The Young, the Old, and the Government: Demographics and Fiscal Multipliers*

Henrique S. Basso
Banco de España

Omar Rachedi
Banco de España

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Abstract

We document that fiscal multipliers depend on the age structure of the population. Using the variation in military spending and birth rates across U.S. states, we show that local fiscal multipliers increase with the share of young people in total population. We rationalize this fact with a parsimonious life-cycle open-economy New Keynesian model with credit market imperfections and age-specific labor supply elasticities. The model explains 65% of the relationship between local fiscal multipliers and demographics. We use the model to study the implications of population aging, and find that nowadays U.S. national fiscal multipliers are 36% lower than in 1980.

Key Words: Life-cycle, Population Aging, Fiscal Policy.

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*Addresses: henrique.basso@bde.es and omar.rachedi@bde.es. The views expressed in this paper are those of the authors and do not necessarily represent the views of the Banco de España or the Eurosystem. We thank Florian Bilbiie, Jesus Fernandez-Villaverde, Basile Grassi, Alexandre Janiak, Matthias Kredler, Alisdair McKay, Claudio Michelacci, Nicola Pavoni, William Peterman, Emiliano Santoro, Harald Uhlig, Ernesto Villanueva, Minchul Yum, and presentation participants at the Pontificia Universidad Catolica de Chile, University of Copenhagen, Bank of Canada, Federal Reserve Bank of New York, Stony Brook University, CEFER at the Bank of Lithuania, the RES Conference in Bristol, the IBEO Workshop in Alghero, the CEF Conference in New York, the EEA-ESEM Meeting in Lisbon, the Money Macro and Finance Conference in London, the Macro Banking and Finance Workshop in Milan, the ESCB Research Cluster Workshop on "Medium and long-run challenges for Europe", and the SAEe Meeting in Barcelona for useful comments and suggestions.

1 Introduction

Every time a government considers a plan of fiscal stimulus or fiscal consolidation, there is a strong debate among policymakers, journalists, and economists on the effectiveness of such a policy. This effectiveness is often summarized by the size of the fiscal multiplier, which measures by how much output expands following a rise in government spending. Nevertheless, fiscal multipliers are not constant structural parameters, but rather they depend on the characteristics of the economy.

This paper sheds light on a novel determinant of the size of fiscal multipliers: the age structure of an economy. We study a panel of output, government military spending, and demographic characteristics across U.S. states and document that local fiscal multipliers rise with the share of young people in total population. We show that a parsimonious open-economy life-cycle New Keynesian model with credit market imperfections and age-specific labor supply elasticities explains 65% of the link between local fiscal multipliers and demographics. Then, we use the model to study the implications of population aging and find that nowadays U.S. national fiscal multipliers are 36% lower than in 1980.

We focus on the differences across U.S. states to uncover the causal effect of demographics on fiscal multipliers. The identification comes from the cross-state variation in the share of young people in total population. As states' age structure can respond to government spending shocks through migration flows, we exploit the heterogeneity in fertility across U.S. states and instrument the share of young people with lagged birth rates. Then, we identify the government spending shocks by using the geographical distribution of government military spending, as in Nakamura and Steinsson (2014). Usually, the literature on national military spending and fiscal multipliers identifies government spending shocks by assuming that the U.S. do not embark in a war when national output is low (Barro, 1981; Barro and Redlick, 2011; Ramey, 2011). Instead, we refer to a much weaker exogeneity restriction and posit that the U.S. do not embark in a war when the output of a specific state is lower than the output of all the other states.

In our benchmark regression, the size of fiscal multipliers depends positively on the share of young people (aged 20 - 29) in total population: increasing the share of young people by 1% above the average share across U.S. states raises the local output fiscal multiplier by 3.1%, from 1.51 up to 1.56. These estimates imply an inter-quantile range of output fiscal multipliers across U.S. states that varies between 1.27 and 1.65. We run a comprehensive battery of robustness checks and find that the age sensitivity of local fiscal multipliers is always highly economically and statistically significant.

To rationalize the link between demographics and fiscal multipliers, we build a life-cycle open-economy New Keynesian model with credit market imperfections and age-specific labor supply elasticities. We consider a staggered price setting model with two countries that belong to a monetary union. The household sector has a life-cycle structure, whereby individuals face three stages of life: young, mature, and old. Following Gertler (1999), we define a framework in which the optimal choices of the individuals within each age group aggregate linearly. Although this approach reduces the relevance of differences *within* age groups, it allows us to emphasize the heterogeneity *across* age groups and incorporate nominal rigidities and open economy interactions into a tractable environment. In this way, our model extends a standard two-country New Keynesian economy with a rich life-cycle structure.

The model features credit market imperfections. Households can trade capital and bonds but cannot perfectly smooth consumption because markets are incomplete. In the baseline model, we restrict further households' borrowing capacity with an ad-hoc constraint which does not allow any borrowing at all. Then, we consider age-specific differences in labor supply such that the labor supply elasticity varies exogenously across the three age groups. In the empirically relevant case, young and old workers have a higher labor supply elasticity than mature workers. These differences capture the fact that in the data the hours worked by young and old households are much more volatile than the hours worked by mature households.

In the model government spending triggers a negative income effect for the households, which smooth consumption by working longer hours. The rise of labor - and thus the size of fiscal multipliers - depends on households' labor supply elasticity and marginal propensity to consume. Moreover, price rigidities define a demand channel through which government spending raises even further employment and output.

How can demographics alter fiscal multipliers? The link is twofold. First, the high labor supply elasticity of young workers makes young employment much more responsive to government spending shocks than the employment of mature workers. Second, an economy with relatively more young households features a stronger demand channel. Since young households face a hump-shaped labor income over the life-cycle, they want to borrow and smooth lifetime consumption. Yet, this mechanism is limited by the presence of credit market imperfections, which boost the marginal propensity to consume of young households well above the one of mature households, as it is in the data.[1] Consequently, as the proportion of young workers increases, both labor and output react more sharply to a fiscal shock.

In the quantitative analysis, the model explains 92% of the size of fiscal multipliers and 65% of the link between fiscal multipliers and demographics: increasing the share of young people by 1% above the average share across U.S. states raises the local output fiscal multiplier by 2%, from 1.39 up to 1.42. The age sensitivity of local multipliers depends mostly on credit market imperfections. Indeed, when we eliminate the differences in the labor supply elasticity, the age sensitivity drops just by 10%, from 2% to 1.8%. Instead, when we also remove the ad-hoc borrowing constraint and let young households to borrow, the age sensitivity equals 0.9%. Hence,

[1] Young households have a number of characteristics associated with a higher marginal propensity to consume. For instance, young households own much less liquid assets than older households and the marginal propensity to consume depends negatively on the amount of liquid assets (Kaplan et *al.*, 2014; Misra and Surico, 2014).

even in absence of the ad-hoc borrowing constraint, the lack of complete markets in a life-cycle setting can generate the age sensitivity of local multipliers.

Does the link between demographics and fiscal multipliers exist also at the national level? Although our evidence shows that the effect of demographics on fiscal multipliers at the state level is economically and statistically significant, this result does not necessarily imply that demographics alter also national multipliers.[2] We evaluate in the model the effects of government spending on *national* output and find that demographics still matter: increasing the share of young people by 1% raises the national output fiscal multiplier by 1.1%.

Finally, we study the implications of the U.S. population aging for fiscal policy. After the post-World War II baby boom, the demographic structure of the U.S. population has progressively shifted towards older ages: the share of young people in total population plummeted by 30% from 1980 to 2015. Once we feed this shift in population shares into our model, we find that nowadays the national output fiscal multiplier is 36% lower than forty years ago. Since most advanced economies are experiencing a gradual population aging, the model suggests that over time fiscal policy could become a relatively less effective tool to spur economy activity.[3]

This paper is related to the literature that focuses on the implications of demographics for long-run trends (Krueger and Ludwig, 2007; Aksoy et *al.*, 2015; Carvalho et *al.*, 2016), and short-term fluctuations (Jaimovich and Siu, 2009; Wong, 2016). The implications of demographics for the aggregate effects of fiscal policy have been highlighted by Anderson et *al.* (2016), Janiak and Santos-Monteiro (2016), and Ferraro and Fiori (2016). Our paper differs from this strand of the literature on two main dimensions. First, we focus on the elasticity of output to fiscal

[2]Nakamura and Steinsson (2014) and Chodorow-Reich (2017) show that local fiscal multipliers consider the local impact of federally financed policies and wash out any monetary policy response to government spending. Both features make local multipliers larger than national ones. On the other hand, local multipliers are dampened by expenditure switching and import leakage effects that do not take place at the national level.

[3]This result refers to the effectiveness of fiscal policy in normal times. The literature has highlighted cases in which fiscal multipliers are very high, e.g., when the economy is at zero lower bound (Christiano et al., 2011; Woodford, 2011) or there is slack in the economy (Auerbach and Gorodnichenko, 2012; Rendahl, 2016).

shocks. Following Nakamura and Steinsson (2014) and Chodorow-Reich (2017), we exploit the heterogeneity across U.S. states and estimate the causal effect of demographics on fiscal multipliers.[4] Second, we build a quantitative model that can be used as a laboratory to measure the effects of changes in the age structure of the economy on fiscal multipliers.

2 Empirical Evidence

This Section shows that local fiscal multipliers depend on demographics: fiscal multipliers are larger in states with higher shares of young people in total population.

We study a panel of output, government military spending, and demographic characteristics across U.S. states. To estimate the effect of government spending on output - and how this effect depend on the age structure of each state - we use the variation across U.S. states in both military buildups and birth rates. This procedure identifies the *local fiscal multiplier*, which is a federally-financed open-economy relative multiplier. This multiplier estimates the response of output in a specific state (say, California) relative to the response of output of all the other U.S. states when the federal government spends one extra dollar in California, and this dollar is financed by taxing individuals in all U.S. states.

2.1 Data

We build a data set of government military spending, output, and demographic characteristics across the 50 U.S. states and the District of Columbia at the annual frequency from 1967 until 2015.

We complement the data on the geographical distribution of military spending of Nakamura and Steinsson (2014) with information from the Statistical Abstract

[4]Anderson et *al.* (2016) and Ferraro and Fiori (2016) derive the responses of consumption and unemployment across age groups to national fiscal shocks identified with the narrative approach of Romer and Romer (2010).

of the U.S. Census Bureau and the website usaspending.org of the U.S. Office of Management and Budget. The data cover any procurement of the U.S. Department of Defense above 10,000$ up to 1983, and above 25,000$ from 1983 on.[5]

State output is the state GDP series of the Bureau of Economic Analysis (BEA). State employment is taken from the Current Employment Statistics of the Bureau of Labor Statistics (BLS). The data on state population and births rates are from the Census Bureau. The data on births rates are from 1930 onwards. The birth rates of Alaska and Hawaii are available only from 1960 onwards. The data on the state demographic structure by age, race, and sex are from the Survey of Epidemiology and End Results of the National Cancer Institutes.

2.2 Econometric Specification

We estimate the causal effect of demographics on local output fiscal multipliers using the following panel regression:

$$\frac{Y_{i,t} - Y_{i,t-2}}{Y_{i,t-2}} = \alpha_i + \delta_t + \beta \frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} + \gamma \frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} \left(D_{i,t} - \bar{D} \right) + \zeta D_{i,t} + \epsilon_{i,t} \quad (1)$$

where $Y_{i,t}$ denotes per capita output in state i at time t, $G_{i,t}$ refers to per capita federal military spending allocated to state i at time t, $D_{i,t}$ is the log-share of young people over total population in state i at time t, $\bar{D} = \sum_i \sum_t D_{i,t}$ is the average log-share of young people, α_i is a state fixed effect, and δ_t denotes time fixed effects. The fixed effects capture any state-specific trend in output, government spending, and demographics, and control for aggregate shocks, such as variations in the national monetary policy stance.[6]

In the baseline regression we consider the share of young people as the ratio of

[5]Nakamura and Steinsson (2014) show that military procurements tend not to be subcontracted to firms in different states from the original recipient.

[6]Following Nakamura and Steinsson (2014), we consider two-year changes in output and government spending to capture in a parsimonious way the dynamic effects of fiscal policy.

20-29 years old white males over the total population of white males. We focus on the white male population to avoid that the different trends across U.S. states in the labor participation of female workers and workers of other racial groups could be confounding factors that spuriously drive the effect of changes in the age structure of the population on the size of local fiscal multipliers. In the robustness checks, we show that our results do not change if we consider either all males or the entire population of 20-29 years old individuals.[7]

In Equation (1) the coefficient β denotes the local output fiscal multiplier: it defines by how much a 1% increase in federal government spending raises output per capita in a state with the average share of young people. The parameter γ is associated to our regressor of interest, which is the interaction between changes in federal government spending and the share of young people in total population. This parameter defines how fiscal multipliers vary with the age structure of a state: when the share of young people rises by 1% above the average, the fiscal multiplier increases from β up to $\beta + \gamma$.

We also estimate the effect of government spending on state employment rate with a similar regression, in which the dependent variable is the growth rate of state employment rate $E_{i,t}$:

$$\frac{E_{i,t} - E_{i,t-2}}{E_{i,t-2}} = \alpha_i + \delta_t + \beta \frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} + \gamma \frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} \left(D_{i,t} - \bar{D}\right) + \zeta D_{i,t} + \epsilon_{i,t}. \quad (2)$$

We identify government spending shocks following the approach of Nakamura and Steinsson (2014), which exploits the heterogeneous sensitivity of states' military procurements to an increase in federal military spending.[8] This IV strategy implies a first stage regression in which per capita state military procurement (as a fraction

[7]Appendix A.4 shows that lagged births rates are a more relevant instrument for the share of young white males rather than for either the share of young males or the share of overall young people.

[8]E.g., federal military spending as a fraction of national GDP dropped by 1.5% following the U.S. withdrawal from Vietnam. The withdrawal had large heterogeneous effects across U.S. states: in California federal military procurements as a fraction of the state GDP decreased by 2.5%, while Illinois experienced a drop of just 1%.

of per capita state GDP) is regressed against the product of per capita national military spending (as a fraction of per capita national GDP) and a state fixed effect:

$$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} = \alpha_i + \delta_t + \eta_i \frac{G_t - G_{t-2}}{Y_{t-2}} + \zeta X_{i,t} + \epsilon_{i,t} \tag{3}$$

where $X_{i,t}$ includes the instruments for both the share of young people and its interaction with the changes in government spending. The coefficient η_i captures the heterogeneous exposure of each state to a rise in federal military spending.

The use of military spending to estimate national fiscal multipliers follows the work of Barro (1981), Barro and Redlick (2011), and Ramey (2011), among many others. This strand of the literature considers national military spending as exogenous. The implicit assumption is that the U.S. do not embark in a war because national output is low. Our instrumenting approach relies on a much weaker exogeneity restriction: we posit that the U.S. do not embark in a war because the output of a specific state is lower relatively to the output of all the other states.

Then, we evaluate whether the effects of government spending shocks on output and employment depend on states' age structure. The panel dimension of the data is crucial to identify the link between demographics and fiscal multipliers. Since our baseline regression features state and time fixed effects, the identification comes from the cross-state variation - and its changes over time - in the share of young people in total population. At any point of time, there is a large dispersion across states in the share of young people. For instance, in 2015 the share of young people ranges between the 11.9% of Maine and the 22.6% of D.C. Moreover, the relative ranking across states has been changing over time. As an example, in 1980 New York had the fourth lowest share of young people in the U.S. Yet, in 2015 the share of young people of New York has become the tenth highest in the U.S.

States' age structure would not be exogenous to government spending shocks if they trigger migration flows.[9] To avoid any concern on the endogeneity of demographics, we follow Shimer (2001) and instrument the share of young people with lagged birth rates.[10] This IV strategy allows us to identify the causal effect of states' age structure on fiscal multipliers. In our baseline specification, we instrument the share of young people with 20-30 year lagged birth rates: we use the average birth rate between 1940 and 1950 to instrument the share of young people in 1970.[11] Our implicit exclusion restriction posits that, conditional on state and time fixed effects, whatever determines the cross-sectional variation in births rates has no other long lasting effect on the size of fiscal multipliers 20-30 years later. This IV approach would not be valid if the sensitivity to federal government shocks - i.e., η_i of Equation (3) - is related to states' age structure. We find that in the data the correlation between states' demographic structures and sensitivity to federal government shocks is -0.03, corroborating our identification approach.

2.3 Results

Table 1 reports the results of the benchmark regressions estimated using instrumental variables for both military spending and the share of young workers.

Column (1) refers to the regression in which the dependent variable is the change in output per capita. The first entry shows that the local output fiscal multiplier for a state with an average share of young people (e.g., Massachusetts and Nevada) is statistically significant at the 1% level and equals 1.51. Also the estimated value of the parameter γ associated with the interaction term is highly statistically significant, with a p-value of 0.005. The value of the estimated parameter points out that the effect of demographics on local output fiscal multipliers is also highly eco-

[9] Blanchard and Katz (1992) show that state migration reacts to shocks. We find that although total population does not change following government spending shocks, the population of young people does rise.

[10] Appendix A.4 shows that lagged birth rates explain the bulk of the variability of the age structure of the population across states and time.

[11] The birth rates for Alaska and Hawaii start in 1960. The results do not change if we consider either an unbalanced panel of birth rates, or we use 10 year lagged birth rates for Alaska and Hawaii.

nomically significant: increasing the share of young people by 1% above the average raises output fiscal multipliers by 3.1%, from 1.51 up to 1.56. These estimates imply an inter-quantile range of output fiscal multipliers across U.S. states that varies between the values of 1.27 in Ohio and 1.65 in Arizona.

Table 1: Response to a Government Expenditure Shock across U.S. States

	(1) Output per Capita	(2) Employment Rate
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}}$	1.511***	1.095***
	(0.406)	(0.215)
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}} \times \left(D_{i,t} - \bar{D}\right)$	0.047***	0.034***
	(0.016)	(0.011)
$D_{i,t}$	0.002***	0.001
	(0.001)	(0.001)
R^2	0.374	0.621
N. Observations	2374	2374

Note: The table reports the estimates of a panel IV regression across U.S. states using data from 1967 to 2015 at an annual frequency. In regression (1) the dependent variable is the change in output per capita. In regression (2) the dependent variable is the change in employment rate. The independent variables are the change in per capita state government spending (as a fraction of per capita state GDP), $\left(G_{i,t} - G_{i,t-2}\right)/Y_{i,t-2}$, the log-share of young people (aged 20-29) in total population, $D_{i,t}$, and the interaction between the change in per capita state government spending (as a fraction of per capita state GDP) and the log-share of young people, $\left[\left(G_{i,t} - G_{i,t-2}\right)/Y_{i,t-2}\right] \times \left(D_{i,t} - \bar{D}\right)$. In both regressions, changes in per capita state government spending (as a fraction of per capita state GDP) are instrumented with the product of state fixed effects and the change in per capita national government spending (as a fraction of per capita national GDP). The share of young people is instrumented with 20-30 year lagged birth rates. We include time and state fixed effects in all regressions. Robust standard errors clustered at the state level are reported in brackets. *** indicates statistical significance at the 1%.

Column (2) displays the results of the regression in which the dependent variable is the change in the employment rate. For a state with an average share of young people, the local employment fiscal multiplier equals 1.10. Demographics affect also the local employment fiscal multiplier: increasing the share of young people by 1% in absolute terms above the average raises employment fiscal multipliers by 3.1%, from 1.10 up to 1.13.

To assess whether the age sensitivity of local fiscal multipliers hinges on a particular econometric specification, we run a comprehensive battery of robustness checks. Table 2 reports the first of alternative specifications for the estimates of both the local output fiscal multiplier (Panel A) and the local employment fiscal multiplier (Panel B). In either case, the first column displays the results of the baseline regression. The following columns show the results for the OLS regression, the "partial" IV regression in which we instrument state government spending but we do not instrument the share of young people, an IV regression in which we use a different measure of young people (those aged 15-29), and an IV regression in which we use a different measure of birth rates (25 years lagged birth rates). Finally, we estimate the fiscal multipliers in regressions in which the share of young people is computed over the entire male population and the overall population, rather than using only the white male population.

The "partial" IV regression yields an estimated coefficient of the interaction between changes in government spending and the share of young people which is larger for the response of output (and smaller for the response of the employment rate) than in the baseline IV regression. This difference could be driven by the endogenous reaction of states' migration flows to a government spending shock. If migration raises the population, then it would boost further the change in output, while dampening the response of the employment rate. In Appendix A.2 we confirm this conjecture by showing that although total population does not change following a fiscal shock, the population of young people does rise. This evidence strengthen the relevance of instrumenting the share of young people to avoid any endogeneity concern driven by state migration flows.

Columns (4) and (5) show that the relationship between demographics and fiscal multipliers does not hinge on a specific definition of the young group or a specific instrumenting strategy. Finally, columns (6) and (7) show that the estimated effect of a change in demographics on fiscal multipliers becomes even larger when

Table 2: Response of Output & Employment Rate to Government Shocks - Robustness Checks

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Baseline	Baseline	No IV Birth Rates	Share Age 15-29	Birth Rates 25 Year Lag	All Men	Men & Women
	IV	OLS	"Partial" IV	IV	IV	IV	IV
			Panel A. Response of Output				
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}}$	1.511***	0.109	1.515***	1.251***	1.451***	1.664***	1.613***
	(0.409)	(0.112)	(0.468)	(0.394)	(0.396)	(0.432)	(0.435)
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}} \times (D_{i,t}-\bar{D})$	0.047***	0.011*	0.067**	0.051**	0.051***	0.066**	0.060**
	(0.017)	(0.006)	(0.028)	(0.024)	(0.017)	(0.028)	(0.025)
$D_{i,t}$	0.002***	0.002***	0.002***	0.003***	0.001	0.002**	0.002**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
R^2	0.374	0.390	0.330	0.382	0.411	0.362	0.364
N. Observations	2374	2397	2397	2374	2366	2374	2374
			Panel B. Response of Employment Rate				
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}}$	1.095***	0.180**	1.046***	0.959***	1.097***	1.091***	1.075***
	(0.215)	(0.076)	(0.236)	(0.210)	(0.210)	(0.226)	(0.220)
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}} \times (D_{i,t}-\bar{D})$	0.034***	0.001	0.025**	0.038**	0.035***	0.038**	0.039**
	(0.011)	(0.005)	(0.010)	(0.016)	(0.010)	(0.017)	(0.016)
$D_{i,t}$	0.001	0.001	0.001	0.001**	0.001	0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
R^2	0.621	0.635	0.590	0.627	0.627	0.625	0.624
N. Observations	2374	2397	2397	2374	2366	2374	2374

Note: The table reports the estimates of panel regressions across U.S. states from 1967 to 2015 at an annual frequency. In Panel A the dependent variable is the change in output per capita. In Panel B the dependent variable is the change in the employment rate. If not stated otherwise, the independent variables are the change in per capita state government spending (as a fraction of per capita state GDP), $(G_{i,t}-G_{i,t-2})/Y_{i,t-2}$, the share of young people (aged 20-29) in total population, $D_{i,t}$, and the interaction between the change in per capita state government spending (as a fraction of per capita state GDP) and the share of young people, $[(G_{i,t}-G_{i,t-2})/Y_{i,t-2}] \times (D_{i,t}-\bar{D})$. In the IV regressions, state-specific changes in per capita state government spending (as a fraction of per capita state GDP) are instrumented with the product of state fixed effects and the change in per capita national government spending (as a fraction of per capita national GDP). The share of young people is instrumented with 20-30 year lagged birth rates. Regression (1) displays the results of the benchmark IV regressions. Regression (2) shows the results of the regression estimated by OLS. In regression (3) we instrument state government spending but we do not instrument the share of young people. In regression (4) we use the share of the people aged 15-29 in total population as independent variable. In regression (5) we instrument the share of young people with 25 year lagged birth rates. In regression (6) we compute the share of young people not focusing only on white men, but rather on all men. In regression (7) we compute the share of young people not focusing only on white men, but rather on the entire population of young men and women. We include time and state fixed effects in all the regressions. Robust standard errors clustered at the state level are reported in brackets. *, **, and *** indicate statistical significance at the 10%, 5% and 1%, respectively.

computing the share of young people over either the entire male population or the overall population: a 1% increase in the share of young people rises the size of fiscal multipliers by around 3.7% - 4%. This pattern is consistent with the fact that white males have a much lower elasticity of labor supply than females and individuals of other racial groups.

The effect of demographics on fiscal multipliers could be biased by potential confounding factors which are highly correlated with changes in states' age structures. Yet, the link between demographics and fiscal multipliers holds after we introduce additional national-level variables (interacted with state fixed effects), such as the change in the oil price, the households' debt to GDP ratio, the federal debt to GDP ratio, Ramey (2011) and Ramey and Zubairy (2018)'s series on news about future increases in government spending, and the real interest rate. The age sensitivity of local fiscal multipliers keeps holding even when we introduce additional state-level variables, such as the change in house price, per capita federal personal taxes, the unemployment rate, and per capita unemployment benefits. Finally, we use data from the Current Population Survey (CPS) of the Bureau of Labor Statistics to compute measures of skilled labor and female labor participation at the state level and find that the addition of these variables does not alter the economic and statistical significance of the effect of demographics on fiscal multipliers. All these robustness checks are reported in Appendix A.1.

3 The Model

We build a two-country New Keynesian model with a rich, and yet tractable, life-cycle structure. The two countries - a home and a foreign economy - belong to a monetary union, with a unique Taylor rule which responds to union-level inflation and output gap. In the union there is also a federal government which purchases final consumption goods subject to spending shocks. The government finances its expenditures by levying lump-sum taxes on the households and issuing bonds.

In each country, the household sector has a life-cycle structure whereby individuals face three stages of life: young, mature, and old. All the individuals supply labor, accumulate assets, and consume. The model features credit markets imperfections and age-specific labor supply, such that the labor supply elasticity varies exogenously across age groups.

The two countries differ only in the relative size of the population. Hereafter we just describe the home country. The variables and parameters of the foreign economy are distinguished by a star superscript.

3.1 Households

In each country there is a continuum of households that belong to three different age groups: young agents (y), mature agents (m), and old agents (o). The demographic structure in the home country is described by the measures of young agents $N_{y,t}$, mature agents $N_{m,t}$, and old agents $N_{o,t}$ such that $N_{y,t} + N_{m,t} + N_{o,t} = N_t$. The total population of the monetary union is $N_{U,t} = N_t + N_t^\star$.

Agents move through the three different groups of households in a life-cycle manner as in Yaari (1965) and Blanchard (1985). In the home country, in each period $\omega_n N_{y,t}$ new young agents are born and enter the economy. At any given point of time, households face an idiosyncratic probability to change age groups in the following period: young agents become mature with a probability $1 - \omega_y$, mature agents become old with a probability $1 - \omega_m$, and old agents die and leave the economy with a probability $1 - \omega_o$.

We can define the law of motion of population across the three age groups as

$$N_{y,t+1} = \omega_n N_{y,t} + \omega_y N_{y,t}, \tag{4}$$

$$N_{m,t+1} = (1 - \omega_y) N_{y,t} + \omega_m N_{m,t}, \text{ and} \tag{5}$$

$$N_{o,t+1} = (1 - \omega_m) N_{m,t} + \omega_o N_{o,t}. \tag{6}$$

Over the lifetime each individual faces three idiosyncratic shocks: the transition from young to mature, the transition from mature to old, and the exit from the economy. Although agents are born identical, the idiosyncratic uncertainty would generate a distribution of ex-post heterogeneous households. Following Gertler (1999), we define a framework in which the optimal choices of the individuals within each age group aggregate linearly. This approach reduces the relevance of differences heterogeneity *within* age group but it allows us to emphasize the heterogeneity *across* age groups and incorporate nominal rigidities and open economy interactions into a tractable environment. In this way, our model extends a standard two-country New Keynesian economy with a rich life-cycle structure.

First, we introduce a perfect annuity market which insures old agents against the risk of death. Old agents transfer their investment in capital and bonds to financial intermediaries, which pay back the proceedings only to surviving households. Free entry and perfect competition in the annuity market guarantee a premium to the return on investment which compensates old agents for the risk of death.

Second, we assume that households are risk neutral. In this way, the uncertainty on the labor income dynamics due to the transition from young to mature and from mature to old does not affect optimal choices. Nevertheless, we keep a motive for consumption smoothing by assuming that individual preferences belong to the Epstein and Zin (1989) utility family, such that risk neutrality coexists with a positive elasticity of intertemporal substitution.

At time t the agent i of the age group $z = \{y, m, o\}$ chooses consumption $c^i_{z,t}$, labor supply $l^i_{z,t}$, capital $k^i_{z,t+1}$, and nominal bonds $b^i_{z,t+1}$ to maximize

$$\max_{c^i_{z,t},\, l^i_{z,t},\, k^i_{z,t+1},\, b^i_{z,t+1}} v^i_{z,t} = \left\{ \left(c^i_{z,t} - \chi_z \frac{{l^i_{z,t}}^{1+\frac{1}{\nu_z}}}{1+\frac{1}{\nu_z}} \right)^{\eta} + \beta \mathbb{E}_t [v^i_{z',t+1} \mid z]^{\eta} \right\}^{1/\eta} \tag{7}$$

$$\text{s.t.} \quad P_t c^i_{z,t} + P_{I,t} k^i_{z,t+1} + P_{I,t} \varphi^i_{z,t+1} + b^i_{z,t+1} + P_t \tau^i_{z,t} = \dots$$

$$\dots = a^i_{z,t} + W_t \xi_z l^i_{z,t} + (1-\tau_d) d^i_{z,t} \mathbb{I}_{\{z=m\}} \tag{8}$$

$$\begin{cases} a^i_{z,t} = P_{I,t}(1-\delta) k^i_{z,t} + R_{k,t} k^i_{z,t} + R_{n,t} b^i_{z,t} & \text{if } z = \{y, m\} \\ a^i_{z,t} = \frac{1}{\omega_z} \left[P_{I,t}(1-\delta) k^i_{z,t} + R_{k,t} k^i_{z,t} + R_{n,t} b^i_{z,t} \right] & \text{if } z = \{o\} \end{cases} \tag{9}$$

$$k^i_{z,t+1} = (1-\delta) k^i_{z,t} + x^i_{z,t} - \varphi^i_{z,t+1} \tag{10}$$

$$k^i_{z,t+1} \geqslant 0,\ b^i_{z,t} \geqslant 0 \tag{11}$$

$$c^i_{z,t} = \left[\lambda^{1/\psi_c} {c^i_{H,z,t}}^{\frac{\psi_c - 1}{\psi_c}} + (1-\lambda)^{1/\psi_c} {c^i_{F,z,t}}^{\frac{\psi_c - 1}{\psi_c}} \right]^{\frac{\psi_c}{\psi_c - 1}} \tag{12}$$

$$x^i_{z,t} = \left[\lambda^{1/\psi_I} {x^i_{H,z,t}}^{\frac{\psi_I - 1}{\psi_I}} + (1-\lambda)^{1/\psi_I} {x^i_{F,z,t}}^{\frac{\psi_I - 1}{\psi_I}} \right]^{\frac{\psi_I}{\psi_I - 1}} \tag{13}$$

where β is the time discount factor and χ_z denotes the weight of leisure in the utility. The parameter $(1-\eta)^{-1}$ denotes the elasticity of intertemporal substitution, which drives households' motive to smooth consumption. Finally, ν_z is the labor supply elasticity, which varies exogenously across age groups. Since the utility function displays consumption-labor complementarities,[12] the response of labor to a government spending shock depends uniquely on the labor supply elasticity: when the labor supply is constant across age groups, all households have the same labor response.

In the budget constraint, each household purchases consumption goods $P_t c^i_{z,t}$, and invests in capital $P_{I,t} k^i_{z,t+1}$ and nominal bonds $b^i_{z,t+1}$. Capital investment is

[12] Nakamura and Steinsson (2014) show that consumption-labor complementarities are required to match the level of the local fiscal multiplier. Gnocchi et *al.* (2016) study data on time use to document that the complementarity between consumption and hours worked is indeed an empirically relevant driver of the response of labor to a government spending shock. Bilbiie (2011) shows that the consumption-labor complementarities can rationalize a positive national consumption fiscal multiplier if prices are not flexible.

subject to convex adjustment costs $\varphi^i_{z,t+1}$. Equation (9) defines the total nominal return on assets $a^i_{z,t}$. If the household is either young or mature, the total nominal return on assets equals the sum of the nominal return on capital and the nominal return of bonds. Instead, the return on assets for old households equals the return granted by the annuity market, that is, the return on assets divided by the survival probability of an old agent ω_o. Households also pay a lump-sum tax $\tau^i_{z,t}$.

Each household earns a nominal labor income $W_{z,t}\xi_z l^i_{z,t}$, where ξ_z denotes the age-specific efficiency units of hours worked. These parameters allow us to calibrate the model to match the hump-shaped pattern of labor income over the life-cycle. Finally, we assume that mature agents own the firms and therefore receive firms' nominal dividends, which are taxed at a proportional rate τ_d.

Equation (11) denotes the ad-hoc borrowing constraints that restrict the households from going short in capital and bonds. In equilibrium, the constraints bind only for young individuals. Given the hump-shaped pattern of labor income over the life cycle, young individuals would like to borrow and smooth consumption but are prevented from doing so. In the quantitative analysis, we also consider a version of the model which abstracts from the ad-hoc constraint on bonds. Even in this case, in our life-cycle setting a non-contingent bond is not sufficient to ensure perfect consumption smoothing across generations.[13]

Equations (12) and (13) show that households consumption $c^i_{z,t}$ and investment $x^i_{z,t}$ combine final goods produced in both the home and foreign country. The parameter λ captures the degree of home bias of the economy, that is, the amount of home produced goods consumed by households in the home economy. The optimal amount of home goods and foreign goods purchased by households in the home

[13]Gordon and Varian (1988) show that in a overlapping generations economy markets are complete only if young individuals can trade with unborn generations. This missing market prevents an efficient risk allocation across generations.

economy equal respectively

$$c^i_{H,z,t} = \lambda \left(\frac{P_{H,t}}{P_t}\right)^{-\psi_c} c^i_{z,t}, \qquad x^i_{H,z,t} = \lambda \left(\frac{P_{H,t}}{P_{I,t}}\right)^{-\psi_I} x^i_{z,t} \tag{14}$$

and

$$c^i_{F,z,t} = (1-\lambda) \left(\frac{P_{F,t}}{P_t}\right)^{-\psi_c} c^i_{z,t}, \qquad x^i_{F,z,t} = (1-\lambda) \left(\frac{P_{F,t}}{P_{I,t}}\right)^{-\psi_I} x^i_{z,t} \tag{15}$$

where $P_{H,t}$ denotes the price of home produced goods, $P_{F,t}$ is the price of foreign produced goods, ψ_c is the elasticity of substitution across home and foreign produced consumption goods, and ψ_I is the elasticity of substitution across home and foreign produced investment goods. The price indexes of consumption P_t and investment $P_{I,t}$ are respectively defined as

$$P_t^{1-\psi_c} = \left[\lambda {P_{H,t}}^{1-\psi_c} + (1-\lambda){P_{F,t}}^{1-\psi_c}\right] \tag{16}$$

$$P_{I,t}^{1-\psi_I} = \left[\lambda {P_{H,t}}^{1-\psi_I} + (1-\lambda){P_{F,t}}^{1-\psi_I}\right]. \tag{17}$$

Appendix C shows in detail the problems of young, mature, and old agents.

3.2 Production

In each country the production sector is split into one competitive final goods firm and a continuum $j \in [0,1]$ of intermediate producers under monopolistic competition. In the home country, the final goods firm produces domestic output Y_t with a CES aggregator of the different varieties of the intermediate producers

$$Y_t = \left(\int_0^1 {Y_t^j}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{18}$$

where Y_t^j denotes the output produced by the intermediate producer j at time t, and ε is the elasticity of substitution across varieties. The final good firm is perfectly competitive and takes as given the price of the goods produced by the intermediate producers $P^j_{H,t}$. The isoleastic demand of each variety and the price level of the home country $P_{H,t}$ equal respectively

$$Y^j_t = \left(\frac{P^j_{H,t}}{P_{H,t}}\right)^{-\varepsilon} Y_t, \text{ and} \tag{19}$$

$$P_{H,t} = \left(\int {P^j_{H,t}}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}. \tag{20}$$

The foreign country has the same structure with the only difference that it produces output $Y^\star_t$ at a production price $P_{F,t}$.

The intermediate firms produce the differentiated varieties

$$Y^j_t = {L^j_t}^{1-\alpha} {K^j_t}^{\alpha} \tag{21}$$

using labor L^j_t, hired at the nominal wage W_t, and physical capital K^j_t, rented from home residents at the nominal gross rate $R_{k,t}$. Then, nominal dividends D^j_t equal

$$D^j_t = P^j_{H,t} Y^j_t - W_t L^j_t - R_{k,t} K^j_t. \tag{22}$$

The firms decide the optimal amount of capital and labor to hire in the following cost minimization problem

$$\min_{K^j_t, L^j_t} \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} Q^m_{t,s} \left(W_s L^j_s + R_{k,s} K^j_s\right)\right\}, \tag{23}$$

where $Q^m_{t,s}$ denotes the stochastic discount factor of the mature agents between period t and period $s \geq t$. Given firms' nominal marginal costs Φ^j_t, the cost minimization problem implies the following first-order conditions for labor and capital

$$W_t = \Phi^j_t (1-\alpha) \frac{Y^j_t}{L^j_t} \qquad \text{and} \qquad R_{k,t} = \Phi^j_t \alpha \frac{Y^j_t}{K^j_s}. \tag{24}$$

With respect to the firms' price setting behavior, we introduce a nominal price rigidity à la Calvo (1983), such that firms can reset their prices with a probability $1 - \zeta$. This probability is independent and identically distributed across firms, and constant over time. As a result, in each period a fraction ζ of firms cannot reset their prices and maintain the prices of the previous period, whereas the remaining fraction $1 - \zeta$ of firms are allowed to set freely their prices. The properties of the Calvo price friction imply that the aggregate price level follows the law of motion

$$P_{H,t}^{1-\varepsilon} = (1 - \zeta) \, P_{H,t}^{\#\ 1-\varepsilon} + \zeta P_{H,t-1}^{1-\varepsilon}. \tag{25}$$

where the optimal reset price $P_{H,t}^{j,\#}$ for a firm that can change its price is

$$P_{H,t}^{j,\#} = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \zeta^s Q_{t,t+s}^m \Phi_{t+s}^j P_{H,t+s}^{\varepsilon} P_{t+s}^{-1} Y_{t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \zeta^s Q_{t,t+s}^m P_{H,t+s}^{\varepsilon} P_{t+s}^{-1} Y_{t+s}}. \tag{26}$$

3.3 Government

In the monetary union there is a government that constitutes of a monetary authority and a fiscal authority. On the monetary side, the government sets the nominal interest rate $R_{n,t}$ following a Taylor rule that reacts to the inflation rate of the monetary union $1 + \pi_t^u \equiv \frac{P_{u,t}}{P_{u,t-1}}$, where $P_t^u \equiv N_t P_t + N_t^\star P_t^\star$, and the gap between the output of the monetary union $Y_t^u \equiv Y_t + Y_t^\star$ and the output of an economy with flexible prices $Y_t^{u,F}$

$$\frac{R_{n,t}}{\bar{R}} = \left[\frac{R_{n,t-1}}{\bar{R}} \right]^{\psi_R} \left[(1 + \pi_t)^{\psi_\pi} \left(\frac{Y_t^u}{Y_t^{u,F}} \right)^{\psi_Y} \right]^{1-\psi_R} \tag{27}$$

where $\bar{R}$ is the steady-state nominal interest rate, ψ_R denotes the degree of interest rate inertia, and ψ_π and ψ_Y capture the degree at which the nominal interest rates respond to inflation and output gap, respectively.

On the fiscal side, the federal government purchases home goods $G_{H,t}$ and foreign goods $G_{F,t}$. The government finances its expenditures with the revenues of a one-period non-contingent bond $B_{g,t}$, that yields a nominal gross interest rate $R_{n,t}$, a nominal lump-sum tax levied in the home country T_t and in the foreign country $T_t^\star$, and the proceeds from dividend taxation $\tau_d(D_{m,t} + D_{m,t}^\star)$. The government budget constraint reads

$$P_{H,t}G_{H,t} + P_{F,t}G_{F,t} + B_{g,t+1} = B_{g,t}R_{n,t} + P_tT_t + P_t^\star T_t^\star + \tau_d(D_{m,t} + D_{m,t}^\star) \qquad (28)$$

where $T_t = \int_0^{N_{y,t}} \tau_{y,t}^i \,\mathrm{di} + \int_0^{N_{m,t}} \tau_{m,t}^i \,\mathrm{di} + \int_0^{N_{o,t}} \tau_{o,t}^i \,\mathrm{di}$, and $D_{m,t} = \int_0^{N_{m,t}} d_{m,t}^i \,\mathrm{di}$. Analogous expressions apply for $T_t^\star$ and $D_{m,t}^\star$.

Government expenditures $G_{H,t}$, and $G_{F,t}$ are exogenous and follow first order autoregressive processes

$$\log G_{H,t} = (1-\rho_G)\log G_{H,SS} + \rho_G \log G_{H,t-1} + \epsilon_{G_H,t}, \qquad (29)$$

and

$$\log G_{F,t} = (1-\rho_G)\log G_{F,SS} + \rho_G \log G_{F,t-1} + \epsilon_{G_F,t}, \qquad (30)$$

where $G_{H,SS}$ and $G_{F,SS}$ are the steady-state values of government spending in each country, ρ_G denotes the persistence of the processes, $\epsilon_{G_H,t}$ is a spending shock in home goods, and $\epsilon_{G_F,t}$ is a spending shock in foreign goods. These shocks are independent and identically distributed following a Normal distribution $N(0,1)$.

We assume that the government follows a fiscal rule which determines the response of debt and tax to the exogenous changes in government spending:

$$\frac{\widehat{B}_{g,t+1}}{Y_{SS}^u} = \rho_{bg}\frac{\widehat{B}_{g,t}}{Y_{SS}^u} + \phi_G\frac{\widehat{P_{H,t}G_{H,t}}}{Y_{SS}^u} + \phi_G\frac{\widehat{P_{F,t}G_{F,t}}}{Y_{SS}^u} + \phi_T\frac{\widehat{P_tT_t}}{Y_{SS}^u} + \phi_T\frac{\widehat{P_t^\star T_t^\star}}{Y_{SS}^u} \qquad (31)$$

where Y_{SS}^u denotes the steady-state value of the output of the monetary union, and $\hat{Z}_t \equiv Z_t - Z_{SS}$ denotes the absolute deviation from steady-state. The parameters ρ_{bg},

ϕ_G, and ϕ_T control to what extent debt and tax finance an increase in government spending and how long the government takes to raise taxes to bring government debt back to the steady state level. For instance, when $\phi_G = 0$, $\rho_{bg} = 0$, and $\phi_T = 0$, spending is fully financed through taxes. As ϕ_G and ρ_{bg} increase, government spending becomes partially financed through debt. As ϕ_T increases, debt levels above steady-state trigger tax adjustments.

3.4 Closing the Model

Our setup allows us to derive optimal policies for each individual that can be aggregated linearly within each age-group. For instance, we can define total young consumption, mature consumption, and old consumption of goods produced in the home economy as

$$C_{H,y,t} = \int_0^{N_{y,t}} c^i_{H,y,t}\,\mathrm{di}, \quad C_{H,m,t} = \int_0^{N_{m,t}} c^i_{H,m,t}\,\mathrm{di}, \quad \text{and} \quad C_{H,o,t} = \int_0^{N_{o,t}} c^i_{H,o,t}\,\mathrm{di},$$

such that the overall total consumption equals $C_{H,t} = C_{H,y,t} + C_{H,m,t} + C_{H,o,t}$. The same applies to all the variables of the model. Appendix C shows that the life-cycle setup of the model allows for a simple linear aggregation within age groups.

Bonds move freely across countries, and the clearing of the market implies that the supply of government bonds equals the sum of individual positions across countries, that is $B_{g,t} = B_t + B^\star_t = B_{y,t} + B_{m,t} + B_{o,t} + B^\star_{y,t} + B^\star_{m,t} + B^\star_{o,t}$. Instead, we assume that labor and physical capital are immobile.[14] The clearing of the rental markets of capital implies $K_t = K_{y,t} + K_{m,t} + K_{o,t}$ and $K^\star_t = K^\star_{y,t} + K^\star_{m,t} + K^\star_{o,t}$. The labor markets clear when $L_t = \xi_y L_{y,t} + \xi_m L_{m,t} + \xi_o L_{o,t}$ and $L^\star_t = \xi_y L^\star_{y,t} + \xi_m L^\star_{m,t} + \xi_o L^\star_{o,t}$.

Then, the resource constraint of the home economy posits that output is split into the consumption of the home goods of the households of both countries, the

[14] In the empirical analysis we instrument of the share of young people with lagged birth rates to wash out the effect of migration on local fiscal multipliers. Accordingly, we set that labor is immobile in the model. When we do not control for migration flows in the data, the age sensitivity of local fiscal multipliers is even larger.

investment of both countries, and the goods purchased by the government, net of the adjustment costs of capital $Y_t = C_{H,t}+C^{\star}_{H,t}+G_{H,t}+X_{H,t}+X^{\star}_{H,t}-\varphi_t$, where φ_t denotes the sum of the adjustment costs bore by all agents in the home economy. Similarly, for the foreign economy we have that $Y^{\star}_t = C_{F,t} + C^{\star}_{F,t} + G_{F,t} + X_{F,t} + X^{\star}_{F,t} - \varphi^{\star}_t$.

4 Quantitative Analysis

4.1 Calibration

In the calibration exercise, we discipline the life-cycle dynamics by matching some salient facts on the demographics of the U.S. population and the life-cycle pattern of labor income. Throughout the calibration, we set one period of the model to correspond to one quarter.

The calibration of the set of parameters that govern the demographic and life-cycle structure of the model is reported in Table D.11 in the Appendix. We first set the size of the home economy to $N/N^u = 0.1$, which is roughly the relative population size of California. We define young households as the individuals between 20 and 30 years old, mature households are the individuals between 30 and 65 years old, and old households are the individuals above 65 years old. Then, we define the parameters that control the law of motion of age group populations to match the average share of young people in total population between 1967 and 2015, the average share of old people in total population between 1967 and 2015, the average number of years that an individual spends as young (10 years), the average number of years that an individual spends as mature (35 years). Matching these moments yields a birth rate of new young agents of $\omega_n = 0.0274$, a probability of the transition from young to mature of $1-\omega_y = 0.0250$, a probability of the transition from mature to old of $1-\omega_m = 0.0071$, and a death probability for an old agent of $1-\omega_o = 0.0274$.

We define the relative disutility of working for mature individuals such that their steady-state hours worked equal 0.35. This condition yields $\chi_m = 308.02$. Then, we define the relative disutility of working for young and old individuals such that their hours worked equal 0.324 and 0.08, respectively. These moments are derived by multiplying the steady-state hours worked by mature individuals with the employment rate of either young or old individuals relative to the employment rate of mature individuals.[15] These conditions yield the values of $\chi_y = 5.40$ and $\chi_o = 33.79$. The efficiency unity of hours worked across the age groups are calibrated such that the model is consistent with the life-cycle dynamics of labor earnings. First, we normalize the efficiency unity of hours of mature agents and set $\xi_m = 1$. Then, we use CPS data and find that the labor income of individuals between 20 and 29 years equals on average 68% of the labor income of individuals between 30 and 64 years. Consequently, we set $\xi_y = 0.68$. We follow the same procedure for the labor income of individuals above 65 years and find that $\xi_o = 0.72$.

The calibration of the labor supply elasticity is key to generate the age-specific differences in labor supply, which is one of the mechanisms of the model to rationalize the age-sensitivity of fiscal multipliers. Since Hall (2009) shows how the size of fiscal multipliers depends positively on the labor supply elasticity, we opt for a conservative calibration approach: we discipline the values of the labor supply elasticity by following the evidence on the micro elasticity provided by the literature. The meta-analysis of quasi-experimental studies carried out by Chetty et *al.* (2013) computes a mean of the intensive margin Frisch elasticity of 0.54. However, these studies tend to focus on groups with weak attachment to the labor force, such as single mothers or workers near retirement. Since we are after the elasticity of mature white male workers, which feature a much lower elasticity than the rest of the workers, we choose a value of $\nu_m = 0.2$, which is at the lower end of the Frisch elasticity estimates and at the average of the Hicksian elasticity estimates in Chetty

[15]The average employment rate of young individuals between 1970 and 2015 equals 76.44%. The employment rate of mature individuals equals 83.57%. The employment rate of old individuals equals 19.09%.

et *al.* (2013).

We calibrate the labor supply elasticity of old workers following Rogerson and Wallenius (2013), who point out that only elasticities above 0.75 can rationalize the observed retirement behavior from full-work. Accordingly, we calibrate the elasticity of old workers to $\nu_o = 0.75$. Finally, we calibrate the labor supply elasticity of young workers such that the weighted-average elasticity of the economy equals 0.4. Again, we choose a value which is slightly lower than the 0.54 provided by Chetty et *al.* (2013) to wash out the influence of groups with weak attachment to the labor force. This procedure yields an elasticity of young workers of $\nu_y = 0.71$, which is slightly lower than the elasticity of old workers. Interestingly, this relative ranking between elasticities across age groups is consistent with the evidence of Jaimovich and Siu (2009), which document that the volatility of hours of young and old workers is much higher than the volatility of hours of mature workers, with the volatility of old workers being the highest among all age groups.[16,17]

The calibration of the set of parameters of the New Keynesian structure of the model is reported in Table D.10 in the Appendix. We set the time discount factor to $\beta = 0.995$, whereas we fix $\eta = -9$ to define an elasticity of intertemporal substitution which equals 0.1, at the lower end of the empirical estimates (see Hall, 1988).

The capital depreciation rate is set to the standard value of $\delta = 0.025$, which

[16] Our calibration choice for the labor supply elasticity across age groups is consistent with French (2005), Jaimovich and Siu (2009), Rogerson and Wallenius (2009), Erosa et *al.* (2016), Janiak and Monteiro (2016), Karabarbounis (2016), Peterman (2016), who find that young and old individuals have higher labor supply elasticities than mature individuals. The differences across age groups in the labor supply elasticity are also motivated by the response of hours to a government spending shock in the data. In Appendix A.3, we compute hours worked by state for all workers, young workers (workers between 20 and 29 years old), and older workers (workers above 30 years old) using CPS data from 1977 to 2015. When we estimate the hours worked local fiscal multiplier, we find that total hours increase following a government spending shock. Since the sample starts only in 1977, we lose the first ten years of our baseline sample, which implies that the uncertainty around the local multiplier estimates becomes rather large. Then, we compare the estimates of the local fiscal multiplier for the hours worked by young and older workers. Although the large standard deviations make the estimates not to be statistically different from zero, the point estimate of the local multiplier for the hours worked by the young is 2.5 times larger than the point estimate of the local multiplier for the hours worked by the rest of the population.

[17] Although we cannot discount the possibility that the labor supply elasticity changes with population aging, throughout the paper we assess the effect of aging conditional on a constant labor supply elasticity over time.

implies a 10% annual depreciation rate. Instead, for the capital adjustment costs we do the following. First, we posit that the adjustment costs for an individual i in the age group z at time t equal $\varphi^i_{z,t+1} = \frac{\varphi}{2}\left(\frac{k^i_{z,t+1}}{k^i_{z,t}} - \vartheta_z\right)^2 k^i_{z,t}$. The parameter ϑ_z captures the life-cycle dynamics of capital accumulation and it is pinned down such that no adjustment cost is paid at steady-state. In the baseline calibration, young households do not own capital and therefore do not bear adjustment costs. The average quarterly capital accumulation rate for mature households is 0.72%, which implies $\vartheta_m = 1.0072$, whereas old households on average deplete capital, and they do so at a quarterly rate of -0.19%, such that $\vartheta_o = 0.9981$. Then, we set $\kappa = 135$ such that the two-year national fiscal multiplier for investment equals -0.9, which coincides with the estimate of Blanchard and Perotti (2002).

Regarding the consumption and investment bundles, there are three parameters to be calibrated: the home bias λ, the elasticity of substitution across home and foreign consumption goods ψ_c, and the elasticity of substitution across home and foreign investment goods ψ_i. Following Nakamura and Steinsson (2014) we set the home bias to $\lambda = 0.69$ and the elasticity of substitution across home and foreign consumption goods to $\psi_c = 2$. Then, we impose that the elasticity of substitution across home and foreign investment goods equals the one of consumption goods, that is, $\psi_i = \psi_c$. We set the elasticity of substitution across varieties to $\epsilon = 9$, which implies a markup of 12.5%, in the ball park of the estimates used in the literature of New Keynesian models. The capital share in the production function is set to $\alpha = 0.32$, and the Calvo price parameter to $\zeta = 0.75$, which implies that on average firms adjust their prices every 12 months.

Regarding the fiscal setting of the economy, we first fix the proportional tax on dividends to $\tau_d = 0.9394$. Since dividends are then redistributed in a lump-sum fashion to all households, this proportional rate implies that mature households receive 60% of the overall dividends of the economy. Then, we set the steady-state value of government spending to output ratio to $\frac{G_{H,SS}+G_{F,SS}}{Y^U_{SS}} = 0.2$. This value coincides with the average ratio of total government spending to output observed in the data from 1960 to 2016. The persistence of the government spending shock is calibrated to $\rho_G = 0.933$, which matches the persistence of the military procurement data, as computed by Nakamura and Steinsson (2014). Finally, we calibrate the fiscal rule parameters. We calibrate the three parameters ρ_{bg}, ϕ_G, and ϕ_T to match the inertia observed in the data in the response of government debt to a government spending shock. First, we posit that following a government spending shock the ratio of government deficit to debt issuance is u-shaped, with a trough after 6 quarters. Second, throughout the first 8 quarters, new debt issuance covers on average 70% of total deficit. Third, after the trough, debt issuance starts decreasing and after 16 quarters government debt is progressively repaid through an increase in lump-sum taxation. This procedure yields the following parameters: $\rho_{bg} = 0.95$, $\phi_G = 4.5$, and $\phi_T = 0.01$.

We set the Taylor rule parameters following the estimates of Clarida et *al.* (2000): the inertia parameter equals $\psi_R = 0.8$, the degree of response to the inflation rate is $\psi_\pi = 1.5$, and the degree of response to the output gap is $\psi_Y = 0.2$.

4.2 Results

We start by studying to what extent the model can explain the age sensitivity of local fiscal multipliers, contrasting theoretical and empirical estimates. This analysis attempts to validate the quantitative appeal of our model and measure the relevance of its different channels. Then, we evaluate whether also national fiscal multipliers depend on the age structure of the population.

4.2.1 Demographics and Local Fiscal Multipliers

What is the effect of a change in the age structure of the economy on the size of local fiscal multipliers in the model? We address this question by replicating the same empirical analysis carried out in Section 2 with the simulated data of our model. In the simulation, we consider the effect of federally-financed increases in (wasteful) government spending in each of the two economies: we shock the economy with innovations to government spending in home goods $G_{H,t}$ and innovations to government spending in foreign goods $G_{F,t}$. These purchases are financed at the federal level, partially through bonds and partially through lump-sum taxes on all the households of the monetary union.

We proceed in two steps. In the first one, we estimate the local output fiscal multiplier in a model in which both economies are symmetric in the shares of population across age groups, which are calibrated to average values observed between 1967 and 2015. To do so, we estimate the following panel regression:

$$\frac{Y_{i,t} - Y_{i,t-2}}{Y_{i,t-2}} = \alpha_i + \delta_t + \beta \frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} + \epsilon_{i,t}, \qquad i \equiv \{H, F\}.$$

This first step yields the model counterpart of the coefficient β of the regression (1), that is, the size of local multipliers for a state with an average share of young people in total population. In the second step, we change the age structure of the home economy by increasing the share of young people by 1%. Then, we estimate again the local fiscal multiplier as before. The difference in the size of the local output fiscal multiplier between the second and the first step yields the model counterpart of the coefficient γ of the regression (1), which defines how local multipliers vary with the age structure of an economy.

Table 3 reports the results of this exercise. In the data, the local output fiscal multiplier for a U.S. state with an average share of young people in total population is 1.51. A 1% increase in the share of young people raises the multiplier by 3.1%, up to 1.56. In the model, the local output fiscal multiplier for a U.S. state with an average share of young people in total population is 1.39. A 1% increase in the share of young people raises the multiplier by 2%, up to 1.42. Hence, the model can account for 92% of the size of fiscal multipliers and 65% of the link between fiscal multipliers and demographics.

Table 3: Local Output Fiscal Multiplier - Data vs. Model

		Data	Model
Average Local Output Fiscal Multiplier	β	1.511	1.392
Sensitivity of Local Output Fiscal Multiplier with States' Age Structure	γ	0.047	0.027
Δ Local Output Fiscal Multiplier of 1% Increase in Share Young People	γ/β	3.1%	2%

Note: The Table reports the results of the estimation of the local output fiscal multiplier in the data and in the model. The first row reports the estimated value of the local output fiscal multiplier for a U.S. state with an average share of young people in total population. The second row reports how a 1% increase in the share of young people rises the size of the local output fiscal multiplier. The last row computes the age sensitivity of local output fiscal multiplier.

What is the contribution of the age-specific labor supply elasticities, the ad-hoc borrowing constraint, and the incomplete markets for the quantitative implications of the model on the age sensitivity of the local fiscal multiplier? We measure the relevance of these channels by comparing the baseline model with two counterfactual economies. The first one, the "Constant Labor Supply Elasticity", refers to a version of the baseline economy in which we eliminate the age-specific differences in the labor supply elasticities and set a unique elasticity across the three age groups: we set the labor supply elasticity to the weighted average value of the baseline economy, that is, $\nu_y = \nu_m = \nu_o = 0.4$. The second economy we consider is the "No Borrowing Constraint", where we eliminate the ad-hoc constraint and let all young households to borrow. This version of the model builds on the previous one and therefore features a constant labor supply elasticity across the three age groups. Table 4 reports the age sensitivity of the local fiscal multipliers in all these specifications.

When we consider a homogeneous labor supply elasticity across age groups, the age sensitivity of the local output fiscal multiplier shrinks by just 10%, from

2% to 1.8%. Hence, in the model the age sensitivity of local multipliers depends mostly on credit market imperfections. To understand which kind of credit market imperfection is at the root of the age sensitivity of local multipliers, we isolate the contribution of the ad-hoc borrowing constraint and incomplete markets on our results. When we eliminate completely the ad-hoc constraint, the age sensitivity of local multipliers drops from 1.8% to 0.9%. Thus, the ad-hoc borrowing constraint accounts for 45% of the quantitative prediction of the model, confirming the key role of the fraction of hand-to-mouth households for understanding the effectiveness of fiscal policy highlighted by Gali et al. (2007) and Kaplan and Violante (2016). Nevertheless, in the "No Borrowing Constraint" economy the age-sensitivity is still positive and equals 0.9%, which implies that incomplete credit markets account for 45% of the prediction of the model on the link between fiscal multipliers and demographics. This result highlights that even in the absence of age-specific labor supply and the ad-hoc borrowing constraint, the lack of complete markets in a life-cycle setting can still generate local multipliers that depend on the age structure of the economy.

Table 4: Age Sensitivity of Local Output Fiscal Multiplier - Channels

	Data	Baseline Model	Constant Labor Supply Elasticity	No Borrowing Constraint
Δ Local Output Fiscal Multiplier of 1% Increase in Share Young People	3.1%	2%	1.8%	0.9%

Note: The Table reports the results of the age sensitivity of local output fiscal multiplier in the data and in different versions of the "Baseline Model". The "Constant Labor Supply Elasticity" builds on the "Baseline Model" and eliminates the age-specific labor supply elasticity such that all individuals in the model have the same labor supply elasticity. The "No Borrowing Constraint" considers a version of the model in which all individuals in the model have the same labor supply elasticity and no household faces a borrowing constraint.

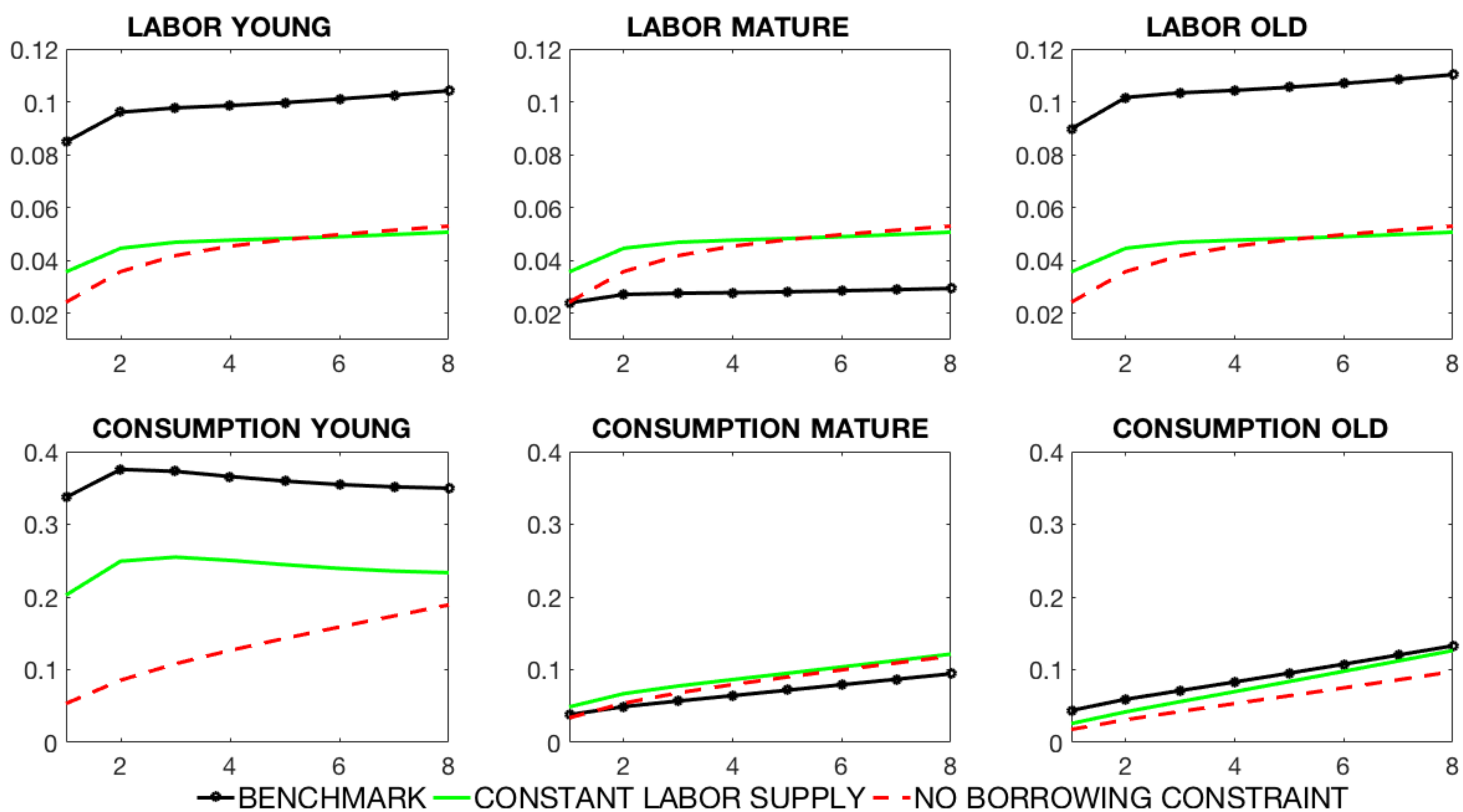
To shed further light on the contribution of each age group on the age-sensitivity of local fiscal multipliers, we report in Figure 1 the individual cumulative responses of labor and consumption across the three different age groups in the benchmark model and in the two counterfactual economies. The Figure shows that following a government spending shock the responses of young and old households labor are on impact four times larger than the response of labor of mature households. These dynamics are driven by the age-specific labor supply elasticities as the utility function we consider features no wealth effect on labor supply: the high elasticity of young and old workers makes young and old employment much more responsive to government spending shocks than the employment of mature workers. When we remove the differences in the labor supply elasticities in the "Constant Labor Supply Elasticity" economy, then the responses of labor across age groups coincide.

Not only the response of labor, but also the one of consumption displays sizable differences across age groups. Although consumption always rise following a government spending shock due to the complementarity between consumption and leisure in the utility function, the response of the young households is the largest one. The response of consumption of old households is slightly higher than the response of consumption of mature households, and much lower than the one of young households. As old households work only few hours, they experience a mild positive income effect, which then translates into a lower consumption response compared to young households. Overall these implications of the model are consistent with the evidence of the literature on the response of age-group consumption to tax changes. Although Shapiro and Slemord (1995), Johnson et *al.* (2006), Agarwal et *al.* (2007), Anderson et *al.* (2016) use different approaches to identify the consumption response to tax shocks, they all conclude that young households display a much larger consumption response than older individuals.[18]

The consumption response of young households is still larger than the response

[18] Also Kaplan and Violante (2010), Berger *al.* (2018), and Carroll et *al.* (2018) provide model-based evidence pointing out that the marginal propensity to consume is highest among young households. Young households display a larger marginal propensity because they are more likely to be liquidity constrained, as documented in Jappelli (1990), Kaplan et *al.* (2014), and Misra and Surico (2014).

Figure 1: Labor and Consumption Cumulative Responses by Age Groups.



The figure plots the cumulative responses of individual labor and individual consumption by age groups. The first row reports the cumulative responses of working hours of an individual young, an individual mature, and an individual old household. The second row reports the cumulative responses of consumption of an individual young, an individual mature, and an individual old household. In each plot we report the cumulative responses under four different scenarios. The dashed line corresponds to the "Baseline Economy". The continuous line corresponds to the "Constant Labor Supply Elasticity" economy. The dotted line corresponds to the "No Borrowing Constraint" economy.

of older individuals even when we eliminate the age-differences in the labor supply elasticity and the borrowing constraint. Indeed, in our model the age-sensitivity of local fiscal multipliers hinges also on the market incompleteness, that boosts the marginal propensity to consume of young agents. The relevance of incomplete markets - above and beyond the fraction of borrowing constrained agents - for the size of fiscal multipliers is also highlighted by Brinca et *al.* (2016), Ferriere and Navarro (2017), and Hagedorn et *al.* (2017). In these papers, markets are incomplete because the idiosyncratic labor income risk is uninsurable and there is no state-contingent bonds. In our environment the lack of complete markets is also rooted in the overlapping generation structure of the model. In equilibrium, given the interest rate and the amount of bonds traded, young agents cannot borrow sufficiently to smooth consumption in the face of a hump-shaped labor income dynamics over the life-cycle.[19] As a result, the marginal propensity to consume of young households is above the one of mature households, as it is in the data. Hence, an economy with relatively more young households features a stronger demand channel.

4.2.2 Demographics and National Fiscal Multipliers

Does the link between demographics and fiscal multipliers persist also at the national level? Although our evidence shows that the effect of demographics on fiscal multipliers at the state level is economically and statistically significant, this result does not necessarily imply that demographics alter also national multipliers. Indeed, Nakamura and Steinsson (2014), Fahri and Werning (2016), and Chodorow-Reich (2017) show that local fiscal multipliers consider the local effect of federally financed policies and wash out any monetary policy response to government spending. Both features make local multipliers larger than national ones. On the other hand, local multipliers are dampened by expenditure switching and import leakage effects that

[19]The inability to trade bonds/write contracts with the agents that are unborn prevents the (current) young agents from accessing additional asset markets to perfectly smooth consumption.

do not take place at the national level.

We evaluate the role of the age structure on the size of national fiscal multipliers through the lenses of the model. To do so, we estimate national multipliers in the following exercise. First, we consider a symmetric increase in government spending in both the home and the foreign economy. Similarly to our definition of national output Y_t^U, we define national government spending as sum of government spending in the home economy and government spending in the foreign economy, that is, $G_t^u = G_{H,t} + G_{F,t}$. Hence, now we consider an increase in national (wasteful) government spending which is financed by all the individuals in the monetary union.

We estimate the national output multiplier β_N as

$$\frac{Y_t^u - Y_{t-2}^u}{Y_{t-2}^u} = \beta_N \frac{G_t^u - G_{t-2}^u}{Y_{t-2}} + \epsilon_t.$$

Again, we proceed in two steps. First, we estimate β_N by running the regression on the simulated data from the model which is calibrated to the average population shares observed in the U.S. between 1967 and 2015. Then, we change the age structure of the economy by increasing the share of young people in the overall union by 1% and estimate again β_N. The difference between the estimates of the second and the first step yields the age sensitivity of national output fiscal multipliers. Following the same procedure, we also estimate the age sensitivity of the national consumption, investment, and employment fiscal multiplier.

Table 5 reports the results of this exercise. In the model a 1% increase in the share of young people raises the national output fiscal multiplier by 1.1%, from 0.94 up to 0.95. It also raises the consumption multiplier by 1.3% and the employment multiplier by 1%. Instead, the investment multiplier barely changes following an increase in the share of young people.

Although the age sensitivity is lower than for the case of local multipliers, it is still highly economically significant: changes in the age structure of an economy affect fiscal multipliers also at the national level. In Appendix B we validate the prediction of the model on the link between demographics and national fiscal multipliers. We estimate a SVAR on both a panel of developed countries and a panel of developing countries and identify government spending shock with a Choleski ordering à la Blanchard and Perotti (2002). In either case, we show that the long-run national output fiscal multiplier is indeed larger in countries with higher shares of young people in total population.

Table 5: National Fiscal Multipliers

	Output	Consumption	Investment	Employment
Avg. National Fiscal Multiplier	0.93	0.84	−0.90	1.38
Δ National Fiscal Multiplier of 1% Increase in Share Young People	1.1%	1.3%	−0.1%	1%

Note: The Table reports the results of the estimation of the national fiscal multipliers in the model. We consider the two-year output fiscal multiplier, the two-year output consumption fiscal multiplier, the two-year investment fiscal multiplier, and the two-year employment fiscal multiplier. The first row reports the estimated value of the national fiscal multipliers. The second row computes the age sensitivity of national fiscal multipliers.

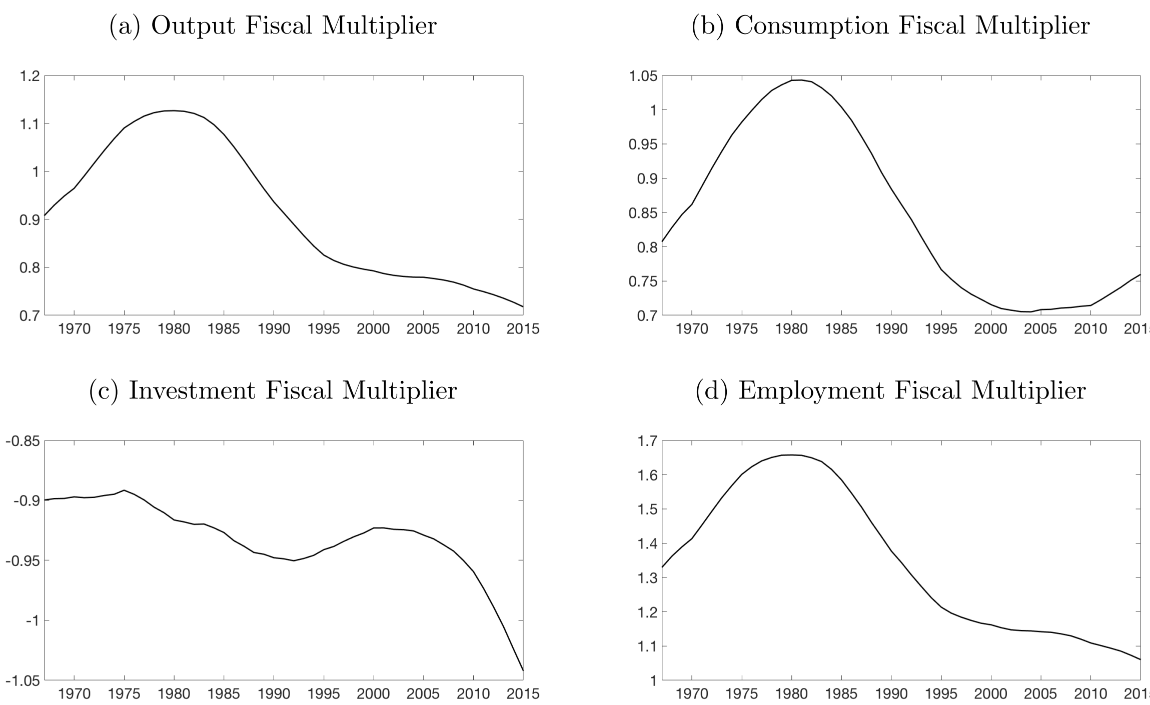
5 Population Aging and Fiscal Multipliers

After the World War II, the demographic structure of the U.S. population has been changing dramatically over time. For instance, the onset of the baby boomers raised the share of young people by 22% between 1967 and 1980. From 1980 on, the U.S. population has progressively shifted towards older ages. Indeed, from 1980 to 2015 the share of young people has shrunk by 30%.

What are the implications of the aging of the U.S. population on the effectiveness of fiscal policy? We address this question by feeding the model with the entire path of population shares observed from 1967 until 2015, and then compute national fiscal multipliers through the lenses of the model. Figure 2 shows the results of this exercise.

The output fiscal multiplier was 0.96 in 1970 and increased up to 1.13 in 1980, when the effect of the baby boom on the share of young people was the greatest. As the share of young people progressively shrinks, the multiplier starts decreasing, drops below 1 in 1988, and reaches a value of 0.72 in 2015. Hence, the model predicts that over the last forty years the size of the output fiscal multipliers went down by 36%. Interestingly, even if we remove both the age-specific labor supply elasticity and the ad-hoc borrowing constraint, the model predicts that from 1980 to 2015 the national output fiscal multipliers has decreased by 29%, from 1.08 down to 0.77.

Figure 2: Fiscal Multipliers from 1967 until 2015.



Note: This graph reports two-year national fiscal multipliers in a sequence of versions of the baseline model, which are calibrated to the changes in the population shares observed in the U.S. from 1967 to 2015. Panel (a) plots the two-year national output fiscal multiplier. Panel (b) plots the two-year national consumption fiscal multiplier. Panel (a) plots the two-year national investment fiscal multiplier. Panel (a) plots the two-year national employment fiscal multiplier.

A similar pattern characterizes also the consumption and employment multiplier:

over the last forty years the consumption fiscal multiplier decreased by 27% (from 1.04 down to 0.76) whereas the employment fiscal multipliers experienced a drop of 36% (from 1.65 down to 1.06). Instead, the investment fiscal multipliers shrinks very mildly over time until the early 2000s. Then, as the share of old people increases dramatically over the last 10 years, the investment fiscal multiplier becomes more and more negative, reaching a value of -1.04% in 2015. Indeed, as the increase in consumption of old households is larger than for mature households, the shift towards a rising share of old people boosts the consumption multiplier while decreasing even further the investment multiplier.

These results are consistent with the empirical evidence of Blanchard and Perotti (2002) and Bilbiie et *al.* (2008) on the reduction of the size of fiscal multipliers over time. Both papers show that fiscal multipliers in the recent decades are smaller than what they used to be during the 1960s and 1970s. From this perspective, our model provides a rationale of this empirical finding, by linking the process of aging of the U.S. population to the observed reduction in the effectiveness of fiscal policy.

Although over the recent decades the age structure of the U.S. population has already experienced a remarkable shift towards older ages, the population aging is not expected to decelerate. The United Nations project that by 2080 the share of old people will be around 30% and the share of young people will drop a further 20% from 2015 to 2080. To assess the implications of these changes, we feed our model with the projected shares of young, mature, and old people in the U.S. population in 2080, and compute the output fiscal multiplier. The model predicts that in 2080 the output fiscal multiplier will equal 0.88. Hence, in 2080 the output fiscal multiplier will be 58% lower than in 1980, and 33% lower than in 2015.

Our model predicts that in U.S. over the future decades fiscal policy would become a relatively less effective tool for spurring economy activity. Since most economies are experiencing a similar process of population aging, our results suggest that the reduction in the effectiveness of fiscal policy is a global phenomenon. This result has to be interpreted with two caveats. First, our analysis refers to the effectiveness of fiscal policy in normal times, abstracting from cases in which there is slack in the economy or the stance of monetary policy changes. Second, although fiscal policy - intended in the classical form of purchasing goods from the private sector - becomes less effective in spurring economic activity due to population aging, fiscal interventions targeted to specific age groups could be still highly expansionary. To this end, a new class of model as ours could be used as a laboratory to design and evaluate the effects of such policies.

6 Conclusion

This paper shows that the age structure of an economy determines the effectiveness of fiscal policy such that fiscal multipliers are larger in economies with higher shares of young people in total population.

First, we identify the causal effect of a change in demographics on the size of fiscal multipliers using the variation across U.S. states in government spending and lagged birth rates. We find that a 1% increase in the share of people between 20 and 29 years in total population raises the local fiscal multipliers by 3.1%.

Second, to rationalize this finding we build a tractable life-cycle open-economy New Keynesian model with credit market imperfections and age-specific labor supply elasticities. The model can explain 65% of the link between demographics and local fiscal multipliers: in the model a 1% increase in the share of people between 20 and 29 years in total population raises the local fiscal multipliers by 2%. Demographics affect the size of fiscal multipliers also at the national level. Indeed, a 1% increase in the share of young people raises the *national* fiscal multipliers by 1.1%.

Third, we use the model to study the implications of population aging for the effectiveness of fiscal policy. Over the recent decades, the demographic structure of the U.S. population has progressively shifted towards older ages: the share of young people in total population plummeted by 30% from 1980 to 2015. Once we feed this shift in population shares into our model, we find that nowadays national fiscal multipliers are 36% lower than forty years ago. This result suggests that the process of population aging could dampen over time the effectiveness of fiscal policy.

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A Local Fiscal Multipliers: Further Evidence

A.1 Additional National-Level and State-Level Controls

In Section 2 we have documented the causal effect of demographics on fiscal multipliers, such that fiscal multipliers are larger in economies with higher shares of young people in total population. This result could be biased by potential confounding factors which are highly correlated with changes in states' age structures. In this Section we address this issue and report a comprehensive battery of robustness checks for the estimates of both the output fiscal multiplier and the employment fiscal multiplier.

First, we define a number of national controls, such as the oil price (the annual average spot price of West Texas Intermediate), households' debt to GDP (the ratio of the credit market instruments - liability - of the households and nonprofit organizations from the Financial Accounts of the U.S. over the series of national GDP provided by the BEA), federal debt to GDP (the ratio of the total public debt from the U.S. Office of Management and Budget over the series of national GDP provided by the BEA), the military news variable of Ramey (2011) and Ramey and Zubairy (2018), and the real interest rate (the difference between the effective federal funds rate from the St. Louis Federal Reserves FRED database and the change in the Consumer Price Index for all urban consumers from the BLS).

Second, we consider state-level controls, such as the house price (provided by the U.S. Federal Housing Finance Agency from 1975 on), per capita real income (provided by the BEA), per capita real federal personal taxes (provided by the BEA), the unemployment rate (provided from the BLS from 1976 on), and per capita real unemployment benefits (provided by the BLS).

Table A.1 shows the estimate of the output fiscal multiplier - and its relationship with the share of young people in total population - in a number of alternative specifications of the baseline regression in which we add each time an additional

Table A.1: Response of Output to a Government Shock across U.S. States - Robustness Checks

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	National Controls					State Controls			
	Oil Price	Households' Debt	Federal Debt	Real Interest Rate	Ramey News	House Price	Personal Taxes	Unempl. Rate	Unempl. Benefits
	IV	IV	IV	IV	IV	IV	IV	IV	IV
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}}$	1.311***	1.661***	1.511***	1.500***	1.508***	0.795***	1.468***	0.627	1.500***
	(0.333)	(0.451)	(0.443)	(0.395)	(0.416)	(0.398)	(0.401)	(0.412)	(0.406)
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}} \times (D_{i,t} - \bar{D})$	0.039**	0.065***	0.041**	0.048***	0.039**	0.066***	0.048***	0.049***	0.047***
	(0.015)	(0.022)	(0.017)	(0.017)	(0.018)	(0.016)	(0.016)	(0.016)	(0.016)
$D_{i,t}$	0.001**	0.002**	0.002***	0.001**	0.002***	0.001***	0.001**	0.001**	0.002***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
R^2	0.446	0.371	0.397	0.405	0.389	0.441	0.378	0.451	0.375
N. Obs.	2374	2374	2374	2374	2374	2031	2374	2031	2374

Note: The table reports the estimates of panel regressions across U.S. states using data from 1967 to 2015 at an annual frequency, in which the dependent variable is the change in per capita real output. In all regressions, the independent variables are the change in per capita government spending, $(G_{i,t} - G_{i,t-2})/Y_{i,t-2}$, the log-share of young people (aged 20-29) in total population, $D_{i,t}$, and the interaction between the change in per capita government spending and the share of young people, $[(G_{i,t} - G_{i,t-2})/Y_{i,t-2}] \times (D_{i,t} - \bar{D})$. State-specific changes in state per capita government spending (as a fraction of state per capita GDP) are instrumented with the product of state fixed effects and the change in national per capita government spending (as a fraction of national per capita GDP). The share of young people is instrumented with 20-30 year lagged birth rates. In regressions (1) - (5) we include one additional national-level control to the benchmark specification, which we interact with state-fixed effects. Regression (1) includes the log-difference of the real oil price. Regression (2) includes households' debt to GDP ratio. Regression (3) includes federal debt to GDP ratio. Regression (4) includes the level of the real interest rate. Regression (5) includes Ramey government spending news variable. In regressions (6) - (9) we include one additional state-level control to the benchmark specification. Regression (6) includes the log-difference of the real house price. Regression (7) includes per capita real households' federal taxes. Regression (8) includes the unemployment rate. Regression (9) includes per capita real unemployment benefits. Robust standard errors clustered at the state level are reported in brackets. *, **, and *** indicate statistical significance at the 10%, 5% and 1%, respectively.

Table A.2: Response of Employment Rate to a Government Shock across U.S. States - Robustness Checks

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	National Controls					State Controls			
	Oil Price	Households' Debt	Federal Debt	Real Interest Rate	Ramey News	House Price	Personal Taxes	Unempl. Rate	Unempl. Benefits
	IV	IV	IV	IV	IV	IV	IV	IV	IV
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}}$	1.104***	1.070***	1.025***	1.069***	1.073***	0.416**	1.071***	0.325	1.084***
	(0.207)	(0.013)	(0.216)	(0.211)	(0.222)	(0.220)	(0.218)	(0.220)	(0.217)
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}} \times (D_{i,t}-\bar{D})$	0.033***	0.040***	0.034***	0.032***	0.035***	0.043***	0.035***	0.035***	0.035***
	(0.011)	(0.013)	(0.011)	(0.011)	(0.011)	(0.008)	(0.011)	(0.009)	(0.011)
$D_{i,t}$	0.001	0.001	0.001	0.001	0.001	-0.001	0.001	-0.001**	0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
R^2	0.630	0.635	0.641	0.639	0.625	0.723	0.624	0.726	0.622
N. Obs.	2374	2374	2374	2374	2374	2031	2374	2031	2374

Note: The table reports the estimates of panel regressions across U.S. states from 1967 to 2015, at an annual frequency, in which the dependent variable is the change in employment rate. In all regressions, the independent variables are the change in per capita government spending, $(G_{i,t}-G_{i,t-2})/Y_{i,t-2}$, the share of young people (aged 20-29) in total population, $D_{i,t}$, and the interaction between the change in per capita government spending and the log-share of young people, $[(G_{i,t}-G_{i,t-2})/Y_{i,t-2}] \times (D_{i,t}-\bar{D})$. State-specific changes in state per capita government spending (as a fraction of state per capita GDP) are instrumented with the product of state fixed effects and the change in national per capita government spending (as a fraction of national per capita GDP). The share of young people is instrumented with 20-30 year lagged birth rates. In regressions (1) - (5) we include one additional national-level control to the benchmark specification, which we interact with state-fixed effects. Regression (1) includes the log-difference of the real oil price. Regression (2) includes households' debt to GDP ratio. Regression (3) includes federal debt to GDP ratio. Regression (4) includes the level of the real interest rate. Regression (5) includes Ramey government spending news variable. In regressions (6) - (9) we include one additional state-level control to the benchmark specification. Regression (6) includes the log-difference of the real house price. Regression (7) includes per capita real households' federal taxes. Regression (8) includes the unemployment rate. Regression (9) includes per capita real unemployment benefits. Robust standard errors clustered at the state level are reported in brackets. *, **, and *** indicate statistical significance at the 10%, 5% and 1%, respectively.

Table A.3: Local Fiscal Multipliers: Skilled Workers & Female Labor Participation

	(1)	(2)	(3)	(4)
	Skilled Workers	Young Skilled Workers	Female Workers	Young Female Workers
	IV	IV	IV	IV
		Panel A. Response of Output		
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}}$	1.125**	1.177**	1.147**	1.138**
	(0.480)	(0.478)	(0.477)	(0.470)
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}} \times \left(D_{i,t} - \bar{D}\right)$	0.070***	0.070***	0.071***	0.067***
	(0.018)	(0.018)	(0.017)	(0.017)
$D_{i,t}$	0.001*	0.001**	0.001*	0.001**
	(0.001)	(0.001)	(0.001)	(0.001)
R^2	0.348	0.351	0.349	0.352
N. Observations	1982	1982	1982	1982
		Panel B. Response of Employment Rate		
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}}$	0.537*	0.611**	0.581*	0.591**
	(0.298)	(0.300)	(0.301)	(0.298)
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}} \times \left(D_{i,t} - \bar{D}\right)$	0.045***	0.046***	0.046***	0.045***
	(0.010)	(0.011)	(0.011)	(0.011)
$D_{i,t}$	-0.001*	-0.001	-0.001	-0.001
	(0.001)	(0.001)	(0.001)	(0.001)
R^2	0.664	0.660	0.660	0.663
N. Observations	1982	1982	1982	1982

Note: The table reports the estimates of panel regressions across U.S. states from 1967 to 2015 at an annual frequency. In Panel A the dependent variable is the change in output per capita. In Panel B the dependent variable is the change in the employment rate. If not stated otherwise, the independent variables are the change in per capita state government spending (as a fraction of per capita state GDP), $(G_{i,t} - G_{i,t-2})/Y_{i,t-2}$, the share of young people (aged 20-29) in total population, $D_{i,t}$, and the interaction between the change in per capita state government spending (as a fraction of per capita state GDP) and the log-share of young people, $[(G_{i,t} - G_{i,t-2})/Y_{i,t-2}] \times \left(D_{i,t} - \bar{D}\right)$. In the IV regressions, state-specific changes in per capita state government spending (as a fraction of per capita state GDP) are instrumented with the product of state fixed effects and the change in per capita national government spending (as a fraction of per capita national GDP). The share of young people is instrumented with 20-30 year lagged birth rates. Regression (1) includes as an additional independent variables states' share of skilled workers. Regression (2) includes states' share of skilled workers compute over the young population. Regression (3) includes states' share of female workers. Regression (4) includes states' share of female workers compute over the young population. We include time and state fixed effects in all the regressions. Robust standard errors clustered at the state level are reported in brackets. *, **, and *** indicate statistical significance at the 10%, 5% and 1%, respectively.

control variables. For the national level variables, we build state-specific values by interacting the variable with the state fixed effects. Table A.2 reports a similar battery of robustness checks for the employment fiscal multiplier. In all cases the estimated coefficient on the interaction between state government spending and the log-share of young people is always highly statistically and economically significant. Actually, the introduction of additional controls alters the level of fiscal multipliers but not the sensitivity of multipliers to states' age structure.

Table A.3 considers a further set of robustness check in which we include as additional controls the share of skilled workers and female workers in each state. We build these measures using CPS data from 1977 on. Again, the link between demographics and fiscal multipliers is not driven by recent trends in the U.S. labor market.

A.2 Population Response to Government Spending Shocks

Table A.4 studies the response of state population to a government spending shock. In this case, we estimate a simplified regression in which we consider as independent variable just the change in state government spending:

$$\frac{Pop_{i,t} - Pop_{i,t-2}}{Pop_{i,t-2}} = \alpha_i + \delta_t + \beta \frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} + \epsilon_{i,t}$$

where $Pop_{i,t}$ denotes the population of state i at time t. In particular, we consider four different definitions of population: (*i*) overall population, (*ii*) young population (i.e., people between 20 and 29 years old), (*iii*) mature population (i.e., people between 30 and 64 years old), and (*iv*) old population (i.e., people above 65 years old). Given data availability on the disaggregation of total population across age groups, this set of regressions uses annual data from 1969 until 2015.

Column (1) of Table A.4 shows that the overall population does not change fol-

Table A.4: Response of Population to a Government Spending Shock Across U.S. States

	(1) Overall Population IV	(2) Young Population IV	(3) Mature Population IV	(4) Old Population IV
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}}$	-0.179	1.145***	-0.398	-0.070
	(0.303)	(0.399)	(0.403)	(0.212)
R^2	0.611	0.654	0.584	0.790
N. Observations	2295	2295	2295	2295

Note: The table reports the estimates of panel regressions across U.S. states from 1969 to 2015 at an annual frequency. In Column (1) the dependent variable is the state overall white male population. In Column (2) the dependent variable is the state white male young population (aged 20-29). In Column (3) the dependent variable is the state white male mature population (aged 30-64). In Column (4) the dependent variable is the state white male old population (aged 65+). The independent variable is the change in per capita state government spending (as a fraction of per capita state GDP), which is instrumented with the product of state fixed effects and the change in per capita national government spending (as a fraction of per capita national GDP). We include time and state fixed effects in all the regressions. Robust standard errors clustered at the state level are reported in brackets. *, **, and *** indicate statistical significance at the 10%, 5% and 1%, respectively.

lowing a government spending shock. Yet, this aggregate result compounds different dynamics of the populations by age group. On the one hand, column (2) shows that the young population does rise following a fiscal shock. On the other hand, columns (3) and (4) show that mature and old population shrink following a government spending shock, even though this effect is not statistically significant.

These results are consistent with the findings of the literature on the sensitivity of state population to shocks. On the one hand, Blanchard and Katz (1992) show that state migration flows are important transmission mechanisms of changes in state unemployment rates over time. On the other hand, Nakamura and Steinsson (2014) find that overall state population does not react to government spending shocks at short horizon. Our results emphasize that although overall population may not change following a fiscal shock, this aggregate pattern masks heterogenous reactions in the population of different age groups.

This evidence validates our approach in instrumenting the share of young people with lagged birth rates. Indeed, as the young population does react to fiscal shocks, using raw log-shares of the young people in total population would also capture the endogenous reaction of states' age structure to government spending shocks. Hence, instrumenting the log-share of young people with lagged birth rates is key to identify the causal effect of demographics on the size of fiscal multipliers.

A.3 Hours Worked Response to Government Spending Shocks

In the model we assume that the labor supply elasticity varies exogenously across age groups. In the calibration exercise, the labor supply elasticity of young and old individuals to be larger than the labor supply elasticity of mature individuals. This feature reflects the fact that hours worked by young and old workers are much more volatile than hours worked by mature workers.

In this Section, we show that our calibration choice is also consistent with the response of hours worked to a government spending shock in the data. To do so, we use CPS data to build a measure by state of hours worked by all workers, hours worked by young workers (i.e., workers between 20 and 29 years old), and hours worked by older workers (i.e., workers above 30 years old). Our measure of hours worked equals the per-worker amount of hours worked, in which we focus only on non self-employed workers that are employed in the private sector.

Then, we use all these measures to estimate the effect of government spending on state hours worked with a regression in which the dependent variable is the growth rate of state hours worked $N_{i,t}$:

$$\frac{N_{i,t} - N_{i,t-2}}{N_{i,t-2}} = \alpha_i + \delta_t + \beta \frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} + \epsilon_{i,t} \tag{1}$$

Again, we instrument state military spending with a first-stage regression in which the independent variable is the product of a state fixed effect and the change in national military spending. Since CPS data start in 1977, we are left with 1887 observations, which is a substantial reduction in the sample size with respect our benchmark analysis, that spans from 1967 to 2015.

Table A.5: Response of Hours Worked to a Government Spending Shock Across U.S. States

	(1)	(2)	(3)
	All Workers	Young Workers	Older Workers
	IV	IV	IV
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}}$	0.656**	1.036	0.449
	(0.294)	(0.711)	(0.407)
R^2	0.176	0.100	0.121
N. Observations	1887	1887	1887

Note: The table reports the estimates of panel regressions across U.S. states from 1969 to 2015 at an annual frequency. In Column (1) the dependent variable is state hours worked by all workers. In Column (2) the dependent variable is state hours worked by young workers (i.e., workers between 20 and 29 years old). In Column (3) the dependent variable is state hours worked by older workers (i.e., workers above 30 years old). The independent variable is the change in per capita state government spending (as a fraction of per capita state GDP), which is instrumented with the product of state fixed effects and the change in per capita national government spending (as a fraction of per capita national GDP). We include time and state fixed effects in all the regressions. Robust standard errors clustered at the state level are reported in brackets. ** indicates statistical significance at the 10%, 5% and 1%, respectively.

Column (1) of Table A.5 shows that hours worked increase following a government spending shock. When we consider the disaggregated measures of hours worked by young and older workers, the uncertainty around the estimates is so large that we cannot reject the hypothesis that the local multiplier is zero in either case. Nevertheless, the point estimates of Column (2) and (3) highlight that the hours worked by young and older workers increase following a government spending shock, but with different sensitivities. Indeed, the point estimate of the response of hours worked by young workers is 2.5 times larger than the point estimate of the response of hours worked by older workers. Although we do acknowledge that these estimates are surrounded by a high degree of uncertainty, these results - together with the findings of Jaimovich and Siu (2009) - are consistent with our modeling approach of having the labor supply elasticity of young individuals to be higher than the labor supply elasticity of older workers.

A.4 Relevance of Birth Rates

In the baseline regression we instrument the share of young people in total population with lagged birth rates. This approach aims at avoiding any endogeneity of states' age structure with respect to government spending shocks. In particular, states' age structure would not be exogenous to government spending shocks if they trigger migration flows.[20] The use of lagged birth rates as an instrument imposes an identifying exclusion restriction which posits that, conditional on state and time fixed effects, whatever determines the cross-sectional variation in birth rates has no other long lasting effect on the size of fiscal multipliers 20-30 years later.

In this Section we study the relevance of lagged birth rates as an instrument for the share of young people in total population, by reporting the results of the first-stage regression of the share of young people on lagged birth rates. We consider four different cases for the share of young white males, the share of young males, and the share of overall young people: (*i*) we regress the raw share of young people on the raw series of lagged birth rates and both time and state fixed effects; (*ii*) we regress the residual series of the raw share of young people on the residual series of the raw series of lagged birth rates. Each residual variable is derived by taking the residuals of a regression in which the dependent variable is is either the share of young people or the lagged birth rates and the independent variables are state and time fixed effects; (*iii*) we regress the log-share of young people on the series of lagged birth rates in logarithm and both time and state fixed effects; (*iv*) we regress the residual series of the log-share of young people on the residual series of the series

[20]Blanchard and Katz (1992) show that state migration reacts to shocks. We find that although total population does not change following government spending shocks, the population of young people does rise.

Table A.6: First Stage Regression Share of Young White Males on Lagged Birth Rates

	(1) Share Young People	(2) Share Young People Residuals	(3) Share Young People Log	(4) Share Young People Log, Residuals
Lagged Birth Rates	0.317*** (0.062)			
Lagged Birth Rates (Residuals)		0.317*** (0.014)		
Lagged Birth Rates (Log)			0.509*** (0.064)	
Lagged Birth Rates (Log, Residuals)				0.509*** (0.018)
State FE	YES	NO	YES	NO
Year FE	YES	NO	YES	NO
R^2	0.938	0.176	0.934	0.259
N. Observations	2374	2374	2374	2374

Note: The table reports the results of the first-stage regression in which the share of young white males (aged 20-29) in the total white male population is regressed on 20-30 year lagged birth rates. In column (1) the raw share of young people is regressed on the raw series of lagged birth rates, in addition to state and year fixed effects. In column (2) the residual series of the raw share of young people is regressed on the residual series of the raw series of lagged birth rates. Each residual variable is derived by taking the residuals of a regression in which the dependent variable is the raw series and the independent variables are state and time fixed effects. In column (3) the log-share of young people is regressed on the series of lagged birth rates in logarithm, in addition to state and year fixed effects. In column (4) the residual series of the log-share of young people is regressed on the residual series of the raw series of lagged birth rates in logarithm. Each residual variable is derived by taking the residuals of a regression in which the dependent variable is the log series and the independent variables are state and time fixed effects. Robust standard errors clustered at the state level are reported in brackets. *** indicates statistical significance at the 1%.

Table A.7: First Stage Regression Share of Young Males on Lagged Birth Rates

	(1) Share Young People	(2) Share Young People Residuals	(3) Share Young People Log	(4) Share Young People Log, Residuals
Lagged Birth Rates	0.280*** (0.062)			
Lagged Birth Rates (Residuals)		0.280*** (0.013)		
Lagged Birth Rates (Log)			0.446*** (0.059)	
Lagged Birth Rates (Log, Residuals)				0.446*** (0.017)
State FE	YES	NO	YES	NO
Year FE	YES	NO	YES	NO
R^2	0.913	0.159	0.915	0.228
N. Observations	2374	2374	2374	2374

Note: The table reports the results of the first-stage regression in which the share of young males (aged 20-29) in the total male population is regressed on 20-30 year lagged birth rates. In column (1) the raw share of young people is regressed on the raw series of lagged birth rates, in addition to state and year fixed effects. In column (2) the residual series of the raw share of young people is regressed on the residual series of the raw series of lagged birth rates. Each residual variable is derived by taking the residuals of a regression in which the dependent variable is the raw series and the independent variables are state and time fixed effects. In column (3) the log-share of young people is regressed on the series of lagged birth rates in logarithm, in addition to state and year fixed effects. In column (4) the residual series of the log-share of young people is regressed on the residual series of the raw series of lagged birth rates in logarithm. Each residual variable is derived by taking the residuals of a regression in which the dependent variable is the log series and the independent variables are state and time fixed effects. Robust standard errors clustered at the state level are reported in brackets. *** indicates statistical significance at the 1%.

Table A.8: First Stage Regression Share of Young People on Lagged Birth Rates

	(1) Share Young People	(2) Share Young People Residuals	(3) Share Young People Log	(4) Share Young People Log, Residuals
Lagged Birth Rates	0.262*** (0.057)			
Lagged Birth Rates (Residuals)		0.262*** (0.012)		
Lagged Birth Rates (Log)			0.427*** (0.057)	
Lagged Birth Rates (Log, Residuals)				0.427*** (0.016)
State FE	YES	NO	YES	NO
Year FE	YES	NO	YES	NO
R^2	0.921	0.159	0.922	0.226
N. Observations	2374	2374	2374	2374

Note: The table reports the results of the first-stage regression in which the share of young people (aged 20-29) in the total population is regressed on 20-30 year lagged birth rates. In column (1) the raw share of young people is regressed on the raw series of lagged birth rates, in addition to state and year fixed effects. In column (2) the residual series of the raw share of young people is regressed on the residual series of the raw series of lagged birth rates. Each residual variable is derived by taking the residuals of a regression in which the dependent variable is the raw series and the independent variables are state and time fixed effects. In column (3) the log-share of young people is regressed on the series of lagged birth rates in logarithm, in addition to state and year fixed effects. In column (4) the residual series of the log-share of young people is regressed on the residual series of the raw series of lagged birth rates in logarithm. Each residual variable is derived by taking the residuals of a regression in which the dependent variable is the log series and the independent variables are state and time fixed effects. Robust standard errors clustered at the state level are reported in brackets. *** indicates statistical significance at the 1%.

of lagged birth rates in logarithm. Each residual variable is derived by taking the residuals of a regression in which the dependent variable is either the log-share of young people or the logged lagged birth rates and the independent variables are state and time fixed effects.

Table A.6 reports the results on the first-stage regressions for the share of young white males, Table A.7 reports the results on the first-stage regressions for the share of young males, and Table A.8 reports the results on the first-stage regressions for the share of overall young people. The results indicate that in all cases the lagged birth rates are a relevant instrument for the current share of young people in total population, as the relative coefficient on the instrument is always highly statistically significant at the 1% level. Moreover, when we use state and time fixed effects, the R^2 of the regressions ranges between 91% and 94%. Even in the case we use the residual series and we abstract from the state and time fixed effects, the R^2 still ranges between 22% and 24%. Hence, birthrates in a state do have a predictive power for the future age composition in that state.

Furthermore, comparing the results of Tables A.6-A.8, we find that lagged birth rates are a more relevant instrument for the share of young white males than for the share of young males or the share of all young people. Indeed, the regressions with the share of young white males feature the highest values for the R^2.

B National Fiscal Multipliers

The fact that at the state level demographics have an effect on fiscal multipliers which is statistically and economically significant does not necessarily imply that the same applies also at the national level. In this Section we provide some suggestive evidence showing that also national fiscal multipliers depend on demographics. To do so, we run a SVAR à la Blanchard and Perotti (2002) on both a panel of developed countries and a panel of developing countries. In either case, we show that the long-run national output fiscal multiplier is larger in countries with higher shares of young people in total population.

B.1 Data

We take the data from Ilzetzki et *al.* (2013). These authors compiled an unbalanced panel on government spending, GDP, current account, real effective exchange rate, and interest rates at quarterly frequency from 1960Q1 until 2009Q4 for 19 developed countries and 25 developing countries.[21] Then, we take the data on the demographic structure of each country from the World Population Prospects prepared by the Population Division of the Department of Economic and Social Affairs of the United Nations Secretariat. The data on demographics are at the annual frequency from 1950 on.

B.2 Econometric Specification

We estimate fiscal multiplier using a SVAR system as in Blanchard and Perotti (2002), such that

$$AX_{i,t} = \sum_{k=1}^{K} C_k X_{i,t-k} + BU_{i,t}$$

where $X_{i,t}$ is a vector that consists of the logarithm of real government expenditure, the logarithm of real GDP, the ratio of the real current account balance over GDP, and the log difference of the real effective exchange rate of country i. To identify government spending shocks, we follow the identification assumption of Blanchard and Perotti (2002): we assume that government spending reacts to changes in the other macroeconomic variables with the delay of a quarter. This assumption defines

[21] The developed countries are Australia, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Israel, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom and the United States. The developing countries are Argentina, Botswana, Brazil, Bulgaria, Chile, Colombia, Croatia, Czech Republic, Ecuador, El Salvador, Estonia, Hungary, Latvia, Lithuania, Malaysia, Mexico, Peru, Poland, Romania, Slovakia, Slovenia, South Africa, Thailand, Turkey, and Uruguay.

a Cholesky decomposition in which government spending is ordered first. For the selection of the lag structure of the panel SVAR we follow Ilzetzki et *al.* (2013) by choosing $K = 4$ lags. The results do not change if we choose a number of lags between 1 and 8.

To identify the role of demographics on fiscal multipliers, we do the following. First, we take all the developed countries and split them in two sets: 9 countries with high shares of young people in total population, and 10 countries with low share of young people in total population. Second, we estimate the SVAR system on the two different panels and compare the results. Then, we repeat the same exercise for the developing countries. In this case, we find 11 countries with high shares of young people and 14 countries with low shares.[22,23]

Finally, we follow Ilzetzki et *al.* (2013) and define the long-run output fiscal multiplier as $\frac{\sum_{t=0}^{\infty}(1+r_i)^{-t}\Delta Y_{i,t}}{\sum_{t=0}^{\infty}(1+r_i)^{-t}\Delta G_{i,t}}$, where $t = 0$ denotes the date in which the government expenditure shock occurs, and r_i is the median of the country specific nominal interest rate.

B.2.1 Results

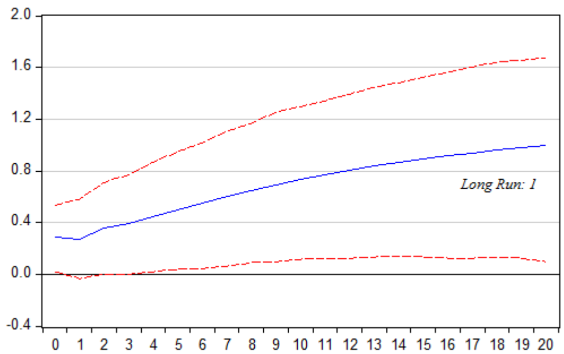
Figure B.1 reports the response of national output to an increase in government spending in both developed countries and developing countries. We also report the estimates of the long-run fiscal multiplier. Panel (a) shows the response in developed countries with high shares of young people in total population whereas Panel (b)

[22]We consider developed and developing countries separately because Ilzetzki et *al.* (2013) show that national fiscal multipliers in developed countries are large and positive, while in developing countries are large and negative. The results of Ilzetzki et *al.* (2013) suggest that other factors (e.g., the exchange rate policy rule, the degree of trade openness, and the level of public debt) could be explaining the differences in fiscal multipliers across our sets of countries.

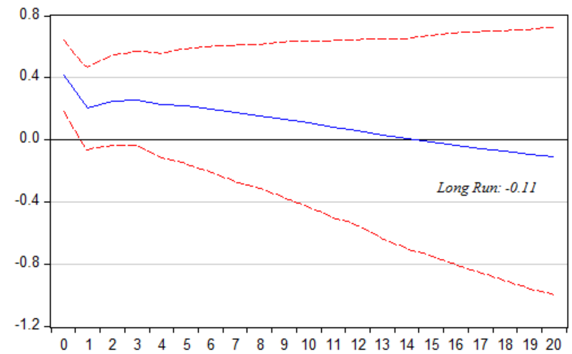
[23]Table B.9 reports countries' average share of young people (age 20-29) over total population computed from 1970 to 2010. We show how we group the countries in the set with high shares of young people and the set with low shares of young people. In the case of developed countries, the nine countries with high shares of young people have shares in the range of 15%-15.6%. Instead, the ten countries with low shares of young people have shares in the range of 13.5%-14.7%. In the case of developing countries, the eleven countries with high shares of young people have shares in the range of 16.4%-17.2%. Instead, the fourteen countries with low shares of young people have shares in the range of 14.7%-15.9%.

Figure B.1: National Fiscal Multipliers and Demographics.

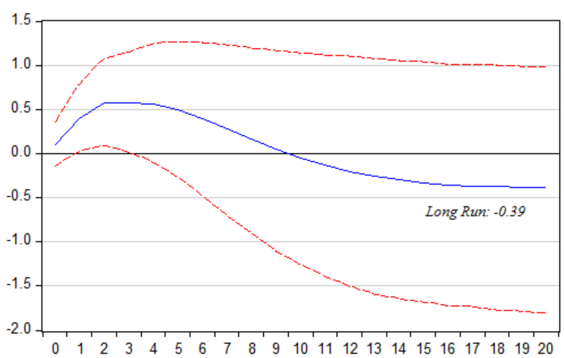
(a) High Income Countries - High Share of Young



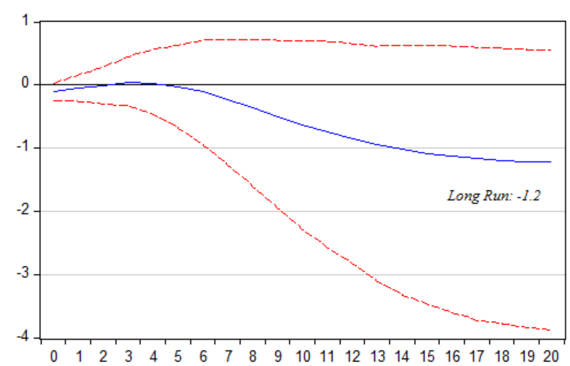
(b) High Income Countries - Low Share of Young



(c) Low Income Countries - High Share of Young



(d) Low Income Countries - Low Share of Young



Note: Panel (a) plots the cumulative national fiscal multipliers over twenty quarters following a government expenditure shock in a panel of nine high income countries with high shares of young people (i.e., age 20-29) in total population. Panel (b) plots the cumulative national fiscal multipliers in a panel of eleven high income countries with low shares of young people in total population. Panel (c) plots the cumulative national fiscal multipliers in a panel of eleven low income countries with high shares of young people in total population. Panel (d) plots the cumulative national fiscal multipliers in a panel of fourteen low income countries with low shares of young people in total population. In each Panel, the dotted lines display 90% confidence bands. The data on government expenditures and real GDP at quarterly frequency from 1960 until 2009 across 19 high income countries and 25 low income countries is from Ilzetzki et *al.* (2013).

plots the response in developed countries with low shares of young people in total population.

Although the impact response is similar across groups, in countries with low shares of young people the fiscal multipliers becomes statistically insignificant from zero from the first quarter on, leading to a long-run multiplier of -0.11. Instead, in countries with high shares of young people the fiscal multiplier is always statistically significant and the long-run multiplier equals 1.

Panel (c) and Panel (d) report the same set of results for developing countries. As already pointed out in Ilzetzki et *al.* (2013), fiscal multipliers in developing countries tend to be negative. Nevertheless, we find again that fiscal multipliers vary with the demographic structure of the countries. In the developing countries with high shares of young workers the impact responses are positive for the first ten periods, and interestingly the point estimate of the cumulative fiscal multiplier after two quarters is around 0.5, and is statistically different from zero. Then, the responses turn into negative values and as a result the long-run multiplier is -0.39. Instead, in the panel of developing countries with low shares of young people fiscal multipliers are much smaller. The impact responses are always negative and in the long-run the multiplier drops down to -1.2

Table B.9: Demographic Structure Across Countries

Average Share of Young People (Age 20-29) in Total Population			
Developed Countries		Developing Countries	
High Shares of Young People	Low Shares of Young People	High Shares of Young People	Low Shares of Young People
United States 15.0%	Sweden 13.5%	Mexico 16.4%	Hungary 14.7%
Portugal 15.0%	United Kingdom 13.9%	Ecuador 16.5%	Czech Republic 14.7%
Netherlands 15.1%	Norway 13.9%	Chile 16.6%	Bulgaria 14.9%
Greece 15.1%	Belgium 14.1%	Malaysia 16.6%	Uruguay 15.1%
Australia 15.2%	Denmark 14.1%	Peru 16.8%	Latvia 15.1%
Spain 15.5%	Germany 14.1%	Colombia 16.9%	Estonia 15.1%
Israel 15.6%	France 14.3%	Turkey 16.9%	Croatia 15.1%
Canada 15.6%	Ireland 14.3%	Botswana 17.0%	Lithuania 15.5%
Iceland 15.6%	Italy 14.5%	South Africa 17.1%	Slovenia 15.6%
	Finland 14.7%	Thailand 17.1%	Romania 15.8%
		Brazil 17.2%	Argentina 15.7%
			El Salvador 15.9%
			Poland 15.9%
			Slovakia 15.9%

Note: The table reports the average share of young people (age 20-29) over total population in percentage terms from 1970 until across both developed countries and developing countries.

C More on the Household Sector

In this Section we provide the maximization problems and the optimal conditions for each age group separately. We show that the optimal decisions of each individual are linear in wealth, so we can linearly aggregate the optimal choices of individuals within each age group to form a representative agent for each of the three age groups. For the sake of exposition, we derive the aggregation results only for the home economy. Nevertheless, the aggregation of the optimal choices of households within each age group in the foreign economy follows the same procedure. We derive all the problems and first-order conditions in real terms. We denote $\tilde{b}^j_{z,t} = \frac{b^j_{z,t}}{P_t}$ as the real bond-holdings of an individual i in the age group z at time t, $\tilde{a}^j_{z,t} = \frac{a^j_{z,t}}{P_t}$ is the real total return on assets of an individual i in the age group z at time t, $r_{k,t} = \frac{R_{k,t}}{P_t}$ is the real return on capital, and $w_t = \frac{W_t}{P_t}$ is the real wage. Finally, as in our calibration, we set $\psi_c = \psi_I$ such that $P_t = P_{I,t}$.

C.1 Old Agents

Assuming interior solutions for capital and bond holdings, the decision problem of an old agent i is

$$\max_{c^i_{o,t}, l^i_{o,t}, k^i_{o,t+1}, \tilde{b}^i_{o,t+1}} v^i_{o,t} = \left\{ \left(c^i_{o,t} - \chi_o \frac{{l^i_{o,t}}^{1+\frac{1}{\nu_o}}}{1+\frac{1}{\nu_o}} \right)^\eta + \beta \omega_o \mathbb{E}_t [v^i_{o,t+1}]^\eta \right\}^{1/\eta}$$

subject to

$$c^i_{o,t} + k^i_{o,t+1} + \tilde{b}_{o,t+1} + \frac{\varphi}{2} \left(\frac{k^i_{o,t+1}}{k^i_{o,t}} - \vartheta_o \right)^2 \frac{k^i_{o,t}}{\omega_o} = \tilde{a}^i_{o,t} + w_t \xi_o l^i_{o,t} - \tau^i_{o,t}$$

$$\tilde{a}^i_{o,t} = \left\{ k^i_{o,t} \left[(1-\delta) + r_{k,t} \right] + \tilde{b}^i_{o,t} \frac{R_{n,t}}{1+\pi_t} \right\} \left(\frac{1}{\omega_o} \right).$$

The first order and envelop theorem conditions are

$$l_{o,t}^{i} = \left(\frac{\xi_o w_t}{\chi_o}\right)^{\nu_o}$$

$$\left(c_{o,t}^{i} - \chi_o \frac{{l_{o,t}^{i}}^{1+\frac{1}{\nu_o}}}{1+\frac{1}{\nu_o}}\right)^{\eta-1} = \frac{\beta\omega_o}{\left(1+\frac{\varphi}{\omega_o}\left(\frac{k_{o,t+1}^{i}}{k_{o,t}^{i}} - \vartheta_o\right)\right)} \mathbb{E}_t \frac{\partial v_{o,t+1}^{i}}{\partial k_{o,t+1}^{i}} {v_{o,t+1}^{i}}^{\eta-1} = \dots$$

$$\dots = \beta\omega_o \mathbb{E}_t \frac{\partial v_{o,t+1}^{i}}{\partial \tilde{b}_{o,t+1}^{i}} {v_{o,t+1}^{i}}^{\eta-1}$$

$$\frac{\partial v_{o,t}^{i}}{\partial k_{o,t}^{i}} = {v_{o,t}^{i}}^{1-\eta} \left(c_{o,t}^{i} - \chi_o \frac{{l_{o,t}^{i}}^{1+\frac{1}{\nu_o}}}{1+\frac{1}{\nu_o}}\right)^{\eta-1} \times \dots$$

$$\dots \times \frac{[(1-\delta) + r_{k,t}] - \frac{\varphi}{2}\left(\frac{k_{o,t+1}^{i}}{k_{o,t}^{i}} - \vartheta_o\right)^2 + \varphi\left(\frac{k_{o,t+1}^{i}}{k_{o,t}^{i}} - \vartheta_o\right)\frac{k_{o,t+1}^{i}}{k_{o,t}^{i}}}{\omega_o}$$

$$\frac{\partial v_{o,t}^{i}}{\partial \tilde{b}_{o,t}^{i}} = {v_{o,t}^{i}}^{1-\eta} \left(c_{o,t}^{i} - \chi_o \frac{{l_{o,t}^{i}}^{1+\frac{1}{\nu_o}}}{1+\frac{1}{\nu_o}}\right)^{\eta-1} \frac{R_{n,t}}{(1+\pi_t)\,\omega_o}.$$

Combining these conditions above gives the Euler equation

$$\left(c_{o,t}^{i} - \chi_o \frac{{l_{o,t}^{i}}^{1+\frac{1}{\nu_o}}}{1+\frac{1}{\nu_o}}\right) = \mathbb{E}_t \left(\frac{\beta R_{n,t+1}}{1+\pi_{t+1}}\right)^{1/(1-\eta)} \left(c_{o,t+1}^{i} - \chi_o \frac{{l_{o,t+1}^{i}}^{1+\frac{1}{\nu_o}}}{1+\frac{1}{\nu_o}}\right).$$

Then, we conjecture that retirees consume a fraction of all their assets (i.e., the sum of financial assets and the present value of human capital gains, net of taxes and adjustment costs), such that

$$c_{o,t}^{i} = \varepsilon_t \varsigma_t \left[\tilde{a}_{o,t}^{i} + HC_{o,t}^{i} - T_{o,t}^{i} - ADJ_{o,t}^{i}\right]$$

where ς_t is the marginal propensity to consume of mature agents and therefore ε_t measures the wedge in the marginal propensity to consume between mature agents and old agents. For instance, if $\varepsilon_t > 1$, the old agents have a higher marginal propensity to consume than mature agents. Then, $\tilde{a}_{o,t}^{i}$ denotes the assets of the individual i of the old age group, $HC_{o,t}^{i}$ defines the value of the human capital of an old agent, $T_{o,t}^{i}$ defines the present value of taxes for agent i, and $ADJ_{o,t}^{i}$ is the present value of the adjustment costs incurred by agent i.

The no-arbitrage condition on investment posits that the expected return on capital should equalize the expected return on bonds, that is,

$$\mathbb{E}_t\left(\frac{R_{n,t+1}}{1+\pi_{t+1}}\right)=\mathbb{E}_t\left(\frac{(1-\delta)+r_{k,t+1}-\frac{\varphi}{2}\left(\frac{k^i_{o,t+2}}{k^i_{o,t+1}}-\vartheta_o\right)^2+\varphi\left(\frac{k^i_{o,t+2}}{k^i_{o,t+1}}-\vartheta_o\right)\frac{k^i_{o,t+2}}{k^i_{o,t+1}}}{\left[1+\frac{\varphi}{\omega_o}\left(\frac{k^i_{o,t+1}}{k^i_{o,t}}-\vartheta_o\right)\right]}\right) \tag{C.1}$$

Rearranging the budget constraint and our guess on consumption yields to the following law of motion for total assets of the old agent i

$$\tilde{a}^i_{o,t+1}=\frac{R_{n,t+1}}{(1+\pi_{t+1})\,\omega_o}\Big(\tilde{a}^i_{o,t}(1-\varepsilon_t\varsigma_t)+w_t\xi_o l^i_{o,t}-\varepsilon_t\varsigma_t HC^i_{o,t}-\tau^i_{o,t}+\varepsilon_t\varsigma_t T^i_{o,t}-adj^i_{o,t}+\varepsilon_t\varsigma_t ADJ^i_{o,t}\Big).$$

where $adj^i_{o,t}=\left(1-\frac{(1-\delta+r_{k,t+1})(1+\pi_{t+1})}{R_{n,t+1}}\right)k^i_{o,t+1}+\frac{\varphi}{2}\left(\frac{k^i_{o,t+1}}{k^i_{o,t}}-\vartheta_o\right)^2\frac{k^i_{o,t}}{\omega_o}$. The condition above, together with the Euler equation and the guess for consumption, gives

$$\begin{aligned}
&\varepsilon_t\varsigma_t\left[a^i_{o,t}+HC^i_{o,t}-T^i_{o,t}-ADJ^i_{o,t}\right]-\chi_o\frac{{l^i_{o,t}}^{1+\frac{1}{\nu_o}}}{1+\frac{1}{\nu_o}}=\dots\\
&\quad\dots=\left(\frac{\beta R_{n,t+1}}{1+\pi_{t+1}}\right)^{\frac{1}{\eta-1}}\left(\varepsilon_{t+1}\varsigma_{t+1}\left[\frac{R_{n,t+1}}{(1+\pi_{t+1})\,\omega_o}\left(a^i_{o,t}(1-\varepsilon_t\varsigma_t)+(w_t\xi_o l^i_{o,t}-\varepsilon_t\varsigma_t HC^i_{o,t}-\dots\right.\right.\right.\\
&\quad\quad\left.\left.\left.\dots-\tau^i_{o,t}+\varepsilon_t\varsigma_t T^i_{o,t}-\mathrm{adj}^i_{o,t}+\varepsilon_t\varsigma_t ADJ^i_{o,t}\right)+HC^i_{o,t+1}-T^i_{o,t+1}-ADJ^i_{o,t+1}\right]-\chi_o\frac{{l^i_{o,t+1}}^{1+\frac{1}{\nu_o}}}{1+\frac{1}{\nu_o}}\right).
\end{aligned}$$

Collecting terms we have that

$$\begin{aligned}
1-\varepsilon_t\varsigma_t&=\frac{\beta(R_{n_t+1}/\,(1+\pi_{t+1}))^{\frac{1}{1-\eta}}\omega_o}{(R_{n_t+1}/\,(1+\pi_{t+1}))}\frac{\varepsilon_t\varsigma_t}{\varepsilon_{t+1}\varsigma_{t+1}}\\
T^i_{o,t}&=\tau^i_{o,t}+\frac{(1+\pi_{t+1})\,\omega_o}{R_{n,t+1}}T^i_{o,t+1}\\
ADJ^i_{o,t}&=\mathrm{adj}^i_{o,t}+\frac{(1+\pi_{t+1})\,\omega_o}{R_{n,t+1}}ADJ^i_{o,t+1}
\end{aligned}$$

$$HC^i_{o,t} = w_t \xi_o l^i_{o,t} + \frac{(1+\pi_{t+1})\,\omega_o}{R_{n,t+1}} HC^i_{o,t+1} + \frac{(1-\varepsilon_t\varsigma_t)}{\varepsilon_t\varsigma_t}\frac{1}{1+\frac{1}{\nu_o}} \times \dots$$

$$\dots \times \left(w_t \xi_o l^i_{o,t} - \left(\frac{\beta R_{n,t+1}}{1+\pi_{t+1}}\right)^{\frac{1}{\eta-1}} w_{t+1}\xi_o l^i_{o,t+1} \right).$$

We can also show that $v^i_{o,t} = (\varepsilon_t\varsigma_t)^{\frac{-1}{\eta}} \left(c^i_{o,t} - \chi_o \frac{(l^i_{o,t})^{1-\frac{1}{\nu_o}}}{1-\frac{1}{\nu_o}} \right)$.

C.2 Mature Agents

The decision problem of a mature agent i, assuming interior solutions for capital and bond holdings, is

$$\max_{c^i_{m,t}, l^i_{m,t}, k^i_{m,t+1}, b^i_{m,t+1}} v^i_{m,t} = \left\{ \left(c^i_{m,t} - \chi_m \frac{{l^i_{m,t}}^{1+\frac{1}{\nu_m}}}{1+\frac{1}{\nu_m}} \right)^\eta + \beta \mathbb{E}_t[\omega_m v^i_{m,t+1} + (1-\omega_m) v^i_{o,t+1}]^\eta \right\}^{1/\eta}$$

subject to

$$k^i_{m,t+1} + \tilde{b}^i_{m,t+1} + c^i_{m,t} + \frac{\varphi}{2}\left(\frac{k^i_{m,t+1}}{k^i_{m,t}} - \vartheta_m\right)^2 k^i_{m,t} = a_{jw,t} + w_t l^i_{m,t} + (1-\tau_d) d^i_{m,t} - \tau^i_{m,t}$$

$$\tilde{a}^i_{m,t} = k^i_{m,t}((1-\delta) + r_{k,t}) + \tilde{b}^i_{m,t}\frac{R_{nt}}{1+\pi_t}.$$

First order conditions and envelop theorem are

$$l^i_{m,t} = \left(\frac{w_t}{\chi_m}\right)^{\nu_m}$$

$$\left(c^i_{m,t} - \chi_m \frac{{l^i_{m,t}}^{1-\frac{1}{\nu_m}}}{1-\frac{1}{\nu_m}} \right)^{\eta-1} = \frac{\beta}{\left(1+\varphi\left(\frac{k^i_{m,t+1}}{k^i_{m,t}} - \vartheta_m\right)\right)} \mathbb{E}_t[\omega_m v^i_{m,t+1} + (1-\omega_m) \times \dots$$

$$\dots \times v^i_{o,t+1}]^{\eta-1} \left[\omega_m \frac{\partial v^i_{m,t+1}}{\partial k^i_{m,t+1}} + (1-\omega_m)\frac{\partial v^i_{o,t+1}}{\partial k^i_{m,t+1}} \right]$$

$$\left(c_{m,t}^i - \chi_m \frac{{l_{m,t}^i}^{1-\frac{1}{\nu_m}}}{1-\frac{1}{\nu_m}}\right)^{\eta-1} = \beta \mathbb{E}_t[\omega_m v_{m,t+1}^i + (1-\omega_m) v_{o,t+1}^i]^{\eta-1} \times \dots$$
$$\dots \times \left[\omega_m \frac{\partial v_{m,t+1}^i}{\partial \tilde{b}_{m,t+1}^i} + (1-\omega_m)\frac{\partial v_{o,t+1}^i}{\partial \tilde{b}_{m,t+1}^i}\right]$$
$$\frac{\partial v_{mt}^i}{k_{m,t}^i} = {v_{m,t}^i}^{1-\eta}\left(c_{m,t}^i - \chi_m \frac{{l_{m,t}^i}^{1-\frac{1}{\nu_m}}}{1-\frac{1}{\nu_m}}\right)^{\eta-1}\left((1-\delta) + r_{k,t} - \frac{\varphi}{2}\left(\frac{k_{m,t+1}^i}{k_{m,t}^i} - \vartheta_m\right)^2 + \dots\right.$$
$$\left.\dots + \varphi\left(\frac{k_{m,t+1}^i}{k_{m,t}^i} - \vartheta_m\right)\frac{k_{m,t+1}^i}{k_{m,t}^i}\right)$$
$$\frac{\partial v_{m,t}^i}{\tilde{b}_{m,t}^i} = {v_{m,t}^i}^{1-\eta}\left(c_{m,t}^i - \chi_m \frac{{l_{m,t}^i}^{1-\frac{1}{\nu_m}}}{1-\frac{1}{\nu_m}}\right)^{\eta-1}\frac{R_{n,t}}{1+\pi_t}.$$

Since individuals are risk neutral, they select the same (conditional) asset profile independently of whether they are mature agents or old agents. The only difference lies in the different discounting due to the probability of death, which agents take into account as they become old. Hence, we have that $\frac{\partial \tilde{b}_{o,t}^i}{\partial \tilde{b}_{m,t}^i} = \frac{1}{\omega_o}$. We can then use the solution to the old agents' problem to determine $\frac{\partial v_{o,t+1}^i}{\partial \tilde{b}_{m,t+1}^i}$.

The no-arbitrage condition on investment posits that the expected return on capital should equalize the expected return on bonds, that is,

$$\mathbb{E}_t\left(\frac{R_{n,t+1}}{1+\pi_{t+1}}\right) = \mathbb{E}_t\left(\frac{(1-\delta) + r_{k,t+1} - \frac{\varphi}{2}\left(\frac{k_{m,t+2}^i}{k_{m,t+1}^i} - \vartheta_m\right)^2 + \varphi\left(\frac{k_{m,t+2}^i}{k_{m,t+1}^i} - \vartheta_m\right)\frac{k_{m,t+2}^i}{k_{m,t+1}^i}}{\left(1+\varphi\left(\frac{k_{m,t+1}^i}{k_{m,t}^i} - \vartheta_m\right)\right)}\right) \tag{C.2}$$

Combining the conditions above, and using the conjecture that

$$v_{m,t}^i = (\varsigma_t)^{\frac{-1}{\eta}}\left(c_{m,t}^i - \frac{\chi_m}{1-\nu_m}{l_{m,t}^i}^{1-\frac{1}{\nu_m}}\right)$$

gives the Euler equation

$$\left(c^i_{m,t} - \chi_m \frac{{l^i_{m,t}}^{1-\frac{1}{\nu_m}}}{1-\frac{1}{\nu_m}}\right) \mathbb{E}_t \left(\beta \mathfrak{z}_{t+1} \frac{R_{m,t+1}}{1+\pi_{t+1}}\right)^{1/(1-\eta)} = \mathbb{E}_t \Bigg[\omega_m \left(c^i_{m,t+1} - \chi_m \frac{{l^i_{m,t+1}}^{1-\frac{1}{\nu_m}}}{1-\frac{1}{\nu_m}}\right) + \dots$$

$$\dots + (1-\omega_m)\varepsilon_{t+1}^{\frac{-1}{\eta}} \left(c^i_{o,t+1} - \chi_o \frac{{l^i_{o,t+1}}^{1-\nu_o}}{1-\nu_o}\right)\Bigg]$$

where $\mathfrak{z}_{t+1} = (\omega_m + (1-\omega_m)\varepsilon_{t+1}^{\frac{\eta-1}{\eta}})$.

We conjecture that mature agents consume a fraction of all assets (which include the financial assets, the present value of their human capital gains, profits, and transfers), such that

$$c^i_{m,t} = \varsigma_t \left[\tilde{a}^i_{m,t} + HC^i_{m,t} - T^i_{m,t} + \tilde{D}^i_{m,t} - ADJ^i_{m,t}\right].$$

As before, we can write the law of motion of total assets of mature agent i as

$$\tilde{a}^i_{m,t+1} = \frac{R_{n,t+1}}{1+\pi_{t+1}} \left(\tilde{a}^i_{m,t}(1-\varsigma_t) + w_t l^i_{m,t} - \varsigma_t HC^i_{m,t} + (1-\tau_d)d^i_{m,t} - \varsigma_t \tilde{D}^i_{m,t} - \tau^i_{m,t} + \varsigma_t T^i_{m,t} - adj^i_{m,t} + \varsigma_t ADJ^i_{m,t}\right).$$

where, $adj^i_{m,t} = \left(1 - \frac{(1-\delta+r_{k,t+1})(1+\pi_{t+1})}{R_{n,t+1}}\right) k^i_{m,t+1} + \frac{\varphi}{2}\left(\frac{k^i_{m,t+1}}{k^i_{m,t}} - \vartheta_m\right)^2 k^i_{m,t}$

Following the same procedure as before we have that

$$\left(\varsigma_t \left[\tilde{a}^i_{m,t} + HC^i_{m,t} - T^i_{m,t} + \tilde{D}^i_{m,t} - ADJ^i_{m,t}\right] - \chi_m \frac{{l^i_{m,t}}^{1-\frac{1}{\nu_m}}}{1-\frac{1}{\nu_m}}\right) (\beta \frac{R_{n,t+1}}{1+\pi_{t+1}} \mathfrak{z}_{t+1})^{\frac{1}{1-\eta}} = \dots$$

$$\dots = \omega_m \Bigg(\varsigma_{t+1} \Bigg[\frac{R_{n,t+1}}{1+\pi_{t+1}} (\tilde{a}^i_{m,t}(1-\varsigma_t) + w_t l^i_{m,t} - \varsigma_t HC^i_{m,t} + (1-\tau_d)d^i_{m,t} - \varsigma_t \tilde{D}^i_{m,t} - \tau^i_{m,t} + \varsigma_t T^i_{m,t} - \dots$$

$$\dots - \mathrm{adj}^i_{m,t} + \varsigma_t ADJ^i_{m,t}) + HC^i_{m,t+1} - T^i_{m,t+1} + \tilde{D}^i_{m,t+1} - ADJ^i_{m,t+1}\Bigg] - \chi \frac{{l^i_{m,t}}^{1-\frac{1}{\nu_m}}}{1-\frac{1}{\nu_m}}\Bigg) + \dots$$

$$\dots + \varepsilon_{t+1}^{\frac{-1}{\eta}}(1-\omega_m)\big(\varepsilon_{t+1}\varsigma_{t+1}\big[\frac{R_{n,t+1}}{1+\pi_{t+1}} \times (\tilde{a}^i_{m,t}(1-\varsigma_t) + w_t l^i_{m,t} - \varsigma_t H^i_{m,t} + (1-\tau_d)d^i_{m,t} - \varsigma_t \tilde{D}^i_{m,t} - \dots$$

$$\dots - \tau^i_{m,t} + \varsigma_t T^i_{m,t} - \mathrm{adj}^i_{m,t} + \varsigma_t ADJ^i_{m,t}) + HC^i_{o,t+1} - T^i_{o,t+1} - ADJ^i_{o,t+1}\big] - \chi_o \frac{{l^i_{o,t+1}}^{1-\frac{1}{\nu_o}}}{1-\frac{1}{\nu_o}}).$$

Collecting terms and simplifying we have that

$$
\begin{aligned}
\varsigma_t &= 1-\frac{\varsigma_t}{\varsigma_{t+1}}\frac{(\beta(R_{n_t+1}/(1+\pi_{t+1}))\mathfrak{z}_{t+1})^{1/(1-\eta)}}{(R_{n,t+1}/(1+\pi_{t+1}))\mathfrak{z}_{t+1}}\\
T^i_{m,t} &= \tau^i_{m,t}+\frac{\omega_m}{(R_{n,t+1}/(1+\pi_{t+1}))\mathfrak{z}_{t+1}}T^i_{m,t+1}+\frac{(1-\omega_m)\varepsilon_{t+1}^{(\eta-1)/\eta}}{(R_{n,t+1}/(1+\pi_{t+1}))\mathfrak{z}_{t+1}}T^i_{o,t+1}\\
ADJ^i_{m,t} &= \mathrm{adj}^i_{m,t}+\frac{\omega_m}{(R_{n,t+1}/(1+\pi_{t+1}))\mathfrak{z}_{t+1}}ADJ^i_{m,t+1}+\frac{(1-\omega_m)\varepsilon_{t+1}^{(\eta-1)/\eta}}{(R_{n,t+1}/(1+\pi_{t+1}))\mathfrak{z}_{t+1}}ADJ^i_{o,t+1}\\
\tilde{D}^i_{m,t} &= (1-\tau_d)d^i_{m,t}+\frac{\omega_m}{(R_{n,t+1}/(1+\pi_{t+1}))\mathfrak{z}_{t+1}}\tilde{D}^i_{m,t+1}\\
HC^i_{m,t} &= w_t l^i_{m,t}+\frac{\omega_m}{(R_{n,t+1}/(1+\pi_{t+1}))\mathfrak{z}_{t+1}}HC^i_{m,t+1}+\frac{(1-\omega_m)\varepsilon_{t+1}^{\frac{\eta-1}{\eta}}}{(R_{n,t+1}/(1+\pi_{t+1}))\mathfrak{z}_{t+1}}HC^i_{o,t+1}+\dots\\
&\quad \dots+\frac{(1-\varsigma_t)}{\varsigma_t}\left[\frac{w_t l^i_{m,t}}{1+\frac{1}{\nu_m}}-(\beta\mathfrak{z}_{t+1}R_{n,t+1}/(1+\pi_{t+1}))^{1/(\eta-1)}\times\dots\right.\\
&\quad \left.\dots\times\left(\omega_m\frac{w_{t+1}l^i_{m,t+1}}{1+\frac{1}{\nu_m}}+(1-\omega_m)\varepsilon_{t+1}^{-1/\eta}\frac{w_{t+1}\xi_o l^i_{o,t+1}}{1+\frac{1}{\nu_o}}\right)\right].
\end{aligned}
$$

C.3 Young Agents

The solution to the problem of a young agent i is similar to the mature agents. Since individuals are risk neutral, they select the same (conditional) asset profile independently of whether they are mature agents or young agents, that is $\frac{\partial\tilde{b}^i_{m,t}}{\partial\tilde{b}^i_{y,t}}=1$. As such we have that

$$
\tilde{a}^i_{y,t+1}=\frac{R_{n,t+1}}{1+\pi_{t+1}}\left(\tilde{a}^i_{y,t}(1-\varsigma_t)+\xi_y w_t l^i_{y,t}-\varsigma_t HC^i_{y,t}-\tau^i_{y,t}+\varsigma_t T^i_{y,t}-adj^i_{y,t}+\varsigma_t ADJ^i_{y,t}\right).
$$

where, $adj^i_{y,t}=\left(1-\frac{(1-\delta+r_{k,t+1})(1+\pi_{t+1})}{R_{n,t+1}}\right)k^i_{y,t+1}+\frac{\varphi}{2}\left(\frac{k^i_{y,t+1}}{k^i_{y,t}}-\vartheta_y\right)^2 k^i_{y,t}$, and

$$
\begin{aligned}
c^i_{y,t} =&\varepsilon_{y,t}\varsigma_t\left[\tilde{a}^i_{y,t}+HC^i_{y,t}-T^i_{y,t}-ADJ^i_{y,t}\right]\\
1-\varepsilon_{y,t}\varsigma_t =&\frac{\varepsilon_{y,t}\varsigma_t}{\varsigma_{t+1}}\frac{(\beta_y(R_{n,t+1}/(1+\pi_{t+1}))\mathfrak{z}_{y,t+1})^{1/(1-\eta)}}{(R_{n,t+1}/(1+\pi_{t+1}))\mathfrak{z}_{y,t+1}}
\end{aligned}
$$

$$
\begin{aligned}
T^i_{y,t} =& \tau^i_{y,t} + \frac{\omega_m \varepsilon^{(\eta-1)/\eta}_{y,t+1}}{(R_{n,t+1}/\left(1+\pi_{t+1}\right))\mathfrak{z}_{y,t+1}} T^i_{y,t+1} + \frac{(1-\omega_m)}{(R_{n,t+1}/\left(1+\pi_{t+1}\right))\mathfrak{z}_{y,t+1}} (T^i_{m,t+1} - \tilde{D}^i_{m,t+1}) \\
ADJ^i_{y,t} =& adj^i_{y,t} + \frac{\omega_m \varepsilon^{(\eta-1)/\eta}_{y,t+1}}{(R_{n,t+1}/\left(1+\pi_{t+1}\right))\mathfrak{z}_{y,t+1}} ADJ^i_{y,t+1} + \frac{(1-\omega_m)}{(R_{n,t+1}/\left(1+\pi_{t+1}\right))\mathfrak{z}_{y,t+1}} (ADJ^i_{m,t+1}) \\
HC^i_{y,t} =& \xi_y w_t l^i_{y,t} + \frac{\omega_m \varepsilon^{(\eta-1)/\eta}_{y,t+1}}{(R_{n,t+1}/\left(1+\pi_{t+1}\right))\mathfrak{z}_{y,t+1}} HC^i_{y,t+1} + \frac{(1-\omega_m)}{(R_{n,t+1}/\left(1+\pi_{t+1}\right))\mathfrak{z}_{y,t+1}} HC^i_{m,t+1} + \dots \\
& \dots + \frac{(1-\varepsilon_{y,t}\varsigma_t)}{\varepsilon_{y,t}\varsigma_t} \left(\frac{\xi_y w_t l^i_{y,t}}{1+\frac{1}{\nu_y}} - \left(\mathfrak{z}_{y,t+1}\beta_y R_{n,t+1}/\left(1+\pi_{t+1}\right)\right)^{1/(\eta-1)} \times \dots \right. \\
& \left. \dots \times \left(\omega_m \varepsilon^{\frac{-1}{\eta}}_{y,t+1} \frac{\xi_y w_{t+1} l^i_{y,t+1}}{1-\frac{1}{\nu_y}} + (1-\omega_m) \frac{w_{t+1} l^i_{m,t+1}}{1-\frac{1}{\nu_m}} \right) \right).
\end{aligned}
$$

where $\mathfrak{z}_{y,t+1} = (\omega_m \varepsilon^{(\eta-1)/\eta}_{y,t+1} + (1-\omega_m))$ and $\varepsilon_{y,t}$ measures the wedge in the marginal propensity to consume between mature agents and young agents.

C.4 Aggregation

In this Section we show that we can linearly aggregate the optimal choices of individuals across each age group, such that for a variable $x_{z,t}$ we have that $x_{z,t} = \int_0^{N_{z,t}} x^i_{y,t}\,\mathrm{di}$.

Firstly we must ensure that at steady state adjustment costs are zero. Given the arbitrage conditions (C.1), (C.2), and its counterpart for the young problem, we have that the ratio of capital for any agent within a type is constant, which is to say that $\frac{k^i_{y,t+1}}{k^i_{y,t}} = \frac{\hat{k}_{y,t+1}}{\hat{k}_{y,t}}$, $\frac{k^i_{m,t+1}}{k^i_{m,t}} = \frac{\hat{k}_{m,t+1}}{\hat{k}_{m,t}}$, and $\frac{k^i_{o,t+1}}{k^i_{o,t}} = \frac{\hat{k}_{o,t+1}}{\hat{k}_{o,t}}$, where $\frac{\hat{k}_{y,t+1}}{\hat{k}_{y,t}}$, $\frac{\hat{k}_{m,t+1}}{\hat{k}_{m,t}}$, and $\frac{\hat{k}_{o,t+1}}{\hat{k}_{o,t}}$ define given age-group specific values for the ratio of physical capital holdings over time. Then given that individuals are born with no capital at steady state we have that

$$
k_{y,t+1} = \int_0^{N_{y,t}} k^i_{y,t+1} = \int_0^{N_{y,t+1}} k^i_{y,t+2} = k_{y,t+2} = k_{y,SS}
$$

As the young individuals who become mature are selected randomly

$$
k_{y,SS} = \int_0^{N_{y,t+1}} k^i_{y,t+2} = \int_0^{\omega_y N_{y,t+1}} \frac{\hat{k}_{y,t+1}}{\hat{k}_{y,t}} k^i_{y,t+1} = \omega_y \frac{\hat{k}_{y,t+1}}{\hat{k}_{y,t}} \int_0^{N_{y,t}} k^i_{y,t+1} = \omega_y \frac{\hat{k}_{y,t+1}}{\hat{k}_{y,t}} k_{y,SS}
$$

Hence, $\frac{\hat{k}_{y,t+1}}{\hat{k}_{y,t}} \mid_{SS} = \frac{1}{\omega_y}$. For mature agents we have that

$$k_{m,t+1} = \int_0^{N_{m,t}} k^i_{m,t+1} = \int_0^{N_{m,t+1}} k^i_{m,t+2} = k_{m,t+2} = k_{my,SS}$$

where

$$\begin{aligned} k_{m,SS} &= \int_0^{N_{m,t+1}} k^i_{m,t+2} = \int_0^{\omega_m N_{m,t+1}} \frac{\hat{k}_{m,t+1}}{\hat{k}_{m,t}} k^i_{m,t+1} + \int_0^{(1-\omega_y)N_{y,t+1}} \frac{\hat{k}_{y,t+1}}{\hat{k}_{y,t}} k^i_{y,t+1} = \dots \\ \dots &= \omega_m \frac{\hat{k}_{m,t+1}}{\hat{k}_{m,t}} \int_0^{N_{m,t}} k^i_{m,t+1} + (1-\omega_y) \frac{\hat{k}_{y,t+1}}{\hat{k}_{y,t}} \int_0^{N_{y,t}} k^i_{y,t+1} = \dots \\ \dots &= \omega_m \frac{\hat{k}_{m,t+1}}{\hat{k}_{m,t}} k_{m,SS} + \frac{(1-\omega_y)}{\omega_y} k_{y,SS}. \end{aligned}$$

As a result, we have that

$$\frac{\hat{k}_{m,t+1}}{\hat{k}_{m,t}} \mid_{SS} = \frac{1}{\omega_m} \left(1 - \frac{(1-\omega_y)}{\omega_y} \frac{k_{y,SS}}{k_{m,SS}} \right).$$

Analogously, we have that

$$\frac{\hat{k}_{o,t+1}}{\hat{k}_{o,t}} \mid_{SS} = \frac{1}{\omega_o} \left(1 - (1-\omega_m) \frac{\hat{k}_{m,t+1}}{\hat{k}_{m,t}} \mid_{SS} \frac{k_{m,SS}}{k_{o,SS}} \right).$$

Thus, if we set

$$\vartheta_y = \frac{1}{\omega_y}$$
$$\vartheta_m = \frac{1}{\omega_m} \left(1 - \frac{(1-\omega_y)}{\omega_y} \frac{k_{y,SS}}{k_{m,SS}} \right)$$
$$\vartheta_o = \frac{1}{\omega_o} \left(1 - (1-\omega_m) \vartheta_m \frac{k_{m,SS}}{k_{o,SS}} \right)$$

we ensure that at steady state capital adjustment costs are zero. At steady state agents accumulate or reduce capital at a constant rate while within a group $z \in \{y, m, o\}$. Nonetheless, as individuals transition across groups through their life cycle, the aggregate capital holdings of each group remain constant and no adjust cost of capital is paid.

Ensuring that at steady state adjustment costs are zero is important for aggregation since the only non-linear term in the consumption decision is the quadratic term in the adjustment cost condition. As we solve a linearized version of the model around the steady state this quadratic term disappears such that the choice variables across agents within a group can be easily aggregated to find a condition for each group. Consequently, for instance, the aggregate consumption of all old agents at time t is simply given by

$$c_{o,t} = \varepsilon_t \varsigma_t \left[\tilde{a}_{o,t} + HC_{o,t} - T_{o,t} - ADJ_{o,t}\right].$$

where we excluded the quadratic terms which are irrelevant in a first order approximated solution and thus, $ADJ_{o,t} = \tilde{adj}_{o,t} + \frac{(1+\pi_{t+1})\omega_o}{R_{n,t+1}} ADJ_{o,t+1}$ and $\tilde{adj}^{i}_{o,t} = \left(1 - \frac{(1-\delta+r_{k,t+1})(1+\pi_{t+1})}{R_{n,t+1}}\right) k^{i}_{o,t+1}$. Therefore, the equilibrium conditions can be defined without explicitly incorporating the heterogeneity within age groups.

As some young agents become mature and some mature agents become old every period, when we aggregate and discard the quadratic adjustment terms, the flow of assets are given by

$$\begin{aligned}
k_{y,t+1} + \tilde{b}_{y,t+1} &= \omega_y(\tilde{a}_{y,t} + l_{y,t}\xi_y w_t + \tau_{y,t} - c_{y,t}) \\
\tilde{a}_{y,t} &= k_{y,t}\left[(1-\delta) + r_{k,t}\right] + \tilde{b}_{y,t}\frac{R_{n,t+1}}{1+\pi_{t+1}} \\
k_{m,t+1} + \tilde{b}_{m,t+1} &= \omega_m(\tilde{a}_{m,t} + l_{m,t}w_t + d_{m,t} + \tau_{m,t} - c_{m,t} + \dots \\
&\quad \dots + (1-\omega_m)(\tilde{a}_{y,t} + l_{y,t}\xi_y w_t + \tau_{y,t} - c_{y,t}) \\
\tilde{a}_{m,t} &= k_{m,t}\left[(1-\delta) + r_{k,t}\right] + \tilde{b}_{m,t}\frac{R_{n,t+1}}{1+\pi_{t+1}}
\end{aligned}$$

$$k_{o,t+1} + \tilde{b}_{o,t+1} = \tilde{a}_{o,t} + \xi_o l_{o,t} w_t + \mathrm{tr}_{o,t} - c_{o,t} + \dots$$

$$\dots + (1-\omega_m)(\tilde{a}_{m,t} + l_{m,t} w_t + d_{m,t} + \tau_{m,t} - c_{m,t})$$

$$\tilde{a}_{o,t} = k_{o,t}\left[(1-\delta) + r_{k,t}\right] + \tilde{b}_{o,t} \frac{R_{n,t+1}}{1+\pi_{t+1}}$$

We then define the stochastic discount factor for the mature group as

$$Q_t^m = \beta \mathfrak{z}_{t+1} \frac{\left[\omega_m \left(c_{m,t+1} - \chi_m \frac{{l_{m,t+1}}^{1-\frac{1}{\nu_m}}}{1-\frac{1}{\nu_m}} \right) + (1-\omega_m) \varepsilon_{t+1}^{\frac{-1}{\eta}} \left(c_{o,t+1} - \chi_o \frac{{l_{o,t+1}}^{1-\frac{1}{\nu_o}}}{1-\frac{1}{\nu_o}} \right)\right]^{(1-\eta)}}{\left(c_{m,t} - \chi_m \frac{{l_{m,t}}^{1-\frac{1}{\nu_m}}}{1-\frac{1}{\nu_m}} \right)^{(1-\eta)}}$$

Finally, given that we are interested in a solution under a linear approximation,

$$\begin{aligned}
\left(\frac{k^i_{m,t+1}}{k^i_{m,t}} - \vartheta_m \right) &= \left(\frac{\hat{k}_{m,t+1}}{\hat{k}_{m,t}} - \vartheta_m \right) \approx \vartheta_m \left(\frac{\hat{k}_{m,t+1} - \hat{k}_{m,t+1}\mid_{SS}}{\hat{k}_{m,t+1}\mid_{SS}} - \frac{\hat{k}_{m,t} - \hat{k}_{m,t}\mid_{SS}}{\hat{k}_{m,t}\mid_{SS}} \right) \\
&= \vartheta_m \left(\frac{1}{N_{m,t}} \frac{k_{m,t+1} - k_{m,t+1}\mid_{SS}}{k_{m,t+1}\mid_{SS}} - \frac{1}{N_{m,t-1}} \frac{k_{m,t} - k_{m,t}\mid_{SS}}{k_{m,t}\mid_{SS}} \right) \\
&\approx \vartheta_m \left(\frac{k_{m,t+1} N_{m,t-1}}{k_{m,t} N_{m,t}} - 1 \right)
\end{aligned}$$

then the aggregated arbitrage condition for mature agents becomes

$$\frac{R_{n,t+1}}{1+\pi_{t+1}} = \frac{(1-\delta) + r_{k,t+1} + \varphi \vartheta_m^2 \left(\frac{k_{m,t+1}}{k_{m,t}} \frac{N_{m,t}}{N_{m,t+1}} - 1 \right) \frac{k_{m,t+1}}{k_{m,t}} \frac{N_{m,t}}{N_{m,t+1}}}{\left(1 + \vartheta_m \varphi \left(\frac{k_{m,t}}{k_{m,t-1}} \frac{N_{m,t-1}}{N_{m,t}} - 1 \right) \right)} \tag{C.3}$$

D More on Calibration

This Section reports the values of the entire set of parameters of the model. Table D.10 reports the calibration choices of the block of parameters that comes with the structure of a standard open-economy New Keynesian model. Table D.11 reports the calibration choices of the set of parameters that govern the demographic and life-cycle structure of the model.

Table D.10: Calibration - Standard Parameters

Parameter	Value	Target/Source
Time Discount Factor	$\beta = 0.995$	Standard Value
Elasticity Intertemporal Substitution	$\eta = -9$	EIS = 0.1
Capital Depreciation Rate	$\delta = 0.025$	Standard Value
Capital Adjustment Cost	$\kappa = 135$	Two-Year National Investment Fiscal Multiplier = -0.9
Home Bias in Consumption & Investment	$\lambda = 0.69$	Nakamura and Steinsson (2014)
Elasticity Substitution Home & Foreign Consumption	$\psi_c = 2$	Nakamura and Steinsson (2014)
Elasticity Substitution Home & Foreign Investment	$\psi_i = 2$	$\psi_i = \psi_c$
Elasticity Substitution Across Varieties	$\epsilon = 9$	Standard Value
Capital Share in Production	$\alpha = 0.32$	Standard Value
Calvo Parameter	$\zeta = 0.75$	Standard Value
Dividend Tax Rate	$\tau_d = 0.9394$	Mature Agents Receive 60% Total Dividends
Steady-State Government Spending to Output Ratio	$\frac{G_{H,SS}+G_{F,SS}}{Y^u_{SS}} = 0.2$	Data
Persistence Government Spending Shock	$\rho_G = 0.933$	Data
Inertia of Government Debt	$\rho_{bg} = 0.95$	Dynamic Response to Spending of Government Debt
Response to Spending of Government Debt	$\phi_G = 4.5$	Dynamic Response to Spending of Government Debt
Response to Spending of Taxation	$\phi_T = 0.01$	Dynamic Response to Spending of Taxation
Inertia of Taylor Rule	$\psi_R = 0.8$	Clarida et *al.* (2000)
Taylor Rule Response to Inflation	$\psi_\pi = 1.5$	Clarida et *al.* (2000)
Taylor Rule Response to Output Gap	$\psi_Y = 0.2$	Clarida et *al.* (2000)

Table D.11: Calibration - Demographics & Life-Cycle of Hours and Wages

Parameter	Value	Target
Panel a: Demographics		
Birth Rate of New Young Agents	$\omega_n = 0.0024$	Share of Young in Population
Probability Transition from Young to Mature	$1 - \omega_y = 0.0250$	Avg. Number of Years as Young: 10y
Probability Transition from Mature to Old	$1 - \omega_m = 0.0071$	Avg. Number of Years as Mature: 30y
Death Probability of Old Agents	$1 - \omega_o = 0.0274$	Share of Old in Population
Relative Size Population Home Economy	$N/N^u = 0.1$	Relative Size of California
Panel b: Hours and Wages		
Disutility Labor for Young Agents	$\chi_y = 5.40$	Fraction of Hours Worked = 0.324
Disutility Labor for Mature Agents	$\chi_m = 308.02$	Fraction of Hours Worked = 0.35
Disutility Labor for Old Agents	$\chi_o = 33.79$	Fraction of Hours Worked = 0.08
Efficiency Units of Hours for Young Agents	$\xi_y = 0.68$	Wage Young = 68% Wage Mature
Efficiency Units of Hours for Mature Agents	$\xi_m = 1$	Normalization
Efficiency Units of Hours for Old Agents	$\xi_o = 0.72$	Wage Old = 72% Wage Mature
Labor Supply Elasticity for Young Agents	$\nu_y = 0.71$	Weighted Avg. Labor Supply Elasticity = 0.4
Labor Supply Elasticity for Mature Agents	$\nu_m = 0.2$	Chetty et *al.* (2013)
Labor Supply Elasticity for Old Agents	$\nu_o = 0.75$	Rogerson and Wallenius (2013)