Educational Standards and Parental Investment*

Dyotona Dasgupta† Rajiv Sethi‡

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Abstract

We examine the problem faced by an educational authority tasked with setting a performance standard for a cohort of students who are heterogeneous along two dimensions—parental wealth and capacity for learning. Parents invest in the academic preparation of their children, and children exert effort to improve their performance. The introduction of a standard has two effects that can push in opposite directions. Conditional on preparation, the standard leads some students to raise effort and performance as they strive to meet it. But the standard can also lead to lower parental investment, by setting a floor for the resulting fall in performance. As a result, the introduction of a standard can reduce performance within some subgroups of a population. We show that parents will generally prefer standards that are binding on their children but lower than performance-maximizing, and that heterogeneity in the population can result in lower standards than would be faced by any homogeneous subgroup.

Keywords: Education, Inequality, Academic Standards, Parental Investment

JEL Codes: D63, I24, O15.

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†Jindal Global Business School, O.P. Jindal Global University.

‡Department of Economics, Barnard College, Columbia University and the Santa Fe Institute.

1 Introduction

Consider an educational authority tasked with setting a standard for academic performance, and faced with a population of students that is heterogeneous with respect to prior preparation. Too demanding a standard will cause some students with low levels of preparation to give up, since passing would require a prohibitive amount of costly effort. The result will be low levels of learning in the student population. But if the standard is set too low, passing can be achieved with minimal effort, and learning will again be limited. The authority must confront this trade-off and consider the manner in which students will react to any standard it might set.

In addition, the authority needs to take into account the fact that parents can respond to the standard by adjusting investments in the academic preparation of their children, and that households vary in wealth and hence in their capacity to make such investments.

In this paper, we examine the manner in which parental investments and the efforts of children respond to different standards, and the choice of standard by the educational authority in anticipation of these effects. We do so based on a model with the following features. The academic preparation of children depends on two factors—the amount invested in them by parents, and their own idiosyncratic capacity to learn. We refer to this capacity as ability, while recognizing that it depends on both endowed and environmental factors that are outside the direct control of the household. Academic performance depends on both preparation and effort, where the latter is chosen by the child. The educational authority chooses a standard of performance, and if this is met the child is said to have passed. Children care about performance, about passing, and about the effort expended by them. Parents also care about performance and passing, and in addition care about household consumption, which depends on the amount of wealth not spent on investment in increasing the academic preparation for their children. The educational authority seeks to maximize mean performance in the population of students.

For any given household, define baseline investment, preparation, effort, and performance as the levels of these variables that would arise when there is no standard in place. Conditional on preparation, the introduction of a standard creates an incentive for students to raise effort above baseline, since they care about passing. Given any distribution of preparation, the student population can be partitioned into (at most) three groups: those who exceed the standard, those who meet it exactly, and those who fail to meet it. Those passing easily and those failing will all choose their respective baseline effort levels, while those who meet the standard exactly will typically do so by exceeding baseline effort. By setting the standard judiciously, the authority can stimulate considerable increased effort and thus improve the distribution of achieved performance. That is, conditional on the distribution of preparation, the imposition of a standard will raise effort among some children and leave it unchanged at baseline among others.

However, the introduction of a standard will not leave the distribution of preparation unchanged, since it will affect the incentives of parents to invest. We show that for any given level of parental wealth, there exists a range of ability levels such that the imposition of a standard lowers investment, and hence also lowers preparation. Within this range is a set of ability levels at which even performance falls when the standard is introduced. That is, a low standard can be worse than no standard from a performance perspective. This happens because the presence of a standard sets a lower bound for the performance of students who will make the effort to meet it. As a result, a contraction of investment is less costly for parents—they can increase household consumption without risking too great a fall in performance. Children in such households would perform better if the standard were removed or substantially reduced. Meanwhile, there are other ability levels at which investment, preparation, and performance all rise when the standard is introduced. So the introduction of a standard can raise performance at some ability levels and lower it at others, even among households with the same level of wealth.

We next consider ideal standards from the perspective of the educational authority, parents, and children. Faced with a population of homogeneous households, an educational authority that seeks to maximize mean performance will choose the highest standard that households will meet exactly. This standard maximizes parental investment, preparation, effort, and performance. But the choice of the authority will not generally coincide with the standard considered ideal by parents, who will prefer a somewhat lower standard that their children will also exactly meet. This happens because higher standards induce higher parental investments (conditional on the standard being binding), and this corresponds to lower household consumption. It is possible, however, that the optimal choice of the authority coincides with that of children. Although effort is maximized at this standard, so is parental investment, and hence the opportunity set available to the child is largest at this point.

Finally, we show that when the authority is faced with a heterogeneous population, the standard chosen may be strictly lower than any that would maximize the performance of a homogeneous subgroup. For example, when the population is composed of two groups with differing wealth levels (and homogeneous ability), the standard chosen by the authority may be lower than that which would maximize the performance of either the more affluent or the less affluent group, treated in isolation. Similar results apply when wealth is homogeneous but there are two different ability levels. Heterogeneity in the population thus exerts downward pressure on the standard.

Taken together, these results show just how complex and counterintuitive the effects of changing standards can be when the academic preparation of children depends on parental investment. Intuitions derived from models that neglect parental choices in the determination of academic potential can give rise to misleading policy prescriptions. Our goal here is to make the interactions between parental investment and student effort more transparent, and thus provide a better foundation for the determination of educational policies.

2 Related Literature

The idea that students have agency and respond to incentives in the process of learning dates back at least to Bishop (1985), who observed that learning "occurs when an individual who is *ready and able* to learn, is offered an *opportunity* to learn and makes the *effort* to learn" (1985, p. 1). This idea has been implemented in formal models by Kang (1985), Becker and Rosen (1992), Costrell (1994), and Betts (1998), on which we build.

Kang (1985) formalizes the fundamental insight that a student will invest effort in meeting a standard only if doing so is not too costly. He allows for randomness in performance, so students trade-off costs with expected benefits. For any given student, effort rises with the difficulty level of the standard until a threshold is reached, at which time it drops precipitously.

Costrell (1994) and Betts (1998) both explicitly consider the goals of the standard setting authority, but reach different conclusions regarding the consequences of egalitarian motives. Costrell finds that egalitarian policy makers set standards that are lower than those that would maximize mean income, for intuitive reasons—lower standards allow for those with lowest ability to meet them. But Betts observes that when there is heterogeneity in productivity among those meeting (or failing to meet) a standard, raising standards can lower wage inequality, and thus be favored by an egalitarian authority. This happens because a very demanding standard results in many high ability students failing to meet it, so wage differences between passing and failing pools of students can be diminished.

We assume throughout that performance on a test depends only on costly effort and prior preparation. Ability plays a role in determining preparation, in concert with parental investment, but plays no independent role in determining performance conditional on preparation. This is a standard assumption in the literature on affirmative action, which typically equates ability and preparation and neglects the role of parental investment (Coate and Loury, 1993; Chan and Eyster, 2003; Fryer and Loury, 2013). For some purposes it is important to allow for the possibility that ability affects performance even conditional on preparation, as in the discussion of meritocracy in Sethi and Somanathan (2023). However, we do not explore this pathway here.

Changes in educational conditions have been found to affect parental investment in many different contexts. For example, the Indian state of West Bengal abolished the use of English as a medium of primary school instruction in 1983, which led to increased expenditures on private tutoring (Roy, 2014). Parental investment in Sweden has been found to respond to class size (Fredriksson et al., 2016). And household educational expenditures and non-salary cash grants to schools appear to be substitutes in Zambia (Das et al., 2013).

In a study of test-based admission to more demanding schools in Romania, Pop-Eleches and Urquiola (2013) report that "relative to individuals who just miss scoring above a school cutoff... children who just make it into higher achieving schools receive *less* homework-related help from their parents... Romanian parents may view educational quality and their own effort as substitutes." This is consistent with our finding that parental investment (and even performance) may decline when standards rise as parents expect greater effort from their children.

The importance of parental investment in the determination of academic achievement has been emphasized by many researchers; see for example Todd and Wolpin (2003) for a specification of the cognitive production function. Albornoz et al. (2018) explore a model in which educational outcomes depend on parental investment and student effort, as well as resources made available to schools. As in our model, investment and effort both respond endogenously to changes in policy, but the policies they consider (such as reduced class size) induce greater effort by facilitating learning rather than certifying performance.

Our work may be seen as a contribution to the literature on educational policy, which is broad and deep. This includes work on education financing in the face of residential and occupational sorting (Fernández and Rogerson, 1996; Benabou, 2000), the effects of competitive grading (Becker and Rosen, 1992; Krishna et al., 2022) and tracking (Betts, 2010; Duflo et al., 2011; Card and Giuliano, 2016), and the labor market consequences of changes in standards (Angrist and Lavy, 1997; Somanathan, 1998; Betts and Grogger, 2003; Chakraborty and Bakshi, 2016).

The choice of standards is just one factor among many that affect the distribution of educational outcomes. Blanden et al. (2023) consider a variety of other mechanisms through which educational inequality can arise, including inequality in parental resources and aspirations, assortative mating, environmental influences, and educational inputs. Given the importance of human capital in determining the distribution of earnings, educational inequality has a significant impact on economic inequality writ large. In this sense, our work also contributes to the literature on the intergenerational transmission of economic inequality (Becker and Tomes, 1979; Loury, 1981; Mookherjee and Ray, 2003).

3 The Baseline Model

Consider an Education Authority (EA) facing a continuum of students that is heterogeneous with respect to preparation. The EA is tasked with setting the difficulty $d \in \mathbb{R}_+$ of a test that students benefit from passing. Let $g(\theta)$ denote the density of preparation in the population of students, and $[\theta_{\min}, \theta_{\max}] \subset \mathbb{R}_+$ its support. We treat this distribution as exogenous for the moment, although it will depend on endowed ability and parental investment in a manner to be considered later.

Given the difficulty d of the test, each student chooses some level of effort $e \in \mathbb{R}_+$. The performance of the student, denoted ρ, depends on preparation and effort. In particular,

$$\rho = f(\theta, e),$$

where $\rho \geq 0$, and f is strictly increasing and strictly concave in both arguments. The cost of effort is denoted $c(e)$, assumed to be strictly increasing and strictly convex, with $c(0) = c'(0) = 0$.

A student with performance ρ passes an exam of difficulty d if $\rho \geq d$. That is, the student passes if her performance meets or exceeds the standard. We assume for simplicity that all students can pass an exam of any given difficulty if they exert enough effort. Allowing for tests that are impossible to pass for some students would not change any of our results. As we show below, some students will fail exams even when passing is feasible, since passing would require effort expenditures that are too costly relative to the benefits.

As f is strictly increasing in both arguments, attaining any level of performance ρ requires a unique amount of effort (possibly zero) for any student with preparation θ. Let $\xi(\theta, \rho)$ denote this effort level. The minimum effort required to meet a performance standard d for a student with preparation θ is then $\xi(\theta, d)$, where $\xi(\theta, d) = 0$ if $d \leq f(\theta, 0)$ and

$$d = f(\theta, \xi(\theta, d)) \tag{1}$$

otherwise. Whenever $d > f(\theta, 0)$, the effort $\xi(\theta, d)$ is decreasing in the first argument (better prepared students need to make less effort to pass any given test) and increasing in the second (more effort is required to pass a more difficult test).

Let $p \in \{0, 1\}$ denote the (endogenously determined) test outcome for a student, where $p = 1$ if the student passes and $p = 0$ otherwise.

A student with preparation θ derives utility from better performance, and additionally from passing the test. Such a student will choose effort e to maximize

$$f(\theta, e) + bp - c(e), \tag{2}$$

where the coefficient b captures the benefits of passing.

Choices are made in the following sequence. The EA sets the standard d, with knowledge of the distribution $g(\theta)$ of preparation. Each student then chooses effort level e, which determines their performance ρ and test outcome p. The distribution of performance, over which the EA has preferences, is denoted $h(\rho)$. In principle, the EA may want to minimize the gap between the least and the most prepared students, maximize the mean performance in the population as a whole, maximize the median student's performance, or maximize the lowest level of performance. We focus on the maximization of mean performance below, but first consider the choice of effort for an arbitrarily given standard.

3.1 The Choice of Effort

Given a standard d, a student with preparation θ chooses effort level e to maximize (2). It is useful to first consider the hypothetical effort level chosen optimally in the absence of any standard. Define the *baseline payoff*

$$f(\theta, e) - c(e),$$

and let $e^u(\theta)$ denote the effort level that maximizes this objective function. We call $e^u(\theta)$ the *baseline effort* for a student with preparation θ. For any given θ, strict concavity of f in effort and strict convexity of c together imply that baseline effort is uniquely determined. If a student chooses baseline effort, the resulting level of performance is $f(\theta, e^u(\theta))$; we call this *baseline performance*, denoted by $\rho^u(\theta)$.

If baseline effort is increasing in preparation, then so is baseline performance. However, it is possible for baseline performance to be increasing in preparation even if baseline effort is decreasing; an example is provided below. We next identify a sufficient condition for baseline performance to be increasing in preparation. In this case, for any given standard, preparation levels can be partitioned into (at most) three adjacent intervals corresponding to students who fail, exactly meet, and exceed the standard. The condition is the following:

Definition 1. *The performance function f satisfies* ***increasing differences in effort*** *if, for any $\theta' > \theta$, the difference $\xi(\theta, \rho) - \xi(\theta', \rho)$ is non-decreasing in ρ.*

To interpret this condition, consider the following. If two students with different preparation levels have the same level of performance, the less well-prepared student must be exerting more effort. The above condition states that the extra effort required by the less prepared student is higher when the (common) performance level is higher. The condition is equivalent to the following: at any given level of performance attained by two students with different levels of preparation, the better-prepared student has higher marginal returns to effort.[1] We show below that this condition is sufficient to ensure that baseline performance is increasing in preparation (although baseline effort may be decreasing).

If baseline performance is increasing in preparation, then for any standard d there exists a unique preparation level $\bar{\theta}(d)$ such that baseline performance exceeds the standard if and only if preparation exceeds this threshold. Students with preparation exceeding the threshold will simply choose baseline effort and pass. Those with lower levels of preparation will either choose baseline effort and fail, or choose their minimum passing effort and meet the standard exactly.

Specifically, consider a student with preparation θ facing a standard d. When d is such that $e^u(\theta) \geq \xi(\theta, d)$, the student chooses $e^*(\theta, d) = e^u(\theta)$ as the optimal effort level. Otherwise, she compares the payoff from making enough effort to pass the exam with the payoff from choosing $e^u(\theta)$ and failing. She chooses the former when that gives her a (weakly) higher payoff. Hence:

$$e^*(\theta, d) = \begin{cases} \xi(\theta, d) & \text{if } e^u(\theta) < \xi(\theta, d) \text{ and } d + b - c(\xi(\theta, d)) \geq \rho^u(\theta) - c(e^u(\theta)), \\ e^u(\theta) & \text{otherwise.} \end{cases}$$

Let $\rho^*(\theta, d) = f(\theta, e^*(\theta, d))$ denote the level of performance optimally attained by a student with preparation θ under standard d.

Define $d_{min} = \rho^u(\theta_{\min})$ as the baseline performance of the least prepared student, and $d_{\max}$ as the highest standard that the best prepared student would meet:

$$\rho^u(\theta_{\max}) - c(e^u(\theta_{\max})) = d_{\max} + b - c(\xi(\theta_{\max}, d_{\max})).$$

As long as optimal performance is increasing in preparation, all students would exceed the standard with baseline effort if $d < d_{\min}$, and all would fail if $d > d_{\max}$. Accordingly, we focus on standards between these two extremes. In this case, under the increasing differences in effort condition, the set of students can be partitioned into (at most) three groups: those who exceed the standard (and thus pass), those who meet the standard exactly by choosing the minimum necessary effort, and those who fail to meet the standard and choose baseline effort:

Proposition 1. *If f satisfies increasing differences in effort, then baseline performance is strictly increasing in preparation. Furthermore, for any standard $d \in (d_{min}, d_{max})$, optimal performance is non-decreasing in preparation, and there exist preparation thresholds $\underline{\theta}(d)$ and $\bar{\theta}(d)$ such that $\underline{\theta}(d) < \bar{\theta}(d)$ and*

(a) students with $\theta > \bar{\theta}(d)$ choose baseline effort and exceed the standard;

(b) students with $\theta \in [\underline{\theta}(d), \bar{\theta}(d)]$ choose minimum passing effort;

[1] See Lemma 1 in the Appendix for a formal statement and proof of this claim, as well as proofs of all our other results.

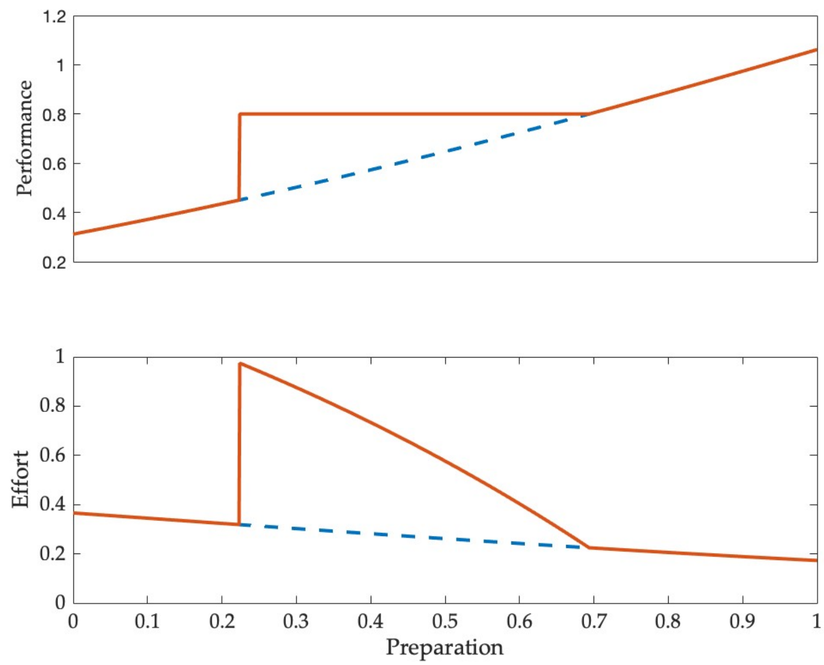

Figure 1: Performance and effort choices; dashed lines show baseline effort and performance.

(c) students with $\theta < \underline{\theta}(d)$ choose baseline effort and fail.

The increasing differences in effort condition is therefore sufficient for optimal choices to depend on preparation in a straightforward and intuitive manner. The condition is also necessary in the following sense: if does not hold, it is possible to find performance and cost functions such that baseline performance (and hence also optimal performance) decrease with preparation over some range.

Note that one or two of the elements in the partition identified in Proposition 1 may be empty. For instance, all students may pass (if the standard is sufficiently low) or all may fail (if it is so high that the levels of effort required to pass are too costly even for the best-prepared students).

The following example illustrates the result for a case where baseline effort is decreasing in preparation, but the increasing differences in effort condition is still satisfied, so baseline performance is increasing in effort, and optimal performance is non-decreasing.[2]

Example 1. *Suppose $f(\theta, e) = \log(\exp(\theta) + e)$, $c(e) = e^2$, $b = 0.5$, and $g(\theta)$ has support $[0, 1]$. Figure*

[2]The performance function in the example satisfies constant differences in effort (a special case of increasing differences under our definition). It also satisfies $f(\theta, e) = f(f(\theta, e_1), e_2)$ for any pair of positive effort levels e_1 and e_2 that sum to e. That is, any sequence of effort levels that has the same sum results in the same level of eventual performance.

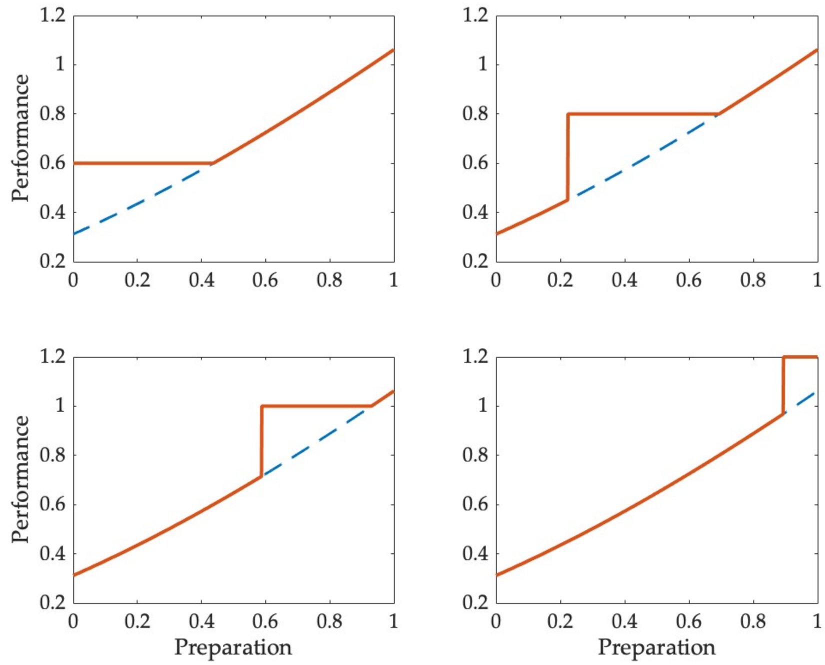

Figure 2: Performance outcomes for values of the testing standard $d \in \{0.6, 0.8, 1.0, 1.2\}$.

1 shows the outcomes (top panel) and optimal effort choices (lower panel) when $d = 0.8$*, in which case* $\underline{\theta} \approx 0.22$ *and* $\bar{\theta} \approx 0.69$*. Figure 2 shows outcomes for four different values of* d*.*

To this point we have considered the response of students to an arbitrarily given standard. The next step is to consider the goals of the EA and the equilibrium choice of standard.

3.2 The Choice of Standard

The choice of standard in equilibrium will take into account the anticipated student response, and will vary with the objectives of the EA. We focus on the case where the EA seeks to maximize mean achieved performance, given by

$$\int_{\theta_{min}}^{\theta_{max}} f(\theta, e^*(\theta, d))g(\theta)d\theta.$$

Recall that only those with preparation $\theta \in [\underline{\theta}(d), \bar{\theta}(d))$ choose efforts other than $e^u(\theta)$ under standard d, where $\underline{\theta}(d)$ and $\bar{\theta}(d)$ are as in Proposition 1. Hence the mean standard involves a choice of d that maximizes

$$\int_{\theta_{min}}^{\underline{\theta}(d)} f(\theta, e^u(\theta))g(\theta)d\theta + d\int_{\underline{\theta}(d)}^{\bar{\theta}(d)} g(\theta)d\theta + \int_{\bar{\theta}(d)}^{\theta_{max}} f(\theta, e^u(\theta))g(\theta)d\theta. \tag{3}$$

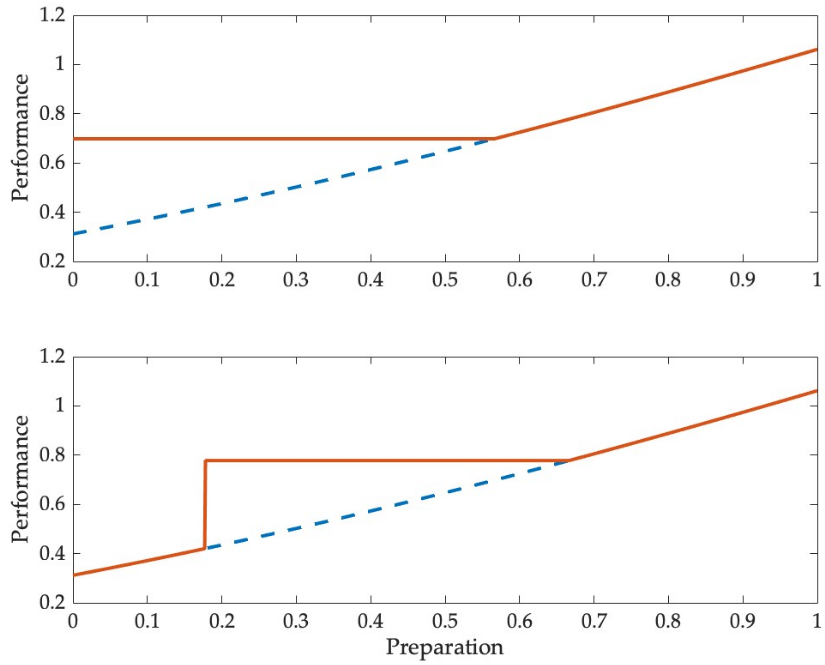

Figure 3: Equilibrium standards when $\beta = 1$ (top) and $\beta = 2$ (bottom).

Let d^* denote the equilibrium standard under this objective function for the EA. This will depend on the distribution of preparation in the population, as illustrated by the following example.

Example 2. *Suppose that all specifications are in Example 1 and that distribution of preparation $g(\theta)$ is a symmetric beta distribution with both shape parameters equal to β. If $\beta = 1$ the optimal standard is $d^* = 0.70$ and all students pass. If $\beta = 2$ the optimal standard is $d^* = 0.78$ and some students fail.*

The example considers two different distributions of preparation, both belonging to the family of symmetric beta distributions, where one distribution is a mean preserving spread of the other. In one case we have uniformly distributed preparation ($\beta = 1$), and here the equilibrium standard is such that even the least prepared students pass, as shown in the top panel of Figure 3. When the distribution of preparation is more concentrated around the mean, the standard is higher and those with lower levels of preparation fail to meet it. This case is shown in the bottom panel of the figure.

When the distribution of preparation is more highly concentrated around the mean, the population is more homogeneous. Intuition suggests that a more tightly clustered preparation distribution allows the EA to choose a test that is better calibrated to the potential of students, with the result that the realized value of mean performance in equilibrium is greater. However, this intuition can fail under certain conditions, as the following example illustrates.

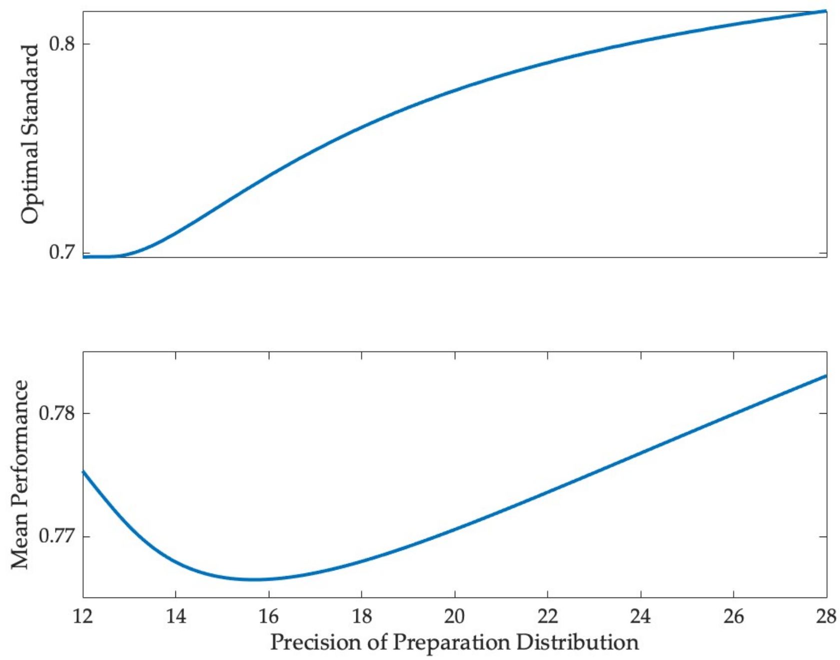

Figure 4: Optimal standards and mean performance for a family of preparation distributions.

Example 3. *Suppose that all specifications are in Example 1 and let preparation be given by a symmetric beta distribution with both shape parameters equal to* $\beta \in [1, 3]$*. Then the precision (inverse variance) of the distributions lies in the range* $[12, 28]$*, while the mean is equal to 0.5 in all cases. The resulting values of the equilibrium standard and the maximum achieved mean performance are shown in Figure 4.*

Figure 4 shows that while the optimal standard is (weakly) increasing, the mean performance attained in equilibrium is not monotonic in the precision of the preparation distribution. For example mean performance is higher when $g(\theta)$ is uniformly distributed, with shape parameters $(1, 1)$, than when the shape parameters are $(2, 2)$, even though the former is a mean-preserving spread of the latter. This failure of monotonicity may be understood as follows. At low levels of precision (starting with the uniform distribution) the equilibrium standard is such that the least prepared students pass. As the preparation distribution becomes more clustered around the mean, the population at the extremes of the preparation distribution becomes thinner. Initially, however, this does not lead to a change of equilibrium standard, so there is no change in performance for those with low levels of preparation. Meanwhile, fewer people exceed the standard easily since the population at the highest levels of preparation is also thinned. This brings mean performance down. Eventually, however, the standard itself is raised and mean performance starts to rise.

To this point, we have treated the distribution of student preparation as exogenous. In practice, parents can invest resources to increase the academic preparedness of their children. Their capacity and willingness to do so will depend on their material conditions, the endowed ability of their children, and the incentives created by the testing standard. We explore these mechanisms next.

4 Parental Investment

The preparation of any given student will depend in part on their own idiosyncratic characteristics, and in part on parental investments. We take the former as exogenously distributed and the latter as subject to a decision.

Consider two dimensions along which students can vary: the wealth of their parents and their level of ability. We use the term ability to encompass all characteristics relevant for success aside from parental investment and effort; these could depend on endowed potential as well as such factors as early childhood nutrition or environmental exposures.

Given any student, let $\alpha \in [\alpha_{min}, \alpha_{max}]$ denote their ability, $\omega > 0$ their parental wealth, and m the (endogenously chosen) level of parental investment. Preparation depends on investment and ability in accordance with

$$\theta = s(\alpha, m) \tag{4}$$

where s is strictly increasing and strictly concave in both arguments.

Parents fully internalize the preferences of their child regarding performance and passing, but do not internalize the child's cost of effort.[3] In addition, they care about household consumption, which is financed by the wealth remaining after the investment m has been chosen.[4] That is, given a standard d, a parent with wealth ω and child with ability α chooses $m \leq \omega$ to maximize

$$v(\omega, \alpha) = u(\omega - m) + f(\theta, e^*(\theta, d)) + bp, \tag{5}$$

where preparation θ itself depends on parental investment m in accordance with (4), and effort e^* is chosen optimally by children in response to the standard, given their preparation, as described in Section 3. The first term is household consumption, the second is child performance, and the third is the value of passing. The last two terms are inherited from the child's objective function (2) with

[3]We consider this to be an empirically plausible specification; parents may even take pleasure in witnessing the goal-oriented expenditure of effort by their child, even though the child would rather avoid the burden. See Albornoz et al. (2018) for a similar approach to parental preferences.

[4]Recall that we have not included household consumption in the objective functions of children. This does not mean that children are indifferent to their levels of consumption, only that they have no direct control over this variable, and preferences are such that their level of consumption (exogenous from their perspective) does not affect effort choices.

preparation now endogenous. The standard d affects parental investment through the anticipated effort choices of children, which affect academic performance and the passing outcome.

Since s is strictly increasing in both arguments, a unique (possibly zero) level of investment is required to provide a child with preparation θ, given the child's ability α. Let $\mu(\alpha, \theta)$ denote this investment level. Since the minimum preparation level required to induce a child to meet standard d is $\underline{\theta}(d)$, the smallest investment required for a parent to ensure that their child (with ability α) passes is $\mu(\alpha, \underline{\theta}(d))$. For expositional purposes we suppress function arguments when the meaning is clear, and refer to μ as *minimum passing investment*. This is zero if the child's ability is such that she would make the effort to pass the exam even without parental investment. That is $\mu = 0$ if $s(\alpha, 0) \geq \underline{\theta}(d)$. Otherwise, $\mu > 0$ and given by the solution to

$$s(\alpha, \mu) = \underline{\theta}(d).$$

To characterize optimal investment for any given values of parental wealth, child ability, and the educational standard, it is useful to first consider the hypothetical investment level chosen when there is no standard in place. Let this *baseline parental investment* be denoted $m^u(\alpha, \omega)$, which maximizes

$$u(\omega - m) + \rho^u(s(\alpha, m)), \tag{6}$$

where ρ^u is the performance level when a child with preparation $\theta = s(\alpha, m)$ chooses baseline effort as defined in Section 3. Let $\theta^u = s(\alpha, m^u)$ denote the baseline preparation. For any given levels of parental wealth and child ability, baseline investment (and hence also baseline preparation, effort, and performance) are all generically unique without any further assumptions on the parental objective function.

In the absence of a standard, more affluent parents invest more conditional on child ability, and hence endow their children with higher levels of preparation:

Proposition 2. *Baseline investment is strictly increasing in parental wealth at any level of child ability.*

Proposition 2 is very intuitive. Given concavity of utility in household consumption, the utility cost of any given investment is lower for a wealthier parent. As a result, *in the absence of a standard*, children of more affluent parents will receive higher levels of investment than equally able children of less affluent parents. Under the conditions of Proposition 1, therefore, they also have higher performance when there is no standard in place.

The effect of child ability on baseline preparation (holding constant parental wealth) is more complicated. To explore this, we introduce a condition analogous to Definition 1 but with reference to investment rather than effort:

Definition 2. *The preparation function s satisfies* ***increasing differences in investment*** *if, for any* $\alpha' > \alpha$*, the difference* $\mu(\alpha, \theta) - \mu(\alpha', \theta)$ *is non-decreasing in* θ*.*

That is, the preparation function satisfies increasing differences in investment if, for any pair of child ability levels $\alpha' > \alpha$, the extra investment required for type α (relative to that required for type α') to reach preparation θ is increasing in θ. This condition is sufficient to ensure that baseline preparation is strictly increasing in child ability given parental wealth:

Proposition 3. *If s satisfies increasing differences in investment, then baseline preparation is strictly increasing in child ability at any given level of parental wealth.*

We have shown earlier (see Proposition 1) that at any given level of preparation, students will choose either baseline effort or minimum passing effort. With endogenous preparation, parents choose either baseline investment or minimum passing investment:

Proposition 4. *Suppose f and s satisfy increasing differences in effort and investment respectively. Then the optimal choice of each parent is either baseline investment or minimum passing investment.*

That is, facing any standard d, a parent having wealth ω and a child with ability α will choose to invest either m^u or μ, depending on which of these yields the higher payoff:

$$m^* = \begin{cases} \mu & \text{if } u(\omega - \mu) + d + b \geq u(\omega - m^u) + \rho^u(\theta) + bp \\ m^u & \text{otherwise} \end{cases}$$

where θ is given by (4) and $p = 1$ if $\rho^u \geq d$ and $p = 0$ otherwise.

For any given standard and level of parental wealth, the following result describes the qualitative properties of equilibrium investments, effort, preparation, and performance.[5]

Proposition 5. *Suppose f and s satisfy increasing differences in effort and investment respectively. Given any standard and parental wealth, there exist ability thresholds* $\underline{\alpha}$ *and* $\hat{\alpha}$ *such that* $\underline{\alpha} < \hat{\alpha}$ *and:*

(a) students with $\alpha > \hat{\alpha}$ *receive baseline investment and exceed the standard;*

(b) students with $\alpha \in (\underline{\alpha}, \hat{\alpha})$ *receive minimum passing investment and meet the standard;*

(c) students with $\alpha < \underline{\alpha}$ *receive baseline investment and fail.*

Furthermore, there exist ability thresholds α^* *and* $\bar{\alpha}$ *with* $\underline{\alpha} < \alpha^* < \bar{\alpha} < \hat{\alpha}$ *such that investment and preparation are both below baseline when ability is in* $(\alpha^*, \hat{\alpha})$*, and performance is below baseline when ability is in* $(\bar{\alpha}, \hat{\alpha})$*. Conditional on the standard d, the thresholds* $\underline{\alpha}$*,* α^**,* $\bar{\alpha}$*, and* $\hat{\alpha}$ *are all decreasing in* ω*.*

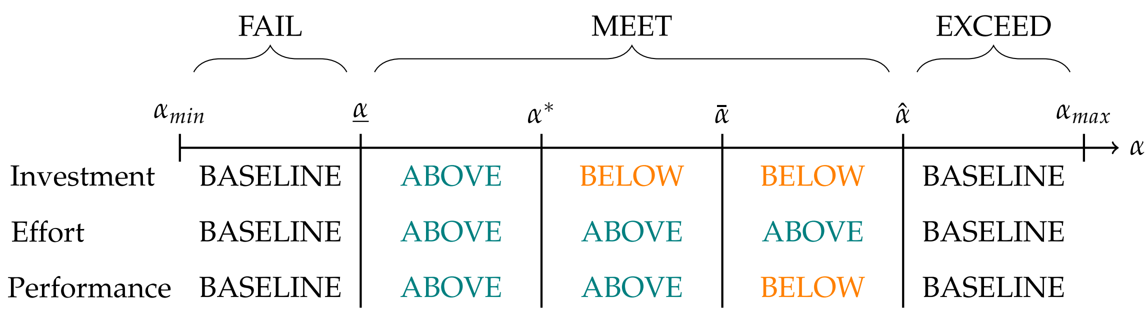

Figure 5: Investment, effort, and outcomes as ability varies, for given parental wealth and standard.

Proposition 5 is illustrated schematically in Figure 5, and numerically in Figures 6 and 7, based on the following specification:

Example 4. *The objective function for parents is* $5(\omega - m)^{0.2} + \rho + bp$*, that for students is* $\rho + bp - 5e^2$*,* $\theta = \sqrt{\alpha m}$*,* $\rho = \theta^{0.8} e^{0.2}$*;* $b = 0.01$*;* $d = 0.1$*;* $\omega = 1$*.*

At the lowest range for ability, investment, preparation, effort, and performance are all at baseline levels and the student fails to meet the standard. As ability rises, a point is reached at $\underline{\alpha}$ where investment jumps discontinuously above baseline so that preparation can reach $\underline{\theta}$, which induces the student to exert minimum passing effort and thus meet the standard. Further increases in ability leave preparation and effort unchanged, but investment declines, as less investment is needed to reach the preparation threshold $\underline{\theta}$. Eventually, at ability α^*, investment declines below baseline even as effort and performance are maintained and the standard continues to be met exactly.

The most interesting shift occurs at $\bar{\alpha}$, where even *performance* drops below baseline. This happens because when baseline preparation is slightly above $\bar{\theta}$ (which would lead to performance exceeding the standard) parents find it optimal to choose much lower than baseline investment levels, thus raising household consumption, while lowering performance only to the level of the standard. Eventually a point is reached at $\hat{\alpha}$ where the sacrifice in performance is too great, and all variables return to baseline levels.

There are two implications of this analysis that are worth highlighting. First, in contrast with the case of exogenous preparation, it is possible for the imposition of a standard to lower performance at some ability levels. This happens because the standard induces student effort, which parents anticipate, and which allows parents to lower investment substantially while lowering

[5] In the statement of the result (and in the discussion that follows), we have suppressed the dependence of ability and performance thresholds on the standard d and parental wealth ω for expositional clarity.

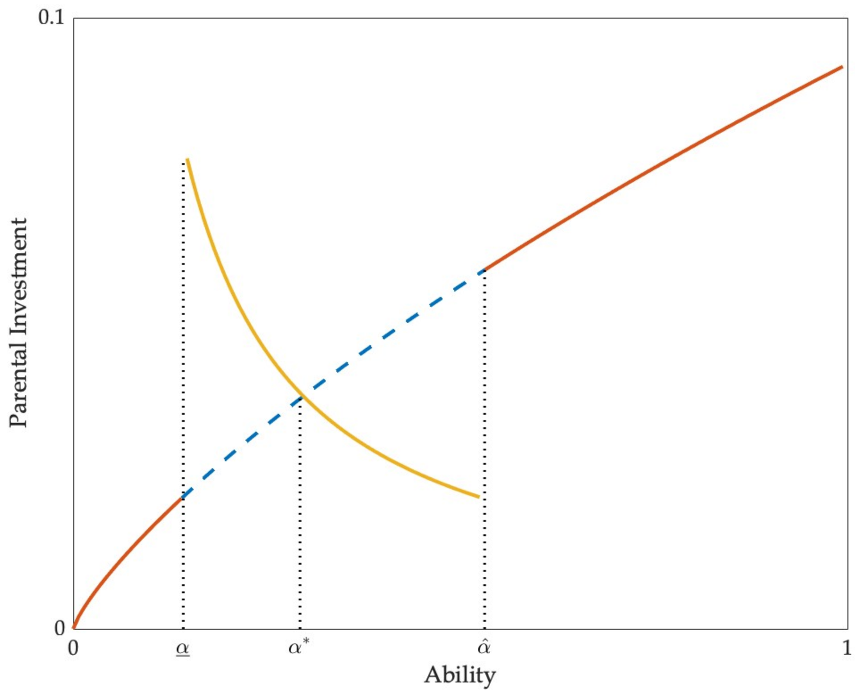

Figure 6: Optimal parental investment as a function of child ability for given parental wealth.

performance only to the level of the standard. So that imposition of a standard raises performance at some ability levels but lowers it at others.

Second, at any given wealth level, there are levels of preparation and performance that are never observed even if ability is continuously distributed. This is because deviations from baseline investment all lead to the same level of preparation, which is the lowest necessary for the standard to be met. Parents with higher ability students make smaller expenditures to get there, so end up with greater household consumption. Moreover, the upper bound $\hat{\theta}$ of this range of unobserved preparation levels exceeds $\bar{\theta}$ (the preparation level at which baseline performance is equal to the standard). This is what leads some students to achieve lower than baseline performance in the presence of the standard.

Thus, the first part of Proposition 5 identifies an ability interval such that a parent of given wealth chooses investments that are calibrated to induce their child to meet the standard exactly. That is, when ability falls within this interval, the parent chooses the minimum investment necessary to make passing worthwhile for the child. The last part of the proposition establishes how this interval varies with parental wealth.

Proposition 5 tells us that wealthier parents invest enough to allow their children to meet the

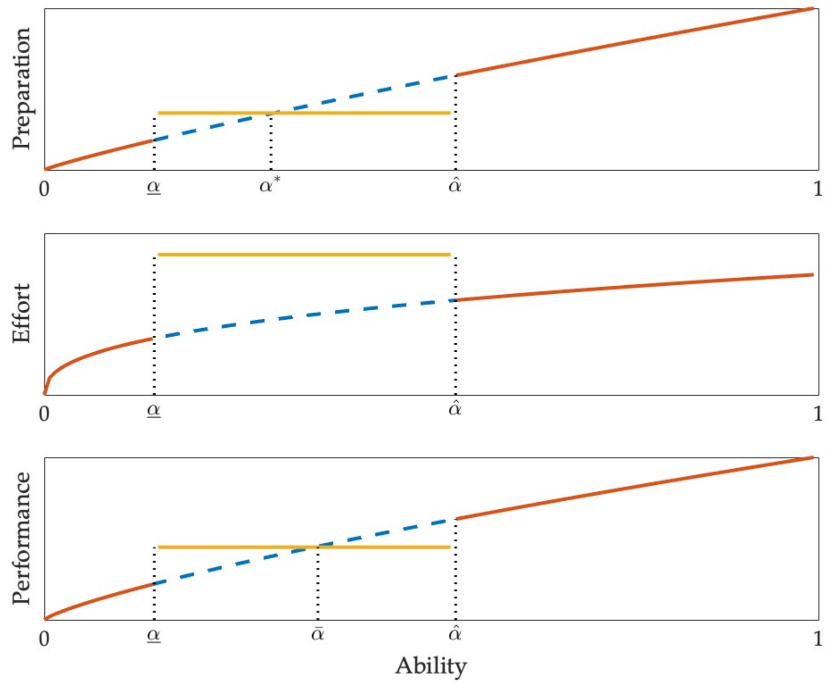

Figure 7: Preparation, effort and performance as function of ability with endogenous parental investment.

standard at lower levels of ability. If two children have the same ability but different parental wealth, the one from a more affluent household may pass while the other fails. This happens because the former receives enough investment to make the effort required to pass worthwhile, while the latter does not. More generally, children of wealthier parents will have higher performance and higher rates of passing at any given ability level. This alone is not surprising. Less obvious is the fact that children of wealthier parents exert more *effort* at certain levels of ability, because parents invest enough to make this effort worthwhile.

The analysis in this section deals with the manner in which parents and students respond to a given standard. We next consider how a given household responds to changes in the standard, and which standards children, parents, and the educational authority would consider ideal.

5 Ideal Standards

In this section we consider the manner in which investment, effort performance, and the welfare of parents and children changes as the standard varies, holding constant parental wealth and child ability. We show that for any given household, there will be a range of standards that result in an increase in performance relative to baseline. Less intuitively, there will also be a range of standards such that performance will drop *below* baseline, so the introduction of a standard within this range will result in decreased achievement by the child.

Consider a given household with characteristics (α, ω). Since parental wealth and child ability are given, and baseline choices are independent of the standard, baseline investment, effort, and performance remain constant as the standard varies. Let D denote the set of standards that are exactly met, and let $\underline{d}$ and $\bar{d}$ respectively denote the infimum and supremum of this set.[6] All standards below $\underline{d}$ will be exceeded and those above $\bar{d}$ will fail to be met. We call D is set of *binding standards*. We show below that there exist standards within this set at which both investment and performance are *below* baseline, that $\bar{d}$ is the highest level of performance that a policy can induce (and hence the standard chosen by the EA if all households have the same characteristics), that the ideal standard for parents is binding but lies below $\bar{d}$, and that within the set of binding standards the preferences of children and the EA can be aligned.

Proposition 6. *Suppose f and s satisfy increasing differences in effort and investment respectively. Then, for any given household, there exist sets of standards $D_l \subset D$ and $D_h \subset D$ such that*

(a) if $d \in D_l$ performance is below baseline, and

(b) if $d \in D_h$ performance is above baseline.

Furthermore, there exist standards $\tilde{d}$ and d^* with $\underline{d} < \tilde{d} < d^* < \bar{d}$ such that $\tilde{d}$ is equal to baseline performance ρ^u and minimum passing investment is equal to baseline investment at d^*. Parental utility is maximized at a binding standard strictly below $\bar{d}$. Among binding standards, child utility is highest at $\bar{d}$.

At the threshold standards $\underline{d}$ and $\bar{d}$ parents are indifferent between two different values of investment, but their children are not. In this case, we break ties in favor of children, so the standard $\underline{d}$ is exceeded, and the standard $\bar{d}$ is exactly met. An immediate consequence of the result is the following:

Corollary 1. *Suppose f and s satisfy increasing differences in effort and investment respectively. A mean performance maximizing educational authority faced with a homogeneous population will choose $\bar{d}$, the highest standard that will be exactly met.*

We discuss the case of a heterogeneous population in Section 6 below, and show that the authority may choose a standard that is strictly lower than the highest that any homogeneous subgroup would meet.

[6] We show below that our tie-breaking rule in favor of children's preferences implies that $\bar{d} \in D$ (and is therefore the highest standard that will be exactly met). However, $\underline{d} \notin D$ so there is no lowest standard that is exactly met. If D is an interval, $\underline{d}$ is the highest standard that will be exceeded.

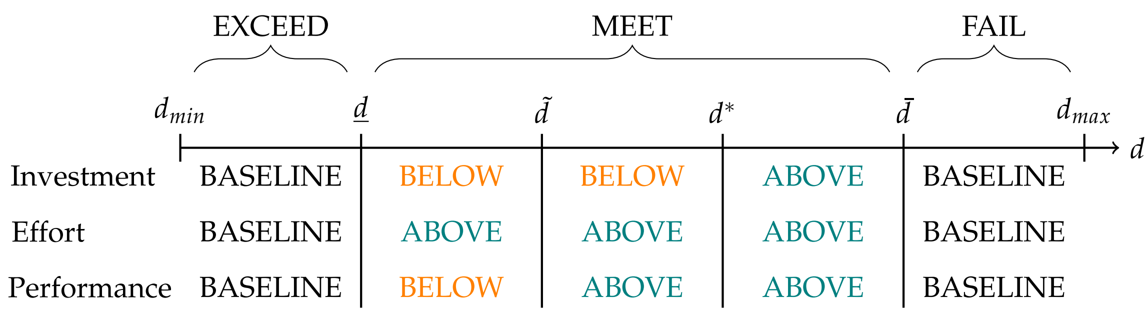

Figure 8: Investment, effort, and outcomes as ability varies, for given parental wealth and standard.

To illustrate Proposition 6, consider the following numerical specification:

Example 5. *The objective function for parents is* $5(\omega - m)^{0.2} + \rho + bp$*, that for students is* $\rho + bp - 5e^2$*,* $\theta = \sqrt{\alpha m}$*,* $\rho = \theta^{0.8} e^{0.2}$*;* $b = 0.01$*;* $\alpha = \omega = 1$*. In this case the binding set is the interval* $(0.17, 0.35]$*, parental welfare is maximized at* $d = 0.25$*, and investment, effort, performance, and child welfare are all maximized at* $d = 0.35$*.*

The manner in which investment, effort, and performance vary with the standard in this example is illustrated schematically in Figure 8 and numerically in Figure 9. Very low values of the standard are not binding; investment, effort, and performance are all at baseline and the child passes comfortably. As the standard rises, a point is reached where investment drops and effort rises discontinuously, while performance drops *below* baseline. This is the start of the binding interval, within which the standard is exactly met.

Why do parents choose below baseline investment at standards between $\underline{d}$ and $\tilde{d}$, even though they could secure greater performance by maintaining investment at baseline? They do so because the reduction of investment frees up resources for household consumption. Why do they not do this at standards below $\underline{d}$? Because if they did, performance would drop too much, as the child will expend only enough effort to meet the standard. It is at $\underline{d}$ that the trade-off between these forces shifts in favor of lower investment. The parent is effectively buying greater household consumption with reduced child performance, while the standard itself sets a bound to the resulting drop in performance.

As the standard rises further, it becomes more challenging to meet, and parents increase investment to ensure that it is indeed met. At standard $\tilde{d}$ performance returns to the baseline level, though investment is still below baseline. That is, baseline performance is met at $\tilde{d}$ with below baseline investment and above baseline effort. Further increases in the standard lift performance above baseline through increases in investment. At d^*, investment reaches baseline levels, and continues to rise until $\bar{d}$ is reached. At this point there is a discontinuous drop in investment,

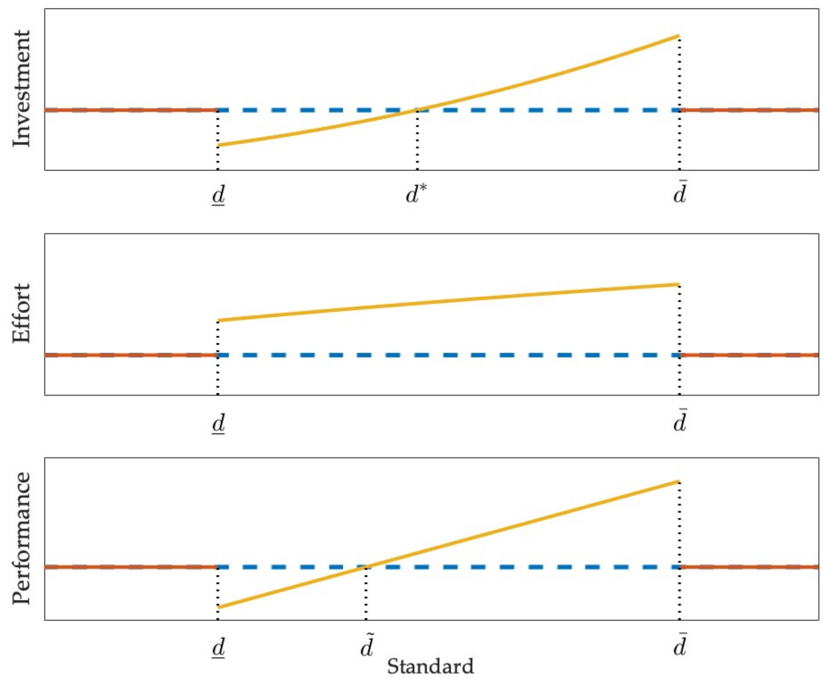

Figure 9: Investment, effort, and performance as the standard varies.

effort, and performance back to baseline levels, as parents and children give up on attempting to meet the standard.

Although standards below and above the binding interval have the same levels of investment, effort, and performance, they do not have the same levels of welfare for either parent or child. This is because the standard is exceeded when it lies below $\underline{d}$, and remains unmet when it lies above $\bar{d}$. The last part of Proposition 6 establishes that parental well-being is maximized at a standard within the binding interval, and that within this interval, the utility of the child is increasing in the standard.

Figure 10 shows how the utility of parent and child varies with the standard, based on the specification in Example 5. As stated in the result, the ideal standard from the perspective of the parent lies within the binding interval. The ideal standard from the perspective of the child either lies at the top of this interval (as shown in the figure) or lies in the range of standards that are low enough to be exceeded with baseline choices. That is, a performance maximizing EA faced with a homogeneous population will choose a standard higher than that considered optimal by parents, but possibly identical to that considered optimal by children. This is no longer the case when households are heterogeneous, as we show in Section 6 below—in the presence of heterogeneity

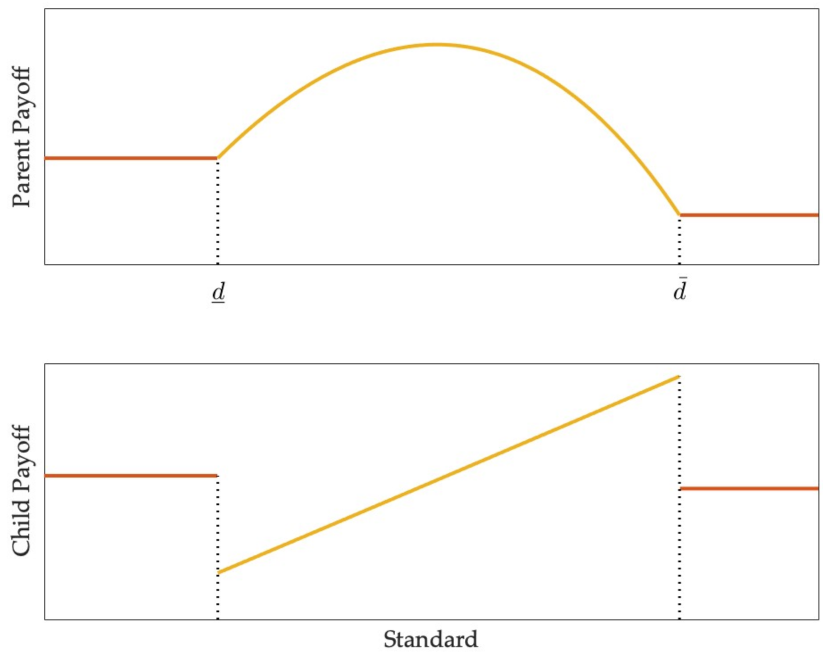

Figure 10: Parent and child welfare as the standard varies.

in wealth or ability the authority may choose a standard lower than that preferred by any parent or child.

Note that the utility of the child falls discontinuously as the binding interval is entered, since investment and performance both drop discontinuously, and the child is induced to make greater effort to meet the standard. The child also suffers a discontinuous drop in utility upon exit from the binding interval. Although effort drops here, so does investment and performance, and the child moves from passing to failing.

In this section so far we have considered a single household with given levels of wealth and child ability. We next consider the choices of an educational authority faced with a heterogeneous population of households.

6 Heterogeneous Households

When faced with a population that is heterogeneous with respect to child ability or parental wealth, an educational authority could well choose a standard that does not maximize the performance of any subgroup within the population. In this section we show that heterogeneity can exert downward pressure on the standard in the following sense—the chosen standard can be lower than any that would be chosen if the authority were facing a homogeneous subgroup.

Consider a population with homogeneous ability but two wealth classes ($j = l, h$) with $\omega_h > \omega_l$. The population share of those with low wealth is denoted ϕ. For each wealth class j there are threshold standards $\underline{d}_j$, $\bar{d}_j$ as defined in the previous section.

If ϕ is sufficiently close to zero or one, a performance-maximizing educational authority will choose the standard that maximizes performance in the more prevalent of the two groups. When the population is more balanced, however, it is possible for the authority to choose a standard that lies below both $\bar{d}_h$ and $\bar{d}_l$. In this case the authority may choose a standard that will be exceeded by those in the affluent group, and met exactly by those in the less affluent group. Raising the standard slightly from this level will raise performance in the less affluent group but will cause those in the more affluent group to lower performance discontinuously, moving from baseline performance above the standard to below baseline performance at the standard. The following example illustrates.

Example 6. *The objective function for parents is* $5(\omega - m)^{0.2} + \rho + bp$*, that for students is* $\rho + bp - 5e^2$*,* $\theta = \sqrt{\alpha m}$*,* $\rho = \theta^{0.8}e^{0.2}$*;* $b = 0.01$*;* $\alpha = 1$*. Wealth levels are* $\omega_l = 0.27$ *and* $\omega_h = 1$*. Then the optimal choice of the authority is (i)* $\bar{d}_h = 0.346$ *if* $\phi < 0.643$*, (ii)* $\underline{d}_h = 0.167$ *if* $\phi \in (0.643, 0.870)$*, and (iii)* $\bar{d}_l = 0.175$ *if* $\phi > 0.870$*.*

In this example the standard is lowest when the population composition is in an intermediate range. Figure 11 illustrates this for the case of $\phi = 0.7$, when the optimal standard from the EA perspective is $\underline{d}_h = 0.167$. This standard is exactly met by the less wealthy group, and is the highest standard exceeded by the more wealthy group. A small increase in standard above this level would cause both groups to meet it exactly, lowering mean performance in the population. And a substantial increase to $\bar{d}_h$, which is the highest standard that the wealthier group would meet, would not raise mean performance to the level attained at $\underline{d}_h$.

That is, the performance-maximizing standard in a heterogenous group may be lower than that in either of the homogeneous subgroups. In order for this to happen, it is necessary that $\underline{d}_h < \bar{d}_l$, which means that the binding intervals for the two groups must have a nonempty intersection. This in turn requires that the two wealth levels not be too far apart.

Next consider heterogeneity in ability, with all households having the same parental wealth. Specifically, consider two ability levels ($j = l, h$) with $\alpha_h > \alpha_l$. The population share of those with low ability is denoted π. For each ability class j there are threshold standards $\underline{d}_j$, $\bar{d}_j$ as defined earlier; here $\underline{d}_j$ is the highest standard that those in ability group j will exceed, and $\bar{d}_j > \underline{d}_j$ is the

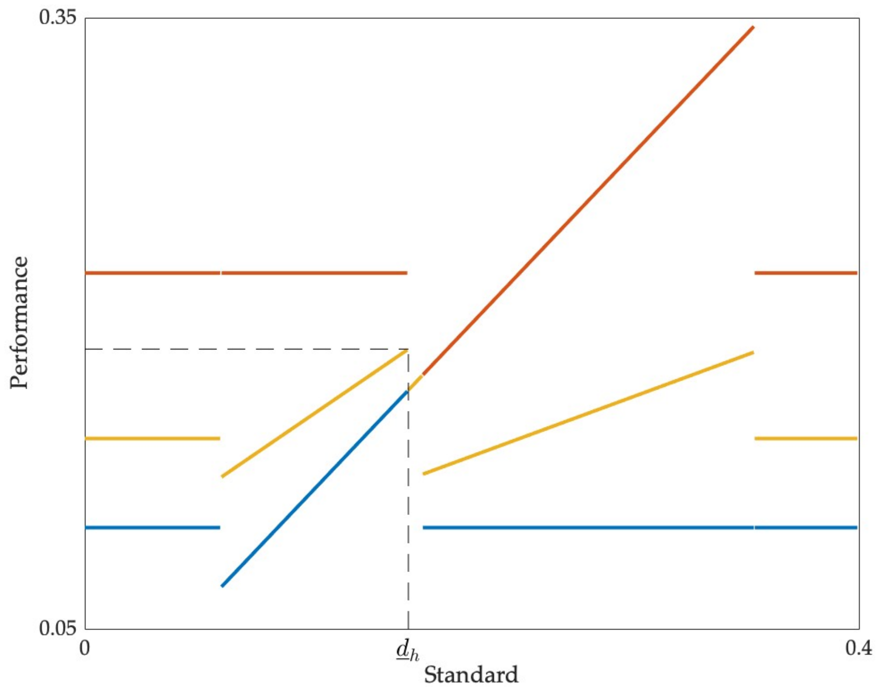

Figure 11: Optimal standard and mean performance when low wealth population share is 70 percent.

highest standard that those in ability group j will meet exactly.

As in the case of heterogeneous wealth, if the population share of one of the groups is close to one, a performance-maximizing educational authority will choose the standard that maximizes performance in this group. Otherwise, it is possible for the authority to choose a standard that lies below both $\bar{d}_h$ and $\bar{d}_l$. The following example illustrates.

Example 7. *The objective function for parents is* $5(\omega - m)^{0.2} + \rho + bp$, *that for students is* $\rho + bp - 5e^2$, $\theta = \sqrt{\alpha m}$, $\rho = \theta^{0.8} e^{0.2}$; $b = 0.01$; $\omega = 1$. *Ability levels are* $\alpha_l = 0.35$ *and* $\alpha_h = 1$. *The optimal choice of the authority is (i)* $\bar{d}_h = 0.346$ *if* $\pi < 0.645$, *(ii)* $\underline{d}_h = 0.167$ *if* $\pi \in (0.645, 0.835)$, *and (iii)* $\bar{d}_l = 0.177$ *if* $\pi > 0.835$.

What these examples show is that heterogeneity across households in either wealth or ability can give rise to lower standards than those that would maximize performance in either group in isolation. That is, heterogeneity sometimes exerts downward pressure—but never upward pressure—on standards.

7 Conclusions

Parents and children share certain goals in common, but don't have identical preferences. Both care about academic performance and meeting standards, but parents must consider other demands on household resources when contemplating educational investments, and children directly experience the burdens of study time and effort. Educational authorities have their own priorities, among them being a concern for the distribution of achieved academic performance in the populations they serve.

Each of these parties have different means available for furthering their objectives. Parents choose investments in boosting the academic preparation of their children, children have some autonomy in choosing study effort, and educational authorities can set standards for passing that shape the incentives faced by parents and children.

In this paper, we have considered the interplay of these incentives. We have shown how investment, preparation, effort, and performance respond to a given standard, and how authorities can set standards taking these responses into account. While intuition suggests that the introduction of standards will raise parental investment and the performance of children, this is true only under certain conditions, and for some households investment and performance can decline. The authority can avoid this outcome by careful choice of standard when facing a homogeneous population, but may be unable to avoid it in the face of population heterogeneity.

There are several directions in which the model presented here could be extended, and many open questions remain. Bishop (2006, p. 912) has observed that "learning requires active participation of the learner," and students decide "whether to skip school, how much effort to devote to each course and whether to help or obstruct the learning of others." While we have considered the efforts of students to raise their own performance, we have not examined the possibility that there may be incentives to assist in or sabotage the learning of others. More generally, we have neglected peer effects, which are known to matter a great deal in educational settings. We have also neglected policy options other than a uniform standard applied to the population at large, and it would be worth considering additional policy levers such as tracking and competitive grading.

There are other factors that may be worth incorporating into the model. When there are multiple children in each household, the standard might affect whether resources are evenly divided among them or whether one of them receives the lion's share. The level of parental wealth as well as the degree of ability heterogeneity within the household are likely to affect such decisions. Children who receive substantial parental investments may feel an obligation to make more effort, especially when parental resources are strained. The value of passing may be increasing in the standard—meeting a challenging standard may be more satisfying than exceeding a trivial one. The objective of the authority could be endogenized by taking account of the political process that gives rise to membership of educational boards and their oversight. These are all potentially interesting and feasible extensions.

The issues of curriculum design and testing standards are vitally important and hotly debated in academic and policy circles. Modifications to the mathematics curriculum in San Francisco over the past decade, while motivated by egalitarian objectives, have failed to close achievement gaps, and similar proposals at the state level are facing intense backlash (Hong, 2021; Fortin, 2021). It is difficult to make sense of these effects without taking into account the interacting incentives faced by parents, children, and educational authorities when households are heterogeneous along multiple dimensions. While our contributions here are theoretical, our hope is that the framework is rich enough to productively address some of these pressing policy concerns.

Appendix

The following result provides an alternative statement of the increasing differences in effort condition, and is useful for our subsequent arguments.

Lemma 1. *The performance function f satisfies increasing differences in effort if and only if for any $\theta' > \theta$, $f(\theta', e') = f(\theta, e)$ implies $f_2(\theta', e') \geq f_2(\theta, e)$.*

Proof. Define $\Psi(\theta, \rho, \xi) = f(\theta, \xi) - \rho$ and note by the definition of ρ that $\Psi(\theta, \rho, \xi) = 0$. Since f is strictly increasing in both arguments and Ψ is differentiable, we can define a differentiable function $\xi(\theta, \rho)$ that satisfies

$$\frac{\partial \xi}{\partial \rho} = -\frac{\partial \Psi / \partial \rho}{\partial \Psi / \partial \xi} = \frac{1}{f_2(\theta, \xi)}.$$

Now suppose $\theta' > \theta$ are given, and define

$$\Delta(\rho) = \xi(\theta, \rho) - \xi(\theta', \rho).$$

Then Δ is non-decreasing in ρ (increasing differences in effort) if and only if

$$f_2(\theta, e) \leq f_2(\theta', e'),$$

where $e = \xi(\theta, \rho)$ $e' = \xi(\theta', \rho)$. □

Proof of Proposition 1. Suppose f satisfies increasing differences in effort and $\theta_1 > \theta_2$. By definition, $\rho^u(\theta_2) = f(\theta_2, e^u(\theta_2))$ and by the first order condition for optimality,

$$f_2(\theta_2, e^u(\theta_2)) = c'(e^u(\theta_2)). \tag{7}$$

Define e_1 as the unique effort level that satisfies

$$f(\theta_1, e_1) = f(\theta_2, e^u(\theta_2)).$$

Note that $e_1 < e^u(\theta_2)$ since f is increasing in both arguments. Hence, for any $e \leq e_1$,

$$c'(e) < c'(e^u(\theta_2))$$

by convexity of c. As f satisfies increasing differences in effort, we have

$$f_2(\theta_1, e_1) \geq f_2(\theta_2, e^u(\theta_2)) = c'(e^u(\theta_2))$$

from Lemma 1 and (7). Hence, for any $e \leq e_1$,

$$f_2(\theta_1, e) > c'(e)$$

by concavity of f. Hence no effort level $e \le e_1$ can be optimal for type θ_1, and we have $e^u(\theta_1) > e_1$. This implies $\rho^u(\theta_1) > \rho^u(\theta_2)$, so baseline performance is strictly increasing in preparation.

If $\rho^u(\theta_{\max}) < d$, set $\bar{\theta}(d) = \theta_{\max}$; otherwise let $\bar{\theta}(d)$ be the unique value of θ such that $\rho^u(\theta(d)) = d$. A unique solution exists since $d > d_{\min} = \rho^u(\theta_{\min})$ and baseline performance is strictly increasing in preparation. Note that $\rho^u(\theta) > d$ if and only if $\theta > \bar{\theta}(d)$. All those with preparation $\theta > \bar{\theta}(d)$ will choose baseline effort and pass (this set will be empty if $\bar{\theta}(d) = \theta_{\max}$).

If $\rho^u(\theta_{\min}) - c(e^u(\theta_{\min})) < d + b - c(\xi(\theta_{\min}, d))$ then set $\underline{\theta}(d) = \theta_{\min}$; otherwise define $\underline{\theta}(d)$ as the unique value of θ such that a student is indifferent between baseline effort (which results in a failure to meet the standard) and minimum passing effort (we establish uniqueness of this threshold below). That is:

$$d + b - c(\xi(\underline{\theta}, d))) = \rho^u(\underline{\theta}) - c(e^u(\underline{\theta})). \tag{8}$$

It is easily verified that a solution exists for any $d < d_{\max}$ and that $\underline{\theta}(d) < \bar{\theta}(d)$.

We now show that any student with preparation $\theta < \underline{\theta}(d)$ strictly prefers $\rho^u(\underline{\theta})$ to d. Consider a student with preparation $\theta < \underline{\theta}(d)$, and define e_θ as the effort required by type θ to meet the baseline performance of type $\underline{\theta}(d)$:

$$f(\theta, e_\theta) = \rho^u(\underline{\theta}) = f(\underline{\theta}, e^u(\underline{\theta})).$$

Since f is strictly increasing in both arguments and $d > \rho^u(\underline{\theta})$, we have

$$\xi(\theta, d) > \xi(\underline{\theta}, d) > e^u(\underline{\theta})$$

and

$$\xi(\theta, d) > e_\theta > e^u(\underline{\theta}).$$

Since f satisfies increasing differences in effort and $d > \rho^u(\underline{\theta})$,

$$e_\theta - e^u(\underline{\theta}) \le \xi(\theta, d) - \xi(\underline{\theta}, d)$$

The above inequality, together with $e_\theta < \xi(\theta, d)$ and strict convexity of c, then implies

$$c(e_\theta) - c(e^u(\underline{\theta})) < c(\xi(\theta, d)) - c(\xi(\underline{\theta}, d)).$$

Hence

$$c(\xi(\underline{\theta}, d)) - c(e^u(\underline{\theta})) < c(\xi(\theta, d)) - c(e_\theta)$$

and so

$$(\rho^u(\underline{\theta}) - c(e_\theta)) - (d + b - c(\xi(\theta, d))) > (\rho^u(\underline{\theta}) - c(e^u(\underline{\theta}))) - (d + b - c(\xi(\underline{\theta}, d))) = 0$$

where the last equality follows from (8). Hence any student with preparation $\theta < \underline{\theta}(d)$ strictly prefers $\rho^u(\underline{\theta})$ to d, and hence also prefers baseline performance $\rho^u(\theta) < \rho^u(\underline{\theta})$ to d. This proves the uniqueness of $\underline{\theta}(d)$, and also shows that any student with $\theta \in [\underline{\theta}(d), \bar{\theta}(d)]$ chooses minimum passing effort and meets the standard exactly. □

Proof of Proposition 2. Consider two children with the same ability α and parental wealth ω_1 and ω_2, where $\omega_1 > \omega_2$. Let m_i and θ_i denote the optimally chosen baseline investment and preparation levels in household $i = 1, 2$. We want to show that $m_1 > m_2$. In the absence of a standard, parents maximize $u(\omega - m) + \rho^u(\theta)$, where $\theta = s(\alpha, m)$. Since $\alpha_1 = \alpha_2$, θ_i depends only on m_i. Hence, from strict concavity of u and the first order condition for optimality we can immediately rule out $m_1 = m_2$ and $\theta_1 = \theta_2$. Now suppose, by way of contradiction, that $m_1 < m_2$ and define $\delta = m_2 - m_1 > 0$. From the optimality conditions for parents:

$$u(\omega_1 - m_1) + \rho^u(\theta_1) > u(\omega_1 - m_2) + \rho^u(\theta_2)$$

and

$$u(\omega_2 - m_2) + \rho^u(\theta_2) > u(\omega_2 - m_1) + \rho^u(\theta_1).$$

Hence

$$u(\omega_1 - m_1) - u(\omega_1 - m_1 - \delta) > \rho^u(\theta_2) - \rho^u(\theta_1) > u(\omega_2 - m_1) - u(\omega_2 - m_1 - \delta).$$

This contradicts strict concavity of u since $\omega_1 - m_1 > \omega_2 - m_1$. Hence $m_1 > m_2$ as claimed. □

Proof of Proposition 3. Suppose s satisfies increasing differences in investment. Consider two parents with same wealth ω and children with respective abilities α_1, α_2, where $\alpha_1 > \alpha_2$. Let m_i and θ_i denote the optimally chosen baseline investment and preparation levels in household $i = 1, 2$. We want to show that $\theta_1 > \theta_2$.

We first show $\theta_1 \geq \theta_2$. Suppose not and $\theta_1 < \theta_2$. Define

$$\mu_1 \equiv \mu(\alpha_1, \theta_2) \text{ and } \mu_2 \equiv \mu(\alpha_2, \theta_1).$$

Since s is strictly increasing in both arguments,

$$\mu_2 > m_1. \tag{9}$$

As s satisfies increasing differences in investment and $\theta_1 \leq \theta_2$ we have

$$m_2 - \mu_1 \geq \mu_2 - m_1.$$

Hence

$$m_2 - \mu_2 \geq \mu_1 - m_1. \tag{10}$$

From the optimal choices of parents

$$u(\omega - m_1) + \rho^u(\theta_1) > u(\omega - \mu_1) + \rho^u(\theta_2)$$

and

$$u(\omega - m_2) + \rho^u(\theta_2) > u(\omega - \mu_2) + \rho^u(\theta_1).$$

Hence

$$u(\omega - m_1) - u(\omega - \mu_1) > \rho^u(\theta_2) - \rho^u(\theta_1) > u(\omega - \mu_2) - u(\omega - m_2). \tag{11}$$

We shall show that the above inequality contradicts the concavity of u. Note that for any $x > y$ and $a > b$, concavity of u implies

$$u(x) - u(x - b) < u(y) - u(y - a).$$

Setting $x = \omega - m_1$, $b = \mu_1 - m_1$, $y = \omega - \mu_2$, and $a = m_2 - \mu_2$, and using (9) to verify $x > y$ and (10) to verify that $a > b$, we obtain

$$u(\omega - m_1) - u(\omega - \mu_1) < u(\omega - \mu_2) - u(\omega - m_2),$$

which contradicts (11).

To complete the proof, we show that $\theta_1 \neq \theta_2$. Suppose not and $\theta_1 = s(\alpha_1, m_1) = s(\alpha_2, m_2) = \theta_2 = \theta$. Since $\alpha_1 > \alpha_2$ and s is strictly increasing in both arguments and satisfies increasing differences in investment, we have $m_1 < m_2$ and

$$s_2(\alpha_1, m_1) > s_2(\alpha_2, m_2). \tag{12}$$

Here we are using the version of increasing differences identified in Lemma 1, but applied to investment rather than effort. Recall that baseline investment maximizes $u(\omega - m) + \rho^u(\theta)$. Using the first order conditions for optimality of investment, we obtain:

$$\begin{aligned} u'(\omega - m_1) &= \frac{d\rho^u}{d\theta} s_2(\alpha_1, m_1), \\ u'(\omega - m_2) &= \frac{d\rho^u}{d\theta} s_2(\alpha_2, m_2). \end{aligned}$$

Hence, using (12), we obtain

$$u'(\omega - m_1) > u'(\omega - m_2),$$

which contradicts the strict concavity of u. □

Proof of Proposition 4. We prove that any $m^* \neq \{m^u, \mu\}$ cannot be optimum. Suppose, by way of contradiction, that there exists a parent with wealth ω and child ability α such that $m^* \neq \{m^u, \mu\}$.

Let $s(\alpha, m^*) = \theta^*$. We show that this parent has an incentive to deviate. Since $\theta = \underline{\theta}$ at the unique investment level μ, $m^* \neq \{m^u, \mu\}$ implies that $\theta^* \neq \underline{\theta}$. Hence we consider $\theta^* \in [\theta_{\min}, \theta_{\max}] \setminus \underline{\theta}$.

If $\theta^* < \underline{\theta}$, the child chooses baseline effort and fails (by Proposition 1). Parental utility is then $u(\omega - m^*) + \rho^u(\theta^*)$, which is maximized at m^u. Hence $m^* \neq m^u$ cannot be optimal.

If $\theta^* \in (\underline{\theta}, \bar{\theta}]$, the child chooses minimum passing effort and meets the standard exactly (by Proposition 1). But the same outcome can be ensured by investing $\mu < m^*$ at a lower consumption cost for the parent, so m^* cannot be optimal.

If $\theta^* > \bar{\theta}$, investment m^* results in a pass. If baseline investment also results in a pass, then baseline investment (which maximizes baseline parental utility) yields a higher payoff than investment m^* since

$$u(\omega - m^u) + \rho^u(\theta^u) + b > u(\omega - m^*) + \rho^u(\theta^*) + b. \tag{13}$$

Hence m^* cannot be optimal.

Finally, if $\theta^* > \bar{\theta}$ and baseline investment results in a failure to meet the standard, then by Proposition 3, we must have $\alpha < \bar{\alpha}$, where $\bar{\alpha}$ is the ability level at which the baseline payoff of a parent with wealth ω is maximized by choosing investment $\bar{m}^u$, resulting in preparation equal to precisely $\bar{\theta}$. That is,

$$s(\bar{\alpha}, \bar{m}^u) = \bar{\theta}.$$

Given Propositions 1 and 3, this ability level is unique, and $\theta^u < \bar{\theta}$. We show that $\bar{\theta}$ is strictly preferred by type (α, ω) to θ^*, so m^* cannot be optimal for this type.

Define m and $\bar{m}^*$ as follows:

$$s(\alpha, m) = s(\bar{\alpha}, \bar{m}^u) = \bar{\theta}$$

and

$$s(\bar{\alpha}, \bar{m}^*) = s(\alpha, m^*) = \theta^*.$$

Since s is strictly increasing in both arguments, we have $m > \bar{m}^u$ and hence

$$\omega - m < \omega - \bar{m}^u.$$

From the definition of baseline investment at $\bar{\alpha}$:

$$u(\omega - \bar{m}^u) + \rho^u(\bar{\theta}(d)) > u(\omega - \bar{m}^*) + \rho^u(\theta^*),$$

which implies

$$u(\omega - \bar{m}^u) - u(\omega - \bar{m}^*) > \rho^u(\theta^*) - \rho^u(\bar{\theta}(d)). \tag{14}$$

Furthermore, as s satisfies increasing differences in investment and $\theta^* > \bar{\theta}(d)$, we have

$$m - \bar{m}^u < m^* - \bar{m}^*,$$

which implies

$$(\omega - m) - (\omega - m^*) > (\omega - \bar{m}^u) - (\omega - \bar{m}^*).$$

Due to strict concavity of u we have

$$u(\omega - m) - u(\omega - m^*) > u(\omega - \bar{m}^u) - u(\omega - \bar{m}^*)$$

which, together with (14), implies

$$u(\omega - m) - u(\omega - m^*) > u(\omega - \bar{m}^u) - u(\omega - \bar{m}^*) > \rho^u(\theta^*) - \rho^u(\bar{\theta}(d)).$$

Hence we have

$$u(\omega - m) + \rho^u(\bar{\theta}(d)) + b > u(\omega - m^*) + \rho^u(\theta^*) + b.$$

Showing that m^* cannot be optimal. Hence $m^* \in \{m^u, \mu\}$ as claimed. $\square$

Proof of Proposition 5. Let (d, ω) be given, and define $\underline{\alpha}$ as the value of α such that a parent with wealth ω and child with ability α is indifferent between baseline investment (which results in preparation $\theta < \underline{\theta}$ and a failure to meet the standard) and minimum passing investment. That is,

$$u(\omega - \underline{\mu}) + d + b = u(\omega - \underline{m}^u) + \underline{\rho}^u, \tag{15}$$

where $\underline{\mu}$, $\underline{m}^u$, and $\underline{\rho}^u$ are minimum passing investment, baseline investment, and baseline performance at ability $\underline{\alpha}$ and arguments have been dropped for expositional clarity. If there is no solution to the above equation, then set $\underline{\alpha} = \alpha_{\min}$. Assume for the moment that $\underline{\alpha}$ is uniquely determined; we shall show that this must be the case.

Consider any $\alpha < \underline{\alpha}$ and define m and μ as follows

$$s(\alpha, m) = s(\underline{\alpha}, \underline{m}^u) = \underline{\theta}^u,$$

and

$$s(\alpha, \mu) = s(\underline{\alpha}, \underline{\mu}) = \underline{\theta},$$

where $\underline{\theta}^u$ is baseline preparation at ability $\underline{\alpha}$. As s is strictly increasing in both the arguments we have

$$\underline{m}^u < m,$$

which implies

$$\omega - \underline{m}^u > \omega - m.$$

Furthermore, as $\underline{\theta} > \underline{\theta}^u$, and s satisfies increasing differences in investment we have

$$m - \underline{m}^u < \mu - \underline{\mu}$$

which implies

$$(\omega - \underline{m}^u) - (\omega - \underline{\mu}) < (\omega - m) - (\omega - \mu).$$

Strict concavity of u implies

$$u(\omega - \underline{m}^u) - u(\omega - \underline{\mu}) < u(\omega - m) - u(\omega - \mu). \tag{16}$$

Combining (15) and (16) we obtain

$$u(\omega - m) + \underline{\rho}^u > u(\omega - \mu) + d + b.$$

Thus a parent with wealth ω and child ability α strictly prefers $\underline{\theta}^u$ to $\underline{\theta}$, and hence, from the definition of baseline preparation, must prefer baseline preparation $\theta^u(\alpha, \omega) < \underline{\theta}^u$ to $\underline{\theta}$. This proves the uniqueness of $\underline{\alpha}$.

Recall the definition of $\bar{\alpha}(d, \omega)$ and note that the utility of a parent with wealth ω and child with ability $\bar{\alpha}$ from investing m^u is

$$u(\omega - m^u) + d + b.$$

The utility of this parent from the minimum passing investment is strictly higher than the baseline. Accordingly, define the threshold of child ability $\hat{\alpha}(d, \omega)$.

$$u(\omega - \hat{m}^u) + \hat{\rho}^u + b = u(\omega - \hat{\mu}) + d + b$$

where $\hat{\mu}$, $\hat{m}^u$, and $\hat{\rho}^u$ are minimum passing investment, baseline investment, and baseline performance at ability $\hat{\alpha}$. That is the parent with characteristics $(\hat{\alpha}, \omega)$ facing standard d is indifferent between baseline investment (which results in preparation higher than $\bar{\theta}$ and induces the child to exceed the standard) and the minimum passing investment $\hat{\mu}$. If there is no solution to the above equation, then set $\hat{\alpha} = \alpha_{\max}$.

From the above discussion we can observe that $\hat{\alpha} > \bar{\alpha}$. For any $\alpha \in [\bar{\alpha}, \hat{\alpha})$, the parent strictly prefers to invest $\mu(\alpha, \underline{\theta})$ to $m^u(\alpha, \omega)$. The child's preparation is thus below baseline, resulting in performance below baseline, and the child meets the standard exactly by putting the minimum passing effort.

Define $\hat{\theta}$ as the preparation level such that

$$\hat{\theta} = s(\hat{\alpha}, \hat{m}^u).$$

Observe that $\hat{\theta}(d, \omega)$ is the only threshold of preparation which depends on parental wealth. Furthermore, $\hat{\theta} > \bar{\theta}$ and equilibrium preparation does not lie in the range $(\underline{\theta}, \hat{\theta})$.

Finally, define the threshold α^* as:

$$s(\alpha^*, m^u(\alpha^*, \omega)) = \underline{\theta},$$

where $m^u(\alpha^*,\omega)$ is baseline investment at ability α^* and parental wealth ω. Note that at any $\alpha \in [\underline{\alpha},\hat{\alpha})$, the parent chooses minimum passing investment and the child exerts minimum passing effort. At any $\alpha \in [\underline{\alpha},\alpha^*)$, the parent's investment is above baseline and at any $\alpha \in (\alpha^*,\hat{\alpha})$, investment is below baseline.

We now show that conditional on the standard d, these thresholds are decreasing in ω. Let d be given, and consider a parent with characteristics (α,ω). From Proposition 1, we know that $\underline{\theta}$ and $\bar{\theta}$ are unique (given d). Recall that α^* is the ability level at which $\theta^u = \underline{\theta}$, and $\bar{\alpha}$ is the ability level at which $\theta^u = \bar{\theta}$. From Propositions 2 and 3, we know that θ^u is strictly increasing in α and ω. Hence α^* and $\bar{\alpha}$ are decreasing in ω.

Next we show that $\underline{\alpha}$ is decreasing in ω. Consider two households with wealth ω_1 and ω_2, ability thresholds $\underline{\alpha}_1$ and $\underline{\alpha}_2$, and children with common ability $\alpha = \underline{\alpha}_2$. Suppose, by way of contradiction, that $\underline{\alpha}_1 \geq \underline{\alpha}_2 = \alpha$ and $\omega_1 > \omega_2$. Let m_i^u and θ_i^u denote the optimally chosen baseline investment and preparation levels in household with wealth ω_i, where $i = 1, 2$. By Proposition 2, we know that $m_1^u > m_2^u$ and $\theta_1^u > \theta_2^u$. As $\underline{\alpha}_1 \geq \alpha$, the parent with wealth ω_1 (weakly) prefers baseline investment to minimum passing investment μ (which is common to both households):

$$u(\omega_1 - \mu) + d + b \leq u(\omega_1 - m_1^u) + \rho_1^u, \tag{17}$$

where ρ_1^u is the baseline performance of the household with wealth ω_1. Note that

$$u(\omega_2 - m_1^u) - u(\omega_2 - \mu) > u(\omega_1 - m_1^u) - u(\omega_1 - \mu) \geq d + b - \rho_1^u$$

where the first inequality comes from the strict concavity of u and that $\omega_1 > \omega_2$, and the second inequality from (17). Hence

$$u(\omega_2 - m_1^u) + \rho_1^u > u(\omega_2 - \mu) + d + b.$$

That is, the parent with wealth ω_2 strictly prefers θ_1^u to $\underline{\theta}$. Since this parent is indifferent between θ_2^u and $\underline{\theta}$ at child ability $\alpha = \underline{\alpha}_2$, this implies θ_1^u is strictly preferred to θ_2^u, contradicting the fact that θ_2^u maximizes baseline utility for this type. Hence $\underline{\alpha}$ is decreasing in ω.

Finally, we show that $\hat{\alpha}$ is decreasing ω. Suppose not, and for $\omega_1 > \omega_2$

$$\hat{\alpha}_1 \geq \hat{\alpha}_2 > \bar{\alpha}_2 > \bar{\alpha}_1.$$

Here the thresholds correspond to the wealth levels indicated by the subscripts, as before. Consider two parents with wealth ω_1 and ω_2 respectively and children with common ability $\alpha = \hat{\alpha}_2$. Let m_i^u and θ_i^u denote the baseline investment and preparation levels corresponding to ability α for parents with wealth ω_i, where $i = 1, 2$. Since $\alpha \leq \hat{\alpha}_1$, the parent with wealth ω_1 and child ability α prefers minimum passing investment to baseline investment:

$$u(\omega_1 - \mu) + d + b \geq u(\omega_1 - m_1^u) + \rho_1^u + b. \tag{18}$$

Furthermore, since $\alpha = \hat{\alpha}_2$, we have

$$u(\omega_2 - \mu) + d + b = u(\omega_2 - m_2^u) + \rho_2^u + b. \tag{19}$$

Strict concavity of u and $\omega_1 > \omega_2$ imply

$$u(\omega_2 - \mu) - u(\omega_2 - m_2^u) > u(\omega_1 - \mu) - u(\omega_1 - m_2^u). \tag{20}$$

Using (19) and (20), we obtain

$$\rho_2^u - d > u(\omega_1 - \mu) - u(\omega_1 - m_2^u),$$

which, along with (18), implies

$$u(\omega_1 - m_2^u) + \rho_2^u > u(\omega_1 - \mu) + d \geq u(\omega_1 - m_1^u) + \rho_1^u.$$

That is, a parent with wealth ω_1 obtains higher baseline utility at m_2^u than at m_1^u, contradicting the fact that m_1^u maximizes baseline utility for a parent of this type. □

Proof of Proposition 6. Consider a household with characteristics (α, ω) and let m^u, θ^u, and ρ^u respectively denote baseline investment, preparation, and performance for this household. Define $\tilde{d} = \rho^u$; this is the standard that matches baseline performance. Let $\tilde{\mu}$ denote the minimum passing investment at this standard. From Proposition 1, $\theta^u = \bar{\theta}(\tilde{d})$ and $\underline{\theta}(\tilde{d}) < \bar{\theta}(\tilde{d})$. Hence $m^u < \tilde{\mu}$. Since $\tilde{d} = \rho^u$ by definition, these two investment levels give rise to the same performance and passing outcome. Hence parents will strictly prefer that the standard $\tilde{d}$ is exactly met using minimum passing investment rather than baseline investment:

$$u(\omega - \tilde{\mu}) + \tilde{d} + b > u(\omega - m^u) + \rho^u + b.$$

Since the standard $\tilde{d}$ is exactly met, the set $D(\alpha, \omega)$ is non-empty.

Now consider the set of standards $(\tilde{d} - \epsilon, \tilde{d} + \epsilon)$ for some $\epsilon > 0$. If ϵ is sufficiently small, continuity of the u, f, and s functions ensures that at any $d \in (\tilde{d} - \epsilon, \tilde{d} + \epsilon)$, we have

$$u(\omega - \mu) + d + b > u(\omega - m^u) + \rho^u + b > u(\omega - m^u) + \rho^u.$$

Hence all standards $d \in (\tilde{d} - \epsilon, \tilde{d} + \epsilon)$ will be exactly met. Performance at standards in $(\tilde{d} - \epsilon, \tilde{d})$ will be below baseline and performance at standards in $(\tilde{d}, \tilde{d} + \epsilon)$ will be above baseline. That is, the sets D_l and D_h are both nonempty.

Note that $\underline{d}$ is the infimum of D_l and $\bar{d}$ is the supremum of the set D_h. At $\underline{d}$, we must have

$$u(\omega - \underline{\mu}) + \underline{d} + b = u(\omega - m^u) + \rho^u + b$$

where $\underline{\mu}$ is the minimum passing investment for standard $\underline{d}$. Otherwise, if

$$u(\omega - \underline{\mu}) + \underline{d} + b > u(\omega - m^u) + \rho^u + b,$$

it is possible to find a $\delta > 0$ such that at $d = \underline{d} - \delta$

$$u(\omega - \mu) + d + b > u(\omega - m^u) + \rho^u + b$$

contradicting the fact that $\underline{d}$ is the infimum of D. Analogously, at $\bar{d}$, the parent must be indifferent between minimum passing investment and baseline investment, where the latter results in a failure to meet the standard.

Define d^* as the standard at which baseline investment is equal to minimum passing investment, and note that $\tilde{d} < d^* < \bar{d}$. It is clear that baseline investment is below minimum passing investment for $d < d^*$ and above minimum passing investment for $d > d^*$, as claimed.

Next consider parental utility. We have shown that parental utility at standard $\tilde{d}$ is strictly higher than baseline utility. Hence parental utility is maximized at some standard d in D. This cannot happen at $\bar{d}$, since parents are indifferent between minimum passing investment and baseline investment at this standard. Hence parental utility is maximized at some binding standard below $\bar{d}$.

Finally, we prove that within the set of binding standards, the child's utility is maximized at $\bar{d}$. This follows from the fact that parental investment (and hence the child's preparation) is highest at this standard, and the child's utility is strictly increasing in preparation. $\square$

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