Knowledge Generality, Competition and Growth

Chenchuan Shi*

November 21, 2025

[Latest version]

Abstract

This paper studies how the *generality* of knowledge—its applicability across technologies and industries—shapes firms' innovation strategies, market structure, and aggregate growth. I build an endogenous growth model in which firms choose between general and firm-specific R&D while competing for market leadership. General innovations enhance firms' capacity to absorb and apply outside knowledge, creating spillovers within and across industries, whereas firm-specific innovations yield mainly private gains. The model predicts, and the data confirm, that (1) leaders favor firm-specific R&D while followers rely on general innovations to catch up, and (2) the gap in innovation generality between them follows a U-shaped pattern with market concentration. Leveraging variation in the enforceability of non-compete agreements across U.S. states, I provide empirical evidence consistent with the model's spillover mechanisms. The findings point to a novel growth policy: encouraging general R&D, particularly among leading firms, can improve knowledge diffusion and sustain long-run growth.

JEL codes: D22, L13, O30, O40.

*University of Oxford, 10 Manor Rd, Oxford OX1 3UQ, UK, email: chenchuan.shi@nuffield.ox.ac.uk.
I am indebted to Matin Ellison and Petr Sedláček for the guidance and support throughout this project. I would like to thank Steve Bond, Timo Boppart, Andrea Chiavari, Paola Conconi, Sergio de Ferra, Maarten De Ridder, Xavier Jaravel, Alexandre Kohlhas, Hannes Malmberg, Michael McMahon, Silvia Miranda-Agrippino, Jane Olmstead-Rumsey, Morten O. Ravn, Abhijit Tagade, Alex Teytelboym, Florian Trouvain, Joshua Weiss, Zijun Xu, Francesco Zanetti and Wenchuan Zhao and conference and seminar participants of 17th Normac Symposium, Transatlantic Doctoral Conference, London Macro PhD Workshop, CEPR Long-Run Productivity Conference, MMF Annual Conference, Theories and Methods in Macro, Doctoral Workshop on Quantitative Dynamic Economics and University of Oxford for helpful discussions and comments.

1 Introduction

Innovation is a fundamental driver of long-run growth, but not all innovations are alike. Some are firm-specific, benefiting primarily the innovating firm. Others are general, creating spillovers across technologies and industries. This distinction—captured by the notion of *innovation generality* (Trajtenberg et al., 1997), which measures the breadth of an idea's impact—is crucial for understanding how firms strategically respond to knowledge spillovers and, in turn, shape market competition and aggregate growth.

To study how firms choose between general and firm-specific innovations and the aggregate implications of their decisions, I develop a novel and tractable endogenous growth model. In this framework, two firms compete along a technological ladder. The focus on *innovation generality* introduces a key dimension for understanding firms' strategic responses to knowledge spillovers. General innovations, while benefiting rivals, also expand a firm's own capacity for learning and absorption by connecting it to a broader pool of ideas (Cohen and Levinthal, 1990).[1] An illustration comes from the photography industry: firms were able to invent and refine digital cameras by building on semiconductor technology, a general innovation developed outside their field (Fossum, 2020). The choice between general and firm-specific innovation is therefore inherently strategic: it balances the protection of a firm's current market position against the opportunity to exploit the expanding frontier of general knowledge.

More concretely, the model builds on an endogenous growth framework with step-by-step innovations in a leader–follower setting. It features two types of general knowledge spillovers: cross-industry and within-industry. By investing in general innovations, both leaders and followers gain access to cross-industry spillovers that enable them to absorb and apply technologies developed in other sectors. Crucially, these spillovers are not automatic; they require firms to engage in general R&D. Within each industry, spillovers flow from the leader to the follower, with their magnitude increasing in the leader's general R&D effort. Yet followers cannot benefit passively either; they also need to undertake general innovations to absorb and exploit the knowledge produced by leaders. This assumption about within-industry spillovers is later validated empirically through a quasi-natural experiment that leverages policy-driven variation across U.S. states with firm-level data.

The model incorporates a dynamic feedback loop between firms' innovation choices and the aggregate level of general knowledge. General R&D contributes to this shared knowledge base, which in turn expands learning opportunities and increases the returns to such investments. This encourages firms to pursue further general innovations. In this

[1]Importantly, though basic innovations also create broader spillovers, they differ from general innovations in several aspects: basic research is non-applied and mainly conducted by public institutions (e.g., universities) (Akcigit et al., 2020), whereas general research is applied, largely conducted by private firms, and more prevalent in smaller and younger firms.

way, general innovations not only drive growth directly but also reinforce firms' incentives to invest in general R&D—a feedback mechanism largely overlooked in existing growth models.

Building on these mechanisms, the model yields two central theoretical predictions. First, leaders tilt toward firm-specific R&D while followers prefer general R&D: leaders seek to protect their dominance by limiting within-industry spillovers, while followers rely on general innovations to absorb external knowledge and accelerate catch-up. Second, the gap in innovation generality—defined as the share of R&D efforts devoted to general innovations—between the two firms varies non-monotonically with the level of market concentration. When firms compete neck-and-neck, they have the same level of innovation generality. Once a small leader–follower productivity gap emerges, followers have strong incentives to pursue general innovations, as the potential profit gain from overtaking leaders is large, while leaders focus on firm-specific innovations to defend against being displaced. However, when the gap is large, followers have weaker incentives to catch up, and leaders face less pressure to protect their position, narrowing the generality gap. This pattern implies that market structure does not only affect competition in the short run; it shapes the long-run direction of innovation, the extent of knowledge spillovers, and ultimately the pace of growth.

To test these predictions, I construct an empirical measure of firm-level innovation generality using the established indicator of patent generality (Trajtenberg et al., 1997). Patent generality captures the breadth of an innovation's impact: the extent to which it influences subsequent innovations across technological fields.[2] For example, a U.S. patent for a "location-aware application development framework" (granted in the early 2000s) has high generality (score of 0.83), as it enabled diverse applications across industries, from app-based taxi services and logistics to tourism, marketing, and healthcare. By contrast, a narrowly tailored invention such as the "stack clearing device and method" addresses a specific technical problem through a particular implementation and therefore has limited spillover potential and displays low generality (score of 0.18).

The measure of firm-level innovation generality is based on a dataset that integrates patent information with firm-level data from multiple sources. Patent-level data are sourced from the United States Patent and Trademark Office (USPTO) and PatentsView, and are linked to Compustat, with standard firm-level information. The mapping between patents and firm assignees follows Kogan et al. (2017), which provides carefully constructed firm–patent matches. Using average patent generality at the firm level—referred to as innovation generality—as the empirical counterpart to the model, the analysis confirms its key theoretical predictions: Leaders produce less general innovations than followers, and the gap in innovation generality between them follows a U-shaped relationship

[2]Formally, patent generality is defined as $1-\sum_j s_{ij}^2$, where s_{ij} denotes the share of citations to patent i originating from technological class j (Trajtenberg et al., 1997).

with market concentration.

To further validate the model's key assumption of within-industry spillovers, I use a quasi-natural experiment based on variation in state-level enforceability of non-compete agreements. These agreements affect the mobility of R&D workers, which is an important channel of knowledge diffusion (Jaffe et al., 1993; Almeida and Kogut, 1999; Singh and Agrawal, 2011; Stoyanov and Zubanov, 2012; Liu, 2023). Stronger enforcement restricts inventor mobility and thereby reduces the efficiency of within-industry knowledge spillovers. Building on this mechanism, the empirical results show that stricter enforcement encourages industry leaders to produce more general innovations, while discouraging followers from doing so. This aligns with the model's logic: when within-industry spillovers are weakened, leaders shift from firm-specific to general innovations because they face less risk of being overtaken, while followers lose incentives to pursue general innovations as a way to absorb external general knowledge and catch up.

To quantify the effects of innovation generality on growth and study its policy implications, I calibrate the model using U.S. data from 1995–2000, a period characterized by high innovation generality and peak economic growth over the past four decades. The comparative statics yield two key insights. First, reducing the cost of general R&D stimulates growth more effectively than reducing firm-specific R&D cost, as it magnifies spillovers both within and across industries. Second, when within-industry spillovers become highly efficient, aggregate growth may decline, as leaders are strongly discouraged from investing in general innovations. This reduction offsets followers' expansion of general R&D and limits the spillovers available to firms in other industries. Consequently, weak enforcement of non-compete agreements may not necessarily promote growth: if inventors carrying general knowledge can easily move to follower firms, leaders may have little incentive to undertake general innovations in the first place.

Building on the comparative statics, I evaluate the effects of general R&D subsidies. The key finding is that subsidizing leaders' general innovations is typically more effective in promoting aggregate growth than supporting followers', especially in industries where general R&D efficiency is high.[3] Targeting leaders' general R&D strengthens their incentives to pursue general knowledge, which in turn encourages followers to expand their own R&D to absorb it. This increases the aggregate level of general knowledge and further enhances cross-industry spillovers through a feedback loop. By contrast, when subsidies are directed to followers, they increase their general R&D to catch up, but leaders in response scale back their own efforts to avoid being displaced, partly offsetting the aggregate gains.

This policy implication differs from the traditional leader–follower framework in the literature (see, e.g., Liu et al., 2022; Akcigit and Ates, 2023), where supporting follow-

[3] That is, where the cost scale of general R&D is low.

ers is typically viewed as the optimal policy. While subsidizing followers remains valuable—since they are often self-motivated to pursue general innovations and require less monitoring—policies that sharpen leaders' incentives to invest in general R&D can create larger spillovers and stimulate growth more effectively. Practical tools therefore include prize systems that explicitly reward leading firms for developing general innovations and research collaborations that encourage cross-sector knowledge sharing.[4]

Finally, the model provides a lens on the link between innovation generality and secular trends. Since 2000, the U.S. has experienced a marked growth slowdown, coinciding with a sharp decline in aggregate patent generality across most major sectors. Calibrating the model to 2010–2015 data suggests that reduced efficiency of general R&D investment explains about half of the observed growth slowdown, as well as a large fraction of the decline in firm entry and leadership turnover. As general ideas become harder to find, firms shift their focus toward firm-specific innovations, which hinders knowledge diffusion and makes it more difficult for followers to catch up. In other words, slower technological diffusion and lower leadership turnover emerge endogenously from the lower efficiency of general R&D faced by both leaders and followers. In this way, the model connects falling innovation generality to slower diffusion and lower business dynamism—highlighting how long-run growth and market structure are jointly shaped by the scope of innovation.

Related Literature. This paper relates to several strands of literature. First, it contributes to work on heterogeneous innovation and firm dynamics (Klette and Kortum, 2004; Acemoglu and Cao, 2015; Akcigit and Kerr, 2018; Garcia-Macia et al., 2019; Caggese, 2019; Peters, 2020; Acemoglu et al., 2022; Caicedo and Pearce, 2024; Casal, 2025; Fernández-Villaverde et al., 2025). Prior studies distinguish between radical vs. incremental, external vs. internal, and basic vs. applied innovations, as well as differences between incumbents and entrants. Building on this tradition, I focus on innovation generality, which captures the extent of knowledge spillovers across firms. Importantly, while both general and basic innovations create broad spillovers, they differ in several aspects: basic research is non-applied and mainly conducted by public institutions (e.g., universities) (Akcigit et al., 2020), whereas general research is applied, largely conducted by private firms, and more prevalent in smaller and younger firms.

Second, this paper relates to the literature on technological spillovers (Jaffe et al., 1993, 2000; Bloom et al., 2013; Perla and Tonetti, 2014; Arora et al., 2021; Benhabib et al., 2021; Liu and Ma, 2021; Dyévre, 2024; Giroud et al., 2024). Earlier work emphasizes agglomeration effects, where inventor productivity rises with the number of inventors in the same field or location. General innovations, however, create broader spillovers across technological boundaries. To capture this, I introduce spillover mechanisms both within

[4] Recent work shows that cash prizes can complement subsidies by steering innovation toward socially valuable outcomes (see, e.g., Che et al., 2021; Graff Zivin and Lyons, 2021).

and across industries. Unlike research on patenting strategies to block rivals (Argente et al., 2023; Cunningham et al., 2021), my focus is on knowledge spillovers—captured through innovation generality—as a strategic margin.

Finally, the paper contributes to the literature on the recent U.S. productivity slowdown (Liu et al., 2022; Aghion et al., 2023; De Ridder, 2024; Olmstead-Rumsey, 2025). Existing research highlights weaker spillovers from leaders or declining innovation quality of followers as mechanisms that widen firm inequality and dampen growth (Akcigit and Ates, 2023; Olmstead-Rumsey, 2025). Different from prior studies, this paper focuses on the role of general R&D efficiency in shaping firms' strategic innovation decisions and in driving the slowdown in growth and business dynamism.

The remainder of the paper is organized as follows. Section 2 develops the theoretical framework and derives predictions about knowledge generality. Section 3 introduces the empirical measure of innovation generality. Section 4 quantifies the model and tests its predictions. Section 5 examines the policy implications and the role of knowledge generality in driving the post-2010 growth slowdown. Section 6 concludes.

2 A Growth Model with Knowledge Generality

To study how firms adjust their innovation strategies in response to knowledge spillovers and the aggregate implications of their decisions, I develop an endogenous growth model in which firms choose between general and firm-specific innovations. The model is set in continuous time and features a continuum of heterogeneous markets, which differ in the efficiency of general R&D investment. In each market, two firms compete along a technological ladder, following the framework of Aghion et al. (2001), Liu et al. (2022), and Akcigit and Ates (2023). I begin by describing household preferences, then outline firms' pricing and innovation decisions, define the equilibrium, and finally present the theoretical predictions about innovation generality.

2.1 Household Preferences

The representative household owns firms in the economy and provides production workers L inelastically. At each instance t, the representative household decides its consumption across a continuum of heterogeneous duopoly markets indexed by j, and maximizes their utility:

$$\max_{y_a(t,j),y_b(t,j)} \exp\Big\{\int_0^1 \ln[y_a(t,j)^{\frac{\sigma-1}{\sigma}} + y_b(t,j)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} dj\Big\}$$
$$\text{s.t.} \int_0^1 \big[p_a(t,j)y_a(t,j) + p_b(t,j)y_b(t,j)\big]\,dj = \Pi(t) + W(t)L \tag{1}$$

where $y_i(t,j)$ is the demand for firm i's product in market j, $p_i(t,j)$ is the price of firm i's product, with $i \in \{a,b\}$. The term $W(t)$ is the equilibrium wage of production workers, and $\Pi(t)$ represents the aggregate economy-wide profits.[5] The elasticity of substitution across the two varieties within each market is measured by σ.

2.2 Firms

In each market, two incumbent firms compete for leadership. I first describe their pricing decisions and then their innovation choices.

Pricing. Firm i in market j has the following production function:

$$y_i(t,j) = \gamma^{z_i(t,j)} l_i(t,j) \tag{2}$$

where $y_i(t,j)$ is firm i's output, $\gamma^{z_i(t,j)}$ is firm i's productivity, and $l_i(t,j)$ is the number of production workers it employs. Firms improve their productivity through step-by-step innovations, with γ the innovation step size. Henceforth, the indices j and t are dropped to simplify the notation. In each market, the two firms engage in Bertrand competition by setting their prices p_i and solve:

$$\begin{gathered} \max_{p_i} (p_i - W\gamma^{-z_i}) y_i \\ \text{s.t. } \; p_a y_a + p_b y_b = \Pi + WL \;\; \text{and} \;\; y_a/y_b = (p_a/p_b)^{-\sigma} \end{gathered} \tag{3}$$

The pricing problem can be analyzed based on the gap between the leader and the follower, denoted as $s = |z_a - z_b|$. When $s > 0$, one firm acts as a temporary leader while the other follower. If $s = 0$, the two firms are considered to be neck-and-neck. Solving (3) yields the price ratio $\rho_s = p_s/p_{-s}$ between the leader and the follower, which satisfies:

$$\rho_s^{\sigma} = \gamma^{-s} \frac{\sigma \rho_s^{\sigma-1} + 1}{\sigma + \rho_s^{\sigma-1}} \tag{4}$$

Let π_s represent the leader's profit, normalized by the total income $(\Pi + WL)$, and π_{-s} the normalized follower's profit. We then have a pair of profits fully characterized by the distance s:

$$\pi_s = \frac{\rho_s^{1-\sigma}}{\sigma + \rho_s^{1-\sigma}}, \;\; \pi_{-s} = \frac{1}{\sigma \rho_s^{1-\sigma} + 1} \tag{5}$$

[5] It is important to note that the R&D investments are paid directly from firm profits, $\Pi(t)$, and are paid back to the representative households. Hence they do not appear on the right hand side of budget constraint (1).

This pair of profits indicates that the leader earns more profit than the follower for all $s \geq 1$, and its profit increases with s.

Innovation Types: General and Firm-Specific. To improve their productivity, firms allocate their resources between two types of innovations, general and firm-specific. Unlike other forms of heterogeneous innovations[6], general and firm-specific innovations differ in their contribution to knowledge spillovers and how they shape a firm's capacity to absorb outside ideas.

General innovations create spillovers that are widely accessible, benefiting not only the innovating firm but also its competitors and even firms in other industries. Firms nevertheless still have strong incentives to pursue them, as general innovations expand absorptive capacity—the ability to recognize, learn from and build on external general knowledge. By contrast, firm-specific innovations are narrowly tailored to a firm's own operations, products, or processes. Their benefits remain largely internal and are difficult for competitors to imitate or appropriate.

These contrasting features—broad spillovers and absorptive gains from general innovation versus narrow, private returns from firm-specific innovation—play a central role in shaping firms' strategic choices in response to knowledge spillovers.

Innovation Decisions. I use the superscript (or subscript) k to index different types of markets, which differ in the efficiency of general R&D investment.[7]

A firm that is currently in the leadership position incurs a firm-specific R&D cost $c_\eta(\eta_s^k)$ and a general R&D cost $c_\lambda^k(\lambda_s^k)$ in exchange for a Poisson rate $(\eta_s^k + \Phi\lambda_s^k)$ to improve its productivity z_s by one step. Here, η_s^k and λ_s^k represent the leader's firm-specific and general R&D efforts, respectively. Similarly, the follower chooses its own firm-specific and general R&D efforts, denoted by η_{-s}^k and λ_{-s}^k. Both types of effort contribute to firm-level productivity, but their growth payoffs differ.

I introduce two forms of spillovers arising from general innovations: cross-industry and within-industry spillovers.

Cross-industry Spillovers. By investing in general innovations, both firms gain access to cross-industry knowledge spillovers, captured by Φ, which is determined endogenously by the aggregate level of general R&D efforts in equilibrium:

$$\Phi \equiv \Phi(\Lambda) \tag{6}$$

[6]For example, radical versus incremental, external versus internal, and basic versus applied (Klette and Kortum, 2004; Acemoglu and Cao, 2015; Akcigit and Kerr, 2018; Garcia-Macia et al., 2019; Caggese, 2019; Peters, 2020; Acemoglu et al., 2022).

[7]Specifically, higher efficiency of general R&D investment corresponds to a lower cost scale.

where Λ is the aggregate level of general R&D efforts of all incumbents across all types of markets.

Importantly, these spillovers are not freely available: firms need to undertake general innovations themselves in order to absorb and apply the technologies developed in other industries.[8]

Within-industry Spillovers. Within an industry, spillovers flow from the leader to the follower, and their magnitude increases with the leader's general R&D effort, captured by $f(\lambda_s^k)$. However, as with cross-industry spillovers, followers cannot benefit passively: they should also invest in general innovations to absorb and exploit the knowledge created by the leader.

The state $s(k)$ transitions over time interval Δ according to:

$$s(k, t+\Delta) = \begin{cases} s(k,t)+1 & \text{with probability } \Delta \cdot (\eta_s^k + \Phi\lambda_s^k), \\ s(k,t)-1 & \text{with probability } \Delta \cdot (\eta_{-s}^k + \Phi f(\lambda_s^k)\lambda_{-s}^k + h), \\ s(k,t) & \text{otherwise.} \end{cases}$$

In the first line, the term $\Phi\lambda_s^k$ indicates that the leader's general knowledge production benefits from cross-industry spillovers, Φ. In the second line, the term h is the fixed catch-up rate for the follower, a standard and important assumption to ensure the convergence of the model. The term $\Phi f(\lambda_s^k)\lambda_{-s}^k$ reflects that the follower's general innovations benefit from two sources: cross-industry spillovers, Φ, and within-industry spillovers, $f(\lambda_s^k)$, from the leader. As the leader invests more in general knowledge production, the follower gains greater resources to learn and imitate. However, to effectively absorb this external general knowledge, the follower should also engage in general innovations. In Section 4, I empirically validate the spillover mechanism through a quasi-natural experiment that leverages policy-driven variation across U.S. states together with firm-level data.

Denote v_s^k, v_{-s}^k and v_0^k as the normalized value functions for the leader, the follower and the neck-and-neck firms.[9] Then the dynamic optimization problem is independent

[8] As an example of the cross-industry spillovers, artificial intelligence (AI) tools were initially invented and advanced mainly within high tech sectors. Then these tools (e.g., natural language processing and machine learning) were adopted and integrated by other sectors, such as finance (e.g., fraud detection), healthcare (e.g., medical imaging), manufacturing (e.g., automation), education (e.g., adaptive learning platforms), entertainment (e.g., video recommendations), etc.

[9] Consistent with the firms' profits (π_s and π_{-s}), the value functions (v_s^k, v_{-s}^k and v_0^k) are normalized by the total income ($\Pi + WL$). Note that along the balanced growth path, they grow at the same rate.

of time t, and can be characterized by the following HJB equations:

$$\begin{aligned} rv_s^k &= \max_{\eta_s^k,\lambda_s^k\geq 0}\{\pi_s - c_\eta(\eta_s^k) - c_\lambda^k(\lambda_s^k) + [\eta_{-s}^k + \Phi f(\lambda_s^k)\lambda_{-s}^k + h](v_{s-1}^k - v_s^k) \\ &\quad +(\eta_s^k + \Phi\lambda_s^k)(v_{s+1}^k - v_s^k) - \tau^k v_s^k\} \qquad (7) \\ rv_{-s}^k &= \max_{\eta_{-s}^k,\lambda_{-s}^k\geq 0}\{\pi_{-s} - c_\eta(\eta_{-s}^k) - c_\lambda^k(\lambda_{-s}^k) + [\eta_{-s}^k + \Phi f(\lambda_s^k)\lambda_{-s}^k + h](v_{-s+1}^k - v_{-s}^k) \\ &\quad +(\eta_s^k + \Phi\lambda_s^k)(v_{-s-1}^k - v_{-s}^k) - \tau^k v_{-s}^k\} \qquad (8) \\ rv_0^k &= \max_{\eta_0^k,\lambda_0^k\geq 0}\{\pi_0 - c_\eta(\eta_0^k) - c_\lambda^k(\lambda_0^k) + (\eta_{-0}^k + \Phi\lambda_{-0}^k)(v_{-1}^k - v_0^k) \\ &\quad +(\eta_0^k + \Phi\lambda_0^k)(v_1^k - v_0^k) - \tau^k v_0^k\} \qquad (9) \end{aligned}$$

where r is the growth-adjusted interest rate. The competitor's R&D decisions in state 0 (neck-and-neck competition) are given by η_{-0}^k for firm-specific R&D and λ_{-0}^k for general R&D. Both firms face an endogenous creative destruction rate τ^k, which is determined in equilibrium and depends on market type.

2.3 Entrants

There is a continuum of potential entrants who conduct general R&D and take over a random incumbent in each market. Upon entry, an entrant inherits the productivity level (z) of the incumbent it replaces. Because entrants' innovations are assumed to be inherently general, they can build on the aggregate stock of general knowledge produced by incumbents.[10] Thus, entrants benefit from cross-industry spillovers in the same way as incumbents do through their general innovations. A higher degree of cross-industry spillovers (Φ) therefore leads to a higher entry rate. To achieve an entry rate of $\Phi\lambda_e$, entrants incur an R&D cost:

$$c_e(\lambda_e) = \frac{1}{\theta}(\kappa_e\lambda_e)^\theta \qquad (10)$$

Upon entry, entrants have a probability p_k of entering a market of type k. This probability is known and endogenously determined in equilibrium. The optimization problem for entrants becomes:

$$\max_{\lambda_e} \; -c_e(\lambda_e) + \Phi\lambda_e\sum_k p_k\mathbb{E}\big[\frac{1}{2}(v_s^k + v_{-s}^k)\big] \qquad (11)$$

Hence, the optimal entry rate λ_e satisfies the following condition:

$$\Phi\sum_k p_k\mathbb{E}\big[\frac{1}{2}(v_s^k + v_{-s}^k)\big] = \kappa_e^\theta\lambda_e^{\theta-1} \qquad (12)$$

[10] In the Appendix, I show that empirically entrants exhibit higher innovation generality than incumbents.

2.4 Equilibrium Definition

The equilibrium is characterized by market-dependent creative destruction rates $\{\tau^k\}$, entrants' R&D effort λ_e, the endogenous cross-industry spillover effect Φ, an infinite collection of value functions, and R&D efforts $\{v_s^k, v_{-s}^k, \eta_s^k, \eta_{-s}^k, \lambda_s^k, \lambda_s^k\}_{s=0}^{\infty}$ such that: The incumbents' pricing decisions satisfy (3) and generate $\{\pi_s, \pi_{-s}\}_{s=0}^{\infty}$; The value functions solve the HJB equations (7) through (9); Entrants' decisions satisfy (12); The creative destruction rate satisfies: $\tau^k = \frac{p_k \Phi \lambda_e}{2\chi_k}$, where χ_k is the mass of market of type k; The cross-industry spillover is determined in equilibrium according to equation (6), by the aggregate level of general knowledge produced by incumbents, which can be expressed as:

$$\Lambda = \frac{1}{2}\sum_k \sum_{s=0}^{\infty} \chi_k \mu_s^k (\lambda_s^k + \lambda_{-s}^k) \tag{13}$$

Denote the steady-state probability distribution of the productivity gaps in the type-k market as $\{\mu_s^k\}_0^{\infty}$. The aggregate growth rate g satisfies that for all k:

$$\begin{aligned} g &= \ln(\gamma)\big[\sum_{s=1}^{\infty} \mu_s^k(\eta_s^k + \Phi\lambda_s^k) + 2\mu_0^k(\eta_0^k + \Phi\lambda_0^k)\big] \\ &= \ln(\gamma)\big\{\sum_{s=1}^{\infty} \mu_s^k\big[\eta_{-s}^k + \Phi f(\lambda_s^k)\lambda_{-s}^k + h\big]\big\} \end{aligned} \tag{14}$$

where the transition of the states satisfies:

$$\begin{aligned} 2\mu_0^k(\eta_0^k + \Phi\lambda_0^k) &= \big[\eta_{-1}^k + \Phi f(\lambda_1^k)\lambda_{-1}^k + h\big]\mu_1^k \\ \mu_s^k(\eta_s^k + \Phi\lambda_s^k) &= \big[\eta_{-(s+1)}^k + \Phi f(\lambda_{s+1}^k)\lambda_{-(s+1)}^k + h\big]\mu_{s+1}^k \end{aligned}$$

Equation (14) states that in the steady state, the average growth rate of the leader and neck-and-neck firms should equal the average growth rate of the follower in each type of the market. Along the balanced growth path, all types of markets have the same average growth rate.

2.5 Model Predictions

The model yields two predictions linking leadership, innovation generality, and market concentration, which I test empirically in Section 4. From a theoretical perspective, these predictions also distinguish general and firm-specific innovations from other dimensions of heterogeneous innovation.

Definition: Innovation Generality. I define firm-level innovation generality as the share of general R&D effort in total R&D efforts. For leaders, this is given by:

$$\psi_s^k = \frac{\lambda_s^k}{\lambda_s^k + \eta_s^k} \tag{15}$$

This measure captures, on average, the breadth of impact of the ideas produced by a firm. A higher value of ψ_s^k (ψ_{-s}^k for the follower) indicates that a larger fraction of the leader's innovation effort is directed toward general knowledge, which is then more widely applicable across firms and industries. This concept can be linked to the empirical measure documented in Section 3.

Proposition 1

Suppose the cost functions are given by:

$$c_\eta(\eta) = \tfrac{1}{\theta}(\alpha\eta)^\theta \quad \text{and} \quad c_\lambda^k(\lambda) = \tfrac{1}{\theta}(\kappa_k\lambda)^\theta,$$

with $\theta > 1$.[11] *Assume further that the within-industry spillover function $f(\cdot)$ satisfies $f(x) \geq 1$ and $f'(x) > 0$ for all $x \geq 0$. Then the innovation generality of the follower is higher than that of the leader, i.e.,*

$$\psi_{-s}^k > \psi_s^k \qquad \forall\ s \geq 1.$$

Proof. See Appendix A.1. □

Proposition 1 states that the follower prioritizes general innovations, while the leader prefers firm-specific innovations. This is because engaging in general innovations enhances the follower's capacity of learning and absorbing the general knowledge created by the leader, thereby increasing its probability of catching up. Conversely, while producing general knowledge contributes to the leader's growth and its absorptive capacity for cross-industry spillovers, it also raises the risk of being overtaken by the follower. Therefore, the leader has weaker incentives to invest in general innovations.

This prediction captures the differences in firms' investment patterns between general and basic innovations. Although both types generate greater knowledge spillovers, larger firms tend to invest less in general innovations, while only the very largest firms engage in basic research (Akcigit et al., 2020).

Corollary 1

In the case of neck-and-neck competition (i.e., $s = 0$), the innovation generality of both

[11] Convex cost functions are common in the literature (see, e.g., Bloom et al. (2002); Acemoglu et al. (2018)).

firms is given by:

$$\psi_0^k = \frac{(\Phi\alpha^\theta)^{\frac{1}{\theta-1}}}{(\Phi\alpha^\theta)^{\frac{1}{\theta-1}} + \kappa_k^{\frac{\theta}{\theta-1}}}$$

As the technology gap between the leader and the follower diverges, the innovation generality of both firms converges to the same value:

$$\psi_s^k, \psi_{-s}^k \to \frac{(\Phi\alpha^\theta)^{\frac{1}{\theta-1}}}{(\Phi\alpha^\theta)^{\frac{1}{\theta-1}} + \kappa_k^{\frac{\theta}{\theta-1}}} \quad \textit{when } s \to \infty$$

Proof. See Appendix A.2. □

Corollary 1 states that while the follower's innovation generality is higher than that of the leader (as shown in Proposition 1), the gap between them diminishes as the distance between the leader and the follower diverges, and each firm's innovation generality approaches the level in the neck-and-neck competition. Intuitively, when the distance is small but positive, the follower has strong incentives to produce general knowledge, as the potential profit gain is great if the follower successfully catches up with the leader. In contrast, the leader has strong incentives to focus on firm-specific knowledge, as the potential profit loss from being overtaken is significant. This strategic interaction between the two firms towards general knowledge spillovers implies the following Proposition, which will be tested empirically in Section 4.

Proposition **2**

Define the innovation generality gap as $g_s := \psi_s^k - \psi_{-s}^k$, $s \in \mathbb{N}_0$*. Then the innovation generality gap between the two firms varies non-monotonically with market concentration. Formally, there exist* $a, b, c \in \mathbb{N}_0$ *with* $a < b < c$ *such that:*

$$g_a > g_b \quad \textit{and} \quad g_b < g_c$$

Proof. See Appendix A.3. □

This prediction captures the differences between general and external innovations along the dimensions of firms' R&D allocation and market concentration. The gap in innovation generality varies non-monotonically with market concentration, while the gap in the share of external product lines relative to total product lines decreases monotonically, as larger firms invest disproportionately less in external innovations (Akcigit and Kerr, 2018).

3 Knowledge Generality in the Data

In this section, I introduce an empirical measure of innovation generality, defined as the average breadth of impact of the ideas produced by a firm. To implement this concept, I draw on the established indicator of patent generality (Trajtenberg et al., 1997). While the notion of patent generality was first formalized in 1997, its implications for market competition and economic growth remain relatively underexplored. I begin by outlining the data sources used to measure innovation generality, and then extend its definitions to the firm level.

3.1 Data Sources

I compile a firm-level dataset that integrates information on patents and innovating firms from multiple sources. The core patent-level data come from the United States Patent and Trademark Office (USPTO) and PatentsView.[12] These patent data are linked to Compustat, which contains comprehensive firm-level information. The linkage between patent and firm assignees is based on Kogan et al. (2017), which provides carefully constructed firm–patent matches.[13]

Patent Data. Patent data were sourced from the USPTO and PatentsView. The USPTO provides information on patent application and grant years. Details on patent assignees, citations, and their subclasses were obtained from PatentsView. The analysis focuses on utility patents, which capture the technical innovation and functional aspects of inventions, in contrast to design patents that protect aesthetic designs, and plant patents that protect new plant varieties.

Innovating Firms. To construct a firm sample, I merge the patent dataset with Compustat through the data repository from Kogan et al. (2017), which provides a standardized linkage between firm identifiers across datasets. Firm-level financial and operational variables are obtained from Compustat, while patent-level data allow the calculation of innovation measures, such as generality, at the firm level. The sample covers the period from 1980 to 2018 and includes firms that filed at least one patent in a given year. The sample further excludes agricultural (SIC code 0100-0999), utility (SIC code 4900-4949), financial (SIC code 6000-6799), and non-operating establishments (SIC code 9995).

3.2 Innovation Generality: Construction and Definitions

I use patent generality to construct an empirical firm-level proxy for the model's concept of innovation generality. I begin by defining patent generality and illustrating its con-

[12]PatentsView is an open platform which provides disambiguated data linking patents to unique inventor and assignee entities, and can be accessed through https://patentsview.org.

[13]The data from Kogan et al. (2017) can be accessed at https://github.com/KPSS2017/Technological-Innovation-Resource-Allocation-and-Growth-Extended-Data.

struction with a concrete example. This measure is then extended to represent innovation generality at the firm level.

Definition: Patent Generality. Patent generality measures the breadth of a patent's impact. A higher generality score indicates that the patent influenced subsequent innovations in a wider variety of fields, while a low generality score means that most citations are concentrated in a few fields.

Formally, patent generality is defined as

$$\psi_i = 1 - \sum_j s_{ij}^2 \tag{16}$$

where s_{ij} is the share of citations to patent i coming from patents in technology class j (Trajtenberg et al., 1997). To ensure time consistency, the generality measure is based on citation information within a 5-year window following a patent's grant.[14] In calculating this measure, self-citations are excluded in order to focus on the knowledge spillovers between firms.

Example: Generality Measure Construction. As an example of patent generality construction, the patent with ID 6594666 was granted in 2003 and assigned to Oracle Corporation (CRSP permno 10104) with SIC code 7370 (service-computer programming, data processing). The patent was about "location-aware application development framework", with detailed information shown in Figure 1. Within a 5-year window since the patent grant, it received citations from 63 utility patents across 10 technological classes defined according to Cooperative Patent Classification (CPC) subclasses, as shown in Table 1.[15] The generality score of patent 6594666 is then 0.829, suggesting a relatively broad scope of impact and, therefore, a more general innovation.

By comparison, the patent with ID 6550058, granted in the same year and assigned to International Business Machines Corporation (CRSP permno 12490) in the same industry as Oracle Corporation, was about "stack clearing device and method". It received citations from 10 utility patents spanning only 2 CPC subclasses, resulting in a generality score of 0.180.[16] This indicates that the patent's impact is relatively narrow in scope and therefore more firm-specific.

[14]Note that the definition of generality requires a patent to have at least one forward citation within five years of its grant. In the Appendix, I show that the empirical results in Section 4.2 remain robust when patents with no forward citations are instead assigned a generality score of 0.

[15]In the patent sample, there are 672 CPC subclasses in total.

[16]The detailed patent information and generality score construction can be found in the Appendix.

Figure 1: Detailed Patent Information: Patent ID 6594666

(12) **United States Patent** — **Biswas et al.**

(10) **Patent No.:** **US 6,594,666 B1**

(45) **Date of Patent:** **Jul. 15, 2003**

(54) **LOCATION AWARE APPLICATION DEVELOPMENT FRAMEWORK**

(75) Inventors: **Prabuddha Biswas**, Nashua, NH (US); **Raja Chatterjee**, Nashua, NH (US)

(73) Assignee: **Oracle International Corp.**, Redwood Shores, CA (US)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 234 days.

(21) Appl. No.: **09/669,503**

(22) Filed: **Sep. 25, 2000**

(51) **Int. Cl.**^{7} **G06F 7/00**; G06F 17/00

(52) **U.S. Cl.** **707/100**; 707/102

(58) **Field of Search** 707/100, 102

(56) **References Cited**

U.S. PATENT DOCUMENTS

5,948,040 A * 9/1999 DeLorme et al. 701/201

OTHER PUBLICATIONS

Oracle Spatial User's Guide and Reference, Release 8.1.6, Feb. 1999, 322 pages, Oracle Corporation (Oracle Part No. A77132–01) USA.

Oracle8i interMedia Audio, Image, and Video User's Guide and Reference, Release 8.1.5, Feb. 1999, 616 pages, Oracle Corporation (Oracle Part No. A67299–01) USA.

Oracle8i intermedia Locator, User's Guide and Reference, Release 8.1.5, Feb. 1999,, 42 pages, Oracle Corporation (Oracle Part No. A67298–01) USA.

* cited by examiner

Primary Examiner—Safet Metjahic
Assistant Examiner—Cindy Nguyen
(74) *Attorney, Agent, or Firm*—Sanjay Prasad

(57) **ABSTRACT**

A shareable application program interface (API) infrastructure which is used in combination with a relational database to provide data storage and processing functions for location-aware objects, including particularly mobile objects whose current position is periodically updated by a position determining system. Client and service tables in the relational database are used to store the current point location, and other data, representing virtual objects, including mobile objects. A region table stores that describing the geometry and characteristics of geographical regions having defined boundaries within which the client and service objects reside. For each client, the set of services used by that client is recorded in a client profile database table. The services available on the system which are position-dependent have a geographical location associated with them. The API makes available an assortment of location dependent processing functions which may be used by location aware applications.

22 Claims, 2 Drawing Sheets

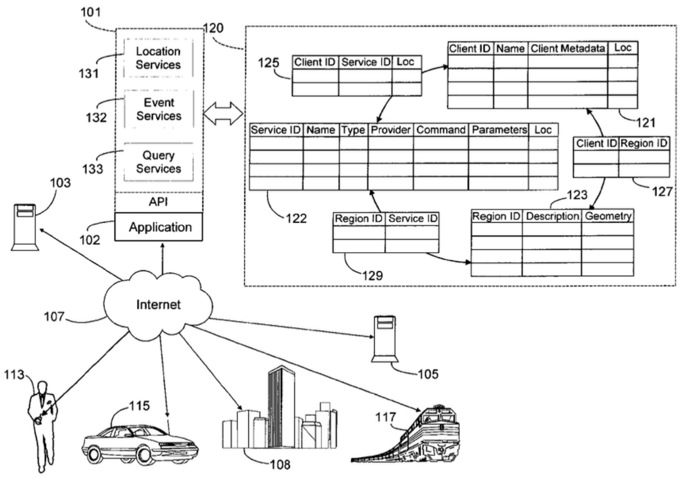


Table 1: Generality Score Construction: Patent 6594666

CPC subclasses	Definitions	Citations
$G01C$	measuring, surveying, navigation, gyroscopic instruments	8
$G06F$	electric digital data processing	19
$G06Q$	ICT for admin., comm., fin., manag. or superv. purposes	3
$G08G$	traffic control system	3
$G09B$	signaling or calling systems	1
$H04B$	transmission	1
$H04L$	transmission of digital information	6
$H04M$	telephonic communication	4
$H04W$	wireless communication networks	12
$Y10S$	technical subjects covered by former XRACs and digests	6

Note: The first and second column list the CPC subclasses the citing patents are from and their definitions. The third column lists the number of citing patents that belong to the subclass.

Definition: Firm-level Innovation Generality. Patent generality is then aggregated to the firm-year level as an indicator of firm innovation generality, by averaging among all patents that are produced by firm f in year t:

$$\psi_{f,t} = \frac{1}{n}\sum_{i=1}^{n}\psi_{i,t} \tag{17}$$

where $\psi_{i,t}$ is the generality score of patent i which was applied for in year t and assigned to firm f.[17] This measure of firm-level innovation generality serves as an empirical proxy for its theoretical counterpart in the model, that is, ψ_s^k for the leader and ψ_{-s}^k for the follower.

Note that the measure of firm-level innovation generality in (17) is the simple average of patent generality scores. In the Appendix, I construct a cost-adjusted version of this measure, using the real value of innovation from Kogan et al. (2017) to approximate patent cost (e.g., R&D expenditure or effort). The empirical results in Section 4.2 remain robust under this alternative specification.

4 Parameterization and Model Validation

In this section, I parameterize the model and test its predictions on the relationship between leadership, innovation generality, and market concentration, using the empirical measure of knowledge generality developed in the previous section. Then I provide evidence supporting the mechanism of within-industry general knowledge spillovers, thereby validating a central assumption of the model.

4.1 Parameterization

The baseline model is calibrated to the 1995-2000 period, a time characterized by high levels of innovation generality and total factor productivity growth in the US. In Section 5, I compare the baseline model with the counterfactual, where general ideas become harder to find to reflect the post-2010 economy, marked by a lower aggregate generality and lower growth. Table 2 provides an overview of the parameter values and their sources. The first part of the table lists parameters set externally, and the second part shows parameters set via internal calibration. For tractability, I assume that there are two types of industries that differ in their efficiency in conducting general R&D investments, corresponding to the type-k markets in the model. Specifically, industries whose average patent generality between 1995 and 2000 lies above the median across all industries are classified as having high general R&D efficiency, while those with an average patent generality below the

[17] I use the application year rather than the grant year to more precisely capture the timing of knowledge production.

median are classified as having low efficiency. The superscript H represents high-type (in general R&D efficiency), while L denotes low-type.

Data Sources. The calibration relies on data from several key sources. Data on the aggregate growth rate are calculated based on Fernald (2015). Data on firm entry are from the Business Dynamics Statistics (BDS). Firm-level innovation generality and R&D to sales ratio, as well as the turnover rate of industry leadership, are calculated by linking patent data from the USPTO and PatentsView, with Compustat firm data, based on the KPSS data repository. The National Center for Science and Engineering Statistics (NCSES) provides information on the R&D to GDP ratio. To ensure consistency, all moments used in the calibration are calculated as averages over the period of 1995–2000.

Functional Forms. Two functions govern the efficiency of cross- and within-industry knowledge spillovers, $\Phi(\cdot)$ and $f(\cdot)$. Following León-Ledesma and Satchi (2019) in their analysis of technology adjustment and balanced growth, I adopt exponential functional forms to measure these spillovers:

$$\Phi(\Lambda) = \exp(\hat{\phi}\Lambda) \tag{18}$$

$$f(\lambda_s^k) = \exp(\hat{f}\lambda_s^k) \tag{19}$$

where Λ is the aggregate level of general R&D efforts:

$$\Lambda = \frac{1}{2}\sum_k \sum_{s=0}^{\infty} \chi_k \mu_s^k (\lambda_s^k + \lambda_{-s}^k) \tag{20}$$

External Calibration. The growth-adjusted interest rate, which is equivalent to the discount rate, is set at $r = 0.02$. Innovation elasticity is set to $\theta = 2$, in line with the microeconometric evidence on innovation (Bloom et al., 2002; Acemoglu et al., 2018). The value of within-sector elasticity of demand, σ, is from De Loecker et al. (2021).

Internal Calibration. The remaining parameters are jointly calibrated using the simulated method of moments, where selected model-generated moments, $\mathbf{m}(\upsilon)$, are compared with their empirical counterparts, $\hat{\mathbf{m}}$, by minimizing the following objective function:

$$\big(\mathbf{m}(\upsilon) - \hat{\mathbf{m}}\big)' \mathbf{\Omega}^{-1} \big(\mathbf{m}(\upsilon) - \hat{\mathbf{m}}\big) \text{ where } \upsilon = \{\gamma, \kappa^H, \kappa^L, \alpha, \kappa_e, \hat{\phi}, \hat{f}, h\}$$

where $\mathbf{\Omega}$ contains weighted squares of the data moments on the main diagonal and zeros elsewhere.[18] While the parameters are calibrated jointly, I provide a discussion on each parameter in relation to the targeted moment it is most closely associated with.

[18] Details about the simulated method of moments are shown in the Appendix.

Model Performance: Targeted Moments. In the model, three parameters are linked to firm-level generality: general R&D cost scale of high type (in R&D efficiency) κ^H, general R&D cost scale of low type κ^L and the efficiency of within-industry knowledge spillovers $\hat{f}$. These parameters are calibrated using three moments related to patent generality: the average patent generality of leaders and followers in high-type industries (SIC 4-digit) and the average patent generality of leaders in low-type industries, based on citation information within a 5-year window following a patent's grant. Specifically, firms with the largest sales in their industries are classified as leaders, while all remaining firms are followers. The innovation step size γ, firm-specific R&D cost scale α, and catch-up rate h are pinned down using three additional moments: an annual aggregate TFP growth rate of 1.66% based on data from Fernald (2015), a median R&D costs to sales ratio of 4.0% and a leadership turnover rate of 23% in high-type industries calculated from the firm sample constructed by linking patent and firm data.[19] The entry cost, κ_e is determined using the average firm entry rate of 11.6% from BDS.[20] Finally, the efficiency of cross-industry knowledge spillovers is pinned down by matching the average ratio of cross-industry to within-industry backward citations of patents.[21] Table 3 documents how the model aligns with the targeted moments.

Model Performance: Untargeted Moments. In addition to the targeted moments, the model also matches three additional moments: average innovation generality of followers and leadership turnover rate in low-type industries and the R&D expenditure to GDP ratio from NCSES. Consistent with the data, the model-implied leadership turnover rate is higher in industries with more efficient general R&D investments.

4.2 Testing the Model's Predictions on Innovation Generality

Using the firm-level dataset for the years 1980 to 2018 constructed in Section 3, I study the patterns of innovation generality within markets. In particular, I test the model predictions from Propositions 1 and 2, that is, whether there exist gaps in innovation generality between leaders and followers and how these gaps vary with the level of market concentration.

Test of Proposition 1: Leaders have lower innovation generality.

[19]The leadership turnover rate is calculated as the fraction of industries with a leading firm different from the one in the last year. The time series of leadership turnover rates for both industry types is provided in the Appendix.

[20]The empirical entry rate is defined as the number of new firms over the number of incumbent firms. Hence the model-implied entry rate equals $\frac{\Phi\lambda_e}{2}$, as in each market, there are two incumbents.

[21]Self-citations are excluded to focus on the knowledge diffusion between firms. Details about the calculation of this ratio in the model are provided in the Appendix.

Table 2: Parameter Values

Parameter	Description	Value	Source
r	Growth-adjusted interest rate	0.02	Standard
σ	Within-sector elasticity of demand	5.75	De Loecker et al. (2021)
θ	Innovation elasticity	2	Bloom et al. (2002)
γ	Innovation step size	1.67	Internal calibration
κ^H	General R&D cost scale, high-type	11.63	Internal calibration
κ^L	General R&D cost scale, low-type	14.06	Internal calibration
α	Firm-specific R&D cost scale	10.59	Internal calibration
κ_e	Entry cost	4.01	Internal calibration
$\hat{f}$	Efficiency of within-industry spillovers	24.90	Internal calibration
$\hat{\phi}$	Efficiency of cross-industry spillovers	45.06	Internal calibration
h	Catch-up rate	0.03	Internal calibration

Note: The first part of the table lists parameters set externally. The second part of the table shows parameters set via internal calibration.

Table 3: Model Fit

	Model	Data
Targeted Moments		
Average innovation generality - Leader, high-type	0.50	0.52
Average innovation generality - Follower, high-type	0.56	0.56
Average innovation generality - Leader, low-type	0.41	0.41
Median R&D to sales ratio	3.7%	4.0%
Aggregate growth rate	1.66%	1.66%
Entry rate	11.6%	11.6%
Leadership turnover, high-type	17%	23%
Average relative backward citations	3.25	3.17
Untargeted Moments		
Average innovation generality - Follower, low-type	0.46	0.46
Leadership turnover, low-type	14%	15%
R&D expenditure to GDP ratio	2.7%	2.5%

Note: Data on innovation generality, R&D to sales ratio, leadership turnover and relative backward citations are calculated from the firm sample constructed by linking USPTO, PatentsView and Compustat. Data on aggregate growth rate are based on Fernald (2015). Data on entry rate are from BDS. R&D to GDP ratio is collected from NCSES. All moments used for calibration are calculated as averages over the 1995–2000 period.

First, I investigate whether there exists a difference in the average innovation generality between leaders and followers from the following OLS estimation:

$$\psi_{f,t} = \underset{(s.e.\ 0.003)}{-\ 0.028} \cdot \mathbb{1}(Leader)_{f,t} + \delta_{j,t} + \epsilon_{f,t} \tag{21}$$

where $\psi_{f,t}$ represents the average patent generality of firm f in year t. $\mathbb{1}(Leader)_{f,t}$ is an indicator function, which equals one if firm f is the leader (i.e., firm with the largest sales) in industry j (with at least two patenting firms) defined according to 4-digit Standard Industrial Classification (SIC) code in year t.[22] $\delta_{j,t}$ captures industry-year fixed effects. I further control for the log of average forward citations at the firm level, where forward citations are counted within a 5-year window following the patent's grant. This accounts for patent quality that may be correlated with the generality score. Holding the leadership or not accounts for about 0.13 of the standard deviation of firm-level innovation generality. This fact is consistent with the model's prediction in Proposition 1.

Note that the estimation results are based on the whole sample from 1980 to 2018. In the US between 1995 and 2000, the average generality gap between leaders and followers was 0.04 in high-type industries and 0.05 in low-type industries, and the parameterized model yields corresponding gaps of 0.06 and 0.05.

Test of Proposition 2: The gap in innovation generality between the two firms varies non-monotonically with market concentration.

Next, based on the firm sample from 1980 t0 2018, I study the relationship between gaps in innovation generality and market concentration, by conducting the following estimation:

$$\psi^L_{j,t} - \psi^F_{j,t} = \underset{(s.e.\ 0.098)}{-\ 0.357} \cdot HHI_{j,t} + \underset{(s.e.\ 0.080)}{0.229} \cdot HHI^2_{j,t} + \delta_j + \delta_t + \epsilon_{j,t} \tag{22}$$

where $\psi^L_{j,t}$ and $\psi^F_{j,t}$ represent average patent generality of the leader and follower in industry j in year t, respectively. $HHI_{j,t}$ is Herfindahl–Hirschman index (HHI), defined as the sum of the squared market shares of all firms in industry j in year t, which is a standard measure of market concentration.[23] The term $HHI^2_{j,t}$ is the square of $HHI_{j,t}$. δ_j and δ_t capture industry fixed effects and year fixed effects, respectively. I further control for the total number of firms in the industry. This estimation indicates that generality gaps follow a U-shaped pattern with respect to market concentration, consistent with the theoretical prediction in Proposition 2.[24] Figure 2 illustrates the U-shaped pattern of the parameterized model. For each level of market concentration, the figure shows the weighted average of the generality gap across the two industry types.

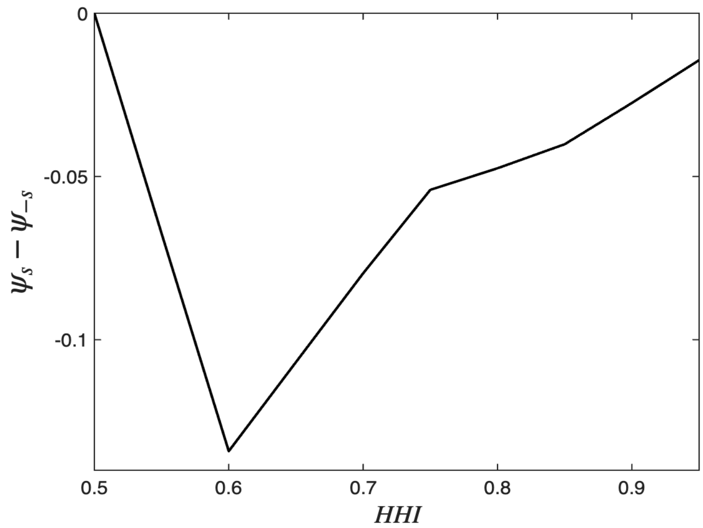
To conclude, these two estimations show that there is a discrepancy in innovation generality, defined as the average patent generality, between leaders and followers. Lead-

[22]This definition of market leader is common in the literature (see e.g., Kroen et al., 2021; Olmstead-Rumsey, 2025). The results are robust if firms whose sales are above the top 5 percentile are considered as leaders. See Appendix.

[23]For industries with multiple followers, I use the median value of the innovation generality among followers. The results remain robust if the mean value is used. See Appendix.

[24]The bin plot of this empirical relationship is shown in Figure 8 in the Appendix.

Figure 2: Model-implied Generality Gaps and Market Concentration



Note: The figure plots the model-implied relationship between gaps in innovation generality, $(\psi_s - \psi_{-s})$, and market concentration, measured by HHI.

ers prefer innovations with lower generality, while followers tend to produce more general innovations, consistent with Proposition 1. However, the gaps in innovation generality between the leader and the follower have a U-shaped relationship with the level of market concentration, in line with Proposition 2.

4.3 Model Validation: Within-industry Spillovers

The central mechanism behind firms' strategic innovation decisions is within-industry general knowledge spillovers. This mechanism has two key elements: (1) general knowledge flows from leaders to followers, and (2) absorbing these spillovers requires followers themselves to engage in general knowledge production. The model therefore predicts that when within-industry spillovers are more efficient, leaders will shift toward firm-specific innovations to protect their advantage, while followers will invest more in general innovations to absorb external general knowledge and catch up. Conversely, when within-industry spillovers weaken, leaders become more willing to produce general knowledge, while followers are discouraged from doing so.

To validate this mechanism, I draw on a quasi-natural experiment that leverages variation in state-level enforcement of non-compete agreements, which serves as a proxy for the efficiency of within-industry spillovers. The logic behind this is that non-compete agreements restrict workers from competing against their former employer within a certain period, thereby directly affecting the mobility of R&D workers—a key channel for knowledge diffusion (Jaffe et al., 1993; Almeida and Kogut, 1999; Singh and Agrawal, 2011; Stoyanov and Zubanov, 2012; Liu, 2023). Stronger enforcement of non-competes limits labor mobility and thereby reduces the efficiency of within-industry knowledge spillovers.

State-level Enforceability of Non-compete Agreements. I use a state-year index of non-compete enforceability, first developed by Bishara (2011) and later extended by Marx (2022), which incorporates judicial and legislative decisions affecting the enforceability of non-compete agreements for all workers. The state-level changes in non-compete enforceability can be treated as plausibly exogenous legal shocks (see, e.g., Marx et al., 2009; Samila and Sorenson, 2011). Following Reinmuth and Rockall (2025) and Ma et al. (2025), I rely on the normalized version of this index to measure the strength of enforcement across states, to ensure the interpretability of regression coefficients. To capture major policy shifts, I construct a binary treatment variable, NCA, which indicates whether a state has recently experienced a significant change in enforceability—defined as a year-over-year change in the index greater than 0.02—following the clean treatment approach in Reinmuth and Rockall (2025) and Ma et al. (2025).[25] The variable equals 1 if the change represents a strengthening of enforcement, -1 if it represents a weakening, and 0 otherwise. Using this definition, I identify 16 major changes between 1991 and 2014, of which 11 involved stronger enforcement.[26]

Estimation. To examine how firms respond to changes in the efficiency of within-industry general knowledge spillovers, proxied by shifts in non-compete agreement enforceability, I estimate the following difference-in-differences specification:

$$\psi_{f,t} = \beta_N NCA_{s,t} + \beta_L \mathbb{1}(Leader)_{f,t} + \beta_{N,L}\left[NCA_{s,t} \times \mathbb{1}(Leader)_{f,\tau}\right] + \delta_{j,t} + \delta_s + \epsilon_{f,t} \quad (23)$$

where $\psi_{f,t}$ is the average patent generality of firm f in year t. The variable $NCA_{s,t}$ indicates the direction of the last major reform in the enforceability of non-compete agreements in the state s, where firm f's headquarters is located. It equals 1 if the last reform reflects stronger enforcement, -1 if it reflects weaker enforcement, and 0 otherwise. The indicator $\mathbb{1}(Leader)_{f,t}$ equals one if firm f is an industry leader in year t, capturing contemporaneous differences in innovation generality between leaders and followers. The interaction term $\left[NCA_{s,t} \times \mathbb{1}(Leader)_{f,\tau}\right]$ identifies whether firms that were industry leaders at the time of the reform (i.e., τ) respond differently to changes in non-compete enforceability.[27] Finally, $\delta_{j,t}$ and δ_s represent industry-year and state

[25] Note that the results remain robust when a significant change is alternatively defined as a larger or smaller shift in the index. See Appendix.

[26] Data on state-level enforceability of non-compete agreements are available for the period 1991–2014. Accordingly, the combined firm sample is restricted to this period.

[27] As a robustness check, I reconstruct the leadership indicator in two alternative ways: (1) using predetermined leader status measured one or two periods before the treatment year, to mitigate potential post-treatment bias; and (2) using a measure of persistent leadership, defined as maintaining leader

fixed effects, respectively, which absorb time-varying industry-specific shocks and time-invariant heterogeneity across states. The key coefficients of interest are β_N and $\beta_{N,L}$: β_N captures the effect of changes in non-compete enforceability on followers; the coefficient $\beta_{N,L}$ captures the differential effect of changes in non-compete enforceability on leaders relative to followers.

Table 4 reports the estimated coefficients, with standard errors clustered at the state level. Importantly, β_N is negative and statistically significant, and $\beta_{N,L}$ is positive and statistically significant. Moreover, the magnitude of $\beta_{N,L}$ is larger than that of β_N (i.e., $\beta_{N,L} + \beta_N > 0$). This indicates that when non-compete enforcement strengthens, leaders shift toward more general innovations, whereas followers tilt toward more firm-specific innovations. These findings align with the model's logic of within-industry knowledge spillovers: when spillovers weaken, leaders can safely produce more general innovations without fear of being overtaken, while followers lose incentives to produce general innovations as a means of absorbing external knowledge and catching up.

Table 4: Within-industry General Knowledge Spillovers

β_N	-0.016^{**}
	(0.008)
$\beta_{N,L}$	0.037^{***}
	(0.008)
Industry-Year FE	✓
State FE	✓
R-squared	0.454
Observations	23,764

Standard errors are clustered at the state level.
*** p<0.01, ** p<0.05, * p<0.1

Note: This table shows the results from estimation (23).

In the parameterized model, reducing the coefficient for within-industry knowledge spillover efficiency ($\hat{f}$) from 24.9 to 8.4 leads to a rise in the average innovation generality of leaders across the two industry types from 0.452 to 0.472 (an increase of 0.020). Meanwhile, the average innovation generality of followers declines from 0.508 to 0.492 (a decrease of 0.016). These patterns align closely with the estimation results reported in column (3) of Table 4, where leaders' innovation generality increases by ($\beta_{N,L} + \beta_N$) (0.021) to a stricter enforcement of non-compete agreements, while followers' innovation generality falls by β_N (0.016).

status from the time of the reform (i.e., τ) through period t. See Appendix for details.

4.4 Firms' Innovation Decisions and Within-industry Spillovers

Having validated the model's within-industry general knowledge spillovers, I next examine how leaders and followers adjust their innovation decisions in response to these spillovers, based on the parametrized model.

Without loss of generality, I focus on the high-type industries. Figure 3 shows the general and firm-specific R&D of the leader and follower in each state s, the stationary distribution, and the extent of within-industry knowledge spillovers, which is measured by the additional growth that followers gain through within-industry knowledge spillovers, and can be expressed as:

$$\Phi\big[f(\lambda_s^H) - 1\big]\lambda_{-s}^H = \Phi\big[\exp(\hat{f}\lambda_s^H) - 1\big]\lambda_{-s}^H \tag{24}$$

$$\approx \Phi\hat{f}\lambda_s^H\lambda_{-s}^H \tag{25}$$

where $\Phi(\Lambda) = \exp(\hat{\phi}\Lambda)$ measures the external effects of cross-industry general knowledge, $\hat{f}$ governs the efficiency of within-industry knowledge spillovers, λ_s^H and λ_{-s}^H are the general R&D efforts of the leader and the follower, respectively.

As predicted in Proposition 2, when the distance between leaders and followers is small but positive, followers have strong incentives to pursue general innovations to absorb the external general knowledge produced by leaders, as the potential profit gains are substantial. Conversely, leaders prioritize firm-specific knowledge to mitigate the risk of being overtaken. However, as the distance widens, these incentives weaken.

In state $s = 0$, the within-industry spillovers is muted by definition (i.e., $f(0) = 1$). At $s = 1$, the extent of within-industry knowledge spillovers reaches its peak, as the follower invests heavily in general R&D to absorb the external general knowledge and improve its productivity z_{-s}. As the distance s increases, the probability of the follower overtaking the leader decreases. Therefore, the leader faces less competition and reduces its innovation efforts, allocating resources more evenly between general and firm-specific R&D. Meanwhile, as the follower falls further behind, its probability of catching up declines, weakening its incentive to invest in R&D.

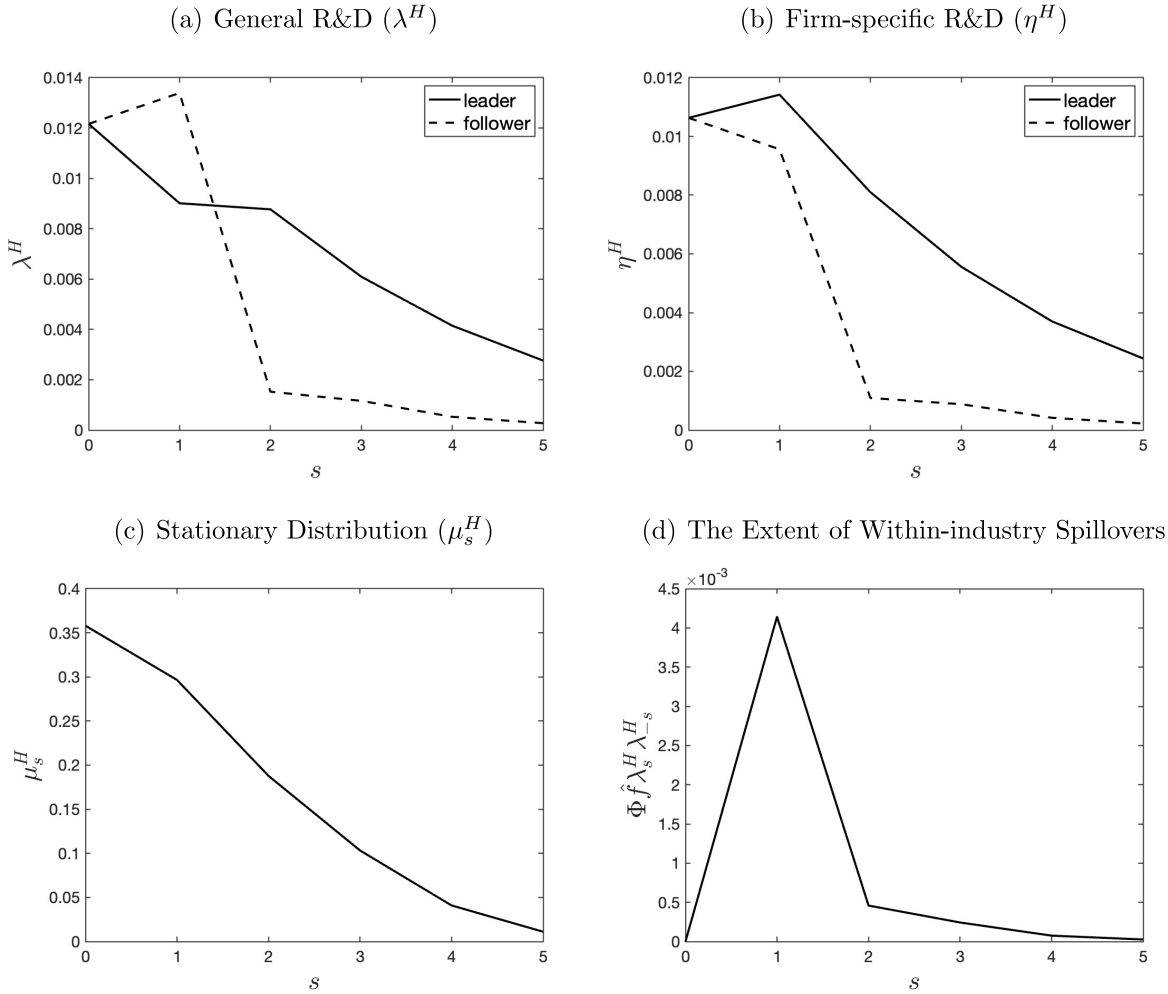
4.5 Market Concentration and Innovation Generality

After studying firms' innovation decisions, I turn to the link between innovation generality and market concentration. Note that the level of market concentration in the model has a one-to-one relationship with the distance between the leader and the follower.

$$HHI_s = \Big(\frac{\rho_s^{1-\sigma}}{\rho_s^{1-\sigma}+1}\Big)^2 + \Big(\frac{1}{\rho_s^{1-\sigma}+1}\Big)^2$$

with ρ_s the price ratio between the leader and the follower, which satisfies $\rho_s^\sigma = \gamma^{-s}\frac{\sigma\rho_s^{\sigma-1}+1}{\sigma+\rho_s^{\sigma-1}}$.

Figure 3: R&D Efforts, Spillovers and Distribution (High-type Industries)



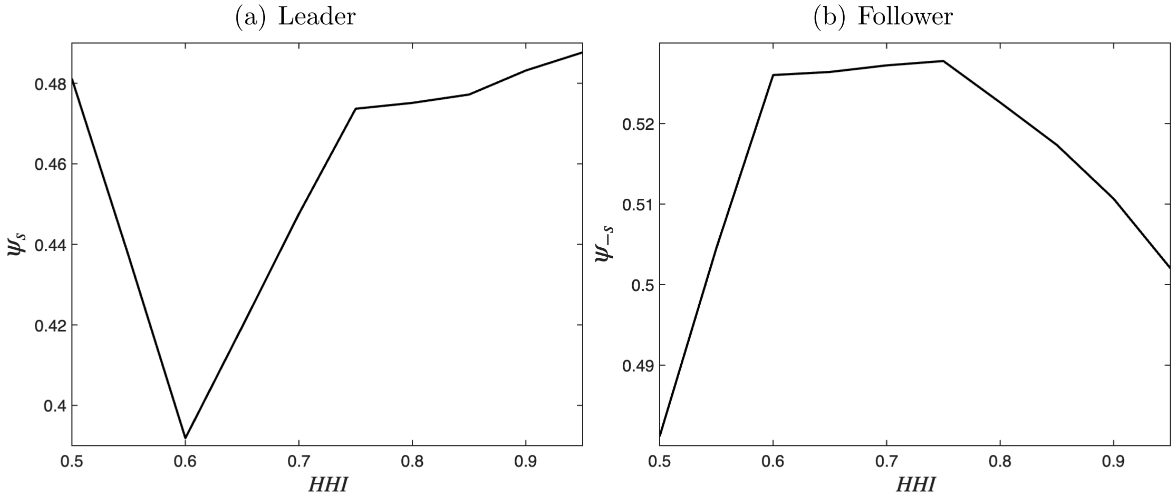
Note: Panel (a) plots general R&D efforts of the leader (solid line) and the follower (dashed line) in each state. Panel (b) plots the corresponding firm-specific R&D efforts. Panel (c) plots stationary distribution of productivity gap s. Panel (d) plots the extent of within-industry knowledge spillovers.

As predicted in Proposition 2, the generality of leaders' and followers' innovations varies systematically with concentration. In the parametrized model, leaders exhibit a U-shaped relationship between innovation generality and concentration, while followers display a hump-shaped pattern, as shown in Figure 4.

To compare these patterns in the data, I investigate the empirical relationship between market concentration and innovation generality of leaders and followers, separately, following estimation (22) from the test of Proposition 2:

$$\psi_{j,t}^L = \underset{(s.e.\ 0.070)}{-\ 0.194} \cdot HHI_{j,t} + \underset{(s.e.\ 0.057)}{0.137} \cdot HHI_{j,t}^2 + \delta_j + \delta_t + \epsilon_{j,t} \tag{26}$$

Figure 4: Innovation Generality and Market Concentration



Note: Panel (a) plots the relationship between the leader's innovation generality, ψ_s, with level of market concentration, measured by HHI. Panel (b) plots that of the follower. Both figures plot the weighted average of innovation generality across the two industry types.

$$\psi^F_{j,t} = \underset{(s.e.\ 0.071)}{0.164} \cdot HHI_{j,t} - \underset{(s.e.\ 0.058)}{0.092} \cdot HHI^2_{j,t} + \delta_j + \delta_t + \epsilon_{j,t} \tag{27}$$

where $\psi^L_{j,t}$ and $\psi^F_{j,t}$ represent average patent generality of the leader and follower. The estimation results are qualitatively consistent with the model and reinforce its mechanism: leaders limit general knowledge production to preserve their advantage, while followers rely on it to absorb within-industry spillovers and narrow the technological gap.[28] These incentives are strongest when the leader–follower distance is small (i.e., $s = 1$).

5 Policy Implications and Secular Trends

In this section, I study the policy implications based on the parameterized model and secular trends of knowledge generality. I begin with the comparative statics of aggregate growth, which show that growth is more responsive to general R&D costs than to firm-specific R&D costs, suggesting that subsidies for general R&D are more effective.[29] Then I evaluate the impact of general R&D subsidies, and consider the effects of improving the efficiency of general knowledge spillovers.[30] Finally, I document the secular trends in

[28]The results are robust if further controlling for the log of average forward citations at the firm level. See Appendix.

[29]Note that along the BGP, total output, total income, economy-wide profits and wage for production workers all grow at the same rate, g.

[30]The analysis focuses on inefficiencies from knowledge spillovers rather than distortions from firms' pricing. In this setting, policies that maximize aggregate growth also maximize welfare, since firms'

U.S. aggregate growth and innovation generality, and discuss how they are related.

5.1 Comparative Statics of Aggregate Growth

Before discussing the policy implications, I provide comparative statics on the key factors that impact growth. Figure 5 shows how the aggregate growth rate responds to a reduction in R&D costs for all firms across all industries, or an increase in the efficiency of general knowledge spillovers.

Decrease in R&D Costs. Panels (a) and (b) plot the growth elasticity with respect to (a decline in) general and firm-specific R&D cost scale, respectively. From panel (a), we observe that lowering the general R&D cost scale (i.e., κ_k) across all industries consistently increases economic growth. When incumbents engage more in general innovations, they experience faster firm-level growth. In addition, this increases the level of aggregate general knowledge (i.e., Λ), which further reinforces cross-industry spillovers through a feedback loop (i.e., $\Phi(\Lambda)$).

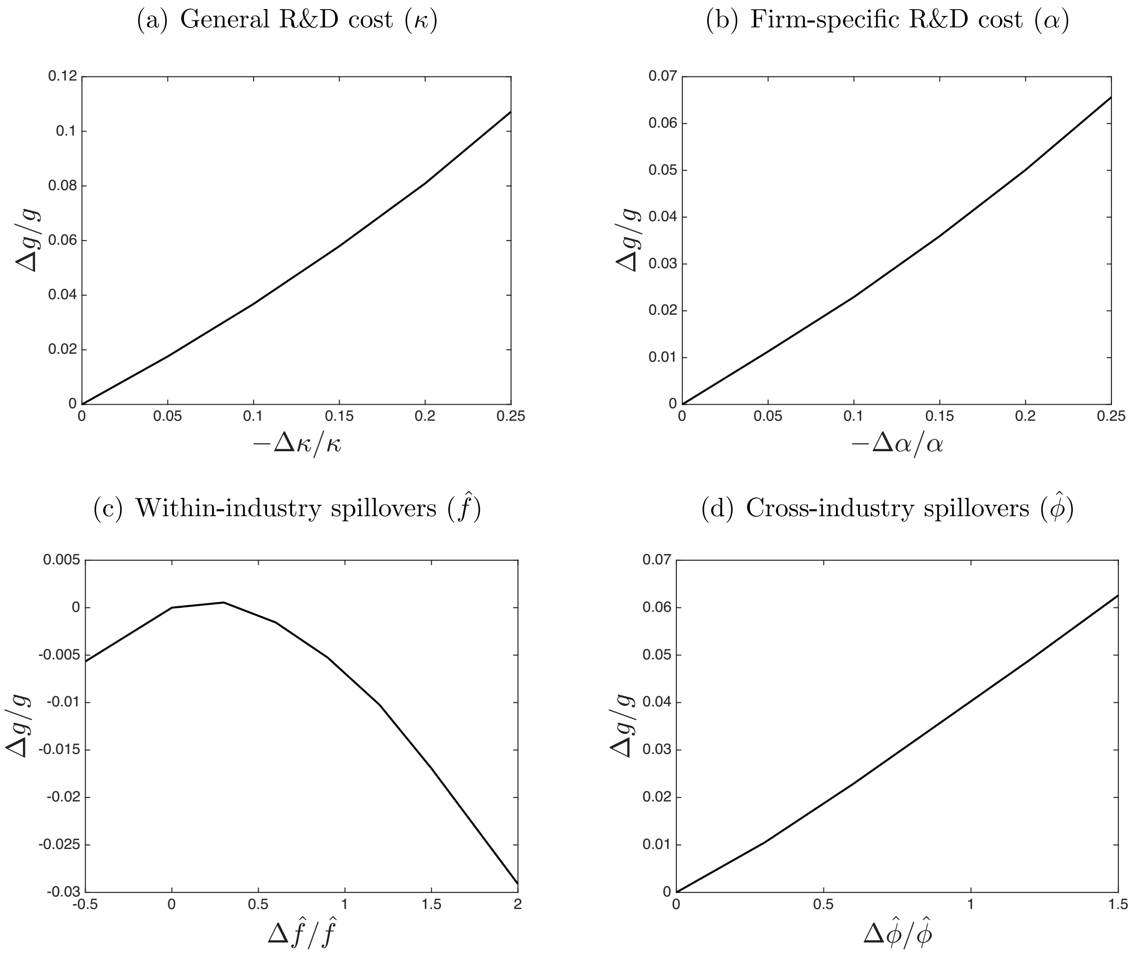
Reducing firm-specific R&D cost scale (i.e., α) also leads to higher aggregate growth but by a much lower magnitude, as shown in panel (b). While incumbents intensify their firm-specific R&D efforts, the benefits are partially offset by reduced cross-industry knowledge spillovers, due to a decline in aggregate general knowledge. Quantitatively, the growth elasticity with respect to general R&D costs is nearly twice that of firm-specific R&D costs.

Improvement in the efficiency of general knowledge spillovers. In traditional models without knowledge generality, improving the efficiency of knowledge spillovers, represented by a higher constant catch-up rate, directly leads to faster aggregate growth (see e.g., Liu et al., 2022; Akcigit and Ates, 2023). However, when innovation generality is taken into account, the effects of more efficient within-industry knowledge spillovers may turn negative for growth, as firms adjust their innovation strategies in response to the extent of knowledge diffusion.

Panels (c) and (d) plot the growth elasticity with respect to (an increase in) the efficiency of within- and cross-industry general knowledge spillovers. Panel (c) suggests that the relationship between aggregate growth and the efficiency of within-industry spillovers (i.e., $\hat{f}$) follows an inverted U-shape. When within-industry spillovers become highly efficient, aggregate growth may decline, as leaders are strongly discouraged from investing in general innovations. This reduction offsets followers' expansion of general R&D, reducing the aggregate level of general knowledge and limiting the extent of cross-industry spillovers.

innovation expenditures ultimately return to households through R&D employment.

Figure 5: Growth Elasticity w.r.t. R&D Costs and Efficiency of Spillovers



Note: Panel (a) and (b) plot the elasticity of growth with respect to the general and firm-specific R&D cost scale. Panel (c) and (d) plot the elasticity of growth with respect to the efficiency of knowledge spillovers within- and cross-industry. For costs, changes in growth are shown with a decrease in costs. For spillovers, changes in growth are shown with an increase in the efficiency of spillovers.

In contrast, enhancing the efficiency of cross-industry knowledge spillovers (i.e., $\hat{\phi}$) leads to higher growth, as shown in panel (d). This is because it encourages incumbents to conduct general innovations, which improves the extents of both within- and cross-industry knowledge spillovers.

5.2 Policy Implications

The comparative statics indicate that growth is more elastic with respect to general R&D costs than to firm-specific R&D costs, implying that subsidies for general R&D are more effective.

5.2.1 General R&D Subsidies for High-type Industries

The first policy experiment relates to R&D subsidies and follows a traditional approach, assuming a fixed corporate tax rate δ for all firms across all industries. I begin with the case where subsidies are directed toward general innovations in high-type industries, and then compare it with the scenario of targeting general innovations in low-type industries.

Denote the implied R&D subsidies ν_L^H for leaders, ν_F^H for followers and ν_0^H for neck-and-neck firms. Then the HJB equations (7)-(9) can be rewritten as:

$$\begin{aligned}
rv_s^H &= \max_{\eta_s^H,\lambda_s^H\geq 0} \big\{(1-\delta)\pi_s - c_\eta(\eta_s^H) - (1-\nu_L^H)c_\lambda^H(\lambda_s^H) \\
&\quad +[\eta_{-s}^H + \Phi f(\lambda_s^H)\lambda_{-s}^H + h](v_{s-1}^H - v_s^H) + (\eta_s^H + \Phi\lambda_s^H)(v_{s+1}^H - v_s^H) - \tau^H v_s^H\big\} \\
rv_{-s}^H &= \max_{\eta_{-s}^H,\lambda_{-s}^H\geq 0} \big\{(1-\delta)\pi_{-s} - c_\eta(\eta_{-s}^H) - (1-\nu_F^H)c_\lambda^H(\lambda_{-s}^H) \\
&\quad +[\eta_{-s}^H + \Phi f(\lambda_s^H)\lambda_{-s}^H + h](v_{-s+1}^H - v_{-s}^H) + (\eta_s^H + \Phi\lambda_s^H)(v_{-s-1}^H - v_{-s}^H) - \tau^H v_{-s}^H\big\} \\
rv_0^H &= \max_{\eta_0^H,\lambda_0^H\geq 0} \big\{(1-\delta)\pi_0 - c_\eta(\eta_0^H) - (1-\nu_0^H)c_\lambda(\lambda_0^H) \\
&\quad +(\eta_{-0}^H + \Phi\lambda_{-0}^H)(v_{-1}^H - v_0^H) + (\eta_0^H + \Phi\lambda_0^H)(v_1^H - v_0^H) - \tau^H v_0^H\big\}
\end{aligned}$$

subject to a budget constraint:

$$\sum_{k=\{H,L\}}\sum_{s=0}^{\infty}\delta(\pi_s + \pi_{-s})\mu_s^k = 2\nu_0^H c_\lambda^H(\lambda_0^H)\mu_0^H + \sum_{s=1}^{\infty}\left[\nu_L^H c_\lambda^H(\lambda_s^H) + \nu_F^H c_\lambda^H(\lambda_{-s}^H)\right]\mu_s^H \tag{28}$$

This budget constraint says that the overall corporate taxes collected are used to cover all the general R&D subsidies among high-type industries. I consider two schemes: subsidizing leaders (and neck-and-neck firms) only, or subsidizing followers (and neck-and-neck firms) only. Table 5 shows the results for each scenario.

Surprisingly, subsidizing general R&D for leaders is almost always more beneficial than supporting followers, a result that differs from the policy implications of the traditional leader–follower framework (see e.g., Liu et al., 2022; Akcigit and Ates, 2023). As shown in the table, implementing a 15% tax rate and using the revenue to subsidize general innovations by leaders in high-type industries raises the aggregate growth rate from 1.66% to 1.82%. In contrast, directing the subsidies to followers results in only two-thirds of that growth improvement. This is because when followers receive general R&D subsidies, they focus more on producing general knowledge, which aids their catch-up process. Anticipating this, leaders reduce their general R&D efforts to maintain their dominance. As a result, the overall positive impact of general R&D subsidies on aggregate general knowledge is mitigated, resulting in limited improvement in the cross-industry spillovers. Conversely, subsidizing leaders' general R&D encourages them to invest more in general innovations, which, in turn, motivates followers to enhance their own R&D efforts to

Table 5: General R&D Subsidies for High-type Industries

Tax rate	δ	0.1	0.15	0.2	0.25
Scheme 1: Subsidizing Leaders					
Growth rate	g	1.76%	1.82%	1.88%	1.95%
Average Generality - Leaders	ψ_s^H	0.65	0.71	0.77	0.83
Average Generality - Followers	ψ_{-s}^H	0.60	0.61	0.62	0.63
Scheme 2: Subsidizing Followers					
Growth rate	g	1.73%	1.77%	1.80%	1.80%
Average Generality - Leaders	ψ_s^H	0.48	0.47	0.47	0.47
Average Generality - Followers	ψ_{-s}^H	0.67	0.71	0.74	0.79

Note: The first row shows the fixed tax rates for all incumbent firms. Scheme 1 reports counterfactuals when only leaders and neck-and-neck firms in high-type industries are subsidized. The calculation of ψ_{-s}^H excludes the innovation generality of neck-and-neck firms. Scheme 2 reports counterfactuals when only followers and neck-and-neck firms in high-type industries are subsidized. The calculation of ψ_s^H excludes the innovation generality of neck-and-neck firms.

absorb the increasing external general knowledge. This feedback effect further pushes up the aggregate level of general R&D efforts, and benefits all incumbents through cross-industry spillovers (i.e., Φ).

Comparison with General R&D Subsidies in Low-type Industries. For comparison, I provide a discussion when general innovations in low-type industries are subsidized instead. Table 6 provides the results. The first part shows the aggregate growth rate, average innovation generality when general innovations of leaders and neck-and-neck firms in low-type industries are subsidized. The second part shows the results when only followers and neck-and-neck firms in low-type industries are subsidized.

Subsidizing general R&D innovations in low-type industries still promotes growth, and targeting leaders is more effective than targeting followers—consistent with the results for high-type industries. However, the growth gains are substantially smaller. For instance, implementing a 15% tax rate and subsidizing leaders' general innovations in low-type industries increases the aggregate growth rate from 1.66% to 1.75%, which is half the improvement observed in the high-type case. This is because, for the same level of subsidies, incumbents in low-type industries produce less general knowledge and create weaker cross-industry spillovers.

To conclude, policies aimed at targeting general R&D of leaders in high-type industries can maximize knowledge spillovers and stimulate aggregate growth efficiently. This policy implication differs from the traditional leader–follower framework in the literature (see, e.g., Liu et al., 2022; Akcigit and Ates, 2023), where subsidizing followers is typically regarded as the optimal policy. Supporting followers remains valuable, as they are often self-motivated to engage in general innovations and require less monitoring. However, di-

Table 6: General R&D Subsidies for Low-type Industries

Tax rate	δ	0.1	0.15	0.2	0.25
Scheme 1: Subsidizing Leaders					
Growth rate	g	1.72%	1.75%	1.78%	1.81%
Average Generality - Leaders	ψ_s^L	0.54	0.60	0.65	0.70
Average Generality - Followers	ψ_{-s}^L	0.50	0.51	0.51	0.52
Scheme 2: Subsidizing Followers					
Growth rate	g	1.71%	1.72%	1.74%	1.75%
Average Generality - Leaders	ψ_s^L	0.39	0.39	0.39	0.38
Average Generality - Followers	ψ_{-s}^L	0.57	0.61	0.64	0.67

Note: The first row shows the fixed tax rates for all incumbent firms. Scheme 1 reports counterfactuals when only leaders and neck-and-neck firms in low-type industries are subsidized. The calculation of ψ_{-s}^L excludes the innovation generality of neck-and-neck firms. Scheme 2 reports counterfactuals when only followers and neck-and-neck firms in low-type industries are subsidized. The calculation of ψ_s^L excludes the innovation generality of neck-and-neck firms.

recting subsidies toward general innovations in leading firms can foster aggregate growth more efficiently, as it strengthens their incentives to pursue knowledge with broader applicability and amplifies both within- and cross-industry spillovers. At the same time, because leaders tend to focus on firm-specific innovations to protect their market position, careful oversight of subsidy allocation is required. A more detailed discussion of the policy design is provided in section 5.2.3.

5.2.2 Improving the Efficiency of Cross-industry Spillovers

The comparative statics suggest that improving the efficiency of within-industry general knowledge spillovers does not necessarily increase growth, since it reduces leaders' incentives to undertake general innovations, which limits the spillovers available to firms in other industries. This implies that weak non-compete agreements do not necessarily improve growth. If inventors with general knowledge can easily move to follower firms, industry leaders may be discouraged from pursuing such innovations in the first place.

Instead, policies that improve the efficiency of cross-industry general knowledge spillovers can foster aggregate growth. For example, interdisciplinary research centers that promote collaboration across sectors can help firms adapt advances from other fields into new products, production methods, and business models within their own industries.

5.2.3 Discussion

Since the empirical measure of patent generality depends on forward citations, it has limited practical use for policies like R&D subsidy allocation. Instead, we need measures available prior to patent grant that can predict the generality of a patent.

Patent Originality. The originality measure is built upon patents' backward citations, and is formally defined as:

$$o_i = 1 - \sum_j c_{ij}^2$$

where c_{ij} is the fraction of citations that patent i makes that belong to patent class j. A high originality score indicates that the patent cites previous patents in a wide set of technologies, whereas a low score implies a narrow range of fields the patent is built on (Hall et al., 2001; Babina et al., 2023).

Importantly, patent generality is positively and significantly associated with its originality score:

$$\psi_{i,t} = \underset{(s.e.\ 0.001)}{0.223} \cdot o_{i,t} + \delta_f + \delta_t + \epsilon_{i,t} \tag{29}$$

where $\psi_{i,t}$ is patent i's generality score, and $o_{i,t}$ is its originality score, with t its application year. δ_f and δ_t represent firm fixed effects and application year fixed effects, respectively.

Therefore, patent originality, which is based on backward citations, can serve as a predictor of how general the knowledge is. However, firms may strategically manipulate backward citations to inflate the perceived originality of their patents. To discourage such behavior, a penalty system can be introduced. For instance, if a patent is later found to have deliberately excluded key citations or include irrelevant citations, a reduction in subsidy for general innovations can be applied to the same firm.

Skill Complementarity. In addition to patent originality scores, human capital can also serve as a predictor of knowledge generality. For example, research teams composed of inventors from diverse backgrounds may generate more general knowledge due to skill complementarity.

Other criteria, such as a high self-cite rate and long patent claims possibly imply that the patent is less general and of narrower scope.[31]

Prize Instead of Subsidy. Admittedly, identifying general innovations before a patent is granted can be challenging. An alternative policy instrument—mathematically similar to an R&D subsidy but differing in timing—is a prize system to encourage general innovations. Recent work shows that cash prizes can complement subsidies by steering innovation toward socially valuable outcomes (see, e.g., Che et al., 2021; Graff Zivin and Lyons, 2021). Crucially, unlike subsidies, which are provided upfront, prizes are granted after the innovation is developed, thus reducing the opportunity for firms to strategically

[31] Akcigit and Ates (2023) use self-cite rate and length of patent claims to infer the strategic use of patents to limit the scope of spillovers to competitors.

manipulate backward citations to influence funding decisions. For instance, the King's Award for Innovation in the United Kingdom recognizes firms that deliver outstanding technological advancements. A prize system that rewards leading firms for conducting general innovations can enhance both within- and cross-industry knowledge spillovers, thereby stimulating aggregate growth.

5.3 (General) Ideas Become Harder to Find

This part quantifies the impact of the post-2010 decline in general R&D efficiency on aggregate growth and business dynamism.

Trends in Average Patent Generality and TFP Growth. In their seminal paper, Bloom et al. (2020) show the trends in rising research effort and declining research productivity (i.e., ideas are getting harder to find). Building on their findings, I provide evidence that general ideas, in particular, are becoming harder to find. Figure 6 shows the trend in average patent generality by industry type and aggregate productivity growth in the US. Panel (a) presents the generality measure based on citation information within 5-year periods following a patent's grant. Panel (b) plots the annual TFP growth based on Fernald (2015), and is smoothed using an HP filter with an annual smoothing parameter of 100. The average patent generality reached its peak prior to 2000, followed by a consistent decline.[32] The growth of TFP shows a similar trend.

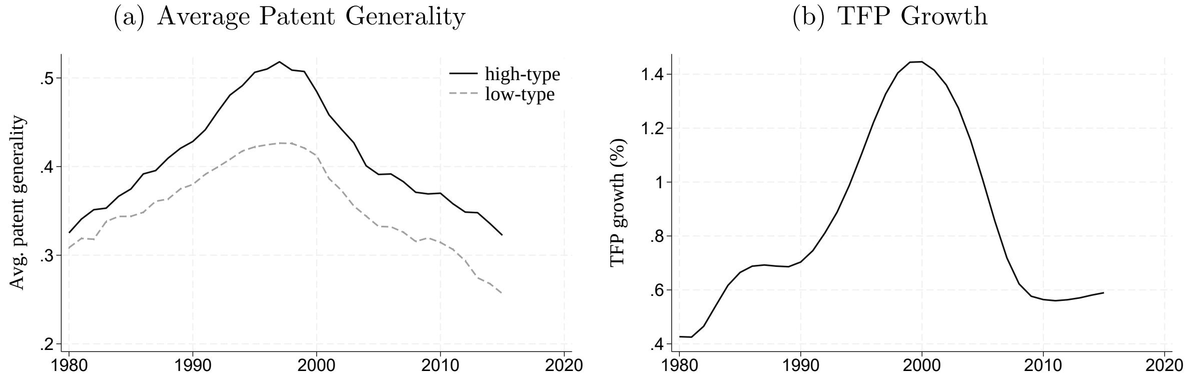
This observed trend in aggregate patent generality is broadly shared across most major sectors (see Figure 9 in Appendix B.4). A similar pattern also appears in the average leadership turnover rate (see Figure 11 in Appendix B.6).

Calibration to Post-2010 Data. During the 2010-2015 period, the average innovation generality of leaders and followers declined to 0.34 and 0.38 in high-type industries, and to 0.30 and 0.33 in low-type industries. Meanwhile, the average TFP growth rate dropped to 0.98%. To align the model with these post-2010 statistics, I apply the simulated method of moments, as in the previous calibration. The parameters adjusted include the general R&D cost scale in high- and low-type industries (κ^H and κ^L), the firm-specific R&D cost scale (α), the efficiency of within- and cross-industry spillovers ($\hat{f}$ and $\hat{\phi}$), as listed in Table 7. In particular, there is a reduction in the efficiency of both general and firm-specific R&D investments, in line with Bloom et al. (2020).

The first half of Table 8 lists the targeted moments, while the second half reports the untargeted ones. The lower efficiency of general R&D investments—especially in high-type industries—reduces incumbents' incentives to produce general knowledge. The

[32]Note that this pattern is not driven by changes in the overall number of forward citations, the trend of which is shown in Figure 10 in Appendix B.5.

Figure 6: Average Patent Generality and Aggregate Growth



Note: Panel (a) plots the average patent generality in the patent sample from 1980 to 2015. The black solid line and the gray dashed line show the average patent generality in high- and low-type industries, respectively. Panel (b) plots the annual productivity growth using Fernald (2015). The plot is smoothed using an HP filter with an annual smoothing parameter of 100.

resulting decline in aggregate general knowledge discourages entry and makes it harder for followers to catch up. Together, these forces generate a lower leadership turnover rate. Since the efficiency decline is more pronounced in high-type industries, the model predicts a larger reduction in turnover there, consistent with the data. Moreover, this decline in leadership turnover arises endogenously from the reduced efficiency of R&D investments faced by both leaders and followers, rather than from a decrease in the fixed catch-up rate (h).

Table 7: Parameter Values: Post-2010

Parameter	Description	Value	Source
κ^H	General R&D cost scale, high-type	21.09	Internal calibration
κ^L	General R&D cost scale, low-type	23.32	Internal calibration
α	Firm-specific R&D cost scale	14.52	Internal calibration
$\hat{f}$	Efficiency of within-industry spillovers	28.50	Internal calibration
$\hat{\phi}$	Efficiency of cross-industry spillovers	55.04	Internal calibration

Note: The table shows parameters that match the post-2010 US economy and are set via internal calibration.

Decomposition of the Decline in Aggregate Growth. Next, I analyze the channels underlying the post-2010 slowdown in U.S. aggregate growth. Table 9 presents the decomposition results. Column (1) reports the decomposition method, with channels added sequentially. Consistent with the parameter adjustments in Table 7, I consider three channels: less efficient general R&D in high-type industries (an increase in κ^H), less efficient general R&D in low-type industries (an increase in κ^L), less efficient firm-

Table 8: Model Fit: Post-2010

	Model	Data
Targeted Moments		
Average innovation generality - Leader, high type	0.35	0.34
Average innovation generality - Follower, high type	0.37	0.38
Average innovation generality - Leader, low-type	0.31	0.30
Aggregate growth rate	0.98%	0.98%
Leadership turnover, high-type	12%	11%
Untargeted Moments		
Average innovation generality - Follower, low-type	0.33	0.33
Entry rate	8.80%	9.86%
Leadership turnover, low-type	11%	10%
Average relative backward citations	3.73	4.27

Note: Data on innovation generality, leadership turnover rate and relative backward citations are calculated from the firm sample constructed by linking USPTO, PatentsView and Compustat. Data on aggregate growth rate are based on Fernald (2015). Data on entry rate are from BDS. All moments used for calibration are calculated as averages over the 2010–2015 period.

specific R&D (an increase in α). Column (2) presents the model-implied growth rate, and Column (3) reports the contribution of the accumulated channels to the change in aggregate growth. The decline in the overall efficiency of general R&D accounts for half of the growth slowdown, with around 60% of this effect driven by less efficient general R&D in high-type industries.

Table 9: Decomposition of the Decline in Aggregate Growth

	Implied g	Contribution
Less eff general R&D - high-type only	1.46%	30%
Less eff general R&D - all industries	1.32%	51%
Less eff general + firm-specific R&D	0.98%	100%

Note: The first column shows the decomposition method. The second column reports the implied aggregate growth rate. The third column shows the accumulation of the contribution to the change in aggregate growth.

Innovation Generality and Business Dynamism. As predicted by the model, the efficiency of general R&D is closely tied to business dynamism through within- and cross-industry spillovers, as well as entrants' reliance on general knowledge. Since 2000, the U.S. economy has experienced a marked decline in entry rate and leadership turnover (Bessen et al., 2020; Olmstead-Rumsey, 2025). This decline, according to the calibrated model, is primarily driven by a reduction in aggregate general knowledge production, which hinders diffusion from incumbents to entrants and, in turn, lowers the endogenous entry rate. At the same time, lower general R&D efficiency reduces leadership turnover in both types of industries: leaders are less likely to be replaced by entrants, and followers struggle to catch up. Table 10 summarizes how the decline in general R&D efficiency across both industry types contributes to the reduction in business dynamism between the pre-2000 and post-2010 periods. The first column lists the dimensions of business dynamism, and the second column reports the model-implied values given the lower general R&D efficiency in the post-2010 period. The third column reports its contribution, by comparing the baseline model (calibrated to pre-2000 U.S. economy) with a counterfactual in which only the general R&D cost scale is updated to its post-2010 calibrated value, while all other parameters remain at their baseline levels.

Table 10: Changes in Dynamism due to Less efficient General R&D (in %)

	Implied Values	Contribution
Entry rate	9.06%	91%
Leadership turnover, high-type	13%	86%
Leadership turnover, low-type	12%	65%

Note: Data on entry rate are from BDS. Data on leadership turnover rate are calculated from the firm sample constructed by linking USPTO, PatentsView and Compustat. The second column reports the model-implied values based on the lower efficiency of general R&D investment in post-2010. The third column shows its contribution to the change in business dynamism.

6 Conclusion

This paper studies the role of knowledge generality in shaping market structure, knowledge diffusion, and long-run growth. By distinguishing between general and firm-specific innovations, the model yields two predictions: (1) leaders often limit general knowledge production to preserve their market position, while followers rely on it to absorb external knowledge and narrow the technological gap; and (2) the gap in innovation generality between the two firms varies non-monotonically with the level of market concentration. Using firm-level data, I provide empirical evidence consistent with these patterns. A quasi-natural experiment based on variation in state-level enforceability of non-compete agreements validates the model's key assumption of within-industry spillovers. The quantitative analysis further shows that the post-2010 decline in the efficiency of general R&D investment can account for half of the slowdown in U.S. growth and a large portion of the decline in business dynamism. It also points to a distinct policy implication: well-designed support for general R&D—particularly when directed at leading firms—can amplify spillovers both within and across industries, and sustain long-run growth. Future research could build on this framework in several directions. One promising avenue is to explore the micro-level determinants of generality—such as inventor networks, team diversity, or research collaborations.

References

Acemoglu, Daron, Ufuk Akcigit, Harun Alp, Nicholas Bloom, and William Kerr (2018) "Innovation, Reallocation, and Growth," *American Economic Review*, 108 (11), 3450–91.

Acemoglu, Daron, Ufuk Akcigit, and Murat Alp Celik (2022) "Radical and Incremental Innovation: The Roles of Firms, Managers, and Innovators," *American Economic Journal: Macroeconomics*, 14 (3), 199–249.

Acemoglu, Daron and Dan Cao (2015) "Innovation by entrants and incumbents," *Journal of Economic Theory*, 157, 255–294.

Aghion, Philippe, Antonin Bergeaud, Timo Boppart, Peter J Klenow, and Huiyu Li (2023) "A Theory of Falling Growth and Rising Rents," *The Review of Economic Studies*, 90 (6), 2675–2702.

Aghion, Philippe, Christopher Harris, Peter Howitt, and John Vickers (2001) "Competition, Imitation and Growth with Step-by-Step Innovation," *The Review of Economic Studies*, 68 (3), 467–492.

Akcigit, Ufuk and Sina T. Ates (2023) "What Happened to US Business Dynamism?" *Journal of Political Economy*, 131 (8), 2059–2124.

Akcigit, Ufuk, Douglas Hanley, and Nicolas Serrano-Velarde (2020) "Back to Basics: Basic Research Spillovers, Innovation Policy, and Growth," *The Review of Economic Studies*, 88 (1), 1–43.

Akcigit, Ufuk and William Kerr (2018) "Growth Through Heterogeneous Innovations," *Journal of Political Economy*, 126 (4).

Almeida, Paul and Bruce Kogut (1999) "Localization of Knowledge and the Mobility of Engineers in Regional Networks," *Management Science*, 45 (7), 905–917.

Argente, David, Salome Baslandze, Douglas Hanley, and Sara Moreira (2023) "Patents to Products: Product Innovation and Firm Dynamics," working paper.

Arora, Ashish, Sharon Belenzon, and Lia Sheer (2021) "Knowledge Spillovers and Corporate Investment in Scientific Research," *American Economic Review*, 111 (3), 871–98.

Babina, Tania, Alex Xi He, Sabrina T Howell, Elisabeth Ruth Perlman, and Joseph Staudt (2023) "Cutting the Innovation Engine: How Federal Funding Shocks Affect University Patenting, Entrepreneurship, and Publications," *The Quarterly Journal of Economics*, 138 (2), 895–954.

Benhabib, Jess, Jesse Perla, and Christopher Tonetti (2021) "Reconciling Models of Diffusion and Innovation: A Theory of the Productivity Distribution and Technology Frontier," *Econometrica*, 89 (5), 2261–2301.

Bessen, James E., Erich Denk, Joowon Kim, and Cesare Righi (2020) "Declining Industrial Disruption," working paper, Boston Univ. School of Law, Law and Economics Research Paper 20-28.

Bishara, Norman D. (2011) "Fifty Ways to Leave Your Employer: Relative Enforcement of Covenants Not to Compete, Trends, and Implications for Employee Mobility Policy," *University of Pennsylvania Journal of Business Law*, 13.

Bloom, Nicholas, Charles I. Jones, John Van Reenen, and Michael Webb (2020) "Are Ideas Getting Harder to Find?" *American Economic Review*, 110 (4), 1104–44.

Bloom, Nicholas, Mark Schankerman, and John Van Reenen (2013) "Identifying Technology Spillovers and Product Market Rivalry," *Econometrica*, 81 (4), 1347–1393.

Bloom, Nick, Rachel Griffith, and John Van Reenen (2002) "Do R&D tax credits work? Evidence from a panel of countries 1979–1997," *Journal of Public Economics*, 85, 1–31.

Caggese, Andrea (2019) "Financing Constraints, Radical versus Incremental Innovation, and Aggregate Productivity," *American Economic Journal: Macroeconomics*, 11 (2), 275–309.

Caicedo, Santiago and Jeremy Pearce (2024) "Need for Speed: Quality of Innovations and the Allocation of Inventors," working paper.

Casal, Lucía (2025) "Lock-In and Productive Innovations: Implications for Firm-to-Firm Innovation Pass-Through," working paper.

Che, Yeon-Koo, Elisabetta Iossa, and Patrick Rey (2021) "Prizes versus Contracts as Incentives for Innovation," *The Review of Economic Studies*, 88 (5), 2149–2178.

Cohen, Wesley and Daniel Levinthal (1990) "Absorptive Capacity: A New Perspective on Learning and Innovation," *Administrative Science Quarterly*, 35, 128–152.

Cunningham, Colleen, Florian Ederer, and Song Ma (2021) "Killer Acquisitions," *Journal of Political Economy*, 129 (3), 649–702.

De Loecker, Jan, Jan Eeckhout, and Simon Mongey (2021) "Quantifying Market Power and Business Dynamism in the Macroeconomy," working paper, National Bureau of Economic Research.

De Ridder, Maarten (2024) "Market Power and Innovation in the Intangible Economy," *American Economic Review*, 114 (1), 199–251.

Dyévre, Arnaud (2024) "Public R&D Spillovers and Productivity Growth," working paper.

Fernald, John G. (2015) "Productivity and Potential Output before, during, and after the Great Recession," *NBER Macroeconomics Annual*, 29 (1), 1–51.

Fernández-Villaverde, Jesús, Yang Yu, and Francesco Zanetti (2025) "Defensive Hiring and Creative Destruction," CESifo Working Paper 11753.

Fossum, Eric R. (2020) "The Invention of CMOS Image Sensors: A Camera in Every Pocket," in *2020 Pan Pacific Microelectronics Symposium (Pan Pacific)*, 1–6.

Garcia-Macia, Daniel, Chang-Tai Hsieh, and Peter J. Klenow (2019) "How Destructive Is Innovation?" *Econometrica*, 87 (5), 1507–1541.

Giroud, Xavier, Ernest Liu, and Holger Mueller (2024) "Innovation Spillovers across U.S. Tech Clusters," working paper.

Graff Zivin, Joshua and Elizabeth Lyons (2021) "The Effects of Prize Structures on Innovative Performance," *AEA Papers and Proceedings*, 111, 577–81.

Hall, Bronwyn H, Adam B Jaffe, and Manuel Trajtenberg (2001) "The NBER Patent Citation Data File: Lessons, Insights and Methodological Tools," Working Paper 8498, National Bureau of Economic Research.

Jaffe, Adam B., Manuel Trajtenberg, and Michael S. Fogarty (2000) "Knowledge Spillovers and Patent Citations: Evidence from a Survey of Inventors," *The American Economic Review*, 90 (2), 215–218.

Jaffe, Adam B., Manuel Trajtenberg, and Rebecca Henderson (1993) "Geographic Localization of Knowledge Spillovers as Evidenced by Patent Citations," *The Quarterly Journal of Economics*, 108 (3).

Klette, Tor Jakob and Samuel Kortum (2004) "Innovating Firms and Aggregate Innovation," *Journal of Political Economy*, 112 (5), 986–1018.

Kogan, Leonid, Dimitris Papanikolaou, Amit Seru, and Noah Stoffman (2017) "Technological Innovation, Resource Allocation, and Growth," *The Quarterly Journal of Economics*, 132 (2), 665–712.

Kroen, Thomas, Ernest Liu, Atif R Mian, and Amir Sufi (2021) "Falling Rates and Rising Superstars," Working Paper 29368, National Bureau of Economic Research.

León-Ledesma, Miguel A and Mathan Satchi (2019) "Appropriate Technology and Balanced Growth," *The Review of Economic Studies*, 86 (2), 807–835.

Liu, Ernest and Song Ma (2021) "Innovation Networks and RD Allocation," Working Paper 29607, National Bureau of Economic Research.

Liu, Ernest, Atif Mian, and Amir Sufi (2022) "Low Interest Rates, Market Power, and Productivity Growth," *Econometrica*, 90 (1), 193–221.

Liu, Jingnan (2023) "Worker Mobility, Knowledge Diffusion, and Non-Compete Contracts," working paper.

Ma, Song, Wenyu Wang, and Yufeng Wu (2025) "Steering Labor Mobility Through Innovation," working paper, Fisher College of Business.

Marx, Matt (2022) "Employee Non-compete Agreements, Gender, and Entrepreneurship," *Organization Science*, 33 (5), 1756–1772.

Marx, Matt, Deborah Strumsky, and Lee Fleming (2009) "Mobility, Skills, and the Michigan Non-Compete Experiment," *Management Science*, 55 (6), 875–889.

Olmstead-Rumsey, Jane (2025) "Market Concentration and the Productivity Slowdown," working paper.

Perla, Jesse and Christopher Tonetti (2014) "Equilibrium Imitation and Growth," *Journal of Political Economy*, 122 (1), 52–76.

Peters, Michael (2020) "Heterogeneous Markups, Growth, and Endogenous Misallocation," *Econometrica*, 88 (5), 2037–2073.

Reinmuth, Kate and Emma Rockall (2025) "Innovation through Inventor Mobility: Evidence from Non-Compete Agreements," working paper.

Samila, Sampsa and Olav Sorenson (2011) "Noncompete Covenants: Incentives to Innovate or Impediments to Growth," *Management Science*, 57 (3), 425–438.

Singh, Jasjit and Ajay Agrawal (2011) "Recruiting for Ideas: How Firms Exploit the Prior Inventions of New Hires," *Management Science*, 57 (1), 129–150.

Stoyanov, Andrey and Nikolay Zubanov (2012) "Productivity Spillovers across Firms through Worker Mobility," *American Economic Journal: Applied Economics*, 4 (2), 168–98.

Trajtenberg, Manuel, Rebecca Henderson, and Adam Jaffe (1997) "University Versus Corporate Patents: A Window On The Basicness Of Invention," *Economics of Innovation and New Technology*, 5 (1), 19–50.

A Model Details

A.1 Proof of Proposition 1

The first-order conditions for the leader and follower can be written as

$$\begin{aligned}
c'_\eta(\eta_s^k) &= v_{s+1}^k - v_s^k, \\
(c_\lambda^k)'(\lambda_s^k) &= \Phi f'(\lambda_s^k)\,\lambda_{-s}^k\,(v_{s-1}^k - v_s^k) + \Phi\,(v_{s+1}^k - v_s^k), \\
c'_\eta(\eta_{-s}^k) &= v_{-s+1}^k - v_{-s}^k, \\
(c_\lambda^k)'(\lambda_{-s}^k) &= \Phi f(\lambda_s^k)\,(v_{-s+1}^k - v_{-s}^k).
\end{aligned}$$

Using the convex cost forms $c_\eta(\eta) = \frac{1}{\theta}(\alpha\eta)^\theta$ and $c_\lambda^k(\lambda) = \frac{1}{\theta}(\kappa_k\lambda)^\theta$ with $\theta > 1$, we have $c'_\eta(\eta) = \alpha^\theta\eta^{\theta-1}$ and $(c_\lambda^k)'(\lambda) = \kappa_k^\theta\lambda^{\theta-1}$. Hence,

$$\left(\frac{\lambda_{-s}^k}{\eta_{-s}^k}\frac{\eta_s^k}{\lambda_s^k}\right)^{\theta-1} = \frac{(v_{s+1}^k - v_s^k)\,\Phi f(\lambda_s^k)}{\Phi f'(\lambda_s^k)\lambda_{-s}^k(v_{s-1}^k - v_s^k) + \Phi(v_{s+1}^k - v_s^k)} = \frac{f(\lambda_s^k)}{1 + f'(\lambda_s^k)\lambda_{-s}^k\,\dfrac{v_{s-1}^k - v_s^k}{v_{s+1}^k - v_s^k}}.$$

Since $v_{s+1}^k - v_s^k > 0$, $v_{s-1}^k - v_s^k < 0$, $f'(\cdot) > 0$, and $\lambda_{-s}^k \geq 0$, we have

$$f'(\lambda_s^k)\lambda_{-s}^k\,\frac{v_{s-1}^k - v_s^k}{v_{s+1}^k - v_s^k} \leq 0$$

This means that

$$\left(\frac{\lambda_{-s}^k}{\eta_{-s}^k}\frac{\eta_s^k}{\lambda_s^k}\right)^{\theta-1} \geq f(\lambda_s^k) > 1$$

Because $\theta > 1$, this implies

$$\frac{\lambda_{-s}^k}{\eta_{-s}^k}\frac{\eta_s^k}{\lambda_s^k} > 1 \quad\Longrightarrow\quad \frac{\lambda_{-s}^k}{\eta_{-s}^k} > \frac{\lambda_s^k}{\eta_s^k}.$$

By the definition of $\psi_\cdot^k = \frac{\lambda_\cdot^k}{\lambda_\cdot^k + \eta_\cdot^k}$, this inequality is equivalent to $\psi_{-s}^k > \psi_s^k$, as claimed.

□

A.2 Proof of Corollary 1

In the case of neck-and-neck competition ($s = 0$), the innovation generality of both firms is

$$\psi_0^k = \frac{\kappa_k^{-\frac{\theta}{\theta-1}} \left[\Phi(v_1^k - v_0^k)\right]^{\frac{1}{\theta-1}}}{\kappa_k^{-\frac{\theta}{\theta-1}} \left[\Phi(v_1^k - v_0^k)\right]^{\frac{1}{\theta-1}} + \alpha^{-\frac{\theta}{\theta-1}} (v_1^k - v_0^k)^{\frac{1}{\theta-1}}}$$
$$= \frac{(\Phi\alpha^\theta)^{\frac{1}{\theta-1}}}{(\Phi\alpha^\theta)^{\frac{1}{\theta-1}} + \kappa_k^{\frac{\theta}{\theta-1}}}.$$

Next, consider the limit as the technology gap $s \to \infty$. From the pair of profits (5), we have $\pi_s \to 1$ and $\pi_{-s} \to 0$. This implies

$$v_s^k \to \frac{1}{r+\tau}, \quad v_{-s}^k \to 0,$$

and hence

$$|v_{-s+1}^k - v_{-s}^k| \to 0, \quad |v_{s-1}^k - v_s^k| \to 0, \quad |v_{s+1}^k - v_s^k| \to 0 \qquad \text{as } s \to \infty.$$

The follower's FOC for general innovations is

$$\kappa_k^\theta (\lambda_{-s}^k)^{\theta-1} = \Phi f(\lambda_s^k)(v_{-s+1}^k - v_{-s}^k).$$

Since the right-hand side vanishes as $s \to \infty$, we have $\lambda_{-s}^k \to 0$. Therefore, the innovation generality of the leader can be written as

$$\psi_s^k = \frac{\kappa_k^{-\frac{\theta}{\theta-1}} \left[\Phi f'(\lambda_s^k)\lambda_{-s}^k (v_{s-1}^k - v_s^k) + \Phi(v_{s+1}^k - v_s^k)\right]^{\frac{1}{\theta-1}}}{\kappa_k^{-\frac{\theta}{\theta-1}} \left[\Phi f'(\lambda_s^k)\lambda_{-s}^k (v_{s-1}^k - v_s^k) + \Phi(v_{s+1}^k - v_s^k)\right]^{\frac{1}{\theta-1}} + \alpha^{-\frac{\theta}{\theta-1}} (v_{s+1}^k - v_s^k)^{\frac{1}{\theta-1}}}.$$

Dividing numerator and denominator by $(v_{s+1}^k - v_s^k)^{\frac{1}{\theta-1}}$ yields

$$\psi_s^k = \frac{\Phi^{\frac{1}{\theta-1}} \kappa_k^{-\frac{\theta}{\theta-1}}}{\Phi^{\frac{1}{\theta-1}} \kappa_k^{-\frac{\theta}{\theta-1}} + \alpha^{-\frac{\theta}{\theta-1}} \left[\frac{f'(\lambda_s^k)\lambda_{-s}^k (v_{s-1}^k - v_s^k)}{v_{s+1}^k - v_s^k} + 1\right]^{-\frac{1}{\theta-1}}}.$$

As $s \to \infty$, we use $|v_{s-1}^k - v_s^k| \to |v_{s+1}^k - v_s^k|$ and the fact that $f'(0)$ is finite, to obtain

$$\psi_s^k \to \frac{(\Phi\alpha^\theta)^{\frac{1}{\theta-1}}}{(\Phi\alpha^\theta)^{\frac{1}{\theta-1}} + \kappa_k^{\frac{\theta}{\theta-1}}}.$$

The leader's FOC for general innovations is

$$\kappa_k^{\theta}(\lambda_s^k)^{\theta-1} = \Phi f'(\lambda_s^k)\lambda_{-s}^k(v_{s-1}^k - v_s^k) + \Phi(v_{s+1}^k - v_s^k).$$

Since the right-hand side vanishes as $s \to \infty$, it follows that $\lambda_s^k \to 0$. Hence the innovation generality of the follower is

$$\psi_{-s}^k = \frac{\kappa_k^{-\frac{\theta}{\theta-1}}\left[\Phi f(\lambda_s^k)(v_{-s+1}^k - v_{-s}^k)\right]^{\frac{1}{\theta-1}}}{\kappa_k^{-\frac{\theta}{\theta-1}}\left[\Phi f(\lambda_s^k)(v_{-s+1}^k - v_{-s}^k)\right]^{\frac{1}{\theta-1}} + \alpha^{-\frac{\theta}{\theta-1}}(v_{-s+1}^k - v_{-s}^k)^{\frac{1}{\theta-1}}}.$$

Dividing numerator and denominator by $(v_{-s+1}^k - v_{-s}^k)^{\frac{1}{\theta-1}}$ gives

$$\psi_{-s}^k = \frac{\kappa_k^{-\frac{\theta}{\theta-1}}[\Phi f(\lambda_s^k)]^{\frac{1}{\theta-1}}}{\kappa_k^{-\frac{\theta}{\theta-1}}[\Phi f(\lambda_s^k)]^{\frac{1}{\theta-1}} + \alpha^{-\frac{\theta}{\theta-1}}}.$$

Since $f(0) = 1$, letting $s \to \infty$ yields

$$\psi_{-s}^k \;\to\; \frac{(\Phi\alpha^{\theta})^{\frac{1}{\theta-1}}}{(\Phi\alpha^{\theta})^{\frac{1}{\theta-1}} + \kappa_k^{\frac{\theta}{\theta-1}}}.$$

Thus, in the limit as $s \to \infty$, the innovation generality of the leader and follower converge to the same value. $\square$

A.3 Proof of Proposition 2

From Proposition 1, we have $g_s < 0$ for all $s \geq 1$. By construction, we have $g_0 = \psi_0^k - \psi_{-0}^k = 0$.

Suppose, for contradiction, that g_s is not non-monotonic. Since $g_0 = 0 > g_1$, g_s cannot be (strictly) increasing; hence it must be non-increasing on $\mathbb{N}$. Formally, for all $a, b, c \in \mathbb{N}$ with $a < b < c$,

$$g_a \;\geq\; g_b \;\geq\; g_c.$$

In particular, for every $s \geq 1$,

$$g_s \;\leq\; g_1 \;<\; 0,$$

so $\limsup_{s\to\infty} g_s \leq g_1 < 0$. This contradicts Corollary 1, which states $\lim_{s\to\infty} g_s = 0$. Therefore, g_s must be non-monotonic.

Note that market concentration in the model increases strictly with the distance between the leader and the follower.

$$HHI_s = \Big(\frac{\rho_s^{1-\sigma}}{\rho_s^{1-\sigma}+1}\Big)^2 + \Big(\frac{1}{\rho_s^{1-\sigma}+1}\Big)^2$$

with ρ_s the price ratio between the leader and the follower, which satisfies $\rho_s^\sigma = \gamma^{-s} \frac{\sigma\rho_s^{\sigma-1}+1}{\sigma+\rho_s^{\sigma-1}}$. Therefore, the innovation generality gap also varies non-monotonically with the level of market concentration. □

A.4 Computation Algorithm

Solving the Balanced Growth Path.

1. Guess the entrants' R&D effort λ_e, the aggregate general R&D efforts Λ and the probability of entering a market of type k, i.e., p_k.
2. The creative destruction rate of market k is given by: $\tau^k = \frac{p_k \Phi \lambda_e}{2\chi_k}$.
3. Solve the incumbents' innovation decisions. Compute and update the aggregate general R&D efforts Λ.
4. Verify the entrants' decisions from (12).
5. Calculate the average growth rate in each market and check if it is equalized from (14).
6. If not converge, update values of λ_e, Λ and $\{p_k\}$, and repeat the process.

General R&D Subsidies.

1. Guess an implied subsidy ν.
2. Repeat the process in "Solving the Balanced Growth Path".
3. Check the budget constraint (28). If not converge, repeat the process.

A.5 Details about Simulated Method of Moments

In Section 4, I apply the simulated method of moments for the internal calibration. In particular, the objective function can be written as:

$$\min_{\upsilon} \sum_{j=1}^{J} w_j \Big(\frac{m(\upsilon_j) - \hat{m}(\upsilon_j)}{\hat{m}(\upsilon_j)}\Big)^2$$

where υ is the vector of calibrated parameters, $m(\upsilon_j)$ and $\hat{m}(\upsilon_j)$ are the model-generated and empirical moments, respectively. I set the weights w_j such that the moments regarding innovation generality, aggregate growth and entry rates, are weighted 5 times more than other moments.

A.6 Details about the Average Relative Backward Citations

In the model, the average ratio of cross-industry to within-industry backward citations is calculated as:

$$\frac{\sum_k \sum_{s=0}^{\infty} \chi_k (\Phi - 1) \mu_s^k (\lambda_s^k + \lambda_{-s}^k)}{\sum_k \sum_{s=0}^{\infty} \chi_k \mu_s^k \left[f(\lambda_s^k) - 1 \right] \lambda_{-s}^k}$$

which is essentially the ratio of external effect of cross-industry general knowledge spillovers to the within-industry spillovers, weighted by the industry size and firm-level growth.

B Empirical Analysis: Additional Exercises and Robustness

B.1 Detailed Information on Patent 6550058

Figure 7: Detailed Patent Information: Patent ID 6550058

(12) **United States Patent**
Wynn

(10) **Patent No.: US 6,550,058 B1**
(45) **Date of Patent: Apr. 15, 2003**

(54) **STACK CLEARING DEVICE AND METHOD**

(75) Inventor: **Allen C. Wynn**, Round Rock, TX (US)

(73) Assignee: **International Business Machines Corporation**, Armonk, NY (US)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

(21) Appl. No.: **09/497,606**

(22) Filed: **Feb. 3, 2000**

(51) **Int. Cl.**^{7} **G06F 9/45**
(52) **U.S. Cl.** **717/158**; 717/154; 717/130
(58) **Field of Search** 717/152, 157, 717/158, 159, 124, 127, 128, 130, 131, 132, 133, 154; 711/100, 103; 712/202; 713/189, 193, 194, 200, 201, 185

(56) **References Cited**

U.S. PATENT DOCUMENTS

3,889,243	A		6/1975	Drimak	
4,485,435	A		11/1984	Sibley	
4,751,636	A		6/1988	Sibley	
5,193,180	A	*	3/1993	Hastings	717/163
5,293,385	A	*	3/1994	Hary	714/38
5,628,016	A	*	5/1997	Kukol	717/114
6,009,258	A	*	12/1999	Elliott	703/22
6,085,029	A	*	7/2000	Kolawa et al.	714/38
6,189,141	B1	*	2/2001	Benitez et al.	717/153
6,260,187	B1	*	7/2001	Cirne	717/110
6,351,843	B1	*	2/2002	Berkley et al.	709/332
6,385,727	B1	*	5/2002	Cassagnol et al.	713/193
2001/0056539	A1	*	12/2001	Pavlin et al.	713/193

OTHER PUBLICATIONS

"Secure Deletion of Data from Magnetic and Solid–State Memory", Gutmann, Peter, Department of Computer Science, University of Auckland, Jul. 1996, retrieved from http://www.cs.auckland.ac.nz/~pgut001/pubs/secure_del.html, May 21, 2002.*

* cited by examiner

Primary Examiner—Tuan Q. Dam
Assistant Examiner—Mary J. Steelman
(74) *Attorney, Agent, or Firm*—Robert H. Frantz; David A. Mims, Jr.

(57) **ABSTRACT**

A method for removing residual data from a computer program stack prior to returning control to a calling or controlling process with system and method for automatic inclusion thereof into software application programs at the time of production of executable code. Two methods, one for removing residual data from a relatively small stack frame and another for removing residual data from a large stack frame, are automatically inserted into application program code during an enhanced compiling method. Two compiler controls allow a software designer to globally include the stack cleaning feature in all code being produced, or to selectively include the stack cleaning feature into certain indicated modules, code areas, or procedures.

36 Claims, 3 Drawing Sheets

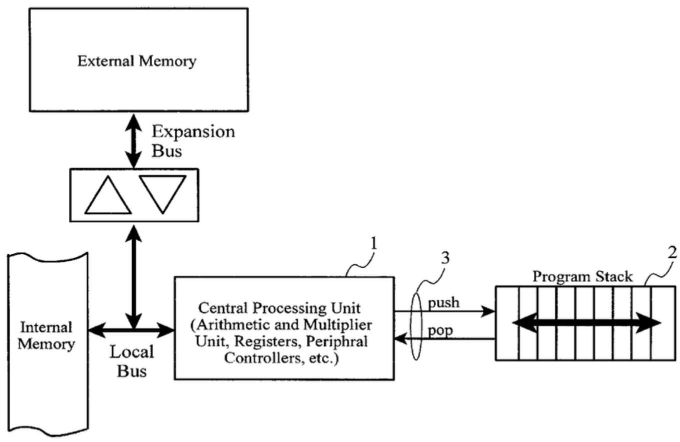


Table 11: Generality Score Construction: Patent 6550058

CPC subclasses	Definitions	Citations
G06F	electric digital data processing	9
G06Q	ICT for admin., comm., fin., manag. or superv. purposes	1

Note: The first and second column list the CPC subclasses the citing patents coming from and their definitions. The third column lists the number of citing patents that belong to the subclass.

B.2 Heterogeneity in Innovation Generality Across Industries.

There exists substantial heterogeneity in innovation generality across industries. Table 12 reports examples of average industry-level patent generality in the United States during 1995–2000.

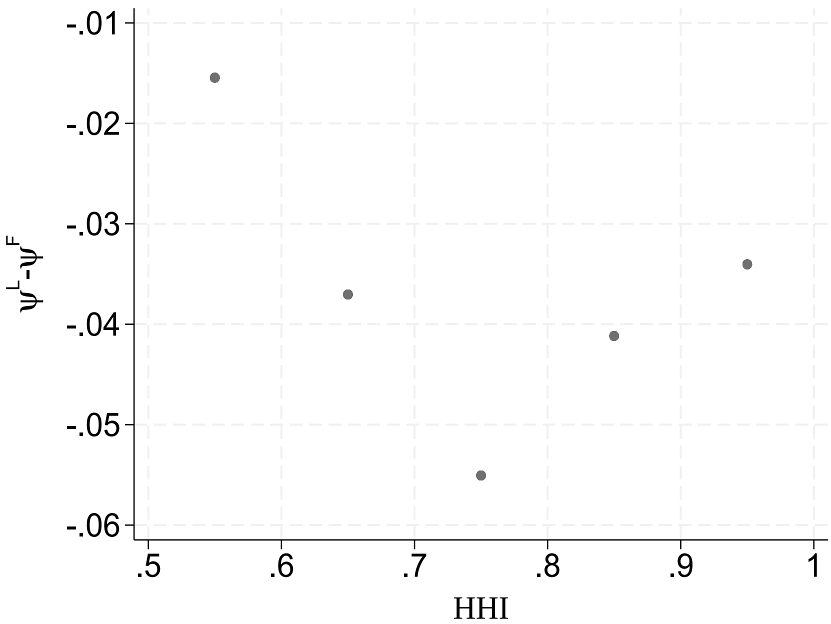
Table 12: Heterogeneity in Innovation Generality Across Industries

SIC code	**Details**	**Avg. Patent Generality**
7323	Credit Reporting Services	0.71
4832	Radio Broadcasting Stations	0.71
1731	Electrical Work	0.70
7331	Direct Mail Advertising Services	0.70
8734	Testing Laboratories	0.67
3443	Fabricated Plate Work	0.31
3221	Glass Containers	0.29
3532	Mining Machinery and Equipment	0.28
5093	Scrap and Waste Materials	0.24
5944	Jewelry Stores	0.06

Note: The first column reports industries' 4-digit SIC codes, the second provides their detailed definitions, and the third presents industry-level patent generality for 1995–2000.

B.3 Empirical Relationship Between Gaps in Innovation Generality and Market Concentration

Figure 8: Empirical Relationship: Gaps in Generality and Market Concentration

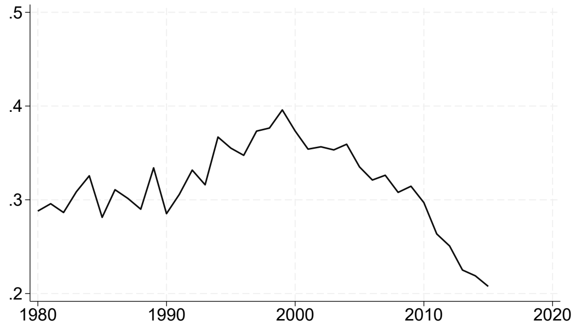


Note: This figure plots the empirical relationship between gaps in innovation generality, i.e., $(\psi^L - \psi^F)$, and market concentration, measured by HHI. The innovation generality gap is taken as the median value in each bin of HHI (with an interval of 0.1). The sample ranges from 1980 to 2015.

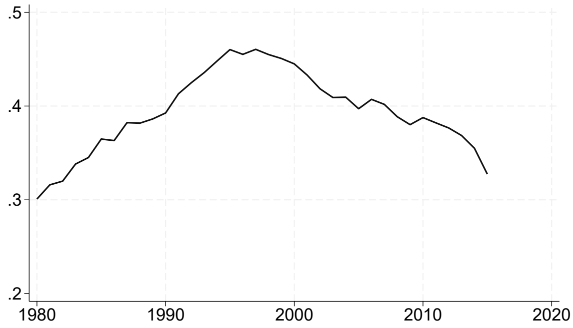
B.4 Average Patent Generality Across Sectors

Figure 9: Average Patent Generality Across Sectors

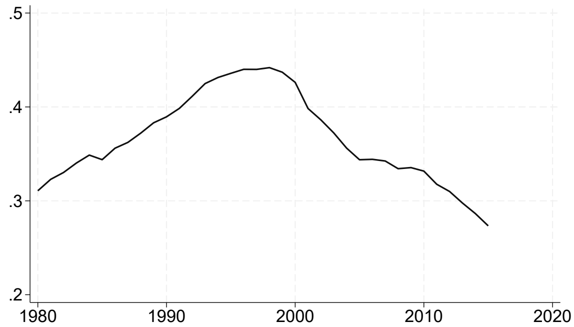
(a) Mining and Construction: SIC Codes 1000-1799



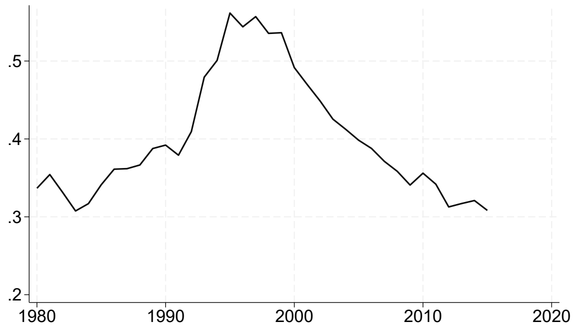
(b) Manufacturing I: SIC Codes 2000-2999



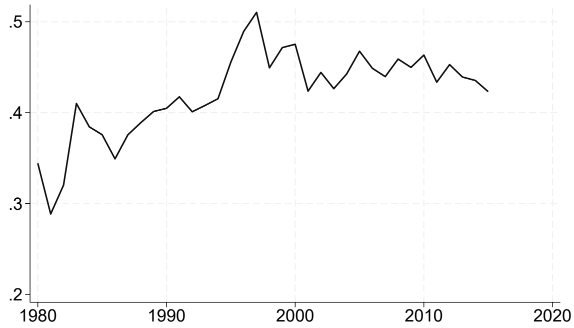
(c) Manufacturing II: SIC Codes 3000-3999



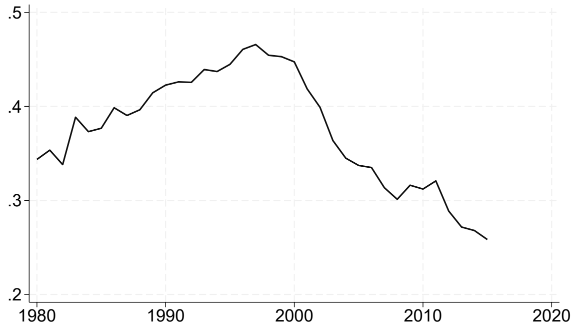
(d) Transportation, Communications, Electric, Gas and Sanitary service: SIC Codes 4000-4999



(e) Trade: SIC Codes 5000-5999



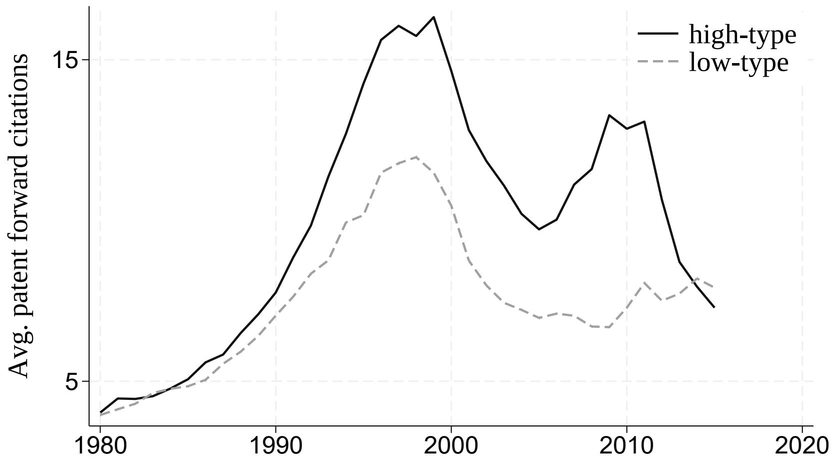
(f) Services: SIC Codes 7000-8999



Note: The figure shows the average patent generality across sector according to one-digit SIC codes, based on citation information within 5-year periods following a patent's grant. SIC codes 2000-2999 generally represent manufacturing industries related to food, textiles, wood, paper, chemicals, and petroleum products, while codes 3000-3999 include fabricated metal products, machinery, electrical equipment, transportation equipment, and instruments.

B.5 Average Patent Forward Citations

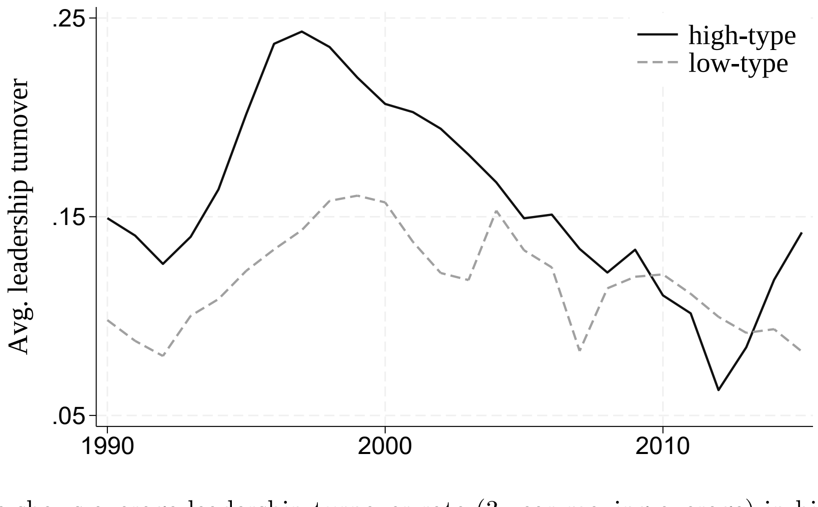
Figure 10: Average Patent Forward Citations (5-year window)



Note: The figure shows the average number of forward citations received within five years of the patent grant for patents in high- and low-type industries from 1980 to 2015. The classification of industry types follows the definitions provided in the main text.

B.6 Average Leadership Turnover

Figure 11: Average Leadership Turnover



Note: The figure shows average leadership turnover rate (3-year moving average) in high- and low-type industries from 1990 to 2015, respectively.

B.7 Robustness Check: Tests of Propositions

Test 1: Definition of Leaders. If leaders are defined as firms whose sales are above the top 5 percentile in their industries, the result of the estimation (21) in the main text becomes:

$$\psi_{f,t} = \underset{(s.e.\ 0.002)}{-\ 0.030} \cdot \mathbb{1}(Leader)_{f,t} + \delta_{j,t} + \epsilon_{f,t}$$

In this specification, holding leadership or not accounts about 0.14 of the standard deviation of firm-level innovation generality.

Test 2: Using Mean Generality Instead of Median. The estimation (22) can be written as:

$$\psi^L_{j,t} - \psi^F_{j,t} = \gamma HHI_{j,t} + \zeta HHI^2_{j,t} + \delta_j + \delta_t + \epsilon_{j,t}$$

Table 13: Robustness Check: Test of Proposition 2

	(1)	(2)	(3)	(4)
	Median	Median	Mean	Mean
γ	-0.221^{**}	-0.357^{***}	-0.210^{**}	-0.314^{***}
	(0.086)	(0.098)	(0.084)	(0.103)
ζ	0.156^{**}	0.229^{***}	0.143^{**}	0.198^{**}
	(0.080)	(0.080)	(0.072)	(0.078)
Number of firms		✓		✓
R-squared	0.135	0.139	0.135	0.139
Observations	5,542	5,542	5,542	5,542

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 13 reports the results under different specifications. In (1), I use the median value of the innovation generality of followers without controlling for the number of firms in the industry. (2) is the baseline estimation in the main text. In (3), I use the mean value of the innovation generality of followers to represent $\psi^F_{j,t}$ in industries with multiple followers, instead of using median value. In (4), I use the mean value and further control for the number of firms in the industry.

Test 2: Measurement of Market Concentration. If instead HHI is constructed using all firms in CRSP-Compustat, regardless of having patent applied or not, the results are as follows:

$$\psi^L_{j,t} - \psi^F_{j,t} = \underset{(s.e.\ 0.091)}{-\ 0.326} \cdot HHI_{j,t} + \underset{(s.e.\ 0.092)}{0.207} \cdot HHI^2_{j,t} + \delta_j + \delta_t + \epsilon_{j,t}$$

Note that in this case, leaders are defined as firms with the highest sale among all firms that patent within the same industry (4-digit SIC code). The controls are the same as in the main text.

B.8 Robustness Check: Alternative Generality Score Construction

Including patents with zero forward citations. Note that the definition of patent generality in the main text requires a patent to have at least one forward citation within five years of its grant. Among all utility patents assigned to publicly listed U.S. firms between 1980 and 2018, approximately 26.6% have no forward citations within this five-year window. In this analysis, rather than excluding these patents as in the main text, I assign them a generality score of 0.

Test 1: Based on the alternative generality score, the result of the estimation (21) becomes:

$$\psi_{f,t} = \underset{(s.e.\ 0.002)}{-\ 0.034} \cdot \mathbb{1}(Leader)_{f,t} + \delta_{j,t} + \epsilon_{f,t}$$

Test 2: Based on the alternative generality score, the result of the estimation (22) becomes:

$$\psi^L_{j,t} - \psi^F_{j,t} = \underset{(s.e.\ 0.099)}{-\ 0.292} \cdot HHI_{j,t} + \underset{(s.e.\ 0.081)}{0.178} \cdot HHI^2_{j,t} + \delta_j + \delta_t + \epsilon_{j,t}$$

Cost-adjusted firm-level innovation generality. Note that in the main text, firm-level innovation generality is defined as the simple average of patent generality scores. In this analysis, I adjust this measure by incorporating the value of each patent, based on Kogan et al. (2017). The cost-adjusted measure is defined as follows:

$$\widetilde{\psi}_{f,t} = \frac{\sum_{i=1}^{n} \psi_{i,t}/\xi_{i,t}}{\sum_{i=1}^{n} 1/\xi_{i,t}} \tag{30}$$

where $\psi_{i,t}$ denotes the generality score of patent i applied for in year t, and $\xi_{i,t}$ represents the value of patent i, deflated to 1982 (million) dollars using the CPI (Kogan et al., 2017).

The basic idea is that if each patent's "cost" (e.g., R&D expenditure or effort) can be approximated by its real value, then this measure reflects on average how general the firm's innovations are, after accounting for how costly they are to produce.

Test 1: Based on the alternative generality score, the result of the estimation (21) becomes:

$$\widetilde{\psi}_{f,t} = \underset{(s.e.\ 0.003)}{-\ 0.026} \cdot \mathbb{1}(Leader)_{f,t} + \delta_{j,t} + \epsilon_{f,t}$$

Test 2: Based on the alternative generality score, the result of the estimation (22) becomes:

$$\widetilde{\psi}^{L}_{j,t} - \widetilde{\psi}^{F}_{j,t} = \underset{(s.e.\ 0.101)}{-\ 0.310} \cdot HHI_{j,t} + \underset{(s.e.\ 0.082)}{0.199} \cdot HHI^2_{j,t} + \delta_j + \delta_t + \epsilon_{j,t}$$

B.9 Robustness Check: Definition of Major Changes in Non-compete Enforceability

Table 14 shows the estimation results of equation (23) when a significant change in state-level non-compete enforceability is alternatively defined as a 0.01 shift in the normalized index. Using this definition, I identify 19 major changes between 1991 and 2014, of which 14 involved stronger enforcement.

Table 14: Within-industry Spillovers: Smaller shift

β_N	-0.015^{*}
	(0.008)
$\beta_{N,L}$	0.033^{***}
	(0.009)
Industry-Year FE	✓
State FE	✓
R-squared	0.454
Observations	23,764

Standard errors are clustered at the state level.
*** p<0.01, ** p<0.05, * p<0.1

Note: This table shows the results from estimation (23).

Table 15 shows the estimation results of equation (23) when a significant change in state-level non-compete enforceability is alternatively defined as a 0.06 shift in the normalized index. Using this definition, I identify 14 major changes between 1991 and 2014, of which 10 involved stronger enforcement.

Table 15: Within-industry Spillovers: Larger shift

β_N	-0.016^{*}
	(0.008)
$\beta_{N,L}$	0.041^{***}
	(0.008)
Industry-Year FE	✓
State FE	✓
R-squared	0.454
Observations	$23,764$

Standard errors are clustered at the state level.
*** p<0.01, ** p<0.05, * p<0.1

Note: This table shows the results from estimation (23).

B.10 Robustness Check: Pre-determined Leadership

Table 16 shows the estimation results of equation (23) when the leadership indicator in the interaction term is constructed using pre-determined leader status measured one or two periods prior to the treatment year. This is to avoid treatment bias: If policy affects competition dynamics, leadership could shift as a result of the reform.

Table 16: Within-industry Spillovers: Pre-determined Leadership

	$\mathbb{1}(Leader)_{f,\tau-1}$	$\mathbb{1}(Leader)_{f,\tau-2}$
β_N	-0.015^{*}	-0.015^{*}
	(0.008)	(0.008)
$\beta_{N,L}$	0.030^{***}	0.032^{***}
	(0.010)	(0.012)
Industry-Year FE	✓	✓
State FE	✓	✓
R-squared	0.454	0.454
Observations	$23,764$	$23,764$

Standard errors are clustered at the state level.
*** p<0.01, ** p<0.05, * p<0.1

Note: This table shows the results from estimation (23).

B.11 Robustness Check: Persistent Leadership

Table 17 shows the estimation results of equation (23) when the leadership indicator in the interaction term is based on a measure of persistent leadership, defined as maintaining leader status from the time of the reform (i.e., τ) through period t. This approach accounts for potential leadership turnover over time.

Table 17: Within-industry Spillovers: Persistent Leadership

β_N	-0.016^{**}
	(0.008)
$\beta_{N,L}$	0.044^{***}
	(0.008)
Industry-Year FE	✓
State FE	✓
R-squared	0.454
Observations	23,764

Standard errors are clustered at the state level.
*** p<0.01, ** p<0.05, * p<0.1

Note: This table shows the results from estimation (23).

B.12 Additional Stylized Facts

Fact 3: Entrants have higher innovation generality.

I show entrants have higher innovation generality compared with incumbents using the following estimation:

$$\psi_{f,t} = \underset{(s.e.\ 0.002)}{0.033} \cdot \mathbb{1}(Entrant)_{f,t} + \delta_{j,t} + \epsilon_{f,t} \tag{31}$$

where $\psi_{f,t}$ represents the average patents generality of firm f in year t. $\mathbb{1}(Entrant)_{f,t}$ is an indicator function, which equals one if firm f is an entrant (i.e., within 5 years since first appeared in Compustat) in industry j in year t. $\delta_{j,t}$ captures industry-year fixed effects.

Fact 4: General patents have lower self-cite rates.

Furthermore, I study how general knowledge produced by a firm relies on firm's own knowledge pool from the following estimation:

$$s_{i,t} = \underset{(s.e.\ 0.001)}{-0.031} \cdot \psi_i + \delta_f + \delta_t + \epsilon_{i,t} \tag{32}$$

where $s_{i,t} \in [0,1]$ is patent i's self-cite rate. Self-cite rate is the share of backward citations made to patents with the same assignee (i.e., same firms) (Akcigit and Kerr, 2018). Patents having higher self-cite rates are more dependent on firm's own technology pools, and hence are more firm-specific in nature. As before, ψ_i measures generality of patent i. δ_f and δ_t are firm fixed effects and application year fixed effects. Patent-level generality explains about 0.14 of the standard deviation in the self-cite rate. This suggests that general knowledge relies less on firm-specific knowledge.

B.13 Robustness Check: Generality and Market Concentration

Measurement of Market Concentration. Following estimation (26) and (27) in the model validation section, I use HHI constructed using all firms in CRSP-Compustat, regardless of having patent applied or not, the results are as follows:

$$\psi^L_{j,t} = \underset{(s.e.\ 0.060)}{-0.124} \cdot HHI_{j,t} + \underset{(s.e.\ 0.057)}{0.088} \cdot HHI^2_{j,t} + \delta_j + \delta_t + \epsilon_{j,t} \tag{33}$$

$$\psi^F_{j,t} = \underset{(s.e.\ 0.069)}{0.229} \cdot HHI_{j,t} - \underset{(s.e.\ 0.069)}{0.191} \cdot HHI^2_{j,t} + \delta_j + \delta_t + \epsilon_{j,t} \tag{34}$$

Controlling for Citation Counts. If I further control for the log number of average forward citations (within 5-year window since the patent grant) received by patents produced by firm f in year t, the results are as follows:

$$\psi^L_{j,t} = -\underset{(s.e.\ 0.058)}{0.153} \cdot HHI_{j,t} + \underset{(s.e.\ 0.047)}{0.116} \cdot HHI^2_{j,t} + \delta_j + \delta_t + \epsilon_{j,t} \tag{35}$$

$$\psi^F_{j,t} = \underset{(s.e.\ 0.071)}{0.164} \cdot HHI_{j,t} - \underset{(s.e.\ 0.058)}{0.092} \cdot HHI^2_{j,t} + \delta_j + \delta_t + \epsilon_{j,t} \tag{36}$$