Abstract. This work presents an equilibrium model of diversification through merger formation. Due to moral hazard problems, poorly capitalized firms are credit rationed and may seek to alleviate the incentive problem (and thereby raise external funds) by either merging, employing a monitor or a combination of the two. Within this setting, the effects on merger activity of different kinds of capital tightening are studied. In particular, credit crunches, collateral squeezes and savings squeezes are analyzed. One of the main results is that conglomerate merger activity increases during times of economic expansion and is positively related to aggregate output. Furthermore, the model offers a rationale for diversification that is immune to the diversification neutrality result and furthermore explains why diversified companies trade at a discount relative to their non-diversified counterparts.

Keywords: Mergers, merger waves, diversification, diversification discount, financial intermediation, capital tightening.

JEL Classification: L16, G34.

1. Introduction

Corporate diversification through merger has a long history, both as a fact of life and as a subject of analysis by financial economists. Received wisdom is less than favorable in judging the benefits of diversification, for broadly two reasons. The first may be termed the diversification neutrality result and poses that any diversification possibilities that corporations might have, will, in a perfect capital market, already have been exhausted through shareholders’ individual portfolio choices. The second is the so-called diversification discount, i.e. the finding that diversified corporations have, during some periods, traded at a discount relative to their non-diversified counterparts. Together, these two findings have prompted scholars to conclude that corporate diversification is at best neutral, but more likely, a value destroying strategy.¹

Despite these concerns, corporate diversification continues to be strong, a finding that should prompt further study into the causes and consequences of diversification. The current paper seeks to fill this gap in the theory by showing how diversification may be useful despite the diversification neutrality result and that corporate diversification may be optimal and yet consistent with diversified firms trading at a discount relative to non-diversified firms. Furthermore, by embedding the analysis of diversification within an equilibrium framework, it is also possible to address two further issues. First, it is possible to characterize firms

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¹A leading corporate finance textbook even goes as far as characterizing diversification as one of three main “Dubious Reasons for Mergers” (see Brealy and Myers, 2003, p. 934).
that choose to diversify. Second, it is possible to derive the extent of diversification in the economy. As a consequence of this last finding, it can be determined under which conditions conglomerate merger activity should be prevalent and thereby address the timing of mergers and merger waves.

The current work thus contributes to two hitherto separate literatures, namely that one corporate diversification and the diversification discount and to that on merger waves. Accordingly, the following two subsections will briefly outline existing work on those two literatures and place the current work within them.

1.1. The Diversification Debate. The central question of the diversification debate is why corporations are active in seemingly disparate lines of business. At face value, diversification can be explained by the fact that when pooling income streams that are less than perfectly positively correlated, the resulting income stream is less volatile than were the constituent income streams. Such a reduction of riskiness could potentially be beneficial. This view of corporate diversification was challenged by numerous scholars, such as Levy and Sarnat (1970) and others. Their basic insight is that any reduction in risk that a firm may achieve through diversification can be replicated by the individual shareholders through an appropriately chosen portfolio. Shareholders may even achieve such diversification more cheaply than the firm can. Loosely speaking, what is being criticized by this literature is diversification as an activity that is pursued because it directly benefits shareholders by reducing their exposure to risk, or in other words, a pure risk-reduction motive for corporate diversification.

Beginning with Lewellen (1970), numerous authors considered another rationale for diversification. These include Higgins (1970), Lintner (1971), Rubinstein (1973), Melnik and Pollatschek (1973), Higgins and Schall (1975) and Lee (1977). According to this strand of literature, the view of diversification as a means to decrease shareholders’ exposure to risk is unnecessarily narrow in that corporate diversification and the resulting decrease in riskiness could increase the combined entity’s debt capacity. I will term this the debt capacity motive for diversification. The main problem with the debt capacity motive is that it suffers from some of the same drawbacks as that of the pure risk-reduction motive. Specifically, this strand of literature has ignored the fact that in perfectly functioning capital markets, firms’ access to credit (and hence their debt capacity) should be constrained only by the value of their projects. In other words, under the same conditions where the diversification neutrality result has bite, increasing debt capacity through diversification should not be a concern in the first place.

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2 A conglomerate merger is best viewed as a combination of two separate legal entities into one single entity, further characterized by the fact that the two firms are not active in the same industry, broadly defined, or vertically related by being active in the same supply chain. It should be noted that there are immense difficulties in empirically distinguishing pure conglomerate (or diversifying) mergers from vertical mergers (see e.g. Goudie and Meeks, 1982, who provide a description of firms engaged in diversifying mergers). Discrimination between the two kinds of mergers must be based on a case by case review which is prohibitive for large data sets. For a review of descriptive work on diversifying firms, see also Montgomery (1994) and references therein.

3 Actually, Levy and Sarnat (1970) themselves concede that a debt capacity motive may exist.

4 Shleifer and Vishny (1992), Fluck and Lynch (1999), Amihud and Lev (1981) and Hermalin and Katz (2000) consider other motives for corporate diversification than the ones mentioned above. Cerasi and Daltung (2002) study similar issues to the ones considered in this paper, but their focus is slightly different, concentrating on overload costs to diversification and their relation to delegation.
Another apparent problem with diversification is the finding that diversified firms trade at a discount relative to their non-diversified counterparts. A number of authors describe the existence of such a diversification discount, e.g. Lang and Stulz (1994), Berger and Ofek (1995, 1996), Servaes (1996), Lins and Servaes (1999). One problem with interpreting the existence of a diversification discount is one of causality. If indeed diversified firms trade at a discount, is that prima facie evidence that diversification per se destroys value? In fact, the causality could be reversed if one finds that what makes firms trade at a discount is also what makes them diversify. In a nutshell, what must be recognized is that firms choose to diversify, i.e. the diversification decision itself is endogenous.

A number of recent contributions make exactly this point. Villalonga (2000), Lamont and Polk (2001), Maksimovic and Phillips (2002), Campa and Kedia (2002), Kruse et al. (2002), Yang (2005) and Akbulut and Matsusaka (2005) all revisit the value of diversified firms while explicitly taking into account the endogeneity of the diversification decision. Their results suggest that when properly acknowledging this endogeneity, the diversification discount may disappear altogether or even turn into a premium. Similarly, their work also suggests that there is a strong element of self-selection to the group of diversified firms. In particular, such firms tend to do poorly prior to diversification, are more heavily leveraged and are generally more liquidity constrained. Cabral (2003) explicitly considers the endogeneity of the diversification decision in a model of reputation.

1.2. Aggregate Merger Activity. One of the most salient features of M&A activity is that it displays a marked wave pattern which roughly follows the business cycle. The literature has, until recently, been characterized by a lack of theoretical explanations for this phenomenon. Financial theory has mostly taken a single firm as the object of analysis without embedding the merger formation decision in an equilibrium environment. The industrial organization literature has in turn focused almost exclusively on merger formation within explicit oligopoly models, which have turned out to be somewhat unsuitable for the analysis of merger waves.

In view of this lack of theory, recent developments have been encouraging. A number of theories have been proposed, each focusing on different aspects of M&A activity. Jovanovic and Rousseau (2004) focus on mergers as responses to reorganization possibilities brought about by changes in technology. Rhodes-Kropf and Viswanathan (2004) show how overvaluation may lead to increases in merger activity. These theories are non-strategic in the sense that no merger is, per se, prompted by other mergers in the industry or the economy. Toxvaerd (2004) considers a model in the alternative category, focusing on strategic aspects of the merger decision and on how such strategic considerations can prompt firms to decide to merge at the same time.

The current paper adds to the first (non-strategic) strand of literature by identifying another channel through which aggregate variables can prompt increases in M&A activity. In the economy under consideration, poorly capitalized firms rely on financial intermediaries in order to raise funds for projects. By merging with unrelated firms, a firm may also boost its debt capacity. In turn, this means that the equilibrium extent of merger activity is a function of the amount of funds available from financial intermediaries. In equilibrium, when intermediary capital is plentiful, more firms will become active in the economy, many of them becoming so by forming conglomerates.
1.3. Overview and Results. The present model builds on work by Holmström and Tirole (1997). They consider the effects of capital tightening on an economy in which cash-poor firms rely on financial intermediation to finance their projects. I enrich their model by allowing firms either to seek the assistance of a financial intermediary, or to engage in a diversifying merger, or (if they so desire) any combination of the two. Assuming that projects of a merged firm are independent, I draw on a diversification result found in Diamond (1984) and show that by diversifying, a firm may relax incentive constrains and thus facilitate funding.

In a nutshell, the aim of the analysis is to consider how shifts in aggregate demand and supply of capital affect funding possibilities for cash-poor firms and in turn how the firms’ incentives to engage in diversifying mergers are affected by such changes in funding possibilities.

Two distinct scenarios are considered. In the first, the supply of uninformed capital is infinitely elastic and the return on this capital is thus exogenously given. In this scenario, equilibrium is completely characterized by equilibrium in the market for intermediary capital. I find that an increase in intermediary capital (i.e. the opposite of a credit crunch) decreases intermediaries’ rate of return. In turn, this increases economic activity in the real sector of the economy. This increase is brought about by an increase in the amount of diversified firms. In the second scenario, the supply of uninformed capital is modeled by means of a standard, increasing supply function. In this scenario, equilibrium must obtain in both markets for capital, informed and uninformed. The two kinds of capital are found to be gross substitutes which introduces the possibility of multiple equilibria. It turns out that for the chosen parameterization, there is a unique equilibrium which is also stable. I find that in equilibrium, credit crunches and savings squeezes increase both intermediaries’ returns and interest rates, while collateral squeezes decrease both. When rates of return and interest rates decrease, economic activity increases, as does the extent of merger activity. The comparative statics predictions of the model are therefore robust across scenarios.

Last, because the firms that choose to diversify are those with lowest net worth, namely those firms that also rely on costly financial intermediation, the average value of diversified firms is found to be lower than that of richer non-diversified ones that access the financial markets directly.

Section 2 contains a presentation of the basic model and the derivation of a number of intermediate results needed for the analysis in subsequent sections. Building on the results contained in Section 2, in Section 3, I discuss the parameterization of the model and derive expressions and properties of the aggregate demand functions for informed and uninformed capital respectively. In Section 4, equilibria of the model are characterized under two alternative scenarios, that of infinitely elastic supply of uninformed capital and that of elastic supply. This section also presents comparative statics results and discusses their relation to stylized facts of merger activity. In Section 5, the equilibrium value of firms is discussed and contrasted with the literature on the diversification discount. Last, Section 6 offers a discussion and concluding remarks.

2. The Model

The economy under consideration is populated by three separate classes of risk neutral agents. There is a continuum of firms making up the real sector of the economy. These firms can each undertake a project requiring up front investment $I > 0$. Each firm is characterized
by its initial holding of assets $K > 0$. Assume that the assets are uniformly distributed along an interval $[0, L]$ of firms. If a project is undertaken, the outcome depends on the unobservable effort exerted by the entrepreneur managing it. Specifically, the entrepreneur can choose high, intermediate or low effort, leading to a probability of success $p_H$ if effort is high and $p_L$ if effort is intermediate or low, where $p_L < p_H$. Why switching between intermediate and low effort does not change the probability of success will become apparent below, where the role of intermediaries is discussed. For convenience, define $\Delta p \equiv p_H - p_L$.

In case of success, the investment returns $R > 0$, while a failed project returns zero. To introduce moral hazard into the firm’s problem, assume that the entrepreneur enjoys private benefit $S > 0$ from exerting low effort, $s < S$ with $s > 0$ from exerting intermediate effort and zero from exerting high effort. The private benefit can be thought of as an opportunity cost of working on the project. Alternatively, the choice of the agent can be thought of as the choice between three different projects, a good project $G$, an intermediate project $s$ and a bad project $S$. The table below summarizes the three versions of the project in terms of private benefit and success probability:

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<tr>
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<th>$G$</th>
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<tr>
<td>$p$</td>
<td>$p_H$</td>
<td>$p_L$</td>
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<tr>
<td>$z$</td>
<td>0</td>
<td>$S$</td>
<td>$s$</td>
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It should be noted immediately that the firm will always choose an $S$ project over an $s$ project, since it yields higher private benefit without lowering the probability of success.

If a firm’s assets are such that $K < I$, then it needs to raise an amount $I - K > 0$ externally in order to realize the project. Assuming that the choice of effort is unobservable to outsiders, there is a moral hazard problem which must be taken into account when deriving the optimal contract vis-a-vis outside investors. Since moral hazard is trivial when the agent is risk neutral, a limited liability constraint is imposed such that the agent must have a non-negative return even in case of failure.

Turning to the financial sector, many small uninformed investors invest in projects in return for adequate compensation. The market for uninformed capital is assumed to be competitive. Furthermore, these investors can only observe the outcome of the project, not the agent’s effort. As regards the aggregate supply of uninformed capital, two scenarios will be considered. In the first, there is an infinite amount supplied at rate $\gamma$. In the second, uninformed capital is supplied according to an upward-sloping supply function $S(\gamma)$.

Last, there are informed investors (or monitors). The informed investors can be thought of as financial intermediaries such as venture capitalists or large banks. These investors can, at a personal and unverifiable cost, inspect a firm’s project and reduce the private benefit from shirking. Specifically, the monitor can, at cost $c > 0$, rule out the $S$ project, effectively reducing the monitored firm’s options to the $G$ and $s$ projects. Thus the informed investors’ monitoring activity is also subject to moral hazard, a fact which is reflected in the monitors’ incentive scheme. Only monitors have access to the monitoring technology and are assumed to operate in a competitive market. It should be noted at this stage that the firms and the monitors may, if they so wish, invest in the market for uninformed capital on the same terms as the uninformed investors.

The market for intermediation deserves some further discussion. First note that since monitoring activity is subject to moral hazard, the problem of providing incentives to monitor is very much like that of providing the firm with incentives to behave diligently. In the same
way that firms may relax financing constraints by diversifying, a financial intermediary may diversify through its choice of projects to monitor. This is exactly the case considered by Diamond (1984), where he shows that if there are no overload costs to monitoring projects, the intermediary can diversify away all risk by monitoring an unbounded number of firms. This would have the effect that the intermediary could get away with monitoring firms without having itself a financial stake in the monitored firms. Since an integral part of the current analysis is to determine the effects of aggregate capital held by the intermediary sector on aggregate merger activity, it is assumed for simplicity, like in Holmström and Tirole (1997), that projects monitored by an intermediary are perfectly correlated. This means that the amount of monitoring activity is bounded by the intermediary sector’s asset holdings.

Denote by $K_F$ and $K_M$ the aggregate amounts of firm capital and monitoring capital respectively. For later use, note that the aggregate amount of firm capital is given by

$$K_F = \int_0^L KdU(K) = L^22^{-1}$$

In the full version of the model, firms are allowed to either stay independent, do so with a combination of direct and indirect financing, merge with another firm, or do so with a combination of direct and indirect finance. In the following subsections, each option will be analyzed in turn.

2.1. Direct Finance. As noted above, the firm will never choose an $s$ project if his project choice is unobservable. The relevant options for the firm are thus given by the following table:

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<th>G</th>
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<td>p</td>
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<td>z</td>
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Consider the entrepreneur’s incentives when writing the optimal debt contract. Letting $R_F$ and $R_U$ denote the firm’s and the (uninformed) investor’s return in case of success, a sharing rule is given by

$$R_F + R_U = R$$

Assuming that it is indeed optimal to induce the agent to exert high effort (choosing probability $p_H$), the incentive compatibility constraint is $p_H R_F \geq p_L R_F + S$, or

$$R_F \geq \frac{S}{\Delta p}$$

Define pledgeable income as the maximum amount that can be pledged to investors while maintaining the right incentives for the agent. This amount is given by

$$p_H \left( R - \frac{S}{\Delta p} \right)$$

5With a slight abuse of notation, $R_F$, $R_M$ and $R_U$ will denote the rewards to the firm, the monitor and the uninformed investors respectively, across scenarios. Although the magnitude of these rewards will vary across the sections of the paper, this notation is only employed in intermediate calculations and should cause no confusion in context.

6For a more detailed exposition of the basic building blocks of the model, i.e. the results contained in sections 2.1, 2.2 and 2.3, see Tirole (2005).
Letting $\gamma$ be the rate of return on the market for uninformed capital, it follows immediately that the firm obtains finance if and only if

$$p_H \left( R - \frac{S}{\Delta p} \right) \geq \gamma (I - K)$$

where $p_H R_U = \gamma (I - K)$ is the minimum return to uninformed investors. Define the following threshold for net worth:

$$A(\gamma) \equiv I - \frac{p_H}{\gamma} \left( R - \frac{S}{\Delta p} \right)$$

For $K < A(\gamma)$, the firm cannot raise external finance, while it can for $K \geq A(\gamma)$. Last, note that $A(\gamma)$ is an increasing function of $\gamma$.

### 2.2. Indirect Finance

Now, revert to the setup with three versions of the project as given in the table below:

<table>
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<tr>
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<th>$G$</th>
<th>$S$</th>
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<tbody>
<tr>
<td>$p$</td>
<td>$p_H$</td>
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<tr>
<td>$z$</td>
<td>0</td>
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</table>

Assume that a monitor or financial intermediary can rule out the $S$ project at personal (and unverifiable) cost $c > 0$. To induce the monitor to monitor, he must be provided with the right incentives. Letting $R_M$ denote the return to the monitor in case of success, the relevant incentive constraint is given by

$$R_M \geq \frac{c}{\Delta p}$$

which is akin to the incentive constraint of the agent. Given that the monitor monitors, the agent’s incentive constraint is

$$R_F \geq \frac{s}{\Delta p}$$

and the sharing rule is the following three-way split:

$$R_F + R_U + R_M = R$$

Monitoring is useful because it reduces the agent’s opportunity cost of choosing the $G$ project, not because the private benefit changes from $S$ to $s + c$.

Note that the monitor earns a strictly positive rent from monitoring, since

$$p_H R_M - c = \frac{p_H c}{\Delta p} - c = \frac{p_L c}{\Delta p} > 0$$

But the monitor’s incentives are provided solely by the expectation of receiving $R_M$ in case of success and consequently the firm can regulate the investor’s return by requiring him to make some contribution $I_M(\beta)$ towards the funding of the project. Defining the return on monitoring capital by

$$\beta = \frac{p_H R_M}{I_M(\beta)}$$
it follows from the monitor’s incentive constraint that
\[ I_M(\beta) \geq \frac{pHc}{\beta \Delta p} \]

Since \( \gamma \) is the rate of return on the market for uninformed capital, the minimum acceptable return on monitoring capital is implicitly given by
\[ \frac{pHc}{\Delta p} - c = \gamma I_M(\beta) \]
or, rewriting, by
\[ \beta = \frac{\gamma pH}{pL} > \gamma \]

In turn, this implies that the monitor must contribute at least
\[ I_M(\beta) \geq \frac{pLc}{\gamma \Delta p} \]
towards the funding of the project. Last, since monitoring capital is expensive \((\beta > \gamma)\), it is easy to see that an entrepreneur will avoid monitoring whenever possible and in case monitoring is essential, he will use the minimum possible amount of monitoring capital.

Assuming that the monitor is induced to monitor (and thus reduces the agent’s private benefit to \( s \)) and substituting for the incentive constraints, yields pledgeable income
\[ pH \left( R - \frac{s + c}{\Delta p} \right) \]

Since the expected return to uninformed investors must be at least \( pH R_U = \gamma (I - K - I_M(\beta)) \), their individual rationality constraint is
\[ pH \left( R - \frac{s + c}{\Delta p} \right) \geq \gamma (I - K - I_M(\beta)) \]

Thus the cut-off level of net worth over which the firm can obtain funding with the aid of an intermediary is
\[ B(\gamma, \beta) \equiv I - I_M(\beta) - \frac{pH}{\gamma} \left( R - \frac{s + c}{\Delta p} \right) \]

Again, for \( K \geq B(\gamma, \beta) \) the firm can obtain finance with the assistance of a monitor, while for \( K < B(\gamma, \beta) \), it cannot. Last, \( B(\gamma, \beta) \) is increasing in both arguments.

Note that as long as \( s + c \geq S \), employing a monitor actually decreases the firm’s debt capacity but that the decrease in pledgeable income is compensated by the monitor’s contribution towards the project.

2.3. Diversification by Merger and Direct Finance. Now revert to the version of the model with only the \( G \) and the \( S \) projects available (again, the \( s \) project is a strictly dominated choice) and consider two firms with assets \( K' \) and \( K'' \) respectively. Let \( K = (K' + K'')/2 \), so the firms have an average net worth \( K \) each. Consider a merger between the two firms. It now has two projects and net worth \( 2K \). Assume that the two projects
managed by the merged unit are independent (probabilistically).\textsuperscript{7} To ensure high effort on both projects, it must be worthwhile for the manager to give up $2S$ (which he would enjoy by shirking on both projects). The relevant quantities are summarized in the following table:

<table>
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<tr>
<th>$p$</th>
<th>$p_H^2$</th>
<th>$p_H p_L$</th>
<th>$p_L^2$</th>
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<tbody>
<tr>
<td>$z$</td>
<td>0</td>
<td>$S$</td>
<td>$2S$</td>
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</table>

The following result is very useful in the analysis that follows:

**Lemma 1.** The optimal debt contract pays the entrepreneur $R_F$ if there is success on both projects and zero otherwise.

**Proof.** The optimal contract that induces the agent to choose $(G, G)$ is sought. Three different possibilities must be considered: The agent chooses $(G, G)$, he chooses $(S, G)$ or $(G, S)$, or he chooses $(S, S)$. Let $R_i$ denote reward if there is success on $i = 0, 1, 2$ projects. First observe that $R_0 = 0$ is trivially optimal (otherwise it would be more expensive to give incentives to choose $G$ projects). This reduces the set of incentive compatibility constraints to the following inequalities:

\[
p_H^2 R_2 + 2p_H(1 - p_H)R_1 \geq p_H p_L R_2 + (p_H + p_L - 2p_H p_L)R_1 + S \quad (1)
\]

\[
p_H^2 R_2 + 2p_H(1 - p_H)R_1 \geq p_L^2 R_2 + 2p_L(1 - p_L)R_1 + 2S \quad (2)
\]

The first inequality (1), ensures that $(G, G)$ dominates $(G, S)$ or $(S, G)$. The second, (2), ensures that $(G, G)$ dominates $(S, S)$.

Next, consider a contract $\{R_2, R_1, 0\}$ that satisfies the two incentive constraints. It must be shown that there exists a contract $\{R_F, 0, 0\}$ that also satisfies the incentive constraints and that

\[p_H^2 R_F = p_H^2 R_2 + 2p_H(1 - p_H)R_1\]

which just states that the agent is indifferent between these two contracts. It is useful to rewrite the constraints as

\[p_H R_2 + (1 - 2p_H)R_1 \geq \frac{S}{\Delta p}\]

\[(p_H + p_L)R_2 + 2(1 - p_H - p_L)R_1 \geq \frac{2S}{\Delta p}\]

First consider changes $\Delta R_2 > 0$ and $\Delta R_1 < 0$ that leave (1) unaffected, i.e. such that $p_H \Delta R_2 + (1 - 2p_H)\Delta R_1 = 0$, or

\[
\Delta R_2 = \left(\frac{2p_H - 1}{p_H}\right) \Delta R_1
\]

Pledgeable income is given by

\[2p_H^2 R + 2p_H(1 - p_H)R - p_H^2 R_2 - 2p_H(1 - p_H)R_1\]

\textsuperscript{7}It will not be specified how the two owners of a merged firm divide the return to their investments. It can either be assumed that they divide it according to a sharing rule determined by their relative contributions, or that one of the managers buys out the other from the outset. What is important is that after the merger they act as if they were one.
The impact on pledgeable income brought about by the changes $\Delta R_2$ and $\Delta R_1$ is thus

$$-p_H^2 \Delta R_2 - 2p_H(1-p_H)\Delta R_1$$

To determine the sign of this change, substitute for $\Delta R_2$ to get

$$-p_H^2 \left(\frac{2p_H - 1}{p_H} \right) \Delta R_1 - 2p_H(1-p_H)\Delta R_1 = -\Delta R_1 p_H > 0$$

Thus these changes increase pledgeable income. It must be checked that such a change in $R_2$ and $R_1$ that leaves (2) unaffected also increases pledgeable income. It must be the case that $(p_H + p_L)\Delta R_2 - 2(p_H + p_L - 1)\Delta R_1 = 0$, or

$$\Delta R_2 = \left(\frac{2(p_H + p_L - 1)}{p_H + p_L} \right) \Delta R_1$$

Thus the change in pledgeable income is

$$-p_H^2 \left(\frac{2(p_H + p_L - 1)}{p_H + p_L} \right) \Delta R_1 - 2p_H(1-p_H)\Delta R_1 = -\Delta R_1 \left(\frac{2p_H p_L}{p_H + p_L} \right) > 0$$

This completes the proof.

Thus it is not restrictive to consider contracts where the agent is paid only when there is success on both projects. The constraint ensuring that the agent prefers to work on both to shirk on both implies the constraint that the agent prefer to work on both to work on only one project. To see this, rewrite the two conditions as

$$R_F \left(\frac{p_H + p_L}{2} \right) \geq \frac{S}{\Delta p}$$

$$R_F p_H \geq \frac{S}{\Delta p}$$

Since $p_H > p_L$, the second constraint is satisfied if the first one is. The relevant incentive compatibility constraint is thus

$$R_F \geq \frac{2S}{(p_H + p_L)\Delta p}$$

The expected payoff to the agent is then

$$p_H^2 R_F = \left(\frac{p_H}{p_H + p_L} \right)^2 2p_H S \frac{\Delta p}{\Delta p} = 2p_H(1-d)S \frac{\Delta p}{\Delta p}$$

where

$$d = \frac{p_L}{p_H + p_L} \in ]0, 1/2[$$

is a measure of the effectiveness of diversification.

Since uninformed investors must earn at least $p_H R_U = \gamma (2I - 2K)$, the two projects get jointly funded if

$$2p_H \left( R - \frac{(1-d)S}{\Delta p} \right) \geq \gamma (2I - 2K)$$
Again, this defines a threshold of average net worth per project:

\[ C(\gamma) \equiv I - \frac{p_H}{\gamma} \left( R - \frac{(1 - d)S}{\Delta p} \right) \]

For \( K < C(\gamma) \), the merged firm cannot obtain funding, while it can for \( K \geq C(\gamma) \). Last, \( C(\gamma) \) is an increasing function of \( \gamma \).

Note that in contrast to the case in which monitoring is employed, diversification by merger actually boosts the firms’ joint debt capacity.

2.4. Diversification by Merger and Indirect Finance. Now the above model is extended in a straightforward way, by allowing firms to merge and to employ monitors on either or both of a merged firm’s projects. The table below shows the probabilities of success \( p \) on both projects and private benefits \( z \) from different choices of projects. For example, if the agent chooses \((G, s)\), the probability of success of both projects is \( p = p_H p_L \) and the agent earns benefit \( z = s \) from the project which he shirks on.

<table>
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<tr>
<th></th>
<th>( G, G )</th>
<th>( G, S )</th>
<th>( G, s )</th>
<th>( S, S )</th>
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<tr>
<td>( p )</td>
<td>( p_H^2 )</td>
<td>( p_H p_L )</td>
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<tr>
<td>( z )</td>
<td>0</td>
<td>( S )</td>
<td>( s )</td>
<td>( 2S )</td>
<td>( S + s )</td>
<td>( 2s )</td>
</tr>
</tbody>
</table>

As in the previous sections, conditions on net worth for the merged unit that ensure direct or indirect finance are now derived. While there are now more combinations to consider, the analysis is only slightly complicated by this. Recall that in the absence of monitoring possibilities, the optimal contract for a merged firm pays it \( R_F \) if there is success on both projects and zero otherwise. Now consider how a merged firm with average per project net worth \( K \). As noted above, the monitor can (if provided the incentives to do so) identify and rule out \( S \) projects at personal cost \( c > 0 \) per project. Thus, it costs him \( c \) to rule out the \((S, S)\) project combination and \( 2c \) to rule out the \((S, s)\) or \((s, S)\) project combinations. When writing the optimal contract with monitoring, the following result is useful:

**Lemma 2.** (i) The optimal debt contract with monitoring on both projects (complete monitoring) pays the agent and the monitor \( R_F \) and \( R_M \) respectively in case of success of both projects and zero otherwise. (ii) The optimal debt contract with monitoring on only one project (partial monitoring) pays the agent \( R_F \) in case of success of both projects and zero otherwise, while the monitor earns \( R_M \) in case of success on the monitored project.

**Proof.** (i) Given that the monitor is provided incentives to monitor both projects, the agent is facing the same problem as a merged firm in the absence of monitoring possibilities, with \( s \) substituted for \( S \). Next, consider the monitor’s incentives to monitor. When writing the optimal contract for the monitor, the following three possibilities must be compared: monitor both projects, monitor just one project or monitor no projects. It is immediately clear that the problem of providing the right incentives for the monitor to monitor is essentially the same as the problem of providing incentives to the entrepreneur to choose two good projects. The result follows immediately.

(ii) In the absence of monitoring possibilities, the merged firm had to compare the following three options: \((GG)\), \((G, S)\) or \((S, G)\) and \((S, S)\). Now consider the following setup:

\(^8\)As before, \( K = (K' + K'')/2 \) where \( K' \) and \( K'' \) are the merging firms’ respective net worth.
A monitor is assigned to monitor a single project, say project 1. When writing the contract with partial monitoring, the following four alternatives are available: the agent works on both, the agent shirks on both, the agent works on the monitored project and shirks on the non-monitored project and last, he shirks on the monitored project and works on the non-monitored project. It is immediately clear that if the agent were to shirk on a single project, he would always choose to shirk on the non-monitored project. This leaves three relevant alternatives: work on both (yielding zero private benefit), shirk on both (yielding private benefit $S + s$) or work on the monitored and shirk on the non-monitored project (yielding private benefit $S$). But note that the problem is then essentially the same as that under complete monitoring, except that the value of shirking is reduced only for one instead of two projects (namely the monitored one). The result follows immediately.

While the basic structure of the optimal debt contract is the same under complete and partial monitoring, the differences in the incentive compatibility constraints influence the rents to the different parties. In particular, the required contribution from the monitor will change in a way that will be shown shortly.

**Complete Monitoring.** First note that, as in the case of merger without the possibility of monitoring, the constraint ensuring that work on both projects dominates shirking on both implies the constraint that working on both dominates working on only one. The optimal contract for the monitor under complete monitoring is thus $\{R_M, 0, 0\}$ with the reward in case of two successes given by

$$R_M = \frac{2c}{(p_H + p_L)\Delta p}$$

Letting $R_U$ denote the reward to the uninformed lenders in case of success, a sharing rule is given by

$$R_F + R_U + R_M = R$$

The expected return to the monitor is

$$p_H^2 R_M - 2c = p_H^2 \left( \frac{2c}{(p_H + p_L)\Delta p} \right) - 2c = \frac{2cp_Ld}{\Delta p} > 0$$

Note again that the monitor receives a positive expected rent from monitoring. Let $I_M(\beta)$ denote the monitor’s investment in the firm (per project). Assuming that the market for monitoring capital is competitive, the monitor must break even when signing the contract. He is thus required to invest at least

$$I_M(\beta) \geq \frac{p_H(1-d)c}{\beta \Delta p}$$

in the firm. As the outside opportunity to monitoring is to invest in the market for uninformed capital, it follows from familiar steps that

$$I_M(\beta) \geq \frac{dplc}{\gamma \Delta p}$$
Also, a little algebra shows that
\[ \beta = \frac{\gamma p_H^2}{p_L^2} > \gamma \]
so the firm will use the minimum possible amount of informed capital. Note that the return on monitoring capital with merger and complete monitoring \( \gamma p_H^2/p_L^2 \) is larger than \( \gamma p_H/p_L \), which was the return on monitoring capital in the absence of merger possibilities.

Since the monitor is induced to monitor both projects, the agent’s opportunity cost of choosing \( (G, G) \) is \( 2s < 2S \). In turn, this implies that the pledgeable income is given by
\[
2p_H R - p_H^2 R_F - p_H^2 R_M = 2p_H \left( R - \frac{(1 - d)(s + c)}{\Delta p} \right)
\]
Since the uninformed investors must obtain at least \( p_H R_U = 2\gamma (I - K - I_M(\beta)) \), their individual rationality constraint is
\[
p_H \left( R - \frac{(1 - d)(s + c)}{\Delta p} \right) \geq \gamma (I - K - I_M(\beta))
\]
Consequently, the cut-off level for a merged firm to obtain finance with monitoring is given by
\[
D(\gamma, \beta) \equiv I - I_M(\beta) - \frac{p_H}{\gamma} \left( R - \frac{(1 - d)(s + c)}{\Delta p} \right)
\]
Note that this amount is required per project. For \( K < D(\gamma, \beta) \), a merged firm with complete monitoring cannot obtain financing while it can for \( K \geq D(\gamma, \beta) \). Last, \( D(\gamma, \beta) \) is increasing in both arguments.

**Partial Monitoring.** The relevant incentive compatibility constraints ensuring that the firm chooses two good projects are now given by
\[
R_F \geq \frac{S}{p_H \Delta p}
\]
\[
R_F \geq \frac{S + s}{(p_H + p_L) \Delta p}
\]
The first constraint ensures that working on both dominates working on only the \( s \) project (and thus shirking on the \( S \) project), while the second ensures that working on both dominates shirking on both. Unlike the case of complete monitoring, it is not generally true under partial monitoring that if one of the constraints is satisfied, then so is the other constraint.

A little algebra shows that the condition separating the two cases where this is indeed the case is
\[
\frac{p_H}{p_L} \geq \frac{S}{s} \equiv \sigma
\]
Note that \( \sigma \) is a measure of the effectiveness of monitoring in reducing moral hazard in the firm’s choice of effort.

The two cases will be analyzed separately. In both cases, the monitor’s incentive compatibility constraint is given by
\[
R_M \geq \frac{c}{\Delta p}
\]
as was the case under monitoring of a single firm without the possibility of merger. Consequently, in order to regulate the monitor’s rents, he is required to make a contribution towards the funding of the firm amounting to

\[ I_M(\beta) \geq \frac{p_{HC}}{\beta \Delta p} = \frac{p_{LC}}{\gamma \Delta p} \]

Since the contract for the monitor under merger and partial monitoring is essentially identical to that under monitoring in the absence of merger, it follows that the required return on the monitor’s contribution is given by

\[ \beta = \frac{p_H}{p_L} > \gamma \]

Note that this is lower than the required return on monitoring capital under complete monitoring.

**Case I.** First consider the case where \( \sigma \in [1, p_H/p_L] \). The incentive compatibility constraint for the agent is thus given by

\[ R_F \geq \frac{S + s}{(p_H + p_L) \Delta p} \]

The merged firm’s pledgeable income is given by

\[ 2p_H R - p_H^2 R_F - p_H R_M = 2p_H \left( R - \frac{(1 - d)(S + s) + c}{2 \Delta p} \right) \]

Again letting \( K = (K' + K'')/2 \) denote the merged firm’s average per project net worth, the firm is funded only if

\[ 2p_H \left( R - \frac{(1 - d)(S + s) + c}{2 \Delta p} \right) \geq \gamma (2I - 2K - I_M(\beta)) \]

where \( p_H R_U = \gamma (2I - 2K - I_M(\beta)) \) is the minimum acceptable return to uninformed investors. Rearranging, the sought threshold on average net worth per project becomes

\[ E(\gamma, \beta) \equiv I - \frac{I_M(\beta)}{2} - \frac{p_H}{\gamma} \left( R - \frac{(1 - d)(S + s) + c}{2 \Delta p} \right) \]

For \( K < E(\gamma, \beta) \), a merged firm with partial monitoring cannot obtain financing while it can for \( K \geq E(\gamma, \beta) \). Last, \( E(\gamma, \beta) \) is increasing in both arguments.

**Case II.** Next, consider the case where \( \sigma \in [p_H/p_L, \infty] \). In this case, the relevant incentive constraint for the agent is given by

\[ R_F \geq \frac{S}{p_H \Delta p} \]

Pledgeable income is then given by

\[ 2p_H R - p_H^2 R_F - p_H R_M = 2p_H \left( R - \frac{S + c}{2 \Delta p} \right) \]
and funding is secured if

\[ 2p_H \left( R - \frac{S + c}{2\Delta p} \right) \geq \gamma (2I - 2K - I_M(\beta)) \]

where again \( p_H R_U = \gamma (2I - 2K - I_M(\beta)) \) is the minimum acceptable return to uninformed investors. From this, the relevant threshold on average per project net worth is given by

\[ F(\gamma, \beta) \equiv I - \frac{I_M(\beta)}{2} - \frac{p_H}{\gamma} \left( R - \frac{S + c}{2\Delta p} \right) \]

For \( K < F(\gamma, \beta) \), a merged firm with partial monitoring cannot obtain financing while it can for \( K \geq F(\gamma, \beta) \). Last, \( F(\gamma, \beta) \) is increasing in both arguments.

3. Aggregate Demand

To sum up the analysis so far, the different thresholds are represented in the following list:

- \( A(\gamma) \) : no merger, no monitoring
- \( B(\gamma, \beta) \) : no merger, monitoring
- \( C(\gamma) \) : merger, no monitoring
- \( D(\gamma, \beta) \) : merger, complete monitoring
- \( E(\gamma, \beta) \) : merger, partial monitoring (Case I)
- \( F(\gamma, \beta) \) : merger, partial monitoring (Case II)

In order to determine the aggregate demand for uninformed and monitoring capital, the ranking of the different thresholds must be established. Unfortunately, only a few of these rankings are unambiguous. Most depend on the parameter values. As shall be seen below, only one range of parameter values is relevant for the current analysis and this range of values yields a unique ranking of the thresholds.

First, the cutoffs on net worth are ranked as follows, as functions of the monitoring cost:

**Lemma 3.** The rankings of the cutoffs for capital are given by

\[
\begin{align*}
A(\gamma) & \geq B(\gamma, \beta) \quad \text{for} \quad c \leq k_4 \\
A(\gamma) & \geq C(\gamma) \quad \text{for all} \quad c \\
A(\gamma) & \geq D(\gamma, \beta) \quad \text{for} \quad c \leq k_5 \\
A(\gamma) & \geq E(\gamma, \beta) \quad \text{for} \quad c \leq k_6 \\
A(\gamma) & \geq F(\gamma, \beta) \quad \text{for} \quad c \leq k_{AF} \\
B(\gamma, \beta) & \geq C(\gamma) \quad \text{for} \quad c \geq k_1 \\
B(\gamma, \beta) & \geq D(\gamma, \beta) \quad \text{for all} \quad c \\
B(\gamma, \beta) & \geq E(\gamma, \beta) \quad \text{for} \quad c \geq k_9 \\
B(\gamma, \beta) & \geq F(\gamma, \beta) \quad \text{for} \quad c \geq k_{BF} \\
C(\gamma) & \geq D(\gamma, \beta) \quad \text{for} \quad c \leq k_3 \\
C(\gamma) & \geq E(\gamma, \beta) \quad \text{for} \quad c \leq k_3 \\
C(\gamma) & \geq F(\gamma, \beta) \quad \text{for} \quad c \leq k_{CF} \\
D(\gamma, \beta) & \geq E(\gamma, \beta) \quad \text{for} \quad c \geq k_3 \\
D(\gamma, \beta) & \geq F(\gamma, \beta) \quad \text{for} \quad c \geq k_{DF}
\end{align*}
\]
where

\[
\begin{align*}
  k_0 &= \frac{p_H ((1-d)S - (1+d)s)}{\Delta p} \\
  k_1 &= \frac{p_H ((1-d)S - s)}{\Delta p} \\
  k_2 &= S - s \\
  k_3 &= \frac{p_H (1-d)(S - s)}{\Delta p} \\
  k_4 &= \frac{p_H (S - s)}{\Delta p} \\
  k_5 &= \frac{p_H (1-d)(S - s) + p_L (1-d)S}{\Delta p} \\
  k_6 &= \frac{p_H (1-d)(S - s) + 2p_L (1-d)S}{\Delta p} \\
  k_{AF} &= \frac{p_H S}{\Delta p} \\
  k_{BF} &= \frac{p_H (S - 2s)}{\Delta p} \\
  k_{CF} &= \frac{(1-2d)p_H S}{\Delta p} \\
  k_{DF} &= \frac{(1-d)(p_H + p_L)S - 2(1-d)p_H s}{\Delta p}
\end{align*}
\]

**Proof.** The rankings follow from straightforward but lengthy algebra. In turn, the ranking of cost cutoffs depends on the magnitude of the parameter \(\sigma\). Each case will be analyzed in turn.

### 3.1. Case I.

First consider the case where \(\sigma \in [1, p_H/p_L]\). In this case, the relevant threshold for partial monitoring is \(E(\gamma, \beta)\) and the only relevant cutoffs for the monitoring cost is the sequence \(k_i, i = 0, \ldots, 6\). Straightforward algebra shows that in Case I, the ranking is given as follows:

\[
k_6 \geq k_5 \geq k_4 \geq k_3 \geq k_2 \geq k_1 \geq k_0
\]

First, note that for \(c > k_4\), \(A(\gamma) \leq B(\gamma, \beta)\) and thus monitoring is too costly to be socially useful. This is because for monitoring costs in this range, firms would not demand any informed capital even if the rate of return for monitors was so low that it just allowed them to break even. Thus the condition \(c \leq k_4\) is imposed. Next, note that if \(c < k_2\), the firms’ pledgeable income (and thus their ability to raise uninformed capital) would be boosted by employing a monitor. In turn, this means that monitors would be able to monitor firms without having to inject any of their own funds into the monitored projects. Since the amount of informed capital is one of the important quantities with respect to which comparative statics will be performed, this possibility is excluded.11 It is therefore assumed

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11 For completeness, it should be noted that the equivalent conditions when comparing merger with complete monitoring to merger without monitoring and merger with partial monitoring to merger without monitoring, are that \(c > S - s\) and that \(c > (1-d)(S - s)\), respectively. The former condition coincides with the condition \(c > k_2\) while the latter is implied by it.
that $c \geq k_2$. This leaves the interval $[k_2, k_4]$. But for $c \geq k_3$, $C(\gamma) \leq E(\gamma, \beta)$, which means that it takes lower per project net worth to obtain funding as stand-alone projects than with partial monitoring of a merged firm. This means that there would be no demand for partial monitoring from merged firms, since monitoring reduces pledgeable income. This leaves the interval $[k_2, k_3]$. But for $c \leq k_3$, it is the case that $E(\gamma, \beta) \leq D(\gamma, \beta)$, which means that it takes lower per project net worth to obtain funding for a merged firm with partial monitoring than it does with complete monitoring, in turn implying that no merged firms would demand complete monitoring. In conclusion, Case I is ruled out since for $\sigma \in [1, p_H/p_L]$, not all markets can be simultaneously active.

### 3.2. Case II.

Next, turn to case II, where $\sigma \in [p_H/p_L, \infty]$. In this case, the relevant threshold for partial monitoring is $F(\gamma, \beta)$. In contrast to case I where most of the critical thresholds coincide, case II breaks down to four further subcases.

Case II.a: $\sigma \in [(p_H + p_L)/p_L, (p_H + 2p_L)/p_L]$. In this subcase, the cutoffs for monitoring cost $c$ are ranked as follows:

$$k_6 > k_{AF} > k_5 > k_4 > k_{DF} > k_3 > k_{CF} > k_2 > k_1 > k_{BF} > k_0$$

The same arguments as those for case I above reduces the relevant range to $c \in [k_2, k_3]$.\(^\text{12}\)

But note that for $c \geq k_{CF}$ it is the case that $F(\gamma, \beta) \geq C(\gamma)$ which means that it is easier for a merged firm to get funded without monitoring than with partial monitoring. Thus, there would be no demand for partial monitoring from merged firms. This leaves the interval $c \in [k_2, k_{CF}]$ as the only viable parameter range for this case.

Case II.b: $\sigma \in [(p_H + 2p_L)/p_L, (2p_H + p_L)/p_L]$. In this subcase, the cutoffs for monitoring cost $c$ are ranked as follows:

$$k_6 > k_{AF} > k_5 > k_4 > k_{DF} > k_3 > k_{BF} > k_1 > k_2 > k_{CF} > k_0$$

The same arguments as those in cases I and II.a rule this case out since for this range of $\sigma$ it is the case that $k_{CF} \leq k_2$.

Case II.c: $\sigma \in [(2p_H + p_L)/p_L, (2p_H + 2p_L)/p_L]$. In this case, the cutoffs are ranked as follows:

$$k_6 > k_{AF} > k_5 > k_4 > k_{DF} > k_{BF} > k_3 > k_1 > k_2 > k_0 > k_{CF}$$

The same arguments as those in cases I and II.a rule this case out since for this range of $\sigma$ it is the case that $k_{CF} \leq k_2$.

Case II.d: $\sigma \in [(2p_H + 2p_L)/p_L, \infty]$. In this case, the cutoffs are ranked as follows:

$$k_6 > k_{AF} > k_5 > k_4 > k_{DF} > k_{BF} > k_3 > k_1 > k_0 > k_2 > k_{CF}$$

The same arguments as those in cases I and II.a rule this case out since for this range of $\sigma$ it is the case that $k_{CF} \leq k_2$.

\(^\text{12}\)For completeness, it should be noted that the condition on $c$ that ensures that pledgeable income under partial monitoring in Case II is not larger than pledgeable income under merger with no monitoring is that $c \geq (1 - 2d)S$. But this is automatically satisfied in the parameter range under consideration, since then $k_2 > (1 - 2d)S$. 
3.3. Parameterization. In conclusion, there is only one set of parameter ranges under which all markets are simultaneously active, namely that given by case II.a. In what follows, the following assumptions will be imposed:

**A1** \[ \sigma \in \left[ \frac{p_H}{p_L}, \frac{p_H + p_L}{p_L} \right] \]

**A2** \[ c \in \left[ S - s, \frac{p_H S}{p_H + p_L} \right] \]

**A3** \[ \Delta p (p_H R - \gamma I) \leq p_H (1 - d) (s + c) - d p_L c \]

**A4** \[ \Delta p (p_H R + \gamma L - \gamma I) \geq p_H S \]

**A5** \[ \frac{\beta \gamma}{\sigma} \geq \left( \frac{p_H}{p_L} \right)^2 \]

**A6** \[ p_H R - \gamma I > 0 > p_L R - \gamma I + S \]

Assumptions A1-A2 correspond to the parameter range of case II.a discussed above. Assumptions A3-A4 ensure that no firm can obtain financing without supplying own funds and that the richest firm can obtain funding without monitoring. Assumption A5 ensures that the minimum return on monitoring capital makes informed investors willing to provide complete monitoring of merged firms. Assumption 6 ensures that a project is only viable if high effort is exerted.

Under assumptions A1-A4, the ranking of the thresholds on net worth are given by

\[ L \geq A(\gamma) \geq B(\gamma, \beta) \geq C(\gamma) \geq F(\gamma, \beta) \geq D(\gamma, \beta) \geq 0 \]

Now the aggregate demand can be established. The demand for monitoring capital is given by

\[ D_M(\gamma, \beta) = \left[ F(\gamma, \beta) - D(\gamma, \beta) \right] \frac{p_H (1 - d) c}{\beta \Delta p} + \left[ C(\gamma) - F(\gamma, \beta) \right] \frac{p_H c}{\beta \Delta p} \tag{3} \]

Next, the net demand for uninformed capital is given by

\[
D_U(\gamma, \beta) = \int_{C(\gamma)}^{L} (I - \alpha K) dK + \int_{F(\gamma, \beta)}^{C(\gamma)} \left( I - \alpha K - \frac{p_H c}{\beta \Delta p} \right) dK \\
+ \int_{D(\gamma, \beta)}^{F(\gamma, \beta)} \left( I - \alpha K - \frac{p_H (1 - d) c}{\beta \Delta p} \right) dK + \int_{0}^{D(\gamma, \beta)} (I - \alpha K) dK \\
= L \left( \frac{2I - \alpha L}{2} \right) - \frac{p_H c}{\beta \Delta p} \left[ C(\gamma) - dF(\gamma, \beta) - (1 - d) D(\gamma, \beta) \right] \tag{4}
\]

In this demand function, the funds that firms with either \( K \geq I \) or \( K \geq I - I_M(\beta) \) reinvest in the market for uninformed capital have been netted out. Furthermore, each firm’s assets

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13 Assumptions A3 and A4 are simply the inequalities \( D(\gamma, \beta) \geq 0 \) and \( L \geq A(\gamma) \), respectively.

14 Note that firms are assumed to only merge within groups. Thus, a rich firm will not use excess funds to fund a poor firm. I consider this setting to ensure that there will be positive demand for financial intermediation. Furthermore, such settings in which cash-rich firms fund poor firms has been considered elsewhere in the literature, e.g., by Fluck and Lynch (1999). To the extent that such mergers do occur, the current analysis will tend to understate the magnitude of merger activity.
K have been scaled by a factor $\alpha \geq 1$ so the effects of an increase in collateral can be studied. Last, note that firms with $K \in [B(\gamma, \beta), A(\gamma)]$ could choose to remain independent and raise funds with the aid of an intermediary. Since intermediation is costly and no cost to mergers has been assumed, it is natural to suppose that these firms will indeed choose to diversify.

For clarity, the slopes of the demand equations will be studied separately.

$$\frac{\partial D_M(\gamma, \beta)}{\partial \gamma} = \frac{c \left[ 2p_H^2 + pHPL + \gamma \right] - S \left[ 2p_H^2 + pHPL + \gamma \right] + 2p_H^2 s}{2(\beta + \gamma)^2 \Delta p^2}$$  \hspace{1cm} (5)

$$\frac{\partial D_M(\gamma, \beta)}{\partial \beta} = \frac{c \left[ (\beta - 2\gamma) \beta \left[ 2p_H^2 + pHPL + \gamma \right] - S \left[ 2p_H^2 + pHPL + \gamma \right] + 2p_H^2 s \right]}{2(\beta + \gamma)^2 \Delta p^2}$$  \hspace{1cm} (6)

$$\frac{\partial U(\gamma, \beta)}{\partial \gamma} = \frac{-c \left[ 2p_H^2 + pHPL + \gamma \right] - S \left[ 2p_H^2 + pHPL - \gamma \right] + 2p_H^2 s}{2(\beta + \gamma)^2 \Delta p^2}$$  \hspace{1cm} (7)

$$\frac{\partial U(\gamma, \beta)}{\partial \beta} = \frac{-c \left[ (\beta - 2\gamma) \beta \left[ 2p_H^2 + pHPL + \gamma \right] - S \left[ 2p_H^2 + pHPL - \gamma \right] + 2p_H^2 s \right]}{2(\beta + \gamma)^2 \Delta p^2}$$  \hspace{1cm} (8)

Under assumptions A1-A2, (5) is positive while (7) is negative. Next, define the cutoff $c(\gamma, \beta)$ by

$$c(\gamma, \beta) = \left( \frac{\beta}{\beta - 2\gamma} \right) \left( \frac{2p_H^2 + pHPL - \gamma}{2p_H^2 + pHPL + \gamma} \right) S - 2p_H^2 s$$

and note that under assumptions A1-A2, the second factor is positive. For $\beta \leq 2\gamma$, (6) is negative while (8) is positive. For $\beta > 2\gamma$, the signs of (6) and (8) depend on the magnitude of $c$. A sufficient condition for (6) to be negative and for (8) to be positive is that $c > c(\gamma, \beta)$.

The following assumption is imposed:

\textbf{A7} If $\beta > 2\gamma$, then $c < c(\gamma, \beta)$.

4. Equilibrium and Comparative Statics

Two scenarios will be analyzed, each differing in its assumption about the market for uninformed capital. In the first, it is assumed that there is an infinite supply at rate $\gamma$ while in the second, uninformed capital is supplied according to an upward-sloping supply function $S(\gamma)$. The following definitions and results shall prove useful in what follows:

\textbf{Lemma 4.} Let $A(\gamma, \beta) \equiv L - D(\gamma, \beta)$, $P(\gamma, \beta) \equiv C(\gamma) - F(\gamma, \beta), C(\gamma, \beta) \equiv F(\gamma, \beta) - D(\gamma, \beta)$,

$M(\gamma, \beta) \equiv A(\gamma) - D(\gamma, \beta)$ be the sets of firms who are active, partially monitored, completely monitored, merged respectively. Then, it follows from inspection that

$$\frac{\partial A(\gamma, \beta)}{\partial \beta} < 0, \quad \frac{\partial P(\gamma, \beta)}{\partial \beta} < 0, \quad \frac{\partial C(\gamma, \beta)}{\partial \beta} < 0, \quad \frac{\partial M(\gamma, \beta)}{\partial \beta} < 0$$  \hspace{1cm} (9)

Furthermore, under assumptions A1-A4,

$$\frac{\partial A(\gamma, \beta)}{\partial \gamma} < 0, \quad \frac{\partial P(\gamma, \beta)}{\partial \gamma} > 0, \quad \frac{\partial C(\gamma, \beta)}{\partial \gamma} < 0, \quad \frac{\partial M(\gamma, \beta)}{\partial \gamma} < 0$$  \hspace{1cm} (10)

\textsuperscript{15} For $\beta < 2\gamma$, a sufficient condition for these signs is that $c > c(\gamma, \beta)$, but for such magnitudes of the returns it is the case that $c(\gamma, \beta) < 0$ and the sufficient condition is then implied by assumptions A1-A2.

\textsuperscript{16} If $\beta$ is not too large compared to $\gamma$, then assumption A6 is implied by assumption A2. This is the case if $\frac{\beta}{\gamma} \leq \left( \frac{2p_H^2 + pHPL + \gamma}{2p_H^2 + pHPL - \gamma} \right) S$. For completeness, it should be noted that this inequality is consistent with assumption A5.
4.1. Infinitely Elastic Supply. In this scenario, equilibrium is described entirely by equilibrium in the market for monitoring capital, which is described by the equation

$$e_M(\gamma, \beta) \equiv D_M(\gamma, \beta) - K_M = 0$$

(11)

Under the maintained assumptions, demand for monitoring capital is downward-sloping. Since

$$\lim_{\beta \to 0} D_M(\gamma, \beta) = \infty \quad \text{and} \quad \lim_{\beta \to \infty} D_M(\gamma, \beta) = 0,$$

continuity and monotonicity imply both the existence and uniqueness of equilibrium in the market for informed capital.

From (11), it is immediately clear that an increase in the amount of monitoring capital decreases the equilibrium rate of return $\beta$. In turn, inequalities (9) imply the following:

**Proposition 5.** In a credit crunch, (i) the rate of return on informed capital $\beta$ increases, (ii) the set of merged firms decreases, (iii) the set of firms with partial monitoring decreases, (iv) the set of firms with complete monitoring decreases and (v) the set of all active firms decreases.

As the proposition shows, a credit crunch decreases the set of active firms by squeezing poorly capitalized firms out of the market, much as in Holmström and Tirole (1997). This decrease of active firms is brought about in part by a decrease in the set of merged firms, both partially and completely monitored ones. This observation accords well with the stylized fact that merger activity is to be expected to increase in times of economic expansion.

4.2. Elastic Supply. When the supply of uninformed capital is not infinitely elastic, an equilibrium condition for the market for uninformed capital must be added. Equilibrium in the market for uninformed capital is described by the equation

$$e_U(\gamma, \beta) \equiv D_U(\gamma, \beta) - S(\gamma) = 0$$

(12)

Equilibrium in the two markets obtains when (11) and (12) are satisfied.

Note for later use that

$$\lim_{\gamma \to 0} D_U(\gamma, \beta) = \infty \quad \text{while} \quad \lim_{\gamma \to \infty} D_U(\gamma, \beta) = \lambda < 0 \quad \text{where} \quad \lambda \text{ is a finite constant.}^{17}$$

Next, the two equilibrium conditions will be studied in $(\gamma, \beta)$-space. By the implicit function theorem and the signs of (5)-(8), the slopes of the equilibrium conditions are given by

$$\left. \frac{d\beta}{d\gamma} \right|_{e_M(\gamma, \beta) = 0} = -\frac{\partial}{\partial \gamma} D_M(\gamma, \beta) > 0$$

(13)

$$\left. \frac{d\beta}{d\gamma} \right|_{e_U(\gamma, \beta) = 0} = -\frac{\partial}{\partial \gamma} D_U(\gamma, \beta) - S'(\gamma) > 0$$

(14)

Note that uninformed and informed capital are gross substitutes. Since both equilibrium conditions (11) and (12) are upward-sloping in $(\gamma, \beta)$-space, there are potentially multiple equilibria. This means that comparative statics predictions may be reversed when moving from one equilibrium to another. It turns out that under the assumed parametrization, this is not the case.

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$^{17}$That $\lambda$ is a finite constant follows directly from taking the limit. To see that it must be negative, note that for $\gamma > p_H R/I$ the net value of projects is negative and so demand cannot be positive. In fact, for $\gamma$ above this level, it pays for firms to simply supply all their assets on the market for uninformed capital.
Existence, Uniqueness and Stability of Equilibrium (to be completed). To show that there is at most one equilibrium, it must be shown that there is at most one pair \((\gamma, \beta)\) such that (11)-(12) hold. This is the case if one of the functions is everywhere steeper than the other in \((\gamma, \beta)\)-space. That this is the case is now shown:

**Proposition 6.** (i) There exists a unique equilibrium and (ii) the equilibrium is stable

**Proof.** The condition for uniqueness to obtain and a necessary condition for that equilibrium to be stable is that

\[
\frac{d\beta}{d\gamma} \bigg|_{eM(\gamma, \beta)=0} < \frac{d\beta}{d\gamma} \bigg|_{eU(\gamma, \beta)=0}
\]

This condition is equivalent to

\[
S'(\gamma) > \frac{\frac{\partial}{\partial \gamma} D_U(\gamma, \beta) \frac{\partial}{\partial \beta} D_M(\gamma, \beta) - \frac{\partial}{\partial \gamma} D_M(\gamma, \beta) \frac{\partial}{\partial \beta} D_U(\gamma, \beta)}{\frac{\partial}{\partial \gamma} D_M(\gamma, \beta)}
\]  

(15)

Since \(\frac{\partial}{\partial \gamma} D_M(\gamma, \beta) = -\frac{\partial}{\partial \gamma} D_U(\gamma, \beta)\) and \(\frac{\partial}{\partial \beta} D_M(\gamma, \beta) = -\frac{\partial}{\partial \beta} D_U(\gamma, \beta)\), condition (15) reduces to \(S'(\gamma) > 0\), which holds by assumption. Stability of the equilibrium then follows from the signs of (5)-(8) \(\blacksquare\)

Next, I will determine how the equilibrium conditions shift in response to changes in collateral, savings and aggregate monitoring capital.

**Credit Crunch.** In a credit crunch, the aggregate amount of monitoring capital \(K_M\) decreases. This in effect increases the required rate of return on informed capital \(\beta\), thus shifting the function (11) in \((\gamma, \beta)\)-space leftward. In any stable equilibrium, this shift increases both \(\gamma\) and \(\beta\).

**Savings Squeeze.** In a savings squeeze, the supply function \(S(\gamma)\) shifts rightward for any given value of \(\beta\). In consequence, equilibrium on the market for uninformed capital is now at a higher rate of return \(\gamma\). This means that the function (12) in \((\gamma, \beta)\)-space shifts rightward. In any stable equilibrium, this shift increases both \(\gamma\) and \(\beta\).

**Collateral Squeeze.** In a collateral squeeze, the firm’s asset holdings are eroded. This corresponds to a decrease in \(\alpha\) in the current setting.\(^{18}\) Such a decrease in collateral shifts the demand function (4) leftward, thereby decreasing the return on uninformed capital \(\gamma\) for any given level of \(\beta\). This has the effect of shifting the function (12) leftward. There is no effect of changes in \(\alpha\) on the demand for informed capital (3). In any stable equilibrium, this shift decreases both \(\gamma\) and \(\beta\).

Since the two rates of return \(\gamma\) and \(\beta\) both change as there are shifts in the equilibrium conditions, it is potentially difficult to determine the resulting changes in the different sets of firms under consideration. But the fact that \(\gamma\) and \(\beta\) move together in equilibrium makes it possible to say something further. In equilibrium, \(\gamma\) and \(\beta\) must simultaneously satisfy (11) and (12). It then follows from the implicit function theorem that \(\beta\) can be viewed as a function of \(\gamma\). The next step is to determine the overall effect on the different sets of agents of changes in \(\gamma\), taking into account both the direct effect through \(\gamma\) and the indirect effect through \(\beta\). Since \(\gamma\) and \(\beta\) move together, the overall effect can be determined whenever these two effects are of the same sign. From inequities (9)-(10), the following can be established:

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\(^{18}\)Note that the decrease in assets is proportional to each firm’s initial assets.
Proposition 7. In equilibrium, a credit crunch or a savings squeeze (i) increases the rate of return on uninformed capital $\gamma$ and the rate of return on informed capital $\beta$, (ii) decreases the set of merged firms, (iii) decreases the set of firms with complete monitoring and (v) decreases the set of all active firms. The effects on the set of firms with partial monitoring are ambiguous. In equilibrium, a collateral squeeze has the opposite effects as those in (i)-(v).

This proposition shows that, with the exception of the set of partially monitored firms, the comparative statics results obtained under the assumption of infinitely elastic supply of uninformed capital are robust to the extension to a general equilibrium setting.

5. Diversification and the Value of Firms

In this section, the value of firms undertaking diversifying mergers will be analyzed and the findings will be related to the diversification debate outlined in the introduction.

Let the net value of a firm, as a function of initial asset holding $K$, be defined as $V(K) = p_H R_F - \gamma K$. A little algebra shows that this value is given as follows:

$$V(K) = \begin{cases} 
0 & \text{for } K \in [0, D(\gamma, \beta)] \\
 p_H R - \gamma I - c & \text{for } K \in [D(\gamma, \beta), F(\gamma, \beta)] \\
 p_H R - \gamma I - c/2 & \text{for } K \in [F(\gamma, \beta), C(\gamma)] \\
 p_H R - \gamma I & \text{for } K \in [C(\gamma), L] 
\end{cases}$$

The average value of merged firms is thus given by

$$V^M(K) = \frac{[F(\gamma, \beta) - D(\gamma, \beta)] (p_H R - \gamma I - c)}{A(\gamma) - D(\gamma, \beta)} + \frac{[C(\gamma) - F(\gamma, \beta)] (p_H R - \gamma I - c/2)}{A(\gamma) - D(\gamma, \beta)} + \frac{[A(\gamma) - C(\gamma)] (p_H R - \gamma I)}{A(\gamma) - D(\gamma, \beta)}$$

In turn, the average value of independent (non-diversified) firms is given by

$$V^I(K) = p_H R - \gamma I$$

Clearly, the average value of diversified firms is lower than that of non-diversified firms, i.e. $V^M(K) < V^I(K)$. But note that this “discount” of diversified firms is not in fact an effect of the diversification per se, but rather a reflection of the fact that those firms who choose to diversify tend to be low value firms.

How does this finding relate to the diversification debate? In the current setting, the decision to diversify through mergers is indeed endogenous, which means that I can characterize the firms that will choose such diversification in equilibrium. These are exactly the firms that must rely on costly financial intermediation that decreases firm value. The considered environment is rich enough to be able to differentiate between firms along more than one dimension, which enables us to drive a wedge between the diversification and the existence of a diversification discount (although of course, there is no wedge between the decision to diversify and the discount in the value of diversified firms). Existing literature cannot make such distinctions, as the only feature that differentiates between diversifying and non-diversifying firms in existing models is diversification itself.
6. Discussion

The present paper considered an equilibrium model in which cash-poor firms are credit rationed due to moral hazard problems. Two ways of alleviating the incentive problem were considered, namely diversifying mergers (which boost debt capacity) and the employment of a financial intermediary (which is costly but reduces the incentive problem). It was shown that in equilibrium, the poorest firms will opt for a combination of mergers and intermediation. This has several implications. First, diversifying firms are on average worth less than their non-diversifying counterparts. Second, when the cost of financial intermediation is low, as should be expected at the height of the business cycle, more firms become active and many of these will be diversified firms. Thus this model predicts that conglomerate merger activity should be procyclical, a time series feature of M&A activity previously established in the empirical literature. In short, a consequence of the current analysis is that the diversification neutrality result may be irrelevant while the existence of a diversification discount, when properly taking account of the endogeneity of the diversification decision, is illusory.

References


