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The Psychology of the Monty Hall Problem: Discovering Psychological Mechanisms for Solving a Tenacious Brain Teaser

Stefan Krauss
Max Planck Institute for Human Development

X. T. Wang
University of South Dakota

The Monty Hall problem (or three-door problem) is a famous example of a "cognitive illusion," often used to demonstrate people's resistance and deficiency in dealing with uncertainty. The authors formulated the problem using manipulations in 4 cognitive aspects, namely, natural frequencies, mental models, perspective change, and the less-is-more effect. These manipulations combined led to a significant increase in the proportion of correct answers given by novice participants, largely because of the synergy of frequency-based formulation and perspective change (Experiments 1, 2). In a training study (Experiment 3) frequency formulation and mental models, but not Bayes's rule training, showed significant positive transfer in solving related problems.

For 28 years, Monty Hall hosted a game show on American television called "Let's Make a Deal." Contestants on this show were often confronted with a dilemma in which they had to decide whether to stick with an initial choice or switch to an alternative. What contestants should do in this situation sparked a heated public debate in 1991, after a reader of *Parade* magazine asked the following question (see vos Savant, 1997), today known as the *Monty Hall problem* or the *three-door problem*:

> Suppose you're on a game show and you're given the choice of three doors. Behind one door is a car; behind the others, goats. You pick a door, say, Number 1, and the host, who knows what's behind the doors, opens another door, say Number 3, which has a goat. He then says to you, "Do you want to switch to Door Number 2?" Is it to your advantage to switch your choice?[1]

In three of her weekly columns (vos Savant, 1990a, 1990b, 1991), vos Savant[2] attempted to convince readers that switching is to the contestant's advantage. First, she declared, "Yes, you should switch. The first door has a 1/3 chance of winning, but the second door has a 2/3 chance." Then she explained her statement by asking readers to visualize one million doors: "Suppose there are a million doors, and you pick number 1. Then the host, who knows what's behind the doors and will always avoid the one with the prize, opens them all except door number 777,777. You'd switch to that door pretty fast, wouldn't you?"

Responses to these columns were numerous, passionate, and, in some cases, vitriolic. Many of vos Savant's disbelievers had Ph.D.s and worked in the field of statistics. A Ph.D. from the University of Florida wrote, "Your answer to the question is in error. But if it is any consolation, many of our academic colleagues have also been stumped by this problem." A member of the U.S. Army Research Institute responded thusly to her second attempt to convince readers of the correct solution: "You made a mistake, but look at the positive side. If all those Ph.D.s were wrong, the country would be in some very serious trouble." Some people even offered to wager large sums of money (e.g., $20,000) on their belief that switching has no advantage. In addressing these replies, vos Savant wrote, "Gasp! If this controversy continues, even the postman won't be able to fit into the mailroom. I'm receiving thousands of letters, nearly all insisting that I'm wrong, including the Deputy Director of the Center for Defense Information and a Research Mathematical Statistician from the National Institutes of

Stefan Krauss, Max Planck Institute for Human Development, Berlin, Germany; X. T. Wang, Psychology Department, University of South Dakota.

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Correspondence concerning this article should be addressed to Stefan Krauss, Max Planck Institute for Human Development, Lentzeallee 94, 14195 Berlin, Germany. E-mail: krauss@mpib-berlin.mpg.de

[1] In the following, we refer to this as the *standard version* of the problem. In the real game show, Monty Hall played several variations of this setting (see Friedman, 1998). But it is important to note that the discussion about the problem started only after vos Savant's columns appeared in *Parade*. Readers there were explicitly referred to the version posed by the inquisitive reader, and no mention was made of the real game show.

[2] Marilyn vos Savant's column in *Parade* magazine is called "Ask Marilyn." According to the *Guinness Book of World Records* of 1991 (Newport, 1991) she was, at the time of the controversy, said to be the person with the highest IQ in the world (IQ: 228), and readers could ask her whatever they wanted. In 1997, she summarized the exploding discussion about the Monty Hall problem in her book *The Power of Logical Thinking*.

Health! Of the letters from the general public, 92% are against our answer, and of the letters from universities, 65% are against our answer . . . . But math answers aren't determined by votes" (vos Savant, 1997, p. 10). Vos Savant's account of the public discussion makes it clear that not only is it difficult to find the correct solution to the problem, but that it is even more difficult to make people accept this solution.

Previous Research

Although this seemingly simple problem had previously been discussed in the statistical literature (Selvin, 1975a, 1975b), it was only after vos Savant's columns that it was discussed at length; for example, in the *Skeptical Inquirer* (Frazier, 1992; Posner, 1991). *The New York Times* also reported on the debate in a front-page story (Tierney, 1991). These discussions have verified vos Savant's conclusion that the mathematically correct solution is for the contestant to switch, providing that the rules of the game show are as follows: Monty Hall has in any case to reveal a goat after the contestant's first choice, and he cannot open the door chosen by the contestant.

In von Randow's (1993) book about the Monty Hall problem, the German science journalist described how he shifted his interest from mathematical to psychological issues after he realized that switching is indeed better. He raised the following three questions (p. 9): Why were so many people, even those who were highly educated, deceived? Why are so many of them still convinced of the wrong answer? Why are they so enraged?

Similarly, Piattelli-Palmarini (as cited in vos Savant, 1997, p. 15) remarked that "no other statistical puzzle comes so close to fooling all the people all the time . . . . The phenomenon is particularly interesting precisely because of its specificity, its reproducibility, and its immunity to higher education." He went on to say "even Nobel physicists systematically give the wrong answer, and . . . *insist* on it, and are ready to berate in print those who propose the right answer." In his book *Inevitable Illusions: How Mistakes of Reason Rule Our Minds* (1994), Piattelli-Palmarini singled out the Monty Hall problem as the most expressive example of the "cognitive illusions" or "mental tunnels" in which "even the finest and best-trained minds get trapped" (p. 161).

Experimental psychologists have used the Monty Hall problem to study various psychological aspects of human probabilistic reasoning and decision making.[3] In fact, before the Monty Hall problem became so well-known, Shimojo and Ichikawa (1989) investigated a problem that is mathematically equivalent, namely, the problem of the three prisoners (see Experiment 3). Shimojo and Ichikawa examined the beliefs of participants experimentally, whereas Falk (1992), for instance, looked at the same issue theoretically. The main aim of both lines of work was to provide explanations for people's reasoning errors in this kind of problem. Granberg and Brown (1995) later conducted the first comprehensive experimental study of the Monty Hall problem. They presented participants with the standard version of the Monty Hall problem with slight changes in wording and found that only 13% of them correctly chose to switch doors.

Until now, all experimental studies have had similar results: The vast majority of participants think that switching and staying are equally good alternatives and decide to stay. Falk (1992) calls the belief in the equiprobability of the two remaining alternatives "uniformity belief" (p. 202). But if most participants see no reason to favor one option over the other, why do a vast majority decide to stick to their original choice? To answer this question, Granberg and Brown (1995) gave a new group of participants hypothetical histories of choices made by previous contestants (e.g., "contestant switched and lost" or "contestant stayed and lost") and asked how they would feel in the described situations. Participants reported that they would feel worse if they switched from a door with the car behind it than if they stuck to a door with a goat behind it because, in the first case, they had picked the winning door but then decided against it. Gilovich, Medvec, and Chen (1995) showed that people can be guided by a confident confederate to either the stay or the switch decision in the Monty Hall problem. This suggests that even though naive participants have a rather strong tendency to stay when left to their own devices, this tendency may be counteracted by introducing additional psychological components.

To date, efforts to encourage people to solve the Monty Hall problem with mathematical insight have not been very successful. Expressed in terms of the percentage of participants who switch, even the most encouraging findings (e.g., Aaron & Spivey, 1998) have not exceeded 30%.

Present Approach and Objectives

Most of the research on the Monty Hall problem has focused on beliefs that might lead to the mathematically incorrect choice. We are instead interested in the mental processes that lead to correct reasoning. Despite the difficulties people have with the Monty Hall problem, there are people who do find the correct solution intuitively. This naturally leads to two questions: Which reasoning processes are used by these few successful problem solvers? Providing we identify these mechanisms, how can we develop appropriate ways to represent and explain the brain-teaser such as to eliminate the typical resistance to the switch decision? In the brain-storming phase preceding the experiments, we confronted colleagues and students with the problem and later discussed their intuitions with them. This led us to the insight that the reasoning processes of successful problem solvers have a common denominator, the essence of which is expressed in Figure 1. Note that Monty Hall's behavior in Arrangement 3 of Figure 1 is not specified ("no matter what Monty Hall does"). Most representations found in the literature consist of more than three single arrangements and specify Monty Hall's behavior in each arrangement (see, for instance, Table 1). We will later demonstrate why ignoring Monty Hall's behavior in Arrangement 3 turns out to be a crucial building block of an intuitive solution.

By performing a "mental simulation" of the three possible arrangements (i.e., considering the whole sequence of actions specified by each arrangement in Figure 1), one can see that in two

[3] Generally, there are two kinds of experiments related to the Monty Hall problem. First, one can present participants with a written version of the problem and ask them to make (and justify) a decision on it. Second, one can let people play the game repeatedly with feedback (e.g., against a computer) and investigate how they change their behavior in response to the outcome. In this article, we focus only on experiments of the first kind. For experiments of the second kind see, for example, Friedman (1998) or Granberg and Dorr (1998).

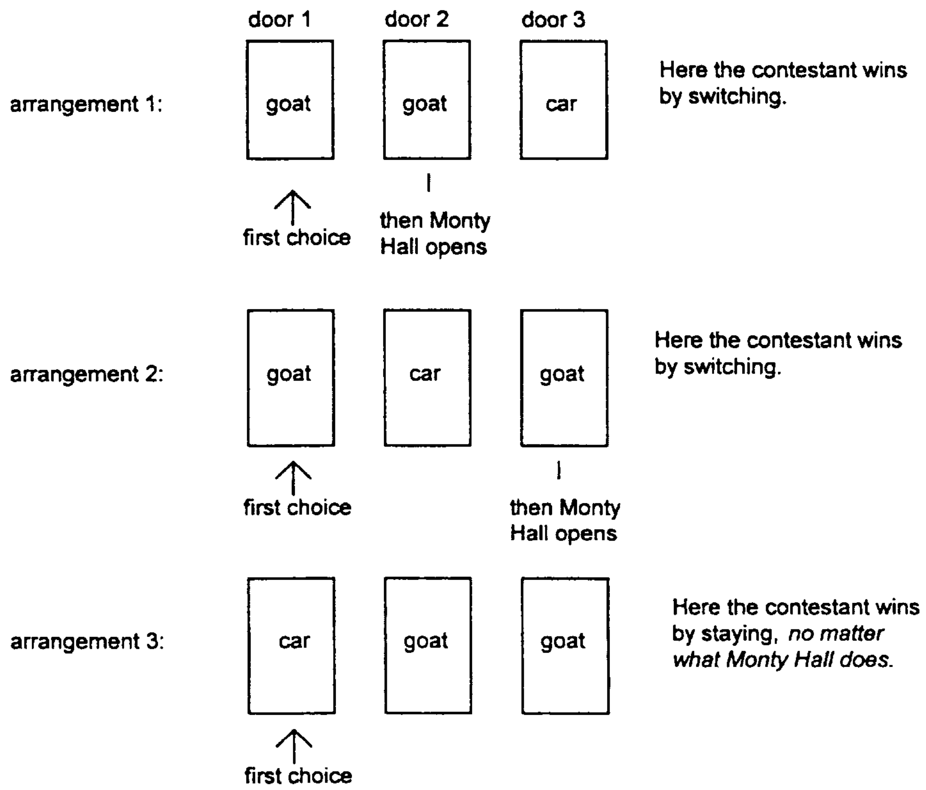

Figure 1. Explanation of the solution to the Monty Hall problem: In two out of three possible car–goat arrangements the contestant would win by switching; therefore she should switch.

out of three possible arrangements, the contestant would win the car by switching (namely, in Arrangements 1 and 2). Let us identify some interrelated features in Figure 1 and express them in terms of psychological elements:

1. Rather than reasoning with probabilities, one has to count and compare frequencies.

2. These frequencies correspond to possible arrangements of goats and cars behind the doors. One has to compare the number of arrangements in which the contestant would win the car by switching to the number in which she would win by staying.

Table 1
Mental Model Representation of the Monty Hall Problem

Mental model	Door 1 (chosen door)	Door 2	Door 3
1	car	open	
2	car		open
3		car	open
4		car	open
5		open	car
6		open	car

Note. Based on mental models from Johnson-Laird et al. (1999).

3. One has to consider the possible arrangements as they would appear from behind the doors.

4. One has to ignore the last piece of information provided in the standard version (Monty Hall opens Door 3). Taking this information for granted would eliminate Arrangement 1 in Figure 1, because the host will not open a door concealing a car.

Item 4 demands some clarification: Although, semantically, Door 3 in the standard version is named merely as an example ("Monty Hall opens another door, *say,* number 3"), most participants take the opening of Door 3 for granted and base their reasoning on this fact. In a pretest we gave participants (N = 40) the standard version, asking them to illustrate their view of the situation described by drawing a sketch. After excluding four uninterpretable drawings, we saw that 35 out of the remaining 36 participants (97%) indeed drew an open Door 3, and only a single participant (3%) indicated that other constellations also remain possible according to the wording of the standard version. The assumption that only Door 3 will open is further reinforced by the question that follows: "Do you want to switch to Door Number 2?" Note that, once formed, this assumption prevents the problem solver from gaining access to the intuitive solution illustrated in Figure 1.

Each of the four features specified above has theoretical underpinnings eliciting correct responses from naive participants. In the following section, we discuss these four features in terms of four psychological elements, namely, natural frequencies, mental models, perspective change, and the less-is-more effect.

Natural Frequencies

To find the correct solution to the Monty Hall problem, one needs to consider the three arrangements in Figure 1 and to reason in terms of frequencies, for example, "in one—or two—of three arrangements . . . ." One of the characteristics of intuitive probabilistic reasoning seems to be that it is often done in terms of frequencies rather than probabilities. Gigerenzer and Hoffrage (1995) have shown experimentally that representing probabilistic information in natural frequencies rather than in probabilities helps participants to solve Bayesian reasoning problems. For instance, they asked participants to assess the probability that a woman in her 40s has breast cancer, given she has received a positive mammography result. In one version, the relevant information was given in probabilities (e.g., "the probability of a woman in her 40s having breast cancer is .01"); in the other version, the same information was expressed in frequencies ("10 out of every 1,000 women in their 40s have breast cancer"). The results of Gigerenzer and Hoffrage's study suggest that the format of information can either facilitate or hinder reasoning. Currently, there is a lively debate on frequency formats (for detailed discussion on their theoretical underpinning and on the differences between normalized frequencies and natural frequencies, see Evans, Handley, Perham, Over, & Thompson, 2000; Girotto & Gonzales, 2001; Hoffrage, Gigerenzer, Krauss, & Martignon, 2002).

Information format may also be an important factor affecting the search for a mathematically correct solution to the Monty Hall problem. Gigerenzer and Hoffrage's (1995) proposal for improving probabilistic reasoning by translating single-event probabilities (i.e., consider the case of a single woman) into natural frequencies (i.e., consider a whole sample of women) is readily applicable to the Monty Hall problem. Aaron and Spivey (1998) presented the Monty Hall problem in both probability and frequency versions to different groups of participants.[4] In one of their experiments 12% of participants given the probability version gave the correct answer, whereas 29% of participants given the frequency version did. This suggests an advantage of reasoning with frequencies rather than probabilities. A disadvantage of Aaron and Spivey's treatment is that the wording no longer has much in common with the standard version of the problem. The participants in that study were given a pictorial presentation of the problem and answered a series of 11 questions that were posed in terms of either probabilities or frequencies. In the frequency condition, for instance, the participants were instructed to imagine 30,000 game shows and asked frequency questions such as "Of the 30,000 rounds in which the player chooses door 1, in how many of them is the car actually behind door 1?" Aaron and Spivey's formulation of all 11 questions is rather verbose, and their manipulation thus looks heavy-handed.

In contrast to the frequency procedure used by Aaron and Spivey (1998), the approach shown in Figure 1 does not involve imagining multiple rounds. Instead, it uses the concept of frequencies in the actual context of a single-shot game.

Mental Models

Following the diagram in Figure 1, one has to count and compare conditional outcomes of possible arrangements (e.g., if the car is actually behind Door 2, I would win by switching) for a single game show. This sort of case-based mental simulation relates to Johnson-Laird's (1983) work on the dynamics of logical reasoning. According to his theory, people reason about logical problems, for example, syllogisms, by constructing mental models. Recently, Johnson-Laird, Legrenzi, Girotto, Legrenzi, and Caverni (1999) extended this theory to probabilistic reasoning—including reasoning about the Monty Hall problem. In a section of their article, entitled *Pedagogy of Bayesian Reasoning*, they suggested six mental models, which are illustrated in Table 1, that people might use to represent the Monty Hall problem in an intuitive way. In Table 1, the word *open* indicates the door that Monty Hall opens after the contestant chooses door 1, and the word *car* indicates the door behind which the car is actually located. The rows correspond to the mental models, each of which represents a possible situation of the Monty Hall problem, given that the contestant first chooses Door 1. If the car actually is behind Door 1, Monty Hall can open either Door 2 (Mental Model 1) or Door 3 (Mental Model 2). Johnson-Laird et al.'s Mental Models 1 and 2 correspond to our Arrangement 3 (Figure 1). Note that we get by with one arrangement because we do not take Monty Hall's behavior into account. Because they do consider his behavior, Johnson-Laird et al. have to construct two models for the case that the car is behind Door 1. Because all car positions a priori are equally probable, Johnson-Laird et al. also have to construct two mental models for each of the other car–goat arrangements, though in these arrangements, Monty Hall does not have to decide which door to open. According to Johnson-Laird et al., once the six mental models have been constructed, one can take into account the door actually opened by Monty Hall: Assume Monty Hall has opened Door 3. This would render Models 1, 5, and 6 impossible. Considering the remaining models—2, 3, and 4—reveals that switching would win in two of the three models. Johnson-Laird et al. did not run an empirical study on whether people actually use these mental models to solve the Monty Hall problem. Although they admitted that the artificial replication of models (Model 3 corresponds to Model 4 and Model 5 corresponds to Model 6) might be difficult to grasp, they proposed that these six mental models can serve as a means of explaining the problem.

It is interesting that Marilyn vos Savant also used six mental models in her second attempt to explain the Monty Hall problem (vos Savant, 1997, p. 8). Her models had a 3×2 structure, in which the dimensions were the three possible locations of the car and the two possible choice strategies (i.e., stay vs. switch). Yet, as we have learned, this approach did not convince her readers.

Both vos Savant (1997) and Johnson-Laird et al. (1999) suggested six models to explain the Monty Hall problem. We argue that the three-model representation of Figure 1 is a more effective way of presenting the problem.

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[4] Although the format of the standard version of the Monty Hall problem is not obviously determined, it clearly does not ask for frequencies. The question "Is it to your advantage to switch?" refers rather to a single-event probability, that is, to the possible outcome of one specific game show.

Perspective Change

The three mental models in Figure 1 are constructed as if one were standing behind the doors and could see each possible arrangement of goats and car. This perspective, which is that of the game show host, makes it possible to imagine what the host would have to do, contingent upon which door the car is behind. Taking the contestant's perspective, in contrast, may block participants' "view" of the three possible arrangements behind the doors. The idea of investigating the impact of changing perspective on human reasoning has been applied with different aims and in different reasoning tasks (e.g., Fiedler, Brinkmann, Betsch, & Wild, 2000; Gigerenzer & Hug, 1992; Wang, 1996).

With respect to the Monty Hall problem, we suggest the following theoretical connection between a perspective change and the structure of Bayes's rule: Assuming that the contestant first chooses Door 1 and that Monty then opens Door 3 (the standard version), the probability that the car is behind Door 2 can be expressed in terms of Bayes's rule as follows:

$$p(C_2 \mid M_3) = \frac{p(M_3 \mid C_2) \cdot p(C_2)}{p(M_3 \mid C_1) \cdot p(C_1) + p(M_3 \mid C_2) \cdot p(C_2) + p(M_3 \mid C_3) \cdot p(C_3)}, \tag{1}$$

where C_i = car behind door i; $i = 1, 2, 3$; and M_3 = Monty opens Door 3. Note that in the standard version, $p(M_3 \mid C_3) = 0$.

The theoretical connection between the perspective change and the structure of Bayes's rule is apparent: When calculating a conditional probability of an arbitrary event A given a condition B, that is, $p(A \mid B)$, Bayes's rule stipulates that one has to consider the inverse conditional probabilities $p(B \mid A)$ and $p(B \mid \bar{A})$. For the Monty Hall problem, this means that to judge $p(C_2 \mid M_3)$, one has to insert the three conditional probabilities—$p(M_3 \mid C_3)$, $p(M_3 \mid C_2)$, and $p(M_3 \mid C_1)$—into Bayes's rule. The cognitive process for assessing these three probabilities is independent of the behavior of the contestant but relies on imagining Monty Hall's behavior in all three arrangements. Thus, a Bayesian solution of the problem—whether a formal one based on Bayes's rule or an intuitive one based on Figure 1—focuses on the behavior of the host rather than on that of the contestant. Consequently, the change from the contestant's perspective to Monty Hall's perspective corresponds to a change from non-Bayesian to Bayesian thinking.

Less-Is-More Effect

A common question encountered by both users and providers of information concerns the optimal amount of information that should be used or provided. Goldstein and Gigerenzer (1999) have reported empirical evidence indicating that sometimes "knowing less is more." A clear example provided by the authors is the use of the *recognition heuristic*, which exploits the potential of recognition to help people make inferences. When a situation requires people to infer which of two objects has a higher value on some criterion (e.g., which is faster, higher, stronger), the recognition heuristic can be stated in the following simple terms: If one of the two objects is recognized and the other is not, then infer that the recognized object has a higher value. One of the surprising findings presented by the authors was that Germans were better able than Americans to judge which of two cities in the United States (e.g., San Diego and San Antonio) had the larger population. Why? The German participants, many of whom did not know of San Antonio, could use the recognition heuristic (e.g., infer that San Diego has a larger population than San Antonio because they recognized the former but not the latter). The recognition heuristic is not only frugal in its use of information, it actually *requires* a lack of knowledge to work. This research finding shows that, under certain conditions, a counterintuitive *less-is-more effect* emerges, in which a lack of knowledge is actually beneficial for inference.

Regarding the door opened by Monty Hall, a participant solving the three-door problem may have two possible states of knowledge: First, she may merely know that after her first choice Monty Hall will open another door revealing a goat, or, second, she may already have learned the number of this door. Note that participants are only able to provide the intuitive solution (see Figure 1) if the specific number of the door that Monty Hall actually opens is not taken into account. The easiest way to make sure that participants' reasoning processes are not impeded by knowing which door Monty Hall opens is simply not to give them this information. The corresponding formulation would be "Monty Hall now opens another door and reveals a goat." Although the cognitive situation here differs from that in the recognition heuristic, the underlying principle is the same: Having less information can be beneficial for inference.

The issue of "door information" is of great relevance for the cognitive processes to solve the Monty Hall problem. Before inserting the four psychological elements into the wording of the problem, let us take a closer look at the different possible scenarios of the problem based on different "door information." Since we learned from the pretest that the formulation "say, number 3" is interpreted psychologically as "Door 3 is open," we will refer to expressions such as that used in the standard version ("you pick a door, say, number 1, and the host opens another door, say, number 3") as *specifications* of doors.

No-Door Scenario

If no door were specified in the formulation of the Monty Hall problem (*no-door scenario*), that is, neither that chosen by the contestant nor that chosen by Monty Hall, then there are no restrictions on the participant's mental simulation of the game show. The contestant's three possible choices and the three possible locations of the car would then yield a total of nine possible arrangements. In Figure 2, which illustrates these arrangements, we label the rows that denote the actual car location by numbers and the columns that denote first choice by letters. For instance, the arrangement in which the car is behind Door 3 and the contestant first chooses Door 1 is labeled A3. Figure 2 illustrates that a contestant would win the prize in six of the nine possible arrangements by switching doors, but in only three of the nine arrangements (A1, B2, and C3) by sticking with the door initially chosen. Hence, switching affords a better chance of winning. Not specifying a door in the wording would thus allow participants to use an intuitive representation that is likely to lead to the correct response. However, this may be suboptimal because it would be difficult to simulate all nine scenarios mentally.

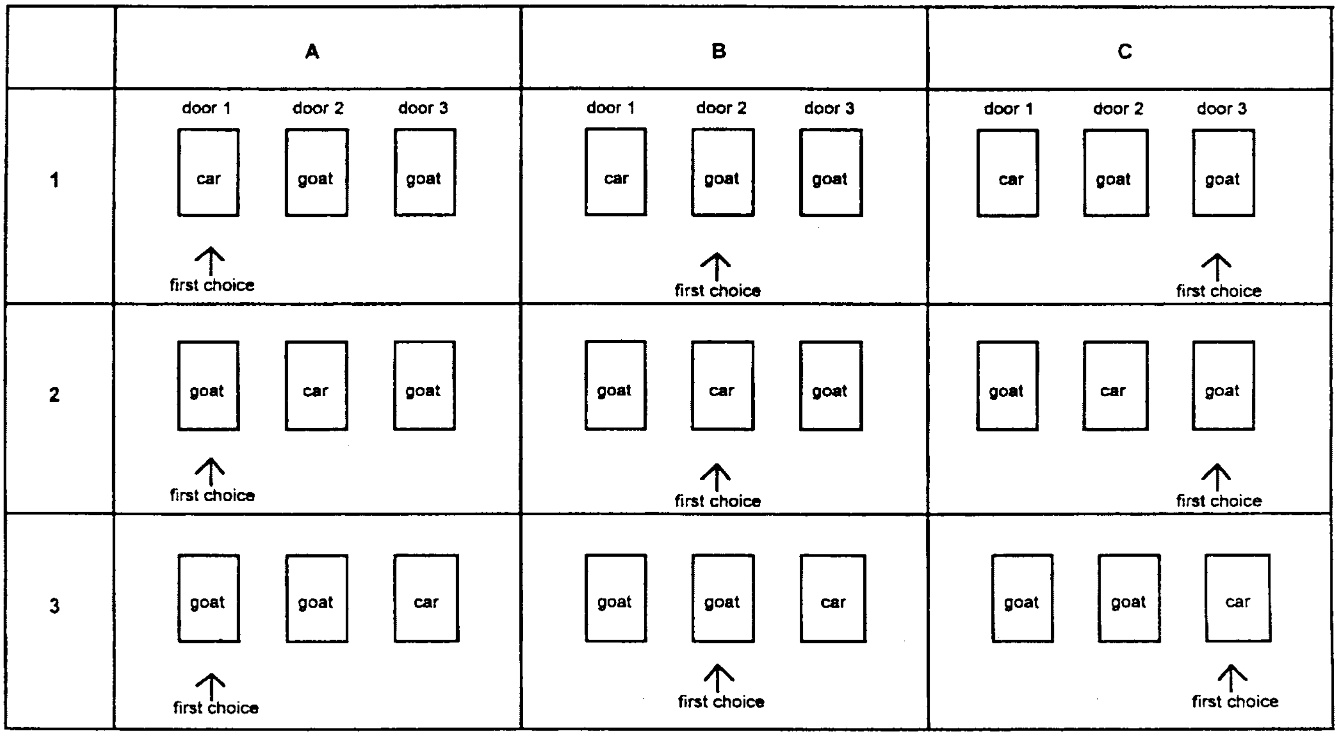


Figure 2. The nine possible arrangements in a no-door scenario.

One-Door Scenarios

If the contestant's first choice is specified (one-door scenario), then only three arrangements remain possible. If, for instance, the wording is such that the contestant chooses Door 1, then only arrangements A1, A2, and A3 remain (see left column of Figure 2). By switching, the contestant would win in two of the three arrangements (A2 and A3) and lose in only one arrangement (A1).

A second type of one-door scenario that may lead to a similar path of intuitive thinking would entail specifying the location of the car. The arrangements can then be illustrated by any row in Figure 2. For example, the third row would represent the three possible first choices of a contestant when the car is specified to be behind Door 3. One can see that the contestant would win in two of three arrangements by switching (A3 and B3), and in only one arrangement by staying (C3). Thus, whether the door specified is the contestant's first choice or the car position, one-door scenarios allow participants to use just one column or one row of Figure 2 to gain an insight into the correct solution.

In a nutshell, both no-door and one-door scenarios allow unrestricted mental simulations, thereby making the counting and comparison of the frequency of wins possible, given that the contestant switches or stays. Another crucial advantage of all no-door and one-door scenarios is that one does *not* have to think about the behavior of Monty Hall in the cases in which he can choose which door to open: In a one-door scenario in which the contestant has chosen Door 1 first, a correct and sufficient chain of reasoning might take the following form:

> If the car is actually behind Door 3, then Monty Hall must open Door 2, and I win by switching (A3).
>
> If the car is actually behind Door 2, then Monty Hall must open Door 3, and I win by switching (A2).
>
> If the car is actually behind Door 1, then I win by sticking to my first choice, no matter what Monty Hall does (A1).

In the single arrangements of a one-door scenario, Monty Hall's behavior is either predetermined (here: A3, A2) or irrelevant for the decision (A1). The problem becomes cumbersome, however when the door opened by Monty Hall is specified in addition to the contestant's first choice, as is the case in *two-door scenarios*.

Two-Door Scenarios

The additional specification of the door opened by Monty Hall in the standard version of the problem leaves only two of the three arrangements in the left column of Figure 2 (A1 and A2). A3 is impossible because Monty Hall cannot open the door concealing the car. As a result, one cannot simply count and compare the frequencies of winning, given that the contestant switches or stays; instead, one has to reason in probability terms to reach the Bayesian solution. That is, Monty Hall's opening Door 3 has a lower probability in A1 than in A2, because in A1 he could have opened either Door 2 or Door 3, whereas in A2 he had to open Door 3. Thus, one has to make assumptions about what Monty Hall would do in A1 and estimate the probability that Monty Hall would open Door 3 rather than Door 2. Some authors have argued that participants' assumptions about Monty Hall's strategy in A1 may affect their probability judgments, and that the lack of information about this strategy in the standard version may therefore help to explain participants' poor performance on the problem (cf. von Randow, 1993).

To illustrate this "strategy" argument, we use the standard version, in which the contestant chooses Door 1. Taking the left column of Figure 2, let us consider one arrangement after the other. This means considering the three conditional probabilities $p(M_3 \mid C_3)$, $p(M_3 \mid C_2)$, and $p(M_3 \mid C_1)$ in accordance with Equation 1:

> Arrangement A3: According to the wording "the host . . . opens another door . . . which has a goat," A3 is no longer possible, which means that the probability $p(M_3 \mid C_3) = 0$.
>
> Arrangement A2: It also follows from the wording that the probability that Monty Hall opens Door 3, given the contestant first chooses Door 1, is unity, that is, $p(M_3 \mid C_2) = 1$.
>
> Arrangement A1: Investigating arrangement A1 reveals that $p(M_3 \mid C_1)$ reflects Monty Hall's strategy.

We now consider three different strategies that he might use concerning arrangement A1:

First, if one assumes that Monty Hall's strategy is to choose randomly when he has a choice, then the probability of his opening Door 3, $p(M_3 \mid C_1)$, equals 1/2. The probability of the contestant's winning by switching (to Door 2) can now be expressed in terms of Bayes's rule:

$$p(C_2 \mid M_3) = \frac{p(M_3 \mid C_2) \cdot p(C_2)}{p(M_3 \mid C_1) \cdot p(C_1) + p(M_3 \mid C_2) \cdot p(C_2) + p(M_3 \mid C_3) \cdot p(C_3)} = \frac{1 \cdot 1/3}{1/2 \cdot 1/3 + 1 \cdot 1/3 + 0 \cdot 1/3} = \frac{2}{3}. \quad (2)$$

Thus, assuming Monty Hall uses this random strategy, the probability of the contestant winning by switching is indeed equal to what it is in the no-door and one-door scenarios, namely, 2/3.

Second, if one assumes that Monty Hall's strategy is to open Door 3 whenever possible, then $p(M_3 \mid C_1)$ equals 1, and the probability of the contestant winning by switching changes to 1/2:

$$p(C_2 \mid M_3) = \frac{p(M_3 \mid C_2) \cdot p(C_2)}{p(M_3 \mid C_1) \cdot p(C_1) + p(M_3 \mid C_2) \cdot p(C_2) + p(M_3 \mid C_3) \cdot p(C_3)} = \frac{1 \cdot 1/3}{1 \cdot 1/3 + 1 \cdot 1/3 + 0 \cdot 1/3} = \frac{1}{2}. \quad (3)$$

Third, if one assumes that Monty Hall's strategy is to open Door 2 whenever possible, then $p(M_3 \mid C_1)$ is 0, and the probability of the contestant winning by switching would become 1:

$$p(C_2 \mid M_3) = \frac{p(M_3 \mid C_2) \cdot p(C_2)}{p(M_3 \mid C_1) \cdot p(C_1) + p(M_3 \mid C_2) \cdot p(C_2) + p(M_3 \mid C_3) \cdot p(C_3)} = \frac{1 \cdot 1/3}{0 \cdot 1/3 + 1 \cdot 1/3 + 0 \cdot 1/3} = 1. \quad (4)$$

As demonstrated, different assumptions about Monty Hall's strategy indeed lead to different Bayesian solutions. Note that these different solutions are possible only in two-door scenarios such as the standard version. Taking Monty Hall's strategy into account not only can lead to different solutions, but also forces one to reason in terms of probabilities. Furthermore, there is no intuitive diagram that can reflect Monty Hall's strategy appropriately. The advantage of the no-door and one-door scenarios, in which Monty Hall's behavior is not specified, is that participants do not need to consider the possible strategies that Monty Hall might use.

Are There Possible Effects of Incomplete Information?

After the Monty Hall problem became famous, many questions on possible effects of incomplete information in the standard version arose. Besides not mentioning Monty Hall's strategy (1), the standard version refers neither to the exact rules of the game show (2) nor to the a priori probability distribution of car and goats (3; cf. Mueser & Granberg, 1999; Nickerson, 1996; von Randow, 1993).

1. The standard version provides no information about Monty Hall's strategy. Is the problem therefore mathematically underspecified and insoluble? The answer is no, because the standard version does not ask for a probability, but for a decision. The general Bayes's rule for the standard version of the Monty Hall problem in the absence of information about Monty Hall's strategy is:

$$p(C_2 \mid M_3) = \frac{p(M_3 \mid C_2) \cdot p(C_2)}{p(M_3 \mid C_1) \cdot p(C_1) + p(M_3 \mid C_2) \cdot p(C_2) + p(M_3 \mid C_3) \cdot p(C_3)} = \frac{1 \cdot 1/3}{p(M_3 \mid C_1) \cdot 1/3 + 1 \cdot 1/3 + 0 \cdot 1/3} = \frac{1}{p(M_3 \mid C_1) + 1}, \quad (5)$$

where $p(M_3 \mid C_1)$ is a "strategy" parameter that can vary between 0 and 1.

Because the strategy-dependent probability $p(M_3 \mid C_1)$ varies between 0 and 1, the conditional probability $p(C_2 \mid M_3)$ can vary only between 0.5 and 1 (see Equation 5). Therefore, whatever strategy one assumes Monty Hall to use, the conclusion is that the contestant should switch. Only if Monty Hall always opens Door 3, that is, $p(M_3 \mid C_1) = 1$ (an assumption for which the wording of the problem does not provide the slightest support) do staying and switching afford the contestant equal chances of winning. Given all other assumptions about Monty Hall's strategy (an infinite set of possible strategies), switching is better than staying. Thus, Equation 5 implies that switching is better even in two-door scenarios, regardless of Monty Hall's strategy. Even after clarifying this mathematical question, a psychological question remains: Does the lack of information on Monty Hall's strategy hinder participants in choosing the right alternative? According to Ichikawa (1989, p. 271), letting participants know Monty Hall's strategy does not help them find the correct solution either.

2. The conditional probabilities $p(M_3 \mid C_1)$, $p(M_3 \mid C_2)$, and $p(M_3 \mid C_3)$ describe Monty Hall's behavior in different arrangements. This behavior can be influenced either by his personal strategy or by the official rules of the game show (in the standard version, the implicit rule is "after the contestant chooses a door,

Monty Hall has to open another door and reveal a goat"). If the rule were, instead, that the host has to reveal a goat if the contestant first chooses the "car door" and should otherwise do nothing, then $p(M_3 \mid C_2) = 0$, which makes the probability of winning by switching 0 (see Equation 1). Nickerson (1996) writes that "without information or an assumption about the host's behavior, the situation is ambiguous, and the question of whether one should switch is indeterminate" (p. 420). Consequently, most experimental psychologists insert the intended rule "Monty has to open another door and reveal a goat" into the standard version to avoid criticism about ambiguity in the wording. But this does not seem to help participants: Although Granberg and Brown (1995) stressed this rule, they observed only 13% switch decisions.

3. As we have seen, we cannot be sure of the values of the conditional probabilities $p(M_3 \mid C_1)$, $p(M_3 \mid C_2)$, and $p(M_3 \mid C_3)$ in the standard version, because we know neither the complete rules of the show nor Monty Hall's personal strategies. What about the remaining terms in Equation 2, namely, $p(C_1)$, $p(C_2)$, and $p(C_3)$? One may wonder whether the car was randomly placed behind one of the three doors. In other words, is the assumption of an equal a priori distribution $p(C_1) = p(C_2) = p(C_3) = 1/3$ justified? Perhaps the car is more likely to be placed behind Door 1 because it is closest to the entrance of the stage.

A formulation of the Monty Hall problem providing all of this missing information and avoiding possible ambiguities of the expression "say, number 3" would look like this (mathematically explicit version):

> Suppose you're on a game show and you're given the choice of three doors. Behind one door is a car; behind the others, goats. The car and the goats were placed randomly behind the doors before the show. The rules of the game show are as follows: After you have chosen a door, the door remains closed for the time being. The game show host, Monty Hall, who knows what is behind the doors, now has to open one of the two remaining doors, and the door he opens must have a goat behind it. If both remaining doors have goats behind them, he chooses one randomly. After Monty Hall opens a door with a goat, he will ask you to decide whether you want to stay with your first choice or to switch to the last remaining door. Imagine that you chose Door 1 and the host opens Door 3, which has a goat. He then asks you "Do you want to switch to Door Number 2?" Is it to your advantage to change your choice?

Even though the Bayesian solution (Equation 2) is now wholly justified, fleshing out the problem in this manner would fail to foster insight into its mathematical structure. The problem is that people still do not have access to an intuitive solution (such as that illustrated as Figure 1). We argue that most of the criticisms of the standard version regarding its unstated assumptions are mathematically relevant, but not psychologically relevant, because participants still assume the intended rules, even if those rules are not stated explicitly.

Evidence supporting this claim comes from the observation that the vast majority of people wrongly regard the stay and switch choices as equally likely to result in winning. Let us give examples of how assumptions different from the intended ones would make this "uniformity belief" in the standard version impossible:

1. If participants assumed that Monty Hall's strategy is to always open the middle door whenever possible, they would know that it was not possible for Monty Hall to open the middle door because it had the car. Thus they would not conclude equiprobability of the remaining alternatives, but rather would switch to Door 2.

2. If participants assumed the game show rule is that Monty Hall only reveals a goat when the first choice is a car, they too would not follow the uniformity belief, but rather have an obvious reason to stay.

3. If participants did not assume the a priori distribution $p(C_1) = p(C_2) = p(C_3) = 1/3$, they again would not have any reason to come up with an equiprobable a posteriori distribution.

In sum, when solving the standard version, in which the required assumptions are not made explicit, people seem to assume the intended scenario anyway. Along the same lines, vos Savant (1997) observed, "Virtually all of my critics understood the intended scenario. I personally read nearly three thousand letters (out of the many additional thousands that arrived) and found nearly every one insisting simply that because two options remained (or an equivalent error), the chances were even. Very few raised questions about ambiguity, and the letters actually published in the column were not among those few" (p. 15).

In short, people seem to struggle not with the ambiguity of the standard version's assumptions, but with the mathematical structure of the scenario. As we will see in the next section, what blocks correct intuitive reasoning is not a lack of information, but a lack of the right information representation.

Intuitive Versions of the Monty Hall Problem

To formulate wordings of the Monty Hall problem that should elicit the correct solution, we take into consideration the four psychological elements discussed earlier, as well as the discussion on missing information. The four psychological elements were (1) perspective change, (2) the less-is-more effect, (3) mental models, and (4) natural frequencies. We incorporated these elements by means of the following manipulations:

1. We manipulated perspective by asking participants to "imagine you are the host of this game show" instead of assigning them the role of the contestant.

2. We used a one-door scenario (this means *not* specifying the number of the door opened by Monty Hall). Relative to the two-door scenario in the standard version, this can be considered an incorporation of a less-is-more effect. The beneficial lack of information about the door opened by Monty Hall would allow participants to make an intuitive sketch (e.g., Figure 1) and to reason in terms of frequencies instead of probabilities.

3. We explicitly mentioned the three possible arrangements of goats and car behind the doors to participants to prime the relevant mental models (i.e., A1, A2, and A3).

4. We asked participants to state the frequencies with which the contestant would win by switching and by staying: "In how many of the three possible arrangements would

the contestant win the car after the opening of a 'goat-door,'"

if she stays with her first choice (Door 1)?

in ____ out of 3 cases

if she switches to the last remaining door?

in ____ out of 3 cases

Various versions of the Monty Hall problem can be constructed by incorporating combinations of these manipulations. Note that not all of the possible resulting versions are meaningful. For example, without the less-is-more manipulation, none of the other manipulations can work: In a two-door scenario, only two arrangements are possible and—as we have seen—considering just two arrangements can never lead to an intuitive, correct solution, regardless of whether the right perspective is taken or an intuitive frequency question is provided. Therefore, we consider the less-is-more manipulation to be a "basic" manipulation that is required before the others are implemented.

In two experiments (Experiments 1 and 2) we tested seven meaningful and theoretically relevant versions. All versions had similar layouts (see Figures 3 and 4). Our prediction was that the more manipulations are incorporated in the wording of the Monty Hall problem, the better the performance of participants becomes. Furthermore, we expected that, when all manipulations are incorporated (we call such a version a *guided intuition version*), the mathematical structure will become accessible to humans' reasoning and participants' performance will be significantly improved.

As the control version (see Figure 3), we used an unambiguous variant of the standard version. Note that our control version contained the following additional features: We included the rules of the game show to give total clarification of the intended scenario and to guarantee comparability with other studies. Furthermore, to reduce variance in participants' assumptions, we eliminated the word *say* when specifying the door opened by Monty Hall. To describe the scenario more vividly, we added a diagram of the three doors. Finally, because we were interested not only in participants' actual decisions, but also in how many participants who switched had genuine mathematical insight into the problem, we asked participants to justify their decisions. The impact of these additional changes, which are not the four intended manipulations, can be assessed by comparing participants' performance in our control version (see Figure 3) with that usually obtained with standard versions.

Figure 4 illustrates a guided intuition version, which incorporates all four psychological elements into our control version. To examine the impact of incorporating certain combinations of the four manipulations on participants' performance, we conducted two experiments, one in Germany (Experiment 1) and one in the United States (Experiment 2). Table 2 gives an overview of all versions of the Monty Hall problem tested in Experiments 1 and 2.

Experiment 1

Method

In this experiment, we had three groups of participants. We compared the German control (C-Ger) version (see Figure 3 for English translation) with the German guided intuition (GI-Ger) version of the Monty Hall problem (see Figure 4 for English translation). We also tested a version (1D version) in which only the less-is-more manipulation was incorporated by specifying only one door (see Appendix A). For the labels of each experimental group in Experiment 1 and Experiment 2, see Table 2.

After excluding 14 participants who reported that they had already heard of the Monty Hall problem, we were left with 135 students (47 men and 88 women) whose average age was 24.7 years. Participants were students of different disciplines and were recruited from various universities in Berlin. They were tested at the Max Planck Institute for Human Development in small groups of up to 5 people. Each participant received only one version of the Monty Hall problem. There were 67 participants in the control condition and 34 participants in each of the other conditions. After the experiment, every participant received a payment of 10 Deutsche Mark (approximately $5).

To classify our participants' justifications, we first divided participants into "stayers" and "switchers." We then further classified the switchers into the following three groups according to their justifications for switching: (a) participants who gave correct justifications and exhibited full insight into the mathematical structure; (b) participants who had the right intuition but could not provide a mathematically correct proof; and (c) participants who switched randomly, meaning that they regarded switching and staying as equally good alternatives.

The criteria for correct justification were strict: We counted a response as a correct justification only if the 2/3 probability of winning was both reported and comprehensibly derived. This could, for instance, be fulfilled by applying one of the algorithms reported above (Equation 1, Table 1, or Figure 1), or by any other procedure indicating that the participant had fully understood the underlying mathematical structure of the problem. The right intuition category contained all the switchers who believed that switching was superior to staying but failed to provide proof for this.[5] Finally, a participant was assigned to the "random switch" category if she thought it made no difference if she switched or stayed. Participants' protocols in this latter category revealed the uniformity belief, yet were not followed by the more popular choice of staying.

Interestingly, the stayers did not give differentiated justifications. None of the stayers maintained that staying might be better from a mathematical point of view.

As a check on the reliability of these classifications, a coder trained to have a thorough understanding of the problem classified responses from 10 participants in each condition (stripped of information indicative of the condition). The coder agreed with our initial classification of 93% of the responses.

Results

The results of Experiment 1 are summarized in Table 3. A Pearson's chi-square test of the rate of switch choices indicated a reliable difference between conditions, $\chi^2(2, N = 135) = 14.531$, $p < .01$. Follow-up comparisons indicated that the rate of switching was significantly lower in the control (C-Ger) group than in the guided intuition (GI-Ger) group, $\chi^2(1, N = 101) = 14.529$, $p < .01$. The one-door (1D) group lay between the C-Ger group and the GI-Ger group. However, the differences in the rate of switching between the 1D group and the C-Ger group, $\chi^2(1, N = 101) = 3.462$, $p < .07$, and between the 1D group and the GI-Ger group, $\chi^2(1, N = 68) = 2.885$, $p < .09$, were not significant at the alpha level of .05.

[5] A prototypical response in this category, for instance, is "If I stay, the probability of winning remains at .33; if I switch it increases to .50. Somehow it seems that switching pays."

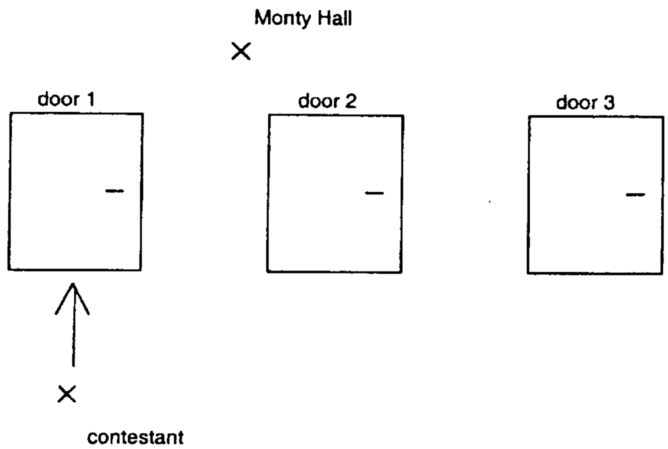
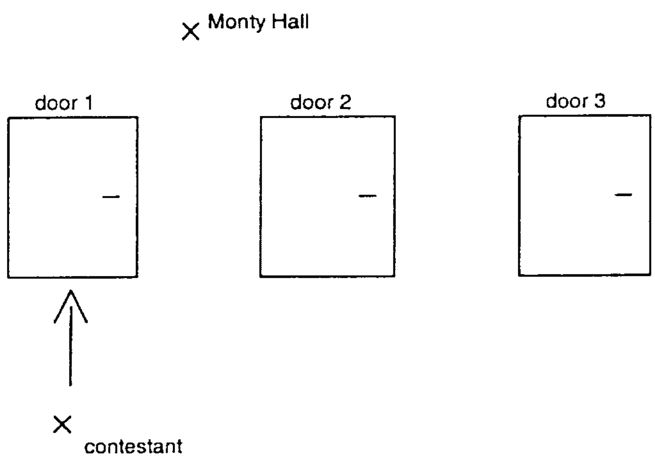
	LET'S MAKE A DEAL
The rules of the game show:	There is a game show called "Let's Make a Deal," where a contestant is allowed to choose one of three closed doors. Behind one door is the first prize, a new car; behind each of the other doors is a goat. After the contestant has chosen a door, the door remains closed for the time being. According to the rules of the game, the game show host, Monty Hall, who knows what is behind the doors, now has to <u>open one of the two unchosen doors and reveal a goat</u>. After Monty Hall shows a goat to the contestant, he asks the contestant to decide whether she or he wants to stay with the first choice or to switch to the last remaining door.
	<u>Task:</u>
Perspective of the contestant:	Imagine you are the contestant and you do not know which door the car is behind.
Specifying the first choice:	You now randomly choose a door, say, number 1.
Diagram of three doors:	
Specifying the door opened by Monty Hall:	In accordance with the rules of the game, Monty Hall then opens door 3 and shows you a goat. Now he asks you whether you want to stay with your first choice (door 1) or to switch to door 2.
Asking just for a decision:	After Monty Hall has opened a "goat-door," what should you do? ___ stay ___ switch
Asking for justification:	<u>Important:</u> Please tell us in writing what went on in your head when you made your decision. You may use sketches, etc., to explain your answer. Please also tell us if you were already familiar with this game ___ (Yes) ____ (No) and knew what the correct answer should be ____ (Yes) ____ (No).

Figure 3. Control version of the Monty Hall problem (the left column was not provided to participants).

More important than simply getting participants to switch rather than stay is getting them to provide correct justifications for their decision to switch. Data on the rate of correct justifications for switch choices were analyzed with pairwise Fisher exact probability tests (due to the small number of correct responses in the control and one-door conditions), accompanied by Cohen's effect size h for the difference of proportions (Cohen, 1988, pp. 179–213). According to Cohen, h = .20 denotes a small effect size, h = .50 a medium effect size, and h = .80 a large effect size. The tendency for correct justification to be given more often in the

The rules of the game show:	**LET'S MAKE A DEAL** There is a game show called "Let's Make a Deal," where a contestant is allowed to choose one of three closed doors. Behind one door is the first prize, a new car; behind each of the other doors is a goat. After the contestant has chosen a door, the door remains closed for the time being. According to the rules of the game, the game show host, Monty Hall, who knows what is behind the doors, now has to open one of the two unchosen doors and reveal a goat. After Monty Hall shows a goat to the contestant, he asks the contestant to decide whether she or he wants to stay with the first choice or to switch to the last remaining door.
Perspective change to the perspective of Monty Hall:	**Task:** Imagine you are Monty Hall, the host of this game show, and you know which door the car is behind.
Specifying the first choice:	The contestant chooses a door, say, number 1.
Diagram of three doors:	



Not specifying the door opened by Monty Hall (less-is-more manipulation):	In accordance with the rules of the game, you then open another door and show the contestant a goat. Now you ask him/her whether he/she wants to stay with the first choice (door 1) or to switch to the last remaining door.
Motivating the construction of mental models:	The car may be behind any of the three doors.
Frequency question:	In how many of these three possible arrangements would the contestant win the car after you open a "goat-door," - if she/he stays with the first choice (door 1)? in ___ out of 3 cases - if she/he switches to the last remaining door? in ___ out of 3 cases What should the contestant therefore do? ___ stay ___ switch
Asking for justification:	Important: Please tell us in writing what went on in your head when you gave your answers. You may use sketches, etc., to explain your answers. Please also tell us if you were already familiar with this game ___ (Yes) ____ (No) and knew what the correct answer should be ____ (Yes) ____ (No).

Figure 4. Guided intuition version of the Monty Hall problem (the left column was not provided to participants).

Table 2
Versions of the Monty Hall Problem Tested in Experiments 1 and 2

Version		Manipulation
C-Ger	(Control version in the German experiment; Experiment 1)	No manipulation
C-US	(Control version in the U.S. experiment; Experiment 2)	No manipulation
1D	(One-door version; Experiment 1)	Less-is-more
1D + F	(One-door plus frequency version; Experiment 2)	Less-is-more and frequency simulation
1D + P	(One-door plus Monty's perspective version; Experiment 2)	Less-is-more and perspective change
GI-Ger	(Guided intuition version in the German experiment; Experiment 1)	Full manipulation
GI-US	(Guided intuition version in the U.S. experiment; Experiment 2)	Full manipulation

one-door condition than in the control condition was not statistically reliable (p = .10, one-tailed; h = .36). The rate of correct justifications in the guided intuition group was reliably greater than that in the control condition (p = .0001, one-tailed; h = .98) or the one-door condition (p = .01, one-tailed; h = .62). Thus, the guided intuition manipulation significantly improved understanding of the rationale for switching yielding large effect sizes.

Regarding the other two categories of switch choices, the rate of random switch between the three experimental groups was not significantly different (see Table 3). The rate of switching based on the right intuition was significantly lower in the control condition than in the one-door condition (p = .04, one-tailed) or the guided intuition condition (p = .04, one-tailed).

Discussion

The percentage of switch choices in the guided intuition condition (59%), with 38% correct justifications, sets a new standard in the literature on the Monty Hall problem. Most studies on the Monty Hall problem report only the percentage of switch decisions, which is usually around 10–15%. We expect that, given a standard or similar version of the Monty Hall problem, the percentage of participants with the correct mathematical insight would be much lower. This assumption is supported by the finding that only 3% of participants in the control condition solved the problem by mathematically correct reasoning.

Mueser and Granberg (1999) obtained more than 70% switch decisions by offering participants an additional monetary incentive if they switched. Yet, their experiments did not aim to qualify as an attempt to foster insight into the mathematical structure of the problem. Hell and Heinrichs (2000) obtained 65% switch decisions by investigating a variant of the problem with 30 doors, in which 28 doors were opened after the first choice. Note that increasing the number of doors changes the problem's structure. That is, by opening all doors except 2 (e.g., the first chosen and Door Number 21) the probability of winning by switching to Door 21 is now 97%. Burns and Wieth (2002) demonstrated that participants reason better when the problem is presented in a way that encourages the correct representation of causality. By transforming the story of the Monty Hall problem into a box-competition scenario, they obtained 50% switch decisions, of which 15% revealed a full understanding.

Studies using simulation of multiple trials also achieved a remarkable improvement in performance after several rounds of feedback (Friedman, 1998; Granberg & Brown, 1995). However, in these repeated game settings, a good performance is possible without insight into the mathematical structure of the Monty Hall problem: The feedback provided in the first rounds might convince participants that switching is better, yet, they may not necessarily know *why*. Our approach, in contrast, is to increase performance

Table 3
Parameters for the Three Experimental Groups in Experiment 1, Including Percentages of Switch Choice

Parameter	C-Ger	1D	GI-Ger
Participants (N)	67	34	34
Less-is-more effect	No	Yes	Yes
Specified door(s)	First choice and door opened by Monty Hall	First choice	First choice
Perspective	Contestant	Contestant	Monty Hall
Frequency question for mental models ("frequency simulation")	No	No	Yes
Switch choice (%)			
Random switch	16	15	9
Right intuition	2	12	12
Correct justification	3	12	38
Total	21	38	59

Note. C-Ger = control version (Germany), no manipulation; 1D = one-door version (less-is-more manipulation); GI-Ger = guided intuition with all four manipulations.

and insight in the notorious original problem, but not by changing the number of doors or the number of rounds.

In Table 3, as in the following tables displaying empirical results, the mental model and the frequency manipulations are combined. The reason for this is that both elements are theoretically and practically connected to each other: On the one hand, the frequency question alone automatically evokes the construction of mental models, because these are the instances to be counted. On the other hand, building mental models can only be half of the process that leads to a problem's solution. The correct answer can be reached only if the mental models are then counted and compared with respect to their outcomes. In the following, we refer to this combined manipulation as *frequency simulation*. We choose the word *simulation* instead of *model* because, as shown in Figure 1, just building the three models is not enough. Each model first has to be "simulated" (i.e., considering the whole sequence of actions specified by the model) until the outcome becomes apparent. As we know, such a cognitive procedure within a model is not intended in Johnson-Laird's (1983) notion of mental models.

The significant improvement observed when all elements are applied together motivates the analysis of these underlying elements and their individual impact on participants' performance. For example, at first glance the frequency question seems to be rather a heavy-handed hint on how to solve the problem. Yet, as Experiment 2 will reveal, this seemingly powerful hint does not work effectively if a certain perspective is not provided at the same time.

Experiment 2

A key question concerning the findings of Experiment 1 is whether we need all four conceptual manipulations to foster people's insight into the structure of the Monty Hall problem. Is any one of the manipulations more crucial than the others? Are there synergistic effects of the manipulations? In a second experiment conducted in the United States (Experiment 2), we examined four different versions that were designed to partition the effects of the four crucial manipulations.

Method

Participants in Experiment 2 were students recruited from the University of South Dakota. After excluding 2 participants who reported that they had already heard of the Monty Hall problem, we were left with a total of 137 participants (96 women and 41 men) with an average age of 22.7 years. Participants were randomly assigned to four different conditions and were tested in a classroom with 10–30 students in each session. Each participant received only one version of the Monty Hall problem. As in Experiment 1, participants in Experiment 2 were asked to give a written justification for their choices. Participants were rewarded with extra course credit. The four groups received the following versions of the problem (see Table 2 for the labels of these experimental groups).

1. Control (C-US): An English version of the C-Ger scenario used in Experiment 1.

2. One-door plus frequency simulation (1D + F): A one-door scenario in which the first choice is specified (less-is-more manipulation). The participants were asked to take the contestant's perspective and make the stay–switch decision after answering the frequency question (frequency simulation).

3. One-door plus perspective change (1D + P): A one-door scenario (less-is-more manipulation) in which the position of the car is specified. The participants were asked to take Monty Hall's perspective (perspective change) and make the stay–switch decision without first having answered the frequency question.

4. Guided intuition (GI-US): A one-door scenario (less-is-more manipulation) in which the position of the car is specified. The participants were asked to take Monty Hall's perspective (perspective change) and make the stay–switch decision after answering the frequency question (frequency simulation). Because all four manipulations are incorporated in this version, it constitutes a second version of the guided intuition condition (parallel to GI-Ger).

Note that in the 1D + P and GI-US conditions, the one-door scenario is incorporated by specifying the position of the car instead of the contestant's first choice. In reference to Figure 2, this specification requires the participants to reason "row-wise," and the frequency question now requires them to imagine the three possible first choices of the contestant, rather than the three different car–goat arrangements.

The procedure and criteria for classifying participants' justifications were the same as described in the method section of Experiment 1. In Experiment 2, a trained student who had a thorough understanding of the Monty Hall problem but was unaware of the experimental conditions classified the participants' responses independently. The independent coder agreed with our initial classification of 94% of the responses.

Results

The results of Experiment 2 are presented in Table 4. A Pearson's chi-square test of the rate of switch choices did not reach the alpha level of .05, $\chi^2(9, N = 137) = 6.98, p = .07$. However, the

Table 4
Parameters for the Three Experimental Groups in Experiment 2, Including Percentages of Switch Choice

Parameter	C-US	1D + F	1D + P	GI-US
Participants (*N*)	35	34	34	34
Less-is-more effect	No	Yes	Yes	Yes
Specified door(s)	First choice and door opened by Monty Hall	First choice	Car position	Car position
Perspective	Contestant	Contestant	Monty Hall	Monty Hall
Frequency question for mental models ("frequency simulation")	No	Yes	No	Yes
Switch choice (%)				
Random switch	17	6	3	6
Right intuition	6	12	17	12
Correct justification	0	9	9	32
Total	23	26	29	50

Note. C-US = control version (United States), no manipulation; 1D + F = one-door version plus frequency simulation; 1D + P = one-door plus perspective change; GI-US = guided intuition with all four manipulations.

overall Pearson chi-square test of the frequencies of three types of justifications indicated a reliable difference, $\chi^2(3, N = 137) = 25.40$, $p < .01$. Of the three types of justifications for switch choices, namely, random switch, right intuition, and correct justification, only the rate of correct justifications showed a significant difference between the four experimental conditions, $\chi^2(3, N = 137) = 18.21$, $p < .01$.

The guided intuition version elicited the highest rate of correct justifications (32%), comparable to that found in Experiment 1. For analyzing the rates of correct justifications, we used a Fisher's exact probability test accompanied by Cohen's effect size h for differences between proportions. A Fisher's exact probability test revealed that the 32% switch choice with a mathematically correct justification in the guided intuition condition was significantly higher than the corresponding 0% in the control condition ($p = .0002$, one-tailed; $h = 1.2$).

Monty Hall's perspective alone (1D + P) and frequency simulation alone (1D + F) both had a rate of 9% correct justifications, significantly lower than the 32% rate found in the guided intuition condition (Fisher's exact probability two-tailed test, $p < .04$, $h = .59$). The 9% rate of correct justifications in the 1D + P and 1D + F conditions was not a statistically reliable improvement on the 0% found in the control condition (Fisher's exact probability two-tailed test, $p < .12$, $h = .61$). However, the effect sizes for both comparisons (no manipulation vs. two manipulations and two manipulations vs. full manipulation) yielded values of approximately $h = .6$ (between medium and large effect).

Discussion

In our further discussion of Experiment 2, we consider the results of Experiments 1 and 2 together. The similar performance in the two identical control versions suggests that it would be reasonable to look at the results collapsed across the experiments. Figure 5 illustrates the results for all seven versions, ordered by observed percentage of switch decisions. The difference between the performances on the two control versions (on the left of Figure 5) and that on the two guided intuition versions (on the right of Figure 5) demonstrates the strong impact of simultaneously incorporating all four manipulations. The improved grasp of the mathematical structure of the task in the GI-Ger and the GI-US condition clearly can be attributed to the four psychological manipulations. The similar performance in both guided intuition conditions—as well as analysis of participants' protocols—suggests that the kind of one-door scenario implemented (specifying first choice vs. specifying car position) makes no notable difference.

Note that our control condition was a modification of the standard version: We specified the rules of the game show, included a diagram of the three doors, deleted the ambiguous word *say* when specifying which door Monty Hall opens, and asked participants to justify their choice. Approximately 22% of participants chose to switch in our control condition, which is slightly greater than the switch rates typically reported with standard control conditions. However, the percentage of participants who gained a mathematically correct insight into the problem's structure given these nonexperimental changes (3% in Experiment 1 and 0% in Experiment 2) was very low.

The synergistic effect of the combination of the perspective change and the frequency simulation is particularly intriguing. The perspective change alone (1D + P) and the frequency simulation alone (1D + F) both failed to facilitate understanding: The rates of correct justifications in these two conditions were not significantly

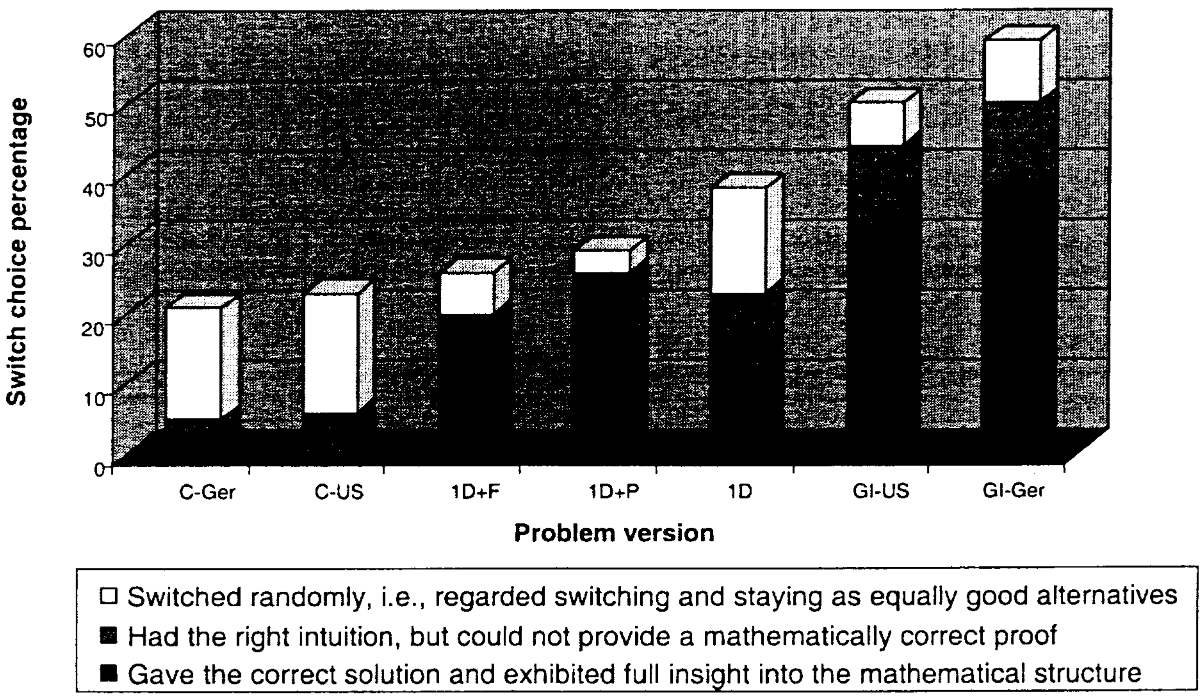


Figure 5. Complete results of Experiment 1 and Experiment 2, ordered according to the percentages of switch choice in each condition. See Table 2 for definitions of each problem version.

different from that in the 1D condition. Thus the better performance of participants in the guided intuition versions can be attributed neither to the perspective change nor to the frequency simulation alone. In fact, there seems to be a synergistic effect between the frequency simulation and the perspective change, as both manipulations have to be applied simultaneously.

The effects of perspective change and frequency simulation appear to be essentially additive when measured in terms of switch-choice total but appear to be superadditive for mathematically correct justifications. To examine the superadditivity of the effects of perspective change and frequency simulation, we compared the rate of correct justifications collapsed across the two guided intuition groups (i.e., the GI-Ger and GI-US conditions) with double the rate of correct justifications collapsed across the two single manipulation groups (i.e., 1D + P and 1D + F). In other words, we conducted a chi-square test of the rate of correct justifications, comparing 24/68 with 12/68. The result showed a synergistic effect (superadditivity) of perspective change and frequency simulation. The facilitation effect of the combined manipulation in the guided intuition condition was significantly higher than the doubling of the average effect of the two single manipulations, $\chi^2(1, N = 136) = 5.44$, $p = .02$.

The trichotomous categorization (see Figure 5) also shows that the counterintuitively higher switch rate in the 1D condition as compared with the 1D + P and the 1D + F conditions is in fact due to a relatively high proportion of participants who had no mathematical insight but switched in accordance with the uniformity belief (empty bars).

In our tables presenting the results of Experiments 1 and 2 (Tables 3 and 4), the two theoretical elements mental model and natural frequencies were collapsed into the frequency question for mental models, in short, the frequency simulation. Johnson-Laird et al. (1999) claimed that the mental model concept provides another theory of probabilistic reasoning that differs from the natural frequency approach (Gigerenzer & Hoffrage, 1995, 1999). It is true that both elements stress different aspects of knowledge representation: On the one hand, the term *natural frequencies* emphasizes external information representation. In nature, we observe frequencies of outcomes rather than probabilities, and our minds should be adapted to this kind of information. Thus the natural frequency approach provides an ecological explanation for why humans are good at dealing with frequencies. On the other hand, the term *mental models* stresses internal information representation. Yet, when considering the actual reasoning process, it emerges that natural frequencies and mental models are deeply intertwined: The frequency question ("in how many of these three possible arrangements . . . ") is answered by counting arrangements, which actually are mental models.

Experiment 3

In Experiments 1 and 2, people's intuition was guided by the combined manipulations. Participants in both experiments had to choose and justify the correct solution on their own; no attempts were made to convince disbelievers of the correct solution. Vos Savant's (1997) experiences show that even entire demonstrations of how to solve the problem—including presentation of the correct solution—can fail to break people's resistance. One of the indicators of insightful understanding of a problem is the ability to transfer one's knowledge about its resolution to other, similar problems.

The questions addressed in Experiment 3 are pedagogical in nature: Can a guided intuitive training in the Monty Hall problem effectively be transferred to similar problem-solving situations? Which way of demonstrating the solution to the Monty Hall problem would be most effective? Consider, for instance, the following two problems:

The One Goat and Two Cars Problem

Imagine that there are two cars and one goat behind the doors. The contestant chooses a door. The host then has to open another door and show a car to the contestant. Now the contestant can decide whether she wants to stay with her first choice or switch to the last remaining door. (She is not allowed to take the car behind the opened door.) What should the contestant do?

Problem of the Three Prisoners

Tom, Dick, and Harry are awaiting execution while imprisoned in separate cells in some remote country. The monarch of that country arbitrarily decides to pardon one of the three, but the name of the lucky man is not announced immediately, and the warden is forbidden to inform any of the prisoners of his fate. Dick argues that he already knows that at least Tom or Harry must be executed, thus convincing the compassionate warden that by naming one of them he will not be violating his instructions. The warden names Harry. Did this change the chances of Dick and Tom being freed?

Both problems are variants of the Monty Hall problem. In the first problem, the contestant should choose to stay, because in two out of three arrangements the first choice is a car. In the problem of the three prisoners, the chance of Dick being freed remains one third, whereas the chance of Tom being freed increases to two thirds. Previous studies have shown that participants' beliefs and justifications regarding the three prisoners problem do not differ from the corresponding ones in the Monty Hall problem (Falk, 1992; Ichikawa & Takeychi, 1990). The majority of participants think that the chances of Dick and Tom being freed are the same.

In a pilot study, we provided the participants in Experiment 1 with one of three different explanations of how to solve the Monty Hall problem (after they had returned their response sheets). One was based on Figure 1 (frequency simulation explanation, FS), the second on Table 1 (mental models explanation, MM), and the third on Equation 1 (Bayes's rule explanation, BR). To investigate the relative effects of these explanations, we presented the participants with four new problems. One problem (the one goat– two cars problem) was given immediately afterward, and the other three problems were presented 10 weeks later. One of these three problems was the three-prisoners problem, the other two were extended Monty Hall problems by using four doors (see Appendix B). All problems were presented in their standard version, involving none of the manipulations used in Experiments 1 and 2. The results of the pilot study presented in Table 5 suggest that the three training explanations all have the potential to facilitate problem solving in related situations. A better controlled experiment was needed to further examine the possible transfer effects of these training methods.

The following features were not controlled in the pilot study:

Table 5
Participants' Performance (Percentage of Correct Justifications) Immediately and 10 Weeks After Training in the Pilot Study

Problem	Training type		
	FS (N = 22)	BR (N = 25)	MM (N = 21)
	Immediately after training		
1 goat–2 cars	82	49	42
	10 weeks after training		
3 goats–1 car	73	60	29
2 goats–2 cars	73	28	43
3 prisoners	46	44	14

Note. FS = frequency simulation explanation; BR = Bayes's rule explanation; MM = mental models explanation.

1. Participants were given different versions of the Monty Hall problem before training.
2. The follow-up problems were presented in constant order.
3. Because the three-prisoners problem was the last to be presented, it was obvious that this problem, which deviates from the familiar game show content, could also be solved with the trained scheme.
4. When tested after training, frequency simulation and mental models had to be retrieved from memory, but Bayes's rule was available on paper.
5. There was no control condition (i.e., solving test problems without previous training).

In Experiment 3, the above issues were controlled.

Method

Participants in Study 3 were 120 students (76 females and 44 males) recruited from universities in Berlin, whose average age was 23.1 years. The participants were paid 10 euros (about $10) for approximately 1 hr of work. Participants were randomly divided into four groups with 30 participants in each group: the FS group, the MM group, the BR group, and the control group. In the FS condition, participants were taught how to solve the Monty Hall problem by constructing the frequency simulation illustrated in Figure 1. In the MM condition, the six mental models adapted from Johnson-Laird et al. (1999) and illustrated in Table 1 were used. In the BR condition, we taught participants how to use Bayes's rule to attain the correct solution. In the control group, participants were presented with the original Monty Hall problem and then asked to solve the other test problems without further explanation. After excluding 10 participants who reported that they had already heard of the Monty Hall problem, we were left with a total of 110 participants.

Before the respective training, all participants were presented with the control version of the Monty Hall problem (see Figure 3). To test possible transfer effects of training, we used the three-prisoners problem and a four-door version of the Monty Hall problem called the two goats–two cars problem (see Appendix B). To conceal the applicability of the received explanations to the three-prisoners problem, we also included a distractor problem (the Duncker Tumor problem) that could not be solved by applying the explanations learned in the training session. The order of presentation of the follow-up problems, including the Duncker Tumor problem, was counterbalanced. All explanation sheets, including the Bayes's rule explanation, had to be returned to the experimenter before any of the follow-up problems were presented. To minimize memory effects, we presented all test problems immediately after training.

In Experiment 3, a trained student who had a thorough understanding of the Monty Hall problem but was unaware of the experimental conditions classified all participants' responses independently. The independent coder agreed with the initial classification of 94% of the responses.

Results

The results of Experiment 3 are summarized in Table 6. In the following analysis, we focus on the rate of correct solutions with mathematically correct justifications. Before training, none of the 110 participants managed to solve the Monty Hall problem and give a correct justification for the switch choice. This poor performance (0% of mathematically correct solution) remained the same in the control (no training) condition when participants were presented with the two test problems (i.e., the four-doors and three-prisoners problem), showing no ad hoc understanding of the test problems at all.

A Pearson's chi-square test showed a significant difference in the rate of correct solutions between the control, BR, MM, and FS

Table 6
Percentage of Mathematically Correct Solutions Given for the Monty Hall Problem Before Training and for Related Problems After Training in Experiment 3

Problem	Control and training conditions			
	Control (no training; n = 24)	FS (n = 28)	BR (n = 28)	MM (n = 30)
	Before training			
Monty Hall problem	0	0	0	0
	After training			
2 goats–2 cars	0	46	4	30
3 prisoners	0	18	0	10

Note. FS = frequency simulation explanation; BR = Bayes's rule explanation; MM = mental models explanation.

conditions for both the four-doors problem, $\chi^2(3, N = 110) = 23.96, p < .01$, and the three-prisoners problem, $\chi^2(3, N = 110) = 9.06, p < .03$. Follow-up comparisons indicated that the rate of correct solutions after BR training was not better than in the control group. For both test problems, the rate of correct solutions was nearly the same in the control and BR condition (see Table 6). Thus, contrary to the results of the pilot study, the findings of Experiment 3 suggest that training with Bayes's rule does not help participants to solve related problems.

In contrast, FS training and, to a lesser degree, MM training helped participants to solve the test problems. A Fisher's exact probability one-tailed test indicated that the FS group had a significantly higher rate of correct solutions than the control group in both the four-doors test problem ($p < .01$, $h = 1.49$) and the three-prisoners problem ($p < .04$, $h = 0.88$), both yielding large effect sizes. Compared with the control group, the MM group had a significantly higher rate of correct solutions for the four-doors problem ($p < .01$, $h = 1.16$), but not for the three-prisoners problem ($p = .16$, $h = 0.64$). However, the advantage of the FS training over the MM training in terms of the rate of correct solutions (46% vs. 30% for the four-doors problem; 18% vs. 10% for the three-prisoners problem) was not statistically reliable ($p = .15$, $h = 0.33$ and $p = .31$, $h = 0.23$. respectively).

Discussion

The findings of Experiment 3 can be further reviewed by taking into account the results of the pilot study. This combined analysis offers several suggestions on how Bayesian reasoning can be fostered and improved.

First, the positive transfer effects of BR training in the pilot study suggest that participants were able to use and even to generalize Bayes's rule (e.g., extend the denominator from three to four summands as required in the four-doors problems), as long as they had their BR explanation sheets handy while working on the test problems. In consideration of this, the lack of effects of BR training observed in Experiment 3 suggests that participants failed to recall the entire Bayes formula rather than failed to transfer such a rule-learning to the test problems.

Second, an interesting finding of Experiment 3 is that it was easier for the participant to solve the four-doors (two goats–two cars) problem than the three-prisoners problem. Across the three training conditions, the average rate of succeeding in solving the four-doors problem was 26.7%, whereas this rate was 9.3% in solving the three-prisoners problem. From a mathematical point of view, the four-doors problem should be more difficult to solve because its underlying structure differs from that of the training problem. To solve the four-doors problem, the mathematical structure of the problem must be constructed by expanding the structure of the training problem (i.e., adding arrangements in FS, adding models in MM, or adding summands in the denominator of BR). To solve the three-prisoners problem, in contrast, one only needs to replace the original content of the training problem with a new content, that is, the three prisoners correspond to the three doors, the fates of the prisoners correspond to the objects behind the doors, and so on.

It is interesting to note that this finding is contrary to some previous findings in the field of problem solving. According to Reed (1993), there are four types of test problems in terms of similarities of stories and similarities of solution procedures. An *equivalent* test problem has the same story content and solutions as the training example. A *similar* test problem has the same story content but a different solution. An *isomorphic* test problem has a different story content but the same solution as the example. And finally, an *unrelated* test problem differs in both dimensions. A review of the literature in the fields of problem solving and educational psychology suggests that it is easier to solve isomorphic problems than similar problems (see Reed, 1993; Renkl, 1997). In contrast, in Experiment 3, the similar problem (the four-doors problem) was much easier to solve than the isomorphic problem (the three-prisoners problem). A reason for this difference might be that the transfer from the Monty Hall problem to the three prisoners problem requires that humans (the prisoners) have to be mapped to doors. Ross (1987) found that "the details of the correspondence, in terms of the object mappings, are a crucial part of how the earlier example is used" (p. 630), and that "in particular, if similar objects were included in the study and test problems, at test, subjects tended to assign them to the same variables that they had been assigned to in the study problem" (p. 632). Thus, one could speculate that the following isomorphic problem will be solved by participants: A prisoner, who is fated to die, gets a last chance. In one out of three scrolls is a certificate of amnesty. He can choose one of the scrolls. After he has made his choice, the warden opens another scroll, which is unwritten. The warden asks whether the prisoner wants to stay with his first choice or switch to the remaining scroll.

In sum, regarding Bayesian reasoning, once a match between a training problem and a test problem has been identified, people seem to be able to transfer frequency-based Bayesian training methods to new problems (see also Krauss, Martignon, Hoffrage, & Gigerenzer, 2002; Sedlmeier & Gigerenzer, 2001).

Theoretical Implications and Conclusion

Bayesian reasoning can be fostered either through the *formulation* of a problem (e.g., guided intuition versions of Experiments 1 and 2) or through an *explanation* (e.g., FS training in Experiment 3). Studying how a combined manipulation of cognitive elements improves problem solving is of practical interest, and finding evidence of synergistic relationships between these cognitive elements is of theoretical value.

An information format such as natural frequency presentation that is effective in one problem context may be ineffective in another. However, its effectiveness may depend on the perspective taken by the problem solver. In the case of the Monty Hall problem, the contestant perspective often inhibits the problem solver from using frequency information correctly. By combining the two psychological elements (i.e., frequency simulation and perspective change), the inhibitory relation becomes multiplicative facilitation.

In particular, the frequency simulation and perspective change only improve participants' performance if presented in combination (psychological dependency). The frequency simulation requires the less-is-more manipulation as a precondition to present insightful information, because only in a one-door scenario can the frequency counting be applied to the relevant mental models (mathematical dependency). In other words, not implementing the less-is-more manipulation would lead to the uniformity belief, as participants would be limited to only two arrangements, instead of the three possible ones.

The superadditive effects of the frequency simulation and perspective change showcase the importance of information presentation in reasoning and problem solving. Our multidimensional approach of testing dynamic relationships between cognitive factors may enable us to interpret the seemingly futile results of single-factor manipulation and achieve a fuller understanding of the cognitive mechanisms involved in our reasoning and problem solving.

A second theoretical implication of our findings concerns a mathematical foundation for perspective change. Taking Monty Hall's perspective opens a pathway to the insight that the game show host has no choice in two out of three arrangements: Whenever the contestant first chooses a goat, Monty Hall has to reveal the other goat, and the contestant wins by switching. Taking the game show host's perspective is to take a Bayesian view: The question posed in the standard version ("Is it to your advantage to switch your choice?") corresponds to the left side of Equation 1, that is, $p(C_2 \mid M_3)$. To calculate this conditional probability with Bayes's rule, one has to assess the ingredients of the right side of Equation 1. Although clear on the equal distribution of the car's position, that is $p(C_1) = p(C_2) = p(C_3) = 1/3$, the conditional probabilities $p(M_3 \mid C_1)$, $p(M_3 \mid C_2)$, and $p(M_3 \mid C_3)$ remain to be assessed. In our view, this is the step that requires looking at the possible arrangements through Monty Hall's eyes: What is the probability that Monty Hall will open Door 3, given that the car actually is behind Door 1, Door 2, or Door 3? Arriving at the correct solution requires the participant to detect the constraints imposed on Monty Hall. Focusing on his behavior leads to a straightforward Bayesian solution, be it the formal one (Bayes's rule) or an intuitive one (in accordance with Figure 1). Does the beneficial effect of changing perspectives on the understanding of Bayes's rule generalize to different situations? This approach might have great potential for other Bayesian reasoning problems.

Indeed, a training based on the manipulations of the guided intuition versions (FS) helped the participants in solving related problems (Experiment 3). The positive transfer effects were significant for the FS and MM but not BR training. The superiority of the FS training over the MM training was evident in the case of solving a test problem that had a greater degree of difficulty (i.e., the three-prisoners problem). Across all trainings, content similarity between a training problem and a test problem appeared to be more helpful than mathematical similarity in transferring learning into new situations.

In conclusion, a remarkable proportion of naive participants can gain full insight into the structure of the Monty Hall problem when elements from the cognitive psychologists' toolbox are applied. During the last 10 years, the claim has persisted that there is no way to break most people's resistance to grasping the mathematical structure of the problem. In hindsight, one might think it obvious that many participants will understand the problem when presented with the guided intuition version. However, in view of the long tradition of vain attempts to explain the problem, the crucial progress made in the present experiments was to identify these versions, which have not been considered in previous research. Note that our manipulations do not "destroy a fascinating cognitive illusion," but that—as we learned from our participants—the Monty Hall problem only displays its whole fascination when one realizes that switching is indeed better.

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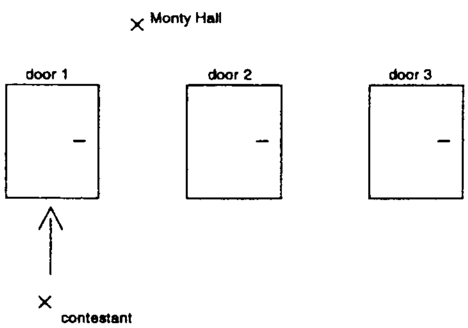
Appendix A

Experiment 1: Version 1D
LET'S MAKE A DEAL

In America there is a game show called "Let's Make a Deal." The contestant is allowed to choose one out of three closed doors. Behind one door is the first prize, a car. Behind the other two doors are goats. Monty Hall (the host of the game show) asks the contestant to choose one door. After the contestant has chosen a door, the door remains closed for the time being, because the rules of the game show require that the host (who actually knows where the car is) first opens one of the other two doors and shows a goat to the contestant. Now the contestant can again decide whether she wants to stay with her first choice or whether she wants to switch to the last remaining door.

Task:

Imagine you are the contestant and you don't know which door the car is behind. You chose a door, say, number one.



In accordance with the rules of the game, Monty Hall then opens another door and shows you a goat. Now he asks you whether you want to stay with your first choice (Door 1) or to switch to the last remaining door.
What should you do?___stay___switch

Important:

Please tell us in writing what went on in your head when you made your decision. You may use sketches, etc., to explain your answer.

Please also tell us if you were already familiar with this game___(Yes) ___(No) and knew what the correct answer should be___(Yes)___(No).

(Appendixes continue)

Appendix B

Problems A–D

One Goat-Two Cars Problem

Imagine that two cars and one goat are now behind the doors. After the contestant has chosen a door, the door remains closed for the time being. The game show host now has to open one of the two unchosen doors and reveal a car. After Monty Hall has shown a car to the contestant, he asks her to decide whether she wants to stay with the first choice or switch to the last remaining door. (The car behind the open door cannot be chosen.)

Probability of winning by switching: 1/3

One Goat-Three Cars Problem

A contestant in a game show is allowed to choose one of four doors. Behind one door is the first prize, a car. Behind the other three doors are three goats. Monty Hall asks the contestant to choose one door. After the contestant has chosen a door, the door remains closed for the time being, because the rules of the game require that the host (who knows where the car is) first opens two of the other three doors and reveals two goats to the contestant. Now the contestant can decide whether she wants to stay with the first choice or switch to the last remaining door.

Probability of winning by switching: 3/4

Two Goats-Two Cars Problem

A contestant in a game show is allowed to choose one of four doors. Behind two doors are cars. Behind the other doors are goats. Monty Hall asks the contestant to choose one door. After the contestant has chosen a door, the door remains closed for the time being, because the rules of the game require that the host (who knows where the cars are) first opens two of the other three doors and reveals a goat and a car to the contestant. Now the contestant can decide whether he wants to stay with his first choice or switch to the last remaining door.

Probability of winning by switching: 1/2

Problem of the Three Prisoners

Imagine three prisoners (A, B, and C) sitting in three different cells. They know that one of them will be set free; the other two will be executed. The only person present who already knows the identity of the lucky prisoner is the guard. He is not allowed to inform the prisoners about their fate. Therefore one of the prisoners (A) asks him, "Please name at least one of the others who will be executed." The guard thinks for a while and says, "Prisoner C will be executed." Does this change the chances of Prisoner A being set free?

Probability of Prisoner A being set free: 1/3

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