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**IMPLICATIONS OF HETEROGENEITY IN CASH IN  
ADVANCE MODELS**

**ABSTRACT**

This paper explores the properties of asset prices in the cash-in-advance economy with heterogeneous agents. It modifies the standard representative agent cash-in-advance asset pricing model with Swenson timing to incorporate ex-post heterogeneous agents subject to idiosyncratic productivity shocks. Such modification is justified by the fact that, as it was shown by Giovanni and Labadie (1991), representative agent cash-in-advance asset pricing models are unable to match empirical regularities concerning inflation interest rates and stock returns observed in the actual US economy. In particular representative agent cash-in-advance models generate too low equity premium, only occasional correlation between inflation and real interest rates and inflation and real stock returns and ability of nominal interest rates to almost perfectly predict nominal stock returns. My results imply that heterogeneous agent cash-in-advance model performs better in matching regularities observed in the actual data, since it generates much higher equity premium and quite persistent negative correlation between inflation and real interest rate and inflation and real returns on stocks as it is the case in the US economy. Moreover nominal interest rates are no longer good predictors of nominal stock returns which is also true for the real economy

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# 1. Introduction

There has been quite a big body of literature in cash in advance models which have been used to study the underlying role of money and monetary policy in general equilibrium framework. The basic logic of cash in advance models can be described by the idea that certain consumption goods can be bought using only cash and so agents need to hold money balances in advance to be able to pay for these goods in the beginning of each period. Thus the holding of money is purely motivated by need of purchasing consumption goods or in other words by transaction services of money. However if on one hand households derive utility from transaction services of money, on the other hand they incur losses if higher inflation reduces the purchasing power of money. Thus higher inflation in some sense plays the role of tax on consumption. The cash in advance models have been used to address and analyze a wide range of issues such as empirical regularities relating to inflation asset prices and nominal and real rates of return, optimality of monetary policy and Friedman rule, impact of variability of inflation rates and growth rates of money etc.

The cash in advance model was originally developed in 1982 by Lucas. In his paper he introduced the approach to model the role of money in general equilibrium framework as providing liquidity or transactions services rather than directly giving utility. For this purpose he imposed additional cash in advance constraint implying that households should hold certain money in advance to be able to buy cash good. Lucas assumed that assets market opens first, but in 1985 Swenson redeveloped the model using the assumption that goods market opens first and this timing became more conventional in the literature. Lucas and Stockey (1987) developed methods for verifying existence of and explicitly calculating

the competitive equilibrium in the cash in advance economy. Cooley and Hansen (1989) incorporated money into RBC models using cash in advance constraint and analyzed impact of variability of inflation rates and growth rates of money.

Giovanni and Labadie (1991) developed and simulated cash in advance models of money and asset prices. These models were also calibrated by them and used to study certain empirical regularities observed in actual US economic data. In particular their research concentrated on average level of stock returns and returns on nominal bonds, covariation of realized interest rate real interest rate and real returns with inflation, and ability of nominal interest rates to predict inflation and nominal stock returns. In their paper authors find that data produced by representative agent cash in advance models cannot match many of the features of actual data related to stock returns and inflation. In particular the model leads to only occasional correlation between real returns on stocks and inflation as well as no correlation between real interest rate and inflation, while in data these are quite large and persistent. Also the data suggests that nominal interest rates are very poor predictors of stock returns while the data obtained from the models in contrary indicates that nominal interest rates are quite good predictors of stock returns. Finally they observe much lower equity premium than in actual data. One of the extensions they suggest in the end of the paper is to go beyond representative agent framework incorporating heterogeneity and incomplete markets into the model, but they mention that this is not feasible given current state of computational methods.

The other line of research connected with cash in advance models concentrated on analyzing the optimality of Friedman rule and monetary policy in cash in advance framework. Models introduced in this type of literature already incorporate certain degree

of heterogeneity. In particular Stefania Albanesi (2005) studied the structure and time consistency of optimal monetary policy in an economy with cash in advance constraint and two types of agents, differing according to their earning ability and asset holdings. In her paper she finds that even in the case of commitment it can be optimal to deviate from Friedman rule, and optimal monetary and fiscal policies are time consistent in this economy. Also, Shuon Shi (1999) examines redistributive role of expansionary monetary policy, in an economy where there two type of agents who move across different markets, namely buyers and seller, and where both within and cross market frictions are operational. He finds that when both frictions are operational optimal monetary policy requires growth rate of money that exceeds the Friedman rule.

The review presented above points out that although some cash-in-advance models with heterogeneous agents already appeared in the literature, no research has explored how heterogeneous agent cash-in-advance asset pricing model performs with respect to reflecting empirical regularities concerning inflation, real and nominal returns on risky assets, interest rates and equity premium. Thus in my research I'm going to modify Swenson type cash-in-advance representative agent asset pricing model analyzed in Giovanni and Labadie(1991) to include heterogeneous agents with individual idiosyncratic productivity shocks and incomplete markets. Then I 'm going to test similar to Giovanni and Labadie (1991) how well the data which one gets from simulating the model reflects the empirical regularities concerning inflation, real and nominal returns on risky assets, interest rates and equity premium observed in the actual data.

At this point it is important to provide some intuition on why the heterogeneous agent economy with incomplete markets might help to reconcile the properties of the asset prices

in the model economy with those in the actual data. The important feature of the representative agent asset pricing models analyzed in Giovanni and Labadie (1991) is the fact that both the stock returns and bond returns (interest rates) are driven by common stochastic discount factor and common money supply shocks. That explains the ability of nominal interest rates to predict nominal stock returns and small equity premium that is small difference between risky asset's returns and risk-free asset returns. The introduction of idiosyncratic productivity shocks which creates the economy with incomplete markets is expected to become a major driving force of stock returns and increase their volatility which will break the strong link between nominal interest rates and nominal stock returns and increase the equity premium. Also looking at the summary statistics on inflation and real stock returns presented in Giovanni and Labadie (1991) one can note that the inflation rate generated by the model has nearly the same standard deviation as it's counterpart in the data while the stock returns from the model display much lower variation for reasonable values of risk aversion than their counterparts from the data. Thus, the higher uncertainty in the economy with idiosyncratic shock might help to generate stock returns with volatility closer to the real one and provide the negative correlation between real stock returns and inflation rate.

Since in my research I am going to modify representative agent model of asset prices according to heterogeneous agent assumption, I will rely on several asset pricing papers with heterogeneous agents. For instance, Lucas (1980) and Mailath and Sandroni(2003), represent quite successful attempts to incorporate heterogeneity into asset pricing models. Both of these papers consider heterogeneous asset pricing models with finite number of agents and their types. However, Lucas paper can be considered the benchmark model of

endowment economy with heterogeneous agents while the second paper also develops heterogeneous agent asset pricing model but uses it for studying issues related to asymmetric information in the asset markets. Kubler and Schmedders (2002, 2003) in their papers exploring the properties of recursive equilibrium in the economies with incomplete asset markets and infinite time construct generalized version of Lucas asset pricing model with ex ante heterogeneous agents having individual shocks to preferences. However, their model is based on trading securities which are somewhat different from shares of productive assets which I have in my benchmark model of Giovanni and Labadie (1991). In my research I will rely primarily on the model presented in Altug (2006). In this paper the author makes a comprehensive analysis of economies with complete and incomplete markets and presents several models corresponding to different trading arrangements such as complete contingent claims trading, securities market and also equities trading. The paper also presents an asset pricing model of equity trading, which includes heterogeneous agents with idiosyncratic productivity shocks trading shares on their stream of earnings. In my research I will apply this idea to cash-in-advance asset pricing model, derive the equilibrium equity prices and then simulate the model to find out how well this modified model matches the regularities of actual data.

## **2. The model**

Let's assume, that there are two types of infinitely lived agents in the economy and let  $N_i$  denote the number of agents of type  $i$  for  $i=1,2$ . There is no ex-ante heterogeneity between the agents but they are ex-post heterogeneous due to different actual productivities. The agents in the economy are hit by idiosyncratic productivity shock. A

productivity shock follows a stochastic process which is a collection of random variables  $(s(t,w), t \in T)$  defined on a probability space  $(\Omega, \Phi, P)$ , in which  $T = \{0, 1, 2, \dots\}$  and  $s : T \times \Omega \rightarrow S$ , where  $S$  is the state space which is defined as  $S = \{1, 2\}$ . For a fixed  $\omega$   $s(\cdot, \omega)$  is the sample path of realization and for fixed  $t$   $s(t, \cdot)$  is a random variable. Assume  $s$  follows first order Markov process with transition matrix  $Q(s, s')$ . Each agent whether type 1 or type 2 can produce  $a$  units of consumption good per one unit of labor when he is productive, but he can also suffer random spells of lower productivity in case of which he produces only  $f$  units of output per unit of labor employed. Thus the history of productivity is what makes the agent ex-post heterogeneous. When  $s(t,w)=1$ , type 1 agent is more productive while type 2 agent suffers lower productivity spell and produces less. When  $s(t,w)=2$ , type 2 agent is more productive but type 1 is not so productive. Define a function  $\theta : S \rightarrow \{a, f\}$  indexed by  $i$ .

If  $s(t,w)=1$  then :

$$\theta_{1,t}(s) = a$$

$$\theta_{2,t}(s) = f \quad \text{where } a > f$$

If  $s(t,w)=2$  then :

$$\theta_{1,t}(s) = f$$

$$\theta_{2,t}(s) = a$$

The production function for agent type  $i$  is given by:

$$y_i = \theta_i * l_i \quad \text{where } l_i \text{ is the labor supply}$$

The agents buy and sell shares and pay dividends on their labor earnings streams. Let's

denote by  $z_i^j$  shares of the agent j-th earning stream held by agent i. The agent also buys and sells claims to his own earnings stream. The price of the share of earning stream of agent j is at time t is denoted by  $Q_t^j$ . The dividends paid by the agent j for one share on his labor earnings stream are denoted by  $d_{j,t}$ . The agents can also buy and sell nominal bonds which are denoted by  $B_t$  and which yield a gross nominal interest rate denoted by  $R_t$ . Bonds in this case are also risky assets since their nominal value is affected by monetary shocks.

The purchase of consumption goods is subject to cash-in-advance constraint that is the individuals have to hold certain money balances in advance to be able to buy the consumption good which can be paid for only by cash. The assumption about timing here corresponds to the Swenson timing under which the goods trade occurs before asset markets open. The holdings of real money balances of agent i are denoted by  $M_{i,t}^d$ .

Finally, each period the all households in the economy receive monetary transfers from the authorities denoted by  $M_t$ , which follow the law of motion given by  $M_t = \gamma_{t-1} * M_{t-1}$ . For now I assume that money transfers are distributed between the agents of both types equally.

The timing of transactions in this economy is the following. Agents begin the period by buying consumption goods in the goods market using the real money balances held from the previous period. Next they learn the realizations of productivity shocks  $s(t,\omega)$  and monetary shocks  $\gamma$  for the current period. Then they receive their monetary transfers and dividends from shares bought in the previous period and the value of their shareholdings. Consumers use these resources to obtain money balances and shares for the next period.

The preferences of the representative agent of type  $i$  are given by:



$$U^i = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [U(c_{i,t}) - W(l_{i,t})] \right\}$$

where  $l_{i,t}$  stands for the labor input.

For simplicity and tractability I assume that  $W(l_{i,t}) = \frac{l_{i,t}^{1+\varepsilon}}{1+\varepsilon}$ . Utility function is standard

and taken from Giovanni and Labadie(1991) and is given by

$$U(c_{i,t}) = \frac{(c_{i,t})^{1-\sigma}}{1-\sigma}$$

Finally let's denote by  $\pi_t = \frac{1}{P_t}$  the inverse of the price level at time t.

Thus, the representative type i agent chooses stochastic sequences of consumption, labor, money balances and shares of labor earning streams, correspondingly  $\{c_{i,t}, l_{i,t}, M_{i,t+1}^d, z_{i,t+1}^j, z_{i,t+1}^i\}$  to maximize:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [U(c_{i,t}) - W(l_{i,t})] \right\}$$

subject to

$$c_{i,t} \leq M_{i,t}^d \cdot \pi_t \quad (\text{cash-in-advance constraint}) \quad (1)$$

$$M_{i,t+1}^d \cdot \pi_t + Q_t^j \cdot z_{i,t+1}^j + Q_t^i \cdot z_{i,t+1}^i + B_{i,t+1} \cdot \pi_t \leq (Q_t^j + d_{j,t}) \cdot z_{i,t}^j + \theta_{i,t} \cdot l_{i,t} + \frac{(\gamma_t - 1) \cdot M_t}{N_i + N_j} \cdot \pi_t + M_{i,t}^d \cdot \pi_t - c_{i,t} + (Q_t^i + d_{i,t}) \cdot z_{i,t}^i - d_{i,t} \cdot (N_i \cdot z_{i,t}^i + N_j \cdot z_{j,t}^i) + R_t \cdot B_{i,t} \cdot \pi_t$$

$$(\text{current period budget constraint}) \quad (2)$$

$$l_{i,t} \geq 0, \quad c_{i,t} \geq 0 \quad \text{for } i=1,2$$

### 3. Definition and characterization of equilibrium

Value function of this problem is given by the following:

$$V_i(z_i^i, z_i^j, s, B_i, M_i^d) = \max_{\{c_i, l_i, M_i^d, z_i^i, z_i^j\}} [U(c_i) - W(l_i) + \beta * EV(z_i^i, z_i^j, s, B_i, M_i^d)] \quad (3)$$

subject to (1) and (2).

*Stationary recursive equilibrium in this economy is a collection of decision rules*

$$c_i(z_i^i, z_i^j, s, B_i, M_i^d), l_i(z_i^i, z_i^j, s, B_i, M_i^d), M_i^d(z_i^i, z_i^j, s, B_i, M_i^d), z_i^i(z_i^i, z_i^j, s, B_i, M_i^d)$$

$$z_i^j(z_i^i, z_i^j, s, B_i, M_i^d) \text{ and pricing functions } Q^i : Z \times S \rightarrow R_+, Q^j : Z \times S \rightarrow R_+ \text{ and interest}$$

rates  $R_t$  s.t.

1) given prices, decision rules solve (3) subject to (1), (2) and

2) Markets clear:

$$a) N_i \cdot z_i^i + N_j \cdot z_j^i = 1$$

$$N_i \cdot z_i^j + N_j \cdot z_j^j = 1 \text{ (shares market)}$$

$$b) N_i \cdot y_i + N_j \cdot y_j = N_i \cdot c_i + N_j \cdot c_j \text{ (goods' market)}$$

$$\text{where } y_i = \theta_i(s) \cdot l_i(z_i^i, z_i^j, M_i^d, s)$$

$$c) N_i \cdot M_i^d + N_j \cdot M_j^d = M \text{ (money market)}$$

$$d) N_i \cdot B_i + N_j \cdot B_j = 0 \text{ (bonds market)}$$

**The first conditions of the problem for the agent of type i are given by:**

$$(c_i)^\sigma = \lambda_i + \mu_i$$

$$(l_i)^\varepsilon = \lambda_i \cdot \theta_i$$

$$\lambda_i \cdot \pi = \beta E[(\lambda_i^{\cdot} \cdot R^{\cdot} \cdot \pi^{\cdot})]$$

$$\lambda_i \cdot \pi = \beta E[(\lambda_i^{\cdot} + \mu_i^{\cdot}) \cdot \pi^{\cdot}]$$

$$\lambda_i \cdot Q^i = \beta E[\lambda_i^{\cdot} \cdot (Q^{i^{\cdot}} + d_i^{\cdot} - N_i \cdot d_i)]$$

$$\lambda_i \cdot Q^j = \beta E[\lambda_i^{\cdot} \cdot (Q^{j^{\cdot}} + d_j^{\cdot})]$$

where  $x^{\cdot}$  denotes the next period value

If the shares' market clears, all the labor income produced will be distributed through shares so all the output will be used to pay the dividends in equilibrium the following holds

$$d_i = \theta_i \cdot l_i$$

Another point is that under optimal allocation all budget is consumed and thus I can use the budget constraint with equality. And using the last equality and market clearing condition for shares' market I can cancel out some parts which leaves me with

$$c_i = (Q^j + d_j) \cdot z_i^j + \frac{(\gamma - 1) \cdot M}{N_i + N_j} \cdot \pi + (M_i^{d^{\cdot}} - M_i^d) \cdot \pi + (Q^i + d_i) \cdot z_i^i + \\ + R \cdot B_i \cdot \pi - Q^j \cdot z_i^j - Q^i \cdot z_i^i - B_i^{\cdot} \cdot \pi$$

The similar set of conditions can be derived from maximization problem of agent j. Thus the full set of conditions defining the equilibrium in this economy is given by the following:

$$(c_i)^{\sigma} = \lambda_i + \mu_i$$

$$(l_i)^{\varepsilon} = \lambda_i \cdot \theta_i$$

$$\lambda_i \cdot \pi = \beta E[(\lambda_i^{\cdot} \cdot R^{\cdot} \cdot \pi^{\cdot})]$$

$$\lambda_i \cdot \pi = \beta E[(\lambda_i + \mu_i) \cdot \pi]$$

$$\lambda_i \cdot Q^i = \beta E[\lambda_i \cdot (Q^i + d_i - N_i \cdot d_i)]$$

$$\lambda_i \cdot Q^j = \beta E[\lambda_i \cdot (Q^j + d_j)]$$

$$d_i = \theta_i \cdot l_i$$

$$c_i = (Q^j + d_j) \cdot z_i^j + \frac{(\gamma - 1) \cdot M}{N_i + N_j} \cdot \pi + (M_i^d - M_i^d) \cdot \pi + (Q^i + d_i) \cdot z_i^i +$$

$$+ R \cdot B_i \cdot \pi - Q^j \cdot z_i^j - Q^i \cdot z_i^i - B_i \cdot \pi$$

$$(c_j)^\sigma = \lambda_j + \mu_j$$

$$(l_j)^\varepsilon = \lambda_j \cdot \theta_j$$

$$\lambda_j \cdot \pi = \beta E[(\lambda_j \cdot R) \cdot \pi]$$

$$\lambda_j \cdot \pi = \beta E[(\lambda_j + \mu_j) \cdot \pi]$$

$$\lambda_j \cdot Q^j = \beta E[\lambda_j \cdot (Q^j + d_j - N_j \cdot d_j)]$$

$$\lambda_j \cdot Q^i = \beta E[\lambda_j \cdot (Q^i + d_i)]$$

$$d_j = \theta_j \cdot l_j$$

$$c_j = (Q^j + d_j) \cdot z_j^j + \frac{(\gamma - 1) \cdot M}{N_i + N_j} \cdot \pi + (M_j^d - M_j^d) \cdot \pi + (Q^i + d_i) \cdot z_j^i +$$

$$+ R \cdot B_j \cdot \pi - Q^j \cdot z_j^j - Q^i \cdot z_j^i - B_j \cdot \pi$$

$$N_i \cdot z_i^i + N_j \cdot z_j^j = 1$$

$$N_i \cdot z_i^j + N_j \cdot z_j^j = 1$$

$$N_i \cdot M_i^d + N_j \cdot M_j^d = M$$

$$N_i \cdot \theta_i \cdot l_i + N_j \cdot \theta_j \cdot l_j = N_i \cdot c_i + N_j \cdot c_j$$

$$N_i \cdot B_i + N_j \cdot B_j = 0$$

This defines the system of 21 equations in 21 unknowns but it can be further simplified before log-linearizing it and running simulations. After some manipulations the system is presented in the following way:

$$1) \quad \frac{(l_i)^\varepsilon}{\theta_i} \cdot \pi = \beta E \left[ \frac{(l_i')^\varepsilon}{\theta_i'} \cdot R' \cdot \pi' \right]$$

$$2) \quad \frac{(l_i)^\varepsilon}{\theta_i} \cdot \pi = \beta E [(c_i')^\sigma \cdot \pi']$$

$$3) \quad \frac{(l_i)^\varepsilon}{\theta_i} \cdot Q^i = \beta E \left[ \frac{(l_i')^\varepsilon}{\theta_i'} \cdot (Q^{i'} + \theta_i' \cdot l_i' - N_i \cdot \theta_i \cdot l_i) \right]$$

$$4) \quad \frac{(l_i)^\varepsilon}{\theta_i} \cdot Q^j = \beta E \left[ \frac{(l_i')^\varepsilon}{\theta_i'} \cdot (Q^{j'} + \theta_j' \cdot l_j') \right]$$

5)

$$c_i = (Q^j + \theta_j \cdot l_j) \cdot z_i^j + \frac{(\gamma - 1) \cdot M}{N_i + N_j} \cdot \pi + (M_i^d - M_i^d) \cdot \pi + (Q^i + \theta_i \cdot l_i) \cdot z_i^i + \cdot \\ + R \cdot B_i \cdot \pi - Q^j \cdot z_i^j - Q^i \cdot z_i^i - B_i' \cdot \pi$$

$$6) \quad \frac{(l_j)^\varepsilon}{\theta_j} \cdot \pi = \beta E \left[ \frac{(l_j')^\varepsilon}{\theta_j'} \cdot R \cdot \pi' \right]$$

$$7) \quad \frac{(l_j)^\varepsilon}{\theta_j} \cdot \pi = \beta E [(c_j')^\sigma \cdot \pi']$$

$$8) \quad \frac{(l_j)^\varepsilon}{\theta_j} \cdot Q^j = \beta E \left[ \frac{(l_j')^\varepsilon}{\theta_j'} \cdot (Q^{j'} + \theta_j' \cdot l_j' - N_j \cdot \theta_j \cdot l_j) \right]$$

$$9) \quad \frac{(l_j)^\varepsilon}{\theta_j} \cdot Q^i = \beta E \left[ \frac{(l_j')^\varepsilon}{\theta_j'} \cdot (Q^{i'} + \theta_i' \cdot l_i') \right]$$

10)

$$c_j = (Q^j + \theta_j \cdot l_j) \cdot z_j^j + \frac{(\gamma - 1) \cdot M}{N_i + N_j} \cdot \pi + (M_j^d - M_j^d) \cdot \pi + (Q^i + \theta_i \cdot l_i) \cdot z_j^i + \\ + R \cdot B_j \cdot \pi - Q^j \cdot z_j^j - Q^i \cdot z_j^i - B_j \cdot \pi$$

$$11) \quad N_i \cdot z_i^i + N_j \cdot z_j^i = 1$$

$$12) \quad N_i \cdot z_i^j + N_j \cdot z_j^j = 1$$

$$13) \quad N_i \cdot M_i^d + N_j \cdot M_j^d = M$$

$$14) \quad N_i \cdot \theta_i \cdot l_i + N_j \cdot \theta_j \cdot l_j = N_i \cdot c_i + N_j \cdot c_j$$

Thus after some simplification I arrive at a system of 14 equations with 14 unknowns

$(c_i, c_j, Q^i, Q^j, l_i, l_j, z_i^i, z_i^j, z_j^i, z_j^j, M_i^d, M_j^d, B_i, B_j)$ . The next step is to derive

deterministic steady state and loglinearize the model around this steady state.

The steady state of the model is given by the condition that for all variables  $x_i = x_i^{ss}$ .

Using this one can derive steady state given by the following conditions:

$$R^{ss} = \frac{1}{\beta}$$

$$\frac{(l_i^{ss})^\varepsilon}{\theta_i^{ss}} = \beta \cdot (c_i^{ss})^\sigma$$

$$Q^{i,ss} = \frac{(\theta_i^{ss} \cdot l_i^{ss} - N_i \cdot \theta_i^{ss} \cdot l_i^{ss})}{1 - \beta}$$

$$Q^{j,ss} = \frac{(\theta_j^{ss} \cdot l_j^{ss} - N_j \cdot \theta_j^{ss} \cdot l_j^{ss})}{1 - \beta}$$

$$c_i^{ss} = \theta_j^{ss} \cdot l_j^{ss} \cdot z_i^{j,ss} + \theta_i^{ss} \cdot l_i^{ss} \cdot z_i^{i,ss} + (R^{ss} - 1) B_i^{ss} \cdot \pi^{ss}$$

$$\frac{(l_j^{ss})^\varepsilon}{\theta_j^{ss}} = \beta \cdot (c_j^{ss})^\sigma$$

$$c_j^{ss} = \theta_j^{ss} \cdot l_j^{ss} \cdot z_j^{j,ss} + \theta_i^{ss} \cdot l_i^{ss} \cdot z_j^{i,ss} + (R^{ss} - 1) \cdot B_j^{ss} \cdot \pi^{ss}$$

$$N_i \cdot z_i^{i,ss} + N_j \cdot z_j^{i,ss} = 1$$

$$N_i \cdot z_j^{i,ss} + N_j \cdot z_j^{j,ss} = 1$$

$$N_i \cdot M_i^{d,ss} + N_j \cdot M_j^{d,ss} = M$$

$$N_i \cdot \theta_i^{ss} \cdot l_i^{ss} + N_j \cdot \theta_j^{ss} \cdot l_j^{ss} = N_i \cdot c_i^{ss} + N_j \cdot c_j^{ss}$$

$$N_i \cdot B_i^{ss} + N_j \cdot B_j^{ss} = 0$$

There are also two other conditions which are coming from the fact that in a steady with

positive interest rate there is no sense for the households to hold more money balances than they need to buy consumption since this implies positive opportunity cost for them. In other words cash in advance constraint can be used with equality and thus:

$$c^{j,ss} = M_j^{d,ss} \cdot \pi^{ss}$$

$$c^{i,ss} = M_i^{d,ss} \cdot \pi^{ss}$$

Also in the steady state budget constraints reduce to the forms given by 5-th and 7-th formula since I am looking for stationary solution in case of which money holding of households change only because of the money printing of monetary authority and thus the transfer received by each individual in steady state should be equal to the change in money holdings. And by the same logic of stationary steady state product of shares with prices on both sides of the budget constraint as well as bonds cancel out thus leaving one with the given equations. Thus steady state values are defined by the given system of steady state equations.

Last step of solving this model is deriving log-linearized equations which will make the model ready for simulations. The system of log-linearized equations is given by the following:

$$1) \quad \varepsilon \cdot \tilde{l}_{i,t} - \tilde{\theta}_{i,t} + \tilde{\pi}_t = E_t[\tilde{R}_{t+1} + \tilde{\pi}_{t+1} + \varepsilon \cdot \tilde{l}_{i,t+1} - \tilde{\theta}_{i,t+1}]$$

$$2) \quad \varepsilon \cdot \tilde{l}_{i,t} - \tilde{\theta}_{i,t} + \tilde{\pi}_t = E_t[\tilde{\pi}_{t+1} + \sigma \cdot \tilde{c}_{i,t+1}]$$

3)

$$\varepsilon \cdot \tilde{l}_{i,t} - \tilde{\theta}_{i,t} + \tilde{Q}_t^i = E_t[\varepsilon \cdot \tilde{l}_{i,t+1} - \tilde{\theta}_{i,t+1} + \frac{Q_i^{ss}}{Q_i^{ss} + \theta_i^{ss} \cdot l_i^{ss} - N_i \cdot \theta_i^{ss} \cdot l_i^{ss}} \cdot \tilde{Q}_{i,t+1} \frac{\theta_i^{ss} \cdot l_i^{ss}}{Q_i^{ss} + \theta_i^{ss} \cdot l_i^{ss}} \cdot (\tilde{l}_{i,t+1} + \tilde{\theta}_{i,t+1}) - \frac{N_i \cdot \theta_i^{ss} \cdot l_i^{ss}}{Q_i^{ss} + \theta_i^{ss} \cdot l_i^{ss} - N_i \cdot \theta_i^{ss} \cdot l_i^{ss}} \cdot (\tilde{l}_{i,t} - \tilde{\theta}_{i,t})]$$



4)

$$\varepsilon \cdot \tilde{l}_{i,t} - \tilde{\theta}_{i,t} + \tilde{Q}_t^j = E_t[\varepsilon \cdot \tilde{l}_{i,t+1} - \tilde{\theta}_{i,t+1} + \frac{Q_j^{ss}}{Q_j^{ss} + \theta_j^{ss} \cdot l_j^{ss}} \cdot \tilde{Q}_{j,t+1} \frac{\theta_j^{ss} \cdot l_j^{ss}}{Q_j^{ss} + \theta_j^{ss} \cdot l_j^{ss}} \cdot (\tilde{l}_{j,t+1} + \tilde{\theta}_{j,t+1})]$$

5)

$$\begin{aligned} c_i^{ss} \cdot \tilde{c}_i &= Q^{j,ss} \cdot z_i^{j,ss} \cdot (\tilde{z}_{i,t}^j - \tilde{z}_{i,t+1}^j) + \theta_j^{ss} \cdot l_j^{ss} \cdot z_i^{j,ss} \cdot (\tilde{\theta}_{j,t} + \tilde{l}_{j,t} + \tilde{z}_{i,t}^j) + \frac{\gamma-1}{N_i + N_j} \cdot M^{ss} \cdot \pi^{ss} \cdot (\tilde{M}_t + \tilde{\pi}_t) + \\ &+ M_i^{d,ss} \cdot \pi^{ss} \cdot (\tilde{M}_{i,t+1}^d - \tilde{M}_{i,t}^d) + Q^{i,ss} \cdot z_i^{i,ss} \cdot (\tilde{z}_{i,t}^i - \tilde{z}_{i,t+1}^i) + \theta_i^{ss} \cdot l_i^{ss} \cdot z_i^{i,ss} \cdot (\tilde{\theta}_{i,t} + \tilde{l}_{i,t} + \tilde{z}_{i,t}^i) + R^{ss} \cdot B_i^{ss} \cdot \pi^{ss} \cdot (\tilde{R}_t + \tilde{B}_{i,t} + \tilde{\pi}_t) - \\ &- B_i^{ss} \cdot \pi^{ss} \cdot (\tilde{B}_{i,t+1} + \tilde{\pi}_t) \end{aligned}$$

$$6) \varepsilon \cdot \tilde{l}_{j,t} - \tilde{\theta}_{j,t} + \tilde{\pi}_t = E_t[\tilde{R}_{t+1} + \tilde{\pi}_{t+1} + \varepsilon \cdot \tilde{l}_{j,t+1} - \tilde{\theta}_{j,t+1}]$$

$$7) \varepsilon \cdot \tilde{l}_{j,t} - \tilde{\theta}_{j,t} + \tilde{\pi}_t = E_t[\tilde{\pi}_{t+1} + \sigma \cdot \tilde{c}_{j,t+1}]$$

8)

$$\begin{aligned} \varepsilon \cdot \tilde{l}_{j,t} - \tilde{\theta}_{j,t} + \tilde{Q}_t^j &= E_t[\varepsilon \cdot \tilde{l}_{j,t+1} - \tilde{\theta}_{j,t+1} + \frac{Q_j^{ss}}{Q_j^{ss} + \theta_j^{ss} \cdot l_j^{ss} - N_j \cdot \theta_j^{ss} \cdot l_j^{ss}} \cdot \tilde{Q}_{j,t+1} \frac{\theta_j^{ss} \cdot l_j^{ss}}{Q_j^{ss} + \theta_j^{ss} \cdot l_j^{ss}} \cdot (\tilde{l}_{j,t+1} + \tilde{\theta}_{j,t+1}) - \\ &- \frac{N_j \cdot \theta_j^{ss} \cdot l_j^{ss}}{Q_j^{ss} + \theta_j^{ss} \cdot l_j^{ss} - N_j \cdot \theta_j^{ss} \cdot l_j^{ss}} \cdot (\tilde{l}_{j,t} - \tilde{\theta}_{j,t})] \end{aligned}$$

9)

$$\varepsilon \cdot \tilde{l}_{j,t} - \tilde{\theta}_{j,t} + \tilde{Q}_t^i = E_t[\varepsilon \cdot \tilde{l}_{j,t+1} - \tilde{\theta}_{j,t+1} + \frac{Q_i^{ss}}{Q_i^{ss} + \theta_i^{ss} \cdot l_i^{ss}} \cdot \tilde{Q}_{i,t+1} \frac{\theta_i^{ss} \cdot l_i^{ss}}{Q_i^{ss} + \theta_i^{ss} \cdot l_i^{ss}} \cdot (\tilde{l}_{i,t+1} + \tilde{\theta}_{i,t+1})]$$

10)

$$\begin{aligned} c_j^{ss} \cdot \tilde{c}_j &= Q^{j,ss} \cdot z_j^{j,ss} \cdot (\tilde{z}_{j,t}^j - \tilde{z}_{j,t+1}^j) + \theta_j^{ss} \cdot l_j^{ss} \cdot z_j^{j,ss} \cdot (\tilde{\theta}_{j,t} + \tilde{l}_{j,t} + \tilde{z}_{j,t}^j) + \frac{\gamma-1}{N_i + N_j} \cdot M^{ss} \cdot \pi^{ss} \cdot (\tilde{M}_t + \tilde{\pi}_t) + \\ &+ M_j^{d,ss} \cdot \pi^{ss} \cdot (\tilde{M}_{j,t+1}^d - \tilde{M}_{j,t}^d) + Q^{i,ss} \cdot z_j^{i,ss} \cdot (\tilde{z}_{j,t}^i - \tilde{z}_{j,t+1}^i) + \theta_i^{ss} \cdot l_i^{ss} \cdot z_j^{i,ss} \cdot (\tilde{\theta}_{i,t} + \tilde{l}_{i,t} + \tilde{z}_{j,t}^i) + R^{ss} \cdot B_j^{ss} \cdot \pi^{ss} \cdot (\tilde{R}_t + \tilde{B}_{j,t} + \tilde{\pi}_t) - \\ &- B_j^{ss} \cdot \pi^{ss} \cdot (\tilde{B}_{j,t+1} + \tilde{\pi}_t) \end{aligned}$$

$$11) \quad N_i \cdot z_i^{i,ss} \cdot \tilde{z}_i^i = -N_j z_j^{i,ss} \cdot \tilde{z}_j^i$$

$$12) \quad N_i \cdot z_i^{j,ss} \cdot \tilde{z}_i^j = -N_j z_j^{j,ss} \cdot \tilde{z}_j^j$$

$$13) \quad N_i \cdot M_i^{d,ss} \cdot \tilde{M}_{i,t+1}^d + N_j \cdot M_j^{d,ss} \cdot \tilde{M}_{j,t+1}^d = M^{ss} \cdot \tilde{M}_t$$

$$14) \quad N_i \cdot \theta_i^{ss} \cdot l_i^{ss} \cdot (\tilde{\theta}_{i,t} + \tilde{l}_{i,t}) + N_j \cdot \theta_j^{ss} \cdot l_j^{ss} \cdot (\tilde{\theta}_{j,t} + \tilde{l}_{j,t}) = N_i \cdot c_i^{ss} \cdot \tilde{c}_{i,t} + N_j \cdot c_j^{ss} \cdot \tilde{c}_{j,t}$$

where  $\tilde{x}$  denotes the deviation of the variable from steady state.

## 4. Simulations and Results

After having derived the log-linearized equations 1-14 which fully describe the economy the next step is to simulate the model for different values of stochastic states (shocks). In my model there are two kinds of shocks namely idiosyncratic productivity shocks ( $\theta_1, \theta_2$ ) and monetary shock ( $\gamma$ ). Another stochastic variable is  $M$  that is amount of money printed but information about it is embodied in  $\gamma$ . Similar to Giovanni and Labadie(1991) I use Tauchen's quadrature method for fitting Markov process by discretizing the state space, that is space of  $(\theta_1, \theta_2, \gamma)$ . Tauchen's quadrature allows to construct discrete probability model that approximates conditional density function over the state space. The grid points and discrete probability weights in this method are chosen optimally using method of moments. Thus the method gives conditional transition probabilities and a state-space grid for  $(\theta_1, \theta_2, \gamma)$ . The specific of my model is that each of the  $\theta$ s can take on only two values that is  $a$  corresponding to high productivity state and  $f$  corresponding to low productivity state. Thus through Tauchens quadrature I get several grid points with a values which  $a$  and  $f$  can take on, but preserving the inequality  $a > f$  for each point. In my simulation I use 16

realizations corresponding to low productivity state and 16 realizations corresponding to high productivity state. Also since each period two productivity shocks ( $\theta_1, \theta_2$ ) are hitting the economy and for each of them there are 32 possible realizations this gives 64 possible realizations for productivity shocks. Also I use 32 possible realizations of monetary shocks which in total gives 96 possible realizations of both monetary and productivity shocks. I simulate the model for large number of agents of both types setting  $N_i = N_j = 1000$ . Also I simulate the model for 10 combinations of remaining parameters, namely discount rate ( $\beta$ ), relative risk aversion ( $\gamma$ ) and parameter defining the disutility from labor in the utility function ( $\epsilon$ ).

Using the transition matrices, state vectors and different combinations of parameters and simulating the log-linearized equations 1-14 by Matlab (using SimulEditor software to generate the code) I get a series of realizations of all endogenous variables. This series is then used to study the properties of nominal stock returns nominal interest rates and inflation generated by the model.

#### 4a) Equity premium

The first of regularities discussed in the introduction was the behavior of the equity premium. Equity premium is defined as the difference between the rate of return on risky asset and the risk free rate of return. For calculating asset returns let's rewrite equations (1),(4) and (9) can be rewritten in the following way:

$$1 = \beta \cdot E_t \left[ \frac{(l_{1,t+1})^\epsilon \cdot (\theta_{1,t+1})^{-1}}{(l_{1,t})^\epsilon \cdot (\theta_{1,t})^{-1}} \cdot R_{t+1}^n \right]$$

$$1 = \beta \cdot E_t \left[ \frac{(l_{1,t+1})^\epsilon \cdot (\theta_{1,t+1})^{-1}}{(l_{1,t})^\epsilon \cdot (\theta_{1,t})^{-1}} \cdot R_{2,t+1}^r \right]$$

$$1 = \beta \cdot E_t \left[ \frac{(l_{2,t+1})^\varepsilon \cdot (\theta_{2,t+1})^{-1}}{(l_{2,t})^\varepsilon \cdot (\theta_{2,t})^{-1}} \cdot R_{1,t+1}^r \right]$$

where  $R_{1,t+1}^r$ ,  $R_{2,t+1}^r$  and  $R_{t+1}^n$  are real rates of return correspondingly on stocks of labor earnings of type 1 and type 2 agent and nominal bonds. In similar way one can derive risk free rate that is rate of return on indexed bonds which will be given by reciprocal of marginal rate of substitution between future and current wealth. Then using the above formulas one can write equity premium as:

$$E_t[R_{t+1}^n] - R_t^f = - \frac{\text{cov} \left[ \frac{(l_{i,t+1})^\varepsilon \cdot (\theta_{i,t+1})^{-1}}{(l_{i,t})^\varepsilon \cdot (\theta_{i,t})^{-1}}, \frac{R_{t+1} \cdot \pi_{t+1}}{\pi_t} \right]}{E_t \left[ \frac{(l_{i,t+1})^\varepsilon \cdot (\theta_{i,t+1})^{-1}}{(l_{i,t})^\varepsilon \cdot (\theta_{i,t})^{-1}} \right]}$$

$$E_t[R_{2,t+1}^r] - R_t^f = - \frac{\text{cov} \left[ \frac{(l_{1,t+1})^\varepsilon \cdot (\theta_{1,t+1})^{-1}}{(l_{1,t})^\varepsilon \cdot (\theta_{1,t})^{-1}}, R_{2,t+1}^r \right]}{E_t \left[ \frac{(l_{1,t+1})^\varepsilon \cdot (\theta_{1,t+1})^{-1}}{(l_{1,t})^\varepsilon \cdot (\theta_{1,t})^{-1}} \right]}$$

$$E_t[R_{1,t+1}^r] - R_t^f = - \frac{\text{cov} \left[ \frac{(l_{2,t+1})^\varepsilon \cdot (\theta_{2,t+1})^{-1}}{(l_{2,t})^\varepsilon \cdot (\theta_{2,t})^{-1}}, R_{1,t+1}^r \right]}{E_t \left[ \frac{(l_{2,t+1})^\varepsilon \cdot (\theta_{2,t+1})^{-1}}{(l_{2,t})^\varepsilon \cdot (\theta_{2,t})^{-1}} \right]}$$

where the first equation defines the equity premium for nominal bonds and R is gross nominal interest rate ,the second equation defines the equity for the stocks of the agents of type 2 and the third equation defines the equity premium stocks of the agent of type 1.

Now using the series received from simulations of the model and the last three equations I can calculate the equity premium for each asset. To simplify the exposition when getting the real rate of return for each type of the stocks I take their average to approximate the economy wide rate of returns on stocks. Assumption about the equal number of agents of each type in the economy allows for such simplification. Table 1 summarizes the sample means of real stock returns, real interest rates and equity premium obtained both from actual data (taken from Giovanni and Labadie (1991)) and simulations of the model presented in this paper. For comparison I also include the same statistics from the Giovanni and Labadie results:

TABLE 1: Sample means of stock returns interest rates and resulting equity premium (the data is the same set as in Giovanni and Labadie (1991))

RETURNS			
	Real returns on stocks(average)	Real interest rate	Equity premium
Data	7,34	1,02	6,32
My model ( $\rho, \varepsilon, \sigma$ )			
(0,01;0,2;0,5)	4,96	1,46	3,5
(0,01;0,4;1)	6,2	2,48	3,72
(0,01;0,6;2)	8,67	4,43	4,23
(0,01;0,8;5)	14,65	10,28	4,37
(0,01;0,9;10)	24,95	18,92	6,03
(0,03;0,2;0,5)	6,78	3,68	3,1
(0,03;0,4;1)	8,23	4,71	3,52
(0,03;0,6;2)	10,61	6,78	3,83
(0,03;0,8;5)	17,42	12,67	4,65
(0,03;0,9;10)	24,18	21,5	2,68
Benchmark model ( $\rho, \gamma$ )			
(0,01;0,5)	2,67	1,44	1,23
(0,01;1)	3,55	2,45	1,1
(0,01;2)	5,3	4,46	0,85
(0,01;5)	10,35	10,22	0,12
(0,01;10)	17,69	18,8	-1,11
(0,03;0,5)	4,71	3,65	1,05
(0,03;1)	5,62	4,69	0,94
(0,03;2)	7,44	6,73	0,71

(0,03;5)	12,67	12,64	0,03
(0,03;10)	20,32	21,49	-1,17

$$* \rho = \frac{1}{\beta} - 1 ,$$

\*\*  $\gamma$  is the risk aversion parameter in the benchmark model and the benchmark model is the cash-in-advance Swenson timing model simulated in Giovanni and Labadie (1991)

The table demonstrates that according to my expectations modified model performs better in matching equity premium. The heterogeneity and incomplete market structure incorporated into the model does not affect much the real interest rate since this variable is primarily affected by monetary shocks. But incomplete markets environment increases uncertainty in the economy and thus the riskiness of stocks which drives up their rate of return. Thus, under all combinations of parameters heterogeneous agent model generates equity premium closer to the one observed in reality, than its representative agent counterpart. Another point is whether model presented here exactly matches actual equity premium. The above table shows that the model presented with relatively simple heterogeneous structure is not able to fully match the equity premium. It comes close to matching observed equity premium under low  $\rho$  and quite relative risk aversion (equal to 10). This is rather extreme parametrization taking into the fact that relative risk aversion observed in reality is around 2.

In summary the heterogeneous agent model presented here performs better in matching the actual equity premium than the standard representative agent model, but it still does not

provide the perfect match to the data. Most probably the model with more realistic structure having continuous distribution of heterogeneous agents can provide better results, but computationally it is much more demanding and is left for the future research.

## **b) Correlation between inflation and ex-post (realized) real stock returns and real interest rates**

Another important issue is how the model performs with respect to observed correlations between realized inflation and real stocks returns and real interest rates. Giovanni and Labadie (1991) found that the benchmark model provides only occasional negative correlation between realized inflation and realized real returns on stocks as well as inflation and realized real interest rates, and the correlation coefficients are not big in magnitude. However in the actual data both of these correlations are marked and persistent and coefficients are high.

Table 2 presents the results of regressing nominal stock returns obtained from simulating the model on inflation rates as well as the results of regressing realized real interest rates on inflation. For comparison I also repeat the results obtained in Giovanni and Labadie (1991). The results show that heterogeneous agent model provides a better fit to the data also in this respect. The model provides persistent negative correlations between inflation and real interest rates as well as inflation and real returns on stocks. Also the correlation coefficients are quite close to the one obtained from actual data nearly for all combinations of parameters which indicates that the model does quite a good job in matching regularities concerning inflation real interest rates and real return on stocks.

**TABLE2 Correlation between inflation and real interest rates and  
inflation and real return on stocks**

	Real returns on stocks	Real interest rate
	Slope Coefficient(Rsquared)	Slope coefficeint(Rsquared)
Data	-0.78(0.06)	-0.93(0.76)
My model ( $\rho, \epsilon, \sigma$ )		
(0,01;0,2;0,5)	-0.65(0.05)	-0.82(0.65)
(0,01;0,4;1)	-0.68(0.05)	-0.85(0.74)
(0,01;0,6;2)	-0.68(0.057)	-0.92(0.69)
(0,01;0,8;5)	-0.75(0.06)	-0.75(0,79)
(0,01;0,9;10)	-0.90(0.04)	-0.86(0.57)
(0,03;0,2;0,5)	-0.73(0.08)	-0.85(0.73)
(0,03;0,4;1)	-0.63(0.15)	-0.74(0.72)
(0,03;0,6;2)	-0.68(0.12)	-0.90(0.76)
(0,03;0,8;5)	-0.71(0.18)	-0.81(0,79)
(0,03;0,9;10)	-0.6(0.04)	-0.67(0.57)
Benchmark model ( $\rho, \gamma$ )		
(0,01;0,5)	-0.12(0.02)	-0.51(0.69)
(0,01;1)	-0.12(0.02)	-0.46(0.59)
(0,01;2)	-0.11(0.03)	-0.36(0.37)
(0,01;5)	-0.12(0.03)	-0.16(0.04)
(0,01;10)	0.52(0.11)	-0.31(0.05)
(0,03;0,5)	-0.22(0.06)	-0.42(0.64)
(0,03;1)	-0.19(0.05)	-0.37(0.52)
(0,03;2)	-0.13(0.04)	-0.26(0.26)
(0,03;5)	0.01(0.00)	0.03(0.00)
(0,03;10)	0.28(0.03)	-0.01(0.00)

$$* \rho = \frac{1}{\beta} - 1 ,$$

\*\*  $\gamma$  is the risk aversion parameter in the benchmark model and the benchmark model is the cash-in-advance Swenson timing model simulated in Giovanni and Labadie (1991)

### c) Interest rates as the predictors of inflation and nominal stock returns

Finally the last important test of the model is the ability of nominal interest rates to predict inflation and nominal stock returns. The intuition that because of increased riskiness of



stock and increased volatility of stock returns the close link between nominal interest rates and nominal returns on stocks will break up is confirmed by the results shown in table 3. In contrary to the standard model nominal interest are rather poor predictors of nominal interest rates as it is the case in the data. The situation with inflation is similar though the nominal interest rates are still better predictors of inflation than it is in the data. Thus the model is not able to match data in this respect but still performs better than the representative agent model.

**TABLE3 Correlation between inflation and real interest rates and inflation and real return on stocks**

	Inflation	Nominal stock returns
	Slope Coefficient(Rsquared)	Slope coefficeint(Rsquared)
Data	0.24(0.02)	-0.20(0.00)
My model ( $\rho, \varepsilon, \sigma$ )		
(0,01;0,2;0,5)	0.45(0.15)	-0.23(0.02)
(0,01;0,4;1)	0.31(0.13)	-0.15(0.05)
(0,01;0,6;2)	0.39(0.22)	-0.1(0.001)
(0,01;0,8;5)	0.16(0.09)	-0.3(0,02)
(0,01;0,9;10)	0.4(0.21)	-0.26(0.001)
(0,03;0,2;0,5)	0.35(0.12)	-0.16(0.06)
(0,03;0,4;1)	0.41(0.15)	-0.11(0.01)
(0,03;0,6;2)	0.32(0.08)	-0.14(0.008)
(0,03;0,8;5)	0.26(0.24)	-0.22(0,03)
(0,03;0,9;10)	0.33(0.18)	-0.13(0.002)
Benchmark model ( $\rho, \gamma$ )		
(0,01;0,5)	1.37(0.67)	0.96(0.20)
(0,01;1)	1.23(0.66)	0.96(0.29)
(0,01;2)	1.01(0.64)	0.96(0.51)
(0,01;5)	0.62(0.52)	0.95(0.96)
(0,01;10)	0.31(0.21)	0.93(0.82)
(0,03;0,5)	1.33(0.76)	0.98(0.29)
(0,03;1)	1.20(0.76)	0.98(0.40)
(0,03;2)	1.00(0.74)	0.97(0.64)
(0,03;5)	0.63(0.65)	0.96(0.98)
(0,03;10)	0.33(0.32)	0.94(0.82)

$$* \rho = \frac{1}{\beta} - 1 ,$$

## 5) Conclusion

In this research I have explored the properties of stock returns, interest rates and inflation in a cash-in-advance asset pricing model modified to include heterogeneous agents and incomplete markets. Based on representative agent cash-in-advance asset pricing model with Swenson timing, I have built the economy with two types of heterogeneous agents subject to idiosyncratic productivity shocks. Afterwards I have solved and simulated the model and explored how the realizations of stock returns interest rates and inflation obtained from this model fit the properties of actual data underlined in Giovanni and Labadie (1991). I have found that heterogeneous model performs better in matching empirical regularities concerning interest rates stock returns and inflation than the representative agent model tested in Giovanni and Labadie (1991) and in some aspects comes quite close to matching properties of actual data. In particular the model generates higher equity premium than the representative agent model, provides quite significant and persistent correlations between inflation and ex-post (realized) real stock returns and inflation and real interest rates and gives coefficient of correlation quite close to those observed in the data. Finally, nominal rates are no longer good predictors nominal stock returns which is also consistent with data.

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