Privatization Policy and Dynamics of the Retail Market in a Transitional Economy

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Abstract

This paper develops a theoretical framework in which the impact of different privatization strategies on the dynamics of the retail market in a transitional economy can be analyzed. The analysis shows that rational behavior of the monopolistic manufacturer under uncertainty of demand naturally implies organization of the retail market. The equilibrium characteristics of unrestricted (competitive) and restricted (monopolistic) retail markets are analyzed, and the theoretical results derived are applied to study the effects of various sequences of privatization on the dynamics of the retail market in a transitional economy. Our findings indicate that privatizing the manufacturer first is always at least as good as privatizing the retailers first because (unlike the strategy of privatizing the retail sector first) it neither decreases the number of firms in the retail market nor the profitability of the state owned firms in the transition period (in the unrestricted retail market both the number of retail firms and the profitability of state owned firms increase).

Keywords: Privatization, imperfect information, demand uncertainty, risk aversion, retail market.

JEL Classification: P00, D73.

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1. Introduction

The problem of privatization and the relative advantages of private ownership and central planning has been intensively discussed in the economic literature (see, e.g., Hayek, 1945; and Tirole, 1991). Current economic opinion favors privatization (see Kikeri et al., 1992; and Glaeser and Scheinkman, 1996). However, the next question: how to privatize industries (what is the optimal speed of the privatization process, or what should be privatized first) is still a subject of economic discussion. What economists do agree on is that privatization is a slow process and only a few enterprises can be privatized at a given time (Aghion and Tirole, 1993; Katz and Owen, 1993; Schmidt and Schnitzer, 1993; and Glaeser and Scheinkman, 1996).

Taking into account that all industries cannot be privatized instantaneously, we focus on the problem of sequencing privatization (i.e., we would like to clarify what should be privatized first: the upstream supplier or the retail sector?). In particular, we are interested in the impact of different sequences of privatization on the profitability of the supplier and retail firms and on the size of the retail market in the period of transition to market economy.

We begin by setting out four “stylized facts” that are intended to characterize the situation in transitional economies:

1. The firms inherited from the pretransition period are highly monopolistic. Bennet and Dixon (1995) emphasize that in the pretransition period, industrial production in Eastern European and CIS countries was highly concentrated (partly for economies of scale and partly to simplify the central planner resource allocation problem). They point out that:

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1 See, for example, Aghion and Tirole 1993; Katz and Owen, 1993; and Murphy et al. 1992.
2 See, e.g., Husain and Sahay (1992) for the justification of the importance of sequencing of privatization.
3 “Stylized facts” 1 and 2 have been adopted from Bennet and Dixon (1995).
In the USSR, for example, the IMF et al. (1991) note that, for a breakdown of 344 industries product groups, the largest producer made 50-75% of the output in 83 cases, and more than 75% in 126 cases. (the literature on the topic stresses that similar relationships can also be found in other Eastern European economies, see, e.g., Landesmann and Szekely, 1991). Moreover, the break-up of large firms in a short period is difficult and not always technically feasible (see Bennet and Dixon, 1995; and Kroll, 1991). Consequently, in a transitional economy highly monopolistic structures of industries prevail.

2. The firms in a transitional economy (also state-owned firms) are using commercial criteria with control of their production and pricing decisions. The characteristic feature of the transitional economy is the lack of a central planner. Schmidt and Schnitzer (1993) argue that before privatization firms are usually run as joint stock companies, owned by the state and controlled by a board of directors (selected by a government agency) which supervises the management and induces profit maximizing behavior. Therefore, in a transitional economy even state owned firms take into account market criteria; in particular, they attempt to exploit their monopolistic positions to earn high profits (see, e.g., Hare and Revesz, 1992; or Bennet and Dixon, 1995).

3. State-owned firms in a transitional economy show a certain degree of risk aversion. In the centrally planned economy, where all decisions were made by a central planner, an individual firm did not care about the risk associated with variable market conditions. In a transitional economy, where firms are obliged to use commercial criteria, the manager's income is usually linked to the firm’s performance by a system of bonuses and/or stock

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options (also, if the firm is doing badly the manager can be replaced). Consequently, even if the owner of the firm (i.e., the state) can be considered risk neutral, the managers of state-owned firms are not less concerned about the risk associated with the variability of market conditions, and surely they don’t like the risk (i.e., are risk averse).

4. Privately-owned firms are more risk averse than state-owned firms in a transitional economy. Economists agree that one of the primary advantages of decentralized, private ownership is the ability of the firm to learn and process information, and, consequently, the main difference between state-owned and privatized firms is in their incentives and ability to acquire and use information in order to decrease uncertainty about the environment (see, for example, Hayek 1945; and Glaeser and Scheinkman, 1995). Examination of the behaviour of firms under imperfect information (i.e., under demand uncertainty) indicates that only risk averse firms devote resources to market analysis and information processing. Moreover, it shows that more risk averse firms acquire and process more information (see Cukrowski, 1996). Thus, private firms which acquire and use more information can be considered as more risk averse than state-owned enterprises.

Furthermore, we assume that market demand in a transitional economy is not deterministic, but stochastic (i.e., the relationship between quantities demanded and market prices randomly varies from period to period), and that the firm’s beliefs about the sales price are summarized in a subjective probability distribution (the firm cannot affect its characteristics; i.e., the firm is not able to predict changes in demand or decrease the range of possible variations). For the sake of simplicity we focus on the behavior of the risk averse monopolistic producer operating in the single commodity market.

In Section 2 the optimal behaviour of the monopolistic firm operating in a stochastic

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environment is analyzed. Section 3 shows that optimal behavior of the monopolistic manufacturer under uncertainty of demand naturally implies organization of the retail market. Sections 4 and 5 examine how the retail market can be organized and what are the equilibrium characteristics of unrestricted (competitive) and restricted (monopolistic) retail markets. In Section 6, taking into account that changes in the ownership structure in the privatization period increase risk aversion of the manufacturer and/or the retail firms, we employ the theoretical results derived in Sections 4 and 5 to analyze the impact of various privatization strategies on the dynamics of the retail market in a transitional economy.

We realize that the information gain from changes in the attitude towards risk in the economy may be less important in some cases than other potential gains from privatization (such as, for example, improved financial incentives or clearly defined property rights). The concern of the paper, however, is to emphasize the existence of a path of causation — from privatization, to changes in retail markets via changes in attitudes towards risk. In particular, we aim to show that the change in the ability to process and use information (and by association, the change in the attitude towards risk) is the main factor affecting the evolution of the retail market in the period of transition to the market economy.

2. The firm facing uncertain demand

To set up a formal model which can explain dynamics of the retail sector in the period of transition to the market economy, consider a single commodity market in which stochastic demand comes from a large number of identical sources, N (one can think of these sources as shops, or even consumers). Assume that demand in each individual source i (i=1,2,...,N) can be described by the following implicit demand relationship:

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See Radner and Van Zandt (1992) for a similar model of market demand.
\[ q_i = q_i(p) + \eta_i, \quad (2.1) \]

where

\[ q_i \quad (q_i \geq 0) \text{ is a quantity demanded at price } p \quad (p \geq 0), \]

\( \eta_i \) denotes independent, identically distributed random variables specified by probability density functions (for the sake of simplicity assume that the probability distribution of random variables \( \eta_i \) is normal with the mean value equal to zero and the variance \( \sigma_i^2 \)).

The restrictions placed on (2.1) are that, for any particular value of \( \eta_i \), the relationship between \( p \) and \( q_i \) is downward sloping and that larger values of \( \eta_i \) are associated with greater demand (Leland, 1972; Lim, 1980). Thus, demand in each source can be expressed as either

\[ q_i = q_i(p, \eta_i), \quad \partial q_i(p, \eta_i)/\partial p < 0 \quad \text{and} \quad \partial q_i(p, \eta_i)/\partial \eta_i > 0, \quad (2.2) \]

or

\[ p = p(q_i, \eta_i), \quad \partial p(q_i, \eta_i)/\partial q_i < 0 \quad \text{and} \quad \partial p(q_i, \eta_i)/\partial \eta_i > 0. \quad (2.3) \]

Define \( \eta_i^o \equiv E[\eta_i] \), where \( E \) is an expectation operator. Then for any values of \( p \) and \( q_i \) and sufficiently concentrated distributions of \( \eta_i \) (see Samuelson, 1970, or Lim, 1980), we can approximate \( q_i(p, \eta_i) \) and \( p(q_i, \eta_i) \) around \( \eta_i^o \) as

\[ q_i(p, \eta_i) = q_i(p, \eta_i^o) + (\eta_i - \eta_i^o) \frac{\partial q_i(p, \eta_i^o)}{\partial \eta_i}, \quad (2.4) \]

\[ p(q_i, \eta_i) = p(q_i, \eta_i^o) + (\eta_i - \eta_i^o) \frac{\partial p(q_i, \eta_i^o)}{\partial \eta_i}, \quad (2.5) \]

where \( \partial q_i(p, \eta_i^o)/\partial \eta_i \) and \( \partial p(q_i, \eta_i^o)/\partial \eta_i \) denote partial derivatives of the demand and inverse demand functions with respect to random variables \( \eta_i \) evaluated at their expected values, \( \eta_i^o \) (\( i=1,2,\ldots,N \)).

For any given price \( p=P \quad (P \geq 0) \), the real value of the total demand faced by the firm (a sum of individual demands coming from all sources), \( Q(P, \eta_1, \eta_2, \ldots, \eta_N) \), can be represented as
\[ Q(P, \eta_1, \eta_2, \ldots, \eta_N) = \sum_{i=1}^{N} \left[ q_i(P, \eta_i^o) + (\eta_i - \eta_i^o) \frac{\partial q_i(P, \eta_i^o)}{\partial \eta_i} \right] - \sum_{i=1}^{N} q_i(P, \eta_i^o) \frac{\partial q_i(P, \eta_i^o)}{\partial \eta_i} \sum_{i=1}^{N} (\eta_i - \eta_i^o). \] (2.6)

Taking expectation we obtain

\[ \mathbb{E}[Q(P, \eta_1, \eta_2, \ldots, \eta_N)] = \sum_{i=1}^{N} q_i(P, \eta_i^o) - Q(P, \eta^o), \] (2.7)

where

\[ \eta^o = \sum_{i=1}^{N} \eta_i^o. \] (2.8)

Note that for any \( i (i=1,2,\ldots,N) \), and \( p=P \), we have

\[ \frac{\partial q_i(p, \eta_i^o)}{\partial \eta_i} = Q_2(P, \eta^o), \quad \frac{\partial q_i(p, \eta_i^o)}{\partial \eta_i} = P_2(Q, \eta^o), \] (2.9, 2.10)

and

\[ \frac{\partial^2 q_i(p, \eta_i^o)}{\partial p \partial \eta_i} = Q_{1,2}(P, \eta^o), \quad \frac{\partial^2 q_i(p, \eta_i^o)}{\partial q_i \partial \eta_i} = P_{1,2}(Q, \eta^o), \] (2.11, 2.12)

where \( Q_2(P, \eta^o), P_2(Q, \eta^o) \) denote partial derivatives with respect to the second argument\(^7\) evaluated in \( \eta^o \), and \( Q_{1,2}(P, \eta^o), P_{1,2}(Q, \eta^o) \) are cross-partial derivatives evaluated in \( \eta^o \). Thus, for a given price \( P \), the random deviation from the expected total quantity demanded can be determined as

\[ Q_2(P, \eta^o) \eta, \] (2.13)

where

\(^7\) Henceforth, numerical subscripts will denote partial derivatives unless otherwise specified.
\[ \eta = \sum_{i=1}^{N} (\eta_i - \eta_i^\circ), \quad (2.14) \]

Since the distribution of random variables \( \eta_i \) (i=1,2,...,N) is normal, the random deviation from the expected total quantity demanded is normally distributed with the mean value equal to zero and the variance equal to \( Q_2(P,\eta^\circ)\sigma^2 \).

Similarly, the total random deviation from price \( P \) corresponding to the expected total quantity demanded \( Q \), is

\[ P_2(Q,\eta^\circ)\eta, \quad (2.15) \]

and the distribution of this random deviation is normal with the mean value equal to zero and the variance \( P_2(Q,\eta^\circ)^2\sigma^2 \).

Since the probability distribution of the random deviation is normal (i.e., symmetric) we can use an approximation and simplify the analysis by saying that with probability 1/2, the firm faces the expected inverse demand curve described as

\[ P(Q,\eta^\circ) = P(Q,\eta^\circ) + \int_{-\infty}^{0} \frac{\eta}{\sqrt[2]{2\pi N \sigma^2}} e^{-\frac{\eta^2}{2P_2(Q,\eta^\circ)^2\sigma^2}} d\eta = \]

\[ - P(Q,\eta^\circ) - P_2(Q,\eta^\circ) \sqrt{\frac{N}{2\pi}} \sigma. \quad (2.16) \]

and with probability 1/2 the expected inverse demand curve specified as
Consequently, with probability 1/2 the firm earns low expected value of profit

\[
\Pi(Q, \eta^o) = QP(Q, \eta^o) - C(Q) - F ,
\]

(2.18)

where \(C(Q)\) denotes the deterministic variable cost, \(F\) stands for a fixed cost; and with probability 1/2 the firm earns high expected value of profit

\[
\Pi(Q, \eta^o) = Q\bar{P}(Q, \eta^o) - C(Q) - F .
\]

(2.19)

Note that for any given \(Q\), the expected value of profit \(E[\Pi(Q, \eta)]\) equals the profit when \(\eta\) equals its expected value \(\eta^o\), i.e., \(E[\Pi(Q, \eta)] = \Pi(Q, \eta^o)\).

Assuming that the decision on the volume of output to be produced must be made prior to the sales date, at which time the market price becomes known, and that the firm seeks to maximize the expected utility from profit,\(^8\) the firm's objective function can be approximated as

\[
\bar{P}(Q, \eta^o) - P(Q, \eta^o) + \int_0^{\infty} \frac{\eta}{\sqrt{2\pi N\sigma^2}} e^{-\frac{\eta^2}{2P_2(Q, \eta^o)^2 N\sigma^2}} d\eta = -
\]

\[
= P(Q, \eta^o) + P_2(Q, \eta^o) \sqrt{\frac{N}{2\pi}} \sigma .
\]

(2.17)

8 We implicitly assume that managers of the firm maximize the expected utility of their shareholders, and the decisions in the firm are made by a group of decision-makers with sufficiently similar preferences to guarantee the existence of a group-preference function (see, for example, Sandmo 1971, for a detailed discussion). This ensures that the behavior of the firm under demand randomness obeys the axioms of the Neuman-Morgenstern utility theory.
\[
\begin{align*}
\max_Q E[U(\Pi)] &= \frac{1}{2} U[\Pi(Q, \eta^\circ)] + \frac{1}{2} U[\Pi(Q, \eta^\circ)] - (2.20) \\
\frac{1}{2} U[ \Pi(Q, \eta^\circ) - QP_2(Q, \eta^\circ) \sqrt{\frac{N}{2\pi}}] + \frac{1}{2} U[ \Pi(Q, \eta^\circ) + QP_2(Q, \eta^\circ) \sqrt{\frac{N}{2\pi}}] ,
\end{align*}
\]

where

\[\Pi(Q, \eta^\circ) = Q P(Q, \eta^\circ) - C(Q) - F \quad (2.21)\]
is the profit function of the firm if the inverse demand curve is \(P(Q, \eta^\circ)\), i.e., when there is no uncertainty of demand (for any fixed value of \(\eta^\circ\), \(\Pi(Q, \eta^\circ)\) is assumed to be strictly concave in \(Q\)). \(U(\Pi)\) is a function which reflects the attitude of the firm towards risk.\(^9\)

Given that firms are managed according to the wishes of their owners who are usually asset holders, we assume in the analysis which follows that the firm exhibits risk averse behavior (see Leland, 1972).\(^10\)

The first order condition to the optimization problem above can be represented as

\[
\frac{1}{2} U'(\Pi) \{ (MR-MC) - \frac{\partial [QP_2(Q, \eta^\circ)]}{\partial Q} \sqrt{\frac{N}{2\pi}} \} + \\
\frac{1}{2} U'(\Pi) \{ (MR-MC) - \frac{\partial [QP_2(Q, \eta^\circ)]}{\partial Q} \sqrt{\frac{N}{2\pi}} \} = 0 , \quad (2.22)
\]

where

\[MR - MC = P(Q, \eta^\circ) + QP_1(Q, \eta^\circ) - dC(Q)/dQ \quad (2.23)\]

\(^9\) Strictly concave utility function (\(dU(\Pi)/d\Pi>0\) and \(d^2U(\Pi)/d\Pi^2<0\)) corresponds to risk-averse behavior, linear utility (\(dU(\Pi)/d\Pi>0\) and \(d^2U(\Pi)/d\Pi^2=0\)) describes risk-neutrality, and the strictly convex utility function (\(dU(\Pi)/d\Pi>0\) and \(d^2U(\Pi)/d\Pi^2>0\)) corresponds to risk preference.

\(^10\) Sandmo (1971) and Leland (1972), for example, assumed risk aversion; Dreze and Gabsewicz (1967) and Smith (1969) assumed risk neutrality, but perhaps for the sake of simplicity as no justification was given (see Leland, 1972, for detailed discussion).
is the difference between the value of marginal revenue and marginal cost if the demand is known with certainty.

The second derivative of the objective function with respect to Q can be represented as

$$\frac{1}{2} \left[ U'(\Pi)\Pi_{1,1} + U''(\Pi)\Pi^2_1 \right] + \frac{1}{2} \left[ U'(\bar{\Pi})\bar{\Pi}_{1,1} + U''(\bar{\Pi})\bar{\Pi}^2_1 \right].$$

(2.24)

Rearranging we get

$$\frac{1}{2} \left[ U'(\Pi)\Pi_{1,1} + U'(\bar{\Pi})\Pi_{1,1} \right] + \frac{1}{2} \left[ U''(\Pi)\Pi^2_1 - U''(\bar{\Pi})\bar{\Pi}^2_1 \right].$$

(2.25)

Taking into account that the firm is risk averse (i.e., $U'(\Pi)>0$ and $U''(\Pi)<0$), the second term in the expression above is always negative. If

$$\Pi < \bar{\Pi},$$

then the first term is always smaller than

$$\frac{1}{2} U'(\Pi) \left[ \Pi_{1,1} + \Pi_{1,1} \right],$$

(2.26)

if the expression in the brackets is negative. On the other hand, if

$$\Pi \geq \bar{\Pi},$$

then the first term in (2.25) is not greater than
if the expression in the brackets is negative. Taking into account that 

\[ \Pi_{1,1} + \bar{\Pi}_{1,1} < 0 \] by the assumption above, the expression (2.25) is always negative. This implies that the second order condition holds for the risk averse firm.

If we represent the first order condition as 

\[ MR-MC = \sigma \left[ \frac{N}{2\pi} \right] \left( \frac{\partial[QP_2(Q,\eta^o)]}{\partial Q} \right) \frac{1-U'(\Pi)/U'(\Pi)}{1+U'(\Pi)/U'(\Pi)} , \]  

we can conclude that the optimal output of the firm facing uncertain demand is the same as the output without uncertainty if MR-MC=0, smaller if MR-MC>0, and greater otherwise. Thus, the risk-averse firm produces less than it would under certainty if the right-hand side of the condition (2.29) is positive, i.e., if the partial derivative of the marginal revenue with respect to \( \eta \) evaluated at \( \eta^o \) (a marginal risk premium):\(^{11}\)

is greater than zero, the same as the output without uncertainty if this partial derivative is

\[ \left[ 1-U'(\Pi)/U'(\Pi) + 1+U'(\Pi)/U'(\Pi) \right] \]

is positive.

\(^{11}\) Note that for risk averse firm
equal to zero, and more otherwise.

In general, all outcomes are possible and the sign of the marginal risk premium should be analyzed for each particular form of stochastic demand. Note, however, that if the stochastic demand function satisfies the “principle of increasing uncertainty”\(^\text{12}\) (Leland, 1972), then the output of the firm is always smaller than it would be without uncertainty (a necessary and sufficient condition for this principle to hold is the same sign of the marginal revenue and the marginal risk premium).\(^\text{13}\) On the other hand, if the stochastic demand function does not satisfy the principle of increasing uncertainty, then the partial derivative of the marginal revenue with respect to \(\eta\) evaluated at \(\eta^o\) is equal to zero (or is negative) and the risk-averse firm produces the same as (respectively, more than) it would produce without uncertainty of demand.\(^\text{14}\)

The optimal behavior of the monopolistic firm is illustrated by the example below.

Example 2.1.

Consider a monopolistic firm producing a single commodity with constant marginal cost \((c>0)\). Assume that the profit function without uncertainty of demand is strictly concave, and that the firm under study is risk averse (i.e., the firm’s utility function is increasing and

\[
MR_2(Q, \eta) = \frac{\delta [QP_2(Q, \eta)]}{\partial Q}, \quad (2.30)
\]

\(^{12}\) The *principle of increasing uncertainty* states that the riskiness (or dispersion) of total revenue increases if total expected revenue increases (see Leland, 1972, for a detailed discussion).

\(^{13}\) See Leland (1972), Appendix.

\(^{14}\) Note that if the firm is risk neutral,

\[
[1 - U'(\Pi)/U(\Pi)]/[1 - U'(\bar{\Pi})/U(\bar{\Pi})]
\]

in expression (2.29) equals zero, and, consequently, a risk neutral firm facing uncertain demand always produces the same quantity as it would produce without uncertainty.
strictly concave in $\Pi$). Suppose that total demand for the commodity produced comes from $N$ identical sources of stochastic demand. Assume that demand curve in each individual source $i$ ($i=1,2,\ldots,N$) is given by
\[ f(q_i,p,\eta_i) = A - p - B q_i + \eta_i = 0, \]
where $\eta_i$ are independent, normally distributed random variables with the mean value $(\eta_i^0)$ equal to zero and the variance $\sigma_i^2=\sigma^2>0$; i.e., demand is linear with additive normally distributed random term (note that the principle of increasing uncertainty is satisfied). The total inverse demand curve can be represented as
\[ P(q_1,q_2,\ldots,q_N,q_1,\eta_1,\eta_2,\ldots,\eta_N) = A - B \sum_{i=1}^{N} q_i + \sum_{i=1}^{N} \eta_i, \]
or
\[ P(Q,\eta_1,\eta_2,\ldots,\eta_N) = A - B Q + \sum_{i=1}^{N} \eta_i, \]
where $Q$ denotes the total quantity demanded.

Probability distribution of the total random deviation from price $P$ is normal with the mean value equal to zero and the variance $N\sigma^2$.$^{15}$

Taking into account that, for the monopolistic firm facing deterministic linear demand (when market demand equals to the expected market demand), marginal revenue (MR) equals $A-2BQ$, and rearranging the first order condition of the firm’s maximization problem, we can determine (numerically) the optimal quantity produced from the following expression

\[ 15 \text{ Note that } P_2(Q,\eta^0)=1, \text{ where } \eta^0=\eta_1+\eta_2+\ldots+\eta_N. \]
where \( \frac{A-c}{2B} \) is the optimal output of the monopolistic firm if there is no uncertainty of demand, i.e., when market demand equals to the expected market demand.

The relationship between optimal quantities supplied and monopolistic prices with and without uncertainty of demand is presented in Fig. 2.1.

To summarize: the optimal output of the risk averse quantity-setting monopolistic firm operating in a single commodity market with uncertain demand deviates from the optimal

\[
Q = \frac{A-c}{2B} - \frac{\sigma}{2B} \sqrt{\frac{N}{2\pi}} \frac{1-U'(\Pi)/U'(\Pi)}{1+U'(\Pi)/U'(\Pi)},
\]

where \( \frac{A-c}{2B} \) is the optimal output of the monopolistic firm if there is no uncertainty of demand, i.e., when market demand equals to the expected market demand.
Fig. 2.1. The relationships between optimal quantities supplied and expected prices with and without uncertainty of demand, in the monopolistic market analyzed in example 2.1; $Q^*_{\sigma>0}$ and $P(Q^*_{\sigma>0} \eta^0)$ correspond respectively to the monopolistic output and expected monopolistic price with uncertainty of demand; $Q^*_{\sigma=0} = (A-c)/2B$ is the monopolistic output without uncertainty and $P(Q^*_{\sigma=0} \eta^0)$ is a corresponding price; rectangles (KLMN) and (KORS) correspond to monopolistic profit without demand uncertainty, and the expected profit of the monopolistic firm with uncertainty of demand, respectively.
output without demand uncertainty (in particular, it is always smaller if the principle of increasing uncertainty is satisfied). Consequently, the optimum expected value of profit of the risk averse firm is always smaller than it would be without uncertainty of demand. Moreover, the deviation of the optimal monopolistic output with uncertainty of demand from the monopolistic output without uncertainty of demand increases with the variance of each individual demand, the number of sources of demand, and the degree of risk aversion (see expression 2.29).

The most important result for the analysis which follows is that under demand uncertainty, the optimal output supplied to the market by the risk averse producer is not a random but deterministic variable, smaller (if the principle of increasing uncertainty is satisfied) than it would be under certainty.

3. Market policy of the monopolistic producer

Consider a monopolistic single commodity market in which total demand comes from N identical sources of uncertain demand (as presented in Section 2). To focus directly on the problem, assume that demand in each individual source i (i=1,2,...,N) satisfies the principle of increasing uncertainty and that transactions, transportation and storage costs are equal to zero. Moreover, as in the preceding section, assume that the behavior of the monopolistic manufacturer and retail firms under demand uncertainty obeys axioms of the Neuman-Morgenstern utility theory, and that all these firms are risk averse (U’(Π)>0 and U’’(Π)<0).

The manufacturer has two options: (1) to sell goods directly to final consumers, or (2) to sell the output produced to retail firms, which resell the goods to final demanders.

Assuming that retail firms can freely enter and exit the retail market, each individual risk averse, expected utility maximizing, retail firm will be willing to operate in the market
only if its expected utility from profit is at least equal to the utility of some benchmark activity \((b > 0)\). Since the expected utility from profit of each individual risk averse firm operating in the market is nonnegative (i.e., greater or equal to \(b\)), its expected value of profit is nonnegative as well. This implies that the retail market can be created only if the expected value of profit of the retail sector as a whole is positive, i.e., if the manufacturer sells goods to retail firms at lower prices than the expected price to final consumers for the same quantity of output. The following result shows that the risk averse producer facing uncertain demand always has incentives to offer goods to retail firms at a lower price than the expected price at which he could offer goods to final consumers.

**PROPOSITION 3.1.** *Rational behavior of the risk averse monopolistic firm facing uncertain demand implies that it is always willing to offer goods to retail firms at lower prices than to final demanders.*

**Proof of Proposition 3.1.** If the risk averse monopolistic manufacturer facing uncertain demand sells goods directly to final demanders, it earns random profit with the optimal expected value \(E[\Pi(Q_{\sigma^0}^*)]\), which is smaller than the maximum expected value of monopolistic profit. Risk aversion implies that the monopolist always prefers deterministic profit over random profit with the same or even slightly higher expected value. Note that if the output produced by the monopolistic manufacturer is delivered to final demanders through the retail sector, then demand faced by the manufacturer is not uncertain (the optimal quantity supplied to final consumers by the expected utility maximizing retail firm, and, consequently,

\[^{16}\text{See Applebaum and Katz (1986).}\]
\[^{17}\text{\(Q_{\sigma^0}^*\) denotes an optimal quantity supplied by the risk averse monopolistic firm under uncertainty.}\]
demanded from the manufacturer, is deterministic)\textsuperscript{18}. Thus, for any given optimum expected utility from random profit (E\{U[\Pi(Q_{\sigma>0})]\}), the price (P^0(Q)) at which the risk averse manufacturer would be willing to offer goods to retailers should satisfy the following condition

\[ E[U(P^0(Q) - C(Q) - F)] \geq E[U[\Pi(Q_{\sigma>0})]]. \tag{3.1} \]

For the risk averse firm, the utility level corresponding to deterministic profit, specified by the left hand side of the expression (3.1), is always greater than expected optimal utility from random profit E\{U[\Pi(Q_{\sigma>0})]\}. Then, it follows that the price (P^0(Q)), at which the risk averse manufacturer would be willing to offer goods to retailers, can be approximated from the following arbitrage condition

\[ P^0(Q) - C(Q) - F - E[\Pi(Q_{\sigma>0})]. \tag{3.2} \]

where Q is the output of the manufacturer, P^0(Q) denotes the price at which the manufacturer offers quantity Q to retail firms, C(Q) denotes the variable cost of the manufacturer, and F stands for its fixed cost. The condition above states that the deterministic profit of the monopolistic manufacturer dealing with retail firms (the left hand side) should be at least equal to the expected value of profit that the monopolist would earn if it sells goods directly to final demanders (the right hand side). Therefore, the monopolist is always better off if it sells quantity Q to retail firms at price

\textsuperscript{18} See Section 2.
per each unit of output than if the retail market is not created.

Note that the expected value of profit of the monopolistic firm (\(E[\Pi(Q)]\)) is continuous and strictly concave function of Q (i.e., \(d^2E[\Pi(Q)]/dQ^2<0\)), positive for \(Q \in (0,Q^*)\), where \(Q^*\) is the optimal competitive output without uncertainty (\(E[\Pi(Q)]\) achieves its maximum if the output produced equals to the optimal monopolistic output without uncertainty, \(Q_{\sigma=0}^*\)). Moreover, the maximum expected value of profit is greater than the optimal expected value of profit under uncertainty (i.e., \(E[\Pi(Q_{\sigma=0}^*)]>E[\Pi(Q_{\sigma>0}^*)]\)). Consequently, there exists an interval (say, \((Q_A,Q_B)\), where \(Q_A=Q_{\sigma>0}^*\) and \(Q_B=Q_{\sigma=0}^*\)), for which \(E[\Pi(Q)]>E[\Pi(Q_{\sigma>0}^*)]\). Thus, for any \(Q \in (Q_A,Q_B)\), the monopolistic supplier can offer goods to retail firms at price \(P^{0}(Q)\), which is lower than the expected price to final consumers (\(P(Q,\eta^0)\)) for the same quantity demanded, and earn deterministic profit equal to the expected value of random profit, which it could earn trading directly with final demanders (see Fig.3.2). Since the monopolistic supplier is risk averse, it will always choose this option.

Note that if the monopolistic manufacturer does not deal with final demanders but with intermediate firms, then its profit depends on both the vertical (i.e., the relationships between the monopolistic manufacturer and retail firms) and horizontal structure of the retail market (i.e., the relationships between retail firms operating in the market). The relationships between the monopolist and retailers can be settled in many different ways; thus, in the present research we focus only on the two extreme cases, i.e., on (1) the unrestricted (competitive) retail market, and on (2) the restricted (monopolistic) retail market.

In the first case, the manufacturer sells goods to a large group of competitive retailers, and it is either unable or not legally allowed to impose any vertical restraints (such as, for

\[
P^{0}(Q) = \frac{E[\Pi(Q_{\sigma>0}^*)]}{Q} + C(Q) + \frac{F}{Q},
\]

(3.3)
example, a franchise fee, exclusive territories, resale price maintenance, etc.). In the second case, the monopolistic supplier imposes vertical restraints in order to form the retail market and to extract profit from the retail firms (i.e., uses a franchise fee to extract profit from the retail sector and supplies goods to the limited number of monopolistic retailers where each individual retailer operates in its exclusive territory).

QED.
Fig. 3.2. The offer curve of the monopolistic producer (note areas of the shadow rectangle and the rectangle specified by a solid thick line are equal) \(^{19}\)

\(^{19}\) Linear demand and constant marginal cost are chosen for the sake of clarity.
4. Unrestricted (competitive) retail market

Consider the case when the manufacturer sells goods to a large group of competitive retailers and assume that it does not impose any vertical restraints (i.e., assume that the monopolistic manufacturer is willing to sell to perfectly competitive retail firms any given quantity of output \( Q \) at price \( P^{0}(Q) \)). Economic equilibrium on such a market (i.e., the equilibrium number of the retail firms and the equilibrium quantity supplied to final demanders) is characterized by Propositions 4.1 and 4.2.

PROPOSITION 4.1. An equilibrium number of retail firms operating in an unrestricted retail market exists and is finite.

Proof of Proposition 4.1. The number of firms in the retail market is determined by free entry and exit, such that in equilibrium the expected utility of being in the industry is equal to the utility of some benchmark activity \( (b) \), i.e., is positive (note that the expected utility from profit of a risk averse firm operating in the market is positive when its expected value of profit is positive as well). On the other hand, the maximum expected value of profit of the retail sector equals the difference between the maximum \( (E[\Pi(Q_{\sigma=0}^{*})]) \) and the optimum \( (E[\Pi(Q_{\sigma>0}^{*})]) \) expected value of profit of the risk averse monopolistic firm facing uncertain demand, and, consequently, is finite. This implies that there is always a finite number of firms which could earn positive expected value of profit in the retail market. Consequently, in equilibrium, the number of firms operating in the unrestricted (competitive) retail market is finite. Note, however, that if the expected value of profit for which expected utility from profit equals \( b \) is greater than the difference between the maximum \( (E[\Pi(Q_{\sigma=0}^{*})]) \) and the optimum \( (E[\Pi(Q_{\sigma>0}^{*})]) \) expected value of profit of the risk averse monopolistic firm facing

\[20\text{ See Section 3.}\]
uncertain demand, then a retail market is not created, i.e., the equilibrium number of firms equals zero.

QED.

For the analysis which follows, an important implication of Proposition 4.1 is that for a given value of the utility level of a benchmark activity \((b)\), the equilibrium number of firms in the unrestricted (competitive) retail market increases with the total expected value of profit of the retail sector.

The following result characterizes the optimal quantity supplied to final demanders through unrestricted (competitive) retail market.

**PROPOSITION 4.2.** The equilibrium quantity supplied to final consumers through the unrestricted (competitive) retail market (with \(H^*\) retail firms) equals the optimal output of the monopolistic firm without uncertainty \((Q_{\sigma=0}^*)\).

**Proof of Proposition 4.2.** It follows from Proposition 4.1 that if the retail market is created, then a finite number of retail firms \((1 \leq H^* < \infty)\) operates in it. Free entry and exit condition implies that if the equilibrium number of firms in the unrestricted retail market equals \(H^*_e\), then the following is true

\[(H^*+1)E[\Pi_R(q^b)] > E[\Pi(Q_{\sigma=0}^*)] - E[\Pi(Q_{\sigma>0}^*)] \geq H^* E[\Pi_R(q^b)], \quad (4.1)\]

where \(q^b\) is the quantity supplied by a single retailer for which the expected utility from profit equals \(b\), and \(E[\Pi_R(q^b)]\) is the expected value of profit of a single retail firm supplying quantity \(q^b\). Thus, if \(E[\Pi(Q_{\sigma=0}^*)]-E[\Pi(Q_{\sigma>0}^*)]=H^*E[\Pi_R(q^b)]\), then each firm supplies \(q^b=Q_{\sigma=0}^*/H^*\); i.e., the output supplied to final demanders by each individual firm \((q^b)\) equals the expected utility of some benchmark activity \(b\) (the total quantity supplied to final...
consumers equals \( q^bH^* = Q_{\sigma=0}^* \). If \( E[\Pi(Q_{\sigma=0}^*)] - E[\Pi(Q_{\sigma>0}^*)] > H^*E[\Pi_r(q^b)] \), then each individual firm increases the quantity supplied to final demanders up to \( Q_{\sigma=0}^*/H^* \), and earns profit which corresponds to expected utility from profit greater than \( b \) (note that each individual retail firm always has incentives to increase the quantity supplied, i.e., expected profit of each individual firm \( q[P(qH^*, \eta^0) - P(0)(qH^*)] \) increases with \( q \), if \( Q_{\sigma=0}^* < qH^* < Q_{\sigma=0}^* \)).

QED.

The last result implies that the organization of unrestricted retail market changes the distribution of welfare in the economy. In particular, it decreases the expected value of the deadweight loss of the economy and increases the expected value of consumer’s surplus. The monopolistic supplier is also better off since it changes random profit to sure profit with the same expected value.

5. Restricted (monopolistic) retail market

In the analysis presented in the preceding section we assumed that the manufacturer does not impose any vertical restraints and supplies goods to perfectly competitive retail firms. In this section we will consider the case when the monopolistic supplier imposes restrictions (vertical restraints) on the retail market in order to remove part of the retail sector’s profit (recall from Section 3 that under demand uncertainty the expected value of profit left to the retail sector is positive).

In this case, the objective of the manufacturer is to organize the retail market in such a way that maximizes its total value of profit (i.e., the sum: \( E[\Pi(Q_{\sigma=0}^*)] + \pi_r \), where \( E[\Pi(Q_{\sigma=0}^*)] \) is the value of deterministic profit which it earns if the retail sector is created, and \( \pi_r \) is the value of the profit transfer from the retail sector). The optimal policy of the
manufacturer towards the retail sector is considered below.

Let $K$ be the number of identical monopolistic retail firms operating in the market and $Q$ be the total output of the monopolistic supplier. Assume that the market is equally divided between retail firms; i.e., each retail firm faces stochastic demand coming from $N/K$ sources and is a monopolist in its exclusive territory.\(^{21}\)

The quantity of output supplied to final demanders by each individual retail firm equals $Q/K$, and its profit is

\[
\Pi^{Q/K}_R (Q/K, \eta_{N/K}) = \frac{Q}{K} [P^{Q/K}(Q/K, \eta_{N/K}) - P^{(0)}(Q)]
\]

(5.1)

where $P(Q/K, \eta_{N/K})$ denotes price in the exclusive market (with $N/K$ final demanders) of the monopolistic retail firm corresponding to quantity $Q/K$, $P^{(0)}(Q)$ is price to the retail sector.

Denote the expected utility from profit of a single retail firm as

\[
E[U_R(\Pi^{Q/K}_R (Q/K, \eta_{N/K}))] = \frac{1}{2} U_R(\Pi^{Q/K}_R (Q/K, \eta_{N/K})) + \frac{1}{2} U_R(\Pi^{Q/K}_R (Q/K, \eta_{N/K}^0))
\]

(5.2)

where

\[
\Pi^{Q/K}_R (Q/K, \eta_{N/K}^0) = \frac{Q}{K} [P^{Q/K}(Q/K, \eta_{N/K}^0) - P^{(0)}(Q)]
\]

(5.3)

\[
\Pi^{Q/K}_R (Q/K, \eta_{N/K}^0) = \frac{Q}{K} [\Pi^{Q/K}_R (Q/K, \eta_{N/K}^0) - P^{(0)}(Q)]
\]

(5.4)

\(^{21}\) For the sake of simplicity, it is assumed that $N/K$ is an integer number.
For any given $\pi_T$ a single retail firm supplying quantity $Q/K$ operates in the market if its expected utility from profit at least equals to the utility level of some benchmark activity ($b$), i.e., if

$$\frac{1}{2} U_R[\Pi^{NJK}(Q/K, \eta_{NJK}^\circ) - \frac{\pi_T}{K}] + \frac{1}{2} U_R[\Pi^{NJK}(Q/K, \eta_{NJK}^\circ) - \frac{\pi_T}{K}] \geq b,$$

(5.7)

where $\pi_T/K$ is the value of the profit transfer from a single retailer to the upstream monopolistic firm.

Taking into account that the manufacturer maximizes its total profit ($E[\Pi(Q_{\sigma=0})] + \pi_T$), setting the optimal number of monopolistic firms ($K$) in the retail market (i.e., the number of exclusive territories), and the optimal volume of output supplied ($Q$), the objective of the monopolistic producer is

$$\text{Max}_{K \in \{1, 2, \ldots\}, Q, \sigma} \left( E[\Pi(Q_{\sigma=0})] + \pi_T \right),$$

(5.8)

s.t.

Constraint (5.9) states that the expected utility from profit left to each individual monopolistic retail firm should be at least equal to the utility level of some benchmark
activity \( (b) \); otherwise, the retail firm doesn’t enter the market.

PROPOSITION 5.1. The optimal number of retail firms in the restricted (monopolistic) market \( (K^*) \) exists and is finite.

Proof of Proposition 5.1. Note that if the restricted retail market is not created (i.e., \( K^* = 0 \)), then the monopolistic supplier is even worse off than it would be without any restraints. Consequently, the rational behavior of the monopolistic firm implies that the number of firms in the restricted market is positive, i.e., \( K^* \geq 1 \).

Note that the maximum value of the profit transfer \( (\pi_T) \) which satisfies condition (5.9) with equality can be represented as a function of \( Q \) and \( K \). Consequently, for any given total quantity supplied to final demanders \( (Q) \), the optimal number of retail firms in the market depends on the shape of this function (in particular, if \( \pi_T \) is an increasing function of \( K \), then the optimum number of firms in the monopolistic retail market doesn’t exist). To determine the pattern of changes in \( \pi_T \) in response to changes in the number of firms in the retail market, assume for the time being that \( K \) is a continuous variable, and consider a function \( G(K, \pi_T(K)) = 0 \) specified as

\[
G(K, \pi_T(K)) = \frac{1}{2} U_R [\Pi^{N|K}_R (Q/K, \eta_{N|K}^o) - \pi_T] + \frac{1}{2} U_R [\Pi^{N|K}_R (Q/K, \eta_{N|K}^o) - \pi_T] \geq b,
\]

(5.9)

(5.10)

Consequently, by the implicit function theorem, the first derivative of \( \pi_T(K) \) with respect to \( K \) is

\[
G(K, \pi_T(K)) = \frac{1}{2} U_R [\Pi^{N|K}_R (Q/K, \eta_{N|K}^o) - \pi_T] + \frac{1}{2} U_R [\Pi^{N|K}_R (Q/K, \eta_{N|K}^o) - \pi_T] - b = 0
\]

(5.10)

Function \( G(K, \pi_T(K)) \) is continuously differentiable with respect to \( K \) \( (K > 0) \) and \( \pi_T \) \( (\pi_T \geq 0) \). Consequently, by the implicit function theorem, the first derivative of \( \pi_T(K) \) with respect to \( K \) is

It should be clear that the partial derivative of \( G(K, \pi_T(K)) \) with respect to \( \pi_T \) is negative for
\[
\frac{d\pi_T}{dK} = -\frac{\partial G/\partial K}{\partial G/\partial \pi_T},
\]

(5.11)

\(\pi_T \geq 0\), i.e., \(\partial G/\partial \pi_T < 0\). The partial derivative of \(G(K, \pi_T(K))\) with respect to \(K\) \((K > 0)\) equals

\[
\frac{\partial G}{\partial K} = -\frac{1}{2} U_R' \left[ \Pi_R^{NK} (Q/K, \eta_{NK}^{o}) - \pi_T \right] \frac{\partial \left[ \Pi_R^{NK} (Q/K, \eta_{NK}^{o}) - \pi_T \right]}{\partial K} +
\]

\[
\frac{1}{2} U_R' \left[ \Pi_R^{NK} (Q/K, \eta_{NK}^{o}) - \pi_T \right] \frac{\partial \left[ \Pi_R^{NK} (Q/K, \eta_{NK}^{o}) - \pi_T \right]}{\partial K}.
\]

(5.12)

Considering the form of the utility function of the risk averse firm, the partial derivative of \(G(K, \pi_T(K))\) with respect to \(K\) is always smaller than

\[
-\frac{1}{2} U_R' \left[ \Pi_R^{NK} (Q/K, \eta_{NK}^{o}) - \pi_T \right] \frac{\partial \left[ \Pi_R^{NK} (Q/K, \eta_{NK}^{o}) + \Pi_R^{NK} (Q/K, \eta_{NK}^{o}) - 2\pi_T \right]}{\partial K}
\]

(5.13)

if

\[
\frac{\partial \left[ \Pi_R^{NK} (Q/K, \eta_{NK}^{o}) + \Pi_R^{NK} (Q/K, \eta_{NK}^{o}) - 2\pi_T \right]}{\partial K}
\]

is negative.

Taking into account expressions (5.3-5.6) and rearranging them we can represent expression (5.14) as
Note that

\[ \frac{\partial}{\partial K} \left\{ \frac{2Q}{K} \left[ P^{N|K}(\frac{Q}{K}, \eta_{N|K}^*) - P^{(0)}(Q) \right] - \frac{2\pi_T}{K} \right\} \]

\[ = -\frac{2Q}{K^2} \left[ P^{N|K}(\frac{Q}{K}, \eta_{N|K}^*) - P^{(0)}(Q) \right] - \frac{2Q^2}{K^3} \frac{\partial P^{N|K}(Q/K, \eta_{N|K}^*)}{\partial (Q/K)} + \frac{2\pi_T}{K^2} \]

\[ = -\frac{2Q}{K^2} \left\{ [Q(P^{N|K}(\frac{Q}{K}, \eta_{N|K}^*) - P^{(0)}(Q) - \pi_T] + \frac{Q^2}{K} \frac{\partial P^{N|K}(Q/K, \eta_{N|K}^*)}{\partial (Q/K)} \right\}, \]

(5.15)

Note that

\[ P^{N|K}(Q/K, \eta_{N|K}^*) = P(Q, \eta^*) \]

and, consequently, the expression in square brackets is non-negative if the retail market exists (i.e., if \(K \geq 1\)). Since the first derivative of the expected inverse demand curve with respect to the quantity of output supplied is negative, the second term in the expression above is negative. This implies that if \(K\) goes to infinity the expression above goes to zero from below. That means that starting from sufficiently large \(K\), the expression (5.15) is negative. Therefore, the partial derivative of \(G(K, \pi_T(K))\) with respect to \(K\) is always negative for sufficiently large \(K\). Consequently, for sufficiently large \(K\), \(\pi_T(K)\) decreases with \(K\). Note that the expression (5.15) continuously decreases with \(K\), i.e., crosses zero once or never (it is always negative). If the expression (5.15) is always negative (\(\partial G/\partial K\) is always negative as well), then \(\pi_T(K)\) is continuously decreasing in \(K\) (in this case, the optimal number of retail firms in the market \(K^*\) equals 1). If the expression (5.15) equals zero for

\[ \text{Note that if } K \text{ increases the market demand in each individual exclusive territory decreases, and, consequently, the first derivative of the expected inverse demand function in the exclusive territory } \frac{\partial P^{N|K}(Q/K, \eta_{N|K}^*)}{\partial (Q/K)} \text{ does not change with } K. \]
certain $K^*<\infty$ ($\partial G/\partial K$ is positive for $K<K^*$, equals zero for $K=K^*$, and is negative, otherwise), then $\pi_T(K)$ has a single maximum at $K=K^*<\infty$ (in this case the optimum number of firms in the market $K^*=\arg\{\max\{\pi_T([K^*]),\pi_T([K^\circ])\}\}$, where, brackets $[\ ]$ and $[\ ]$ denote rounding down and up to the nearest integer, respectively).

QED.

The optimal volume of output supplied to final demanders through restricted (monopolistic) retail market is characterized by the Proposition below.

**PROPOSITION 5.2.** The optimal quantity supplied to final consumers through restricted (monopolistic) retail market is equal to the optimal output of the monopolistic firm without uncertainty ($Q_{a=0}^*$).

**Proof of Proposition 5.2.** Consider the pattern of changes in the maximum value of the profit transfer $\pi_T$, in response to changes in the total quantity of output supplied to final demanders $Q$. Recall that the maximum value of $\pi_T$, which satisfies condition (5.9) with equality, can be represented as a function of $Q$ and $K$, and consider a function $W(Q,\pi_T(Q))=0$ specified as

\[
W(Q,\pi_T(Q)) = \frac{1}{2} \frac{U_R[\Pi_{NR}^K(Q,K^\circ)]}{K} - \frac{\pi_T}{K} - 1 - \frac{1}{2} \frac{U_R[\Pi_{NR}^K(Q,K^\circ)]}{K} - \frac{\pi_T}{K} - b = 0 .
\]

Function $W(Q,\pi_T(Q))$ is continuously differentiable with respect to $Q$ ($Q>0$) and $\pi_T$ ($\pi_T\geq0$). Consequently, by the implicit function theorem, the first derivative of $\pi_T(Q)$ with respect to $Q$ is

\[
\frac{d\pi_T}{dQ} = -\frac{\partial W/\partial Q}{\partial W/\partial \pi_T} .
\]

Note that partial derivative of $W(Q,\pi_T(Q))$ with respect to $\pi_T$ is negative for $\pi_T\geq0$. i.e., $\partial W/\partial \pi_T<0$. The partial derivative of $W(Q,\pi_T(Q))$ with respect to $Q$ ($Q>0$) equals...
Since the utility function is strictly concave, the partial derivative of $W(Q, \pi_t(Q))$ with respect to $Q$ is never smaller than

$$\frac{\partial W}{\partial Q} = \frac{1}{2} U'_R \left[ \Pi^{N/K}_R \left( \frac{Q}{K}, \eta_{N/K}^{\circ} \right) - \frac{\pi_T}{K} \right] \frac{\partial \left[ \Pi^{N/K}_R \left( \frac{Q}{K}, \eta_{N/K}^{\circ} \right) - \frac{\pi_T}{K} \right]}{\partial Q}$$

(5.18)

and never greater than

$$\frac{1}{2} U'_R \left[ \Pi^{N/K}_R \left( \frac{Q}{K}, \eta_{N/K}^{\circ} \right) - \frac{\pi_T}{K} \right] \frac{\partial \left[ \Pi^{N/K}_R \left( \frac{Q}{K}, \eta_{N/K}^{\circ} \right) - \frac{\pi_T}{K} \right]}{\partial Q}$$

(5.19)

if the partial derivative in the expression above is positive. Taking expressions (5.3-5.6) and rearranging them we can represent the partial derivative under study as

$$\frac{\partial}{\partial Q} \left( \frac{2Q}{K} \left[ P^{N/K}(Q, \eta_{N/K}^{\circ}) - P^{(0)}(Q) \right] - \frac{2\pi_T}{K} \right)$$

(5.21)

Note that for any integer $K (K \geq 1)$, the expected price if the quantity $Q/K$ is supplied to the exclusive market (with $N/K$ demanders) is the same as the expected price for quantity $Q$ in the large market (with $N$ demanders)$^{23}$

$$P^{N/K}(Q/K, \eta_{N/K}^{\circ}) = P(Q, \eta^{\circ})$$

(5.22)

Therefore, the partial derivative under consideration can be represented as

$^{23}$ See Section 2 for details.
The expression above represents the first derivative of the expected value of profit of the retail sector with respect to the total quantity supplied (Q), multiplied by 2/K. The expected value of profit of the retail sector reaches its maximum (E[Π(Qσ=0⁺)]-E[Π(Qσ>0⁺)]-πₜ) if the quantity of output supplied Q equals Qσ=0⁺ (at this point the first derivative of the expected profit of the retail sector with respect to Q equals zero). Therefore, for all Q smaller than Qσ=0⁺, the partial derivative specified by the expression (5.23) is positive. This implies that \( \frac{\partial W}{\partial Q} \) is always positive for Q<Qσ=0⁺, and equals zero if Q=Qσ=0⁺ (using similar arguments one can show that it is negative, otherwise, i.e., for Q>Qσ=0⁺). Given that \( \frac{\partial W}{\partial \pi_t} \) is always negative, \( \frac{d\pi_t}{dQ} \) is positive if Q<Qσ=0⁺, equals zero if Q=Qσ=0⁺, and is negative, otherwise. Thus, for any K the maximum value of the profit transfer \( \pi_T \) corresponds to the total quantity supplied Q⁰⁺=Qσ=0⁺.

QED.

The optimal policy of the monopolistic manufacturer towards monopolistic retail sector can be implemented using the following set of policy instruments:

\{limited number of retailers (K⁺) and territorial protection, quantity forcing (Q⁺=Qσ=0⁺), franchise fee (Ff⁺=πT⁺/K⁺)\}.

One can show that if the monopolistic firm imposes vertical restrictions on the competitive retail market, the optimal number of firms operating in the market (H⁺) equals one, and the optimal quantity supplied to final demanders Q⁺=Qσ=0⁺ (i.e., there is a single firm supplying quantity Qσ=0⁺, and earning profit for which the expected utility equals \( b \)). Therefore, the restricted retail market with competitive firms can be considered as a particular
case of the restricted monopolistic retail market (with $K^*=1$).\textsuperscript{24}

6. Dynamics of the retail market in the privatization period

To show the dynamics of the retail market in the period of transition to a market economy, consider a monopolistic single commodity market with stochastic demand (as presented in Section 2), and for the sake of simplicity, assume that there are no transaction costs (as in Section 3). Furthermore, divide the transition time path into the following three periods:

1. pre-privatization period - all firms are state-owned (a little risk-averse) and take into account commercial criteria,

2. privatization period - firms are either state-owned (a little risk averse) or privately-owned (more risk averse) and behave according to commercial criteria,

3. post-privatization period - all firms are privately-owned (more risk averse than state-owned) and take into account commercial criteria.

To simplify the analysis, assume that commercial criteria imply that managers of firms operating in the market maximize profit of their owners (private-shareholders or the state).

The risk aversion of the manufacturer and retail firms in the pre-privatization period naturally leads to the organization of the retail market with a finite number of firms.\textsuperscript{25}

Considering that at the beginning of the transition period all firms (the monopolist and retailers) are state-owned, we can assume that the horizontal organization of the retail market (if it is created) maximizes total expected value of profit of all state owned firms (i.e.,

\textsuperscript{24} Note that the restricted monopolistic market with exclusive territories is always as good (or better, if $K^*>1$) for the manufacturer as the restricted competitive market.

\textsuperscript{25} Similarly, in the market economy when all firms are privately-owned (i.e., more risk averse than state-owned firms) there will be a certain, finite number of firms operating in the market.

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monopolistic manufacturer and retailers). Thus, we can focus on two types of retail market: unrestricted (competitive) market, and restricted (monopolistic) market with K exclusive territories. Moreover, for the sake of simplicity, we assume that the monopolist does not change the horizontal structure of the retail market during the transition period.

Since there are two different types of firms: monopolistic manufacturer and retail firms, the following two privatization time paths: \((t_1,t_2)_{M-R}\) and \((t_1,t_2)_{R-M}\) (where \(t_1, t_2 \in \mathbb{R}_+\), and \(t_1 < t_2\)) are possible:

- \(M-R\): \(t_1\) - privatization of the monopoly, \(t_2\) - privatization of the retail sector,
- \(R-M\): \(t_1\) - privatization of the retail sector, \(t_2\) - privatization of the monopoly.

Both strategies lead to the same final state (i.e., to the market economy); however, their transitions paths are not the same but depend upon the type of retail market.

Consider first the pattern of changes in the equilibrium number of firms and the profitability of the monopolistic producer and retail firms operating in unrestricted (competitive) retail market in response to changes in the ownership structure in the privatization period.

**Proposition 6.1.** In the unrestricted (competitive) retail market the privatization of the monopolistic firm increases the expected value of profit of the retail sector, decreases profit of the monopolistic firm, and increases or doesn’t change the equilibrium number of retail firms.

**Proof of Proposition 6.1.** To simplify the analysis, assume that the number of firms in the unrestricted (competitive) market \((H)\) as a continuous variable. As we have already admitted (see Section 1, *stylized fact* 4) privatization makes the privatized firm more risk averse. Since the deviation of the optimal output of the monopolistic firm under uncertainty of
demand from its optimal output without uncertainty increases with risk aversion (see Section 2, expression (2.29)), the optimal expected value of profit of the monopolistic firm under uncertainty decreases if the firm becomes more risk averse (i.e., the optimal expected value of profit of the more risk averse firm is smaller than the less risk averse firm). Thus, the offer curve of the more risk averse firm is located closer to the origin, and, consequently, for any given quantity supplied, the difference between the expected market price and the offer price to retailers is greater for the more risk averse firm than it would be for the less risk averse firm. Taking into account that the optimal quantity supplied to final demanders by the unrestricted (competitive) retail sector \( (Q') \) equals the optimal output of the monopolistic firm without uncertainty of demand \( (Q_{\sigma=0}^*) \), the maximum total expected value of profit of the retail sector increases (consequently, profit of the monopolistic firm decreases).\(^{26}\) Since only a significant increase in the expected profit of the retail sector is sufficient to cover the utility level of the benchmark activity of the additional retail firm \( (b) \), the equilibrium number of firms in the retail market increases only if an increase in the expected value of profit of the retail sector is large enough; otherwise, it remains unchanged.

QED.

PROPOSITION 6.2. In the unrestricted (competitive) retail market the privatization of the retail sector either destroys the retail market, or, if after privatization the retail market exists, it decreases or doesn’t change the equilibrium number of retail firms and leaves the expected value of profit of the retail sector and profit of the monopolistic firm unaffected.

\(^{26}\) Note that the sum of the monopolistic profit and the expected profit of the retail sector equals the maximum monopolistic profit without uncertainty of demand.
Proof of Proposition 6.2. Assume first, that the number of firms in the competitive market (H) is a continuous variable. As mentioned above, the expected value of profit for which the expected utility equals the utility level of some benchmark activity (b) is positive (for the risk averse firm) and increases if the firm becomes more risk averse. Taking into account that privatization makes retail firms more risk averse, the expected value of profit for which the expected utility equals b increases (note, however, that neither the expected value of profit of the retail sector nor the profit of the monopolistic firm changes). Since the equilibrium number of firms H (assuming that H is a continuous variable) in the unrestricted (competitive) retail market is inversely proportional to the expected value of profit for which the expected utility of a single retailer equals b (which increases if the retail firms are privatized), the privatization of the retail sector decreases the value of H. However, if we drop the assumption about continuity of H, and consider H as an integer number, then the equilibrium number of firms in the retail market decreases only if an increase in the expected value of profit for which the expected utility of a single retail firm equals b is large enough; otherwise, it remains unchanged. Note, however, that if the equilibrium number of firms equals zero (i.e., if the retail market disappears), the expected value of profit of the retail sector decreases to zero, and the profit of the monopoly changes from deterministic to random (note, however, that the expected value of monopolistic profit remains unchanged).

QED.

The next results explain the pattern of changes in the optimum number of firms and in the profitability of the monopolistic producer and retail firms operating in a restricted (monopolistic) retail market in response to changes in the ownership structure in the privatization period.
PROPOSITION 6.3. *In the restricted (monopolistic) retail market the privatization of the monopolistic supplier doesn’t affect the optimum number of firms (exclusive territories) in the retail market, the profit of the monopolistic supplier or the expected value of profit of the retail sector (i.e., changes nothing).*

Proof of Proposition 6.3. Suppose that there are $K$ identical monopolistic retailers in the market (each of them faces stochastic demand coming form $N/K$ sources). As in Section 5, assume that the number of firms in the market ($K$) is a continuous variable (for the sake of simplicity we will treat $N/K$ as an integer number). Under such assumptions the optimal number of monopolistic firms in the retail market corresponds to $K^*$ ($K^* \geq 1$), for which the expression (5.15) equals zero (see Proof of Proposition 5.1). Privatization of the monopolistic supplier makes the supplier more risk averse, and, consequently, decreases its optimal expected value profit ($E[\Pi(Q_{\sigma>0}^*)]$); i.e., the offer curve of the monopoly shifts closer to the origin. Therefore, for any given quantity supplied to final demanders by an individual monopolistic retail firm the expected value of this firm profit increases. This increase, however, can be immediately captured by an increase in the profit transfer to the monopolistic supplier ($\pi_T$). The maximum value of the additional profit transfer equals the change in $E[\Pi(Q_{\sigma>0}^*)]$. Thus, the value of the first term in the expression (5.15) doesn’t change, and consequently, the optimal number of firms in the retail market doesn’t change either. Since the sum of the monopolistic profit and the expected profit of the retail sector equals to the maximum monopolistic profit without uncertainty of demand, the expected value of profit of the retail sector remains unchanged.

QED.

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27 Recall that the optimal quantity of output supplied to final demanders is equal to the optimal monopolistic output without uncertainty ($Q_{\sigma>0}^*$).
PROPOSITION 6.4. In the restricted (monopolistic) retail market the privatization of the retail sector either destroys the retail market, or if after privatization the retail market still exists, it increases the expected value of profit of the retail sector, decreases the profit of the monopolistic supplier and doesn’t affect the optimum number of firms (exclusive territories) in the retail market.

Proof of Proposition 6.4. As mentioned above, privatization of the retail sector makes retail firms more risk averse, and, consequently, increases the expected value of profit for which the expected utility of each individual firm is equal to the utility of some benchmark activity \((b)\). If this increase is large enough, then it can exceed the maximum expected value of profit of the retail sector, and, consequently, no retail firm can operate in the market. In this case, the retail market disappears; i.e., the monopolistic supplier deals directly with final demanders, earns random profit with the expected value \((E[\Pi(Q_{\sigma_n}^*)])\), and the expected value of profit and the number of firms in the retail sector decrease to zero. However, if after privatization of the retail sector, the retail market exists (i.e., \(K \geq 1\)), then the optimal number of monopolistic retailers in the market corresponds to such value of \(K^*\), for which the expression (5.15) equals zero. Note that this expression doesn’t depend on the utility function (and, consequently, on the risk aversion) of a single retail firm. It implies that privatization of the retail sector (which makes retail firms more risk averse) leaves the number of retail firms in the market unaffected. However, if retail firms become more risk averse then the expected value of profit for which the expected utility of a single firm equals \(b\), increases.

Given that profit of each individual firm operating in the market has to be non-negative, the expected value of profit of the retail sector increases (note that \(K^*\) is constant). Since the sum of the monopolistic profit and the expected profit of the retail sector is equal to the maximum monopolistic profit without uncertainty of demand, the profit of the monopolistic supplier
Changes in unrestricted (competitive) and restricted (monopolistic) retail markets resulting from different privatization strategies are illustrated by the example below.

Example 5.1.

Consider the effects of privatization strategies defined above (i.e., M-R and R-M), on the dynamics of the retail sector, assuming that in all periods analyzed the retail market exists. In particular, assume that there are $H_1^*, H_2^*, H_3^*$ ($H_1^*, H_2^*, H_3^* \geq 1$) firms in the unrestricted (competitive) retail market in pre-privatization, privatization, and post-privatization periods, respectively, and $K$ ($K \geq 1$) retail firms in the restricted (monopolistic) market in the pre-privatization period. Denote profit of the monopolistic firm in subsequent periods as $\Pi_1^M, \Pi_2^M$ and $\Pi_3^M$, and the expected profit of the retail sector as $E(\Pi_1^R), E(\Pi_2^R)$ and $E(\Pi_3^R)$, respectively. Moreover, in order to focus on the differences in privatization period, assume that in both considered privatization strategies (M-R and R-M), the initial and final market equilibria are the same.

We begin from the privatization strategy: M-R ($t_1$ - privatization of the monopolistic supplier, $t_2$ - privatization of the retail sector).

At $t_1$ (i.e., at the beginning of the privatization period):

- in the unrestricted (competitive) retail market the following changes occur (see Proposition 6.1): the number of retail firms in the market changes from $H_1^*$ to $H_2^*$, where $H_2^* \geq H_1^*$ (if the change in risk aversion of the monopolist resulting from privatization is not significant then $H_2^* = H_1^*$; otherwise, $H_2^* \gt H_1^*$), profit of the monopolistic firm decreases from
$\Pi_1^M$ to $\Pi_2^M$, and the expected value of profit of the retail sector increases from $E(\Pi_1^R)$ to $E(\Pi_2^R)$;

- in the restricted (monopolistic) retail market with exclusive territories nothing changes (i.e., $K_2=K_1$, $\Pi_2^M=\Pi_1^M$, $E(\Pi_2^R)=E(\Pi_1^R)$, see Proposition 6.3).

At $t_2$ (the end of the privatization period):

- in the unrestricted (competitive) retail market the number of retail firms in the market changes from $H_2^*$ to $H_3^*$, where $H_3^* \leq H_2^*$ (if the change in the risk aversion of the retail firms resulting from privatization is not significant, then $H_3^*=H_2^*$, otherwise $H_3^* < H_2^*$), and profit of the monopolistic firm, as well as the expected value of profit of the retail sector, doesn’t change (i.e., $\Pi_2^M=\Pi_3^M$, and $E(\Pi_2^R)=E(\Pi_3^R)$, see Proposition 6.2);

- in the restricted (monopolistic) retail market (assuming that after privatization the retail market exists), the number of retail firms in the market doesn’t change ($K=\text{const}$), profit of the monopolistic firm decreases from $\Pi_2^M$ to $\Pi_3^M$, and the expected value of profit of the retail sector increases from $E(\Pi_2^R)$ to $E(\Pi_3^R)$ (see Proposition 6.4).

Consider now privatization strategy: R-M ($t_1$ - privatization of the retail sector, $t_2$ - privatization of the monopolistic firm).

At $t_1$ (the beginning of the privatization period):

- in the unrestricted (competitive) retail market the number of retail firms in the market changes from $H_1^*$ to $H_2^*$, where $H_2^* \leq H_1^*$ (if the change in the risk aversion of the retail firms resulting from privatization is not significant, then $H_2^*=H_1^*$; otherwise, $H_2^* < H_1^*$), and profit of the monopolistic firm, as well as the expected value of profit of the retail sector, doesn’t change (i.e., $\Pi_1^M=\Pi_2^M$, and $E(\Pi_1^R)=E(\Pi_2^R)$, see Proposition 6.2);

- in the restricted (monopolistic) retail market (assuming that after privatization the retail market exists) the number of retail firms in the market doesn’t change ($K=\text{const}$), profit
of the monopolistic firm decreases from $\Pi_1^M$ to $\Pi_2^M$, and the expected value of profit of the retail sector increases from $E(\Pi_1^R)$ to $E(\Pi_2^R)$ (see Proposition 6.4).

At $t_2$ (the end of the privatization period):

- in the unrestricted (competitive) retail market the following changes occur (see Proposition 6.1): the number of retail firms in the market changes from $H_2^*$ to $H_3^*$, where $H_3^* \geq H_2^*$, (if the change in risk aversion of the monopolist resulting from privatization is not significant, then $H_3^* = H_2^*$; otherwise, $H_3^* > H_2^*$), profit of the monopolistic firm decreases from $\Pi_2^M$ to $\Pi_3^M$, and the expected value of profit of the retail sector increases from $E(\Pi_2^R)$ to $E(\Pi_3^R)$;

- in the restricted (monopolistic) retail market with exclusive territories nothing changes (i.e., $K_3 = K_2$, $\Pi_3^M = \Pi_2^M$, $E(\Pi_3^R) = E(\Pi_2^R)$, see Proposition 6.3).

Changes in the number of firms operating in the retail market, and profitability of the monopolistic firm and the retail sector are represented in Fig.6.1 (unrestricted retail market), and Fig.6.2 (restricted monopolistic market).

7. Conclusion

The analysis of the monopolistic manufacturer in a stochastic environment shows that rational behavior of the monopolist under uncertainty of demand naturally implies organization of the retail market. In particular, in the presence of uncertainty the risk averse manufacturer is always willing to offer goods produced to retail firms at lower prices than to final demanders (in order to change random profit to deterministic profit). This explains why retail firms can earn profit, and, consequently, why the retail market can be organized. This paper shows that equilibrium characteristics of the retail market depend upon vertical and horizontal arrangements; however, in both types of retail market considered, i.e., in the
unrestricted (competitive) and restricted (monopolistic) retail markets, the total output supplied to final consumers is equal to the optimal output of the monopolistic firm without uncertainty, and an equilibrium number of firms exists and is finite. The equilibrium
Fig. 6.1. Changes in the number of firms (H), the expected value of profit of the retail sector (E(\(\Pi^R\))) and the profit of the monopolistic firm (\(\Pi^M\)) in the unrestricted (competitive) retail market. The solid line corresponds to the M-R strategy, and the dashed line corresponds to the R-M strategy (\(\Pi_{\sigma=0}^*\) is the optimal monopolistic profit without uncertainty of demand, \(\Pi_{\sigma=0}^*=\Pi^M+E(\Pi^R)\)).
Fig. 6.2. Changes in the number of firms (K), the expected value of profit of the retail sector 
(E(\Pi^R)) and the profit of the monopolistic firm (\Pi^M) in the restricted (monopolistic) retail 
market. The solid line corresponds to the M-R strategy, and dashed line corresponds to the 
R-M strategy (\Pi^{\sigma=0}_* is the optimal monopolistic profit without uncertainty of demand, 
\Pi^{\sigma=0}_* = \Pi^M + E(\Pi^R)).
characteristics of the restricted (monopolistic) market imply that in a world with imperfectly informed firms, vertical restrictions may be profitably imposed by an upstream manufacturer. In particular, the analysis shows that under demand uncertainty, the main indicators of social welfare (i.e., deadweight loss, and consumer and producer surplus) in the restricted (monopolistic) market are the same as in the unrestricted (competitive) retail market (the only difference is in the distribution of producer surplus among the monopolistic supplier and retail firms). This result contributes to contemporary economic debate concerning the legal status of vertical restraints (in the US, for example, franchise fees are presumptively legal, but exclusive territories after being illegal per se, are now subject to rule of reason) and supports the claim (presented, for example, by Williamson, 1975; and Mathewson and Winter, 1983) that there is no economic reason per se for the illegality of vertical restraints.28

Based on the equilibrium characteristics of the retail markets, the impact of different privatization strategies (M-R and R-M) on the dynamics of unrestricted (competitive) and restricted (monopolistic) retail markets in the transition period has been examined. The results derived show that:

1. Privatization strategy: M-R (the first: the monopolist, the second: the retail sector), increases or doesn’t change the number of retailers in the market or the total expected profit of retail firms (note that retailers are state-owned in the privatization period), and decreases or doesn’t change the profit of the monopolistic manufacturer (which is privately-owned).

2. Privatization strategy: R-M (the first: the retail sector, the second: the monopolist), increases or doesn’t change the total expected profit of retail firms (which are privately-owned) and decreases or doesn’t change the number of retail firms in the market and the expected value of the monopolistic profit (note that the monopolist is state-owned in the

28 The opposite view is presented in e.g., by Rey and Tirole (1986).
privatization period).

Consequently, privatizing the monopolist first (strategy M-R) is always at least as good as privatizing the retailers first (strategy R-M), because during the privatization period it neither harms the development of the competitive retail market nor decreases the profit of the state owned firms or the number of firms in the retail market.\footnote{This conclusion is consistent with the result presented by Glaeser and Scheinkman (1996) derived from a comparative analysis of informational gains from different privatization strategies.}
References


