The Origins of Global Imbalances

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In this paper we study the endogenous response of unequally developed regions to a drop in investment and trade costs in a general equilibrium model. The response is characterized by a rise in foreign direct investment in underdeveloped region and increased consumption in the developed one, leading to trade imbalances between regions. We hereby propose that the investment and trade costs decline has the potential to originate this century global imbalances.

KEYWORDS: global imbalances, export, FDI.

1. INTRODUCTION

The already large and growing literature on current global imbalances identifies the following roots of their formation: (1) macroeconomic policies and technological structural breaks in the U.S. (Bems et al., 2007; Roubini and Setser, 2005) leading to excess liquidity and housing boom; (2) global saving glut (Bernanke, 2005) partly due to inadequate provision of health and social insurance in emerging Asia; (3) world shortage of financial assets (Caballero et al., 2006) or differences in quality of assets across regions (Mendosa et al., 2007) leading to increased demand for the U.S. assets (the share of U.S. assets in financial wealth of the rest of the world has tripled over a decade: from 6-7% in early 1990’s to 17-18% at the beginning of 2000’s), and (4) the exchange rate policy in emerging Asia (Ahearne et al., 2007, Obstfeld, Rogoff (2005)) leading to foreign reserves accumulation in Asia and distortions in international trade competitiveness.

However, in our view these factors did play an important role in deepening the imbalances but account rather for secondary factors, since they were present long before the start of the current episode of current global imbalances (such as high propensity to save in Asia or sound financial system in the U.S.). Surely, macroeconomic policies played a role but the current episode of imbalances seems to be much more resilient to the standard adjustment mechanism (the exchange rate changes) that worked well in the previous episode from the 1980s. This leads us to search for a more fundamental and structural changes that could trigger today’s global imbalances.
According to our view, and some other authors - see Dooley et al. (2004), the primary factors that stood behind the origination of this century global imbalances are the global integration forces represented by the decline in investment and trade costs between developed and developing world regions.

The current episode of global imbalances is distinct from the previous ones in some particularly important aspects. In contrast to the imbalances in the 1980s, when equally developed and financially relatively independent, at least from the point of view of foreign direct investment, world regions were involved (the U.S., Europe, and Japan), the current episode involves mainly the U.S. and emerging Asia. As of 2007, the U.S. current account deficit accounts for 80% of the world current account deficit - countries that record current account deficit. While China plus Japan and oil exporting countries plus Russia account for 40% and 50% of the world current account surplus, respectively. Thus, it concerns unequally developed and the fastest growing world regions (by respective regional standards) with tight portfolio and foreign direct investment linkages. For instance, in 2002-2006, China reported 9.9% vs. 8.0% of Emerging Asia, Russia recorded 6.4% vs. 5.3% of Central and Eastern Europe, and the U.S. grew 2.9% compared to 2.5% of Advanced Countries.

China and in fact substantial part of emerging Asia introduced ‘new policies’ only after the 1980s. Following the Maoist policies in the 1960-1970s, China opened for rapid science and technology development. The search for exploiting ‘capabilities’ - a combination of cheap labor and technological advantage, as Sutton (2007) puts it, resulted in a massive and signifying inflow of foreign direct investment from developed countries, in particular led by the U.S. Continuously declining duties and transportation costs had added additional momentum for the regional development. Between 1980-2001, the total trade costs (duties and transportation costs) had fallen from 11 to 5% of customs value, see Baier, Bergstrand (2001). Similarly, Gust et al. (2006) report that, since the late 1980s, tariffs had fallen by 3% in developed countries and by 10% in developing countries.

In order to investigate whether a fall in investment and trade costs between unequally developed world regions could trigger macroeconomic ad-
justments similar to the current global imbalances, we develop a three regions general equilibrium model. In particular, we applied the two-country framework of Ghironi, Melitz (2005), extended by Bruha, Podpiera (2007), which by construction allows for simulation of endogenous effects from investment and trade cost declines, to a three regions setting. Our modeling framework thus can be viewed as a model with trade and cross border asset ownership, such as by Caballero et al. (2006), enriched by heterogeneous firms and investment and trade costs.

In our exercise we compare two steady states, i.e. before and after the drop in investment and trade costs of two relatively equally developed regions (the U.S., and the Eurozone and Japan) where the U.S. total factor productivity is higher than in the Eurozone and Japan and the one underdeveloped region (emerging Asia, where the total factor productivity is initially low). We find that the endogenous mechanisms act in an intuitive direction. The developed fast growing country would invest via foreign direct investment in the underdeveloped region, while would increase consumption, leading to the current global trade imbalances between these regions. This could explain the world development after the 1980s when the U.S. invested in the emerging Asia and its own growth was driven by consumption, while the source of growth in emerging Asia was the net export. In relation to the slower growing developed region (the Eurozone and Japan), the direct investment to emerging Asia was significant but due to slow growth of consumption, the imbalances were very moderate.

The slow-growth developed countries experience less trade imbalances than fast growing developed countries from a financial and trade world liberalization (world production, input, and product market integration). From the policy implications point of view, to the extent the world integration stands behind the global trade imbalances, the imbalances are natural convergence symptoms after liberalization of investment and trade that will eventually end with more balanced economic development across world regions. Our findings thus support the argument pursued by Richardson (1995) about trade liberalization reducing inequality.

In the remainder of this paper, we present first the three regions model,
which is followed by its calibration. We then describe the results of simulations and summarize the major findings in conclusion.

2. MODEL

The model is a discrete-time perfect-foresight dynamic general equilibrium model. The model consists of three regions, the region $U$ stands for the U.S., the region $A$ is the emerging Asian countries and the region $E$ is Eurozone and Japan. The model is a multi-regional version of the model by Ghironi, Melitz (2005) or Brüha, Podpiera (2007).

2.1. Households

Each region is populated by a representative competitive household who has recursive preferences over discounted stochastic streams of period utilities. The period utilities are derived from consumption of the region $i$ final good $C_i^t$. Labor is immobile factor and is supplied inelastically, we normalize the labor supply in each region to unity. The labor-market clearing condition (see section 2.3) then determines the real wage $\bar{w}^i$.

The intertemporal utility function:

$$\bar{U}_t = \sum_{\tau \geq 0} \beta^\tau u(C_{t+\tau}^i),$$

where $0 < \beta < 1$, and $u$ is the increasing and concave momentary function, then determines the intertemporal pricing kernel:

$$\mathcal{K}_{it}^\tau = \beta^\tau u'(C_{t+\tau}^i)/u'(C_t^i).$$

The households can invest in the following two types of instruments:

- An internationally traded real bonds (denominated in the currency of - say - $U$ region); we denote $B_{it}$ the holding of the bond by the household $i$ at time $t$. The real interest rate (in $U$ currency) is denoted as $r^U$.
- Stocks of domestic and foreign firms. We denote by $n_{ij}^t$ the number of firms, which are located in the region $i$ and owned by the household $j$. 
Households face the quadratic adjustment costs when investing in either type of asset. The adjustment costs are denominated in the asset currency. The household budget constraint reads as

\[ \eta U_i B_t = (1+r_{t-1}U)\eta U_i B_{t-1} + (\Psi i - C_i) + \sum_j \left[ \eta^{ji} \left( n^{ji} \tilde{P}^{ji} - \frac{\Psi^{ji}}{2} \left( \nu^{ji} \right)^2 \right) \right] - \eta U_i \Psi B^2 B^2_t + T_i, \]

where \( \eta^{ji} \) is the real exchange rate between currencies \( i \) and \( j \), \( C_i \) is the real consumption of the household \( i \), \( \tilde{P}^{ji} \) is the expected real profit of a \((i, j)\) firm\(^1\) (profits and costs are denominated in the currency of the firm’s location), \( n^{ji} \) is the stock of \((i, j)\) firms, \( \nu^{ji} \) are new entrants and \( T_i \) are lump-sum payments of all adjustment costs to the household \( i \).

Using the definition of the pricing kernel, the asset portfolio held by the household \( i \) satisfies:

\[ \nu^{ji} = \frac{1}{\Psi^{ji}} \left[ \sum_{\tau \geq 0} (1-\delta)^\tau K^{\tau} n^{ji}_{t+\tau} \tilde{P}^{ji}_{t+\tau} \right], \]

and

\[ B_t = \frac{1}{\Psi_B} \left[ \frac{\eta U_i^{t+1}}{\eta U_i^{t+1}} K^{1}(1+r_U^{t+1}) - 1 \right]. \]

The dynamics of the number of firms is given as:

\[ n^{ij}_t = (1-\delta)n^{ij}_{t-1} + \nu^{ij}_t. \]

### 2.2. Firms

In a region \( i \) there is a large number of \((i, j)\) firms. Firms ex-post differ by the total factor productivity: upon entry, they draw a shock \( z \) from a distribution \( G(z) \). This shock determines the idiosyncratic part of the firm productivity. At the end of each period, there is an exogenous probability that a firm is hit by an exit shock \( \delta \), which is assumed to be independent on aggregate as well as individual states. Hit firms shut down.

The only production factor is labor. The productivity has two components: (a) idiosyncratic component \( z_t \), which is i.i.d. across firms and which

\(^1\)Henceforth, to economize on notations we will use the following terminology: a firm located in the region \( i \) and owned by the household \( j \) will be referred to as an \((i, j)\) firm.
follows distribution $G(z)$ introduced above, and (b) the common component $Z_t$. The total factor productivity $Z_t^j$ pertains to the ownerships and depends on the calendar time and not on the time of firm entry (the time of entry is henceforth called *vintage*) or the location.

We thus assume that the final output of the firm $l$ owned by household $j$ is given as: $q_{lt} = z_l Z_t^j \ell_t$, where $\ell_t$ is labor hired. Labor is a variable input, which can be hired on a period-by-period basis. Let $p_{lt}$ denote the price of a good produced by a firm $l$. We further assume that prices are denominated in the currency of the market of sale.

Firms located in region $i$ may sell products to the market $k$ only if special costs are sunk. These costs are paid on period-by-period basis and are denoted as $c_{ijk}$.

Let $P_{ij}^{lt}$ denote a t-period real operating profit of a $(i, j)$ firm. The real profit is given as follows:

$$P_{ij}^{lt} = \left[ 1_{it}^{ij} \kappa_{it}^{ij} \frac{\eta_{it}^{ki}}{1 + t^{ik}} p_{lt}^{k*} \right] Z_t^j z_l \ell_t - \mathbb{W}_t^{i} \ell_t - \sum_k 1_{it}^{ijk} c_{ijk},$$

where $0 \leq \kappa_{it}^{ijk} \leq 1$ is the share of product $q_{lt}$ sold at the market $k$, and $t^{ik} \geq 0$ represents unit iceberg exporting costs for exporting from region $i$ to region $k$, $1_{it}^{ijk}$ is the indicator whether an $(i, j)$ firm with productivity $z_l$ exports to the market $k$, and $\mathbb{W}_t^{i}$ is the real wage prevailing in the region $i$.

We set $t^{ii} = 0$ and it must hold that $\forall l, t: \sum_k 1_{it}^{ijk} c_{ijk} = 1$.

We assume that firm’s manager maximizes the expected stream of discounted profits. The discounting respects the ownerships. Thus the value of the profit stream of a $(i, j)$ firm of vintage $t$ and enjoying the idiosyncratic productivity level $z_l$ is:

$$V_{ij}^{t}(z_l) = \max_{\{\ell_t\},\{1_{it}^{ij}\}} \sum_{\tau = 0}^{\infty} (1 - \delta)^{\tau} \mathcal{K}_{jt}^{\tau} \eta_{jt}^{ij} p_{lt}^{k*} \mathbb{P}_{lt+\tau}^{ij},$$

where $\phi_{ij}$ is the fixed cost of setup of a $(i, j)$ firm.

The ex-ante expected profit (which is by the law of large numbers and perfect foresight also the ex-post average profit) of a $(i, j)$ firm is given as $\overline{P}_{it}^{ij} = \int P_{it}^{ij} dG(l)$. Similarly for the value of a new entrant, we define: $\overline{V}_{it}^{ij} \equiv \int V_{it}^{ij}(z_l) dG(l)$. 

Thus, the sequencing starts with the households’ decisions about the number of new entrants in each region, i.e., a household \( j \) determines the number of \((i, j)\) new entrants given by (2.2)
\[ \text{2} \] The fixed cost of the firm setup is paid. Then, each new entrant draws a productivity level from the distribution \( G \). Then labor demand, export decisions, and production (of both entrants and incumbents) take place. At the end of the period, some firms experience the exit shock and shut down.

We assume that the final goods \( Q_t^i \) are composed from the individual varieties available in the market \( i \) using a consistent aggregation. The appendix A derivates the optimal production behavior under the Dixit-Stiglitz aggregation with the parameter of the intratemporal substitution \( \theta \). This aggregation is used in calibration.

\[ 2.3. \text{ General-equilibrium Closure} \]

The general equilibrium is a sequence of prices and quantities, such that all agents optimize and the following conditions are satisfied:

- goods market clear;
- labor markets clear;
- balances-of-payments are in equilibrium;
- the consistency of portfolios is satisfied.

The implications of these conditions are described in the Appendix B.

3. CALIBRATION

There are two kinds of parameters: constant and variable. The constant parameters include parameters of the production and utility functions. We follow Ghironi, Melitz (2005) in setting their values (for yearly frequency). Thus, the momentary utility function is assumed to be of the constant-relative-risk-aversion form with the parameter of intertemporal substitution \( \varepsilon \), which takes the conventional value 2; the parameter \( \beta \) is equal to 0.95. The value of the parameter of intratemporal substitution \( \theta \) is calibrated at 3.8; and the probability of the exit shock \( \delta \) is set to 0.1.

\[ ^2 \text{This can be rewritten – using (2.4) – as } \nu_{ij}^t = \Psi_{ij}^{-1} \tilde{V}_{ij}^t. \]
Further, following Ghironi, Melitz (2005), the parameter of adjustment cost is set to 1% for investment in bonds. The same number is used for domestic investment adjustment costs (i.e., when the household invests in a firm located in his region). Thus, $\Psi_B = \Psi_{ii} = 0.01$. Adjustment costs related to FDI between the U and E regions is set 10 times higher than investments within the region of residence (i.e., $\Psi_{ij}/\Psi_{ii} = 10$ for $i \neq j, i \neq A, j \neq A$). This reflects the notion that engaging in FDI is more difficult than investments within the region of residence due to the need of acquiring expertise with the foreign legal and business environment and culture.

The adjustment costs related to FDI between the U.S (Eurozone and Japan) and emerging Asia is taken to be a transitory parameter. By its fall, we model a sharp increase in the investment opportunities in emerging Asia since the 1980s. For the year 1985, we set its value to a large number $\Psi_{ij}/\Psi_{ii} = 10^4$ for $i \neq j, i = A$ or $j = A$, while for the year 2005 it reaches the value of FDI adjustment costs between advanced regions, i.e. $\Psi_{ij}/\Psi_{ii} = 10$ for all $i \neq j$. The calibration of the sharp fall in FDI adjustment costs is used to explain the value of the FDI inflows to the emerging Asia.

Similarly, the iceberg costs are used to model the increase in trade-openness. For the year 2005, we set $t_{ij} = 0.05$ for $i \neq j$ and this value holds for trade between the U.S. and Europe for the year 1985 too. On the other hand, we assume that the trade between the advanced regions and emerging Asia was twice more costly back in the 1985. In particular, we set $t_{ij} = 0.10$ in 1985 for $i \neq j, i = A$ or $j = A$. The magnitude of the fall in iceberg costs between the advanced regions and emerging Asia reflects the evidence that between 1980-2001, the total trade costs (duties and transportation costs) had fallen from 11 to 5% of customs value, see Baier, Bergstrand (2001).

We assume that the TFP grew by 3% annually in the U region (the U.S.). Over the period 1985-2005, the TFP seems to grew slower in the E region (Eurozone and Japan) and faster in the A region (emerging Asia). To

\[3\text{In the 1980s the value of the U.S. (European) FDI in Asia was about 2% (1%) of the Asian GDP, while in the late 1990s and the early 2000s, the value of the U.S. (European) FDI was 5% (3%) of the Asian GDP. Our calibration of } \Psi_{ij} \text{ replicates these numbers.}\]

The overview of the parameter values is given in Table I.

However, since we do not account for all factors influencing the size of the current account imbalances in our model (especially the role of Asia’s exchange rate policies), we do not expect to fit the data perfectly by the model. We aim to show whether the decline in investment and trade costs would lead to qualitatively consistent development as observed in reality.

4. SIMULATION

The simulation was carried out on a calibrated model according to the values in Table I. We start by simulating the change in the world economy between 1985 and 2005 as a response to a simultaneous decline in investment and trade costs. In order to distinguish the relative importance of decline in investment and trade costs, we perform an additional simulation only with a decline in investment costs. Figure I contains the main results. The bars in the figure represent the actual change in main macroeconomic aggregates (Data), the results for simulation of a simultaneous decline in investment and trade costs (Model), and the results for simulation of a fall in investment costs only (Model w.t.).

The figure displays eight variables. The first row shows the percentage change between 1985 and 2005 in the real GDP and real domestic final demand in the three regions. Our experiment mimic the change observed in data by assumptions about the productivity growth. The next row displays the percentage changes between the two years in export and import. The third row displays the percentage change in the real value of imports from emerging Asia to the two regions and the change in FDI to Asia from the two advanced regions. Finally, the last row displays the change in the real exchange rate and the change in trade balance in the three modeled regions. The trade-balance figure is the only, which is not represented by percentage changes but by a change in percentage points.

It is apparent that our quantitative experiment with a decline in investment and trade costs can genuinely generate the observed trade imbalances
between the advanced and emerging world regions. The intuition behind the results is the following. The most advanced region (the U.S.) has a strong incentives to engage in the FDI. These FDI are financed along the transition dynamics by the debt, which leads to the current-account deficit.

The effect on the emerging economy is symmetric. It has the incentive to attract the FDI. Since the FDI to the emerging Asia is mainly export-oriented (due to large market in the advanced countries and due to decline in trade costs), it leads to a large trade balance surplus.

It is worth noting the effects for the second advanced region (E). Although its productivity is close to the first advanced region (U), but slightly smaller, the region does not exhibit so large imbalances. Our simulations suggest that the increase in the economic performance in E region could indeed alleviate the current macroeconomic imbalances, which corresponds to finding in the literature e.g., [Laxton et al. (2005)]. Although the usual mechanism is the increase of the demand for the U.S. exports, we consider the demand mechanism as secondary. On the contrary, our model would suggest that the elevated growth potential in E region would at least partially crowd out the demand for the U assets which would spread the imbalances pattern more evenly over the advanced regions.

The results of simulation further hint at certain inconsistency in the size of real effective exchange rate developments. In particular, the real exchange rate between the U and A currencies turned out to be overvalued. The appreciation of the U currency between 1985 and 2005 was stronger in reality than in the model’s outcome. In relation to the E region, the U currency has depreciated during the two decades significantly more than would be consistent with decline in investment and trade costs. Thus our exercise suggest that beyond the structural factors of investment and trade costs decline that potentially lead to origination of global imbalances, the actual dynamics was driven by other supporting factors, such as rigid exchange rate policies in the A region.

And finally, from the comparison of the two model simulations, i.e. (Model) and (Model w.t.), we can see that the decline in investment costs is the dominant factor driving the changes.
The qualitative results are quite robust with respect to alternative numerical values. Quantitatively, the most important parameter is the parameter of the intratemporal substitution $\theta$. A higher value of the parameter would cause a drop in the trade increase and therefore in the FDI investment from the advanced regions to A region. This can be expected since the high values of $\theta$ mean that goods are close substitutes and therefore the gains from trade (and from the export-platform FDI) are lower.

5. CONCLUSION

In this paper we propose that the current century global imbalances could have been triggered by a decline in investment and trade costs. Our motivation consists in the two major distinctions between the current and previous episodes of global imbalances. Namely, the fact that the current episode involves unequally developed world regions with significant investment linkages and the involved regions grow above their regional benchmarks. These two symptoms are common to integrating and converging underdeveloped countries to advanced counterparts. As a matter of fact, the emerging Asia opened to foreign capital inflows only starting in the 1980s, which could be taken as a point of reference. Therefore, we use a three region model with unequally developed regions to simulate the effects of a decline in investment and trade costs on external balances. We calibrate the model to the U.S., Eurozone and Japan, and emerging Asia and show that the decline in investment and trade costs between 1985 and 2005 genuinely produces trade imbalances between the three regions similar to those observed in reality. Generally, the effects generated by a decline in investment costs dominate those induced by the trade cost decline. Therefore we conclude that the decline primarily in investment costs triggered economic integration between developed and underdeveloped world regions and likely originated the current episode of global imbalances.

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APPENDIX A: DERIVATION OF THE OPTIMAL PRODUCTION PLAN

In the part of the paper, we characterize the optimal production plan of firms under the particular market structure – the Dixit-Stiglitz market structure.

A.1. Market Structure

The Dixit-Stiglitz approach is used to model the market structure. The final good $Q^i$ in the region $i$ is composed of a continuum of intermediate goods, some of them are produced in the region $i$ and some are imported. There is an imperfect substitution among these goods with the parameter $\theta$ of intratemporal substitution. The aggregate good in the domestic region is defined as:

\[
Q^i_t = \left( \sum_m \sum_j n^{mj}_t \int_{\Omega^{mji}} (q_{lt})^{\frac{\theta-1}{\theta}} G(dl) \right)^{\frac{1}{\theta-1}},
\]

where, $q_{lt}$ is the output of the firm $l$; $\Omega^{mji}$ denotes the set of products of firms located in the region $m$, owned by the household $j$, eligible to sell its products to the market $i$. The sets $\Omega^{mji}$ are further characterized in Section A.2 below. The market structure implies the following definition of the region $i$ aggregate price index:

\[
P^i_t = \left( \sum_m \sum_j n^{mj}_t \int_{\Omega^{mji}} (p_{lt})^{1-\theta} G(dl) \right)^{\frac{1}{1-\theta}},
\]

where $p_{lt}$ is the price of products of firm $l$ at time $t$.

The CES market structure implies that the demand for individual firm products in the market $i$ satisfies:

\[
q_{lt} = \left( \frac{p_{lt}}{P^i_t} \right)^{-\theta} Q^i.
\]

A.2. Optimal Production Plans

In this subsection, we derive the optimal production plans. The timing protocol is described in Section 2.2.

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4The final good is consumption as well as investment good, so that $Q^i$ can be interpreted as domestic absorption.
Let us derive the optimal production plan for a \((i, j)\) firm. The real cost function associated with the linear production function is given as:

\[
C(q, W_i^t, Z_j^t, z_l) = W_i^t \frac{q}{Z_j^t z_l}.
\]

The Dixit-Stiglitz market structure implies that the price is a mark-up over marginal costs. Thus, given the inverse of the demand function \((A.2)\), the optimal production decision for market \(k\) takes the following form:

\[
\kappa_{ij}^{k,ij_l t} q_{ij_l t} = \left[ \frac{\theta - 1}{\theta} \eta^{k_i}_{l+tm} Z_j^t z_l \right] Q_k^t.
\]

Since \(\sum_k 1_{ij}^{k,i l} = 1\) for all \((i, j)\) and \((l,t)\), the exporting shares, provided that the firm sells in at least one market, satisfy:

\[
\kappa_{ij}^{k,ij_l t} = \frac{1_{ij}^{k,i l} Q_i^t + \sum_{m \neq i} 1_{ij}^{m,i l} \left( \eta^{m_i}_{l+tm} \right)^\theta Q_m^t}{\sum_{m \neq i} 1_{ij}^{m,i l} \left( \eta^{m_i}_{l+tm} \right)^\theta Q_m^t}.
\]

If the firm does not sell to any market, than we can define \(\kappa_{ij}^{k,ij_l t} = 0\) for all \(k\).

To characterize lumpy decisions of firms to sell to a given market, we assume that firms do that, provided that doing so is not loss-making. Lemma \(A.1\) below shows when this will be so.

Define the ‘effective’ demand as follows:

\[
\xi_{ij}^{\ell,ij_l t} \equiv 1_{ij}^{\ell,i l} Q_i^t + \sum_{m \neq i} 1_{ij}^{m,i l} \left( \eta^{m_i}_{l+tm} \right)^\theta Q_m^t.
\]

The total production of a firm can be then written as follows:

\[
q_{ij} = z_l^\theta \left[ \frac{\theta - 1}{\theta} Z_j^t \right]^\gamma Z_i^t \xi_{ij}^{\ell,ij_l t},
\]

and real turnovers on the domestic and the foreign markets, respectively are given by \(z_j^{\theta - 1} \left[ \frac{\theta - 1}{\theta} Z_j^t \right]^\gamma Q_i^t\), and by \(z_i^{\theta - 1} \left( \eta^{m_i}_{l+tm} \right)^\theta \left[ \frac{\theta - 1}{\theta} Z_i^t \right]^\gamma Q_m^t\).

Real production costs read as follows:

\[
C_{ij}^{ij_l t} = z_j^{\theta - 1} \left[ \frac{\theta - 1}{\theta} Z_j^t \xi_{ij}^{\ell,ij_l t} \right]^\gamma W_i^t.
\]

thus, the real operating profit in a period \(t\) is given as:

\[
\mathbb{P}_{ij}^{ij_l t} = W_i z_j^{\theta - 1} \left[ \frac{Z_j^t}{W_i^t} \right]^\gamma \xi_{ij}^{\ell,ij_l t}.
\]

\(^5\text{Define } W_1 \equiv \left[ \frac{\theta - 1}{\theta} \right]^\theta - \left[ \frac{\theta - 1}{\theta} \right]^\gamma = \frac{1}{\theta - 1} \left[ \frac{\theta - 1}{\theta} \right]^\theta.\)
Lemma A.1 The \((i, j)\) firm with the productivity level \(z_l\) sells to the market \(k\) (i.e. \(1_{l_{ij}} = 1\)) provided that:

\[
z_l \geq \frac{W_{i_1}}{Z_l^i} \left[ c_{ij} \left( \frac{Q_k^k \left( \eta_{ij}^{ik} \right)^{\theta}}{1 + t_{ik}^l} \right)^{-1} \right]^{\frac{1}{\theta - 1}},
\]

otherwise it does not sell to the market \(k\), i.e. \(1_{l_{ij}} = 0\).

Proof of Lemma A.1 Since operating profit \(P_{ij}^t\) is linear in quantities sold to the market \(k\) and the lumpy eligibility costs are paid on a period-by-period basis, one concludes that to enter the market \(k\) is not loss-making if:

\[
W_{i_1} z_l^{\theta - 1} \left[ c_{ij} \left( \frac{Q_k^k \left( \eta_{ij}^{ik} \right)^{\theta}}{1 + t_{ik}^l} \right) - c_{ij} \right] \geq 0.
\]

Rearranging of the terms yields the lemma. Q.E.D.

Corollary to Lemma A.1 Define

\[
\bar{z}_{ki}^{ij} = \frac{W_{i_1}}{Z_l^i} \left[ c_{ij} \left( \frac{Q_k^k \left( \eta_{ij}^{ik} \right)^{\theta}}{1 + t_{ik}^l} \right)^{-1} \right]^{\frac{1}{\theta - 1}}.
\]

Thus, the export decision takes the simple form: all \((i, j)\) firms export to the market \(k\) if and only if their productivity \(z_l\) is higher than the cut-off \(\bar{z}_{ki}^{ij}\). This is an extension of Melitz (2003) result to the multi-region setting. Therefore a set of \((i, j)\) firms exporting to the market \(k\) is given by \(1 - G(\bar{z}_{ki}^{ij})\), where \(G\) is the cumulative distribution function for \(z\).

Now, we are able to derive the expected present value of operating-profit flows of a new entrant. The ex-post value of the flow, after the shock \(z_l\) is revealed, is (the value is expressed in the currency of the owner):

\[
V_{ij}^t(z_l) = z_j^{\theta - 1} W_1 \sum_{t=\tau}^{\infty} \eta_{ij}^t \left( \frac{(1 - \delta)^{t - \tau}}{R_{j}^{t - \tau}} \left[ \frac{Z_l^j}{W_l^1} \right]^{\theta - 1} \xi_{lt}^{ij} \sum_k 1_{l_{ij}} \ c_{ij} \right).
\]

The ex-ante expected value \(\tilde{V}_{ij}^t\) satisfies:

\[
(A.3) \quad \tilde{V}_{ij}^t = W_1 \sum_{i=\tau}^{\infty} \eta_{lt} \left( \frac{(1 - \delta)^{t - \tau}}{R_{j}^{t - \tau}} \left[ \frac{Z_l^j}{W_l^1} \right]^{\theta - 1} \int \xi_{lt}^{ij} z_j^{\theta - 1} dG(l) - \int \sum_k 1_{l_{ij}} c_{ij} dG(l) \right).
\]

Note that \(\xi_{lt}^{ij}\) should be inside the integral, since \(\xi_{lt}^{ij}\) depends on the productivity \(z_l\) through the choice of \(1_{l_{ij}}\).
APPENDIX B: A DETAILED OVERVIEW OF GENERAL-EQUILIBRIUM CONDITIONS

The general equilibrium requires that:

- goods market clear;
- labor markets clear;
- balances-of-payments are in equilibrium;
- the consistency of portfolios is satisfied.

B.1. Goods market equilibrium

The final good $Q_i$ is divided between consumption and investments:

\begin{equation}
Q_i = C_i + I_i,
\end{equation}

where $C_i$ is the consumption of household living in the region $i$. The investments $I_t$ consist of eligibility costs $c_{ijk}$ that are spent in the region of production.

Therefore, the aggregate investment in the region $i$ follows:

\[
I_i = \sum_j n_{ij} \sum_k c_{ijk} \int 1^{ijk} G(dl) = \sum_j n_{ij} \sum_k c_{ijk} \left(1 - G(z_{ij})\right).
\]

The second equality follows from Lemma [A.1].

B.2. Labor market equilibrium

A region $i$ is endowed with one unit of labor, which is supplied inelastically. The wage $\mathbb{W}_i$ is set to equate the total labor demand with the inelastic labor supply. Because of linearity of the production functions, it can be easily derived that the total labor demand in the region $i$ is given as $\sum_k \sum_m n_{ik} \int 1^{ikm} \kappa_{ilm} \frac{q_{il}}{z_{il}Z_k} G(dl)$. The first summation is over ownership, the second summation is over markets, and the integral adds firms with different level of idiosyncratic productivity $z_l$.

Therefore the three market clearing conditions read as

\[
1 = \sum_k \sum_m n_{ik} \int 1^{ikm} \kappa_{ilm} \frac{q_{il}}{z_{il}Z_k} G(dl),
\]

for $i \in \{U, E, A\}$. 
B.3. Balance-of-payments equilibrium condition

The balance-of-payments equilibrium condition for the region \( i \) is given as:

\[
\sum_j X_{it}^j + \sum_j \eta_{it}^j n_{it}^j \tilde{P}_{it} + \eta_{it}^j \left( 1 + r_{it}^U \right) B_{it-1} - B_{it} = \sum_k \eta_{it}^k \chi_{it}^k + \sum_k \eta_{it}^k n_{it}^k \tilde{P}_{it}^k
\]

where \( X_{it}^j \) is the value of exports from the region \( i \) to the region \( j \) (expressed in the currency \( i \)). The real export values satisfy\(^6\)

\[
X_{it}^j = \sum_k n_{it}^k \int 1_{it}^{ijk} \kappa_{it}^{ijk} \frac{\tilde{P}_{it}}{\tilde{P}_{it}^k} G(dl) = \sum_k n_{it}^k \kappa_{it}^{ijk} \frac{p_{it}}{\tilde{P}_{it}^k} q_{it} \left( 1 - G(z_{ij}^k) \right).
\]

Note that – by the Walras law – only two of the three balance-of-payments equilibrium conditions are needed.

B.4. The derivation of portfolio consistency

In equilibrium, it must be that \( \sum_i B_{it} = 0 \). This condition determines the real interest rate \( r_{it}^U \).

\(^6\)The trade balance of the region \( i \) is then given as

\[
TB_i^t = \sum_j X_{it}^j - \sum_k \eta_{it}^k \chi_{it}^k.
\]
REFERENCES


**Figure 1.** Simulation results
### TABLE I

**Parametrization of the model**

<table>
<thead>
<tr>
<th>Constant parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The parameter of intratemporal substitution $\theta$</td>
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</tr>
<tr>
<td>The parameter of impatience $\beta$</td>
<td>0.95</td>
</tr>
<tr>
<td>The probability of the deadly shock $\delta$</td>
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<tr>
<td>The parameter of intertemporal substitution $\varepsilon$</td>
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<tr>
<td>The annual US TFP growth $\gamma$</td>
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</tr>
<tr>
<td>Adjustment costs (bond) $\Psi_B$</td>
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</tr>
<tr>
<td>Adjustment costs (domestic firms) $\Psi_{ii}$</td>
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</tr>
<tr>
<td>Adjustment costs (FDI) $\Psi_{ij}$, $i \neq j$, $i \neq A$, $j \neq A$</td>
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</tr>
<tr>
<td>Icebergs $t_{ij}$, $i \neq j$, $i \neq A$, $j \neq A$</td>
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</tr>
</tbody>
</table>

<table>
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<tr>
<th>Transitory parameters</th>
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<th>Value in 2005</th>
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</thead>
<tbody>
<tr>
<td>Icebergs $t_{ij}$, $i \neq j$, $i = A$, or $j = A$</td>
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<td>0.05</td>
</tr>
<tr>
<td>Adjustment costs (FDI) $\Psi_{ij}$, $i \neq j$</td>
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<td>0.10</td>
</tr>
<tr>
<td>$Z^U$</td>
<td>10</td>
<td>$10(1 + \gamma)^{20}$</td>
</tr>
<tr>
<td>$Z^E$</td>
<td>9.5</td>
<td>$9.0(1 + \gamma)^{20}$</td>
</tr>
<tr>
<td>$Z^A$</td>
<td>3.5</td>
<td>$5.0(1 + \gamma)^{20}$</td>
</tr>
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</table>