COORDINATION IN A MOBILE WORLD

Jakub Steiner

CERGE-EI

Charles University
Center for Economic Research and Graduate Education
Academy of Sciences of the Czech Republic
Economics Institute

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1 Introduction

Coordination problems are common in economics (see e. g. Cooper, 1999), though typically they are modelled in isolation. For instance: players decide between a risky investment with returns increasing in the number of investors and a safe outside option which represents all other investment opportunities. In contrast, this paper studies coordination failures in many simultaneously occurring coordination problems and allows mobile players to move among them. More specifically, we consider projects, or coordination problems, and a set of players uniformly matched to projects at the beginning of the game. An outside option of a player who considers investing into project j consists of a search which allows her to join one of the other projects. Thus the outside option value in any coordination problem j is endogenously determined by players' behavior in all the other coordination problems. Similarly, the mass of observers of j considering investment depends on coordination outcomes of all other coordination problems, because players rejecting any other project j' will search and may end up being matched to j.

Intuitively, the outside option value and number of observers influence the coordination outcome of each of the coordination problems. A valuable outside option lowers the attraction of a Pareto dominant but risky equilibrium and hence undermines successful coordination. On the opposite side a high number of observers enhances coordination, as it is easier to find other investors. These two externalities lead to non-trivial comparative statics and welfare effects. Searching players who have rejected project j impose a negative externality on other observers of j and a positive externality on observers of all other projects.

Both these causal links are difficult to analyze under the equilibrium multiplicity of coordination games. We therefore study the model using the global games approach, which uniquely predicts the coordination outcome for a given outside option and number of players. Comparative statics of the global games equilibrium is indeed in line with the causal links.[1] We depart from the standard global game, which we refer to as *static* and use as a benchmark, to build a *mobile* game, in which not only one but many projects are realized, each with economic fundamentals independently drawn from a prior distribution. Players receive an imprecise signal about the project's fundamentals they are matched to, and may move to another project if dissatisfied with the current signal.

[1] Global games were introduced by Carlsson and van Damme (1993) and further developed by Morris and Shin (2003). Heinemann, Nagel and Ockenfels (2004) test the theory experimentally, and although reject the global games threshold prediction, confirm the qualitative features of the predicted comparative statics.

Introduction of search allows us to analyze the welfare effects of increased mobility (decreased search costs). Counterintuitively, welfare is non-monotonic in mobility. The direct non-strategic effect is always positive as, ignoring strategic considerations, reaching a successful project is cheaper. However, the strategic effect is negative: smaller search costs increase the outside option value associated with the search, which undermines successful coordination. Thus some projects that would have succeeded had the search costs been high, fail if search costs are low. This negative strategic effect may outweigh the positive direct effect, and welfare may decrease with mobility.

The mobile game also has a natural *self-regulatory* property. Consider, for instance, a shift in the distribution of economic fundamentals towards poorer states of the world. This decreases the outside option value, as searching results in finding poorer projects. The lower value of the outside option enhances successful coordination, and this positive strategic effect partially counteracts the negative direct effect. Another channel through which the self-regulatory mechanism operates is the increasing mass of players observing each project: the more projects have poor fundamentals, the more players search. This makes coordination attempts more likely to succeed and thus helps to partially counteract the direct effect of the distribution's shift.

Below we show that this self-regulatory mechanism is powerful. Players frequently coordinate successfully on many projects even for distributions of fundamentals that almost preclude coordination in the static game. On the other hand, if the distribution of fundamentals is such that a project almost always succeeds in the static game, the value of the outside option in the mobile game is high and the mass of observers of each project is low as players need not search much. Thus, some coordination failures are to be expected in the mobile game. Because of this self-regulatory mechanism, an intermediate willingness to invest is typical for the mobile game equilibrium.

The above "general equilibrium" effects occur in a number of settings in which players actively choose the coordination problem they will participate in; thus our results can complement many of the existing global games applications. Let us apply the mobility extension to the model of currency attacks of Morris and Shin (1998). Allowing speculators to choose a currency they short-sell makes it possible to assess how the speculators' freedom to choose the currency influences their coordination on attacks. Another prominent example of coordination problems in economics is the game of foreign investors in emerging markets with increasing returns to scale. We can interpret the model as a study of many such markets on which mobile investors operate. Our main result under this interpretation is that welfare is non-monotonic with respect to capital mobility.

Two independent broad streams of literature, global games (Carlsson and van Damme, 1993) and stochastic stability concept (Kandori, Mailath and Rob, 1993), agree that risk dominance rather than Pareto dominance selects the equilibrium in coordination games. The influence of mobility on the coordination outcome has been examined in various papers belonging to the latter stream with the main conclusion that, if players are allowed to move and/or to choose with whom they interact, then a Pareto efficient equilibrium may prevail.[2] Goyal and Vega-Redondo (2005) vary the cost of link formation and find a similar welfare effect to the one we find: welfare is non-monotonic with respect to mobility — the efficient equilibrium prevails only at high cost — while if the cost of the link formation is low the risk dominant equilibrium prevails.

To our knowledge, mobility has not been studied within the global games literature. However, the outside option value is often varied exogenously in many global games applications, which leads to a similar tension between the positive direct effect and the negative strategic effect. Morris and Shin (2004 and section 2.3.1 in 2003) show that an increase in collateral may decrease debt value. The collateral is an outside option of creditors, so its increase undermines their ability to coordinate on (efficient) rolling over of the debt, which may outweigh the positive direct effect. Similarly, Goldstein and Pauzner (2005) study the influence of demand-deposit contracts on bank run probability. While the direct effect of higher short-term payment offered by banks is an increase in risk sharing, the strategic effect is negative as it increases the probability of panic-based bank runs.

Jeong (2003) and Burdett, Imai and Wright (2004) study "break-up" externalities which occur when matched players search for new partners while not taking into account the welfare loss of the abandoned partner. Jeong stresses the possible welfare improvements of mobility restrictions in environments with break-up externalities, which is in line with our main finding. Burdett et al. focus on the multiplicity of equilibria; if matched players search intensively, the partnerships become unstable and intensive search is the best response. While our basic model has a unique equilibrium, we encounter this self-fulfilling prophecy feature for general payoff function in section 5.2. However, though the externality studied in our model is similar to break-up externalities, it is of a subtler form. While break-up externalities relate to players who cooperate on production, the searching player in our model leaves the project *before* production starts and the mere fact that she has

[2]E.g. Oechssler (1997); Mailath, Samuelson and Shaked (2000).

stopped contemplating involvement in the project induces the negative externality on the rest of the project's observers.

Technically, the present paper combines the modelling frameworks of Dasgupta (2005) and Steiner (2005). Dasgupta studies the effects of social learning on coordination failures by allowing players to delay investment and to learn from the behavior of early investors. Players delaying investment stay with the same project in Dasgupta's model, whereas they search for a new project in our model. Though the settings of both models are seemingly similar, the conclusions differ significantly. Social learning, central to Dasgupta's model, turns out to be irrelevant in our model (see section 4). Moreover, while the delay option unambiguously enhances welfare in Dasgupta's model, it may decrease welfare in ours.

Steiner (2005) considers a repeated coordination game in which players, by choosing to invest today, risk their instantaneous payoffs as well as their ability to participate in future projects — unsuccessful investment can cause bankruptcy. The fear of bankruptcy motivates players not to invest, especially just before an expected boom. This negative feedback between tomorrow's and today's coordination success leads to endogenous cycles in the willingness to invest.

Both Steiner (2005) and the model at hand are based on non-trivial effects of the endogenous outside option but they differ in timing[3] and interpretation. Steiner (2005) focuses on cycles endogenously arising in the equilibrium, whereas in this paper we emphasize the self-regulatory properties of the mobile game and particularly the non-monotonicity of welfare with respect to mobility.

In section 2 we describe the mobile game formally. We compute the equilibrium in the limit of precise signals in section 3, analyze its comparative statics, and contrast it to the static game equilibrium. We relegate the analysis of the general case away from the limit to appendix A.1. In section 4 we allow players to observe actions of early investors and find that social learning is irrelevant in the mobile game. We further demonstrate the robustness of the model in section 5 in which we vary the number of search rounds, consider general payoff functions, and allow players to direct their search toward better projects. Section 6 concludes.

[3]In the present model, there exist many projects simultaneously and returns are paid only after all search and investment takes place. In Steiner (2005), there is only one project in each round, and its returns are paid at the end of each round.

2 The Model

We start by describing a simple coordination game in section 2.1 and then briefly introducing the benchmark global game, dubbed static game here, by adding noise to the observation of fundamentals. Then, having set the stage, we describe the mobile game in section 2.2.

2.1 The Static Game

There is a continuum of homogeneous risk-neutral players of measure 1 and one project; each player possesses one (indivisible) token and decides between investing or not investing into the project. The payoff of those who have invested is

$$R(\theta, l) = \begin{cases} 1 - c & \text{if } l \geq \theta, \\ -c & \text{if } l < \theta, \end{cases} \tag{1}$$

where l is the measure of investors and $0 < c < 1$ the sunk cost of investment. We say that the project has succeeded when $l \geq \theta$. The payoff for not investing is normalized to 0. The payoff function (1) is being used for its simplicity as the workhorse of the global games literature;[4] we study general payoff functions in section 5.2. The payoff exhibits strategic complementarity; investment is more attractive if many players invest, which typically leads to equilibrium multiplicity. Clearly the game has, for non-extreme values of θ, two pure strategy equilibria in which nobody, respectively everybody, invests.

Building on Carlsson and van Damme (1993), Morris and Shin (2003) show that the equilibrium multiplicity disappears if we assume noise in observation of the project's parameters. We introduce this standard global games structure in the rest of this paragraph: we refer to θ as a state of economic fundamentals and assume that it is a realization of a random variable Θ distributed according to $N(y, \tau^2)$; we denote c.d.f. of Θ by $\Phi(.)$. The players observe only an imprecise signal $x^i = \theta + \sigma\epsilon^i$ of the state θ which itself is unobserved. The parameter σ describes the size of the noise term. The error terms $\epsilon^i \sim N(0, 1)$ are independent across players. We denote c.d.f. of $N(0, 1)$ by $F(.)$. The random variable corresponding to realization x^i is X^i. Pure strategy is a function $s : \mathbb{R} \to \{0, 1\}$, which maps the signal x^i to an action where 0 corresponds to not investing and 1 to investing. We label this benchmark global game a *static* game and denote it by $\Gamma_S(\sigma)$.

To avoid confusion, it is worth mentioning that the *higher* the value of θ the

[4]The payoff function (1) has been used in Morris and Shin (1999); Dasgupta (2005); Angeletos, Hellwig, and Pavan (2004), and in others.

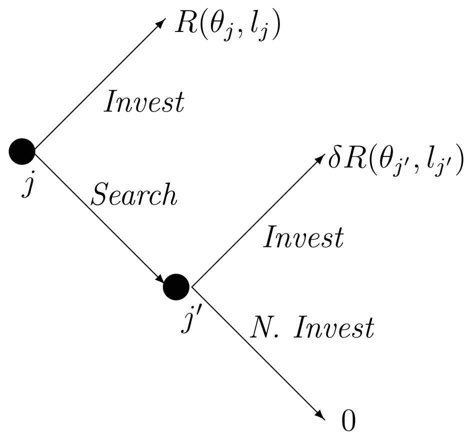

Figure 1: Structure of the mobile game — The player of a mobile game is matched to a project j and decides whether to invest or search. In the latter case, she is randomly matched to another project j' and decides whether to invest or not. (The diagram is not a formal game tree, as it does not depict moves of Nature and simultaneous moves of other players.)

worse the fundamental, as more investors are needed for the success of the project. Some, but not all, global games papers use transformation $\tilde{\theta} = 1 - \theta$ which we do not use here.

2.2 The Mobile Game

The outside option payoff is treated exogenously and normalized to 0 in the static game; our next step is to endogenize it, which we achieve by considering many projects simultaneously and by allowing players to search for another project. More formally, there is a continuum of projects indexed by $j \in [0, 1]$; each project has a state of fundamentals θ_j independently drawn from the distribution with c.d.f. $\Phi(.)$. Each project's state is fixed during the whole game. The game has two rounds. Players are randomly and uniformly matched to the projects at the beginning of round 1. The measure of players observing each project in round 1 after the matching is normalized to 1.[5] At round 1, after the players are matched to the projects, each player i observes a private signal $x_1^i = \theta_{j(i)} + \sigma_1 \epsilon_1^i$ about the fundamentals of project[6] $j(i)$ she is matched to. Each player chooses from:

To Invest into the project she observes in round 1. The player can take no further action afterwards.

[5]There is a continuum of continua of players and thus the total measure of players is undefined. Formally, we should refer to a density, rather than to a measure of investors in a particular project. However, our formal impreciseness does not lead to confusion, because we never refer to a total measure of players in all projects. Occasionally, we will stress that we refer to measure *per* project.

[6]We will omit argument i and write simply j, but let us remember the matching process.

To Search: The player continues to round 2, is randomly matched to another of the existing projects j', and observes signal $x_2^i = \theta_{j'} + \sigma_2 \epsilon_2^i$ about the new project j'. Errors ϵ_1^i and $\epsilon_2^i \sim N(0,1)$ are independent across players and rounds.

Players who have searched decide in round 2 between investing and not investing into their new projects. The payoff of players who have invested into project j depends on its fundamentals θ_j, on the cumulative measure l_j of investment into j, and on the timing $t \in \{1, 2\}$ of the investment:

$$u^i(t, l_j, \theta_j) = \delta^{t-1} R(\theta_j, l_j),$$

see figure 1. Players who have not invested in round 1 nor 2 receive payoff 0.

Note that the return depends on the *cumulative* measure of investments l_j into project j over both rounds. The return on the investment in round 2 is scaled down by the factor $0 < \delta < 1$ which should not be understood as a time discount factor because all payoffs are realized at the same moment, at the end of round 2. Rather, δ models implicit search costs: late investors (in round 2) may find the largest profitable investment opportunities being taken by early investors in round 1. Also, the late investors have less time to realize their investments, thus they will get less involved with the project. Alternatively, instead of discounting by δ, we could model the search cost by a fixed cost q that searching players incur. But, as expensive search cannot be mandatory, we would have to allow players who do not want to invest or to search a third, outside option. The simplicity of the two-action global games framework would be lost. Therefore, to enhance tractability, we model search costs by discounting.[7]

In the basic setting, players in round 2 observing j do not observe the measure of investors from round 1. The information sets of player i are histories of the signals: $I_1^i = \{x_1^i\}$, $I_2^i = \{x_1^i, x_2^i\}$. Later, in section 4, we consider social learning: we allow players to (imprecisely) observe the measure of previous investors, and we find our results to be robust to such a modification, which is in contrast with the study of social learning in the static game done by Dasgupta (2005).

Pure strategy of the mobile game is a pair of functions $a_1(x_1) : \mathbb{R} \to \{0, 1\}$, $a_2(x_1, x_2) : \mathbb{R}^2 \to \{0, 1\}$ that prescribe actions in round 1 and 2 contingent on the observed signals. A *threshold* strategy is a particularly simple pure strategy characterized by two thresholds x_1^*, x_2^* such that a player invests at round $t \in \{1, 2\}$

[7] Players in our setting can always choose not to invest in both rounds which assures 0, and thus we do not have to consider a particular outside option for players who wish not to engage in costly search.

if and only if the observed signal x_t^i is below x_t^*. A *critical* state θ^* is such a state that only projects with $\theta_j < \theta^*$ succeed. Obviously, if players use sufficiently non-monotone strategies, the critical state may not exist. However, we show below that players play threshold strategies and that the critical state exists in equilibrium. We call the whole game a *mobile* game and denote it by $\Gamma_M(\boldsymbol{\sigma})$, where $\boldsymbol{\sigma} \equiv (\sigma_1, \sigma_2)$ describes the size of noise terms in both rounds. We consider different noise sizes in both rounds primarily because it will be helpful in the analysis of social learning; otherwise it does not play any substantial role.

We do not allow players to return to the project observed in round 1 after they have observed a project in round 2. Later, in section 5.1, we study a game with an infinite number of search rounds. In that game, returning to a previously observed project is always suboptimal, and thus we can allow it without any consequences on the equilibrium.

2.2.1 Economic Example

We offer an economic example similar to Dasgupta's (2005) setting. There is a continuum of risky bonds indexed by $j \in [0, 1]$, whose returns increase with measure of investment; bond j returns $e^{r(l_j)(T-t)}$ at time T, where t is the time of investment, l_j is the cumulative investment into j over the whole time period and $r(l) = \underline{r} < 0$ if $l < \theta_j$ and $r(l) = \overline{r} > 0$ if $l \geq \theta_j$.

Investors possessing one dollar observe a random bond at $t = 0$ and the measure of investors per bond is 1. After observing a signal about the bond she is matched to, each investor decides whether to invest one dollar at $t = 0$ to the first bond she observes or whether to search for a new bond. However, the search lasts time t_s after which she observes a signal about the new bond and decides whether to invest at time t_s or not to invest at all. At time T, players who have not invested consume 1 while players who have invested at $t \in \{0, t_s\}$ consume $e^{r(l_j)(T-t)}$. This coincides with our model if $\underline{r}T = -c$, $\overline{r}T = 1 - c$, $\frac{T-t_s}{T} = \delta$ and the utility function is $u(.) = \ln(.)$.

3 Equilibrium

The key observation in the analysis of the mobile game is that each project can be treated as an independent coordination game with parameters induced by players' aggregate behavior in other projects. Let V_2 be the expected payoff in round 2, and n_2 be the measure of players per project continuing to round 2; the values V_2, n_2 are defined for any strategy profile. The interactions of project j's observers constitute a coordination game of two types of players: measure 1 has the outside option δV_2, measure n_2 has the outside option 0, and payoff for investment of all players is described by (1). Observers of each particular project interact in a global game and thus their equilibrium behavior is uniquely determined for any assumed values V_2, n_2: only projects with $\theta_j < \theta^*$ succeed, where the critical state θ^* is a function of V_2, n_2. Moreover, values V_2, n_2 are uniquely determined by θ^*, which leads to a system of equations with a unique solution.

We first analyze the mobile game informally in the limit $\boldsymbol{\sigma} \to \mathbf{0}$. The simplification of the limit case is that players receiving infinitely precise private signals x_t^i neglect their prior beliefs. We defer the formal analysis to appendix A.1, where we explicitly account for both prior distribution and the private signals, and only then take the limit $\boldsymbol{\sigma} \to \mathbf{0}$. We deal only with symmetric equilibrium in threshold strategies in this section and later prove that no other equilibria exist (also in appendix A.1).

The following technical preliminary is needed. Denote $P^i_{\theta^*,t} \equiv Prob(\Theta_j < \theta^*|X_t^i)$, which is a posterior probability player i assigns to the success of the project she is matched to in round t; $P^i_{\theta^*,t}$ is a random variable that depends on the signal X_t^i player i receives.

Lemma 1. *Random variable $P^i_{\theta^*,t}$, conditional on $\Theta_j = \theta^*$ (which is unknown to the players), is distributed uniformly on $[0,1]$ in the limit $\boldsymbol{\sigma} \to 0$.*

Proof[8]: ignoring the prior distribution,

$$P^i_{\theta^*,t} = Prob(\Theta_j < \theta^*|X_t^i) = Prob(X_t^i - E_t^i < \theta^*|X_t^i) = Prob(E_t^i > X_t^i - \theta^*) = 1 - F\left(\frac{X_t^i - \theta^*}{\sigma_t}\right),$$

in round $t \in \{1,2\}$. Then, for $p \in [0,1]$:

$$\begin{aligned} Prob(P^i_{\theta^*,t} < p|\Theta_j = \theta^*) &= Prob\left(1 - F\left(\frac{X_t^i - \theta^*}{\sigma_t}\right) < p|\Theta_j = \theta^*\right) \\ &= Prob\Big(\theta^* + \sigma_t F^{-1}(1-p) < X_t^i|\Theta_j = \theta^*\Big) \\ &= 1 - F\left(\frac{\big(\theta^* + \sigma_t F^{-1}(1-p)\big) - \theta^*}{\sigma_t}\right) = 1 - F\Big(F^{-1}(1-p)\Big) = p, \end{aligned}$$

which is the c.d.f. of the uniform distribution. □ (lemma 1)

Having established lemma 1, we now guess arbitrary equilibrium values $V_2 \geq 0$, $n_2 \geq 0$ and consider the interaction of players who have been matched to project

[8] Lemma 1 is a variation of Morris and Shin's (2003) argument of Laplacian beliefs.

j in round 1 or 2. The players observe x_t^i, form posterior probabilities p_t^i of the project's success that are realizations of $P_{\theta^*,t}^i$ and decide about investing into a lottery with expected payoff[9]

$$(1-c)p_t^i + (-c)(1-p_t^i) = p_t^i - c.$$

A player invests if she prefers such a lottery to δV_2 in round 1 or to 0 in round 2. Thus she invests if $p_1^i - c > \delta V_2$ in round 1 or, if $p_2^i - c > 0$ in round 2. Suppose the state of project j (unknown to the players) happens to be just equal to the critical state, $\theta_j = \theta^*$. Then, knowing the uniform distribution of p_t^i and the trigger probabilities $c + \delta V_2$ and c in round 1 and 2 we can compute the total measure of investors into j. The definition of the critical state implies that the measure of investment is just equal to θ^*:

$$(1 - c - \delta V_2) + n_2(1-c) = \theta^*. \qquad (\textit{Crit.St.})$$

The expected payoff in round 2 is

$$V_2 = (1-c)\Phi(\theta^*), \qquad (\textit{Value})$$

because in the limit of precise signals all observers of projects with $\theta < \theta^*$ successfully invest and receive $1-c$, and other players do not invest and receive 0.

Using the law of large numbers, the measure of players per project not investing in round 1 and thus continuing into round 2 is

$$n_2 = 1 - \Phi(\theta^*). \qquad (\textit{Search})$$

We substitute equation (*Value*) and (*Search*) into (*Crit.St.*) and get

$$(1-c)[2 - (1+\delta)\Phi(\theta^*)] - \theta^* = 0. \qquad (\textit{Modif.Crit.})$$

Equation (*Modif.Crit.*) has a unique solution because its left hand side is continuous, decreases in θ^* and is asymptotically linear in θ^*. Thus there is a unique equilibrium of the mobile game in the class of symmetric equilibria.

We justify the shortcut of computing the equilibrium in the limit $\boldsymbol{\sigma} \to \mathbf{0}$ by computing the symmetric equilibrium out of the limit, for $\boldsymbol{\sigma} > \mathbf{0}$. Moreover, we show that there is no other equilibrium than the symmetric one:

[9]Downsized by δ in round 2.

Proposition 1. *1. There exists $\overline{\sigma}$ such that if $\sigma_1 < \overline{\sigma}$ and $\sigma_2 < \overline{\sigma}$, then: the mobile game $\Gamma_M(\boldsymbol{\sigma})$ has a unique Bayesian Nash equilibrium, it is symmetric, and all players play threshold strategies.*

2. $\theta^*(\boldsymbol{\sigma})$, $V_2(\boldsymbol{\sigma})$, and $n_2(\boldsymbol{\sigma})$ describing the unique equilibrium of $\Gamma_M(\boldsymbol{\sigma})$ converge in the limit $\boldsymbol{\sigma} \to \mathbf{0}$ to solution θ^*, V_2, and n_2 of the system of equations (Crit.St.), (Value) and (Search).

The proof, found in appendix A.1, has a structure typical for the global games literature. We first specify equations for a symmetric equilibrium in threshold strategies and show that these have a unique solution. Then we show, by an argument based on iterated dominance, that no other equilibrium exists: for any assumed equilibrium values V_2, n_2 we find a unique set of fundamentals with which projects succeed. Obviously, a project always succeeds if $\theta_j < 0$ and never does if $\theta_j > 2$ and we iteratively expand intervals of sure success/failure until they meet at the critical state θ^*, which is uniquely determined by the assumed values V_2, n_2 according to a critical state condition. This is a unique candidate for an equilibrium with the assumed values V_2, n_2 and if this truly is an equilibrium, the critical state θ^* has to generate the assumed values V_2, n_2 according to value and search conditions. Thus any equilibrium satisfies all conditions that specify the symmetric equilibrium and hence no other exists.

3.1 Comparative Statics

We examine comparative statics of the equilibrium in the limit $\boldsymbol{\sigma} \to 0$ described by equation (*Modif.Crit.*) with respect to the exogenous parameters c, y, δ:

Corollary 1. *The critical state θ^* decreases in c, δ and increases in y.*

Proof: the left hand side of (*Modif.Crit.*) decreases in c, δ, θ^* and because $\Phi(\theta^*) \equiv F(\frac{\theta^*-y}{\tau})$ it increases in y. The comparative statics of θ^* follows from the implicit function theorem. □ (corollary 1)

In particular, the set of successful projects shrinks with higher mobility, which decreases welfare. We examine the welfare effects in the next step:

Corollary 2. *Comparative statics of welfare with respect to c, δ, and y is as summarized in table 1.*

The ex ante expected payoff V at the beginning of the game is

$$V = (1-c)F\left(\frac{\theta^*-y}{\tau}\right) + \left(1 - F\left(\frac{\theta^*-y}{\tau}\right)\right)\delta(1-c)F\left(\frac{\theta^*-y}{\tau}\right). \quad (2)$$

Parameter q	c	δ	y
Direct effect $\frac{\partial V}{\partial q}$	-	+	-
Strategic effect $\frac{\partial V}{\partial \theta^*}\frac{\partial \theta^*}{\partial q}$	-	-	+
Total effect $\frac{dV}{dq}$	-	-/+	-

Table 1: Overview of welfare effects.

The total welfare effect with respect to parameters $q \in \{c, \delta, y\}$ consists of the direct non-strategic effect $\frac{\partial V}{\partial q}$ and the strategic effect $\frac{\partial V}{\partial \theta^*}\frac{\partial \theta^*}{\partial q}$ via the change of the critical state θ^*, so the proof of corollary 2 consists of computing the derivations, which we omit here. Note that $\frac{\partial V}{\partial \theta^*}$ is unambiguously positive, and hence the sign of the strategic effect is the same as $\frac{\partial \theta^*}{\partial q}$ specified in corollary 1. The total effect of the increase of y is unambiguously negative, despite the fact that the direct and strategic effects are of opposite signs, because derivative $\frac{\partial \theta^*}{\partial y}$ turns out to be smaller than 1 so $\theta^* - y$ decreases with y; hence the measure of successful projects decreases with y.

We summarize both corollaries verbally: increased mobility, measured by higher δ, makes players choosier (see figure 2a) because it increases the value of the outside option δV, and this negative strategic effect may outperform the positive direct effect (see figure 2b). Similarly, a decrease in the average project's quality, higher[10] y, makes players less choosy, as it decreases the outside option value and also increases search activity, which in turn increases the measure of observers of each project. Increase of c causes two strategic effects. A negative strategic effect, which already exists in the static game, makes players choosier because the profits from successful investment decrease, but this effect is partially counteracted by a positive strategic effect in the mobile game: larger c decreases the endogenous outside option value and increases search activity, both of which enhance successful coordination. The negative strategic effect always prevails and $\frac{\partial \theta^*}{\partial c}$ is unambiguously negative.

The comparative statics is simpler to analyze in a limit $\tau \to 0$ for which we obtain a closed form solution.[11] We report the limit solution in appendix A.2, because the expressions, although in principle simple, are tiresome. We find that:

Corollary 3. *Welfare unambiguously decreases with increased mobility (higher δ) in the ordered limit $\tau \to 0, \boldsymbol{\sigma} \to \mathbf{0}$ (such that $\frac{\boldsymbol{\sigma}}{\tau} \to \mathbf{0}$).*

Proof can be found in appendix A.2.

[10]We remind that θ_j is a measure necessary for the success of project j. Hence higher θ_j means worse quality of the project.

[11]We take the ordered limit $\lim_{\tau \to 0, \boldsymbol{\sigma} \to \mathbf{0}}$. This assures that $\frac{\boldsymbol{\sigma}}{\tau} \to \mathbf{0}$ and thus we stay in the equilibrium uniqueness region.

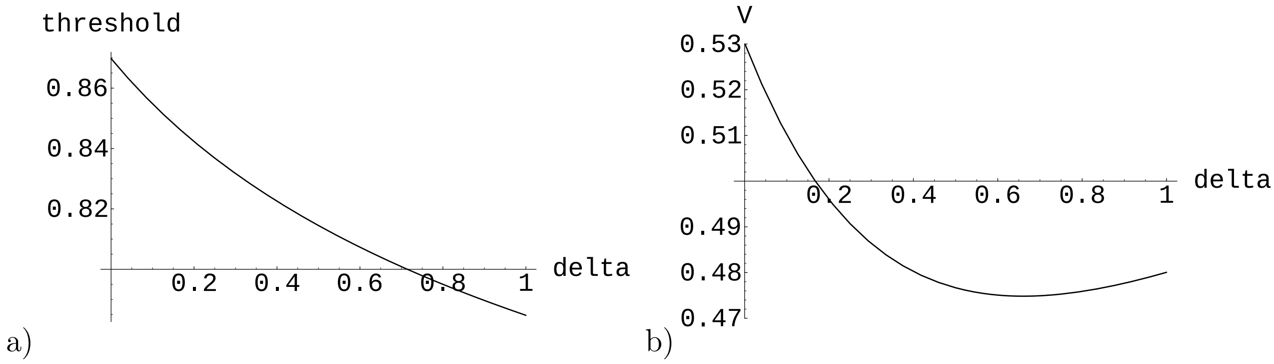

Figure 2: Welfare and comparative statics analysis of the mobile game, parameters: $c = 0.3$, $y = 0.8$, $\tau = 0.1$. **a)** $\theta^*(\delta)$, **b)** $V(\delta)$.

Though we vary mobility exogenously in our model, the finding of the welfare's non-monotonicity with respect to δ implies that, if welfare V decreases with δ, then were the players able to influence their mobility, they could find themselves in a prisoners' dilemma-like situation: each would benefit from a unilateral increase of mobility but a mutual increase would harm all.

3.2 Comparison of the Mobile and the Static Game — the Self-Regulatory Property

The mobile game is constructed in such a way that its outcomes are directly comparable with the static game outcomes because the measure of players per project is the same in both games, the fundamentals are drawn from the same distribution, and players can invest only once in both games. The solution to the static game is described by the following proposition:

Proposition 2. (Morris and Shin) *There exists $\overline{\sigma}$ such that the game $\Gamma_S(\sigma)$ is dominance solvable for all $\sigma < \overline{\sigma}$. The unique strategy surviving iterated elimination of dominated strategies is a threshold strategy*

$$s(x^i) = \begin{cases} 1 & \text{if } x < x^*_\sigma, \\ 0 & \text{if } x > x^*_\sigma, \end{cases}$$

where the threshold x^*_σ converges to $1 - c$ for $\sigma \to 0$.

Proof is in Morris and Shin (2003).[12]

[12] The threshold in the limit $\sigma \to 0$ can be found by informal arguments similar to those behind the equation (*Crit.St.*): only players preferring lottery with expected payoff $(1-c)p^i + (-c)(1-p^i) = p^i - c$ to the safe outside option payoff 0 invest, and because the conditional probabilities

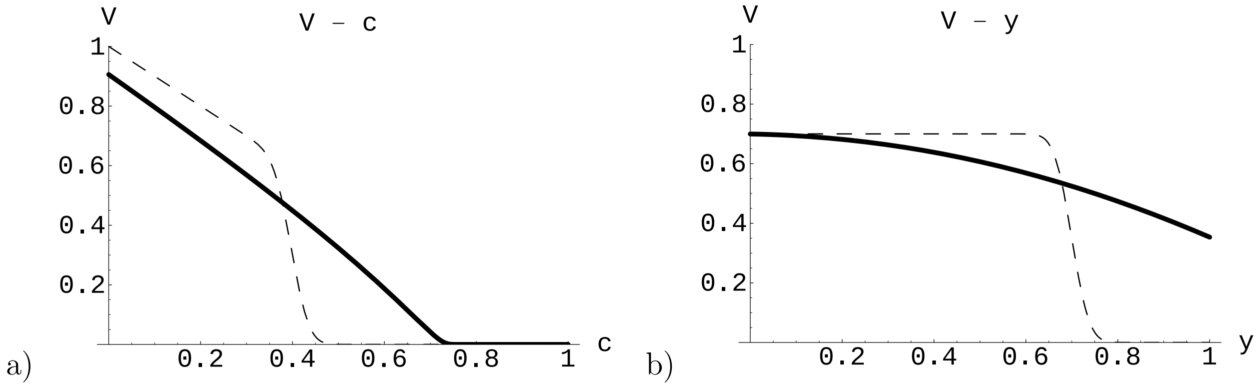

Figure 3: **a)** Comparison of $V(c)$ in the mobile game — thick line, and the static game — dashed line. (parameters: $\delta = 0.9$, $y = 0.6$, $\tau = 0.03$) **b)** Comparison of $V(y)$ in the mobile game — thick line, and the static game — dashed line. (parameters: $c = 0.3$, $\delta = 0.9$, $\tau = 0.03$)

Welfare in the static game is

$$V_{stat} = (1-c)F\left(\frac{\theta^*_{stat} - y}{\tau}\right),$$

and thus, if variance τ of the fundamentals' distribution is small, welfare V_{stat} declines sharply with y or c in the neighborhood of θ^*_{stat}. In contrast, the critical state θ^*_{mob} in the mobile game adjusts to an increase of y or c because players increase their thresholds — they become less choosy. As a result, V decreases with c and y markedly more slowly in the mobile game than in the static game; this self-regulatory property is depicted in figures 3a,b.

Let us further examine the self-regulatory property analytically. Denote by P the equilibrium measure of successful projects $P = \Phi(\theta^*) \equiv F(\frac{\theta^*-y}{\tau})$; the welfare increases in P. Let us compare the dependence of P on y in the case of the static and the mobile game.[13] The derivative $\frac{\partial P_{stat}}{\partial y}$ can be computed straightforwardly in the case of the static game:

$$\frac{\partial P_{stat}}{\partial y} = -\frac{1}{\tau} f\left(\frac{\theta^*_{stat} - y}{\tau}\right),$$

hence for small τ, P_{stat} declines quickly when $y \approx \theta_{stat}$. In the case of the mobile game, we get from equation (*Modif. Crit.*):

$$P_{mob} = \Phi\Big((1-c)[2-(1+\delta)P_{mob}]\Big). \tag{3}$$

p^i of the project's success are distributed uniformly on $[0,1]$ if the state happens to be critical, the mass of players believing that $p^i > c$ is $1-c$ which must coincide with the critical state.

[13]The analysis of dependence of P on c is virtually identical, and hence omitted.

The self-regulatory property is caused by the negative influence of P_{mob} on the right hand side of (3), which is absent in the static case. We express the derivative

$$\frac{\partial P_{mob}}{\partial y} = \frac{-\frac{1}{\tau} f\left(\frac{\theta^*_{mob}-y}{\tau}\right)}{1 + \frac{1}{\tau} f\left(\frac{\theta^*_{mob}-y}{\tau}\right)(1-c)(1+\delta)}.$$

P.d.f. $\frac{1}{\tau} f\left(\frac{\theta^*_{mob}-y}{\tau}\right)$ is both in the numerator and in the denominator, and hence the derivative $\frac{\partial P_{mob}}{\partial y}$ does not diverge even for $\tau \to 0$ and $\theta^*_{mob} \approx y$. In fact, the derivative simplifies to $\frac{-1}{(1-c)(1+\delta)}$ in the limit $\tau \to 0$ and for non-extreme y (see appendix A.2).

3.3 Limit of the Inefficient Search

Next, we examine the mobile game with very inefficient search, when $\delta \to 0$, and show that it does not approximate the static game. Let us consider the mobile game with parameter $\delta \equiv 0$, which is out of the assumed range of $\delta \in (0,1)$ and thus proposition 1 does not hold. Obviously, players are indifferent between investing and not investing in round 2, which creates equilibrium multiplicity. The equilibrium in which nobody invests in round 2 can be associated with the equilibrium of the static game. However, this equilibrium is not approximated by the equilibrium of the mobile game as $\delta \to 0+$. The critical state $\theta^*_0 \equiv \lim_{\delta\to 0+} \theta^*(\delta)$ solves the limit of equation (*Modif.Crit.*):

$$(1-c)[2-\Phi(\theta^*_0)] - \theta^*_0 = 0,$$

the solution of which differs from $\theta^*_{stat} = 1-c$. This can be seen in figure 2a,b: $\lim_{\delta\to 0+} \theta(\delta) > \theta_{stat}$ and also $\lim_{\delta\to 0+} V(\delta) > 0$ whereas welfare of the static game would be virtually 0 for that setting of the parameters.[14] The intuition is that search increases the measure of observers of each project from 1 to $1+n_2$, which enhances successful coordination (also) in round 1. Hence the critical state moves towards worse states and players are matched to a successful project in round 1 more often. Welfare in the mobile and the static game thus differs for purely strategic reasons for small δ and the difference does not disappear even if the gains from investment in round 2 are negligible but positive. Therefore, search options should not be ignored in the analysis of coordination problems even when search is very inefficient.

[14] More precisely, it would be very small, $V \to 0$ in the limit $\tau \to 0$.

4 Social Learning

We have assumed until now that players matched to a project j in round 2 do not observe the measure of investment into j realized in round 1. This assumption is abandoned in this section and we find, somewhat surprisingly, that social learning is irrelevant in the mobile game. The game analyzed in this section remains as the mobile game described in section 2.2 except that, additionally, players matched to j in round 2 observe a signal z^i about the measure of investment $l_{j,1}$ into j in round 1. We assume Dasgupta's (2005) error structure which allows for analytical solution of the game:

$$z^i = F^{-1}(l_{j,1}) + \omega\xi^i, \tag{4}$$

where the error terms $\xi^i \sim N(0,1)$ are independent across players and also independent of the error terms ϵ_t^i of the signals x_t^i. We argue below that ω depicts the informativeness of signal z^i compared to x_1^i; if $\omega = 1$ the two signals have the same informativeness.

A pure strategy is a pair of functions $a_1(x_1) : \mathbb{R} \to \{0,1\}$, $a_2(x_1, x_2, z) : \mathbb{R}^3 \to \{0,1\}$ which prescribe actions in rounds 1 and 2 conditional on the observed signals. We refer to this game as a *learning* game $\Gamma_L(\boldsymbol{\sigma})$. A *monotone* strategy is a pair of functions $a_1(x_1)$, $a_2(x_2, z)$ such that $a_1(.)$ is non-increasing, $a_2(x_2, z)$ is non-increasing in x_2, non-decreasing in z, and does not depend on x_1; hence we omit x_1 from its arguments. We restrict players to monotone strategies in this section. We find that the equilibrium in monotone strategies of the learning game coincides with the unique equilibrium of the mobile game in the limit $\boldsymbol{\sigma} \to \mathbf{0}$. Although we have not ruled out an equilibrium in non-monotone strategies that differs from the equilibrium of the mobile game, we have not found such.

Proposition 3. *The learning game $\Gamma_L(\boldsymbol{\sigma})$ has a unique Bayesian Nash equilibrium in monotone strategies. This equilibrium is symmetric and converges to the equilibrium of the mobile game $\Gamma_M(\boldsymbol{\sigma})$ as $\boldsymbol{\sigma} \to \mathbf{0}$.*

Proof: for any equilibrium in monotone strategies the critical value θ^* must exist because the measure of investors l_j monotonically decreases in θ_j. The existence of the critical state implies that the equilibrium is symmetric, because the maximization problem of each player is identical as it depends only on the common values of V_2, θ^* and on the exogenous parameters, and the best responses are strict.[15]

[15]Precisely, players are indifferent between investing and not investing only when observing threshold signals, which happens with 0 probability.

The measure of early investors is $l_{j,1} = F\left(\frac{x_1^* - \theta_j}{\sigma_1}\right)$ because only those who receive a signal below x_1^* invest. We define $\tilde{z}^i \equiv x_1^* - \sigma_1 z^i$ and because of the assumed error technology $\tilde{z}^i = \theta_j - \sigma_1 \omega \xi^i$. Thus, receiving signal z^i is equivalent to receiving signal $\tilde{z}^i$ about θ_j with error drawn from $N(0, (\sigma_1 \omega)^2)$ and independent of error of signal x_2^i. Finally, players form sufficient statistics $\tilde{x}_2^i$ for θ_j using x_2^i and $\tilde{z}^i$:

$$\tilde{x}_2^i = \frac{\sigma_2^2 \tilde{z}^i + \sigma_1^2 \omega^2 x_2^i}{\sigma_2^2 + \sigma_1^2 \omega^2},$$

with an error term $(\tilde{x}_2^i - \theta_j) \sim N(0, \frac{\sigma_1^2 \sigma_2^2 \omega^2}{\sigma_2^2 + \sigma_1^2 \omega^2})$.

The equilibrium of the learning game therefore corresponds to the unique equilibrium of the mobile game with $\tilde{\boldsymbol{\sigma}} = (\sigma_1, \sqrt{\frac{\sigma_1^2 \sigma_2^2 \omega^2}{\sigma_2^2 + \sigma_1^2 \omega^2}})$. If $(\sigma_1, \sigma_2) \to \mathbf{0}$ then $\tilde{\boldsymbol{\sigma}} \to \mathbf{0}$ as well, so the equilibrium of the learning game converges to the limit equilibrium of the mobile game. □ (proposition 3)

We have found that social learning is irrelevant in the mobile game, whereas in Dasgupta (2005) social learning matters. This difference is due to the mobility present in our mobile game but not in Dasgupta's. Players delaying investment in Dasgupta's game remain in the same project as they were in round 1; they only gain additional information — the signal z^i. In contrast, the motivation to wait (search) in our model is to find a project with better fundamentals. Additional information z^i cannot be the decisive motivation for search, because if signal x_2^i is far away from θ^*, it is a sufficient guideline for the investment decision. The additional signal z^i is useful only if the distance of x_2^i from θ^* is of the order of σ_2, which has negligible probability for $\boldsymbol{\sigma} \to \mathbf{0}$. Hence giving players additional information z^i does not alter the mobile game equilibrium, because players are almost sure they will not need this information in round 2 in the limit $\boldsymbol{\sigma} \to \mathbf{0}$. In contrast, in Dasgupta's game, a player in need of additional information in round 1 knows that this information will be useful in round 2.

5 Robustness

We consider several other modifications of the mobile game and show that the qualitative features of the equilibrium are robust to them. See figure 4 for the relationships among individual modifications. In section 5.1, we let the players search repeatedly. In section 5.2 we generalize the payoff function, and players are able to direct their search towards projects with better fundamentals in section 5.3.

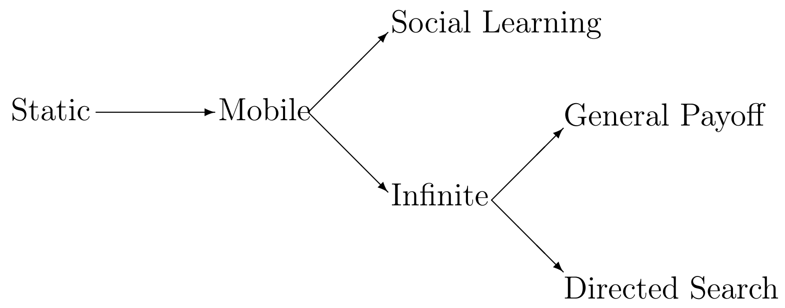

Figure 4: Modifications Structure — We develop the benchmark static game into a network of related models.

5.1 Infinite Number of Search Rounds

The players of the mobile game have only one possibility of search. Are the results robust to a change in the number of search rounds? We examine the following modification: the game remains the same as the mobile game described in section 2.2 except it does not end after round 2. Instead, players decide in infinity of rounds, indexed by $t \in \mathbb{N}$, whether to invest into a currently observed project or to search and continue to round $t+1$. As in the mobile game, players can invest only once, hence they can search only until they invest and afterwards cannot take any further action. The return $R(\theta_j, l_j)$ of a project j depends on its fundamentals θ_j and on the cumulative investment l_j over all rounds. Payoffs of late investors are downsized to $\delta^{t-1}R(\theta_j, l_j)$, t being the time of investment. The payoff of players who never invest is normalized to 0. Player i who has continued to round t receives signal $x^i_t = \theta_j + \sigma\epsilon^i_t$, where j is the project she is matched to in round t and errors ϵ^i_t are independent across players and rounds. For the sake of simplicity, we assume the same value of σ in all rounds. We refer to this game as the *infinite* game.

We sketch the solution of the infinite game in the limit $\sigma \to 0$ in a similar manner as we did for the mobile game in section 3. Let V be the ex ante expected payoff at the beginning of the game and n be a measure of observers of any project cumulative over all rounds; n is a common value for all projects because the search is undirected. Consider the interaction of players matched to a project j in any of the rounds $t \in \mathbb{N}$. All observers of j are in the same situation, except the payoffs of those in round t are linearly re-scaled by factor δ^{t-1}, which does not alter their strategic position. Thus they are of the same type and simultaneously[16] decide between investing, which pays $R(\theta_j, l_j)$, and the outside option, which pays δV. Therefore, observers of j interact in a simple global game.

We denote player i's posterior probability of the project's success by $P^i_{\theta^*} \equiv Prob(\Theta_j < \theta^* | X^i)$ as in section 3 and we reiterate that $P^i_{\theta^*}$ is distributed uniformly

[16]We can treat the decision of players in all rounds as simultaneous because players do not observe the measure of investments from previous rounds.

on $[0,1]$ conditional on $\theta_j = \theta^*$. Player i invests if and only if she prefers the lottery of investment to the outside option:

$$P^i_{\theta^*}(1-c) + (1-P^i_{\theta^*})(-c) > \delta V,$$

which implies the critical mass condition:

$$(1-c-\delta V)n = \theta^*. \qquad (\textit{Crit.st.'})$$

The value condition is

$$V = (1-c)\Phi(\theta^*) + \delta V\left(1-\Phi(\theta^*)\right). \qquad (\textit{Value'})$$

The measure of observers per project in round 1 is 1. Ratio $1-\Phi(\theta^*)$ of the observers are matched to a project with $\theta > \theta^*$ so they continue into round 2. Out of these, the ratio $1-\Phi(\theta^*)$ continue into round 3,...The cumulative measure of observers per project is

$$n = \sum_{t=1}^{\infty}\left(1-\Phi(\theta^*)\right)^{t-1} = \frac{1}{\Phi(\theta^*)}, \qquad (\textit{Search'})$$

and because the search is undirected, each project is observed by the same measure of players.

We substitute (*Value'*) and (*Search'*) into (*Crit.st.'*) and get

$$(1-c)\frac{1-\delta}{[1-\delta+\delta\Phi(\theta^*)]\Phi(\theta^*)} - \theta^* = 0. \qquad (\textit{Modif.Crit.'})$$

Equation (*Modif.Crit.'*) has a unique solution because its left hand side is continuous, decreases and is asymptotically linear in θ^*. Moreover, it decreases in c, δ, θ^* and because $\Phi(\theta^*) \equiv F(\frac{\theta^*-y}{\tau})$, it increases in y. The implicit function theorem implies that θ^* decreases in c, δ and increases in y, exactly as in the mobile game. The welfare effects are also the same as in the mobile game and table 2, which summarizes the signs of the welfare effects, remains valid.

The infinite game has no other equilibrium except the symmetric one: any assumed pair V and n imply a particular simple global game describing the interaction of players observing a project j. This global game has a unique equilibrium, with critical state θ^* depending on V and n according to (*Crit.st.'*). Thus any V and n imply a unique θ^*, and any θ^* implies a unique V and n according to equations (*Value'*) and (*Search'*). Hence any equilibrium has to satisfy the triplet of equations (*Crit.st.'*), (*Value'*) and (*Search'*), which has a unique solution.

As we have mentioned in section 2.2, a player of the infinite game never wishes to return to a project she has observed in an earlier round. If she has considered the expected payoff of some project inferior to search, then she will not change her opinion after any number of search rounds, as she does not learn anything new about the project nor about the underlying distribution of fundamentals. Thus, we could introduce the possibility of returning to earlier projects without any consequences on equilibrium behavior.

5.2 General Payoff Functions

Until now, we have been analyzing games with a particular return function (1). In this section we take first steps in examining the effects of mobility for a general return function. We do the analysis in the framework of the infinite game rather than the mobile game because the former is simpler to analyze, as all the players are of the same type, whereas in the mobile game the players of round 1 and 2 differ in their outside options.

We analyze the same game as the infinite game described in the previous section 5.1, except with a general return function $R(\theta_j, l_j)$. As in the previous section, we want each project to generate a simple global game with a unique equilibrium, conditional on V and n. To assure this, we impose Morris and Shin's (2003) assumptions on $R(\theta_j, l_j)$, slightly modified to fit our setting: let $\mathcal{V}$ denote the positive part of the range of the return function $R(\theta_j, l_j)$.

MS1: *Action Monotonicity*: $R(\theta_j, l_j)$ is weakly increasing in l_j.

MS2[17]: *State Monotonicity*: $R(\theta_j, l_j)$ is weakly increasing in θ_j.

MS3: *Strict Laplacian State Monotonicity*: for any $V \in \mathcal{V}$, $n \in [1, +\infty)$, there exists a unique $\theta^* \in \mathbb{R}$ such that $\frac{1}{n}\int_0^n R(\theta^*, l)dl = \delta V$.

MS4: *Uniform Limit Dominance*: for any $V \in \mathcal{V}$, $n \in [1, +\infty)$, there exist $\underline{\theta}$ and $\overline{\theta}$ and $\epsilon > 0$ such that 1. $R(\theta, l) < -\epsilon + \delta V$ for all $l \in [0, n]$ and $\theta < \underline{\theta}$ and 2. $R(\theta, l) > \epsilon + \delta V$ for all $l \in [0, n]$ and $\theta > \overline{\theta}$.

MS5: *Continuity*: $\int_0^1 g(l)R(x, l)dl$ is continuous with respect to signal x and density $g(.)$.

Proposition 4. *Suppose MS1–MS5 are satisfied. Then, in the limit $\sigma \to 0$, all Bayesian Nash equilibria of the infinite game with the return function $R(\theta_j, l_j)$ are symmetric and in threshold strategies. Variables θ^*, V and n describing the*

[17] Note that $R(\theta_j, l_j)$ as described in (1) is weakly decreasing in θ_j instead of increasing and thus MS2 is formally not satisfied. However, this can be accommodated by introducing $\tilde{\theta} = 1 - \theta$.

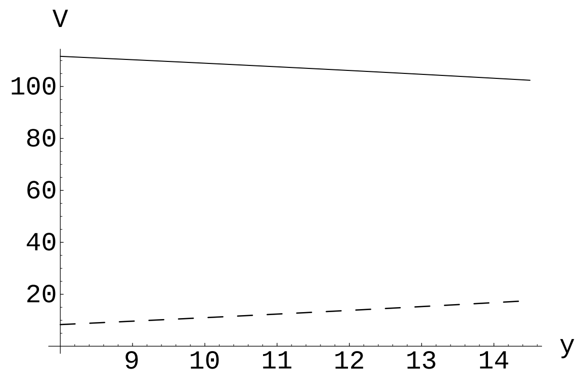

Figure 5: Payoff $R(\theta_j, l_j) = \theta_j - 1 + l^2$ generates equilibrium multiplicity. In one equilibrium, the full line, players invest only in projects of very high quality; hence search activity and measure of investment into successful projects are high, and returns of successful projects are very high. The incentive to search is thus high too. In the other equilibrium, the dashed line, players also invest in projects of medium quality, search activity and measure of investment into successful projects is low, and hence successful projects have only medium returns and the motivation to search is low. Welfare in the first equilibrium decreases with improving distribution of fundamentals. Parameters: $\delta = .9$, $\tau = .01$.

equilibrium satisfy:

$$\frac{1}{n}\int_0^n R(\theta^*, l)dl = \delta V, \qquad (\mathit{Crit.st.g.})$$

$$V = \int_{\theta^*}^{+\infty} R(\theta, n)d\Phi(\theta) + \delta V \Phi(\theta), \qquad (\mathit{Value.g.})$$

$$n = \frac{1}{1 - \Phi(\theta)}. \qquad (\mathit{Search.g.})$$

Proof: values V and n are defined in any equilibrium. For any assumed pair V, n interaction of observers of any particular project is a simple global game with the payoff function $R(\theta_j, l_j)$, the outside option value δV, and the measure of players n. Because of the assumptions MS1 – MS5 and the normality of the errors' distribution, this simple global game satisfies proposition 2.2 in Morris and Shin (2003) and thus has a unique equilibrium with threshold θ^* satisfying equation (*Crit.st.g.*). Moreover, threshold θ^* determines expected value and search activity, which gives equations (*Value.g.*) and (*Search.g.*). □ (proposition 4)

However, proposition 4 does not guarantee equilibrium uniqueness. We have found an example of the return function

$$R(\theta, l) = \theta - 1 + l^2, \qquad (5)$$

for which the system of equations (*Crit.st.g.*), (*Value.g.*) and (*Search.g.*) have multiple solutions (see figure 5). In such a case, each solution represents a symmetric equilibrium in threshold strategies differing in the endogenous values of the outside option and in the search activity and hence also in the threshold.

Though the uniqueness is not guaranteed generally, we pinpoint two simple classes of return functions, for which the equations (*Crit.st.g.*), (*Value.g.*) and (*Search.g.*) have a unique solution and hence the equilibrium uniqueness is guaranteed:

Corollary 4. *Let the return function, satisfying MS1-MS5, be of the form* $R(\theta, l) = p(\theta) + q(l)$ *where* $q(l)$ *is concave and the derivative* $q'(.)$ *exists. Then the game has a unique Bayesian Nash equilibrium.*

Proof is in appendix A.3.

Another class of return functions guaranteeing equilibrium uniqueness is

$$R(\theta_j, l_j) = \begin{cases} \zeta(\theta_j) - c & \text{if } a(\theta_j) < l_j, \\ -c & \text{if } a(\theta_j) \geq l_j, \end{cases} \tag{6}$$

where $a(\theta_j)$ decreases and $\zeta(\theta_j)$ increases in θ_j. This return function generalizes the coordination problem induced by return function (1) and was studied in Morris and Shin's (1998) model of currency attacks.

Corollary 5. *Let the return function be of the form (6) and the derivatives* $a'(.)$, $\zeta'(.)$ *exist. Then the game has a unique Bayesian Nash equilibrium.*

Proof can be found in appendix A.3.

The simple form of the particular return function (1) has allowed us to eliminate integrals in (*Crit.st.g.*) and (*Value.g.*), which is not possible for a general return function, and therefore we do not draw general conclusions about the comparative statics. However, examination of the return function (5) shows that the non-monotonicity of welfare with respect to δ is not a special feature of return function (1); it can be observed also in the case of (5).[18] Moreover, the return function (5) generates non-monotonicity with respect to y (see figure 5). The intuition is that worse distribution of fundamentals increases search activity and hence the measure of observers of each project. Thus the measure of investment into successful projects increases, and this positive strategic effect dominates the negative direct effect because the return steeply increases in the measure of investors.

[18] For instance, for parameters $y = 10$, $\delta = .9$, $\tau = .01$ and for the solution $\theta^* = .997$, $V = 11.0$, $n = 1.67$ welfare V locally decreases with δ.

5.3 Directed Search

To this point we have assumed that search and matching to projects is undirected, and hence that each project has been observed by the same measure of players which, although computationally convenient, is unrealistic. We let the agents direct their search toward better projects in this section. As a consequence, the distribution of projects describing the matching outcome differs from the distribution of physically existing projects.

Let the fundamentals of physically existing projects be distributed according to p.d.f. $\phi(.)$, but let us assume in this section that players are able to influence the matching process such that they are matched to a project drawn from $\psi(.)$, with c.d.f. $\Psi(.)$. We assume return function 1 and use the framework of the infinite game; players can search in an infinite number of search rounds, and each search leads to a project drawn from $\psi(.)$. We also assume that $\phi(.)$ and $\psi(.)$ satisfy the monotone likelihood property, $\frac{\psi(.)}{\phi(.)}$ is decreasing. Accordingly, better projects, characterized by lower θ, are observed by more players than are worse projects. We dub this game a *directed search* game.

Proposition 5. *The directed search game has a unique BNE in the limit $\sigma \to 0$.*

Proof: The measure of observers $o(\theta_j)$ depends on project j's fundamentals θ_j. Let n be the measure of all searchers per project cumulatively over all rounds. The number n would also be the measure of observers of each project in the previous sections, but in this section the observers are distributed unevenly. Value n induces $o_n(\theta_j)$ observers of project j:

$$o_n(\theta_j) = n\frac{\psi(\theta_j)}{\phi(\theta_j)}. \tag{7}$$

Number n and the expected ex ante payoff V are defined for each strategy profile. Interaction among all observers of project j could be formalized as a simple global game in the previous sections. To proceed in the same way here, we need to renormalize the measure of observers in order to avoid its dependence on θ_j. Interaction of $o_n(\theta_j)$ observers described by the return function

$$R(\theta_j, l) = \begin{cases} 1-c & \text{if } l \geq \theta_j, \\ -c & \text{if } l < \theta_j \end{cases}$$

can be equivalently described as the interaction of players with measure 1 and the return function

$$\tilde{R}_n(\theta_j, \tilde{l}) = \begin{cases} 1-c & \text{if } \tilde{l} \geq \frac{\theta_j}{o_n(\theta_j)}, \\ -c & \text{if } \tilde{l} < \frac{\theta_j}{o_n(\theta_j)}, \end{cases}$$

where $\tilde{R}_n(.,.)$ is defined on $\mathbb{R} \times [0,1]$. In other words, we measure investment in relative instead of absolute terms, and modify the return function accordingly. Function $\tilde{R}_n(.,.)$ is non-decreasing in $\tilde{l}$ and non-increasing in θ on its definition range.[19] This modified description of the interaction associated with project j is a global game and satisfies conditions of theorem 2.2 in Morris and Shin (2003). Thus each assumed pair of values n, V generates a unique critical state θ^* according to

$$\int_0^1 \tilde{R}_n(\theta^*, l)dl = \delta V,$$

which can be simplified into

$$\frac{\theta^*}{o_n(\theta^*)} = 1 - c - \delta V. \qquad (\textit{Crit.st.DS.})$$

The critical state θ^* implies values V and n: The value condition is

$$V = (1-c)\Psi(\theta^*) + \big(1 - \Psi(\theta^*)\big)\delta V. \qquad (\textit{Value.DS.})$$

Measure n of searchers per project cumulatively over all rounds is

$$n = \sum_{t=1}^{\infty} \big(1 - \Psi(\theta^*)\big)^{t-1} = \frac{1}{\Psi(\theta^*)}. \qquad (\textit{Search.DS})$$

Note that the value and search condition depend on the distribution describing the matching process, not on the distribution describing the physical occurrence of states.

Substituting (*Value.DS.*), (*Search.DS*) and (7) into (*Crit.st.DS.*) gives:

$$(1-c)\frac{1-\delta}{[1-\delta+\delta\Psi(\theta^*)]\,\Psi(\theta^*)} - \frac{\theta^*}{\frac{\psi(\theta^*)}{\phi(\theta^*)}} = 0, \qquad (\textit{Modif.Crit.DS})$$

which has a unique solution as the left hand side of (*Modif.Crit.DS*) is continuous, positive for $\theta^* \leq 0$, decreasing for $\theta^* > 0$, and negative for sufficiently large θ. □ (proposition 5)

Comparative statics can be computed in the same way as in the case of the

[19]Note the non-monotonicity of $\frac{\theta}{o_n(\theta)}$. However, $\frac{\theta}{o_n(\theta)}$ increases for $\theta \geq 0$ and though it can decrease for $\theta < 0$, it is then always negative and thus smaller than $\tilde{l} \in [0,1]$.

mobile or the infinite game: the left hand side of (*Modif.Crit.DS*) decreases in c, δ and θ^*, hence the solution θ^* decreases in c and δ. The comparative statics thus remains the same as in the case of the mobile and the infinite game. Furthermore, numerical solution of (*Modif.Crit.DS*) shows that welfare is non-monotonic in δ for some parameters.

6 Conclusion

We have studied search among many simultaneous projects, each being a coordination problem. Players, dissatisfied with the signal about the project they currently observe, may search for another of the projects, but search is costly. This mobile game is an expansion of a simple benchmark global game, labelled a static game, from which it inherits equilibrium uniqueness allowing for examination of comparative statics. The mobile game has a "self-regulatory" property: any effects characteristic for the benchmark static game are partially counteracted by a strategic effect in the opposite direction through the endogenous changes of the outside option values and of the mass of observers of each project. Thus the occurrence of coordination failures is notably robust to the changes of exogenous parameters such as the distribution of fundamentals.

The self-regulatory mechanism implies that a project's coordination failure is determined not only by the absolute state of economic fundamentals but also by its relative ranking compared to other projects. This may explain the occurrence of investment crises despite substantial improvements in the distribution of countries' fundamentals over the past decades. In fact, we have found a payoff function for which an improvement in the distribution of fundamentals may increase the amount of coordination failures and decrease welfare. Improvement in the distribution decreases search activity which results in investment scattered among more projects, and this may outweigh the direct positive effects.

Similarly, welfare may decrease with mobility. The positive direct effect of lower search costs is counteracted by a negative strategic effect since lower search costs increase the outside option value, which hampers successful coordination. Again, the strategic effect may prevail and so the welfare is non-monotonic with respect to mobility. The result may be a prisoners' dilemma-like situation. While we have considered a fixed, exogenously given mobility, real investors are able to unilaterally increase their mobility, which could be modelled as an increase of δ. Obviously, any investor would benefit from the unilateral increase, but the collective increase would harm all.

The qualitative features of the comparative statics of the mobile game seem to be robust to modifications; we have considered the possibility of social learning, infinite number of search opportunities, directed search, and general payoff functions satisfying strategic complementarity.

The mobile game is a realistic extension to many static global games applications. For instance, while Morris and Shin (1998) study a coordination game of speculators considering an attack on an isolated currency, the mobile game allows for the incorporation of parallel coordination problems of other currencies. The cost of search can be associated with the cost of acquiring the private signal x_j^i about currency j. We have argued that occurrence of successful coordination always decreases with lower search costs. Thus, the low cost of acquiring private signals about other currencies decreases the occurrence of currency attacks.

The possibility of analyzing the influence of mobility on coordination failures makes the model a useful framework for a study of globalization's consequences. Numerous projects succeed only if many agents coordinate their efforts. Globalization allows people skeptical about the risky project they were matched (born) with, to search for another risky opportunity. On the one hand, higher mobility allows agents to avoid risky projects with bad fundamentals; on the other hand, it lowers their ability to coordinate on risky investments. We show that either effect may prevail under certain circumstances. A firmer connection of the model to globalization processes is an opportunity for future research.

A Appendix

A.1 Proof of Proposition 1

1. We first formulate conditions for symmetric equilibrium in threshold strategies characterized by V_2, n_2, x_1^*, x_2^* and θ^* and later prove that no other equilibrium exists.

Critical state θ^* satisfies a *critical mass* condition: if the realized state of a project j is θ^*, the measure of players investing into j, because they have received a signal below x_t^*, must be precisely θ^*:

$$F\left(\frac{x_1^* - \theta^*}{\sigma_1}\right) + F\left(\frac{x_2^* - \theta^*}{\sigma_2}\right) n_2 = \theta^*. \qquad (crit.st.'')$$

The players combine signals x_t^i and prior beliefs to form a posterior belief about the fundamental θ_j. Both the prior distribution and the distribution of errors are normal distributions and thus the posterior distribution in round $t \in \{1,2\}$ is also a normal distribution $N(e_t(x_t^i, y), u_t^2)$ where $e_t(x,y) \equiv \frac{\sigma_t^2 y + \tau^2 x}{\sigma_t^2 + \tau^2}$ and $u_t^2 \equiv \frac{\sigma_t^2 \tau^2}{\sigma_t^2 + \tau^2}$. Knowing the posterior distribution, we can express the expected payoff for investing into j, conditional on signal x_t^i:

$$(1-c)F\left(\frac{\theta^* - e_t(x_t^i, y)}{u_t}\right) - c\left[1 - F\left(\frac{\theta^* - e_t(x_t^i, y)}{u_t}\right)\right] = F\left(\frac{\theta^* - e_t(x_t^i, y)}{u_t}\right) - c.$$

A player observing x_1^* must be indifferent between investing and the outside option value δV_2. This gives an *indifference 1* condition:

$$F\left(\frac{\theta^* - e_1(x_1^*, y)}{u_1}\right) - c = \delta V_2. \qquad (indif.1)$$

A player observing x_2^* must be indifferent between investing and the outside option which is 0 in round 2. This gives an *indifference 2* condition:

$$F\left(\frac{\theta^* - e_2(x_2^*, y)}{u_2}\right) - c = 0. \qquad (indif.2)$$

The equilibrium value V_2 can be expressed in terms of θ^* as a solution of a nonstrategic maximization problem. Investing into j gives a lottery with expected payoff $Prob(\Theta_j < \theta^* | x_2) - c$ and players invest only if that is greater than 0. This gives a *value* condition:

$$V_2 = E[Max(Prob(\Theta_j < \theta^* | X_2^i) - c, 0)], \qquad (Value")$$

where the expectation is over unconditional distribution of X_2^i.

The unconditional distribution of signal x_1^i is $N(y, \tau^2 + \sigma_1^2)$. Players observing signal $x_1^i > x_1^*$ search, which gives a *search* condition:

$$n_2^* = 1 - F\left(\frac{x_1^* - y}{\sqrt{\tau^2 + \sigma_1^2}}\right). \qquad (Search")$$

We have specified a system of five equations (*crit.st."*), (*indif.1*), (*indif.2*), (*Value"*), and (*Search"*) for five unknowns θ^*, x_1^*, x_2^*, V_2 and n_2. Next, we prove that this system has a unique solution if $\boldsymbol{\sigma}$ is sufficiently small. We express $x_1^* = \chi_1(\theta^*, V_2)$ as a function of θ^* and V_2 from (*indif.1*):

$$\chi_1(\theta^*, V_2) = (1 + \frac{\sigma_1^2}{\tau^2})\theta^* - \frac{\sigma_1}{\tau}\sqrt{\tau^2 + \sigma_1^2} F^{-1}(c + \delta V_2) - \frac{\sigma_1^2}{\tau^2} y, \qquad (8)$$

and $x_2^* = \chi_2(\theta^*)$ as a function of θ^* from (*indif.2*):

$$\chi_2(\theta^*) = (1 + \frac{\sigma_2^2}{\tau^2})\theta^* - \frac{\sigma_2}{\tau}\sqrt{\tau^2 + \sigma_2^2}F^{-1}(c) - \frac{\sigma_2^2}{\tau^2}y. \tag{9}$$

We substitute (8) and (9) into (*crit.st."*):

$$F\left[-\frac{\sqrt{\tau^2+\sigma_1^2}}{\tau}F^{-1}(c+\delta V_2) + \frac{\sigma_1}{\tau^2}(\theta^* - y)\right] +$$
$$F\left[-\frac{\sqrt{\tau^2+\sigma_2^2}}{\tau}F^{-1}(c) + \frac{\sigma_2}{\tau^2}(\theta^* - y)\right] n_2 - \theta^* = 0, \tag{10}$$

and denote the left hand side of (10) as $\Lambda(V_2, n_2, \theta^*)$. The function $\Lambda(V_2, n_2, \theta^*)$ increases in n_2 and decreases in V_2. It also decreases in θ^* for sufficiently small $\boldsymbol{\sigma}$ because the derivative $\frac{\partial \Lambda}{\partial \theta^*}$ is bounded above by $\frac{1}{\sqrt{2\pi}}\frac{\sigma_1+n_2\sigma_2}{\tau^2} - 1 \leq \frac{1}{\sqrt{2\pi}}\frac{max(\sigma_1,\sigma_2)}{\tau^2}2 - 1$ which is negative for small $\boldsymbol{\sigma}$. The function $\Lambda(V_2, n_2, \theta^*)$ can be naturally interpreted as a measure of investment into j when the project's fundamentals θ_j (unknown to players) happens to be θ^* and V_2, n_2, θ^* are equilibrium values.

Our next aim is to eliminate unknowns V_2 and n_2 by expressing them as functions of θ^* in order to express Λ as a one-dimensional function of θ^*. The condition (*Value"*) has the form $V_2 = v(\theta^*)$ but variable $n_2 = \eta(x_1^*)$ is a function of x_1^* according to (*Search"*), so first we have to express x_1^* as a function of θ^*: $x_1^* = \tilde{\chi}_1(\theta^*) \equiv \chi_1(\theta^*, v(\theta^*))$. Function $v(\theta^*)$ increases in θ^* but monotonicity of $\tilde{\chi}_1(\theta^*)$ is not guaranteed:

$$\frac{d\tilde{\chi}_1}{d\theta^*} = \frac{\partial \chi_1}{\partial \theta^*} + \frac{\partial \chi_1}{\partial V_2}\frac{dv}{d\theta^*},$$

because the term $\frac{\partial \chi_1}{\partial \theta^*}$ is positive but the term $\frac{\partial \chi_1}{\partial V_2}\frac{dv}{d\theta^*}$ negative. However, the sign of the derivative $\frac{d\tilde{\chi}_1}{d\theta^*}$ is determined for sufficiently small σ_1 because the two terms are of different orders of magnitude. The term $\frac{\partial \chi_1}{\partial \theta^*} = 1 + \frac{\sigma_1^2}{\tau^2}$ is of order σ_1^0. The derivative $\frac{\partial \chi_1}{\partial V_2}$ is of order $\sigma_1 \tau$, and $\frac{dv}{d\theta^*}$ is of order $\frac{1}{\tau}$ because $v(\theta^*)$ increases from 0 to $1-c$ within the increase of θ^* of order τ. So the term $\frac{\partial \chi_1}{\partial V_2}\frac{dv}{d\theta^*}$ is of order σ_1 and thus it is negligible compared to the first term $\frac{\partial \chi_1}{\partial \theta^*}$ for sufficiently small σ_1. We conclude that $\tilde{\chi}_1(\theta^*)$ increases with θ^* for sufficiently small σ_1.

Condition (*Search"*) specifies that $n_2 = \eta(x_1^*)$ is a decreasing function of x_1^*, so $n_2 = \tilde{\eta}(\theta^*) \equiv \eta(\tilde{\chi}_1(\theta^*))$ increases in θ^*. We can now substitute $V_2 = v(\theta^*)$, $n_2 = \tilde{\eta}(\theta^*)$ into (10) and get the equation with one unknown: $\lambda(\theta^*) \equiv \Lambda(v(\theta^*), \tilde{\eta}(\theta^*), \theta^*) = 0$. Given the monotonicity of $v(\theta^*)$ and $\tilde{\eta}(\theta^*)$ it is easy to check that $\lambda(\theta^*)$ decreases in θ^*. Moreover it is asymptotically linear in θ^* and continuous, therefore the equation $\lambda(\theta^*) = 0$ has a unique solution.

We have found a symmetric equilibrium in threshold strategies and have shown that there is only one of this kind for sufficiently small $\boldsymbol{\sigma}$. Next, we show that, for sufficiently small $\boldsymbol{\sigma}$, no other equilibrium exists: each equilibrium generates values V_2, n_2 and a success set S of all values θ_j for which a project j succeeds. Note that V_2, n_2 and S are known by players in equilibrium.

Let us consider a project j and a random variable $P^i_{S,t} = Prob(\Theta_j \in S|X^i_t)$ that denotes the posterior probability of the project's success after player i observes signal X^i_t. Let $k(V_2, n_2, S, \theta_j)$ be the measure of investors for given V_2, n_2, S and for the state of the project (unknown to players) being θ_j:

$$k(V_2, n_2, S, \theta_j) = Prob(P^i_{S,1} > c + \delta V_2|\theta_j) + n_2 Prob(P^i_{S,2} > c|\theta_j).$$

Note that $k(.)$ increases in S; precisely $S \supseteq S' \Rightarrow k(V_2, n_2, S, \theta_j) \geq k(V_2, n_2, S', \theta_j)$.

Let $m(V_2, n_2, \theta', \theta_j) \equiv k(V_2, n_2, (-\infty, \theta'), \theta_j) - \theta_j$ be the measure of investors net of θ_j in a special case when the success set is an interval, $S = (-\infty, \theta')$. Note that $m(V_2, n_2, \theta^*, \theta^*) \equiv \Lambda(V_2, n_2, \theta^*)$ as $\Lambda(V_2, n_2, \theta^*)$ was formed from condition (*crit.st.”*) and hence it coincides with the definition of $m(V_2, n_2, \theta^*, \theta^*)$.

Next, we assume that V_2, n_2 attain some particular values in equilibrium. We will find that there is a unique success set S compatible with this assumption: surely $S \supseteq (-\infty, 0)$ as the measure of investment $l_j \geq 0$. Moreover $\Lambda(V_2, n_2, 0) > 0$, hence $m(V_2, n_2, 0, 0) > 0$ and because the function m is continuous, there exists $\epsilon > 0$ such that $m(V_2, n_2, 0, \epsilon) > 0$. Value $m(V_2, n_2, 0, \epsilon)$ is a lower bound for the measure of equilibrium investment into a project with $\theta_j = \epsilon$ because the true success set contains $(-\infty, 0)$. Thus a project with $\theta_j = \epsilon$ surely succeeds. We conclude that a project surely succeeds for all $\theta \leq \epsilon$ because $m(V_2, n_2, \theta', \theta)$ decreases in θ. Hence $S \supseteq (-\infty, \epsilon)$. We can iterate this argument in the same manner and expand the interval of sure success further into the region of higher θ_j, up to the minimal θ' for which $m(V_2, n_2, \theta', \theta') = 0$, which is the minimal θ' solving $\Lambda(V_2, n_2, \theta') = 0$.

Symmetric arguments apply from above. The project never succeeds for $\theta > 2$ because 2 is the upper bound of observers of each project. Again, we can expand the interval of infeasible success to (θ'', ∞), where θ'' is the maximal solution of $\Lambda(V_2, n_2, \theta'') = 0$.

$\Lambda(V_2, n_2, \theta)$ decreases in θ for any V_2, n_2 so equation $\Lambda(V_2, n_2, \theta) = 0$ has a unique solution, and therefore $\theta' = \theta''$. Hence any pair V_2, n_2 imply a unique critical state $\theta^*_{V_2,n_2}$ that satisfies equation (10). On the other hand, the critical state θ^* uniquely determines equilibrium values V_2, n_2 as functions $v(\theta^*)$ and $\tilde{\eta}(\theta^*)$. Therefore equilibrium values V_2, n_2 and θ^* must coincide with values of the

	$y < (1-c)(1-\delta)$	$(1-c)(1-\delta) < y < (1-c)2$	$(1-c)2 < y$
θ^*	$(1-c)(1-\delta)$	y	$(1-c)2$
$\Phi(\theta^*)$	1	$\frac{2-2c-y}{(1+\delta)(1-c)}$	0

Table 2: Closed form solution of the mobile game in the ordered limit $\tau \to 0$, $\boldsymbol{\sigma} \to \mathbf{0}$.

unique symmetric equilibrium in threshold strategies and thus no other equilibrium exists.

2. Equations (10), (*Value"*) and (*Search"*) converge to equations (*Crit.St.*), (*Value*) and (*Search*) as $\boldsymbol{\sigma} \to \mathbf{0}$, hence their solution $\theta^*(\boldsymbol{\sigma})$, $V_2(\boldsymbol{\sigma})$, and $n_2(\boldsymbol{\sigma})$ converges to the solution of the latter equation system. □ (theorem 1)

A.2 Limit $\tau \to 0$

We find a closed form solution for the mobile game in the ordered limit $\tau \to 0$, $\boldsymbol{\sigma} \to \mathbf{0}$, where τ and $\boldsymbol{\sigma}$ approach 0 in such a way, that the private signals are much more precise than the prior distribution, $\frac{\boldsymbol{\sigma}}{\tau} \to \mathbf{0}$.

The equilibrium is described by equation (*Modif.Crit.*) which we reproduce here for convenience:

$$(1-c)[2-(1+\delta)\Phi(\theta^*)] = \theta^*.$$

We solve (*Modif.Crit.*) by guessing and verifying:

- $\frac{\theta^*-y}{\tau} \ll 0 \Rightarrow \Phi(\theta^*) \to 0 \Rightarrow \theta^* \to (1-c)2 < y$,
- $\frac{\theta^*-y}{\tau} \gg 0 \Rightarrow \Phi(\theta^*) \to 1 \Rightarrow \theta^* \to (1-c)[1-\delta] > y$,
- $\theta^* \approx y \Rightarrow (1-c)[2-(1+\delta)\Phi(\theta^*)] = y \Rightarrow \Phi(\theta^*) = \frac{2-2c-y}{(1+\delta)(1-c)}$.

Table 2 summarizes the solution of equation (*Modif.Crit.*) in the limit $\tau \to 0$.

Next, we substitute $\Phi(\theta^*)$ into the welfare equation (2) and get a closed form expression for V. Welfare in the extreme regions is 0 respectively $1-c$. Welfare for the medium value of y is

$$V = \frac{2\,c^2\,(1+\delta^2) + c\,[-4+\delta^2\,(-4+y)+y-2\,\delta\,y] + (2-y)\,(1+\delta^2+\delta\,y)}{(1-c)\,(1+\delta)^2}.$$

We can compute $\frac{dV}{d\delta}$ explicitly:

$$\frac{dV}{d\delta} = -\frac{(1-\delta)\,(2-2\,c-y)^2}{(1-c)\,(1+\delta)^3},$$

which is negative for all $\delta \in (0,1)$, so an increase of mobility unambiguously decreases welfare in the limit $\tau \to 0$. $\square$ (lemma 3)

A.3 Proof of Corollaries 4 and 5

Proof of corollary 4: we eliminate unknown V by expressing it from (*Value.g.*) and substituting it into (*Crit.st.g.*). For the sake of simplifying tedious expressions we omit arguments of functions, but let us keep in mind that $n = \frac{1}{1-\Phi(\theta^*)}$ etc. We get

$$\frac{1}{n}\int_0^n R dl = \frac{\delta}{1-\delta\Phi}\int_{\theta^*}^{+\infty} R d\Phi(\theta). \tag{11}$$

We denote the left and right hand side of (11) by $LHS(\theta^*)$ and $RHS(\theta^*)$ and show that they satisfy the single-crossing property: A simple manipulation gives derivatives:

$$LHS'(\theta^*) = \left(-\int_0^n R dl + nR\right)\phi + p', \tag{12}$$

$$RHS'(\theta^*) = \left(-R + nR_l + \underbrace{\frac{\delta}{1-\delta\Phi}\int_{\theta^*}^{+\infty} R d\Phi(\theta)}_{(*)}\right)\frac{\delta\phi}{1-\delta\Phi} \tag{13}$$

We use the equality in (11) and replace the term (*) in (13) by $\frac{1}{n}\int_0^n R dl$. Next, combining (12) and (13) we find the difference of derivatives:

$$LHS'(\theta^*) - RHS'(\theta^*) = p' + \left\{\underbrace{R\left(n(1-\delta\Phi)+\delta\right) - \int_0^n R dl\left(1-\delta\Phi+\frac{\delta}{n}\right) - \delta n R_l}_{(**)}\right\}\frac{\phi}{1-\delta\Phi}.$$

We now show that $LHS'(\theta^*) - RHS'(\theta^*)$ is positive. Derivative p' is non-negative by assumption MS2, fraction $\frac{\phi}{1-\delta\Phi}$ is positive, and the term $(**)$ is positive because

$$(**) = \left(1-\delta\Phi+\frac{\delta}{n}\right)\underbrace{\left(Rn - \frac{R_l n^2}{2} - \int_0^n R dl\right)}_{(I)} + \underbrace{(1-\delta)\frac{n^2}{2}}_{(II)},$$

where part (I) is positive as $R(\theta, l)$ is assumed to be concave with respect to l; part (II) is a residuum of the examined expression and it is positive. Therefore

$LHS(\theta^*)$ crosses $RHS(\theta^*)$ always from below, and the single-crossing property implies uniqueness of the solution. □ (corollary 4)

Proof of corollary 5: equations (*Crit.st.g.*), (*Value.g.*) and (*Search.g.*) simplify into

$$\left(1 - \frac{a(\theta^*)}{n}\right)\zeta(\theta^*) - c = \delta V \tag{14}$$

$$V = \int_{\theta^*}^{+\infty} (\zeta(\theta^*) - c)d\Phi(\theta) + \Phi(\theta^*)\delta V \tag{15}$$

$$n = \frac{1}{1 - \Phi(\theta^*)} \tag{16}$$

After eliminating V and n we get

$$[1 - a(1 - \Phi)]\zeta - c = \frac{\delta}{1 - \delta\Phi}\int_{\theta^*}^{+\infty} (\zeta - c)d\Phi(\theta). \tag{17}$$

We denote the left and right hand side of (17) by $LHS(\theta^*)$ and $RHS(\theta^*)$ and show that they satisfy the single-crossing property: The derivatives are

$$LHS'(\theta^*) = -a'(1 - \Phi)\zeta + a\phi\zeta + (1 - a(1 - \Phi))\zeta', \tag{18}$$

and $LHS'(\theta^*)$ is positive in an equilibrium, because $a' < 0$, $\zeta' > 0$ and in equilibrium $0 < \zeta(\theta^*)$, $0 < a(\theta^*) < n(= \frac{1}{1-\Phi})$.

$$RHS'(\theta^*) = \frac{-\delta}{1 - \delta\Phi}(\zeta - c)\phi + \underbrace{\frac{\delta^2\phi}{(1 - \delta\Phi)^2}\int_{\theta^*}^{+\infty} (\zeta - c)d\Phi(\theta)}_{(*)} \tag{19}$$

We use the equality in (17) and replace the term (*) in (19) by $\frac{\delta\phi}{1-\delta\Phi}[(1 - a(1 - \Phi))\zeta - c]$. A simple manipulation leads to

$$RHS'(\theta^*) = -\frac{\delta a\ (1 - \Phi)\zeta\phi}{1 - \delta\Phi}, \tag{20}$$

and hence $RHS'(\theta^*)$ is negative in equilibrium. Therefore the single-crossing property is satisfied and thus the solution is unique. □ (corollary 5)

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B Summary of the Main Notation

Exogenous parameters:	*Endogenous variables:*
c Sunk cost of investment.	V_2 Expected payoff in round 2.
δ Discount factor.	n_2 Measure of players observing each project in round 2.
σ_t^2 Variance of private signal at t.	x_t^* Threshold signal at round t.
τ^2 Variance of prior distribution.	θ^* Critical state.
y Average state of fundamentals.	l_j Cumulative investment into project j.
θ_j Fundamentals of project j.	
x^i Private signal of player i.	

Games analyzed:

Static game: A benchmark simple global game.

Mobile game: Same as the static game but players are allowed to search once for another project.

Learning game: Same as the mobile game but players in round 2 receive a signal about the amount of early investment from round 1.

Infinite game: Same as the mobile game but players are allowed to search infinitely many times.

General payoff: Same as the infinite game but a general payoff function satisfying strategic complementarity is assumed.

Directed Search game: Same as the infinite game but the search is directed, and hence better projects are observed more often.

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P.O.BOX 882
Politických vězňů 7
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