The Dynamics of Optimal Taxation when Human Capital is Endogenous

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Abstract

This paper characterizes the dynamics of Pareto efficient income taxes in a dynamic economy with unobservable human capital accumulation. I extend the tools and insights developed by Mirrlees (1971) into a dynamic framework. I follow Diamond (1998) by assuming that there are no income effects on labor supply. I show that if the government have access to perfect credit markets then i) the problem of finding the efficient allocation can be decomposed into two relatively simple stages and ii) if the agents have access to perfect capital markets as well then the efficient allocations may be implemented in a competitive equilibrium by using history independent income taxes. I compute the sequence of optimal income taxes and show that the marginal income taxes tend to decrease over time and that the decrease is approximately uniform for all skill levels. I also find that, in terms of aggregate output, the dynamic gains from the adjustment of human capital are about 17 times larger than the static gains from the initial labor supply adjustment.

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1 Introduction

In this paper I study Pareto efficient allocations and optimal taxes in a dynamic heterogeneous agent economy where agents can invest in human capital to augment their productivity. It is assumed that neither their productivity nor human capital are observable by the government. I provide a solution method for this type of problems and solve numerically for the dynamics of efficient allocations and optimal taxes.

By assuming that productivity shocks are permanent, this paper focuses attention on dynamics that is driven by the accumulation of individual human capital. Dynamic private information environments where the dynamics is driven by unobserved individual state variable have not been much studied so far. One reason is that such problems have been relatively hard to solve. In this paper I provide a key result that makes the analysis and computation of such models tractable. The Decomposition Theorem of Section 3 shows that the social planner's problem can be conveniently separated into two related subproblems. The first one is a static problem of redistribution between the agents and the second one is a dynamic problem of finding the efficient sequence of labor supply and human capital. While the first problem captures redistributional aspects of the model, the second one captures all the dynamics present in the model. Moreover, I show that the second problem can be written recursively and so can easily be solved numerically.

Private information frictions imply that marginal utilities will not be equalized across agents. I define a cumulative distortion function to be the cumulative percentage distortion of marginal utility and show that it plays a pivotal role in the analysis of the economy. In particular, the cumulative distortion function provides a connection between the static problem of redistribution and the dynamic problem of finding the efficient labor supply and human capital sequence. For a given agent, all one needs to know to solve for the whole time path of labor supply and human capital allocation is her cumulative distortion.

There are many ways to implement the efficient allocation in a market economy. I argue that one particularly appealing implementation, where agents can freely borrow and save and the government uses income taxes that depend only on current income, may be available. Private savings are essential, because they carry information about agent's past. They can therefore act as a substitute for the direct information about agent's past incomes. Thus, if the implementation with current income taxes and capital markets works, agent's savings are a perfect substitute for the information about her past incomes. The problem with using current income taxes is that the agent may be able to gain by deviating jointly in her consumption, labor supply and human capital investment. I do not rule out this possibility on theoretical grounds. Rather, I rely on numerical implementation verification procedure to show that for my particular parametrization this does not happen.

The implementation is, to some extent, similar to the recursive implementation of Albanesi and Sleet [3] in economies with i.i.d. productivity shocks and no human capital. The main difference is that permanent shocks now allow to simplify the tax structure further by making it independent of individual's savings. This result holds despite the fact that the unobservable human capital accumulation is added to the framework.

Numerical simulations reveal that the shape of the marginal income tax schedule does not change much over time, but the level of optimal marginal tax rates decreases over time. The average marginal tax rates decrease from 33% in the first period to 22.6% in the steady state. The intuition is that in the human capital can be fully adjusted only in the long run. Thus, tax policies in the short run can only provide incentives to supply more labor, while tax policies in the long run can also provide incentives to accumulate human capital. Thus, it is more important to have low marginal tax rates in the long run. The mechanism is therefore somewhat reminiscent of the Chamley-Judd result of zero capital tax in the long run (although here the optimal taxes are still nonzero in the long run).

As a consequence of the tax reform, the aggregate income grows until the new steady state is reached. I compare the static gain in aggregate income, which is the first period gain when only labor supply can be adjusted, and the dynamic gain which is the long run gain when both labor supply and human capital can be adjusted. The dynamic gain turns out to be far more important than the static gain: Aggregate income increases only by 1.7% in the first period but by 29.8 % in the long run. Thus, human capital adjustment appears to be essential when the gains from the tax reform are determined.

Recent research on optimal taxation with private information followed the seminal contributions of Mirrlees ([23],[24],[25]) and extended them to dynamic economies. It has focused primarily on cases when the dynamics of efficient allocations is driven by gradual revelation of private information over time (Albanesi and Sleet [3], Battaglini and Coate[4], Fahri and Werning [10]), when the dynamic is driven by an aggregate state variable (Werning [29]) or both (Golosov, Kocherlakota and Tsyvinski [11], Kocherlakota [21]). In contrast, this paper focuses on a case when the driving force behind the dynamics is an unobservable individual specific state variable, namely human capital. Such environments have been recently studied in a two period setting, namely by Grochulski and Piskorski [13] who study unobservable risky human capital and Albanesi [2], who studies observable risky physical capital. In contrast, this paper abstracts from riskiness of human capital, but allows for fully dynamic environment with infinitely many periods. By assuming that private information shocks are permanent, it also differs from the previously cited literature (with the exception of Werning [29]) by focusing on taxation as a tool of redistribution, rather than the insurance against dynamically evolving shocks. In this sense, it appears to be closer to the original Mirrleesian idea of optimal taxation.

This paper is closely related to Kapicka [19] where I analyze the optimal steady state allocations when human capital is unobservable, the government is restricted to use current income taxes and agents cannot borrow or save. This paper extends these results in two ways. First, no exogenous restrictions on the tax system are imposed. Second, I now solve for the whole transitional dynamics of efficient allocations and not just for a steady state. These additional results come at a cost, however. First, I assume that the government can freely borrow and save at an exogenously given interest rate rather than assuming that the resource constraint must clear in each period. Second, I assume that preferences take a very particular form: there are no income effects on leisure. Such preferences were used recently by Diamond [8] to show that the analysis of the static optimal taxation problem can be much simplified. I show that this specification brings even more benefits in a dynamic setting: it simplifies both the computation of efficient allocations and the problem of their implementation in a market economy.[1] In its substance, the model is also closely related to Diamond and Mirrlees

[1] In addition, Saez [26] shows that, at least in a static setting, the optimal tax system is not so different from the optimal tax system when income effects are nonzero.

[9] who analyze unobservable human capital investments in a static framework. While this framework cannot capture all the aspects of human capital accumulation, it allows them to give exact conditions when endogeneity of human capital will cause the optimal marginal income taxes to decrease.

The paper is organized as follows. Next section sets up the model and introduces a direct mechanism that will be used to solve for the efficient allocations. Section 3 characterizes the efficient allocations. Section 4 partially decentralizes the direct mechanism by introducing credit markets. Section 5 discusses how to implement the efficient allocations in a competitive equilibrium with history independent income taxes. Numerical simulations and computed optimal taxes and allocations are presented in Section 6. Section 7 concludes. The Appendix contains most of the proofs.

2 The Model

Time is discrete, $t \geq 0$. There is a measure 1 of agents in the economy. Each individual is associated with a skill level $\theta \in [\underline{\theta}, \bar{\theta}] = \Theta$. I assume that the skill level does not change over time. Although this is a very restrictive condition, it keeps the model tractable. But income processes are very persistent[2] and so it may approximate reality reasonably well. Distribution of skills is given by a distribution F. I assume that F is twice differentiable and has density $f(\theta)$. The skills are private information of each agent: only she knows her own ability. The skills affect earnings of the agent in a way specified below.

Each agent is endowed with one unit of time. In each period, time can be divided between leisure, work, and time spent by human capital accumulation. Denote working time as l_t and time spent by accumulating human capital by s_t.

Period utility of each person depends on consumption c_t and leisure $1 - l_t - s_t$. I assume that the utility function is such that the income effects on leisure are zero: an individual

[2] Heathcote, Storesletten and Violante [17] find that the autocorrelation of wage shocks is about 0.94, and thus exhibits near random walk behavior.

evaluates consumption and leisure sequences according to

$$\sum_{t\geq 0}\beta^t U(c_t + v(1-l_t-s_t)) \qquad 0<\beta<1.$$

where $U : \mathbf{R}_+ \to \mathbf{R}$ is period utility and $v : [0,1] \to \mathbf{R}$ is utility from leisure. I assume that U and v are continuously differentiable on $\mathbf{R}_{++} \times (0,1]$, strictly increasing and strictly concave. In addition, v is assumed to be bounded by above by some constant $\bar{v}$.

Each individual starts with initial human capital h_0. To reduce the complexity of the model I assume that h_0 is identical for all people and is observed by the government. Agent's human capital at the beginning of period $t+1$ is denoted h_{t+1}. It depends on time spent accumulating it in previous period s_t, previous level of human capital h_t and is given by a human capital accumulation function $G : \mathbf{R}_+ \times [0,1] \to \mathbf{R}_+$:

$$h_{t+1} = G(h_t, s_t).$$

Several assumptions are made about G. It is assumed to be continuously differentiable on $\mathbf{R}_{++} \times (0,1]$, strictly increasing and strictly concave. There is an upper bound on human capital stock $\bar{h}$ that cannot be exceeded even if all time is invested in schooling: $G(\bar{h},1) \leq \bar{h}$. If, on the other hand, no additional human capital investments are made then human capital depreciates over time: $G(h,0) < h$ if $h \in (0,\bar{h}]$. Finally, I assume that $\lim_{s\to 0} G_s(s,h) = +\infty$ for all $h \in (0,\bar{h}]$ and that G_h and $\frac{1}{G_s}$ are bounded on $\left[0,\bar{h}\right] \times [0,1]$. I also define an inverse function $g(h_t, h_{t+1})$ which expresses schooling time required to have next period human capital h_{t+1} when current human capital is h_t.

Human capital affects production abilities of the agent. I assume an efficiency unit specification: a person with human capital h_t, skills θ and working l_t hours produces $y_t = \theta h_t l_t$ at time $t \geq 0$.

It is assumed that the government can borrow or lend at an interest rate $\frac{1}{\beta} - 1$. The government has to finance an exogenous sequence of expenditures with present value E. The government is supposed to maximize the expected lifetime utility of an agent that is yet to draw her ability level from the distribution F.

I define an *allocation* to be a sequence of functions $\sigma = \{c_t, y_t, h_{t+1}\}_{t=0}^{\infty}$ where $c_t : \Theta \to \mathbf{R}_+$ specifies consumption in period t, $y_t : \Theta \to \mathbf{R}_+$ specifies output in period t and $h_{t+1} : \Theta \to$

$\mathbf{R}_+$ specifies the recommended amount of human capital at the beginning of period $t+1$.[3] The allocation must satisfy the feasibility constraint:

$$E + \int_{\Theta} \sum_{t \geq 0} \beta^t c_t(\theta) f(\theta) d\theta \leq \int_{\Theta} \sum_{t \geq 0} \beta^t y_t(\theta) f(\theta) d\theta. \tag{1}$$

Since the agent's skill level is a private information, the social planner needs to elicit agent's type from her. At the beginning of period 0 the agent is asked to report her type to the social planner. She is free to choose any human capital sequence as long as it is feasible. If an agent reports $\hat{\theta}$, then a human capital sequence $\{\hat{h}_{t+1}\}_{t=0}^{\infty}$ is feasible if it satisfies $0 \leq g(\hat{h}_t, \hat{h}_{t+1}) \leq 1 - \frac{y_t(\hat{\theta})}{\theta \hat{h}_t}$ for all $t \geq 0$. Let $\Delta(\hat{\theta})$ be the set of all such feasible human capital sequences. The utility of a θ - type agent who reports $\hat{\theta}$ is given by

$$V_\sigma(h_0, \theta; \hat{\theta}) = \max_{\{\hat{h}_{t+1}\} \in \Delta(\hat{\theta})} \sum_{t \geq 0} \beta^t U[c_t(\hat{\theta}) + v(1 - \frac{y_t(\hat{\theta})}{\theta \hat{h}_t} - g(\hat{h}_t, \hat{h}_{t+1})].$$

The incentive compatibility constraint requires the allocation to be such that θ - type agent prefers to report her own type to any other report and chooses the recommended human capital sequence. Thus, an allocation σ is *incentive compatible* if, for all $\theta \in \Theta$, $\{h_{t+1}(\theta)\}_{t=0}^{\infty}$ maximizes $V_\sigma(h_0, \theta; \theta)$ and

$$V_\sigma(h_0, \theta; \theta) \geq V_\sigma(h_0, \theta; \hat{\theta}) \quad \forall \hat{\theta} \in \Theta \tag{2}$$

Denote the set of all allocations that are both feasible and incentive compatible by Σ^{IF}. The social planner chooses an incentive compatible and feasible allocation to maximize the expected utility of an agent:

$$W(h_0, E) = \max_{\sigma \in \Sigma^{IF}} \int_{\Theta} V_\sigma(h_0, \theta; \theta) f(\theta) d\theta, \tag{3}$$

where $W(h_0, E)$ is the value of the social planner's problem when the initial human capital is h_0 and the present value of government expenditures is E. The *efficient allocation* is the allocation that attains the maximum of (3).

[3] Schooling s_t and labor supply l_t can be recovered by inverting the human capital production function and output production function respectively.

3 Characterizing Efficient Allocations

In this section I will characterize the incentive compatible allocations by using the first order approach. Before doing so, I will show that one can safely restrict attention to only such allocations that involve constant period utility over time. Define period utility $u_t(\theta) = c_t(\theta) + v(1 - \frac{y_t(\theta)}{\theta h_t(\theta)} - g(h_t(\theta), h_{t+1}(\theta))$. Proposition 1, which states the result, will help to simplify the structure of the problem significantly. The proof is in the Appendix.

Proposition 1 *If an allocation σ solves the social planner's problem then for all $\theta \in \Theta$, $u_t(\theta) = u(\theta)$ for some function $u(\theta)$.*

The reason is that with quasilinear utility and interest rate equal to the discount rate there are no costs of transferring utility across time in terms of consumption and everyone prefers constant flow of utility to a time varying one.

Incentive compatible allocations that exhibit the property that period utility is constant over time will be called *constant utility incentive compatible.* From now on, I will restrict attention to such allocations. Hence, Σ^{IF} will now denote the set of all feasible allocations that are constant utility incentive compatible. It will also be easier to think about an allocation in terms of period utility $u(\theta)$ rather than period consumption $c_t(\theta)$ and in terms of labor supply $l_t(\theta) = \frac{y_t(\theta)}{\theta h_t(\theta)}$ rather than income $y_t(\theta)$. Thus, an allocation σ now consists of period utility and sequences of labor supply and human capital, $\sigma = \{u(\theta), l_t, h_{t+1}\}_{t=0}^{\infty}$.

I will now derive two sets of necessary conditions for incentive compatibility: the first-order condition in h_{t+1} and the envelope condition in θ. I use the envelope condition rather than the more usual first-order condition in $\hat{\theta}$ because it is more general and applies even in cases when the allocation is not differentiable in θ.

Proposition 2 *If an allocation is constant utility incentive compatible then*

$$u(\theta) = (1-\beta) \int_0^\theta \sum_{t=0}^{\infty} \beta^t v_t'(\varepsilon) l_t(\varepsilon) \frac{d\varepsilon}{\varepsilon} + u_0 \tag{4}$$

and

$$\frac{v_t'}{G_{s_t}} \geq \beta (v_{t+1}' \frac{G_{h_{t+1}}}{G_{s_{t+1}}} + v_{t+1}' \frac{l_{t+1}}{h_{t+1}}) \qquad = \text{if } g(h_t, h_{t+1}) > 0 \tag{5}$$

where $v_t'(\theta) = v'[1 - l_t(\theta) - g(h_t(\theta), h_{t+1}(\theta))]$ *and* $G_{x_t}(\theta) = G_x[h_t(\theta), g(h_t(\theta), h_{t+1}(\theta))]$.

Proof. See the Appendix. ■

Equation (4) shows how the agent's period utility varies with her type. The variation in period utility is proportional to the informational rent the agent obtains from having a given skill level. The utility of the lowest type agent, who gets no informational rent, is given by u_0. The second equation (5) is the Euler equation in human capital. It equalizes the marginal costs of investing in human capital with the marginal benefits of doing so.

Let Σ^{FOC} be the set of allocations that satisfy the first order conditions (4) and (5) and are feasible. A relaxed social planner's problem is defined as a problem of maximizing the expected utility by choosing an allocation that belongs to the set Σ^{FOC} :

$$\hat{W}(h_0, E) = \max_{\sigma \in \Sigma^{FOC}} \int_{\Theta} V_\sigma(h_0, \theta; \theta) f(\theta) d\theta, \tag{6}$$

where $\hat{W}(h_0, E)$ is the value of the relaxed social planner's problem when the initial human capital is h_0 and the present value of government expenditures is E.

The constraints (4) and (5) are necessary for an allocation to be constant utility incentive compatible, but not sufficient. Therefore, $\Sigma^{IF} \subseteq \Sigma^{FOC}$ and the solution to the relaxed social planner's problem may not be identical to the solution of the social planner's problem. There are two reasons why this may be true. First, an individual could find it profitable to deviate jointly in her choice of human capital and in her choice of report.[4] Second, even if joint deviations were not profitable, the envelope condition (4) might still not be enough to prevent a deviation in the report itself. This problem is well known from the static optimal taxation literature. To make sure that these deviations will not happen, one therefore needs to consider an ex-post incentive compatibility verification procedure similar to the one suggested by Abraham and Pavoni [1]. In section 6.1 I discuss an implementation verification procedure that is in fact stronger than the incentive compatibility verification procedure and is therefore sufficient to verify incentive compatibility.

[4] Kocherlakota [22] argues that similar joint deviations are in some cases profitable in a model where savings are hidden.

3.1 The Lagrangean

Let λ be the Lagrange multiplier on the resource constraint (1) and $\mu(\theta)f(\theta)$ be the Lagrange multiplier on the envelope condition (4). The Lagrangean for the social planner's problem (6) is

$$\mathcal{L}(\sigma,\lambda,\mu)=\int_{\underline{\theta}}^{\overline{\theta}}\sum_{t=0}^{\infty}\beta^t\{U(u)+\lambda[\theta h_t l_t+v_t-u]-\mu[(1-\beta)\int_{\underline{\theta}}^{\theta}v_t' l_t\frac{d\varepsilon}{\varepsilon}+u_0-u]\}fd\theta-\lambda E.$$

where $v_t=v[1-l_t-g(h_t,h_{t+1})]$. Let $\sigma_{l,h}=\{l_t,h_{t+1}\}_{t=0}^{\infty}$ be a sequence of labor supply and human capital allocations. Define Π to be the set of all labor supply and human capital sequences $\sigma_{l,h}(\theta)$ that satisfy the Euler equation in human capital (5) and imply that leisure is between zero and one: $\Pi=\{\sigma_{l,h}(\theta)\text{: } 0\leq g(h_t(\theta),h_{t+1}(\theta))\leq 1-l_t(\theta)$ and (5) holds for all $t\geq 0\}$.[5] The solution to the relaxed social planner's problem is the saddle point of the Lagrangean:

$$\hat{W}(E,h_0)=\max_{\sigma}\min_{\lambda,\mu}\mathcal{L}(\sigma,\lambda,\mu)\qquad s.t.\ \sigma_{l,h}\in\Pi. \tag{7}$$

The first-order conditions in $u(\theta)$ and in u_0 are

$$U'(u) = \lambda-\mu, \tag{8}$$

$$\int_{\Theta}\mu f d\theta = 0. \tag{9}$$

Instead of taking the first-order conditions with respect to $\sigma_{l,h}$, I define a partially optimized Lagrangean $\hat{\mathcal{L}}_{u,\lambda}(\sigma_{l,h})$ where the Lagrange multiplier on the envelope condition μ is eliminated directly by using (8) and (9).[6] The partially optimized Lagrangean is

$$\begin{aligned}\hat{\mathcal{L}}_{u,\lambda}(\sigma_{l,h}) &= \int_{\underline{\theta}}^{\overline{\theta}}\sum_{t=0}^{\infty}\beta^t\{U(u)+\lambda[\theta h_t l_t+v_t-u]+[U'(u)-\lambda][\int_{\underline{\theta}}^{\theta}v_t' l_t\frac{d\varepsilon}{\varepsilon}-u]\}fd\theta-\lambda E\\ &= \int_{\underline{\theta}}^{\overline{\theta}}\sum_{t=0}^{\infty}\beta^t\{\lambda[\theta h_t l_t+v_t]-v_t' l_t\frac{1}{f\theta}\int_{\underline{\theta}}^{\theta}[U'(u)-\lambda]f(\varepsilon)d\varepsilon\}fd\theta+\xi(u)-\lambda E\end{aligned} \tag{10}$$

[5] Writing $\sigma_{l,h}\in\Pi$ will mean that this property holds for all $\theta\in\Theta$.

[6] Since (8) and (9) hold only in the optimum, the value of $\hat{\mathcal{L}}_{u,\lambda}(\sigma_{l,h})$ is not necessarily always identical to the value of the Lagrangean $\mathcal{L}$ evaluated at the same values. What matters, however, is that the two values are the same at the optimum.

where $\xi(u) = \frac{1}{1-\beta}\int_{\underline{\theta}}^{\overline{\theta}}\{U(u) - U'(u)u\}fd\theta$. The second equality is obtained by integrating by parts and using (9).

Define a *cumulative distortion function* $X_{u,\lambda}(\theta)$ to be a function giving, for each agent, the average percentage deviation of marginal utility from the shadow price, the average being taken across all agents with lower skills. The cumulative distortion function is

$$X_{u,\lambda}(\theta) = \frac{1}{\lambda}\int_{\underline{\theta}}^{\theta}[U'(u) - \lambda]fd\varepsilon.$$

The cumulative distortion function appears directly in the Lagrangean (10) and will play an important role later on. It is therefore worthwhile to analyze its properties. I show next that it is a hump-shaped nonnegative function that starts and ends at zero.

Lemma 3 *$X_{u,\lambda}(\theta)$ is positive for all $\theta \in \Theta$. In addition, $X_{u,\lambda}(\underline{\theta}) = X_{u,\lambda}(\overline{\theta}) = 0$.*

Proof. $X_{u,\lambda}(\overline{\theta}) = 0$ is obvious. $X_{u,\lambda}(\underline{\theta}) = 0$ follows from (8) and (9). As follows from (4), u must be increasing in θ. Since U is strictly concave, U' is decreasing in θ. (8) and (9) then implies that $U'(u) - \lambda$ is first positive and then negative. Consequently, $\int_{\underline{\theta}}^{\theta}[U'(u) - \lambda]fd\varepsilon$ is always positive. ■

If the utility allocation u and the Lagrange multiplier λ are optimal, i.e. if they solve (7) then the relaxed social planner's problem can be written as

$$\begin{aligned}
\hat{W}(E, h_0) &= \max_{\sigma_{l,h}\in\Pi}\hat{\mathcal{L}}_{u,\lambda}(\sigma_{l,h}) \\
&= \max_{\sigma_{l,h}\in\Pi}\lambda\int_{\underline{\theta}}^{\overline{\theta}}\sum_{t=0}^{\infty}\beta^t[\theta h_t l_t + v_t - v_t' l_t\frac{1}{f\theta}X_{u,\lambda}(\theta)]fd\theta + \xi(u) - \lambda E \\
&= \lambda\int_{\underline{\theta}}^{\overline{\theta}}\left\{\max_{\sigma_{l,h}(\theta)\in\Pi}\sum_{t=0}^{\infty}\beta^t[\theta h_t l_t + v_t - v_t' l_t\frac{1}{f\theta}X_{u,\lambda}(\theta)]\right\}fd\theta + \xi(u) - \lambda E. \quad (11)
\end{aligned}$$

Thus, the maximization of the partially optimized Lagrangean can be broken into a continuum of separate maximization problems. These maximization problems are mutually connected - but only through the cumulative distortion function. In other words, knowledge of $x = X_{u,\lambda}(\theta)$ is sufficient to compute the optimal sequence of labor supply and human capital stock for a $\theta-$type agent. Let $Q_0(x, \theta, h_0)$ be the value of the maximization problem of such agent, if her distortion is x:

$$Q_0(x, \theta, h_0) = \max_{\sigma_{l,h}(\theta)\in\Pi} \sum_{t=0}^{\infty} \beta^t \{\theta h_t l_t + v_t - v_t' l_t \frac{x}{f\theta}\}. \tag{12}$$

Denote the solution to this problem by a sequence of functions $\sigma_{l,h}^d = \{l_t^d, h_{t+1}^d\}_{t\geq 0}$ where $l_t^d : \mathbf{R}_+ \times \Theta \times [0, \bar{h}] \to [0, 1]$ is the optimal labor supply in period t and $h_{t+1}^d : \mathbf{R}_+ \times \Theta \times [0, \bar{h}] \to [0, \bar{h}]$ is the optimal human capital stock at the beginning of period $t + 1$.

One way to look at this problem is to view it as an individual's problem, which is affected by the cumulative distortion.[7] I will therefore call this problem an *individual's distorted problem*. It will later be shown that the individual's distorted problem can be conveniently written recursively. Thus, once the cumulative distortion function is known, one can solve fairly easily for the optimal human capital and labor supply sequences - and that is the main benefit of the individual's distorted problem.

The formulation (11) would be of little use if, in order to solve (12), one needed to first find the optimal values of u and λ and the only way to do so would be to solve the original problem (7). Fortunately, one can show that this is not necessary. Any value of u and λ that satisfies the envelope condition (4) and the resource constraint (1) will solve the relaxed social problem, as long as $\sigma_{l,h}$ solves the individual's distorted problem. This result is shown in the next theorem. The proof can be found in the Appendix.

Theorem 4 (The Decomposition Theorem) *An allocation σ^*, together with the Lagrange multiplier on the resource constraint λ^*, solves the relaxed social planner's problem if and only it is feasible, satisfies the envelope condition (4) and for all $\theta \in \Theta$,*

$$\sigma_{l,h}^*(\theta, h_0) = \sigma_{l,h}^d(X^*(\theta), \theta, h_0)$$

where $X^*(\theta)$ is the cumulative distortion function satisfying $X^*(\bar{\theta}) = 0$ and

$$X^*(\theta) = \frac{1}{\lambda^*} \int_{\underline{\theta}}^{\theta} [U'(u^*) - \lambda^*] f d\varepsilon.$$

[7] If $x = 0$ then this problem is identical to the individual's problem with no government intervention.

Although the social planner's problem is not a convex problem (because the production function exhibits increasing returns to scale), sufficiency is obtained even with reliance on the first-order condition in u. The reason is that all the nonconvex elements of the problem appear in the individual's distorted problem for which the first-order conditions were not used.

The theorem is called the Decomposition Theorem, because of its main finding - one can decompose the social planner's problem into two related but distinct problems. One problem is to find the utility allocation u or, alternatively, the cumulative distortion function X. This is essentially the problem of redistribution between agents of different skills. The second problem, the individual's distorted problem, is to find the optimal sequence of labor supply and human capital allocations. This is the dynamic problem where the accumulation of human capital plays a major role. These two problems are interrelated. The individual's distorted problem is connected with the problem of redistribution in a very simple way - only through the cumulative distortion function. On the other hand, the redistribution problem is affected by the individual's distorted problem in a much more complex way. In particular, the whole solution to the individual's distorted problem matters for both the resource constraint and for the envelope condition (4). What is important, however, is that the individual's distorted problem is so simple, because it is the dynamic element of the model which is by far the hardest to solve for.

Note that the Theorem does not say how to obtain the efficient allocations in practice. In Section 6 I therefore describe an iterative procedure which, provided it converges to a fixed point, ensures that the optimal allocations are found. I will now turn to a recursive characterization of the individual's distorted problem.

3.2 A Recursive Characterization

The individual's distorted problem involves one endogenous state variable - human capital and two exogenous state variables - individual's skill level θ and the distortion x. Both of the exogenous state variables are constant over time and thus work like a fixed effect. To ensure that the Euler equation (5) holds over time, I introduce an additional co-state variable. Let

$d_t = \frac{v_t'}{G_{s_t}}$ be the marginal disutility from increasing period $t+1$ human capital by one unit. It is also the left-hand side of the Euler equation (5) and will be used to make sure that the Euler equation (5) is satisfied.

Let $\bar{d}$ be the supremum of $\frac{v'}{G_s}$ over h, l and s. Define a value function $Q_{x,\theta} : [0, \bar{h}] \times [0, \bar{d}] \to \mathbf{R}$ to be a solution to the following Bellman equation:

$$\begin{aligned} Q_{x,\theta}(h,d) &= \max_{l,h',d'} \{\theta h l + v - v' l \frac{x}{f\theta} + \beta Q_{x,\theta}(h', d')\} \\ s.t.\ d &\geq \beta v' (\frac{l}{h} + \frac{G_h}{G_s}) \qquad = \text{if } g(h,h') > 0 \end{aligned} \tag{13}$$

where h' and d' satisfy their respective laws of motion

$$h' = G(h,s),\ d' = \frac{v'}{G_s}.$$

The relationship between the dynamic program (13) and the sequence program (12) is slightly complicated by the fact that, as follows from (12), the social planner is not constrained in the first period to deliver any particular value of d. To show the relationship formally, define $\hat{Q}_0(x, \theta, h_0, d_0)$ to be the maximum in the sequence problem (12) where an additional first period constraint $d_0 = \beta v_1'(\frac{l_1}{h_1} + \frac{G_{h_1}}{G_{s_1}})$ is imposed. $\hat{Q}_0$ is the proper counterpart of the Bellman equation (13) in the sequence space.

I will now show the equivalence between this restricted sequence problem and the dynamic program (13). The period return function $\theta h l + v - v' l \frac{x}{f\theta}$ is bounded above by $\theta\bar{h} + \bar{v}$ since v is bounded above by $\bar{v}$ by assumption and both v' and x are positive. Also, for any $(h,d) \in [0,\bar{h}] \times [0,\bar{d}]$ one can choose a triplet l, h', d' in such a way that it satisfies all the constraints of the dynamic program.[8] The assumptions of Theorems 4.2 and 4.4 of Stokey, Lucas and Prescott [27] are therefore satisfied, $\hat{Q}_0(x, \theta, ., .)$ satisfies the Bellman equation (13) and the solution to the sequence problem attains its maximum. The converse is also true, as follows from Exercise 4.3 and Theorem 4.5 of [27].

The value function $Q_0(x, \theta, h_0)$ that solves the sequence problem (12) can then be obtained by finding d that maximizes $Q_{x,\theta}(h,d)$:

$$Q_0(x,\theta,h_0) = \max_{d \in [0,\bar{d}]} Q_{x,\theta}(h_0, d),$$

[8] Setting h' such that $g(h,h') = 0$, $l \in (0,1)$ and $d' = 0$ is an example.

and the optimal policy functions that attain maximum of (13) can be used to generate the solution of the sequence problem (12).

There are many ways to implement the efficient allocation in a market economy. One way to do it is to impose income taxes that depend on the whole lifetime profile of incomes.[9] While this implementation does not require credit markets in order to work, it may be difficult to carry out in practice. I will therefore suggest an alternative implementation, which relies heavily on agent's ability to borrow and save, and which may be available. This implementation is simpler because it only uses income taxes that depend on current income. On the other hand, it may not always be available. I will study the implementation in two steps. Next section will introduce credit markets, into the direct mechanism. Section 5 will then add, on top of that, history independent income taxes and study if the efficient allocation can be implemented in such environment.

4 Credit Markets

I will now partially decentralize the direct mechanism of Section 2 by introducing credit markets. I will show two results. The first one is that unrestricted credit markets give the social planner more options how to achieve a given equilibrium allocation. A whole set of allocations is now equivalent in a sense that the equilibrium will be the same no matter which one is chosen by the social planner. These allocations differ in the timing of resources the social planner allocates to the agents and, as a consequence, in the amount of borrowing and saving the agents undertake on their own. I will argue in the next section that one such allocation is in particular interesting because it may be implementable by current income taxes.

The second result is that when credit markets are introduced and no capital taxes are used, the set of equilibrium allocations is smaller than the set of feasible constant utility incentive compatible allocations. Thus, unrestricted credit markets limit the social planner's ability to redistribute resources to some extent. But the limitations are quite weak and they may not be binding in the optimum.

[9] See Werning [29] for an example of such implementation.

In a *mechanism with credit markets* agents are allowed to engage in trade in credit markets. The interest rate is fixed at $\frac{1}{\beta} - 1$ and no capital taxes are imposed. It is worth stressing, that this is still a direct mechanism in a sense that agents still report their types directly to the social planner. It only differs from the direct mechanism of Section 2 in that it allows unrestricted borrowing and saving.

It is easy to see that when the agents are given the option to trade consumption, they will always choose a constant utility sequence. Therefore, the optimal reporting strategy and the optimal human capital sequence will maximize the present value of consumption plus the present value of utility from leisure. This leads to the following notion of incentive compatibility: An allocation σ is called *incentive compatible with credit markets* if, for each $\theta \in \Theta$,

$$\{\theta, h_{t+1}(\theta)\}_{t=0}^{\infty} \in \arg\max_{\hat{\theta}, \{\hat{h}_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t [u_t(\hat{\theta}) + v(1 - \frac{y_t(\hat{\theta})}{\theta \hat{h}_t} - g(\hat{h}_t, \hat{h}_{t+1}) - v(1 - \frac{y_t(\hat{\theta})}{\hat{\theta} \hat{h}_t} - g(\hat{h}_t, \hat{h}_{t+1})]. \tag{14}$$

Incentive compatibility with credit markets ensures that the agents will report truthfully and choose the recommended human capital sequence. Denote the set of all feasible allocations that are incentive compatible with credit markets by Σ^{IFCM}.

Note that this definition of incentive compatibility with credit markets is weaker than usual in a sense that it does not require the utility sequence $\{u_t\}_{t=0}^{\infty}$ to be the one chosen by the agents (i.e. constant over time). An allocation that is incentive compatible with credit markets therefore does not necessarily maximize agent's utility in the mechanism with credit markets. Rather, for any $\sigma \in \Sigma^{IFCM}$, the equilibrium in a mechanism with credit markets is given by an allocation $\sigma^E = \{\bar{u}(\{u_t\}), y_t, h_{t+1}\}_{t=0}^{\infty}$ where

$$\bar{u}(\{u_t\}) = (1 - \beta) \sum_{t=0}^{\infty} \beta^t u_t. \tag{15}$$

The equilibrium allocation is clearly incentive compatible with credit markets, but in addition requires the utility sequence to be constant over time. The set of all equilibrium allocations Σ^E therefore satisfies $\Sigma^E \subset \Sigma^{IFCM}$.

The set Σ^{IFCM} of all allocations that are incentive compatible with credit markets and feasible can be partitioned according to the equilibrium allocation it implies. Let $\mathcal{P}_{\sigma^*}$ be the set of feasible, incentive compatible allocations that imply that the equilibrium allocation is $\sigma^* \in \Sigma^E$. More formally, $\mathcal{P}_{\sigma^*} = \{\sigma \in \Sigma^{IFCM} : \bar{u}(\{u_t\}) = u^*\}$. It follows from previous discussion that all the allocations $\sigma \in \Sigma^{IFCM}$ that differ from σ^* only in their utility sequence but satisfy (15) belong to $\mathcal{P}_{\sigma^*}$.

From the social planner's perspective, all the allocations that belong to $\mathcal{P}_{\sigma^*}$ are equivalent to σ^*. Instead of selecting the equilibrium allocation σ^* directly, he can choose any allocation $\sigma \in \mathcal{P}_{\sigma^*}$. The agent will then use the credit market to transfer resources across time herself and the equilibrium allocation will be σ^*.

While any equilibrium allocation is also feasible and constant utility incentive compatible, the reverse is not true. An allocation σ that is constant utility incentive compatible satisfies, for any report $\hat{\theta}$ and any human capital sequence $\{\hat{h}_{t+1}\}_{t=0}^{\infty}$,

$$\begin{aligned} u(\theta) &\geq U^{-1}\{(1-\beta)\sum_{t=0}^{\infty}\beta^t U[u(\hat{\theta}) + v(1 - \frac{y_t(\hat{\theta})}{\theta\hat{h}_t} - S(\hat{h}_t,\hat{h}_{t+1}) - v(1 - \frac{y_t(\hat{\theta})}{\hat{\theta}\hat{h}_t} - S(\hat{h}_t,\hat{h}_{t+1})]\} \\ &\leq (1-\beta)\sum_{t=0}^{\infty}\beta^t[u(\hat{\theta}) + v(1 - \frac{y_t(\hat{\theta})}{\theta\hat{h}_t} - S(\hat{h}_t,\hat{h}_{t+1}) - v(1 - \frac{y_t(\hat{\theta})}{\hat{\theta}\hat{h}_t} - S(\hat{h}_t,\hat{h}_{t+1})]. \end{aligned}$$

The second inequality, implied by the Jensen's inequality, is strict as long as the terms on the right hand side are not constant over time. The intuition is that if there are no credit markets, truthtelling may dominate some deviation only because the deviation delivers a nonconstant utility sequence. When credit markets are introduced, the agent can smooth utility over time herself and the deviation becomes strictly preferable. Thus, $\Sigma^E \subseteq \Sigma^{IF}$.

It is easy to see that the equilibrium allocations share with the constant utility incentive compatible allocations the same envelope and first-order conditions. Therefore, if an allocation is constant utility incentive compatible but not equilibrium one, it must be the case that the second-order conditions are different. To put it in a different way, the first order approach will more likely fail to detect equilibrium allocations than constant utility incentive compatible ones. Credit markets thus limit social planner's ability to redistribute resources in a weaker way than in models with dynamically evolving productivity shocks (see Golosov and Tsyvinski [12]) where even the first order conditions are not identical.

The hope is that the limitations of credit markets will not be binding and the efficient allocation σ will satisfy $\sigma \in \Sigma^*$. If this is the case then the main implication of credit markets is that it gives the social planner more options to achieve the equilibrium (and at the same time efficient) allocation. This will become important for the following reason. The efficient allocation is in general not obtainable by history independent income taxes, because history independent income taxes restrict the social planner's ability to transfer resources over time too much. But some other allocation $\tilde{\sigma} \in \mathcal{P}_\sigma$ may be obtainable by current income taxes. Next section will determine the properties of such candidate allocation.

5 Implementation in a Market Economy

In this section I will argue that if people can borrow and save, one may be able to implement the efficient allocations by using income taxes that depend only on *current* income. The restriction to current income taxation will not be justified in general. I will rather establish sufficient conditions that an allocation must satisfy in order to be implementable with current income taxes (Corollary 6). Whether the efficient allocation satisfies these conditions is another question and will be addressed later.

5.1 Limited Record Keeping

I show in Kapicka [19] that an allocation is implementable by current income taxes if and only if it can be represented by a mechanism where the agent is asked to report her type each period, and the social planner is unable to keep track of agents' past reports. The allocation in such mechanism thus depends only on the current report. A mechanism with limited record keeping is clearly inferior to the direct mechanism introduced in Section 2, since it imposes much more severe restrictions on the allocations. An efficient allocation will in general not satisfy the constraints of the mechanism with limited record keeping.

Let $\hat{\theta}^\infty = \{\hat{\theta}_t\}_{t=0}^\infty \in \Theta^\infty$ be a sequence of reports. Define an allocation to be *incentive compatible with credit markets and limited record keeping* if a truthful sequence of reports and the recommended human capital sequence are optimal for each $\theta \in \Theta$,

$$\{\theta^\infty, h_{t+1}(\theta)\}_{t\geq 0} \in \underset{\{\hat{\theta}_t, \hat{h}_{t+1}\}_{t=0}^\infty}{\arg\max} \sum_{t=0}^{\infty} \beta^t [u_t(\hat{\theta}_t) + v(1 - \frac{y_t(\hat{\theta}_t)}{\theta \hat{h}_t} - g(\hat{h}_t, \hat{h}_{t+1}) - v(1 - \frac{y_t(\hat{\theta}_t)}{\hat{\theta} \hat{h}_t} - g(\hat{h}_t, \hat{h}_{t+1})]. \tag{16}$$

The set of all allocations that are feasible and incentive compatible with credit markets and limited record keeping is denoted by $\hat{\Sigma}^{IFCM}$. To highlight the difference between the efficient mechanism and the mechanism with limited record keeping, one should compare (14) with (16). While (14) requires that an alternative report $\hat{\theta}$ must yield lower utility than truthtelling only "on average", (16) requires that such deviation must be unprofitable in any single period. It is therefore much more restrictive than (14).[10] Therefore $\hat{\Sigma}^{IFCM} \subseteq \Sigma^{IFCM}$ but the reverse does not hold.

For any $\sigma \in \hat{\Sigma}^{IFCM}$, the equilibrium in a mechanism with credit markets and limited record keeping is given by an allocation $\sigma^E = \{\bar{u}(\{u_t\}), y_t, h_{t+1}\}_{t=0}^{\infty}$ that, similarly to the mechanism with credit markets, smooths utility over time but keeps income and human capital allocation unchanged. Let $\hat{\Sigma}^*$ be the set of all such equilibrium allocations. Since limited record keeping provides additional restrictions, $\hat{\Sigma}^* \subseteq \Sigma^*$. Notice also that the limited record keeping restrictions imply that the equilibrium allocation will typically not be in $\hat{\Sigma}^{IFCM}$. Thus, credit markets are usually strictly welfare improving.

5.2 Market Economy

Consider now a market economy where the agents can borrow and save at the interest rate $\frac{1}{\beta} - 1$ and face a sequence of income taxes (tax policy) $\mathcal{T} = \{T_t\}_{t=0}^{\infty}$ imposed by the government. It is assumed that a tax function in period t, $T_t : \mathbf{R}_+ \to \mathbf{R}$, depends only on current income. The agent will again choose constant period utility and maximize the present value of consumption plus present value of utility from leisure:

$$\Omega^{\mathcal{T}}(h_0, \theta) = \max_{\{y_t, h_{t+1}\}_{t=0}^{\infty} \in \Delta} \sum_{t=0}^{\infty} \beta^t [y_t - T_t(y_t) + v(1 - \frac{y_t}{\theta h_t} - g(h_t, h_{t+1})].$$

Let $\sigma^{\mathcal{T}} = \{u^{\mathcal{T}}, y_t^{\mathcal{T}}, h_{t+1}^{\mathcal{T}}\}_{t=0}^{\infty}$ be the optimal policy functions, where $y_t^{\mathcal{T}} : \Theta \to \mathbf{R}_+$ and $h_{t+1}^{\mathcal{T}} : \Theta \to \mathbf{R}_+$ is the income and next period human capital chosen by agents in period t and

[10] They are equivalent only when there is no dynamics in the model (for instance in Da Costa and Werning [7]). On the other hand, one can show that show that if an allocation is incentive compatible with credit markets but there is a profitable deviation in a mechanism with credit markets and no memory then it must be a joint deviation in human capital and report. Up to a first order, both mechanisms are therefore equivalent.

$u^{\mathcal{T}} : \Theta \to \mathbf{R}_+$ is the period utility chosen by the agents where $u^{\mathcal{T}}(\theta) = (1-\beta)\Omega^{\mathcal{T}}(h_0, \theta)$. A competitive equilibrium in the market economy consists of the tax policy $\mathcal{T}$ and the optimal policy functions $\sigma^{\mathcal{T}}$ such that the optimal policy functions solve the above problem taking $\mathcal{T}$ as given and the government's present value budget constraint is satisfied.

Next proposition states the taxation principle for history independent income taxes. It shows the equivalence between the set of allocations that are incentive compatible with credit markets and limited record keeping and the set of competitive equilibrium allocations in a market economy. The proof is in the Appendix.

Proposition 5 (Taxation Principle) *An allocation σ^* is an equilibrium allocation in a mechanism with credit markets and limited record keeping if and only if there exists a tax policy $\mathcal{T}$ such that $\sigma^{\mathcal{T}} = \sigma^*$.*

The proof relies on the result in [19] that, if borrowing and saving is not allowed, there is an equivalence between the set of allocations that can be obtained with current income taxes and the set of allocations that are incentive compatible with limited record keeping. It shows that adding borrowing and saving does not change the result, since the agents will trade in the same way in both the market economy and the mechanism with credit markets and limited record keeping. The proposition implies the following:

Corollary 6 *The efficient allocation σ^* is implementable by current income taxes and credit markets if and only if $\sigma^* \in \hat{\Sigma}^E$.*

To find out whether the efficient allocation σ^* is implementable by current income taxes and credit markets, one needs to define a candidate allocation $\tilde{\sigma} \in \mathcal{P}_{\sigma^*}$ and check if $\tilde{\sigma} \in \hat{\Sigma}^{IFCM}$. If this is true, then $\sigma^* \in \hat{\Sigma}^E$. To define such a candidate allocation notice that (16) can be equivalently written as a sequence of forward looking constraints, one for each time period $t \geq 0$:

$$\{\theta^\infty, h_{t+1}(\theta)\}_{j \geq t} \in \arg\max \sum_{j=t}^{\infty} \beta^t [u_j(\hat{\theta}_j) + v(1 - \frac{y_j(\hat{\theta}_j)}{\theta \hat{h}_j} - g(\hat{h}_j, \hat{h}_{j+1}) - v(1 - \frac{y_j(\hat{\theta}_j)}{\hat{\theta} \hat{h}_j} - g(\hat{h}_j, \hat{h}_{j+1})]. \tag{17}$$

An allocation satisfies (17) if and only if it satisfies (16). By applying the envelope theorem to (17) in two consecutive periods and substracting the envelope conditions, one gets that the period utility $\tilde{u}_t(\theta)$ must satisfy

$$\tilde{u}_t(\theta) = \int_0^\theta v_t' l_t^* \frac{d\varepsilon}{\varepsilon} + \int_0^\theta v_t' (\frac{l_t^*}{h_t^*} + \frac{G_{h_t}}{G_{s_t}}) \frac{dh_t^*}{d\theta} d\varepsilon - \beta \int_0^\theta v_{t+1}' (\frac{l_{t+1}^*}{h_{t+1}^*} + \frac{G_{h_{t+1}}}{G_{s_t+1}}) \frac{dh_{t+1}^*}{d\theta} d\varepsilon + u_0. \tag{18}$$

The allocation $\tilde{\sigma} = \{\tilde{u}_t, l_t^*, h_{t+1}^*\}_{t=0}^\infty$ is the candidate allocation that will be examined whether it belongs to $\hat{\Sigma}^{IFCM}$. Note that $\{\tilde{u}_t\}_{t=0}^\infty$ satisfies (15) and so $\tilde{\sigma} \in \mathcal{P}_{\sigma^*}$, as required. The implementation verification procedure I will discuss in Section 6.1 will check whether there is some time period t and some report sequence $\hat{\theta}_t^\infty$ that violates (17). If this is true, then $\tilde{\sigma}$ also violates (16) and the equilibrium in a mechanism with credit markets and limited record keeping differs from σ^*. Otherwise, σ^* can be implemented with history independent income taxes and credit markets.

The implementation verification procedure should not be confused with the incentive compatibility verification procedure. It is in fact much stronger one. If an allocation passes the ex post implementation verification procedure, it passes the ex post incentive compatibility verification procedure. Otherwise one needs to verify the incentive compatibility directly.

6 Numerical Example

In this section I will numerically compute the optimal allocations and show how the welfare maximizing tax reform should look like. The utility function is assumed to be logarithmic and to exhibit a constant elasticity of labor supply,

$$\begin{aligned} U(u) &= \log(u) \\ v(1-n) &= -\frac{n^{1+k}}{1+k} \end{aligned}$$

where the elasticity of labor supply is given by $\frac{1}{k}$. The human capital production function is assumed to be Cobb-Douglas:

$$G(h,s) = (1-\delta)h + \delta h^\alpha s^{1-\alpha}.$$

For the utility function I choose $k = 2$ so that the elasticity of labor supply is 0.5. For the human capital production function I assume the Ben-Porath specification with $\alpha = 0.5$. There is a very diverse evidence regarding depreciation δ. The evidence ranges from 0.0016 to 0.089, with most of the estimates concentrated around 0.04.[11] This is also the value I have chosen for parameter δ. The time period is one year and so the discount factor β is set equal to 0.96.

I will calibrate the distribution of skills in such a way that the steady state distribution of earnings under the current U.S. tax code will resemble the empirical distribution of earnings. There is, however, one problem with this approach. To meaningfully calibrate the model, one needs to assume that the initial human capital stock varies across people. But, why can't the social planner then use the initial distribution of human capital to infer the distribution of skills?[12] When shocks are permanent, this problem is hard to avoid. I suggest the following resolution. Suppose that the tax reform takes place at time 0. The world has started in period $-T$, when human capital was identical across population and people learned about their skills. At this time a constitution was written stating that only income taxes that depend on current income may be used. This provision was not restrictive, provided that the efficient allocation passes the implementation verification procedure. Neither the shape of the income tax function nor capital taxes were specified in the constitution and were chosen by previous governments. In particular, the current U.S. income tax schedule was chosen and savings were not allowed. When the tax reform takes place in period 0, the government must obey the constitution and so can only use income taxes that depend on current income.

Tax return data for 1992 are used for the empirical distribution of earnings. Since the data and hence the implied skills levels not very smooth, I use a double Pareto-Lognormal distribution to approximate the empirical distribution of skills. This distribution combines lognormal distribution with heavy Paretian tails and replicates the empirical distribution reasonably well.[13] When computing the optimal allocations, I discretize the continuum of skills by a vector of 300 skill levels.

[11] See the evidence in Browning, Hansen and Heckman [6] or Trostel [28].

[12] One could then use the differentiated lump-sum taxes, as shown by Berliant and Ledyard [5].

[13] See Jorgensen and Reed [18] for the definition of the distribution.

To calibrate the distribution of skills I first construct the U.S. income tax code. I use the NBER TAXSIM program to construct the effective federal marginal income tax schedule for calendar year 1992. The agent in the model is supposed to represent a household. Thus I restrict attention to married couples with two children. I adjust the income tax schedule by adding state income tax and sales tax as a linear tax. Tax rates for the state income tax and sales tax were obtained by dividing government receipts from these taxes by labor income and consumption respectively and their values were 2.78% and 7.06%. The government finances its expenditures and all the remaining resources are rebated back as lump-sum transfers to balance the budget. It is assumed that government expenditures are equal to 25% of the aggregate income and so the consumption to income ratio is 0.75.

When solving for the efficient allocations I follow the following numerical procedure. I fix the utility allocation u and the Lagrange multiplier on the resource constraint λ. I then compute the cumulative distortion function $X_{u,\lambda}$ and adjust u_0 until this function satisfies $X_{u,\lambda}(\overline{\theta}) = 0$. I solve the dynamic program (13) for each agent and generate the sequence of optimal human capital and labor supply allocations for 250 periods. The envelope condition (4) is then used to compute a new utility allocation Tu. I repeat the procedure until $\|Tu - u\| \leq \varepsilon$ for some error tolerance ε. After that, I check if the resource constraint holds with equality (up to an error tolerance). If not, I update the Lagrange multiplier λ until the resource constraint is satisfied with equality.

Figure 1 plots the optimal marginal tax rates in periods 1, 50, and 250 (when the economy is approximately in a steady state).[14] It shows that marginal income tax rates decrease over time. They decrease more or less uniformly, by the same amount for each income level. The largest decreases are concentrated in the initial periods. The intuition is that it is the future marginal tax rate that drives the investment in human capital. Current marginal tax rates are therefore less important for the investment in human capital. They matter only for current labor supply and the social planner can afford to set them higher.

[INSERT FIGURE 1 HERE]

[14] Marginal income tax schedule in period 1 appears to have several kinks. This is because the current U.S. marginal tax rates are not smooth as well and this property is inherited by the calibrated initial distribution of human capital.

Figure 2 confirms this result by plotting the average optimal marginal income tax rates for all periods. They decrease by more than 10 percentage points, from about 33% to about 23%.

[INSERT FIGURE 2 HERE]

Aggregate income increases in all periods after the tax reform. In the first period, the increase is aggregate income is caused only by the adjustment of labor supply. The static gain turns out to be fairly small, about 1.7% of initial aggregate income. The dynamic gain is the long run increase in aggregate income, when the human capital is fully adjusted. Figure 3 shows that this gain is much larger: in the long run, the aggregate income increases by as much as 29.8% of initial aggregate income. Thus, the dynamic gain is about 17 times larger than the static gain.

[INSERT FIGURE 3 HERE]

Figure 3 also shows how the pattern of aggregate borrowing and saving in the economy. The economy runs a deficit in the first 16 periods. After that, the production abilities of the economy are improved and the economy starts paying off the deficit.

One can decompose the overall gain in aggregate income between the increase in average human capital, increase in average labor supply and the correlation between individual productivity, labor supply and human capital. The simulations show that both human capital and labor supply increase on average by 8.8%. Since individual increases in human capital and labor supply are positively correlated, as well as correlated with individual productivity, the total gain is much larger.

Figure 4 reveals how human capital changes are distributed across population. It depicts the cumulative average human capital for periods 1, 50, and 250 as a fraction of median human capital in period 1. Initially, the low skilled agents have too much human capital and so their human capital depreciates. This is more than compensated by the gain of high skilled agents.

[INSERT FIGURE 4 HERE]

Finally, Figure 5 shows the pattern of individual's savings across time. The high skill agents expect that their human capital and income will increase over time and so prefer to borrow from the future in order to smooth their period utility. The low skill agents find it in general optimal to have exactly the opposite pattern of saving and borrowing. However, their savings rate is not monotone, because there are two opposing effects that determine their labor supply and income. First, their human capital depreciates and so low skill agents tend to decrease their labor supply and income over time. This increases their savings. Second, marginal income taxes decline over time and so their labor supply and income increases, and savings decrease. The first effect is in general stronger but in some time periods the second effect can become dominant. If this is the case, then the savings rate can temporarily increase.[15] The simulations also show that aggregate savings of all individuals are first negative and later positive. Thus, the pattern found for high skilled agents tends to prevail in the aggregate.

[INSERT FIGURE 5 HERE]

6.1 Implementation Verification Procedure

I now describe a procedure that will verify whether the candidate allocation $\tilde{\sigma}$ can be implementable by current income taxes with unrestricted borrowing and saving. Since this test is stronger than the test of incentive compatibility, if the verification procedure succeeds, the allocation will be incentive compatible as well.

The condition to be verified is the incentive compatibility constraint in the mechanism with credit markets and limited record keeping (17). An allocation will pass the implementation verification procedure if the agent chooses a truthful sequence of reports and the recommended sequence of human capital. To check numerically whether (17) holds, I formulate a recursive version of (17). Let $\Omega_t(h, \theta)$ be $\theta-$type agent's value of having human capital h at the beginning of period t. Principle of optimality implies that the value function

[15]The savings rate of low skill agents is in general much lower than the savings rate of high skill agents. Therefore, the savings rate of low skilled agents is multiplied by 100 in Figure 5 to make it comparable to the savings rate of high skill agents.

$\Omega_t : [0, \bar{h}] \times \Theta \to \mathbf{R}$ satisfies

$$\Omega_t(h, \theta) = \max_{\hat{\theta}, h'} \{c_t(\hat{\theta}) + v(1 - \frac{y_t(\hat{\theta})}{\theta h} - S(h, h') + \beta \Omega_{t+1}(h', \theta)\}$$

and that an allocation will pass the implementation verification procedure if

$$\{\theta, h_{t+1}(\theta)\} \in \arg\max_{\hat{\theta}, h'} \{c_t(\hat{\theta}) + v(1 - \frac{y_t(\hat{\theta})}{\theta h_t(\theta)} - S(h_t(\theta), h') + \beta \Omega_{t+1}(h', \theta)\}. \quad (19)$$

The equivalence between the recursive formulation (19) and its sequence counterpart (17) is easy to show as long as $c_t(\hat{\theta})$ and $y_t(\hat{\theta})$ are bounded by above, which in turn happens when the upper bound on the skill distribution $\bar{\theta}$ is finite. Since all numerical simulations have to assume some upper bound no matter what the underlying distribution is, this assumption is innocuous to make.[16]

Let $\{\hat{\theta}_t^v(h, \theta), h_t'^v(h(\theta), \theta)\}_{t=0}^{\infty}$ be the optimal policy functions, where $\hat{\theta}_t^v : [0, \bar{h}] \times \Theta \to \Theta$ is the optimal report policy in period t and $h_t'^v : [0, \bar{h}] \times \Theta \to [0, \bar{h}]$ is the optimal next period human capital policy in period t. The verification procedure will be successful if for $t \geq 0$ and for all $\theta \in \Theta$, $|\hat{\theta}_t^v(h_t(\theta), \theta) - \theta| < \varepsilon$ where ε is the error tolerance.[17]

I discretize the human capital state space by a vector and compute Ω_t for each $t = 1..250$ and each skill level from the skill grid. I allow the agents to choose any feasible level of next period human capital and linearly interpolate the value function between gridpoints. I allow the agents to report any feasible skill level from the skill grid. If the verification procedure fails for a given human capital grid, I will refine the grid until the verification procedure is successful. Obviously, if refining the grid and increasing the support for human capital does not help, then the verification procedure fails and the allocations are not incentive compatible.

I choose the error tolerance ε to be such that the verification procedure is successful even if the agent misses the truthful report by one gridpoint. The reason is that when (18) is computed, one needs to numerically approximate the integral as well as the derivatives of human

[16] I do not show the equivalence formally since it is very similar to the proof of principle of optimality contained in Section 3.2.

[17] It is not necessary to check as well if next period human capital deviates from the recommended one because, conditional on the report being truthful, the Euler equation in human capital is sufficient for the optimum.

capital. This creates numerical errors which may cause slight deviations from the optimum. Given that, the candidate allocation $\tilde{\sigma}$ passes the implementation verification procedure for all skill levels and time periods.[18] Thus, the efficient allocation can be implemented by current income taxes and is also incentive compatible.

7 Conclusions

This paper analyzes efficient allocations in a dynamic economy where private information skill shocks are permanent and human capital is endogenous and unobservable by the government. The main contribution is to provide a tractable framework which can be used to analyze these allocations as well as to provide an algorithm how to compute them numerically. I also discuss the problem of implementation of the efficient allocations in a competitive equilibrium and show it may be possible to do so in a very simple way: with history independent taxes and unlimited borrowing and saving.

Numerical simulations reveal several interesting results: Marginal income taxes should decrease over time and this decrease is more or less uniform over all ranges of income. The production gains from the adjustment of human capital are much larger than the gains from adjustment of labor supply. The economy overall runs a deficit during the first 16 periods to finance the tax reform. Individual savings vary by productivity, high skill agents tend to borrow first, while low skill agents tend to save.

These results rely on several assumptions: The government can freely borrow and save at an exogenously given interest rate, there are no income effects and the skill shocks are permanent. What happens if these assumptions are relaxed?

The assumption of permanent shocks is crucial for one reason: It ensures that the cumulative distortion is constant over time. If private information about skills is revealed only gradually, this result will no longer hold. In such case, there are two sources of dynamics: the dynamics of private information and the dynamics of individual specific state variable. Whether a problem like this can be solved successfully depends on one's ability to write

[18]Deviations by 1 gridpoint occur in less than 0.1% of all periods and skill levels for which the value function was computed.

the Decomposition Theorem recursively. Similar complications arise when the assumption that the government has access to perfect credit markets is relaxed. The main implication is again that the time profile of cumulative distortions will no longer be constant.

When the assumption of no income effects is relaxed, one faces three types of problems. First, the individual's distorted problem is complicated by the fact that it also depends on individual's consumption. Thus, the dimension of the state space increases. Second, one may face additional complications with the implementation since a joint deviation in savings, human capital investment and labor supply may now be profitable. It thus appears less likely that taxes depending only on current income could be used. And, finally, the implementation verification procedure would be harder to compute because one would need to keep track of individual savings explicitly when checking the deviations. All these extensions are nevertheless interesting and are left for future research.

References

[1] Abraham, A. and Pavoni, N. (2004). "Efficient Allocations with Moral Hazard and Hidden Borrowing and Lending." University of Rochester, unpublished manuscript.

[2] Albanesi, S. (2006). "Optimal Taxation of Entrepreneurial Capital with Private Information." Columbia University, unpublished manuscript.

[3] Albanesi, S. and Sleet, Ch. (2003). "Dynamic optimal Taxation with Private Information." Discussion Paper 140, Federal Reserve Bank of Minneapolis.

[4] Battaglini, M. and Coate, S. (2003). "Pareto Efficient Income Taxation with Stochastic Abilities." Princeton University, unpublished manuscript.

[5] Berliant, M. and Ledyard, J. (2004). "Optimal Dynamic Nonlinear Income Taxes with No Commitment." Washington University, unpublished manuscript.

[6] Browning, M., Hansen, L.P. and Heckman, J.J. (2000). "Micro Data and General Equilibrium Models." (In J. B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics* Vol 1a, Chapter 8).

[7] Da Costa, C. and Werning, I. (2005). "On the Optimality of the Friedman Rule with Heterogeneous Agents and Non-Linear Income Taxation." MIT, unpublished manuscript.

[8] Diamond, P. (1998). "Optimal Income Taxation: an Example with a U-Shaped Pattern of Optimal Marginal Tax Rates." *The American Economic Review,* **88**, 83-95.

[9] Diamond, P. and Mirrlees, J.A. (2002). "Optimal Taxation and the Le Chatelier Principle." MIT, unpublished manuscript.

[10] Fahri, E. and Werning, I. (2005). "Inequality, Social Discounting and Progressive Estate Taxation." MIT, unpublished manuscript.

[11] Golosov, M., Kocherlakota, N. and Tsyvinski, A. (2003). "Optimal Indirect and Capital Taxation." *Review of Economic Studies*, 70, 569-588.

[12] Golosov, M. and Tsyvinski, A. (2006). "Optimal Taxation with Endogenous Insurance Markets." NBER Working Paper #11185.

[13] Grochulski, B. and Piskorski, T. (2005). "Optimal Wealth Taxes with Risky Human Capital." Working Paper 05-13, Federal Reserve Bank of Richmond.

[14] Heckman, J.J. (1975). "Estimates of a Human Capital Production Function Embedded in a Life-Cycle Model of Labor Supply." (In N. Terlekyj, ed., *NBER Studies in Income and Wealth*, **40**, New York, Columbia University Press).

[15] Heckman, J.J., Lochner, L. and Taber, Ch. (1998). "Explaining rising wage inequality: Explorations with a Dynamic General Equilibrium Model of Labor Earnings with Heterogeneous Agents." *Review of Economic Dynamics* **1**, 1-58.

[16] Heckman, J.J., Lochner, L. and Taber, Ch. (1998). "Tax Policy and Human-Capital Formation." *The American Economic Review*, **88**, Papers and Proceedings of the Hundred and Tenth Annual Meeting of the American Economic Association, 293-297.

[17] Heathcote, Jonathan, Kjetil Storesletten and Giovanni L. Violante (2004). "The Macroeconomic Implications of Rising Wage Inequality in the United States.", Georgetown University, unpublished manuscript.

[18] Jorgensen, M. and Reed, W.J. (2004). "The Double Pareto-Lognormal Distribution - A New Parametric Model for Size Distributions." *Com. Stats -Theory & Methods*, **33**.

[19] Kapicka, M. (2004). "Optimal Taxation and Human Capital Accumulation." UCSB, unpublished manuscript.

[20] Kotlikoff, L., Smetters, K. and Walliser, J. (2001). "Finding a Way Out of America's Demographic Dilemma." Boston University, unpublished manuscript.

[21] Kocherlakota, N. (2005). "Zero Expected Wealth Taxes: A Mirrlees Approach to Dynamic Optimal Taxation." *Econometrica*, 73, 1587-1621.

[22] Kocherlakota, N. (2004). "Figuring out the impact of hidden savings on optimal unemployment insurance." *Review of Economic Dynamics*, **7**, 541-554.

[23] Mirrlees, J.A. (1971). "An Exploration in the Theory of Optimum Income Taxation." *Review of Economic Studies,* **38**, 175-208.

[24] Mirrlees, J.A. (1976). "Optimal Tax Theory: A Synthesis." *Journal of Public Economics*, **6,** 327-358.

[25] Mirrlees, J.A. (1986). "The Theory of Optimal Taxation." (In K.J. Arrow and M.D. Intriligator, eds., *Handbook of Mathematical Economics*, vol. III, chapter 24).

[26] Saez, E. (2001). "Using Elasticities to Derive Optimal Income Tax Rates." *Review of Economic Studies,* **68**, 205-229.

[27] Stokey, N.l., Lucas, R.E. and Prescott R.C. Recursive Methods in Economic Dynamics. Cambridge, Harvard University Press.

[28] Trostel, P A. (1993). "The Effect of Taxation on Human Capital." *Journal of Political Economy*, **101**, 327-350.

[29] Werning, I. (2001). "Dynamic Optimal Taxation." MIT, unpublished manuscript.

8 Appendix

Proof of Proposition 1. Let λ be the Lagrange multiplier on the resource constraint (1) and $\hat{\mu}(\theta, \hat{\theta})$ be the Lagrange multiplier on the incentive compatibility constraint (2). The necessary first order condition in $c_t(\theta)$ is

$$U'[u_t(\theta)]f(\theta) = \lambda f(\theta) + U'[u_t(\theta)]\int_{\Theta}[\hat{\mu}(\theta, \tilde{\theta}) - \hat{\mu}(\tilde{\theta}, \theta)]d\tilde{\theta}$$

and since nothing but $u_t(\theta)$ depends on time in this equation, $u_t(\theta)$ must be independent of time as well. ■

Proof of Proposition 2. To show that (5) is necessary, differentiate $V(h_0, \theta, \theta)$ w.r.t. h_{t+1}:

$$\begin{aligned} \frac{\partial}{\partial h_{t+1}} \sum_{t \geq 0} \beta^t U[c_t(\theta) + v(1 - \frac{y_t(\theta)}{\theta h_t} - g(h_t, h_{t+1})] &= 0 \\ -U'(u_t)\frac{v_t'}{G_{s_t}} + U'(u_{t+1})v_{t+1}'(\frac{y_{t+1}}{\theta h_{t+1}^2} + \frac{G_{h_{t+1}}}{G_{s_{t+1}}}) &= 0. \end{aligned}$$

Using the fact that u_t is constant over time and that $\frac{y_{t+1}}{\theta h_{t+1}^2} = \frac{l_{t+1}}{h_{t+1}}$, (5) follows.

To show (4), apply the envelope theorem on the incentive compatibility constraint on (2):

$$V_\sigma(h_0, \theta, \theta) = \frac{U[u(\theta)]}{(1-\beta)} = \int_{\underline{\theta}}^{\theta} \sum_0^\infty \beta^t U'(u(\varepsilon))v_t'(\varepsilon)l_t(\varepsilon)\frac{d\varepsilon}{\varepsilon} + V_\sigma(h_0, \underline{\theta}, \underline{\theta}). \tag{20}$$

where $v_t'(\varepsilon) = v'(1 - l_t(\varepsilon) - g(h_t(\varepsilon), h_{t+1}(\varepsilon))$. I will skip the proof that the envelope theorem applies and (20) holds. It is a fairly straightforward modification of the proof that can be found in ([19]) or in Mirrlees ([25]). Rewriting (20), one gets

$$u(\theta) = U^{-1}[(1-\beta)\int_{\underline{\theta}}^{\theta} \sum_0^\infty \beta^t U'(u(\varepsilon))v_t'(\varepsilon)l_t(\varepsilon)\frac{d\varepsilon}{\varepsilon} + U_0].$$

where $U_0 = (1-\beta)V_\sigma(h_0, \underline{\theta}, \underline{\theta})$. I will use the fact that, for any differentiable function $\varphi(\theta)$ one has

$$U^{-1}(\varphi(\theta)) = \int_{\underline{\theta}}^{\theta} U^{-1'}(\varphi(\varepsilon))\varphi'(\varepsilon)d\varepsilon + U^{-1}(\varphi(0)) = \int_{\underline{\theta}}^{\theta} \frac{\varphi'(\varepsilon)}{U'[U^{-1}(\varphi(\varepsilon))]}d\varepsilon + U^{-1}(\varphi(0))$$

I apply this formula for $\varphi(\theta) = (1-\beta)\sum_0^\infty \beta^t \int_0^\theta U'(u(\varepsilon))v_t'(\varepsilon)l_t(\varepsilon)\frac{d\varepsilon}{\varepsilon} + U_0$ which is clearly differentiable in θ. Note that $U'[U^{-1}(\varphi(\theta))] = U'(u(\theta))$ and so both terms involving $U'(u(\theta))$ cancel out. What remains is equation (4), where $u_0 = U^{-1}(\varphi(0)) = U^{-1}(U_0)$. ■

Proof of Theorem 4. Necessity of the conditions is obvious. For sufficiency, let $\sigma^* = \{u^*, l_t^*, h_{t+1}^*\}$, together with $\hat{\lambda}^*$ and $\mu^* = \lambda^* - U'(u^*)$ be an allocation that satisfies the conditions of the theorem. Let $\sigma = \{u_t, l_t, h_{t+1}\}$ be an alternative allocation that solves the relaxed social planner's problem. I will prove sufficiency of the conditions of the theorem by showing that allocation σ^* delivers expected utility at least as high as σ.

Since U is increasing and concave, I have $U(u^*) - U(u) \geq U'(u^*)(u^* - u)$. Hence

$$\int_\Theta [U(u^*) - U(u)] f d\theta \geq \int_\Theta U'(u^*)(u^* - u) f d\theta$$

and so it will suffice to show that the right hand side of the inequality is positive.

To show this, substract first the two corresponding envelope conditions. I have

$$u(\theta) - u^*(\theta) = u(0) - u^*(0) + (1-\beta)\int_{\underline{\theta}}^{\theta} \sum_{t\geq 0}^{\infty} \beta^t (v_t' \frac{l_t}{\varepsilon} - v_t'^* \frac{l_t^*}{\varepsilon}) d\varepsilon.$$

Multiply both sides by $\mu^*(\theta)f(\theta)$ and integrate over Θ :

$$\int_\Theta \mu^*(u - u^*) f d\theta = \int_\Theta \mu^* f \int_{\underline{\theta}}^{\theta} \sum_{t\geq 0}^{\infty} \beta^t (v_t' \frac{l_t}{\varepsilon} - v_t'^* \frac{l_t^*}{\varepsilon}) d\varepsilon$$

since $[U(u(0)) - U(u^*(0))] \int_\Theta \mu^* f d\theta = 0$. Reversing the order of integration, I get

$$\begin{aligned}
\int_\Theta \mu^*(u^* - u) f d\theta &= (1-\beta)\lambda^* \int_\Theta X^*(\theta) \{ \sum_{t\geq 0}^{\infty} \beta^t (v_t'^* \frac{l_t^*}{f\theta} - v_t' \frac{l_t}{f\theta}) \} f d\theta \\
&= (1-\beta)\lambda^* \int_\Theta \sum_{t\geq 0}^{\infty} \beta^t \{ [\theta h_t l_t + v_t - v_t' l_t \frac{X^*(\theta)}{f\theta}] - [\theta h_t^* l_t^* + v_t^* - v_t'^* l_t^* \frac{X^*(\theta)}{f\theta}] \} f d\theta \\
&\quad + (1-\beta)\lambda^* \int_\Theta \sum_{t\geq 0}^{\infty} \beta^t [\theta h_t^* l_t^* + v_t^* - (\theta h_t l_t + v_t)] f d\theta \\
&. \leq (1-\beta)\lambda^* \int_\Theta \sum_{t\geq 0}^{\infty} \beta^t [\theta h_t^* l_t^* + v_t^* - (\theta h_t l_t + v_t)] f d\theta.
\end{aligned}$$

The inequality follows from the fact that $\{l_t^*(\theta), h_{t+1}^*(\theta)\}$ maximizes individual's distorted problem and so the first term on the right hand side of the second equality is smaller or equal to zero. By using the fact that the resource constraint holds with equality for both σ^* and σ and rewriting the right-hand side of the inequality, one gets that

$$\int\limits_{\Theta} \mu^*(u^* - u) f d\theta \leq \lambda^* \int\limits_{\Theta} (u^* - u) f d\theta.$$

I will now use the fact that $\mu^* = \lambda^* - U'(u^*)$ and so

$$\int\limits_{\Theta} [\lambda^* - U'(u^*)](u^* - u) f d\theta \leq \lambda^* \int\limits_{0}^{\bar{\theta}} (u^* - u) f d\theta.$$

Upon cancelling terms and rearranging the terms, I get that

$$\int\limits_{\Theta} U'(u^*)(u^* - u) f d\theta \geq 0$$

which completes the proof. ■

Proof of Theorem 5. Applying Proposition 1 of [19] for the period utility of the form $c + v(l)$ (rather than $U(c + v(l))$) implies that an allocation $\sigma \in \hat{\Sigma}^{IFCM}$ if and only if there is a tax policy $\mathcal{T}$ such that for all $t \geq 0$,

$$\begin{aligned} y_t &= y_t^{\mathcal{T}}, \\ h_{t+1} &= h_{t+1}^{\mathcal{T}}, \\ u_t &= y_t^{\mathcal{T}}(\theta) - T_t(y_t^{\mathcal{T}}(\theta)) + v(1 - \frac{y_t^{\mathcal{T}}(\theta)}{\theta h_{t+1}^{\mathcal{T}}(\theta)} - g(h_t^{\mathcal{T}}(\theta), h_{t+1}^{\mathcal{T}}(\theta)). \end{aligned} \tag{21}$$

That is, there is a 1 to one correspondence between the set of incentive compatible allocations with limited record keeping and the set of allocations implementable with current income taxes and no borrowing and saving.

The equilibrium allocation in a mechanism with credit markets and limited record keeping σ_σ^* satisfies

$$\begin{aligned} u^*(\theta) &= (1-\beta)\sum_{t=0}^{\infty}\beta^t u_t(\theta) \\ &= (1-\beta)\sum_{t=0}^{\infty}\beta^t [y_t^{\mathcal{T}}(\theta) - T_t(y_t^{\mathcal{T}}(\theta)) + v(1 - \frac{y_t^{\mathcal{T}}(\theta)}{\theta h_{t+1}^{\mathcal{T}}(\theta)} - g(h_t^{\mathcal{T}}(\theta), h_{t+1}^{\mathcal{T}}(\theta))] \\ &= (1-\beta)\Omega^{\mathcal{T}}(h_0, \theta) \\ &= u^{\mathcal{T}}(\theta) \end{aligned}$$

where the first equality follows from the fact that σ_σ^* is an equilibrium allocation in a mechanism with credit markets and limited record keeping, the second equality follows from (21) and the last two equalities follow from the agent's problem in a market economy. Since any $\sigma^* \in \hat{\Sigma}^*$ is implied by some $\sigma \in \hat{\Sigma}^{IFCM}$, the result follows. ■

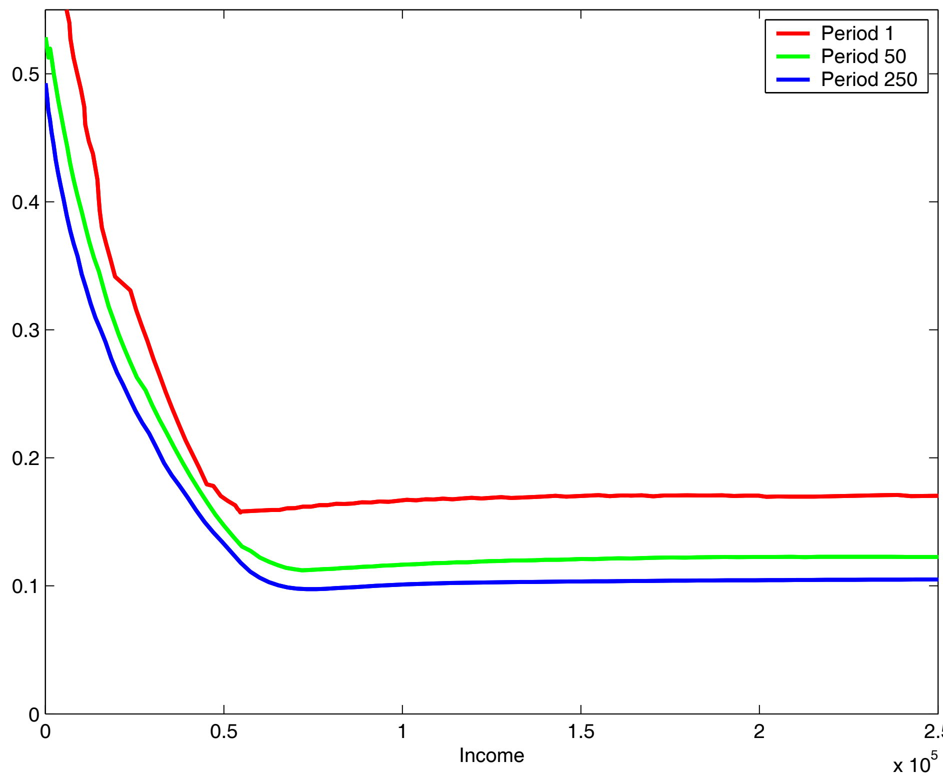

Figure 1: Marginal Income Tax Rates

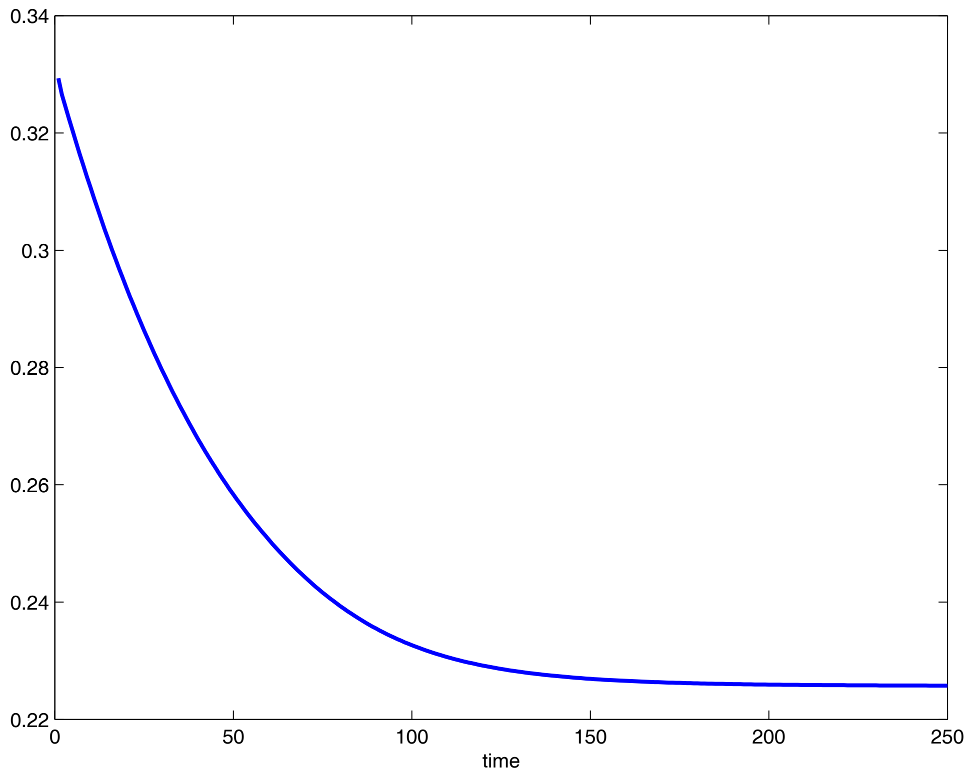

Figure 2: Average Marginal Tax Rates

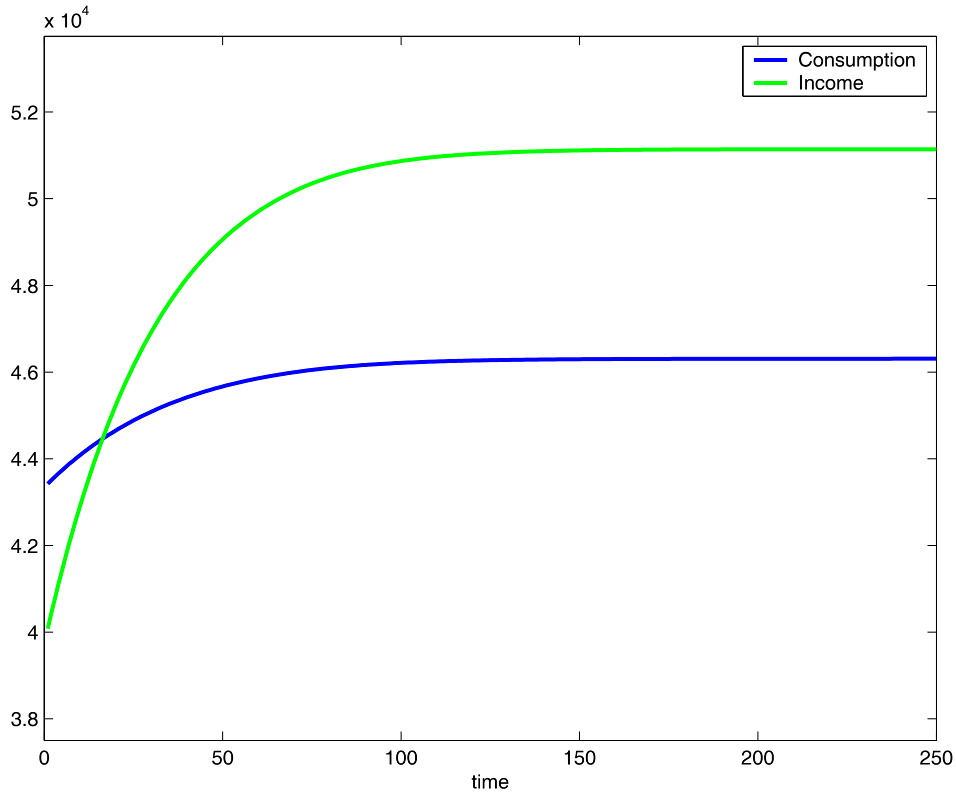

Figure 3: Aggregate Consumption and Income

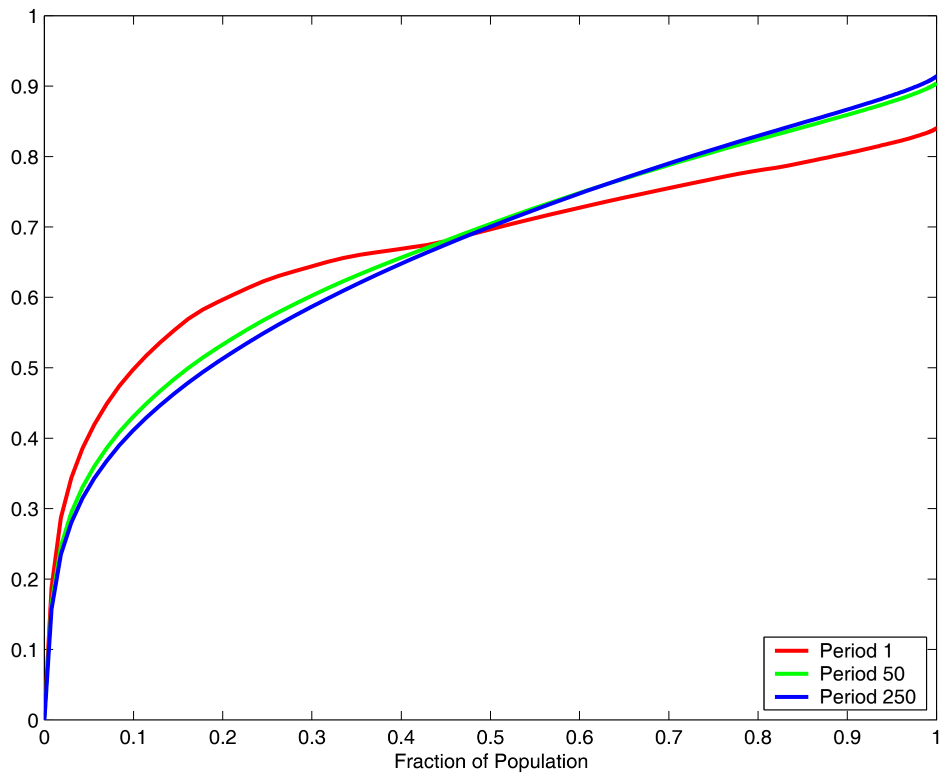

Figure 4: Cumulative Average Human Capital

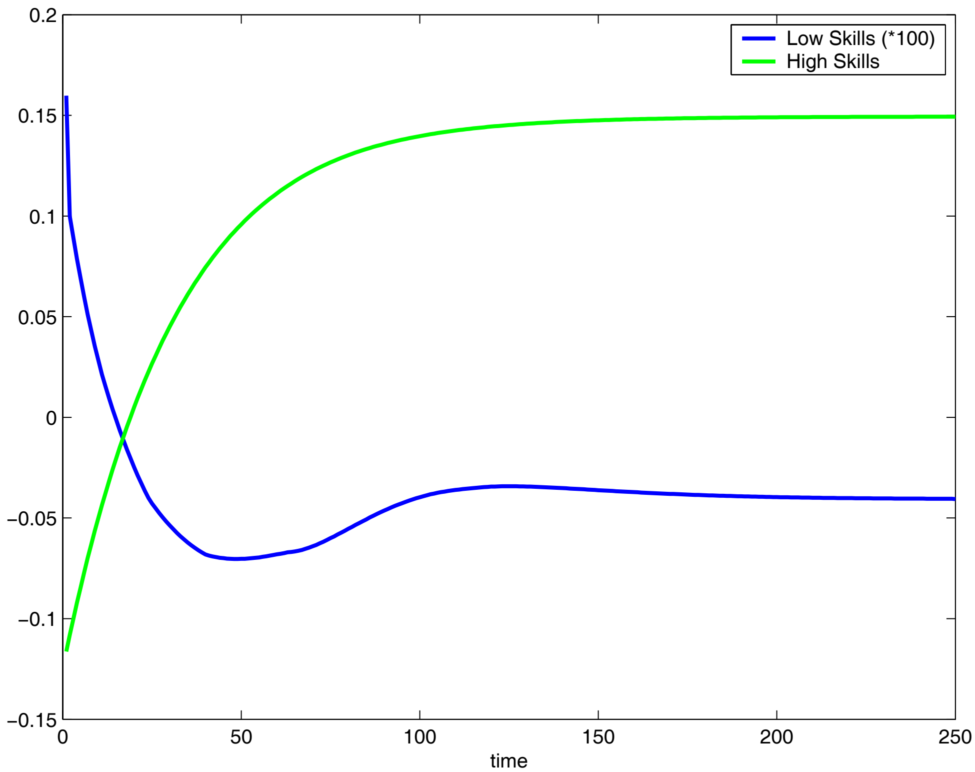

Figure 5: Individual Saving Rates