Retirement Savings Accounts and Human Capital Investment*

TETYANA DUBOVYK†

University of Minnesota and
Federal Reserve Bank of Minneapolis

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ABSTRACT

This paper studies the role of endogenous human capital accumulation in evaluating tax and Social Security policies. It considers two overlapping generations environments with borrowing constraints: one with exogenous human capital and a second with human capital accumulation through time investment. Baseline environments are calibrated to the U.S. tax and Social Security system. This paper analyzes two alternative Social Security systems: (a) voluntary and (b) mandatory retirement savings accounts. The paper finds that the welfare ranking of these alternatives depends on the endogeneity of human capital investment. Both systems are welfare improving when compared to the baseline. However, the system with mandatory (voluntary) accounts leads to lower welfare gains in the endogenous (exogenous) human capital environment. This difference is due to young individuals (i) switching time allocation towards human capital accumulation and (ii) being borrowing constrained under mandatory savings in the endogenous environment.

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†Department of Economics, University of Minnesota, 271 19th Ave South Suite 1035, Minneapolis, MN 55455, USA. E-mail: dubovyk@econ.umn.edu.

1. Introduction

Labor productivity over an individual's life-cycle is modelled in a number of ways. One strand of literature calibrates the life-cycle profile to match the observed earnings profile. This approach is widely used in empirical macroeconomics literature. Example of these papers are Auerbach and Kotlikoff (1987), DeNardi et al. (1999), and Kotlikoff et al. (1999). In the empirical public finance literature, an individual's schooling or labor force participation decisions determine the evolution of labor productivity over the life-cycle. For example, Imai and Keane (2004) show the importance of human capital accumulation in estimating the intertemporal elasticity of labor supply. This paper studies the importance of human capital accumulation decisions in evaluating tax and social security policies.

I analyze two retirement systems alternative to the Pay-As-You-Go (PAYG) social security system. Welfare ranking of these alternatives depends on the endogeneity of human capital accumulation. To evaluate the role of human capital investment, I set up two baseline environments that differ by evolution of labor productivity over an agent's life-cycle. In the environment with exogenous human capital, the labor productivity is exogenously given. In the environment with endogenous human capital, an agent chooses the amount of time allocated to human capital accumulation. In both environments, labor supply is elastic, the retirement decision is exogenous, and negative asset holdings are not permitted. These baseline environments incorporate the stylized version of the U.S. tax and social security system.

There is concern over the financial solvency of the current PAYG system due to demographic changes. A number of reforms have been proposed. These proposals stress the effect of reforms on savings and output. I evaluate two alternative retirement arrangements and consider the welfare effects as well. Under the first alternative, the PAYG social security system is removed and the labor income tax is reduced for the portion used to finance social security benefits. Within this alternative, agents finance their consumption during the retirement years through their own savings. These savings are accumulated on the Voluntary Retirement Savings Accounts (RSA). The second retirement system does not have the PAYG system, and agents are required to contribute a fixed portion of their labor income toward tax-deferred retirement accounts. These accounts are called Mandatory Retirement Savings

Accounts.

Under both alternatives, output and savings are higher than in the baseline. However, there are different welfare implications. Both systems are welfare improving when compared to the baseline. However, the system with Voluntary RSA leads to higher welfare gains in the environment with endogenous human capital investment. These welfare gains are measured by life-time consumption equivalents in comparison to the baseline. On the contrary, the arrangements with Mandatory RSA are the preferred ones in the environment with exogenous human capital. This difference is due to young individuals (i) switching time allocation towards human capital accumulation and (ii) being borrowing constrained under mandatory savings in the endogenous environment.

This paper builds on the quantitative tradition of evaluating tax and social security policies in an overlapping generations (OG) framework started by Auerbach and Kotlikoff (1987). I also incorporate the human capital investment technology proposed by Ben-Porath (1967). The papers closest to my work are Davies and Whalley (1991), Heckman et al. (1999), and Alvarez-Albelo (2004). This paper differs from Davies and Whalley (1991) and Heckman et al. (1999) in two dimensions. First, I consider an elastic labor supply. As a result, time reallocation among different activities has important welfare implications. Second, I study different sets of tax and social security policies. In my models, time investment is an input into human capital accumulation while Alvarez-Albelo (2004) studies human capital enhancement through learning-by-doing, i.e., participation in the market production enhances the human capital from tomorrow on. One of the experiments in Alvarez-Albelo (2004) closely resembles the Voluntary RSA studied in this paper. However, Alvarez-Albelo (2004) does not perform welfare analysis. The environment with exogenous human capital is motivated by and is comparable to DeNardi et al. (1999) and Kotlikoff et al. (1999).

The paper is organized as follows. In section 2, I construct two baseline OG models with exogenous and endogenous human capital decisions. These models incorporate the stylized version of the U.S. tax and social security system. Section 3 discusses the alternative retirement arrangements and welfare implications of each. Section 4 discusses conclusions and extensions for the future research.

2. Baseline Overlapping Generations Models

To quantify the effects of different social security arrangements on savings and welfare, I consider two general equilibrium models with OG structure. Common features between these two economies are the finite and certain life-span of agents, the Cobb-Douglas production technology, and the set of government policies. The agents allocate their time endowment among leisure, market production, and time investment into human capital accumulation. These models differ by the evolution of the human capital profile over the agents' life-cycle and time allocation decisions. In the first model, the agents' time allocation decisions, in particular, the time investment into human capital enhancement, determine the evolution of the human capital and, consequently, the wage income profiles over the life-cycle. This model is called the model with endogenous human capital. The second model has an exogenous age-specific labor productivity profile and is called the economy with exogenous human capital. The traditional macroeconomics policy literature studies the model of this type.

I initially describe the model with endogenous human capital accumulation decision. Then, I explain the features of the model with exogenous human capital.

2.1. Model with Endogenous Human Capital

I start with a description of the demographic structure and preferences. The economy has overlapping generations of agents who live for J adult periods, with ages denoted by $j \in \Im \equiv \{0, ..., J-1\}$. The agents' life-spans are certain. In the first time period, the measure of newly born agents is normalized to 1. The population is constant, and the total population size is J.

A young agent born at period t is endowed with initial levels of physical and human capitals, s_t^t and h_t^t respectively[1]. Each period agents are endowed with one unit of time that can be allocated to leisure, production activities in the market sector, or investment into human capital accumulation. Let $\left\{c_{t+j}^t, l_{m,t+j}^t, l_{h,t+j}^t\right\}$ denote consumption, market hours, and investment hours, respectively, of an agent born at period t (superscript) and at time period

[1]Notational convention for an agent's variables is as follows. The superscript denotes a period when an agent is born, and the subscript is a time period when an allocation takes place. Hence, an agent's age is given by the difference between the subscript and the superscript.

$t+j$ (subscript). The preferences of a young agent born at period t are ordered by

$$(1) \qquad \sum_{j=0}^{J-1} \beta^j u(c^t_{t+j}, 1 - l^t_{m,t+j} - l^t_{h,t+j}),$$

where β is a time preference parameter. Each agent chooses sequences of consumption, market hours, and investment hours to maximize the discounted value of life-time utility subject to its budget constraint,

$$(2) \qquad (1+\tau_c)\, c^t_{t+j} + s^t_{t+1+j} \leq (1-\tau_l)\, w_{t+j} h^t_{t+j} l^t_{m,t+j} + (1 + (1-\tau_k)\, r_{t+j}) s^t_{t+j} + d^t_{t+j}.$$

This constraint must be balanced at each age of the agent's life, i.e., for any $j \in \Im$. The agent's expenditures on consumption and savings in the form of physical capital, s^t_{t+1+j}, must be less or equal to the after-tax income.

The agent born at period t and of age j receives labor income $w_{t+j} h^t_{t+j} l^t_{m,t+j}$, where w_{t+j} is the real wage per efficient unit of labor in terms of the consumption good at period $t+j$. The agent's labor productivity at age j depends on the stock of human capital h^t_{t+j}, which is determined by the undepreciated human capital from the last period and the new human capital accumulation during the last period:

$$(3) \qquad h^t_{t+j} = (1-\delta_h) h^t_{t+j-1} + Q(h^t_{t+j-1}, l^t_{h,t+j-1}).$$

The creation of new human capital depends on its existing level and investment hours and is determined by the function $Q(h, l_h)$. The Q function is increasing in both arguments and has decreasing returns to scale. The agent's savings earn capital income at the real rate of return r_{t+j}. Agents are restricted to have strictly positive amount of savings at all ages

$$(4) \qquad s^t_{t+j} \geq 0.$$

Agents pay taxes on consumption at rate τ_c, labor income at rate τ_l, and capital income net of depreciation at rate τ_k.

The government transfers to the agent born at t and of age j are denoted by d^t_{t+j}. These transfers consist of two components: a lump-sum transfer for agents of all ages, f^t_{t+j}, and social security benefits to retirees, b^t_{t+j},

$$d^t_{t+j} = \begin{cases} f^t_{t+j}, & j = 0, \overline{J}-1, \\ f^t_{t+j} + b^t_{t+j}, & j = \overline{J}, J-1. \end{cases}$$

Agents are entitled to retirement benefits starting with age $\overline{J}$. The amount of social security benefits is the fraction of the average labor income during working periods. This fraction is called a replacement rate, ϕ. social security benefits of the individual born at period t are calculated as

$$b_{t+j}^{t} = \phi \frac{\sum_{i=0}^{\overline{J}-1} w_{t+i} h_{t+i}^{t} l_{m,t+i}^{t}}{\overline{J}}, j = \overline{J}, J-1.$$

The government's budget is balanced every period. The government levies taxes on consumption, labor income, and capital income and uses tax revenue to purchase a wasteful public good, G, and provide two types of transfers. Then, the government's budget constraint at period t is

$$G_t + \sum_{j=0}^{J-1} f_t^{t-j} + \sum_{j=\overline{J}}^{J-1} b_t^{t-j} = \sum_{j=0}^{J-1} \left(\tau_c c_t^{t-j} + \tau_l w_t h_t^{t-j} l_{m,t}^{t-j} + \tau_k r_t s_t^{t-j} \right).$$

At period t, firms hire capital, K_t, and labor, L_t, to produce output with a constant returns-to-scale production technology,

$$Y_t = A K_t^{\theta} L_t^{1-\theta},$$

where A is total factor productivity. The aggregate inputs are determined as

$$\begin{aligned} K_t &= \sum_{j=0}^{J-1} s_t^{t-j}, \\ L_t &= \sum_{j=0}^{J-1} h_t^{t-j} l_{m,t}^{t-j}. \end{aligned}$$

The aggregate feasibility constraint at period t is

$$\sum_{j=0}^{J-1} c_t^{t-j} + K_{t+1} + G_t = A K_t^{\theta} L_t^{1-\theta} + (1-\delta_k) K_t. \tag{5}$$

DEFINITION 1. *A competitive equilibrium is factor prices, (w_t, r_t); aggregate allocations, (K_t, L_t); individual allocations, $\left(\left\{ c_{t+j}^{t}, s_{t+1+j}^{t}, l_{m,t+j}^{t}, l_{h,t+j}^{t}, h_{t+j}^{t} \right\}_{j \in \Im} \right)$ for any generation born at $t \in [1, \infty)$; and government policies, $(\tau_c, \tau_l, \tau_k, \phi, G_t, F_t)$, for any period $t \in [1, \infty)$ such that the following holds: (1) given factor prices and government policies, individual allocations, $\left(\left\{ c_{t+j}^{t}, s_{t+1+j}^{t}, l_{m,t+j}^{t}, l_{h,t+j}^{t}, h_{t+j}^{t} \right\}_{j \in \Im} \right)$, maximize (1) subject to (2)-(4) for each genera-*

tion t; (2) factor inputs are paid the marginal products for any period t :

$$\begin{aligned} w_t &= (1-\theta)\, A K_t^{\theta} L_t^{-\theta}, \\ r_t &= \theta A K_t^{\theta-1} L_t^{1-\theta} - \delta_k; \end{aligned}$$

(3) government's budget is balanced every period; and (4) aggregate and individual allocations satisfy market clearing conditions for any period t.

2.2. Model with Exogenous Human Capital

This model is motivated by the macroeconomics literature that studies OG models with life-cycle labor productivity being exogenously given. To make comparisons to this literature, I modify the model from the previous subsection in the following way. The life-cycle profiles of human capital and investment hours are exogenously fixed at the level of the solution for the model with endogenous human capital under the baseline calibration.

The demographic structure, production technology, the set of government policies, and market clearing conditions of this economy are the same as the one in the model with endogenous human capital decisions. The difference between the two models is in the agents' decisions. Let $\left(\left\{\bar{l}_{h,j}, \overline{h}_j\right\}_{j\in\Im}\right)$ be a fixed life-cycle profiles of investment hours and human capital stock. Introducing a fixed life-cycle profile of investment hours is equivalent to changing the time endowment over the life-cycle. Consequently, the time endowment for each agent is $\left(\left\{1-\bar{l}_{h,j}\right\}_{j\in\Im}\right)$. The preferences of a young agent born at period t are ordered by

$$\sum_{j=0}^{J-1} \beta^j u(c_{t+j}^t, 1 - l_{m,t+j}^t - \bar{l}_{h,j}). \tag{6}$$

Each agent chooses a sequence of consumption and market hours to maximize a discounted value of life-time utility subject to the budget constraint,

$$(1+\tau_c)\, c_{t+j}^t + s_{t+1+j}^t \leq (1-\tau_l)\, w_{t+j} \overline{h}_j l_{m,t+j}^t + (1 + (1-\tau_k)\, r_{t+j}) s_{t+j}^t + d_{t+j}^t. \tag{7}$$

This constraint must be balanced every period of an agent's life, i.e., for any $j \in \Im$. An agent's labor productivity over the life-cycle is predetermined by the profile of human capital. This human capital profile is frequently called the efficiency units profile[2]. The agents are restricted

[2]Examples are Rios-Rull (1996) and DeNardi et al. (1999).

to have positive physical capital asset holdings during all ages,

$$(8) \qquad s_{t+j}^{t} \geq 0.$$

Taxes levied on the agents' income and expenditures and transfer system are the same as in the model with endogenous human capital accumulation. The government's budget constraint and market clearing conditions are as in the model with endogenous human capital accumulation.

Definition 2. *A competitive equilibrium is factor prices,* (w_t, r_t)*; aggregate allocations,* (K_t, L_t)*; individual allocations,* $\left(\left\{c_{t+j}^{t}, s_{t+1+j}^{t}, l_{m,t+j}^{t}\right\}_{j\in\Im}\right)$ *for any generation born at* $t \in [1,\infty)$*; profiles of investment hours and human capital stock* $\left(\left\{\bar{l}_{h,j}, \overline{h}_j\right\}_{j\in\Im}\right)$*; and government policies,* $(\tau_c, \tau_l, \tau_k, \phi, G_t, F_t)$*, for any period* $t \in [1,\infty)$ *such that the following holds: (1) given factor prices, government policies, and profiles* $\left(\left\{\bar{l}_{h,j}, \overline{h}_j\right\}_{j\in\Im}\right)$*, individual allocations,* $\left(\left\{c_{t+j}^{t}, s_{t+1+j}^{t}, l_{m,t+j}^{t}\right\}_{j\in\Im}\right)$*, maximize (6) subject to (7) and (8) for each generation* t*; (2) factor inputs are paid the marginal products for any period* $t:$

$$\begin{aligned} w_t &= (1-\theta)\, A K_t^{\theta} L_t^{-\theta}, \\ r_t &= \theta A K_t^{\theta-1} L_t^{1-\theta} - \delta_k; \end{aligned}$$

(3) government's budget is balanced every period; and (4) aggregate and individual allocations satisfy market clearing conditions for any period t*.*

2.3. Calibration of the Baseline OG Models

I calibrate the baseline economies to the U.S. tax and social security system. The calibration year is 2000. Parameters of demographics, preferences, and technology are the same between the economies with exogenous and endogenous human capital. The parameters of human capital production technology in the economy with endogenous human capital are calibrated to the life-cycle earnings profile. Appendix C provides details on data sources and calculation procedures for all parameters. The parameter values are summarized in Table 1.

A. Parameters for both economies

The demographic structure of the economy is calibrated as follows. Agents enter the economy at age 20, retire at age 65, and die at age 80. Each model period corresponds to 5 years. Hence, the agents are working during the first nine model periods and are retired during the last three model periods. In this section, I report all parameters in annual terms and adjust these parameters accordingly in computations.

The time preference parameter β is calibrated to match the after-tax interest rate of 4.0 percent per year. I assume that the agents' flow utility functions are

$$u(c, 1 - l_m - l_h) = \log c + \alpha \log(1 - l_m - l_h),$$

where α is chosen to match average weekly hours of the population of ages between 20 and 64. Based on Census data, average hours for working age population is 29 hours per week.

The calibration of production technology is standard. Capital income share, θ, is set to 0.333. Depreciation of physical capital, δ_k, is calibrated to match the investment share in GDP. This investment share is equal to 16.9% of GDP in 2000. The resulting depreciation rate is 7.5% and is comparable to the estimates in the literature.

Average effective tax rates are calibrated using the methodology of Mendoza et al. (1994) and are reported in Table 1. The share of government expenditures in output, g, is set to match the corresponding value in NIPA. In 2000, the government consumption expenditures are 14.44 percent of GDP. The replacement rate for social security benefits, ϕ, is calibrated to match the benefit payments from the Old-Age and Survivors Insurance (OASI) Fund. In the calibration year, OASI benefit payments are equal to 4.23 percent of GDP and the resulting replacement rate is $\phi = .195$.

B. Parameters of human capital production technology

I assume the following law of motion for human capital:

$$h_{j+1} = (1 - \delta_h)h_j + B h_j^{\psi_1} l_{h,j}^{\psi_2},$$

where the conditions $B, \psi_1, \psi_2 \geq 0$ and $\psi_1 + \psi_2 \leq 1$ guarantee the decreasing returns to scale. Hence, the life-cycle profile of time investment into human capital is time-independent.

I have to choose five parameters for the human capital production technology: initial stock of human capital, h_0; the depreciation rate of human capital, δ_h; productivity of human capital accumulation, B; weight of human capital stock in new accumulation, ψ_1; and weight of time investment, ψ_2. I calibrate these parameters to match the life-cycle earnings profile, which is constructed using 2000 decennial Census data. I divide the population of ages between 20 and 64 into nine age groups, $j \in \{0, ..., 8\}$. The size of the working age population is denoted by N_t. The measure of earnings is the hourly wage rate, denoted by e_j, $j \in \{0, ..., 8\}$. The average wage rate for the working population, $\overline{e}$, is \$17.24 per hour. This average rate for the working population is calculated using the size of each age group, $n_j(t)$:

$$\overline{e} = \frac{\sum_{j=0}^{8} n_j(t) e_j}{N_t}.$$

To express the wage earnings profile in units comparable to the model, I report the average hourly wage for an age group j as the ratio to the average hourly wage of the working population: $\varepsilon_j = e_j/\overline{e}$, $j = 0, ..., 8$.

Equivalently, the wage rate in the model is $w_t h_j$ and the average wage for the working population is

$$\overline{wh} = \frac{\sum_{j=0}^{8} w_t h_j}{\overline{J}}.$$

I choose parameters of the human capital production function to minimize the distance between the model and data wage hour profiles:

$$\min_{(h_0, \delta_h, B, \psi_1, \psi_2)} \sum_{j=0}^{8} \left(\frac{wh_j}{\overline{wh}} - \frac{e_j}{\overline{e}} \right)^2 .$$

The chosen parameters are reported in Table 1.

2.4. Stationary Equilibrium in both Baseline Environments

Given calibrated parameters, I solve for a stationary equilibrium in the economy with endogenous human capital. The procedure for the numerical algorithm is described in Appendix 5.2. Equilibrium life-cycle profiles of investment hours and human capital are given in Figure 1. Time investment into human capital accumulation is the highest at the beginning of life and exhibits steady decline. I refer to this time investment as investment hours. Hours devoted to market production activities are called market hours. The sum of these two types of time usage are called total production hours. The life-cycle profile of investment hours and technology for human capital production determine the life-cycle labor productivity. Under the baseline calibration, an individual reaches a peak in labor productivity between ages 45 and 49.

Let $\left(\left\{l^*_{h,j}, h^*_j\right\}_{j\in\Im}\right)$ denote equilibrium life-cycle profiles of investment hours and human capital in the environment with endogenous human capital.

PROPOSITION 1. *If* $\left\{\bar{l}_{h,j}\right\}_{j\in\Im} = \left\{l^*_{h,j}\right\}_{j\in\Im}$ *and* $\left\{\overline{h}_j\right\}_{j\in\Im} = \left\{h^*_j\right\}_{j\in\Im}$, *a stationary equilibrium in the environment with exogenous human capital is identical to the one in the environment with endogenous human capital under baseline calibrated parameters.*

Proof. The method of the proof is to compare equilibrium conditions in the two environments. These conditions are derived in the Appendix 5.1. QED

Under the baseline calibration, the equilibria in the two environments are the same both on the aggregate and individual level. The values for various aggregate variables in the baseline environments are given in the second column of Table 3. With a calibrated after-tax interest rate of 4%, the resulting capital-to-output ratio is 2.44. In both environments, agents of working age devote on average 29 hours per week for market production activities.

3. Alternative Retirement Arrangements

I consider two alternative arrangements. First, I analyze an elimination of social security benefits with a corresponding reduction in labor income tax used to finance these benefits. This type of reform is analyzed by Kotlikoff et al. (1999) and DeNardi et al. (1999) in an OG model with life-cycle labor productivity exogenously specified. Due to the precautionary life-cycle saving motive, the agents choose to accumulate assets to finance their retirement. I call this arrangement as Voluntary Retirement Savings Accounts. For assets on this accounts, the return net of depreciation is subject to capital income taxation. Second, I consider a retirement arrangement without social security benefits and with Mandatory Retirement Savings Accounts. Many countries have introduced this retirement system, examples of which are Australia, Chile, and Mexico. Under this system, agents are required to contribute a fixed portion of their income to Mandatory RSA. Contributions to Mandatory

RSA are tax-deferred. Within the environment with exogenous human capital, I compare a stationary equilibrium under the alternative retirement arrangements to the baseline one. The same comparison is conducted within the environment with endogenous human capital.

3.1. Arrangements with Voluntary Retirement Savings Accounts

Under this retirement arrangement, social security benefits are eliminated and the labor income tax is reduced to keep lump-sum transfers the same as in the baseline environment with endogenous human capital. In both environments, the labor income tax is reduced from 27% to 21.09%, whereas tax rates on consumption and capital income and government expenditures as a share of output are kept at the level of the baseline environments. The numerical algorithm of solving for a stationary equilibrium under this alternative is the same as in the baseline.

The welfare gains of replacing PAYG social security system with Voluntary Retirement Savings Accounts are reported in Table 2. I measure the welfare gains by life-time consumption equivalents. This measure determines the percentage increase in the agent's life-time consumption in the baseline environment needed to make her or him indifferent to the alternative retirement arrangements. Within the environment with exogenous human capital, an agent's life-time consumption in the baseline economy must be increased by 7.81% to make her or him indifferent to the policy considered. This measure takes into account the difference in labor supply between two economies. The welfare gains come from the reduction in the distortionary labor income tax.

Environment with exogenous human capital All comparisons in this section are between the economy with Voluntary RSA and the baseline one. To understand the welfare gains, we need to study the reasons behind the increase in the agents' consumption. An individual has three sources of income: labor and capital income and lump-sum transfers from the government. The lump-sum transfers are held the same between the two economies. Both labor and capital incomes are higher in the absence of the PAYG social security system. To analyze this income increase, let us consider the change in wage and interest rates. The factor prices are determined by the stocks of physical capital and labor supply. Because the agents save for their retirement on their own, the amount of savings under the Voluntary

RSA is 16.31% higher. Labor supply is determined by the product of human capital stock and market hours. In the environment with exogenous human capital, the human capital stock is kept fixed and the agents devote more hours to market production, on average 1.62 hours per week more. The capital-to-labor ratio is higher. Consequently, the wage rate is higher by 3.69%, and the interest rate is slightly lower by 0.49 percentage points when the individuals provide for their retirement without public assistance. Even with a slightly lower interest rate, the capital income is higher due to the higher stock of savings, as reported in Figure 5.

The retirement arrangements affect the agents' time allocation throughout the life-cycle. Figure 3 shows time allocation among market production activities, time investment into human capital accumulation, and total production hours. The time endowment is normalized to 100 hours per week. The leisure consumed by the agents is the difference between the time endowment and total production hours. The difference in time allocation between the economy with Voluntary RSA and the baseline one is presented in Figure 4. In the environment with exogenous human capital, the agents are forced to devote part of their time to human capital accumulation. This exogenous profile of investment hours is the same between two economies as shown in the bottom panels of Figures 3 and 4.

The time allocation between two economies differs for three age groups: 20 to 34, 35 to 44, and 45 to 64 year olds. In the first age group, the agents work approximately the same amount in the two economies, because their labor productivity is not very high. Due to the higher after-tax wage rate, the agents still enjoy higher consumption under the Voluntary RSA. The agents of ages 35 to 44 work slightly less under the Voluntary RSA for three reasons. First, the after-tax wage income is higher. Second, the labor productivity is close to the peak one, as seen in Figure 5. Third, the consumption level is close to the desired life-time path. In the third age group, the agents work on average 4 hours per week more. These agents take advantage of high labor productivity and heavily save for retirement. As seen in Figure 5, the agents save more in the absence of the publicly provided social security benefits.

Environment with endogenous human capital Welfare gains from eliminating the social security system are lower in the environment with endogenous human capital. In this environment, agents have an additional margin of adjustment, human capital investment, as compared to the one with exogenous human capital. Therefore, the reduction in distortionary labor income tax leads to lower welfare gains.

An agent can take advantage of the higher after-tax wage rate in two ways: (i) supply more hours for market production and/or (ii) invest more into human capital. Due to the human capital technology, the young agents invest more into human capital and work less for the market production. Even though the young agents are borrowing constrained, the discounted life-time benefit from the human capital investment outweighs the forgone wage income. The comparison of the human capital profiles is in Figure 5. As can be seen in Figure 4, the agents of ages 50 to 64 work for the market production on average 6 hours more. There are two reasons for this. First, they want to take advantage of the high labor productivity. The productivity for the age group 60 to 64 is higher than that for the one 30 to 34. Second, the agents save to finance an increasing stream of consumption during retirement.

Table 3 reports changes in aggregate variables between the baseline economies and the ones under the Voluntary RSA. The interest rate in the environment with this alternative is lower than the one in the baseline environment. Kotlikoff et al. (1999) and DeNardi et al. (1999) report that the output is higher in the steady state under the new system by 12% and 8.7%, respectively, as compared to the initial steady state. I find that in the environment with exogenous and endogenous human capital the output is 8.16% and 11.81%, respectively, higher in the economies with the alternative retirement system as compared to the baseline. The total amount of savings under the new arrangements are higher by 16.31% and 16.91% in the environment with exogenous and endogenous human capital, respectively.

3.2. Arrangements with Mandatory Retirement Savings Accounts

Under this retirement arrangement, the agents are required to contribute a fixed fraction of labor income to Mandatory RSA. These contributions are tax-deferred, accumulation of assets on the Mandatory RSA is tax-exempt, and withdrawals are subject to labor income taxes.

A. Economy Description

I initially describe the economy with endogenous human capital and modifications for the economy with exogenous human capital are given at the end of section . The demographic structure is the same as in the baseline environment.

An agent born at period t and of age j has two types of savings accounts: a voluntary one, $s_{1,t+j}^t$, and a mandatory one, $s_{2,t+j}^t$. An agent starts her/his life with zero assets, $s_{1,t}^t = 0$ and $s_{2,t}^t = 0$. Under this retirement arrangement, the agents are required to contribute a fraction ξ of wage income towards a tax-deferred retirement account.

An agent born at period t chooses a sequence of allocations, $\left\{c_{t+j}^t, s_{1,t+1+j}^t, l_{m,t+j}^t, l_{h,t+j}^t, h_{t+j}^t\right\}_{j\in\Im}$, to maximize the discounted value of life-time utility (1) subject to the following budget constraint:

$$(9) \qquad (1+\tau_c)\, c_{t+j}^t + s_{1,t+1+j}^t \leq (1-\tau_l)\,(1-\xi)\, w_{t+j} h_{t+j}^t l_{m,t+j}^t + (1+(1-\tau_k)\, r_{t+j}) s_{1,t+j}^t + d_{t+j}^t,$$

borrowing constraints:

$$\begin{aligned} s_{1,t+j}^t &\geq 0, \\ s_{2,t+j}^t &\geq 0, \end{aligned}$$

and the law of motion for human capital technology (3).

The accumulation of assets in a mandatory savings account is

$$s_{2,t+1+j}^t = \begin{cases} \xi w_{t+j} h_{t+j}^t l_{m,t+j}^t + (1+r_{t+j})\, s_{2,t+j}^t, & j = 0, \overline{J}-1, \\ (1+r_{t+j})\, s_{2,t+j}^t - b^t, & j = \overline{J}, J-1. \end{cases}$$

The transfers to a household consist of two components: a lump-sum transfer for agents of all ages, f_{t+j}^t, and annuity payments from the mandatory savings account, b^t,

$$d_{t+j}^t = \begin{cases} f_{t+j}^t, & j = 0, \overline{J}-1, \\ f_{t+j}^t + (1-\tau_l)\, b^t, & j = \overline{J}, J-1. \end{cases}$$

During retirement, an agent born at period t receives a constant annuity payments from the Mandatory RSA, b^t. These annuity payments are calculated such that the mandatory savings account is exhausted by the end of agent's life, i.e., $s^t_{2,t+J} = 0$. The annuity payment depends on the accumulation of assets at the beginning of retirement, $s^t_{2,t+\overline{J}}$:

$$b^t = [1 + (1 + r_{t+J-1}) + (1 + r_{t+J-1})(1 + r_{t+J-2})]^{-1} \prod_{z=\overline{J}}^{J-1} (1 + r_{t+z}) s^t_{2,t+\overline{J}}.$$

Asset accumulation on the mandatory savings account at the beginning of retirement is[3]

$$s^t_{2,t+\overline{J}} = \sum_{j=0}^{\overline{J}-1} \xi w_{t+\overline{J}-1-j} h^t_{t+\overline{J}-1-j} l^t_{m,t+\overline{J}-1-j} \prod_{z=1}^{j} \left(1 + r_{t+\overline{J}-z}\right).$$

The government's budget is balanced every period. At period t, the government makes the same lump-sum transfers to all living generations, f_t.

$$G_t + f_t J = \sum_{j=0}^{J-1} \left[\tau_c c^{t-j}_t + \tau_l (1-\xi) w_t h^{t-j}_t l^{t-j}_{m,t} + \tau_k r_t s^{t-j}_{1,t}\right] + \sum_{j=\overline{J}}^{J-1} \tau_l b^{t-j}.$$

Production technology is as in the baseline environment with the capital stock determined by the accumulation of assets on two savings accounts:

$$K_t = \sum_{j=0}^{J-1} s^{t-j}_{1,t} + s^{t-j}_{2,t}.$$

And market clearing conditions are given by (5).

The model with exogenous human capital has the initial human capital stock and investment hours profile as in the baseline economy. The household's problem and retirement arrangements are as in the endogenous environment described above.

B. Calibration and Results

The labor income tax is calibrated to keep lump-sum transfers as the share of output the same between the baseline and Mandatory RSA arrangements in the environment with endogenous human capital. The labor income tax is reduced from 27% in the baseline to 26.05%. The rate of contributions to Mandatory RSA, ξ, is set at 9%.

Welfare gains from introducing these retirement arrangements are reported in Table 4. Table 5 compares aggregate variables between the economies with alternative and baseline retirement arrangements. Figures 6 through 10 compare equilibrium individual allocations at

[3] Here, I define the following product operation: $\prod_{z=1}^{0} \left(1 + r_{t+\overline{J}-z}\right) = 1.$

two stationary equilibria: (i) Mandatory RSA and (ii) baseline. First, I discuss a stationary equilibrium within the environment with exogenous human capital. Then, I explain the role of the endogenous human capital in the second environment considered.

Environment with exogenous human capital The retirement system with Mandatory RSA results in the capital stock being higher by 36.89% as compared to the baseline economy. This retirement arrangement gives the highest welfare gains because the agents enjoy higher consumption and leisure. Since the wage rate is higher by 13.18% as compared to the baseline, an individual can afford to work less and still enjoy the higher consumption.

Environment with endogenous human capital Under this retirement arrangement, the labor income tax is only slightly lower in comparison to the baseline economy. The agents choose to take advantage of the higher wage rate by accumulating more human capital, since the time investment into human capital accumulation at the beginning of life leads to the highest return in accordance with the human capital technology. Due to this technology restriction, agents choose to invest substantially more time during ages 20 to 34 into schooling as compared to the baseline. This increase in investment hours is accompanied by a decrease in market hours. During this period their borrowing constraint binds and is tighter under Mandatory RSA.

Table 6 reports the welfare ranking for two alternative retirement arrangements and two environments that differ by human capital endogeneity. From this table, I conclude that the endogeneity of human capital is important in evaluating tax and social security policies.

4. Conclusion

This paper considers three different retirement systems and analyzes the role of human capital endogeneity in evaluating these policies. I show that incorporating human capital investment into an overlapping generations model is important. Welfare ranking of two alternative retirement arrangements depends on the environment considered. Within the environment with exogenous human capital, a household enjoys the highest welfare when he or she is required to save a fixed fraction of labor income in the tax-deferred Mandatory RSA. These arrangements result in the highest level of savings and the highest wage rate. Hence, the households get the highest level of consumption while enjoying more leisure.

The ranking is different in the environment with endogenous human capital. The retirement arrangements with Voluntary RSA lead to the highest household's welfare. In comparison with the exogenous environment, an agent has an additional margin of adjustment, human capital investment. He or she prefers to accumulate more human capital under the alternative arrangements in order to take advantage of higher wage rates. Due to the human capital technology, time investment early in life earns the highest return. Because young agents are borrowing constrained and shift hours from market production into human capital accumulation, the agents prefer to save during middle ages. The arrangements with Voluntary RSA allow an agent to optimally and simultaneously choose the stocks of physical and human capitals. Hence, these arrangements result in the highest welfare. Under Mandatory RSA, young agents have tighter borrowing constraints when they are forced to save.

I plan to extend my analysis in a number of ways. The level of intertemporal labor supply elasticity is a controversial issue among labor and macro economists. I plan to explore sensitivity of my analysis with different values of labor supply elasticity.

5. Appendix

This appendix consists of three parts. In the first part, I describe a solution to a stationary equilibrium in the environments considered under different retirement arrangements. The second part provides a numerical algorithm to solve for a stationary equilibrium. Data sources and calibration procedures are discussed in the third part.

5.1. Solution for a Stationary Equilibrium

To compare different retirement arrangements, I solve for a stationary equilibrium. For each retirement system, I solve for a set of equations that must be satisfied in a stationary equilibrium. Within each retirement system, I provide details for the economies with endogenous and exogenous human capital accumulation.

A. Retirement arrangements with PAYG system

Environment with endogenous human capital An equilibrium in the environment with endogenous human capital must satisfy Definition 1. To derive equilibrium conditions, I start with a problem of a young agent born at period t. Given initial stocks of physical and human capital, (s_t^t, h_t^t), factor prices, and government policy, an individual chooses $\left(\left\{c_{t+j}^t, s_{t+1+j}^t, l_{m,t+j}^t, l_{h,t+j}^t, h_{t+j}^t\right\}_{j\in\Im}\right)$ to maximize (1) subject to (2)-(4).

To simplify notation, I solve the problem of an agent born at period t and omit the time subscript. In this appendix, a subscript denotes the period of an agent's life. Let λ_j, $j = 0, ..., J-1$, be multipliers on household's budget constraints (2) and μ_j, $j = 0, ..., J-1$, be multipliers on the law of motion of human capital (3). Assume that all non-negativity constraints are satisfied. The Lagrangian function for this problem is

$$\begin{aligned}
& \pounds\left(\left\{c_j, s_{j+1}, l_{mj}, l_{hj}, h_{j+1}, \lambda_j, \mu_j\right\}_{j=0}^{J-1}\right) \\
= \quad & \sum_{j=0}^{J-1} \beta^j u(c_j, 1 - l_{mj} - l_{hj}) \\
& - \sum_{j=0}^{J-1} \lambda_j \left[(1+\tau_c)\, c_j + s_{j+1} - (1-\tau_l)\, w h_j l_{mj} - (1 + (1-\tau_k)\, r) s_j - d_j\right] \\
& - \sum_{j=0}^{J-1} \mu_j \left[h_{j+1} - (1-\delta_h) h_j - G(h_j, l_{hj})\right].
\end{aligned}$$

The following notation for partial derivatives is used:

$$u_{1j} = \frac{\partial u(c_j, 1-l_{mj}-l_{hj})}{\partial c_j}, \quad u_{2j} = \frac{\partial u(c_j, 1-l_{mj}-l_{hj})}{\partial \left(1-l_{mj}-l_{hj}\right)},$$
$$G_{1j} = \frac{\partial G(h_j, l_{hj})}{\partial h_j}, \qquad G_{2j} = \frac{\partial G(h_j, l_{hj})}{\partial l_{hj}}.$$

First-order conditions with respect to an agent's choice variables are

$$\beta^j u_{1j} = \lambda_j, \tag{10}$$
$$\lambda_j = \lambda_{j+1}(1 + (1 - \tau_k)\, r), \tag{11}$$
$$\beta^j u_{2j} = \lambda_j \,(1 - \tau_l)\, w h_j, \tag{12}$$
$$\beta^j u_{2j} = \mu_j G_{2j}, \tag{13}$$
$$\lambda_{j+1}\,(1 - \tau_l)\, w l_{mj+1} = \mu_j - \mu_{j+1}(1 - \delta_h + G_{1j+1}), \tag{14}$$
$$(1 + \tau_c)\, c_j + s_{j+1} = (1 - \tau_l)\, w h_j l_{mj} + (1 + (1 - \tau_k)\, r) s_j + d_j, \tag{15}$$
$$h_{j+1} = (1 - \delta_h) h_j + G(h_j, l_{hj}). \tag{16}$$

I use (10) and (11) to get an intertemporal condition:

$$\frac{u_{1j}}{\beta u_{1j+1}} = (1 + (1 - \tau_k)\, r). \tag{17}$$

Using (10) and (12), I get a condition for an intratemporal trade-off between consumption and leisure:

$$\frac{u_{2j}}{u_{1j}} = (1 - \tau)\, w_j h_j, \tag{18}$$

where $\tau = (\tau_l + \tau_c) \,/\, (1 + \tau_c)$ is a labor tax wedge.

The condition relating investment hours to other variables is derived as follows. Conditions (12) and (13) imply

$$\mu_j = \frac{\lambda_j \,(1 - \tau_l)\, w h_j}{G_{2j}}. \tag{19}$$

Combining (19), (14), and (11), I get an expression that implicitly determines the choice of investment hours:

$$l_{mj+1} = (1 + (1 - \tau_k)\, r) \frac{h_j}{G_{2j}} - \frac{h_{j+1}}{G_{2j+1}} (1 - \delta_h + G_{1j+1}). \tag{20}$$

Given the initial stocks of physical and human capital, s_0 and h_0, factor prices, r and w, and government sector variables, $(\tau_c, \tau_l, \tau_k, \phi, g, f)$, the household's choice variables can be found by solving a system of equations consisting of (17), (18), (20), (15), and (16) for the agent's life periods $j = 0, ..., J-1$. All retirees receive the same benefits, $b = \left(\phi/\overline{J}\right) \sum_{j=0}^{\overline{J}-1} wh_j l_{m,j}$, and agents of all ages get the lump-sum transfers, f.

Because I consider a stationary equilibrium, the market clearing conditions and government's budget constraint become:

$$K = \sum_{j=0}^{J-1} s_j,$$

$$L = \sum_{j=0}^{J-1} h_j l_{m,j}, \tag{21}$$

$$\sum_{j=0}^{J-1} c_j + \delta_k K = (1-g)\, AK^{\theta} L^{1-\theta}, \tag{22}$$

$$gAK^{\theta}L^{1-\theta} + Jf + \left(J - \overline{J}\right) b = \sum_{j=0}^{J-1} \left(\tau_c c_j + \tau_l w h_j l_{m,j} + \tau_k r s_j\right).$$

Factor inputs are paid their marginal products:

$$w = (1-\theta)\, AK^{\theta} L^{-\theta}, \tag{23}$$

$$r = \theta AK^{\theta-1} L^{1-\theta} - \delta_k. \tag{24}$$

Environment with exogenous human capital Within this environment, profiles for investment hours and human capital stock, $\left(\left\{\bar{l}_{h,j}, \bar{h}_j\right\}_{j\in\Im}\right)$, are exogenous for an agent. A household also takes as given an initial endowment of physical capital, government policy variables, and factor prices. In the stationary equilibrium, the household's choice variables must satisfy (17), (18), and (15). The equations describing the government and production sectors and the market clearing conditions are the same as in the environment with endogenous human capital.

B. Retirement arrangements with Voluntary RSA

The PAYG social security system is eliminated by setting the replacement rate for the social security benefits, ϕ, equal to zero. The tax on labor income is recalibrated to keep the lump-sum transfers to households as the share of output the same as compared to the baseline retirement arrangements within the environment with endogenous human capital. Given these two changes in the policy parameters, the set of equations describing a stationary equilibrium is identical to the one under the baseline retirement arrangements.

C. Retirement arrangements with Mandatory RSA

Environment with endogenous human capital Because a household is forced to contribute a fraction of labor income toward the Mandatory RSA, the following variables in the household's problem change. Accumulation of physical capital takes place on two accounts: (i) one with voluntary contributions, $s_{1,j}$, and (ii) one with tax-deferred mandatory accounts, $s_{2,j}$. The endowment of assets on both savings accounts at the beginning of agents' lives is zero. The fraction of mandatory savings contributions is denoted by ξ. A household's budget constraint is

$$(1+\tau_c)\, c_j + s_{1,j+1} \leq (1-\tau_l)\,(1-\xi)\, w h_j l_{m,j} + (1+(1-\tau_k)\, r) s_{1,j} + d_j. \tag{25}$$

The amount of assets accumulated by households of different ages is

$$s_{2,j+1} = \begin{cases} \sum_{z=0}^{j} \xi w_{t+j-z} h_{j-z} l_{m,j-z} (1+r)^z, & j = 0, \overline{J}-1, \\ (1+r)\, s_{2,j} - b, & j = \overline{J}, J-1. \end{cases}$$

At the beginning of retirement, an agent has the following amount of assets on her or his tax-deferred Mandatory RSA:

$$s_{2,\overline{J}} = \sum_{j=0}^{\overline{J}-1} \xi w h_{\overline{J}-1-j} l_{m,\overline{J}-1-j} (1+r)^j .$$

Annuity payments from this savings account are the same for all retirees:

$$b = \left[\sum_{j=0}^{J-\overline{J}-1} (1+r)^j \right]^{-1} (1+r)^{J-\overline{J}} s_{2,\overline{J}},$$

and transfers to households become

$$d_j = \begin{cases} f, & j = 0, \overline{J}-1, \\ f + (1-\tau_l)\, b, & j = \overline{J}, J-1. \end{cases}$$

The labor wedge affecting an intratemporal condition is $\tau = (\tau_l + \tau_c + \xi - \tau_l \xi) \,/\, (1+\tau_c)$. Given initial stocks of physical and human capital, $s_{1,0}$, $s_{2,0}$, and h_0, factor prices, r and w, and government sector variables, $(\tau_c, \tau_l, \tau_k, \xi, \phi, g, f)$, the household's choice variables can be found by solving a system of equations consisting of (17), (18), (20), (25), and (16) for the agent's life periods $j = 0, ..., J-1$.

The market clearing condition for physical capital and the government's budget constraint are

$$\begin{aligned} K &= \sum_{j=0}^{J-1} s_{1,j} + s_{2,j}, \\ G + fJ &= \sum_{j=0}^{J-1} \left[\tau_c c_j + \tau_l \left(1-\xi\right) w h_j l_{m,j} + \tau_k r s_{1,j}\right] + \sum_{j=\overline{J}}^{J-1} \tau_l b. \end{aligned}$$

Market clearing conditions for the labor and the good markets are (21) and (22). The factor prices are determined by (23) and (24).

Environment with exogenous human capital Within this environment, profiles for investment hours and human capital stock, $\left(\left\{\bar{l}_{h,j}, \overline{h}_j\right\}_{j\in\Im}\right)$, are exogenous for an agent. These profiles are kept the same under three retirement arrangements analyzed. The accumulation of assets on and the annuity payments from the Mandatory RSA are as in the environment with endogenous human capital. A household also takes as given an initial endowment of physical capital, government policy variables, and factor prices. In the stationary equilibrium, the household's choice variables must satisfy (17), (18), and (25). The equations describing the government and production sectors and the market clearing conditions are the same as in the environment with endogenous human capital.

5.2. Numerical algorithm

Environment with endogenous human capital I describe a numerical algorithm for finding a stationary equilibrium in the environment with endogenous human capital accumulation under retirement arrangements with PAYG system. Modifications of this algorithm for the economy with exogenous human capital and alternative retirement arrangements are discussed at the end of this subsection.

Since I solve for a stationary equilibrium, I omit the time subscript and consider cohorts alive at a period t. I also incorporate functional choices for the utility and human capital production functions. The values of parameters are given in Table 1. The life-span J is set at 12 periods, and the agents are working for the first $\overline{J} = 9$ periods. I solve for $3J - 1$ equilibrium variables using $3J - 1$ equations.

A. Initial guess for iteration procedure

I provide an initial guess for the following equilibrium variables:

- savings made by different cohorts at a given period[4], $\{s_{j+1}\}_{j=0}^{J-1}$;
- labor supply of different cohorts, $\{l_{mj}\}_{j=0}^{J-2}$;
- human capital stock of different cohorts[5], $\{h_j\}_{j=1}^{\overline{J}+1}$;
- aggregate capital stock, K;
- lump-sum transfers to cohorts of all ages, f.

B. Additional economy variables

Given the set of variables I iterate on, the rest of the economy variables are calculated as:

- during the last period of life, the agents are forced to retire, $l_{mJ} = 0$;
- human capital stock during the last period of life is $h_{J-1} = (1 - \delta_h)\, h_{J-2}$;
- aggregate labor supply is given by (21);
- factor prices are determined by (23) and (24);
- social security benefits and transfers to households are

$$b = \frac{\phi w}{\overline{J}} \sum_{j=0}^{\overline{J}-1} h_j l_{m,j},$$

$$d_j = \begin{cases} f, & j = 0, \overline{J} - 1 \\ f + b & j = \overline{J}, J - 1 \end{cases};$$

- aggregate capital next period is

$$K^{'} = \sum_{j=0}^{J-1} s_{j+1};$$

[4] The last element of the vector is savings of the oldest cohort at the last period of their lives, i.e., $s_J = 0$. The endowment of physical capital is $s_0 = 0$.

[5] Initial human capital stock is a given parameter, h_0.

- consumption of different cohorts, $\{c_j\}_{j=0}^{J-1}$, is determined by the household's budget constraints (15);
- aggregate consumption is $C = \sum_{j=0}^{J-1} c_j$;
- life-cycle profile for investment hours is determined by the human capital production technology:

$$l_{h,j} = \left[\frac{h_{j+1} - (1-\delta_h)\, h_j}{B\, (h_j)^{\psi_1}} \right]^{\frac{1}{\psi_2}}, j = 0, J-1.$$

C. System of equations to iterate on

The algorithm for imposing non-negativity constraints on savings consists of three steps: (1) solve the unconstrained problem; (2) for a cohort j with negative savings, replace an Euler equation for this cohort with the equation $s_{j+1} = 0$; and then (3) solve the modified system of equations. I solve the system of non-linear equations using a Newton-Raphson method as described in Press et al. (1986).

The system of equations for the unconstrained problem is

- Euler equations,

$$\begin{aligned} c_{j+1} &= \beta(1+(1-\tau_k)\, r)c_j, && j = 0, J-2, \\ s_{j+1} &= 0, && j = J-1; \end{aligned}$$

- given the labor tax wedge $\tau = (\tau_l + \tau_c) / (1+\tau_c)$, intratemporal conditions on labor supply are

$$\begin{aligned} \alpha c_j &= (1-\tau) w h_j \left(1 - l_{m,j} - l_{h,j}\right), && j = 0, \overline{J}-1, \\ l_{m,j} &= 0, && j = \overline{J}, J-2; \end{aligned}$$

- intertemporal condition on labor supply is

$$\begin{aligned} l_{m,j+1} B \psi_2 &= (1+(1-\tau_k)\, r)\, (h_j)^{1-\psi_1} (l_{h,j})^{1-\psi_2} \\ &\quad - (h_{j+1})^{1-\psi_1} (l_{h,j})^{1-\psi_2} \left[1 - \delta_h + B\psi_1 (h_{j+1})^{\psi_1 - 1} (l_{h,j})^{\psi_2}\right], \\ j &= 0, \overline{J}; \end{aligned}$$

- market clearing condition for good market:

$$C + K' = (1-g)\, A K^{\theta} L^{1-\theta} + (1-\delta_k) K;$$

- government's budget constraint:

$$gAK^{\theta}L^{1-\theta}+\left(J-\overline{J}\right)b+Jf=\sum_{j=0}^{J-1}\left[\tau_{c}c_{j}+\tau_{l}wh_{j}l_{m,j}+\tau_{k}\left(r-\delta_{k}\right)s_{j}\right].$$

Environment with exogenous human capital The additional set of fixed parameters is the life-cycle profiles for investment hours and human capital stock, $\left(\left\{\bar{l}_{h,j},\overline{h}_{j}\right\}_{j\in\Im}\right)$. The number of equilibrium variables to solve for is $2J+1$. These variables are $\left(\{s_{j+1}\}_{j=0}^{J-1},\{l_{mj}\}_{j=0}^{J-2},K,f\right)$. The set of equations to iterate on is the same as in the environment with endogenous human capital excluding the intertemporal conditions on labor supply.

When I solve for a stationary equilibrium under alternative retirement arrangements, I modify the procedure using the set of equations characterizing each of these alternatives. The solution for a stationary equilibrium is derived in Section 5.1.

5.3. Data Sources and Calibration Procedure

Two main data sources are the Bureau of Economic Analysis (BEA), which publishes the national accounts tables, and the Integrated Public Use Microdata Series (IPUMS) provided by the University of Minnesota (www.ipums.org).

A. Details on NIPA accounts and government expenditures.

The rearrangement of NIPA accounts is specified in Table 7. Because I consider a closed economy, net exports are included into investment expenditures. The details on government expenditure programs are in Table 8.

B. Construction of wage earnings profile from decennial Census data.

To construct the life-cycle wage earnings profile, I use the Integrated Public Use Microdata Series (IPUMS) provided by the University of Minnesota (www.ipums.org). These series are based on the decennial Census surveys collected by the U.S. Bureau of the Census. I construct the wage earnings profile for 2000 and using a sample size of 1 percent of the population. The following variables are extracted from the IPUMS sample:

- PERWT: person's weight;
- AGE: person's age at last birthday;

- UHRSWORK: the number of hours per week that respondents usually worked if they worked during the previous calendar year;
- WKSWORK1: the number of weeks that the respondent worked during the preceding calendar year, including weeks with paid vacation and sick leave;
- INCWAGE: respondent's total pre-tax wage and salary income for the previous calendar year.
- CLASSWKR: class of worker.

In 2000, the top-code for INCWAGE is $175,000 (in contemporary dollars). Observations with INCWAGE above the top-code are assigned values of wage and salary income equal to the *state means* of values above $175,000. Hence, no adjustment for top-coded INCWAGE observations is needed.

I consider the age group 20 to 64 and restrict the sample to full-time full-year workers. These workers are individuals who worked at least 35 hours per week and 40 weeks a year during the last year. I further exclude workers reporting positive hours and zero wage income and self-employed people. The observations are divided into nine age groups:

Age group	00 =	Population 20 to 64 year old
	0 =	From 20 to 24 years
	1 =	From 25 to 29 years
	2 =	From 30 to 34 years
	3 =	From 35 to 39 years
	4 =	From 40 to 44 years
	5 =	From 45 to 49 years
	6 =	From 50 to 54 years
	7 =	From 55 to 59 years
	8 =	From 60 to 64 years

I denote each age group as A_j , where j is a group index and takes integer values from 0 to 9. For an individual i, hourly wage e_i is calculated as the ratio of the annual wage income and annual hours:

$$e_i = \frac{INCWAGE_i}{UHRSWORK_i \cdot WKSWORK1_i}.$$

Average hourly wage for an age group A_j is calculated using personal weights for observations:

$$e_j = \frac{\sum_{i \in A_j} e_i \cdot PERWT_i}{\sum_{i \in A_j} PERWT_i}.$$

To express the wage earnings profile in units comparable to the model, I report the ratio of the average hourly wage for an age group A_j to the average hourly wage of the working population: $\varepsilon_j = e_j/e_{00}$, $j = 0, ..., 8$.

To calculate average weekly hours for population ages 20 to 64, I use the variable $UHRSWORK$. Let $\overline{l_m}$ denote average weekly hours. These hours are then calculated as

$$\overline{l_m} = \frac{\sum_{i \in A_{00}} UHRSWORK_i \cdot PERWT_i}{\sum_{i \in A_{00}} PERWT_i}.$$

In 2000, a person in the age group 20 to 64 on average was working 29 hours per week.

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Table 1: Model Parameters[a].

PARAMETER	EXPRESSION	VALUE
PREFERENCES AND TECHNOLOGY		
Discount factor	β	0.966
Leisure preference parameter	α	1.88
Capital share	θ	0.333
Depreciation rate of physical capital	δ_k	0.075
GOVERNMENT SECTOR		
Tax rate on consumption	τ_c	0.05
Tax rate on labor income	τ_l	0.27
Tax rate on capital income	τ_k	0.40
Share of government expenditures	g	0.1444
Replacement rate for Social Security benefits	ϕ	0.195
HUMAN CAPITAL TECHNOLOGY		
Initial stock of human capital	h_0	0.25
Depreciation rate of human capital	δ_h	0.062
Productivity of human capital accumulation	B	0.6
Weight of human capital stock	ψ_1	0.43
Weight of time investment	ψ_2	0.43

[a]All parameter values are given in annual terms. Since one model period corresponds to five years, the parameters are adjusted in the computations accordingly.

Table 2: Welfare gains in the environments with Voluntary Retirement Savings Accounts as compared to the baseline, measured by life-time consumption equivalents, in %.

	Welfare Gains
Exogenous Human Capital	7.81
Endogenous Human Capital	6.98

Table 3: Aggregate variables in two environments: Baseline and with Voluntary Retirement Savings Accounts[a].

	Value in	Change in % from baseline	
	Baseline	HC Exogenous	HC Endogenous
After-tax interest rate, %	4.00	-0.49[b]	-0.31[b]
Wage rate	41.65	3.69	2.25
Capital stock	38.80	16.31	16.91
Labor supply	1.27	4.31	9.34
Capital-labor ratio	30.53	11.51	6.92
Output	79.34	8.16	11.81
Capital-output ratio	2.44	7.54	4.56
Average market hours	29.00	4.56	3.88

[a]All variables are in annual terms.
[b]Change of interest rate from the baseline one is in percentage points.

Table 4: Welfare gains in the environments with Mandatory Retirement Savings Accounts as compared to the baseline, measured by life-time consumption equivalents, in %.

	Welfare Gains
Exogenous Human Capital	8.42
Endogenous Human Capital	5.01

Table 5: Aggregate variables in two environments: Baseline and with Mandatory Retirement Savings Accounts[a].

	Value in	Change in % from baseline	
	Baseline	HC Exogenous	HC Endogenous
After-tax interest rate, %	4.00	-1.58[b]	-1.48[b]
Wage rate	41.65	13.18	12.19
Capital stock	38.80	36.89	41.20
Labor supply	1.27	-5.62	-0.03
Capital-labor ratio	30.53	45.04	41.24
Output	79.34	6.82	12.15
Capital-output ratio	2.44	28.15	25.90
Average market hours	29.00	-4.29	-5.91

[a]All variables are in annual terms.

[b]Change of interest rate from the baseline one is in percentage points.

Table 6: Welfare gains in the environments with Voluntary/Mandatory Retirement Savings Accounts as compared to the baseline, measured by life-time consumption equivalents.

	Change in % from baseline	
	HC Exogenous	HC Endogenous
Voluntary RSA	7.81	6.98
Mandatory RSA	8.42	5.01

Table 7: National Accounts, Relative to GDP, 2000 (in percentage)

1	GROSS DOMESTIC PRODUCT (NIPA 1.1.5)	100.00
2	Private consumption	68.65
3	Personal consumption expenditures (NIPA 1.1.5)	68.65
4	Investment expenditures	16.91
5	Gross private domestic investment (NIPA 1.1.5)	17.68
6	Gross government investment (NIPA 3.1)	3.10
7	Net exports of goods and services (NIPA 1.1.5)	-3.87
8	Government consumption expenditures (NIPA 3.1)	14.44

Sources are listed in the Appendix 5.3.

Table 8: Government Expenditure Programs, Relative to GDP, 2000 (in percentage).

1	Current expenditures (NIPA 3.1)	29.40
2	Government final consumption expenditure (NIPA 3.1)	14.44
3	Current transfer payments (NIPA 3.1)	10.82
4	Expenditures of OASI and DI fund (ASS 4.A3)	4.23
5	Other expenditures (NIPA 3.1)	4.15

Sources are listed in the Appendix 5.3.

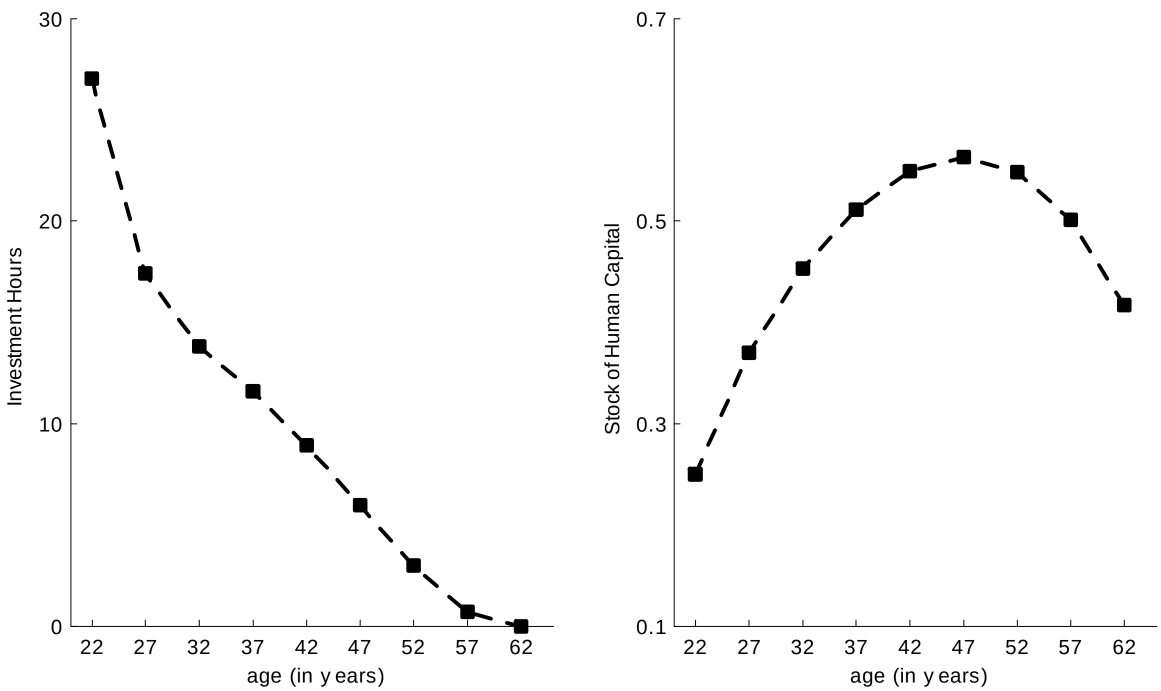

Figure 1: Life-cycle profiles of investment hours and human capital, left and right panels, respectively. These profiles are held fixed in the environment with exogenous human capital.

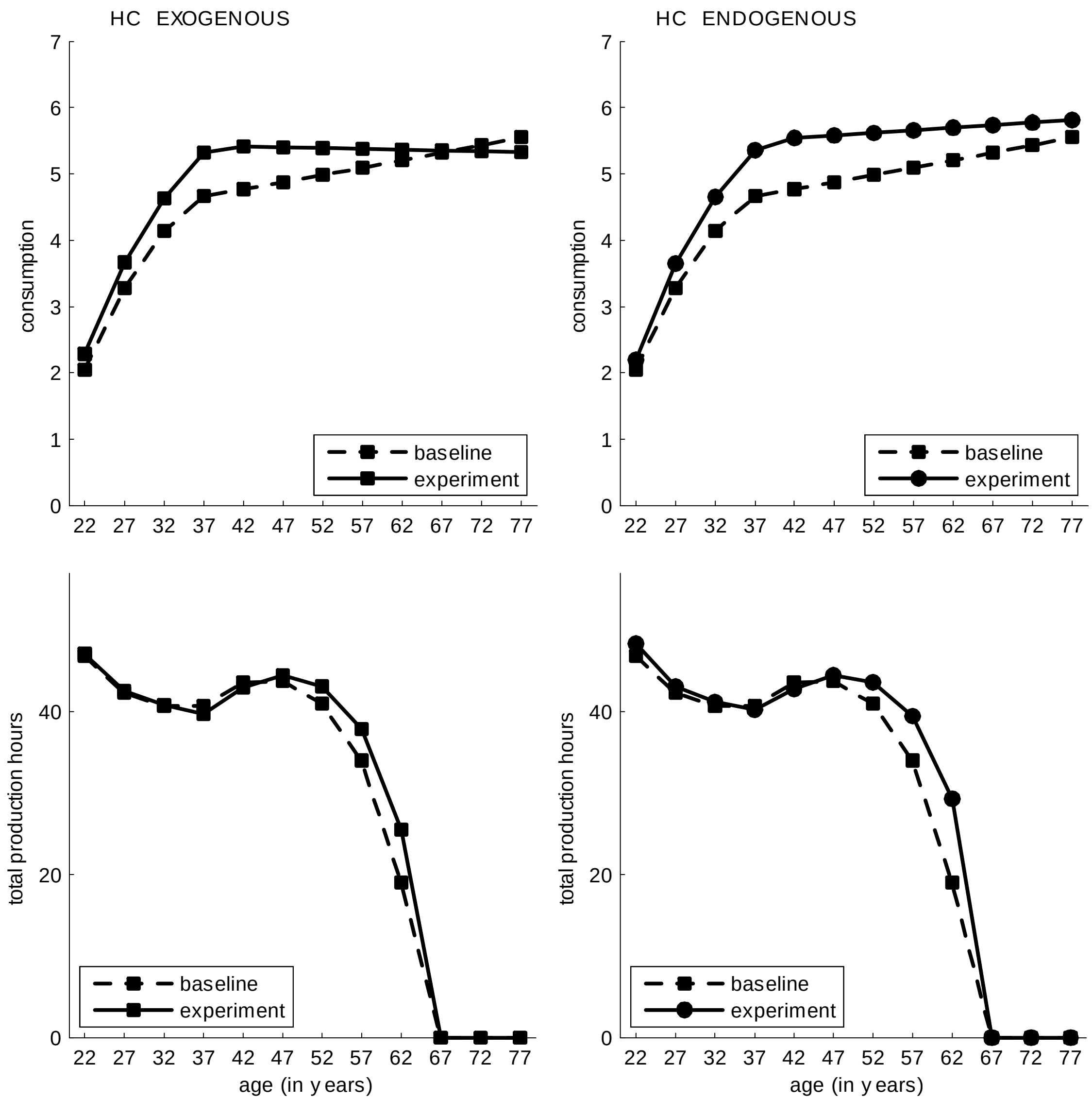

Figure 2: Figures 2 through 5 compare steady states in the environment with Voluntary Retirement Savings Accounts to the one in the baseline environment; variables in these equilibria are denoted as 'experiment' and 'baseline', respectively. Left panel: economies with exogenous human capital. Right panel: economies with endogenous human capital. Top panel: consumption. Bottom panel: total weekly production hours are hours for market production activities plus hours invested into human capital accumulation.

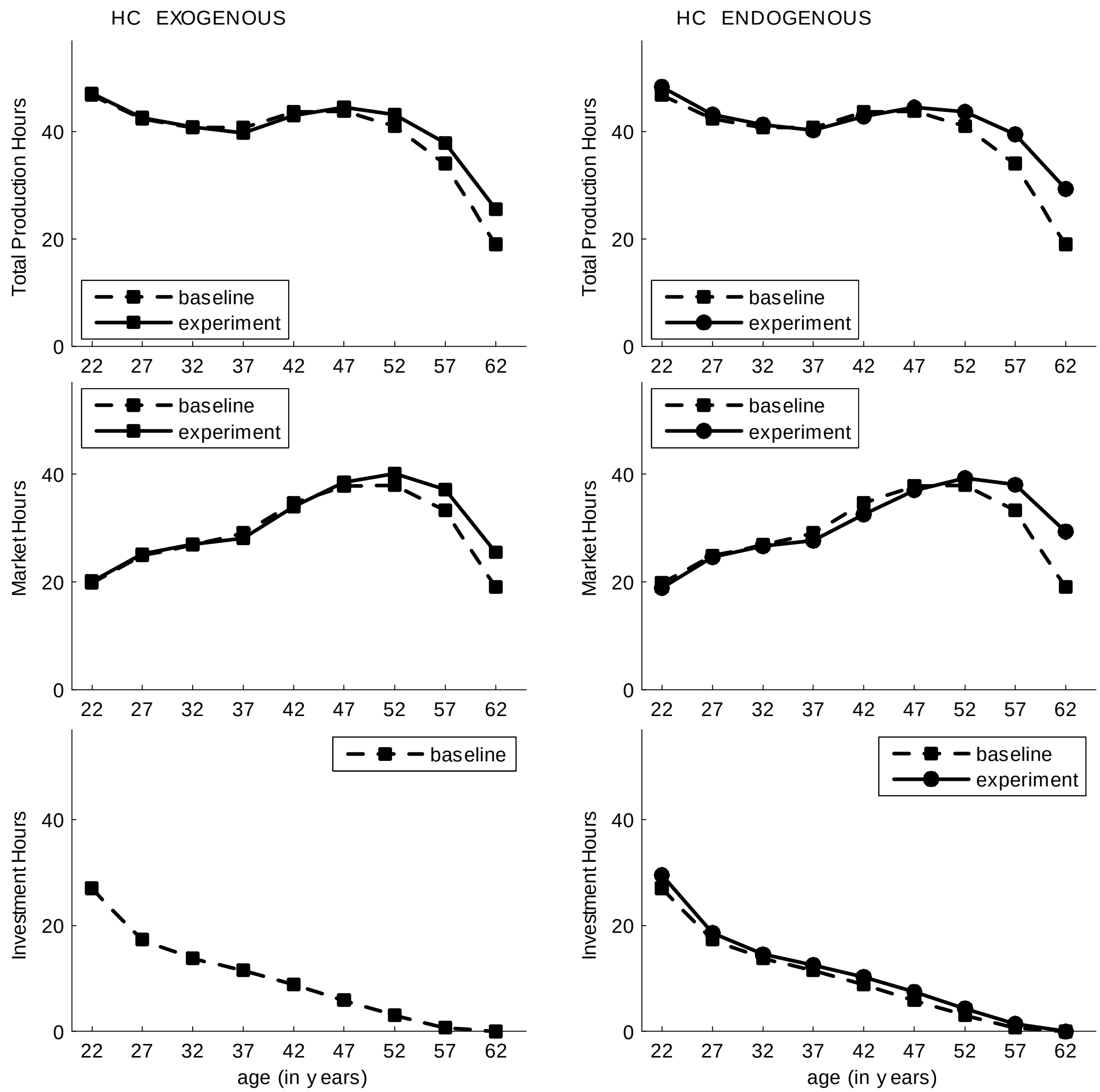

Figure 3: Life-cycle profiles of weekly hours. Top panel: total production hours. Middle panel: hours supplied for market production activities. Bottom panel: hours invested into human capital accumulation.

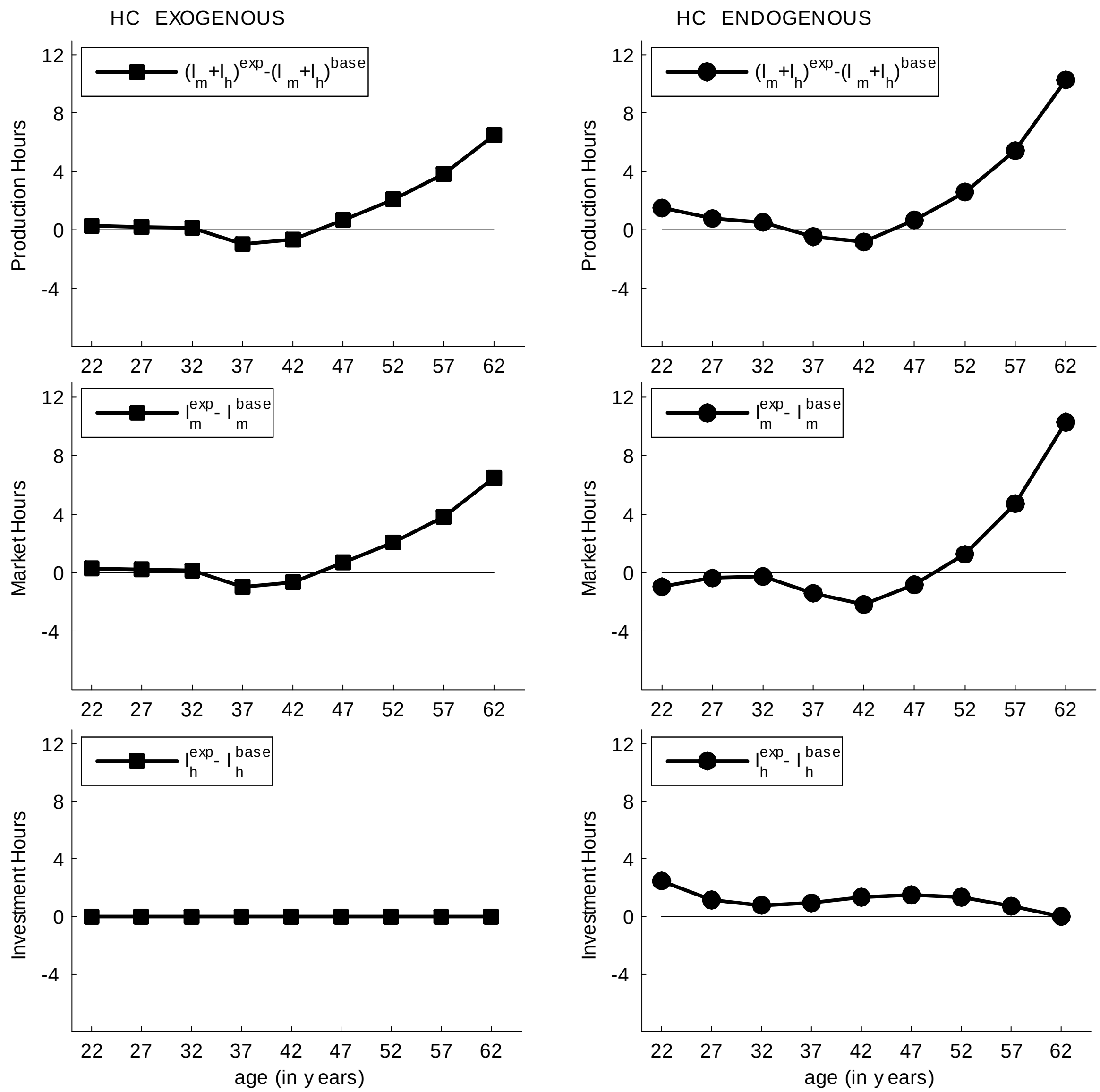

Figure 4: Difference between weekly hours in the environment with Voluntary Retirement Savings Accounts and the baseline one. Top panel: total production hours. Middle panel: market hours. Bottom panel: investment hours.

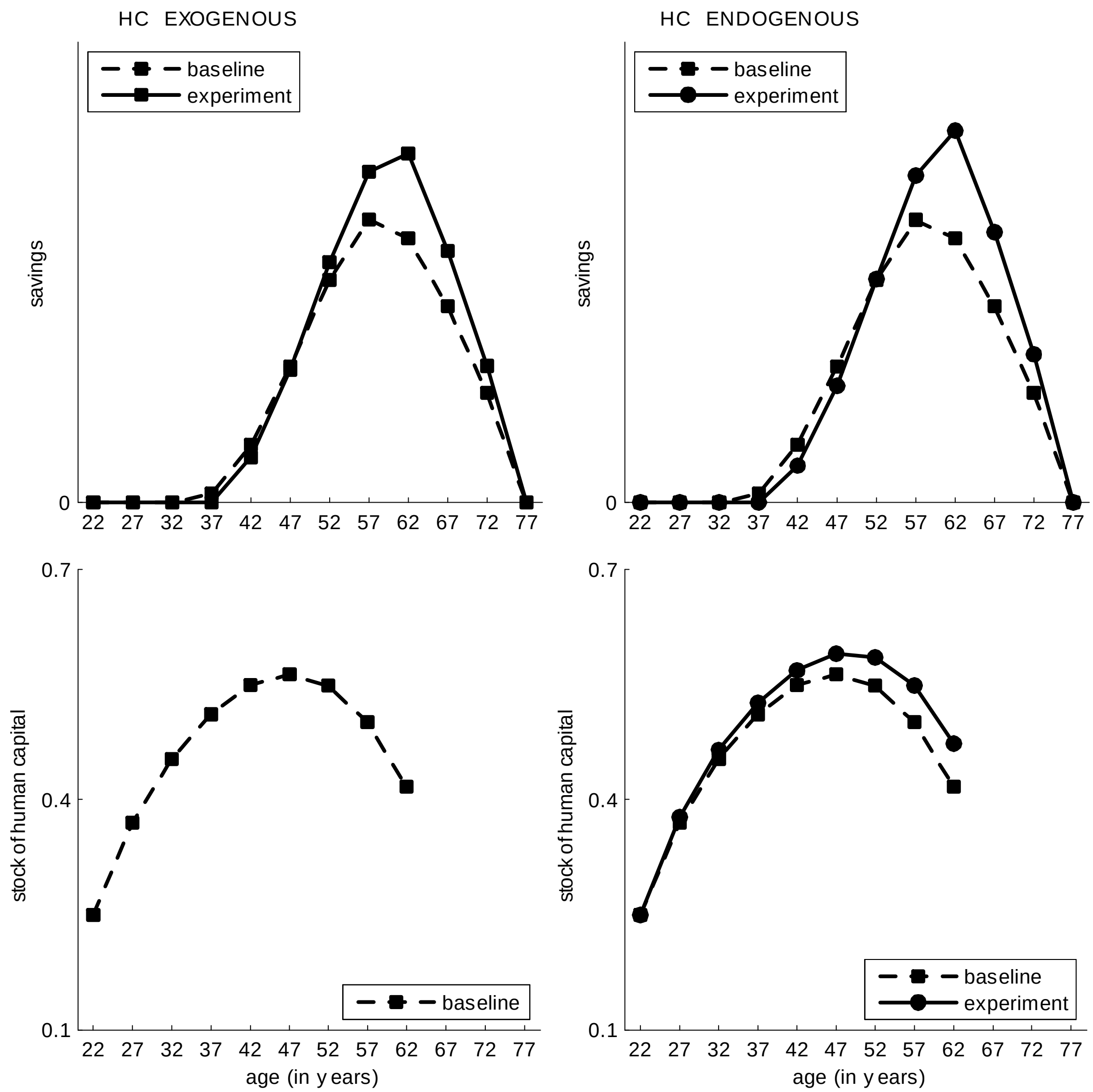

Figure 5: Stocks of physical and human capital in top and bottom panels, respectively.

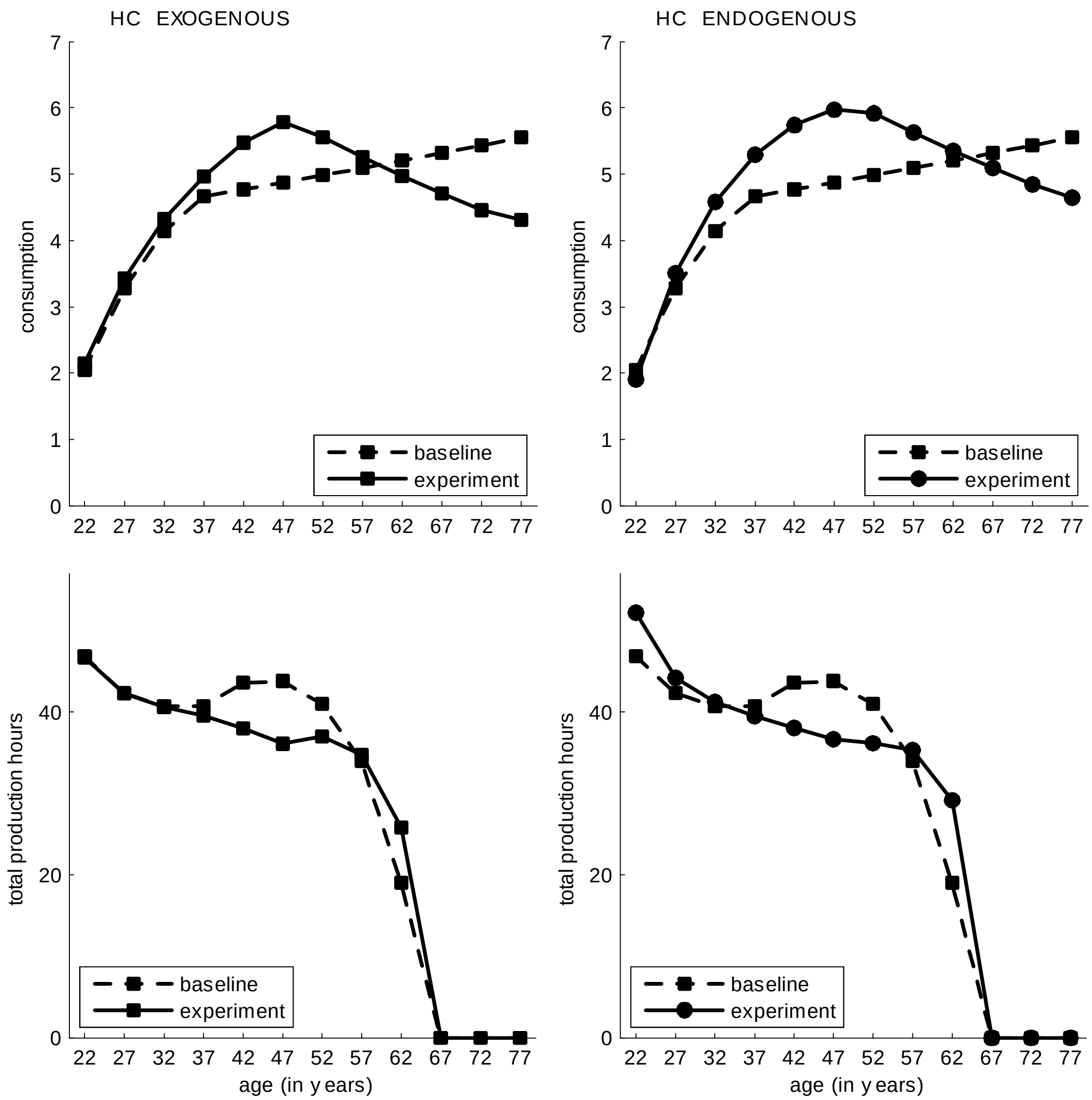

Figure 6: Figures 6 through 10 compare steady states in the environment with Mandatory Retirement Savings Accounts to the one in the baseline environment; variables in these equilibria are denoted as 'experiment' and 'baseline', respectively. Left panel: economies with exogenous human capital. Right panel: economies with endogenous human capital. Top panel: household consumption. Bottom panel: total weekly production hours.

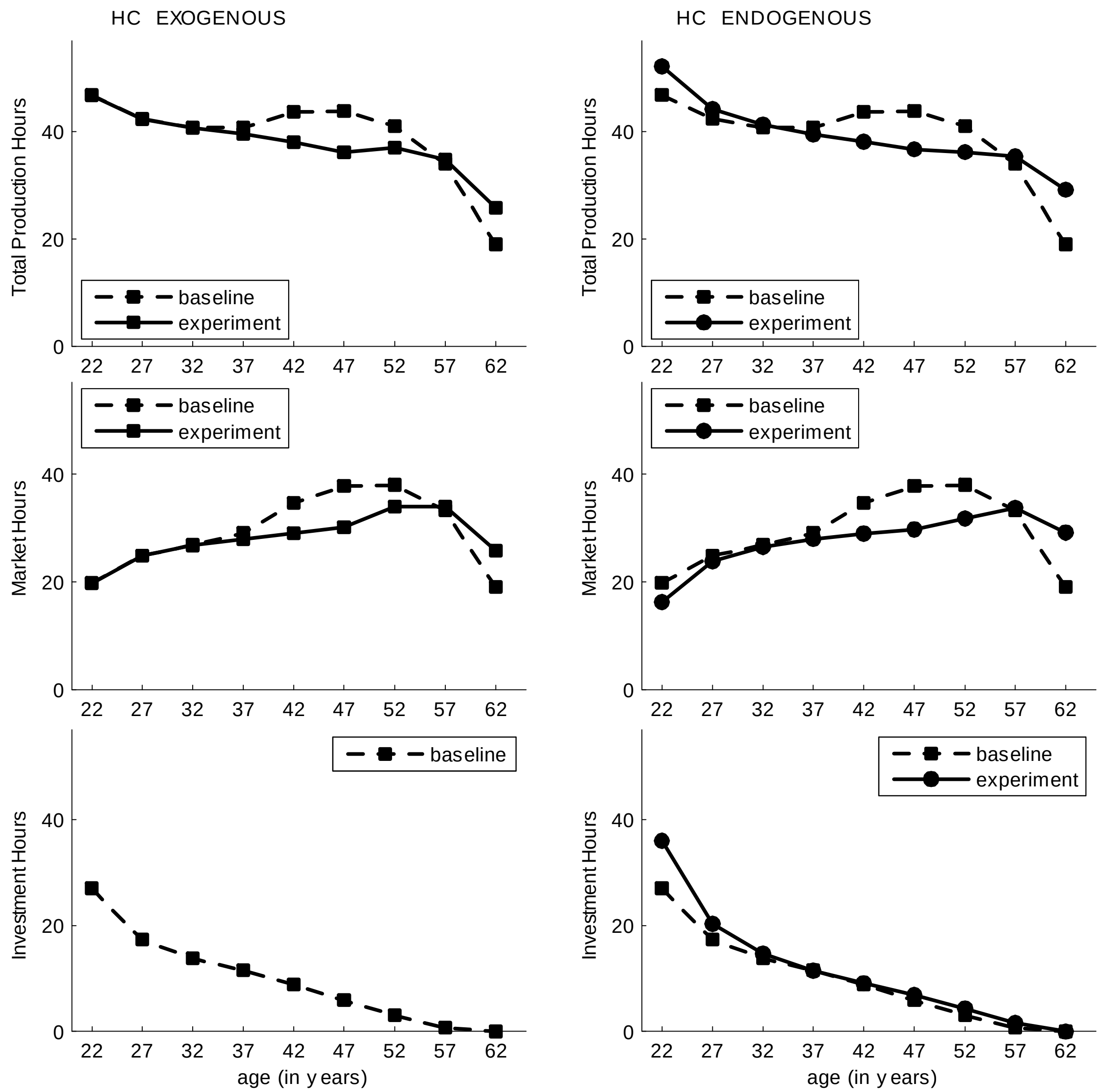

Figure 7: Life-cycle profiles of weekly hours. Top panel: total production hours. Middle panel: hours supplied for market production activities. Bottom panel: hours invested into human capital accumulation.

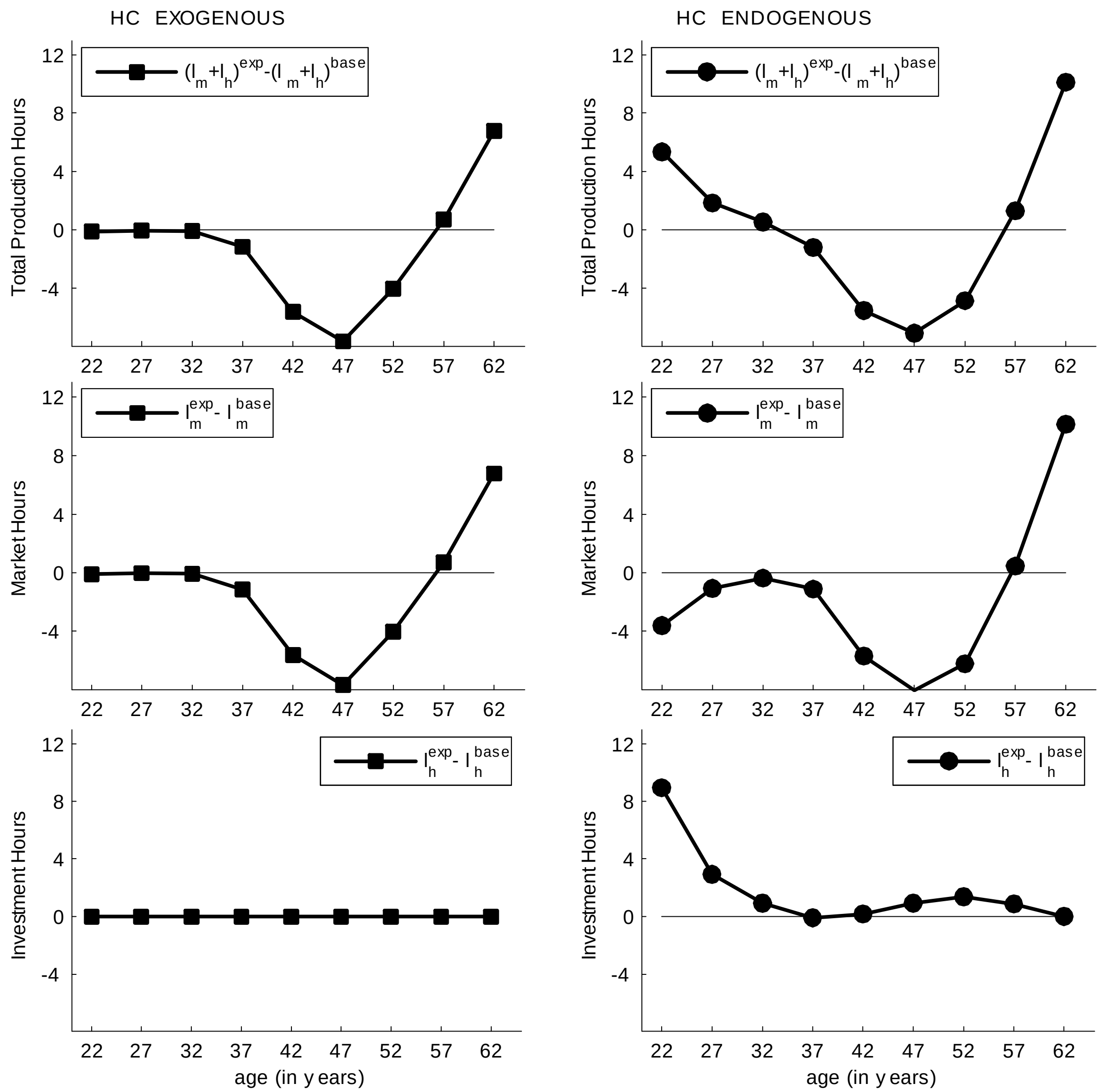

Figure 8: Difference between weekly hours in the environment with Mandatory Retirement Savings Accounts and the baseline one. Top panel: total production hours. Middle panel: market hours. Bottom panel: investment hours.

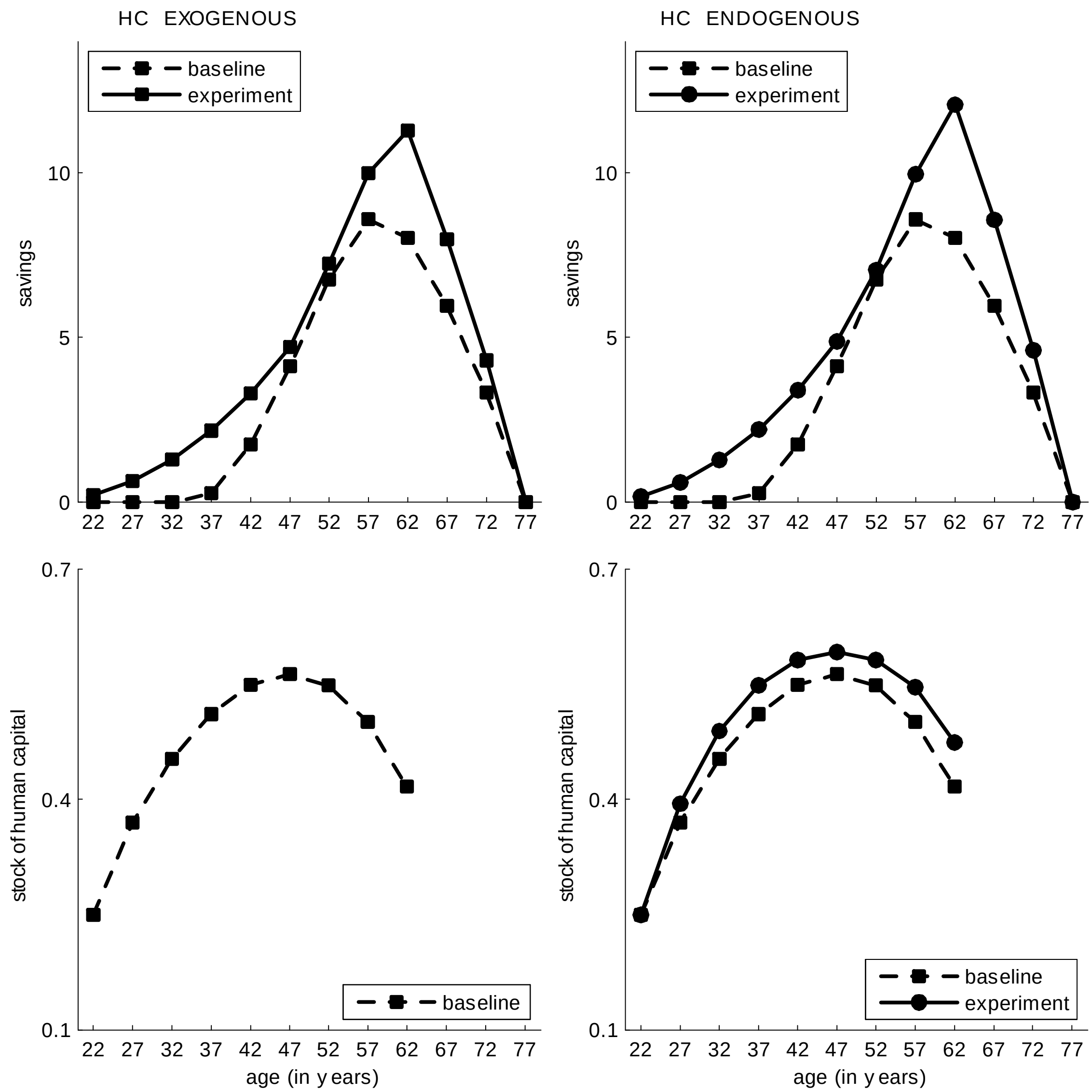

Figure 9: Stocks of physical and human capital in top and bottom panels, respectively. The stock of physical capital in the environment with Mandatory RSA is the sum of voluntary and mandatory savings.

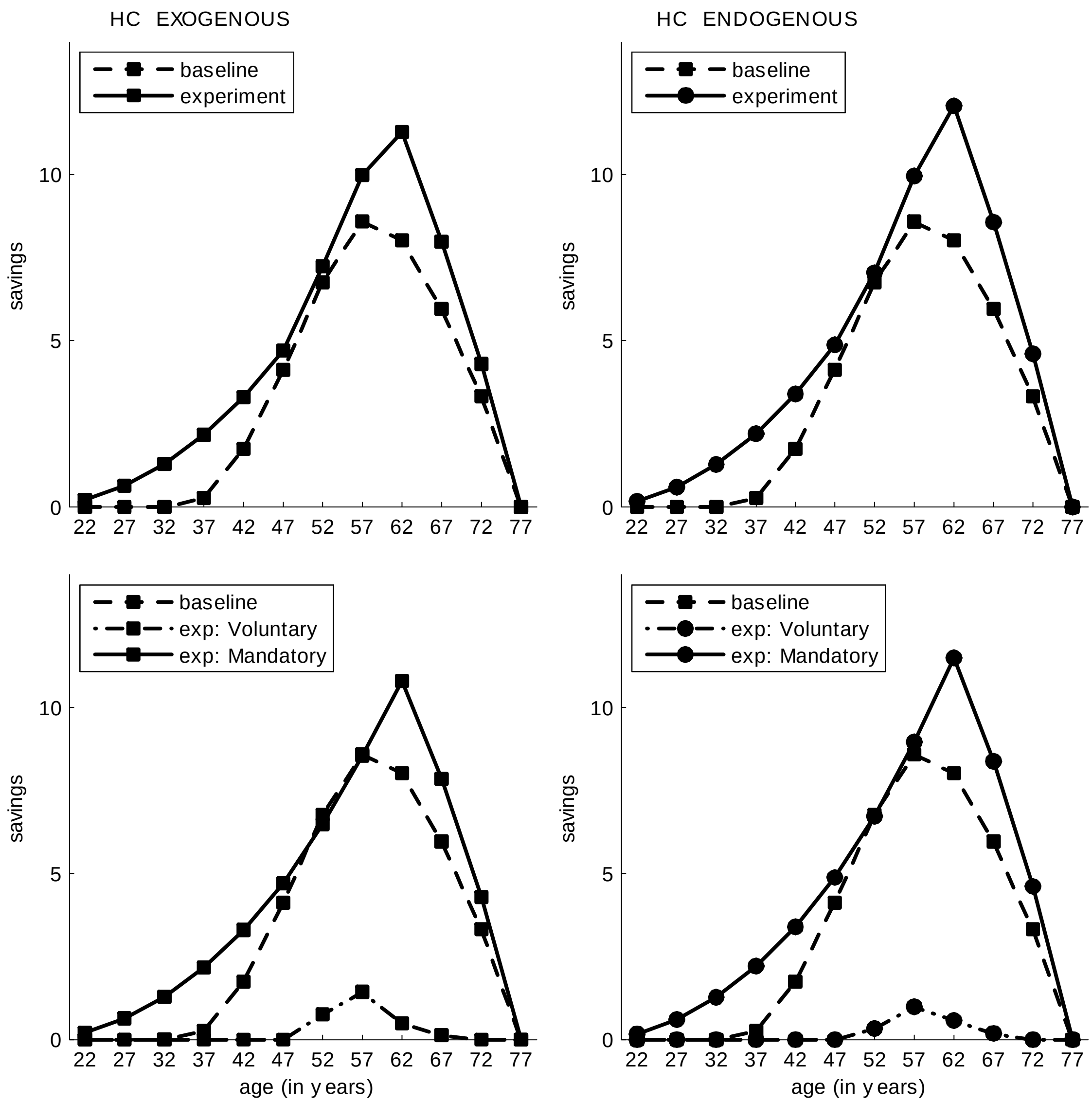

Figure 10: Stock of physical capital. Top panel: total stock. Bottom panel: decomposition into assets held at voluntary and mandatory accounts.