ON THE DYNAMICS OF CORRUPTION

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ABSTRACT: We study corruption in an environment in which two generations of officials overlap in each period. If at the end of his first term an official is found to have been corrupt, he is dismissed, otherwise he is reappointed for a second and final term. We focus on the impact of dismissals on the dynamics of corruption. A unique steady state exists and equilibrium sequences are globally stable and unique. Our main contribution is that equilibrium dynamics are asymmetric: If corruption is above the steady state, it converges monotonically to it; if corruption is below the steady state, it first jumps above the steady state and then converges monotonically to it. We use these findings to study the dynamic response of corruption to changes in the returns from corruption and we investigate some positive and normative implications of our equilibrium characterizations.

KEYWORDS: Corruption dynamics.

JEL CLASSIFICATION NUMBERS: C73, D72, D73, K42.

"Why, you simple creatures, the weakest of all weak things is a virtue that has not been tested in the fire." Mark Twain (1899) The Man that Corrupted Hadleyburg, reprint in Penguin Books, London, 1995, page 46.

1 Introduction

The objective of this paper is to study the dynamic response of corruption to policy changes or to shocks that alter the returns from corruption. We intend to address questions such as the following: (i) Can the corruption opportunities created by privatization programs have long lasting repercussions on the level of corruption? (ii) What are the long run consequences of a brief anti-corruption campaign? (iii) What is the dynamic response of corruption to changes in anti-bribery legislation, such as changes in monitoring or penalties?

In this paper we center our attention on the dynamic repercussions of exogenous dismissal and penalty rules for officials such as politicians, politically appointed bureaucrats, or members of government agencies. Our analysis is based on the following three premises:
1. **Officials have heterogeneous inclinations to be corrupted.** We portray this situation by assuming that the returns to corruption are identical for all officials, but the costs to be involved in a corrupt transaction are heterogeneous.

2. **Officials are long-lived.** This implies that the dismissal of officials found to be corrupt changes the future distribution of officials’ inclinations to corruption. We portray this situation by two generations of officials overlap in each period.

3. **Society responds to the anticipated level of corruption.** We portray this situation by assuming that a principal responds to the anticipated level of corruption in such a way that the returns to corruption are low when the officials’ average corruption cost is low and high when the latter is high. By this we mean to represent the fact that officials are given more freedom when they are less likely to have low corruption costs.

Our main objective is to analyze how changes in the dismissal and penalty rules or shocks to the returns from corruption influence the dynamics of corruption. To do this we characterize the steady state and the dynamics away from the steady state. We find that the dynamics can be characterized by comparisons of the replacement of officials found to have been corrupted with the steady state replacement. We formalize this comparison introducing the notions of excessive replacement and insufficient replacement.

With *excessive replacement* we refer to situations in which relatively honest officials, who would not be corrupt in the steady state, choose to be corrupt and are therefore dismissed and replaced more often than in the steady state. We term this replacement excessive, because some officials who would be honest in the steady state are replaced by other officials who would be corrupt in the steady state.

With *insufficient replacement* we refer to situations in which relatively dishonest officials, who would be corrupt in the steady state, choose not to be corrupt and are therefore dismissed and replaced less often than in the steady state. We term this replacement insufficient, because some officials who would be corrupt in the steady state fail to be replaced by other officials who would be honest in the steady state.

We use these notions to study the dynamic repercussions of different changes. For instance, we analyze the repercussions of a temporary shock to the returns of corruption in the steady state. If the shock temporarily increases the returns to corruption, excessive replacement arises; if it decreases them, insufficient replacement follows. In either case the resulting distribution will have higher frequencies of dishonest officials and this will lead to a higher level of corruption than in the steady state. In other words, the immediate consequence of either shock is adverse.

Our model predicts, as one could imagine, that the level of corruption in a given period is higher than in the steady state when there are more dishonest officials than in the steady state and it is lower when there are fewer. But the fact that society responds to the anticipated level of corruption (in the way described above in point 3), implies that higher corruption is accompanied by excessive replacement and lower corruption by insufficient replacement. This has two important additional implications:

- **Abnormally high corruption is persistent.** Because higher corruption is accompanied by insufficient replacement, changes that temporarily increase corruption generate insufficient replacement and therefore breed future corruption levels that are higher than steady state corruption, although converging to it.

- **Even shocks that may have immediate beneficial effects take a turn for the worse.** Because lower corruption is accompanied by excessive replacement, changes that decrease corruption
below the steady state (such as increases in corruption detection) also generate excessive replacement and cause future corruption to jump above the steady state and then converge to it.

We summarize our findings by affirming that *the response to shocks is adverse*, because any deviation from the steady state eventually leads to more corruption than in the steady state before it converges down to it.

Our results generate a number of testable implications, but there are several reasons that made us refrain from attempting to test the implications of our propositions. The first is that the series of corruption data are still relatively short. The second is that in the paper we analyze the consequences of shocks and it is difficult to identify a shock and at the same time factor out other possibly unobservable shocks. We believe that that when longer series of corruption data will be available it will make sense to test the general prediction of our model that the response to shocks is adverse by verifying that countries that have more volatile corruption indices also have higher mean corruption indices.

But there is a third and more important reason that makes of our model a tool to assist policy design and policy evaluation. As mentioned above, we do not endogenize policy variables, such as penalties or monitoring intensities, but we study their consequences. For this reason we think that our model can be used among other things to think about how to adjust policy parameters in response to exogenous shocks, to decide the timing of policy measures, or to verify whether enforced policies have the desired impact.

For instance, our model predicts that the cleansing effect of a brief anti-corruption campaign is short-lived, but also that the reversion to the initial level will be from higher levels of corruption. Our model also predicts that a permanent increase in corruption deterrence will have its full desired impact only in the long run, because the distribution at the initial stage is characterized by excessive replacement as compared to the distribution in the new steady state. Our model also suggests that to shorten the transition to a new lower steady state, would require an initial corruption deterrence that is higher than the one that will be enforced once the steady state is reached.

Because this paper focuses on given dismissal and penalty rules as the only means to discipline agents, it is best suited to think about the corruption of public officials. But the qualitative nature of our results can be expected to hold in other agency relationships. For this reason we believe that, apart from shedding light on the dynamics of public officials’ corruption, our work also provides a more general contribution as a starting point to think about the implications of discharge in other agency relationships, such as the fiduciary relationship between managers and shareholders or between fund managers and investors.¹

Our focus on dismissals relates our work to the literature on politicians’ career concerns (see, for example, Rogoff and Sibert (1988), Rogoff (1990), or Persson and Tabellini (2000)), but our analysis differs in at least two important respects.

First, the existing literature on politicians career concerns centers its attention on the effects of dismissals for inadequate performance on a politician’s incentives to act in the interest of his constituency. In other words, the existing literature argue that dismissals link current choices (of policies or effort levels) to future outcomes (reelection or not) and reputational concerns link future outcomes to current choices. Our paper reverses the angle of analysis. We study the consequences of dismissals for inadequate performance on the distribution of officials’ inclinations to corruption and therefore on future corruption. In other words we argue that dismissals link current choices

¹See for example, Noe and Rebello (1994).
to future outcomes because dismissals determine the distribution of inclination to corruption of future officials. If officials in their first term were not myopic, it would also be important to study how reputational concerns link future outcomes to current behavior. But because our results are independent of reputational concerns and robust to their introduction, in the current paper we prefer to simplify the analysis and focus on the case in which officials are myopic.

Second, the existing literature on politicians’ career concerns is silent about the future consequences of current shocks. By contrast, we focus our attention on the future consequences of current behavior and our main concern is to study the consequences of changes that alter the dismissal of officials in their early terms directly (e.g., because of a change in the monitoring technology) or indirectly (e.g., because of changes in the returns of corruption or the penalties for corruption). In this sense while the existing literature is mainly concerned with steady state behavior, our paper is mainly concerned with the dynamics away from the steady state.

Our paper is also related to the literature on the dynamics of corruption. Lui (1986) and Andvig and Moene (1990) focus on the consequences of assuming that the monitoring effectiveness is decreasing in the overall level of corruption. Tirole (1996) argues that the loss of value of collective reputations implied by current corruption makes officials more inclined to be corrupt in the future and that, at the same time, the expectation of higher future corruption decreases the current value of reputation and makes current corruption more attractive.

The previous papers assume that the marginal returns from corruption are increasing in (contemporaneous or future) corruption and this strategic complementarity is responsible for the possibility of multiple steady states. Our paper differs because we assume that the marginal returns from corruption are decreasing in contemporaneous corruption and this strategic substitutability implies that our model has a unique steady state. Our paper also differs from previous research on corruption dynamics because we provide a complete characterization of the dynamics away from the steady state.

For its focus on monitoring and termination in a dynamic environment our paper is also related to Ichino and Muehlheusser (2004). They argue that in a dynamic adverse selection setting reducing monitoring increases the probability of misbehavior, but may also increase the probability of early termination of undesirable types of agents. Our paper views the problem from a completely different angle. Rather than analyzing optimal monitoring, we investigate the dynamic implications of changes in the officials’ payoff functions for a given monitoring technology.

Our results are also reminiscent of the findings of Noe and Rebello (1994). They study the dynamics of business ethics when managerial preferences for opportunistic behavior are transmitted from one generation to the next by a given assumed transition rule. Our work differs substantially because we do not assume a transition rule. We simply assume that an official is replaced if his conduct is revealed to be improper and we derive the dynamics from equilibrium behavior.

Lastly, our result that excessive replacement increases corruption seems in line with the classic saying according to which “corruption of the best is worst.” On the other hand our finding that insufficient replacement also increases corruption has no counterpart in the classics of which we are aware, but it could be paraphrased to assert that “honesty of the worst is worst.”

The paper is structured in the following way. Section 2 studies corruption in a static setting. Section 3 turns to the main focus of the paper by presenting the dynamic setting and characterizing the equilibrium dynamics of corruption. Section 4 applies the result of the previous section and studies the dynamics of corruption following permanent or temporary shocks. In section 5 we discuss our main findings and directions for future research. All proofs are presented in the Appendices.
At any point in time a short-lived principal acting in the interest of a constituency sets a discretion level $\lambda$ for a population of mass 1 of officials and officials simultaneously decide whether to be corrupt or not. Half of the officials are young (are in the first period of their life) and the other half are old (are in the second and final period of their life). Young officials are myopic.

When $\lambda$ is high, an official has broad discretion and can make use of it to better serve the interests of the constituency, but also to obtain illegal returns. When $\lambda$ is low, tight regulations make it more difficult for an official to serve the interests of the constituency, but also impair his ability to obtain illegal returns. For this reason we assume that

- The optimal discretion level for the principal is a decreasing function of $\gamma$, the fraction of officials who choose to be corrupt, $\lambda(\gamma)$ with $\lambda'(\gamma) < 0$ for $\gamma \in (0,1)$. An example of this situation is in Celentani and Gauzuza (2002) where a principal sets the value to be assigned to the procurement agent’s quality assessment in a procurement auction.

- The gross return to an official from being corrupt $R(\lambda)$ is increasing in the discretion level $\lambda$ and is identical for all officials.

Officials differ in terms of the cost of being corrupted. A corrupt official faces the prospect of a penalty but he also bears an idiosyncratic cost, $\beta$, regardless of whether he is detected or not.\(^2\) This heterogenous cost component can be given different interpretations, e.g., a moral cost (the disutility an official derives from engaging in an illegal activity) or the cost of arranging a corrupt transaction. To simplify the presentation we will assume that $\beta$ is drawn from the uniform distribution on $[0,1]$ but all our results hold with arbitrary continuous distribution functions. We will refer to $\beta$ as the type or the corruption cost of an official.

Officials are subject to exogenous dismissal and penalty rules that depend on the legal framework. If an official is corrupt, he is detected with probability $\mu \in (0,1)$ and in this case he is inflicted penalty $\phi$.\(^3\) If a young official is detected to be corrupt he is also dismissed.

Summarizing, in any period $t = \ldots, -1, 0, 1, \ldots$:

1. Young officials (born at time $t$) privately learn their types, $\beta$.

2. The principal publicly announces the discretion level $\lambda_t$. Simultaneously officials decide whether to be corrupt or honest. An official who decides to be honest receives a payoff that is normalized to 0. An official of type $\beta$ who decides to be corrupted receives a gross payment of payment $R(\lambda_t)$ and bears a cost of $\beta$.

3. Each corrupt official is discovered with probability $\mu$. When an official is discovered, he is imposed penalty $\phi$. If a young official is discovered, he is also dismissed. All old officials are dismissed and are substituted by officials who will be born in $t+1$. Young officials who were found to be corrupt are dismissed and are substituted with randomly chosen officials of the same age (i.e., born in $t$).

The equilibrium concept that we use is perfect Bayesian equilibria.\(^4\)

\(^2\)The assumption that the heterogeneous corruption cost component $\beta$ is borne regardless of whether the official is detected or not is without loss of generality.

\(^3\)By introducing a penalty for corruption we can normalize the official’s wage to zero without loss of generality. Also, all our results are identical when the provider is also imposed a penalty. In such a case $\phi$ represents the sum of the penalties.

\(^4\)Because an agent’s individual play has no impact on the aggregate outcome, it will not be necessary to specify beliefs after histories that do not belong to the equilibrium path.
2.1 Discussion of the model

- All officials who are corrupt are equally corrupt regardless of beta, therefore mu is independent of beta.

- Young officials who are detected are replaced by agents of the same age is replaced by a new, randomly selected official of the same age. Two remarks
  
  - Dismissing a young official found to have been corrupted is ex post optimal given that in equilibrium an official who is corrupt when young is corrupt with probability 1 when old.
  
  - The hypothesis that a young official dismissed on corruption charges is replaced by an official of the same age is not important and is made only for the sake of simplicity. But it is also interesting to stress that there is anecdotal evidence in its favor.\(^5\)

- To simplify the analysis and to clarify that our results are independent of officials’ career concerns, we will concentrate on the case in which young officials are completely myopic, but our results are robust to the introduction of career concerns.\(^6\)

2.2 An example

A benevolent principal acting in the interest of a constituency sets the quality of a good \(\lambda\) that has to be supplied by a population of measure 1 of risk-neutral external suppliers. It is common knowledge that the cost of supplying quality \(\lambda\) to a supplier is \(C(\lambda)\) and that its reservation level is 0. The principal delegates the task of verifying whether suppliers supply the specified quality to a population of risk-neutral officials of measure 1. Each official observes the activity level supplied by a single supplier and authorizes a payment \(p\) to it. If the supplier supplies quality \(\lambda\) and receives a payment \(p\), the principal obtains utility \(V(\lambda) - p\).

If an official decides to be honest he makes a payment of \(C(\lambda)\) if the suppliers supplied quality \(\lambda\) and 0 otherwise. If an official decides to be corrupt he makes a take-it-or-leave-it bribe demand to the supplier in order to illegally allow it to deliver a lower quality, \(\lambda_C\).

Assume that payoff and cost functions satisfy the following conditions:

**Assumption 1**

1. \(V(\cdot)\) is twice continuously differentiable,

\[
V'(\cdot) > 0, \quad V''(\cdot) < 0, \quad \lim_{\lambda \to 0^+} V'(\lambda) = +\infty, \quad \lim_{\lambda \to +\infty} V'(\lambda) = 0.
\]

2. \(C(\cdot)\) is twice continuously differentiable,

\[
C'(\cdot) > 0, \quad C''(\cdot) \geq 0, \quad \lim_{\lambda \to 0^+} C'(\lambda) < +\infty.
\]

\(^5\)A study conducted by The Economist and the Enrico Mattei Foundation on the 100 most powerful posts in Italy found that in the 5 years following the 1992 Clean Hands investigation campaign the turnover rate has been of 71%, up from 54% in each of the two five-year periods preceding 1992. Despite this higher turnover rate, the study also finds that the average age of people holding those posts went down to just above 59 years, only six months less than 5 years before. Cf. The Economist (1997).

\(^6\)In Celentani and Ganzuza (1999) we have obtained qualitatively identical results for the case in which officials care about the future.
Because we assume that the official has all the bargaining power with respect to the supplier, his gross return from corruption is equal to the supplier’s cost savings,  \( R(\lambda_t) = C(\lambda_t) - C(\lambda_C) \) and is therefore increasing in \( \lambda_t \).

Suppose that the principal expects that a fraction \( \gamma_t \) of the officials are corrupt. If the principal sets quality \( \lambda_t \), a fraction \( 1 - \gamma_t \) of officials require \( \lambda_t \) of their suppliers and pay them \( C(\lambda_t) \). A fraction \( \gamma_t \) of officials require \( \lambda_C \) of their providers but announce that the delivered quality is \( \lambda \) and accordingly pay \( C(\lambda_t) \) to the supplier. Given this, the principal’s best response to a given \( \gamma_t \) is the solution to the following maximization problem

\[
\max_{\lambda_t} (1 - \gamma_t) (V(\lambda) - C(\lambda_t)) + \gamma_t (V(\lambda_C) - C(\lambda_t)) = (1 - \gamma_t) V(\lambda_t) + \gamma_t V(\lambda_C) - C(\lambda_t) \tag{1}
\]

A solution to (1), \( \lambda(\gamma_t) \), is a best response of the principal to a given corruption level \( \gamma_t \).

**Lemma 1** Under Assumption 1, \( \lambda(\gamma_t) \) exists, is unique, positive, and finite and \( \lambda'(\gamma_t) < 0 \).

**Proof:** Appendix A.2.

The Lemma clarifies that the principal chooses a lower activity level with higher corruption, because the expected marginal value of activity level \( \lambda_t \) is decreasing in the fraction of corrupt officials \( \gamma_t \) (\( \lambda_t \) is delivered only with probability \( 1 - \gamma_t \)), whereas its cost is independent of \( \gamma_t \).

### 3 Equilibrium

Consider first the level of corruption in \( t \) for given activity levels \( \lambda_{t-1} \) and \( \lambda_t \). To simplify the presentation we assume that for all \( t = 1, \ldots, \lambda_t > \lambda_{\text{min}} \) and \( \gamma_t < 1 \). Notice first that the type of official who is indifferent between being corrupt or not \( \beta_t(\lambda_t) \) is

\[
\beta_t(\lambda_t) = R(\lambda_t) - \mu \phi.
\]

Because we assume that the initial distribution of officials’ types is uniform, a fraction

\[
\gamma_t^Y = R(\lambda_t) - \mu \phi
\]

of young officials are therefore corrupt in \( t \).

To compute the fraction of officials in the old generation who are corrupt in \( t \geq 1 \), it is necessary to take into account that some officials in that generation were corrupt in \( t - 1 \) and a fraction \( \mu \) of those were dismissed and replaced by “new” old officials drawn from the uniform distribution and living for one period only. Consider the distribution of types of a generation of old officials such that all types below \( \beta' \) were corrupt in their first term. Because of dismissals, the density of officials with type below \( \beta' \) is reduced by \( \mu \). On the other hand, given that \( \mu \beta' \) officials were dismissed and that the same measure of officials, drawn from the uniform distribution, are appointed in replacement, the density \( \mu \beta' \) has to be added for all \( \beta \in [0, 1] \). The new distribution can then be characterized by the following density function, where \( \beta' \in [0, 1] \) and \( \mu \in (0, 1) \)

\[
f(\beta; \beta', \mu) = \begin{cases} 
1 - \mu + \mu \beta' & \beta \in [0, \beta') \\
1 + \mu \beta' & \beta \in [\beta', 1]
\end{cases}
\]

represented in Figure 2, or, equivalently, by the associated distribution function

\[
F(\beta; \beta', \mu) = \int_{0}^{\beta} f(s; \beta', \mu) \, ds = \begin{cases} 
(1 - \mu + \mu \beta') & \beta \in [0, \beta') \\
(1 - \mu + \mu \beta') \beta' + (1 + \mu \beta') (\beta - \beta') & \beta \in [\beta', 1]
\end{cases}
\]

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represented in Figure 3. Notice that \( F(\beta; \beta', \mu) \) first order stochastically dominates the uniform distribution. This clarifies that the replacement of officials found to have been corrupt dynamically sorts officials’ types and leads to a distribution of old that has a lower inclination to corruption than the young, in the sense of first order stochastic dominance.

Letting \( \beta_{t-1} \) denote the type of official who was indifferent between being corrupt or not in \( t-1 \), the fraction of old officials who are corrupt in \( t \geq 1 \) is

\[
\gamma_t^O = F(\beta_t; \beta_{t-1}, \mu).
\] (2)

To avoid unnecessary complications in notation, in what follows we will assume that the indifferent type for both young and old officials belongs to \((0, 1)\). Given this and because \( \beta_{t-1} < \beta_t \) if and only if \( \lambda_{t-1} < \lambda_t \), we can rewrite (2) as

\[
\gamma_t^O = \begin{cases} 
(1-\mu)\beta_t + \mu\beta_{t-1}\beta_t & \text{if } \lambda_{t-1} \geq \lambda_t \\
\beta_t - \mu\beta_{t-1} + \mu\beta_{t-1}\beta_t & \text{if } \lambda_{t-1} < \lambda_t.
\end{cases}
\]

We can therefore compute the mass of corrupt officials in both the old and the young generation in a given period \( t \geq 1, 8 \) for given \( \lambda_{t-1} \) and \( \lambda_t \) as follows

\[
\gamma_t = \frac{1}{2} (\gamma_t^Y + \gamma_t^O) = \begin{cases} 
\frac{1}{2} \{ R(\lambda_t) - \mu \phi \} + (1-\mu + \mu (R(\lambda_{t-1}) - \mu \phi)) (R(\lambda_t) - \mu \phi) & \text{if } \lambda_{t-1} \geq \lambda_t \\
\frac{1}{2} \{ R(\lambda_t) - \mu \phi \} + (1-\mu + \mu (R(\lambda_{t-1}) - \mu \phi)) (R(\lambda_t) - \mu \phi) - \mu (R(\lambda_{t-1}) - R(\lambda_t)) & \text{if } \lambda_{t-1} < \lambda_t.
\end{cases}
\] (3)

Given that the principal is short lived, her best response to a given corruption level is \( \lambda_t = \lambda(\gamma_t) \) as in (8).

Substituting the principal’s best response \( \lambda_t = \lambda(\gamma_t) \) into (3) and recalling (??) we obtain the following first order difference equation in \( \gamma_t \), characterizing equilibrium for \( t \geq 1 \):

\[
[(2-\mu)\Gamma(\gamma_t) + \mu\Gamma(\gamma_{t-1}) \Gamma(\gamma_t) - 2\gamma_t] - \delta [\Gamma(\gamma_{t-1}) - \Gamma(\gamma_t)] (1-\mu\Gamma(\gamma_t))] = 0
\] (4)

where \( \delta = 1 \) if \( \gamma_{t-1} > \gamma_t \), and \( \delta = 0 \) otherwise.

The following Proposition provides a characterization of the steady state.

**Proposition 1**

1. There exists a unique steady state, \( \hat{\gamma} \).

2. The steady state is decreasing in the detection probability, \( \mu \), the penalty, \( \phi \), and the activity level delivered under corruption, \( \lambda_C \).

**Proof:** Appendix A.2.

Steady state corruption is lower than equilibrium corruption in the static model (part 1) because the replacement of officials who have been found to be corrupt at the end of their first term sorts heterogenous officials and leads to a distribution of corruption costs in the generation of old with a lower measure of types with low costs of being corrupt. Part 2 states the straightforward

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7The reason why \( t \) is constrained to be greater than or equal to 1 is that equation (2) provides the mass of old agents who are corrupt in \( t \) using the distribution of old agents’ corruption costs derived from the following assumptions: 1) That the distribution of their corruption costs at birth was the uniform over \([0, 1]\); 2) That all types below \( \beta_{t-1}^* \) were corrupt at the previous date. Because we are interested in equilibrium sequences starting after a variety of exogenous shocks, we allow the generation of old agents in \( t = 0 \) to have corruption costs described by an arbitrary distribution over the interval \([0, 1]\). This implies that equation (2) is only applicable for \( t \geq 1 \).

8To see why \( t \geq 1 \), refer to footnote 7.
results of the comparative statics of the steady state: corruption is higher when the net returns from being corrupt are higher.

The next proposition summarizes the main properties of the dynamics of equilibrium sequences of corruption starting from an initial distribution of old officials’ types. Recall that the indifferent type of official in the steady state is \( \gamma^* = \Gamma (\gamma^*) \) and that therefore the fraction of old officials who are corrupt in the steady state is \( F (\beta^*; \beta^*, \mu) \).

**Proposition 2** Let the initial distribution of old officials at \( t = 0 \) be described by \( G (\beta) \), a continuous cumulative distribution function, and let \( \{\gamma_t^t\}_{t=0}^{\infty} \) denote the equilibrium sequence starting from this distribution.

1. If
   \[
   G (\beta^*) = F (\beta^*; \beta^*, \mu),
   \]
   there exists a unique equilibrium sequence \( \{\gamma_t^t\}_{t=0}^{\infty} \), with \( \gamma_t^t = \gamma^* \), \( t = 0, 1, \ldots \).

2. If
   \[
   G (\beta^*) > F (\beta^*; \beta^*, \mu),
   \]
   there exists a unique equilibrium sequence \( \{\gamma_t^t\}_{t=0}^{\infty} \), with \( \gamma_0 > \gamma_1 > \gamma_2 > \ldots > \gamma^* \) and with
   \[
   \lim_{t \to \infty} \gamma_t = \gamma^*.
   \]

3. If
   \[
   G (\beta^*) < F (\beta^*; \beta^*, \mu),
   \]
   there exists a unique equilibrium sequence \( \{\gamma_t^t\}_{t=0}^{\infty} \), with \( \gamma_0 < \gamma^* \), \( \gamma_1 > \gamma_2 > \ldots > \gamma^* \) and with
   \[
   \lim_{t \to \infty} \gamma_t = \gamma^*.
   \]

**Proof:** Appendix A.3.

Proposition 2 says that to characterize the equilibrium sequence it is necessary to ask the following hypothetical question. Suppose that corruption is as in the steady state, \( \gamma^* \), and therefore that the returns from corruption are as in the steady state; is the measure of old officials who would be corrupt higher in the initial distribution or in the steady state distribution?

When the measure of old officials who would be corrupt in the steady state is higher in the initial distribution than in the steady state distribution, there is a high measure of officials who are inclined to be corrupt and the initial level of corruption is higher than the steady state, \( \gamma_0^0 > \gamma^* \) (part 2). When the measure of old officials who would be be corrupt in the steady state is lower in the initial distribution than in the steady state distribution, there is a low measure of officials who are inclined to be corrupt and the initial level of corruption is lower than in the steady state, \( \gamma_0^0 < \gamma^* \) (part 3).

But Proposition 2 also establishes that the equilibrium sequence following the first period, exhibits levels of corruption higher than in the steady state, \( \gamma_2 > \ldots > \gamma^* \), regardless of whether corruption in \( t \) was higher or lower than in the steady state. This occurs because when corruption differs from its steady state level, it generates a distribution of old officials in the following period with a higher measure of officials with an inclination to be corrupt than in the steady state distribution.

When corruption is higher than in the steady state, this happens because of excessive replacement. Some young officials with a low inclination to be corrupt, who in the steady state would not be corrupt and would therefore not be replaced, choose to be corrupt and are therefore replaced with probability \( \mu \), possibly by officials with higher inclinations to be corrupt. When corruption
is lower than in the steady state, this happens because of insufficient replacement. Some young officials with a high inclination to be corrupt, who in the steady state would be corrupt and would therefore be replaced with probability $\mu$, choose not to be corrupt and are therefore not replaced.

Figure 4 plots the steady state distribution of the old and the distributions of the old following a period in which the returns from corruption were lower ($\Gamma$) or higher ($\Gamma'$) than in the steady state. Both the insufficient replacement and the excessive replacement lead to a distribution with a higher measure of officials willing to be corrupt in the steady state.

Proposition 2 also considers the case in which the initial distribution and the steady state distribution have the same measure of officials who would be willing to be corrupt in the steady state. In this case, corruption remains constant at the steady state (part 1).

Proposition 2 completes the characterization of equilibrium sequences by stating that they are unique and monotonically converging to the steady state.

We do not provide a formal proof here, but it is easy to verify that for any initial distribution, corruption is lower than in the static setting at all dates. This is simply a consequence of the fact that the replacement of officials found to have been corrupt leads to a distribution of old officials with a lower inclination to corruption than the prior distribution.

For the sake of simplicity we have chosen to consider the case in which officials live for two periods but are completely myopic, i.e., they ignore when young the consequences of their actions on their future payoffs. But all our results hold in a setting in which officials care about their future payoffs\(^9\). The main difference with respect to the case described in this paper is that when a nonmyopic young official anticipates a decrease in corruption and therefore an increase in the returns from corruption, he is less inclined to be corrupt than an old official. This implies that as corruption monotonically decreases to the steady state, fewer young officials will be corrupt. Because this lowers the replacement of officials with a high inclination to be corrupt, the convergence to the steady state is slower.

4 The Dynamic Response of Corruption

In this section we want to study the consequences of changes in the returns from corruption or in policy parameters using the characterization results of Proposition 2. The results of this section are useful from two different perspectives. First, they provide predictions on how corruption responds to the given changes. Second, because we also consider changes in the legal framework (monitoring probabilities or corruption penalties), the results offer an evaluation of some temporary or permanent policy changes.

4.1 Response to Permanent Changes

The following Proposition characterizes the dynamic response of corruption to a permanent change in the returns to corruption.

**Proposition 3** Suppose that the system was in the steady state $\gamma'$ at $t = 0$. Consider a change to some parameter other than $\mu$ that implies a new steady state $\gamma^* \neq \gamma'$. Suppose that the change takes place in $t = 1$ and becomes known before players play. Then, the equilibrium sequence $\{\gamma_t\}_{t=1}^{\infty}$ will be such that $\gamma' > \gamma'_1 > \gamma'_2 > \gamma'_3 > \ldots > \gamma^*$ and with $\lim_{t \to \infty} \gamma_t = \gamma^*$.

**Proof:** Appendix A.4.

\(^9\)When agents are nonmyopic the equilibrium is described by a second order difference equation and proofs are substantially more involved. All proofs are available in Celentani and Ganuza (1999).
Proposition 3 considers permanent changes in parameters other than the detection probability \( \mu \), but in Celentani and Ganuza (1999), a previous version of this paper, we show that similar results can also be obtained for permanent changes in \( \mu \).

Proposition 3 says that when a permanent change implies a new steady state, the level of corruption that follows the change is above the new steady state and corruption will then converge to the new steady state, regardless of whether the old steady state was higher or lower than the new one. This result is easily understood recalling the intuition of Proposition 2.

When the change implies that the new steady state is higher, the system inherits a distribution of old officials with insufficient replacement with respect to the new steady state level of corruption. This means that there is a high measure of officials who have a high inclination to be corrupt but who were not corrupt simply because the returns from corruption in their first term were not large enough. Roughly speaking, there are old officials who are not very honest, but who chose not to be corrupt in their first term simply because it wasn’t worth their while. Given that these officials refrained from corruption in their first term, they have not been replaced and the distribution exhibits a larger measure of officials with a high inclination to be corrupt than the new steady state distribution. This means that condition (6) holds and by Proposition 2 corruption will therefore first jump up, above the new steady state, and then will monotonically decrease down to it.

When the change implies that the new steady state is lower, on the other hand, the system inherits a distribution of old officials with excessive replacement with respect to the new steady state level of corruption. As clarified in the discussion of Proposition 2, this means that condition (6) holds and by Proposition 2 corruption will monotonically decrease down to the new steady state.

4.2 Response to Temporary Changes

The following proposition characterizes the dynamics of corruption following a temporary change in the returns from corruption.\(^\text{11}\)

**Proposition 4** Suppose that the system was in the steady state \( \gamma^* \) at \( t = 0 \). Suppose that a temporary shock to some parameter other than \( \mu \) changes the returns from corruption to \( \Gamma(\gamma) \) for period \( t = 1 \) only and becomes known before players play. Then:

1. If the shock increases the returns from corruption, \( \Gamma(\gamma) > \Gamma(\gamma), \forall \gamma \in [0, \hat{\gamma}), \gamma_1 > \gamma_2 > \gamma_3 > \ldots > \gamma^* \) and \( \lim_{t \to \infty} \gamma_t = \gamma^* \).

2. If the shock decreases the returns from corruption, \( \Gamma(\gamma) < \Gamma(\gamma), \forall \gamma \in [0, \hat{\gamma}), \gamma_1 < \gamma^* \), \( \gamma_2 > \gamma_3 > \ldots > \gamma^* \) and \( \lim_{t \to \infty} \gamma_t = \gamma^* \).

**Proof:** Appendix A.5.

Part 1 of Proposition 4 establishes that a temporary increase in the returns from corruption may lead to increased levels of corruption over a long period of time. The reason is that the

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\(^{10}\)Changes in \( \mu \) give rise to slight differences (corruption may be below the steady state for one period only) but significantly more involved formal arguments. For this reason we have chosen not to include the formal analysis of this case.

\(^{11}\)As for the case of Proposition 3, in the following Proposition we ignore the cases in which the change in the profitability of corruption is determined by a change in the monitoring probability unforeseen until the beginning of period \( \tau \). A similar result can be proved using a slightly more involved argument. Notice, however, that Proposition 5 below considers the case of a change in the monitoring probability unforeseen until the end of period \( \tau \).
temporary increase in the returns from corruption leads to excessive replacement. This leads to levels of corruption higher than the steady state in the following periods, even though the adverse effects eventually taper off.

An example of the case considered in part 1 of Proposition 4 is the wave of privatizations of state enterprises in many former communist bloc countries, as well as in India and Mexico. These programs created unusually profitable temporary corruption opportunities. Proposition 4 claims that the negative effects in terms of corruption may well last beyond the privatization period.

Part 2 of Proposition 4, on the other hand, states that the initial decrease in corruption following a temporary decrease in the returns from corruption gives way to subsequent corruption levels above the steady state. The reason for this is that the initial decrease in corruption occurs because some officials (young and old) that would be corrupt in the steady state now choose not to be corrupt. This leads to insufficient replacement and leads to corruption levels in all following periods that are above the steady state and that converge to it.

The next Proposition considers the case of a temporary change in the monitoring probability for one period that takes place at the end of the period, i.e., after officials currently alive have made their choices. We consider this case for two different reasons. First, it is interesting to study the long run effects of short lived anti-corruption campaigns whose main purpose is to purge rather than to deter. An example is an amnesty for firms that denouncing officials who demanded payments that leads to higher detection probabilities of past events. Second, it is also interesting to study the long-run consequences of spells in which detection probabilities were lower than expected possibly as a consequence of underperforming judiciary, or investigative police.

**Proposition 5** Suppose that the system was in the steady state $\gamma^*$ at $t = 0$. Suppose that, unforeseen until the end of period $t = 1$, the probability of detection changes from $\mu$ to $\mu'$ for period $t = 1$ only.

1. If $\mu' > \mu$, then $\gamma_1 = \gamma^*$, $\gamma_2 < \gamma^*$, $\gamma_3 > \gamma_4 > \ldots > \gamma^*$ and $\lim_{t \to \infty} \gamma_t = \gamma^*$.

2. If $\mu' < \mu$, then $\gamma_1 = \gamma^*$, $\gamma_2 > \gamma_3 > \gamma_4 > \ldots > \gamma^*$ and $\lim_{t \to \infty} \gamma_t = \gamma^*$.

**Proof:** Appendix A.6.

Proposition 5 first clarifies that, because the change in the monitoring probability $\mu$ is unforeseen until the end of period 1, play in that period does not change (and therefore $\gamma_1 = \gamma^*$). However, Proposition 5 also takes into account that the change in $\mu$ induces a change in the distribution of old officials’ types in the following period. If $\mu' > \mu$, this distribution will satisfy condition (7), and if $\mu' < \mu$, it will satisfy condition (6) and the stated results on the continuation sequences are straightforward applications of Proposition 2.

Part 1 of Proposition 5 shows that the initial benefits of short anticorruption campaigns, such as the Italian “Clean Hands” of 1992-93, are followed by an increase in corruption above the steady state. Part 2 of Proposition 5 shows that even a short-lived decrease in the detection probability of corrupt agreements has long lasting adverse consequences.

The findings of Proposition 5 contrast with other results that have been proposed. For example, Tirole (1996) considers a period in which a temporary shock to the returns from corruption causes

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13 Any temporary unforeseen change other than a change in the monitoring probability that takes place after agents have decided to be corrupt or not can only have an impact on their payoff, but not on equilibrium play.

14 As mentioned in footnote 5, a study conducted by *The Economist* and the Enrico Mattei Foundation on the 100 most powerful posts in Italy found that in the 5 years following the 1992 Clean Hands investigation campaign dismissals increased substantially. Cf. *The Economist* (1997).
all corruptible agents to be corrupted and argues that an amnesty can be beneficial because it would allow society to stay in the low corruption equilibrium. Notice that an amnesty for Tirole (1996) is a way to make current decisions unobservable, so that in the continuation equilibrium the economy stays in the steady state. By contrast in our model we think of amnesties as situation in which either $\mu = 0$ (in the case in which corrupt public officials are pardoned) or $\mu = 1$ (in the case in which firms are pardoned if they give in their corruption partners). According to Proposition 5 the immediate consequences of these two different types of amnesties are very different, but they both lead to adverse long-run consequences.

5 Discussion

The results in the previous section are only a few examples of the questions that can be addressed within our framework. But out results can be applied to address a number of policy issues. We now want to provide an informal discussion of a few such issues.

First, suppose that a government decides to decrease red tape and that this leads to a decrease of the probability of unveiling corrupt transactions. This leads to a higher corruption steady state, but our results also indicate that the adjustment to the new higher steady state will occur after corruption overshoots. In other words our results suggest that an observed initial increase in corruption should not be viewed as only the first step in a trend of increasing corruption, but should be considered as an overestimate of the final corruption level. Understanding this fact is important, because it allows one to appropriately anticipate the cost of the new legislation in terms of increased corruption and because it makes it possible to evaluate correctly the effects of the new legislation once it has been introduced.

Second, suppose that a temporary change in period $t$ has upset the equilibrium from the steady state and that the government is considering the possibility to modify temporarily its monitoring or penalty rules at the beginning of period $t + 1$ in reaction to this change. What would be the consequences of such temporary changes? We know that regardless of whether the shock increased or decreased corruption in $t$, if nothing changes, corruption in $t + 1$ would be higher than in the steady state. This implies that in $t + 1$ there would be insufficient replacement that would in turn imply future corruption levels higher than the steady state although converging to it. If the government reacts at the beginning of period $t + 1$ by increasing monitoring or penalties, it would reduce corruption in $t + 1$, but it would also cause a more pronounced insufficient replacement which would lead to future levels of corruption that would be even higher than otherwise. If the government, on the other hand, reacts at the beginning of period $t + 1$ by decreasing monitoring or penalties, it would counter insufficient replacement and it would therefore speed up the reversion to the steady state corruption level at the cost of tolerating a higher corruption level in $t + 1$.

Third, suppose that a government has decided to increase permanently corruption detection and/or penalties to achieve a lower steady state corruption level. Should the government simply set the monitoring and the penalties that would lead to new steady state and wait until this is reached? Or could it adjust its policies to shorten the transition to the new steady state? When a permanent change occurs that leads to a lower steady state corruption level, the initial distribution is characterized by excessive replacement and this causes an initial level of corruption in $t + 1$ that would be higher than in the new steady state. This implies that in $t + 1$ there would be insufficient replacement that would in turn imply future corruption levels higher than the steady state, although converging to it. If the government desired to speed up the transition to the new steady state it would therefore need to counter the insufficient replacement by adopting monitoring and penalty rules that are less severe than the ones that it would want to adopt once the steady
state is reached.

The previous discussions clarify that the general result that we obtain that the dynamics of corruption away from the steady state is *adverse* has important implications for policy and policy evaluation. First, the fact that corruption is more likely to be above than below the steady state is important to assess the consequences of policy changes. Second, because higher corruption is accompanied by insufficient replacement, a speedy transition to the steady state is normally eased by measures, such as as *reductions* in monitoring and/or penalties, that to counter insufficient replacement increase rather than decrease current corruption. Third, because corruption is more likely to be above than below the steady state, recurrent random policy changes are likely to have adverse consequences and this suggests that perseverance increases the value of governance rules.\(^{15}\)

Our work is a contribution to the understanding of political or bureaucratic corruption. But we also think that at a more general level it provides a starting point for analyzing how dismissals affects the dynamics of agency relationships. We believe that the ideas presented in this paper may be usefully applied in dynamic agency relationships in which dismissals are the main discipline and sorting device. Our results, for example, may be used to evaluate the consequences of changes in corporate governance rules on business ethics or to analyze the relationships between investors and fund managers.

\(^{15}\)This intuition is confirmed by numerical exercises that we have performed. We have compared two situations. In the first one we allowed for random shocks to monitoring probabilities and corruption penalties. In the second we have considered the case in which monitoring probabilities and corruption penalties were fixed at the average levels of their realizations of the first sequence. We then have performed simulations and we have verified that the average level of corruption in the first case is higher than in the second.
A Appendix

A.1 Proof of Lemma 1

Under Assumption 1 a solution to 1 is characterized by the following first-order condition

$$ (1 - \gamma) V' (\lambda (\gamma)) = C' (\lambda (\gamma)). $$

(8)

Under Assumption 1, a solution to 8 exists, is unique, positive, and finite and

$$ \lambda' (\gamma) = \frac{V' (\lambda (\gamma))}{(1 - \gamma) V'' (\lambda (\gamma)) - C'' (\lambda (\gamma))} < 0. $$

The second order condition

$$ (1 - \gamma) V'' (\lambda (\gamma)) - C'' (\lambda (\gamma)) < 0 $$

is satisfied since $(1 - \gamma) > 0$, $V'' (\lambda (\gamma)) < 0$, and $C'' (\lambda (\gamma)) > 0$. □

A.2 Proof of Proposition 1

(1) Let $e (\gamma) = (2 - \mu) \Gamma (\gamma) + \mu (\Gamma (\gamma))^2 - 2\gamma$. The steady state is given by the solution of $e (\gamma^*) = 0$ for $\gamma^* \in [0, 1]$. Since we assume that $\Gamma (0) > 0$ (otherwise no official would ever want to be corrupt) to show part 1 of the Proposition it suffices to show that $e (0) > 0$, $e (\gamma) < 0$, where $0 < \gamma < 1$ is such that $\Gamma (\gamma) = 0$ or $\lambda (\gamma) = \lambda_{\text{min}}$, and $e' (\gamma) < 0$. It is easy to see that $e (0) = (2 - \mu) \Gamma (0) + \mu (\Gamma (0))^2 > 0$. On the other hand we have

$$ e (\gamma) = -2\gamma < 0. $$

Noticing that $e' (\gamma) = (2 - \mu) \Gamma' (\gamma) + 2\mu \Gamma (\gamma) \Gamma' (\gamma) - 2 < 0$ and that $e (\gamma)$ is continuos, concludes the argument.

The proof of the fact that $\gamma^* < \hat{\gamma}$ follows from $e (\hat{\gamma}) = -\mu \Gamma (\hat{\gamma}) (1 - (\Gamma (\hat{\gamma})) < 0 = e (\gamma^*)$ and $e' (\gamma) < 0$.

(2) To explicitly take into account the impact of the parameters of the problem on the steady state, rewrite the equation of the steady state as $e (\gamma^*, \mu, \phi, \lambda_C) = 0$. Recall that $\Gamma (\gamma) = C (\lambda (\gamma)) - k$ and that $k = C (\lambda_C) + \mu \phi$ which implies that $\Gamma (\gamma) = C (\lambda (\gamma)) - C (\lambda_C) - \mu \phi$. Let $a \in \{\phi, \lambda_C\}$; then we have

$$ \frac{d\gamma^*}{da} = \frac{\frac{\partial e (\gamma^*)}{\partial a}}{\frac{\partial e (\gamma^*)}{\partial \gamma}} = -\frac{\frac{\partial e (\gamma^*)}{\partial a}}{\frac{\partial e (\gamma^*)}{\partial \gamma}}. $$

Since $\frac{\partial e (\gamma^*)}{\partial a} < 0$, $\frac{\partial e (\gamma^*)}{\partial \gamma} > 0$, $\frac{\partial \Gamma (\gamma^*)}{\partial a} < 0$, for all $a \in \{\phi, \lambda_C\}$ we have that $\frac{d\gamma^*}{da} < 0$ for $a \in \{\phi, \lambda_C\}$. Similarly, we have

$$ \frac{d\gamma^*}{d\mu} = -\frac{\frac{\partial e (\gamma^*)}{\partial \mu}}{\frac{\partial e (\gamma^*)}{\partial \gamma}} = -\frac{\frac{\partial e (\gamma^*)}{\partial \mu}}{\frac{\partial e (\gamma^*)}{\partial \gamma}} + \frac{\frac{\partial e (\gamma^*)}{\partial \mu}}{\frac{\partial e (\gamma^*)}{\partial \gamma}} = -\frac{\frac{\partial e (\gamma^*)}{\partial \mu}}{\frac{\partial e (\gamma^*)}{\partial \gamma}} - \left[ \Gamma (\gamma) - (\Gamma (\gamma))^2 \right] < 0 $$

as $\frac{\partial \Gamma (\gamma^*)}{\partial a} < 0$ and $\Gamma (\gamma^*) - (\Gamma (\gamma))^2 > 0$ imply that the numerator is negative. □
A.3 Proof of Proposition 2

To characterize the equilibrium sequence we start by computing corruption in \( t = 0 \). Let 
\[
R(\gamma) = \Gamma(\gamma) + G(\Gamma(\gamma)) - 2\gamma. 
\]
The level of corruption in \( t = 0 \) is then given by the solution of
\[
R(\gamma_0) = 0 \quad (9)
\]
for \( \gamma_0 \in [0,1] \). To see that a solution to (9) exists and is unique, notice that
\( R(\gamma) \) is continuous, \( R(0) = \Gamma(0) + G(\Gamma(0)) > 0 \), \( R(\overline{\gamma}) = -2\overline{\gamma} < 0 \) and
\( R'(\gamma) = \Gamma'(\gamma) \left[ 1 + g(\Gamma(\gamma)) \right] - 2 < 0 \). Recall that the fraction of old agents who are corrupt in the steady state is
\[
F(\beta^*; \beta^*, \mu) = 2\gamma^* - \Gamma(\gamma^*). 
\]
Thus, if \( G(\beta^*) = F(\beta^*; \beta^*, \mu) = 2\gamma^* - \Gamma(\gamma^*), \) then \( R(\gamma^*) = 0 \) and therefore \( \gamma_0 = \gamma^* \). If \( G(\beta^*) = F(\beta^*; \beta^*, \mu) > 2\gamma^* - \Gamma(\gamma^*), \) then \( R(\gamma^*) > 0 \), and given that \( R'(\gamma) < 0 \) and \( R(\gamma_0) = 0 \), we have that \( \gamma_0 > \gamma^* \). Finally, if \( G(\beta^*) = F(\beta^*; \beta^*, \mu) < 2\gamma^* - \Gamma(\gamma^*), \) then \( R(\gamma^*) < 0 \), and given that \( R'(\gamma) < 0 \) and \( R(\gamma_0) = 0 \), we have that \( \gamma_0 < \gamma^* \).

Having characterized corruption in \( t = 0 \), we now move on to the characterization of equilibrium in \( t = 1, 2, \ldots \). Letting
\[
E(\gamma_t, \gamma_{t-1}) = (2 - \mu)\Gamma(\gamma_t) + \mu \Gamma(\gamma_{t-1}) \Gamma(\gamma_t) - 2\gamma_t \quad (10)
\]
\[
E_P(\gamma_t, \gamma_{t-1}) = -\mu [\Gamma(\gamma_{t-1}) - \Gamma(\gamma_t)], \quad (11)
\]
rewrite (4) as
\[
E(\gamma_t, \gamma_{t-1}) + \delta E_P(\gamma_t, \gamma_{t-1}) = 0 \quad (12)
\]
where \( \delta = 1 \) if \( \gamma_{t-1} > \gamma_t \) and \( \delta = 0 \) otherwise.

From (10) we get
\[
\frac{\partial E(\gamma_t, \gamma_{t-1})}{\partial \gamma_t} = [(2 - \mu) + \mu \Gamma(\gamma_{t-1})] \Gamma'(\gamma_t) - 2 < 0 \quad (13)
\]
\[
\frac{\partial E(\gamma_t, \gamma_{t-1})}{\partial \gamma_{t-1}} = \mu \Gamma'(\gamma_{t-1}) \Gamma(\gamma_t) < 0. \quad (14)
\]

From (11) we get
\[
\frac{\partial E_P(\gamma_t, \gamma_{t-1})}{\partial \gamma_t} = \mu \Gamma'(\gamma_t) < 0 \quad (15)
\]
\[
\frac{\partial E_P(\gamma_t, \gamma_{t-1})}{\partial \gamma_{t-1}} = -\mu \Gamma'(\gamma_{t-1}) > 0. \quad (16)
\]

Finally notice that
\[
\frac{\partial [E(\gamma_t, \gamma_{t-1}) + E_P(\gamma_t, \gamma_{t-1})]}{\partial \gamma_{t-1}} = \mu \Gamma'(\gamma_{t-1}) [\Gamma(\gamma_t) - 1] > 0 \quad (17)
\]
since \( \Gamma(\gamma_t) < 1 \) and \( \Gamma'(\gamma_{t-1}) < 0 \).

If \( \gamma_{t-1} < \gamma_t \), \( \delta = 0 \) and from the implicit function theorem and from (13) and (14) we obtain
\[
\frac{\partial \gamma_t}{\partial \gamma_{t-1}} = -\frac{\partial E(\gamma_t, \gamma_{t-1})}{\partial \gamma_{t-1}} < 0. \quad (18)
\]

If \( \gamma_{t-1} > \gamma_t \), \( \delta = 1 \) and from the implicit function theorem and from (15) and (16) we obtain
\[
\frac{\partial \gamma_t}{\partial \gamma_{t-1}} = -\frac{\partial [E(\gamma_t, \gamma_{t-1}) + E_P(\gamma_t, \gamma_{t-1})]}{\partial \gamma_{t-1}} > 0. \quad (19)
\]
Finally, notice that $\gamma_{t-1} = \gamma^*$ implies $\gamma_t = \gamma^*$ and that (12) is continuous in $\gamma_{t-1}$ and $\gamma_t$. From this and (18) and (19), we can plot the phase diagram of the equilibrium system in Figure 5. Figure 5 clarifies that If $\gamma_0' = \gamma^*$, then $\gamma_1' = \gamma_2' = \ldots = \gamma^*$ and that for a given $\gamma_0' \neq \gamma^*$ there exists a unique equilibrium path with $\gamma_1' > \gamma_2' > \ldots > \gamma^*$. Given the equilibrium sequences are monotone and they are bounded below by the steady state, they have to converge to it. □

### A.4 Proof of Proposition 3

Given $\gamma' \neq \gamma^*$, $\beta' \neq \beta^*$, the distribution of old officials' types at time $t = 1$ is $F(\beta; \beta', \mu)$. Consider now $F(\beta; \beta', \mu)$ and $F(\beta; \beta^*, \mu)$. It is easy to see that for both $\beta < \beta^*$ and $\beta > \beta^*$

$$F(\beta'; \beta', \mu) > F(\beta^*; \beta^*, \mu)$$

This is because, if we take $\beta$ as given, $F(\beta; \beta', \mu)$ is increasing in $\beta$ if $\beta < \beta'$ and decreasing otherwise.

$$F(\beta; \beta', \mu) = \begin{cases} (1 - \mu + \mu' \beta') & \beta < \beta' \\ (1 + \mu' \beta - \mu \beta') & \beta > \beta' \end{cases}$$

Hence, the function $F(\beta^*; \beta', \mu)$ is minimized when $\beta' = \beta^*$. Since condition 20 is equivalent to (2), the result that $\gamma_1' > \gamma_2' > \ldots > \gamma^*$ follows from Proposition 2. □

### A.5 Proof of Proposition 4

(1) From Proposition 2 we know that $\gamma_2 > \gamma_3 > \ldots > \gamma^*$. We therefore only need to show that $\gamma_1 > \gamma^*$. To do this suppose that, contrary to the statement, $\gamma_1 < \gamma^*$. $\gamma_1$ will be determined by the following equation:

$$\gamma_1 = \frac{1}{2} \left[ \bar{\Gamma}(\gamma_1) + F(\bar{\Gamma}(\gamma_1); \beta^*, \mu) \right].$$

If $\gamma_1 < \gamma^*$

$$\bar{\Gamma}(\gamma_1) > \Gamma(\gamma_1) > \Gamma(\gamma^*)$$

and

$$F(\bar{\Gamma}(\gamma_1); \beta^*, \mu) > F(\Gamma(\gamma^*); \beta^*, \mu).$$

On the other hand, $\gamma^*$ is characterized by

$$\gamma^* = \frac{1}{2} \left[ \Gamma(\gamma^*) + F(\Gamma(\gamma^*); \beta^*, \mu) \right].$$

It is now easy to see that (24), (22) and (23) provide a contradiction to (21) when $\gamma_0 < \gamma^*$.

(2) From Proposition 2 we know that $\gamma_2 > \gamma_3 > \ldots > \gamma^*$. We therefore only need to show that $\gamma_1 < \gamma^*$. To do this suppose that, contrary to the statement, $\gamma_1 > \gamma^*$, and a contradiction to the hypothesis that $\gamma_1 > \gamma^*$ is reached using the same arguments that were used in the proof of part 1, with all inequalities reversed. □

### A.6 Proof of Proposition 5

Since the change in $\mu$ is unforeseen until the end of period $t = 1$, we have that $\gamma_1 = \gamma^*$. Given this, and given $\mu'$, the distribution of the officials' types will then be $F(\beta; \beta^*, \mu')$. Since it is easy to see that $F(\beta^*; \beta^*, \mu') > F(\beta^*; \beta^*, \mu)$ if $\mu' > \mu$ and $F(\beta^*; \beta^*, \mu') < F(\beta^*; \beta^*, \mu)$ if $\mu' < \mu$, applying Proposition 2 concludes the argument. □
References


Figure 2: Old officials’ density function
Figure 3: Old officials’ distribution function
Figure 4: Old officials’ distribution functions
\[ E(\gamma_t, \gamma_{t-1}) + \delta E_P(\gamma_t, \gamma_{t-1}) = 0 \]

**Figure 5:** Phase diagram