta')^{1-\alpha}]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}$$

subject to:

$$c + s' + b' + \phi \mathbb{I}_{\{s' \neq 0\}} = \omega$$

$$\omega' = \begin{cases} w'l'(1 - \tau^l) + s'(1 + r^s(1 - \tau_s)) + b'(1 + r^b) & \text{if } e = 1 \\ g'_u w'(1 - \tau^l) + s'(1 + r^s(1 - \tau_s)) + b'(1 + r^b) & \text{if } e = 0 \end{cases}$$

$$\mu' = \Gamma(\mu, z, z', d, d')$$

$$(c, b', s') \geq (0, \kappa^b, \kappa^s)$$

where  $\omega$  is the vector of individual wealth of all agents,  $\mu$  is the probability measure generated by the set  $\Omega x E x L$ ,  $\mu' = \Gamma(\mu, z, z', d, d')$  is a transition function and  $'$  denotes the next period.

## 2.10 General equilibrium

The economy-wide state is described by  $(\omega, e; z, \mu, d)$ . Therefore, the individual household policy functions are:  $c^j = g^{c,j}(\omega, e, l; z, \mu, d)$ ,  $b^j = g^{b,j}(\omega, e, l; z, \mu, d)$  and  $s^j = g^{s,j}(\omega, e, l; z, \mu, d)$ , and the law of motion for the aggregate capital is  $K' = \int_S g^{b,j}(\omega, e, l; z, \mu, d) + g^{s,j}(\omega, e, l; z, \mu, d)$ .

A recursive competitive equilibrium is defined by the set of individual policy and value functions  $\{v_R, g^{c,R}, g^{s,R}, g^{b,R}, v_W, g^{c,W}, g^{s,W}, g^{b,W}\}$ , the law of motion for the aggregate capital  $g^K$ , a set of pricing functions  $\{w, R^b, R^s\}$ , government policies in period  $t$ :  $\{\tau^{lb}, \tau^u, \tau^s, \tau^b\}$  and tax rates contingent on the aggregate states in period  $t + 1$ :  $\{\tau'^{lb}, \tau'^u, \tau'^s, \tau'^b\}$ , and forecasting equations  $g^L$ , such that:

1. The law of motion for the aggregate capital  $g^K$  and the aggregate “wage function”  $w$ , given the taxes satisfy the optimality conditions of the firm.
2. Given  $\{w, R^b, R^s\}$ , the law of motion  $\Gamma$ , the exogenous transition matrices  $\{\Pi_z, P_{i_e}, P_{i_l}\}$ , the forecasting equation  $g^L$ , the law of motion for the aggregate capital  $g^K$ , and the tax rates, the policy functions  $\{g^{c,j}, g^{b,j}, g^{s,j}\}$  solve the household problem.
3. Labor, shares and the bond markets clear (the goods market clears by the Walras’ law):

•

$$L = \int_S e l d\mu$$

•

$$\int_S g^{s,j}(\omega, e, l; z, \mu, \delta) d\mu = (1 - \lambda)K'$$

•

$$\int_S g^{b,j}(\omega, e, l; z, \mu, \delta) d\mu = \lambda K'$$

4. The law of motion  $\Gamma(\mu, z, z', \delta, \delta')$  for  $\mu$  is generated by the optimal policy functions  $\{g^c, g^b, g^s\}$ , which are endogenous, and by the transition matrices for the aggregate shocks ( $z$  and  $\delta$ ).<sup>10</sup> Additionally, the forecasting equation for aggregate labor is consistent with the labor market clearing:  $g^L(z', \delta') = \int_S e l d\mu$ .

5. Government budget constraints are satisfied:

$$T_t^{ss} = \frac{L_t}{\Pi_R} w_t L_t \tau^{lss}$$

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<sup>10</sup>  $\mu'$  is given by a function  $\Gamma$ , i.e.  $\mu' = \Gamma(\mu, z, z', \delta, \delta')$

$$\tau_t^u = \frac{1}{1 + \frac{1 - \Pi_u(z)}{\Pi_u(z)\phi}}$$

### 3 Parametrization

Parametrization and calibration mainly follow Algan et al. (2009) and Krueger et al. (2016). The model is calibrated to quarterly frequency. There are two possible realizations for TFP shocks: good and bad state. In addition, capital depreciation shock can also take two possible values. Therefore, there are four possible aggregate states overall. The probability of remaining in the same state is 0.875. A discount factor is calibrated to match the capital-output ratio and interest rate. <sup>11</sup>

**Table 1: Internally-calibrated parameters**

Parameter	Symbol	Value	Target
Discount factor	$\beta$	0.8703	Capital-Output ratio
Subsistence constraint	$\gamma$	0.03	Portfolio choice pattern
Quarterly stock market participation costs	$\phi$	0.0044	Share of households with no equity

**Table 2: Externally-calibrated parameters**

Parameter	Symbol	Value	Target
Intertemporal elasticity of substitution	$\frac{1}{\rho}$	0.5	Capital-Output ratio
Expected depreciation rate	$E(\delta)$	0.033	Equity premium
Chance of not retiring	$\theta$	.994	Average working duration
Chance of not dying	$v$	0.983	Average retirement duration
Tax advantage of debt	$\tau^s$	0.3	Hennessy and Whited (2005)
Capital share	$\Delta$	0.4	Algan et al. (2009)

**Table 3: Parameters to generate a sizable equity premium**

Parameter	Symbol	Value
Risk aversion	$\alpha$	12
Variance of depreciation rate	$\sigma^2(\delta)$	0.001

<sup>11</sup> $\beta$  is relatively low because the agents face high idiosyncratic risk, while having substantial risk aversion, which makes them have high precautionary savings. The variance of the depreciation shocks may not seem large, but depreciation equals, on average, 6.5% in the high state and 0.2% in the low state.

**Table 4: Other parameters**

Parameter	Symbol	Value
Social security tax	$\tau^{lss}$	0.06
Unemployment replacement rate	$\eta$	0.042
Borrowing constraint: bonds	$\kappa^b$	-0.19
Borrowing constraint: stocks	$\kappa^s$	0.0

For the idiosyncratic labor income shocks transition matrix, I use the same values as Pijoan-Mas (2007).

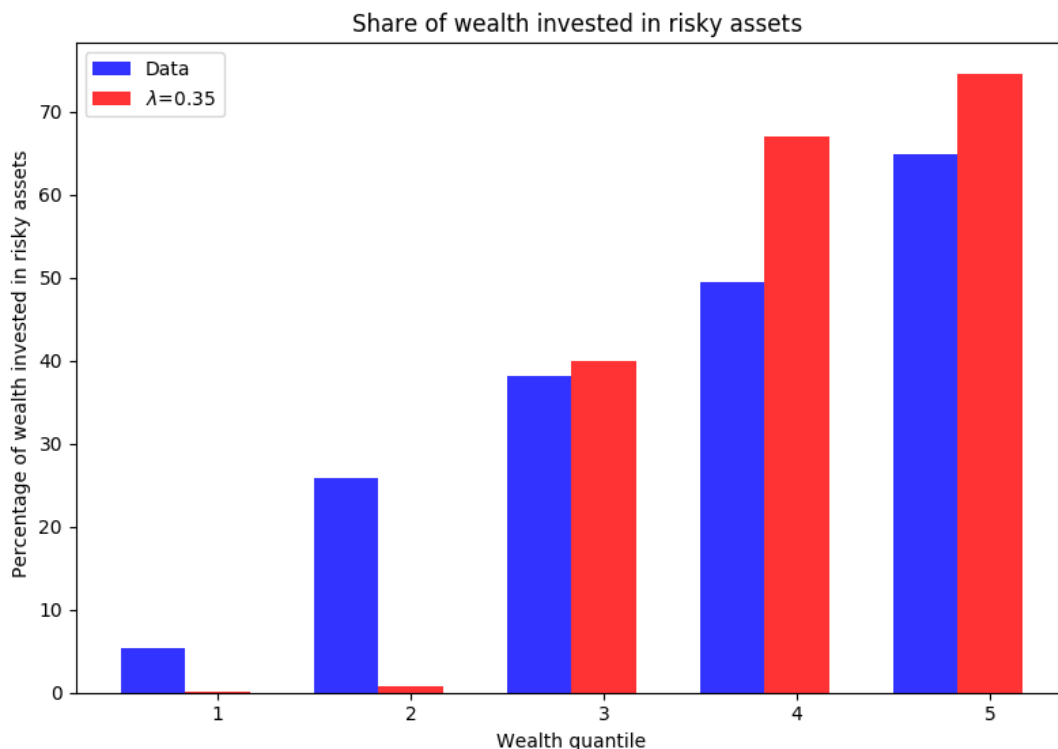
$$\Pi_l = \begin{bmatrix} 0.9850 & 0.0100 & 0.0050 \\ 0.0025 & 0.9850 & 0.0125 \\ 0.0050 & 0.0100 & 0.9850 \end{bmatrix}$$

For the individual labor productivity levels, the following values are used:  $l \in \{36.5, 9.5, 1.2\}$  (they differ slightly from those used by Pijoan-Mas (2007) to account for the fact that, in the model presented here, the household has to pay taxes for social programs). This type of modeling the labor productivity process allows the generation of realistic earnings inequality while keeping the possible number of states relatively low. As in Algan et al. (2009), the expected unemployment duration is set to 1.5 quarters in a good TFP state, and 2.5 quarters in bad TFP state. The unemployment benefits  $g_u$  are set to 4.2% of the average wage in period  $t$ .

Following Krueger et al. (2016), I set  $\theta$  and  $v$  to match the expected work-life length to 40 years, and retirement to 15 years.

Table 5 shows the performance of the model concerning asset pricing and compares it to the values from Algan et al. (2009), which can be considered as a benchmark economy, where the Modigliani-Miller theorem holds approximately. The model asset pricing performance moves much closer to the data, but the classic asset pricing puzzles are still present (excessive bond interest rate, and insufficiently high equity premium).

Figure 1.



Data for portfolio choice are taken from Chang et al. (2018), who use Survey of Consumer Finance.

## 4 Solution method

Following Krusell and Smith (1997, 1998), households consider that the aggregate amount of capital and equity premium in the economy move according to the perceived laws of motion, which depend on the TFP aggregate state and aggregate capital. Instead of using a perceived laws of motion for the bond interest rate, like Krusell and Smith (1997) and Algan et al. (2009), I use the perceived law of motion for the equity premium. This method facilitates computation, because during the course of solving the model, predicting negative equity premium is eliminated if logarithmic rules are used (see Harenberg and Ludwig (2018)). For details about the solution algorithm, see the Appendix. The perceived laws of motion are:

$$\ln K' = a_0(z, \delta) + a_1(z, \delta) \ln K$$

$$\ln P^e = b_0(z, \delta) + b_1(z, \delta) \ln K'$$



For the benchmark economy ( $\lambda = 0.35$ ) the perceived aggregate laws of motion are: In a high TFP and high  $\delta$  state:

$$\ln K' = 0.085 + 0.941 \ln K$$

$$\ln P^e = -4.547 - 0.107 \ln K'$$

In a low TFP and high  $\delta$  state:

$$\ln K' = 0.075 + 0.945 \ln K$$

$$\ln P^e = -4.568 - 0.090 \ln K'$$

In a high TFP and low  $\delta$  state:

$$\ln K' = 0.112 + 0.948 \ln K$$

$$\ln P^e = -4.311 + 0.115 \ln K'$$

In a low TFP and low  $\delta$  state:

$$\ln K' = 0.111 + 0.947 \ln K$$

$$\ln P^e = -4.305 - 0.086 \ln K'$$

The perceived laws of motion predicts the actual movements of capital and equity premium with  $R^2 = 0.99991$  for capital and  $R^2 = 0.99900$  for equity premium.

The average error for the aggregate capital laws of motion is 0.020% percent of the capital stock, while the maximum error is 0.062% of the capital stock.

## 5 Results

The model is calibrated to match the share of households who do not participate in the equity markets in a low leverage economy, and generates 45.9% of non-participating households, which is only slightly less than 46.7%, which is what Chang et al. (2018) observe in the data. In a high leverage economy, this share is only slightly lower, with 45.0% of agents participating in the stock market.<sup>12</sup> Furthermore, the model roughly generates the portfolio choice pattern along the wealth distribution dimension (as shown in Figure 1.) Furthermore, the model does not generate the realistic right tail of the wealth distribution. This is a common issue in standard models in the field.<sup>13</sup> Furthermore, it is questionable how relevant the returns on bonds and market indices are for the saving behavior of the very richest households, who mostly own very diversified equity. Finally, the model generates a Sharpe ratio of 0.33.

I perform an exercise in which the leverage of the economy rises from  $\lambda = 0.35$ , which was the leverage in the US economy in the 1990s, to  $\lambda = 0.48$ , which was the value in 1984.

**Table 5. Quarterly statistics for the benchmark economy**

	$K/Y$	Wealth Gini	Households not owning any equity %	$r^b$	$E(r_{pt}^s) - r_{pt}^b$
Data	7	0.780	46.7	0.23	1.0 - 2.0
Economy $\lambda = 0.35$	7.00	0.5738	45.9	0.36	1.02
Economy $\lambda = 0.48$	7.21	0.5631	45.0	0.24	1.31
Algan et al. (2009) $\lambda = 0.37$	7.01	0.480	41.0	2.4	0.007

<sup>12</sup>Billas et al. (2017) discuss how an increase in stock market participation can influence inequality. On the one hand, it makes poorer households obtain equity premium, but at the same time, poorer households might make bad investment decisions, if they are not sophisticated enough in their stock investment behavior. The authors find that the increased stock market participation has not significantly changed inequality in the US during the 1990s. However, in my model, the latter channel is not existent, since only the representative firm issues equity (which is only subject only to the aggregate risk).

<sup>13</sup>It would be possible, for example, to generate it by introducing a stochastic discount factor in the fashion of Krusell and Smith (1998), but that would complicate the analysis and would be computationally expensive.

**Table 6: Results: Change in asset prices and inequality**

Unconditional moments	$\lambda = 35$ Economy	$\lambda = 48$ Economy
$E(r^{s,p} - r^{b,p})$	1.02%	1.31%
$E(r^{b,p})$	0.36%	0.23%
$E(r^{s,p})$	1.39%	1.54%
Wealth Gini	0.5738	0.5631
$E(K)$	5.76	6.07
Share of wealth by quintiles (%)		
1st	0.92%	0.96%
2st	4.20%	4.65%
3st	11.40%	11.85%
4st	22.14%	22.26%
5st	61.57%	60.48%

**Table 7: Changes in the Wealth shares:**

	Wealth change				
Quintile	1	2	3	4	5
% change	+4.35	+10.71	+3.95	+0.54	-1.77

The change in leverage has important implications for asset prices. Increased leverage means increased variance of equity returns, and consequently, increased equity premium. However, this does not necessarily mean that the portfolios of the households holding equity are riskier; now the bonds are in higher demand, and the households will hold less risky equity and more safe bonds. Overall, the Sharpe ratio in the economy remains approximately the same: 0.33.<sup>14</sup>

The average long-run aggregate capital depends on the leverage. The economy with high leverage  $\lambda = 0.48$  has 5.38% more capital in the long run than the economy with relatively low leverage  $\lambda = 0.35$ . To see the intuition behind this, one can see that the increase in leverage effectively functions as a decrease in the capital tax rate. With the higher leverage, more of the firms' surplus (not profits, but a surplus as defined by surplus=profits+debt interest payment) is distributed as debt interest payments, and less as profits. Since the debt has a beneficial

<sup>14</sup>Not reported in tables. There is a slight change from 0.3258 to 0.3327

tax treatment (debt repayment is tax-deductible), this effectively serves as the reduction in the capital tax rate. Consequently, the effective decrease in the overall capital tax rate leads to an increase in capital accumulation.

Intuitively, the effective reduction in the tax rate, or the effective reduction in the tax base (decrease in profits and increase in debt repayment), leads to a decrease in government revenue. More precisely, the government revenue from capital taxes in the low leverage economy is 1.85%, and in the high leverage economy, it is reduced in the low leverage economy to 1.74%. Furthermore, as a consequence of the capital increase, wages rise by 1.03% in the high leverage economy.

Wealth inequality decreases by 1.07 Gini points. This amounts to 12% of the change in wealth Gini in the observed period. Given that there were many policies and global economic changes during the studied period, this can be considered a significant decrease. The reason that drives the wealth inequality reduction is the different response to the change in the effective tax rate of the poor and rich. Increased leverage implies less corporate tax paid, which in turn implies increased net returns on savings. How will the households' savings respond to the increased interest rate depends on two counteracting effects: substitution and income effects. The substitution effect means a household wants to save more, since the reward for saving in the next period is higher, i.e., households want to substitute the consumption today for consumption in the future. On the other hand, there is an income effect, which means that, since the households are now richer (its savings are now more valuable since they give a higher return), it wants to consume more and save less. The change in wealth inequality will be determined by the size of the two effects for the poor and the rich. The results of the model are the following: first, the substitution effect is stronger than the income effect for all households, meaning that all households save more. Second, the income effect is relatively stronger for the rich, compared to poor households. This occurs because the more wealth the households already have, the stronger the income effect is. This follows from the fact that the income effect of the increased interest rate is multiplicative; the more wealth the households already have, proportionately the more income it will receive if their savings behavior does not

change.<sup>1516</sup>

Additionally, the return on bonds (safe asset) declines, both directly because of the change in leverage: increased supply of the bond would imply lower returns, and (mainly) indirectly through the increased aggregate capital and decreased marginal productivity of capital. The equity premium does not increase as much with the increased leverage as it would in the economy without equity taxation. This is because, absent of taxes, when leverage increases, the variance of equity returns also increases (compared to the low-leverage economy, the returns are even better in the good state, and even worse in the bad state because the firms need to repay the same amount to debt owners, regardless of the performance in the current period). However, when equity returns are taxed, this effect is dampened, because the tax bill will be higher in the good state, and lower in the bad state.

From the utilitarian standpoint, the change of leverage (including the transition path of the economy) is welfare improving from the utilitarian standpoint, and equivalent to the 5.7% permanent increase in consumption. This is hardly unexpected, as the increase in leverage implies a smaller tax base for corporate income tax, and the government revenue collected by corporate income tax is assumed to be wasteful in this model. In addition, the eventual capital accumulation raises wages, which are an important source of income for agents close to the borrowing constraint. Furthermore, as in line with Davila et al. (2012), there is a severe capital under-accumulation in this model. The welfare gains across the household type and wealth can be seen in Appendix C.

To demonstrate the importance of modeling capital taxation and debt tax benefits in extension 1, I perform the same exercise of increasing leverage, but without capital income taxation. As can be seen in tables 8 and 9, the results on the aggregates and inequality are quite small.

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<sup>15</sup>In other words: the intuition behinds result is: the rising returns on savings have substitution effect (consumption today is more expensive) which increases savings, and income effect (asset owners are now getting higher interest/dividends and are effectively richer) which decreases savings (increases consumption today). Since poor households do not own many assets, their income effect is much weaker than for wealthy households, and consequently, they increase their saving rates more than the wealthy households do. In the long run, this results in lower wealth inequality.

<sup>16</sup>The size of this effect, among other factors, depends on the intertemporal elasticity of substitution, which is partially governed by the parameter  $\rho$ . The magnitude of the change of the wealth inequality should increase with the increase of intertemporal elasticity of substitution (decrease in  $\rho$ , for example).

These results show that the “Modigliani-Miller” result of the irrelevance of leverage holds even in a Krusell and Smith (1997) type of economy, as stated in Algan et al. (2009). However, this paper extends the result to the economy in which the equity premium is realistically high, and equity market participation constraints are present. It is important to generate a high equity premium since the Modigliani-Miller result is less surprising in a model where the two assets are not very different from a households’ perspective. In the absence of capital taxation, there are no real effects of the change in leverage because households are able to rebalance their asset holdings such that they can almost perfectly replicate their old portfolios. For example, when the leverage increases, the equity becomes more risky. But at the same time, there is less equity in the economy and there is more of safer bonds. Consequently, households will hold less equity and more bonds, and these two effects will approximately negate each-other. An important feature of this rebalancing is that the bond return remains approximately the same. This means that even the constrained agents, who hold only bonds and cannot rebalance their portfolios, will not be affected by the change in leverage.

### **Decomposition: Holding prices consistent with the levels of aggregate capital from $\lambda = 0.35$ economy**

The decomposition is performed to isolate the effect of higher capital accumulation from the other effects of the increase in leverage. To achieve this, leverage is set to  $\lambda = 0.48$ , but the set of prices is fixed such that they are consistent with the levels of capital in the  $\lambda = 0.35$  economy. Therefore, wages and output will be the same as in the  $\lambda = 0.35$  economy, but the asset prices (returns) will be different as a consequence of the changed leverage (but not as a consequence of the increased capital, as the output, and marginal capital productivity are held to be the same as in  $\lambda = 0.35$  economy). The increase in the aggregate capital (or rather the changes in the interest rate and wages caused by the rise in the aggregate capital) decreases inequality. Disregarding the general equilibrium effects of the increase in aggregate capital, the change of leverage itself decreases inequality by 0.85 Gini points (of total of 1.07). The reason is that an increase in the leverage acts similarly to a decrease in the tax rate on capital income, which ultimately leads to the increased net return on savings. The key is that the poor households respond more strongly to the increased savings returns, meaning that they increase their savings more than the rich households. This is because the income effect of the increased

savings returns is rather weak for the poor households, and relatively strong for the wealthy households since they have a considerable amount of accumulated savings. Furthermore, it increases the supply of the safe asset, in which the equity-constrained households exclusively save.

An increase in the aggregate capital further decreases wealth inequality (for the remaining 0.22 Gini points), as it decreases the interest rate (the primary source of income for the rich), and increases wages (the primary source of income for the poor). It is interesting to observe that the households in the lowest quintile benefit significantly from the increase in aggregate capital, with their wealth share increasing from 0.91% to 0.96%, as they are the ones who mostly rely on wages, which increase with the increase in aggregate capital.

To check that the changes in inequality are indeed driven by a change in the behavior of the rich and poor, and not young (which are poorer on average) and old (which are richer on average) households, I include the Gini indicator excluding retired households and focusing only on working-age households. The same inequality pattern is observed as when the whole population is included. Moreover, when excluding the retired households, the wealth inequality is reduced by 1.34 Gini points, and by 1.13 Gini points when ignoring the change in aggregate capital.

**Table 8: Results: Change in asset prices and inequality**

Unconditional moments	$\lambda = 35$ Economy	Decomposition Economy	$\lambda = 48$ Economy
$E(r^{s,p} - r^{b,p})$	1.02%	1.31%	1.31%
$E(r^{b,p})$	0.36%	0.36%	0.23%
$E(r^{s,p})$	1.37%	1.67%	1.54%
Wealth Gini	0.5738	0.5653	0.5631
Wealth Gini excluding retired households	0.4727	0.4614	0.4593
$E(K)$	5.76	/	6.07
Share of wealth by quintiles (%)			
1st	0.92%	0.91%	0.96%
2nd	4.20%	4.57%	4.65%
3rd	11.40%	11.80%	11.85%
4th	22.14%	22.24%	22.26%
5th	61.57%	60.69%	60.48%



## 6 Extensions

In this section, I analyze four extensions. All four models are recalibrated to match the crucial moments from the data.<sup>17</sup>

First, I consider an economy without capital taxation. The effects of the change in leverage on the wealth inequality are rather small, and even smaller on the aggregate capital. This extension contributes to the literature by showing that the Modigliani-Miller theorem holds “approximately” (Algan et al., 2009) in the borrowing constraint general equilibrium models, even when inequality and equity premium are comparable to those that we observe in the data. Unlike in the case without corporate tax, in the benchmark model, the households cannot approximately replicate their portfolios after the change in equity. This is because the presence of corporate income tax not only changes the value of the firm (and payoffs), but it also changes the riskiness of equity. This means that in the aggregate states with higher equity payoff, the tax bill will be higher than in the states with lower equity payoff, and this reduces the equity payoff variance.

Second, I consider an economy in which there is no capital income taxation, but also assumes that 56% of households are exogenously equity-constrained,<sup>18</sup> i.e., they can only save in bonds. For this economy, I find that the effects of the changes in leverage are quite small on the macroaggregates and wealth distribution, even though a majority of households are portfolio constrained. The intuition behind this result is that, in the absence of the debt tax shield, as long as there is a sufficient number of rich, well-insured households that can adjust their portfolio after the leverage change, the safe interest rate will not change significantly (thus the constrained households will not be significantly affected), and the non-constrained households can almost exactly replicate their portfolios from the low-leverage economy.

Third, I consider an economy in which 4% of households are exogenously allowed to save only in equity. This number is consistent with the share of such households in the Survey of Consumer Finance in 2016. For a household to be considered to invest only in equity, I choose

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<sup>17</sup>Therefore, some differences between variables in different extensions are simply a result of the different calibration.

<sup>18</sup>There are additional 3.3% of households who endogenously chose not to invest in equity.

the threshold value that no more than 10% of their savings should be in safe assets.<sup>19</sup> Now, the exogenously constrained households are directly impacted by the change in the leverage, since the variance of equity returns is changed, and they cannot rebalance their portfolios. However, the amount of this type of household is too small to change the wealth inequality or macroaggregates significantly.

The fourth extension considers a benchmark economy, but in addition, the government revenues from capital income taxes are rebated equally to the households in a lump-sum fashion. The goal is to perform a welfare comparison between the economies with different leverage since in the benchmark economy the additional tax revenue in the low-leverage economy is wasteful. However, there are many ways in which the additional revenue might be used, so naturally the welfare comparison results would change depending on the way in which this revenue is used. In this extension, the budget constraint of the households is as follows:

$$c_{i,t} + s_{i,t+1} + b_{i,t+1} + \phi \mathbb{I}_{\{s_{i,t+1} \neq 0\}} \leq \omega_{i,t}$$

$$\omega_{i,t+1} = \begin{cases} w_{t+1} l_{i,t+1} (1 - \tau_{t+1}^l) + (1 + r_{t+1}^s (1 - \tau_s)) s_{i,t+1} + (1 + r_{t+1}^b) b_{i,t+1} + T_{ls,t+1} & \text{if } e = 1 \\ g_{u,t+1} (1 - \tau_{t+1}^l) + (1 + r_{t+1}^s (1 - \tau_s)) s_{i,t+1} + (1 + r_{t+1}^b) b_{i,t+1} + T_{ls,t+1} & \text{if } e = 0 \end{cases}$$

$$(c_{i,t}, b_{i,t+1}, s_{i,t+1}) \geq (0, \kappa^b, \kappa^s)$$

$T_{ls,t+1}$  is a lump sum subsidy. This subsidy equally redistributes the revenue from the capital income taxes:

$$T_{ls,t+1} = \tau_s r_{t+1}^s K_t (1 - \lambda)$$

As reported in table 12., the increase in leverage leads to a decrease in wealth inequality, exactly as in the benchmark model. Furthermore, when examining the long-run results, welfare still increases in the high-leverage economy. This means that these two results persist even though lump-sum subsidies are reduced as a consequence of the decreased capital tax revenue. Lump-sum subsidies should generally decrease wealth inequality and increase welfare,

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<sup>19</sup>These are mostly the assets held in checking accounts, which are often held for the purpose of liquidity, and not necessarily risk aversion.

as they are financed by capital taxes (which are mostly paid by the wealthy households), and rebated to everyone equally, making the wealth-poor households net recipients and wealthy households net-payers. However, when taking transition into consideration, the welfare gains are neutralized by the transition path, during which the households increase savings and reduce consumption, and the leverage change is roughly welfare neutral. In particular, when the transition is taken into account, an increase in leverage leads to a fall of welfare equivalent to a 0.006% permanent decrease of consumption.

**Table 9: Extension 1: Model without capital taxation: Outcomes in economies with different leverage  $\lambda$**

Unconditional moments	$\lambda = 35$ Economy	$\lambda = 48$ Economy
$E(r^{s,p} - r^{b,p})$	1.82%	2.27%
$E(r^{b,p})$	0.18%	0.17%
Wealth Gini	0.5420	0.5414
$E(K)$	5.79	5.81
Share of wealth by quintile (%)		
1st	1.09%	1.09%
2nd	5.40%	5.43%
3rd	12.78%	12.81%
4th	22.01%	22.03%
5th	58.94%	58.86%

**Table 10: Extension 2: Majority of households (56%) can ex-ante only save in safe assets + no capital taxation**

Unconditional moments	$\lambda = 35$ Economy	$\lambda = 48$ Economy
$E(r^{s,p} - r^{b,p})$	0.69%	0.86%
$E(r^{b,p})$	0.87%	0.87%
Wealth Gini	0.4386	0.4386
$E(K)$	5.86	5.86
Share of wealth by quintile (%)		
1st	2.57%	2.57%
2nd	8.59%	8.59%
3rd	15.96%	15.97%
4th	25.91%	25.92%
5th	49.46%	49.46%

**Table 11: Extension 3: 4% of households can only save in equity + no debt tax shield**

Unconditional moments	$\lambda = 35$ Economy	$\lambda = 48$ Economy
$E(r^{s,p} - r^{b,p})$	1.87%	2.33%
$E(r^{b,p})$	0.10%	0.11%
Wealth Gini	0.5439	0.5474
$E(K)$	5.78	5.75
Share of wealth by quintile (%)		
1st	1.33%	1.11%
2nd	5.14%	5.12%
3rd	12.64%	12.64%
4th	22.02%	22.08%
5th	59.20%	59.37%

**Table 12: Extension 4: Tax revenue from capital taxation if rebated in a lump-sum fashion**

Unconditional moments	$\lambda = 35$ Economy	$\lambda = 48$ Economy
$E(r^{s,p} - r^{b,p})$	1.04%	1.31%
$E(r^{b,p})$	0.40%	0.35%
Wealth Gini	0.5671	0.5585
$E(K)$	5.62	5.77
Share of wealth by quintile (%)		
1st	2.14%	2.44%
2nd	4.37%	4.60%
3rd	11.13%	11.30%
4th	20.60%	20.54%
5th	62.10%	61.45%

## 7 Conclusion

This paper studies the effects of a change in firm leverage on wealth inequality and macroaggregates. The effects are examined in a model with heterogeneous agents, life-cycle, incomplete

markets, and aggregate risk. The analysis focuses on the particular change in the financial policy that occurred in recent US history, when firm leverage increased significantly during the 1980s, which was followed by a steady fall beginning in the early 1990s. I consider a general equilibrium model, that generates a sizable equity premium and a realistic amount of households that are portfolio constrained. Increasing the leverage leads to a decrease in the amount of corporate taxes paid since the debt repayment is tax-deductible. Analysis of the benchmark model shows that the increase in firm leverage leads to an increase in capital accumulation, a decrease in wealth inequality, and a decline in government revenues. The reduction in the inequality amounts to 1.07 Gini percentage points, aggregate capital increases by 5.38%, and government revenue decreases by 0.11% of output. The difference in wealth inequality amounts to the 12% of the increase of the wealth inequality in the US during the relevant period. The reduction in inequality is mainly driven by the fact that poor households increase their savings more than wealthy households as a response to the effective reduction in capital tax, caused by the increase in firm leverage, and indirectly by the increase in capital accumulation which increases wages (the main source of income of poor households) and decreases the returns to capital (the main source of income of the rich). The results of the model imply that increase in firm leverage that occurred in the US during the 1980s did not contribute to the increase in wealth inequality, but on the contrary, it is the reduction in firm leverage since the early 1990s that has contributed to the subsequent increases in the wealth inequality.

Furthermore, the paper shows that, if the model abstracts from capital taxation, an increase in leverage has only minor effects on macroaggregates and inequality, despite having significant implications for asset prices. This is consistent with previous literature that shows that Modigliani-Miller theorem holds either exactly or approximately. Unlike in the past literature, this paper shows that the Modigliani-Miller theorem holds approximately in imperfect market models with borrowing constraint, even if the equity premium is comparable to that observed in the data, wealth inequality is sizable, and a majority of households are portfolio constrained.

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## Appendix A: Numerical algorithm

This appendix briefly describes the solution algorithm used to obtain the solution. The algorithm broadly follows the method used by Krusell and Smith (1997,1998), which replaces the infinite-dimensional wealth distribution with a finite set of moments of wealth distribution, as a state variable.

Furthermore, to solve the individual problem, given the laws of motion for aggregate variables, I use the “endogenous grid method” proposed by Carroll (2006). The method is augmented to allow for two choice variables (amount of savings and the composition of savings between stocks and bonds). Rather than using the perceived aggregate law of motion bond interest rate, I follow Harenberg and Ludwig (2018) and use a perceived law of motion for equity premium. This eliminates the possibility of guessing negative equity premium and facilitates the computation. I use a FORTRAN programming language for the numeric computation, since the computation is intensive, and requires a compile programming language for the run-time of the program to be feasibly short. The symbol  $m$  denotes whether the household is ex-ante constrained in portfolio choice or not. For the benchmark model (unlike the extensions),  $m$  can take only one value, since none of the agents are ex-ante constrained.

1. Guess the law of motion for aggregate capital  $K_{t+1}$  and equity premium  $P_t^e$ . This means guessing the starting 16 coefficients in the following equations (since there are two possible realizations of  $z$  and two for  $\delta$ ):

$$\begin{aligned} \ln K' &= a_0(z, \delta) + a_1(z, \delta) \ln K \\ \ln P^e &= b_0(z, \delta) + b_1(z, \delta) \ln K' \end{aligned}$$

2. Given the perceived laws of motion, solve the individual problem described earlier. In this step, the endogenous grid method (Carroll, 2006) is used. Instead of constructing the grid on the state variable  $\omega$  and searching for the optimal decision for savings  $\tilde{\omega}$ , this method creates a grid on the ”optimal savings amounts  $\tilde{\omega}$ , and evaluates the individual optimality conditions to obtain the level of wealth  $\omega$  at which it is optimal to save  $\tilde{\omega}$ . This way, the root-finding process is avoided, since finding optimal  $\omega$ , given  $\tilde{\omega}$ , involves only the evaluation of a function (households optimality condition). However, the root-finding process is necessary to find the optimal portfolio choice of the household, which is performed after finding the optimal pairs  $\omega$  and  $\tilde{\omega}$ .
3. Simulate the economy, given the perceived aggregate laws of motion. To keep track of wealth, instead of a Monte Carlo simulation, I use the method proposed by Young (2010). For each realized value of  $\omega$ , the method distributes the mass of agents between two grid points:  $\omega_i$  and  $\omega_{i+1}$ , where  $\omega_i < \omega < \omega_{i+1}$ , based on the distance of  $\omega$ , based on Euclidean distance between  $\omega_i$ ,  $\omega$  and  $\omega_{i+1}$ . Do this in the following steps:
  - (a) Set up an initial distribution in period 1:  $\mu$  over a simulation grid  $i = 1, 2, \dots, N_{sgrid}$ , for each pair of efficiency and employment status, where  $N_{sgrid}$  is the number of wealth grid points. Set up an initial value for aggregate states  $z$  and  $d$ .
  - (b) Find the bond interest rate (expected equity premium  $P^e$ ) in the given period, which clears the market for bonds. This is performed by iterating on  $P^e$  (or bond price), until the following equation is satisfied (bond market clears):

$$\sum_j g^{m,b,j}(\omega, e, l; z, d, K, P^e) d\mu = \lambda \sum_j \{g^{m,b,j}(\omega, e, l; z, d, K, P^e) d\mu + g^{m,s,j}(\omega, e, l; z, d, K, P^e) d\mu\}$$

where  $g^{m,b,j}(\omega, e, l; z, d, K, P^e)$  and  $g^{m,s,j}(\omega, e, l; z, d, K, P^e)$  are the policy functions for bonds and shares, where  $j$  denotes the age of the household (working age or retired), that solve the following recursive household maximization problems: Retired households:

$$v_R^m(\omega; z, \mu, \delta, P^e) = \max_{c, b', s'} \left\{ u(c - \gamma)^{1-\rho} + v\beta E_{z', \mu', \delta', P^{e'} | z, \mu, \delta, P^e} [v_R^m(\omega'; z', \mu', \delta', P^{e'})^{1-\alpha}]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}$$

Working-age households:

$$v_W^m(\omega, e, l; z, \mu, \delta, P^e) = \max_{c, b', s'} \left\{ u(c - \gamma)^{1-\rho} + \beta E_{e', l', z', \mu' P^{e'}, \delta' | e, l, z, \mu, \delta, P^e} \left[ (1 - \theta)v_W(\omega', e', l'; z', \mu', \delta', P^{e'})^{1-\alpha} + \theta v_R(\omega', e', l'; z', \mu', \delta', P^{e'})^{1-\alpha} \right]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}$$

where  $v_j$  are the value functions, obtained in step 2. In this step, an additional state variable is included explicitly: expected equity premium  $P^e$ .

- (c) Depending on the realization for  $z'$  and  $d'$ , compute the joint distribution of wealth, labor efficiency and employment status.
  - (d) To generate a long time series of the movement of the economy, repeat substeps b) and c).
4. Use the time series from step 2 and perform a regression of  $\ln K'$  and  $P^e$  on constants and  $\ln K$ , for all possible values of  $z$  and  $d$ . This way, the new aggregate laws of motion are obtained.
  5. Compare the laws of motion from step 4 and step 1. If they are almost identical and their predictive power is sufficiently accurate, the solution algorithm is completed. If not, make a new guess for laws of motion, based on a linear combination of laws from steps 1. and 4. Then, proceed to step 2.

## Details of endogenous grid method implementation

The introduction of the fixed participation cost to the endogenous grid method solution is as follows: First, the grid is fixed over the saving decision choices, rather than over current wealth. Then, using the first-order-conditions (inverted Euler-equations), the current levels of wealth, for which it is optimal to choose a pre-selected amount of savings, are computed.<sup>20</sup> This is conducted for two cases: In the first case, the households are not allowed to save in equity ( $s = 0$ ), and therefore do not pay the fixed participation cost  $\phi$ , and the second case where households are allowed to choose their portfolio freely but must pay the participation cost  $\phi$ . This way, two choice-specific endogenous grids (on current wealth) are obtained, with corresponding value function values. The true value function is the upper envelope of the two choice-specific value functions (for every level of current wealth, the greater of the two choice-specific value functions is chosen). To obtain it, both of the choice-specific value functions defined over choice-specific endogenous grids are interpolated on the exogenous grid on current wealth so that they can be directly compared. Then, for each grid point on the exogenous grid, the maximum value of the two choice-specific value functions is chosen to obtain the actual value function over the exogenous grid of current wealth. Then, the algorithm proceeds to calculate the next iteration in the value function iteration.

A potential complication in the next step algorithm is that the value function might be non-concave in the neighborhood of the threshold where households start to invest in stocks and choose to pay the fixed participation cost. The reason is that the actual value function is an upper envelope of the two choice-specific value functions. In the non-concave part of the value function, the first-order conditions are no longer sufficient for the optimal solution, but only necessary. In other words, there might be multiple local optima in that region. This means that the two different grid points over the savings can produce the same point on the endogenous grid over current wealth. Therefore, it is possible that in the neighborhood of the threshold where households start to invest in stocks, a segment could arise where, instead of the value function, a value correspondence is obtained. This segment implies that for the given value of current wealth, two savings choices satisfy the first order condition. In these segments, similar as in the algorithm of Iskhakov et al. (2017), the savings value which obtains the higher overall utility would be chosen. However, in practice, the non-concavity does not arise. The numerical implementation of the algorithm (cubic and cubic spline interpolated value function defined over a grid on current wealth) cannot register the non-concavity. This is the case even when the grid points in the neighborhood of the threshold are as close as 0.0087% of the average wealth to each other. The reason is that in the neighborhood of the threshold, both choice-specific value functions have very similar curvature, and the fixed participation costs are very small. The concavity of the value function is always checked, if it would arise, in the segments with multiple satisfied first order conditions, the solution with the overall higher resulting value is selected in the style of Iskhakov et al. (2017). However, throughout the solutions, this case does not arise. A similar situation is the case in Bayer et al. (2019), who use liquid/illiquid assets and adjustment costs, and find that there are no non-concavities, even though theoretically, they can arise when using the Fella (2014) algorithm.

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<sup>20</sup>For some values of savings, the obtained current wealth values can be lower than the borrowing constraint. In this case, these grid points are discarded. If the lowest obtained point for current wealth, obtained when the minimum amount of savings is used, is higher than the minimum one implied by the borrowing constraint, then additional grid points are inserted in the region between the borrowing constraint and the lowest obtained point. In this section, the policy function is obtained by using budget constraints with the lowest possible savings. This is in line with the previous papers which use the endogenous grid method (Carroll (2006), Barillas and Fernandez-Villaverde (2007) and Iskhakov et al. (2017)).

## Appendix B: Historical corporate leverage in the US

Figure 3.: Firm leverage in the US



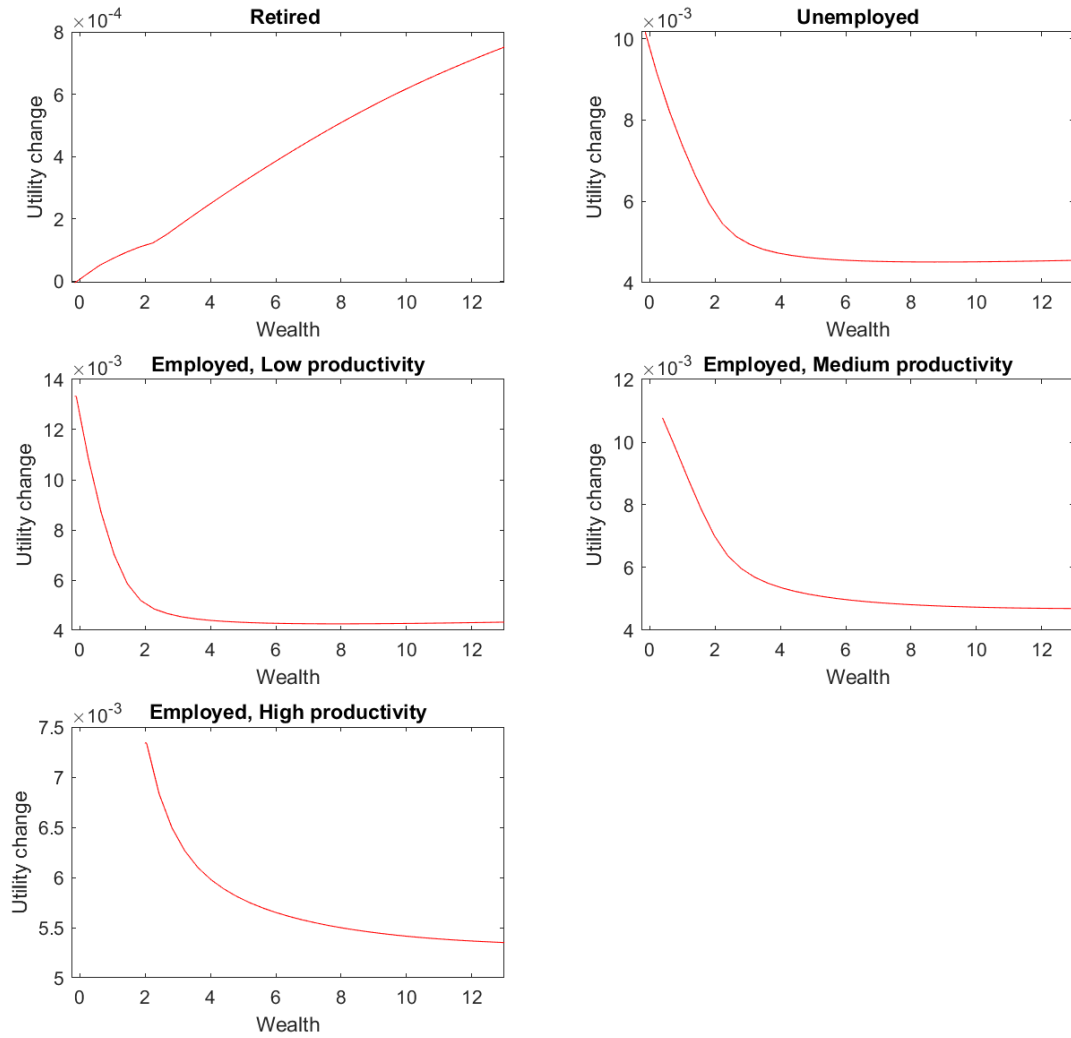
Taken from: Graham et al. (2015)<sup>21</sup>

The historical movement of corporate leverage in the US, taken from Graham et al. (2015), who use data from Moody's Industrial Manuals.

<sup>21</sup><https://corpgov.law.harvard.edu/wp-content/uploads/2015/04/graham-leary-roberts.jpg>

## Appendix C: Welfare changes

Figure 4.: Utility change in the increase in leverage; the change occurs at a high TFP state and a low capital depreciation state



# Appendix D: Policy functions

Figure 5.: Policy functions  $\lambda = 0.35$ , high TFP state, high capital depreciation state

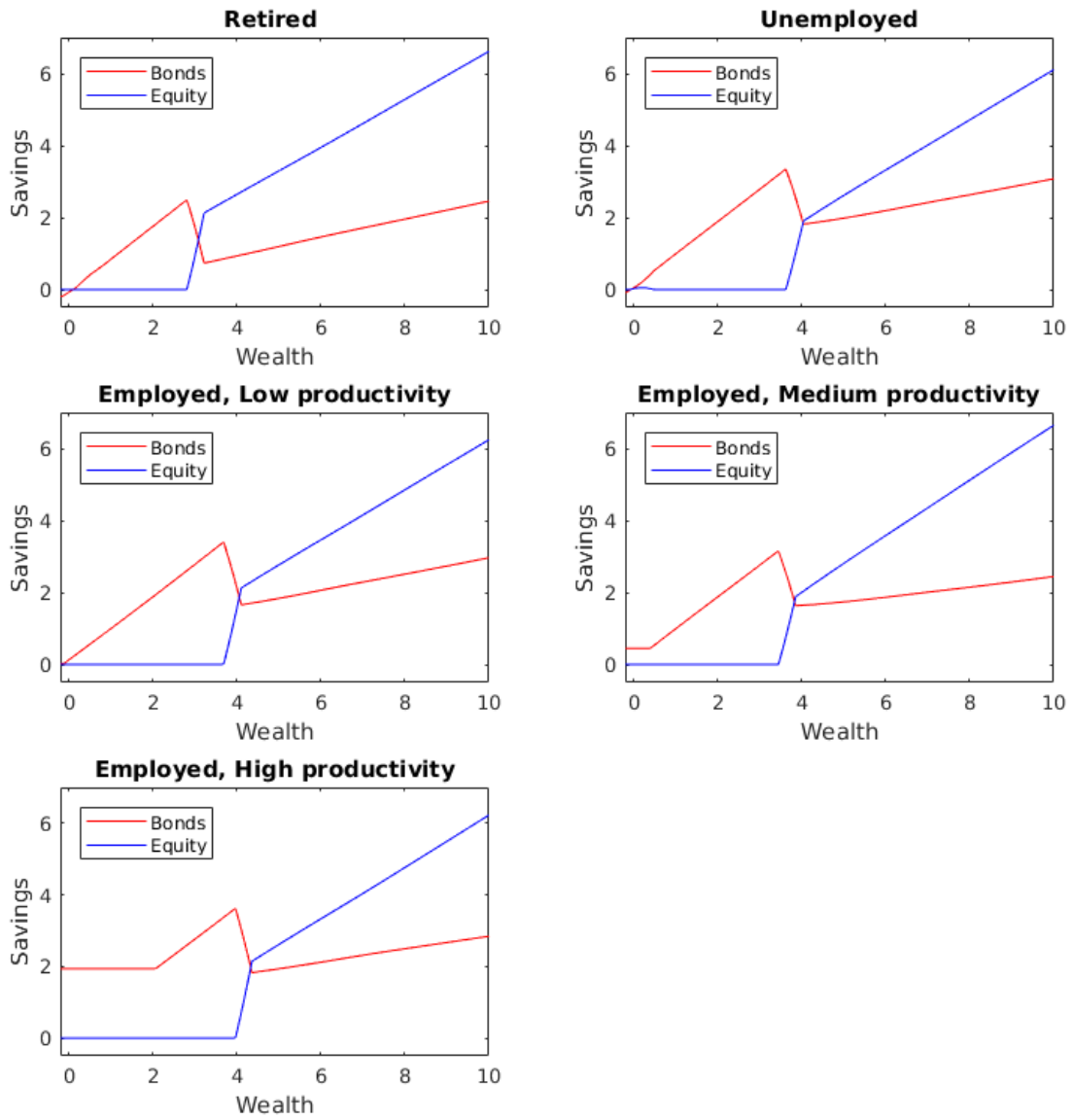
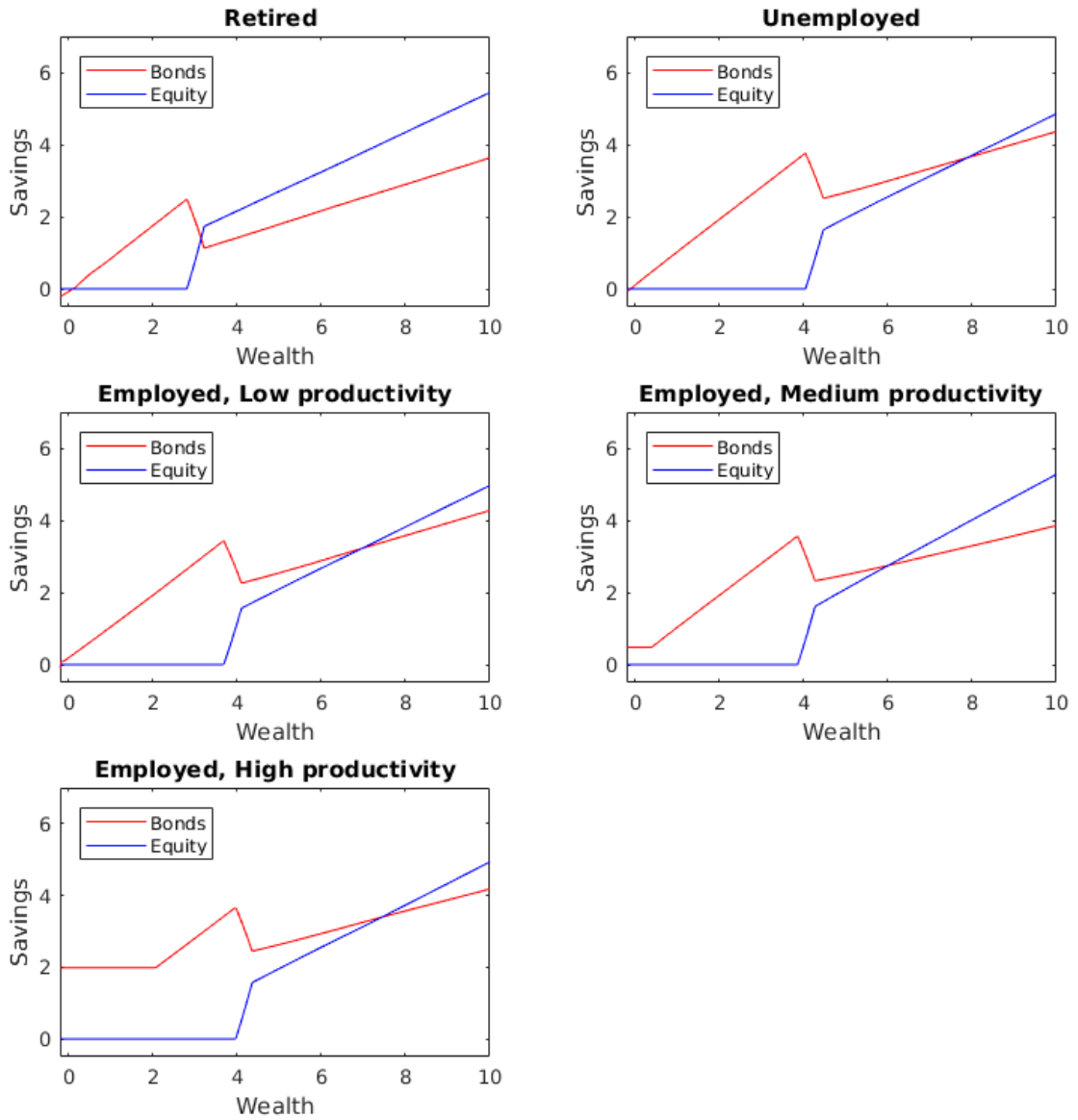
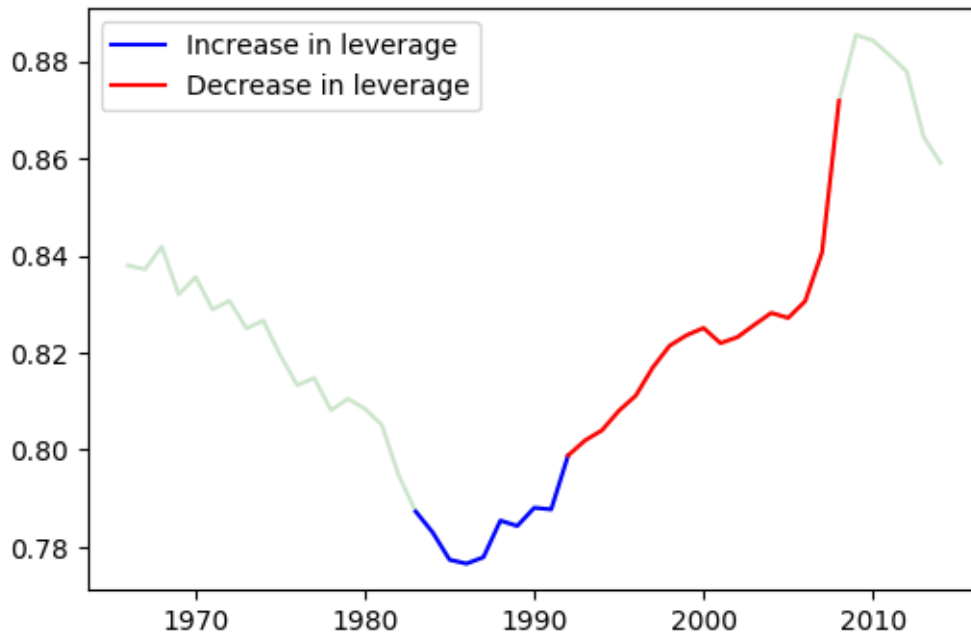


Figure 6.: Policy functions  $\lambda = 0.48$ , high TFP state, high capital depreciation state



## Appendix E: Leverage and wealth inequality

Figure 7.: Net Wealth Inequality and Corporate Leverage in the US



The inequality data is taken from World Inequality Database, which builds on Piketty et al. (2018).<sup>22</sup>

<sup>22</sup>URL=<https://wid.world/data/>

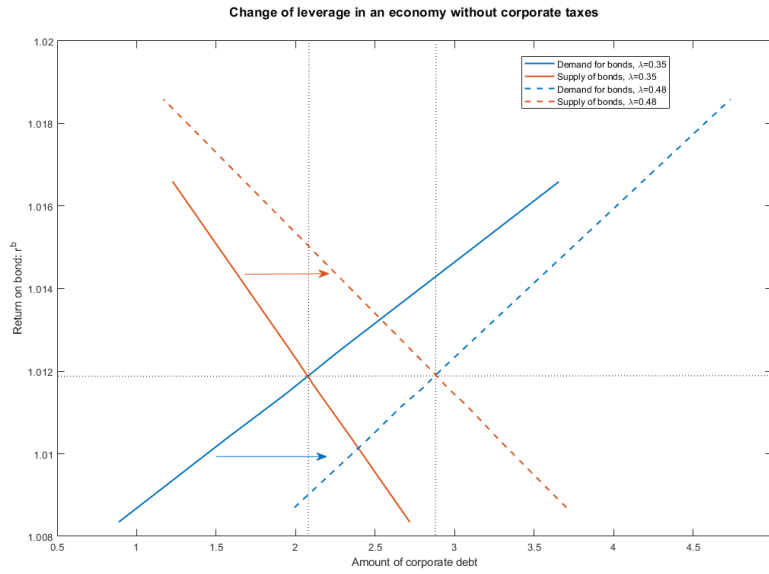


## Appendix F: Supply and demand for bonds

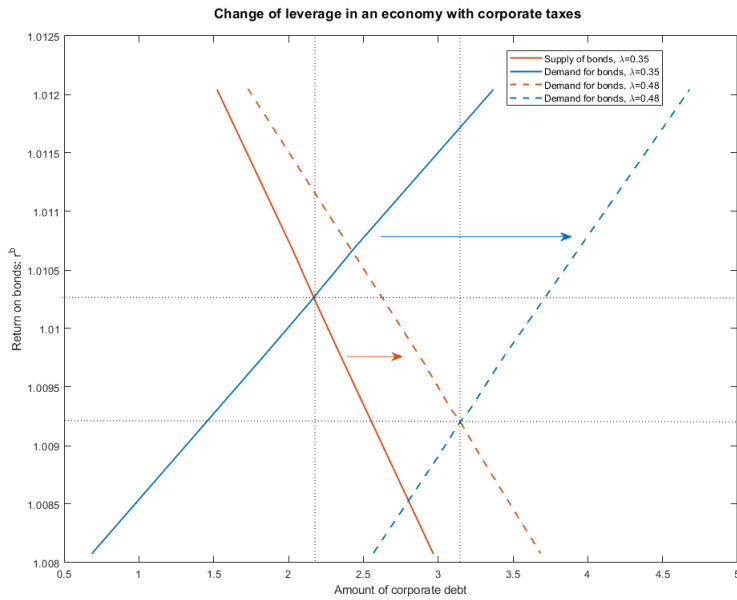
In the model without capital income taxation, the interest rate stays (approximately) the same with the change in leverage. This is because the increase in the supply of bonds is offset by the increase in the demand for bonds resulting from the decreased supply of equity and increase of the equity's riskiness. In the model with capital income taxation, the return on bond falls because in the long run, the aggregate capital rises, and consequently return on both assets (stocks and bonds) falls. This is because, with the Cobb-Douglas production function, increase of aggregate capital decreases its marginal productivity. The effects can be seen on the graph. However, when the leverage suddenly increases, in the short-run, the net return on both bonds and stocks increase.

In the graphs below one can see the change in supply (by the firms) and demand (from the households) of bonds when the leverage changes. The supply of the bond is defined as the sum of savings in stocks multiplied by a fraction  $\frac{\lambda}{1-\lambda}$ .

**Figure 8.: Change in Supply and Demand for Bonds in a model without corporate income taxation**



**Figure 9.: Change in Supply and Demand for Bonds in a model with corporate income taxation**



## Appendix G: Potential reasons for a change in corporate leverage

The scope of this paper is to study the effects of a change in leverage, no matter the underlying reasons that generated it. Therefore, the focus of the paper is on understanding the channel of corporate leverage rather than explaining the overall change in the wealth inequality in the observed period. The increase in wealth inequality in the late 1980s and 1990s in the US is a complex topic, as many factors have contributed to the increase, either directly or indirectly. Determining all the underlying causes of the changes in inequality and leverage would significantly broaden the scope of this paper.

One potential reason is the change in government debt. Government debt and corporate debt are partial substitutes from the point of view of investors. Therefore, an increase in government debt could cause a decrease in corporate debt. However, during the 1980s and early 1990s, both government and corporate debt move in the same direction.<sup>23</sup> Furthermore, Graham et al. (2015) find that the newly created firms in the late 1980s and early 1990s tended to have lower leverage than their incumbent counterparts. Another potential cause for the change in corporate leverage, which would also influence wealth inequality is the decrease in corporate income taxes. However, the corporate tax rate and the tax advantage of debt do not seem to play a role in the change of corporate leverage in the US in the 20th-century (Graham et al., 2015), and the tax advantage of debt in the late 1980s decreased and then increased slightly in the beginning of 1990s. This is the opposite of what one might expect from the theoretical stand-point, as the leverage increased in the late 1980s and decreased in the early 1990s. Furthermore, Graham et al. (2015) state that financialization in the US leads to an increase in issuing both debt and equity.

Another possible reason for the change in leverage during the observed period is financial deregulation. One important act is the 1980 “Depository Institutions Deregulation and Monetary Control Act,” which deregulated the banking sector, which could have contributed to the subsequent increase in leverage.

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<sup>23</sup>According to the data from Graham et al. (2015).

## Abstrakt

Tento článek se zabývá vlivy finanční páky firem na ekonomickou nerovnost a makroekonomické agregáty. Vliv zkoumám s využitím modelu všeobecné tržní rovnováhy s kontinuem heterogenních aktérů, životním cyklem, neúplnými trhy a s idiosynkratickým i agregátním rizikem. Analýza se zaměřuje na konkrétní změnu finanční páky firem, která nastala v USA během osmdesátých let. Finanční páka firem nejdříve významně vzrostla a následně od počátku devadesátých let postupně klesala. Zvýšení finanční páky firem v rozsahu obdobném k situaci v USA v 80. letech v benchmark modelu vede ke zvýšení akumulace kapitálu o 5,38 %, poklesu ekonomické nerovnosti o 1.07 Giniho bodu a poklesu vládních příjmů o 0,11 % HDP. Zvýšení finanční páky firem zvyšuje průměrné výnosy úspor po zdanění, pokud firemní půjčky podléhají zvýhodněnému zdanění. To má za následek zvýšení míry úspor domácností, kdy se zvýšení úspor projevuje disproporcionálně u relativně chudších domácností. V konečném důsledku model implikuje, že zvýšení finanční páky firem nepřispělo ke zvýšení ekonomické nerovnosti v USA v 80. letech, ale spíše naopak. Tedy že snížení páky od počátku 90. let do roku 2008 přispělo k růstu ekonomické nerovnosti. Dále ukazují, že pokud model upouští od daňového zvýhodnění korporátních dluhopisů, pak má změna finanční páky pouze zanedbatelný efekt na makroekonomické agregáty a ekonomickou nerovnost i navzdory tomu, že má významné implikace pro oceňování aktiv. To je konzistentní s předchozími výsledky v literatuře, ukazující, že Modigliani-Millerův teorém přibližně platí v modelu s heterogenními aktéry a nedokonalými trhy.