Perfect Competition and Intra-industry Trade*

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The paper presents a formal analysis which incorporates risk aversion to international trade. It is shown that risk-averse firms operating in perfectly competitive markets with uncertainty of demand tend to diversify markets what gives a basis for international trade in identical commodities between identical countries. If market demand is elastic enough such trade is welfare improving despite efficiency losses due to cross-hauling and transportation costs.

Keywords: Reasons for trade, intra-industry trade, demand uncertainty, risk aversion, market diversification

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1. Introduction

The traditional approach to intra-industry trade is to assume that such trade arises because slightly different commodities are produced and traded to satisfy consumer’s for variety (see Krugman, 1980). Brander (1981) shows that there are reasons to expect two-way trade even in identical products, due to strategic interactions among firms operating in non competitive markets. What is not so widely recognized is that there are reasons to expect international trade in identical commodities (i.e., within a single industry) even if markets are perfectly competitive. This paper is, then, intended to contribute to the theory of trade between similar (or even identical) countries. In particular, countries in the model are assumed to be identical and pattern

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of trade is determined by the interaction of demand uncertainty, risk aversion and perfectly competitive behavior of firms.

2. The model

2.1. Markets

There are two identical countries, called: Home country and Foreign country. In each country a single commodity can be produced and supplied to perfectly competitive and separated markets (re-imports of once exported goods is not possible). The countries are identical, but in both of them there is uncertainty about market demand. We assume that two states of nature (S1 and S2) can occur independently in each country: state S1 with probability \( q \) and state S2 with probability \( 1-q \). Market demand in each particular state is assumed to be identical in both countries, and in the analysis which follows inverse market demands at state S1 and S2 are denoted correspondingly as \( D^{-1}(X) + \lambda \) and \( D^{-1}(X) - \lambda \) \((dD^{-1}(X))/dX < 0\), and \( D^{-1}(X) - \lambda > 0 \), for any \( X \geq 0 \), where \( X \) \((X \geq 0)\) is the total quantity supplied to the market, and \( \lambda (\lambda > 0) \) is a constant parameter, identical in both states of the nature (depending on the state of the nature it shifts expected inverse demand curve up or down).

2.2. Firms

The cost function of any firm in the model is given as

\[
TC(x + x^*) = C \cdot (x + x^*),
\]

where \( TC \) is a total cost, \( x \) and \( x^* \) denote correspondingly the volume of output supplied to the domestic market and exported \((x, x^* \geq 0)\), and \( C \) denotes constant marginal cost \( (\text{to focus directly on the issue no fixed cost is assumed})\). Transport costs are borne by producers. The per unit transport cost equals \( t \) \((t > 0)\) and is the same in both directions.
Following Sandmo (1971) we assume that, in an uncertain world, any decision on the volume of output to be produced must be taken prior to the sales date, at which actual market demand is known. This is because the production and planning period, the period that elapses between the decision to produce a given quantity of output and the time that output is finished and marketed is usually long, and, thus, the firm when it makes its daily output decision, never knows exactly the market conditions in the sales date. The firm’s beliefs about market demand are given by the probabilities of state $S1$ and $S2$. The firm is assumed to be unable to influence this distribution (i.e., to be able to predict market demand). Moreover, we assume that firms are managed according to the wishes of their owners who are typical asset holders, and that the decisions in each firm are made by a group of decision-makers with sufficiently similar preferences to guarantee the existence of a group-preference function, representable by a von Neuman-Morgenstern utility function. Given these conditions we assume risk aversion, so that the utility function of each firm ($U$) is strictly concave and twice differentiable function of profit (i.e., $U(0)=0$, $U'(\pi)>0$, $U''(\pi)<0$, and, $\lim_{\pi \to -\infty}U'(\pi)=\infty$, $\lim_{\pi \to \infty}U'(\pi)=0$).

2.3. Individual output decisions

For the sake of clarity assume that the firm makes its output decisions with sole regard for short-run profits and does not consider the relationship between this output policy and long-run policies for investment and finance (more complete model would make it necessary to draw up a much larger and more detailed list of assumptions about the economic environment of the firm than is needed for the present paper).

Each firm takes market price in each particular state as given and must decide, before the real market price is known, how much of the commodity to produce for domestic consumption and how much to export (i.e., the distribution over domestic consumption and export sales must

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1 See Sandmo (1971) for discussion.
be taken prior to the sales date). Thus each firm takes prices as given and maximizes (with respect to \( x \) and \( x^- \), where \( x, x^- \geq 0 \)) an expected utility from profit:

\[
E[U(\delta)] = q^2U(\pi_c) + q(1-q)U(\pi_c) + q(1-q)U(\pi_c) + (1-q)^2U(\pi_c),
\]

where

\[
\pi_c(x, x^-) = (P + \lambda) x + (P + \lambda) x^- - C(x + x^-) - tx^-, \quad (3)
\]
denotes profit of the firm if state \( S1 \) occurs in both countries;

\[
\pi_c(x, x^-) = (P + \lambda) x + (P - \lambda) x^- - C(x + x^-) - tx^-, \quad (4)
\]
denotes profit of the firm if state \( S1 \) occurs in the home country and state \( S2 \) occurs in the foreign country;

\[
\pi_c(x, x^-) = (P - \lambda) x + (P + \lambda) x^- - C(x + x^-) - tx^-, \quad (5)
\]
denotes profit of the firm if state \( S2 \) occurs in the home country and state \( S1 \) occurs in the foreign country;

\[
\pi_c(x, x^-) = (P - \lambda) x + (P - \lambda) x^- - C(x + x^-) - tx^-, \quad (6)
\]
denotes profit of the firm if state \( S2 \) occurs in both countries (to simplify the notation we will skip the arguments \((x, x^-)\) when referring to the profit functions specified above). Note that in the moment when decision about the output is made the firm knows only that one state, out of four possible combinations of states of the nature, will occur, and, consequently, in it’s objective function takes into account all possible profit functions and probabilities of corresponding states. Since for risk averse utility function the objective function (2) is strictly concave for any \( x \) and \( x^- (x, x^- \geq 0) \), there exists a single pair \((\bar{x}, \bar{x}^-)\) for which the objective function is maximized (note that \( \bar{x} \) and \( \bar{x}^- \) depend on price \( P \)). Taking into account that the countries under study are identical the total volume of output supplied to the market of any country can be represented as \( N \cdot [\bar{x}(P) + \bar{x}^-(P)] \), i.e., depends on the number of firms in the

\(^2\) Sandmo (1971) and Leland (1972) provide detailed justifications for this assumption.
industry \(N\); the equilibrium market prices also depend on \(N\) (i.e., \(P = P(N)\)). Consequently, an equilibrium volume of output supplied to the market by each individual firm can be considered as a function of \(N\), i.e., \(\tilde{x}(N)\) and \(\tilde{x}^*(N)\). The number of firms in the industry \(N\) is determined by free entry and exit, and perfect competition implies that in market equilibrium the expected profit of each individual firm equals to zero, i.e.

\[
E[U[\pi(\tilde{N}, \tilde{x}, \tilde{x}^*)]] = 0. \tag{7}
\]

Solving maximization problem (2) together with the condition (7) one find can equilibrium values \(\tilde{N}, \tilde{x}\) and \(\tilde{x}^*\), and then expected equilibrium market price \(\tilde{P}\) can be obtained from equilibrium market clearing condition.

3. International exchange

Suppose the total equilibrium volume of output supplied to the market is positive i.e., \(\tilde{X} > 0\), then an equilibrium output of a single firm \(\tilde{X} = \tilde{x} + \tilde{x}^* = \tilde{X} / N\) is also positive \((\tilde{X} > 0)\). Representing \(x^* = \tilde{X} - x\), substituting into (3)-(6) and differentiating (2) with respect to \(x\), we get

\[
\frac{d}{dx}E[U(\pi)] = \left[q' \frac{dU}{d\pi}(\pi) + q(1-q) \frac{dU}{d\pi}(\pi) + (1-q)^2 \frac{dU}{d\pi}(\pi) + (1-q)^2 \frac{dU}{d\pi}(\pi) \right]t + 2q(1-q) \frac{dU}{d\pi}(\pi) - \frac{dU}{d\pi}(\pi) \tag{8}
\]

Note that \(\pi_2(x = 0) < \pi_3(x = 0)\). Consequently, \(\frac{dU}{d\pi}(\pi_2(x = 0)) - \frac{dU}{d\pi}(\pi_3(x = 0)) > 0\) and

\[
\frac{d}{dx}E[U(\pi)] > 0, \text{ for } x = 0 \text{ and } x^* = \tilde{X}. \text{ Therefore, the pair } (x = 0, x^* = \tilde{X}) \text{ cannot be optimal, since for any small } \Delta x > 0, \text{ the pair } (x = \Delta x, x^* = \tilde{X} - \Delta x) \text{ gives a higher expected utility level. On the other hand, } \pi_2(x = \tilde{X}) > \pi_3(x = \tilde{X}). \text{ Consequently, for } x = \tilde{X} \text{ and } x^* = 0 \frac{dU}{d\pi}(\pi_2) - \frac{dU}{d\pi}(\pi_3) < 0, \text{ and for sufficiently small } t, \frac{d}{dx}E[U(\pi)] < 0. \text{ Therefore, for}
\]
sufficiently small $t$ the pair $(x = \tilde{x}, x^* = 0)$ cannot be optimal, since there exists such a pair $(x = \tilde{x} - \Delta x, x^* = \Delta x)$, where $\Delta x > 0$, for which the value of the objective function is higher.

Thus, we conclude that for sufficiently small $t$ each firm supplies to both markets (i.e., $\tilde{x} > 0$ and $\tilde{x}^* > 0$). This means that if transportation costs are small enough, equilibrium in a market with uncertain demand involves international trade despite the fact that both countries produce exactly the same commodity in perfectly competitive environments, and there is an obvious loss due to transportation costs. If countries are identical, the situation in the foreign country is symmetric to that in the home country, i.e., the firm located in the home country exports to the foreign country and produces for its domestic market, while the firm in the foreign country exports to the home country and produces for its domestic market.

4. Welfare effects

**Consumer surplus.** Consumer surplus measures the amount a consumer gains from a purchase by the difference between the price he actually pays and the price he would have been willing to pay. Thus, expected consumer surplus equals:

$$E[CS] = \int_{\tilde{P}}^{+\infty} D(z)dz$$

(9)

Taking derivative of (9) with respect to $t$ (at $P = \tilde{P}$, where $\tilde{P}$ denotes expected equilibrium market price), we get:

$$\frac{d}{dt}E[CS] = \frac{d}{dt} \int_{\tilde{P}}^{+\infty} D(z)dz = -D(\tilde{P}) \frac{d\tilde{P}}{dt}.$$  

(10)

The equilibrium values: $\tilde{x}$, $\tilde{x}^*$ and $\tilde{P}$, satisfy the following conditions:

$$\frac{\partial}{\partial x} E[U(\pi)] = 0,$$  

(11)

$$\frac{\partial}{\partial x^*} E[U(\pi)] = 0,$$  

(12)

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\[ E[U(\pi)] = 0. \] (13)

Consider the equilibrium values \( \bar{x}, \tilde{x}^+ \) and \( \tilde{P} \) as functions of \( t \) and differentiate (13) with respect to \( t \). Obviously,

\[
\frac{d}{dt} E[U(\pi)] = \frac{\partial}{\partial x} E[U(\pi)] \frac{d\bar{x}}{dt} + \frac{\partial}{\partial x'} E[U(\pi)] \frac{d\tilde{x}^+}{dt} + \frac{\partial}{\partial P} E[U(\pi)] \frac{d\tilde{P}}{dt} + \frac{\partial}{\partial t} E[U(\pi)] = 0. \quad (14)
\]

Taking into account (11) and (12), the expression above reduces to

\[
\frac{d}{dt} E[U(\pi)] = \frac{\partial}{\partial P} E[U(\pi)] \frac{d\tilde{P}}{dt} + \frac{\partial}{\partial t} E[U(\pi)]. \quad (15)
\]

Plugging

\[
\frac{\partial}{\partial P} E[U(\pi)] = \left[ q^2 U' (\pi_1) + q(1-q)U' (\pi_2) + q(1-q)U' (\pi_3) + (1-q)^2 U' (\pi_4) \right] \cdot (\bar{x} + \tilde{x}^+), \quad (16)
\]

and

\[
\frac{\partial}{\partial t} E[U(\pi)] = -\left[ q^2 U' (\pi_1) + q(1-q)U' (\pi_2) + q(1-q)U' (\pi_3) + (1-q)^2 U' (\pi_4) \right] \cdot \tilde{x}^+. \quad (17)
\]

into (15) and rearranging we get

\[
\frac{d\tilde{P}}{dt} = \frac{\tilde{x}^+}{\bar{x} + \tilde{x}^+}, \quad (18)
\]

and finally

\[
\frac{d}{dt} E[CS] = -D(\tilde{P}) \frac{\tilde{x}^+}{\bar{x} + \tilde{x}^+}. \quad (19)
\]

Therefore, the expected consumer surplus falls if transportation costs increase.

**Producer surplus.** Analogous to the concept of expected consumer surplus is that of expected producer surplus, which is understood as expected aggregate profit of the industry.

Let \( \tilde{\pi}_i = \pi_i (\bar{x}, \tilde{x}^+) \), for \( i = 1,2,\ldots,4 \). In equilibrium the expected producer surplus is determined as
\[ E[PS] = \tilde{N}[q^2\tilde{\pi}_1 + q(1-q)\tilde{\pi}_2 + q(1-q)\tilde{\pi}_3 + (1-q)^2\tilde{\pi}_4] \]  
\hspace{2cm} (20)

Differentiating (20) with respect to \( t \) we get:

\[ \frac{d}{dt} E[PS] = \frac{d\tilde{N}}{dt} E[\pi] + \tilde{N} \frac{d}{dt} E[\pi]. \]  
\hspace{2cm} (21)

Since \( \tilde{N} = \tilde{X} / (\tilde{x} + \tilde{x}^+) \),

\[ \frac{d\tilde{N}}{dt} = \frac{\frac{d\tilde{X}}{dt}(\tilde{x} + \tilde{x}^+) - \tilde{X} \frac{d(\tilde{x} + \tilde{x}^+)}{dt}}{(\tilde{x} + \tilde{x}^+)^2}. \]  
\hspace{2cm} (22)

Taking into account that \( \tilde{X} = D(\tilde{P}) \), differentiating and rearranging we get:

\[ \frac{d\tilde{N}}{dt} = \frac{\frac{dD(\tilde{P})}{dP}(\tilde{x}^+) - D(\tilde{P}) \left( \frac{d\tilde{x}}{dt} + \frac{d\tilde{x}^+}{dt} \right)}{(\tilde{x} + \tilde{x}^+)^2}. \]  
\hspace{2cm} (23)

Bearing in mind (3)–(6) we can represent expected value of equilibrium profit as:

\[ E[\pi] = \left[ q(\tilde{P} - C + \lambda) + (1-q)(\tilde{P} - C - \lambda) \right] \tilde{x} + \left[ q(\tilde{P} - C - t + \lambda) + (1-q)(\tilde{P} - C - t - \lambda) \right] \tilde{x}^+. \]  
\hspace{2cm} (24)

Differentiating (24) with respect to \( t \), and rearranging taking into account (15) we get:

\[ \frac{d}{dt} E[\pi] = \left[ q(\tilde{P} - C + \lambda) + (1-q)(\tilde{P} - C - \lambda) \right] \frac{d\tilde{x}}{dt} + \left[ q(\tilde{P} - C - t + \lambda) + (1-q)(\tilde{P} - C - t - \lambda) \right] \frac{d\tilde{x}^+}{dt}. \]  
\hspace{2cm} (25)

Finally, the change of the expected producer surplus with response to change in transportation costs can be represented as

\[ \frac{d}{dt} E[PS] = \frac{1}{(\tilde{x} + \tilde{x}^+)^2} \left\{ \frac{dD(\tilde{P})}{dP} (\tilde{P}) \tilde{x} \cdot E[\pi] + D(\tilde{P}) \left( \frac{d\tilde{x}}{dt} \tilde{x}^+ - \tilde{x} \frac{d\tilde{x}^+}{dt} \right) \right\}, \]  
\hspace{2cm} (26)

where \( E[\pi] \) is given by (24). Thus, the pattern of changes in the expected producer surplus in response to changes in transportation costs, depends on the shape of demand curve. In particular, expected producer surplus falls as transportation costs increase if
\[
\frac{\left(\frac{d\tilde{x}^+}{dt} - \tilde{x} \frac{d\tilde{x}^-}{dt}\right)}{\tilde{x}^+ E[\tilde{\pi}]} \leq -\frac{dD}{d\tilde{P}}(\tilde{P}) ,
\]

i.e., if market demand is very elastic (the inverse demand curve is flat), or/and per unit transportation costs are negligible \((t)\) is close to zero).

**Total effect.** Under free trade expected welfare is the sum of expected consumer and producer surplus. Consequently, the change in total expected welfare in response to changes in transportation costs is determined as

\[
\frac{d}{dt} E[W] = -D(\tilde{P}) \frac{\tilde{x}^+}{\tilde{x} + \tilde{x}^-} + \frac{1}{(\tilde{x} + \tilde{x}^-)^2} \left\{ \frac{dD}{d\tilde{P}}(\tilde{P}) \tilde{x}^+ E[\tilde{\pi}] + D(\tilde{P}) \left[ \left( \frac{d\tilde{x}^+}{dt} - \tilde{x} \frac{d\tilde{x}^-}{dt} \right) \right] \right\}.
\]

Thus, total expected welfare decreases if transportation costs increase if

\[
\left[ \frac{\left( \frac{d\tilde{x}^+}{dt} - \tilde{x} \frac{d\tilde{x}^-}{dt} \right) \cdot t}{E[\tilde{\pi}]} \right] / \tilde{x}^+ - (\tilde{x} + \tilde{x}^-) < -\frac{dD}{d\tilde{P}}(\tilde{P})
\]

i.e., if market demand is very elastic (the inverse demand curve is flat), per unit transportation costs are negligible \((t)\) is close to zero).

It follows from the above that reduction of transportation costs, which allows countries to extend international exchange, improves expected total welfare if market demand is elastic enough, and decreases expected total welfare in the opposite case. The intuition behind this is that in the case of elastic demand, small reduction of transportation cost significantly increases the total output (decreases a deadweight loss) but at the same time only slightly reduces expected market price. Thus, the total gain from output expansion exceeds losses associated with higher total transportation costs due to cross-hauling.

**4. Conclusion**

The analysis we have just gone through shows that there is justification for the international trade in identical goods even if markets are perfectly competitive. International
exchange of identical commodities (cross-hauling) occurs due to the fact that risk-averse firms operating in perfectly competitive markets with price uncertainty tend to diversify markets. If transportation costs are small enough this gives a basis for international trade between identical countries. If market demand is elastic enough such trade is welfare improving despite efficiency losses due to cross-hauling and transportation cost. Since the firms never know exactly the market demand in the sales date this is a view of trade which appears to be useful in understanding trade among industrial countries.

The basic idea of the paper: risk reduction by market diversification has been adopted from the theory of portfolio choice with risk aversion. However, in the model above we have introduced transportation costs explicitly and we interpret it as a model of international trade. We need to note that the result would be similar if each risk-averse firm operated a plant in the home country and a higher cost plant in a foreign country, without trade taking place. Free trade in shares of the firms would be also a vehicle to diversify risk without incurring transport costs. Similarly, an access to developed insurance market may reduce the effect of described trade mechanism. All of this may affect the patterns of trade and make an empirical analysis not easy.

The present paper just indicates the issue. There are many ways in which this study can be extended and generalized. In particular, we said nothing about trade policy and welfare effects of different policy instruments, but detailed analysis of these and other issues is left for further research.

References

