The Role of Debt and Equity Finance over the Business Cycle

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Abstract

Net equity issuance occurs frequently and is quantitatively important for both small and large publicly traded firms. Moreover, we show that net equity and net debt issuance are positively correlated and both are procyclical for small firms. For large firms net equity issuance is neither cyclical nor correlated with debt issuance. We extend the existing business cycle models with agency costs in two ways. First, we relax the standard assumptions of linearity and full depreciation. Consequently, variables such as the default probability and leverage will depend on firm size. It also means that an increase in net worth reduces the default probability (instead of leaving it unchanged). Second, we relax the standard assumption that firms cannot attract outside equity. In our model, aggregate shocks are propagated as in the model without equity issuance, but in contrast to the standard model they are also magnified and the default rate is countercyclical. Moreover, our model is consistent with the observed cyclical behavior of firms’ financing sources for both small and large firms.

Preliminary, please do not quote or circulate.

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1 Introduction

The idea that changes in agency costs over the business cycle are important to understand the severity and persistence of business cycles has obtained a lot of attention in the literature. There are now several empirical and theoretical papers to support the basic idea. On the empirical side there is the classic paper by Gertler and Gilchrist (1994) that shows that sales of small firms are more sensitive to a monetary tightening than sales of large firms. On the theoretical side there are numerous models in which shocks are propagated in subsequent periods through the net worth channel.\(^1\) That is, an improvement in aggregate productivity increases firms’ net worth, which alleviates the premium on external finance and, consequently, increases lending and firm size.

Incorporating agency problems into dynamic stochastic general equilibrium (DSGE) models improves some empirical predictions of this class of models. But one should not overlook that these models have problems.

- First, in many models there are no defaults occurring in equilibrium, or if defaults do occur, the default rate is procyclical. Both predictions are, of course, at odds with the data.

- Second, it is standard to assume that the technology of the firm that faces agency costs is characterized by linear technology and capital that fully depreciates in one period. Both assumptions are unrealistic. Moreover, linear technology implies that small firms are simply scaled down versions of large firms and all firms have, for example, the same default rate. But the empirical evidence suggests that the cyclical behavior of small and large firms is quite different and that agency costs affect small firms more strongly than large firms.

- Third, although these models are successful in generating a hump-shaped response for real activity (propagation) they are not successful in magnifying external shocks.

\(^1\) Most notably Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), and Bernanke, Gertler, and Gilchrist (1999).
In fact, they frequently generate responses for output that are smaller than those implied by the version of the model without agency problems.

- Fourth, theoretical models typically assume that the owner and manager of the firm are the same agent and that the only type of external finance is debt finance. This means that net worth only increases through retained earnings. One might think that this is not a bad assumption because seasoned equity issues are rare. Firms issue equity frequently, however, and equity issuance is an important financing source for increases in firm size.\(^2\) Note that although seasoned equity issues are indeed rare, there are many other ways through which firms can issue equity.\(^3\)

In this paper we accomplish two things. In the first place we document the cyclical behavior of default rates and financing sources (debt, retained earnings, and equity issues). Not surprisingly, the data show that default rates are countercyclical. To analyze the cyclical behavior of firm finance, we use US data for small and large listed firms.\(^4\) For small firms, we find that net-equity issuance is positively correlated with debt issuance and that both are procyclical. This suggests that net-equity issuance could reinforce the net-worth channel if one allows this channel to operate in the model. For large firms net-equity issuance is not correlated with either debt issuance or the business cycle.

The second contribution of this paper is to construct a DSGE model with a firm problem that avoids the problems discussed above. Our starting point is the standard costly state verification (CSV) problem in which firms with limited net worth borrow funds in a competitive market to finance investment. The modifications we consider are the following.

- We allow firms that face agency problems to have a technology with diminishing returns to capital, whereas the standard assumption in the literature is that technology is linear. The immediate consequence of diminishing returns is that firm

\(^2\text{Fama and French (2005) and Frank and Goyal (2005).}\)

\(^3\text{For example, firms also issue equity in mergers, private placements, convertible debt, warrants, and employee stock options.}\)

\(^4\text{In the appendix we show that the results are very similar when Canadian data is used.}\)
size matters. Also, in models with linear technology changes in net worth do not affect the default rate. With diminishing returns, however, the model generates the appealing property that the default rate is decreasing with net worth.

- The first modification makes it possible to assume that agency costs are present both in the production of consumption and investment commodities. Standard procedure is to assume that consumption is produced with a non-linear technology by firms that do not face agency costs and that only firms that produce investment commodities face agency costs. But changes in the amount of investment commodities produced only has a small effect on aggregate activity. The reason is that a large share of capital is undepreciated capital so that even if investment increases substantially the percentage increase in aggregate capital is much smaller.\(^5\) Moreover, because of diminishing returns an increase in capital will lead to a smaller percentage increase in aggregate output.

- We allow firms to issue equity. We model equity issuance in a simple way by assuming that there is a linear-quadratic cost of issuing equity. The costs of issuing equity are calibrated so that the behavior of equity issuance matches its empirical counterpart. We address the question whether this dampens or reinforces the net-worth channel. For example, it is possible that firms will choose to issue more equity when times are good to take advantage of the increased productivity and by doing so reinforce the net-worth channel. Alternatively, it is possible that firms will issue less equity because it is easier to obtain debt finance. Key in answering this question are the following two aspects. The first aspect is what happens with agency costs when aggregate productivity increases. We show that for standard specifications of technology (but for constant as well as for diminishing returns) that firms would like to issue more equity when aggregate productivity increases. The second aspect is how the cost of issuing equity varies over the cycle. Consistent with the work by Baker and Wurgler (2000) and Lamont and Stein (2006) we allow for equity issuance

\(^5\)Of course, when the presence of agency costs dampen productivity shocks as in Carlstrom and Fuerst (1997) then the limited impact of the investment sector on aggregate real activity is an advantage.
costs to be countercyclical.

With the modified framework we can show the following.

- The model predicts that in response to a positive aggregate productivity shock the
default rate not only declines on impact but also keeps on declining for several
periods.
- The model not only propagates shocks but also generates output responses that are
more than twenty percent larger than those generated by the version of the model
without agency costs.
- The response for small firms is much stronger than the response for large firms. In
particular, the response of output produced by small firms is more than two times
as large as the response of output produced by large firms, which is similar to the
response of output in the model without frictions.
- As in the data, small firms respond to a positive aggregate shock by both increasing
net-equity issuance and debt issuance, whereas large firms mainly increase debt
issuance.

The model considered in the literature is characterized by linear technology and full
depreciation. An increase in aggregate productivity then leads to—keeping everything
else equal—an increase in the default probability. As part of our analysis, we show that
this property remains present when technology is nonlinear (diminishing returns) and
capital does not depreciate fully. For some technologies, however, this is not the case. For
example, when there is a large enough fixed cost then the default rate does decrease when
aggregate productivity increases. In our modified framework with equity issuance we can
generate a countercyclical default rate without relying on these alternative technologies.

The organization of this paper is as follows. In the next section we discuss the cyclical
behavior of default rates and document how the importance of alternative financing sources
move over the business cycle. In Section 3, we discuss the standard (static) costly state
verification problem for different assumptions about technology and analyze whether the
default rate decreases or increases with technology. In Section 4, we compare the responses of aggregate capital and real activity in the static model with and without frictions. In Section 5, we discuss whether equity issuance decreases or increases with aggregate productivity and show that this is related to the question on whether agency costs decrease or increase with aggregate productivity. In Section 6, we present the full dynamic model and in Section 7 we discuss the results. The last section concludes.

2 Cyclical Properties of Financing Sources

The goal of this section is twofold. The first goal is to analyze how asset growth is financed over the business cycle. The financing sources considered are net debt issues, net equity issues and retained earnings. The second goal is to document the business cycle dynamics of default rates using data for publicly traded firms.

2.1 Data

The balance sheet data is from Compustat and consist of annual balance sheet and cash flow statement information from 1971 through 2004. Before 1971 both the coverage as well as the data availability is very incomplete in Compustat. The sample includes firms listed on the three U.S. exchanges (NYSE, AMEX and Nasdaq) with a non-foreign incorporation code. Following standard practise, we exclude financial firms (SIC codes 6000-6999) and utilities (4900-4949). We also exclude firms involved in major mergers (Compustat footnote code AB) from the whole sample. The balance sheet data are transformed to constant dollars using the CPI for all urban consumers.

For the Compustat sample the variables of interest are aggregate net debt issues, aggregate retained earnings and aggregate net equity issues as well as the aggregate increase in assets.\footnote{The aggregate series are size-weighted averages of the firm’s individual series.} We construct the variables as in Fama and French (2005). Hence, net debt issues, $\Delta L$, correspond to the change in liabilities (Compustat data item 181) between years $t$ and $t - 1$. Retained earnings, $\Delta RE$, is the difference between Compustat’s adjusted value of balance sheet retained earnings (36) between years $t$ and $t - 1$. The change
in the book value of assets, $\Delta A$, is the change in assets (6) between years $t$ and $t-1$. The (book) measure of net equity issues, $\Delta SB$, is then given by

$$\Delta SB = \Delta A - \Delta L - \Delta RE.$$  \hspace{1cm} (1)

Fama and French (2005) argue that $\Delta SB$ is a noisy measure of stock issuance.\footnote{Book net equity issues are a noisy measure of stock issues for three reasons: pooling of interest mergers, employee stock options and stock dividends. See Fama and French (2005) for details.} An alternative is the market measure of net equity issues, $\Delta SM$, which is the change in the number of shares between years $t$ and $t-1$ multiplied by the average stock price. Below we report the results for both measures of net equity issues.

To document our stylized facts we further split the sample into two groups, small and large firms.\footnote{In the corporate finance literature is has been argued that to understand firms’ capital structure decisions it is important to look at small and large public firms separately. See, for example, Frank and Goyal (2003) and Fama and French (2005). For our purpose such a distinction is also important, since agency problems are likely to be more important for small firms.} We follow Frank and Goyal (2003) and define small firms as having total assets below the 33\textsuperscript{rd} percentile in each year. The average real book value of total assets for small firms is equal to 30 million (constant 2000) dollars, and the average number of employees is equal to 422 workers. Large firms are defined as having total assets above the 67\textsuperscript{th} percentile in each year. The average size of a large firm corresponds to 4173 million (2000) dollars. In our sample large firms employ, on average, 2200 workers. The group of small firms has an average of 703 observations per year. The group of large firms is slightly bigger with an average of 940 observations per year.

In addition to balance sheet data we also analyze data on debt defaults. The default rate series is from Moody’s. The annual default rate is defined as the number of defaults during a year divided by the number of outstanding issuers at the beginning of the year. Defaults include bankruptcy, missed interest payments and distressed exchange (workout).\footnote{In the period between 1995 and 2003, 77 per cent of defaults by U.S. nonfinancial corporations ended in bankruptcy (Moody’s).} This series does not include defaults on credit market instruments other than corporate bonds. Corporate bonds are the single most important credit market instru-
ment for U.S. nonfarm, nonfinancial corporations. In particular, in 2003, it accounted for 1/3 of total liabilities outstanding (excluding net worth) and close to 3/5 of all credit market instruments.\footnote{Based on calculations using the Flow of Funds Accounts for nonfarm nonfinancial corporate business (Table B.102). Half of liabilities are credit market instruments, defined as corporate bonds, bank loans, commercial paper, municipal securities, other loans and advances, and mortgages. The other half includes trade payables, taxes payable, and miscellaneous liabilities.}

Figure 12 shows the annual default rate with NBER recessions represented by the shaded areas in the figure. The annual default rate series is available at the monthly frequency and covers the period between 1971 and 2004. The figure shows sharp increases during economic downturns. The figure also shows that the behavior of the series in the first and the second half is quite different.

First, in the second half of the series there are on average much more defaults. In particular, whereas from 1986 to 2004 the yearly default rate averaged 2.2 per cent, from 1970 to 1985 the yearly default rate was just 0.6 per cent. One possible explanation is that during the sample period, the use of corporate debt has become much more common, mainly due to deregulation and financial innovation\footnote{Only in 1989 were commercial banks allowed to underwrite corporate bonds. Corporate equity underwriting was permitted for the first time in 1990. Before the Glass-Steagall Act of 1933 prohibited commercial banks from underwriting corporate securities and underwriting was restricted to top-tier investment banks. For more details see Gande, Puri, and Saunders (1999).} and that in the beginning of the sample corporate debt was mainly issued by firms with strong balance sheets that had a solid reputation. Moreover, the number of listed non-financial and non-utility firms in Compustat more than doubles during the period of 1971 through 2004. It is very well possible that the more recently listed companies have higher default rates. Our sample starts with 1530 firms in 1971 and ends with 3128 firms in 2004.

Second, in the second half of the sample the increase in the default series clearly starts before the start of the official recession and decreases only gradually after the end of the recession. Given that defaults were quite low in the first half of the sample, it is perhaps not surprising that during this period defaults only increased noticeably when the economic downturn was most severe, that is, during the official recession. These observations are
Notes: The default rate series is from Moody’s (mnemonic USMDDAIW in Datastream) and it is for all corporate bonds in the US. The plot shows the annual default rate, i.e., the number of defaults during a year divided by the number of outstanding issuers at the beginning of the year, adjusted by the number of rating withdrawals during the year.

also true when we scale the default rate by the amount outstanding of corporate debt, to control for the size of the corporate debt market. Below we report our results for two non-overlapping sample periods, 1970:Q1–1985:Q4 and 1986:Q1–2004:Q4, in addition to the entire sample.

2.2 Stylized facts

We first focus on the balance sheet data. Seasoned offerings of equity are rare, but there are many ways through which firms can issue equity (warrants, convertible debt, options, etc.). Fama and French (2005) and Frank and Goyal (2005) document that firms’ net equity issuance is frequently positive and quantitatively important. For example, on average, 44 per cent of firms made positive net equity issues ($\Delta SB > 0$) in the period between 1971 and 2004. This fraction increased to 54 per cent in the period between 1990 and 2004.
Hence, the first relevant stylized fact is:

**Stylized fact #1:** Non-zero equity issuance is a frequent event at the firm level and quantitatively important.

We now consider the cyclical behavior of the three sources of financing: debt issues, \( \Delta L/A \), retained earnings, \( \Delta RE/A \), and equity issues, \( \Delta SB/A \). We also look at the change in net worth, \( \Delta N = \Delta RE + \Delta SB \). Table 1 summarizes the contemporaneous correlation between the three sources of financing with asset growth and GDP growth. In particular, the comovement between asset growth and net debt issues is 0.85 for large firms and 0.77 for small firms. The comovement between asset growth and net equity issues is 0.27 for large firms and 0.81 for small firms. Thus, with respect to debt issues the comovement with asset growth is relatively homogeneous across firms whereas the comovement between net equity issues and asset growth is not. In particular, the contemporaneous correlation is considerably stronger for small publicly traded firms.

The table also reports the standard deviation of the change in each (scaled) finance source relative to the standard deviation of asset growth. Here we also find a striking difference between small firms and large firms. The relative volatility of debt issues is higher for large firms than for small firms, whereas the opposite is true for changes in net worth, that is, internal and external equity financing. In particular, the standard deviation of \( \Delta N/A \) is 72 per cent of the standard deviation of \( \Delta A/A \) for small firms, whereas for large firms this number is only 48 per cent.

The panels in Figure 2 plot asset growth for the corresponding firm category together with a financing source. The graphs illustrate the correlations results of Table 1. In particular, the top panels shows that net debt issues comove very closely with asset growth for all firm categories, but especially for large firms.

The middle and bottom panel of Figure 2 displays the cyclical behavior of retained earnings and net equity issues with asset growth. A graphical inspection of these plots confirms the result that the comovement between equity issues and asset growth is more evident for small firms.

Figure 2 shows a dramatic increase in asset growth and net debt issues for large firms
Figure 2: Asset growth and Sources of Financing: Cyclical Components

**Left panel:** Cyclical components of asset growth and sources of financing for small firms. **Right panel:** Cyclical components of asset growth and sources of financing for large firms.
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**Notes:** See definitions of the variables in the text. Standard errors are in parenthesis.
that occurred in 1988. First, this was a year of unusual corporate restructuring activity, in particular in the form of leverage buyouts and stock repurchases. Second, a new accounting rule was introduced that required companies to consolidate the balance sheets of their wholly owned subsidiaries (Bernanke, Campbell, and Whited, 1990). This observation has a small effect on our results. For example, eliminating this observation from our sample changes the contemporaneous correlation between asset growth and net debt issues, retained earnings, and net equity issuance for large firms to 0.85, 0.59 and 0.40, respectively.

With respect to the cyclical properties of the three financing sources we find that the comovement between the growth rate of GDP and net debt and between GDP and net equity issues are positive and statistically different from zero for small firms. For large firms, no contemporaneous correlation between GDP and any financing source is significant. Using lagged output growth we do find that the correlation between (lagged) GDP growth and net debt issues is 0.38 and significant. These results lead us to the following observation:

**Stylized fact #2:** Net debt issues and net equity issues are procyclical for small firms. For large firms, net equity issues are acyclical. For large firms, there is some evidence that net debt issues are procyclical but the correlation is not as strong as it is for small firms.

We next investigate the comovement between net equity issues and net debt issues. Once again, we find a striking difference between small and large firms. In particular, the contemporaneous correlation between net equity issues and net debt issues is 0.48 (and significant) for small firms and 0.05 for large firms. Figure 3 documents the comovement for small firms.

This evidence suggests small firms issue debt and equity simultaneously to finance investment. We provide some more evidence of this fact by estimating a vector autoregression (VAR) for small and large firms. Our vector of endogenous variables is $y_t = (\Delta A_t/A_t, \Delta L_t/A_t, \Delta SB_t/A_t)$. We write the system as follows:

$$y_t = A_0 + A_1 y_{t-1} + A_2 y_{t-2} + \varepsilon_t, \quad \text{where} \quad \varepsilon \sim IN(0, \Sigma),$$

(2)
where $IN(0, \Sigma)$ is a three-dimensional independent, normal density with mean zero and a symmetric positive definite covariance matrix $\Sigma$. For the large firms sample we augment the constant $A_0$ with a dummy variable that takes the value 1 if the year is 1988 and 0 otherwise. We also run formal augmented Dickey-Fuller unit-root tests to test for stationarity of the series. Because of our small sample it is difficult to distinguish between stationary and unit-root processes. Still, in the sample for small firms we were able to reject the null hypothesis of a unit root for net equity and net debt issues at a 5 per cent significance level. For the large firms sample we are able to reject the null hypothesis of a unit root for all the three series at a confidence level of 5 per cent.

Figure 4 plots the impulse response function of the three endogenous variables to an impulse in asset growth by assuming shocks to net issues cannot have an immediate impact on asset growth. The top panel reports the response of small firms whereas the bottom panel plots the responses of large firms. We are interested in the immediate response of net issues to an innovation in asset growth. The differences between small and large firms are striking. For small firms both net equity and net debt issues respond significantly to an innovation in asset growth. In particular, net equity issues react even more than net debt issues. In contrast, for large firms only net debt issues reacts to an innovation in asset growth. The response of net equity issues is rather muted. We summarize this evidence
Figure 4: Orthogonalized Impulse Response Functions

**Top panel:** Orthogonalized impulse responses to an innovation on asset growth for the sample of small firms. **Bottom panel:** Orthogonalized impulse responses to an innovation on asset growth for the sample of large firms.
as follows:

**Stylized fact #3:** The comovement between net debt issues and net equity issues is substantial and significant for small firms and we find no comovement for large firms. Evidence from vector autoregressions suggest that small firms make simultaneous net issues of debt and equity.

We now discuss the business cycle fluctuations of debt-to-assets. First, note that, as in other studies such as Rajan and Zingales (1995), we find a strong positive relationship between size and book leverage. In particular, book leverage is 0.43 for small firms and 0.62 for large firms. Second, the contemporaneous correlation between the cyclical component of debt-to-assets with HP-filtered real GDP is 0.53 for small firms and -0.11 for large firms. The comovement for large firms is not robust. In particular, when we use the unweighted average debt-to-assets ratio instead of the weighted average to calculate the correlation with GDP then we find a positive and significant correlation equal to 0.48. This means that the correlation is positive except for the largest firms. For both small and large firms debt-to-assets seem to lag output. The fact that book leverage is higher for large firms is consistent with the idea small firms face some degree of financing constraints. This leads to the forth stylized fact:

**Stylized fact #4:** Large firms have higher debt-to-asset ratios. Debt-to-asset ratios are procyclical for both small and large firms.

Finally, Table 2 shows the cross-correlations between GDP and default rates for two sub-periods, 1971:Q1–1985:Q4 and 1986:Q1–2003:Q4 after both series have been detrended using the HP filter. The contemporaneous correlation between the cyclical components of these two variables is 0.74 in the second sub-period. It is negative but not significant in the first sub-period. For the reasons highlighted above we take the second sub-period as a better proxy for the default rates of all types of debt issues. From this evidence we draw the last main conclusion:

**Stylized fact #5:** Default rates are countercyclical.
Table 2: Cross Correlations Between Default Rate and Real GDP

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<th>$corr(y_{t+i}, \Phi_{\omega_t})$</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971:Q1–2004:Q4</td>
<td></td>
<td>-0.25</td>
<td>-0.34</td>
<td>-0.38</td>
<td>-0.33</td>
<td>-0.26</td>
<td>-0.16</td>
<td>-0.08</td>
</tr>
<tr>
<td>s.e.</td>
<td></td>
<td>(0.14)</td>
<td>(0.14)</td>
<td>(0.13)</td>
<td>(0.13)</td>
<td>(0.14)</td>
<td>(0.15)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>1971:Q1–1985:Q4</td>
<td></td>
<td>-0.20</td>
<td>-0.22</td>
<td>-0.21</td>
<td>-0.11</td>
<td>-0.02</td>
<td>0.07</td>
<td>0.10</td>
</tr>
<tr>
<td>s.e.</td>
<td></td>
<td>(0.17)</td>
<td>(0.16)</td>
<td>(0.16)</td>
<td>(0.16)</td>
<td>(0.17)</td>
<td>(0.17)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>1986:Q1–2004:Q4</td>
<td></td>
<td>-0.46</td>
<td>-0.66</td>
<td>-0.77</td>
<td>-0.74</td>
<td>-0.63</td>
<td>-0.46</td>
<td>-0.27</td>
</tr>
<tr>
<td>s.e.</td>
<td></td>
<td>(0.15)</td>
<td>(0.11)</td>
<td>(0.08)</td>
<td>(0.08)</td>
<td>(0.11)</td>
<td>(0.15)</td>
<td>(0.18)</td>
</tr>
</tbody>
</table>

Notes: The default rate series is from Moody’s (mnemonic USMDDAIW in Datasstream) and it is for all corporate bonds in the US. The default rate series is HP filtered. Real GDP is logged and HP filtered. Standard errors are in parenthesis.

2.3 Data for listed and non-listed firms

In a recent paper, Quadrini and Jermann (2005), report a strong negative correlation between net equity and net debt issues. They use the Federal Reserve Board’s flow of funds data, which also includes data for non-listed firms. This contrasts sharply with our results using firm level data from Compustat that implied a negative correlation for small firms and no correlation for large firms. Therefore, it is important to investigate the difference. We find that the negative correlation is driven by the merger waves of the 1980s and 1990s and the increase in repurchase activity during the same period.

Baker and Wurgler (2000) construct two sets of series for aggregate equity and debt issues. The first set consists of annual totals of equity issues and long-term debt issues between 1927–2004. These series are from the Federal Reserve Bulletin. The second set consists of net changes of equity and debt issues between 1927–1996. These series are from the flow of funds, which is the same source used by Quadrini and Jermann (2005). The advantage of the flow of funds series is that it subtracts repurchases and debt retirements.
The disadvantage is that it also includes exchange issues and retirements associated with merger activity. This is problematic because mergers involve very large transactions and as a result tend to drive the flow of funds series (Baker and Wurgler, 2000). The top panel of Figure 5 shows the importance of mergers and repurchases in the series. The differences between the two equity issues series are dramatic. For example, during the 1980s and 1990s net equity issues and equity issues exhibit very distinct paths. The negative net equity issues in the 1980s and 1990s is in great part explained by the merger waves. This is problematic if one is interested in the correlation between equity and debt issues. During a merger it is often the case the firm issues debt to finance an acquisition and in the process retires the target’s equity. Hence, with this phenomenon occurring in the data, it is not surprising to observe a negative correlation between net debt and equity issues because mergers are one order of magnitude larger than standard finance decisions.

As before we deflate the aggregate finance sources to constant dollars using the CPI for all urban consumers. Using the flow of funds series—that are affected by mergers—we find a negative correlation between net equity and net debt issues equal to -0.26, consistent with the result in Quadrini and Jermann (2005). For the alternative data set, however, we find a positive correlation equal to 0.52, which is consistent with the results we find using Compustat data. The positive correlation between these two series is also documented in the bottom panel of Figure 5 that plots the cyclical (HP-filtered) components.

In summary, we find that the positive correlation between debt and equity issues remains evident when we extend the universe considered to include non-listed firms. The negative correlation observed in the flow of funds data seems driven by the wave of mergers that took place in the 1980s and 1990s.

3 The optimal debt contract and the cyclical behavior of the default rate

The standard framework assumes that production is linear in capital and that capital fully depreciates in one period. This framework has the counterfactual prediction that
Figure 5: Aggregate Equity and Debt Issues

**Top panel:** The blue line (open circles) is unadjusted aggregate equity issues published by the *Federal Reserve Bulletin*. The red line (closed circles) is net equity issues from the Federal Reserve Board’s flow of funds. Both series were downloaded from Jeffrey Wurgler’s website http://pages.stern.nyu.edu/~jwurgler. The data is then deflated to constant billions of dollars using the CPI. **Bottom panel:** Cyclical (HP-filtered) components of aggregate equity issues and debt issues divided by GDP.
the default rate is procyclical. There are two reasons for this prediction. First, keeping net worth constant, an increase in aggregate productivity leads to an increase in the default rate. We will refer to the response of the default rate to changes in aggregate productivity given net worth as $z$-cyclicality. Second, the increase in net worth that follows the increase in aggregate productivity has no effect on the default probability. We will refer to the response of the default rate to changes in net worth keeping aggregate productivity constant as $n$-cyclicality. In the first subsection, we explain these predictions.

In the remainder of this section, we investigate the behavior of the default rate for more general specifications of the production function, including diminishing returns, partial depreciation, and fixed costs. Note that under the assumption of linear technology the one-period debt contract is the optimal type of contract. In this paper, we hold on to the one-period debt contract as the type of contract used to lend funds to the firm, but we do not show what type of contract is the optimal type of contract for different environments. One justification would be that history or simplicity induces borrowers and lenders to use this type of contract. Although we do not endogenize the type of contract in this paper, we do solve for the optimal one-period debt contract in exactly the same way as is done in the standard CSV framework.

In the second subsection, we show that by simply allowing for diminishing marginal returns with respect to capital, the model generates $n$-countercyclicality, that is, an increase in net worth does lead to a reduction in the default rate. In the third subsection, we analyze how the default rate responds to aggregate productivity shocks for a more general production function, that allows for diminishing returns, partial depreciation of capital, and fixed costs. We show that the prediction of the standard model, that the default rate increases with aggregate productivity, carries over to this general production function, unless the firm is small. More specifically, we find that the model can generate $z$-countercyclicality only if the fixed costs are larger than the firm’s net worth. In Section 5, we will show that if we allow for outside equity, the model can generate $z$-countercyclicality.

In the appendix, we consider some alternative, and less standard, production functions.

\footnote{See Townsend (1979).}
that can also generate $z$-countercyclicality.

### 3.1 Procyclical default rates in the standard CSV framework

In this section we describe the standard CSV model, we discuss why net worth has no effect on the default rate, and we explain why productivity has a positive effect.

#### 3.1.1 CSV framework

firms have access to the following technology:

$$z\omega k,$$

where $k$ stands for the amount of capital, $z$ for the aggregate productivity shock (with $z > 0$), and $\omega$ for the idiosyncratic productivity shock (with $\omega \geq 0$). We assume that $z$ is observed at the beginning of the period when the debt contract is written, but that $\omega$ is only observed at the end of the period. The firm’s net worth is equal to $n$ and borrowing occurs through one-period debt contracts.\(^{13}\) That is, the borrower and lender agree on a debt amount, $(k - n)$, and a lending rate, $r^b$. The firm defaults if the resources in the firm are not enough to pay back the amount borrowed plus interest. That is, the firm defaults if $\omega$ is less than the default threshold, $\overline{\omega}$, where $\overline{\omega}$ satisfies

$$z\overline{\omega}k = (1 + r^b)(k - n).$$

If the firm defaults then the lender gets\(^{14}\)

$$z\omega k - \mu zk,$$

where $\mu$ represent bankruptcy costs, which are assumed to be a fraction of revenues. Note that in an economy with $\mu > 0$ default is inefficient and would not happen if the first-best \footnote{If the default costs are the costs the lender pays to verify the realization of the idiosyncratic shock then we have the CSV framework of Townsend (1979) and the one-period debt contract is the optimal type of contract.} \footnote{One can make different assumptions about the bankruptcy costs. Here they are a fraction of expected output. Alternatively, they could be a fraction of actual output, $z\omega k$, or a fraction of the interest payment, $z\overline{\omega}k$. The alternatives do not affect the results discussed here.}
solution could be implemented. Let $\Phi(\omega)$ be the cdf of the idiosyncratic productivity shock. We can then write expected firm income as

$$\int_{-\infty}^{\infty} z\omega k d\Phi(\omega) - (1 - \Phi(\overline{\omega}))(1 + r^b)(k - n).$$

The expected income of the lender is given by

$$(1 - \Phi(\overline{\omega}))(1 + r^b)(k - n) + \int_{-\infty}^{\overline{\omega}} (z\omega k - \mu z\omega) d\Phi(\omega).$$

We follow standard practice and assume that the lender only cares about his expected income. This requires that the lender is either risk neutral or has a well-diversified portfolio. Given values for $n$ and $z$, the financial contract specifies a debt amount, $k - n$, and a lending rate, $r^b$, which then imply a value for $\overline{\omega}$. Alternatively, one can use (4) and define the financial contract as the pair $(k, \overline{\omega})$. If we use (4) then we can write the firm’s expected income as

$$zkF(\overline{\omega}) \text{ with } F(\overline{\omega}) = \int_{\overline{\omega}}^{\infty} \omega d\Phi(\omega) - (1 - \Phi(\overline{\omega}))\overline{\omega},$$

and the lender’s expected revenues as

$$zkG(\overline{\omega}) \text{ with } G(\overline{\omega}) = 1 - F(\overline{\omega}) - \mu \Phi(\overline{\omega}).$$

We assume that the firm is risk neutral. For given values of $n$ and $z$ the values of $(k, \overline{\omega})$ are then chosen to maximize the expected firm income subject to the constraint that the lender must break even. For simplicity we assume in this section that the cost of funds for the lender is equal to zero. The best one-period debt contract is, thus, given by the following maximization problem:

$$\max_{k, \overline{\omega}} zkF(\overline{\omega})$$

s.t. $zkG(\overline{\omega}) = k - n.$

$^{15}$Note that $z$ is known when the contract is written.
The optimal values for $k$ and $\bar{\omega}$ are given by

\[
\frac{zF(\bar{\omega})}{1 - zG(\bar{\omega})} = -\frac{F'(\bar{\omega})}{G'(\bar{\omega})}, \quad \text{and}
\]

\[
k = \frac{n}{1 - zG(\bar{\omega})}.
\]  

Below we will build intuition using the slopes of the iso-profit and zero-profit curves. The slope of the iso-profit curve is given by

\[
\frac{dk}{d\bar{\omega}} = -\frac{kF'(\bar{\omega})}{F(\bar{\omega})},
\]  

and the slope of the zero-profit curve is given by

\[
\frac{dk}{d\bar{\omega}} = \frac{zkG'(\bar{\omega})}{1 - zG(\bar{\omega})}.
\]

The top panel in Figure 6 shows the iso-profit curve for the firm and the zero-profit condition of the lender. Firm profits increase when the default rate decreases, that is, $F'(\bar{\omega}) < 0$. A reduction of the default rate implies that the firm keeps the produced output for a wider range of realizations of $\omega$ (first term in 8). In addition, it reduces the interest payments (second term in 8), since according to (4) a lower value of $\bar{\omega}$ means that the interest payments must be less. A reduction in the default rate, thus, unambiguously increase firm profits. This and the fact that firm profits are increasing with the amount invested implies that the iso-profit curves are upward sloping.

A value of $\bar{\omega}$ equal to zero means that the firm never defaults, not even when it has no resources left. This means that the gross lending rate must be equal to zero. At that lending rate, the lender obviously only breaks even if he doesn’t lend out anything at all. Consequently, $k$ is equal to $n$ when $\bar{\omega}$ is equal to zero. An increase in $\bar{\omega}$ has two effects on bank profits. On the one hand it increases interest payments of those that do not default according to (4), but it also increases the resources lost due to bankruptcy. We assume that in the relevant range $G'(\bar{\omega}) > 0$, that is, the direct effect on banks’ revenues dominates the indirect effect through the increase in lost resources through additional bankruptcies. Consequently, an increase in the default threshold increases the fraction of revenues that
Figure 6: Static CSV Framework: Iso-Profit and Zero-Profit Curves
goes to the bank. The zero-profit curve is then upward sloping. This condition and one
more regularity condition are the assumption we need to prove our propositions.\footnote{The second condition is equivalent to \( \frac{\partial}{\partial \omega} \frac{d\Phi(\omega)}{(1 - \Phi(\omega))} / \partial \theta > 0 \). Such a condition is standard in the literature. For example, Bernanke, Gertler, and Gilchrist (1999) assume that \( \frac{\partial}{\partial \omega} \frac{\omega \Phi(\omega)}{(1 - \Phi(\omega))} / \partial \theta > 0 \), which would be the corresponding condition if bankruptcy costs are—as in Bernanke, Gertler, and Gilchrist (1999)—a fraction of \( z\omega k^\alpha \).}

**Definition 1 (Assumption)** The values of \( k \) and \( \varpi \) are given by (11) and (12). Also
\[
G'(\varpi) > 0,
\]
and
\[
\frac{\partial}{\partial \varpi} \left( -\frac{G'(\varpi)}{F'(\varpi)} \right) < 0.
\]

The two main results of this section are given by the following two propositions.

**Proposition 1** Suppose that Assumption 1 holds. Then \( \frac{d\varpi}{dn} = 0 \).

**Proposition 2** Suppose that Assumption 1 holds. Then \( \frac{d\varpi}{dz} > 0 \).

All proofs are given in the appendix. Proposition 2 is actually a special case of Proposition 4 given below. The next two subsections give the intuition.

### 3.1.2 Changes in net worth and \( n \)-cyclicality

In the standard framework discussed above, changes in net worth have no effect on the
default rate. It only increases the amount invested. Moreover, because of the linearity
assumption the debt-to-asset ratio is independent of firm size. This result is graphically
illustrated in the middle panel of Figure 6. It follows directly from the fact that equation (11) does not depend on \( k \). The intuition is very simple. Because of the linearity, the
slope of the iso-profit curves as well as the zero-profit curve are linear in \( k \). That is, for a
given value of \( \varpi \), the ratio of the slope of the iso-profit to the slope of the zero-profit line
does not change. Since an increase in net worth is simply a parallel shift of the zero-profit
line, this means that the change in net worth doesn’t affect the optimal choice of \( \varpi \). As
discussed below, the linearity of the production function is essential.
3.1.3 Changes in aggregate productivity and \( z \)-cyclicality

In the standard framework, an increase in aggregate productivity increases the default rate which is even less plausible than the effect of an increase of net worth on the default rate. It is easy to understand why the default rate increases with aggregate productivity. An increase in aggregate productivity implies that the zero-profit curve simply rotates upwards. That is, the zero-profit condition implies that an increase in \( \bar{\omega} \) relative to \( k \) has become “cheaper”. But an increase in \( z \) does not affect the slope of the iso-profit curves. Consequently, an increase in \( z \) not only means that the firm reaches a higher iso-profit curve, it also means that there is a movement along the iso-profit curve further increasing both \( k \) and \( \bar{\omega} \). This is graphically illustrated in the bottom panel of Figure 6.

3.2 Diminishing returns and \( n \)-cyclicality

In this section we will show that with one very simple modification to the CSV framework it is possible to achieve that an increase in \( n \) results in a decrease of \( \bar{\omega} \). The modification is to replace the linear technology by a production function with a decreasing marginal product of capital. Suppose that technology is given by

\[
    z\omega k^\alpha
\]

and, thus, satisfies diminishing returns. It is easy to show that with this modification, an increase in net worth reduces the default rate. If the production function satisfies decreasing returns to scale then the slope of the iso-profit curve is still linear in \( k \) (for fixed value of \( \bar{\omega} \)). The slope of the zero-profit curve, however, is not and decreases relative to the slope of the iso-profit curve as \( k \) increases.\(^ {17} \) Consequently, when an increase in net worth pushes the zero-profit curve up, then the relative decrease in the slope of the zero-profit line pushes towards a lower \( \bar{\omega} \) and a lower value of \( k \). So this dampens the direct effect of net worth on \( k \) but one does obtain the desired effect on the default rate.

\(^{17}\) Basically because the cost of an one-unit increase in \( k \) is still equal to one but the benefits are decreasing with \( k \).
Figure 7: Modified CSV Framework: Decreasing Returns to Scale

- Zero-profit curve, \( n = 0.5 \)
- Iso-profit curve, \( n = 0.5 \)
- Zero-profit curve, \( n = 1.5 \)
- Iso-profit curve, \( n = 1.5 \)
It is well known, that the optimal debt contract is not well defined if net worth is equal to zero when \( \alpha = 1 \). The reason is the following. Recall that the optimal value of \( \omega \) does not depend on the value of net worth. If \( zG(\omega) < 1 \), then the bank would not want to lend out any amount. Alternatively, if \( zG(\omega) > 1 \), then the linearity implies that the optimal amount invested is infinite. By introducing diminishing returns the contract would have an interior solution even when net worth is zero.

3.3 More general production functions and \( z \)-cyclicality

In this section, we analyze the response of the default rate when aggregate productivity increases for more general production functions. In particular, we assume that the production function is given by

\[
z\omega k^\alpha + (1 - \delta)k - c,
\]

where \( \delta \) is the depreciation rate and \( c \) is a fixed cost. The other aspects of the problem remain the same, but to simplify the notation we set \( r^b \) equal to zero. The break-even condition of the bank can now be written as follows

\[
zk^\alpha G(\omega) + (1 - \delta)k - c = k - n,
\]

where the definition of \( G(\omega) \) remains the same as before. This can be rewritten as

\[
\frac{z}{\delta} k^\alpha G(\omega) = k - \frac{n - c}{\delta}.
\]

The structure of the break-even condition is, thus, not affected by the modifications introduced in the production function. Partial depreciation, \( \delta < 1 \), means that part of the firm’s resources after production will not be subject to the idiosyncratic productivity shock. These resources help the firm to increase the leverage with the same amount of net worth. An increase in the fixed-cost \( c \) has the same effect as a reduction in the amount of net worth.

The two conditions that determine the optimal values of \( k \) and \( \omega \) are the bank break-even condition, (20), and

\[
\frac{\delta}{z\alpha k^{\alpha-1}} - zG(\omega) = -\frac{\partial F(\omega)}{\partial \omega}/\frac{\partial G(\omega)}{\partial \omega}.
\]
Not surprisingly, an increase in aggregate productivity increases the amount invested, which is formalized in the following lemma.

**Lemma 3** Assume that condition 1 holds. Then $\frac{dk}{dz} > 0$.

The key question, though, is what will happen with the default threshold, $\omega$, and this question is answered by the following proposition.

**Proposition 4** Suppose that Assumption 1 holds. Then $\frac{d\omega}{dz} > 0$ if $n > c$, $\frac{d\omega}{dz} < 0$ if $n < c$, and $\frac{d\omega}{dz} = 0$ if $n = c$.

To understand this proposition use (20) to substitute out $zk^{\alpha - 1}$ from (21). We then get

$$\left(\frac{1}{\alpha} - 1\right) G(\omega) + \frac{n - c}{k} = -\frac{G'(\omega)}{F'(\omega)} F(\omega).$$

(22)

Given Lemma 3 the result of the proposition is straightforward and can be easily understood from Figure 8. This figure plots the right and left hand side as a function of $\omega$, for two values of $k$.\(^{18}\) Note that $z$ only affects this graph by changing $k$ and, thus, $(n - c)/k$. If $n = c$ then the right hand side does not depend on $k$ and (22) pins down $\omega$ independently of $n$. If $n - c > (\prec) 0$ then an increase in $z$ lowers (increases) the right hand side, because $k$ increases. Note that an increase (decrease) in the right hand side would lead to a reduction (increase) in the default probability. Consequently, if $n - c > (\prec) 0$ then an increase in $z$ increases (decreases) the default probability.

This means that the prediction of the standard framework that default rates increase with aggregate productivity carry over to more standard specifications of technology unless the firm is small, that is, when net worth is less than the fixed cost. There is one very simple and not unreasonable modification that will make it possible that $d\omega/dz < 0$ even if $n - c > 0$. In particular, if the fixed cost, $c$, decreases with aggregate productivity, then this would increase the right hand side of Equation (22) and as long as this dominates

\(^{18}\)The right hand side is increasing in $\omega$, because according to Assumption 1 $G'(\omega) > 0$. The left hand side is decreasing in $\omega$, because $F'(\omega) < 0$ and according to Assumption 1 $\partial(-G'(\omega)/F'(\omega))/\partial\omega < 0$. 
Figure 8: Fixed costs and \( z \)-cyclicality

(a) \( n > c \)

(b) \( n < c \)
the downward effect caused by the increase in \( k \) then the default probability would be decreasing with \( k \). We discuss this case in more detail in the appendix.

4 Magnification and \( z \)-cyclicality in the static model

In this section we compare the response of aggregate capital and real activity in the model with and without frictions. Note that the firm’s output, net of bankruptcy costs, is given by

\[
y = zk^\alpha (F(\omega) + G(\omega)),
\]

whereas when there are no frictions real activity, \( \tilde{y} \), is given by \( z\tilde{k}^\alpha \). In the model without frictions, capital is determined from

\[
\alpha z\tilde{k}^{\alpha-1} = \delta.
\]

This means that

\[
\frac{d\tilde{k}}{k} = \frac{1}{1 - \alpha} \frac{dz}{z} \quad \text{and} \quad \frac{d\tilde{y}}{\tilde{y}} = \frac{1}{1 - \alpha} \frac{dz}{z}.
\]

In general, to compare the response of output in the model with frictions to the response of output in the model without frictions one has to look at the response of \( \omega \) and one has to compare the response of capital in the model with frictions, \( k \), with the response of capital in the model without frictions, \( \tilde{k} \). For the production function given by (18), however, one only has to look at the response of \( \omega \).

To understand these statements first consider the case where \( n = c \) and, thus, \( d\omega/dz = 0 \). In this case, \( G(\omega) \) is constant and capital can be solved directly from the break-even condition of the bank. That is, equation (20) gives

\[
\frac{d\tilde{k}}{k} = \frac{1}{1 - \alpha} \frac{dz}{z} = \frac{d\tilde{k}}{\tilde{k}}.
\]

This result is driven by the property that when \( G(\omega) \) is constant and \( n = c \), the break-even condition is identical to the first-order condition in the model without frictions but with \( \alpha \) replaced by \( G(\omega) \). But, of course, this would not be true in general, and for example, it would not be true if production would be given by \( z\omega(k^{\alpha_1} + k^{\alpha_2}) \) with \( \alpha_1 \neq \alpha_2 \).
Thus, for the model considered here, there is a neat link between magnification (or dampening) and \( z \)-countercyclicality (or \( z \)-procyclicality). That is, if \( d\bar{\omega}/dz < 0 \) then \( dk/dz > d\bar{k}/dz \) and \( dy/dz > d\bar{y}/dz \). Shocks are then magnified in the model with frictions. On the other hand, if \( d\bar{\omega}/dz > 0 \) then shocks are dampened. That is, \( dk/dz < d\bar{k}/dz \) and \( dy/dz < d\bar{y}/dz \).

5 Equity issuance and \( z \)-cyclicality

In the dynamic model we consider in this paper we allow the firm to issue equity. A key question is whether the firm would like to issue more or less equity if aggregate productivity in the coming period is higher. In this section, we use a simple two-period extension of the static framework discussed above to derive some analytical results. We will show that the response of equity to an increase in aggregate productivity depends crucially on whether the model displays \( z \)-procyclicality or \( z \)-countercyclicality. That is, if the default probability in(de)creases when aggregate productivity increases then equity in(de)creases as well. The intuition is quite simple. If the model displays \( z \)-procyclicality then an increase in aggregate productivity increases the shadow price of the restriction that the bank has to break-even. Consequently, it is worth it to the firm to add additional equity so it can reduce the increase in agency costs that are caused by the expansion in capital.

The setup in this paper is as follows. At the beginning of the period the firm observes \( z \) and decides how much equity, \( e \), to add to the firm. Issuing equity is subject to an issuance cost, \( \lambda(e) \), that is increasing with the amount of equity issued. We assume that \( \lambda(0) = 0, \lambda'(0) = 0, \lambda'(e) > 0, \) and \( \lambda''(e) > 0 \). We also assume that parameters and the initial value of available funds, \( x \), are such that an internal solution for \( e \) is chosen.\(^{19}\) The firm’s level of net worth is, thus, equal to \( n = x + e \). Given the value of \( n \), the firm borrows funds from a lender using a one-period debt contract and the parameters of the debt contract are chosen using the framework developed above. The equity issuance and

---

\(^{19}\)Below we will allow the firm also to withdraw equity, that is, issue dividends.
debt issuance decisions are represented by the following two maximization problems.

\[
v(z, x) = \max_{e, n} -e - \lambda(e) + w(z, n) \\
\text{s.t.} \quad n = x + e
\]

(26)

\[
w(z, n) = \max_{k, \omega} zk^\alpha F(\omega) \\
\text{s.t.} \quad \frac{zk^\alpha G(\omega)}{\delta} = k - \frac{n-c}{\delta}
\]

(27)

The question is whether \(de/dz\) is negative or positive. Clearly, if \(\partial w(z, n)/\partial n > 0\) then \(de/dz\) would be positive. Using the envelope condition we get that

\[
\frac{\partial w(z, n)}{\partial n} = \zeta,
\]

(28)

where \(\zeta\) is the Lagrange multiplier for the constraint in (27). The value of \(\zeta\) is equal to

\[
\zeta = -\frac{F'(\omega)}{G'(\omega)}.
\]

(29)

According to Assumption 1 we then have that\(^{20}\)

\[
\text{the sign of } \frac{\partial \zeta}{\partial z} \text{ is equal to the sign of } \frac{\partial \omega}{\partial z}
\]

(30)

Thus, if the model displays \(z\)-procyclicality then an increase in \(z\) not only implies an increase in the default probability but also an increase in the shadow price of the constraint. This means that the optimal choice of equity issuance increases.

6 Complete Model

This section describes the full dynamic model. It consists of the following: (i) a unit mass of firms; (ii) a large number of competitive banks that provide one-period loans to firms; and (iii) a representative household that owns the firms.

6.1 Environment

Firms produce goods in this economy and are owned by the representative household (shareholders). Firms can borrow funds from a bank, and at the same time households

\(^{20}\)Note that only a weak assumption on the cdf of \(\omega\) is used.
also provide funds to the firm at an additional cost. Operating profits of the firm are given by

\[ \pi(z, k) = z\omega f(k) - c, \]  

(31)

where \( z \) stands for aggregate productivity (with \( z > 0 \)), \( \omega \) for the i.i.d. idiosyncratic productivity shock (with \( \omega \geq 0 \)), \( k \) for the firm’s capital stock, and \( c \) is a fixed cost. The capital stock depreciates at rate \( \delta \). Further, assume that \( f(\cdot) \) is continuously differentiable, strictly increasing, strictly concave with \( f(0) = 0 \), and satisfying the Inada conditions.

The timing assumptions are as follows. At the beginning of the period aggregate productivity \( z \) is known and the shareholders decide whether to liquidate the firm (take all resources out of the firm), pay out any dividends, or issue equity to the firm. Afterwards the debt contract is written. The value of the idiosyncratic shock, \( \omega \), is observed at the end of the period. The firm then repays the debt plus interest or defaults.

6.2 Debt choice

This economy is populated by a large number of banks that lend funds to firms in a competitive market. Consequently, banks make zero expected profits on each loan they make to a firm. A firm defaults if the resources in the firm are not enough to pay back the amount borrowed plus interest. That is, it is not feasible to attract additional resources to finance the gap. Doing this and avoiding bankruptcy may be worth it if the future earnings potential of the firm is high enough but it wouldn’t be worth it if the same future earnings potential can be achieved if the firm can be restarted with zero (instead of negative) initial funds.

At the beginning of the period the firm borrows \((k - n)\) from one of the banks, where \( n \) is the firm’s net worth. The firm agrees to repay \((1 + r_b)(k - n)\) at the end of the period. The firm then defaults if \( \omega \) is less than the default threshold, \( \bar{\omega} \), where \( \bar{\omega} \) satisfies

\[ z\bar{\omega}k^{\alpha} + (1 - \delta)k - c = (1 + r_b)(k - n). \]  

(32)

If the firm defaults the bank gets

\[ z\omega k^{\alpha} + (1 - \delta)k - c - \mu zk^{\alpha}, \]  

(33)
where $\mu$ represent bankruptcy costs, which are assumed to be a fraction of expected output. Let $\Phi(\omega)$ be the cumulative distribution function of the idiosyncratic shock. We can write the expected income of the bank for each loan it issues as

$$
\int_{0}^{\bar{\omega}} [z\omega k^\alpha + (1 - \delta)k - c] d\Phi(\omega) + [1 - \Phi(\bar{\omega})](1 + r_b)(k - n) - \mu z k^\alpha \Phi(\bar{\omega}).
$$

(34)

As in the static case, the financial contract is given by the $(k, \bar{\omega})$ pair that maximizes the firm’s value subject to the lender being indifferent between loaning the funds or retaining them.

$$
\max_{k, \bar{\omega}} \int_{0}^{\\infty} E[v(z', x)|z]d\Phi
\text{ s.t. } x = z \omega k^\alpha + (1 - \delta)k - c - (1 + r_b)(k - n),
$$

(35)

$$
zk^\alpha G(\bar{\omega}) + (1 - \delta)k - c \geq (k - n).
$$

where

$$
G(\bar{\omega}) = [1 - \Phi(\bar{\omega})]\bar{\omega} + \int_{0}^{\bar{\omega}} \omega d\Phi - \mu \Phi(\bar{\omega}).
$$

(36)

Note that we assume that the opportunity cost for the bank is equal to zero. The function $v(z', x)$ represents the value of the firm at the end of the period when next period’s aggregate shock is equal to $z'$ and resources within the firm are equal to $x$. This continuation value takes into account that at the end of the period the firm could liquidate the firm ($v(z', x) = x$), issue dividends ($n' < x$), or add additional equity ($n' > x$).

Furthermore, the participation constraint of the firm must hold as well. In particular we need that the maximized value in expression (35) to be greater or equal to $E[v(z', n)|z]$. This condition will always be satisfied below. When the lending contract is written the firm has already decided that it isn’t worth liquidating the firm. Because of the fixed cost, however, it may still be better for the firm to produce without a bank contract and set $k$ equal to $n$. Thus, we have to check the participation constraint that the maximized value in expression (35) to be greater or equal to $E[v(z', n)|z]$. We never found this constraint to be violated.
Further, from the financial contract we can simplify expression (35) using the definition of the default threshold given in (32):

\[ x = z(\omega - \bar{\omega})k^\alpha. \]  

(37)

### 6.3 Net Equity Choice and Retained Earnings

In addition to debt financing the firm can also attract outside equity. To consider a more realistic and interesting equilibrium in which both debt and equity co-exist, we assume firms face a linear-quadratic cost of issuing equity. This cost function captures both the cost of underwriting fees and possible adverse selection problems in equity markets. More specifically, the cost of external equity is defined as:

\[ \lambda = \lambda(e; z') \geq 0, \]  

(38)

where \( e \) is new equity issued. When \( e < 0 \) then the firm issues dividends and we assume that in that case \( \lambda(\cdot) \) is equal to zero. Furthermore, we assume that \( \lambda(0; z') = \lambda'(0; z') \) and when \( e > 0 \) that \( \lambda(e; z') > 0 \) and \( \lambda'(e; z') > 0 \). We allow for the possibility that the cost of issuing equity is decreasing with \( z' \). The idea would be that an increase in \( z' \) increases the bargaining position of existing shareholders and makes it easier to attract new equity. Recall that we assume that the firm observes next period productivity, \( z' \), before it decides how much equity to issue in the current period. Furthermore, this term could also represent the so called behavioral or timing the market view (Baker and Wurgler, 2000), in which equity issuance is both sensitive to firm-specific factors as well as aggregate factors, as empirical evidence provided by Lamont and Stein (2006) suggest.

Recall that \( v(z', x) \) is the optimal value of a firm before issuing equity when the productivity value of the coming period is known, \( z' \), and with cash-on-hand equal to \( x \). The firm’s optimization problem can be specified in terms of the following dynamic programming problem:

\[
v(z', x) = \max_{e,n'} -e - \lambda(e, z') + w(z', n') \\
s.t. \quad -e + n' = x,
\]  

(39) (40)
where \( w(z', n') \) is the firm’s optimal value after the equity decision is made. Note that the firm can set \(-e = x\), that is \( n' = 0\). In this case the firm is liquidated and \( v(z', x) = x\).

We focus on the case where \( v(z', 0) > 0\) and it is worthwhile to issue equity even when there is no net worth in the firm. Then liquidation never happens. The reason is that if a firm issues equity the slope of \( w(\cdot) \) must be strictly greater than one (to cover the issuance cost). As \( n' \) increases the slope of \( w(\cdot) \) decreases but never gets below one,\(^{21}\) consequently we always have that \( w(z', n') > n' \).

This value function is defined as follows:

\[
w(z', n') = \frac{1}{1 + r} E \left[ \int_0^{\bar{\omega}'} v(z'', 0) \, d\Phi(\omega) + \int_{\bar{\omega}'}^{\infty} v(z'', x') \, d\Phi(\omega)|z' \right].
\]

Finally, we define the firm’s next period cash-on-hand, \( x' \), as follows:

\[
x' = \begin{cases} 
0 & \text{if } \omega < \bar{\omega}', \\
\omega' k'^{\alpha} + (1 - \delta) k' - c - (1 + r') (k' - n') & \text{if } \omega \geq \bar{\omega}'.
\end{cases}
\]

Bank loans in our model are within-period loans and the opportunity costs is equal to zero. The discount rate is strictly bigger than zero. If instead we would assume that the opportunity cost for banks is equal to \( r \), then firms would never pay out dividends. Because of diminishing returns there would be a maximum level of capital but firms would keep funds inside the firm even if net worth would exceed that level. Firms would simply invest the excess funds at rate \( r \) which is the same as what it would earn outside the relationship, but keeping funds inside the firm serves as a buffer against drops in net worth (and avoiding paying issuance costs). Alternatively, we could assume that the opportunity costs for the banks is equal to \( r \) but that funds deposited by the firm at the bank earn less than \( r \).

### 6.4 Household problem

In this economy there is a representative household which receives dividends of firms and uses it to purchase consumption and shares of firms. Households maximize the expected lifetime utility derived from consumption:

\[
E_0 \sum_{t=0}^{\infty} \beta^t c_t,
\]

\(^{21}\)The slope will be equal to one when it starts paying out dividends.
where $0 < \beta = 1/(1 + r) < 1$ is the discount factor. The household income comes from the dividends of firms. The household’s budget constraint can be written as:

$$
c + \int s'(z', x)p(z', x) \, d\Gamma(z', x) = \int s(z', x)p(z', x) \, d\Gamma(z', x)
+ \int s'(z', x)d(z', x) \, d\Gamma(z', x) + \Pi, \tag{44}
$$

where $s(z', x)$ stands for the fraction of shares owned by the household, $p(z', x)$ for the price of the share and $d(z', x)$ for the dividend. The profits of banks are denoted by $\Pi$. As in Gomes (2001) we assume the payment of dividends occurs right after the shares of the firm are bought.

Taking first-order conditions w.r.t. the accumulation of shares we get:

$$
p(z', x) = d(z', x) + \beta E[p(z'', x')|z']. \tag{45}
$$

where $d(z', x) = -e(z', x) - \lambda(e, z')$.

Proposition 5 In equilibrium $p(z', x) = v(z', x)$.

7 Results

This section demonstrates the role of debt and equity financing to macroeconomic dynamics in a parameterized version of the model presented in Section 6. The model is calibrated to match key empirical observations for corporations included in Compustat. First, we show the model is able to account for the stylized facts discussed in Section 2, except for the positive relation between firm size and debt-to-assets. Next, we demonstrate the importance of equity issues for the magnification and propagation of aggregate shocks, especially for small firms. Finally, we show that if bankruptcy costs are higher than those typically considered in the literature that is also possible to generate a positive relation between firm size and leverage. Moreover, magnification of shocks is substantially larger in this version of the model then in the benchmark version.
7.1 Calibration

The period is one year and so the discount factor, $\beta$, is set equal to 0.95. The aggregate productivity process is first-order Markov:

$$\ln(z') = \gamma(1 - \rho) + \rho \ln(z) + \epsilon', \quad (46)$$

where $\gamma$ represents the unconditional mean and $\epsilon \sim N(0, \sigma_\epsilon)$. The cost of issuing equity is given by the following reduced-form specification:

$$\lambda(e, z') = \lambda_0 z^{-\lambda_1} e^2. \quad (47)$$

where $\lambda_0, \lambda_1 > 0$.

The state variables of the dynamic problem are $(z', x)$. For convenience we discretize the aggregate stochastic process following Tauchen and Hussey (1991). The state variable cash-on-hand, $x$, is discretized into an equally spaced grid and we use linear interpolation to fill in for values in between grid points. The solution of the model is obtained by value function iteration.\(^{22}\) We study the properties of the model by numerical simulation. For this purpose we generate an artificial economy with 5000 firms for 1500 periods. The properties of the model are then obtained from the moments of the simulated data by discarding the first 500 observations.

In addition to the discount factor the model has ten parameters. At the current stage we fix three parameters, $(\sigma_\omega, \sigma_\epsilon, \rho)$, and choose the remaining seven parameters, $(\alpha, \delta, c, \gamma, \mu, \lambda_0, \lambda_1)$, to match the same number of moments calculated from the firm level data. To calibrate these seven parameters we select the following targets: investment-to-assets ratio, operating income-to-assets-ratio, fraction of firms making positive net equity issues, equity-issuance-to-assets of small firms, book leverage, the corporate default rate, and the distribution-to-assets ratio.

Regarding the fixed parameters we set $\sigma_\omega = 0.28$, $\sigma_\epsilon = 0.001$, and $\rho = 0.95$. We are currently investigating ways of calibrating these parameters as well. The calibrated parameters are shown in Table 3 together with the targets.

\(^{22}\)With a smooth function on the cost of issuing equity the model can also be solved by first-order conditions.
Table 3: Calibration

Parameter Values

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<tr>
<th>α</th>
<th>δ</th>
<th>c</th>
<th>γ</th>
<th>μ</th>
<th>λ₀</th>
<th>λ₁</th>
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Targets

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<th></th>
<th>Data</th>
<th>Model (v6_bc1_sc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distribution/Assets</td>
<td>0.028</td>
<td>0.031</td>
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<tr>
<td>Investment/Assets</td>
<td>0.122</td>
<td>0.200</td>
</tr>
<tr>
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<tr>
<td>Net equity issues &gt; 0</td>
<td>54%</td>
<td>50.3%</td>
</tr>
<tr>
<td>Net equity issue/Assets</td>
<td>0.085</td>
<td>0.005</td>
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<tr>
<td>Book leverage</td>
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<td>0.915</td>
</tr>
<tr>
<td>Default rate</td>
<td>0.022</td>
<td>0.031</td>
</tr>
</tbody>
</table>

As we will demonstrate below, the dependence of issuance costs on the aggregate state has a significant impact on the ability of the model to amplify shocks. Therefore it is important to understand the parameterization and the functional form of issuance costs. First, we assume a convex cost of equity issuance for numerical tractability. Second, fixed costs in equity issuance are usually small, as shown by Altinkılıç and Hansen (2000). Hence, our results should not change dramatically if one introduces a reasonable fixed issuance cost. Regarding variable costs we also omit a linear term, again for numerical tractability. Hennessy and Whited (2005) estimate the linear term to be statistically significant, however their empirical measure of equity issuance understates equity issued by Compustat firms (see Fama and French (2005) p. 559 for details). Because the fraction of firms issuing equity in the data on a given year is quite substantial, we argue the quadratic term is appropriate since in the model we have about half of the firms issuing equity every period, in small and large amounts. Finally, we let the quadratic cost term to vary over the business cycle. This is in part based on the evidence provided by Lamont and Stein (2006), which find that equity issuance is more sensitive to aggregates rather than firm-specific variables.

The issuance costs specification, in particular the parameter $\lambda_1$, helps matching two key empirical facts: sizable equity issuance for small firms and smooth distributions for dividend payers. As shown in Section 3, the debt contract exhibits procyclical agency costs of debt and a procyclical default probability. If agency costs of debt are procyclical and there are fixed issuance costs, firms issue equity in response to a positive aggregate shock. By issuing equity the firm is able to mitigate part of the increase in agency costs of debt. The presence of costs of equity issuance that vary inversely with the aggregate state enhances the equity issuance mechanism. Hence, setting $\lambda_1 > 0$ helps increasing the size of equity issuance.

In addition, in response to a positive aggregate shock, dividend payers reduce dividends to partially offset the increase in agency costs of debt. As a result, however, distributions

\footnote{The term $z^{-\lambda_1}$ corresponds to a value between 0.5 and 2.1 for the lowest and highest aggregate state, respectively. Hence, equity issuance costs vary between 5.3 for the least productive state and 1.2 for the most productive state.}
to shareholders fluctuate significantly over the business cycle. This is an unappealing feature to have in the model because in the data dividends are “sticky”. By letting the cost of equity issuance vary inversely with $z$, that is to let $\lambda_1 > 0$, we can also reduce the problem of volatile dividends. The reason is that productivity shocks are persistent. Firms not only worry about agency costs in the current period but also about agency costs in the future. Thus, firms may be careful in issuing dividends because if they obtain a sequence of bad draws they may be forced to issue new equity which is costly. But if the cost of issuing equity is expected to be relatively cheaper in the near future there is less risk of issuing dividends. Consequently, firms may be willing to maintain the same dividend, or even to make distributions to shareholders slightly procyclical.

In summary, the specification with state contingent costs of equity helps makes the model more realistic at least in two dimensions. First, it leads to more equity issuance. Second, it helps smoothing distributions to shareholders.

7.2 Amplification

In this section we investigate the ability of our framework to amplify aggregate shocks. There are two key features of the model: (i) bankruptcy costs (agency costs of debt); and (ii) equity issuance costs. First we study the ability of the model to amplify shocks relative to the frictionless environment, assuming no equity issuance. Figure 9 shows the initial response of capital, default threshold, output and cash-on-hand to a one percent increase in aggregate productivity. The responses of capital and output in the frictionless case are used to normalize all the responses. The size of the responses of capital and output in the economy with bankruptcy costs are less than half of the responses in the frictionless economy. This is not surprising result because with equity fixed the default rate is procyclical, and so also are the agency costs of debt. As a result the cost of external financing is also procyclical, and the responses are somewhat muted. As shown in an earlier section of the paper, this is a robust result and difficult to overturn in the standard costly state verification environment.

In the second experiment we allow firms to adjust net worth by issuing equity. We
Notes: The plots show the percentage deviations of capital, default probability, output and cash-on-hand to a one percent shock in aggregate productivity as a function of the firm’s net worth. The dashed-line represent the responses in the frictionless economy. The solid lines show the response of the economy with bankruptcy costs.
start by assuming a constant cost of equity issuance across the different aggregate states, that is, by setting $\lambda_1 = 0$. With equity issuance the firm is able to increase its net worth if desired. With $\lambda_1 = 0$ then agency costs of debt have to be procyclical for the firm to be willing to issue equity at an additional cost. With the extra net worth inside the firm it is now possible to increase the size of the project by issuing debt. If the increase in equity is strong enough, the default probability may even fall, leading to a lower cost of external funds. With $\lambda_1 = 0$ the amount of equity issued more than doubles the initial response of the firm to the aggregate shock (not reported). However, independently of the distribution of firms, there is still a dampening of shocks in this economy relative to the frictionless economy.

Finally, equity issuance plays a key role in the model when $\lambda_1 > 0$. Figure 10 shows the responses for this case. Note that the responses of capital and output are now considerably more sensitive to shocks than the ones in the frictionless case. This is true for small and medium firms because those are the firms that issue equity more intensively. For those firms the default rate decreases in response to a productivity shock. The intuition is simple. In response to shocks the more constrained firms issue equity and are able to lower the agency costs of debt. The presence of issuance costs that vary inversely with the aggregate state strengthens the equity issuance mechanism. As a result, the default probability becomes countercyclical and the costs of external financing drop. The reduction in agency costs of debt imply that capital and output respond by more than in the frictionless case.

Note that this analysis is useful to provide some intuition on the results of the model, however we are omitting the importance of the distribution of firms in the results. This is considered in the following section of the paper.

### 7.3 Propagation

Another important aspect of the model is the ability of financial frictions to propagate shocks, often known in the literature as the net-worth channel. Carlstrom and Fuerst (1997) show that in the presence of financial frictions, an otherwise standard real business
Notes: The plots show the percentage deviations of capital, default probability, output and net worth to a one percent shock in aggregate productivity as a function of the firm’s cash-on-hand. The dashed-line represent the responses in the frictionless economy. The solid lines show the response of the economy with bankruptcy costs and state contingent costs of equity.
cycle model can exhibit a strong internal propagation mechanism. The caveat is that because the default rate in their model is procyclical propagation is generated not only because net worth increases but also because the initial shock is dampened the most and as aggregate productivity returns to its previous level the dampening diminishes as well.

The purpose of this section is to show and explain why in our model it is possible to generate both amplification and propagation of shocks. The top panel of Figure 11 shows the impulse response of aggregate output for the frictionless economy, the economy with constant costs of issuing equity and the economy with state-dependent costs of issuing equity. The economy with constant costs of issuing equity shows slightly hump-shaped responses of output, however, for several periods the response of output in the economy with financial frictions is still below the response in the frictionless economy.

In contrast the economy with varying costs of issuing equity exhibits both amplification and internal propagation. The bottom panel of Figure 11 shows that the bulk of the response comes from the behavior of small firms. With constant costs of issuing equity the model exhibits some internal propagation, however, shocks are dampened relative to the frictionless case.

The propagation mechanism works as in the model of Carlstrom and Fuerst (1997). Namely, in response to a positive shock small firms issue equity, which allows these firms to borrow more in the subsequent period at a considerably lower cost. Because these firms are not dividend payers, the funds stay in the firm for several periods, which helps to propagate the shocks throughout time.

Regarding the behavior of the default rate Figure 12 shows the response of the default rate to a positive aggregate shock. The model is quite successful in this dimension as well. The specification with varying costs of equity is able to generate the countercyclical and persistent behavior of defaults documented in Section 2. The default rate is countercyclical in this economy because small firms have a strong incentive to issue equity because it is more attractive in good times. The increase in net worth allows the firm to increase borrowing and still reduces the likelihood of default. Without varying costs of equity issuance this mechanism is somewhat muted, because the amount of equity issued is still
Figure 11: Equity Issuance and Output Dynamics

Responses of Aggregate Output

Responses of Aggregate Output for Small Firms
relatively small. As a result, default still increases in response to an increase in $z$.

7.4 Debt-to-Assets and Aggregate Dynamics

The dynamic model is capable of generating, at least, qualitatively, four out of the five stylized facts presented in Section 2. In particular, the model misses stylized fact #4, that is the firm’s decision rules do not display a positive relationship between size and debt-to-assets. We conjecture this is an important property to have in models with financial frictions because it enhances the internal propagation mechanism. The intuition for this conjecture is as follows. The net-worth channel is stronger if debt-to-assets increases with firm size because one additional unit of net-worth would accomplish two things. First, as in the current version it lowers the expected default rate and the firm can borrow at a lower cost (smaller bankruptcy costs). But, if leverage increases it also leads to a even stronger response of investment on the periods subsequent to the shock as firms’ net worth grow. Note that in models with a linear technology, such as Carlstrom and Fuerst (1997), leverage is invariant to the size of the firm.
We are able to generate a positive relation between firm net-worth and book leverage by changing what happens if the firm defaults. First, we assume the firm does not produce any output if \( \omega < \omega \). Second, we assume capital fully depreciates in the event of default (or it is just enough to pay the fixed cost). Under these two assumptions we can rewrite the debt provider’s break-even constraint as follows:

\[
(1 + r_b)(k - n)[1 - \Phi(\omega)] - \mu z k^\alpha \Phi(\omega) = (k - n).
\]

Table 4 compares the targets in the extension with the targets on the benchmark economy. The main difference is regarding the increase in equity issuance for small firms, which is now around 2 per cent, but we still underestimate its empirical counterpart. The top panel in Figure 13 compares the policy rules of debt-to-assets in the benchmark and in the extension economy. The differences are striking. For smaller firms, and the ones facing a more binding constraint we see that debt-to-assets is declining in the benchmark economy, while it is increasing in the extension case. The bottom panel in Figure 13 plots the impulse response of aggregate output in benchmark and extension economies together with the frictionless economy. We can see very clearly that the net-worth channel is considerably more important in the case where debt-to-assets increase with firm size.

8 Conclusions

We have analyzed a dynamic model with heterogenous firms and incomplete markets to understand the role of financial frictions to amplify and propagate aggregate shocks. We show that bankruptcy costs and issuance costs of equity have a great potential in amplifying and propagating aggregate shocks. Evidence of the two mechanisms are well documented stylized facts in macroeconomics and finance: (i) countercyclical default rates; and (ii) a positive relation between debt-to-assets and firm size. Our simulations results suggest that matching these two empirical facts leads to a strong amplification and propagation mechanism.

There are several important extensions to this study. The obvious is to incorporate the two main results of the paper into a computable general equilibrium model to analyze
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<td>Default rate</td>
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Figure 13: Debt-to-Assets and Output Dynamics
important macro questions on monetary and fiscal policy. Another extension is to assess the importance of financial frictions for the reduction in output volatility. In our environment we can provide several potential explanations for the great moderation. The most natural ones that come out of the model are lower bankruptcy costs and/or less variability in equity issuance costs over the business cycle.

A Appendix

In the first appendix, we give an overview of the different approaches used in the literature to describe agency costs. In the second appendix, we describe two production functions that can generate \( z \)-countercyclical default rates. The third appendix contains the proofs of the propositions.

A.1 External finance in the literature

The idea that agency problems cause a wedge between the expected rate that firms pay on outside financing and the opportunity cost of using retained earnings has been well established in the literature.\(^{24}\) In the corporate finance literature there are now several papers that consider models in which there is a choice between debt and external equity (and both are subject to an external finance premium).\(^{25}\) Since GDP is produced mainly in firms that can issue equity\(^{26}\) it is important to allow for external equity in macro models of the business cycle. Almost all DSGE models that incorporate agency problems in firm finance, however, restrict external finance to debt contracts. The first subsection discusses the alternative approaches used. Next, we discuss which agency problem is the basis of our framework and which modifications we introduce to improve key predictions of the model.

Take the money and run One possible problem that a lender may be concerned about is that the borrower takes the funds and either squanders the money or moves to a place on

\(^{24}\) See Jaffee and Stiglitz (1990) for an early survey.

\(^{25}\) See for example Moyen (2004) and Hennessy and Whited (2005).

\(^{26}\) According to the Economic Report of the President, 2003, corporations account for approximately 60 per cent of GDP.
earth where he cannot be harassed by the lender. This is the agency problem considered in Kiyotaki and Moore (1997) (KM henceforth). In particular, KM assume that a borrower will always run away with any funds it could run away with. Consequently, lenders will require collateral and in particular the amount they lend has to be less than the value of the collateral. KM assume that there are two types of producers, farmers and gatherers, and both use the fixed factor (land), which is not only productive but can also serve as collateral. KM choose parameters such that the farmers borrow from the gatherers, are constrained in the amount they can borrow, and at the margin are more productive. Let \( r_l \) be the lending rate, \( k \) the amount of the fixed factor in the hands of the constrained agents, and \( q' \) the value of the collateral in the next period. Then the amount of funds borrowed, \( b \) has to satisfy the following equation.

\[
(1 + r_l)b \leq q'k. \tag{49}
\]

KM show that this framework generates two appealing predictions.

First, an increase in productivity leads to an increase in profits and, thus, an increase in net worth for constrained firms, which in turn makes it possible for the firm to borrow more. This leads to an increase in the scale of production by constrained firms and a further increase in the net worth in the next period. Consequently, a one-time increase in productivity can lead to a persistent increase in the demand of the fixed factor through the so-called net-worth channel. In general equilibrium, an increase in the use of the fixed factor by constrained agents must lead to a reduction in the use by unconstrained agents. But because constrained agents are assumed to be more productive, this reallocation of resources does lead to an increase in aggregate productivity. To understand the effect of net worth on the amount the firm can borrow, consider the budget constraint of the borrower, which is given by

\[
qk = n + b, \tag{50}
\]

where \( n \) is the net worth of the agent. Suppose that the borrowing constraint is binding. Then it is easy to show that
The term in brackets is the real rate of return, that is, the nominal rate of return adjusted for the increase in value of the collateral. The amount a firm can borrow is, thus, determined by the condition that the amount of net worth has to be equal to the real interest payments. This condition implies quite a high leverage ratio. KM also consider a modification of the model in which cultivating the fixed factor requires additional (uncollaterizable) investment. This reduces the leverage factor.

The second appealing prediction of the KM framework is that the value of collateral increases in response to a positive shock. An important component of net worth, \( n \), is the value of the existing fixed factor, \( q k_{-1} \). Consequently, an increase in the demand for this factor leads to an increase in the value of the firm’s net worth and, thus, through the channel described above to a further increase in the demand of the fixed factor. Note that the value of collateral is not only pushed up by the current increase in the demand for the fixed factor, but—since it is a durable asset—also by the increased demand in subsequent periods. The framework, therefore, delivers an appealing within-period and intertemporal multiplier process.

It is hard to believe, however, that a big concern for lenders in developed economies is that managers divert firm resources in a way that none is left for the lender. Indeed, a substantial fraction of business loans does not have collateral. Moreover, the existence of collateralized loans doesn’t mean that this agency problem is important. A collateralized loan is basically a loan that gives the lender priority when bankruptcy occurs and this is advantageous even if bankruptcy occurs for other reasons than diversion of assets.

Furthermore, KM rely on linear production functions and an infinite elasticity of intertemporal substitution. Cordoba and Ripoll (2004) show that if one relaxes those assumptions the effects are quantitatively important only for empirically implausible values. Moreover, if parameters are such that shocks lead to a substantial increase in leverage (and investment) then the corresponding increase in interest rates will be strong as well. This will substantially dampen the effect on net worth in the next period and thus reduce
Kocherlakota (2000) argues that the collateral approach cannot deliver large effects for reasonable parameter values. Here we want to point out that this is true in the framework he considers but the framework of Kocherlakota is quite different from the KM framework. In particular, Kocherlakota assumes that the constrained agents are the only ones that hold the fixed factor. Consequently, there is no reallocation of the fixed factor from less productive to more productive agents as in KM. Whereas in KM the amount of the fixed factor held by constrained agents is the center piece of the analysis, in Kocherlakota it is the investment in the uncollaterizable investment. But this investment is not subject to the leverage effect discussed in the main text. Moreover, it is also not affected by changes in the current value of land. To see why consider the borrowing constraint (assumed to be binding) and the budget constraint used by Kocherlakota.

\[(1 + r^l)b = q'k\]  
\[qk + x = n + b\]

where \(x\) is the amount of uncollaterizable investment. In the Kocherlakota framework farmers use all the land so that \(k\) is constant and equal to the given supply of the aggregate factor. Suppose the aggregate supply is equal to 1. The solution for \(x\) is then given by

\[x = \text{profits} - (1 + r^l)b_{-1} + q'(1 + r').\]  

Note that the leverage effect has disappeared. The direct effect of a change in net worth on investment is equal to one whereas in KM it is huge. Also, note that the current-period land price has disappeared from this equation. In KM an increase in \(q\) increases \(n\) which

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27Kocherlakota (2000) makes a similar point but he doesn't allow for a reallocation of the fixed factor from productive to unproductive agents. As shown in the appendix, with no variation in use of the fixed factor the leverage effect of KM is eliminated.
has such a huge effect on $b$ (because of the leverage effect) that this effect dominates the downward effect on $b$ caused by the decrease in $q'/q$ (keeping $q'$ constant).

**Limited contract enforceability** The contracting environment in KM is very simple. Lending is assumed to occur through one-period lending contracts. Moreover, defaults never occur since loans are fully collateralized. An alternative approach followed in the literature is to design optimal contracts in which the lender commits to a sequence of state contingents capital allocations to the firm and payments to the entrepreneur. Albuquerque and Hopenhayn (2004) and Cooley, Marimon, and Quadrini (2004) use this approach and derive the optimal contract in an environment in which the borrower can divert the assets of the firm as in KM.

An attractive feature of these papers is that they can explain firm growth. In particular, the amount of capital assets under the control of the firm remains constant until the enforcement constraint is binding, that is, when the entrepreneur is tempted to divert the funds. If that happens then the amount of capital is increased and the temptation to divert funds is prevented by making the current relationship more attractive. In Cooley, Marimon, and Quadrini (2004) a positive aggregate shock represents a higher probability that a new project will have high productivity. The aggregate shock doesn’t affect the productivity of existing projects. Consequently, entrepreneurs are more tempted to divert the capital of the existing relationship and start a new project. This means that for more firms the enforcement constraint will be binding and the amount of funds received from the lender will increase. The optimal contract, thus, has the attractive property that a productivity shock is magnified by the increased allocation of funds to firms.

Less appealing is that these additional funds are given because it is exactly during good times that entrepreneurs are more likely to divert capital and entrepreneurs will remain loyal to the existing contract only if it is made more attractive. Another undesirable feature is that firms—as in KM—never default. Also, Quadrini (2004) points out that state-contingent optimal contracts may not be renegotiation proof. That is, it may be possible that both the lender and the entrepreneur would prefer to renegotiate the contract. Quadrini (2004) designs the optimal contract that adds as an additional restriction that it
is renegotiation proof. Interestingly, he shows that (randomized) liquidation may be part of that renegotiation proof contract in an environment in which liquidation would not be part of the optimal contract.\textsuperscript{28}

**Other moral hazard problems** The assumption that the entrepreneur can divert the assets of the firm creates a particular (and quite extreme) moral hazard problem. Besides diverting the assets and leaving nothing to the creditor, there are other—less extreme—entrepreneurial actions that diminish the chance for the lender to get his money back. For example, the revenues could depend on the effort put in by the entrepreneur. Also, the amount of risk chosen by the manager may be hard to control by the lender. One would think that these are more plausible concerns than diverting assets. Although this agency problem has received considerable attention in the literature, there are few papers that have incorporated an unverifiable effort or investment choice into DSGE models. One exception is the paper by den Haan, Ramey, and Watson (2003) in which the outcome of the project is affected by the effort choice of the entrepreneur. The model in den Haan, Ramey, and Watson (2003) has besides this moral hazard problem also a matching friction, so that it takes time for a borrower to find a lender, and an inefficient allocation of funds across lenders. The latter friction implies that lenders who are in a relationship with a borrower may not have enough funds to write a sustainable contract, which stands in sharp contrast with the rest of the literature that assumes that lenders have unlimited access to funds. Unlimited access to funds may not always be an appropriate assumption. For example, bank equity regulation makes the ability of banks to extend loans dependent on the amount of bank equity, which is not easy to adjust.

Another example of a model that incorporates a more realistic moral hazard problem is the DSGE model in Covas (2004) who considers a model in which the manager of the firm chooses the risk of the project undertaken. An important feature of Covas (2004) is that the objectives of the owner and the manager do not necessarily coincide. This stands in sharp contrast with the rest of the papers in the macro literature in which the owner

\textsuperscript{28}Quadrini (2004) also considers an environment in which the scrap value of the firm may exceed the joint value of the contract. In that environment liquidation can be part of the optimal contract as well.
and the manager are typically the same agent.

**Asymmetric information** Uncertainty is an important source of agency problems. One possibility is that there is asymmetric information about the characteristics of the borrower when the contract is written. We are not aware of DSGE models that incorporate this type of agency problem. Given that the probability of drawing a "lemon" may very well increase during a downturn this may be an interesting approach. Another important reason why uncertainty matters is that the outcome of a project is random (even if the borrower puts in the best possible effort). The question arises whether this in itself leads to an increase in the external financing premium. In a classic paper, Townsend (1979) shows that if it is costly for the lender to observe the realization of the shock, then the optimal contract to deal with this agency problem is a debt contract. The relevant cost in the costly-state-verification (or CSV) framework is the cost the lender has to incur to determine the outcome of the realization. In the literature, observed bankruptcy costs are often used to calibrate the magnitude of these verification costs. But this is a bit odd, since the actual verification costs are likely to be much smaller than typical bankruptcy costs. Moreover, we are not aware of any evidence that suggests that the verification itself is very costly.

**One-period debt contracts and uncertain outcomes** Note, however, that the assumption that it is costly to determine how much resources are left in the firm is only needed for the result that the one-period contract is the optimal contract. If there are other reasons why state-contingent lending contracts are not feasible and lending occurs through one-period debt contracts (for example because of habits, convenience, or simplicity) then one can not only use this framework but one can also calibrate the model using observed bankruptcy costs. All equations, including those that determine what contract among all one-period debt contracts is the optimal one, are exactly the same as in the CSV framework. The only difference is that if verification costs are negligible then the model by itself does not imply that the one-period debt contract is the optimal contract.
Properties of the CSV framework  An attractive feature of the optimally chosen one-period debt contract is that the amount lent increases with the net worth of the agent as well as the expected productivity of the firm. Consequently, as shown, for example, in Carlstrom and Fuerst (1997), this framework is helpful in propagating shocks through the so called net-worth channel. A positive shock increases net worth which increases the scale of operations, profits, and, thus, next period’s net worth. This in turn leads to an increase in lending. A not so attractive feature is that defaults increase with aggregate productivity. That is, an increase in aggregate productivity leads to such an increase in the scale of the project that default happens more frequently. Moreover, the assumption of linear production technology implies that the subsequent increase in net worth has no effect on default rates. Bernanke, Gertler, and Gilchrist (1999) assume that the realization of the aggregate productivity is not yet know when the debt contract is determined. This means that an unexpectedly high aggregate productivity shocks results in a lower default rate. But they still have the property that an increase in the expected productivity leads to an expected increase in the default rate.

We discuss the reasons for the procyclical default rate in this framework in more detail in Section 5.1. We also propose two modifications. The first modification is to replace the linear technology production function by a standard production function that satisfies decreasing marginal returns. With this production function, we find that increases in net worth have a negative effect on the default rate. The second modification adds an additional component to the production function. With this specification, it is easy to find parameters such that an increase in aggregate productivity reduces the default rate.

Bernanke, Gertler, and Gilchrist (1999) add adjustment costs of capital to reinforce the net-worth channel. With adjustment costs an increased desire to invest leads to an increase in the price of the capital. This means that a positive productivity shock increases the value of net-worth not only because of the increase in retained earnings but also because of the increase in the market value of the capital stock. We find such a valuation effect

\footnote{Note that with the linear production function typically assumed in this literature, this modification doesn’t affect the result that changes in net worth have no impact on the default rate.}
very plausible and an attractive feature of the model to have. However, an increase in the price of capital because of the difficulty to quickly produce additional capital is likely to be only a part of the increased value of the firm during good times. Moreover, if productivity increases are not embodied in existing capital then the price of existing capital might even go down. In Cooley and Quadrini (2001) net worth of the firm represents the earnings potential of the firm. This is the approach we follow in this paper.

**Predictions of the different frameworks** The agency problems discussed above are quite different. Nevertheless, changes in net worth and productivity have similar effects on real activity, that is, these models all predict that an increase in productivity leads to an increase in real activity, which over time leads to an increase in net worth and a reduction in the external finance premium. There are important differences in the predictions of these different approaches as well. As discussed above, in some frameworks default doesn’t occur and in some frameworks, the default rates are even procyclical. The frameworks also differ in the importance of changes in the risk free or discount rate. If a firm is nothing more than a stock of capital stock and produced commodities can be transformed into investment commodities (either one for one or at a slightly higher rate in the presence of adjustment costs), then there is no direct effect of the discount rate. However, if part of the firm are assets that are in fixed supply then the value of the firm is the discounted value of the earnings generated by this fixed asset and changes in the discount rate are then likely to have strong effects on the value of the firm.

Even across papers that use the same framework we find important differences in the choices needed to implement the problem. For example, in ? the savings decision of the entrepreneur has an interior solution. This means that entrepreneurs withdraw funds from the firm every period. Depending on how the entrepreneur changes his consumption level in response to a shock this can either dampen or reinforce the net worth channel.\footnote{That is, the savings decision affects the price of the commodity produced by the sector with the agency problem. In equilibrium, this price will adjust every period so that the Euler equation of the savings decision has an interior solution.}

\footnote{For example, in Carlstrom and Fuerst (1997) a positive productivity shock does indeed lead to a sharp
Bernanke, Gertler, and Gilchrist (1999) on the other hand choose parameters such that entrepreneurs choose to keep all funds within the firm. Bernanke, Gertler, and Gilchrist (1999) prevent the net worth of firms to go to infinity by assuming that firms exogenously stop operating with some probability in which case the owners consume the net worth of the firm.

Making magnification possible in general equilibrium So far, the profession has not been successful in identifying external shocks that are large enough to explain the observed fluctuations in economic aggregates.\(^\text{32}\) Therefore, the recent literature has tried to build DSGE models in which shocks are magnified and propagated and models with agency problems have been the main contender.\(^\text{33}\)

It is important to realize though that in a general equilibrium this is not enough to magnify and/or propagate shocks. If firms increase the scale of their operation the question arises where the additional resources come from. One possibility is that entrepreneurs decrease their consumption and invest the additional funds in the firm.\(^\text{34}\) In numerous papers, however, the high return on retained earnings forces the consumption level of the entrepreneur to be at a corner so that no further reductions in consumption are possible. Moreover, even if the entrepreneur increases the amount of retained earnings, this still wouldn’t answer the question where the additional funds provided by the lender come from. Some papers simply assume that lenders have unlimited access to funds at an interest rate that is not affected by the shock.\(^\text{35}\) This may be a plausible assumption when one studies the effect of a country-specific shock for a small country with excellent access to international funds. In this case the resources to finance the expansion come from abroad. Interest rates would also be constant if savers have a very high intertemporal reduction in the amount of entrepreneurial consumption but is then followed by a moderate increase but this result may change if nonlinear utility functions for the entrepreneur are considered.

\(^{32}\)See Cochrane (1994), for example.

\(^{33}\)Alternatives are models with labor market frictions such as Andolfatto (1996), Merz (1995), den Haan, Ramey, and Watson (2000).

\(^{34}\)But note that if the risk smoothing motive is strong enough, then entrepreneurs would like to increase their consumption.

\(^{35}\)Papers that make this assumption are Kocherlakota (2000) and Cooley, Marimon, and Quadrini (2004).
elasticity of substitution. In this case the expansion would be financed by a reduction in (the growth of) aggregate consumption. Of course, this does lead to the counterfactual implication that consumption is countercyclical.

In this respect, the analysis of KM is quite interesting. Although KM assume that interest rates are constant, the key factor of production in their analysis is in fixed supply. Consequently, the expansion is financed by a reallocation of the factor from less productive to more productive agents. Unfortunately, Cordoba and Ripoll (2004) show that if one relaxes the assumptions of constant interest rates and constant marginal productivity that it is difficult to generate quantitatively interesting results in the KM framework. Although, Cordoba and Ripoll (2004) focus on the agency problem of KM, the message of their paper is likely to carry over to other environments. For example, if the borrowing constraints are relaxed and intermediaries are more willing to lend out funds, then the risk free rate may have to increase to attract these additional funds from depositors.

Note that in a framework such as CSV that has a procyclical default rate this problem is even worse since it means that more resources are lost due to the increased bankruptcy rate. In fact, in Carlstrom and Fuerst (1997, 1998) this effect is so strong that even in later periods, when the increase in entrepreneurial net worth has reduced agency problems, the output response is only marginally above the output response of the standard real business cycle model.

Consequently, agency costs by themselves may not be enough to generate a framework with quantitatively interesting magnification and propagation effects. In addition, one would need a mechanism through which aggregate resources increase. One mechanism is the increase in labor supply. Empirical estimates of the labor supply elasticities make clear that changes in the intensive margin are likely to be weak but changes in the extensive margin are empirically more plausible. den Haan, Ramey, and Watson (2000) build a labor market matching model into a general equilibrium framework and show that changes in the extensive margin are important in magnifying and propagation shocks.
A.2 Two production functions that generate $z$-countercyclicality

Now we consider specifications in which net revenues of the firm consist on two parts that depend differently on aggregate productivity. There are actually several specifications that work and in this section we illustrate two. In the first specification, the two parts of the production function both represent resources. In the second specification the first part represents a resource whereas the second represents a fixed cost.

A.2.1 Multiple-project specification

If production is given by (17) then an increase in $z$ increases the slope of the zero-profit curve but doesn’t change the slope of the iso-profit curves. For this production function expected firm income, $i(\omega, k; z)$ is given by

$$i(\omega, k; z) = z\omega k^{\alpha} F(\omega)$$

with $F(\omega)$ given by (8). The reason why the slope of the iso-profit condition doesn’t vary with $z$ is that both $\partial i(\cdot)/\partial \omega$ and $\partial i(\cdot)/\partial k$ depend linearly on $z$. Consequently, the value of $z$ drops out of the expression for the slope of the iso-profit curve. One example of a production function for which that is not the case are the following.

$$z\omega k^{\alpha} + a_0 z^{a_1} \omega \text{ with } a_1 > 1$$

The idea here is that the firm’s net revenues consist of different components that (i) depend differently on $z$ ($a_1 \neq 1$) and (ii) depend differently on $\omega$ and/or $k$. The specification used here is just one of several that can accomplish this. One nice feature of this specification is that the combination of $\omega = \bar{\omega}$ and $k = n$ is still on the zero-profit curve and an increase in $z$ simply rotates the zero-profit curve upwards. But the increase in $z$ also increases the slope of the iso-profit curves and it is easy to find values for $a_0$ and $a_1$ such that the default rate decreases. Figure 14 documents the effect of an increase in $z$ on the default rate and the capital lend when $a_0$ is equal to 0.1 and $a_1$ is equal to 8.
Figure 14: Modified CSV Framework: Multiple-Project Specification
A.2.2 Fixed-cost specification

In the second specification, output net of the fixed cost is given by

\[ z\omega k^\alpha - c(z) \]

with \( c(z) = c_0 - c_1 z^2 \).

(57)

It is easy to show that the expression for the expected income of the lender hasn’t changed and the zero-profit condition for the lender is now given by

\[ zk^\alpha G(\omega) - c(z) = k - n \]

(58)

and the expression for \( G(\omega) \) remains unchanged. That is, relative to the case with \( c(\cdot) \) equal to zero, the zero-profit curve shifts down. An increase in \( z \) simply shifts up the zero-profit curve and, thus, has the same effect as an increase in \( n \). That is, in the presence of diminishing returns an increase in \( z \) would decrease the bankruptcy probability. Although, this specification is computationally very easy and quickly delivers the result one has to be aware that now corner solutions are possible. The reason is that at \( \omega = 0 \) there may not be a value for \( k \) bigger than \( n \) such that bank profits are zero. That is, because of the fixed cost, the bank would require \( \omega > 0 \) even if it sets \( k \) equal to \( n \), that is, even if it doesn’t lend anything at all. The contract with \( k = n \) would be an insurance contract. For large values of \( \omega \) the firm pays the bank a fixed sum and when \( \omega \) is so low that the fixed costs are not covered the bank will take the loss. Figure 15 plots zero-profit curves for different values of \( z \) and documents the different starting points of the zero-profit curve.

In the specification an increase in aggregate productivity lowered the fixed costs but did not affect the marginal product of capital. The results will go through if one uses a more standard specification, i.e., \( z\omega k^\alpha - c(z) \), as long as the reduction of \( \omega \) caused by the parallel shift in the zero-profit curve dominates the reduction in \( \omega \) caused by the increase in the slope of the zero-profit curve.
Figure 15: Zero-Profit Curves and the Fixed-Cost Specification
References


