The Demand for Bank Reserves and Other Monetary Aggregates*

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Abstract

The paper starts with Haslag’s (1998) model of the bank’s demand for reserves and reformulates it with a cash-in-advance approach for both financial intermediary and consumer. This gives a demand for a base of cash plus reserves that is not sensitive to who gets the inflation tax transfer. It extends the model to formulate a demand for demand deposits, yielding an M1-type demand, and then includes exchange credit, yielding an M2-type demand. Based on the comparative statics of the model it provides an interpretation of the evidence on monetary aggregates. This explanation relies on the nominal interest as well as technology factors of the banking sector.

E31, E13, O42

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1 Introduction

Modeling the monetary aggregates in general equilibrium has been a challenge. There are some examples such as Chari, Jones, and Manuelli (1996), and Gordon, Leeper, and Zha (1998), who present models that are compared to Base money. Ireland (1995) presents one that he relates to M1-A velocity. And these models have been employed as ways to explain the actual monetary aggregate time series evidence. However McGrattan (1998), for example, argues that the simple linear econometric model in which velocity depends negatively on the nominal interest rate, may do just as well or better in explaining the evidence.

The paper here takes up the topic by modeling a nesting of the aggregates that uses a set of factors that expands from the nominal interest rate by including the production of banking services. Through this approach the productivity factor of banking enters, as well as a cost to using money, sometimes thought of as a convenience cost that ATM machines affect. With this general equilibrium model, and its comparative statics, an explanation similar in spirit to McGrattan (1998), but extended to include these other factors, is provided for the US evidence on monetary base velocity, M1 velocity, and M2 velocity, as well as for the ratios of various aggregates. This represents a more extended explanation than in previous work. And it highlights the limits to a nominal interest rate story, while revealing a plausible role of technological factors in determining the aggregate mix.

The original literature on the welfare cost of inflation, well-represented by Bailey (1992), assumes no cost to banks in increasing their exchange services as consumers flee from currency during increasing inflations.\footnote{The presence of the banking system has no real effect whatever but merely alters the nominal rate of inflation necessary to achieve a given real size of the government budget\cite{P.234, 1992}; in the model here the latter statement is true, but not the former since labor is used up in banking activities, and since reserve requirements affect the real interest rate when there is a non-Friedman optimum rate of interest.} The approach builds upon the more recent literature of Gillman (1993), Aiyagari, Braun, and Eckstein (1998), and Lucas (2000) that assumes resource costs to avoiding the inflation tax by using alternative exchange means. It specifies
production functions of banking products that require real resources. This
gives rise to the role of technological factors in explaining the movement of
aggregates.\(^2\)

The next section reviews Haslag’s (1998) model and shows how it is sen-
sitive to the distribution of the lump sum inflation proceeds. This sensi-
tivity makes tentative the growth effect of inflation with the model. The
demand for reserves can be made insensitive to the distribution of the in-
flation tax transfer by framing it within a model in which the bank must
hold money in advance as in the timing of transactions that is pioneered in
Lucas (1980). This is done in Section 3 using Haslag’s (1998) notation, \(Ak\)
production technology, and full savings intermediation. The resulting real
interest rate depends negatively on the nominal interest rate, and so infla-
tion negatively affects the growth rate, similar in fashion to the central result
also added, to give a model of reserves plus currency; this modeling of the
monetary base is similar to Chari, Jones, and Manuelli (1996) except that
there, as in equation (7), the inflation rate does not effect the real interest
rate.

The paper then expands the model to give a formulation of the demand for
the base plus non-interest bearing demand deposits, or an aggregate similar
to M1.\(^3\) Following a credit production approach used in a series of related

\(^2\)Hicks (1935) seeks a theory of money based on marginal utility, with cash held in
advance of purchases, as Lucas (1980) follows. Hicks shunts aside both Keynes’s alterna-
tive to Fisher’s quantity theory as found in his Treatise (see Gillman (2002) on flaws in
this theory), and considers "Velocities of Circulation" as in Fisher’s quantity theory an
"evasion". He reasons that money use suggests the existence of a friction and that "we
have to look the friction in the face". The "most obvious sort of friction" is "the cost
of transferring assets from one form to another". And Hicks says that we should consider
"every individual in the community as being, on a small scale, a bank. Monetary theory
becomes a sort of generalisation of banking theory." In alignment with Hicks, the agent
in this paper acts as a bank in part, and the bank has costs from creating new instru-
mments such as demand deposits and credit. But in contrast, here velocity is endogenously
determined as a fundamental part of the resulting equilibrium. And Hicks’s and Lucas’s
approaches converge with Fisher’s.

\(^3\)This abstracts from the interest that is earned on some demand deposit accounts
included in the US M1 aggregate, since this interest tends to be of nominal amounts
compared to the savings accounts included in M2.
papers, the paper then adds credit, or interest bearing demand deposits, to give a formulation for an aggregate similar to M2.

2 Sensitivity to Lump Transfers

The model starts with a demand for bank reserves that builds upon Haslag (1998) and Chari, Jones, and Manuelli (1996). In Haslag (1998), all savings funds are costlessly intermediated into investment by the bank. The bank must hold reserves in the form of money. This gives rise to a bank demand for money in order to meet reserve requirements on the savings deposits. The consumer-agent does not use money although the lump sum inflation tax is transferred to the agent. Instead the agent simply holds savings deposits at the bank and earns interest as the bank intermediates all investment. The bank’s return is lowered by the need to use money for reserves. Further, the timing of the model is such that inflation decreases the real return to depositors, and therefore also the growth rate, through the requirement that reserves be held as money.

The following model gives the reported result in Haslag (1998):

$$\Pi_t = P_t(1 + A - \delta)k_t + M_{t-1} - P_t(1 + r_t)d_t. \quad (1)$$

This is stated as a maximization problem with respect to $k_t, M_t, d_t$ and subject to two constraints. The constraints (with equality imposed) are that the sum of capital and last periods real balances equals deposits:


5However to get this result, three changes were made to the model actually published in Haslag (1998), indicating incidental errors in the published paper: the money stock in the profit equation (1) is in time $t-1$, instead of $t$ as published; and the money stock and the price level in equation (2) are in time $t-1$ instead of time $t$ as published. The actual return in the paper as published is that $r_t = (A - \delta)[1 - \gamma_t(1 + g_t)]$, where $g_t$ denotes the balanced-path growth rate; it is independent of the inflation rate.
\[ k_t + M_{t-1}/P_{t-1} = d_t, \]  \hspace{1cm} (2)

and that a fraction \( \gamma_{t-1} \), given in the last period, of time \( t \) deposits is held as real money balances in time \( t-1 \):

\[ M_{t-1}/P_{t-1} = \gamma_{t-1}d_t. \]  \hspace{1cm} (3)

Assuming zero profit this yields through simple substitution the return reported by Haslag (1998):

\[ 1 + r_t = (1 + A - \delta)(1 - \gamma_{t-1}) + \gamma_{t-1}(P_{t-1}/P_t). \]  \hspace{1cm} (4)

The result is sensitive to who gets the lump sum cash transfer from the government. If the transfer instead goes to the bank, the only user of money in the model, then there is no growth effect of inflation. This can be seen in the following way: Let the money supply process be given as in Haslag (1998) as

\[ M_t = M_{t-1} + H_{t-1}, \]

where \( H_{t-1} \) is the lump sum transfer by the government. With the transfer given to the bank, the profit of equation (1) becomes

\[ \Pi_t = P_t(1 + A - \delta)k_t + M_{t-1} + H_{t-1} - P_t(1 + r_t)d_t. \]  \hspace{1cm} (5)

Let the balanced growth rate of the economy be denoted by \( g_t \), and the consumer’s time preference by \( \rho \), whereby the consumer’s problem in Haslag (1998) with log utility gives that

\[ 1 + g_t = (1 + r_t)/(1 + \rho). \]

With this growth rate in mind, the zero profit equilibrium now gives a rate of return to depositors of

\[ 1 + r_t = (1 + A - \delta)(1 - \gamma_{t-1}) + \gamma_t(1 + g), \]  \hspace{1cm} (6)

and there is no inflation tax on the return or on the growth rate.

Alternatively let the profit function be given as equation (5). Then assume that the stock and reserve constraints, equations (2) and (3), are all in terms of current period variables, as in a standard cash-in-advance economy where here the reserve constraint now would look like a Clower (1967)-type
constraint. Then the model is exactly as in Chari, Jones, and Manuelli (1996). This gives the result, also found in Einarsson and Marquis (2001), that

\[ 1 + r_t = (1 + A - \delta)(1 - \gamma_t) + \gamma_t. \]  

(7)

The return is lowered because reserves are idle but there is no inflation tax.

### 3 Models of Monetary Aggregates

#### 3.1 Monetary Base

The financial intermediary has a demand for nominal money, denoted by \( M^r_t \), as created by the need for reserves, with the reserve ratio denoted by \( \gamma \in [0, 1] \). But here, as in Chari, Jones, and Manuelli (1996), the reserve constraint is considered as the bank’s Clower (1967) constraint, and structured accordingly in a fashion parallel to the consumer’s, being that

\[ M^r_t = \gamma P_t d_t. \]  

(8)

In addition the asset constraint adds together the current period real money stock with the current period capital stock to get the current period real deposits. In real terms this is written as

\[ k_t + M^r_t / P_t = d_t. \]  

(9)

And the bank has to set aside cash in advance of the next period’s accounting of the reserve requirement in order to meet any increase in its reserve requirements. The bank has revenue from its return on investment, and costs from payment of interest to depositors, and from any increase in money holdings for reserves.

The technology for the output of goods, as in Haslag (1998), is an \( Ak \) production function, making the current period profit function:

\[ \Pi_t' = P_t (1 + A - \delta) k_t + M^r_t - M^{r+1}_t - P_t d_t (1 + r_t). \]  

(10)
The profit maximization problem is dynamic because of the way in which money enters the bank’s profit function in two different periods, the same dynamic feature of the consumer problem. The competitive bank discounts the nominal profit stream by the nominal rate of interest, and maximizes the time $t$ discounted stream, denoted by $\hat{\Pi}_t$, with respect to the real capital stock, $k_t$, the real deposits, $d_t$, and the money stock used for reserves, denoted by $M^r_t$, and subject to the asset stock and the Clower (1967)-type reserve constraints of equations (8) and (9):

$$\max_{d_s,M^r_{s+1},k_s} \hat{\Pi}_t = \sum_{s=t}^{\infty} \prod_{i=t}^{s-1} \left( \frac{1}{1+R_i} \right)^{s-t} \left\{ [P_s(1+A-\delta)k_s - M^r_s - M^r_{s+1} - P_s(1+r_s)d_s] + \lambda_s [P_s d_s - M^r_s - P_s k_s] + \mu_s [M^r_s - \gamma P_s d_s] \right\}. \quad (11)$$

Assuming a constant money supply growth rate, so that the nominal interest rate is constant over time, the first-order conditions imply that the rate of return is

$$1 + r_t = (1 + A - \delta)(1 - \gamma) - \gamma R_t. \quad (12)$$

Using the Fisher equation of nominal interest rates (presented below), with the above equation, shows that there is a negative effect of inflation on the return. Combined with the consumer’s problem and the derivation of the balanced-growth rate as depending on the real interest rate, inflation therefore causes a negative effect on the balanced-path growth rate.

The bank does not receive any lump sum transfer from the government; the consumer-agent receives it all. However the distribution only affects how much profit the intermediary makes. Since the profit is transferred to the consumer, just as is the lump sum transfer of inflation proceeds, the distribution of the inflation proceeds between the bank and the consumer can be changed without affecting the allocation of resources in the economy. For example, if the intermediary gets part of the inflation proceeds transfer,
by an amount at time \( t \) equal to \( M_{t+1}^r - M_t^r \), then in equilibrium the money terms cancel from the profit function, and \( \Pi_t^r / (P_t k_t) = R_t \left[ \gamma / (1 - \gamma) \right] \). At the Friedman optimum, this profit is zero. \(^6\)

Consider a consumer problem as in Haslag (1998) except that now the consumer uses cash as in Lucas (1980). The problem then includes the setting aside of the consumer’s cash in advance of trading in the next period, denoted by \( M_{t+1}^c \), and the receipt of the lump sum government transfer of inflation proceeds, denoted by \( H_t \).

The consumer’s Clower (1967) constraint is

\[
M_t^c = P_t c_t. \tag{13}
\]

The consumer also makes real (time) deposits, denoted by \( d_t \), with the real return, denoted by \( r_t \), as the form of all savings and wholly intermediated through banks, as in Haslag (1998). This involves choosing the next period deposits \( d_{t+1} \) and receiving as real income \((1 + r_t) \ d_t \). The nominal current period profit of the intermediation bank, \( \Pi_t^r \), is received by the consumer each period as a lump sum income source. This makes the consumer current period budget constraint of income minus expenditures as in the following:

\[
P_t (1 + r_t) d_t + H_t + \Pi_t^r + M_t^c - M_{t+1}^c - P_t c_t - P_t d_{t+1} = 0. \tag{14}
\]

The problem is to maximize the time preference discounted stream of current period utility, where \( \beta \equiv 1/(1 + \rho) \) denotes the discount factor, subject to the income and Clower (1967) constraints:

\[
Max_{c_t, d_{t+1}, M_{t+1}^c} L = \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) + \lambda_t \left[ P_t (1 + r_t) d_t + H_t + \Pi_t^r + M_t^c - M_{t+1}^c - P_t c_t - P_t d_{t+1} \right] + \mu_t \left[ M_t^c - P_t c_t \right] \right\}. \tag{15}
\]

The first-order conditions are

\(^6\)See Bailey (1992) for an early discussion of intermediary earnings during inflation. If current period non-negative profit is required for the bank intermediary to exist, then a transfer to the bank as in the above-described transfer scheme, with \( \Pi_t^r / (P_t k_t) = R_t \left[ \gamma / (1 - \gamma) \right] \), would satisfy this at all inflation rates..
\[ u_{ct} = \lambda_t P_t (1 + \mu_t / \lambda_t), \quad (16) \]

\[ \lambda_t / (\lambda_{t+1} \beta) = (1 + r_{t+1})(1 + \pi_{t+1}) \equiv (1 + R_{t+1}), \quad (17) \]

\[ \lambda_t / (\lambda_{t+1} \beta) = 1 + \mu_{t+1} / \lambda_{t+1}. \quad (18) \]

These imply that

\[ u_{ct} = \lambda_t P_t (1 + R_t), \quad (19) \]

so that the nominal interest rate is the shadow exchange cost of buying a unit’s worth of consumption. Using this later equation to form an Euler equation, then along the balanced-growth equilibrium with log utility it follows that the growth rate of consumption, where \( 1 + g_{t+1} = c_{t+1} / c_t \), is given by

\[ 1 + g_t = \frac{1 + r_t}{1 + \rho}. \quad (20) \]

The demand for money is given by the Clower (1967) constraint, \( M_t^c = P_t c_t \). This standard Lucas (1980) demand function can be thought of as a demand for “currency”, in this, the simplest version of the model. Also note that with \( P_{t+1} / P_t \equiv 1 + \pi_{t+1} \), the first order condition with respect to \( d_t \) implies a Fisher-type equation whereby

\[ 1 + R_{t+1} \equiv \lambda_t / (\lambda_{t+1} \beta) = (1 + \pi_{t+1})(1 + r_{t+1}).^7 \quad (21) \]

The total demand for money is the sum of the bank’s and the consumer’s, and this is set equal to the total money supply as a condition of market clearing in equilibrium:

\[ M_t^r + M_t^c = M_t^b. \quad (22) \]

^7 Including the market for nominal bonds as in Lucas and Stokey (1987) would give \( R_t \) as the price of the bonds and would explicitly derive the Fisher equation.
The total money supply, or market clearing, equation is that this period’s money base, denoted by $M^b_t$ plus the lump sum transfer equal next period’s base supply of money:

$$M^b_t + H_t = M^b_{t+1}.$$  \hspace{1cm} (23)

Assume that the money supply growth rate is constant at $\sigma$, where $\sigma \equiv H_t/M^b_t$.

### 3.2 M1

Now consider an extension in which the consumer suffers a nominal cost of using money that is proportional to the amount of cash used to make purchases. This can be thought of as the "convenience" cost of using money. This can be related to the average amount stolen in robberies by pickpockets, lost by carelessness, and spent on protection against crime and carelessness. And, it can be Karni’s (1974) time costs or Baumol’s (1952) shoe-leather costs. Importantly, these costs can be affected by the availability of bank locations, and now ATM locations.\(^8\) Let this amount be given by $\phi M_t^c$, with $\phi \in [0, 1)$. Second assume that a second bank exists, a bank that supplies only non-interest bearing deposits, denoted by $M^d_{t,s}$, that can be used in exchange. This money can be thought of demand deposits as in the US or as a debit card as is more common in Europe. The bank charges a nominal fee of $P^d_t$ per unit of real deposits, so that it receives from the consumer total such receipts equal to $P^d_t (M^d_{t,s}/P_t)$; and the bank produces these non-interest bearing deposits through a production process. The consumer receives from the deposit bank its nominal profit, denoted by $\Pi^d_t$, and the profit from the intermediation bank, and the lump sum inflation tax transfer.

\(^8\)We are indebted to Bob Lucas for originally suggesting this concept, and to Rowena Pecchenino. Note that these costs are on the consumer side of the problem, while costs of alternative instruments for exchange are on the banking firm side of the problem. The so-called shopping time costs (Lucas 2000) actually compare better to the bank firm costs in this problem, as is shown below in footnote 9. Karni’s and Baumol’s costs are a story more about the costs on the consumer side. The diffusion of ATMs plausibly affects both banking productivity and the consumer’s cost of using money.
The consumer’s demand for the real non-interest bearing deposits is denoted by $M_{dd}^t/P_t$.

The consumer chooses what fraction of purchases to be made with cash, denoted by $a_c^t \in [0, 1]$, and what fraction to be made with non-interest demand deposits, $a_{dd}^t \in [0, 1]$; where

$$a_c^t + a_{dd}^t = 1.$$  \hspace{1cm} (24)

The Clower (1967) constraints becomes

$$M_c^t = a_c^t P_t c_t; \quad (25)$$

$$M_{dd}^t = (1 - a_c^t) P_t c_t. \quad (26)$$

And the consumer problem now is:

$$\max_{c_t, d_{t+1}, M_{c,t+1}, M_{dd,t+1}, a_c^t} L = \sum_{t=0}^{\infty} \beta^t \{ u(c_t) + \lambda_t \left[ P_t (1 + r_t) d_t + H_t + \Pi_t^d + \Pi_{dd}^d + M_c^t + M_{dd}^t \right]$$

$$- M_{c,t+1}^c - \phi M_{c,t+1}^c - (P_{dd}^t / P_t) M_{dd}^t - P_t c_t - P_{dt,t+1} \}$$

$$+ \mu_c^c [M_c^t - a_c^t P_t c_t]$$

$$+ \mu_{dd}^d [M_{dd}^t - (1 - a_c^t) P_t c_t]. \quad (27)$$

The first-order condition with respect to $a_c^t$ gives that $\mu_{dd}^d = \mu_c^c$. In combination with the first-order conditions with respect to the two money stocks, $M_{c,t+1}^c$ and $M_{dd,t+1}^d$, this implies that

$$P_{dd}^t / P_t = \phi. \quad (28)$$

And note that the shadow cost of buying goods with cash now is given by the marginal condition:

$$u_{c_t} = \lambda_t P_t (1 + R_t + \phi), \quad (29)$$

so that the shadow exchange cost now is equal to $R_t + \phi$ instead of only $R_t$ as in the previous subsection. The demands for the cash and for the demand
deposits are given by the Clower (1967) constraints in equilibrium, where the \( a_t \) variable is determined by finding the equilibrium bank supply of demand deposits and setting this equal to the demand for demand deposits.

The original bank, the capital intermediation bank, has the same problem as stated previously. Now consider the specification for the production function of the new bank. With an \( AK \) type production function for the non-interest bearing demand-deposit bank, it can be shown that the equilibrium would not be well defined. If the \( \hat{A} \) parameter equals \( \phi \), then there is no unique equilibrium; and if \( \hat{A} \) equals any other value then there is an equilibrium either with no demand for cash or with no demand for credit. A unique equilibrium is satisfied by specifying a diminishing returns technology whereby there is a margin at which the fixed \( \phi \) is equal to the variable marginal cost of producing the demand deposits. Initially assume that the new demand deposit bank faces the following production function that is diminishing in its capital input. Denoting the shift parameter by \( \hat{A}_{dd} \) and the capital input by \( k_{dd}^t \), and with \( \alpha \in (0, 1) \), let the function be specified as

\[
M_{dd,s}^t / P_t = \hat{A}_{dd}(k_{dd}^t)^\alpha. \tag{30}
\]

With the current period profit, \( \Pi_{dd}^t \), given as the revenue minus the costs, the deposit bank faces the following static profit maximization problem:

\[
\text{Max}_{k_{dd}^t, M_{dd}^t} \Pi_{dd}^t = P_{dd}^t(M_{dd,s}^t / P_t) - P_t r_t k_{dd}^t \\
+ \lambda_t \left[ \hat{A}_{dd}(k_{dd}^t)^\alpha - (M_{dd,s}^t / P_t) \right]. \tag{31}
\]

The first-order conditions imply that

\[
P_{dd}^t / P_t = r_t / [\hat{A}_{dd}\alpha(k_{dd}^t)^\alpha - 1], \tag{32}
\]

which when combined with the consumer’s equilibrium condition (28) gives that

\[
\phi = r_t / [\hat{A}_{dd}\alpha(k_{dd}^t)^\alpha - 1], \tag{33}
\]

11
or, that

\[ k_{dd}^t = (\hat{A}_{dd} \alpha \phi / r_t)^{1/(1-\alpha)}, \]

which gives the supply of demand deposits as

\[ M_{dd,s}^t / P_t = \hat{A}_{dd}^{1/(1-\alpha)} (\alpha \phi / r_t)^{(1-\alpha)} /(ID) \]

As the cost of using money due to security, \( \phi \), goes to zero, the capital used in produced non-interest bearing deposits, or debit cards, also goes to zero, as does the output of such debit. If \( \phi = 0 \), then the consumer uses only cash.

Here \( M_{dd,s}^t / P_t = M_{dd}^t / P_t \) and the M1 aggregate can be represented as follows:

\[ M_{ct}^t + M_{dd}^t \equiv M1A_t. \]

The problem with this specification is that, in the equilibrium, with a constant rate of money supply growth and a positive growth rate \( g_t \), the ratio of \( M_{ct}^t / M_{dd}^t \) is increasing towards infinity. While there may be some trend in this ratio empirically, this trend should be explana

bly by changes in other exogenous factors that determine the ratio; with constant exogenous factors, theoretically the trend should be stable on the balanced growth path. To see that the ratio is not stable, equations (25) and (26) imply that \( M_{ct}^t / M_{dd}^t = a_t^c / (1 - a_t^c) \). The solution for \( a_t^c \) is found by setting equal the supply and demand from equations (26) and (35), giving that

\[ a_t^c = 1 - [\hat{A}_{dd}^{1/(1-\alpha)} (\alpha \phi / r_t)^{(1-\alpha)} / c_t] / [\hat{A}_{dd}^{1/(1-\alpha)} (\alpha \phi / r_t)^{(1-\alpha)} / c_t], \]

or

\[ a_t^c / (1 - a_t^c) = \{ c_t / [\hat{A}_{dd}^{1/(1-\alpha)} (\alpha \phi / r_t)^{(1-\alpha)}] \} - 1. \] By inspection it is clear that with \( c_t \) rising when there is positive growth on the equilibrium path, and with the nominal interest rate being stable given that there is a stationary inflation rate, the ratio \( a_t^c / (1 - a_t^c) \) also rises towards infinity towards a cash-only solution with no demand deposits.
An alternative production function that gives a stationary ratio of $M^c_t/M^{dd}_t$ is one that includes an externality that affects the shift parameter $\hat{A}_{dd}$. In particular let $\hat{A}_{dd} = A_{dd} c_t^{1-\alpha}$, so that the production function is CRS in terms of capital and goods consumption:

$$
\frac{M^{dd,s}_t}{P_t} = A_{dd} c_t^{1-\alpha} (k^{dd}_t)^\alpha.
$$

(37)

This function is a type of “congestion” function that is found in the literature and that Barro and Sala-I-Martin (1995) use in the growth context for government services. It has the property that the share of goods bought with demand deposits, $a^{dd}_t$, is a function of the capital to goods ratio; by equations (24), (26), and (37),

$$
a^{dd}_t = A_{dd} (k^{dd}_t/c_t)^\alpha.
$$

(38)

Substituting the alternative production function into the profit maximization problem of equation (31), with $\hat{A}_{dd} = A_{dd} c_t^{1-\alpha}$, the solution is

$$
k^{dd}_t/c_t = (A_{dd} c_t^{1-\alpha} / r_t)^{1/(1-\alpha)}.
$$

(39)

And from equations (38) and (39), the solution for the equilibrium share of demand deposits is

$$
a^{dd}_t = A^{1/(1-\alpha)}_{dd} (c_t/r_t)^{\alpha/(1-\alpha)}.
$$

(40)

With the production function of equation (37), the balanced-growth path exists and the ratio $M^c_t/M^{dd}_t$ is stationary along it. Stationarity of $M^c_t/M^{dd}_t$ follows directly from above where it is shown that $M^c_t/M^{dd}_t = a^{dd}_t/(1-a^{dd}_t)$. By equation (24) this can be written as $M^c_t/M^{dd}_t = (1-a^{dd}_t)/a^{dd}_t$ and by inspection of equation (40) can be seen to be stationary.

9 Note that if $a^{dd}_t = 1$, and so $a^c_t = 0$, there would be no consumer demand for cash. The monetary equilibrium would still have well-defined nominal prices as long as $\gamma > 0$, so that there was a reserves demand for cash by the intermediation bank. This could then be characterized solely as a “legal restrictions” demand for money. At $a^c_t = 0$, and $\gamma = 0$, and with a positive supply of money, prices may not be well-defined.
3.3 M2

The model can be expanded to its full form by allowing the agent the choice of using costly credit to make purchases, or “exchange credit”, along with cash or non-interest bearing demand deposits. Here the credit is like a credit card, such as the American Express card, rather than a debit card. The agent must pay a fee for this service that is proportional to the amount of the exchange credit; this is like the percentage fee paid by stores using the American Express card (without a roll-over debt feature). Denoting the time $t$ nominal amount of exchange credit demanded by the consumer by $M_{cd}^t$, and the nominal fee by $P_{cd}^t$, the consumer’s expenditure on such fees is given by $(P_{cd}^t/P_t)M_{cd}^t$. The agent also receives the nominal profit of the exchange credit bank, denoted by $\Pi_{cd}^t$. And the consumer must pay off the debt incurred using the exchange credit at the end of the period. But this credit saves the agent from having to set aside money in advance of trading, and so allows avoidance of the inflation tax. And now with three types of exchange, let the share of consumption good purchases made by cash and by non-interest bearing demand deposits remain notated by $a_{ct}^t$ and $a_{dd}^t$, and the share of consumption good purchases made by exchange credit by $a_{cd}^t$, where the shares sum to one:

$$a_{ct}^t + a_{dd}^t + a_{cd}^t = 1. \tag{41}$$

This adds a third Clower (1967) constraint to the consumer’s problem, allowing the three constraints to be written as

$$M_{ct}^t = P_t c_t a_{ct}^t, \tag{42}$$
$$M_{dd}^t = P_t c_t a_{dd}^t, \tag{43}$$
$$M_{cd}^t = P_t c_t (1 - a_{ct}^t - a_{dd}^t). \tag{44}$$

The consumer problem now buys goods with cash or demand deposits as before, but also has a debit of $-a_{cd}^t P_t c_t$ for AMEX purchases, and has a debit of $-(P_{dd}^t/P_t)M_{td}^t$ due to the AMEX fee. This makes the consumer problem
Max
\[ L = \sum_{t=0}^{\infty} \beta^t \{ u(c_t) \} \]
\[ + \lambda_t \left[ P_t (1 + r_t) d_t + H_t + \Pi_t^d + \Pi_t^dd + \Pi_t^c + M_t^c + M_t^dd - M_{t+1}^c \right. \]
\[ - \phi M_{t+1}^c - M_{t+1}^dd - (P_{t+1}^dd / P_t) M_{t+1}^dd - (P_{t+1}^c / P_t) M_{t+1}^c - P_{t+1} c_t - P_t d_{t+1} \]  
\[ \left. + \mu_t^c \left[ M_t^c - a_t^c P_t c_t \right] \right. \]
\[ + \mu_t^d \left[ M_t^dd - a_t^dd P_t c_t \right] \]
\[ + \mu_t^cd \left[ M_t^cd - (1 - a_t^c - a_t^dd) P_t c_t \right] \]  
\[ + \mu_t^cd \left[ M_t^cd - \left( \frac{M_{t+1}^cd}{P_{t+1}} \right) \right] \]  
\[ + \mu_t^cd \left[ M_t^cd - (1 - a_t^c - a_t^dd) P_t c_t \right] . \]  
\[ (45) \]

Denote the name for the exchange credit banking firm as Amex. Amex is assumed to supply the exchange credit, denoted by \( M_{t,s}^{cd} \), using only capital, denoted by \( k_t^{cd} \), in a diminishing returns fashion similar to the technology for the demand deposit bank. While this technology could be given as \( (M_t^{cd,s} / P_t) = \hat{A}_{cd}(k_t^{cd})^\theta \), where \( \hat{A}_{cd} > 0 \) and \( \theta \in (0, 1) \), for a general diminishing returns case, the problem would arise that the equilibrium share of the Amex credit would trend down towards zero if there was a positive growth rate \( g_t \), making infeasible the existence of a balanced-growth path. Therefore consider a technology similar to equation (37), which gives a stable share of exchange credit in purchases. In particular, let the function be specified with a congestion-type externality that affects the shift parameter \( \hat{A}_{cd} \), whereby \( \hat{A}_{cd} = A_{cd} c_t^{1-\theta} \), so that
\[ M_t^{cd,s} / P_t = A_{cd} c_t^{1-\theta} (k_t^{cd})^\theta . \]  
\[ (46) \]

The profit maximization problem is static and given by
\[ \text{Max} \quad \Pi_t^{cd} = P_t^{cd} (M_t^{cd,s} / P_t) - P_t r_t k_t^{cd} \]
\[ + \lambda_t \left[ A_{cd} c_t^{1-\theta} (k_t^{cd})^\theta - (M_t^{cd,s} / P_t) \right] . \]  
\[ (47) \]

The equilibrium conditions of the consumer and Amex bank imply that
\[ R_t + \phi = P_t^{cd} / P_t = r_t / [A_{cd} \theta (k_t^{cd} / c_t)^{\theta - 1}] ; \]  
\[ (48) \]
\[ k_t^{cd} / c_t = [A_{cd} \theta (R_t + \phi) / r_t]^{1/(1-\theta)} . \]  
\[ (49) \]
This means that as the nominal interest rises the Amex bank expands credit supply and \( k_t^{cd}/c_t \) rises in equilibrium.

Equating the supply and demand for the Amex credit, from equations (44) and (46), and using the above equation (48), the share of exchange credit can be found to be

\[
a_t^{cd} = A_{cd}^{1/(1-\theta)} \theta (R_t + \phi)/r_t \theta/(1-\theta),
\]

also rising as the nominal interest rate goes up. And note that by substituting equation (50) into equation (41), so that \( 1 - a_t^{cd} = a_t^c + a_t^{dd} \), and then substituting in equation (40), the solution for \( a_t^c \) is found.\(^{10}\)

Figure 1 illustrates the equilibrium for the credit bank. At the Friedman optimum of \( R = 0 \), some credit would still be provided as long as \( \phi > 0 \). This use of credit at \( R = 0 \) contrasts to zero such use of credit in Gillman (1993), Ireland (1994), and Gillman and Kejak (2002).

The money market clearing condition here is that the demand for the exchange credit equals the supply of the exchange credit. This can also be further aggregated to

\[
M_t^c + M_t^{dd} + M_t^{cd} \equiv M2_t,
\]

and can be considered an aggregate like M2. It includes the monetary base, demand deposits, plus the exchange credit that allows funds to interest during the period, as do certificates of deposit, and is then paid off with “money market mutual funds” invested in short-term government securities. So it

\(^{10}\)Alternatively the exchange credit sector can be kept implicit by having the consumer engage in “self-production” of the exchange credit. This can be done by constraining the consumer’s problem by the technology constraint (46), combining this constraint with equation (44), solving for \( a_t^{cd} \), and using this to substitute in for \( a_t^{cd} \) in the consumer problem (45), with the consumer now choosing \( k_t^{cd} \) instead of \( a_t^{cd} \). This approach would make the revised Clower constraint (44) equal to \( M_t^{cd} = P_t A_{cd} c_t^{-\theta} (k_t^{cd})^\theta \). Setting \( \gamma = 0 \) and \( \phi = 0 \), then \( M_t^c / (P_t c_t) = 1 - a_t^{cd} = a_t^c \), and only this one Clower constraint would be necessary. Now solve this constraint for \( k_t^{cd} \), and it would take a form exactly analogous to a special case of the McCallum and Goodfriend (1987) shopping time constraint, but in capital instead of time, that depends on real money balances and goods in the same direction: \( k_t^{cd} = c_t [1 - (M_t^c / P_t)^{1/\theta} / (A_{cd})^{1/\theta}] \), with \( \partial k_t^{cd} / \partial (M_t^c / P_t) < 0 \), and \( \partial k_t^{cd} / \partial c_t > 0 \) (See Walsh (1998), on shopping time models.)
Figure 1: Equilibrium in the Credit Bank Sector

is a mixed set of non-interest bearing aggregates that suffer the inflation tax, and are traditionally thought of as money-like in nature, and of the Amex credit and money market accounts that avoid the inflation tax, unlike “money”.

4 Changes in Aggregates Over Time

The model of M2 can be used to analyse how subsets of aggregates change according to changes in exogenous factors. In particular the focus is on changes in the money supply growth, $\sigma$, or more simply in the nominal rate of interest since this is given by $R = \sigma + \rho$. Also the focus is on changes in the banking productivity parameters $A_{dd}$ and $A_{cd}$, and the banking cost parameter $\phi$. Comparative statics of these factors are then applied to explain the actual profiles of the velocity of monetary aggregates, and the profiles of their ratios.
4.1 Financial Deregulation and the Increase in Bank Productivity


"Congress passed significant reform legislation in the 1990s. In 1994, the Riegle–Neal Interstate Banking and Branching Efficiency Act repealed the McFadden Act of 1927 and Douglas Amendments of 1970, which had curtailed interstate banking. In particular, the McFadden Act, seeking to level the playing field between national and state banks with respect to branching, had effectively prohibited interstate branch banking. Starting in 1997, banks were allowed to own and operate branches in different states. This immediately triggered a dramatic increase in mergers and acquisitions. The banking system began to consolidate and for the first time form true national banking institutions, such as Bank of America, formed via the merger of BankAmerica and NationsBank." (Guzman 2003).

The 1999 law permitted mergers between banks, brokerage houses, and insurance companies, "allowing banking organizations to merge with other types of financial institutions under a financial holding company structure" (Hoenig 2000).

These banking deregulations can affect the model primarily by increasing the productivity parameters in the banking sector, $A_{cd}$ and $A_{dd}$. Analytically, a proportional tax can be imposed on the banking firms, and then this tax reduced with the advent of deregulating laws. This is equivalent to an increase in the productivity factors.
4.2 Comparative Statics and Comparison to the Evidence

The income velocity of money is defined as income divided by a particular monetary aggregate. The income in the economy typically is consumption plus investment, or consumption plus savings. In the representative agent model investment usually equals savings exactly but here there is a difference due to the cost of intermediation. Investment plus this intermediation cost is equal to savings, and so one way to define income is as consumption plus savings. In the model this is \( c + d \) where \( d \) are the funds the consumer deposits in the financial intermediary for its investment. The velocity of the monetary aggregates can then be defined using \( c + d \), so that the definition of base velocity accordingly is \( (c + d)/MB \).

**Proposition 1** Given \( \sigma > \phi \), and along the balanced path, the base money velocity rises with the nominal interest rate, or \( \partial[(c_t + d_t)/MB_t]/\partial R > 0 \).

**Proof.** The solution for the Base velocity is

\[
(c_t + d_t)/MB_t = [1 + (d_t/c_t)] / \left[ 1 - a_{dd}^t - a_{cd}^t + \gamma (d_t/c_t) \right],
\]

where

\[
a_{dd}^t = A_{dd}^{1/(1-\alpha)}(\alpha \phi/r_t)^{\alpha/(1-\alpha)},
\]

\[
a_{cd}^t = A_{cd}^{1/(1-\theta)}[\theta(R_t+\phi)/r_t]^{\theta/(1-\theta)},
\]

\[
r_t = (A-\delta)(1-\gamma) - \gamma(1+R_t),
\]

and

\[
d_t/c_t = \left( \frac{1 + r_t^{-\theta/(1-\theta)}[A_{cd}\theta(R_t+\phi)]^{1/(1-\theta)}\left[ 1 - \frac{\phi}{r_t} \right] + r_t^{-\alpha/(1-\alpha)}(A_{dd}\alpha\phi)^{1/(1-\alpha)}[1+(1/\alpha)(\frac{\phi}{r_t})] \right) \]

\[
\frac{1+\gamma}{\left[ 1+(1+\gamma)/\left( 1+\gamma \right) \right] + \gamma \sigma}.
\]

Note that the solution of \( d_t/c_t \) requires substituting into the budget constraint of the problem in equation (45), using equations (10), (22), (23), (31), (32), (40), (42), (43), (44), (47), (48), and (50). Assuming \( \sigma > \phi \), it can be seen from inspection that, with \( \partial r_t/\partial R < 0 \), it must be true that \( \partial (d_t/c_t) / R > 0 \). This result and inspection of the \( a_{dd} \) and \( a_{cd} \) terms indicates that \( \partial[(c_t + d_t)/MB_t]/\partial R > 0 \).

Figure 2 shows the post 1959 US base money velocity and the 10 year bond, US Treasury, interest rate. McGrattan (1998) presents such a graph and argues, in her comment on Gordon, Leeper, and Zha (1998), that the nominal interest rate goes a long way to explaining base money velocity.\(^{11}\)

\(^{11}\)McGrattan (1998) argues that the long term rate is better to use than the short term rate that Gordon, Leeper, and Zha (1998) use. "Low frequency movements in velocity are well-explained by low frequency movements in observed interest rates."
And this is the implication of the result of Proposition 1. The difference from McGrattan (1998) is that she uses a simple linear econometric equation, as found in Meltzer (1963) and Lucas (1988), to argue that the nominal interest rate has a direct effect on velocity. Here the velocity is derived analytically to make the point from the general equilibrium perspective.

Comparative statics for the other factors, $A_{cd}, A_{dd},$ and $\phi,$ are ambiguous in general because of the $d_t/c_t$ factor, but holding $d_t/c_t$ constant then all three factors have a positive effect on base velocity. This positive direction of the effect of these factors is also readily apparent in calibrations. While these other factors do not provide any obvious help in interpreting base velocity empirical evidence, they do provide an explanation as based on the model of the evidence on the ratio of reserves to currency.

Figure 3 shows the post 1959 US reserves/currency ratio against the long term interest rate. There is a marked trend down, with a flattening out period during the 1980s, and a rather more pronounced downward direction after 1994. Noting that $M_r^t/M_t^c$ is the model’s notation for the reserves to
currency ratio, the comparative statics are that, given $\sigma > \phi$, it is unambiguous that $\partial\left(\frac{M_r}{M_c}\right)/\partial R > 0$; with $d_t/c_t$ held constant, $\partial\left(\frac{M_r}{M_c}\right)/\partial \phi > 0$, $\partial\left(\frac{M_r}{M_c}\right)/\partial A_{cd} > 0$, and $\partial\left(\frac{M_r}{M_c}\right)/\partial A_{cd} > 0$. Since the US reserves/currency trend is downward, while the effect of the nominal interest is upward in the 1959-1981 period, it appears that the nominal interest plays no role in explaining this ratio. In contrast, a downward trend in the cost of using money, $\phi$, serves well to explain the evidence.

**Proposition 2** Given $\sigma > \phi$, along the balanced growth path, $M1$ velocity rises with the nominal interest rate, or $\partial\left(\frac{(c_t + d_t)/M1_t}{\partial R} > 0$.

**Proof.** $M1$ velocity is defined is given by $(c_t+d_t)/M1_t = [1 + (d_t/c_t)] / \left(1 - a_{cd}^{t}\right)$.

From the proof to proposition 1, with $\sigma > \phi$, then $\partial\left(d_t/c_t\right)/R > 0$, and with $a_{cd}^{t} = A_{cd}^{1/(1-\theta)}[\theta(R_t + \phi)/r_t]^{\theta/(1-\theta)}$, and $\partial a_{cd}^{t}/\partial R > 0$. It follows that $\partial\left((c_t + d_t)/M1_t\right)/\partial R > 0$.

The other comparative statics are that with $d_t/c_t$ constant, then an increase in $A_{cd}$ and $\phi$ cause the $M1$ velocity to go up.
Figure 4 shows the US M1 velocity and the 10-year US Treasury interest rate from 1959 to 2003. The rise in velocity from 1959 to 1981 is consistent with the rise in the nominal interest rate. While still following changes in the nominal interest rate in the 1980s, M1 velocity appears to level off rather than fall during this period by as much as would be expected from the decrease in the nominal interest rate. Deregulation of the 1980s, and an associated increase in $A_{cd}$ presents an explanation of the leveling off of velocity in the 1980s. The striking trend upwards in velocity after 1994, as with the reserves to currency ratio is consistent with an accelerated increase in $A_{cd}$ that can be from the deregulation of interstate branching that led to national branching and the diffusion of ATMs, as well as the banking consolidation because of the 1999 act. Thus the two factors of the nominal interest rates and the banking productivity each play a distinct role in this explanation.\(^{12}\)

A way to see further into the M1 velocity profile is to look at the ratio of its components, currency and demand deposits. Analytically the demand deposit to currency ratio in the model is $\frac{M^{dd}}{M^c}$. Since

$$\frac{M^{dd}}{M^c} = \frac{A_{dd}^{\frac{1}{(1-\alpha)}}(\alpha \phi / r_c)^{\alpha} / (1-\alpha)}{[1 - A_{dd}^{\frac{1}{(1-\alpha)}}(\alpha \phi / r_c)^{\alpha} / (1-\alpha) - A_{cd}^{\frac{1}{(1-\theta)}}(R_t + \phi / r_t)^{\theta} / (1-\theta)]},$$  

the comparative statistics with respect to $R$, $A_{cd}$, $A_{dd}$, and $\phi$ are unambiguously: $\frac{\partial (M^{dd}/M^c)}{\partial R} > 0$, $\frac{\partial (M^{dd}/M^c)}{\partial A_{cd}} > 0$, $\frac{\partial (M^{dd}/M^c)}{\partial A_{dd}} > 0$, and $\frac{\partial (M^{dd}/M^c)}{\partial \phi} > 0$.

Figure 5 shows the US demand deposit to currency ratio, and the 10-year US Treasury interest rate for the same 1959-2003 period. In a first look, the ratio simply trends down. But looking more closely shows a simple trend down, from 1959 to 1981, that levels off in the 1980s, as with M1 velocity, and then moves down steadily post 1994 at an accelerated rate compared to the earlier period.

A downward trend in $\phi$ well explains the downward trend in the demand deposit to currency ratio in a way the nominal interest rate’s pre-1981 upward

\(^{12}\)Ireland (1995) compares US M1-A velocity with 6-month Treasury bill interest rates. He explains velocity as following a continuous upward trend due to financial innovation. To provide evidence on $A_{cd}$, or on financial deregulation is beyond the scope of both Ireland and this paper, resulting instead in an analytic approach. However see Gillman and Otto (2002) for a paper that uses a time series on the productivity of banking to estimate a money demand function similar to this paper’s M1 money demand.
Figure 4: US M1 Velocity and Nominal Interest Rates: 1959-2003

trend and a possible upward trend in $A_{cd}$ and $A_{dd}$ cannot. However the role of $A_{cd}$ and $A_{dd}$ again emerges as the only way to explain the leveling off of the trend in demand deposits to currency in the 1980s, when there was financial deregulation and a surge in $A_{cd}$ and $A_{dd}$. Further the accelerated downward trend in the ratio after 1994 is consistent with an accelerated decrease in $\phi$ because of the ATM diffusion.\footnote{Note that stable deposit to currency ratios were reported by Cagan (1956) for the hyperinflations he studied (an exception was post WWII Hungary that Cagan suggests is due to data problems). This indicates a small role of the nominal interest rate in causing changes in this ratio, and is consistent with the small role given here to the nominal interest rate in explaining the US ratio’s postwar movement.}

Now consider the velocity of the broader aggregate M2. In the model, M2 velocity is defined by $(c_t + d_t) / M2_t$. This is given by 

$$(c_t + d_t) / [c_t (a_{ct}^{dd} + a_{ct}^{cd})] = 1 + (d_t/c_t).$$

The comparative statics of the M2 velocity are therefore as the comparative statics of the ratio of savings to consumption. The effects of $R$, $A_{cd}$, $A_{dd}$, and $\phi$ are ambiguous in general, although with $\sigma > \phi$, it is true as shown above that $\partial (d_t/c_t) / \partial R > 0$. But the
Figure 5: US Demand Deposits to Currency Ratio and Interest Rates: 1959-2003
Figure 6: US M2 Velocity and Nominal Interest Rates: 1959-2003

The \((d_t/c_t)\) factor does not appear to play any significant role in the explanation of base or M1 velocity. Figure 6 indeed shows that US M2 velocity has been remarkably constant relative to the 10-year US Treasury bond rate. Thus the explanation from the model is that the magnitude of changes in \((d_t/c_t)\), because of the factors considered here, is small. It is easy to confirm this with calibrations, although this exercise is not reported. However one aspect of this is worth noting. With a relatively unchanging \(d_t/c_t\) as the explanation for a stable M2 velocity, it is internally consistent with the previous analysis that the comparative statics of \(A_{cd}\), \(A_{dd}\), and \(\phi\), with \(d_t/c_t\) held constant, can be used to explain Base and M1 velocity.

Breaking down the components of M2 is more revealing. Consider the ratio of M2 to M1. In the model this is given by

\[
M2_t/M1_t = \left[ 1 + (d_t/c_t) \right] / \left[ 1 - A_{cd}^{1/(1-\theta)} \left( \theta(R_t + \phi)/r_t \right)^{\theta/(1-\theta)} \right].
\]

**Proposition 3** Given \(\sigma > \phi\), along the balanced growth path, the ratio \(M2_t/M1_t\) rises with an increase in the nominal interest rate, or \(\partial (M2_t/M1_t) / \partial R > 0\).
Proof. As shown in Proposition 1, for $\sigma > \phi$, $\partial (d_t/c_t) / \partial R > 0$. And is then clear from examination that for $\sigma > \phi$, $\partial (M_2 t / M_1 t) / \partial R > 0$.

The other comparative statics with respect to $\alpha_{cd}$ and $\phi$ are ambiguous because of the $d_t/c_t$ factor; holding $d_t/c_t$ constant, the ratio $M_2 t / M_1 t$ rises with each of these. Now consider Figure 7, which shows the US ratio of M2 to M1 from 1959 to 2003, along with the 10-year US Treasury bond rate. Proposition 3 provides a way to explain the upward trend in M2/M1 from 1959 to 1981, and perhaps the fall in M2/M1 from 1990 to 1994. The leveling off of M2/M1 in the 1980s can be explained by financial deregulation and increases in $\alpha_{cd}$; note that the downward change in $R$ during this period, and a downward trend in $\phi$ during this period cannot explain the leveling off of M2/M1, as these factors work to make the ratio go down. The trend upwards after 1994 again can be explained by upward increases in $\alpha_{cd}$ because of national branching being allowed, ATM diffusion, and consolidation.
5 Discussion

The demand for bank reserves that Haslag (1998) put forth helps pave the way for modeling the demand for a range of monetary aggregates. In a sense the Haslag (1998) model as revised here acts as the missing link that ties together conventional money demand functions from the cash-in-advance approach with an analogue to the monetary aggregates widely studied, by adding a bank’s demand for cash reserves. An inflation tax on the deposit rate of return results because, as in the cash-in-advance economies, the intermediation bank must in effect put aside cash-in-advance in order to meet the demands of the reserve requirement.

Because the cash reserve requirement acts as an inflation tax on the intermediated investment, the model implies a type of “inverse Tobin (1965)” effect in which inflation increases cause a decrease in the economy’s capital stock. This effect is focused on by Stockman (1981) in which the Clower (1967) constraint is applied to all investment. Here however, the intermediation bank’s Clower (1967) constraint applies only to the reserve fraction of the investment rather than to all investment as in Stockman and so its inverse Tobin (1965) effect is weaker.

On the basis of the intermediation bank’s demand for reserves plus the imposition of a standard Clower (1967) constraint on the consumer’s purchase of goods, the demand for an aggregate similar to the monetary base, reserves plus currency (cash), is constructed whereby the inflation rate can affect the real return to intermediated investment under an AK technology because of the need to hold cash reserves. This model is extended to include non-interest bearing deposits, in a way that gives an aggregate analogous to M1. The model further is extended to include exchange credit, to give an aggregate analogous to M2. In this fully extended model comparative statics are presented for Base, M1 and M2 velocity, and the ratios of demand deposits to reserves, demand deposits to currency, and M2/M1. With these analytics the empirical evidence on the velocities and ratios are explained, requiring more that only the nominal interest rate.

The models here enable the consumer to choose the least expensive source
of exchange means. As a result, the Clower (1967) constraints are not “exogenously” imposed upon the consumer but rather left as a consumer choice to bind certain fractions of purchases to particular exchange means only to the extent that the particular exchange means is efficient for the consumer to use. This consumer choice amongst alternative means of exchange might be seen as ameliorating the strength of the criticism of the “deep” models of money that the Clower (1967) constraint is exogenously imposed, or even as offering an alternative approach to the search for deep models.14

Note that the model of the exchange credit sets the quantity of credit that is produced equal to the value of the output of the consumption good that is being bought on credit. Aiyagari, Braun, and Eckstein (1998) instead model credit as a service that is produced, and then enters as an input into a production function for credit goods. The credit goods production is Leontieff in its inputs of the credit service and of the value of the consumption goods being bought with the credit. This Leontieff technology in equilibrium implies as a special case the condition that the credit services output equals the value of the of consumption goods being bought with the credit.15 In the paper here, as in Gillman (1993), Ireland (1994), and Erosa and Ventura (2000), there are no credit or cash goods per se, but only the consumption good that can be bought with cash or credit. This in a sense can be thought of as collapsing the Aiyagari, Braun, and Eckstein (1998) -type credit goods and credit services into a single technology called credit, whereby the equilibrium condition that is implied by the special case of the Leontieff technology of Aiyagari, Braun, and Eckstein (1998) is implicitly applied.

The model’s implications for growth are that inflation lowers growth because it lowers the real interest rate, a result supported in Ahmed and Rogers (2000). However, this feature combined with an Ak goods production technology cannot account for the substitution from effective labor to capital,

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14 See Bullard and Smith (2001), and Azariadis, Bullard, and Smith (2000), for example, for an alternative approach to modeling "inside" money, based on a three-period model. They apply this to analyse the optimality of restricting inside money: Gillman (2000) analyses the optimality of such restrictions in a model similar to the paper here.

15 The case is that $q = 1$ in Aiyagari, Braun, and Eckstein (1998) model, using their notation.
as induced by inflation, that Chari, Jones, and Manuelli (1996) describe and that Gillman and Nakov (2003) further elaborate; Gillman and Nakov (2003) find evidence in support of this substitution for the postwar US and UK data. Thus while the Ak model provides easier analytic tractibility, a goods production function with both labor and capital as in Gomme (1993) and Gillman and Kejak (2002) also can account for a negative effect of inflation on growth. And since this approach also involves the inflation-induced labor to capital substitution, it may be useful to nest the models of monetary aggregates within the Gomme (1993) framework.

Gillman and Kejak (2002) go partly in this direction by extending Gomme (1993) so as to include credit, as in Section 3.3 of this paper. One advantage of having monetary aggregates more fully embedded in the King and Rebelo (1990)-type of endogenous growth model is that this provides the channels by which to substitute away from inflation and so make the inflation tax less burdensome to the individual consumer. As Gillman and Otto (2002) show, the Gillman and Kejak (2002) model creates an interest elasticity of money demand that rises in magnitude with inflation. This feature also exists in this model of this paper, and this is the central feature of the Cagan (1956) model. Or as Martin Bailey put it "Cagan’s principal conclusion, indeed, is that the demand for real cash balances ... has a higher and higher elasticity at higher and higher rates of inflation" (Bailey 1992). And Mark and Sul (2002) report recent international panel evidence in support of the Cagan (1956) money demand function. Only with such an elasticity, within the general equilibrium money demand function, are Gillman, Harris, and Matyas (2003) able to show that they can explain international evidence on inflation and growth.

16 See also Jones and Manuelli (1995).
17 Paal and Smith (2000) offer an overlapping generations model in which low inflation can cause a positive effect on growth, while higher inflation causes a negative level. This is supported in the panel evidence of Ghosh and Phillips (1998), Khan and Sendjaji (2000), Judson and Orphanides (1996), and Gillman, Harris, and Matyas (2003) in which a threshold level of inflation is found after which the inflation-growth effect is negative. However the positive effect at low inflation rates is found to be insignificant in these papers. And Gillman, Harris, and Matyas (2003) show that using instrumental variables, the effect of inflation on growth is negative for all positive levels of inflation, across both OECD and APEC regions, as well as in the full sample; Ghosh and Phillips (1998) also find this for
References


a full sample.


