Aggregate consequences of limited contract enforceability*

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Abstract

We study a general equilibrium model in which entrepreneurs finance investment with optimal financial contracts. Because of enforceability problems, contracts are constrained efficient. We show that limited enforceability amplifies the impact of technological innovations on aggregate output. More in general, lower is the degree of contract enforceability and larger is the macroeconomic instability. A key assumption to generate this result is that defaulting entrepreneurs are not excluded from the market.

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1 Introduction

The ability to attract external financing is crucial for the creation of new firms and the expansion of existing ones. For that reason the nature of the financial arrangements between lenders and firms has important consequences for the growth of firms. One important issue in financial contracting is enforceability, that is, the ability of each side to repudiate the contract. This can be an important issue in the financing of firms because projects often involve specific entrepreneurial expertise and might be worth less to investors without the services of managers who initiated them. At the same time the development of such projects may provide managers with experience that is extremely valuable for starting new projects. Limited enforceability conditions the kinds of contracts we are likely to observe and affects the resources that are available for the firm to grow.

Contractual arrangements that are motivated by limited enforceability are most likely to be important for firms that are small and/or young. Albuquerque and Hopenhayn [1] have shown that these considerations can help to explain some of the growth characteristics of small and young firms. We know for example that smaller and younger firms are less likely to distribute dividends and that, conditional on the initial size, they tend to grow faster and experience greater variability of growth. Furthermore, the investment of smaller and younger firms is positively correlated with cash flows and there is at least indirect evidence that they are more likely to be financially constrained.

Even if financial constraints are important at the firm level, it is not obvious that they will have important aggregate consequences. One issue is whether the allocation of resources that results from these contracts reduces welfare significantly compared to a world where contracts are fully enforceable. A related question is whether these constraints cause the economy to be more sensitive to the arrival of new technologies that create better investment opportunities and, in general, whether the economy is more sensitive to aggregate shocks. In this paper we show that the financial constraints that arise because of limited enforceability not only explain the growth characteristics of firms but they are also important for the macro allocation of resources and the propagation of new technologies.

We study a general equilibrium model where entrepreneurs and investors enter into a long-term contractual relationship which is optimal, subject to enforceability constraints.[1] Consequently, financial constraints arise endogenously in the model as a feature of the optimal contract. Our model is closely related to the partial equilibrium model of Marcet and Marimon [18] with two important differences. First, we conduct the analysis in a general equilibrium framework. Second, we do not assume that repudiation leads to market exclusion. Once the contract has been signed and the project has been initiated, the entrepreneur has the ability to start a new investment project by entering into a new contractual relationship. This particular assumption implies that the value of repudiating a contract depends on the whole general equilibrium conditions. In this respect our model differs from most of the macro models with limited enforceability where in case of default the agent reverts to autarky.

Within this framework we show that limited enforceability creates a large amplification mechanism for the macroeconomic impact of new technological innovations. More specifically, our theory predicts that economies in which contracts are less enforceable display greater volatility of output than economies with greater enforceability of contracts. This result provides an explanation for the higher volatility of output observed in developing countries: If we think that the enforcement of contracts in these countries is weaker than in industrialized countries, then the former should display more extreme sensitivity to shocks. This appears to be true in the data as will be shown in the next section.

The amplification result can be explained as follows. In each period there are two types of firms: those that are resource constrained (the enforcement constraint is binding) and those that are unconstrained (the enforcement constraint is not binding). The arrival of a more productive technology increases the value of a new investment project. Because defaulting entrepreneurs are not excluded from the market, the higher value of a new project increases the value of repudiating the existing contract: the entrepreneur can repudiate the existing contract and start the new and more valuable project by entering in a new financial relationship. To prevent the default of constrained firms, the value of the contract for the entrepreneur must increase. By increasing this value, the tightness of the incentive-compatibility constraint is relaxed and more capital is given to the firm (investment boom). Notice that this mech-

[1] This work is closely related to the existing literature on optimal lending contracts with the possibility of debt repudiation. Examples are Alvarez and Jermann [2], Atkeson [3], Kehoe and Levine [13] and Marcet and Marimon [18]. Other papers related to our work are Albuquerque and Hopenhayn [1], Monge [21] and Quintin [24].

anism depends crucially on the assumption that defaulting entrepreneurs are not excluded from the market. With market exclusion the investment boom of constrained firms would not arise.

To explain why a lower degree of enforceability increases the amplification effect let's observe that the mechanism described above is only relevant for constrained firms. Therefore, larger is the fraction of these firms and larger is the aggregate impact of a shock. An important property of the model is that the fraction of constrained firms decline with the degree of contract enforceability. Therefore, when contracts are highly enforceable there are fewer firms that are sensitive to the shock and the economy as a whole will be less responsive to shocks.

There are several contributions that study the importance of financial market frictions for the macro performance of the economy. Examples are Bernanke, Gertler and Gilchrist [4], Carlstrom and Fuerst [6], Cordoba and Ripoll [8], DenHaan, Ramey and Watson [10], Kiyotaki and Moore [14], Smith and Wang [25]. The general conclusion we can draw from these contributions is that financial market frictions do not have quantitatively "large" amplification effects, which is in contrast to our results.

The organization of the paper is as follows. Section 2 shows some evidence about the relationship between contract enforceability and macroeconomic instability. Section 3 describes the model economy and Section 4 characterizes the optimal contract. Section 5 studies the initial conditions of the contract and defines the general equilibrium. The quantitative properties are studied in Section 6.

2 Enforcement and macroeconomic instability

Although there is a large body of literature studying the impact of institutional arrangements on economic growth,[2] little attention has been devoted to studying their impact on growth volatility or macroeconomic instability. Here we present some evidence about the connection between macroeconomic instability and one particular institution, that is, the enforcement of contracts.

Figure 1 relates the standard deviation of per-capita GDP growth to an index of contract enforceability for a cross-section of countries. The index of

[2] See, for example, Demirgüç-Kunt and Levine [9], Knack and Keefer [15], La-Porta, Lopez-de-Silanes, Shleifer and Vishny [17] and Mauro [20].

contract enforceability is compiled by Business Environmental Risk Intelligence and measures "*the relative degree to which contractual agreements are honored and complications presented by language and mentality differences*". (See Knack and Keefer [15]). It takes a value between 0 and 4, with higher scores for superior quality. The data is the average over the period 1980-95 as reported in Demirgüç-Kunt and Levine [9] and it is available for 44 countries. The standard deviation of growth is computed using data from the World Bank Statistical Indicators for the period 1980-2001.

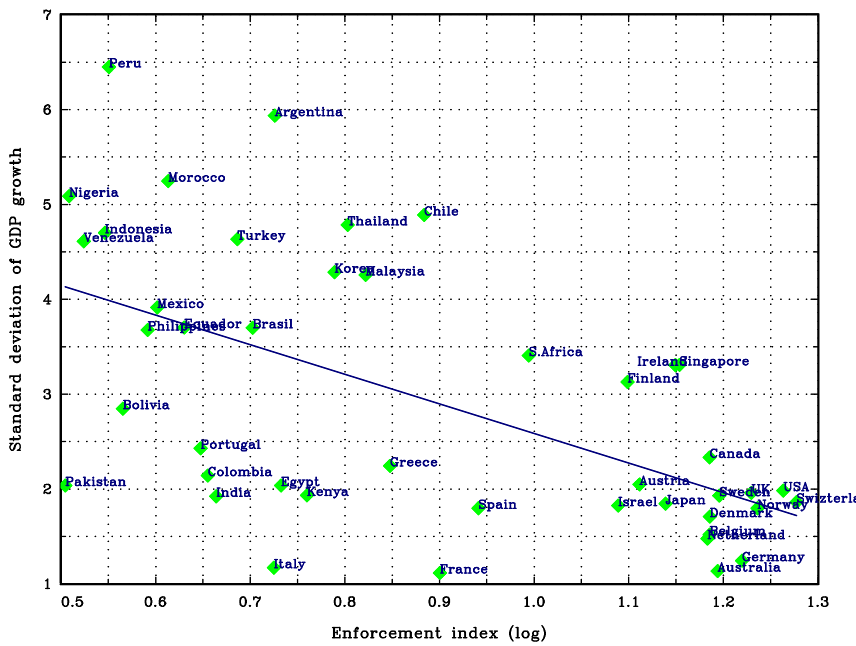

Figure 1: Correlation of contract enforcement and aggregate volatility.

The graph shows a strong negative association between the aggregate volatility and the enforcement of contracts. The correlation is -0.57. This negative association remains significant even if we take into account the development stage of a country. We regressed our index of aggregate volatility (standard deviation of GDP growth) on the log of per-capita GDP and on the log of the enforcement index. The regression results, with standard errors between parenthesis, are reported below:

$$\mathrm{StdGrow} = \underset{(1.91)}{5.84} - \underset{(0.31)}{0.02} \cdot \mathrm{CapGDP} - \underset{(1.12)}{2.73} \cdot \mathrm{Enforce}, \qquad \mathrm{R}^2 = 0.33$$

Therefore, the coefficient of the enforcement index remains negative and statistical significant even if we take into account the development stage of a country.

3 The model

Preferences and skills: The economy is populated by a continuum of agents of total mass 1. In each period a mass $1-\alpha$ of them is replaced by newborn agents and α is the survival probability. A fraction e of the newborn agents have the entrepreneurial skills to manage a firm and become entrepreneurs. The remaining fraction, $1-e$, become workers. Agents maximize:

$$E_0 \sum_{t=0}^{\infty} \left(\frac{\alpha}{1+r}\right)^t \Big(c_t - \varphi(l_t)\Big) \tag{1}$$

where r is the intertemporal discount rate, c_t is consumption, l_t are working hours, $\varphi(l_t)$ is the disutility from working. Utility flows are discounted by $\alpha/(1+r)$ as agents survive to the next period only with probability α. Given the assumption of risk-aversion, r will be the risk-free interest rate earned on assets deposited in a financial intermediary.[3] These assets are denoted by a. The disutility from working satisfies $\varphi(0)=0$, $\varphi'(l)>0$, $\varphi''(l)>0$. Denoting by w_t the wage rate, the supply of labor is determined by the condition $\varphi'(l_t)=w_t$. For entrepreneurs, $l_t=0$ and their utility depends only on consumption.

An agent with entrepreneurial skills has the ability to implement one of the projects available in the period as described below. Entrepreneurial skills fully depreciate if the agent remains inactive. This implies that, as long as the value of a new project is positive, newborn agents with entrepreneurial skills will always undertake a project when young. At the same time, by undertaking a project, an entrepreneur maintains the ability to start new projects in future periods. This assumption simplifies the analysis because it eliminates the possibility that agents with entrepreneurial skills remain inactive and wait for better investment opportunities.

Technology and shocks: There are two types of investment projects identified by the productivity parameter $z \in \{z_L, z_H\}$. There is a large number

[3]On each unit of assets deposited in a financial intermediary, agents receive $(1+r)/\alpha$ if they survive to the next period and zero otherwise.

of projects with low productivity but only a limited number N with high productivity. Given e the number of searching entrepreneurs, the probability of finding the high productive project is $p = \min\{N/e, 1\}$. The arrival of a new technology creates better investment opportunities by increasing the number of high productivity projects, that is, N. We assume that this variable follows some stationary stochastic process that will be specified later in the paper. In this economy expansions are driven by the arrival of more productive projects rather than the improvement of existing ones. In this sense, the economy has the typical features of a model with vintage capital.[4]

An investment project generates revenues according to:

$$F\Big(z; \min\{k\,,\,\xi \cdot l\}\Big) \tag{2}$$

where k is the input of capital, l is the input of labor and $z \in \{z_L, z_H\}$ is the project-specific level of technology. The value of z remains constant for a particular project but differs across projects. The function F is strictly increasing, strictly concave in its second argument and satisfies $F(z; 0) = 0$. Given the Leontif structure of the production function, the capital-labor ratio is equal to the parameter ξ. Therefore, we can rewrite the production function as $F(z; k)$ with the input of labor equal to k/ξ.

The input of capital is chosen one period in advance and it is project-specific. This implies that once used in a project it cannot be reallocated to a different project. Consequently, if the project is liquidated, the liquidation value is zero. If the project remains active, instead, the internal value of capital is $(1-\delta)k$, where δ is the depreciation rate. As long as the difference between z_L and z_H is not too large, this assumption implies that it is never efficient to replace an active project. In the rest of the paper we assume that the difference between z_L and z_H is sufficiently small so that active projects are never replaced.

The last assumption about the revenue technology is that with probability $1-\phi$ the project becomes unproductive. In this case the entrepreneur looses the entrepreneurial skills and becomes a worker.[5]

[4]It may seem counterintuitive to interpret the productivity of new projects as resulting from new technologies when this productivity may actually fall. However, we should interpret the model as a parsimonious representation of a more complex environment in which there is persistent growth and all values of z enhance the productivity frontier. The full specification of this environment would complicate the analysis without changing the basic dynamics studied in this paper.

[5]There are two sources of exogenous liquidation of the firm. The entrepreneur may die

Before continuing, let's define the function $R(z;k,w) = (1-\delta)k + F(z;k) - (k/\xi)w$ to denote the end-of-period resources, if the firm is not liquidated. These are the non-depreciated capital plus the gross production, net of the labor cost. The use of this function will simplify the notational complexity of the analysis that follows.

Financial contract and repudiation: An entrepreneur who starts a new project finances the input of capital by signing a long-term contract with a financial intermediary. The contract is not fully enforceable as the contractual parties can repudiate the contract at any moment. For the intermediary, the repudiation value is zero. The assumption is that in case of repudiation the intermediary will lose the whole value of the contract. Therefore, as long as this value is positive, the intermediary will not repudiate the contract.

For the entrepreneur, the derivation of the repudiation value is more complex. We assume that, if a contract is repudiated, the entrepreneur is able to appropriate (and consume) the revenue generated by the firm. In addition, the entrepreneur can also start a new project by entering into a new contractual relationship. Repudiation, however, also carries with it a cost κ for the entrepreneur. In the absence of such a cost, a financial contract may not exist. This cost can be interpreted as legal punishments that reduce the utility of the entrepreneur. Alternatively, it can be interpreted as a credit that the intermediary carries over to the next contract.

Denote by $\overline{V}(\mathbf{s})$ the value of searching for a new project for the entrepreneur, net of the repudiation cost, where $\mathbf{s}$ denotes the aggregate states of the economy. This function is endogenous in the model and will be derived in section 5. The value of repudiating an active contract can be written as:

$$D(z;k,\mathbf{s}) \;=\; F(z;k) \,+\, \overline{V}(\mathbf{s}) \tag{3}$$

The repudiation value has two components: the value of consuming the current cash-flows, $F(z;k)$, and the external value of searching for a new project net of the repudiation cost, $\overline{V}(\mathbf{s})$. As we will see later in the paper,

with probability $1-\alpha$ or the project becomes unproductive with probability $1-\phi$. The demographic assumption of an exogenous death is introduced for analytical convenience. With this assumption new entrepreneurs are only newborn agents who have no wealth and we do not have to keep track of the distribution of assets among potential entrants. The exogenous probability $1-\phi$ is introduced to generate enough turnover in the distribution of firms. This can also be obtained by reducing the survival probability α but it would require an extremely high death probability of agents.

both components of the repudiation value are important. Without the first component all firms would operate at the optimal scale and the possibility of repudiation plays no role in the transmission of shocks. Without the second component we would still have that some firms are financially constrained, that is, they operate at a suboptimal scale. However, the investment of these firms is not very sensitive to shocks and the model would not generate the amplification result described in the introduction.

4 The optimal financial contract

To simplify the analysis, we first characterize the optimal contract by assuming that the intermediary commits to fulfill any obligations (one-side commitment). After characterizing the optimal contract with one-side commitment, we will show that the value for the intermediary is always non-negative in all parameterizations used in the paper (and therefore, there is no repudiation from the intermediary).

To characterize the optimal contract we use the recursive approach of Marcet and Marimon [19]. This approach studies the optimal contract as the solution to a planner's problem who attributes certain weights to the contractual parties. For the moment we assume that the weights are given. Later we use the assumption of competition in financial markets to determine these weights. The planner takes as given the equilibrium prices and the problem is subject to incentive-compatibility and resource constraints.

Define λ_t the weight assigned to the entrepreneur and, without loss of generality, normalize to 1 the weight assigned to the intermediary. Under the assumption of one-side commitment, the planner's problem is:

$$\max_{\{d_s,\tau_s,k_s\}_{t=0}^{\infty}} \qquad E_t \sum_{s=t}^{\infty} \beta^{s-t}(\lambda_t d_s + \tau_s) \tag{4}$$

subject to

$$\tau_s = -d_s - k_s + \beta E_s R(z;k_s,w(\mathbf{s}_{s+1})) \tag{5}$$

$$E_{s+1} \sum_{j=s+1}^{\infty} \beta^{j-s-1} d_j \geq D(z;k_s,\mathbf{s}_{s+1}) \tag{6}$$

$$d_s \geq 0 \tag{7}$$

The objective (4) defines the surplus of the contract for the planner as the expected discounted value of weighted utilities for the entrepreneur and the intermediary. For the entrepreneur the period utility is the payment d_s. For the intermediary the period utility from the contract is defined as $\tau_s = -d_s - k_s + \beta E_s R(z; k_s, w(\mathbf{s}_{s+1}))$. At the end of period s, the intermediary makes the payment d_s to the entrepreneur and pays for the capital k_s. Then, at the beginning of the next period, it receives the revenue $R(z; k_s, w(\mathbf{s}_s))$. By adopting this particular definition of utility flows, we are able to eliminate the input of capital k as a state variable in the recursive formulation of the contractual problem. Future flows are discounted by $\beta = \alpha\phi/(1+r)$—rather than $1/(1+r)$—because the entrepreneur survives to the next period with probability α and the project remains productive with probability ϕ.

Equation (6) defines the intertemporal participation constraint: the value of continuing the contract for the entrepreneur, starting from next period, cannot be smaller than the value of repudiating it after the realization of the shock. The repudiation value is defined in (3). Notice that the wage variable w is determined by the clearing condition in the labor market, which depends on the aggregate states of the economy.

After writing this problem in Lagrangian form with γ_{s+1} the Lagrange multiplier associated with the incentive compatibility constraint (6), the planner's problem can be rewritten as a saddle-point formulation (see Marcet and Marimon [19]):

$$\min_{\{\mu_{s+1}\}_{s=t}^{\infty}} \max_{\{d_s,\tau_s,k_s\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \beta^{s-t} \Big[\mu_s d_s + \tau_s - (\mu_{s+1} - \mu_s)\beta D(z; k_s, \mathbf{s}_{s+1})\Big] \tag{8}$$

subject to

$$\tau_s = -d_s - k_s + \beta E_s R(z; k_s, w(\mathbf{s}_{s+1})) \tag{9}$$

$$\mu_{s+1} = \mu_s + \gamma_{s+1} \tag{10}$$

$$d_s \geq 0, \quad \mu_t = \lambda_t \tag{11}$$

By Theorem 1 in Marcet and Marimon [19], a solution to the saddle point problem is a solution to the original planner's problem.[6] Of particular

[6]Theorem 1 is a sufficiency theorem and our model clearly satisfies the required assumptions. However, it does not satisfy the convexity assumptions needed to guarantee that all solutions of the original planner's problem can be obtained as solutions to the corresponding saddle point problem: the function $-D(z; \cdot, \mathbf{s})$ fails to be quasiconcave.

interest is the co-state variable μ that evolves according to $\mu_{s+1} = \mu_s + \gamma_{s+1}$. This variable increases when the Lagrange multiplier γ_{s+1} is positive, which happens when the participation constraint (6) is binding.

The above formulation shows how the planner assigns variable weights to the entrepreneur and the intermediary along an accumulation path: it starts with $\mu_t = \lambda_t$ and increases the weight when the enforceability constraint is binding. This property has a very simple intuition. The weight used by the planner determines the value of the contract for the entrepreneur: the larger is this weight, the higher its contract value. When the enforcement condition is binding, the entrepreneur's value is smaller than the repudiation value. To prevent repudiation, the promised value must increase. This is obtained by assigning a larger weight to the entrepreneur, that is, by increasing μ_{s+1}.

From the above formulation, we can rewrite the problem recursively as:

$$W(z; \mathbf{s}, \mu) = \min_{\mu(\mathbf{s}')} \max_{d,k} \Big\{ \mu d + \tau - \beta E[\mu(\mathbf{s}') - \mu] D(z; k, \mathbf{s}') + \beta E W(z; \mathbf{s}', \mu(\mathbf{s}')) \Big\} \tag{12}$$

subject to

$$\tau = -d - k + \beta E R(z; k, w(\mathbf{s}')) \tag{13}$$

$$d \geq 0, \quad \mu(\mathbf{s}') \geq \mu \tag{14}$$

$$\mathbf{s}' \sim H(\mathbf{s}) \tag{15}$$

where the prime denotes the next period variable and H is the distribution function for the next period aggregate states (law of motion), given the current states. The aggregate states are given by the distribution (measure) of firms over the variables z and μ, which we denote by $M(z, \mu)$, and by the number N of new investment projects with high productivity. Therefore, $\mathbf{s} = (N, M)$. Notice that the choice of the next period value of μ is state contingent.

4.1 Characterization of the optimal contract

Conditional on the survival of the firm, the solution to the optimal contract is characterized by the following first order conditions:

$$\mu \le 1, \qquad (= \text{if } d > 0) \tag{16}$$

$$D(z;k,\mathbf{s}') \le \frac{\partial W(z;\mathbf{s}',\mu(\mathbf{s}'))}{\partial \mu(\mathbf{s}')}, \qquad (= \text{if } \mu(\mathbf{s}') > \mu) \tag{17}$$

$$\beta E\left[\frac{\partial R(z;k,w(\mathbf{s}'))}{\partial k} - \Big(\mu(\mathbf{s}') - \mu\Big)\frac{\partial D(z;k,\mathbf{s}')}{\partial k}\right] = 1 \tag{18}$$

with the envelope condition:

$$\frac{\partial W(z;\mathbf{s},\mu)}{\partial \mu} = \begin{cases} d + \beta ED(z;k,\mathbf{s}'), & \text{if} \quad \mu(\mathbf{s}') > \mu \\ d + \beta \frac{\partial W(z;\mathbf{s}',\mu)}{\partial \mu}, & \text{if} \quad \mu(\mathbf{s}') = \mu \end{cases} \tag{19}$$

Conditions (16)-(18) characterize the dynamic properties of a firm induced by an optimal contract. We emphasize three of the properties that are important for understanding the response of the aggregate economy to shocks. At the micro level, these properties are similar to the properties of the partial equilibrium model of Marcet and Marimon [18]. At the aggregate level, however, our results are new as we will see in Section 6.

Property 1 (Pattern of firm growth) *When contracts are fully enforceable, all firms operate at the optimal scale $\bar{k}(z;\mathbf{s})$. With limited enforceability, however, firms are initially small and grow on average until they reach the optimal scale.*

Consider first the economy with full enforceability of contracts. In this economy the enforceability constraints are never binding, and therefore, we have that $\mu(\mathbf{s}') = \mu = \lambda_t$. This implies that condition (18) can be written as $\beta \partial ER(z;k,w(\mathbf{s}'))/\partial k = 1$ which is the condition that defines the optimal input of capital $\bar{k}(z;\mathbf{s})$. Therefore, all firms operate with the optimal scale from the beginning.[7] Condition (16) then shows that, unless $\lambda_t = 1$, the intermediary receives all the rents. Of course, competition in the intermediation sector guarantees that intermediaries and entrepreneurs are equally

[7]The optimal input of capital still depends on the aggregate states because the optimal scale depends on the wage rate which in turn depends on the aggregate demand of labor.

weighted by the planner, that is, $\lambda_t = 1$. In this case the distribution of profits is not determined, meaning that the division of the surplus between the two contractual parties can be obtained with a multiplicity of payment schemes.

Let's consider now the economy in which contracts are not fully enforceable. The fact that enforceability constraints are likely to be binding in the future means—by condition (18)—that the entrepreneur cannot start the contract with the optimal input of capital as in the economy with enforceable contracts. Furthermore, in those periods in which the enforceability constraint is binding, condition (17) is satisfied with equality (and zero payments to the entrepreneur, unless the unconstrained status is reached that period). The pattern of growth will then be determined by this condition. Furthermore, whenever the enforceability constraint is not binding, firms do not grow on average. Once the variable μ reaches the value of 1, the structure of the contract is similar to a contract with full enforceability. At this point the firm is unconstrained.

Property 2 (Asymmetric responses) *Positive shocks tend to have larger impact on the firm investment than negative shocks.*

When there is a positive shock, the repudiation value increases because the value of searching for a new project $\overline{V}(\mathbf{s})$ increases. This would make the incentive compatibility constraint binding which requires an increase in $\mu(\mathbf{s}')$. We would expect the opposite when there is a negative shock. However, when the enforceability constraints is not binding, the value of $\mu(s')$ does not fall below the current value. This is clear from condition (17). Because the next period capital depends on the new value of $\mu(\mathbf{s}')$, this implies that investment is more sensitive to positive shocks than to negative shocks.

Property 3 (Entrepreneur's payments) *Before the firm reaches the unconstrained status $\mu = 1$, the payments to the entrepreneur are zero, that is, $d = 0$. When $\mu = 1$ the optimal contract can be implemented with a multiplicity of compensation schemes.*

This property follows directly from condition (16). The postponement of payments has a simple intuition. Because the input of capital is constrained by the entrepreneur's value, higher promises allow for higher inputs of capital without violating the incentive-compatibility constraint. Therefore, it is optimal to postpone the entrepreneur's payments to the future.

This property derives from the assumption of risk neutrality and is common in several models with financial market frictions such as Albuquerque and Hopenhayn [1], Cooley and Quadrini [7] and Quadrini [22]. With risk averse agents, as in Marcet and Marimon [18], d could be positive also in the constrained status and there is no indeterminacy in the entrepreneur's payments once the firm becomes unconstrained. However, the motive for consumption smoothing does not completely eliminate the incentive for higher savings. See Cagetti and De Nardi [5] and Quadrini [23].

5 Value of a new firm and general equilibrium

The analysis conducted in the previous section takes as given the initial weight λ_t, that is, the relative weight assigned to the entrepreneur in a new contract signed at time t. We now derive this weight.

Define $V(z;\mathbf{s},\mu)$ the value of the contract for the entrepreneur after consumption. This value depends on the technology level of the project, z, the aggregate states, $\mathbf{s}$, and the individual state μ:

$$V(z;\mathbf{s},\mu) \quad = \quad \beta \begin{cases} D(z;k,\mathbf{s}'), & \text{if} \quad \mu(\mathbf{s}') > \mu \\ \\ d(\mathbf{s}',\mu) + V(z;\mathbf{s}',\mu), & \text{if} \quad \mu(\mathbf{s}') = \mu \end{cases} \tag{20}$$

The definition of $V(z;\mathbf{s},\mu)$ can be explained as follows. The current value after the current payment is the expected discounted value of the next period value before the new payment. If in the next period the incentive compatibility constraint is binding ($\mu(\mathbf{s}') > \mu$), the value of the contract for the entrepreneur is the repudiation value. If it is not binding, then the individual state does not change ($\mu(\mathbf{s}') = \mu$) and the next period value is equal to the payment $d(\mathbf{s}',\mu)$ plus the continuation value $V(z;\mathbf{s}',\mu)$.

Given the entrepreneur's value, we can derive the end-of-period value for the intermediary (before capital accumulation) as follows:

$$B(z;\mathbf{s},\mu) \quad = \quad W(z;\mathbf{s},\mu) - \mu\Big[d(\mathbf{s},\mu) + V(z;\mathbf{s},\mu)\Big] \tag{21}$$

Appendix A.1 provides the formal derivation of $B(z;\mathbf{s},\mu)$.

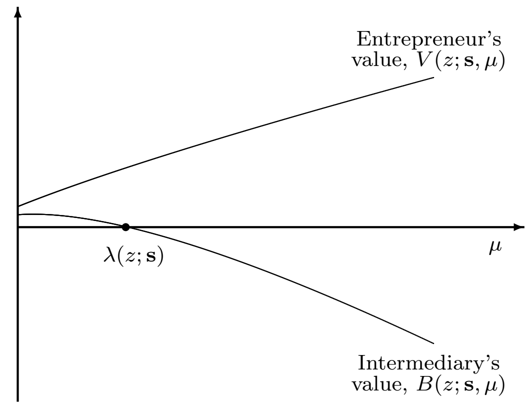

Figure 2: Determination of $\lambda(\mathbf{s})$

Finally, the surplus of the contract, denoted by $S(z;\mathbf{s},\mu)$, is the sum of the entrepreneur's and intermediary's values, that is:

$$S(z;\mathbf{s},\mu) \quad = \quad V(z;\mathbf{s},\mu) + B(z;\mathbf{s},\mu) \tag{22}$$

Assuming competition among intermediaries, the initial entrepreneur's weight λ is determined as:

$$\begin{aligned} \lambda(z;\mathbf{s}) \; &= \; \arg\max_{\mu} \;\; V(z;\mathbf{s},\mu) \\ \textbf{s.t.} \quad & B(z;\mathbf{s},\mu) \;\geq\; 0 \end{aligned} \tag{23}$$

If an optimal contract exists, it satisfies $B(z;\mathbf{s},\lambda(z;\mathbf{s})) = 0$. This is because the function V is strictly increasing for all $\mu < 1$. This can be easily verified by looking at equation (20). The qualification that an optimal contract exists is necessary because for a particular project z and state $\mathbf{s}$ the intermediary may not break even.

The determination of $\lambda(z;\mathbf{s})$ is shown in Figure 2. This figure plots the functions $V(z;\mathbf{s},\mu)$ and $B(z;\mathbf{s},\mu)$ for given aggregate states $\mathbf{s}$ (and z). The initial μ, that is, $\lambda(z;\mathbf{s})$, is given by the point in which the value of the contract for the intermediary crosses the horizontal axis. This is the point that maximizes the value of the contract for the entrepreneur, without violating the participation constraint for the intermediary.

Given the initial weights, the value of searching for a new investment project, net of the repudiation cost, is:

$$\overline{V}(\mathbf{s}) = (1-p)\cdot V(z_L;\mathbf{s},\lambda(z_L;\mathbf{s})) + p\cdot V(z_H;\mathbf{s},\lambda(z_H;\mathbf{s})) - \kappa \qquad (24)$$

where $p = \min\{N/e, 1\}$ is the probability of finding the high productivity project. This probability depends on the number N of high productive projects, which is stochastic, and by the number of searching entrepreneurs e (which is constant in the model).

The function $\overline{V}(\mathbf{s})$ determines the repudiation value $D(z;k,\mathbf{s})$ which we have taken as given in the previous analysis. To solve for the equilibrium we have to solve for a non-trivial fixed point problem. In general we can think of this fixed point as the solution to a mapping T that maps a set of functions $\overline{V}(\mathbf{s})$ into itself, that is,

$$\overline{V}^{j+1}(\mathbf{s}) = T(\overline{V}^{j})(\mathbf{s}) \qquad (25)$$

Given $\overline{V}^{j}(\mathbf{s})$ the function that determines the values of searching for new projects in future periods, the mapping T returns the value of searching for a new project today $\overline{V}^{j+1}(\mathbf{s})$. This mapping embeds all the general equilibrium properties of the model, that is, the clearing conditions in the labor and financial markets. The definition of a general equilibrium follows:

Definition 5.1 (Recursive equilibrium) *A recursive competitive equilibrium is defined as a set of functions for (i) labor supply $l(\mathbf{s})$ and consumption $c(\mathbf{s},a)$ from workers; (ii) entrepreneur's payment $d(\mathbf{s},\mu)$, investment $k(\mathbf{s},\mu)$ and policy $\mu(\mathbf{s}') = \psi(\mathbf{s},\mu)$; (iii) initial weights $\lambda_z(\mathbf{s})$; (iv) value of searching $\overline{V}(\mathbf{s})$ (v) wage $w(\mathbf{s})$; (vi) aggregate demand of labor from firms and aggregate supply from workers; (vii) aggregate investment from firms and aggregate savings from workers and entrepreneurs (intermediated by financial intermediaries); (viii) distribution function (law of motion) for the next period states $\mathbf{s}' \sim H(\mathbf{s})$; (ix) mapping T. Such that: (i) the household's decisions are optimal; (ii) the entrepreneur's payments, investment and policy ψ satisfy the optimality conditions of the financial contract (conditions (16)-(18)); (iii) the initial weights solve problem (23); (iv) the value of searching is the fixed point of T; (v) the wage clears the labor market; (vi) the capital market clears (investment equals savings); (vii) the distribution function for the next period states is consistent with the dynamics induced by the optimal contracts and the stochastic process for the arrival of high productive projects.*

Proving the existence of an equilibrium is equivalent to proving the existence of a fixed point in (25). This is a difficult task because $\overline{V}(\mathbf{s})$ is a function of the whole distribution of firms. However, the existence and uniqueness of a steady state equilibrium characterized by an invariant distribution of firms and by constant values of $\overline{V}(\mathbf{s})$ and $w(\mathbf{s})$ can be easily established. This is formally stated in the next proposition.

Proposition 5.1 *There exists a unique steady-state equilibrium.*

Proof 5.1 *Appendix A.2.*

5.1 Intermediary's renegotiation

Before turning to the study of the quantitative properties of the model, we derive here some conditions under which the value of the contract for the intermediary is always positive, and therefore, there is no repudiation. As for the proof of the existence of a general equilibrium with aggregate uncertainty, it is difficult to find these conditions for any possible realization of the shock. Therefore, we will concentrate on the steady state equilibrium. The results should hold for moderate deviations from the steady state.

Repudiation can take place at the beginning of the period (before any payment) or at the end of the period. The value of the contract for the intermediary at the beginning of the period is:

$$(1-\delta-w/\xi)k_{-1}+F_z(k_{-1})-d+B(\mu) \tag{26}$$

where the function $B(\mu)$ is defined in (21). This is the sum of the current payment from the firm (or to the firm if negative), $(1-\delta-w/\xi)k_{-1}+F_z(k)-d-k$, and the continuation value $k+B(\mu)$. Notice that the current payment is net of the new investment k.

The value of the contract after the current payment, and therefore, the value that the intermediary will loose if it repudiates the contract at the end of the period is:

$$k+B(\mu) \tag{27}$$

Notice that the function $B(\mu)$ could be negative. However, what matters for repudiation is $k+B(\mu)$.

The next proposition establishes a condition under which (26) and (27) are always positive in a steady state equilibrium.

Proposition 5.2 *If* $1 - \delta - w/\xi > 0$, *(26) and (27) are always positive in the steady state and the intermediary will never repudiate the contract.*

Proof 5.2 *Appendix A.3.*

This is only a sufficient condition and it is easily satisfied as long as the depreciation rate and the labor costs are not too high. This condition is always satisfied in all the numerical exercises conducted in this paper.

6 Contrasting economies with and without contract enforceability

In this section we study the properties of the economy numerically. After the parameterization of the model in Section 6.1, Section 6.2 studies some of the steady state properties and Section 6.3 the response of the economy to shocks. The welfare losses associated with limited contract enforceability are evaluated in Section 6.4.

6.1 Parameterization

The period in the economy is one year and the intertemporal discount rate (equal to the interest rate), is set to $r = 0.04$. The survival probability is $\alpha = 0.99$. The disutility from working takes the form $\varphi(l) = \pi \cdot l^{\frac{1+\nu}{\nu}}$, where ν is the elasticity of labor. In the baseline model we set $\nu = 1$ which is the value used often in business cycle studies. We will also report the results for other values of ν. After fixing ν, the parameter π is chosen so that one third of available time is spent working. The mapping from π to the working time will be described below.

The production function is specified as $F_z(k) = z \cdot k^{\theta}$. The parameter θ is assigned the value of 0.95. We will then conduct a sensitivity analysis for this parameter. The production technology becomes unproductive with probability $1 - \phi = 0.04$. Associated with the 1 percent probability that the entrepreneur dies, the exit probability of firms is about 5 percent.

We would like the steady state of the economy to have a capital-output ratio of 2.8 and a labor income share of 0.6. These indices are complicated functions of the whole distribution of firms. However, because most of the aggregate output is produced by unconstrained firms, we can choose these parameters so that these numbers are reproduced by unconstrained firms, that is, $\bar{k}/[E(z)\bar{k}^{\theta}] = 2.8$ and $(w/\xi)\bar{k}/[E(z)\bar{k}^{\theta}] = 0.6$. After normalizing the capital stock of unconstrained firms to $\bar{k} = 1$, a value of 2.8 for the capital-output ratio implies $E(z) = 0.4$. This pins down the mean of z.

The capital input of unconstrained firms is $\bar{k} = \left(\frac{\beta\theta E(z}{1-\beta(1-\delta-w/\xi)}\right)^{\frac{1}{1-\theta}} = 1$. This is equal to 1 because we have normalized $\bar{k} = 1$. After observing that the term w/ξ is equal to the ratio between the labor share and the capital-output ratio, that is, $w/\xi = 0.6/2.8$, the above condition implies $\delta = 0.028$. Because in each period about 5% of firms exit the market and the capital of these firms fully depreciates, the depreciation rate for the aggregate stock of capital is about 0.078.

Given the parameterization of the production sector, and the implied value of w/ξ, the model generates a stationary distribution of firms and an aggregate demand of labor. The parameter κ affects the size of new firms. We set κ so that the initial stock of capital for new firms is about 25 percent the size of incumbent firms. Then the capital-labor ratio ξ and the utility parameter π are determined so that in the steady state equilibrium each worker spends 1/3 of available time working and unconstrained firms employ a certain number of workers n. This implies $\xi = \bar{k}/l = 1/(n \cdot 0.33) = 0.0033$. However, the size of unconstrained firms is not important. Given ξ we are able to determine the steady state wage rate w (remember that $w/\xi = 0.6/2.8)$). Then to pin down the parameter π we consider the worker's first order condition in the supply of labor, that is, $\nu\pi l^{\nu-1} = w$. Given ν and $l = 0.33$, this condition pins down π. Finally, the mass of new firms (newborn agents with entrepreneurial skills, e) is such that the aggregate supply of labor is equal to the aggregate demand. The full set of parameter values are in table 1.

Table 1: Parameter values.

Intertemporal discount rate	$r = 0.040$
Disutility from working $\varphi(l) \equiv \pi \cdot l^{\nu}$	$\nu = 1.100, \pi = 0.001$
Survival probability of agents	$\alpha = 0.990$
Survival probability of projects	$\phi = 0.960$
Production function $F_z(k) \equiv zk^{\theta}$,	$\theta = 0.95, \bar{z} = 0.357, \xi = 0.003$
Depreciation rate	$\delta = 0.028$
Cost of repudiation	$\kappa = 0.087$

6.2 Steady state properties

Before studying the response of the economy to the arrival of new technologies, it would be instructive to examine some features of the optimal contract when there is no aggregate uncertainty ($z_L = z_H$) and the economy is in the steady state equilibrium. These properties will be helpful for understanding the behavior of the economy when there is aggregate uncertainty.

Figure 3 plots the size distribution of firms in the economies with full enforceability (panel a) and limited enforceability (panel b). When contracts are fully enforceable, the input of capital always maximizes the surplus of the firm independently of the division of this surplus. Therefore, all firms operate with the optimal scale.

In the economy with limited enforceability, surviving firms employ different inputs of capital. The initial size of firms is small but after entrance they grow in size until they reach the optimal size. Consequently we observe a concentration of firms in the largest class. If the wage rate in the economy with limited enforceability was the same as in the economy with full enforceability, the optimal input of capital employed by unconstrained firms would be equal to the capital employed by the firms operating in the full enforcement economy. However, because with limited enforceability some of the firms are constrained, the demand of labor and the wage rate are smaller. This implies that the optimal size of firms is larger when contracts are not fully enforceable.

The first panel of Figure 4 plots the values of a new contract for the entrepreneur and the intermediary. These are the functions $V(z;\mu)$ and $V(z;\mu)$ defined in (20) and (21). As can be seen from the figure, the entrepreneur's value increases with μ while the value for the intermediary is decreasing in μ. The assumption of competition in financial markets then implies that the initial value of the contract for the intermediary is zero as we have seen in Section 5.

To show that the intermediary never repudiates the contract, the second panel of Figure 4 plots the values of an active contract for the intermediary in the two possible stages of renegotiation: at the beginning of the period and at the end of the period. These two values were defined in (26) and (27). As can be seen from the figure, the continuation values are always positive for our parameterization.

The third panel of Figure 4 plots the entrepreneur's value as a share of the total firm's value after investment, that is, $k + S(z;\mu)$. We interpret this share as an indicator of the insider's ownership. This share declines as the firm expands in size, which is consistent with the dynamics of insider's ownership observed in the data. This feature, however, depends on the particular parameterization of the model and cannot be generalized.

The last panel of Figure 4 plots the growth rate of capital (conditional on survival) for firms of different size. The growth rate is decreasing in the size of the firm and more in general it is bigger for constrained (small) firms. Because new projects are implemented by new firms which are initially constrained, this feature of the optimal contract is important for understanding the propagation of new technologies.

6.3 Contract enforceability and the diffusion of new technologies

We assume that the number of high productive projects N is independently and identically distributed as a uniform in the interval $[0, e]$. This implies that the probability of finding the high productive project is also uniformly distributed in the interval $[0, 1]$. After simulating the economy for a long sequence of $N = 0.5e$, we consider the arrival of a new technology that increases the number N to e. The increase in N is only for one period and starting in the next period it reverts to its mean $0.5e$. The economy will then converge to the same equilibrium it was before the arrival of the new technology.[8] After conducting this exercise, we will also consider the case in which the arrival of a new technology increases N persistently.

Figure 5 plots the response of aggregate output in the economies with full and limited enforceability and shows that limited enforceability has large amplification effects.

To describe the mechanism that generates the amplification result, let's describe first how the new technology propagates in the economy with full enforceability of contracts. In this economy the expansion of aggregate output derives in part from the increase in the productivity of old and new firms and in part from the increase in employment. The productivity of new firms increases because they implement more productive projects. The productivity of old firms increases because they reduce the scale of production after the wage increase.

The mechanism described above is also present in the economy with limited enforceability. In this economy, however, aggregate output receives an

[8] Although we solve the economy for a particular sequence of N, agents solve a stochastic dynamic problem where they expect N, and therefore p, to be random.

additional impulse from the expansion of constrained old firms. In fact, after the arrival of the new technology, the repudiation value of constrained firms increases. To prevent default, the value promised to the entrepreneur must increase. This relaxes the tightness of the incentive compatibility constraints and more capital can be given to these firms. Therefore, the impact of the new technology is to lessen the tightness of the financial constraints.

This mechanism can be easily illustrated using the first order condition (17). For constrained firms, that is, for firms with $\mu < 1$, this condition is approximately equal to:[9]

$$F(z;k_{-1}) + \overline{V}(\mathbf{s}) \leq \beta E\Big\{F(z;k) + \overline{V}(\mathbf{s}')\Big\} \tag{28}$$

The new technology increases $\overline{V}(\mathbf{s})$ but, because temporary, it will have only a marginal impact on $E\overline{V}(\mathbf{s}')$. Therefore, the left-hand-side increases more than the right-hand-side. Because k_{-1} is given, the new capital k must increase. This equation also shows the asymmetric response to positive and negative shocks. When there is a negative shock that induces a large fall in $\overline{V}(\mathbf{s})$, the state variables μ does not declines and the above condition will be satisfied with the inequality sign.

Figure 5 also shows that the amplification effect is very persistent. This is because the shock shifts to the right the whole distribution of constrained firms. Once this shift has taken place, it takes several periods to converge back to the limiting distribution.

The size of the amplification depends on the degree of contract enforceability determined by the default cost κ. Figure 6 plots the impulse responses when the repudiation cost κ takes higher values. A very large value of κ is equivalent to the case of market exclusion where the repudiation value becomes $D(z;k,\mathbf{s}) = F(z;k)$. As can be seen from the figure, the impact of the new technology declines as we increase κ. In the case of market exclusion there is no amplification. Therefore, the lower the degree of contract enforceability (the lower is κ) and the larger is the macro impact of new technologies.

The reason a higher value of κ dampens the amplification effect is because it reduces the fraction of constrained firms. This in turn derives from the fact that the initial size of new firms is larger when κ is bigger. As a result

[9] This condition would hold exactly if the incentive-compatibility constraint is always binding for constrained firms. This is not a bad approximation because the incentive-compatibility constraint is only occasionally non-binding.

of this, firms reach the unconstrained status faster and in each period there is a smaller fraction of constrained firms. Because, the amplification effect derives from the reaction of constrained firms, smaller is the fraction of these firms and smaller is the impact of the shock on aggregate output. In the limiting equilibrium of the baseline economy, about 40 percent of firms are constrained.

In the case of market exclusion, the amplification effect completely disappears. This is due to the fact that in this case the repudiation value is no longer affected by the shock. It is important to notice that with market exclusion the amplification effect would disappear even if we keep constant the fraction of constrained firms. Therefore, the key mechanism leading to our results is the assumption that defaulting entrepreneurs are not excluded from the market. This modeling feature differentiates our model from previous general equilibrium models with limited enforceability where default always leads to autarky.

Figure 7 reports the impulse responses for alternative values of ν. When the supply of labor is rigid (high values of ν), the shock has a smaller impact on aggregate output. This is because the expansion of constrained firms is compensated by the contraction of unconstrained firms in response to a larger increase in the wage rate. However, the elasticity of labor affects the magnitude of the output response not only in the economy with limited enforceability but also in the full enforcement economy. Therefore, the amplification result is independent of this elasticity.

The above results are based on the assumption that the new technology increases the number of high productive projects only temporarily. A different way to think about the arrival of a new technology is when it increases the productivity of all new projects. We can think of this type of innovations as general purpose technologies. To capture this idea we now assume that the economy has been in the state $N = 0$ for a long period of time. Then, unexpectedly there is the arrival of a new technology which increases N to e persistently and all projects implemented after that period will have high productivity. Figure 8 plots the response of aggregate output for the economy with full and limited contract enforceability.

With limited enforceability the convergence to the higher long-run level of output is slower. Therefore, limited enforceability delays the diffusion of this type of technologies and there is no amplification. To see why in this case the impact of the new technology is not amplified, consider again equation (28).

This equation describes the investment behavior of constrained firms, which are the source of the amplification result shown above. As in the previous case, the new technology increases $\overline{V}(\mathbf{s})$. The difference, however, is that now the term $E\overline{V}(\mathbf{s}')$ also increases. Therefore, the increase in the left-hand-side is mostly compensated by the increase in the right-hand-side. As a result, the input of capital k increases only modestly.

The delaying effect derives from the dynamics of new firms. After the arrival of the new technology, output increases because the new firms are more productive. However, in contrast to the economy with enforceable contracts, the impact on output will be gradual because the new firms are initially small. As these firms grow in size, the contribution of their higher productivity to aggregate output increases. In contrast, when contracts are fully enforceable, new firms start from the beginning with the optimal scale.

This result can be considered a possible explanation of why the new Information Technologies of the 1970s took a long time to display their full potential on productivity as pointed out in Greenwood and Jovanovic [11]. If we interpret the IT revolution as the permanent arrival of a new technology, our theory provides a financial explanation for its sluggish diffusion. The financial explanation complements the sluggish diffusion induced by the vintage structure of investment projects.

Our theory is also consistent with the empirical evidence about the better performance of new listed firms after the IT revolution as shown in Hobijn and Jovanovic [12]. In our model this feature derives from the vintage structure and is consistent with the view expressed in the above paper: the reason new firms outperformed incumbent firms is because they were less dependent on old technologies, and therefore, they were more flexible.

6.4 Efficiency losses from limited contract enforceability

In this final section we evaluate the welfare losses induced by limited contract enforceability. In evaluating these losses we assume that $z_L = z_H$ and there is no aggregate uncertainty.

To evaluate the welfare consequences of limited contract enforceability we conduct the following experiment. Starting from the steady state of the economy with limited enforceability, we assume that all contracts become enforceable (including the existing ones). This unanticipated change brings the economy to a new equilibrium in which all firms employ the same input of capital. The transition dynamics takes only one period.

Table 2 reports the welfare gains from the transition to the fully enforcement economy. These gains are computed as the increase in every period consumption necessary to make all agents indifferent between staying in the economy with limited enforceability (but with the consumption increase) or making all contracts fully enforceable (and starting the transition). Table 2 also reports some key statistics of the steady state competitive allocations in the two versions of the economy.

Table 2: Welfare losses from limited contract enforceability.

	Limited Enforceability	Full Enforceability
	$\nu = 1.1$	
Average size (capital)	0.807	0.887
Working time	0.330	0.363
Steady state output loss	8.88%	
Steady state welfare loss	1.85%	
Welfare gain from transition		0.54%
	$\nu = 2.0$	
Average size (capital)	0.807	0.820
Working time	0.330	0.335
Steady state output loss	1.79%	
Steady state welfare loss	0.65%	
Welfare gain from dynamics		0.40%
	$\nu = 3.0$	
Average size (capital)	0.807	0.814
Working time	0.330	0.333
Steady state output loss	1.08%	
Steady state welfare loss	0.53%	
Welfare gain from transition		0.40%

Different values of the parameter ν are considered: the larger the elasticity of labor (the smaller the value of ν) and the larger is the effect of contract enforceability on the macro allocation of resources. This is because, when the supply of labor is elastic, the increase in labor allows previously constrained firms to expand production, and therefore, aggregate consumption. On the other hand, when the supply of labor is not elastic, the expansion of previously constrained firms is compensated by the contraction of unconstrained firms after the wage increase. Consequently, the increase in production and consumption can not be large. Independently of the value of ν, the welfare gains are in the order of 0.5 percent of consumption. The table also reports the comparison between the steady state welfare levels (which neglects the transition). Looking only at the steady state welfare, the gains from contract enforceability are larger and they increase with the elasticity of the labor supply. The reason is because the shift to the new equilibrium requires a large initial investment cost that is not considered in the steady state welfare calculation. The welfare consequences of limited contract enforceability are also studied in Quintin [24].

7 Conclusion

We have studied an economy in which entrepreneurs finance investment by entering into long-term relationships with financial intermediaries. Contracts are not fully enforceable and the incentive compatibility requirement makes investment dependent on the repudiation value of the entrepreneur, which is only binding for small and fast growing firms.

Limited enforceability has important implication for the macro-economy in two dimensions. On the one hand, it impairs the efficient allocation of resources with significant welfare consequences. On the other, it creates a large amplification mechanism for the diffusion of new technologies. A key assumption to generate the amplification result is that contract repudiation does not preclude entrepreneurs from signing new contracts (no market exclusion). This amplification would not arise if entrepreneurs were excluded from the market, which is the assumption made in most of the macro models with limited enforceability.

A Appendix: Analytical proofs

A.1 Derivation of equation (21)

Consider the recursive formulation (12). After adding and subtracting $\mu[d(\mathbf{s},\mu) + V(z;\mathbf{s},\mu)]$ and $\mu(\mathbf{s}')[d(\mathbf{s}',\mu(\mathbf{s}')) + V(z;\mathbf{s}',\mu(\mathbf{s}'))]$, (12) can be rewritten as:

$$\begin{aligned} W(z;\mathbf{s},\mu) - \mu\Big(d + V(z;\mathbf{s},\mu)\Big) &= -\mu V(z;\mathbf{s},\mu) - \beta E[\mu(\mathbf{s}') - \mu]D(z;k,\mathbf{s}') \\ &+ \beta E\mu(\mathbf{s}')\Big(d(\mathbf{s}',\mu(\mathbf{s}')) + V(z;\mathbf{s}',\mu(\mathbf{s}'))\Big) \\ &+ \tau + \beta E\Big[W(z;\mathbf{s}',\mu(\mathbf{s}')) - \mu(\mathbf{s}')\Big(d(\mathbf{s}',\mu(\mathbf{s}')) \\ &+ V(z;\mathbf{s}',\mu(\mathbf{s}'))\Big)\Big] \end{aligned} \tag{29}$$

We want to show that $-\mu V(z;\mathbf{s},\mu) - \beta E[\mu(\mathbf{s}') - \mu]D(z;k,\mathbf{s}') + \beta E\mu(\mathbf{s}')(d(\mathbf{s}',\mu(\mathbf{s}')) + V(z;\mathbf{s}',\mu(\mathbf{s}')))$ is zero. This term is the expected value of the function

$$\begin{aligned} g(\mathbf{s},\mu,\mathbf{s}') &= -\mu V(z;\mathbf{s},\mu) - \beta[\mu(\mathbf{s}') - \mu]D(z;k,\mathbf{s}') \\ &\quad +\beta\mu(\mathbf{s}')(d(\mathbf{s}',\mu(\mathbf{s}')) + V(z;\mathbf{s}',\mu(\mathbf{s}'))) \end{aligned} \tag{30}$$

This function can be rewritten as:

$$g(\mathbf{s},\mu,\mathbf{s}') = \begin{cases} -\mu\Big[V(z;\mathbf{s},\mu) - \beta D(\mathbf{s}',k,\mu(\mathbf{s}'))\Big], & \text{if} \quad \mu(\mathbf{s}') > \mu \\ \\ -\mu\Big[V(z;\mathbf{s},\mu) - \beta\Big(d(\mathbf{s}',\mu(\mathbf{s}') + V(z;\mathbf{s}',\mu)\Big)\Big], & \text{if} \quad \mu(\mathbf{s}') = \mu \end{cases} \tag{31}$$

Given (20), the expected value of $g(\mathbf{s},\mu,\mathbf{s}')$ is zero. Therefore, equation (29) can be written as:

$$\begin{aligned} W(z;\mathbf{s},\mu) - \mu\Big(d + V(z;\mathbf{s},\mu)\Big) &= \tau + \beta E\Big[W(z;\mathbf{s}',\mu(\mathbf{s}')) \\ &\quad -\mu(\mathbf{s}')\Big(d(\mathbf{s}',\mu(\mathbf{s}')) + V(z;\mathbf{s}',\mu(\mathbf{s}'))\Big)\Big] \end{aligned} \tag{32}$$

This is a Bellman's equation with current return τ, that is, the payment to the intermediary. Therefore, $B(\mathbf{s},\mu) = W(z;\mathbf{s},\mu) - \mu\Big(d + V(z;\mathbf{s},\mu)\Big)$.

A.2 Proof of proposition 5.1

We first prove two lemmas that will be used in the general proof.

Lemma A.1 *Assume that the wage w is constant and z takes only one value. Then the mapping T defined in (25) has a unique fixed point $\overline{V}$.*

Proof A.1 *Given the continuity of T, it is sufficient to show that this mapping is monotone decreasing and takes values in a bounded set. Consider $V_1 < V_2$. The optimal solution associated with $\overline{V} = V_2$ is also feasible (although not optimal) when $\overline{V} = V_1$. In fact, constraint (6) will not be violated if we replace $\overline{V} = V_2$ with $\overline{V} = V_1$. Therefore, the initial value of the contract for the entrepreneur under $\overline{V} = V_1$ must be at least as big as the value under $\overline{V} = V_2$. Because there is some contingency in which the solution under $\overline{V} = V_2$ is binding when $\overline{V} = V_2$ but it is not binding if we replace V_2 with V_1, then we can find another contract (or allocation) that is feasible under $\overline{V} = V^1$ and increases the value of the entrepreneur without changing the value of the intermediary. Therefore, the T is monotone decreasing.*

That the mapping takes values in a bounded set derives from the participation constraints for entrepreneur and the intermediary. If $\overline{V} + \kappa < 0$ the entrepreneur will not participate in the contract. At the same time $\overline{V} + \kappa$ cannot be greater than the value of the surplus when the firm is unconstrained. *Q.E.D.*

Lemma A.2 *Given a constant w, there exists a unique invariant distribution of firms M.*

Proof A.2 *It is sufficient to show that the transition function satisfies the conditions of Theorem 12.12 in Stokey and Lucas [26] (monotonicity and mixing condition).* *Q.E.D.*

According to Lemma A.1, for a constant w there exists a unique fixed point $\overline{V}$ and the solution to the contractual problem is well defined. Lemma A.2 then establishes that for a given w there exists a unique invariant distribution of firms with associated aggregate demand of labor. If we increase w the demand of labor associated with the new invariant distribution decreases. On the other hand, the supply of labor—implicitly defined by $\varphi'(l) = w$—is increasing in w. This implies that there exists a unique value of w that clears the labor market and defines the unique steady state equilibrium. *Q.E.D.*

A.3 Proof of proposition 5.2

Let's observe first that, if (27) is positive, then (26) is also positive. This derives from the fact that (27) is just the next period discounted value of (26). Therefore, in the following proof we simply prove the positiveness of (27).

Because there is a unique and monotone relation between the costate variable μ and the input of capital k, we can use k as the state variable of the optimal contract. The surplus function can be written as:

$$S(k) = -k + \beta\Big[(1-\delta-w/\xi)k + F(k) + S(k')\Big] \tag{33}$$

Before the firm reaches the unconstrained status, the law of motion of capital is given by equation (17) which is satisfied with equality given that in a steady state there is no uncertainty. Because the input of capital never decreases, the surplus function must be increasing in k until it reaches $\bar{k}$.

From equation (20) we see that the value of the contract for the entrepreneur (before the firm has reached the unconstrained status and $\mu < 1$) is:

$$V(k) = \beta F(k) + \beta\overline{V} \tag{34}$$

The value for the intermediary is simply the difference between the surplus and the entrepreneur's value, that is:

$$B(k) = -k + \beta\Big[(1-\delta-w/\xi)k + S(k') - \overline{V}\Big] \tag{35}$$

Adding k to both sides we get:

$$k + B(k) = \beta\Big[(1-\delta-w/\xi)k + S(k') - \overline{V}\Big] \tag{36}$$

The left-hand-side is the continuation value for the intermediary at the end of the period as defined in (27). Because k' is increasing in k and $S(k')$ is increasing in k', the condition $1-\delta-w/\xi > 0$ guarantees that $k+B(k)$ is increasing in k. Therefore, to prove that the intermediary never repudiates the contract it is sufficient to show that $k + B(k) > 0$ for the smallest firms, that is, the new entrants. This follows from the participation constraint of the intermediary $B(k_0) \geq 0$. *Q.E.D.*

B Appendix: Numerical procedure

Steady state: The steps to solve for the steady state equilibrium are as follows:

1. We guess the wage w.
2. We guess the value of a new project V^0.
3. Given w and V^0, we solve the contract on a grid of points for μ. Because μ never decreases, we use a backward procedure starting from $\mu = 1$. Grid points are joined with step-wise linear functions.

4. Using the zero profit condition for the intermediary we find the value of a new contract for the entrepreneur. If this value is different from the initial guess V^0, we restart the procedure from step 2 until convergence.

5. After solving for the optimal contract and finding the investment rules, we iterate on the distribution of firms until we find the invariant distribution.

6. Given the invariant distribution of firms, we check the equilibrium in the labor market and we restart the procedure from step 1 with an updated wage rate. We continue iterating until the labor market clears.

Aggregate shocks: For each grid point of μ and for each z, we parameterize three factors: the expected change in μ, $E(\mu(\mathbf{s}') - \mu)$, the expected value of the contract for the entrepreneur at the beginning of next period, $E\tilde{V}(z; k, \mathbf{s}', \mu(\mathbf{s}'))$, and the expected next period surplus, $ES(\mathbf{s}', \mu(s'))$. The term $E(\mu(\mathbf{s}') - \mu)$ allows us to use equation (18) while the term $E\tilde{V}(z; k, \mathbf{s}', \mu(\mathbf{s}'))$ allows us to use equation (17). It can be shown, in fact, that $\partial W(\mathbf{s}, \mu)/\partial\mu = d + \beta E\tilde{V}(z; k, \mathbf{s}', \mu(\mathbf{s}'))$. The term $E\tilde{V}(z; k, \mathbf{s}', \mu(\mathbf{s}'))$ is also necessary to determine $V(z; \mathbf{s}, \mu)$, that is, the value of a new contract for the entrepreneur. Finally, the term $ES(\mathbf{s}', \mu(s'))$ allows us to determine the current surplus of a new contract. By subtracting $V(z; \mathbf{s}, \mu)$ we can then determine the value of a new contract for the intermediary.

The three factors are parameterized with linear functions of the following variables: (a) the fraction of high productive projects p; (b) the mean value of μ for low productivity firms $\int \mu M_{z_L}(\mu)$; (b) the mean value of μ for high productivity firms $\int \mu M_{z_H}(\mu)$. The last two variables are proxies for the distribution of firms. The detailed steps to solve for the equilibrium are as follows:

1. We guess the coefficients of the parameterized functions.

2. Given the parameterized functions, we solve and simulate the model.

3. We estimate by regression the coefficients of the parameterized functions using the simulated data as in Krusell and Smith [16]. These estimates are used as new guesses for these functions and the procedure is restarted from the previous step until convergence.

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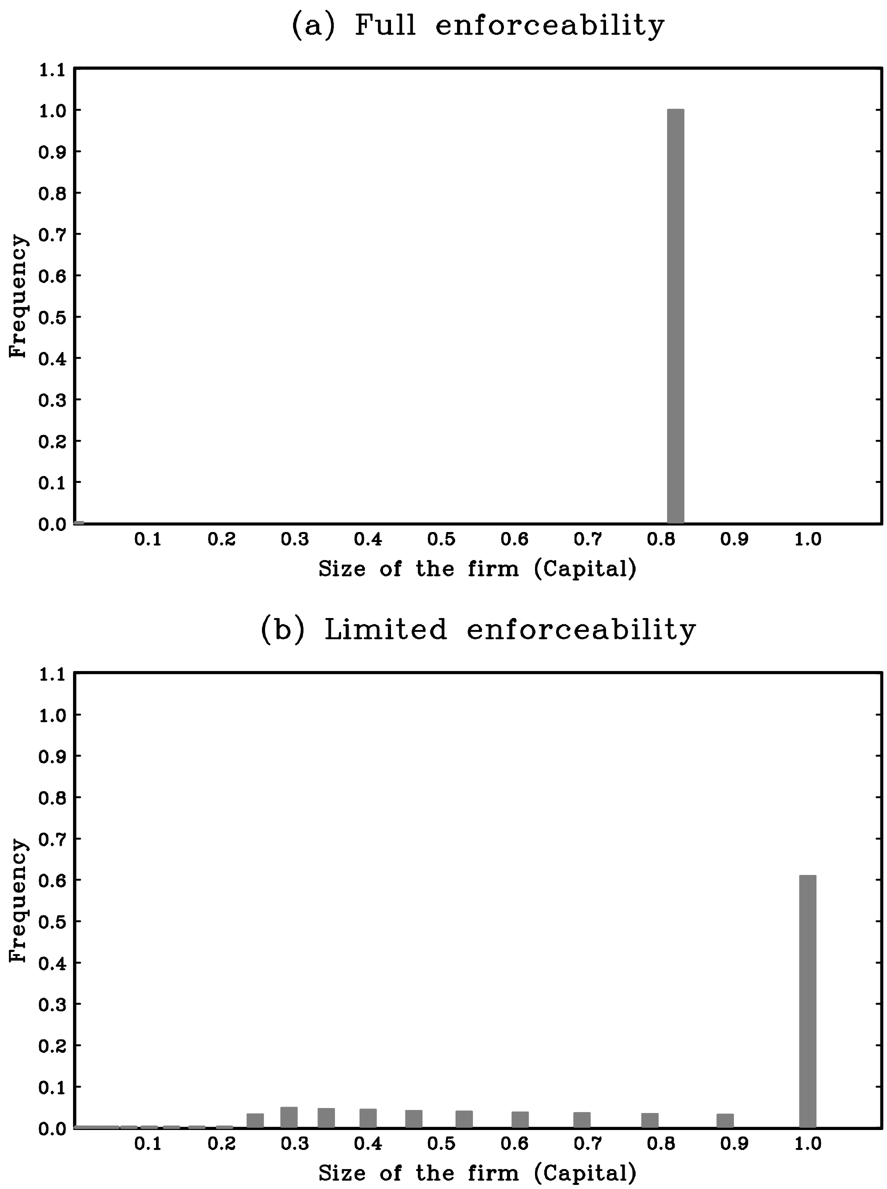

Figure 3: Steady state distribution of firms

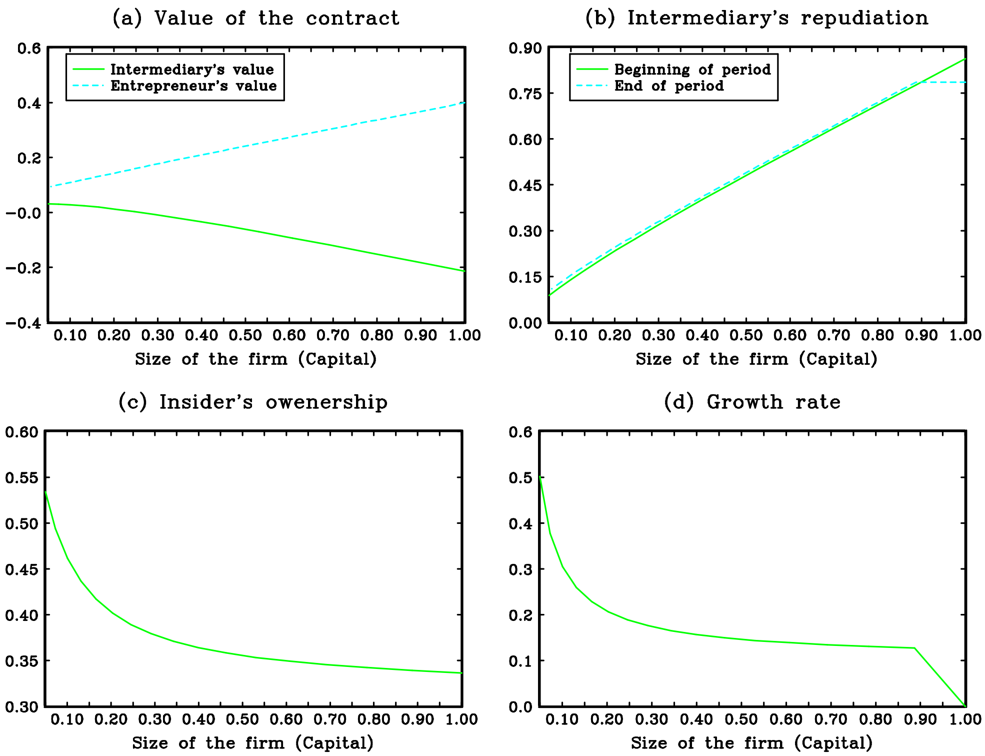

Figure 4: Property of the contract in the steady state

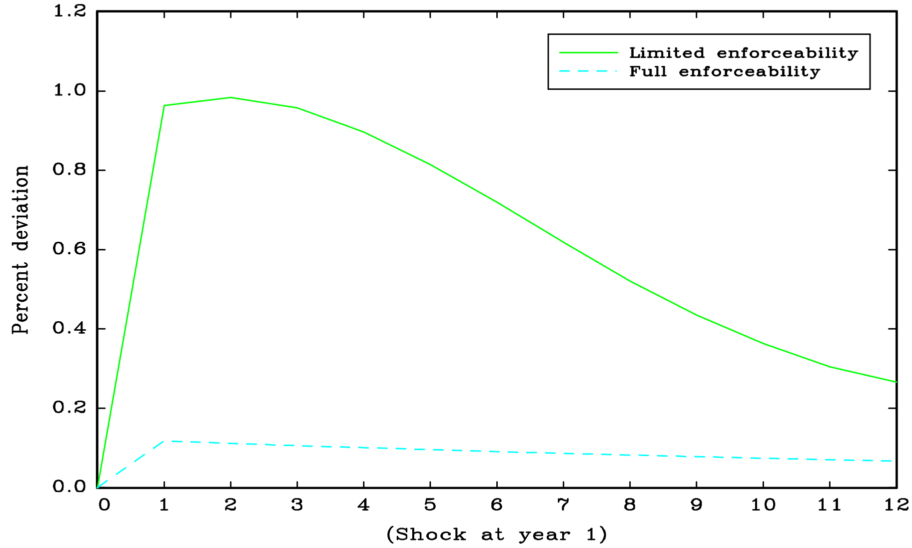

Figure 5: Impulse response to the arrival of a new technology

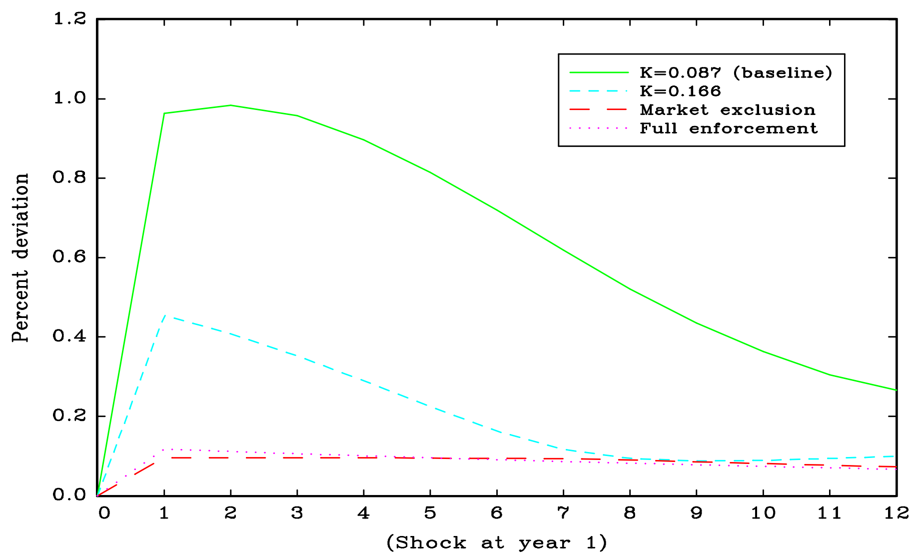

Figure 6: Sensitivity analysis with respect to the repudiation cost

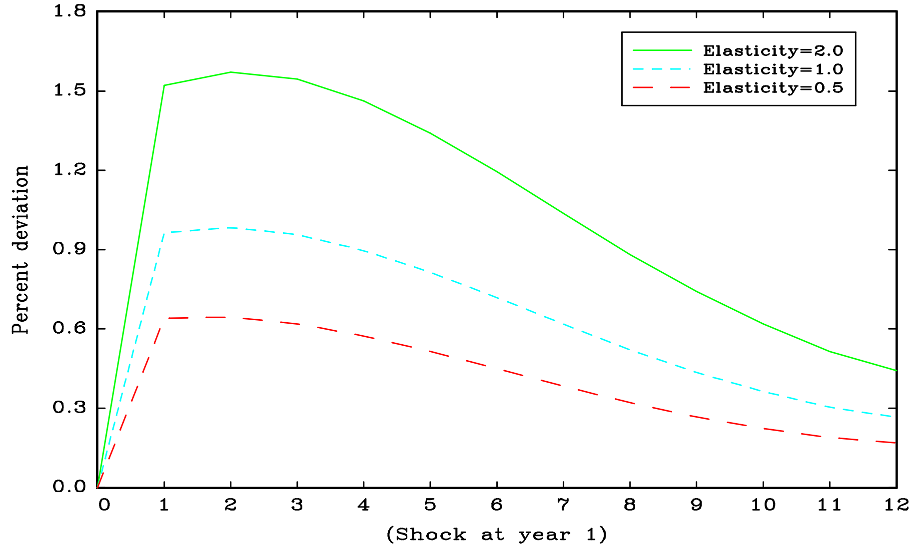

Figure 7: Sensitivity analysis with respect to the repudiation cost

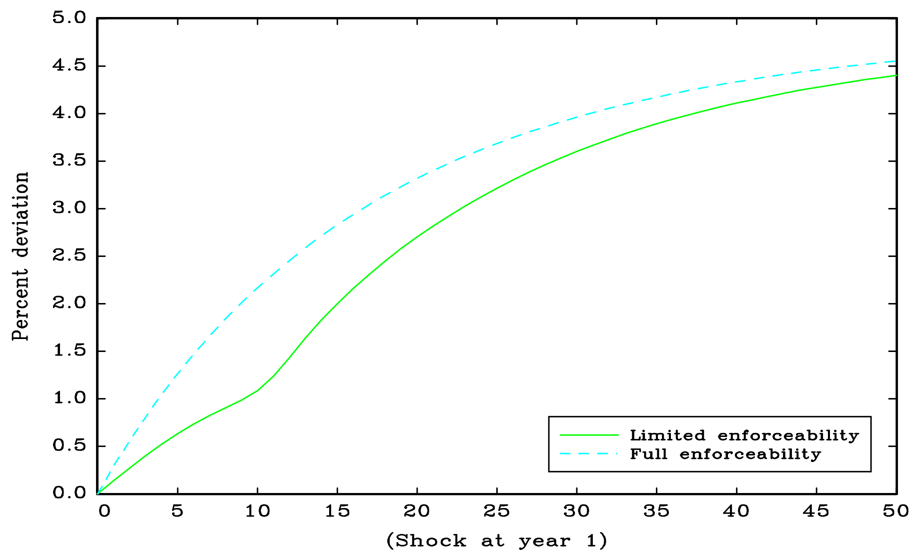

Figure 8: Impulse response to the arrival of new technology