Dynamic contests with fatigue

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PRELIMINARY DRAFT (please do not distribute)

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Abstract
We study experimentally symmetric “best-of-\((2n-1)\)” multi-stage contest series in which two players compete repeatedly until one of them wins \(n\) times. We introduce history dependence into the players’ probabilities of winning at a given stage, termed “fatigue.” Fatigue materializes if a player’s total effort in previous stages is different from her opponent’s. We find that subjects react strongly to treatments with different parameterizations of fatigue. In contrast to the often reported burnout behavior, subjects strategically choose low levels of effort a significant portion of time. We also observe strong dependence of effort on the players’ standing in the series.

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JEL Classification: C73, C90, D21

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1 Introduction

In competitive environments agents are rewarded on the basis of relative performance. Such environments are often used for incentive provision in firms (e.g., Lazear and Rosen 1981, Lazear 1999, Bognanno 2001). They also arise naturally in various other contexts including sports (Szymanski, 2003), R&D competition (Taylor, 1995), elections (Klumpp and Polborn, 2005), and rent-seeking (Lockard and Tullock, 2001).

Competition often occurs dynamically in several stages. Some institutions involve gradual elimination of agents (“up-or-out rules;” Rosen 1986, O’Flaherty and Siow 1995), while in others agents compete repeatedly, either continuously or in discrete stages, until one of them reaches a desired target, as in patent races (Harris and Vickers 1985, 1987). Additional examples readily come from sports: in a tennis match a player needs to be the first to win a pre-specified number of sets; in a MLB/NBA/NHL best-of-7 series, the team that wins four games first is the winner of the series.

One of the central questions in the literature on dynamic contests is how effort is allocated across stages. In this paper, we study best-of-$(2n - 1)$ repeated contests, in which the player who wins $n$ stage contests first is the winner of the series. For such games, it was shown theoretically that equilibrium strategies may become complex even for relatively simple and well-understood stage contests (see Konrad and Kovenock 2005, 2006a, 2006b). In particular, players’ choices of effort may depend on their standing in the series.

Empirical evidence on such dependence is somewhat controversial. Ferrall and Smith (1999) use playoff series data from professional baseball, basketball, and hockey, and find that teams do not strategically allocate their effort and play as well as they can regardless of their standing in the series. Amegashie et al. (2007) demonstrate a related effect experimentally: in a two-stage contest with elimination players “burn out” by using all their resources in the first stage. At the same time, Taylor and Trogdon (2002) find evidence of the NBA teams strategically losing games to obtain a better draft position for the following season. McFall et al. (2006) show that, with the introduction of the season-ending Grand Prize, golfers who win early in the season have stronger incentives to exert high effort later on and, as a result, are more likely to keep winning.

Strategic choice of effort in dynamic contests can be explained by several major factors. One is the state dependence of a player’s continuation value. Even if the costs of effort are symmetric, the continuation value is higher for the winning player as her probability of winning the entire series is higher. This mechanism predicts the “giving-up” behavior, with losing players decreasing their effort and, in extreme cases, dropping out of the contest. Another factor is the dependence of continuation value on effort. For example,
if higher effort is more costly a player may choose lower effort to “save energy.” As a consequence, players will start the contest with low effort and increase effort as the race progresses. Finally, low effort can be expected if a player has an incentive to lose the competition, for example, due to corruption or in order to improve her position in the future.²

It is a challenging task to isolate empirically the various factors determining players’ choices of effort in contests. The main reason is that, in many cases, effort is unmeasurable, and performance itself (e.g., score or output) is the only observable proxy for effort. It is also difficult to control for players’ abilities, valuations of winning, and outside options. A laboratory experiment with controlled effort and induced values is, therefore, a natural alternative research methodology.

In this paper, we study experimentally best-of-(2n − 1) multi-stage contests of two players. We use a stylized model-induced design, with players choosing one of the two levels of effort, “high” or “low,” at each stage (this makes our model similar to Fudenberg et al. 1983). The probability of a player winning at a given stage depends on her and her opponent’s effort at that stage, and also on the complete history of both players’ effort choices. There are no direct effort costs, and the outside option payoff is zero, i.e. giving up or losing is irrational in any state. Thus, history dependence in the winning probabilities, termed fatigue, is the only mechanism through which it may be strategically optimal to choose low effort.

Our stage game is a 2×2 simultaneous-move game with a dominant strategy (high effort). Without fatigue, the whole series turns into a finitely repeated such game, in which case the only Nash equilibrium is for both players to choose high effort at all stages (“burn out”). In the presence of fatigue, the payoff matrix of the stage game acquires history dependence. Although choosing high effort is still the dominant strategy in every stage game, choosing low effort at stage $t$ as a response to high effort results in uniformly higher winning probabilities at stage $t + 1$. The presence of fatigue makes strategic effort choices meaningful. By explicitly calculating the payoff from a simple deviation strategy, we show theoretically that the burnout behavior becomes sub-optimal when fatigue is sufficiently strong.

Static and dynamic contests have been analyzed experimentally by a number of researchers (e.g., Davis and Reilly 1998, Anderson and Stafford 2003, Öncüler and Croson 2004, Schmitt et al. 2004, Amaldoss and Rapoport 2005, Amegashie et al. 2007). Most studies find significant overdissipation of effort in comparison with the equilibrium predic-

²In this case we could say that, implicitly, low effort increases the player’s continuation value, i.e. overall the situation looks as if high effort is too costly.
3 In contrast, in our experiment subjects demonstrate, if anything, underdissipation of effort by responding strongly to treatments with different parameterizations of fatigue. We also find strong dependence of effort on the player’s position in the series: (i) subjects tend to choose higher effort as the end of the series approaches; (ii) subjects tend to choose higher effort if they are winning or losing in the process.

The rest of the paper is organized as follows. In Section 2, we present a theoretical model. In Section 3, we discuss the experimental design and formulate our hypotheses. In Section 4, we present and discuss the experimental results. Section 5 concludes.

2 Theory

2.1 The model

There are two \textit{ex ante} identical risk-neutral players, 1 and 2. When matched in a stage contest, they can exert one of the two levels of effort, “high” and “low,” henceforth respectively denoted as 1 and 0. Let $p_{xy}$ denote the probability for player 1 to win such a contest, where $x$ stands for the effort of player 1, and $y$ stands for the effort of player 2, with $x, y \in \{0, 1\}$. The players are identical, which implies

$$p_{00} = p_{11} = \frac{1}{2}.$$ 

Also, $p_{01} + p_{10} = 1$, therefore, without loss of generality

$$p_{10} = \frac{1 + a}{2}, \quad p_{01} = \frac{1 - a}{2},$$

where $p_{10} - p_{01} = a \in [0, 1]$ is the \textit{advantage parameter}, which characterizes the relative advantage in the winning probability for the player exerting high effort provided her opponent’s effort is low.

\[\text{\textsuperscript{3}}\text{The model of Schmitt \textit{et al.} (2004) is similar to ours in that allows for history dependence in winning probabilities. There, effort is partially “carried over” to the next stage, and the effective effort in period } t \text{ is a discounted sum of efforts from all previous periods. Schmitt \textit{et al.} (2004) show theoretically and confirm experimentally that an increase in the discount factor leads to more effort being expended in earlier periods. The key difference between the model of Schmitt \textit{et al.} (2004) and our model is that in their model past effort affects the winning probability positively through carryovers, while in our model it affects the winning probability negatively through fatigue. The other important difference is that in Schmitt \textit{et al.} (2004) the number of stages is fixed in advance, and a prize is awarded to the winner after every stage, while in our model the number of stages is endogenous and a single prize is awarded to the winner of the series overall. However, one of our results – that effort is lower in earlier periods – is qualitatively consistent with Schmitt \textit{et al.} (2004).}\]
Consider the following “best-of-(2n − 1)” game consisting of at least \( n \) and at most \( 2n − 1 \) stages (how many stages are actually played is determined in the course of the game). At every stage the players simultaneously choose their effort levels in the contest described above, after which the winner of the stage contest is determined. The player who wins \( n \) stage contests first is the winner of the whole series, and gets a payoff of 1, while the other player gets a payoff of 0.

Generally, fatigue can be described as a reduction in a player’s probability of winning after exerting high effort. However, if both players exert high effort each should experience the same reduction, i.e. no comparative advantage will arise. We assume, therefore, that fatigue only materializes at stage \( t + 1 \) if at stage \( t \) one of the players exerted high effort while her opponent exerted low effort. These considerations can be formalized as follows.

Let \( p_{x'y'|xy} \) denote the probability for player 1 to win at stage 2 given effort levels \( x \) and \( y \) at stage 1:

\[
p_{x'y'|xy} = p_{x'y'} - \frac{(x - y)f}2.
\]

Here \( f \in [0, 1] \) is the fatigue parameter. Equation (1) states that if the players expended the same effort at stage 1, then there is no fatigue-related advantage at stage 2; if their effort levels at stage 1 have been different, there is an additional advantage of \( f \) in the winning probability for the player who exerted low effort.

Equation (1) can be rationalized as follows. Suppose \( w(x, y) \) is the underlying contest success function of the stage game that gives the probability of player 1 winning as a function of her and her opponent’s efforts, \( x \) and \( y \). Fatigue can be modeled as a small reduction in the actual effort relative to the exerted effort. For example, if players 1 and 2 exert efforts \( x \) and \( y \) at stage 1, and the same effort \( x' \) at stage 2, their actual efforts at stage 2 will be \( x' - \mu x \) and \( x' - \mu y \), where \( \mu \) is a small parameter. Then player 1’s probability of winning at stage 2 is

\[
w(x' - \mu x, x' - \mu y) \approx \frac12 - w_x(x', x')\mu x - w_y(x', x')\mu y = \frac12 - \mu w_x(x', x')(x - y),
\]

where we used the first-order Taylor expansion and the fact that \( w_x(x', x') + w_y(x', x') = 0 \) for any contest success function.

We assume that fatigue accumulates linearly across stages. This is a very stylized assumption, but we adopt it to have the simplest possible parameterization that still captures the effect. Let \( p_{x'y''|xy,x'y'} \) denote the probability for player 1 to win at stage 3.

\footnote{This term is inherited from Ferrall and Smith (1999). Such games have been also called “multi-battle races” (Konrad and Kovenock, 2006).}
given effort levels $xy$ at stage 1 and $x'y'$ at stage 2:

$$p_{x''y''|xy,x'y'} = p_{x'y'} - \frac{(x + x' - y - y')f}{2}. \quad (2)$$

Equation (2) states that fatigue accumulates in the course of the game, and the additional advantage is given by the difference in the total past efforts exerted. More generally, at stage $t$

$$p_{x_t y_t|x_1 y_1,\ldots,x_{t-1} y_{t-1}} = p_{x_t y_t} - \frac{F_t}{2}, \quad F_t = \sum_{k=1}^{t-1} (x_k - y_k), \quad t \geq 2. \quad (3)$$

Here $F_t$ is the net accumulated effort of player 1. The probabilities given by Eq. (3) must be between 0 and 1 in all cases, which imposes the restriction $a + (2n - 1)f \leq 1$. One immediate consequence of this restriction is that $f < 1/(2n - 1)$, i.e. only relatively small levels of fatigue can be considered. Let $S_n = \{(a, f) : a \geq 0, f \geq 0, a + (2n - 1)f \leq 1\}$ denote the set of admissible values of the model parameters.

We will now explore subgame-perfect Nash equilibria (SPNE) in this game.

2.2 The case of no fatigue

We first consider the case of no fatigue, $f = 0$, and show that the only SPNE is the one in which both players unconditionally exert high effort at all stages. In fact, this statement is trivial. At any stage, each player can strictly increase her probability of winning by switching from low to high effort, no matter what the outcomes of previous stages are, or what the opponent’s effort is. The game in this case is simply a finitely repeated game with a dominant strategy whose only SPNE is for both players to choose high effort at every stage (burn out). The players’ equilibrium expected payoffs are $(1/2, 1/2)$.

2.3 Equilibria in the presence of fatigue

In this section we show that choosing high effort at all stages (both players choosing the burnout strategy) is not a SPNE in the presence of fatigue at least for some values of parameters $a$ and $f$.

The state of the game can be described by a pair of scores $(i, j)$, which are the numbers of wins players 1 and 2 have, starting with $(0, 0)$. State $(i, j)$ becomes $(i + 1, j)$ if player 1 wins, and $(i, j + 1)$ if player 2 wins. The game ends when $\max\{i, j\} = n$ is first reached. The final score is $(n, j)$ with $j < n$ (player 1 wins), or $(i, n)$ with $i < n$ (player 2 wins).
Proposition 1. The number of paths from state \((0,0)\) to state \((i,j)\) is

\[ m_{i,j} = \frac{(i + j)!}{i!j!}. \]  

(4)

Proof. The number of paths \(m_{i,j}\) should satisfy the two-dimensional linear recurrence relation,

\[ m_{i+1,j} = m_{i,j} + m_{i+1,j-1}. \]

with the boundary conditions \(m_{i,0} = 1\) and \(m_{0,j} = 1\). This problem has a unique solution. It is easy to see that expression (4) satisfies both the recurrence relation and the boundary conditions, i.e. it is the solution. Q.E.D.

Proposition 2. There are parameterizations such that both players choosing the burnout strategy is not a SPNE.

Proof. Suppose player 2 adopts the burnout strategy, and consider the following deviation strategy of player 1: choose low effort at stage 1 and high effort thereafter. We will show that the deviation strategy yields a payoff \(\pi_1 > 1/2\) for some values of \(n, a,\) and \(f\). The winning probability for player 1 is \((1-a)/2\) at stage 1 and \((1+f)/2\) thereafter.

The probability for player 1 to win the series with score \((n,j)\) is given by the probability of reaching state \((n-1,j)\) and winning in that state:

\[
P_{(n,j)} = \left[ \frac{1-a}{2} \left( \frac{1+f}{2} \right)^{n-2} \left( \frac{1-f}{2} \right)^{j} m_{n-2,j} \right. \\
+ \left. \frac{1+a}{2} \left( \frac{1+f}{2} \right)^{n-1} \left( \frac{1-f}{2} \right)^{j-1} m_{n-1,j-1} \right] \left( \frac{1+f}{2} \right).
\]  

(5)

This equation can be explained as follows. There are two ways of reaching state \((n-1,j)\). First, player 1 can win at stage 1, and then win \(n-2\) matches and lose \(j\) matches. The probability of this happening multiplied by \(m_{n-2,j}\), the number of paths going from state \((1,0)\) to state \((n-1,j)\), is the first term in Eq. (5). Second, player 1 can lose at stage 1, and then win \(n-1\) matches and lose \(j-1\) matches. The probability of this needs to be multiplied by \(m_{n-1,j-1}\), the number of paths going from state \((0,1)\) to state \((n-1,j)\), which is represented by the second term in Eq. (5).

Player 1 can win with any score \((n,j)\), \(0 \leq j \leq n-1\). The expected payoff of player 1 is, therefore,

\[
\pi_1 = \sum_{j=0}^{n-1} P_{(n,j)}.
\]  

(6)

Figure 1 shows the dependence of \(\pi_1\) on \(f\) for \(a = 0.2\) and \(n = 4\). As seen from Figure 1, \(\pi_1\)
Figure 1: The dependence of player 1’s expected payoff, $\pi_1$, on the fatigue parameter, $f$, for $a = 0.2$ and $n = 4$. The solid line shows $\pi_1(f)$, Eq. (6), when player 1 chooses the deviation strategy. The dashed line shows $\pi_1 = 1/2$ corresponding to both players burning out. The range of $f$ is determined by $S_4$.

exceeds 1/2 for $f$ above a certain threshold value. This proves that both players burning out is not a SPNE at least for some parameterizations. Q.E.D.

3 The experiment

3.1 Preliminaries

The experiment was conducted in the Experimental Social Sciences Laboratory at Florida State University (XS/FS). Subjects were FSU undergraduate students recruited using a web-based announcement system. None of the subjects participated in more than one session of the experiment. All participants received a $10 show-up fee in addition to their experimental earnings, $10.11 on average. There were 4 sessions, 68 subjects total, with 10, 10, 24, and 24 subjects per session, respectively. The experiment was conducted using the z-Tree software (Fischbacher 2007). The experimental instructions are given in Appendix A.

3.2 Experimental design

Each experimental session consisted of two unrelated parts. In the first part, subjects’ risk aversion was measured using the methodology developed by Holt and Laury (2002). In the second part, subjects were paired randomly and competed in multi-stage matches. They stayed in fixed pairs through the match and were randomly re-paired for every new
match.\textsuperscript{5}

The second part of the experiment consisted of 4 consecutive treatments. All subjects went through all the treatments in the same order. This allows us to pool all the data together and use within-subjects analysis. Obvious sides effects of this arrangement are (i) possible order effects and (ii) possible learning across treatments. To mitigate both effects, we changed parameters across treatments nonmonotonically (see below). Additionally, the very structure of the best-of-$(2n - 1)$ contest game suppresses learning because the score is reset to 0-0 automatically at the beginning of each match (and it is the dependence of effort on the score we are primarily interested in). Our analysis shows that, largely, learning across matches did not occur.

Each treatment consisted of 8 multi-stage matches. In treatment 1, subjects played 8 best-of-7 matches ($n = 4$); in treatments 2 and 4, subjects played 8 best-of-3 matches ($n = 2$); in treatment 3, subjects played 8 best-of-11 matches ($n = 6$). Timing of all decisions was synchronized so that subjects could not start the next match until everyone completed the current match. Thus, overall the second part of the experiment consisted of $8 \times 7 + 8 \times 3 + 8 \times 11 + 8 \times 3 = 192$ decision periods. Some subjects did not make decisions in some periods because their matches were over, and they were waiting for all other pairs to finish their matches.

At the beginning of each match subjects were reminded that they are playing against a new randomly selected opponent. Matches were timed in terms of rounds and consisted of at least $n$ and at most $2n - 1$ rounds. At the beginning of a round, each subject was informed about the current score, the remaining number of won rounds needed to win the match (for herself and for the opponent), the history of effort choices and results in previous rounds (for herself and for the opponent), the accumulated level of fatigue (for herself and for the opponent), the values of parameters $a$ and $f$, and the probabilities of winning for each choice of effort by herself and the opponent (in the form of a matrix of winning probabilities). Subjects then made decisions by choosing “high” or “low” effort. Before the actual decision periods they practiced by playing against automated decision makers randomly choosing high or low effort with probability 1/2.

Within each treatment, parameters $a$ and $f$ changed after 4 matches, with matches 1 through 4 being the low advantage/high fatigue (LA/HF) matches, and matches 5 through 8 being the high advantage/low fatigue (HA/LF) matches.\textsuperscript{6} In the LA/HF matches, $a$ was set at 0.2 while $f$ was chosen to be maximal allowed by the model (with downward rounding, if necessary), $f = (1 - a)/(2n - 1)$. In the HA/LF matches, $a$ was set at 0.6

\textsuperscript{5}We are aware of the “independent observations” issue with random re-matching. There are also potential effects of learning across matchings. We address these issues in our estimation.

\textsuperscript{6}There was no fatigue in treatment 4, see below.
and \( f \) was set twice lower than in the corresponding LA/HF case. The parameters’ values for all treatments are summarized in Table 1.

Thus, each subject played 32 matches against randomly changing opponents. Parameters \( a \) and \( f \) changed every 4 matches, while the length of the match (the number of rounds necessary to win, \( n \)) changed every 8 matches.

Treatments 4 (the no fatigue treatment) is a control treatment. Recall that high effort is the dominant strategy in the stage game, and both players always choosing high effort is the only equilibrium in the absence of fatigue. Moreover, unlike in the finitely repeated Prisoner’s Dilemma, this equilibrium generates a Pareto optimal outcome. Thus, the frequency of low effort choices in treatment 4 can serve as a measure of subjects’ “error” or “irrationality.” We cannot distinguish between different sources of error. These include poor understanding of the game as well as possible non-monetary incentives, such as boredom or real fatigue.

Another independent measure of error is the frequency of low effort choices in rounds when the score is \((n - 1, n - 1)\). In such rounds, choosing high effort is the dominant strategy.

### 3.3 Hypotheses

We test the following *ceteris paribus* hypotheses.

**Hypothesis 1.** Low effort will be chosen in treatments with fatigue and will not be chosen in treatments without fatigue.

**Hypothesis 2.** Higher asymmetry advantage (parameter \( a \)) leads to higher effort choices.

**Hypothesis 3.** Higher cost of fatigue (parameter \( f \)) leads to lower effort choices.

**Hypothesis 4.** Low effort is more likely to be chosen in longer matches (larger \( n \)).

**Hypothesis 5.** Within a match, low effort is more likely to be chosen in earlier than in later rounds.

Additionally, we explore how effort choices depend on the relative number of accumu-
Table 2: Frequency of high effort for the low advantage/high fatigue and high advantage/low fatigue parameterizations in each of the 4 treatments. The \( p \)-values correspond to the null hypotheses of the frequency being the same within subjects in each treatment.

<table>
<thead>
<tr>
<th>Parameterization</th>
<th>Treatment 1</th>
<th>Treatment 2</th>
<th>Treatment 3</th>
<th>Treatment 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low advantage/high fatigue</td>
<td>0.54</td>
<td>0.56</td>
<td>0.56</td>
<td>0.92</td>
</tr>
<tr>
<td>High advantage/low fatigue</td>
<td>0.79</td>
<td>0.78</td>
<td>0.79</td>
<td>0.95</td>
</tr>
<tr>
<td>( p )-value, paired ( t )-test</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.084</td>
</tr>
</tbody>
</table>

4 Experimental results

4.1 Summary statistics

The first and foremost question is whether subjects ever choose low effort at all. The answer is “yes:” Table 2 shows the frequency of high effort choices for each parameterization. The reported frequencies are well below unity, except for treatment 4. As expected, subjects tend to choose low effort more often in the LA/HF parameterizations.

In order to statistically compare the frequencies of high effort choices, we computed the frequency of high effort for each subject in each treatment and parameterization separately. This produced 68 within-subjects pairs of observations for each treatment and parameterization and allowed us to run paired \( t \)-tests on the null hypotheses of the frequencies of high effort being the same within each treatment. This is done by computing the difference between the frequencies of high effort for each subject (thus, the unobserved heterogeneity is differenced out) and running the \( t \)-test on the difference being equal to zero. The resulting \( p \)-values are reported in Table 2.

As seen from Table 2, in treatments 1 through 3 high effort is chosen significantly more often for HA/LF parameterizations. In treatment 4, the frequencies of high effort are still different but the difference is not significant at 5% level.

Interestingly, the frequency of high effort practically does not change across treatments 1–3 within each parameterization. This points at the absence of significant learning and order effects across treatments. We discuss other tests for learning below.

In treatment 4, subjects almost always choose high effort. Out of 68 subjects, only 13 ever chose low effort in treatment 4. Out of those 13, 2 subjects chose low effort only once, 4 subjects – two times, 3 subjects – three times, and 4 subjects – four times out of about 20 choices of effort they made. There is still a tendency to choose high effort
more often in the HA/LF parameterization, but the difference is not as significant as in other treatments. The frequency of low effort choices in treatment 4, about 0.08 at most, provides an estimate of the role of “irrational” motives in subjects’ behavior in this experiment.

4.2 Dynamics of average effort

In this section we explore the time dependence of average effort and potential learning effects. A match is a fundamentally dynamic game. From one round to the next, the state of the game changes in two ways: (i) the score changes, making one of the players closer to winning the match; (ii) the accumulated fatigue levels may change, thereby changing the matrix of winning probabilities in the next round stage game. It is, therefore, natural to expect nonstationarity of average effort within a match. This should lead to periodically repeating local trends in average effort, with the beginning of each local trend corresponding to the first round of a match. Potentially there can also be nonstationarity across matches due to learning. For example, subjects may learn over time that low effort is more productive than they thought at the beginning of the experiment. This would lead to a global trend in average effort.

Figure 2 shows the time dependence of average effort separately for each parameterization. The rows of panels in Figure 2 from top to bottom correspond to treatments 1 through 4. In each row, the left panel shows the time dependence of average effort for matches 1 through 4 (the LA/HF parameterization), while the middle panel shows the time dependence of average effort for matches 5 through 8 (the HA/LF parameterization). The right panel in each row shows the dependence of average effort on the round within matches; it is obtained by averaging across matches for each parameterization.

In the left and middle panels of Figure 2, we indeed observe periodic local trends. Each local trend starts with the first round of a match. For example, in treatment 1 consisting of 8 best-of-7 matches, 7 is the period of average effort “oscillations.” Similarly, in treatments 2, 3, and 4, the period of oscillations is 3, 11, and 3, respectively.

The oscillations of average effort in the left and middle panels are remarkably periodic. There are no obvious global trends across matches. This is another indicator of the absence of significant learning across matches, at least in the aggregate. Thus, it is legitimate to treat matches as independent and superimpose them for each parameterization. The result of such aggregation is the right panel in each row of Figure 2 showing the dependence of average effort on the round within a match. In treatments 1 through 3, there is a clear upward trend; there is no trend in treatment 4. Average effort is always larger for the HA/LF parameterizations (again with the exception of treatment 4 where the difference
Figure 2: Average effort as a function of time period for each parameterization. Rows from top to bottom correspond to treatments 1 through 4. The left and middle columns correspond to the low advantage/high fatigue and high advantage/low fatigue parameterizations, respectively. In the right column, average effort is shown as a function of round, with 4 consecutive matches within each parameterization lumped together.
Table 3: Frequency of high effort in round $2n - 1$ for the low advantage/high fatigue and high advantage/low fatigue parameterizations in each of the 4 treatments.

<table>
<thead>
<tr>
<th></th>
<th>Treatment</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2n - 1$</td>
<td></td>
<td>7</td>
<td>3</td>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td>Low advantage/high fatigue</td>
<td></td>
<td>0.81</td>
<td>0.86</td>
<td>0.91</td>
<td>0.93</td>
</tr>
<tr>
<td>High advantage/low fatigue</td>
<td></td>
<td>0.91</td>
<td>0.89</td>
<td>0.94</td>
<td>0.97</td>
</tr>
</tbody>
</table>

is not significant), in line with the aggregate results of Table 2. The slope, however, is greater for the LA/HF parameterizations, so that effort levels become very close in the final rounds.

As mentioned above, in round $2n - 1$ high effort is the dominant strategy. Thus, the frequency of low effort choices in these rounds is another measure of the degree to which “irrational” motives play a role in subjects’ behavior. Table 3 shows the frequency of high effort in rounds $2n - 1$ for each parameterization. Out of 68 subjects, 31 subjects chose low effort at least once in round $2n - 1$. They did this 86 times (about 1.3 times per subject, on average) out of 872 total choices in these circumstances. As seen from Table 3, there is evidence of some learning across treatments, as the frequencies of high effort tend to increase over time (with the only exception of the HA/LF parameterization in going from treatment 1 to treatment 2). As we do not observe any obvious tendency to increase effort globally over time in Figure 2, we conclude that this learning is local; in some sense, subjects learn to be more rational.

4.3 Match length

A best-of-$(2n - 1)$ match can end with a score ranging from $(n, 0)$ to $(n, n - 1)$ in favor of the winning player. If the probability of winning is 1/2 for both players in all rounds (e.g., if both players choose the burnout strategy), the probability for a match to end with score $(n, j)$ is

$$P_{(n,j)}^{(0)} = \frac{(n - 1 + j)!}{(n - 1)!j!} \left(\frac{1}{2}\right)^{n-1+j}.$$  

Figure 3 shows the distribution of possible final scores for each treatment (solid bars) together with probabilities $P_{(n,j)}^{(0)}$ (patterned bars). As seen from the Figure, there is no significant effect of fatigue on the distribution of the length of matches. Longer matches are more likely, on average, than shorter matches, which reflects the fact that competing players are identical.
Figure 3: The distribution of match length (in rounds) by treatment (the upper/lower two panels correspond to treatments 1,2/3,4). The solid bars show the experimentally observed distribution. The patterned bars show the distribution calculated under the assumption of equal probability of winning in every round, $P_{(n,j)}^{(0)}$. The final scores of the corresponding matches are shown above the bars.
4.4 Scores

One of the central questions of this paper is how a subject’s effort choice in a given round depends on her standing in the match, i.e. on the current score. Score \((i, j)\) (where we assume that player 1 has \(i\) points and player 2 has \(j\) points) can be characterized by two alternative variables, round \(r = i + j + 1\) and lead \(l = i - j\). Round ranges from 1 to \(2n - 1\); it is the local time variable used in the right column of panels in Figure 2. Lead \(l\) ranges from \(-(n - 1)\) to \((n - 1)\); it is the relative score variable showing how far player 1 is ahead of player 2 in terms of wins. Variables \(r\) and \(l\) uniquely determine the state of the game, with \(i = (r + l - 1)/2\) and \(j = (r - l - 1)/2\).

The dependence of average effort on \(r\) is shown in Figure 2 and discussed above. The dependence of average effort on \(l\) is shown in Figure 4. As seen from Figure 4, average effort depends on \(l\) nonmonotonically. It is the lowest for \(l = 0\) and tends to increase with \(|l|\). Thus, there is no evidence of the giving-up behavior. Moreover, there is a tendency to increase one’s effort when falling behind.
Table 4: Regression results for coefficients in Eq. (7) (with robust standard errors in parentheses). All the estimates except for those marked with n are significant at 5% level.

4.5 Regression analysis

In order to accurately test the hypotheses formulated in Section 3.3, we use regression analysis. The panel structure of the data allows us to control for the individual heterogeneity in a relatively general manner. Our population model is based on the information available to subjects in the experiment. In the linear specification, the basic model is

\[ e_{it} = \mathbf{x}_{it} \beta + c_i + \epsilon_{it}, \quad \mathbf{x}_{it} = (n_t, a_t, f_t, r_t, l_{it}, l_{it}^2, F_{it}, F_{it}^2). \]  

(7)

Here \( e_{it} \) is the effort level of subject \( i \) in decision period \( t \); \( n_t \) is the number of rounds a player needs to win in order to win the series; \( a_t \) and \( f_t \) are the advantage and fatigue parameters in a given match; \( r_t \) is the current round in the series; \( l_{it} \) and \( l_{it}^2 \) are the number of rounds player \( i \) is ahead of her opponent, and its square (to capture the nonmonotonicity observed in Figure 4; \( F_{it} \) and \( F_{it}^2 \) are the number of net rounds of fatigue accumulated by player \( i \), and its square; \( \beta \) is a vector of coefficients; \( c_i \) is the unobserved individual-specific effect; \( \epsilon_{it} \) is the zero-mean error term.

The regression results for Eq. (7) are shown in Table 4. We start with estimating
Eq. (7) by pooled OLS and fixed effects. The results are very similar, with the only difference being the coefficient on the accumulated fatigue $F_{it}$. While pooled OLS gives a positive and significant estimate, fixed effects produce a negative and insignificant one. This is an indication of a correlation between $F_{it}$ and the unobserved effect. Recall that $F_{it}$ is defined as $F_{it} = \sum_{r=1}^{t-1}(e_{ir} - e_{jr})$, where $j$ is the opponent of player $i$ in the current match, and the summation starts from round 1 of the match. Thus, $F_{it}$ contains player $i$’s past effort choices and is, therefore, indeed, correlated with $c_i$.

Thus, in the presence of $F_{it}$ among the regressors, the FE estimator is, generally, not consistent. We, therefore, exclude $F_{it}$ from the regression and repeat the pooled OLS and FE estimation. We also exclude $l_{it}$ whose coefficient is not significant in both regressions. The resulting estimates are very similar to each other and to those in the first two regressions indicating that (i) the excluded variables are not correlated with the remaining regressors and (ii) the unobserved effect is not correlated with the remaining regressors.

Effort $e_{it}$ is a binary variable, therefore a natural alternative to linear models is to use a nonlinear binary response model. We provide the results for the pooled probit and FE logit regressions. The coefficients are hard to interpret but their signs and significance levels are completely in line with those from the linear regressions.

The results support all the hypotheses formulated in Section 3.3. Subject react to exogenous parameters $a$, $f$, and $n$ as predicted. Additionally, their behavior changes depending on their standing in the series.

5 Concluding remarks

In this paper we explore experimentally the strategic choices of effort in dynamic contests. In our model-driven design, the only factor determining such choices is the history dependence in the winning probabilities of the stage contest, termed fatigue. There are no outside options and no explicit effort costs, i.e. the continuation value is positive in any state. As a result, giving up or losing is irrational. At the same time, non-burnout behavior is optimal.

We observe that subjects react to treatments in a way consistent with the model predictions. The key prediction of the model is that, being aware of fatigue, players optimize their choice of effort by choosing non-burnout strategies. This is in contrast with other models where strategic choice of effort is related to effort costs. Effort costs, however, are hard to define and measure in environments such as sports or the labor market. It is therefore not clear how to interpret the empirical findings that report
burnout behavior in these contexts.

Obvious extensions of this work include considering heterogeneous players and going from a completely model-driven setting to a more realistic setting by conducting a similar experiment with real tasks (and hence, real fatigue).

References


Appendix A

The following experimental instructions have been given to participants and read out loud.

**Experimental instructions**

**Part 1 - distributed at the beginning of the experiment**

Thank you for participating in today’s experiment. I will read through a script to explain to you the nature of today’s experiment as well as how to navigate the computer interface with which you will be working. I will be using this script to make sure that all sessions of this experiment receive the same information, but please feel free to ask questions as they arise. We ask that you please refrain from talking with other participants or looking at their monitors during the experiment. If you have a question or problem, please raise your hand and one of us will come to you. We also ask that you please do not press any keys or mouse buttons until we invite you to do so.

Today’s experiment will consist of two different parts. I will go through the instructions for part 1 and then you will make your decisions for that part and then I will go through the instructions for part 2. Your earnings from both parts will be summed to generate your final earnings. All monetary amounts you will see in this experiment are denominated in US dollars and cents. At the end of the experiment you will be paid your earnings with a check.

**INSTRUCTIONS PART 1**

In each round of this series you will be asked to make a choice between two lotteries that will be labeled A and B. There will be a total of 10 rounds and after you have made your choice for all 10 rounds, one of those rounds will be randomly chosen to be played. Lottery A will always give you the chance of winning a prize of $2.00 or $1.60, while lottery B will give you the chance of winning $3.85 or $0.10. Each decision round will involve changing the probabilities of your winning the prizes. For example in round 1, your decision will be represented on the screen in front of you:

Your decision is between these two lotteries:

**Lottery A:** A random number will be drawn between 1 and 100. You will win
- $1.60 if the number is between 1-90 (90% chance)
- $2.00 if the number is between 91 and 100 (10% chance)

**Lottery B:** A random number will be drawn between 1 and 100. You will win
- $0.10 if the number is between 1 and 90 (90% chance)
- $3.85 if the number is between 91 and 100 (10% chance)

If you were to choose lottery B and this turns out to be the round actually played, then the computer will generate a random integer between 1 and 100 with all numbers being equally likely. If the number drawn is between 1 and 90, then you would win $0.10 while if the number is between 91 and 100, then you would win $3.85. Had you chosen lottery A then if the number drawn were between 1 and 90 you would win $1.60 while a number between 91 and 100 would earn you $2.00.

All of the other 9 choices will be represented in a similar manner. Each will give you the probability of winning each prize as well as translate that probability into the numerical range the random number has to be in for you to win that prize.
At the end of the 10 choice rounds, you will be asked to press a button that will allow the computer to determine your payment. When you do so, the computer will randomly pick one of the 10 rounds to base your payment on, remind you of the choice you made in that round and draw the random number between 1 and 100 to determine your earnings.

Are there any questions before you begin making your decisions?

We ask that you follow the rules of the experiment and in particular we again ask that you do not talk with other participants or look at their screens during the experiment. Anyone who violates the rules may be asked to leave the experiment with only the $10.00 show-up fee.

You will now start the sequence of 10 choices. You will be able to go through the choices at your own pace, but we will not be able to continue the experiment until everyone has completed this series.

Part 2 - distributed after part 1 is completed

INSTRUCTIONS PART II

We now start the second section of the experiment. In this section, the computer program will be randomly sorting you into pairs, that is, each of you will have an opponent from among other people in the room. You will not know who your opponent is. You will compete with your opponent in several rounds, and then will be re-matched with another opponent.

Now please turn to your monitors. What you should see on the screen is the setting that you will be working with during the entire experiment. Please do not make any choices at this point. We will start with several rounds of practicing in order to familiarize you with the interface and the rules. In these practice rounds you will play against the computer. Later, during the real experiment, you will play against an opponent chosen randomly from among the people in the room.

Please do not press any keys or mouse buttons until invited to do so. In this experiment you will be playing a series of artificial matches. You can think about this game as a sport match against an equally strong opponent. For example, think about a best-of-5-sets tennis match, in which you need to be the first to win three sets in order to win the whole match. If you and your opponent put in the same effort, you have equal chances of winning a set. If you put in a larger effort than your opponent, your chances of winning this set increase, but you get more tired, therefore your chances of winning the next set decrease.

Likewise, each match played in the experiment will consist of a sequence of rounds. In each round you should choose one of the two levels of fictitious “effort” that you would like to put into the competition. These effort levels are labeled HIGH and LOW in the right part of the screen. A choice of effort is made by selecting a radio button with a mouse. Your opponent will also choose either HIGH or LOW, but her/his choice will not be known to you until the round is over.

In every round you can either WIN or LOSE, similarly to winning or losing a set in a tennis match. The outcome will be randomly generated by the computer, but the probabilities in this generation will depend on your and your opponent’s choices of effort. These probabilities will be given to you before you make your choice in the 2×2 table that you can see in the left upper part of the screen. The rows of the table correspond to your effort choices, while the columns of the table correspond to your opponent’s effort choices. As you can see, in this round, if both of you choose the same effort (both choose HIGH - upper left cell, or both choose LOW - lower right cell), the probability of winning is 50%. If you choose HIGH but your opponent chooses LOW (upper right cell), your probability of winning is more than 50%. If, on the contrary, you choose LOW and your opponent chooses HIGH (lower left cell), your probability of winning is
below 50%, and your opponent's probability of winning is above 50%.

Remember, this is a practice round, and you are playing against a computer. Now, please choose HIGH or LOW effort for the first round by selecting a radio button, and press the ACCEPT button to confirm your choice. Please do this now. You will see the results of this round in the table in the lower left part of the screen. It shows your effort choice, your opponent’s effort choice, your probability of winning, and if you actually won or lost as a result of the random number generation. You will also see the current score in the match above the effort choice radio buttons. During actual experiment please remember to always press ACCEPT button to confirm your choice. You can also accept by pressing Alt-A on the keyboard if that is more convenient for you.

You need to wait until everybody makes a choice, and then we will continue to the next round. Please do not make any choices at this point.

You and the computer have made effort choices in round 1, and now you see the table of winning probabilities for the next round. If you and the computer chose the same level of effort in the first round (both HIGH or both LOW), the table will look exactly the same as before. However, if you and the computer chose different levels of effort in the previous round, the probabilities of winning must have changed.

Specifically, if you chose HIGH and the computer chose LOW in the previous round, your probabilities of winning in the next round have decreased everywhere in the table. That is, you got “tired” relative to the computer. If you chose LOW and the computer chose HIGH, your probabilities of winning have increased. That is, the computer got “tired” relative to you. This will always be the case for future rounds: if in some round you and your opponent choose different levels of effort, the one who chooses LOW will have her/his winning probabilities increased in the next round, while the one who chooses HIGH will have her/his winning probabilities decreased in the next round.

The amount by which the winning probabilities may increase or decrease in the next round will not change during the match. This amount is shown as the PROBABILITY COST OF FATIGUE below the table with the winning probabilities.

This practice round is a best-out-of-9 match. That is, you need to win in 5 rounds in order to win the whole match. Starting from round 3, your time to make a choice will be limited by 30 seconds. During the practice match you will have 30 seconds to make a decision in each round. In the actual experiment you will have 15 seconds to make a decision in each round. To actually submit your decision you must press the ACCEPT button. If you do not press ACCEPT within 15 seconds, your choice in that round will be made at random by the computer (HIGH or LOW with probabilities 50/50). Your choice will be discarded and replaced by a random computer’s choice. So, if you don’t want the computer to make the choice for you, please make your choice within 15 seconds and press the ACCEPT button (or Alt-A on the keyboard). The remaining time will always be shown in the upper right corner of the screen.

Now please proceed making your choices until either you or the computer wins in 5 rounds. You may try different levels of effort to see how the probabilities of winning change.

Now you are done with the practice match. The result of the match is shown to you.

Are there any questions about how these procedures work?

In the actual experiment, you will play a number of matches with changing opponents. All matches will start in the same way as the practice match, but the initial winning probabilities and the probability cost of fatigue will vary. Also, the number of rounds you need to win in order to win a match will vary.

When a change occurs, you will be alerted by an inscription.
The experiment will consist of four treatments. At the end of the experiment, only one of the four treatments will be chosen at random with equal probabilities 1/4 to generate your actual dollar earnings.

Are there any questions about how the experiment works?

If you have a question during the experiment, please raise your hand, and one of us will assist you.

We will now begin this phase of the experiment. We again ask that you do not talk with other participants or look at their screens during the experiment. Anyone who violates the rules may be asked to leave the experiment with only the $10.00 show-up fee.

Please remember about the 15 seconds time limit to make a decision, and please press the ACCEPT button (or Alt-A) to actually submit your decision.

These treatments were announced as they started:

TREATMENT 1
This treatment involves 8 “best-of-7” matches. You need to win 4 rounds in order to win a match. Should this treatment be chosen in the end for your earnings, you will get $2 for each won match.

TREATMENT 2
This treatment involves 8 “best-of-3” matches. You need to win 2 rounds in order to win a match. Should this treatment be chosen in the end for your earnings, you will get $2 for each won match.

TREATMENT 3
This treatment involves 8 “best-of-11” matches. You need to win 6 rounds in order to win a match. Should this treatment be chosen in the end for your earnings, you will get $2 for each won match.

TREATMENT 4
This treatment involves 8 “best-of-3” matches. Note: there is no fatigue in this treatment. You need to win 2 rounds in order to win a match. Should this treatment be chosen in the end for your earnings, you will get $2 for each won match.