Education Systems, Growth and Welfare

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Abstract

This paper focuses on the implications of Decentralized Education (DE) and Centralized Public Education (CPE) for growth and welfare in an overlapping generations model. Individuals choose learning time, consumption and human capital. Under DE, there is no government intervention, while in CPE, human capital is augmented by public education expenditures financed by a distortionary income tax, where the latter is chosen by a benevolent fiscal authority. CPE is welfare superior to DE for moderate/strong preferences over human capital bequests and medium/high elasticities of human capital with respect to average public education spending, average and parental human capital. So, even if we abstract from equity considerations, education policy may be supported on welfare grounds.

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1 Introduction

In Europe, North America and elsewhere, there is currently a strong debate on alternative ways of financing activities that have public good features and involve positive externalities, since competitive equilibria are typically inefficient in such cases. Education is one of the most important activities with public good characteristics, since human capital accumulation plays a crucial role in the process of economic growth and income distribution in modern economies (Nehru, Swanson & Dubey, 1995). As a result, we have seen significant government involvement in the funding and provision of education worldwide. Usually, primary and secondary education are mandatory and provided by the government and higher education is heavily subsidized (e.g. by deducting educational spending from taxable income).

Education can also be viewed as a mechanism of intergenerational transfers, since it usually takes place at the beginning of the life cycle and is financed by resources provided by the old generation. These transfers are altruistically motivated and affect economic growth, income distribution and welfare through their impact on human capital accumulation (Barro, 2001). Altruistic decisions lead generally to inefficient outcomes and parental decisions about offspring education, which ignore the impact of individual human capital accumulation on economy-wide human capital, are a classic example of such decisions. This view of education agrees with Lucas’ (1988) view that human capital is a “social activity, involving groups of people in a way that has no counterpart with the accumulation of physical capital”.

This work complements three strands of literature on: a) human capital as an engine of growth (Lucas 1988, Azariadis & Drazen, 1990, Temple 2001, Doppelhofer, G., Miller R.I & X. Sala-i-Martin, 2004); b) alternative ways of financing

\footnote{Roughly 5\% of GDP is transferred to the young generation through public education in the OECD countries.}

In this paper, we compare the general equilibrium implications of three education systems, Decentralized Education (DE), Centralized Education (CE) and Centralized Public Education (CPE) using an overlapping generations model based on Glomm-Ravikumar (1992). All systems are characterized by average human capital externalities for each individual, i.e the return to individual education increases with average human capital. In DE, each agent maximizes welfare by treating average human capital as a public good, while CE differs only in that a social planner internalizes average human capital externalities. Under CPE, there is also education policy, i.e individual human capital accumulation is enhanced by public education expenditures financed by a distortionary income tax. The latter is chosen by a benevolent fiscal authority, which maximizes the utility of the representative old agent accounting for human capital externalities. We focus on DE and CPE, which are second-best environments, since in the former market fails due to externalities, while the latter involves also policy failures due to distortionary income taxes.

In the analysis, we use a richer human capital accumulation specification than other studies (Glomm, G. & B. Ravikumar, 1992, Cardak, 1999, 2004b, Preston, 2003) including time devoted to education, average human capital, average public education spending (in CPE) and parental human capital. We also incorporate average human capital transfers, consumption and leisure in the utility function and assume that agents put different weight on the various components of utility.

\footnote{We should note here that there is no conclusive evidence on the choice of the production function for human capital (De La Croix, D & P. Michel, 2002).}
in contrast with most of the literature (Glomm, G. & B. Ravikumar, 2001, Zhang 1996). The presence of average human capital bequests, average human capital and average public education spending guarantees that our results are not influenced by scale effects, which characterize some of the existing work (Glomm, G. & B. Ravikumar, 1998, Blankenau, W.F & N.B Simpson, 2004) and are ambiguous empirically at best. Finally, we endogenize the tax rate, that is sometimes taken as exogenous (Kaganovich, M. & I. Zilcha, 1999, Cardak, 2005).

We characterize human capital decisions and optimal economic policy and compare mainly DE and CPE in terms of endogenous variables and welfare. We derive analytical results, which are mostly ambiguous depending on parameter values, therefore we conduct numerical simulations based on well-accepted parameter values in the empirical literature to obtain a ranking of the education systems.

The main finding is that CPE is welfare superior to DE for moderate/strong preferences over human capital bequests and medium/high elasticities of human capital with respect to average public education spending, average and parental human capital in a neoclassical growth economy. The inverse holds for the remaining parameter values.

The rest of the paper proceeds as follows. Section 2 analyzes the models without education policy, i.e DE and CE. Section 3 studies CPE. Section 4 examines the dynamics of DE and CPE. Section 5 compares the two systems. Section 6 concludes the paper. Details are contained in the Appendix.

2 Private education

2.1 Decentralized education

The analysis will be pursued in the context of an overlapping generations economy, comprised of $N$ two-period-lived agents. Each generation consists of identical
individuals, so it is characterized by a representative agent.\textsuperscript{3} In the second period of life, each individual has one child, so population growth is zero.\textsuperscript{4}

Agents derive utility from leisure when young and consumption and average human capital passed on to the offspring when old. Average human capital corresponds to the education quality transferred to the young generation in Glomm, G. & B. Ravikumar (1992), but is more general in the sense that it incorporates all factors that influence human capital evolution. In other words, parents are happier if their children live in a more knowledge-based economy, since that enhances their learning possibilities, therefore their income (see equations (3), (15) below). This formulation accords with the ever-increasing importance of knowledge since the beginning of the information revolution two decades ago.\textsuperscript{5} So, the first linkage between generations is average human capital bequests, which reflect education-inclined altruism of the parents toward children referred in the literature as ”joy of giving” (or ”warm glove”), because parents have a preference for giving to their descendants.\textsuperscript{6}

The second channel through which parents and children are connected is the stock of parental human capital, which affects children’s learning, since a young individual inherits partially the human capital of the parents, i.e there is intergenerational transmission of ability, knowledge, skills and way of thinking within the family that does not operate through formal schooling but at home (informal

\textsuperscript{3}The solution concept of symmetric equilibria has been widely used in both the public economics and game-theoretic literatures (Persson, T. and G. Tabellini, 1994).

\textsuperscript{4}For an examination of the impact of variable population growth on economic growth see e.g. De La Croix, D. & M. Doepke, 2004.

\textsuperscript{5}The assumption that parents gain utility from average human capital bequethed to their children, simplifies the analysis, but the results would not change in any important way if individual human capital was used instead.

\textsuperscript{6}Generally, reasons for bequests are altruism on behalf of the parents, provision of incentives such that their heirs behave according to what parents believe is appropriate and accidental death of retired individuals who are unable to buy actuarially fair annuities. In the absence of a bequest motive it would be difficult to explain why even very wealthy individuals maintain large asset balances at death (Azariadis, 1993).
Parental human capital might also affect children’s human capital through the quality of parental tutoring (Belzil, C. & J. Hansen, 2003, Restuccia, D. & C. Urrutia, 2004). So, the more educated parents are, the more help they are likely to give to their offspring and the more educated the latter are likely to be.\(^7\)

Furthermore, average economy-wide human capital generates a positive externality in the accumulation of private human capital, acting as an input to private human capital evolution, i.e. there is cross-individual spillover. This externality can be rationalized in several ways. For example, it has been argued that education contributes to a stable and democratic society, inculcates acceptable social values and behavioural norms, lowers crime, thus law enforcement costs and promotes social cohesion (Hanushek, 2002).\(^8\)

Also, one unit of time is available for each agent in every period. During the first period, time is divisible in leisure and schooling, while in the second period all time is supplied in the labour market. The benefit of an increase in schooling in the first period is higher human capital (income) in the second period and the cost is lower leisure in the first period, therefore there is a trade-off between income and leisure.

In this framework, under DE, each agent optimizes with respect to leisure when young \((n_t)\) and consumption when old \((c_{t+1})\)\(^9\), taking average human capital \((\overline{H}_{t+1})\) as given. So, he/she maximizes the utility function:

\[
U^{DE} = \alpha \ln (n_t) + \ln (c_{t+1}) + b \ln (\overline{H}_{t+1}) \tag{1}
\]

\(^7\)We could also include private education spending in human capital accumulation in the spirit of Cardak (2004a). However, this would not change the results qualitatively.

\(^8\)Furthermore, positive effects of educational attainment on civic engagement in the form of voter participation and support for free speech as well as quality of civic knowledge as measured by newspaper readership have been confirmed empirically (Dee, 2004).

\(^9\)The consumption of the children is included in the parents’ consumption.

\(^{10}\)The superscript DE stands for decentralized education.
subject to

\[ h_{t+1} = c_{t+1} \]  \hspace{1cm} (2)

\[ h_{t+1} = A (1 - n_t)^{\beta} H_t^{\gamma} h_t^{\delta} \]  \hspace{1cm} (3)

where \( n_t \in [0, 1] \) and \((1 - n_t)\) stands for time devoted to education in period \( t \). Also, \( h_{t+1} \) is individual human capital in \( t + 1 \), and \( H_t, h_t \) are predetermined and correspond to average human capital and parental human capital in \( t \) respectively.\(^{11}\)

The last element of the utility function reflects ad hoc altruism, i.e “joy of giving”. The welfare from leaving a bequest depends on the size of the bequest and \( b \), the degree of parental altruism towards children. Parameter \( \alpha \) reflects preference for leisure. These parameters are assumed constant over time, i.e they pass from parents to children, assuming that preferences are transmitted across generations.

Furthermore, given the assumption that agents supply one unit of labour inelastically in the second period, \( h_{t+1} \) stands for income and the wage rate. Equation (2) is the budget constraint of the representative family, showing that income is devoted entirely to consumption. Relation (3) is a Cobb-Douglas production function for human capital, where \( A > 0 \) is a technological parameter that stands for total factor productivity in the human capital formation technology and is affected by the organization of schools, attitudes to learning etc. Also, \( \beta, \gamma, \delta \in (0, 1) \) are the elasticities of learning with regard to time devoted to education, initial

\(^{11}\)Regarding human capital accumulation, empirical studies show that the quality of education, measured e.g by the student/teacher ratio, term length or relative pay of teachers, influences positively the rate of return of individuals to education, therefore their future income (Card, D. & A.B Krueger, 1992). Furthermore, time spent on human capital investment is expected to have a positive effect on school performance.
average human capital and parental human capital respectively, so that all factors exhibit diminishing returns. So, human capital accumulation depends positively on time devoted to schooling \((1 - n_t)\), average human capital in \(t\) and parental human capital.

The first-order conditions define the equilibrium as follows:

\[
\begin{align*}
n_t^{DE} &= \frac{\alpha}{\alpha + \beta} \\
\end{align*}
\]

\[
\begin{align*}
h_{t+1}^{DE} = c_{t+1}^{DE} &= A \left( \frac{\beta}{\alpha + \beta} \right)^\beta \bar{H}_t^{\delta} h_t^{\gamma+\delta} = A \left( \frac{\beta}{\alpha + \beta} \right)^\beta h_t^{\gamma+\delta} = B h_t^{\gamma+\delta} \\

H_{t+1}^{DE} &= Nh_{t+1}^{DE} = NA \left( \frac{\beta}{\alpha + \beta} \right)^\beta h_t^{\gamma+\delta} \\
\text{where } B &= A \left( \frac{\beta}{\alpha + \beta} \right)^\beta.
\end{align*}
\]

Equation (4) indicates that learning/leisure choices are constant over time and independent of the parental attributes \((h_t)\), because the income and substitution effects balance each other perfectly.\(^{13}\) The stronger the preference over leisure \((\alpha)\) and the smaller the elasticity of human capital accumulation with regard to time devoted to it \((\beta)\), the higher is optimal leisure and the lower learning time. Also, consumption, income and total human capital in period \(t + 1\) depend positively on learning efficiency \((A)\) and inherited human capital \((h_t)\). Finally, income, consumption and aggregate human capital depend negatively on the preference over leisure \((\alpha)\).

### 2.2 Social planner’s problem

In order to check if the outcome of the decentralized education system presented above is Pareto optimal, we study the social planner’s problem. The planner

\(^{12}\)The second-order conditions hold.

\(^{13}\)This is due to log-linear preferences.
internalizes the positive externalities of average human capital on welfare and private human capital accumulation. Thus, he/she optimizes with respect to $n_t$ and $c_{t+1}$ the utility function:

$$U^{CE} = \sum_{i=1}^{N} \left[ a \ln (n_t) + \ln (c_{t+1}) + b \ln (\bar{H}_{t+1}) \right] \quad (7)$$

subject to

$$\sum_{i=1}^{N} h_{t+1} = \sum_{i=1}^{N} c_{t+1} \quad (8)$$
$$\sum_{i=1}^{N} h_{t+1} = \sum_{i=1}^{N} \left[ A (1 - n_t)^{\beta} \bar{H}_t h_t^\delta \right] \quad (9)$$

The first-order conditions give the following:\textsuperscript{16}

$$n_t^{CE} = \frac{a}{\alpha + (1 + b) \beta} \quad (10)$$
$$h_{t+1}^{CE} = c_{t+1}^{CE} = A \left[ \frac{(1 + b) \beta}{\alpha + (1 + b) \beta} \right]^{\beta} \bar{H}_t h_t^\delta = A \left[ \frac{(1 + b) \beta}{\alpha + (1 + b) \beta} \right]^{\beta} h_t^{\gamma + \delta} \quad (11)$$
$$H_{t+1}^{CE} = Nh_{t+1}^{CE} = NA \left[ \frac{(1 + b) \beta}{\alpha + (1 + b) \beta} \right]^{\beta} h_t^{\gamma + \delta} \quad (12)$$

Equations (10)-(12) give similar results to relations (4)-(6). The basic difference comes from the fact that leisure and time devoted to education differ between CE and DE. Specifically, optimal learning time is a positive function of the preference parameter over average human capital bequests ($b$) under CE, which is

\textsuperscript{14}The superscript CE stands for centralized education.

\textsuperscript{16}The second-order conditions hold.
not the case under DE, since the social planner takes into account the human capital externalities on welfare and human capital accumulation contrary to DE. As a result, time devoted to learning, human capital and consumption are higher under CE compared to DE.

Consequently, the decentralized solution (DE) does not coincide with the social optimum (CE), therefore DE represents a second-best environment.

3 Centralized public education

We now assume that the human capital stock of individual agents is augmented by the government in the form of public education expenditures on e.g. libraries, schools, teachers’ and university professors’ salaries and training (Papke, 2004). These expenditures are financed by a distortionary tax on $t+1$ income. A possible interpretation of the simultaneous presence of private and public factors in the production of human capital (they are complements), is that the majority of public education spending finances primary and secondary education, while private expenditure finances mainly preschool/tertiary education and on the job-training.

Events take place in two stages. First, a centralized fiscal authority chooses the tax rate and the associated level of government expenditures. Second, agents choose consumption and leisure (therefore time devoted to education) taking economic policy as given. However, we solve the problem backward.

3.1 Household behaviour

The representative agent born in period $t$ chooses $n_t$, $c_{t+1}$ taking average human capital in $t$, $t+1$ and the income tax rate as given to maximize lifetime utility as follows:
\[ U^{CPE} = \alpha \ln(n_t) + \ln(c_{t+1}) + b \ln(\bar{H}_{t+1}) \] \hspace{1cm} (13)

subject to

\[ c_{t+1} = (1 - \tau)h_{t+1} \] \hspace{1cm} (14)

\[ h_{t+1} = A (1 - n_t)^\beta \bar{H}_t \bar{G}_{t+1}^\xi h_t^\delta \] \hspace{1cm} (15)

Equation (14) states that consumption equals net (after-tax) income. Relation (15) represents the human capital production function, which differs compared with the same function under DE only in that it incorporates average public education spending \((\bar{G}_{t+1})\).

Equations (14)-(15) imply that

\[ c_{t+1} = (1 - \tau)A (1 - n_t)^\beta \bar{H}_t \bar{G}_{t+1}^\xi h_t^\delta \] \hspace{1cm} (16)

The first-order conditions give:\(^18\)

\[ n_{t}^{CPE} = \frac{\alpha}{\alpha + \beta} \] \hspace{1cm} (17)

\[ c_{t+1}^{CPE} = (1 - \tau)A \left( \frac{\beta}{\alpha + \beta} \right)^\beta \bar{H}_t \bar{G}_{t+1}^\xi h_t^\delta \] \hspace{1cm} (18)

\[ h_{t+1}^{CPE} = A \left( \frac{\beta}{\alpha + \beta} \right)^\beta \bar{H}_t \bar{G}_{t+1}^\xi h_t^\delta \] \hspace{1cm} (19)

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\(^{17}\)The superscript CPE stands for centralized public education.

\(^{18}\)The second-order conditions are also satisfied.
The results are analogous to those obtained under DE. The differences between DE and CPE are as follows: a) Individual consumption is lower than human capital in CPE in contrast with DE due to the imposition of the income tax; b) human capital accumulation is augmented by government education expenditures in CPE, which are not present under DE.

### 3.2 Government budget constraint

The government runs a balanced budget. It uses revenues from a proportional income tax \( \tau \) and allocates them to spending on human capital \( G_{t+1} \).

Given that there are \( N \) private agents, the government budget constraint and average education spending are respectively:

\[
G_{t+1} = \tau H_{t+1} = \tau Nh_{t+1} \quad (20)
\]

\[
\bar{G}_{t+1} = \frac{\tau H_{t+1}}{N} = \frac{\tau Nh_{t+1}}{N} = \tau h_{t+1} \quad (21)
\]

Equations (20)-(21) imply that economic policy is summarized by \( \tau \).

### 3.3 Optimal economic policy

To endogenize economic policy, it is sufficient to determine the independent policy instrument \( \tau \). A centralized fiscal authority, that maximizes the utility of the representative household given in (13) with respect to \( \tau \) acts as a benevolent Stackelberg leader vis-a-vis the private sector by taking into account the private equilibrium.

Substituting (17)-(19), (21) into (13) and differentiating with respect to \( \tau \), the optimal tax rate is the following:\[19\]

\[
\tau = \frac{\zeta(1 + b)}{\zeta(1 + b) + 1 - \zeta} \quad (22)
\]

\[19\]The second-order condition of the problem holds.
The tax rate is constant over time due to log-linear preferences and Cobb-Douglas learning technology and depends positively on the strength of the preferences over average human capital bequests ($b$) and the elasticity of human capital with regard to average public education expenditures ($\zeta$). This is expected, since the higher the utility agents derive from providing bequests to their descendants and the more productive are public education expenditures in terms of human capital accumulation, the more money individuals will be willing to pay for the subsidization of human capital accumulation, which implies a higher optimal tax rate.

### 3.4 Competitive decentralized equilibrium

The Competitive Decentralized Equilibrium (CDE) is defined as the set of allocations $(n_t, c_{t+1}, h_{t+1}, \tau)$ such that: (i) households maximize utility given economic policy; (ii) markets clear; (iii) the government budget constraint is satisfied.

Taking into account our theoretical framework and using (17)-(19), (21)-(22) we get the following:

**Proposition 1** In a symmetric competitive decentralized equilibrium, optimal leisure, consumption, individual human capital, total human capital and average public education spending are respectively:

\begin{align*}
n_t^{CPE} &= \frac{\alpha}{\alpha + \beta} \\
\end{align*}

\begin{align*}
c_{t+1}^{CPE} &= \left( \frac{1 - \zeta}{\zeta(1 + b) + 1 - \zeta} \right) A^{\frac{1}{1 - \zeta}} \left( \frac{\beta}{\alpha + \beta} \right)^{\frac{\beta}{1 - \zeta}} \left( \frac{\zeta(1 + b)}{\zeta(1 + b) + 1 - \zeta} \right) h_{t+1}^{\frac{\gamma + \delta}{1 - \zeta}} h_t^{1 - \zeta} \\
h_{t+1}^{CPE} &= A^{\frac{1}{1 - \zeta}} \left( \frac{\beta}{\alpha + \beta} \right)^{\frac{\beta}{1 - \zeta}} \left( \frac{\zeta(1 + b)}{\zeta(1 + b) + 1 - \zeta} \right) h_{t+1}^{\frac{\gamma + \delta}{1 - \zeta}} h_t^{1 - \zeta} = Ch_t^{\frac{\gamma + \delta}{1 - \zeta}} \\
\end{align*}
\[ H_{t+1}^{CPE} = NH_{t+1}^{CPE} = NA^{\frac{1}{\alpha + \beta}} \left( \frac{\beta}{\alpha + \beta} \right)^{\frac{\beta}{1 + \zeta}} \left( \frac{\zeta(1 + b)}{\zeta(1 + b) + 1 - \zeta} \right)^{\frac{1}{1 + \zeta}} h_t^{\frac{2 + b}{1 - \zeta}} \] (26)

\[ \overline{G}_{t+1} = A^{\frac{1}{1 + \zeta}} \left( \frac{\zeta(1 + b)}{\zeta(1 + b) + 1 - \zeta} \right)^{\frac{1}{1 + \zeta}} \left( \frac{\beta}{a + \beta} \right)^{\frac{\beta}{1 + \zeta}} h_t^{\frac{2 + b}{1 - \zeta}} \] (27)

where \( C = A^{\frac{1}{1 + \zeta}} \left( \frac{\beta}{\alpha + \beta} \right)^{\frac{\beta}{1 + \zeta}} \left( \frac{\zeta(1 + b)}{\zeta(1 + b) + 1 - \zeta} \right)^{\frac{1}{1 + \zeta}}. \)

4 Dynamics

In this section we study the dynamic path of human capital (income) under DE and CPE.

Regarding individual human capital, in the context of DE, we conclude by equation (5):

i) If \( (\gamma + \delta) \in (0, 1) \), the economy converges monotonically to a unique steady-state level of human capital \( h_{sDE} = B^{\frac{1}{1 - \gamma}} \).

ii) If \( \gamma + \delta = 1 \), the human capital of every family exhibits long-run or endogenous growth/decay at rate \( B \). One can distinguish three subcases:

a) \( B = 1 \). Then \( h_{t+1} = h_t \), therefore the representative agent stays at his/her initial human capital.

b) \( B < 1 \). There is monotonic convergence to a steady-state human capital equal to zero.

c) \( B > 1 \). All families experience long-run growth at rate \( B \).

iii) If \( \gamma + \delta > 1 \), the representative family converges to one of two steady-state human capital levels equal to 0 and \( B^{\frac{1}{1 - \gamma}} \), depending on initial conditions, i.e. the equilibria are unstable. Specifically, if \( h_0 < B^{\frac{1}{1 - \gamma}} \), then \( h_t \to 0 \), when \( h_0 > B^{\frac{1}{1 - \gamma}} \), \( h_t \to \infty \) and if \( h_0 = B^{\frac{1}{1 - \gamma}} \) human capital rests at its initial condition.

The cases of primary economic interest are i) and iic) and the conclusions may be summarized as follows:
Proposition 2 a) If \((\gamma + \delta) \in (0, 1)\), the representative family’s human capital converges monotonically to the steady-state level \(B^{1/(1-\gamma-\delta)}\); b) If \(\gamma + \delta = 1\) and \(B > 1\) the agents’ income exhibits long-run growth.

Concerning the dynamics of total human capital, they are qualitatively the same with those of individual capital, the only difference being that total human capital is by definition higher than individual human capital.

By equations (5), (25) we conclude that the dynamics of DE and CPE are qualitatively the same. The differences are: i) dynamics depend on \((\gamma + \delta + \zeta)\), \(C\) under CPE instead of \((\gamma + \delta)\), \(B\) in DE; ii) steady-state individual human capital is \(h^{CPE}_s = C \frac{1-\delta-\zeta}{1-\gamma} \) under the former and \(h^{DE}_s = B \frac{1-\delta-\zeta}{1-\gamma} \) in the latter; iii) growth in CPE is affected also by \(b\), \(\zeta\), compared to DE, since these parameters determine the tax rate, therefore average education spending, which affects human capital accumulation under CPE.

As a conclusion, the dynamics of CPE and DE allow for a variety of growth paths, including the neoclassical and endogenous growth as subcases.

5 Comparison of education systems

5.1 Analytical results

In what follows, we compare DE and CPE. The purpose is to examine the trade-off between market failures associated with DE, due to non-internalization of the average human capital externalities, and government failures under CPE arising from the government attempt to correct the resulting inefficiency by using distorting policy instruments, i.e income taxes. So, we compare the two systems in terms of leisure/time devoted to education, individual and total human capital, consumption and welfare. The results, which can be obtained analytically, are summarized as follows:

Proposition 3 a) Leisure and time devoted to education are identical in both DE
and CPE; b) human capital, consumption and welfare are higher under DE than CPE if and only if the relations (28), (29), (30) hold respectively:

\[
D = A \frac{Q}{R} \left( \frac{h^{DE}}{h^{CPE}} \right) - 1 > 0 \quad (28)
\]

\[
E = A \frac{Q}{1 - \zeta} \left( 1 - \zeta \right)^{-1} P^{-\frac{1}{1+\beta}} h^{DE} - \frac{1}{R^{1+\beta}} - 1 > 0 \quad (29)
\]

\[
F = A \frac{Q}{1 - \zeta} \left( 1 - \zeta \right)^{-1} P^{-\frac{1}{1+\beta}} h^{DE} - \frac{1}{R^{1+\beta}} - 1 > 0 \quad (30)
\]

where \( P = \frac{a}{a+\beta}, \ Q = \zeta (1 + b), \ R = \zeta (1 + b) + 1 - \zeta, \ S = (1 + b) (\gamma + \delta) \).

It follows that we can obtain general results only for leisure and time devoted to education, which are identical due to the fact that the representative old agent maximizes the same utility function under the two regimes, while policy does not affect the education/leisure choice. For human capital, consumption and welfare we derived sufficient conditions for the ranking of DE and CPE. So, we can not generally tell if the cost of tax distortions outweighs the benefit from the subsidization of human capital accumulation regarding human capital and consumption under CPE relative to DE. Therefore, since human capital and consumption are arguments of the utility function, the welfare comparison of the two systems is impossible analytically. However, a welfare analysis is useful even in the context of a representative agent model as ours, since it summarizes the effects of changes in the model’s parameters on utility, which the social planner aims at maximizing through policy. In light of that, we resort to numerical analysis.

5.2 Numerical analysis

5.2.1 Steady-state analysis

We compare DE and CPE in the neoclassical growth case, where there is a unique steady-state (case (i) in section 4). Since both education systems involve second-best environments and there is no a priori guidance from our theoretical model
as to which of them performs better in terms of human capital, consumption and welfare, we evaluate the expressions D, E, F for a range of values of the model’s parameters to verify if conditions (28)-(30) hold.

The baseline values for $\alpha, \beta, \gamma, \delta, \zeta$ were chosen based among others on the work of Psacharopoulos (1985), Magoula, T. & G. Psacharopoulos (1997), Acemoglu, D. & J. Angrist (1999), Cardak (2004b), Glomm, G. & B. Ravikumar (2001), Soares (2003). The baseline values for $\alpha, \beta$ (strength of preferences over leisure and elasticity of human capital accumulation with regard to time devoted to education respectively) are set such that the optimal learning time is in line with estimates of time devoted to schooling work. Also, $\alpha, b$ (the latter corresponds to preferences for human capital bequests) are selected so that the agents put weights on consumption, leisure and human capital bequests equal to 60%, 25% and 15% respectively (the weights are normalized so that the coefficient of consumption equals 1). The benchmark values for the elasticities of human capital accumulation with regard to average human capital, average public education spending and parental human capital ($\gamma, \zeta, \delta$ respectively) correspond to the average estimates provided in the literature except for the latter for which the range of values used is so wide that we assign a relatively low value to satisfy the condition for neoclassical growth. Furthermore, $A$ (productivity of human capital accumulation with respect to all inputs) is such that annual growth rate is positive. Initial (period 0) human capital ($h_0$) was set arbitrarily. As a result, the baseline parameter values are $\alpha = 0.42$, $b = 0.25$, $\beta = 0.2$, $\gamma = 0.15$, $\delta = 0.45$, $\zeta = 0.1$, $A = 3$, $h_0 = 5$.

For these values, the optimal learning time is 32.3% of the total time endowment and the optimal tax rate equals 12.2%. Also, we get $D < 0$, $E > 0$, $F > 0$, therefore $h_s^{DE} < h_s^{CPE}$, $H_s^{DE} < H_s^{CPE}$, $c_s^{DE} > c_s^{CPE}$, and $U_s^{DE} > U_s^{CPE}$.

So, CPE is superior to DE in terms of human capital (income), but DE out-
performs CPE with respect to consumption and welfare in the steady-state. Consequently, the positive welfare effects of human capital subsidization by the government are outweighed by the distortionary tax effects on consumption implying a net welfare loss due to education policy.

Following the computations for the baseline parameter values, we perform a sensitivity analysis with regard to the basic parameters to check how robust are the benchmark results for plausible parameter configurations. The ranges of values used are such that $\alpha \in [0.08, 0.58]$, $b \in [0.08, 0.58]$, $\beta \in [0.1, 0.3]$, $\gamma \in [0.05, 0.25]$, $\delta \in [0.35, 0.55]$, $\zeta \in [0.01, 0.2]$ and are broad enough to include most empirically plausible parameter values.

As a result, the optimal time devoted to education ranges from 14.7% to 78.9% depending on $\alpha$, $\beta$ and the second-best tax rate can be as low as 1.1% and as high as 28.3% according to the values of $b$, $\zeta$. Subsequently, we evaluate $D$, $E$, $F$ and the most important results are presented in figure 1.

Regarding the preference parameter over human capital bequests ($b$), CPE is superior to DE in terms of human capital except for low values of $b$. Also, CPE is welfare superior to DE for high values of $b$. Furthermore, the differential in favour of CPE with regard to both variables increases with $b$. This is expected, because the higher the weight agents give to human capital bequests, the more heavily they will be willing to invest in human capital accumulation through public education spending. Therefore human capital and consumption increase, implying higher welfare under CPE relative to DE ($b$ does not affect human capital and welfare in DE). These effects outweigh the negative impact of higher $b$, through the higher optimal tax rate, on consumption and welfare.

With respect to the sensitivity of human capital with regard to average public education spending ($\zeta$), CPE outperforms DE for moderate and high values of $\zeta$ regarding human capital and welfare. The more effective education expendi-
ture is in stimulating human capital accumulation, i.e. the higher $\zeta$ is, the more favorable the comparison is for CPE (except for very low values of $\zeta$). This is plausible, because a higher elasticity of knowledge creation with respect to government spending implies a faster accumulation of human capital for a given amount of public resources, therefore a higher welfare, since it more than compensates for the negative impact of the higher tax rate on consumption due to the higher $\zeta$ under CPE ($\zeta$ does not play a role under DE).

As for the elasticity of human capital accumulation with respect to learning time ($\beta$), CPE is superior to DE in terms of human capital with the exception of very high values of $\beta$, while DE is always welfare superior to CPE. The differential between the two systems increases with $\beta$ in favour of DE. This is expected, since a higher $\beta$ implies a higher reduction in the marginal contribution of learning time to human capital accumulation under CPE relative to DE. This means that human capital declines faster in CPE than DE and the same holds for welfare both directly and indirectly through lower consumption, since the latter is a fraction of human capital.

The human capital and welfare differential between DE and CPE exhibit similar qualitative behaviour with $\zeta$ when the elasticities with respect to initial average human capital ($\gamma$) and parental human capital ($\delta$) vary except that the differentials decrease monotonically. The rationale for this behaviour is the same as that mentioned in the analysis referring to $\zeta$. It should be noted that as the level of education and development of a country increases, $\gamma$ declines according to empirical studies. Therefore, CPE seems more appropriate for primary/secondary education and poorer countries.

The difference of the two education systems in terms of consumption behaves in the same way as the welfare differential. This is expected, since consumption is the dominant determinant of welfare.
The steady-state average public education spending \((\bar{G}_s = \tau h_s^{CPE})\) depends positively on \(b, \zeta\), since these increase both the optimal tax rate and steady-state human capital \((h_s^{CPE})\) except for low values of \(\zeta\) (in the latter case, the positive effect of \(\zeta\) on \(\tau\) outweighs the negative effect on \(h_s^{CPE}\), so \(\bar{G}_s\) rises). Also, \(\bar{G}_s\) increases with \(\gamma, \delta\), due to the positive impact of these parameters on \(h_s^{CPE}\). Finally, a higher \(\beta\) decreases \(\bar{G}_s\), since it reduces \(h_s^{CPE}\).

5.2.2 Transitional dynamics

To explore the relation between DE and CPE further, we will study the transitional dynamics of human capital and welfare under the two regimes. In this framework, we conduct simulations using the benchmark parameter values, i.e \(\alpha = 0.42, b = 0.25, \beta = 0.2, \gamma = 0.15, \delta = 0.45, \zeta = 0.1, A = 3, h_0 = 5\), assuming each time period of individuals’ life lasts 35 years.

We observe that human capital (income) reaches a steady-state \(h_s^{DE} = 8.85\) in 13 periods in DE and \(h_s^{CPE} = 9.08\) in 17 periods under CPE. The faster rate of convergence in DE is due to a higher average growth rate (0.13% yearly) in relation to CPE (0.1%) (see figure 2 for the human capital dynamics in the two systems). As a result, per capita income converges faster at a lower steady-state under DE compared with CPE. So, education policy increases steady-state income at the expense of a slower rate of convergence. If we look more carefully at the transition path, we realize that starting from the same initial income, DE is superior to CPE up to period 5, in period 6 income is identical in the two regimes and CPE outperforms DE afterwards. Moreover, the difference in favour of DE rises in the periods 1-2 and then gradually falls until CPE outstrips DE income.

However, figure 3 shows that DE converges to a higher steady-state welfare than CPE confirming the findings of the steady-state analysis. The time required for convergence is 9 periods under DE and 11 periods in CPE. The faster con-
vergence in terms of welfare of DE relative to CPE is expected, since per capita income converges faster under DE than CPE, consumption is a fraction of income and both determine welfare.\textsuperscript{20} Also, consumption dynamics seem to increase the welfare convergence rate in both education systems, since steady-state welfare is attained faster than what human capital dynamics would imply. Finally, DE is welfare superior to CPE in all time periods.

Summarizing the above findings, education policy implies a higher steady-state income but lower welfare compared to no government intervention. However, one should always keep in mind that these results are sensitive to the parameter values employed, as the steady-state analysis presented above clearly showed.

6 Conclusions

This paper has focused on the implications of decentralized education and centralized public education systems for growth and welfare. We studied education, because human capital accumulation is a fundamental source of long-run growth in modern economies and government intervention in the education sector is widespread. The ultimate objective was to welfare rank DE and CPE, so that a formal judgement can be made regarding their desirability as alternative ways of organizing education.

Our study shows that time spent on learning and leisure are identical in the two regimes.

However, none of the institutional settings dominates the rest for all parameter values in terms of human capital, consumption, welfare and growth. In light of this, we resorted to numerical methods. As a result, in the steady-state CPE fares better than DE in terms of welfare for moderate/strong preferences over human capital bequests and medium/high elasticities of human capital with respect to

\textsuperscript{20}Leisure, the third component of the utility function, is identical in the two regimes.
average public education expenditures, average and parental human capital in a neoclassical growth economy. The inverse holds for the remaining parameter values.

So, even when we abstract from equity considerations as we do in this work, education policy may be supported on welfare grounds. These results, if combined with the strong capacity of public education to reduce income inequality when asymmetric equilibria ex-post are studied (e.g. Cardak, 1999) justify a prominent role for public education.

We close with possible extensions. First, we could analyze an economy where individuals in each generation are heterogeneous, allowing us to study the impact of more elaborate education policies, e.g. means-tested vouchers, on income distribution. Furthermore, we might study the case of progressive income taxation and uncertainty with respect to characteristics of human capital accumulation, e.g. length of life, innate abilities etc. We leave these extensions for the future.

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References


Figure 1: Sensitivity of $h_s^{DE} - h_s^{CPE}$, $U_s^{DE} - U_s^{CPE}$ wrt $b, \zeta, \beta$
Figure 2: Dynamics of human capital in DE, CPE

![Graph showing dynamics of human capital](image)

Figure 3: Dynamics of welfare under DE, CPE

![Graph showing dynamics of welfare](image)