Bargaining, Search and Outside Options

Anita Gantner*
Department of Economics
University of California, Santa Barbara

Preliminary Version
January 18, 2002

Abstract
This paper studies a bargaining model between a buyer and a seller, where both agents have incomplete information about the opponent’s valuation for the good to be traded, and where the buyer’s outside option is to buy via search. The model extends the Chatterjee and Samuelson (1987) model by introducing an outside option which is modelled as a standard sequential search process, where the buyer can choose to search and return to bargaining at any time. We distinguish two regimes for the search process: In Regime I, the expected return from search is used as an outside option. Here we find that the option to return to the bargaining table is redundant and only the search parameters are relevant for the buyer’s decision to quit the bargaining partner and start search. In Regime II, the buyer has to use actual offers as a valid outside option. Here the results show how the conditions to start search and to continue search differ.

Keywords: Bargaining, Incomplete Information, Outside Option, Search
JEL Classification: C78, D83, C61

*Department of Economics, University of California, Santa Barbara, CA 93106, Email: gantner@econ.ucsb.edu. I would like to thank Ted Bergstrom, Ken Binmore, John McCall, Chris Proulx, Cheng-Zhong Qin and Larry Samuelson for helpful comments. Special thanks to Morten Bech for discussions and encouragement.
1 Introduction

One of the most basic situations in economic analysis is when a buyer and a seller trade one unit of an indivisible good, both agents seeking to maximize their individual surplus. A simple setting like this can answer important questions like: When will the agents trade? What will the price be? Who will get the highest surplus? Of course, it depends on the exact circumstances, or the “rules of the game”, how these questions will be answered. For example, if a seller and a buyer meet and bargain, the analysis will be different from the situation where a buyer can choose to accept one of many non-negotiable price offers from different sellers, which he receives at different times.

In a bargaining situation, it also depends on the rules of bargaining and the available information about agents and their strategies how the equilibrium price is determined and how the surplus is allocated. Rubinstein’s (1982) model (or some modified version thereof) is commonly used to describe the bargaining procedure where two agents alternate in proposing prices and the good is traded as soon as one party accepts a proposal. This model has been extended in order to analyze equilibrium strategies when there is incomplete information (see for example Fudenberg/Tirole (1983) for a two-period model with one-sided and two-sided incomplete information, Rubinstein (1985) for one-sided incomplete information with an infinite time horizon, Chatterjee and Samuelson (1987) for two-sided incomplete information in an infinite time horizon model).

On the other hand, when prices are posted and non-negotiable, and buyers can look around for the best offer, a search theoretic approach using Bellman’s principle of dynamic optimality is useful to answer the fundamental question of price, surplus and timing of trade. Elementary search models and their applications, as presented in Lippman and McCall (1976) and (1981), show how to find the optimal reservation price and how to derive the optimal stopping rule when an agent, who seeks to maximize his surplus, faces a given distribution of price offers and search is costly. In a more recent paper, Arnold and Lippman (1998) analyze a seller’s choice between posting a price and bargaining in a model with incomplete information about buyers’ valuations and their bargaining abilities.

How can bargaining and search models be connected? There are two obvious ways to do this:
(i) The game starts in a search process where an agent is looking for another agent to trade with, that is, a particular match is not yet formed. Once two agents are matched, they start to bargain over the price. Mortensen (1982) is one of the early models that incorporate bargaining in a search and matching model, following the game theoretic approaches of papers by Diamond and Maskin (1979) and Mortensen (1978). These models, however, put less emphasis on the bargaining process. They assume that agreements are instantaneous where the available surplus is divided in a predetermined way. A natural candidate is, of course, Nash’s axiomatic bargaining solution, according to which the surplus is divided equally. Rubinstein and Wolinsky (1985) treat the bargaining problem with the strategic approach, which, as they remark, “constitutes an attempt to look into the bargaining black-box”, hence complementing the above mentioned literature. However, their matching technology is not modelled explicitly, and searching simply means considering a fixed probability of meeting an agent of the opposite type.

(ii) The game starts with the bargaining process of a particular pair of matched agents. Here, at least one agent has an outside option, which is to leave the current bargaining partner in order to look for a higher surplus. This can be modelled as a search process. Applying the outside option principle (see Binmore (1985), Shaked and Sutton (1984)), the paper by Bester (1985) looks at a model where a buyer chooses a bargaining partner from a set of heterogeneous producers at random. The buyers can choose to quit their current partner and search for another seller. Since switching is costly and all consumers are identical, each buyer meets exactly one seller in equilibrium and the “right” price is proposed immediately. A model with an explicit search theoretic approach is Baucells and Lippman (1999), where a seller knows a specific buyer with whom he can bargain over a good. The seller’s outside option is to sell the good via search, that is, by accepting one of the incoming non-negotiable offers described by a given probability distribution. The Nash bargaining solution is applied to solve the bargaining problem, which results in an equal split of the surplus. Their focus is not on the bargaining process but the impact of the buyer’s availability on the payoffs to both agents.

Looking at the cited literature, it seems that the models that use an ex-

---

1Equivalently, agents of opposite type meet with certainty in a market model.
plicit search theoretic approach do not give much attention to the bargaining process, while in the models with an outside option that take a closer look in the "bargaining black-box", negotiation takes place immediately and the outside option only affects the division of the surplus, but not the timing.

The "no-search solution" of bargaining problems is not completely satisfying, given that in real-life bargaining situations immediate trade is rarely observed or even expected. Bargainers often hesitate to take the first offer; they search, hoping to get a better outcome than if they just accepted the first offer. The bargaining models described above seem to lack some important features that make search more likely. Intuitively, the result of no search in equilibrium may change once we allow for heterogeneity in buyers’ valuation and sellers’ cost. For example, if a buyer is matched with a high-cost seller, it might be worth for him to search a little longer to find a low-cost seller who can offer him a better price even though search is costly.

This paper analyzes a situation where a buyer and a seller are matched and they start bargaining. The bargaining process is described by a version of Rubinstein’s alternating offers game. Neither agents knows the opponent’s exact valuation for the good to be traded. The buyer has an outside option which is to buy the good via search by accepting one of the non-negotiable offers that are assumed to arrive in accordance with a given distribution. Thus, the model interlaces a search process and a bargaining process.

The paper is organized as follows: Section 2 outlines the complete bargaining - search model. In section 3, we will look at the bargaining problem taking the outside option as given, hence solving a two-sided incomplete information bargaining problem with an outside option. Having solved the bargaining problem with a given outside option, the solution will be incorporated in the search theoretic analysis in section 4. The goal of this paper is to solve a search problem by explicitly considering the bargaining problem involved, which captures important elements of a bargaining process, including incomplete information about agents’ valuation and an outside option.
2 Motivation of the Model: An Example

Suppose buyer $B$ seeks to buy a house and he happens to know a specific seller $S$ who is willing to sell his house. This house has exactly the features $B$ is looking for and $B$ doesn’t know of any other suitable house for sale at this time. Both $B$ and $S$ are imperfectly informed about the other agent’s valuation for the house. $S$ is not sure whether the particular features of this house are very important for $B$, in which case $B$ would be willing to pay a high price $v^h$, or if they are of minor importance, in which case $B$ is only willing to pay a low price $v^l$, where $v^l < v^h$. $B$, on the other hand, is not sure whether $S$ considers his house to be of high value $c^h$ or low value $c^l$, which determines the lowest price $S$ would be willing to accept.

The bargaining between $B$ and $S$ is modelled according to Rubinstein’s alternating-offers procedure with the following modification: In each round, $S$ can either quit or offer a price $p$. In the same round, $B$ responds with one of three choices: accepting $p$, rejecting $p$ and making a counteroffer $p’$, or opting out. The outside option is to buy via search. Search is modelled in the standard way: non-negotiable offers $y$ arrive according to a Poisson process with arrival rate $\lambda > 0$ and $E[y] < \infty$. The time interval between successive arrivals of offers is distributed exponentially.

The game form of the bargaining-search game is illustrated in Figure 1. Let $G$ denote a subgame starting in the bargaining phase and $N$ denote a subgame starting in the search phase. In the bargaining phase, $B$ immediately responds to $S$’s offer $p$. If he accepts $p$, the game is over. If he rejects $p$ and makes a counteroffer, the bargaining proceeds to the next round, which takes $\Delta$ time units and another (identical) subgame $G$ starts. If $B$ opts out, the subgame $N$ starts immediately, where $B$ searches until he locates an outside offer. During this time, there are no decisions to be made by either player. When $B$ locates an outside offer $y$ at time $t$, he has to choose between one of three options: he can accept $y$ and the game is over, or he can continue search and another (identical) subgame $N$ starts, or he can return to bargain with $S$, which will take him $\Delta$ time units and bring him back to the subgame $G$.

The payoffs to the players are now described. Let $r$ be the common rate of time preference for both players. For notational convenience of the discount factor, let $e^{-r\Delta}$ be denoted by $\delta$. If the game is terminated by an agreement
The Bargaining-Search Game

Figure 1: The Bargaining-Search Game
between \( B \) and \( S \) over a price \( p \) at time \( t + \Delta \), \( B \) receives \( (v - p)\delta e^{-rt} \) for \( v = v^h, v^l \) depending on whether he has a high valuation or a low valuation for the house. \( S \) receives \( (p - c)\delta e^{-rt} \) for \( c = c^h, c^l \) depending on his cost. If the game is terminated by \( B \) accepting an outside offer \( y \) at time \( t \), \( B \) receives a payoff of \( (v - y)e^{-rt} \) for \( v = v^h, v^l \) depending on the buyer’s valuation, and \( S \) receives zero. In case \( B \) and \( S \) perpetually disagree or if \( B \) searches forever, the payoffs are zero for each player.

Muthoo (1995) looks at a similar game that interlaces a bargaining game with a search process, however, in his model agents have complete information. He shows that the option of returning to the old bargaining partner after having searched for some finite time does not affect the unique subgame perfect equilibrium. In other words, the outcome does not depend on whether or not a bargainer is allowed to choose to return to the negotiating table. He concludes that this result depends crucially on the complete information assumption and the stationarity of the move-structure of the game.

In this paper, the solution of the bargaining-search game with incomplete information about agents’ valuations shall be approached by looking at the two processes separately at first and connecting them later. In the next section we will analyze how a given outside option changes the equilibrium strategies in the bargaining model with two-sided incomplete information.

3 The Bargaining Problem

The bargaining-search problem described in section 2 includes an explicit bargaining process, which will be modelled according to the following characteristics:

- each agent is uncertain about the valuation of the opponent
- there are no exogenous restrictions on the duration of the game
- each agent can quit the negotiations at the end of each period
- the buyer can purchase the good via search (outside option)
- the gains from bargaining depend on the characteristics of the agents and their outside options.
A model with these characteristics can be described as a non-cooperative bargaining problem with two-sided incomplete information and outside option. The following analysis is based on the Chatterjee and Samuelson (1987) model of bargaining with two-sided incomplete information, but additionally, it has a fixed outside option for the buyer. To point it out again, the idea of this paper is to solve the "bigger" problem where the bargaining solution is incorporated in the search problem. However, first we shall see how the outside option changes the equilibrium strategies in the Chaterjee and Samuelson model.

With complete information, the introduction of an outside option has been studied in the literature, e.g. the "outside option principle" in Binmore (1985) when the Nash bargaining solution is applied, Rubinstein and Wolinsky (1985), which incorporates an outside option in an alternating-offers bargaining process, or Chatterjee and Lee (1993), where there is incomplete information about the outside option.

When modelled explicitly, the outside option can be described by an arrival of offers according to a Poisson process with some given arrival rate and cumulative distribution function (see e.g. Baucells and Lippmann (1999), Muthoo (1995)). In the following subsection, we will treat it as a given value, \( \bar{M}_N \), and solve the bargaining problem as such. Afterwards, the value of \( \bar{M}_N \) will be modelled as the value of buying the good via search, including the option to return to bargaining. Thus, \( \bar{M}_N \) will be determined endogenously from the optimal search policy.

### 3.1 The Bargaining Model

Consider a bargaining game between two types of agents, a seller \( S \) and a buyer \( B \). \( S \) is endowed with one indivisible unit of a good. He can be one of two possible types: a high-cost seller with a valuation of \( c^h \) for the good or a low-cost seller with a valuation of \( c^l \). Similarly, \( B \) has either a high valuation of \( v^h \) or a low valuation of \( v^l \) for the good. We will assume that \( c^l \leq v^l < c^h \leq v^h \). Since both the low-cost seller and the high-valuation buyer can trade with bargaining partners of any type and hence have some flexibility in the price they can offer, they shall be called flexible agents. The high-cost seller and the low-cost buyer shall be called inflexible agents. The following table summarizes the agents' type and valuation:
At time 0, the buyer’s prior probability that the seller is flexible is $\pi_S$, and $1 - \pi_S$ is his probability that the seller is inflexible. For the seller, his prior probability that the buyer is flexible is $\pi_B$ and the probability that the buyer is inflexible is $1 - \pi_B$. The priors $\pi_S$ and $\pi_B$ are assumed to be given exogenously and are common knowledge. Agents update their beliefs according to Bayes’ rule.

The bargaining procedure is as follows: In round 1, the seller first offers a price $p$, at which he is willing to exchange the good. Then the buyer makes a counteroffer. If the two offers are compatible, that is, mutually acceptable trade is possible, the game ends with trade. The seller’s payoff is $p - c^h$ if he is a high-cost seller and $p - c^l$ if he is a low-cost seller. The buyer’s payoff from the purchase is $v^h - p$ if he is a high-valuation buyer and $v^l - p$ if he is a low-valuation buyer. If the two offers are not compatible, the buyer can choose between two possible moves: He may quit the seller and take his outside option, yielding a payoff of $M_N$ for the buyer, with $0 \leq M_N < \infty$, and zero for the seller. Or, he may choose to continue bargaining with the seller and offer $p’$. In this case, bargaining proceeds to the next round, which takes $\Delta$ units of time and hence, payoffs are discounted by a common discount factor $\delta$ and the sequence of offers begins again.\(^2\)

Chatterjee and Samuelson (1987) examine a similar game of two-sided incomplete information without an outside option. They restrict the offers to come from the set $\{v^l, c^h\}$, that is, there are only two possible strategies for each agent: $v^l$ is the low price offer and $c^h$ is the high price offer. They justify the restriction to the set $[v^l, c^h]$ by the idea that if there are known inflexible agents who are willing to trade at their valuation, accepting any offer outside this interval is dominated by trading with these inflexible agents. The further restriction to the two-element set $\{v^l, c^h\}$ implies that gains from trade will go entirely to either the buyer or the seller and seems

\(^2\)This process differs slightly from Rubinstein’s alternating offers game, where players alternate in offer periods. However, as Chatterjee and Samuelson (1987) remark, this changes only the calculation of $\tilde{\pi}_S$ and $\tilde{\pi}_B$ later on, but leaves the rest of the analysis unaffected.
quite strong in its implications. As they show, this game has a unique Nash equilibrium. In the potentially infinite horizon game, bargaining proceeds only for a finite but endogenously determined number of periods, since the nonzero probability that the opponent is inflexible fixes some limit beyond which a flexible bargainer will never continue.

In Chatterjee and Samuelson (1988), the model without restriction on the offers is examined. Unlike the restricted model, there are multiple equilibria, including the one which shares the features of the restricted-offers case. The authors favor the latter as selection of a unique equilibrium by arguments of plausibility. They emphasize that the multitude of equilibria does not alter the model's qualitative results and implications for bargaining. In view of these findings, the restricted-offers case shall be considered in this paper, in order to simplify the analysis while retaining the important aspects of the model. In the following, since there are only these two strategies available for each agent, we shall call $c_h$ the high price offer and $v_l$ the low price offer.

The immediate implications of the setup are that if $S$ offers the high price and $B$ the low price, no trade will occur; only if both agents offer the high price or both offer the low price, trade may occur. The case where $S$ offers the low price and $B$ offers the high price does not have to be considered, since if $S$ offers the low price, $B$ will certainly have no incentive to respond with a high price offer and vice versa. If both agents are inflexible, offering their true valuation is dominant, hence an inflexible $S$ always offers the high price and an inflexible $B$ always offers the low price. Since an inflexible buyer will never trade with an inflexible seller, he will surely take his outside option, once he concludes that his opponent is an inflexible agent as well.

### 3.2 Equilibrium Analysis

The analysis will closely follow the Chatterjee and Samuelson (1987) model, however, in the present model the buyer's outside option has to be considered when looking at equilibrium strategies. We expect this to influence the equilibrium strategies, since we know that flexible agents will try to masquerade as inflexible ones, and the outside option may help the buyer to reveal the seller's identity.

In our model, since for inflexible agents offering their true valuation is dominant (otherwise they would have a negative payoff), we only have to

10
consider the inflexible buyer’s choice between offering the low price and opting out. Thus, the strategies of the flexible agents are of particular interest, since they have the full strategic possibilities.

Let

- \( n_S \) be the first round where the seller offers the low price \( v^l \)
- \( n_B \) be the first round where the buyer offers the high price \( c^h \) or opts out

Then

- a pure strategy for the flexible seller consists of a decision to offer the high price for \( n_S - 1 \) rounds and to offer the low price in \( t = n_S \).
- a pure strategy for the flexible buyer consists of a decision to offer the low price for \( n_B - 1 \) rounds and to offer the high price or opt out in \( t = n_B \).

We will consider both the pure strategy and mixed strategy case. If pure strategies are played, we can have only two cases: either \( n_S \leq n_B \) or \( n_S > n_B \). The former implies that \( n_S = 1 \) and \( n_B = 1 \). This is true since delay is costly, and if a buyer does not offer the high price or opt out before \( n_S \), the best a flexible seller can do is offer the low price as soon as possible, which is in round 1. If a seller then offers the high price in round 1, a flexible buyer infers that the seller must be inflexible and hence offer the high price or opts out himself in round 1, hence \( n_B = 1 \). A similar argument shows that \( n_S > n_B \) implies that \( n_B = 1 \) and \( n_S = 2 \).\(^3\)

**case I: Pure Strategy Equilibrium with \( n_S = 1 \) and \( n_B = 1 \)**

Intuitively, for the seller to offer the low price already in round 1, he must think that the buyer is not very likely to accept the high price later on and hence it is not worth delaying trade to future rounds in hope for the high surplus. This is the case either if \( B \)'s outside option is more attractive or because \( B \) is an inflexible agent. In the latter case, we will get some critical probability for the buyer being flexible, above which the seller should offer the low price immediately.

To show that \( n_S = 1 \) and \( n_B = 1 \) are equilibrium strategies, we have to show that, given \( n_B = 1 \), the flexible seller cannot do better by offering the high price in round 1. If \( n_B = 1 \), the flexible buyer either offers the

\(^3\)For a formal argument, see Chatterjee and Samuelson (1987).
high price or opts out in round 1. In order for him to opt out, it has to be that $\bar{M}_N > v^h - c^h$, that is, the value of the outside option is greater than the surplus he would get if he reveals his type and accepts the seller’s high price. This shall be called a “good” outside option. Given that the flexible buyer has a good outside option, the flexible seller can do no better in pure strategies than reveal his type in round 1 by offering the low price $v^l$, since the high price $c^h$ would give him a payoff of zero if the buyer he faces is flexible and opts out. In case the seller faces an inflexible buyer, he certainly cannot trade by offering $c^h$, hence offering $v^l$ would be optimal independent of the hard buyer’s outside option value.\footnote{Even though the two types of buyers can have different values for their outside options, it is not very interesting to consider the inflexible buyer’s outside option, since the only case where it might affect strategies is when the flexible buyer’s outside option is bad and the inflexible buyer’s is good. This should give the seller an incentive to make a weak offer to ensure that he trades with the buyer. However, since the inflexible buyer’s surplus is at most zero from trading with the seller, we assume that he will choose to opt out if his $\bar{M}_N$ is good, since it offers him more than zero.}

Given that the outside option is “bad”, that is, $\bar{M}_N < v^h - c^h$, $n_B = 1$ means that the flexible buyer will offer the high price to the seller in round 1. Then the flexible seller’s best response will be to offer the low price and reveal his type in round 1 only if offering $v^l$ and receiving $v^l - c^l$ in round 1 is better than offering the high price $c^h$, when the buyer will accept this in round 1 with probability $\pi_B$ and with probability $1 - \pi_B$ the game continues to the next round:

$$\pi_B (c^h - c^l) + \delta (1 - \pi_B) (v^l - c^l) \leq v^l - c^l \tag{1}$$

which gives a boundary for $\pi_B$

$$\pi_B \leq \frac{(v^l - c^l) (1 - \delta)}{c^h - c^l - \delta (v^l - c^l)} \equiv \bar{\pi}_B \tag{2}$$

The expression on the LHS of (1) follows from the fact that round 2 is reached only if the buyer is inflexible, which causes the seller to reveal his type in round 2, and hence yielding the expected payoff of $\delta (1 - \pi_B) (v^l - c^l)$ for the seller. The boundary $\bar{\pi}_B$ identifies when a prior $\pi_B$ is sufficiently low for a flexible seller to reveal his type in the first round of bargaining. Hence,
$n_S = 1$ and $n_B = 1$ are equilibrium strategies if the outside option is good or if $\pi_B < \bar{\pi}_B$. In the equilibrium with trade, the flexible seller offers the low price ($n_S = 1$) and his payoff will be $v^l - c$. The flexible buyer receives $v^h - v^l$ and the inflexible buyer receives 0. Notice that neither of the two types of buyers can be made better off from trading with $S$ than in this case where the seller offers $v^l$. Since the priors and the outside option are common knowledge, if $\pi_B < \bar{\pi}_B$ an observed high price offer from a seller in round 1 will lead a buyer to the conclusion that the seller must be inflexible.

As long as there is a bad outside option and the seller offered $c^h$ in round 1, a flexible buyer cannot improve upon making a revealing offer. An inflexible buyer cannot trade with an inflexible seller, thus he will certainly opt out in round 1. The game in case I ends after the first round, however, not necessarily with trade. Search is possible if at least one of the agents is inflexible.

**case II: Pure Strategy Equilibrium with $n_S = 2$ and $n_B = 1$**

Intuitively, an equilibrium where the seller does not immediately reveal his type will exist if the buyer’s outside option is bad and if $S$ thinks that $B$ is likely to be flexible. On the other hand, in order for the flexible buyer to offer the high price in round 1, his prior that $S$ is inflexible should be sufficiently high. Thus, the equilibrium will be determined by both a critical level of $\pi_B$ and $\bar{\pi}_S$.

From case I we know that if $B$’s outside option is bad and $\pi_B > \bar{\pi}_B$, the flexible seller will not reveal his type in round 1. The optimal strategy for the flexible seller depends on what the flexible buyer does, thus the latter has to be considered first.

The following condition for a flexible buyer to offer the high price in round 1 reflects the fact that if the game proceeds to round 2, we know that $n_S > n_B \Rightarrow n_S = 2$ and therefore the flexible seller will offer $v^l$ in round 2:

$$\delta \pi_S (v^h - v^l) + \delta (1 - \pi_S) (v^h - c^h) \leq v^h - c^h \quad (3)$$

The boundary for $\pi_S$ is then given by

\footnote{If his outside option is bad, he will search forever.}
\[ \pi_S \leq \frac{(v^h - c^h)(1 - \delta)}{\delta(c^h - v^l)} \equiv \tilde{\pi}_S \quad (4) \]

Hence, if \( \pi_B > \tilde{\pi}_B \) and \( \pi_S \leq \tilde{\pi}_S \), the flexible buyer is better off offering the high price in round 1, given that he has a bad outside option, and \( n_S = 2 \) and \( n_B = 1 \) are equilibrium strategies.

If the outside option is good, the flexible buyer would opt out rather than offering the high price in round 1. Since a seller will be left with a payoff of zero if the buyer opts out, the flexible seller will choose to reveal his type and offer the low price in round 1, even though he thinks it is likely that the buyer he faces is flexible. As described in case I, a good outside option helps the buyer to unravel the seller’s type and get the higher surplus in case the seller is flexible. The inflexible buyer remains in the game as long as he has some positive probability that the seller is flexible. In equilibrium, he will know this by round 2: for the flexible buyer, the game is over after round 1 \( (n_B = 1) \) and hence, a flexible seller’s optimal strategy must be \( n_S = 2 \). If a seller has not offered the low price by round 2, an inflexible buyer should opt out in round 2.

In the pure strategy equilibrium considered so far, the outside option is only taken if at least one of the bargaining parties is inflexible. The inefficient case where the outside option is chosen even though both players are flexible but hide behind the incomplete information does not occur since the outside option is common knowledge and therefore helps the buyer to reveal the seller’s type. Also, the outside option only influences decisions in round 1: it can make the flexible seller change his optimal strategy from offering the high to offering the low price. Since the equilibrium in pure strategies requires the flexible buyer to end the game in round 1, the outside option affects the pure equilibrium strategies only in round 1.\(^7\) Things can be different in a mixed strategy equilibrium. As we know from the game without outside options, bargaining will continue for an endogenously determined number of rounds when agents randomize. The role of the outside option is not necessarily trivial then.

\textit{case III: Mixed Strategy Equilibrium}

\(^7\)This is true for the fixed outside option. Later on, we will consider the option to return to bargaining after searching for some time.
Before we look at the mixed strategies, some logical conclusions for the Nash Equilibrium from the Chatterjee and Samuelson model should be stated here.

- A flexible agent will never continue the game indefinitely, i.e., there exists some round $T$ beyond which a flexible agent will not proceed.\(^8\) To see the idea behind this, take the case of a flexible seller who will wait to offer the low price in round $t$ only if the buyer will offer the high price in $t$ with sufficiently high probability, so that it doesn’t pay for $S$ to offer the low price already in $t-1$. Since the probability of $B$ offering the high price cannot be bounded away from zero in every period, it must eventually become arbitrarily small. Then there must be some $T$ such that the flexible seller will offer the low price and the game ends. A similar argument holds for $B$ where there is some $T$ beyond which he will not continue the game.

- A flexible agent will immediately make the offer that gives him the low surplus if he infers that the opponent is inflexible. Following the above logic, if round $T$ is reached, by which a flexible agent would have ended the game, his opponent infers that the agent is inflexible and makes the for him disadvantageous offer himself. A direct implication is then that the game must have a finite horizon if at least one of the players is flexible. In other words, the potentially infinite horizon game will have a finite horizon.

If both agents have low priors that the opponent is flexible, that is, if $\pi_B > \bar{\pi}_B$ and $\pi_S > \bar{\pi}_S$, there is no equilibrium in pure strategies in the game without outside options. The reason is that since inflexible agents want to identify themselves as inflexible and flexible agents want to pretend they are inflexible, both agents would give the same signal. Thus, there is no new information to update the priors, and from the first round the prior probability that the opponent is inflexible is not sufficiently high, since both $\pi_S > \bar{\pi}_S$ and $\pi_B > \bar{\pi}_B$. Hence, pretending to be an inflexible agent might not lead to a success and is therefore not an equilibrium strategy. On the other hand, it is also not an equilibrium strategy for a flexible agent to offer the for him disadvantageous price with certainty, since this would lead to a perfect

\(^8\)For a formal proof, see Chatterjee and Samuelson (1987).
distinction of the two types and thus make a deviation therefrom profitable. How should flexible agents randomize?

In the Chatterjee and Samuelson model, the mixed strategy equilibrium is constructed in the following way: Each randomization is determined such that the previous agent is indifferent between the revealing and concealing offer. This supports the previous agent’s randomization. Thus, we get a sequence of probabilities until the first round $T$ where the probabilities exceed one. This identifies the round $T$ by which a flexible agent has stopped in equilibrium, where $T$ is determined endogenously.

The question in the present model is how the outside option changes this reasoning. Does the seller have to make the buyer indifferent between staying in the game and taking the outside option in every round? Or does he have to randomize such that the buyer is indifferent between revealing his type and the maximum of the following two: outside option and expected payoff of randomizing as in the game without outside options? To analyze this, the following notation will be useful: In the game without outside options, let $q^i_S$ be a seller’s probability that he plays a pure strategy with $n_S = i$ and $q^i_B$ be a buyer’s probability that he plays a pure strategy with $n_B = i$, where $\sum_{i=1}^{\infty} q^i_S = 1$ and $\sum_{i=1}^{\infty} q^i_B = 1$. Then $\{q^i_S\}$ shall denote a flexible seller’s mixed strategy and $\{q^i_B\}$ shall denote a flexible buyer’s mixed strategy.

Let $E^i_S$ = expected payoff to a flexible seller from playing a pure strategy with $n_S = i$

$E^i_B$ = expected payoff to a flexible buyer from playing a pure strategy with $n_B = i$

Hence

$$E^i_S = \sum_{i=1}^{t-1} \pi_B q^i_B (c^h - c^l) \delta^{i-1} + [\pi_B (1 - \sum_{i=1}^{t-1} q^i_B) + (1 - \pi_B)](v^l - c^l) \delta^{t-1} \tag{5}$$

and $V^t_S = \sum_{i=t}^{\infty} q^i_S E^i_S$, which is the expected payoff to $S$ from the remainder of the game, given that round $t$ has been reached and no agent has offered the for him disadvantageous price so far. In other words, $V^t_S = \sum_{i=t}^{\infty} q^i_S E^i_S$ is the flexible seller’s present value in round 1 if he plays the mixed strategy.
A sequential equilibrium in the game without outside options consists of mixed strategies \( \{q^i_S\} \) and \( \{q^i_B\} \) such that

\[
V_t^i(\{q^i_S\}, \{q^i_B\}) \geq V_t^i(\{q^i_S\}, \{q^i_B\}) \quad \forall \{q^i_S\} \quad (6)
\]

\[
V_t^i(\{q^i_S\}, \{q^i_B\}) \geq V_t^i(\{q^i_S\}, \{q^i_B\}) \quad \forall \{q^i_B\} \quad (7)
\]

for all \( t \) and for consistent beliefs \( \pi^i_B, \pi^i_S \). There are two types of mixed strategy equilibria: one where the seller randomizes first, i.e. \( q^i_S > 0 \), and one where the buyer randomizes first, i.e. \( q^i_B = 0 \). In the former, the round \( T_B \), by which a flexible buyer has made a weak offer in equilibrium, is found by setting \( E^1_S = E^2_S, E^3_S, ..., E^{T_B-1}_S = E^{T_B}_S \) and getting \( q^1_B, q^2_B, q^3_B, ..., q^{T_B-1}_B, q^{T_B}_B \) until the sum of the probabilities are equal to or exceed unity \((\sum_{i=1}^{T_B} q^i_B \geq 1)\).

This determines \( T_B \), and it also determines \( T_S \), which can only be equal to \( T_B \) or \( T_B + 1 \) (from the buyer’s randomization setting \( E^{T_B+1}_S = E^{T_B}_S \)). The other type of equilibrium is where the seller offers the high price in round 1 \((q^1_S = 0)\) and the buyer randomizes first, analogous to the reasoning described above.

We shall now consider the randomization including the outside option.

Suppose \( V^1_B \) is just the present value in round 1 from playing the mixed strategy profile \( \{q^i_B\} \) as in the game without outside options. In a mixed strategy equilibrium without outside options, we know that \( E^1_B = E^{T_B-1}_B \) as long as \( q^i_S > 0 \), that is, as long as the seller has not offered the low price, he randomizes in order to make the buyer indifferent between revealing his type in the current and the previous round.\(^9\) Then \( V^1_B \) can be written as

\[
V^1_B = \sum_{t=1}^{\infty} q^t_B E^t_B = E^1_B \sum_{t=1}^{\infty} q^t_B = E^1_B = \pi^i_S q^1_S (\nu^h - \nu^l) + (1 - \pi^i_S q^1_S) (\nu^h - \nu^l) \quad (8)
\]

On the other hand, for the game with an outside option for the buyer, define

\(^9\)Equilibria where \( T_S = T_B \) and \( T_S = T_B + 1 \) can both exist only if the randomization process gives \( E^{T_B}_S = E^{T_B+1}_S \), which is possible only for a set of games of measure zero.

\(^{10}\)If \( q^i_S = 0 \) and \( t > 1 \), the flexible seller will make a tough offer in all subsequent rounds: \( q^t_S = 0 \forall t > t \). In other words, the game has ended if \( q^t_S = 0 \). For a formal argument see Chatterjee and Samuelson (1987), proposition 2(ii).
that is, $\mathcal{E}^t_B$ is the flexible buyer’s expected payoff from playing a pure strategy with $n_B = t$. Since he can choose to opt out, he will get the maximum of the two values, $\bar{M}_N$ and $v^h - c^h$, in round $t$. In the first term of (9) we need not consider the outside option, since given that the buyer starts to randomize, it has to be that $\bar{M}_N < v^h - v'$. Redefining $V_B^1$ for the game with an outside option, we have

$$V_B^1 = \sum_{t=1}^{\infty} q^t_B \mathcal{E}^t_B = \pi_S q^1_S (v^h - v') + (1 - \pi_S q^1_S) \max \{ \bar{M}_N, v^h - c^h \}$$

(10)

**Proposition 1** If the flexible buyer’s outside option is good, that is, if $\bar{M}_N > v^h - c^h$, there is no equilibrium in mixed strategies. Bargaining ends after round 1.

**Proof.** If $v^h - v' > \bar{M}_N > v^h - c^h$, we have

$$\mathcal{E}^1_B = \pi_S q^1_S (v^h - v') + (1 - \pi_S q^1_S) \bar{M}_N = V_B^1 > \bar{M}_N$$

(11)

There are two possible mixed strategy equilibria: one where $q^1_S > 0$ and one where $q^1_S = 0$. First, suppose there exists and equilibrium with $q^1_S > 0$. From (10) it is obvious that the buyer would be better off entering the randomization than opting out, since $\forall \ q^1_S > 0$ we have $V_B^1 > \bar{M}_N$. A randomization requires $q^1_B$ to be such that $\mathcal{E}^1_S = \mathcal{E}^2_S$, and since the seller gets zero if the buyer opts out, this gives:

$$v' - c' = \pi_B q^1_B (0) + [(\pi_B (1 - q^1_B)) + 1 - \pi_B^t] (v' - c') \delta$$

(12)

This can only be true for $q^1_B = 0$. But if $q^1_B = 0$, we know that $q^t_B = 0 \forall t > 1$, which means that the buyer will always offer the low price. We know that the seller gets zero if the buyer opts out, thus there is no reason for the seller to start randomizing, since he will never receive a high price offer from the buyer. It is impossible for him to receive $c^h - c'$, since $q^t_S = 0 \forall t \geq 1$. Since…

\footnote{i.e. we need not consider $\max \{ v^h - v', \bar{M}_N \}$.}
the only offer that the buyer will accept from the seller is \( v' \) and delay is costly, the flexible seller can do no better than reveal his type in round 1 and offer the low price. The outside option makes it possible for the buyer to receive the full gains from trade in the first round, even though the seller has a relatively high probability that the buyer is flexible. The option to choose \( \bar{M}_N > v^h - c^h \) makes trade instantaneous and favorable for the buyer.

Now, suppose there exists an equilibrium in mixed strategies with \( q^1_S = 0 \). This means that the buyer would start to randomize in round 1, and his expected payoff from randomizing would be

\[
V^1_B = \sum_{t=1}^{\infty} q^t_B \mathcal{E}^t_B = \mathcal{E}^1_B = \bar{M}_N
\] (13)

In other words, the buyer would have to be indifferent in each round between choosing \( \bar{M}_N \) and continuing the randomization, with an expected payoff of \( \bar{M}_N \). The seller, on the other hand, can get at most \( v' - c^h \) by revealing his type, since otherwise the buyer opts out, leaving him with a payoff of zero. Again, the seller would prefer to get \( v' - c^h \) as early as possible, since delay is costly. Therefore, it cannot be an equilibrium where \( q^t_S = 0 \) for any \( t \). The seller reveals his type in round 1 even though he has a relatively high prior that the buyer is flexible. We conclude that a good outside option eliminates the mixed strategy equilibrium. \( \square \)

**Proposition 2** In equilibrium, the flexible buyer never takes the outside option in any round \( t > 1 \).

**Proof.** Suppose the outside option \( \bar{M}_N \) is not taken in round 1. For this to be the case, it must be that \( \bar{M}_N \) is less than the flexible buyer’s expected value in round 1 from the remainder of the game, \( V^1_B \). To find the equilibrium strategies, first suppose that \( \bar{M}_N < v^h - c^h \). This implies that \( V^1_B = V^1_B \), and we have

\[
V^1_B = \pi_S q^1_S (v^h - v') + (1 - \pi_S q^1_S)(v^h - c^h) = E^1_B > \bar{M}_N
\] (14)

Then we know from \( \bar{M}_N < E^1_B \) that \( \bar{M}_N < E^t_B \) for any \( t > 1 \) as long as the buyer randomizes \( (q^t_S > 0) \), since \( E^t_B = E^{t-1}_B \) is precisely the condition to make the buyer indifferent between stopping and continuing the randomization. But then the outside option is never taken. The randomizing strategies are
as in the game without outside options. If \( T_R \) is reached and no agent has made the (for him) disadvantageous offer so far, the buyer will not opt out since we have \( \tilde{M}_N < v^h - c^h \), hence he still prefers to offer the high price in \( T_R \).

Now, suppose that \( \tilde{M}_N > v^h - c^h \), that is the outside option is good. By Proposition 1 the game ends after round 1, either by the buyer opting out or accepting the low price offered by the seller. We conclude that the outside option is either taken in the first round or never. □

What is the result of going through all these different cases? There are some clear answers: the outside option changes the equilibrium strategy of the seller and it helps the buyer to get the whole surplus more often, if the threat of quitting is credible, which confirms our intuition. It changes the strategic situation and incentives for the flexible seller, it reduces the possibilities for him to hide behind the incomplete information. Also, the duration of the game is shorter whenever the outside option is good (case III).

4 The Search Model

After having solved the bargaining problem taking the value of the outside option as given, we can now look at the complete problem, which includes both the bargaining and the search process as described in section 2. The outside option for the buyer in the above bargaining model is to start searching for a better price offer.

Since outside option price offers are not always available, let us assume that these price offers \( y \) arrive according to a Poisson process with a given arrival rate \( \lambda \) and a cumulative distribution function \( F(y) \). These offers are non-negotiable. Payoffs are discounted at the continuous time rate \( r > 0 \), reflecting the cost of search. Following the standard search theory approach, given that the price offer \( p \) has been located, the return from search \( M^j_N(p) \) to the buyer of type \( j, j = h, l \), without an option to leave the search process is given by the following Bellman equation:

\[
M^j_N(p) = \int_0^\infty e^{-rt} \int_0^\infty \max\{M^j_N(p), v^j - y\} dF(y) \lambda e^{-\lambda t} dt
\]  

(15)
since the probability of receiving exactly one offer when price offers come from
a Poisson distribution is $\lambda e^{-\lambda}$. Thus, when the first offer $y$ arrives at time
t, the optimal search policy would be to choose the higher expected payoff
resulting from the following two choices: accepting offer $y$ and receiving the
payoff $v^j - y$ or continuing the search with a payoff of $M^j_N$, where the latter
is again his value function given by (15).

In order to solve the complete bargaining-search game as given in Figure
1, the results from the previous section for the bargaining problem with two-
sided incomplete information with a given outside option for the buyer will
be used. When $G$ denotes the subgame starting at the bargaining phase,
and $N$ denotes the subgame starting at the search phase, let $M_G$ be the
maximum equilibrium payoff to the buyer from the subgame $G$ and $M_N$ be
the maximum equilibrium payoff to the buyer from the subgame $N$. We will
allow for the buyer to return to the bargaining table once he started search,
and we will see if this option of returning to the old bargaining partner is ever
taken in this game with incomplete information. As Muthoo (1995) shows
for a split-the-pie game with complete information, in equilibrium, a player
will never choose to return to the bargaining table if he has an outside option
to search for better offer.

We will approach the solution by going through the three cases of section
2.2, which describe the bargaining equilibrium that is defined by the proba-
bilities $\pi_S$ and $\pi_B$ relative to $\bar{\pi}_S$ and $\bar{\pi}_B$. Following Bauccells and Lippmann
(1999), we will distinguish two regimes: First, we consider the case where
the expected return from search is used as an outside option. In the second
regime, the buyer will have to use actual offers to negotiate with the seller.
It will be shown that this has an important impact for the solution of the
game.

\footnote{If $k$ is the number of offers received, then the probability $f_P(k + 1; \lambda) = f_P(k; \lambda) \frac{\lambda}{k+1}$.
Since for $k = 0$, $f_P(0; \lambda) = e^{-\lambda}$, we have $f_P(1; \lambda) = \lambda e^{-\lambda}$, and the expected time for the
first offer is $\int_0^\infty \lambda e^{-\lambda} dt$.}
4.1 Regime I: Symmetric Information about the Outside Option

case I: \( \pi_B < \bar{\pi}_B \)

The solution of the complete bargaining-search game will always depend on the value of the outside option. If there is symmetric information about the outside option, we assume that the Seller and the Buyer know the parameters of the distribution of the outside offers. Then the buyer can use his expected value from search as his outside option and we have \( \pi_B < \bar{\pi}_B \), we know from case I of the bargaining equilibrium that the flexible seller’s optimal strategy is to reveal his type immediately, irrespective of the value of the outside option. Then

\[
M_G = \max \{ v^h - v^l, M_N \} \tag{16}
\]

When \( \mathcal{B} \) follows an optimal search policy, the value of the outside option \( M_N \), i.e., the maximum expected payoff for the game starting at \( N \), is found by applying the techniques of dynamic programming. Bellman’s equation is

\[
M_N = \int_0^\infty [e^{-rt} \int_0^\infty \max \{ \delta M_G, M_N, v^h - y \} dF(y)] \lambda e^{-\lambda t} dt \tag{17}
\]

since, according to the game structure of Figure 1, we leave the buyer the choice to go back to bargain with the seller after he started the search. We assume that this will take \( \Delta \) units of time, therefore payoffs from the game starting again at \( G \) at time \( t + \Delta \) are discounted by \( \delta \). The solution to the bargaining part as given in (16). To solve (16) and (17) simultaneously, first suppose that \( \delta M_G > M_N \). If this is the case, then it must be that

\[
M_G = \max \{ v^h - v^l, M_N \} = v^h - v^l \tag{18}
\]

since otherwise we would have \( \delta M_G = \delta M_N < M_N \), which contradicts our assumption. Knowing that the seller cannot offer a lower price than \( v^l \), the best that the buyer can get from returning to bargaining is again \( v^h - v^l \), but now discounted by the amount of time he spent searching and the cost of returning to the bargaining table. Will the buyer ever go back to bargain with the seller once he started the search? In order to have an incentive to do this, it must be that

22
that is, \( \delta M_G > M_N \). But if this is true, then also \( M_G > M_N \). And since
\( M_G = \max\{v^h - v^l, M_N\} \), it must be that \( M_G = v^h - v^l \). But then the buyer
would never start the search. Thus, as long as the seller offers \( v^l \) irrespective
of the value of the outside option (which is always true in case I), the buyer
will, in equilibrium, never go back to bargain with the seller once he chose
to start the search.

Then \( M_N \) is reduced to
\[
M_N = \int_0^\infty e^{-rt} \int_0^\infty \max\{M_N, v^h - y\} dF(y) \lambda e^{-M_G} dt
\]
(19)
since we know that going back to the bargaining table is not included in the
optimal path. When \( p \) is the “offer in hand”, i.e. the currently available
price offer, the return from search is
\[
M_N = \int_0^\infty e^{-rt} [M_N \int_0^\infty dF(y) + \int_p^\infty (v^h - y) dF(y)] \lambda e^{-M_G} dt
\]
(21)
which can be simplified to
\[
M_N = \frac{\lambda}{r + \lambda} [M_N (1 - F(p)) + \int_0^p (v^h - y) dF(y)]
\]
(22)
We are looking for an optimal reservation price, \( p^* \), that makes the buyer
indifferent between accepting and continuing search for one more round. Re-
arranging (22), we get
\[
M_N = v^h - p = \frac{\int_0^p (v^h - y) dF(y)}{\frac{\lambda}{r + F(p)}}
\]
(23)
Following the arguments of standard search theory, e.g. Lippman and
McCall (1976), there is a unique optimal reservation price, \( p^* \), that solves
the above equation. Then the optimal search policy given that the buyer is
in subgame \( N \) is to
- stop search if \( p \leq p^* \) and
- continue search if \( p > p^* \), where \( p^* \) is the solution to

\[
v^h - p^* = \frac{\int_0^{p^*} (v^h - y) dF(y)}{\frac{x}{\lambda} + F(p^*)}
\]  

(24)

This implies that the maximum payoff for the buyer in the complete search-bargaining game is

\[
M_G = \max \{ M_N, v^h - v^l \} = \max \{ v^h - p^*, v^h - v^l \}
\]  

(25)

The optimal strategy for a flexible buyer in the bargaining-search game is

- if \( p^* < v^l \), with \( p^* \) given by (24), then a flexible buyer should opt out, i.e. start search. Following the optimal search policy, he should stop the search if he finds a price offer \( p \leq p^* \) and he should continue to search as long as he receives offers \( p > p^* \).

- if \( p^* > v^l \) then the game ends in the bargaining phase in the first round. The buyer accepts the flexible seller’s offer \( v^l \). There is no search in this case.

\textit{case II: } \pi_B > \bar{\pi}_B \text{ and } \pi_S < \bar{\pi}_S

In section 3.2 describing the bargaining equilibrium we found that, unlike case I, in case II the seller’s strategy depends on the outside option for the buyer. If it is “good”, the flexible seller will offer \( v^l \) and the buyer gets the high surplus, while if it is “bad”, he will offer \( c^h \) and get the highest possible surplus himself. Thus \( M_G \) is not known if \( M_N \) is not known. In order to determine \( M_N \), the value of the outside option, we set up the Bellman equation again:

\[
M_N = \int_0^\infty e^{-rt} \left[ \int_0^\infty \max \{ \delta M_G, M_N, v^h - y \} dF(y) \right] \lambda e^{-\lambda t} dt
\]

The seller will reveal his type as a flexible agent if the buyer’s outside option is “good”, that is, if \( M_N > v^h - c^h \) and the buyer’s maximum payoff from the game starting in the bargaining phase \( G \) is

\[
M_G = \max \{ v^h - v^l, M_N \}
\]
as in (16), whereas if the outside option is “bad”, that is $M_N < v^h - c^h$, the flexible seller will offer only $c^h$ and the maximum payoff from the game starting in $G$ is

$$M_G = \max \{ v^h - c^h, M_N \} = v^h - c^h$$

The difference to case I is that here, in case II, it seems that the buyer might have an incentive to come back to the bargaining table if he could locate an outside offer lower than $c^h$, but higher than $v^d$, which would persuade the flexible seller to offer him $v^d$ when he comes back. Suppose, then, that $\delta M_G > M_N$, i.e., the buyer prefers to return to the bargaining table rather than to continue search. But in order for the buyer to have opted out, it must be that the expected value of search, $M_N$, is greater than his surplus from the seller’s current offer in the first bargaining round, $v^h - c^h$. In other words, the buyer has a good outside option. But then the flexible seller changes his strategy from offering $c^h$ to offering $v^d$ in round 1 of the bargaining phase, since if $M_N > v^h - c^h$, the buyer would search until he locates a price that is less or equal to his optimal reservation price, which must be lower than $c^h$. This would give the seller a payoff of zero, thus he prefers to offer $v^d$ in round 1. Again, in equilibrium, $\delta M_G > M_N$ implies that $M_G > M_N$ and there is no search.

However, a this point, neither $M_G$ nor $M_N$ are known, thus it seems that we do not know when the no-search strategy applies. But the only possibility that search is optimal for the buyer is when $\delta M_G < M_N$. The optimal reservation price is then given by (24), which only depends on the search parameters. Then the optimal search policy for the game starting at $N$ is to start search if $v^h - p^* > \delta M_G = \delta(v^h - v^d)$ or $p^* < v^h(1 - \delta) + \delta v^d$, since otherwise it cannot be that $M_N > M_G$, and to continue search as long as the price offer $p > p^*$. Whenever the current offer $p < p^*$, the buyer should accept the offer. His expected payoff from search, i.e. the expected payoff from the game starting in $N$ following the optimal search policy, is $v^h - p^*$. If $p^*$ is known, $M_N = v^h - p^*$ is known and therefore the complete bargaining-search game starting at $M_G$ can be solved:

- if $p^* < c^h$, which we call a “good” outside option, then the highest possible payoff the buyer can get is

$$M_G = \max \{ v^h - v^d, v^h - p^* \}$$

25
since the good outside option implies that the flexible seller, knowing that this is the only offer that ensures that the buyer possibly accepts the deal with him, will offer \( v' \). Then the buyer will start the search (if and) only if \( p^* < v' \), which makes the above condition \( p^* \leq v^h(1 - \delta) + \delta v' \) redundant. If \( v^h > p^* > v' \) then the buyer will choose to accept the seller’s offer \( v' \).

- If the outside option is bad, that is, if \( p^* > c^h \), then the buyer is left with the low surplus \( v^h - c^h \) from bargaining with the seller, there is no search in this case.

Again, the option to return to the bargaining table is never taken in equilibrium, since search is only started if \( p^* < v' \), and the buyer has no incentive to go back to the seller once he started search. Comparing case II to case I, as long as \( \pi_S < \bar{\pi}_S \) and \( p^* < c^h \), the seller’s prior probability for the buyer being flexible is irrelevant. He will offer \( v' \) if he is flexible. Only if the optimal reservation price is higher that the seller’s tough offer, it will depend on \( \pi_B \) what the seller’s strategy is and to which agent the high surplus goes.

case III: \( \pi_B > \bar{\pi}_B \) and \( \pi_S > \bar{\pi}_S \)

Looking at case III of the bargaining equilibrium where \( \pi_B > \bar{\pi}_B \) and \( \pi_S > \bar{\pi}_S \), we found that there is an equilibrium in mixed strategies if the outside option is bad and in pure strategies where the seller offers \( v' \) in round 1 if the outside option is good. The question is again, when do we know that the outside option is good (which is decisive for the solution of the game), since \( M_N \) is also depending on \( M_G \). The value of the outside option is again given by (17). We proceed in an analogous way to the cases before. To find the optimal strategies for the bargaining-search game, suppose \( \delta M_G > M_N \). Can there be an incentive for the buyer to return to the bargaining table after he started search? The argument is analogous to the previous cases: If the outside option is good, the seller offers \( v' \) in round 1 and we have \( M_G = \max\{v^h - v', M_N\} = v^h - v' \). There is no search in this case. If the outside option is bad, \( M_G = \max\{V_B^1, M_N\} \), where \( V_B^1 \) is the buyer’s expected payoff from the mixed strategy equilibrium as described in the previous section. Using our assumption \( \delta M_G > M_N \), we have \( M_G = V_B^1 \) and the buyer does not search.

To complete the case, if \( \delta M_G \leq M_N \), we have \( \delta M_G = \delta \max\{v^h - v', M_N\} \) with a good outside option and \( \delta M_G = \delta(v^h - c^h) \) with a bad outside option.
The latter implies no search if \( p^* > c^h \), while for the former, \( p^* \) is as defined in (24).

The analysis of the bargaining-search game leads to the following conclusions: For all possible values of \( \pi_B, \pi_S, \bar{\pi}_B, \bar{\pi}_S \), which define the bargaining equilibrium, there is one optimal reservation price for the game starting at \( N \), which depends only on the distribution of the incoming offers \( F(y) \), the arrival rate \( \lambda \) and the discount rate \( r \). Using the expected return from search as a valid outside option implies that the option of returning to the bargaining table after having started search is redundant, which coincides with Muthoo’s (1995) result for a bargaining and search game with complete information. The move-structure of the game remains stationary. Since this means that once search is started the buyer remains in the search process, the optimal reservation price does not depend on what the seller can offer. The decision whether to start search is made by first comparing \( p^* \) to \( c^h \) in order to find out if the outside option is “good” or “bad” (except for case I, where the flexible seller always offers \( v^l \)). In the end, search is only started if \( p^* < v^l \), but never to induce the bargaining partner to offer a lower price.

4.2 Regime II: Asymmetric Information about the Outside Option

Using the expected return from search a valid outside option, the previous section showed that the bargaining-search game is stationary in its structure. This is exactly the reason for the result that it is never optimal to return to the bargaining table once search is started. In equilibrium, agents in both games with complete and incomplete information should not be searching with the intention to go back to bargaining. This result is somewhat surprising, given the fact that one might also think about improving the bargaining position when opting out, and not necessarily about leaving the bargaining partner forever. But in Regime I the buyer opts out without ever returning, which makes a game with the option to return to the old bargaining partner identical to one without such option.

The key of this result lies in the stationary structure of the bargaining-search game. The seller’s equilibrium strategy in the subgame \( G \) does not change after the buyer has opted out. This, on the other hand, is explained by what was considered a “valid” outside option in the previous section.
the expected value of search. This value is at the same time the incentive for the buyer to start the search process and his threat for the seller. Now suppose that the seller does not have any information about the distribution of the buyer’s outside option. This can happen if the buyer cannot credibly communicate his information about the search parameters. The buyer then has to use an actual offer as a valid outside option. This can be particularly relevant in a job search situation, where a currently employed worker, who prefers to stay with his employer but wants a raise in his salary, has to bring an actual offer from another employer in order to have a credible outside option.

This changes the structure of the bargaining-search game. Intuitively, it should be worth for the buyer now to come back to the bargaining table with an actual offer in order to receive a better price from the seller he was bargaining with earlier, as long as that price the seller can offer him is still better than the actual outside offer or the expected value of search. This section will show that there is an equilibrium where \( B \) searches for some time and returns to bargaining, if actual offers only can be used as a valid outside option.

Let \( Y \) be the realization of the random variable, in other words \( Y \) shall denote an actual outside offer located by the buyer after he opted out. A buyer will have an incentive to opt out, if the highest payoff he can get from the game starting at \( N \) is higher than accepting the seller’s offer. The expected value from the game starting at \( N \) is now defined as

\[
M_N = \int_0^\infty e^{-rt} \left[ \int_0^\infty \max \{ \delta M^t_G, M_N, v^b - y \} dF(y) \right]
\]

with \( M^t_G \) denoting the expected maximum payoff the buyer can receive from the subgame starting at \( M_G \) after he searched for \( t \) time units. We will show that the bargaining subgames are not identical anymore, hence the time index. We will approach the solution of this game similarly to the previous one, i.e. go through the three cases of the bargaining equilibrium in order to find a solution for the complete bargaining-search game.

**case I**

From regime I we can see that **case I** is trivial as far as the treatment of the outside option is concerned. Irrespective of the value of the outside
option the equilibrium strategy for the seller is to offer \( v^l \) and hence the buyer will only opt out if \( p^* < v^l \). In Regime II where he has to use actual offers, this will not change, since the seller’s equilibrium strategy will not change over time, hence \( M_G^t = M_G \).

case II

This case is more interesting, since here we have a bargaining equilibrium where in the game without outside options both seller types offer the same price, \( v^h \), and the buyer cannot identify a flexible seller in round 1. As shown in the previous section, only a good outside option changes the flexible seller’s equilibrium strategy to offering \( v^l \). However, if only actual offers can be used as a credible threat, the buyer would first have to start search in order to locate an offer. He will opt out if the expected discounted value from search, including the option to go back to bargaining, is higher than \( v^h - c^h \), the low payoff from bargaining that he is offered in the first round in case II. \( M_N \) is as given in (26), and we have to determine \( M_G^t \), the expected maximum payoff from returning to bargaining considering the probability of finding an offer better than \( c^h \) at time \( t \) when the first outside offer has been located.

\[
M_G^t = \max \{M_N, (1 - \pi_S)(v^h - c^h) + \pi_S[(1 - F(c^h))(v^h - c^h) + F(c^h)(v^h - v^l)] \} \quad (27)
\]

To solve (26) with \( M_G^t \) as given in (27), suppose first that \( \delta M_G^t > M_N \). Then we know that the expected surplus from returning to the (unknown type of) seller is

\[
M_G^t = (1 - \pi_S)(v^h - c^h) + \pi_S[(1 - F(c^h))(v^h - c^h) + F(c^h)(v^h - v^l)]
\]

\[= v^h - c^h + \pi_S F(c^h)(c^h - v^l) \]

and the highest possible price \( \bar{p} \) that would be acceptable from search is then given by

\[
v^h - \bar{p} = \delta M_G^t = \delta[v^h - c^h + \pi_S F(c^h)(c^h - v^l)]
\]

\[
\bar{p} = v^h(1 - \delta) + \delta[c^h - \pi_S F(c^h)(c^h - v^l)] \quad (29)
\]
Then the expected return from search is

\[ M_N = \frac{\lambda}{r + \lambda} \int_0^\infty \max \{ \delta M_G^t, v^h - y \} dF(y) \]
\[ = \frac{\lambda}{r + \lambda} \left[ \int_0^p (v^h - y) dF(y) + (1 - F(p))(v^h - \bar{p}) \right] \]  \hspace{1cm} (30)

To proceed, we will use the following result from standard search theory, which states some properties of the payoff function \( M_N(\cdot) \):

**Lemma 1** If the expected payoff the buyer can achieve by following an optimal search policy is

\[ M_N(p) = \frac{\lambda}{r + \lambda} \left[ \int_0^p (v^h - y) dF(y) + (1 - F(p))M_N(p) \right] \]

Then the payoff function \( M_N(\cdot) \) has the following properties:

1. \( v^h - p^* = M(p^*) \)
2. \( p^* < p < \infty \Rightarrow M(p) > v^h - p \)
3. \( 0 < p < p^* \Rightarrow M(p) < v^h - p \)

Then since (30) is true given our assumption \( M_N < \delta M_G^t \), or \( M_N < v^h - \bar{p} \), using Lemma 1 we can conclude that

\[ M_N(\bar{p}) = \frac{\lambda}{r + \lambda} \left[ \int_0^p (v^h - y) dF(y) + (1 - F(\bar{p}))(v^h - \bar{p}) \right] < v^h - \bar{p} \]  \hspace{1cm} (31)

if and only if \( \bar{p} < p^* \).

Now suppose that \( \delta M_G^t < M_N \) to complete the solution of (26). In expectation, the buyer won’t go back to the seller, but follow his optimal search policy giving him an optimal reservation price \( p^* \). In this case we have

\[ M_N = \frac{\lambda}{r + \lambda} \int_0^\infty \max \{ M_N, v^h - y \} dF(y) \]  \hspace{1cm} (32)

and \( p^* \) is given again by (24). The return from search is \( M_N = v^h - p^* \), which is independent of the bargaining parameters. Again, this is true if \( \delta M_G^t < M_N = v^h - p^* \). Now \( \delta M_G^t < M_N \) implies that

\[ \text{...}

30
\[
\delta [v^h - c^h + F(c^h)(c^h - v^l)(1 - \pi_S)] < M_N = v^h - p^*
\]
where the LHS is just a linear combination of \(v^h - c^h\) and \(v^h - v^l\). Now we put the complete solution together:

The first assumption, \(\delta M_G^t > M_N\) is possible iff

\[
v^h - p^* < \delta M_G^t
\]

or

\[
p^* > \bar{p}
\]

whereas the second assumption, \(\delta M_G^t < M_N\) is possible iff

\[
\delta M_G^t < v^h - p^*
\]

or

\[
p^* < \bar{p}
\]

This shows that there is a unique solution. We conclude that the conditions and hence the motivation to start search and the condition to continue search differ: The buyer opts out in case II as long as \(c^h > \bar{p}\). Then two cases are possible: Either \(p^* > \bar{p}\), which means the buyer will (in expectation) return to the bargaining table and he just uses the outside option to locate an offer in order to renegotiate with the seller make the seller change his offer. Or, \(p^* < \bar{p}\), which means that the buyer will continue search according to his optimal search policy.

**case III**

Using actual offers as an outside option, the seller has no incentive here to offer the low price \(v^l\). Thus, we have to compare the expected value from playing the mixed strategy in the bargaining game to the expected return from search in order to see when the buyer opts out, that is, check if \(M_N > \mathcal{V}_{B}^{1}\). To determine the value of \(M_N\), we use the result of case II: \(\delta M_G^t > M_N\) iff \(\bar{p} < p^*\), and thus the buyer will opt out if \(\delta M_G^t > \mathcal{V}_{B}^{1}\), or

\[
\delta [(v^h - c^h)(1 - \pi_S F(c^h)) + (v^h - v^l)\pi_S F(c^h)] > (v^h - v^l)\pi S q_S^1 + (v^h - c^h)(1 - \pi_S q_S^1)
\]
which just says that the buyer should opt out if the probability of locating a price offer lower than \( c^h \) (considering discounting) is higher than the probability that the seller offers the low price \( v^l \) in the first round in his mixed strategy. He will return to renegotiate with the seller after locating the first outside offer and in expectation, the flexible seller will offer him the low price when he returns. Again, the outside option is chosen in order to change the seller’s optimal strategy, not because the expected return from search is high.

On the other hand, \( \delta M^p_G < M_N \) iff \( \bar{p} > p^* \), and then the buyer will opt out if

\[
v^h - p^* > \mathcal{V}^1_B = (v^h - c^h)(1 - \pi_S q^1_S) + (v^h - v^l)\pi_S q^1_S \tag{39}
\]
\[
p^* < c^h + \pi_S q^1_S (c^h - v^l) \tag{40}
\]

and using Proposition 1, we conclude that the buyer will never return to the bargaining table. The mixed strategy equilibrium remains a solution to the bargaining-search problem if \( p^* > c^h + \pi_S q^1_S (c^h - v^l) \).

What is different from the result of Regime I? When expected offers are a valid outside option, for any \( \pi_B \) and \( \pi_S \), an optimal reservation price \( p^* < c^h \) induces the flexible seller to offer the low price \( v^l \) in the first round, and unless the reservation price from search is not too low \( (p^* > v^l) \), the buyer doesn’t opt out. There will not be a renegotiation in equilibrium under Regime I. In Regime II, where actual offers have to be used as a valid outside option, the buyer actually will opt out and start to search if the seller thinks he is likely to be flexible \( (\pi_B > \pi_S) \). In expectation, he will go back to the bargaining table after locating the first outside offer if \( p^* > \bar{p} \), since the expected price he will have to pay to the seller after renegotiating is lower than the optimal reservation price from search. The outside offer might or might not help him to renegotiate, this depends on the realization of the located price, which is a random variable in this model.

Notice that under Regime II it is possible that the buyer searches for more than one outside offer and then returns to the bargaining table. This happens when the realization of the first price offer located through search is not sufficiently low to induce the flexible seller to offer \( v^l \) when they renegotiate. The buyer will then continue search until he locates an offer \( Y \) such that \( v^h - Y < \delta(v^h - c^h) \), or \( Y < v^h(1 - \delta) + \delta c^h \). He will still return to the
bargaining table as long as the reservation price from search, $p^*$ is higher than the expected price $\bar{p}$ he will have to pay the seller.

5 Conclusion

This paper studies a noncooperative bargaining model between a buyer and a seller, where both agents have incomplete information about the opponent's valuation for the good to be traded, and where the buyer's outside option is to buy via search. The model interlaces a standard sequential search process and a version of Rubinstein's alternating-offers bargaining process.

First, the bargaining process with two-sided incomplete information is analyzed, taking the outside option as given. Incorporating the results of the bargaining process in the complete bargaining-search game, we solve for the optimal bargaining and search strategies when the outside option for the buyer consists of searching for a better price offer, and when the buyer can choose to return to bargaining with the seller once he started search. Two different regimes for the search process are distinguished:

- Regime I: Symmetric information about the outside option, i.e. the expected return from search can be used as a credible outside option and

- Regime II: Asymmetric information about the outside option, i.e. the buyer has to use actual offers to (re)negotiate with the seller.

As shown in the paper, this distinction has an important impact on the solution of the bargaining-search game. In Regime I, the condition to opt out depends on the optimal reservation price, which is a function only of the distribution parameters of the outside offers and the discount rate, but not on the bargaining parameters, in particular not on the seller's offer. The option to return to the bargaining table is redundant, which coincides with Muthoo's (1995) result for a game with complete information. Thus, search is never started to induce the bargaining partner to offer a lower price, but with the intention to follow the optimal stopping rule as in a pure search process.

In Regime II, when actual outside offers have to be used as the outside option in the bargaining process, the stationarity of the move-structure of
the game is not preserved and hence, a renegotiation with the old bargaining partner is possible in equilibrium. An important result is that the conditions when to start search and when to continue search differ. The unique solution determines whether the buyer opts out in order to locate an outside offer and to use it for renegotiation with the seller, or whether the buyer opts out with the intention to continue search according to the optimal search police without ever returning to the seller.

This model is a basic framework for a larger variety of bargaining-search problems. It is easy to think about other regimes that would be worth consideration in a similar setting, for example if the player without outside option is available only temporarily. Another extension would be a situation where the player with the outside option is incompletely informed about the distribution function of outside offers. In this case, search would involve learning and the equilibrium strategies would be non-stationary. These considerations could be particularly useful in e-commerce using programmed bidding agents, when incoming information cannot be processed fast enough and/or strategies are too complex for human traders.

References


