RATIONAL INATTENTION AND REAL HETEROGENEITY

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Abstract

We use the NY Fed's Survey of Consumer Expectations to document a systematic relationship between U.S. households' macroeconomic expectations and their hand-to-mouth status. We rationalize our findings by introducing rational inattention in an environment that resembles a Two-Agent New Keynesian model. Real heterogeneity leads households of different types to choose distinct signals, even when facing identical marginal costs of attention. The model calibrated with microdata from the Survey of Consumer Finances delivers predictions regarding households' expectations that are consistent with those measured in the data. Using vintages of finance survey, we show that variations in the fraction and characteristics of hand-to-mouth households affects how the economy responds to aggregate shocks through endogenous attention allocation. Furthermore, neglecting this channel can lead to erronous conclusions about the effects of economic policies.

Keywords: information choice, rational inattention, business cycles, heterogenous agents, monetary policy, borrowing limit, transfer policies

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1. Introduction

Heterogeneous Agent (HA) models have emerged as a workhorse in macroeconomics over the last decade¹. In these models, heterogeneity typically involves a nondegenerate wealth distribution and/or varying access to credit markets. Such distributions act as additional state variables, which can either dampen or amplify the economy's response to aggregate shocks.

While these models offer valuable insights into how real heterogeneity affects the economy, they typically assume that all agents share the same information and form identical expectations. This contrasts with survey data, which show significant heterogeneity in firms' and households' beliefs (see e.g., Carroll (2003), Weber et al. (2022)).

In this paper, we use microdata from the Survey of Consumer Expectations to document that households' expectations are not only heterogeneous, but that their precision also correlates with economic characteristics. In particular, we provide new evidence of a systematic relationship between households' expectations and their hand-to-mouth status. Hand-to-mouth households are a key feature of HA models, referring to individuals with low liquidity and a high marginal propensity to consume. When we identify these households in survey data, we find that their expectations are systematically less precise than those of non-hand-to-mouth households.

Our empirical findings suggest that introducing informational frictions resulting in uniform information processing across households would cannot address the shortcomings of HA models relative to measured expectations. Real heterogeneity among agents should also translate into differences in how they form expectations.

Heterogenous expectation formation remains largely unexplored in the HA litterature. A notable exception is Mitman et al. (2022), who, in a model \dot{a} la Krusell and Smith (1998), study how heterogeneity affects information choices. In their setup, incentives to acquire information depend on a household's wealth and employment status. They refer to this as the attention channel, which is shown to play a significant role in shaping the economy's response to shocks and policy changes.

In this paper, we contribute to the same overarching research question: how does real heterogeneity across households shape endogenous information acquisition, and what are the implications of households' optimal decisions to acquire information for aggregate dynamics?

To rationalize our empirical results and study their consequences on macroeco-

¹See Violante (2021) for a survey on how HA New Keynesian (HANK) models have reshaped our understanding of the effects of monetary and fiscal policy. We focus on an environment that resembles this class of HA models.

nomic dynamics, we specify a physical environment that resembles a Two-Agent New Keynesian (TANK) model with a continuum of consumption varieties and a Central Bank that sets the nominal interest rate. Within the model, a fixed share of house-holds are hand-to-mouth², and therefore Ricardian equivalence does not hold. When we introduce inertial responses to shocks, arising from real or informational frictions, heterogeneity in household responses affects the dynamics of aggregate variables and inequality measures.

Following Sims (1998, 2003), we assume that decision-makers within the model are rationally inattentive such that information is costly in terms of attention. We implement Rational Inattention (RI) by approximating the per-period payoffs functions with quadratic losses and linearizing the optimal decision rules and other relevant equilibrium conditions around the nonstochastic steady state.³ This approach allows us to frame the attention problems in the Linear Quadratic Gaussian (LQG) form, as studied in Afrouzi and Yang (2021) and Miao, Wu, and Young (2022).

We refer to the combination of the physical environment and attention problems described above as an RI-DSGE⁴. This class of models is challenging to solve because it requires finding a fixed point where each agent's chosen signals are optimal given the signals chosen by all others. Few applications exist in the literature, with the most notable being Maćkowiak and Wiederholt (2015) and Maćkowiak and Wiederholt (2023), but none solve a model with more than two groups of inattentive decision-makers. In this paper, we partition households into two groups, such that our model's equilibrium depends on three different attention problems.

Within our RI-DSGE model, decision-makers face a fundamental tradeoff where receiving more informative signals raises both the expected payoff and incurred attention costs. Real heterogeneity among households affects this tradeoff and leads to different attention allocation decisions through two distinct channels. First, the relative impact of losses due to suboptimal actions depends on a household's steadystate marginal utility of consumption. This creates a level effect, where mistakes by wealthier households have a smaller impact on their per-period utility, giving them incentive to cut their attention effort.. Second, households differ in the number of decisions they make each period, with optimizing households making an additional saving decision when compared to the hand-to-mouth. This introduces another source

²Debortoli and Galí (2024) provides a detailed discussion of how well the TANK framework approximates the dynamics of HANK models, where the share of hand-to-mouth households varies.

³Given that the TANK framework is only valid for small shocks around the non-stochastic steady-state such that effects on the share of hand-to-mouth households remain limited, we believe this does not introduce a significant additional source of approximation.

⁴A Dynamic Stochastic General Equilibrium (DSGE) populated with RI decision-makers.

of heterogeneity in attention allocation, as the value of attention increases with the number of actions. Optimizing households have an incentive to exert more attention effort.

In this setup, one cannot determine a priori which of the first or second channels will dominate, nor which type of household, hand-to-mouth or optimizing, will produce the most accurate conditional forecasts. We calibrate the model's non-stochastic steady state using microdata from the *Survey of Consumers Finances* (SCF), equalize marginal attention costs across all household types, and let them optimally design their information structure. We then evaluate the model's predictions regarding expectations by comparing them with our empirical results obtained using the SCE's measured expectations. One thing worth mentioning at this point is that we are not embedding our model's physical environment with an information acquisition theory specifically designed to match the relationship between households' expectations and income observed in the data. Instead, we are testing whether inattention can generate those same patterns, all while stacking the deck against us by specifying identical marginal costs of information.

As a starting point, we solve the model with inattentive firms and households with perfect information (PI). At the fixed point of this economy, aggregate variable responses resemble those in an environment where firms update prices with a Calvo probability. We then determine the optimal attention strategies for a measure-zero group of inattentive households of both types, examining how hand-to-mouth status influences optimal signal design and expectations across different marginal attention costs. Finally, we solve for the general equilibrium when all households and firms face rational inattention.

Next, we refine the definition of hand-to-mouth households, focusing on those at their credit limit, which introduces an exposure to the nominal interest rate absent in our benchmark calibration. Then, we explore alternative calibrations of the model using different SCF vintages. This exercise allows us to assess how changes in the characteristics of hand-to-mouth and optimizing households over time have influenced the economy's response to aggregate shocks.

Finally, we study the model's dynamics through policy experiments. We consider two different exercises: (i) the effects of a more aggressive monetary policy and (ii) a transfer policy that redistributes dividends from optimizing households to handto-mouth households. In both counterfactual scenarios, the attention channel plays a significant role.

To the best of our knowledge, this paper is the first to bridge the HA and RI literatures in this manner. Song and Stern (2020) solve an RI-DSGE in which a fraction of firms are more attentive than others. However, in their model, attention

allocation differs solely due to heterogeneous marginal information costs. In contrast, households in our framework choose different attention strategies even when faced with identical marginal information costs.

In Mitman et al. (2022), the environment is fully non-linear, and decision-makers must forecast not only the entire wealth distribution but also the distribution of households' higher-order beliefs. To simplify optimal information design, the authors restrict agents to a finite set of signals about the state of the economy, each with a monetary cost. In comparison, decision-makers in our model design optimal signals themselves and incur a utility cost⁵. However, we work with a linearized environment such that optimal decisions can be derived from Kalman filtering. It's not clear which set of assumptions is superior, so we view our approach as complementary to theirs. Moreover, their work is set in an RBC environment, while ours has New Keynesian flavors, which further distinguishes our contributions.

The remainder of the paper is structured as follows. Section 2 presents empirical evidence on the relationship between wealth heterogeneity and the quality of household forecasts. Section 3 outlines the model's physical environment. Section 4 states the attention problems faced by decision-makers within the model. Section 5 characterizes the equilibrium. Section 6 first determines the attention strategies of each household type under partial equilibrium, then computes the fixed point of the entire RI economy and presents the resulting dynamics. Section 8 and Section 9 study extensions and policy experiments of the model. Section 10 concludes.

2. Empirical Evidence

In the spirit of Mitman et al. (2022), we examine the empirical relationships between the accuracy of households' expectations and their economic characteristics. We use microdata from the SCE⁶, a large panel survey held by the New-York Fed that collects monthly expectations from heterogeneous households. We merge the SCE with its supplemental survey on households' spending, which includes data on consumption behavior. Our sample for survey data spans the period from 2013M8 to 2024M1, which are compared to their actual outcomes 12 months later.

An important factor influencing the dynamics of HA models (and their tractable counterparts) is the presence of hand-to-mouth households⁷, with others smoothing

⁵This is arguably a more realistic approach, as it does not artificially bias information acquisition toward richer households.

⁶The SCE releases are available at https://www.newyorkfed.org/microeconomics/sce.

⁷In the former, this arises endogenously, while in the latter, households are assigned a type ex-ante.

consumption through savings. We are interested in whether these heterogeneous household profiles also lead to differences in how they form expectations.

Identifying hand-to-mouth households typically requires detailed microdata on finances, which the SCE does not collect⁸. However, in the SCE supplemental survey on spending, the same households are asked the following multiple-choice question,

- Q: Now imagine that next year you were to find yourself with 10% less household income. What would you do?
- 1. Cut spending by the whole amount
- 2. Not cut spending at all, but cut my savings by the whole amount
- 3. Not cut spending at all, but increase my debt by borrowing the whole amount
- 4. Cut spending by some and cut savings by some
- 5. Cut spending by some and increase debt by some.

It turns out that whenever a household answers with option 1, it reveals its hand-to-mouth status. The reasoning is straightforward. After a decrease in income, the household does not smooth consumption⁹ This could be due to various factors, such as already reaching the credit limit, endogenous decisions driven by borrowing costs, or a lack of access to the credit market. For this exercise, we do not need to distinguish between these causes.

When estimating whether being hand-to-mouth affects forecast accuracy in the data, we focus on forecasts for variables that are present in our theoretical model, namely inflation and the nominal interest rate.

The SCE collects households' point forecasts for inflation 12 months ahead, as well as a subjective probability when asked if the interest rate on their savings account will be higher in 12 months from now. We measure accuracy using the absolute value of forecast errors. For inflation, this is straightforward, as we have access to both point forecasts and realized outcomes. For the interest rate, the challenge lies in the

⁸In particular, the SCE supplemental survey on household finances lacks data on credit limits.

⁹An equivalent question is asked for the hypothetical case of an income increase, but it is not relevant here, as it could change the status of all hand-to-mouth households. In contrast, a decrease in income does not affect hand-to-mouth households at the credit limit; it could only lead hand-to-mouth households with no debt and access to credit to start borrowing (options 3 and 5). This decision is an arbitrage that depends on borrowing costs, and whether a 10% income loss would induce that transition is debatable. However, including options 3 and 5 would risk significantly polluting our estimates of hand-to-mouth status.

fact that the true probability is unobservable¹⁰. We make the assumption that if a respondent were asked, "Will the interest rate be higher in 12 months from now?" and reported a subjective probability above 50%, they would answer "yes" to that hypothetical question. This allows us to assess accuracy similarly to inflation, as we can compute the difference between interest rates 12 months apart.

We measure inflation as the year-over-year growth rate of the Consumer Price Index, expressed in percentage points. The interest rate variable is set to one if the Federal Funds Effective Rate has increased over the same month in consecutive years, and zero otherwise¹¹.

The SCE collects information on respondent characteristics that may influence forecast quality beyond their hand-to-mouth status, such as numeracy and education. We control for these factors in the analysis below.

Table 1 reports the results of regressions of the absolute value of forecast errors for inflation and the interest rate on a binary variable indicating whether a household is hand-to-mouth, both with and without controls. The pattern that emerge is clear. Even after controlling for education and numeracy, a household's status as hand-tomouth significantly affects the accuracy of its forecasts, resulting in larger forecast errors than optimizing households. These results cannot be explained by factors related to information-processing ability.

In Appendix A, we show that we can obtain relationships that interpret similarly when partitioning households according to their income or their reported probability of defaulting on debt payments in the next 3 months. It is reasonable to believe that these characteristics are correlated with hand-to-mouth status¹². However, we also show that when we run the same regression as in Table 1, including all these variables, hand-to-mouth status remains a significant predictor of larger forecast errors.

In the remainder of the paper, we use a theoretical model with heterogeneous agents, endogenous information acquisition, and New Keynesian elements to (i) examine whether the interaction between a household's status as hand-to-mouth and inattention can explain our empirical results, and (ii) explore the implications of this attention channel for both positive and normative questions.

¹⁰Mitman et al. (2022) approximate it using an average from the Survey of Professional Forecasters (SPF), but this approach reduces the sample size since the SPF is quarterly.

¹¹Both time series are monthly and sourced from https://fred.stlouisfed.org.

 $^{^{12}}$ We document that hand-to-mouth households have lower wage income in Appendix F, and a high probability of defaulting is essentially an indirect way to identify hand-to-mouth households for which the credit limit is binding.

	Infl	ation	Nominal Rate		
	(1)	(2)	(3)	(4)	
Optimizing	-	-	-	-	
Hand-to-mouth	1.541^{***}	0.835^{***}	0.038^{***}	0.028^{***}	
	(0.264)	(0.265)	(0.003)	(0.003)	
High School	-	_	-	_	
Some College	-	-3.717^{***}	-	-0.014^{***}	
-	-	(0.465)	-	(0.004)	
College	-	-6.353^{***}	-	-0.076^{***}	
	-	(0.450)	-	(0.004)	
Low Numeracy	-	-	-	-	
High Numeracy	-	-4.781^{***}	-	-0.053^{***}	
	-	(0.305)	-	(0.003)	
Observations	112,937	112,937	112,937	112,937	
F Statistic	34.096	170.837	227.397	390.29	
R^2	0.003	0.008	0.279	0.288	
Controls	no	yes	no	yes	
Time Fixed Effects	yes	yes	yes	yes	

Table 1: Expectations Accuracy and Hand-to-mouth Status

Notes: Column (1) shows estimates from a regression of the absolute value of inflation errors on the household hand-to-mouth status. Estimates are relative to optimizing households, those that do not qualify as hand-to-mouth in the SCE spending survey. Column (2) adds controls to the regression specification: education level and numeracy of the respondent, as well as time fixed effects. Columns (3) and (4) perform the same analysis for interest rate forecast errors. Robust standard errors in parentheses, * p < 0.1, ** p < 0.05, *** p < 0.01. Sample: 2013M8-2024M1.

3. Model: Physical Environment

In this section, we describe all the features of the economy, excluding the attention problems. The environment resembles a standard, discrete-time, TANK model (see e.g., Bilbiie (2008, 2020) and Debortoli and Galí (2024)). It features a continuum of firms producing differentiated varieties of goods, a double continuum of households, with one group being hand-to-mouth and the other optimizing, a competitive labor market, and a central bank setting the nominal interest rate. Time is discrete, with periods corresponding to quarters

3.1. Hand-to-mouth Households. There is a continuum $j^h \in [0, 1]$ of hand-tomouth households representing a constant fraction ϕ of total households. Household j^h seeks to maximize its expected discounted sum of period utility. The discount factor is $\beta \in (0, 1)$ and the period utility function is

$$U(C_t(j^h), \bar{L}_t(j^h)) = \frac{C_t(j^h)^{1-\gamma} - 1}{1-\gamma} - \varphi \frac{\bar{L}_t(j^h)^{1+\psi}}{1+\psi}$$
(1)

where

$$C_t(j^h) = \left(\int_0^1 C_t(i,j^h)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}.$$
(2)

Here, $C_t(j^h)$ is a composite consumption index, $C_t(i, j^h)$ is the consumed quantity of variety *i* and $\bar{L}_t(j^h)$ denotes the labor supplied. The parameter γ is the inverse of the intertemporal elasticity of substitution, φ scales labor disutility, ψ is the inverse of the Frisch elasticity and $\theta > 1$ is the preference parameter for the elasticity of substitution between varieties.

The flow budget constraints in period t reads

$$\int_{0}^{1} P_{t}(i)C_{t}(i,j^{h})di = Z_{t}(j^{h})\bar{L}_{t}(j^{h})W_{t} - P_{t}(R_{t-1}-1)\tilde{B}^{h} + D_{t}^{h} - T_{t}^{h}$$
(3)

where $P_t(i)$ is the price of variety *i*, W_t is the nominal wage rate, T_t is a nominal lump-sum tax, $Z_t(j^h)$ is an idiosyncratic exogenous process affecting household j^h 's effective labor supply, \tilde{B}^h is an exogenously specified constant amount of debt denominated in real bonds¹³ and D_t^h are dividends accrued from firms' ownership or through transfers¹⁴.

¹³This introduces exposure to the nominal interest rate, similar to the one faced by households at the borrowing limit in a HANK model.

 $^{^{14}...}$

Effective labor is defined as the product between household j^h 's labor supply and the idiosyncratic process $Z_t(j^h)$ such that

$$L_t(j^h) := Z_t(j^h)\bar{L}_t(j^h).$$

$$\tag{4}$$

For simplicity, we refer to $Z_t(j^h)$ as the idiosyncratic income shock.

In each period, household j^h chooses its consumption vector $\{C_t(i, j^h)\}_{i \in [0,1]}$ and its labor supply $L_t(j^h)$ taking as given exogenous shocks, the vector of prices for consumption varieties, the wages rate, the nominal interest rate and all aggregate quantities.

3.2. Optimizing Households. There is a continuum $j^o \in [0, 1]$ of optimizing households representing a constant fraction $1 - \phi$ of total households. Household j^o owns nominal bonds paying a rate set by the Central Bank that can be used to smooth consumption across periods and seek to maximize the expected discounted sum of its period utility. The discount factor and the functional form of period utility are the same as those of hand-to-mouth households described above.

Household j^o flow budget constraint reads

$$\int_0^1 P_t(i)C_t(i,j^o)di + B_t(j^o) = R_{t-1}B_{t-1}(j^o) + Z_t(j^o)\bar{L}_t(j^o)W_t + D_t^o - T_t^o.$$
 (5)

Here, $B_t(j^o)$ denotes nominal bond holdings and R_t is the gross nominal interest rate paid on period t-1 nominal bond holdings. The remaining variables are analogues of variables defined in Section 3.1.

We make the assumption that $B_t(j^o) > 0$ always holds for all optimizing households. This will allow us to write down Equation (5) in terms of logged variables¹⁵ and also effectively rules out Ponzi schemes.

In each period, household j^o chooses its consumption vector $\{C_t(i, j^o)\}_{i \in [0,1]}$, its nominal bonds holdings $B_t(j^o)$ and its labor supply $L_t(j^o)$ taking as given exogenous shocks, the vector of prices for consumption varieties, the wages rate, the nominal interest rate and all aggregate quantities.

3.3. Firms. There is a continuum $i \in [0, 1]$ of firms. Firm *i* produces a differentiated variety of the consumption good using the production function

$$Y_t(i) = e^{a_t} e^{a_t(i)} L_t(i)^{\alpha}.$$
 (6)

¹⁵In turn, it becomes straightforward to approximate an household utility flow in terms of logdeviations from the non-stochastic steady-state.

Here, $L_t(i)$ is the quantity of labor used by firm *i* for production, while a_t represents aggregate technology and $a_t(i)$ captures firm-specific technology. The parameter $\alpha \in (0, 1]$ denotes the elasticity of output with respect to labor.

Firm i seeks to maximize the discounted¹⁶ sum of its period nominal profits (or dividends) given by

$$D_t(i) = (1 + \tau_P)P_t(i)Y_t(i) - W_t L_t(i)$$
(7)

where τ_P is a production subsidy.

In each period, firm *i* sets a price $P_t(i)$ for its variety and demands quantity $L_t(i)$ of effective labor taking as given exogenous shocks, the vector of prices set by other firms, aggregate demand, the wage rate, the nominal interest rate and all aggregate quantities. Each firm commits to supplying any quantity of its consumption variety demanded, at the price it sets.

3.4. Government. The government consist of a monetary and a fiscal authority. The Central Bank sets the nominal rate according to a Taylor rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\phi_{y^*}} \right]^{1-\rho_R} e^{v_t}$$
(8)

where $\Pi_t := (P_t/P_{t-1})$ denotes the inflation rate, P_t is a price index, (Y_t/Y_t^*) is the output gap defined as the ratio between actual output and its value that would prevail under PI and v_t is a monetary policy shock. Variables without index refer to steady-state values. The parameters ρ_R , ϕ_{π} and ϕ_{y^*} control the degree of inertia and the strength of the response of the monetary policy.

The government budget constraint in period t is

$$T_t + B_t = R_{t-1}B_{t-1} + \tau_P \int_0^1 P_t(i)Y_t(i)di.$$
(9)

To finance interest on nominal bonds, the government can either collect lumpsum taxes or issue new bonds. Following common practice, monetary policy is active and fiscal policy is passive in the sense of Leeper (1991).

The production subsidy is set to correct distortions arising from market power in the non-stochastic steady-state such that

 $^{^{16}}$ Formally, firms use a stochastic discount factor in terms of households' consumption flows to value profits across periods. The definition of the stochastic discount factor's functional form can be found in Appendix E.1.

$$\tau_P = \frac{\tilde{\theta}}{\tilde{\theta} - 1} - 1 \tag{10}$$

where $\theta > 1$ is the model implied elasticity of substitution between varieties of goods, which is strictly smaller than the preference parameter θ whenever households are inattentive.

We allow the government to design transfer and taxation schemes with two additional parameters, ϑ_T and ϑ_D , which can be thought as fixed policy instruments.

The government's total revenue from taxation in a given period read

$$T_t = (1 - \phi)T_t^o + \phi T_t^h.$$
 (11)

The government sets the share of its total tax revenue charged to each type of household through the parameter ϑ_T such that

$$(1-\phi)T_t^o = (1-\vartheta_T)T_t, \quad \phi T_t^h = \vartheta_T T_t.$$
(12)

Similarly, total revenue from dividends is given by

$$D_t = (1 - \phi)D_t^o + \phi D_t^h \tag{13}$$

and assuming the government can enforce redistribution through the parameter ϑ_D , we obtain

$$(1-\phi)D_t^o = (1-\vartheta_D)D_t, \quad \phi D_t^h = \vartheta_D D_t.$$
(14)

Lastly, we need to close the model on the fiscal side to guarantee a unique equilibrium. This can be achieved either by assuming the government commits to a specific path for real bond holdings with $\lim_{T\to\infty} \log(\tilde{B}_T/B) = 0$ or by specifying a rule that relates real taxes to last period's real bond holdings and current period subsidy payments.

3.5. Shocks. In this economy there are four exogenous processes: aggregate monetary policy shocks , $\{v_t\}$, aggregate technology, $\{a_t\}$, firm-specific technology, $\{a_t(i)\}_{i \in [0,1]}$, and two households' specific labor income shocks, $\{z_t(j^h)\}_{j^h \in [0,1]}$ and $\{z_t(j^o)\}_{j^o \in [0,1]}$. I assume that monetary policy are *i.i.d.* Gaussian innovations and that all other shocks follow independent stationnary Gaussian first-order autoregressive processes. Innovations affecting the economy at time *t* can be collected in the vector

$$\boldsymbol{\varepsilon}_{t} = \left(\varepsilon_{t}^{v}, \varepsilon_{t}^{a}, \{\varepsilon_{t}^{a}(i)\}_{i \in [0,1]}, \{\varepsilon_{t}^{z}(j^{h})\}_{j^{h} \in [0,1]}, \{\varepsilon_{t}^{z}(j^{o})\}_{j^{o} \in [0,1]}\right)'$$
(15)

The cross-sectional mean of firm-specific stochastic technology processes is zero.

Household-specific stochastic processes for income shocks are subject to the following normalizations

$$(1-\phi)\int_0^1 Z_t(j^o)dj^o + \phi\int_0^1 Z_t(j^h)dj^h = 1$$
(16)

which allow for unconditional means of idiosyncratic income shocks to differ across household types.

The cross-sectional averages of household-specific shocks are equal to their unconditional means.

3.6. Aggregation. Aggregate composite consumption and labor supply are defined as weighted integrals over the continuum of each household type

$$C_t = (1 - \phi) \int_0^1 C_t(j^o) dj^o + \phi \int_0^1 C_t(j^h) dj^h$$
(17)

and

$$L_t^s = (1 - \phi) \int_0^1 L_t(j^o) dj^o + \phi \int_0^1 L_t(j^h) dj^h$$
(18)

Similarly, agregate demand for consumption variety i is given by

$$C_t(i) = (1 - \phi) \int_0^1 C_t(i, j^o) dj^o + \phi \int_0^1 C_t(i, j^h) dj^h$$
(19)

Aggregate output, labor demand and dividends are obtained by integrating over the continuum of firms

$$Y_t = \int_0^1 Y_t(i)di, \quad L_t^d = \int_0^1 L_t^d(i)di \quad D_t = \int_0^1 D_t(i)di.$$
(20)

Bonds and taxes are aggregated in proportion to each household type's share

$$B_t = (1 - \phi) \int B_t(j^o) dj^o + \phi B^h, \ T_t = (1 - \phi) \int T_t^o dj^o + \phi \int T_t^h dj^h$$
(21)

Lastly, we assume that the price index can always be written as

$$1 = \int_0^1 d_P\left(\hat{P}_t(i)\right) di \tag{22}$$

where d_P is some twice continuously differentiable function. Notice that this functional nests the index that would prevail under perfect information ¹⁷ and yields an identical expression once log-linearized.

3.7. Notation. The relative price of consumption variety i and the relative consumption of variety i by household j are denoted

$$\hat{P}_t(i) = \frac{P_t(i)}{P_t}, \ \hat{C}_t(i,j) = \frac{C_t(i,j)}{C_t(j)}.$$
(23)

The real wage rate is given by

$$\tilde{W}_t = \frac{W_t}{P_t},\tag{24}$$

and aggregate real fiscal variables and dividends are given by

$$\tilde{B}_t = \frac{B_t}{P_t}, \quad \tilde{T}_t = \frac{T_t}{P_t} \quad \tilde{D}_t = \frac{D_t}{P_t}.$$
(25)

Household specific real bonds holdings, real taxes and real dividends for each type of households are defined analogously.

3.8. Non-Stochastic Steady-State. The non-stochastic steady state is defined as an equilibrium of the economy in the absence of shocks, with the property that real quantities, relative prices, the nominal rate, and inflation remain constant over time. In the following, variables without time-subscript denotes steady-state values.

In the non-stochastic steady state, the first-order condition and the period budget constraint for hand-to-mouth household j^h are

$$Z^{h}\tilde{W} = \varphi(\bar{L}^{h})^{\psi}(C^{h})^{\gamma}, \qquad (26)$$

$$C^{h} = Z^{h} \overline{L}^{h} \widetilde{W} - (R-1) \widetilde{B}^{h} + \widetilde{D}^{h} - \widetilde{T}^{h}, \qquad (27)$$

the first-order conditions for optimizing household j^o are

$$Z^{o}\tilde{W} = \varphi(\bar{L}^{o})^{\psi}(C^{o})^{\gamma}, \qquad (28)$$

$$\frac{R}{\Pi} = \frac{1}{\beta},\tag{29}$$

the optimality condition for relative consumption for both household types is

¹⁷For example, when households have PI,
$$1 = \int_0^1 \hat{P}_t(i)^{1-\theta} di$$
, $d_P := (\cdot)^{1-\theta}$ and $\tilde{\theta} = \theta$.

$$\hat{C}(i,j) = \hat{P}(i)^{-\theta}, \qquad (30)$$

and firm i's first order condition is

$$\hat{P}(i) = \frac{\hat{W}}{\alpha} (\hat{P}(i)^{-\theta} C)^{\frac{1-\alpha}{\alpha}}.$$
(31)

Equation (31) implies that all firms set the same price. Equation (30) then implies each households consume the same relative quantity of each consumption variety. Thus, all firms produce the same output, and since all firms have the same productivity, all firms have the same labor input. Aggregate effective labor determines aggregate output, which implies the market clearing real wage and aggregate dividends. Equation (26) and Equation (28) imply values for consumption.

The optimizing households' Euler equation, Equation (29), determines the real interest rate but not R and Π individually. Thus, we will assume a constant price level such that $\Pi = 1$ and $R = \beta^{-1}$ and posit an initial value, P_{-1} , for the price level.

Given initial values for nominal bonds held by optimizing households, $B_{-1}^o = B_{-1}(j^o) \quad \forall j^o \in [0,1]$ and debt B^h owed by the hand-to-mouth households, fiscal variables are uniquely determined in the non-stochastic steady state. The reason is that real bond holdings, B_{-1}^o/P_{-1} , are a quantity that must remain constant, and this can only be the case if the government runs a balanced budget in real terms. Thus, real lump-sum taxes must equate the sum of real interest and subsidy payments.

Given values for the unconditional mean ratio of income shocks, Z^o/Z^h , the nonstochastic steady-state labor¹⁸, \bar{L} , and real debt-to-output ratios, \tilde{B}^h/Y and \tilde{B}^o/Y , the remaining non-stochastic steady-state variables can be computed.

3.9. Perfect Information Equilibrium. Given the physical environment described above, we can define and characterize the equilibrium that arises when all decision-makers have full knowledge of the economy's history up to the current period. Both are detailed in Appendix B.

4. Model: Attention Problems

Decision-makers subject to RI face a fundamental tradeoff where processing more information requires more attention, which is costly, leading them to focus on some pieces of information and disregard others. This section first describes the maximization problem faced by decision-makers in the RI economy.

¹⁸If not specified otherwise, we will always assume that \bar{L} , is identical across household types.

The attention problems have a standard LQG-RI form, characterized by a quadratic objective, linear constraints, and Gaussian innovations. The steps for transforming the objectives from Section 3 into quadratic functions are detailed below.

A decision-maker in the RI economy maximizes by choosing a costly attention strategy consisting of signals about the state of the economy. Formally, his problem reads

$$\max_{\boldsymbol{\Gamma},\boldsymbol{\Sigma}_{\boldsymbol{\nu}}} \left\{ \sum_{t=0}^{\infty} \beta^{t} E_{-1} \left[\frac{1}{2} (\boldsymbol{x}_{t} - \boldsymbol{x}_{t}^{*})' \boldsymbol{H}_{\boldsymbol{x}} (\boldsymbol{x}_{t} - \boldsymbol{x}_{t}^{*}) \right] - \lambda \sum_{t=0}^{\infty} \beta^{t} I(\boldsymbol{\xi}_{t}; \boldsymbol{S}_{t} | \mathcal{I}_{t-1}) \right\}$$
(32)

subject to

$$\boldsymbol{x}_t^* = \boldsymbol{G}\boldsymbol{\xi}_t \tag{33}$$

$$\boldsymbol{\xi}_{t+1} = \boldsymbol{F}\boldsymbol{\xi}_t + \boldsymbol{\varepsilon}_{t+1} \tag{34}$$

$$\mathcal{I}_t(i) = \mathcal{I}_{-1} \cup \{ \boldsymbol{S}_0, ..., \boldsymbol{S}_t \}$$
(35)

$$\boldsymbol{S}_t = \boldsymbol{\Gamma} \boldsymbol{\xi}_t + \boldsymbol{\nu}_t \tag{36}$$

$$I(\boldsymbol{\xi}_t; \boldsymbol{S}_t | \mathcal{I}_{t-1}) = H(\boldsymbol{\xi}_t | \mathcal{I}_{t-1}) - H(\boldsymbol{\xi}_t | \mathcal{I}_t).$$
(37)

$$\boldsymbol{x}_t = E[\boldsymbol{x}_t^* | \mathcal{I}_t] \tag{38}$$

with

$$\mathcal{I}_{-1} \mid \boldsymbol{\Gamma}, \boldsymbol{\Sigma}_{\boldsymbol{\nu}} \tag{39}$$

The vector \boldsymbol{x}_t contains the decision-maker's actions and the vector \boldsymbol{x}_t^* contains the actions they would take under PI. The first term appearing in Equation (32) is the per-period payoff (i.e. losses incurred from suboptimal actions) and has a quadratic form with weighting matrix $\boldsymbol{H}_{\boldsymbol{x}}$.

The second term in Equation (32) is a known quantity representing the discounted sum of information costs. The per-period information cost consists of the product between the marginal cost of attention, λ , and the per-period information flow.

Equation (33) defines a linear mapping between the current state, $\boldsymbol{\xi}_t$, and the vector of optimal actions. Given that structural shocks are Gaussian by assumption,

we know that there exists at least one representation for the state vector for which this equality holds exactly.

Equation (34) describes the evolution of the state vector between two consecutive periods where ε_{t+1} follows a white noise vector process with covariance Σ_{ε} and F is matrix with eigenvalues that may lie outside the unit circle. Thus, the state-space¹⁹ for $\boldsymbol{\xi}_t$ and \boldsymbol{x}_t^* , described by eqs. (38) to (34), is linear with Gaussian innovations, but stationnarity is not imposed²⁰.

The decision-maker's information set in the current period is decscribed by Equation (35). It consists of the initial information, \mathcal{I}_{-1} , and all signals received up to and including the current period.

Equation (36) describes the signal received in period t. This equation posits that the signal loads on the period t state vector according to the matrix Γ plus ν_t , a white noise vector process with diagonal covariance matrix Σ_{ν}^{21} .

Equation (37) measures the per-period information flow as the difference in entropy about the state before and after observing the signal in period t. This essentially quantifies the per-period uncertainty reduction.

Equation (38) describes how the decision-maker optimally selects x_t according to his information set. Given the linear Gaussian structure, the decision-maker applies the Kalman filter to optimally infer x_t^* from any sequence of noisy signals.

Equation (39) states that the initial information set is not entirely exogenous, but at least some of its characteristics depend on the attention strategy chosen by the decision-maker. The exact relationship is described below.

The decision-maker optimizes freely²² over the matrices Γ and Σ_{ν} to maximize the difference between the expected discounted sum of per-period payoffs and information costs. The fundamental tradeoff is that receiving more informative signals raises both the expected payoff and information costs. All decisions regarding the attention strategy that maximize Equation (32) are made in period -1, with eqs. (38) to (39) taken as given.

I make the standard assumption that the initial information set, \mathcal{I}_{-1} , depends on the chosen matrices Γ and Σ_{ν} . Specifically, given its chosen attention strategy the decision-maker receives a long sequence of signals that places him at the steady-state

¹⁹The relevant state vector in the presence of information frictions may be infinite-dimensional, solving this problem numerically requires some level of approximation.

²⁰For the maximization problem to be well-defined, all we need is the conditional second moments to be finite which does not require stationnarity.

²¹It can be shown that a signal loading on the current state plus a vector of *i.i.d.* Gaussian noise, as described in Equation (36), is of the optimal form given the decision-maker problem defined by Equations (32) to (37). For a formal proof, see Maćkowiak, Matějka, and Wiederholt (2018).

 $^{^{22}}$ The rank of these matrices determining the total number of signals is endogenous.

of the Kalman filter in period -1. This assumption ensures that the problems of choosing $\{\Gamma_t, \Sigma_{\nu,t}\}_{t=0}^{\infty}$ sequentially or Γ and Σ_{ν} once and for all in period -1 are equivalent.

Lastly, all noise in signals is assumed to be idiosyncratic, meaning that realizations of ν_t are independent across firms and households and sum to zero in the cross-section. We use this property when aggregating individual decisions.

4.1. Firms. Firm *i*'s expected discounted sum of period profits is approximated with a second-order log Taylor expansion around the non-stochastic steady-state. The derivation can be found in Appendix E.1.

The resulting matrix featured in firm i's objective function is

$$\boldsymbol{H}_{\boldsymbol{x}}^{f} = -(C^{o})^{-\gamma} \tilde{W} L\left[\frac{\tilde{\theta}(\tilde{\theta} + \alpha(1 - \tilde{\theta}))}{\alpha^{2}}\right].$$
(40)

Firm i's vector of choice variables is

$$\boldsymbol{x}_t(i) = \left(p_t(i)\right)' \tag{41}$$

and its vector of optimal decisions is

$$\boldsymbol{x}_{t}^{*}(i) = \left(p_{t} + \frac{\frac{1-\alpha}{\alpha}}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}}c_{t} + \frac{1}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}}\tilde{w}_{t} - \frac{\frac{1}{\alpha}}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}}(a_{t} + a_{t}(i))\right)'.$$
(42)

4.2. Hand-to-mouth Households. Hand-to-mouth household j^{h} 's expected discounted sum of period utilities is approximated using a second-order log Taylor expansion. The derivation can be found in Appendix E.2.

The matrix in the expression quantifying household j^h 's losses due to suboptimal actions is given by

$$\boldsymbol{H}_{\boldsymbol{x}}^{h} = -(C^{h})^{1-\gamma} \begin{bmatrix} (\gamma + \psi) & 0 & \cdots & 0\\ \vdots & \frac{2}{\theta} di & \cdots & \frac{1}{\theta} di\\ \vdots & \vdots & \ddots & \vdots\\ \vdots & \frac{1}{\theta} di & \cdots & \frac{2}{\theta} di \end{bmatrix}.$$
(43)

Here, the continuum of consumption varieties types is treated as a finite sum and di is a weight.

Household j^h 's vector of choice variables is

$$\boldsymbol{x}_t(j^h) = \left(\bar{l}_t(j^h), \quad \hat{c}_t(i, j^h), \quad \cdots \right)', \tag{44}$$

its vector of optimal decisions is

$$\boldsymbol{x}_{t}^{*}(j^{h}) = \begin{pmatrix} \frac{z_{t}(j^{h}) + \tilde{w}_{t} - \gamma c_{t}^{*}(j^{h})}{\psi} \\ -\theta(p_{t}(i) - p_{t}) \\ \vdots \end{pmatrix},$$
(45)

with

$$c_t^*(j^h) = \omega_W^h(z_t(j^h) + \tilde{w}_t + \bar{l}_t^*(j^h)) - \omega_B^h r_{t-1} + \omega_D^h \tilde{d}_t^h - \omega_T^h \tilde{t}_t^h.$$
(46)

4.3. Optimizing Households. Household j^{o} 's expected discounted sum of period utilities is approximated using a second-order log Taylor expansion, with an additional step required to obtain an expression that reduces to a pure tracking problem²³. The derivation can be found in Appendix E.3 and Appendix E.4.

The matrix in the expression quantifying household j^{o} 's losses due to suboptimal actions is given by

$$\tilde{\boldsymbol{H}}^{o}{}_{\tilde{\boldsymbol{x}}} = -(C^{o})^{1-\gamma} \begin{bmatrix} \gamma & 0 & 0 & \cdots & 0\\ 0 & \omega_{W}\psi & 0 & \cdots & 0\\ 0 & 0 & \frac{2}{\theta}di & \cdots & \frac{1}{\theta}di\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \frac{1}{\theta}di & \cdots & \frac{2}{\theta}di \end{bmatrix}.$$
(47)

Household j^{o} 's vector of choice variables is

$$\tilde{\boldsymbol{x}}_t(j^o) = \begin{pmatrix} c_t(j^o), & \bar{l}_t(j^o), & \{\hat{c}_t(i,j^o)\} & \cdots \end{pmatrix}'$$
(48)

and its vector of optimal decisions is

$$\tilde{\boldsymbol{x}}_{t}^{*} = \begin{pmatrix} E_{t}(j^{o}) \left[-\frac{1}{\gamma} (r_{t} - \pi_{t+1}) + c_{t+1}^{*}(j^{o}) \right] \\ \frac{z_{t}(j^{o}) + \tilde{w}_{t} - \gamma c_{t}^{*}(j^{o})}{\psi} \end{pmatrix}$$
(49)

with $c_{t+1}^*(j^o)$ defined accordingly to Equation (108).

Notice that while consumption and labor supply decisions are usually considered interdependent, the change of variables performed in Appendix E.4 effectively decouples them in the attention problem. The intuition is that for any given level of consumption, a household can always adjust its labor supply to be on its intratemporal optimality condition, regardless of consumption mistakes.

²³In a pure tracking problem, the objective is independent of the relationship between today and tomorrow's mistakes. In other words, the state is purely exogenous, and all that matters for the decision-maker is to keep $\mathbf{x}_t(j)$ as close as possible to $\mathbf{x}_t^*(j)$.

5. Model: Equilibrium

The equilibrium of the inattentive economy, consisting of the physical environment from Section 3 and the attention problems from Section 4, is a fixed-point where each decision-maker's signals are optimal, given the signals selected by all others.

In this section, we first define an approximation of the equilibrium in a neighborhood of the non-stochastic steady-state, where variables are expressed in terms of log-deviations from this point ²⁴. We then outline a numerical procedure to solve for the economy's fixed-point.

Definition 1. For any sequence of realizations for the exogenous innovations, $\{\varepsilon_t\}_{t=0}^{\infty}$, a Sequential Rational Inattention Competitive Equilibrium (SRICE) is:

- An allocation, $\Omega(i) := \{ \mathbf{S}_t(i), y_t(i), l_t^d(i), \{ p_t(i) \}_{i \in [0,1]} \}_{t=0}^{\infty}$ for every firm $i \in [0,1]$.
- An allocation, $\Omega(j^h) := \{ \mathbf{S}_t(j^h), c_t(j^h), l_t^s(j^h), \{\hat{c}_t(i, j^h)\}_{i \in [0,1]} \}_{t=0}^{\infty}$ for every hand-to-mouth household $j^h \in [0, 1]$.
- An allocation, $\Omega(j^o) := \{ \mathbf{S}(j^o), c_t(j^o), l_t(j^o), b_t^d(j^o), \{ \hat{c}_t(i, j^o) \}_{i \in [0,1]} \}_{t=0}^{\infty}$ for every optimizing household $j^o \in [0,1]$.
- An allocation, $\Omega_G := \{b_t^s, t_t\}_{t=0}^{\infty}$ for the government.
- A set of prices $\{p_t, w_t, r_t\}_{t=0}^{\infty}$.
- Stationnary distributions of households and firms over idiosyncratic innovations and noise.

such that

- 1. Given $\{\Omega(j^h)_{j^h \in [0,1]}, \Omega(j^o)_{j^o \in [0,1]}, p_t, w_t\}_{t=0}^{\infty}$, a firm's allocation, $\Omega(i)$, solves Section 4.1's attention problem $\forall i \in [0,1]$.
- 2. Given $\{\Omega(i)_{i\in[0,1]}, \Omega(j^o)_{j^o\in[0,1]}, p_t, r_t, w_t, l_t\}_{t=0}^{\infty}$, a hand-to-mouth household allocation, $\Omega(j^o)$, solves Section 4.2's attention problem $\forall j^h \in [0,1]$.
- 3. Given $\{\Omega(i)_{i \in [0,1]}, \Omega(j^h)_{j^h \in [0,1]}, p_t, r_t, w_t, l_t\}_{t=0}^{\infty}$, an optimizing household allocation, $\Omega(j^h)$, solves Section 4.3's attention problem $\forall j^o \in [0,1]$.

²⁴For example $c_t(j) := \ln(C_t(j)/C)$.

- 4. Monetary policy satisfies the specified Taylor rule, fiscal and transfer policies satisfy their respective rules and given interest and subsidies payments, the government runs a balanced budget in real terms.
- 5. All markets clear $\forall t \geq 0$:
 - (a) $y_t = c_t$
 - (b) $l_t^s = l_t^d$
 - (c) $b_t^s = b_t^d$
 - (d) $y_t(i) = c_t(i) \ \forall i \in [0, 1]$
- 6. Aggregate price index is given by $\forall t \geq 0$:

(a)
$$p_t = \int_0^1 p_t(i) di$$

- 7. Aggregate quantities are given by $\forall t \geq 0$:
 - $\begin{aligned} (a) \ y_t &= \int_0^1 y_t(i) di \\ (b) \ c_t &= (1 \phi) (C^o/C) \int_0^1 c_t(j^o) dj^o + \phi (C^h/C) \int_0^1 c_t(j^h) dj^h \\ (c) \ l_t^s &= (1 \phi) \int_0^1 l_t^s(j^o) dj^o + \phi Z^h \int_0^1 l_t^s(j^h) dj^h \\ (d) \ l_t^d &= \int_0^1 l_t^d(i) di \\ (e) \ c_t(i) &= (1 \phi) (C(i)^o/C(i)) \int_0^1 c_t(i, j^o) dj^o + \phi (C(i)^h/C(i)) \int_0^1 c_t(i, j^h) dj^h \\ (f) \ Bb_t^s &= (1 \phi) B^o \int_0^1 b_t^d(j^o) dj^o \end{aligned}$

5.1. Computing the Aggregate Equilibrium. We now describe an iterative procedure that can be used to solve for SRICE numerically. It can be shown²⁵ that decisions affecting cross-sectional efficiency, namely relative consumption, can be disregarded when solving for aggregate dynamics. We focus on this simplified problem.

The dynamics of the inattentive economy can be summarized by the stochastic processes governing the price level, real wage, and consumption for both household types. These equilibrium processes can be determined as follows.

First, we make guesses concerning the $MA(T)^{26}$ representations of the stochastic processes for p_t , \tilde{w}_t , c_t^h and c_t^o . Second, given these guesses, we compute the optimal

²⁵See Briand (2025)

²⁶Where T is a large integer.

price for firm *i* and approximate the resulting dynamics with a finite VARMA process. Third, we solve the attention problem in Section 4.1 and compute the implied stochastic processes for the aggregate price level and labor demand as $p_t = \int_0^1 p_t(i)di$ and $l_t^d = \int_0^1 l_t^d(i)di$. Fourth, we compute optimal labor supply for each type of households, given the processes for the price level and the guesses for c_t^h , c_t^o and \tilde{w}_t . We approximate their joint dynamics with a finite VARMA process. Fifth, we solve the attention problem in Section 4.2 and Section 4.3 and compute the implied stochastic processes for each household type's consumption as $c_t^s = \int_0^1 c_t(j^s)ds$, aggregate consumption as $c_t = (1 - \phi)\frac{C^o}{L}\int_0^1 c_t(j^o)dj^o + \phi\frac{C^h}{L}\int_0^1 c_t(j^h)dj^h$, and aggregate labor supply as $l_t^s = (1 - \phi)\frac{Z^o \tilde{L}}{L}\int_0^1 \bar{l}_t^s(j^o)dj^o + \phi\frac{Z^h \tilde{L}}{L}\int_0^1 \bar{l}_t^s(j^h)dj^h$. Sixth, we compute the process for the real wage implied by the equation for dividends. Seventh, we compare the MA(T) representations of the stochastic processes for p_t , \tilde{w}_t , c_t^h and c_t^o with their initial guesses. If any of the processes differ by more than a prespecified tolerance criterion, we update them using linear extrapolation and repeat the procedure starting from step two until convergence is achieved. If convergence is achieved, we verify that labor demand equals labor supply, if it does we have found a fixed-point.

When solving the attention problems, we use the algorithm from Afrouzi and Yang (2021), which iterates on the first-order condition. To approximate stochastic processes as finite VARMA, we apply Han, Tan, and Wu (2022)'s projection method in the frequency domain. Non-stationary processes are first differenced, and an additional row is added to the state-space representation to restore integration. For more details, see Briand (2025).

6. Model: Dynamics

This section studies the dynamics of inattentive economies and the accuracy of heterogenous households' expectations. We first calibrate the model using microdata from the SCF²⁷, a survey on households' finances independent from the SCE used in Section 2 to document the relationship between hand-to-mouth status and expectations.

We begin by solving the model with inattentive firms and households that have perfect information. The model's solution is summarized by the stochastic processes for the endogenous variables. We then solve for the optimal attention strategies for a measure-zero of inattentive hand-to-mouth and optimizing households at the fixed point. This provides intuition about the behavior of different households while abstracting from their general equilibrium effects. We conclude the section by solving

²⁷Available at https://www.federalreserve.gov/econres/scfindex.htm.

for the model's fixed point when both firms and households are inattentive.

6.1. Calibration. The model's parameters are divided into five blocks, with their values listed in Table 2. The first block contains the model's deep structural parameters, while the second includes the policy parameters for the Taylor rule. Most parameters follow conventional values from the business cycle literature, except for those related to household heterogeneity and the distribution of aggregate dividends and taxes, which we calibrate using financial microdata from the SCF. Details on the identification of hand-to-mouth households are provided in Appendix F, our estimated fraction is consistent with Kaplan and Violante (2014). The computation of tax and dividend shares is discussed in Appendix G, our results for the latter matching the value suggested by Debortoli and Galí (2024).

The third block specifies parameters for the exogenous processes. Parameters governing aggregate technology shocks are calibrated by regressing the growth rate of Fernald (2014)'s Total Factor Productivity²⁸ on its own lag. The variance of the monetary policy shock is determined in a model-consistent manner by inverting a log-linearized Taylor rule, Equation (8), using quarterly data from 1960Q1 to 2019Q4²⁹ on the Federal Funds rate (quarterly average), the Gross Domestic Product Implicit Price Deflator, Real Gross Domestic Product, and Real Potential Gross Domestic Product.

The fourth block specifies values for the non-stochastic steady state, from which we compute the ratios entering each household type's budget constraint. We set gross labor supply to one-third for both household types, while the remaining parameters are calibrated based on our partitioning of households into hand-to-mouth and optimizing types. Details are provided in Appendix G. Our value for the ratio of income shocks' unconditional means is close to the one suggested by Debortoli and Galí (2024).

The fifth and last block sets the marginal cost of attention for firms and households. These values are chosen to be small relative to non-stochastic steady-state quantities and more importantly are equalized between hand-to-mouth and optimizing households. Hence, any difference in their attention allocation can strictly be attributed to real heterogeneity.

6.2. Rational Inattention by Firms. We begin by solving the model with inattentive firms and households with perfect information. Figures illustrating the resulting dynamics and comparing them with other models can be found in Appendix

²⁸Quarterly, 1960Q1-2019Q4, Total Factor Productivity growth adjusted for capacity utilization. Data available at https://www.johnfernald.net/TFP.

²⁹The time series are sourced from https://fred.stlouisfed.org.

Parameter	Interpretation	Value		
	Structural parameters			
β	Households discount factor	0.99		
γ	Inverse of intertemporal elasticity of subs.	1.0		
ψ	Inverse of Frisch elasticity	1.0		
α	Labor share in the production function	2/3		
$ ilde{ heta}$	Model's elasticity of subs. between cons. varieties	6.0		
ϕ	Fraction of HtM households in the economy	0.24		
	Policy parameters			
ρ_R	Nominal interest rate smoothing	0.9		
ϕ_{π}	Nominal interest rate response to inflation	1.5		
ϕ_{y^*}	Nominal interest rate response to output gap	0.125		
ϑ_D	Fraction of aggregate dividends received by HtM	0.0		
ϑ_T	Fraction of aggregate taxes paid by HtM	0.08		
	Exogenous processes			
ρ_A	Persistence of aggregate technology	0.9		
$100\sigma_A$	S.D. of aggregate technology shocks	0.8		
$100\sigma_V$	S.D. of monetary policy shocks	0.08		
	Non-stochastic steady-state			
\bar{L}	Labor supply	1/3		
Z^o/Z^h	Unconditional mean ratio of income shocks	2.22		
\tilde{B}^{h}/Y	Real debt-to-output ratio for HtM	0.0		
\tilde{B}^o/Y	Real bond holdings-to-output ratio for Opt.	2.06		
	Rational inattention parameters			
λ^h	HtM marginal cost of attention	$()/100,000 \times (C^h)^{1-\gamma}$		
λ^o	Opt. hhs marginal cost of attention	$()/100,000 \times (C^{o})^{1-\gamma}$		
λ^f	Firms marginal cost of attention	$10/100,000 \times (C^o)^{-\gamma} \tilde{W}L$		

Table 2: Benchmark parametrization

Note: Parameter values for hand-to-mouth and optimizing households are calibrated using the 2020 wave of the SCF. The remaining non-stochastic steady-state ratios follow.

H. As in a standard TANK model, inflation in the inattentive economy is persistent, leading to a hump-shaped response of aggregate output. Under our benchmark calibration, rationally inattentive firms process 1.90 *bits* of information per quarter. Price adjustments in the inattentive economy occur more in the immediate aftermath of the shocks compared to the TANK model, where we set the average price duration to four quarters. In this case, we could increase firms' marginal cost of attention to match the Calvo model, as firms respond equally to both shocks. However, in general, this is not the case, as firms may allocate more attention to shocks that induce greater volatility and losses.

For our benchmark calibration, the two-agent and representative-agent models with inattentive firms exhibit similar dynamics due to compensating effects between the optimal responses of heterogeneous households. This fixed point is of little interest for the questions studied in this paper, but in the next section, it will serve as a starting point for analyzing the behavior of inattentive households. We will also use it to assess the effects of inattentive households on the general equilibrium.

6.3. Rational Inattention by a Measure Zero of Households. The solution for the economy with inattentive firms, derived in Section 6.2, serves as a starting point to understand how households subject to rational inattention choose to pay attention. We assume that for both types, a measure-zero (atomistic) fraction of households are inattentive, and their decisions do not affect the equilibrium. This allows us to solve for their optimal attention strategy at the fixed-point of the economy without disrupting it.

We solve the households' attention problems for different values of the marginal cost of attention³⁰ and compute the expected absolute nowcast errors. The results are presented in Figure 1.

Figure 3 and Figure 4 present what can be viewed as a decomposition of the expectational errors, showing the impulse response functions of the nowcasts for inflation and the nominal rate in response to technological and monetary policy shocks.

Hand-to-mouth households are significantly better at forecasting inflation than the nominal rate. Up to a certain threshold for the marginal cost of attention, their expectations of inflation are more accurate than those of optimizing households. The latter form expectations that are significantly more accurate in response to technological shocks than to monetary policy shocks. These results can be rationalized by examining the households' optimal response to shocks, which are precisely the ones

³⁰These values are an order of magnitude smaller than for firms. This is because the matrix weighting losses for households feature lower values.

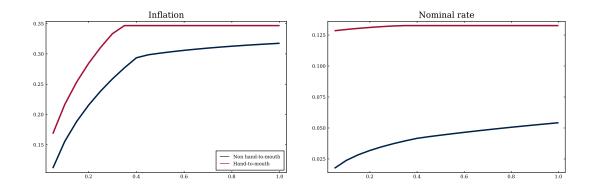


Figure 1: Nowcasts' Average Absolute Errors

Note: The x-axis represents the households' marginal cost of attention multiplied by $(C)^{1-\gamma}/100,000$, and the y-axis shows the mean absolute nowcast errors computed from the posterior covariance matrix.

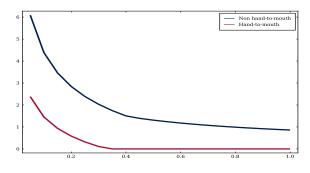


Figure 2: Information flows

Note: The x-axis represents the households' marginal cost of attention multiplied by $(C)^{1-\gamma}/100,000$, and the y-axis shows the information flow quantified in *bits*.

plotted in Figure 5b and Figure 6b.

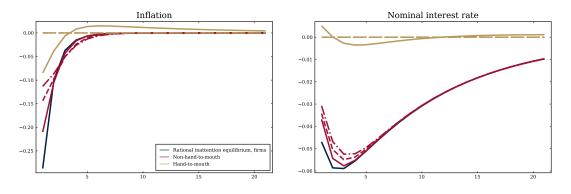


Figure 3: Expectations to a Technological Shock

Note: The solid blue line are the general equilibrium responses. Red and gold lines represent households' beliefs. Solid lines correspond to $\lambda = 0.2$, dashed lines to $\lambda = 0.4$, and dash-dot lines to $\lambda = 0.6$.

First, note that for both shocks, hand-to-mouth households' optimal labor supply is very similar, making it unnecessary to distinguish between the shocks³¹. As a result, they design a single signal that captures a linear combination of the shocks with approximately equal weights. Since both shocks generate similar inflation dynamics, their signal effectively tracks this variable. In contrast, because the shocks push the nominal rate in opposite directions, their signal leads to poor forecasts as it does not distinguish between technological and monetary policy innovations.

Second, the stochastic process describing hand-to-mouth optimal labor supply is not persistent in response to either shock. Consequently, information is valuable only insofar as it enables a prompt reaction. This feature results in a sharp cutoff in information processing, beyond a certain level of marginal attention cost, it becomes optimal to forgo information acquisition entirely.

Turning our attention to optimizing households, as mentioned in Section 4.3, this type makes two decisions each period that are orthogonal to each other. For sufficiently low marginal costs of attention, their optimal attention strategy is based on two signals: one is informative about optimal consumption but contains virtually no information regarding optimal labor supply, and vice versa. After a certain threshold of attention costs, it becomes optimal to condense the information into a single signal.

³¹This would hold true even with a sign flip.

Unlike hand-to-mouth, optimizing households' optimal decisions are not symmetric across shocks. While both shocks induce similar processes for optimal labor supply, consumption moves in opposite directions. As a result, they design their signal(s) to better distinguish which shock has occurred, leading them to forecast the nominal rate more accurately than hand-to-mouth households at any value of marginal attention costs.

Optimal consumption for optimizing households responds more strongly to technological shocks than to monetary policy shocks. As a result, they allocate their attention in an asymmetric way, giving more weight to technological innovations, which induce larger losses from suboptimal decisions. This behavior explains why, for certain marginal attention costs, their inflation forecasts are less accurate than those of hand-to-mouth households. Specifically, this occurs because the latter do not skew their attention toward either shock, as their optimal decisions exhibit similar volatility in response to both shocks, and inflation volatility is roughly evenly distributed between the two types of innovations.

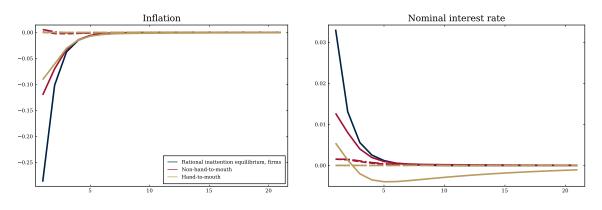


Figure 4: Expectations to a Monetary Policy Shock

Note: The solid blue line are the general equilibrium responses. Red and gold lines represent households' beliefs. Solid lines correspond to $\lambda = 0.2$, dashed lines to $\lambda = 0.4$, and dash-dot lines to $\lambda = 0.6$.

Household inattention also leads to muted labor supply and consumption decisions, which in turn have real effects. We now focus on the model where all households are subject to rational inattention, examining the general equilibrium effects and the implications for expectations.

6.4. General Equilibrium Dynamics. Coming Soon.

7. Alternative Calibrations

In the previous section, our calibration relied on the most recent SCF wave, using the median values within each household group. Here, we examine the implications of alternative calibration approaches.

7.1. Hand-to-mouth Households at the Credit Limit. First, we narrow our definition of hand-to-mouth households within the same SCF wave, targeting only those for which the credit limit is binding. This modification introduces an exposure to nominal interest rate fluctuations that is absent in the benchmark calibration.

7.2. Effects of Time-Varying Households' Characteristics. Second, we recalibrate the model using earlier SCF waves. This allows us to assess whether shifts in household characteristics over time have altered the economy's response to aggregate shocks.

8. Extensions

We previously assumed that households choose their labor supply based on the market-clearing wage. In this section, we introduce alternative assumptions about how the labor market operates and examine the impacts on the economy's response to aggregate shocks and households expectations.

8.1. Monopolistic Competition. Coming Soon.

9. Policy Experiments

9.1. Monetary Policy Coming Soon.

9.2. Transfer Policy Coming Soon.

10. Conclusions.

Heterogeneity is modeled in a stylized way within our model, where agents are ex-ante partitioned into two categories: hand-to-mouth and optimizing households. These characteristics are permanent. There is also a single liquid asset that can be used to smooth consumption. Therefore, our model abstracts from some features of HA model that the literature has stressed as playing an important role in those models such as precautionnary savings or "wealthy" hand-to-mouth households. Conceptually, we view our model as a useful approximation for shocks that are not too large, where the wealth distribution and precautionary motives have small effects.

However, these simplifications do not imply that our framework cannot approximate the behavior of different household types in terms of attention. For instance, a household might commit to an attention strategy and update it only when a large shock causes movement across the wealth distribution. We find this hypothesis plausible, and it is reminiscent of how other economic behaviors, such as investment or price adjustments, can exhibit discontinuities.

Remaining conclusions coming soon.

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Appendix A. Additional Empirical Evidence

The SCE directly classifies households into three income groups. It also asks the following question to each respondent every month they are polled,

Q: What do you think is the percent chance that, over the next 3 months, you will NOT be able to make one of your debt payments (that is, the minimum required payments on credit and retail cards, auto loans, student loans, mortgages, or any other debt you may have)?

We define a household as having a high probability of default if it reports a 70% or higher chance of being unable to make the minimum payment on its debt. This concept is closely related to the "financially fragile" households discussed in Lusardi, Schneider, and Tufano (2011), which can be viewed as a subgroup of hand-to-mouth households. Table 3 reports the results on how these households' characteristics are related to expectations accuracy.

It is worth mentioning that the effects of these two sets of regressors should be interpreted differently. The effects of income could have been confounded with handto-mouth status when not included, as the two are correlated. On the other hand, a high probability of default likely targets a subgroup within the hand-to-mouth population, those at their credit limit.

Appendix B. Perfect Information Equilibrium

We define a perfect information environment as one in which every decision-maker has rational expectations and knows the complete history of the economy up to and including the current period. An approximation³² of the perfect information equilibrium in a neighborhood of the non-stochastic-steady is defined as

Definition 2. For any sequence of realizations for the exogenous innovations, $\{\varepsilon_t\}_{t=0}^{\infty}$, a Sequential Perfect Information Competitive Equilibrium (SPICE) is:

- An allocation, $\Omega(i) := \{y_t(i), l_t^d(i), \{p_t(i)\}_{i \in [0,1]}\}_{t=0}^{\infty}$ for every firm $i \in [0,1]$.
- An allocation, $\Omega(j^h) := \{c_t(j^h), l_t^s(j^h), \{\hat{c}_t(i, j^h)\}_{i \in [0,1]}\}_{t=0}^{\infty}$ for every hand-tomouth household $j^h \in [0, 1]$.
- An allocation, $\Omega(j^o) := \{l_t(j^o), b_t^d(j^o), \{\hat{c}_t(i, j^o)\}_{i \in [0, 1]}\}_{t=0}^{\infty}$ for every optimizing household $j^o \in [0, 1]$.

³²All lowercase variables represent log-deviations from the non-stochastic steady-state.

	Inflation			Interest rate		
	(1)	(2)	(3)	(4)	(5)	(6)
Optimizing	-	_	-	-	-	-
Hand-to-mouth	-	-	0.733^{***}	-	-	0.027^{***}
	-	-	(0.265)	-	-	(0.002)
Low Pr. of default	-	-	-	-	-	_
High Pr. of default	-	10.957***	10.312***	-	0.016^{**}	0.005
-	-	(0.718)	(0.721)	-	(0.007)	(0.007)
Income $< 50K$	-	-	-	-	-	-
Income $\in [50K, 100K]$	-2.803^{***}	-	-2.481^{***}	-0.021^{***}	-	-0.020^{***}
	(0.322)	-	(0.323)	(0.003)	-	(0.003)
Income $> 100K$	-3.535^{***}	-	-3.528^{***}	-0.060^{***}	-	-0.059^{***}
	(0.353)	-	(0.354)	(0.003)	-	(0.003)
High School	-	-	-	-	-	-
Some College	-3.452^{***}	-3.839^{***}	-3.568^{***}	-0.010^{**}	-0.014^{***}	-0.010^{**}
0	(0.466)	()0.465)	(0.465)	(0.004)	(0.004)	(0.004)
College	-5.358^{***}	-6.309^{***}	-5.328^{***}	-0.062^{***}	-0.076^{***}	-0.059^{***}
0	(0.461)	(0.449)	(0.461)	(0.004)	(0.004)	(0.004)
Low Numeracy	-	-	-	-	-	-
High Numeracy	-4.280^{***}	-4.509^{***}	-3.982^{***}	-0.046^{***}	-0.054^{***}	-0.044^{***}
~ v	(0.310)	(0.305)	(0.310)	(0.004)	(0.003)	(0.003)
F Statistic	158.002	226.9113	143.619	352.482	390.29	269.096
R^2	0.009	0.010	0.011	0.289	0.288	0.290

Table 3: Expectations Accuracy Across Economic Characteristics

Notes: Columns (1), (2), and (3) show estimates from a regression of the absolute errors in expectations about inflation 12 months ahead on income brackets, probability of defaulting, and both of these plus hand-to-mouth status. Columns (4), (5), and (6) show estimates from a regression of errors in expectations about whether the interest rate has increased compared to 12 months prior, on the same regressors. All regressions are based on 112,937 observations, include time-fixed effects, and control for education and numeracy. Robust standard errors in parentheses, * p < 0.1, ** p < 0.05, *** p < 0.01. Sample: 2013M8-2024M1.

- An allocation, $\Omega_G := \{b_t^s, t_t\}_{t=0}^{\infty}$ for the government.
- A set of prices $\{p_t, w_t, r_t\}_{t=0}^{\infty}$.
- Stationnary distributions of households and firms over idiosyncratic innovations.

such that

- 1. Given $\{\Omega(j^h)_{j^h \in [0,1]}, \Omega(j^o)_{j^o \in [0,1]}, p_t, w_t\}_{t=0}^{\infty}$, a firm's allocation, $\Omega(i)$, maximizes its discounted sum of nominal profits.
- 2. Given $\{\Omega(i)_{i\in[0,1]}, \Omega(j^o)_{j^o\in[0,1]}, p_t, r_t, w_t, l_t\}_{t=0}^{\infty}$, a hand-to-mouth household allocation, $\Omega(j^o)$, maximizes its discounted sum of period utilities.
- 3. Given $\{\Omega(i)_{i\in[0,1]}, \Omega(j^h)_{j^h\in[0,1]}, p_t, r_t, w_t, l_t\}_{t=0}^{\infty}$, an optimizing household allocation, $\Omega(j^h)$, maximizes its discounted sum of period utilities.
- 4. Monetary policy satisfies the specified Taylor rule, fiscal and transfer policies satisfy their respective rules and given interest and subsidies payments, the government runs a balanced budget in real terms.
- 5. All markets clear $\forall t \geq 0$:
 - (a) $y_t = c_t$
 - (b) $l_t^s = l_t^d$
 - (c) $b_t^s = b_t^d$
 - (d) $y_t(i) = c_t(i) \ \forall i \in [0, 1]$
- 6. Aggregate price index is given by $\forall t \geq 0$:
 - (a) $p_t = \int_0^1 p_t(i) di$
- 7. Aggregate quantities are given by $\forall t \geq 0$:

$$(a) \ y_t = \int_0^1 y_t(i)di$$

$$(b) \ c_t = (1 - \phi)(C^o/C) \int_0^1 c_t(j^o)dj^o + \phi(C^h/C) \int_0^1 c_t(j^h)dj^h$$

$$(c) \ l_t^s = (1 - \phi) \int_0^1 l_t^s(j^o)dj^o + \phi \int_0^1 l_t^s(j^h)dj^h$$

$$(d) \ l_t^d = \int_0^1 l_t^d(i)di$$

$$(e) \ c_t(i) = (1 - \phi)(C(i)^o/C(i)) \int_0^1 c_t(i,j^o)dj^o + \phi(C(i)^h/C(i)) \int_0^1 c_t(i,j^h)dj^h$$

$$(f) \ Bb_t^s = (1 - \phi)B^o \int_0^1 b_t^d(j^o)dj^o$$

B.1. Optimality Conditions. Hand-to-mouth household j^h solves a static maximization problem that yields the following first-order condition for labor supply

$$\varphi \bar{L}_t(j^h)^{\psi} C_t(j^h)^{\gamma} = Z_t(j^h) \tilde{W}_t.$$
(50)

Optimizing household j^o solves a dynamic maximization problem, whose firstorder conditions imply an Euler equation given by

$$C_t(j^o)^{-\gamma} = \beta E_t(j^o) \left[C_{t+1}(j^o)^{-\gamma} \left(\frac{R_t}{\Pi_{t+1}} \right) \right]$$
(51)

and a labor supply condition

$$\varphi \bar{L}_t (j^o)^{\psi} C_t (j^o)^{\gamma} = Z_t (j^o) \tilde{W}_t.$$
(52)

Firm i solves a static price-setting problem that yields the following first-order condition for optimal pricing

$$\hat{P}_t(i) = \frac{\tilde{W}_t}{\alpha e^{a_t} e^{a_t(i)}} \left(\frac{\hat{P}_t(i)^{-\theta} C_t}{e^{a_t} e^{a_t(i)}}\right)^{\frac{1-\alpha}{\alpha}}.$$
(53)

B.2. Linearized Equilibrium Conditions Given that idiosyncratic shocks sum to zero in the cross-section, we can omit them and drop the indexes i, j^h , and j^o when solving for the aggregate dynamics.

After log-linearizing the equilibrium conditions around the non-stochastic steadystate, we obtain the following system of equations

By Walras' law, we omit the optimizing households' budget constraint from the set of equilibrium conditions. However, it is useful to state it explicitly, as it is used in the algorithm that solves for the equilibrium of the inattentive economy.

$$C^{o}c_{t}^{o} + \tilde{B}^{o}\tilde{b}_{t}^{o} = \frac{\tilde{B}^{o}}{\beta}(r_{t-1} - \pi_{t} + \tilde{b}_{t-1}^{o}) + \tilde{W}\bar{L}Z^{o}(\tilde{w}_{t} + \bar{l}_{t}^{o}) + \tilde{D}^{o}\tilde{d}_{t}^{o} - \tilde{T}^{o}\tilde{t}_{t}^{o}$$
(54)

where

$$\tilde{t}_t^o = \tilde{t}_t,\tag{55}$$

$$\tilde{d}_t^o = \tilde{d}_t,\tag{56}$$

and

$$\tilde{B}\tilde{b}_t = (1-\phi)\tilde{B}^o\tilde{b}_t^o \tag{57}$$

No.	Interpretation	Equation
	L	1
i.	Opt. hhs. Euler eq.	$c_t^o = E_t[c_{t+1}^o] - \frac{1}{\gamma}E_t[r_t - \pi_{t+1}]$
ii.	Opt. hhs. labor supply	$ ilde{w}_t = \psi \overline{l}_t^{h'} + \gamma c_t^h$
iii.	HtM. hhs. labor supply	$\tilde{w}_t = \psi \tilde{l}_t^o + \gamma c_t^o$
iv.	HtM. budget constr.	$c_t^h = \omega_W^h(\tilde{w}_t + \bar{l}_t^h) + \omega_D^h \tilde{d}_t^h - \frac{\omega_B^h}{\beta} r_{t-1} - \omega_T \tilde{t}_t^h$
v.	Firms' optimal pricing	$\tilde{w}_t = \frac{\alpha - 1}{\alpha} y_t + \frac{1}{\alpha} a_t$
vi.	Taylor rule	$r_t = \phi_R r_{t-1} + (1 - \phi_R) \phi_\pi \pi_t$
vii.	Agg. tech.	$a_t = \rho_A a_{t-1} + \varepsilon_t^a$
viii.	Agg. consumption	$Cc_t = (1 - \phi)C^o c_t^o + \phi C^h c_t^h$
ix.	Agg. labor supply	$Ll_t = (1-\phi)Z^h \bar{L}^h \bar{l}^h_t + \phi Z^o \bar{L}^o \bar{l}^o_t$
х.	Agg. output	$y_t = a_t + \alpha l_t$
xi.	Agg. budget constr.	$y_t = c_t$
xii.	Agg. real dividend	$\tilde{D}\tilde{d}_t = (1 + \tau_P)Yy_t - \tilde{W}L(\tilde{w}_t + l_t)$
xiii.	Agg. real taxes	$\tilde{T}\tilde{t}_t = R\tilde{B}(r_{t-1} - \pi_t + \tilde{b}_{t-1}) + \tau_P Y y_t$
xiv.	HtM's real taxes	$ ilde{t}^h_t = ilde{t}_t$
XV.	HtM's real dividends	$ ilde{d}^h_t = ilde{d}_t$

Table 4: Log-linear conditions for the perfect information economy

Note: Equation (xiii) describes a fiscal condition that stabilizes real bond holdings at a constant level.

B.3. Characterization of the Aggregate Equilibrium

Definition 3. Given the same initial bonds holdings for optimizing households and the following non-explosive sequence of real bond holdings

$$\lim_{s \to \infty} E_t [\beta^{s+1} (\tilde{b^o}_{t+s+1} - \tilde{b^o}_{t+s})] = 0,$$
(58)

an equilibrium of the PI economy is a solution to the system of equations collected in Table 4.

Appendix C. Models with Frictions

We also introduce alternative models with similar physical environments that incorporate frictions commonly used in the business cycle literature. These models are presented below and are also used for comparison with the RI models in the main text.

C.1. Calvo Prices. The model with Calvo pricing assumes that in any given period, a firm has an unconditional probability of $(1 - \iota_P)$ to reoptimize its price. The equilibrium conditions of the Calvo model are obtained by replacing the firms' optimal pricing equation in Table 4 with a New-Keynesian Phillips curve

$$\pi_t = \kappa m c_t + \beta E_t[\pi_{t+1}] \tag{59}$$

where

$$\kappa = \frac{(1 - \iota_P)(1 - \beta \iota_P)}{\iota_P} \frac{1 - \alpha}{1 - \alpha + \alpha \iota_P}$$
(60)

and

$$mc_t = \tilde{w}_t + \frac{1-\alpha}{\alpha} y_t - \frac{1}{\alpha} a_t \tag{61}$$

represent the slope of the Phillips curve and the real marginal cost, respectively. A complete derivation of these equilibrium conditions can be found in Galí (2015).

We also modify the Taylor rule to account for the presence of an output gap due to price rigidity

$$r_t = \phi_R r_{t-1} + (1 - \phi_R) [\phi_\pi \pi_t + \phi_{y^*} (y_t - y_t^*)].$$
(62)

Here, y_t^* represents the natural level of output, which is the level of economic activity that would prevail under perfect information and in the absence of frictions.

Given the modified log-linear conditions, a characterization of this economy's equilibrium consists of a flexible price block, identical to the one in Definition 3, and a staggered price block in which the optimal pricing condition is replaced with Equation (59) and where the Taylor rule is given by Equation (62).

C.2. Sticky wages. The model with sticky wages ...

Appendix D. Non-Stochastic Steady-State

The following non-stochastic steady-state relationships are useful for approximating the objective functions of households and firms.

Combining firm i's optimal pricing condition with its production function yields

$$\hat{P}(i) = \tilde{W} \frac{1}{\alpha} (\hat{P}(i)^{-\theta} C)^{\frac{1}{\alpha} - 1}$$

$$\hat{P}(i) = \tilde{W} \frac{C(i)^{\frac{1}{\alpha}}}{\alpha C(i)}$$

$$\hat{P}(i) = \tilde{W} \frac{L(i)}{\alpha C(i)}$$

$$\alpha C = \tilde{W}L.$$
(63)

Rearranging the labor supply condition for both household types yields

$$\varphi \bar{L}^{\psi} = C^{-\gamma} Z \tilde{W}$$

$$\varphi \bar{L}^{1+\psi} = C^{-\gamma} Z \tilde{W} \bar{L}$$

$$\varphi \bar{L}^{1+\psi} = C^{1-\gamma} \frac{Z \tilde{W} \bar{L}}{C}$$

$$\varphi \bar{L}^{1+\psi} = C^{1-\gamma} \omega_{W}$$
(64)

where \overline{L} and Z take their respective values for each household type in steady state.

Appendix E. Approximation of the Objective Functions

E.1. Firms' Objective First, we guess that model-implied demand for consumption variety i is³³

 $^{^{33}}$ We can prove that optimal attention allocation yields a demand function of that form.

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\hat{\theta}} C_t.$$
(65)

Second, we substitute the demand function into the expression for period nominal profits

$$D_t(i) = (1 + \tau_P) P_t(i) \hat{P}_t(i)^{-\theta} C_t - W_t L_t(i)$$
(66)

Third, we replace the labor input using the production function

$$D_t(i) = (1 + \tau_P) P_t(i) \hat{P}_t(i)^{-\tilde{\theta}} C_t - W_t \left(\frac{\hat{P}_t(i)^{-\theta} C_t}{e^{a_t} e^{a_t(i)}}\right)^{\frac{1}{\alpha}}$$
(67)

Next, we assume that, in period -1, households who own the firms value profits at time t using the following discount factor

$$Q_{-1,t} = \beta^t \Lambda(\{C_t(j^o)\}_{j^o \in [0,1]}, \{C_t(j^h)\}_{j^h \in [0,1]}) \frac{1}{P_t}$$
(68)

where the functional $\Lambda(\cdot)$ is twice continuously differentiable and satisfies

$$\Lambda(\{C_t(j^o)\}_{j\in[0,1]}, \{C_t(j^h)\}_{j^h\in[0,1]}) = (1-\vartheta_D)(C^o)^{-\gamma} + \vartheta_D(C^h)^{-\gamma}$$
(69)

in the non-stochastic steady-state ³⁴.

We multiply eq. (67), the period nominal profits, by the stochastic discount factor (exempt of β^t) which yields

$$\Lambda(\{C_t(j^o)\}_{j^o \in [0,1]}, \{C_t(j^h)\}_{j^h \in [0,1]}) \left[(1+\tau_P) \hat{P}_t(i)^{1-\theta} C_t - \left(\frac{\hat{P}_t(i)^{-\theta} C_t}{e^{a_t} e^{a_t(i)}}\right)^{\frac{1}{\alpha}} \tilde{W}_t \right].$$
(70)

We denote eq. (70) the real period profits function.

We rewrite this expression in terms of log-deviations around the non-stocastic steady-state

³⁴In the non-stochastic steady state, $\Lambda(\cdot)$ is simply the weighted average of the marginal utility of consumption for both types of households, weighted by the fraction of aggregate dividends they are entitled to.

$$\Lambda \left(\{ C^{o} e^{c_{t}(j^{o})} \}_{j^{o} \in [0,1]}, \{ C^{h} e^{c_{t}(j^{h})} \}_{j^{h} \in [0,1]} \right) \left[\frac{\theta}{\theta - 1} \frac{\tilde{W}L}{\alpha} \left\{ (1 - \phi) \int_{0}^{1} e^{(1 - \theta)\hat{p}_{t}(i) + c_{t}(j^{o})} dj^{o} + \phi \int_{0}^{1} e^{(1 - \theta)\hat{p}_{t}(i) + c_{t}(j^{h})} dj^{h} \right\} - \tilde{W}L\alpha Y e^{\frac{\theta}{\alpha}\hat{p}_{t}(i) - \frac{1}{\alpha}(a_{t} + a_{t}(i)) + \tilde{w}_{t}} \left\{ (1 - \phi) \int_{0}^{1} e^{\frac{c_{t}(j^{o})}{\alpha}} dj^{o} + \phi \int_{0}^{1} e^{\frac{c_{t}(j^{h})}{\alpha}} dj^{h} \right\}$$

$$(71)$$

Let x_t denote the variables appearing in firm *i*'s real period profit function that the firm can affect and ζ_t the vector of variables that are taken as given

$$\boldsymbol{x}_t = (\hat{p}_t(i))' \tag{72}$$

$$\boldsymbol{\zeta} = \left(a_t, a_t(i), \tilde{w}_t, \{c_t(j^h)\}_{j \in [0,1]}, \{c_t(j^o)\}_{j \in [0,1]}\right)'$$
(73)

We define \mathcal{F}^f as the functional obtained from multiplying the period real profit function, eq. (71), by β^t and summing over all t from zero to infinity.

We let $\tilde{\mathcal{F}}^f$ denote the second-order approximation of that functional around the non-stochastic steady-state

$$E_{-1}(i) \left[\tilde{\mathcal{F}}^{f} \left(\boldsymbol{x}_{0}, \boldsymbol{\zeta}_{0}, \boldsymbol{x}_{1}, \boldsymbol{\zeta}_{1}, \cdots \right) \right]$$

$$(74)$$

$$\approx \mathcal{F}^{f}(\mathbf{0},\mathbf{0},\cdots) + E_{-1}(i) \left[\sum_{t=0}^{\infty} \beta^{t} \left(\mathbf{h}_{x}^{\prime} \mathbf{x}_{t} + \mathbf{h}_{\zeta}^{\prime} \boldsymbol{\zeta}_{t} + \frac{1}{2} \mathbf{x}_{t}^{\prime} \mathbf{H}_{x} \mathbf{x}_{t} + \mathbf{x}_{t}^{\prime} \mathbf{H}_{x\zeta} \boldsymbol{\zeta}_{t} + \frac{1}{2} \boldsymbol{\zeta}_{t}^{\prime} \mathbf{H}_{\zeta} \boldsymbol{\zeta}_{t} \right) \right]$$

where the vectors h_x and h_{ζ} are first derivatives with respect to x_t and ζ_t evaluated at the non-stochastic steady-state respectively. Similarly, H_{xx} , $H_{\zeta\zeta}$ and $H_{x\zeta}$ are matrices of second order derivatives evaluated at the non-stochastic steady-state.

We can show that under some regularity conditions eq. (74) converges to a finite element in \mathcal{R} along with each of its components³⁵.

The process defining firm's i vector of optimal actions, noted \boldsymbol{x}_t^* , is defined by the following requirement

$$\boldsymbol{h}_{\boldsymbol{x}} + \boldsymbol{H}_{\boldsymbol{x}\boldsymbol{x}}\boldsymbol{x}_{t}^{*} + \boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}\boldsymbol{\zeta}_{t} = 0. \tag{75}$$

 $^{^{35}}$ See Maćkowiak and Wiederholt (2015) for the formal proof.

The requirement above implies the same equations as the log-linearization of the optimality conditions in Appendix B.

Next, we define firm i's objective function as the losses incurred from suboptimal actions which reads

$$E_{-1}(i) \left[\tilde{\mathcal{F}}^{f} \left(\boldsymbol{x}_{0}, \boldsymbol{\zeta}_{0}, \boldsymbol{x}_{1}, \boldsymbol{\zeta}_{1}, \cdots \right) \right] - E_{-1}(i) \left[\tilde{\mathcal{F}}^{f} \left(\boldsymbol{x}_{0}^{*}, \boldsymbol{\zeta}_{0}, \boldsymbol{x}_{1}^{*}, \boldsymbol{\zeta}_{1}, \cdots \right) \right]$$
(76)
$$= E_{-1}(i) \left[\sum_{t=0}^{\infty} \beta^{t} \left(\boldsymbol{h}_{\boldsymbol{x}}(\boldsymbol{x}_{t} - \boldsymbol{x}_{t}^{*}) + \frac{1}{2} \boldsymbol{x}_{t} \boldsymbol{H}_{\boldsymbol{x}} \boldsymbol{x}_{t} - \frac{1}{2} \boldsymbol{x}_{t}^{*} \boldsymbol{H}_{\boldsymbol{x}} \boldsymbol{x}_{t}^{*} + (\boldsymbol{x}_{t} - \boldsymbol{x}_{t}^{*}) \boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}} \boldsymbol{\zeta}_{t} \right) \right]$$

Lastly, we use eq. (75) to substitute for the term $H_{x\zeta}\zeta_t$ in eq. (76). After rearranging we obtain

$$E_{-1}(i) \left[\tilde{\mathcal{F}}^{f} \left(\boldsymbol{x}_{0}, \boldsymbol{\zeta}_{0}, \boldsymbol{x}_{1}, \boldsymbol{\zeta}_{1}, \cdots \right) \right] - E_{-1}(i) \left[\tilde{\mathcal{F}}^{f} \left(\boldsymbol{x}_{0}^{*}, \boldsymbol{\zeta}_{0}, \boldsymbol{x}_{1}^{*}, \boldsymbol{\zeta}_{1}, \cdots \right) \right]$$
$$= \frac{1}{2} (\boldsymbol{x}_{t} - \boldsymbol{x}_{t}^{*}) \boldsymbol{H}_{\boldsymbol{x}} (\boldsymbol{x}_{t} - \boldsymbol{x}_{t}^{*})$$
(77)

where

$$\boldsymbol{H}_{\boldsymbol{x}} = -(C^{o})^{-\gamma} \tilde{W} L \left[\frac{\theta(\theta + \alpha(1-\theta))}{\alpha^{2}} \right]$$
(78)

and

$$\boldsymbol{x}_{t}^{*} = \left(p_{t} + \frac{\frac{1-\alpha}{\alpha}}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}}c_{t} + \frac{1}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}}\tilde{w}_{t} - \frac{\frac{1}{\alpha}}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}}(a_{t} + a_{t}(i))\right)'.$$
(79)

Notice that we used $\hat{p}_t(i) - \hat{p}_t^*(i) = p_t(i) - p_t^*(i)$ so that firms choose $p_t(i)$, their price in level, instead of $\hat{p}_t(i)$, their relative price. The only matrix of second-order derivatives needed to formulate firm *i*'s attention problem in Section 4.1 is H_x .

E.2. Hand-to-Mouth Households' Objective First, we substitute the consumption aggregator into the flow budget constraint to get

$$C_t(j^h) \left(\int_0^1 P_t(i) \hat{C}_t(i, j^h) di \right) = Z_t(j^h) W_t \bar{L}_t(j^h) - (R_{t-1} - 1)B^h + D_t^h - T^h.$$
(80)

Second, we isolate composite consumption and divide both the numerator and denominator by the price index

$$C_{t}(j^{h}) = \frac{Z_{t}(j^{h})\tilde{W}_{t}\bar{L}_{t}(j^{h}) - (R_{t-1}-1)\tilde{B}^{h} + \tilde{D}_{t}^{h} - \tilde{T}^{h}}{\int_{[0,1)}\hat{P}_{t}(i)\hat{C}_{t}(i,j^{h})di + \hat{P}_{t}(1)\left(1 - \int_{[0,1)}\hat{C}_{t}(i,j^{h})^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}}di}.$$
(81)

Notice that we partially relax mathematical rigor by treating the integral as a finite sum and di as a weight³⁶.

Third, we substitute the expression for consumption in the period utility function

$$U(C_{t}(j^{h}), \bar{L}_{t}(j^{h})) = \frac{1}{1 - \gamma} \left(\frac{Z_{t}(j^{h})\tilde{W}_{t}\bar{L}_{t}(j^{h}) - (R_{t-1} - 1)\tilde{B}^{h} + \tilde{D}_{t}^{h} - \tilde{T}^{h}}{\int_{[0,1)} \hat{P}_{t}(i)\hat{C}_{t}(i, j^{h})di + \hat{P}_{t}(1)\left(1 - \int_{[0,1)} \hat{C}_{t}(i, j^{h})^{\frac{\theta - 1}{\theta}}di\right)^{\frac{\theta}{\theta - 1}}di} \right)^{\frac{\theta}{\theta - 1}}di$$
$$- \frac{1}{1 - \gamma} - \varphi \frac{\bar{L}_{t}(j^{h})^{1 + \psi}}{1 + \psi}.$$
(82)

Next, we rewrite this expression in terms of log-deviations around the non-stocastic steady-state

$$U(C_{t}(j^{h}), \bar{L}_{t}(j^{h})) = \frac{(C^{h})^{1-\gamma}}{1-\gamma} \left(\frac{\omega_{W}^{h} e^{z_{t}(j^{h}) + \tilde{w}_{t} + \bar{l}_{t}(j^{h})} - \frac{\omega_{B}^{h}}{\beta} e^{r_{t-1}} + \omega_{B}^{h} + \omega_{D}^{h} e^{\tilde{d}_{t}^{h}} - \omega_{T}^{h} e^{\tilde{t}_{t}^{h}}}{\int_{[0,1)} e^{\hat{p}_{t}(i) + \hat{c}_{t}(i,j^{h})} di + e^{\hat{p}_{t}(1)} \left(1 - \int_{[0,1)} e^{\frac{\theta-1}{\theta} \hat{c}_{t}(i,j^{h})} di\right)^{\frac{\theta}{\theta-1}} di} \right)^{1-\gamma} - \frac{1}{1-\gamma} - \frac{(C^{h})^{1-\gamma}}{1+\psi} \omega_{W}^{h} e^{(1+\psi)\bar{l}_{t}(j^{h})}$$

$$(83)$$

where ω_W^h , ω_B^h , ω_D^h , ω_T^h denote the following steady-state ratios

 $^{^{36}}$ Throughout this section, we use this approach to ensure there is always a free variable when working with equality conditions. Alternatively, we could assume a finite number of households and firms, as in Maćkowiak and Wiederholt (2015), but this would make the relationship between aggregate and individual variables dependent on the size of the economy.

$$\left(\omega_W^h, \omega_B^h, \omega_D^h, \omega_T^h\right) = \left(\frac{Z^h \tilde{W} \bar{L}^h}{C^h}, \frac{\tilde{B}^h}{C^h}, \frac{\tilde{D}^h}{C^h}, \frac{\tilde{T}^h}{C^h}\right).$$
(84)

Let x_t denote the variables appearing in the period utility function that the hand-to-mouth households can affect and ζ_t the vector of variables that are taken as given

$$\boldsymbol{x}_{t} = \left(\bar{l}_{t}(j^{h}), \{\hat{c}_{t}(i, j^{h})\}_{i \in [0, 1)}\right)'$$
(85)

$$\boldsymbol{\zeta}_{t} = \left(z_{t}(j^{h}), \tilde{w}_{t}, r_{t-1}, \tilde{t}^{h}_{t}, \tilde{d}^{h}_{t}, \{ \hat{p}_{t}(i) \}_{i \in [0,1]} \right)'$$
(86)

We define \mathcal{F}^h as the functional resulting from multiplying the period utility function, Equation (83), by β^t and summing over all t from zero to infinity.

We let $\tilde{\mathcal{F}}^h$ denote the second-order approximation of that functional arond the non-stochastic steady-state

$$E_{-1}(j^{h}) \left[\tilde{\mathcal{F}}^{h} \left(\boldsymbol{x}_{0}, \boldsymbol{\zeta}_{0}, \boldsymbol{x}_{1}, \boldsymbol{\zeta}_{1} \cdots \right) \right]$$

$$= \mathcal{F}^{h} \left(\boldsymbol{0}, \boldsymbol{0}, \cdots \right) + E_{-1}(j^{h}) \left[\sum_{t=0}^{\infty} \beta^{t} \left(\boldsymbol{h}_{\boldsymbol{x}}^{\prime} \boldsymbol{x}_{t} + \boldsymbol{h}_{\boldsymbol{\zeta}}^{\prime} \boldsymbol{\zeta}_{t} + \frac{1}{2} \boldsymbol{x}_{t}^{\prime} \boldsymbol{H}_{\boldsymbol{x}} \boldsymbol{x}_{t} + \boldsymbol{x}_{t}^{\prime} \boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}} \boldsymbol{\zeta}_{t} + \frac{1}{2} \boldsymbol{\zeta}_{t}^{\prime} \boldsymbol{H}_{\boldsymbol{\zeta}} \boldsymbol{\zeta}_{t} \right)$$

$$(87)$$

$$(87)$$

$$(87)$$

$$(87)$$

$$(87)$$

where the vectors h_x and h_{ζ} are first derivatives with respect to x_t and ζ_t evaluated at the non-stochastic steady-state respectively. Similarly, H_x , H_{ζ} and $H_{x\zeta}$ are the matrices of second order derivatives evaluated at the non-stochastic steady-state.

Under some regularity conditions eq. (87) and each of its elements converge to a finite element in \mathcal{R} .

The process defining the vector of optimal actions for household j^h , noted \boldsymbol{x}_t^* , is defined by the following requirement

$$\boldsymbol{h}_{\boldsymbol{x}} + \boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}_{t}^{*} + \boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}\boldsymbol{\zeta}_{t} = 0. \tag{89}$$

Next, we defined household j^h 's objective function as the losses incurred from suboptimal actions which reads

$$E_{-1}(j^{h})\left[\tilde{\mathcal{F}}^{h}\left(\boldsymbol{x}_{0},\boldsymbol{\zeta}_{0},\boldsymbol{x}_{1},\boldsymbol{\zeta}_{1},\cdots\right)\right]-E_{-1}(j^{h})\left[\tilde{\mathcal{F}}^{h}\left(\boldsymbol{x}_{o}^{*},\boldsymbol{\zeta}_{0},\boldsymbol{x}_{1}^{*},\boldsymbol{\zeta}_{1},\cdots\right)\right]$$
(90)
$$=E_{-1}(j^{h})\left[\sum_{t=0}^{\infty}\beta^{t}\left(\boldsymbol{h}_{\boldsymbol{x}}(\boldsymbol{x}_{t}-\boldsymbol{x}_{t}^{*})+\frac{1}{2}\boldsymbol{x}_{t}\boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}_{t}-\frac{1}{2}\boldsymbol{x}_{t}^{*}\boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}_{t}^{*}+(\boldsymbol{x}_{t}-\boldsymbol{x}_{t}^{*})\boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}\boldsymbol{\zeta}_{t}\right)\right]$$

Lastly, we use eq. (89) to substitute for $H_{x\zeta}\zeta_t$ in eq. (90). After rearranging we obtain

$$E_{-1}(j^{h}) \left[\tilde{\mathcal{F}}^{h} \left(\boldsymbol{x}_{0}, \boldsymbol{\zeta}_{0}, \boldsymbol{x}_{1}, \boldsymbol{\zeta}_{1}, \cdots \right) \right] - E_{-1}(j^{h}) \left[\tilde{\mathcal{F}}^{h} \left(\boldsymbol{x}_{0}^{*}, \boldsymbol{\zeta}_{0}, \boldsymbol{x}_{1}^{*}, \boldsymbol{\zeta}_{1}, \cdots \right) \right]$$
(91)
$$= E_{-1}(j^{h}) \sum_{t=0}^{\infty} \frac{1}{2} (\boldsymbol{x}_{t} - \boldsymbol{x}_{t}^{*}) \boldsymbol{H}_{\boldsymbol{x}} (\boldsymbol{x}_{t} - \boldsymbol{x}_{t}^{*})$$

where

$$\boldsymbol{H}_{x} = -(C^{h})^{1-\gamma} \begin{bmatrix} (\gamma + \psi) & 0 & \cdots & 0\\ 0 & \frac{2}{\theta} di & \cdots & \frac{1}{\theta} di\\ \vdots & \vdots & \ddots & \vdots\\ 0 & \frac{1}{\theta} di & \cdots & \frac{2}{\theta} di \end{bmatrix},$$
(92)

$$\boldsymbol{x}_{t}^{*} = \begin{pmatrix} \frac{z_{t}(j^{n}) + w_{t} - \gamma c_{t}^{*}(j^{n})}{\psi} \\ -\theta(p_{t}(i) - p_{t}) \\ \vdots \end{pmatrix},$$
(93)

and

$$c_t^*(j^h) = \omega_W^h(z_t(j^h) + \tilde{w}_t + \bar{l}_t^*(j^h)) - \omega_B^h r_{t-1} + \omega_D^h \tilde{d}_t^h - \omega_T^h \tilde{t}_t^h$$
(94)

The only matrix of second-order derivatives needed to formulate the household j^h 's attention problem in Section 4.2 is H_x .

E.3. Optimizing Households' Objective The first few steps are identical to those employed in the derivation of hand-to-mouth households j^h 's objective, except that in the period t budget constraint, bonds from period t - 1 appear, and these are a variable that optimizing household j^o can affect.

We start by expressing consumption as a function of real variables and substitute in the period utility function. We get

Next, we rewrite the above expression in terms of log-deviations from the non-stochastic steady-state

$$U(C_{t}(j^{o}), \bar{L}_{t}(j^{o})) = \frac{(C^{o})^{1-\gamma}}{1-\gamma} \left(\frac{\frac{\omega_{B}^{o}}{\beta} e^{r_{t-1}-\pi_{t}+\tilde{b}_{t-1}(j^{o})} - \omega_{B}^{o} e^{\tilde{b}_{t}(j^{o})} + \omega_{W}^{o} e^{z_{t}(j^{o})+\tilde{w}_{t}+\bar{l}_{t}(j^{o})} + \omega_{D}^{o} e^{\tilde{d}_{t}^{o}} - \omega_{T}^{o} e^{\tilde{t}_{t}^{o}}}{\int_{[0,1]} e^{\hat{p}_{t}(i)+\hat{c}_{t}(i,j^{o})} di + e^{\hat{p}_{t}(1)} \left(1 - \int_{[0,1]} e^{\frac{\theta-1}{\theta}\hat{c}_{t}(i,j^{o})} di\right)^{\frac{\theta}{\theta-1}} di} \right)^{1-\gamma}} - \frac{1}{1-\gamma} - \frac{(C^{o})^{1-\gamma}}{1+\psi} \omega_{W}^{o} e^{(1+\psi)\bar{l}_{t}(j^{o})}$$
(96)

where $\omega_B^o, \omega_D^o, \omega_W^o, \omega_T^o$ are the following non-stochastic steady-state ratios

$$\left(\omega_W^o, \omega_B^o, \omega_D^o, \omega_T^o\right) = \left(\frac{Z^o \tilde{W} \bar{L}^o}{C^o}, \frac{\tilde{B}^o}{C^o}, \frac{\tilde{D}^o}{C^o}, \frac{\tilde{T}^o}{C^o}\right).$$
(97)

Let x_t denote the variables appearing in the period utility function that optimizing household j^o can affect and let the vector ζ_t denote the variables that are taken as given such that

$$\boldsymbol{x}_{t} = \left(\tilde{b}_{t}(j^{o}), \bar{l}_{t}(j^{o}), \{\hat{c}_{t}(i, j^{o})\}_{i \in [0, 1)}\right)',$$
(98)

and

$$\boldsymbol{\zeta}_{t} = \left(r_{t-1}, \pi_{t}, \tilde{w}_{t}, \tilde{d}_{t}^{o}, \tilde{t}_{t}^{o}, \{ \hat{p}_{t}(i) \}_{i \in [0,1]} \right)'.$$
(99)

Additionally, we define a vector \boldsymbol{x}_{-1} of the same length as \boldsymbol{x}_t , which includes the variable $\tilde{b}_{-1}(j^o)$, the only variable not present in either \boldsymbol{x}_t or $\boldsymbol{\zeta}_t$

$$\boldsymbol{x}_{-1} = \left(\tilde{b}_{-1}(j^o), 0, \cdots\right)'.$$
 (100)

We define \mathcal{F}^o as the functional resulting from multiplying the period utility function, eq. (96), by β^t and summing over all t from zero to infinity.

Letting $\tilde{\mathcal{F}}^o$ denote the second-order Taylor approximaton of this functional evaluated at the non-stochastic steady-state, we get

$$E_{-1}(j^{o}) \left[\tilde{\mathcal{F}}^{o} \left(\boldsymbol{x}_{-1}, \boldsymbol{x}_{0}, \boldsymbol{\zeta}_{0}, \boldsymbol{x}_{1}, \boldsymbol{\zeta}_{1}, \cdots \right) \right]$$
(101)
$$= \begin{bmatrix} \mathcal{F}^{o} \left(\boldsymbol{0}, \boldsymbol{0}, \boldsymbol{0}, \cdots \right) \\ + \sum_{t=0}^{\infty} \beta^{t} \begin{pmatrix} \boldsymbol{h}_{2}^{t} \boldsymbol{x}_{t}^{t} + \boldsymbol{h}_{\zeta}^{t} \boldsymbol{\zeta}_{t} \\ + \frac{1}{2} \boldsymbol{x}_{t}^{t} \boldsymbol{H}_{\boldsymbol{x},-1} \boldsymbol{x}_{t-1} + \frac{1}{2} \boldsymbol{x}_{t}^{t} \boldsymbol{H}_{\boldsymbol{x}} \boldsymbol{x}_{t} + \frac{1}{2} \boldsymbol{x}_{t}^{t} \boldsymbol{H}_{\boldsymbol{x},1} \boldsymbol{x}_{t+1} \\ + \frac{1}{2} \boldsymbol{\zeta}_{t}^{t} \boldsymbol{H}_{\boldsymbol{\zeta}} \boldsymbol{\zeta}_{t} + \frac{1}{2} \boldsymbol{\chi}_{t}^{t} \boldsymbol{H}_{\boldsymbol{x},1} \boldsymbol{\zeta}_{t+1} \\ + \frac{1}{2} \boldsymbol{\zeta}_{t}^{t} \boldsymbol{H}_{\boldsymbol{\zeta}} \boldsymbol{\zeta}_{t} + \frac{1}{2} \boldsymbol{\zeta}_{t}^{t} \boldsymbol{H}_{\boldsymbol{\zeta}\boldsymbol{x},-1} \boldsymbol{x}_{t-1} + \frac{1}{2} \boldsymbol{\zeta}_{t}^{t} \boldsymbol{H}_{\boldsymbol{\zeta}\boldsymbol{x}} \boldsymbol{x}_{t} \end{pmatrix} \right]$$
$$+ \beta^{-1} \left(\boldsymbol{h}_{-1}^{\prime} \boldsymbol{x}_{-1} + \frac{1}{2} \boldsymbol{x}_{-1}^{\prime} \boldsymbol{H}_{-1} \boldsymbol{x}_{-1} + \frac{1}{2} \boldsymbol{x}_{-1}^{\prime} \boldsymbol{H}_{-1x} \boldsymbol{x}_{0} + \frac{1}{2} \boldsymbol{x}_{-1}^{\prime} \boldsymbol{H}_{-1\zeta} \boldsymbol{\zeta}_{0} \right) \end{bmatrix}$$

Here,

We can show that under regularity conditions Equation (101) and each of its elements converge to a finite elements in \mathcal{R} .

The process defining the vector of optimal actions for household j^{o} , noted \boldsymbol{x}_{t}^{*} , is defined by the following requirement

$$E_t(j^o)[h_x + H_{x,-1}x_{t-1}^* + H_xx_t^* + H_{x,1}x_{t+1}^* + H_{x\zeta}\zeta_t + H_{x\zeta,1}\zeta_{t+1}] = 0.$$
(102)

We can rearrange eq. (102) to obtain the following expression

$$E_t(j^o)[(\boldsymbol{x}_t - \boldsymbol{x}_t^*)'(\boldsymbol{h}_{\boldsymbol{x}} + \boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}\boldsymbol{\zeta}_t + \boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta},1}\boldsymbol{\zeta}_{t+1})] = -E_t(j^o)[(\boldsymbol{x}_t - \boldsymbol{x}_t^*)'(\boldsymbol{H}_{\boldsymbol{x},-1}\boldsymbol{x}_{t-1}^* + \boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}_t^* + \boldsymbol{H}_{\boldsymbol{x},1}\boldsymbol{x}_{t+1}^*)].$$
(103)

Next, using eq. (103) and $\mathbf{x}_{-1}^* = \mathbf{x}_{-1}$, and after some rearrangement, we derive an expression that quantifies the losses household j^o incurs due to suboptimal actions. We define this expression as household j^o 's objective function

$$E_{-1}(j^{o}) \left[\tilde{\mathcal{F}}^{o} \left(\boldsymbol{x}_{0}, \boldsymbol{\zeta}_{0}, \boldsymbol{x}_{1}, \boldsymbol{\zeta}_{1}, \cdots \right) \right] - E_{-1}(j^{o}) \left[\tilde{\mathcal{F}}^{o} \left(\boldsymbol{x}_{0}^{*}, \boldsymbol{\zeta}_{0}, \boldsymbol{x}_{1}^{*}, \boldsymbol{\zeta}_{1}, \cdots \right) \right]$$
(104)
$$= E_{-1}(j^{h}) \sum_{t=0}^{\infty} \left[\frac{1}{2} (\boldsymbol{x}_{t} - \boldsymbol{x}_{t}^{*}) \boldsymbol{H}_{\boldsymbol{x}} (\boldsymbol{x}_{t} - \boldsymbol{x}_{t}^{*}) + (\boldsymbol{x}_{t} - \boldsymbol{x}_{t}^{*}) \boldsymbol{H}_{\boldsymbol{x},1} (\boldsymbol{x}_{t+1} - \boldsymbol{x}_{t+1}^{*}) \right]$$

where

$$\boldsymbol{H}_{\boldsymbol{x}} = -(C^{o})^{1-\gamma} \begin{bmatrix} \gamma \omega_{B}^{2} (1+\frac{1}{\beta}) & -\gamma \omega_{B} \omega_{W} & 0 & \cdots & 0 \\ -\gamma \omega_{B} \omega_{W} & \omega_{W} (\omega_{W} \gamma + \psi) & 0 & \cdots & 0 \\ 0 & 0 & \frac{2}{\theta} di & \cdots & \frac{1}{\theta} di \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \frac{1}{\theta} di & \cdots & \frac{2}{\theta} di \end{bmatrix}, \quad (105)$$
$$\boldsymbol{H}_{\boldsymbol{x},1} = (C^{o})^{1-\gamma} \begin{bmatrix} \gamma \omega_{B}^{2} & -\gamma \omega_{B} \omega_{W} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix}. \quad (106)$$

and

$$\boldsymbol{x}_{t}^{*} = \begin{pmatrix} \frac{1}{\beta} (r_{t-1} - \pi_{t} + \tilde{b}_{t-1}^{*}(j^{o})) + \frac{\omega_{W}^{o}}{\omega_{B}^{o}} (z_{t}(j^{o}) + \tilde{w}_{t} + \bar{l}_{t}^{*}(j^{o})) + \frac{\omega_{D}^{o}}{\omega_{B}^{o}} \tilde{d}_{t}^{o} - \frac{\omega_{T}^{o}}{\omega_{B}^{o}} \tilde{t}_{t}^{o} - \frac{1}{\omega_{B}^{o}} c_{t}^{*}(j^{o}) \\ \frac{z_{t}(j^{o}) + \tilde{w}_{t} - \gamma c_{t}^{*}(j^{o})}{\psi} \\ -\theta(p_{t}(i) - p_{t}) \\ \vdots \end{pmatrix}$$
(107)

with

$$c_t^*(j^o) = E_t(j^o) \left[-\frac{1}{\gamma} (r_t - \pi_{t+1}) + c_{t+1}^*(j^o) \right] \text{ and } \omega_B^o > 0.$$
 (108)

E.4. Change of Variable. Equation (104) does not have the standard form of the objective in a dynamic attention problem because of the intertemporal interaction term. We therefore perform a change of variable that allows us to write down optimizing household j^o attention problem as a pure tracking problem.

We specifically focus on the 2 by 2 upper left elements of H_x and $H_{x,1}$. The remaining terms relative to cross-sectional efficiency are unaffected by the following manipulations.

Equation (104) then reads

$$E_{-1}(j)\sum_{t=0}^{\infty} \left[\frac{1}{2}(\bar{\boldsymbol{x}}_t - \bar{\boldsymbol{x}}_t^*)\bar{\boldsymbol{H}}_{\boldsymbol{x}}(\bar{\boldsymbol{x}}_t - \bar{\boldsymbol{x}}_t^*) + (\bar{\boldsymbol{x}}_t - \bar{\boldsymbol{x}}_t^*)\bar{\boldsymbol{H}}_{\boldsymbol{x},1}(\bar{\boldsymbol{x}}_{t+1} - \bar{\boldsymbol{x}}_{t+1}^*)\right]$$
(109)

with

$$\bar{\boldsymbol{x}}_t = \left(\tilde{b}_t(j^o), \bar{l}_t(j^o)\right)',\tag{110}$$

$$\bar{\boldsymbol{H}}_{\boldsymbol{x}} = -C^{1-\gamma} \begin{bmatrix} \gamma \omega_B^2 (1+\frac{1}{\beta}) & -\gamma \omega_B \omega_W \\ -\gamma \omega_B \omega_W & \omega_W (\omega_W \gamma + \psi) \end{bmatrix},$$
(111)

and

$$\bar{\boldsymbol{H}}_{\boldsymbol{x},1} = C^{1-\gamma} \begin{bmatrix} \gamma \omega_B^2 & -\gamma \omega_B \omega_W \\ 0 & 0 \end{bmatrix}$$
(112)

Substituing eqs. (110) to (112) in eq. (109), we obtain

$$C^{1-\gamma}E_{-1}(j)\sum_{t=0}^{\infty}\beta^{t} \begin{bmatrix} \gamma\omega_{B}^{2}(1+\frac{1}{\beta})(\tilde{b}_{t}(j^{o})-\tilde{b}_{t}^{*}(j^{o}))^{2} \\ -2\gamma\omega_{B}\omega_{W}(\tilde{b}_{t}(j^{o})-\tilde{b}_{t}^{*}(j^{o}))(\bar{l}_{t}(j^{o})-\bar{l}_{t}^{*}(j^{o})) \\ +\omega_{W}(\gamma\omega_{W}+\psi)(\bar{l}_{t}(j^{o})-\bar{l}_{t}^{*}(j^{o}))^{2} \\ +\gamma\omega_{B}^{2}(\tilde{b}_{t}(j^{o})-\tilde{b}_{t}^{*}(j^{o}))(\tilde{b}_{t+1}(j^{o})-\tilde{b}_{t+1}^{*}(j^{o})) \\ -\gamma\omega_{B}\omega_{W}(\tilde{b}_{t}(j^{o})-\tilde{b}_{t}^{*}(j^{o}))(\bar{l}_{t+1}(j^{o})-\bar{l}_{t+1}^{*}(j^{o})) \end{bmatrix}$$
(113)

Substracting household j^o linearized budget constraints evaluated at $\bar{\bm{x}}_t$ and $\bar{\bm{x}}_t^*$ we get

$$\omega_B(\tilde{b}_t(j^o) - \tilde{b}_t^*(j^o)) = \frac{\omega_B}{\beta}(\tilde{b}_{t-1}(j^o) - \tilde{b}_{t-1}^*(j^o)) - (c_t(j^o) - c_t^*(j^o)) + \omega_W(\bar{l}_t(j^o) - \bar{l}_t^*(j^o))$$
(114)

We can define the right-hand-side of eq. (114) as a new variable proportional to mistakes in real bond holdings that reads

$$\Delta_t = \omega_B(\tilde{b}_t(j^o) - \tilde{b}_t^*(j^o)).$$
(115)

Morevover, we can decompose Δ_t into two components, one reflecting mistakes in consumption and another specific to errors in labor supply such that

$$\Delta_t^c = \frac{1}{\beta} \Delta_{t-1}^c - (c_t(j^o) - c_t^*(j^o))$$
(116)

and

$$\Delta_t^{\bar{l}} = \frac{1}{\beta} \Delta_{t-1}^{\bar{l}} + \omega_W(\bar{l}_t(j^o) - \bar{l}_t^*(j^o))$$
(117)

with $\Delta_{-1}^c = 0$ and $\Delta_{-1}^{\bar{l}} = 0$. By assumptions, we also have $\tilde{b}_{-1}(j^o) - \tilde{b}_{-1}^*(j^o) = 0$. Therefore, mistakes in real bond holdings read

$$\Delta_t = \Delta_t^c + \Delta_t^{\bar{l}}.\tag{118}$$

Substituing eq. (115) and eq. (118) in eq. (113) yields

$$C^{1-\gamma}E_{-1}(j)\sum_{t=0}^{\infty}\beta^{t}\begin{bmatrix} -\frac{1}{2}\begin{pmatrix} \gamma(1+\frac{1}{\beta})(\Delta_{t}^{c}+\Delta_{t}^{\bar{l}})^{2} \\ -2\gamma\omega_{W}(\Delta_{t}^{c}+\Delta_{t}^{\bar{l}})(\bar{l}_{t}(j^{o})-\bar{l}_{t}^{*}(j^{o})) \\ +\gamma\omega_{W}^{2}(\bar{l}_{t}(j^{o})-\bar{l}_{t}^{*}(j^{o}))^{2}+\psi\omega_{W}(\bar{l}_{t}(j^{o})-\bar{l}_{t}^{*}(j^{o}))^{2} \end{pmatrix} \\ +\gamma(\Delta_{t}^{c}+\Delta_{t}^{\bar{l}})(\Delta_{t+1}^{c}+\Delta_{t+1}^{\bar{l}}) \\ -\gamma\omega_{W}(\Delta_{t}^{c}+\Delta_{t}^{\bar{l}})(\bar{l}_{t+1}(j^{o})-\bar{l}_{t+1}^{*}(j^{o})) \end{bmatrix}.$$
(119)

Next, we use eq. (117) to substitute for the term $(\bar{l}_{t+1}(j^o) - \bar{l}_{t+1}^*(j^o))$ and the first term that contains $(\bar{l}_t(j^o) - \bar{l}_t^*(j^o))$ in eq. (119), we obtain

$$C^{1-\gamma}E_{-1}(j)\sum_{t=0}^{\infty}\beta^{t}\begin{bmatrix} \gamma(1+\frac{1}{\beta})(\Delta_{t}^{c}+\Delta_{t}^{\bar{l}})^{2} \\ -2\gamma(\Delta_{t}^{c}+\Delta_{t}^{\bar{l}})(\Delta_{t}^{\bar{l}}-\frac{1}{\beta}\Delta_{t-1}^{\bar{l}}) \\ +\gamma(\Delta_{t}^{\bar{l}}-\frac{1}{\beta}\Delta_{t-1}^{\bar{l}})^{2}+\psi\omega_{W}(\bar{l}_{t}(j^{o})-\bar{l}_{t}^{*}(j^{o}))^{2}) \\ +\gamma(\Delta_{t}^{c}+\Delta_{t}^{\bar{l}})(\Delta_{t+1}^{c}+\Delta_{t+1}^{\bar{l}}) \\ -\gamma(\Delta_{t}^{c}+\Delta_{t}^{\bar{l}})(\Delta_{t+1}^{\bar{l}}-\frac{1}{\beta}\Delta_{t}^{\bar{l}}) \end{bmatrix}.$$
(120)

Rearranging, we obtain

$$C^{1-\gamma}E_{-1}(j)\sum_{t=0}^{\infty}\beta^{t}\begin{bmatrix} -\frac{\gamma}{2}(1+\frac{1}{\beta})(\Delta_{t}^{c})^{2}+\gamma\Delta_{t}^{c}\Delta_{t+1}^{c}\\ +\gamma\left[\Delta_{t}^{\bar{l}}\Delta_{t+1}^{c}-\frac{1}{\beta}\Delta_{t-1}^{\bar{l}}\Delta_{t}^{c}\right]\\ +\frac{\gamma}{2\beta}\left[(\Delta_{t}^{\bar{l}})^{2}-\frac{1}{\beta}(\Delta_{t-1}^{\bar{l}})^{2}\right]\\ -\frac{\omega_{W}\psi}{2}(\bar{l}_{t}(j^{o})-\bar{l}_{t}^{*}(j^{o}))^{2}\end{bmatrix}.$$
(121)

We can rewrite the first first two terms as follows

Substituing in eq. (121), we get

$$C^{1-\gamma}E_{-1}(j)\sum_{t=0}^{\infty}\beta^{t}\begin{bmatrix} -\frac{\gamma}{2}(c_{t}(j^{o})-c_{t}^{*}(j^{o}))^{2} \\ +\frac{\gamma}{2\beta}\left[(\Delta_{t}^{c})^{2}-\frac{1}{\beta}(\Delta_{t-1}^{c})^{2}\right] \\ +\frac{\gamma}{2\beta}\left[(\Delta_{t}^{\bar{l}})^{2}-\frac{1}{\beta}(\Delta_{t-1}^{\bar{l}})^{2}\right] \\ +\gamma\left[\Delta_{t}^{c}(c_{t+1}(j^{o})-c_{t+1}^{*}(j^{o}))-\frac{1}{\beta}\Delta_{t-1}^{c}(c_{t}(j^{o})-c_{t}^{*}(j^{o}))\right] \\ -\frac{\omega_{W}\psi}{2}(\bar{l}_{t}(j^{o})-\bar{l}_{t}^{*}(j^{o}))^{2} \end{bmatrix}.$$
(122)

 $\begin{array}{l} \text{Comparing terms in consecutive periods, using } \lim_{T \to \infty} \beta^T E_{-1}(j^o) \left[(\Delta_T^c)^2 \right] = \lim_{T \to \infty} \beta^T E_{-1}(j^o) \left[(\Delta_T^{\bar{l}})^2 \right] \\ = \lim_{T \to \infty} \beta^T E_{-1}(j^o) \left[\Delta_T^c \Delta_{T+1}^{\bar{l}} \right] = \lim_{T \to \infty} \beta^T E_{-1}(j^o) \left[\Delta_T^c(c_{T+1}(j^o) - c_{T+1}^*(j^o)) \right] \text{ and } \Delta_{-1}^c = \\ \Delta_{-1}^l \bar{l} = 0 \text{ yields} \end{array}$

$$-C^{1-\gamma}E_{-1}(j)\sum_{t=0}^{\infty}\beta^{t}\left[\frac{\gamma}{2}(c_{t}(j^{o})-c_{t}^{*}(j^{o}))^{2}+\frac{\omega_{W}\psi}{2}(\bar{l}_{t}(j^{o})-\bar{l}_{t}^{*}(j^{o}))^{2}\right].$$
 (123)

Under matricial form we get,

$$E_{-1}(j)\sum_{t=0}^{\infty}\beta^{t}\left[\frac{1}{2}(\tilde{\boldsymbol{x}}_{t}-\tilde{\boldsymbol{x}}_{t}^{*})'\tilde{\boldsymbol{H}}_{\boldsymbol{x}}(\tilde{\boldsymbol{x}}_{t}-\tilde{\boldsymbol{x}}_{t}^{*})\right].$$
(124)

where

$$\tilde{\boldsymbol{H}}_{\boldsymbol{x}} = -C^{1-\gamma} \begin{bmatrix} \gamma & 0\\ 0 & \omega_W \psi \end{bmatrix}, \qquad (125)$$

$$\tilde{\boldsymbol{x}}_t = \left(c_t(j^o), \bar{l}_t(j^o)\right)', \qquad (126)$$

and

$$\tilde{\boldsymbol{x}}_{t}^{*} = \begin{pmatrix} E_{t}(j^{o}) \left[-\frac{1}{\gamma} (r_{t} - \pi_{t+1}) + c_{t+1}^{*}(j^{o}) \right] \\ \frac{z_{t}(j^{o}) + \tilde{w}_{t} - \gamma c_{t}^{*}(j^{o})}{\psi} \end{pmatrix}$$
(127)

with $c_{t+1}^*(j^o)$ defined accordingly to eq. (108).

Appendix F. Estimating the Fraction of Hand-to-mouth

We estimate the share of hand-to-mouth in the economy using the SCF, a triennial survey conducted by the Federal Reserve Board that collects detailed information on household finances in the United States. It covers a representative sample of over 4,500 households and provides data on income, assets and debts. All variables are nominal and expressed in 2022 dollars.

First, we define the conditions under which a household classifies as hand-tomouth

Definition 4. A household is considered hand-to-mouth if it either has

- zero liquid wealth
- or its credit limit is binding.

These are standard in the literature and derive from the endogenous behavior of households in HA incomplete-market models. As shown in Kaplan and Violante (2014), a household at a kink in its budget constraint, either at zero liquid wealth or the credit limit, exhibits a strong propensity to consume, which typically translates into allocating all of its period income to consumption and debt payments (if applicable), but none to savings. Hand-to-mouth households in our simplified framework display the exact same behavior.

Kaplan, Violante, and Weidner (2014) propose identifying hand-to-mouth households in the data using two inequality conditions,

1.
$$0 \le m_t(i) \le \frac{w_t(i)}{2f}$$

2. $m_t(i) \ge 0$ and $m_t(i) \le \frac{w_t(i)}{2f} - \underline{m}_t(i)$

where $m_t(i)$ represents net liquid wealth, $w_t(i)$ denotes monthly income, and $f \geq 1$ is the frequency at which a household receives payments during a month³⁷. The first inequality identifies households with zero liquid wealth, while the second identifies those at their credit limit.

We estimate monthly income by dividing reported annual income by 12. To measure net liquid wealth, we use SCF microdata on households' finances. The composition of liquid assets and debt is defined in Table 5. We follow Kaplan and Violante (2014) and inflate the value of transactional accounts (LIQ) by a factor of

³⁷Income is measured monthly because, when f = 1, it represents the lowest plausible payment frequency. For more details on the estimator, see Kaplan, Violante, and Weidner (2014)

1.05 to account for cash holdings that are not reported in the SCF. We also only count as debt revolving credit card debt (i.e., credit card balances that are not repaid in full at every payment). This avoids including as debt purchases made through credit cards in between regular payments (see e.g., Telyukova (2013)). To do so, we multiply credit card balances after the last payment (CCBAL) by an indicator function that takes the value one if respondents answered either "sometimes" or "almost never" to the question "After the last payment, roughly what was the balance still owed on these accounts?". To ensure consistency with our model, we exclude households with zero wage income, those with a respondent under 22 or over 79 years old, and those above the 95th income percentile.

Category	Mnemonic
Liquid Assets	
All types of transaction account Directly held pooled investment funds Directly held stocks	LIQ NMMF STOCKS
Liquid Debt	
Credit card balances after last payment	CCBAL

Table 5: Assets and Debt in the SCF

Note: This is a conservative definition of liquid wealth, the same as in Kaplan and Violante (2014). One could extend it to include assets with a lesser degree of liquidity.

Table 6 reports our estimates of the fraction of hand-to-mouth households.

In this section, we omit the measurement of illiquid wealth and therefore do not identify "wealthy" hand-to-mouth. This does not imply that we believe illiquid wealth to be irrelevant. However, incorporating multiple assets and adjustment costs (accessing illiquid wealth often incurs penalties) would introduce additional layers of complexity into the RI-DSGE framework, which we leave for future research.

Appendix G. Calibration

We use our estimates of hand-to-mouth households in Appendix F to calibrate parameters of the model. For our benchmark parametrization, we rely on the 2022 and most recent wave of the SCF. We use the fraction of hand-to-mouth households estimated assuming a monthly pay frequency, even though approximately half of

	Week	Bi-Week	Month
All hand-to-mouth			
SCF 1989	0.128	0.117	0.293
SCF 1992	0.176	0.237	0.325
SCF 2001	0.127	0.179	0.275
SCF 2010	0.170	0.241	0.341
SCF 2022	0.120	0.179	0.273
Hand-to-mouth not at their credit limit			
SCF 1989	0.661	0.183	0.208
SCF 1992	0.079	0.131	0.204
SCF 2001	0.066	0.110	0.192
SCF 2010	0.104	0.165	0.249
SCF 2022	0.071	0.121	0.202
Hand-to-mouth at their credit limit			
SCF 1989	0.062	0.066	0.085
SCF 1992	0.098	0.106	0.121
SCF 2001	0.061	0.069	0.083
SCF 2010	0.066	0.076	0.092
SCF 2022	0.049	0.058	0.071

Table 6: Estimates of Hand-to-mouth Households

Note: Entries are fractions of the total households with positive wage income and below the 95^{th} income percentile. The labels Week, Bi-Week, and Month refer to the assumptions on the frequency of pay.

	Wage Income	Income	Net Liquid Wealth
All Hand-to-mouth			
q1	28,103	37,831	-134
q2	49,721	$59,\!450$	420
q3	$85,\!392$	$98,\!362$	1,535
Hand-to-mouth not at their Credit Limit			
q1	28,101	36,751	294
q2	50,802	$59,\!450$	900
q3	$87,\!553$	103767	2,098
Hand-to-mouth at their Credit Limit			
q1	28,103	39,993	-5,305
q2	48,641	$59,\!450$	-1,980
q3	$75,\!663$	90,796	-576
Non Hand-to-mouth			
q1	46,479	59,450	4,967
q2	$86,\!472$	$104,\!848$	21,000
q3	$151,\!327$	$182,\!674$	103,425

Table 7: Statistics from SCF 2022

Note: q1, q2, and q3 denote the first, second (median), and third quantiles, respectively.

households report being paid bi-weekly. We justify this choice because our estimator provides a lower bound. It misses households transitioning into hand-to-mouth status and fails to account for committed consumption expenditures, such as rent, that typically occur at the end of a period.

First, we measure wage income and net liquid assets within each group. For both variables, we take the median. Wage income is directly reported in the survey, while net liquid assets are computed for each household using the definition provided in Table 5. Our model implies that the unconditional mean of income shocks can be measured directly from the wage income ratio.

The survey does not provide sufficient information to compute consumption expenditure directly. Therefore, we rely on a proxy. Following the strategy used by Maćkowiak and Wiederholt (2015), we first apply the tax rate schedule³⁸ to median total income within each group. Next, we compute the total savings required to maintain median (liquid) net worth constant at an annual inflation rate of 2.5%. Consumption expenditure is obtained by subtracting median savings from after-tax median income for each group. Using consumption expenditures and the share of hand-to-mouth households, we estimate quarterly nominal output, which allows us to compute the ratio of bond holdings to output.

We calibrate the share of aggregate taxes paid by the hand-to-mouth by computing the ratio of their median taxes to the weighted sum of the median taxes paid by each group. The share of dividends received by the hand-to-mouth group is not directly observable. We approximate it by computing the fraction of total business equity owned by hand-to-mouth households relative to the total business equity in the economy.

Appendix H. Additional Figures

³⁸Specifically, we apply the tax rate for "married filing jointly". Tax schedule for a given year can be found at https://www.irs.gov/pub/irs-prior.

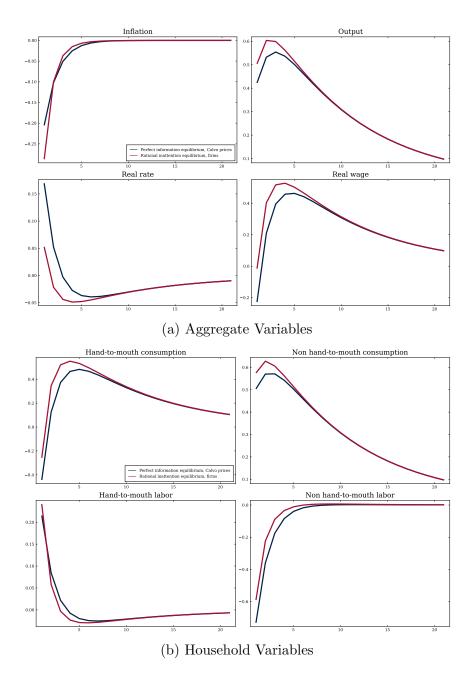


Figure 5: Impulse Responses to a Technology Shock Note: IRFs computed from the models' MA representations.

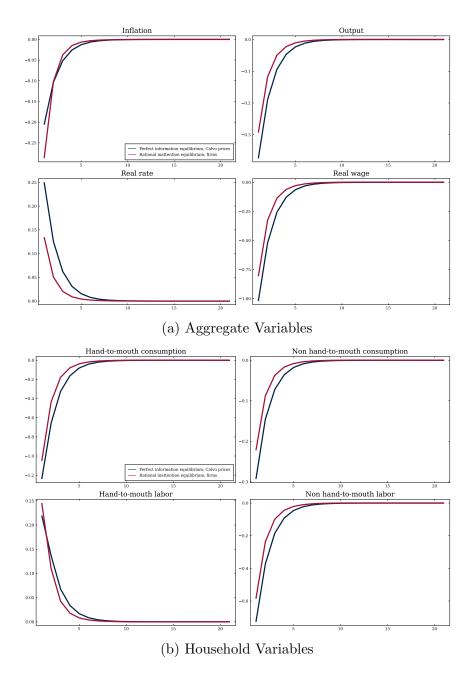


Figure 6: Impulse Responses to a Monetary Policy Shock Note: IRFs computed from the models' MA representations.

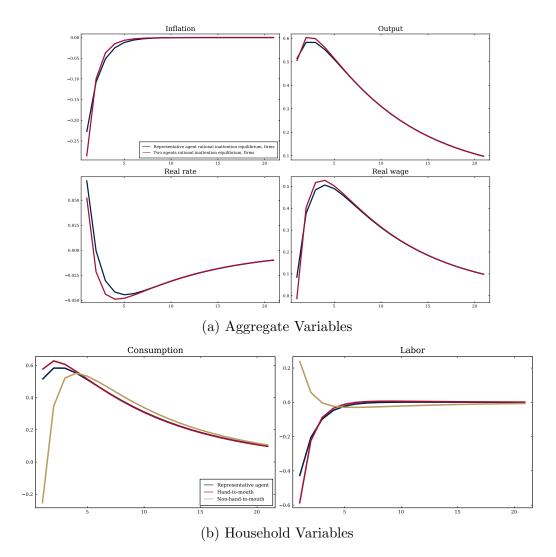


Figure 7: Impulse Responses to a Technology Shock Note: IRFs computed from the models' MA representations.

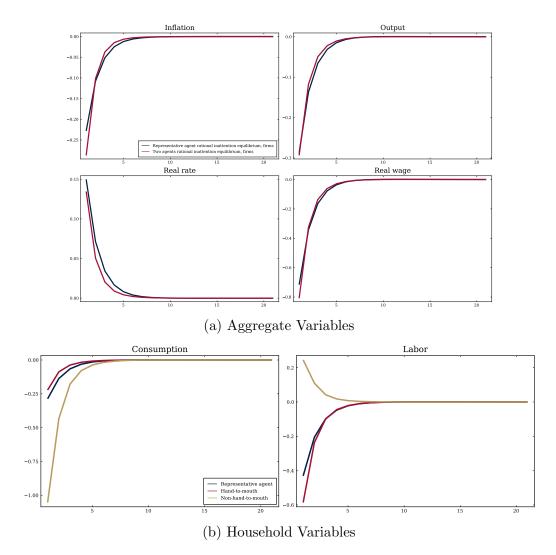


Figure 8: Impulse Responses to a Monetary Policy Shock Note: IRFs computed from the models' MA representations.

Parameters	Interpretation	
	Structural Parameters	
ϕ	Fraction of HtM to all households	0.27
ϑ_T	Fraction of aggregate taxes paid by HtM	0.15
	NSSS ratios	
Z^o/Z^h	Unconditional mean ratio of income shocks	1.74
$\frac{Z^o/Z^h}{\tilde{B^h}/Y}$	Real debt-to-output ratio for HtM	-0.0
$\tilde{B^o}/Y$	Real bond holdings-to-output ratio for Opt.	1.05

Table 8: Parameters calibrated based on the 2022 SCF

We cannot use the household-specific ratios observed in the data directly, as in Maćkowiak and Wiederholt (2015), due to how they are interdependent within the model.