The choice of exchange rate regime and speculative attacks*

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Abstract

We develop a framework for studying the choice of exchange rate regime in an open economy where the local currency is vulnerable to speculative attacks. The optimal regime is determined by a policymaker who trades off the loss from nominal exchange rate uncertainty, against the cost of maintaining a given regime. This cost is affected in turn by the likelihood of a speculative attack. Searching for the optimal regime within the class of exchange rate bands, we show that the optimal regime is either a peg (a zero-width band), a free float (an infinite-width band), or a non degenerate finite width band. In the latter case, the exchange rate is allowed to move freely only within a band set around some center rate. We examine the determinants of the optimal band width and show, among other things, that, ceteris paribus, lower costs of moving across currencies induce policymakers to set more flexible exchange rate systems. This lowers, in turn, the likelihood of financial crises. More generally the framework of the paper can be used to shed new light on the recent world wide trend towards a bipolar system of exchange rate arrangements.

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1 Introduction and summary

Except for some key currencies like the US Dollar, the British Pound, the Japanese Yen, the German Mark, and nowadays the Euro, most currencies are not allowed to freely float against other currencies. This statement was obviously true during the heydays of the Bretton Woods system which explicitly pegged the prices of numerous currencies to the US Dollar. In parallel with the strong evolution of world capital markets and the associated removal of restrictions on capital flows since the early 70’s, the fraction of countries classified by the IMF as having pegs followed an ever decreasing steady trend.1 But even after the demise of the Bretton Woods system, policymakers in many countries continue to, either explicitly or implicitly, engage in policies whose objective is to limit nominal exchange rate volatility.

Such policies are not maintained under all circumstances. In the presence of sufficiently large runs on the domestic currency, policymakers suspend their intervention policy and allow the exchange rate to float. They often reinstate their intervention policy at a new level of the exchange rate, after the turbulence in currency markets has subsided. For example, evidence presented in Calvo and Reinhart (2000) and McKinnon (2001) suggests that prior to the 1997/1998 East Asian crisis, the authorities of the involved economies tightly limited exchange rate variability with respect to the US Dollar. During the crisis years their currencies were allowed to freely depreciate but the post crisis exchange rate regime, since 1999 again exhibits high frequency pegging to the US Dollar much like in the pre-crisis period.

More generally, limitations on the variability of exchange rates vary over a broad spectrum along several dimensions. The most salient dimensions are the range over which the domestic currency is allowed to fluctuate, the strength of commitment to maintain the exchange rate within the desired range or band, the extent to which authorities explicitly preannounce their intention to limit exchange rate fluctuations, and the choice of reference currency (or basket of currencies). At one extreme of the first, second and third dimensions are countries with a strong, explicitly preannounced currency boards type, commitment to tightly peg to a key currency. Examples are Hong-Kong and Argentina which are committed to maintain a fixed parity with the US Dollar. But there are other, less extreme, forms of limitations on exchange rate variability like exchange rate bands.

During the 90’s, a number of countries, including Chile, Finland, Israel, Mexico, Norway, and Sweden adopted unilateral, explicitly preannounced, exchange rate bands of varying width rather than pegs. Under this regime, policymakers preannounce their intention to intervene and maintain exchange rates within a zone of prespecified width.

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1 Calvo and Reinhart (2000), Table 1.
around a center rate. Here, the preannounced limitation on variability is not as tight as in the case of a preannounced peg. Although policymakers in some countries try to maintain the exchange rate within some band this is not always explicitly preannounced. For instance, although the Chinese government does not admit it, the remarkable stability of the Yuen in terms of the US Dollar since the mid 90’s is consistent with the view that the Chinese authorities deliberately maintained the rate of exchange within some range.

The recent work of Calvo and Reinhart (2000) suggests that, as a positive matter, most small open economies have implicit or explicit limitations on exchange rate variability. The preceding opening remarks also suggests that there are substantial cross country and over time variations in the extent of those limitations and that they are sometimes abandoned under duress. This paper proposes a simple framework for analyzing some of the factors that determine the stringency of those limitations in a world inhabited by imperfectly informed currency speculators facing policymakers who are able or willing to defend the exchange rate only as long as the exchange rate that would obtain in the absence of intervention does not deviate too much from the current rate. This is done by endogenizing the choice of exchange rate band and by identifying factors that influence the policymakers’ willingness to maintain the band in the face of varying circumstances.

An important propagation mechanism of currency crises is what each individual speculator believes about what the other speculators believe about whether policymakers will, or will not, use its resources to defend the currency. On the other hand, the amount of resources that policymakers are willing to commit to defending a peg or a band obviously depends on how many speculators decide to run on the currency. Our framework explicitly incorporates this simultaneous interaction by combining elements of currency crises models of the Morris and Shin (1998) type with a framework in which the width of the band is a choice variable.

A basic premise of our framework is that exporters, importers, as well as borrowers

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2Note that a peg can be thought of as a band with zero width while a free float can be thought of as a band with infinite width.

3The IMF provides a formal classification of exchange rates systems into pegs, free floats, managed floats and several additional, less salient, categories. This classification takes at face value that countries actually do what they say they do. After further scrutiny Calvo and Reinhart (2000) conclude that the actual exchange rate behavior of many countries which are classified as “managed floats” or even “free floats” by the IMF is too low to be consistent with such flexible classifications.

4Most of the existing literature on currency bands offers little guide about the policy tradeoffs involved in the choice of band width. While some of the earlier work on exchange rate target zones (e.g., Williamson, 1985; Frenkel and Goldstein, 1986; and Williamson and Miller, 1987) partially dealt with these important policy choices, the literature on target zones from the early 90’s takes the existence of bands and their width as exogenous (surveys appear in ch. 1 and 2 of Krugman and Miller, 1992, and in Svensson, 1992). Three recent exceptions are Cukierman, Kiguel, Leiderman, and Spiegel (1999), Cukierman and Spiegel (1999), and Koren (2000). However neither of those papers explicitly considers the simultaneous interaction between the choice of band width and the behavior of speculators.
and lenders in foreign currency denominated financial assets dislike nominal exchange rate volatility, and that policymakers internalize at least part of this aversion. The recent empirical findings of Calvo and Reinhart (2000) are consistent with this supposition. But leaning against the trends of free exchange markets is not without costs. To defend a currency under attack, policymakers have to use up foreign exchange reserves or raise domestic interest rates. If they decide to avoid those costs by suspending an existing peg or band they incur a loss from a reduced credibility about their resolve to moderate future exchange rate fluctuations. Depending on whether the first or the second cost is larger, policymakers decide to either defend the currency or to allow a realignment of the currency.

During the three decades since the breakdown of the Bretton Woods System there has been a worldwide trend towards the gradual dismantling of restrictions on capital flows making it successively less costly for banks, firms and households to shift between financial assets denominated in domestic and foreign currencies. The analytical framework in the paper makes it possible to relate the endogenous choice of band width in the face of potential currency speculation to the transaction cost of switching between currencies. A main prediction of our model is that this reduction in transaction costs should lead some countries to flexibilize their exchange rate systems. This is because, for a given exchange rate regime, speculative attacks become more likely following a reduction in the transaction cost. This, in turn, makes it more difficult for policymakers to maintain a peg or a narrow exchange rate band.

In particular our model predicts that in the face of the reduction in the transaction cost of switching between currencies, due to capital market liberalization, policymakers in countries with non-degenerate exchange rate bands will increase the width of their band; policymakers with pegs with moderate aversion to exchange rate uncertainty will switch to narrow bands, and policymakers with a strong aversion to exchange rate uncertainty will continue to peg. The actual changes that occurred in the exchange rate systems of many countries during the last two decades are broadly consistent with this prediction. IMF documents suggest that there has been a general trend towards the flexibilization of exchange rate arrangements at least till the beginning of the nineties.

But there are also indications that, during the last decade, the flexibilization is not as dramatic as reported by the IMF (Dornbusch and Park, 1999; Calvo and Reinhart, 2000; and McKinnon, 2000). Some countries like the East Asian economies that were involved in the 1997/98 crisis are back to high frequency exchange rate pegging, as used to be the case prior to the crisis. And, following successful stabilization of inflation...

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5Some prominent economists also consider exchange rate volatility to be undesirable as can be seen from the recent following statement by Mundell (2000, p. 339): "There are, however, two pieces of unfinished business. The most important is the dysfunctional volatility of exchange rates that could sour international relations in time of crisis". 
in the early 90’s, small open economies like Argentina, Estonia, Lithuania, Bulgaria and Bosnia chose to signal their commitment to a fixed exchange rates by instituting currency boards. Using recent reclassification of types of exchange rate regimes by the IMF, Fischer (2001) reports that between 1991 and 1999 there has been a shift away from interemediate regimes to either hard pegs, or floating regimes (the bipolar view). In conjunction with some reasonable additional premises, the analytical results in this paper can be used to understand the trend of the last decade reported in Fischer (2001). This issue is taken up in the concluding reflections section.

The above discussion raises an interesting question about the fundamental factors that induce cross country variations in the degree of aversion to exchange rate volatility. The main factors appear to be: 1. the fraction of financial assets and liabilities denominated in foreign exchange, 2. the importance of foreign trade, 3. the fraction of trade that is invoiced in foreign exchange (McKinnon, 2000; Gyfason, 2000; and Wagner, 2000). 4. the fraction of capital flows that are invoiced in foreign exchange. In particular, the policymakers of relatively open economies with a large fraction of foreign capital flows, and/or a large fraction of trade that is invoiced in foreign exchange are likely to be characterized by strong aversion to exchange rate variability. Our paper predicts that these countries are likely to continue to peg even after substantial liberalization of restrictions on capital flows have taken place.

The paper also contains an analysis of the effects of reductions in the transaction cost of switching between currencies, due to capital market liberalization, on the likelihood of financial crises (speculative attacks). Taking the transaction costs of moving between currencies as a proxy for the extent of capital market liberalization, we show that, holding the width of the exchange rate fixed, liberalization increases the likelihood of a crisis. However, liberalization also induces policymakers to adopt wider bands, and when this effect is taken into account, the overall probability of speculative attacks decreases. The last result implies that, contrary to conventional wisdom, a ”Tobin tax” may not necessarily reduce the probability of a currency crisis, while liberalization of international financial flows may achieve that. A ”Tobin tax”, however, may be beneficial from the point of view of policymakers, as it provides them more ”freedom” in managing the exchange rate.

Interestingly, speculators’ decision regarding whether or not to attack the band depends on how dependable policymakers are in their eyes. We show that a good reputation induces the speculators to attack the band over a smaller range of shocks. This, in turn, induces policymakers to set up narrower bands. Hong-Kong’s currency board fits into this ”box” of the model. Since the peg has never been abandoned in the past,

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6About thirty years ago Tobin proposed to impose a tax on currency transfers on the ground that it will reduce the likelihood of financial crises. A recent practically oriented evaluation of the pros and cons of such a tax appears in Berglund et. al. (2001).
Hong-Kong’s currency board has good reputation, which prevents speculative attacks and allows the authorities to maintain the peg under a wide set of circumstances. There is thus a ”virtuous circle” between good reputation and the performance of the currency board.

The paper is organized as follows. Section 2 presents the basic structure. Section 3 characterizes the equilibrium behavior of speculators given a pre-existing band as well as the (prior) choice of band width by policymakers. Section 4 characterizes the equilibrium properties of the exchange rate band (whether it is a peg, a band or a float, and, if it is a band, of what width) and presents comparative static results with respect to the degree of aversion to exchange rate volatility and with respect to an increase in the variability of the exchange rate under laissez faire. Section 5 analyses the effects of capital account liberalization. Section 6 extends the analysis to the case in which speculators are uncertain about the seriousness of the policymaker’s resolve to maintain the band (i.e., imperfect reputation). All proofs (except for the proof of Lemma 2) are in the Appendix.

2 The model

Consider an open economy in which the initial level of the exchange rate (defined as the number of units of domestic currency per one unit of foreign currency) is denoted by $e^{-1}$. Absent policy interventions, the nominal exchange rate, $e$, fluctuates stochastically in the exchange rate markets. The fluctuations of $e$ reflect various shocks to the current account and to the capital account of the balance of payments. For the purpose of this paper, it turns out that it is more convenient to work with the laissez faire rate of change in $e$, $x \equiv (e - e^{-1})/e^{-1}$, rather than directly with $e$. We assume that the value of $x$ is drawn from a distribution function $f(x)$ on $\mathbb{R}$ with c.d.f. $F(x)$ and that, once it realizes, $x$ persists for some time.\footnote{We assume that the distribution of $x$ has an unbounded support mainly for convenience. This assumption is not essential and can be relaxed although this would require some additional assumptions to ensure that various parameters of the model are not too large relative to the bounds of the support of $x$.}

**Assumption 1**: $f(x)$ is unimodal with a mode at $x = 0$. That is, $f(x)$ is increasing for all $x < 0$ and decreasing for all $x > 0$.

Assumption 1 states that large rates of change in the exchange rate (i.e., large devaluations when $x > 0$ and large appreciations when $x < 0$) are less likely than small changes. This is a realistic assumption and, as we shall see later, it is responsible for some of the results of the paper.
A basic premise of our framework is that policymakers dislike nominal exchange rate variability and may choose to set bands or pegs in order to reduce it. The reasons for this premise are discussed at the beginning of the next subsection. But leaning against the trends of free exchange rate markets is costly. To defend a currency under attack, policymakers have to deplete their foreign exchange reserves (Krugman, 1979) or (as demonstrated time and again during the currency crises of the 90’s) have to put up with substantially higher domestic interest rates (Obstfeld, 1996). If they decide to avoid those costs by realigning an existing peg or band, they incur a loss from reduced credibility about their resolve to moderate exchange rate fluctuations in the future. We denote the present value of this loss by $\delta$.

The existence of speculators who find it profitable to bet against the currency when they believe that policymakers may allow a realignment complicates the task of moderating exchange rate variability. Defending a currency under attack, although often technically feasible, is not a route that is followed under all circumstances since it conflicts with other policy objectives like having reasonably smooth domestic interest rates and maintaining a sufficiently high level of foreign exchange reserves (Obstfeld and Rogoff, 1995).

Even when they ultimately allow a realignment, policymakers initially try to maintain a pre-existing peg or band. We allow for such behavior by postulating that, following realizations of $x$ that are outside the boundaries of the band there are two phases. In the first (possibly short) phase the policymaker always defends the band. During this phase, speculators might attack the band. In the second phase he is in a position to decide whether to continue to maintain the band or not. He makes this decision by comparing the cost of continuing to maintain the band to the cost, $\delta$, of abandoning it. We assume that the larger the disequilibrium he tries to maintain in this phase, and the larger the number of speculators that have recently run on the currency in the previous phase, the more costly it is, in terms of alternative objectives, to continue to defend an existing peg or band. Specifically, normalizing the mass of speculators to 1, and using $\alpha$ to denote the fraction of speculators that have attacked the band in the immediately preceding stage, the cost of intervention in the exchange rate market as a function of the (freely floating) rate of change in the nominal exchange rate, $x$, and of the extent of recent speculation, is given by

$$C(x, \alpha) = \begin{cases} 
  x - \pi + \alpha, & x \geq \pi, \\
  0, & \pi \leq x \leq \pi, \\
  \pi - x + \alpha, & x \leq \pi.
\end{cases} \tag{2.1}$$

where, given $e_{-1}$, $\pi$ and $\pi$ are the maximal rates of devaluation and of appreciation that are consistent with the existing exchange rate band. The function $C(x, \alpha)$ reflects the

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8This may be due to small, but not infinitesimally small, lags in the arrival of information and in decision making.
idea that the cost of continuing to defend the band increases with the size of the gap between the freely floating rate of change in the exchange rate, $x$, and the rate of change that the policymaker permits (either $\pi$, or $\pi$, depending on whether $x$ is negative of positive) and with the fraction of speculators who chose to attack the band in the first phase.

The notion underlying the positive relation between the fraction of active speculators in the first phase and the cost of defending the band in the second phase is that, the larger this fraction, the smaller the resources available to continue to defend the band, and the higher therefore the cost associated with this policy. For simplicity we assume that $\alpha$ enters the cost function additively. The middle line in the definition of $C(x, \alpha)$ states that the policymaker does not intervene when the exchange rate is inside the band so that the cost of intervention is zero. If the policymaker allows the exchange rate to move outside the band, he avoids the cost $C(x, \alpha)$ and incurs instead the present value loss of diminished future credibility, $\delta$.

2.1 The exchange rate band

A basic premise of the paper is that policymakers dislike expected nominal exchange rate volatility. This "fear of floating" has recently been extensively documented at the empirical level by Calvo and Reinhart (2000). At the conceptual level there are several reasons to believe this to be the case. First, domestic exporters and importers dislike exchange rate volatility because it introduces uncertainty into their cash flows. Second, domestic borrowers and lenders in foreign exchange denominated financial instruments face higher exchange rate risks when nominal exchange rate volatility is higher. By increasing the foreign exchange risk premium, an increase in exchange rate volatility reduces international flows of goods and of financial capital. Policymakers internalize at least part of this aversion to exchange rate variability and have, therefore, an incentive to limit exchange rate volatility by means of pegs or bands. But as we saw above the operation of bands is not costless from the point of view of policymakers. We formalize

\textsuperscript{9}Admittedly, some of those risks may be insured by means of future currency markets. However, except perhaps for some of the major key currencies, such markets are largely non-existent, and when they do exist the insurance premia are likely to be prohibitive.

Rose (2000) presents evidence suggesting that countries with the same currency trade substantially more than comparable countries with their own currencies. Lee (1999) presents evidence from US import markets that is consistent with the view that exchange rate volatility depresses demand for imported consumer durables.
the trade-off facing policymakers by postulating that their objective function is

\[ V = -AE|\pi| - E[min\{C(x, \alpha), \delta\}], \quad A > 0. \tag{2.2} \]

The first component of \( V \) represents the policymaker’s aversion to uncertain fluctuations in the nominal exchange rate. The latter is measured in terms of the absolute value of \( \pi \) (i.e., by the magnitude of nominal depreciations or appreciations). The second component of the objective function represents the policymaker’s cost of adopting an exchange rate band. This cost is either equal to \( C(x, \alpha) \) if the policymaker defends the band in the second phase or to \( \delta \) if defending the band is too costly. The parameter \( A \) represents the relative importance that the policymaker assigns to exchange rate stability. The higher is \( A \), the more concerned is the policymaker with exchange rate stability and is more willing to incur costs in order to maintain it. Note that we assume that the policymaker is equally averse to expected depreciations and appreciations. This assumption can be easily relaxed at the cost of more notation.

The parameter \( A \) is likely to vary substantially across economies depending on factors like the degree of openness of the economy, its size, the fraction of financial assets and liabilities of domestic producers and consumers that is denominated in foreign exchange, and the fraction of foreign trade that is invoiced in foreign exchange. Other things being the same, the residents of a small open economy value stability of the nominal exchange rate more than the residents of large, relatively closed, economies like the US or the Euro area. Hence the parameter \( A \) is larger in small open economies than in large, relatively closed, economies.

In general, there are various conceivable institutional arrangements for limiting exchange rate volatility. In this paper we search for an optimal institutional arrangement within the class of bands. This class is quite broad and includes pegs (bands of zero width) and free floats (bands of infinite width) as special cases. Under this class of arrangements, the policymaker sets an exchange rate band \([e, \bar{e}]\) around the preexisting nominal exchange rate, \( e \). The nominal exchange rate, \( e \), is then allowed to move freely

\footnote{We think of this objective function mostly as a positive description of how rational policymaker’s might approach the problem of choosing the band width.}

\footnote{For simplicity, we assume that the first phase is very short relative to the second phase, so that the cost of defending the band in the first phase is negligible. Most of the results in the paper go through when this assumption is relaxed.

Note that if the policymaker does not defend the band, he does not incur an additional cost as a result of the actual, perfectly anticipated, volatility in the exchange rate. This is because in our framework, only unexpected volatility has a negative effect on economic activity. Anticipated volatility, by contrast, has no such adverse effect. More generally, variability and uncertainty do not always coincide. A fuller discussion of the difference between variability and uncertainty appears in Cukierman and Wachtel (1982).}

\footnote{One important reason for this relative preferences structure is that the pass-through from changes in the exchange rate to domestic prices is swifter and stronger in small open economies than in large and relatively closed economies.}
within the band in accordance with market forces, but once it reaches the boundaries of the band, the policymaker is committed to intervene and keep it from moving outside the band.\footnote{This intervention can be operationalized by buying or selling foreign currency in the market, by changing the domestic interest rate or by doing some of both. However, as we shall see below, the cost of intervention may be too high in which case the policymaker exits the band and suffers a reputation loss, $\delta$.} The exchange rate band induces a permissible range of rates of change in the exchange rate, $[\bar{\pi}, \pi]$, where $\bar{\pi} = (e - e_{-1})/e_{-1} < 0$ and $\pi = (e - e_{-1})/e_{-1} > 0$. Within this range, the domestic currency is allowed to appreciate if $x \in [\bar{\pi}, 0)$, and allowed to depreciate if $x \in [0, \pi)$. In other words, $\bar{\pi}$ is the maximal rate of appreciation and $\pi$ is the maximal rate of depreciation that the exchange rate band permits.\footnote{Note that when $\bar{\pi} = \pi = 0$ the band reduces to a peg and when $\bar{\pi} = -\infty$, and $\pi = \infty$ the band becomes a free float.}

2.2 Speculators

We assume that the policymaker is facing a continuum of speculators with a total mass of one. When the exchange rate reaches the upper or the lower boundaries of the band, each speculator independently observes a noisy signal on the exchange rate that would prevail under laissez faire. Specifically, we assume that the signal obtained by speculator $i$ is given by

$$\theta_i = x + \varepsilon_i,$$

where $\varepsilon_i$ is a white noise, independent across speculators, and distributed uniformly on the interval $[-\varepsilon, \varepsilon]$. In what follows, we shall focus on the case where $\varepsilon$ is small so that the signals that speculators observe are "almost perfect." This information structure leads to a unique Bayesian equilibrium in our model, in spite of the fact that the decision of each individual speculator about whether to attack the currency depends on what he believes about the beliefs of other speculators.\footnote{In traditional models of currency crises, there are typically multiple equilibria (e.g., Obstfeld, 1986 and 1997). The fact that very small amounts of noise can lead to a unique equilibrium in models of self-fulfilling beliefs was first observed by Carlsson and van Damme (1993). Other papers (e.g., Morris and Shin, 1998, and Goldstein and Pauzner, 2000) achieve a unique equilibrium for larger amounts of noise, but assume that the distribution of the fundamentals in the economy ($x$ in our paper) is uniform. In our paper we only assume that $f(x)$ is unimodal but do not assume a particular distribution function. For a survey of related literature, see Morris and Shin (2001).}

The conditional density of $x$ given a signal $\theta_i$ is given by:

$$f(x \mid \theta_i) = \frac{f(x)}{F(\theta_i + \varepsilon) - F(\theta_i - \varepsilon)}. \quad (2.4)$$
Based on the signal that he observes, each speculator decides whether or not to attack the currency in the first phase. If the current exchange rate is at the lower bound of the band, each speculator can shortsell the foreign currency at the current (high) price of $e$ and then buy the same amount on the market to clear his position. If the policymaker fails to defend the band and the exchange rate falls to $e < e_0$, the speculator’s profit from shortselling is $e - e$. Denoting by $t$ the nominal transaction cost associated with exchanging domestic currency with foreign currency or vice versa, the speculator’s net payoff is $e_0 - e - t$. On the other hand, if the exchange rate stays at $e$, the net payoff of the speculator is simply $-t$. Likewise, if the current exchange rate is at the upper bound of the band, each speculator can buy the foreign currency at the current (low) price of $e$. If the policymaker exits the band and the exchange rate jumps to $e > e$, the speculator’s profit is $e - e$. Given the nominal transaction cost $t$, the speculator’s net payoff in this case is $e - e - t$. Again, if the policymaker successfully defends the band, the speculative attack fails, and the net payoff of the speculator is $-t$. If a speculator decides not to attack the band his payoff is 0.

Note that since $x = (e - e_{-1})/e_{-1}$, the payoff from attacking the lower bound of the band can be written as $e - (1 + x)e_{-1} - t$, and the payoff from attacking the upper bound of the band can be written as $(1 + x)e_{-1} - e - t$. Since $x$ has an unbounded support, then so long as $e_0 > -\infty$ and $e < \infty$ (i.e., the exchange rate regime is not a float), there are sufficiently low realizations of $x$ for which the payoff from attacking the lower bound of the band is positive and sufficiently high realizations of $x$ for which the payoff from attacking the upper bound of the band is positive. We now make the following assumption on $t$:

**Assumption 2:** The real transaction cost, $\frac{t}{e_{-1}}$, is small relative to $\delta$ (the future credibility loss associated with realignments) in the sense that $\frac{t}{e_{-1}} < \delta$.

As will become clear below, Assumption 2 ensures that speculators will always wish to attack the band if they believe that $x$ is sufficiently high to induce the policymaker to exit the band. This rules out the (uninteresting) possibility that speculators do not wish to attack the band even if they know that the policymaker is not going to defend it.

### 2.3 The sequence of events and the structure of information

The sequence of events unfolds as follows:

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It is shown later that speculators attack only in the first phase, as from the second phase and on there are no further changes in the level of the exchange rate. Hence an attack in the second phase cannot be beneficial to them.
Stage 1: The policymaker announces a band around the existing nominal exchange rate, $e_{-1}$. The band induces a permissible range of rates of change in the exchange rate, $[\pi, \overline{\pi}]$, beyond which the policymaker may intervene and prevent the exchange rate from moving outside the band.

Stage 2: The "free float" random shock, $x$, realizes and persists over the remaining stages of the game. There are now two possible cases.

If $x \in [\pi, \overline{\pi}]$, the nominal exchange rate is allowed to freely move within the boundaries of the band and is, therefore, determined by market forces. Since $x$ persists the exchange rate immediately adjusts to $(1 + x)e_{-1}$ and remains at this value for the remainder of the game.

By contrast, if $x < \pi$ or $x > \overline{\pi}$, the policymaker intervenes in the exchange rate market and always keeps the exchange rate from moving outside the band. As a result, the exchange rate is either at the upper or at the lower bound of the band. This is the initial defense phase discussed above. Simultaneously each speculator gets a noisy signal, $\theta_i$, on $x$ and decides whether or not to attack the band. Those decisions determine the fraction, $\alpha$, of speculators who decide to attack the band.

Stage 3: The policymaker observes $x$ and $\alpha$, and evaluates the total cost of continuing to defend the band. If the policymaker decides to continue to defend the band, the exchange rate stays at the boundary of the band and the policymaker incurs the cost $C(x, \alpha)$. If the policymaker decides to exit the band, the exchange rate moves to its freely floating rate so the induced rate of change in the exchange rate is $x$ and the policymaker bears a loss of future credibility that has a present value of $\delta$.

This sequence of events is consistent with the observation that, at least initially, governments invariably try to defend the currency and with the notion that, through this initial intervention phase, they get more precise information about fundamentals and about the desirability of persisting with the intervention. Although speculators are imperfectly informed, they realize that the policymaker will either defend the band or realign it, depending on the fraction of speculators he believes will attack the band. Hence each speculator tries to estimate this fraction and uses this estimate in deciding whether or not to attack the currency.

\[17\] A possible rationale for this initial phase of unconditional defense is that in it, the policymaker does not yet know $x$, nor the fraction of speculators who are going to attack the currency.
3 The equilibrium

In this section we characterize the perfect Bayesian equilibrium of the model. To this end, we solve the model backwards. First, whenever $x < \pi$ or $x > \pi$, then given the measure of speculators that have attacked in stage 2, $\alpha$, the policymaker makes a decision, in stage 3, on whether or not to continue to maintain the band. Second, given the signals that they observe in stage 2, speculators decide whether or not to attack the band. When $x \in [\pi, \pi]$, the policymaker does not intervene in the exchange rate market as the exchange rate moves within the band. Finally, in stage 1, before $x$ is realized, the policymaker sets the exchange rate regime.

3.1 The choice between defending the currency and realigning

In stage 3, whenever $x < \pi$ or $x > \pi$, the policymaker needs to decide whether to defend the currency and keep the exchange rate at the lower bound of the band if $x < \pi$, or at the upper bound of the band if $x > \pi$. Given the measure of speculators who have attacked the band in stage 2, $\alpha$, the cost of defending the band is $C(x, \alpha)$. If the policymaker does not defend the band and allows the exchange rate to be realigned, he incurs a future credibility loss whose present value is $\delta$. Hence, the policymaker defends the band against a speculative attack if and only if $C(x, \alpha) \leq \delta$. Using equation (2.1), this inequality implies that the set of states for which the policymaker will defend the band is

$$\pi + \alpha - \delta \leq x \leq \pi, \text{ and } \pi \leq x \leq \pi - \alpha + \delta. \quad (3.1)$$

When $x < \pi + \alpha - \delta$ or $x > \pi - \alpha + \delta$, the cost of defending the band is too large, so the policymaker allows the exchange rate to be realigned. Note that as $\alpha$ increases (more speculators attack the band in the initial phase), the set of realizations of $x$ for which the exchange rate is realigned expands.

3.2 Speculative attacks

When $x \in [\pi, \pi]$, the exchange rate is determined solely by market forces. Hence, speculators cannot gain from attacking the currency and their payoff is 0. In contrast, when $x < \pi$ or $x > \pi$, speculators know that, for some realizations of $x$, the policymaker will, at least initially, intervene in the foreign exchange market in order to prevent the exchange rate from moving outside the band. In such cases, the exchange rate no longer reflects market forces and speculators may choose to attack the band in the hope of making a profit in case there is a realignment.
If the exchange rate reaches the upper bound of the band, all speculators realize that $x \geq \pi$. If a speculator decides to attack the band, his net payoff is $e - \pi - t$ if there is a realignment and $-t$ otherwise. Recalling that the policymaker defends the upper bound of the band if and only if $x \leq \pi - \alpha + \delta$, and noting that $e = (1 + x)e_{-1}$ and $\pi = (1 + \pi)e_{-1}$, the net payoff of a speculator who attacks the upper bound of the band is $(x - \pi)e_{-1} - t$ if $x > \pi - \alpha + \delta$, and $-t$ if $\pi \leq x \leq \pi - \alpha + \delta$. Analogously, if the exchange rate reaches the lower bound of the band, the net payoff of a speculator who attacks the lower bound of the band is $(\pi - x)e_{-1} - t$ if $x < \pi + \alpha - \delta$, and $-t$ if $\pi + \alpha - \delta \leq x \leq \pi$.

Speculators do not observe $x$ directly and therefore decide whether or not to attack the band on the basis of the signals that they observe and in anticipation of the policymaker’s decision on whether or not to defend the band. For instance, if the exchange rate is at the upper bound of the band, speculators anticipate that a speculative attack will succeed if and only if $x > \pi - \alpha + \delta$. But since speculators make their decisions simultaneously, each speculator has to estimate the value of $\alpha$ when he decides whether or not to engage in currency speculation. Thus, his decision has to be based on his belief regarding the behavior of other speculators.\footnote{This is analogous to Keynes’ ”betting on a beauty contest” parable, where each individual does not try to find the most beautiful candidate but rather guess which candidate is held by the majority of others to be the most beautiful.}

In the following Lemma, we prove that in the limit as $\varepsilon \to 0$, all speculators adopt the same equilibrium threshold strategy, whereby each speculator attacks the upper bound of the band if and only if the signal that he observes exceed some threshold (which is the same for all speculators). The proof of the lemma, as well as the proofs of most other results, is in the Appendix.

**Lemma 1** Suppose that speculators have almost perfect information, i.e., $\varepsilon \to 0$. Then, when the exchange rate reaches the upper bound of the band, there exists a unique perfect Bayesian equilibrium, such that each speculator attacks the band if and only if the signal that he observes exceed some threshold $\theta^*$. Likewise, if the exchange rate reaches the lower bound of the band, then in equilibrium, each speculator attacks the band if and only if the signal that he observes is below the threshold $\theta^*$.

The proof of Lemma 1 is based on an iterative elimination of dominated strategies. The idea is as follows: the signal that speculator $i$ observes is distributed on the interval $[x - \varepsilon, x + \varepsilon]$. Therefore, if speculator $i$ observes a very high signal such that $\theta_i > \theta^* \equiv \pi + \delta + \varepsilon$, the speculator correctly infers that $x$ must be above $\pi + \delta$. At this level, the cost of defending the upper bound of the band exceeds the associated benefit even if no speculators attack the band. Hence, the speculator correctly infers that the policymaker is surely going to exit the band, so the net payoff from attacking it is $(x - \pi)e_{-1} - t$.\footnote{This is analogous to Keynes’ ”betting on a beauty contest” parable, where each individual does not try to find the most beautiful candidate but rather guess which candidate is held by the majority of others to be the most beautiful.}
But since $x > \pi + \delta$, the net payoff is at least $\delta e_{-1} - t$, which is strictly positive by Assumption 2. Therefore it is a dominant strategy for a speculator who observes a signal above $\overline{\theta}$ to attack the upper bound of the band.\footnote{The strategy to attack the upper bound of the band if $\theta_i > \overline{\theta}$ is dominant because it is optimal no matter what other speculators are going to do.} But now, a speculator who observes a signal just slightly below $\overline{\theta}$ realizes that a large fraction of other speculators must have observed signals above $\overline{\theta}$ and will surely attack the band. From that, this speculator concludes that the policymaker will exit the band (the cost of defending the band when many speculators attack it exceeds the associated benefit) so attacking the band is again an optimal strategy. This chain of reasoning can proceed further where each time we lower the critical signal above which a speculator will attack the upper bound of the band, given that speculators who observe even higher signals surely attack it.

When speculator $i$ observes a low signal such that $\theta_i < \underline{\theta} \equiv \pi + \frac{t}{e_{-1}} - \varepsilon$, he correctly infers that $x < \pi + \frac{t}{e_{-1}}$. Consequently, even if the policymaker will surely exit the band, the payoff from attacking it is negative as $(x - \pi)e_{-1} - t < \left(\left(\pi + \frac{t}{e_{-1}}\right) - \pi\right)e_{-1} - t = 0$. Hence, it is a dominant strategy for speculator $i$ not to attack the band after observing a signal below $\underline{\theta}$. But then, a speculator who observes a signal just slightly above $\underline{\theta}$ will infer that a large fraction of other speculators must have observed signals below $\underline{\theta}$ and will surely not attack the band. Hence, this speculator concludes that the policymaker will successfully defend the band (only a few speculators might attack the band so the cost of defending is small) so it is optimal not to attack the band. Again, we can proceed with this chain of reasoning, each time raising the critical signal below which a speculator will not attack the upper bound of the band.

In the limit as $\varepsilon \to 0$, the critical signal above which speculators attack the band coincides with the critical signal below which speculators do not attack the band. This establishes the existence of a unique threshold signal, $\overline{\theta}$, such that all speculators attack the upper bound of the band if and only if they observe signals above $\overline{\theta}$. Similar arguments apply in the case where the exchange rate reaches the lower bound of the band and establish the existence of a unique threshold signal, $\underline{\theta}$, such that all speculators will attack the lower bound of the band if and only if they observe signals below $\underline{\theta}$.

Having established that all speculators adopt the same threshold strategy, we are now ready to characterize the two thresholds, $\overline{\theta}$ and $\underline{\theta}$.

**Lemma 2** Suppose that $\varepsilon$ goes to 0. Then,
(i) the two thresholds, $\overline{\theta}$ and $\theta^*$, are given by $\overline{\theta} = \pi + r$, and $\theta^* = \pi - r$, where
$$r = \sqrt{\frac{t}{e - 1} + \frac{(\delta - 1)^2}{4} + \frac{\delta - 1}{2}}.$$  

(ii) Let $\pi(\overline{\theta})$ ($\pi(\theta^*)$) be the critical value of $x$ above (below) which the policymaker realigns when each speculator attacks the currency if and only if he gets a signal above $\overline{\theta}$ (below $\theta^*$). Then $\pi(\overline{\theta})$ goes to $\overline{\theta}$ and $\pi(\theta^*)$ goes to $\theta^*$. Consequently, $\alpha = 0$ if $\theta^* \leq x \leq \overline{\theta}$ and $\alpha = 1$ otherwise.

**Proof.** Part (i): We begin with $\overline{\theta}$. Suppose that the exchange rate is at the upper bound of the band and suppose absent intervention, the rate of change in the exchange rate is $x < \overline{\theta} - \varepsilon$. Recalling that the signals that speculators observe are drawn from the interval $[x - \varepsilon, x + \varepsilon]$, it is clear that the highest signal that a speculator can observe in this case is less than $\overline{\theta}^*$. Hence, no speculator will attack the band so $\alpha = 0$. On the other hand, if $x > \overline{\theta}^* + \varepsilon$, then the lowest signal that a speculator can observe is above $\overline{\theta}$. Hence, all speculators will attack the band and $\alpha = 1$. In intermediate cases where $\overline{\theta}^* - \varepsilon \leq x \leq \overline{\theta}^* + \varepsilon$, some speculators will observe signals above $\overline{\theta}$ and will attack the band while others will observe signals below $\overline{\theta}$ and will not attack the band. Given that $\varepsilon_i$ is distributed uniformly on the interval $[-\varepsilon, \varepsilon]$, the density of speculators who observe signals above $\overline{\theta}$ and attack the band is $\frac{x - (\overline{\theta} - \varepsilon)}{\varepsilon}$. In sum, given $x$ and given $\overline{\theta}$, the fraction of speculators who choose to attack the upper bound of the band is given by:

$$\alpha(x, \overline{\theta}) = \left\{ \begin{array}{ll}
0, & x < \overline{\theta} - \varepsilon, \\
\frac{x - (\overline{\theta} - \varepsilon)}{\varepsilon}, & \overline{\theta} - \varepsilon \leq x \leq \overline{\theta} + \varepsilon, \\
1, & x > \overline{\theta} + \varepsilon.
\end{array} \right. \quad (3.2)$$

Given $\alpha(x, \overline{\theta})$, the cost that the policymaker incurs when defending the upper bound of the band against a speculative attack is $C(x, \alpha(x, \overline{\theta}))$, where $C(x, \cdot)$ is given by equation (2.1). Since the policymaker incurs a future credibility loss, $\delta$, if a realignment takes place, it follows that his optimal policy is to defend the band so long as $C(x, \alpha(x, \overline{\theta})) \leq \delta$, and exit it if $C(x, \alpha(x, \overline{\theta})) > \delta$. Since $\alpha(x, \overline{\theta})$ is weakly increasing in $x$, a realignment will occur if and only if $x$ is above some threshold level $\pi(\overline{\theta})$. Note that in a perfect Bayesian equilibrium, $\pi(\overline{\theta})$ cannot be below $\overline{\theta} - \varepsilon$, because then, the number of speculators that attack the band at $x = \pi(\overline{\theta})$ is zero, and the policymaker should strictly prefer not to exit the band (unless $\pi(\overline{\theta}) \geq \pi + \delta$, but this means that $\overline{\theta}$ is above $\pi + \delta + \varepsilon$, which is a contradiction to the fact that speculators have a dominant strategy to attack the band when they observe signals above $\pi + \delta + \varepsilon$). Similarly, one can show that $\pi(\theta^*)$
cannot be above $\overline{\theta} + \epsilon$. Then, using equations (3.1) and (3.2), we get that when the exchange rate reaches the upper bound of the band, a realignment takes place if and only if

$$x > \pi(\overline{\theta}) \equiv \frac{\epsilon(2\pi + 2\delta - 1) + \overline{\theta}}{2\epsilon + 1}. \quad (3.3)$$

Next, consider the decision problem that speculator $i$ faces after observing the signal $\theta_i$. Given that $\theta_i$ is drawn from the interval $[x - \epsilon, x + \epsilon]$, the speculator realizes that $x$ is distributed on the interval $[\theta_i - \epsilon, \theta_i + \epsilon]$, and its conditional density is $f(x \mid \theta_i)$ as given by equation (2.4). But, since the speculator anticipates that the policymaker will defend the band whenever $x < x(\theta^*)$, it follows that he expects a net payoff of $(x - \pi)e_{-1} - t$ if $x > \pi(\overline{\theta})$ and $-t$ if $x < \pi(\overline{\theta})$. Lemma 1 implies that, in equilibrium, speculators attack the band if and only if they observe signals above $\theta^*$. A speculator that observes exactly $\theta^*$ is indifferent between attacking the band and not attacking the band. Using this indifference condition, we get the following equation:

$$\int_{\pi(\overline{\theta})}^{\overline{\theta} + \epsilon}(x - \pi)e_{-1}f(x \mid \overline{\theta})dx - t = 0. \quad (3.4)$$

Substituting the expression for $f(x \mid \theta_i)$ from equation (2.4) into equation (3.4) and letting $\epsilon$ go to 0, we get:

$$\lim_{\epsilon \to 0} \frac{\int_{\pi(\overline{\theta})}^{\overline{\theta} + \epsilon}(x - \pi)e_{-1}f(x)dx}{\pi(\overline{\theta}) - \pi(\overline{\theta} - \epsilon)} = t. \quad (3.5)$$

Substituting the expression for $\pi(\overline{\theta})$ from equation (3.3), using L’Hôpital’s rule, and recalling from equation (3.3) that as $\epsilon$ goes to 0, $\pi(\overline{\theta})$ goes to $\overline{\theta}$, we obtain:

$$(\overline{\theta} - \pi)\left[1 - \delta + (\overline{\theta} - \pi)\right] = \frac{t}{e_{-1}}. \quad (3.6)$$

Solving this equation for $\overline{\theta}$ yields the expression in the statement of the proposition. The characterization of $\theta^*$ is completely analogous.

**Part (ii):** When $\epsilon$ goes to 0, equation (3.3) shows that $\pi(\overline{\theta})$ goes to $\overline{\theta}$ and equation (3.2) shows that $\alpha = 0$ if $x \leq \overline{\theta}$ and $\alpha = 1$ if $x > \overline{\theta}$. Applying the same logic, when $\epsilon$ goes to 0, to the lower bound of the band it can be similarly shown that $\pi(\underline{\theta})$ goes to $\underline{\theta}$, $\alpha = 0$ if $x \geq \underline{\theta}$, and $\alpha = 1$ if $x < \underline{\theta}$. Q.E.D.

The unique equilibrium obtained here contrasts with second generation models of currency crises such as those of Obstfeld (1996, 1997) in which multiple equilibria are
possible. The reason for the difference is that in that framework the only constraint on expectations is that they be consistent with the equilibrium that arises, given those expectations. By contrast, in our framework, it is also required that each speculator estimate the information available to other speculators based on his personal signal. As a consequence there is less freedom in choosing the range of possible beliefs that each speculator may entertain about the beliefs of others, and therefore about each speculator estimate of the likelihood that the policymaker will defend the currency. Since the beliefs of speculators about \( x \) and about other speculators’ beliefs are tied together by the realization of \( x \), the realization of this fundamental leads all of them to believe either that all of them believe that the policymaker will abandon the band, or leads all of them to believe that all of them believe that he will defend the band. Given the realization of \( x \) one of those beliefs about the beliefs of others is inconsistent with a conditional rational inference about the beliefs of others, which rules out multiple equilibria.21

Having solved for the behavior of speculators, we turn next to the implications of this behavior for the exchange rate band. Recall that we are interested in cases where \( \varepsilon \rightarrow 0 \). Then, as equation (3.3) shows, \( \pi(\bar{\theta}') \rightarrow \bar{\theta}' \) and \( \pi(\bar{\theta}) \rightarrow \theta^* \). This implies in turn that the exchange rate band gives rise to two ranges of effective commitment (REC) such that if \( x \) falls inside one of these ranges, the policymaker defends the band. The positive REC is equal to \([\pi, \bar{\theta}']\); when \( x \in [\pi, \bar{\theta}'] \), the policymaker defends the currency and keeps the exchange rate from moving above \( \pi \). The negative REC is equal to \([\theta^*, \pi]\); when \( x \in [\theta^*, \pi] \), the policymaker defends the currency and keeps the exchange rate from moving below \( \pi \). When \( x \) falls outside the band and the two RECs, the policymaker exits the band and despite his earlier announcement, tolerates a realignment. On the other hand, when \( x \) falls inside the band, i.e., when \( x \in [\pi, \bar{\pi}] \), the policymaker allows the exchange rate to move freely in accordance with market forces.

Since \( \bar{\theta}' = \bar{\pi} + r \) and \( \theta^* = \bar{\pi} - r \), we can express the positive REC as \([\pi, \bar{\pi} + r]\), and the negative REC as \([\pi - r, \pi]\). Lemma 2 indicates that \( r \) depends on \( t \), which is the transaction cost associated with speculative attacks, and on \( \delta \) which is the present value of the loss from future diminished credibility triggered by a realignment. \( r \) is independent of the boundaries of the band, \( \bar{\pi} \) and \( \pi \). This means that the actual size of the two ranges of \( x \) at which the policymaker is committed to intervene in the exchange rate market does not depend on how wide the band is. What the policymaker can do is to shift the two RECs either closer to or away from \( 0 \) by choosing \( \bar{\pi} \) and \( \pi \) appropriately.

21 By contrast, in the absence of this additional constraint multiple equilibria do arise for some ranges of \( x \). To see that consider, for example, the case in which the realization of \( x \) is such that \( x - \bar{\pi} - \delta = -0.5 \). If it is assumed that it is common knowledge that all speculators believe that the policymaker will defend the currency it indeed pays him to do that. Hence, this strategy and the postulated speculators’ beliefs constitute an equilibrium. By the same token if it is assumed that it is common knowledge that all of them believe that the policymaker is not going to defend the band and that \( \alpha = 1 \) so that \( C(x, \alpha) = x - \bar{\pi} + 1 > \delta \), it indeed does not pay the policymaker to defend the band. Hence, having a speculative attack and a realignment is also an equilibrium for the same value of \( x \).
Straightforward comparative statics analysis reveals that \( r \) increases with \( t \) and with \( \delta \). These properties are intuitive since they imply that a realignment is less likely when it is more costly for speculators to attack the band and when the policymaker is more averse to realignments.

The discussion is now summarized in the following proposition:

**Proposition 1** The exchange rate band gives rise to two ranges of effective commitment (REC), a positive REC defined by \( [\pi, \pi + r] \), and a negative REC defined by \( [\pi - r, \pi] \), where \( r \) is defined in Lemma 2. The two RECs partition the support of \( x \) into five regions.

- When \( x \) falls inside the positive REC, the policymaker defends the currency and prevents the exchange rate from moving above \( \pi \).
- When \( x \) falls inside the negative REC, the policymaker defends the currency and prevents the exchange rate from moving below \( \pi \).
- When \( x \) falls below the negative REC, or above the positive REC, or inside the band, the policymaker lets the exchange rate move freely in accordance with market forces.
- The width of the two RECs, \( r \), increases with \( t \) and with \( \delta \) but is independent of the boundaries of the band, \( \pi_\text{L} \) and \( \pi_\text{H} \).

### 3.3 The choice of band width

We now turn to the policymaker’s objective function. Note first that the expected variability of the exchange rate around the current level can be written as follows:

\[
E \left| \pi \right| = - \int_{-\infty}^{\pi - r} x f(x) dx - \int_{\pi - r}^{\pi} \pi f(x) dx - \int_{\pi}^{0} x f(x) dx + \int_{0}^{\pi} x f(x) dx + \int_{\pi}^{\pi + r} \pi f(x) dx + \int_{\pi + r}^{\infty} x f(x) dx.
\]

Equation (3.11) shows that the existence of a band has a moderating effect on \( |\pi| \) only inside the two RECs. Over these two regions, the policymaker is expected to keep the exchange rate from moving outside the boundaries of the band.
Second, using equation (2.1) and Lemma 2, the cost of intervention in the exchange rate market is $\pi - x$ if $x \in [\pi - r, \pi]$, and $x - \pi$ if $x \in [\pi, \pi + r]$. When either $x < \pi - r$ or $x > \pi + r$, there are realignments so the policymaker incurs a loss, $\delta$. Hence, using equations (2.2) and (3.11), the expected payoff of the policymaker, given $\pi$ and $\bar{\pi}$, is

$$V = A \left[ \int_{-\infty}^{\pi-r} x f(x) dx + \int_{\pi-r}^{\pi} \pi f(x) dx + \int_{\pi}^{0} x f(x) dx 
- \int_{0}^{\pi} x f(x) dx - \int_{\pi}^{\pi+r} \pi f(x) dx - \int_{\pi+r}^{\infty} x f(x) dx \right] 
- \int_{-\infty}^{\pi-r} \delta f(x) dx - \int_{\pi-r}^{\pi} (\pi - x) f(x) dx - \int_{\pi}^{\pi+r} (x - \pi) f(x) dx - \int_{\pi+r}^{\infty} \delta f(x) dx. \tag{3.12}$$

The first two lines in equation (3.12) represent the policymaker’s loss from exchange rate uncertainty. The first and last expressions in the third line represent the policymaker’s loss from realignments that involve either a downward adjustment in the exchange rate (the first term in the third line) or an upward adjustment (the last term in the third line). Finally, the second and third terms in the third line of equation (3.12) represent the cost that the policymaker incurs when he maintains the exchange rate within the band over the negative REC (the second term) and the positive REC (the third term) respectively.

The policymaker chooses the boundaries of the band, $\pi$ and $\bar{\pi}$, so as to maximize his expected payoff. The first order conditions for an interior solution to the policymaker’s problem (i.e., for $-\infty < \pi < \bar{\pi} < \infty$) are:

$$\frac{\partial V}{\partial \pi} = -[r(A - 1) + \delta] f(\pi - r) + (A - 1) \int_{\pi-r}^{\pi} f(x) dx = 0, \tag{3.13}$$

and,

$$\frac{\partial V}{\partial \bar{\pi}} = [r(A - 1) + \delta] f(\bar{\pi} + r) - (A - 1) \int_{\bar{\pi}+r}^{\bar{\pi}} f(x) dx = 0. \tag{3.14}$$

We prove in the Appendix that if $f''(x) \leq 0$ and if $A > 1$, then given Assumption 1, $V$ is globally concave in $\pi$ and $\bar{\pi}$, so equations (3.13) and (3.14) are sufficient for a unique maximum. The first term in the second line of (3.13) is the marginal effect of $\pi$ on exchange rate variability. Since by Assumption 1, $f(x) > f(\pi - r)$ for all $x \in [\pi - r, \pi]$, this term is positive and represents the marginal benefit from raising $\pi$. This marginal
When is the exchange rate a peg, a float, or a band - and if a band, of what width?

We begin this section by studying the detailed properties of the equilibrium exchange rate regime using equations (3.13) and (3.14).

Proposition 2 In equilibrium, the exchange rate band has the following properties:

(i) **Free float:** If \( A \leq 1 \), then \( \pi = -\infty \) and \( \pi = \infty \), so the optimal regime is a free float.

(ii) A nondegenerate band: If

\[
1 < A < A(-r) \equiv 1 + \frac{\delta}{\int_{-\infty}^{0} \left[ \frac{f(x)}{f(-r)} - 1 \right] dx}.
\]
then $-\infty < \pi < 0$. Likewise, if

$$1 < A < \overline{A}(r) \equiv 1 + \frac{\delta}{\int_0^r \left[ \frac{f(x)}{f(r)} - 1 \right] dx},$$

then $0 < \bar{\pi} < \infty$. Hence, the optimal regime is a nondegenerate band.

(iii) A peg: If $V$ is concave in $\pi$ and in $\bar{\pi}$, and $A > \text{Max}\{\underline{A}(r), \overline{A}(r)\}$, then $\pi = \bar{\pi} = 0$, so the optimal regime is a peg.

(iv) Symmetry: If $f(x)$ is symmetric around 0, so that $f(-x) = f(x)$ for all $x$, the band will be symmetric around 0 in the sense that $-\bar{\pi} = \bar{\pi}$.

Proposition 2 shows that depending on the policymaker’s aversion to nominal exchange rates variability, the optimal regime can either be a peg, a free float, or a band. When the policymaker is not too concerned with nominal exchange rate variability, i.e., $A \leq 1$, he sets a free float and avoids the cost of maintaining a band. On the other hand, if the policymaker is sufficiently concerned with nominal exchange rate variability, i.e., $A > \text{Max}\{\underline{A}(r), \overline{A}(r)\}$, he minimizes it by adopting a peg.\(^{22}\) In intermediate cases, the policymaker balances the two objectives, namely limiting exchange rate uncertainty and minimizing the cost of intervention by setting a nondegenerate band. Now intervention occurs only when $x$ falls inside the negative or the positive RECs. Part (iv) of the proposition states that a sufficient condition for the band to be symmetric is that the distribution of shocks, $f(x)$, is symmetric around 0 (i.e., under laissez faire, depreciations and appreciations are equally likely).\(^{23}\)

Next, we turn to comparative statics analysis for the case in which the policymaker’s problem has a unique interior solution, i.e., $-\infty < \pi < 0 < \bar{\pi} < \infty$. As Proposition 2 indicates, this requires $A$ to be above 1 but not by ”too much.”

**Proposition 3** Suppose that the policymaker’s problem has a unique interior solution. Then:

\(^{22}\)Note that a peg does not mean that the exchange rate is fixed under all circumstances. When the absolute value of $x$ exceeds $r$, the policymaker abandons the peg and the exchange rate is realigned. Nonetheless, under a peg, the policymaker maintains stability over the range of “small” shocks where $x \in [-r, r]$. Given Assumption 1, such small shocks are more likely than big ones, so when $A$ is large, it is optimal for the policymaker to eliminate these shocks by adopting a peg.

\(^{23}\)Note that the symmetry of the band is in terms of the permissible rates of evolutions and devaluations of the exchange rate rather than in terms of the gap of the upper and lower bounds of the band from the center rate (i.e., the symmetry is in terms of $x$ rather than $\epsilon$).
(i) \( \bar{\pi} \) decreases and \( \pi \) increases with \( A \).

(ii) The probability that a speculative attack will occur increases with \( A \).

Proposition 3 says that the policymaker sets a narrower band and thereby allows the exchange rate to move freely within a narrower range around the center rate, as he becomes more concerned with exchange rate stability (i.e., as \( A \) increases). Part (ii) of the proposition shows that as a result of this tightening of the band, the likelihood of a speculative attack increases. Note that as Proposition 2 shows, when \( A \) goes above \( A^1(r) \) and \( A^2(r) \), the optimal band width becomes 0 so the optimal regime is a peg. On the other hand, when \( A \) falls below 1, the optimal band width becomes infinite implying that the optimal regime is a free float. Given that a substantial part of international trade is invoiced in US Dollar (McKinnon, 1979), it is likely that policymakers of a key currency country like the US are going to be less sensitive to nominal exchange rate volatility and therefore have a smaller \( A \) than policymakers in small open economies. Therefore, our model predicts that the US, Japan, and the Euro should be on a float, while Hong-Kong, Panama, Estonia, Lithuania, Bulgaria, and Argentina should be on either pegs, currency boards, or even full dollarization. This indeed happens to be the case.

Next, we examine how the exchange rate band changes when more extreme realizations of \( x \) become more likely. This comparative statics exercise involves shifting probability mass from realizations of \( x \) that are either inside the band or inside the two REC's and therefore do not lead to realignments to realizations that are either below the negative REC or above the positive REC and therefore lead to realignments. Specifically, suppose that the density function \( g(x) \) is obtained from the original density function \( f(x) \) by shifting probability mass from the center of \( f(x) \) towards the tails of \( f(x) \), while keeping the expectation of \( x \) and its mode constant. Moreover, suppose that \( g(x) \) lies above \( f(x) \) for all \( x < \bar{\pi} - r \) and all \( x > \bar{\pi} + r \), where \( \pi \) and \( \bar{\pi} \) are the solutions to the policymaker’s problem under the original density function \( f(x) \). Hence \( g(x) \) cuts \( f(x) \) exactly twice: once from above at \( x = \pi - r \), and once from below at \( x = \pi + r \).

**Proposition 4** Suppose that \( f(x) \) and \( g(x) \) are two density functions with a mode at 0 and equal expected values. Moreover, suppose that \( g(x) \) lies above \( f(x) \) for all \( x < \bar{\pi} - r \) and all \( x > \bar{\pi} + r \), where \( \pi \) and \( \bar{\pi} \) are the solutions to the policymaker’s problem under the original density function \( f(x) \). Then, the policymaker adopts a wider band under \( g(x) \) than under \( f(x) \).

Intuitively, when more extreme realizations of \( x \) become more likely, the policymaker is more likely to bear the loss of future credibility associated with realignments. There-
fore, the policymaker widen the band and thereby lowers the likelihood that a costly realignment will take place.

5 The effects of reductions in the cost of switching between currencies

During the last three decades there has been a world-wide gradual lifting of restrictions on currency flows and on related capital account transactions. One consequence of this trend, in terms of the model, is a reduction in the nominal transaction cost, $t$. The model can therefore be used to answer questions about the connection between restrictions on capital flows, the optimal exchange rate regime and the likelihood of currency crises. This section presents several propositions that address these questions. The first proposition concerns the relationship between the parameter $t$ and the size of the set of countries that choose pegs.

**Proposition 5** The bounds $\underline{A}(-r)$ and $\overline{A}(r)$ above which the policymaker adopts a peg decrease with $t$. As a consequence, when $t$ decreases, the policymaker adopts a peg for a narrower range of values of $A$.

The proposition predicts that liberalization of the capital account, as characterized by a reduction in $t$, should lead to a narrowing of the set of countries that maintain pegs. It also implies that, in spite of this trend, countries with strong preference for stability of the exchange rate (e.g., small open economies with relatively large shares of foreign denominated trade and capital flows as well as emerging markets) will continue to peg even in the face of capital market liberalization. By contrast, countries with intermediate preference for exchange rate stability (e.g., more financially mature economies with a larger fraction of domestically denominated debt and capital flows) will move from pegs to bands. Proposition 5 seems to be consistent with casual evidence. Two years following the 97/98 currency crises in the Far East, most emerging market countries in that region are back on pegs (Calvo and Reinhart, 2000, and McKinnon, 2001). And, in spite of the convertibility that was established in 1991, Argentina has been maintaining a peg to the US Dollar for the last decade. On the other hand, following the EMS currency crisis at the beginning of the 90’s, the system of cooperative pegs that had existed prior to the crisis was replaced by wide bands until the formation of the EMU at the beginning of 1999.

We turn next to the effect of $t$ in case the optimal regime is a band.
Proposition 6 Suppose that the policymaker’s problem has a unique interior solution. Then:

(i) $\pi$ decreases and $\bar{\pi}$ increases with $t$.

(ii) The probability that a speculative attack will occur increases with $t$.

Part (i) of Proposition 6 says that the policymaker sets a narrower band as restrictions on the capital account become more severe (i.e., as $t$ increases). The mechanics of this comparative static result follow from Proposition 1 which shows that an increase in $t$ leads to an increase in $r$ and hence widens the two RECs. Basically, as the transaction cost that speculators incur when they attack the band increases, the policymaker is able to defend the band for a broader set of shocks. In other words, the band becomes more effective in guaranteeing exchange-rate stability. The policymaker’s reaction to this is to set a narrower band and to allow the exchange rate to move freely only within a narrower range around the center rate.

Part (ii) of Proposition 6 implies that on balance, restrictions of free capital markets raise the likelihood of a currency crisis. This is the outcome of two opposing effects. First, holding the band width constant, an increase in $t$ leads to a wider range of effective commitment and hence to a lower probability of speculative attacks. This effect already appears in the recent literature on international financial crises (e.g., Morris and Shin, 1998). But when $t$ increases, the width of the band does not remain constant; part (i) of the proposition shows that when $t$ increases, policymakers attempt to achieve more ambitious stability objectives by narrowing the width of the band. This in turn, raises the probability of speculative attacks. A-priori, it is not clear which effect is larger and hence whether an increase in $t$ leads to a higher or to a lower probability of speculative attacks. However, part (ii) of Proposition 6 shows that given Assumption 1 ($f(x)$ is unimodal) and given that $A > 1$, the second effect always dominates.

The more general lesson from part (ii) of Proposition 6 is that there are cases (our model is an example), in which higher, policy imposed, costs of capital flows lead to more speculative attacks. This occurs because the exchange rate band tends to be narrower under those circumstances. Thus, when the endogeneity of the exchange rate regime is taken into consideration, strong restrictions on capital flows may increase rather than decrease the likelihood of currency crises. One implication of this result is that a ”Tobin tax” may not necessarily achieve the purpose for which it has been proposed.

Proposition 7 Policymakers achieve higher equilibrium values of objectives, $V$, when $t$ is higher.
Proposition 7 implies that policymakers have an incentive to impose capital account restrictions and thereby raise the transactions costs $t$, despite the fact that, at least within the framework of this model, this may increase the likelihood of currency crises. The reason why policymakers prefer to impose capital account restrictions is that speculative attacks impose a constraint on the policymaker that needs to be taken into account in choosing the optimal exchange rate regime. By raising $t$, the policymaker lowers the incentive to mount a speculative attack and is therefore able to relax this constraint, at least somewhat. It should be noted however that Proposition 7 does not provide a complete account of the desirability of imposing capital account restrictions, since the benefits of free capital mobility are not taken into account in our model.

6 The exchange rate band and the policymaker’s reputation

In practice, there is typically considerable uncertainty about the commitment ability of policymakers. In this section, we examine how this uncertainty affects the optimal exchange rate regime. To this end, we assume that there are two possible types of policymakers. The first type, to which we refer as dependable, is identical to the policymaker that we considered so far. The second type, to which we refer as opportunistic, differs from the dependable type in that he does not bear the cost $\delta$ if the exchange rate is realigned in the last stage of the game. Although both policymaker types defend the currency in the penultimate, second stage, an opportunistic policymaker has no incentive to continue to defend it after the initial defense stage. So, when he is in office a realignment always takes place in the last stage of the game when $x$ falls outside the exchange rate band. We assume that speculators assign a probability $\beta$ to the policymaker’s type being dependable. In what follows we will interpret $\beta$ as a measure of the policymaker’s “reputation,” and will examine how the optimal exchange rate regime is affected by changes in $\beta$. As a point of reference, it should be noted that the analysis so far referred to the case where $\beta = 1$.

Before we begin the analysis, we first modify Assumption 2 as follows:

Assumption 3: The real transaction cost, $\frac{t}{e-1}$, is small relative to $\delta$ but not too small in the sense that $\delta(1 - \beta) < \frac{t}{e-1} < \delta$.

Assumption 3 ensures that speculators will always wish to attack the band if they

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24 Even an opportunistic policymaker that does not intend to persist in defending the band needs to put the initial (short) defense stage on “automatic pilot” in order to prevent the public from separating him, already in the first stage (when the band is announced), from his dependable counterpart.
believe that $x$ is sufficiently high to induce the dependable policymaker to exit the band but never wish to attack the band if they believe that $x$ is sufficiently low to induce the dependable policymaker to defend the band.

With Assumption 3 in place, we now examine how the presence of an opportunistic policymaker affects the decisions of speculators on when to attack the band.

Lemma 3 Suppose that $\varepsilon$ goes to 0. Then,

(i) speculators will attack the upper bound of the band if and only if they observe signals above $\theta_{\beta}^*$. They will attack the lower bound of the band if and only if they observe signals below $\theta_{\beta}^*$. The two thresholds, $\theta_{\beta}^*$ and $\theta_{\beta}^*$, are given by $\theta_{\beta}^* = \pi + r^\beta$, and $\theta_{\beta}^* = \pi - r^\beta$, where

$$r^\beta = \sqrt{\frac{t}{\beta e^{-1}} + \frac{(\delta - \frac{1}{2})^2}{4} + \frac{\delta - \frac{1}{2}}{2}}.$$  

(ii) $r^\beta$ increases with $\beta$.

The speculators’ behavior implies that the positive REC is now given by $[\pi, \pi + r^\beta]$ while the negative REC becomes $[\pi - r^\beta, \pi]$. Since $r^\beta$ increases with $\beta$, it follows that when the policymaker’s reputation is imperfect (i.e., $\beta < 1$), the two RECs become narrower relative to the perfect reputation case (i.e., when $\beta = 1$). The intuition behind this result is that when $\beta < 1$, speculators believe that with some probability the policymaker is not going to defend the band. Hence, the expected gain from attacking the band is now larger. As a result, a dependable policymaker finds it more difficult to defend the band.

Next, we examine the impact of the policymaker’s reputation on the choice of exchange rate regime. We begin with the expected variability in the exchange rate. When the policymaker’s reputation is imperfect, it is anticipated that with probability $\beta$ the policymaker will be dependable and will defend the band against speculative attacks whenever $x$ falls inside the two RECs, and with probability $1 - \beta$ the policymaker will be opportunistic and will not defend the band at all. Hence, the expected variability in

\[25\] This is analogous to a result in Cukierman and Liviatan (1991) in the context of a Barro-Gordon (1983) inflation bias equilibrium in which the public is uncertain about the dependability of the policymaker. Cukierman and Liviatan show that the lower the reputation of a (dependable) policymaker the less ambitious is his inflation target.
the exchange rate is:

\[
E^\beta |\pi| = - \int_{-\infty}^{-\pi} xf(x) dx - \int^{-\pi}_{\pi} \beta f_{x} + (1-\beta)x f(x) dx - \int_{\pi}^{0} xf(x) dx \quad (4.2)
\]

\[
+ \int_{0}^{\pi} xf(x) dx + \int^{\pi+r}_{\pi} \beta f_{x} + (1-\beta)x f(x) dx + \int_{\pi+r}^{\infty} xf(x) dx.
\]

Note that \(\beta\) affects \(E^\beta |\pi|\) both through its effect on the width of the two RECS and its effect on the expected change in the exchange rate inside the two RECS which is now a linear combination of \(\pi\) and \(x\) inside the negative REC and \(\pi\) and \(x\) inside the positive REC.

Given \(E^\beta |\pi|\), the expected payoff of a dependable policymaker becomes,

\[
V^\beta = A \left[ \int_{-\infty}^{-\pi-r^\beta} xf(x) dx + \int_{-\pi+r^\beta}^{-\pi} \beta f_{x} + (1-\beta)x f(x) dx + \int_{-\pi}^{0} xf(x) dx 
- \int_{0}^{\pi} xf(x) dx - \int^{\pi+r}_{\pi} \beta f_{x} + (1-\beta)x f(x) dx - \int_{\pi+r}^{\infty} xf(x) dx \right] 
- \int_{-\infty}^{-\pi-r^\beta} \delta f(x) dx - \int_{-\pi-r^\beta}^{\pi} (\pi-x) f(x) dx - \int^{\pi+r}_{\pi} (x-\pi) f(x) dx - \int_{\pi+r}^{\infty} \delta f(x) dx. \quad (4.3)
\]

We do not need to specify the expected payoff of the opportunistic policymaker because the decision problem of such a policymaker is simple: given that an opportunistic policymaker does not intend to defend the band, he always wishes to adopt the same band that his dependable counterpart adopts in order to favorably affect the expectations for the variability of the exchange rate.

A dependable policymaker chooses the boundaries of the band, \(\pi\) and \(\pi\), so as to maximize his expected payoff. The first order conditions for an interior solution are given by:

\[
\frac{\partial V^\beta}{\partial \pi} = - [r^\beta(\alpha \pi - 1) + \delta] f_{x}(\pi-r^\beta) + (\alpha \pi - 1) \int_{\pi-r^\beta}^{\pi} f(x) dx 
= \alpha \pi \int_{\pi-r^\beta}^{\pi} [f(x) - f(\pi-r^\beta)] dx - \int_{\pi-r^\beta}^{\pi} [f(x) - f(\pi-r^\beta)] dx + \delta f(\pi-r^\beta) = 0, \quad (4.4)
\]

and,

\[
\frac{\partial V^\beta}{\partial \pi} = [r^\beta(\alpha \pi - 1) + \delta] f(\pi+\pi^\beta) - (\alpha \pi - 1) \int_{\pi}^{\pi+r^\beta} f(x) dx 
= -\alpha \pi \int_{\pi}^{\pi+r^\beta} [f(x) - f(\pi+r^\beta)] dx + \int_{\pi}^{\pi+r^\beta} [f(x) - f(\pi+r^\beta)] dx + \delta f(\pi+r^\beta) = 0. \quad (4.5)
\]
As in the case where $\beta < 1$, it can be shown that if $f''(x) \leq 0$ and $A\beta > 1$, then given Assumption 1, $V^{\beta}$ is globally concave in $\pi$ and $\overline{\pi}$ so equations (4.4) and (4.5) are sufficient for a unique maximum. The next proposition characterizes the optimal exchange rate regime when the policymaker has imperfect reputation.

**Proposition 8** In equilibrium, the exchange rate band has the following properties:

(i) **Free float:** If $A\beta \leq 1$, then $\underline{\pi} = -\infty$ and $\overline{\pi} = \infty$, so the optimal regime is a free float.

(ii) **A nondegenerate band:** If

$$1 < A\beta < A(-r^\beta) \equiv 1 + \frac{\delta}{\int_{-r^\beta}^0 \left[ \frac{f(x)}{f(-r^\beta)} - 1 \right] dx},$$

(4.6)

then $-\infty < \pi < 0$. Likewise, if

$$1 < A\beta < \overline{A}(r^\beta) \equiv 1 + \frac{\delta}{\int_0^{r^\beta} \left[ 1 - \frac{f(x)}{f(-r^\beta)} \right] dx},$$

(4.7)

then $0 < \underline{\pi} < \infty$. Hence, the optimal regime is a nondegenerate band.

(iii) **A peg:** If $V^{\beta}$ is concave in $\underline{\pi}$ and in $\overline{\pi}$, and $A\beta > \text{Max}\{A^\beta, \overline{A}^\beta\}$ then $\underline{\pi} = \overline{\pi} = 0$, so the optimal regime is a peg.

(iv) **The width of the band:** Suppose that the policymaker’s problem has a unique interior solution. Then, $\bar{\pi}$ decreases towards 0 and $\pi$ increases towards 0 as $\beta$ increases towards 1, implying that as the policymaker’s reputation improves, he adopts a tighter band.

Parts (i)-(iii) of Proposition 8 modify the corresponding parts of Proposition 2 for the case where the policymaker’s reputation is imperfect. Part (iv) of Proposition 8 says that as the policymaker’s reputation improves, the exchange rate band becomes tighter. This implies that a good reputation induces the policymaker to adopt a more ambitious goal. Hong-Kong’s currency board fits into this ”box” of the model. Since the peg has never been abandoned in the past, Hong-Kong’s currency board has good reputation, which induces the authorities to defend the peg under a wider set of circumstances than is the case under a lower reputation level.
7 Concluding reflections

Drawing on data from the IMF Annual Report 2000, Fischer (2001) reports evidence yielding support to the view that, during the last decade, exchange rate regimes in the world have been gradually moving towards a bipolar system. In particular this data shows that, since the beginning of the nineties till 1999, there has been a shift away from intermediate exchange rate regimes to either hard pegs or to freely floating regimes. Hard pegs include currency boards (like Lithuania and Hong-Kong) and countries that have no national currency (like Panama). In terms of our framework, Fischer’s hard pegs are characterized not only by the fact that the width of the band is zero, but also by the fact that the commitment to defend the peg is relatively strong.

In what follows we discuss the set of restrictions on the exogenous parameters of our framework needed to deliver the trends reported by Fischer and related issues. Our framework features three main exogenous parameters: The relative importance attributed by policymakers to avoidance of nominal exchange rate uncertainty (A), the present value of the future costs of abandoning an existing band (δ) and the real cost to speculators of switching between currencies (T = t−1). In section 6 the framework is extended to also include an exogenous parameter, β, that characterizes the reputation of policymakers for dependability.

Proposition 2 implies that countries with relatively large values of A will have pegs and proposition 1 implies that countries with relatively large values of δ will have wider ranges of effective commitment implying that they will defend the currency under a wider set of realizations of the free market rate of change in the exchange rate, x. Thus, our framework delivers the prediction that countries characterized by relatively high values of both A and δ will institute hard pegs. Small open economies with large shares of financial assets and liabilities, and of international trade invoicing in foreign exchange are likely to be characterized by both high values of A and of δ. The reason is that a country with a high value of A is likely to put a higher value on future reputation since it values the ability to reduce exchange rate uncertainty at low cost more, not only in the present but also in the future. Those observations, in conjunction with the analytical results of the paper imply that countries like Panama, Hong-Kong, Lithuania and numerous other small open economies are likely to institute hard pegs. The evidence presented in Fischer’s paper is consistent with this prediction.

Liberalization of world capital markets is likely to affect several of the exogenous parameters of our framework. On one hand, by reducing T, it increases the ease with which speculators can move between currencies and makes speculation easier. On the

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26This statement is based on the presumption that a current realignment reduces the future reputation of policymakers.
other hand, by opening most economies to wider, foreign exchange denominated assets and liabilities, it makes the reduction of nominal exchange rate uncertainty in both the present and the future more valuable. This takes the form of larger values of both \( A \) and \( \delta \). Proposition 6 implies that the reduction in \( T \) induces the policymakers of all countries to move towards more flexible exchange rate arrangements and propositions 1 and 3 imply that the increase in \( A \) and \( \delta \) induces them to move in the opposite direction — towards hard pegs. In the absence of additional information the combined effect of liberalization appears, therefore, to be ambiguous.

But the magnitude of the effect of capital market liberalization, via the increase in \( A \) and \( \delta \) varies across countries. It is likely to be large for small open economies whose currencies are not used much for either capital account or current account transaction in world markets, and to be small or even negligible for large key currency economies. Hence, the first effect is likely to be dominant in large, relatively closed blocks, and the second is likely to be dominant in small open economies. It follows that, ceteris paribus, the liberalization of world capital markets should induce relatively large currency blocks to move towards more flexible exchange rate arrangements while pushing small open economies in the opposite direction. The upshot is that, given an additional hypothesis about the relative magnitude of capital market liberalization on \( A \) and \( \delta \) in different countries, the analytical results of this paper provide an explanation for the recent trend towards a bipolar system of exchange rate arrangements.

Finally, the analysis in section 6 implies that countries which have developed a good reputation for dependability are more likely to maintain hard pegs since (by lemma 3 and proposition 8) they are predicted to have both wider ranges of effective commitment and narrower bands. There is thus, over time, a virtuous circle between good reputation and the likelihood that a hard peg will be maintained. Hong-Kong appears to be a case in point.
8 Appendix

Proof of Lemma 1: We analyze the behavior of the policymaker and the speculators after the exchange rate reaches the upper bound of the band. We show that in this case there exists at the limit as $\varepsilon \to 0$, a unique perfect Bayesian equilibrium in which speculators attack the band if and only if they observe signals above a unique threshold signal $\theta^*$. The proof for the case where the exchange rate reaches the lower bound of the band is analogous.

We start with a few notations. From equation (3.1) we know that the policymaker defends the upper bound of the band if and only if $x \leq \pi - \alpha + \delta$. Using this expression, let

$$
\alpha^*(x) = \begin{cases} 
0, & \text{if } \pi - x + \delta < 0, \\
\pi - x + \delta, & \text{if } 0 \leq \pi - x + \delta \leq 1, \\
1 & \text{if } \pi - x + \delta > 1,
\end{cases} \quad (A-1)
$$

be the critical measure of speculators below which the policymaker will defend the upper bound of the band when the laissez faire rate of change in the exchange rate is $x$. Using the definition of $\alpha^*(x)$, the net payoff from attacking the upper bound of the band is:

$$
v(x, \alpha) = \begin{cases} 
(x - \pi) e_{-1} - t, & \text{if } \alpha \geq \alpha^*(x), \\
-t, & \text{if } \alpha < \alpha^*(x).
\end{cases} \quad (A-2)
$$

We establish two properties of $v(x, \alpha)$. First, $v(x, \alpha)$ is weakly increasing in $\alpha$. This is so because by assumption, $x \geq \pi$, implying that $(x - \pi) e_{-1} - t \geq -t$. Thus, when $\alpha$ increases, $v(x, \alpha)$ is more likely to be given by the top line in (A-2) which exceeds the bottom line. Second, $v(x, \alpha)$ is weakly increasing in $x$, and strictly increasing in $x$ if $v(x, \alpha) \geq 0$. The reason for this is that the top line in (A-2) is strictly increasing in $x$, and since $\alpha^*(x)$ is weakly decreasing in $x$. Note that when $v(x, \alpha) \geq 0$, $v(x, \alpha)$ must be given by the top line in (A-2) and hence is strictly increasing in $x$.

Now, let $\alpha_i(x)$ be speculator $i$’s belief about the measure of speculators who will attack the band for each level of $x$). We will say that the belief $\alpha'_i(x)$ is lower (higher) than $\alpha_i(x)$ if $\alpha'_i(x) \leq \alpha_i(x)$ ($\alpha'_i(x) \geq \alpha_i(x)$) for all $x$ with strict inequality for at least one $x$.

The decision of speculator $i$ on whether or not to attack the band depends on the signal $\theta_i$ that the speculator observes and on the speculator’s belief, $\alpha_i(x)$. Since $\theta_i = x + \varepsilon_i$, where $\varepsilon_i \sim U[-\varepsilon, \varepsilon]$, and using equation (2.4), the net expected payoffs of speculator $i$ from attacking the upper bound of the band is:

$$
h(\theta_i, \alpha_i(x)) = \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} v(x, \alpha_i(x)) f(x \mid \theta_i) dx = \frac{\int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} v(x, \alpha_i(x)) f(x) dx}{F(\theta_i + \varepsilon) - F(\theta_i - \varepsilon)}. \quad (A-3)
$$
We establish three properties of \( h(\theta_i, \alpha_i(x)) \). First, since \( \theta_i \) affects only the boundaries of integration in the numerator of \( h(\theta_i, \alpha_i(x)) \) and since that \( F(\cdot) \) is a continuous function, it follows that \( h(\theta_i, \alpha_i(x)) \) is continuous in \( \theta_i \). Second, recalling that \( v(x, \alpha) \) is weakly increasing in \( \alpha \), it follows that if \( \alpha_i(x) \) is lower (higher) than \( \alpha_i(x) \), then \( h(\theta_i, \alpha_i(x)) \leq (\geq) h(\theta_i, \alpha_i(x)) \) for all \( \theta_i \). Third, note that:

\[
\frac{\partial h(\theta_i, \alpha_i(x))}{\partial \theta_i} = \frac{\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} f(x)dx \left[ v(\theta_i + \varepsilon, \alpha_i(\theta_i + \varepsilon)) f(\theta_i + \varepsilon) - v(\theta_i - \varepsilon, \alpha_i(\theta_i - \varepsilon)) f(\theta_i - \varepsilon) \right]}{(F(\theta_i + \varepsilon) - F(\theta_i - \varepsilon))^2} - \frac{\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} v(x, \alpha_i(x)) f(x)dx \left[ f(\theta_i + \varepsilon) - f(\theta_i - \varepsilon) \right]}{(F(\theta_i + \varepsilon) - F(\theta_i - \varepsilon))^2} = \frac{v(\theta_i - \varepsilon, \alpha_i(\theta_i - \varepsilon)) f(x)dx}{(F(\theta_i + \varepsilon) - F(\theta_i - \varepsilon))^2}. \tag{A-4}
\]

Recalling from above that \( v(x, \alpha) \) is weakly increasing in both \( x \) and \( \alpha \), it follows that if \( \alpha_i(x) \) is non-decreasing in \( x \), then \( h(\theta_i, \alpha_i(x)) \) is weakly increasing in \( \theta_i \). Next, we show that whenever it is nonnegative, \( h(\theta_i, \alpha_i(x)) \) must be strictly increasing in \( \theta_i \). To this end, note that since \( h(\theta_i, \alpha_i(x)) \) is the expected value of \( v(x, \alpha_i(x)) \) when \( x \in [\theta_i - \varepsilon, \theta_i + \varepsilon] \), then \( h(\theta_i, \alpha_i(x)) \geq 0 \) implies that there exists at least one value of \( x \) in the interval \([\theta_i - \varepsilon, \theta_i + \varepsilon]\) for which \( v(x, \alpha_i(x)) > 0 \) (otherwise \( h(\theta_i, \alpha_i(x)) \leq 0 \)). Since we showed above that \( v(x, \alpha) \) is strictly increasing in \( x \) if \( v(x, \alpha) \geq 0 \), it follows that \( v(x, \alpha) \) is strictly increasing in \( x \) for at least one value of \( x \in [\theta_i - \varepsilon, \theta_i + \varepsilon] \). But since \( v(x, \alpha) \) is weakly increasing in \( x \) and strictly increasing in \( x \) for at least one value of \( x \), it follows that \( v(\theta_i + \varepsilon, \alpha_i(\theta_i + \varepsilon)) > v(\theta_i - \varepsilon, \alpha_i(\theta_i - \varepsilon)) \). Consequently, \( h(\theta_i, \alpha_i(x)) \) is strictly increasing in \( \theta_i \) whenever \( h(\theta_i, \alpha_i(x)) \geq 0 \).

In equilibrium, the strategy of speculator \( i \) is to attack the upper bound of the band if \( h(\theta_i, \alpha_i(x)) > 0 \) and not attack it if \( h(\theta_i, \alpha_i(x)) < 0 \). Moreover, the equilibrium belief of speculator \( i \), \( \alpha_i(x) \), must be consistent with the equilibrium strategies of all other speculators (for short we will simply say that in equilibrium, the belief of speculator \( i \) is consistent). To characterize the equilibrium strategies of speculators, we first show that there exists a range of sufficiently large signals for which speculators have a dominant strategy to attack the band and likewise, there exists a range of sufficiently small signals for which speculators have a dominant strategy not to attack the band. Then, we use an iterative process of elimination of dominated strategies to establish the existence of a unique signal, \( \bar{\theta} \), such that speculator \( i \) attacks the upper bound of the band if and only if \( \theta_i > \bar{\theta} \).

Suppose that speculator \( i \) observes a signal \( \theta_i > \bar{\theta} \equiv \bar{\pi} + \bar{\delta} + \varepsilon \). Then speculator \( i \) realizes that \( x > \bar{\pi} + \bar{\delta} \). Using equation (A-1), this means that \( \alpha^*(x) = 0 \) so the
policymaker is surely going to exit the band. By equation (A-2), the net payoff from attacking the band is therefore \( v(x, \alpha) = (x - \pi) e_{-1} - t \), for all \( \alpha \). But since \( x > \pi + \delta \), it follows that \( v(x, \alpha) > \delta e_{-1} - t \) for all \( \alpha \), which is strictly positive by Assumption 2. Hence, the net expected payoff of the speculator is such that \( h(\theta_i, \alpha_i(x)) > 0 \) for all \( \theta_i > \overline{\theta} \) and all \( \alpha_i(x) \), implying that it is a dominant strategy for a speculator who observes a signal above \( \overline{\theta} \) to attack the upper bound of the band. Similarly, suppose that speculator \( i \) observes a signal \( \theta_i < \overline{\theta} \equiv \pi + \frac{t}{e_{-1}} - \varepsilon \) (since we focus on the case where \( \varepsilon \to 0 \) and since \( t > 0 \), such signals are observed with a positive probability whenever \( x > \pi \)), the speculator realizes that \( x < \pi + \frac{t}{e_{-1}} \). Consequently, even if the policymaker surely exits the band, the payoff from attacking it is negative as \( v(x, \alpha) = (x - \pi) e_{-1} - t < \left( (\pi + \frac{t}{e_{-1}}) - \pi \right) e_{-1} - t = 0. \) This implies in turn that \( h(\theta_i, \alpha_i(x)) < 0 \) for all \( \theta_i < \overline{\theta} \) and all \( \alpha_i(x) \), so it is a dominant strategy for speculator \( i \) not to attack the band after observing a signal below \( \overline{\theta} \).

Now, we start an iterative process of elimination of dominated strategies from \( \overline{\theta} \), in order to expand the range of signals for which speculators will surely attack the band. To this end, let \( \alpha(x, \theta) \) represent speculator \( i \)'s belief regarding the measure of speculators who will attack the band for each level of \( x \), when the speculator believes that all speculators will attack the upper bound of the band if and only if they observe signals above some level \( \theta \). Since \( \varepsilon_i \sim U[-\varepsilon, \varepsilon] \), it follows that

\[
\alpha(x, \theta) = \begin{cases} 
0, & \text{if } x < \theta - \varepsilon, \\
\frac{x - (\theta - \varepsilon)}{\theta - \varepsilon}, & \text{if } \theta - \varepsilon \leq x \leq \theta + \varepsilon, \\
\frac{1}{2}, & \text{if } x > \theta + \varepsilon.
\end{cases}
\]  

(A-5)

The iterative process of elimination of dominated strategies works as follows. Above, we already established that \( h(\theta_i, \alpha_i(x)) > 0 \) for all \( \theta_i > \overline{\theta} \) and all \( \alpha_i(x) \). But since \( h(\theta_i, \alpha_i(x)) \) is continuous in \( \theta_i \), it follows that \( h(\overline{\theta}, \alpha_i(x)) \geq 0 \) for all \( \alpha_i(x) \), and in particular for \( \alpha_i(x) = \alpha(x, \overline{\theta}) \). Thus, \( h(\overline{\theta}, \alpha(x, \overline{\theta})) \geq 0 \). Note that since in equilibrium, the beliefs of speculators are consistent, only beliefs that are higher than or equal to \( \alpha(x, \overline{\theta}) \) can hold in equilibrium (because all speculators attack the band when they observe signals above \( \overline{\theta} \)). Thus, we say that \( \alpha(x, \overline{\theta}) \) is the "lowest" belief on \( \alpha \) that can hold in equilibrium.

Let \( \overline{\theta}^1 \) be the value of \( \theta_i \) for which \( h(\theta_i, \alpha(x, \overline{\theta})) = 0 \). That is, \( h(\overline{\theta}^1, \alpha(x, \overline{\theta})) \equiv 0 \). Note that \( \overline{\theta}^1 \leq \overline{\theta} \), and that \( \overline{\theta}^1 \) is defined uniquely because we showed above that \( h(\theta_i, \alpha_i(x)) \) is strictly increasing in \( \theta_i \) whenever \( h(\theta_i, \alpha_i(x)) \geq 0 \). Using the second and third properties of \( h(\theta_i, \alpha_i(x)) \) and recalling that \( \alpha(x, \overline{\theta}) \) is the lowest consistent belief on \( \alpha \), it follows that \( h(\theta_i, \alpha_i(x)) > 0 \) for any \( \theta_i > \overline{\theta}^1 \) and any consistent belief \( \alpha_i(x) \). Thus, in equilibrium, speculators must attack the band if they observe signals above \( \overline{\theta}^1 \). As a result, \( \alpha(x, \overline{\theta}^1) \) becomes the lowest consistent belief on \( \alpha_i(x) \).

Starting from \( \overline{\theta}^1 \), we can now repeat the process along the following steps (these steps are similar to the ones that were used in order to establish \( \overline{\theta}^1 \)). First, note that since
$$h(\bar{\theta}^1, \alpha(x, \bar{\theta})) \equiv 0$$ and since \(\alpha(x, \theta)\) is weakly decreasing with \(\theta\) and \(h(\theta_i, \alpha_i(x))\) is weakly increasing with \(\alpha_i(x)\), it follows that \(h(\bar{\theta}^1, \alpha(x, \bar{\theta}^1)) \geq 0\). Second, find a \(\theta_i \leq \bar{\theta}^1\) for which \(h(\theta_i, \alpha(x, \bar{\theta}^1)) = 0\), and denote it by \(\bar{\theta}^2\). Using the same arguments as above, \(\bar{\theta}^2\) is defined uniquely. Third, since \(\alpha(x, \bar{\theta}^1)\) is the lowest consistent belief on \(\alpha_i(x)\) and using the second and third properties of \(h(\theta_i, \alpha_i(x))\), it follows that speculators must attack the band if they observe signals above \(\bar{\theta}^2\). The lowest possible belief on \(\alpha_i(x)\) becomes \(\alpha(x, \bar{\theta}^2)\).

We repeat the process over and over again (each time lowering the value of \(\theta\) above which speculators will attack the upper bound of the band), until we reach step \(n\) where \(\bar{\theta}^{n+1} = \bar{\theta}^n\), implying that the process cannot continue further. Let \(\bar{\theta}^\infty\) denote the value of \(\theta\) at which the process stops. (Clearly, \(\bar{\theta}^\infty \leq \bar{\theta}\). By definition, speculators will attack the band if they observe signals above \(\bar{\theta}^\infty\). Since \(\bar{\theta}^\infty\) is the point where the process stops, it must be the case that \(h(\bar{\theta}^\infty, \alpha(x, \bar{\theta}^\infty)) = 0\) (otherwise, we can find some \(\theta_i < \bar{\theta}^\infty\) for which \(h(\theta_i, \alpha(x, \bar{\theta}^\infty)) = 0\), and the iterative process could have been continued further).

Starting a similar iterative process from \(\theta\) and following the exact same steps, we also obtain a signal \(\bar{\theta}^\infty(\geq \bar{\theta})\) such that speculators will never attack the band if they observe signals below \(\bar{\theta}^\infty\). At this signal, it must be the case that \(h(\bar{\theta}^\infty, \alpha(x, \bar{\theta}^\infty)) = 0\). Since we proved that in equilibrium speculators attack the upper bound of the band if they observe signals above \(\bar{\theta}^\infty\) and do not attack it if they observe signals below \(\bar{\theta}^\infty\), it must be the case that \(\bar{\theta}^\infty \geq \bar{\theta}^\infty\).

The last stage of the proof involves showing that \(\bar{\theta}^\infty = \bar{\theta}^\infty\). We do that for the case where \(\varepsilon \to 0\). First, note that \(\bar{\theta}^\infty\) is defined by the equation \(h(\bar{\theta}^\infty, \alpha(x, \bar{\theta}^\infty)) = 0\). Using equations (A-3) and (A-5), this equality can be written as

$$\int_{\bar{\theta}^\infty - \varepsilon}^{\bar{\theta}^\infty + \varepsilon} v(\theta, x, \bar{\theta}^\infty) f(x) \, dx \quad F(\bar{\theta}^\infty + \varepsilon) - F(\bar{\theta}^\infty - \varepsilon) = 0. \quad (A-6)$$

Using the equality \(\alpha = \frac{x - (\bar{\theta}^\infty - \varepsilon)}{2\varepsilon}\) to change variables in the integration, equation (A-6) can be written as:

$$\frac{2\varepsilon}{F(\bar{\theta}^\infty + \varepsilon) - F(\bar{\theta}^\infty - \varepsilon)} \int_0^1 v(\bar{\theta}^\infty + 2\varepsilon\alpha - \varepsilon, \alpha) f(\bar{\theta}^\infty + 2\varepsilon\alpha - \varepsilon) \, d\alpha = 0. \quad (A-7)$$

At the limit as \(\varepsilon \to 0\), this equation becomes \(\int_0^1 v(\bar{\theta}^\infty, \alpha) \, d\alpha = 0\) (by L'Hôpital’s rule, the denominator approaches \(f(\bar{\theta}^\infty)\) as \(\varepsilon \to 0\)). Similarly, note that \(\bar{\theta}^\infty\) is defined
by the equation \( h(\bar{\theta}^\infty, \alpha(x, \bar{\theta}^\infty)) = 0 \), which at the limit as \( \varepsilon \to 0 \), can be written as \( \int_0^1 v(\bar{\theta}^\infty, \alpha)d\alpha = 0 \). Now, assume by way of negation that \( \bar{\theta}^\infty > \bar{\theta}^\infty \). Since \( v(x, \alpha) \) is weakly increasing in \( x \) and strictly increasing in \( \alpha \) when \( v(x, \alpha) \geq 0 \), it follows that \( \int_0^1 v(\bar{\theta}^\infty, \alpha)d\alpha > \int_0^1 v(\bar{\theta}^\infty, \alpha)d\alpha \) (the strict inequality follows because the equation \( \int_0^1 v(\bar{\theta}^\infty, \alpha)d\alpha = 0 \) implies that \( v(\bar{\theta}^\infty, \alpha) > 0 \) for at least some values of \( \alpha \)). This inequality contradicts the fact that \( \int_0^1 v(\bar{\theta}^\infty, \alpha)d\alpha = 0 \) and \( \int_0^1 v(\bar{\theta}^\infty, \alpha)d\alpha = 0 \).

Using the notation \( \bar{\theta} \equiv \bar{\theta}^\infty = \bar{\theta}^\infty \), we proved that at the limit as \( \varepsilon \to 0 \), there exists a unique threshold signal, \( \bar{\theta} \), such that all speculators will attack the upper bound of the band if and only if they observe signals above \( \bar{\theta} \). Q.E.D.

**Sufficient conditions for the policymakers’ problem to be globally concave in \( x \):**

Using equation (3.13), we get

\[
\frac{\partial^2 V}{\partial \bar{\pi}^2} = -[r(A-1) + \delta] f'(\bar{\pi} - r) + (A-1) [f(\bar{\pi}) - f(\bar{\pi} - r)] \quad (A-8)
\]

\[
= -\delta f'(\bar{\pi} - r) - r(A-1) \left[ f'(\bar{\pi} - r) - \frac{f(\bar{\pi}) - f(\bar{\pi} - r)}{r} \right].
\]

By Assumption 1, the first term on the second line of (A-8) is negative. Now, if \( A > 1 \) and \( f''(\cdot) \leq 0 \), the second term on the second line of (A-8) is nonpositive so \( \frac{\partial^2 V}{\partial \bar{\pi}^2} < 0 \), implying that \( V \) is concave in \( \bar{\pi} \).

Likewise, using equation (3.14), we get

\[
\frac{\partial^2 V}{\partial \bar{\pi}^2} = [r(A-1) + \delta] f'(\bar{\pi} + r) - (A-1) [f(\bar{\pi} + r) - f(\bar{\pi})] \quad (A-9)
\]

\[
= \delta f'(\bar{\pi} + r) + r(A-1) \left[ f'(\bar{\pi} + r) - \frac{f(\bar{\pi} + r) - f(\bar{\pi})}{r} \right].
\]

By Assumption 1, the first term on the second line of (A-9) is negative. Now, if \( A > 1 \) and \( f''(\cdot) \leq 0 \), the second term on the second line of (A-9) is nonpositive so \( \frac{\partial^2 V}{\partial \bar{\pi}^2} < 0 \), implying that \( V \) is concave in \( \bar{\pi} \). Q.E.D.

**Proof of Proposition 2:** (i) By Assumption 1, \( f(x) > f(\bar{\pi} - r) \) for all \( x \in [\bar{\pi} - r, \bar{\pi}] \). Hence, if \( A \leq 1 \), \( \frac{\partial V}{\partial \bar{\pi}} < 0 \) for all \( \bar{\pi} < 0 \), implying that the policymaker will push \( \bar{\pi} \) all the way to \(-\infty\). Likewise, by Assumption 1, \( f(x) > f(\bar{\pi} + r) \) for all \( x \in [\bar{\pi}, \bar{\pi} + r] \); if \( A \leq 1 \), then \( \frac{\partial V}{\partial \bar{\pi}} > 0 \) for all \( \bar{\pi} > 0 \), implying that the policymaker will push \( \bar{\pi} \) all the way to \( \infty \).
(ii) To establish that $\pi < 0$, it is sufficient to show that evaluated at $\pi = 0$, $\frac{\partial V}{\partial \pi} < 0$ (the policymaker will not push $\pi$ all the way up to 0). Using equation (3.13) we obtain that

$$\frac{\partial V}{\partial \pi}\bigg|_{\pi=0} = -\delta f(-r) + (A - 1) \int_{-r}^{0} [f(x) - f(-r)] dx$$

(A-10)

$$= - \int_{-r}^{0} [f(x) - f(-r)] dx \left[ \frac{\delta}{\int_{-r}^{0} \left[ f(x) \right] dx} + 1 - A \right].$$

By Assumption 1, the integral term outside the square brackets on the second line of equation (A-10) is negative. If $A < \overline{A}(-r)$ then the term in square brackets is positive, so it is optimal to set $\pi < 0$. To show that $\pi > 0$, it is sufficient to show that evaluated at $\pi = 0$, $\frac{\partial V}{\partial \pi} > 0$ (the policymaker will increase $\pi$ above 0). Using equation (3.14) we obtain that,

$$\frac{\partial V}{\partial \pi}\bigg|_{\pi=0} = \delta f(r) - (A - 1) \int_{0}^{r} [f(r) - f(x)] dx$$

(A-11)

$$= \int_{0}^{r} [f(r) - f(x)] dx \left[ \frac{\delta}{\int_{0}^{r} \left[ f(r) \right] dx} + 1 - A \right].$$

By Assumption 1, the integral term outside the square brackets on the second line of equation (A-11) is positive. If $A < \overline{A}(r)$ then the square bracketed term is positive. Hence, it is optimal to set $\pi > 0$.

(iii) If $V$ is concave in $\pi$, then a sufficient condition for $\pi = 0$ is that, evaluated at $\pi = 0$, $\frac{\partial V}{\partial \pi} \geq 0$ (the policymaker would like to push $\pi$ all the way up to 0). From part (ii) of the proposition it is obvious that this occurs when $A > \overline{A}(-r)$. Likewise, if $V$ is concave in $\pi$, then a sufficient condition for $\pi = 0$ is that, evaluated at $\pi = 0$, $\frac{\partial V}{\partial \pi} \leq 0$ (the policymaker would not like to increase $\pi$ above 0). From part (ii) of the proposition it is obvious that this is the case when $A > \overline{A}(r)$.

(iv) If $f(x)$ is symmetric around 0, then for $0 < a < b$,

$$f(x) = f(-x), \quad \text{and} \quad \int_{-b}^{-a} f(x) dx = \int_{a}^{b} f(x) dx. \quad (A-12)$$

Using these properties, equation (3.13) can be written as:

$$\delta f(-\pi + r) - (A - 1) \int_{-\pi}^{-\pi+r} [f(x) - f(-\pi + r)] dx = 0. \quad (A-13)$$

Replacing $\pi$ with $-\bar{\pi}$ in the last expression we obtain equation (3.14), implying that $\bar{\pi} = -\bar{\pi}$. Q.E.D.
Proof of Proposition 3: (i) The proof follows by straightforward differentiation of equations (3.13) and (3.14) and by using Assumption 1.

(ii) Speculative attacks on the lower bound of the band occur when \( x < \bar{\pi} - r \) and speculative attacks on the upper bound of the band occurs when \( x > \bar{\pi} + r \). Therefore, the probability of an attack is \( F(\pi - r) + (1 - F(\pi + r)) \). A straightforward differentiation of this expression along with part (i) of the proposition establish the result. Q.E.D.

Proof of Proposition 4: Let \( \pi^f \) and \( \bar{\pi}^f \) be the solutions to the policymaker’s maximization problem when the density function is \( f(x) \) and let \( \pi^g \) and \( \bar{\pi}^g \) be the corresponding solutions when the density function is \( g(x) \). \( \bar{\pi}^f \) is defined by equation (3.13) with \( g(x) \) replacing \( f(x) \). Now, let’s evaluate \( \frac{\partial V}{\partial \pi} \) when the density is \( g(x) \) at \( \pi^f \):

\[
\frac{\partial V}{\partial \pi} \bigg|_{\pi = \pi^f} = -\delta g(\pi^f - r) + (A - 1) \int_{\pi^f - r}^{\pi^f} [g(x) - g(\pi^f - r)] \, dx
\]

\[
= -\delta f(\bar{\pi}^f - r) + (A - 1) \int_{\bar{\pi}^f - r}^{\bar{\pi}^f} [g(x) - f(\bar{\pi}^f - r)] \, dx \quad (A-14)
\]

\[
< -\delta f(\bar{\pi}^f - r) + (A - 1) \int_{\bar{\pi}^f - r}^{\bar{\pi}^f} [f(x) - f(\bar{\pi}^f - r)] \, dx = 0,
\]

where the first equality follows because by assumption, \( g(\pi^f - r) = f(\pi^f - r) \), the inequality follows because \( f(x) \) lies above \( g(x) \) for whenever \( x > \bar{\pi}^f - r \), and the second equality follows from equation (3.13). Since \( \frac{\partial V}{\partial \pi} \bigg|_{\pi = \pi^f} < 0 \), it follows that \( \pi^f > \pi^g \). The proof that \( \bar{\pi}^f < \bar{\pi}^g \) is analogous. Hence, \( \pi^g < \pi^f < \bar{\pi}^f < \bar{\pi}^g \), so the band becomes wider under \( g(x) \). Q.E.D.

Proof of Proposition 5: \( A(-r) \) and \( A(r) \) are given respectively by equations (3.15) and (3.16). Differentiating these expressions with respect to \( t \) yields:

\[
\frac{\partial A(-r)}{\partial t} = -\frac{\delta f'(-r) \int_{-r}^{0} \frac{f(x)}{f(-r)} \, dx}{(\int_{-r}^{0} \left[ \frac{f(x)}{f(-r)} - 1 \right] \, dx)^2} \frac{\partial r}{\partial t} < 0,
\]

and

\[
\frac{\partial A(r)}{\partial t} = \frac{\delta f'(r) \int_{0}^{r} \frac{f(x)}{f(r)} \, dx}{(\int_{0}^{r} \left[ 1 - \frac{f(x)}{f(r)} \right] \, dx)^2} \frac{\partial r}{\partial t} < 0.
\]

The signs of those expressions follow from the fact that, by Proposition 1, \( \frac{\partial r}{\partial t} > 0 \) and since by Assumption 1, \( f'(r) < 0 \) and \( f'(-r) > 0 \). Part (iii) of Proposition 2 implies that
the policymaker prefers to set a peg when \( A > \max\{A(r), \overline{A}(r)\} \). Since both \( A(r) \) and \( \overline{A}(r) \) fall with \( t \), this condition is more likely to hold when \( t \) is larger. Q.E.D.

**Proof of Proposition 6:** (i) Proposition 1 shows that \( r \) decreases when \( t \) decreases. Straightforward differentiation of equations (3.13) and (3.14) and use of Assumption 1 show that \( \pi \) decreases and \( \overline{\pi} \) increases when \( r \) decreases. Hence a decrease in \( t \) leads to an increase in \( \overline{\pi} \) and to a decrease in \( \pi \).

(ii) Recall from part (ii) of Proposition 3 that the probability of an attack is \( F(\overline{\pi} - r) + (1 - F(\pi + r)) \). Differentiating this expression with respect to \( t \) yields:

\[
\frac{\partial}{\partial t} (F(\overline{\pi} - r) + (1 - F(\pi + r))) = f(\overline{\pi} - r) \left[ \frac{\partial \pi}{\partial r} \frac{\partial r}{\partial t} - \frac{\partial r}{\partial t} \right] - f(\pi + r) \left[ \frac{\partial \pi}{\partial r} \frac{\partial r}{\partial t} + \frac{\partial r}{\partial t} \right]
\]

\[
= \left[ f(\overline{\pi} - r) \left[ \frac{\partial \pi}{\partial r} - 1 \right] - f(\pi + r) \left[ \frac{\partial \pi}{\partial r} + 1 \right] \right] \frac{\partial r}{\partial t}.
\]

By Proposition 1, \( \frac{\partial r}{\partial t} > 0 \). Hence it is sufficient to establish that \( \frac{\partial \pi}{\partial r} > 1 \) and \( \frac{\partial \overline{\pi}}{\partial r} < -1 \). Using equation (3.13), it follows that

\[
\frac{\partial \pi}{\partial r} = \frac{\frac{\partial^2 V}{\partial r \partial \pi}}{\frac{\partial^2 V}{\partial \pi^2}} = -\frac{((A - 1) r + \delta) f'(\overline{\pi} - r) - [r(A - 1) + \delta] f'(\overline{\pi} - r) + (A - 1) [f(\pi) - f(\overline{\pi} - r)]}{[r(A - 1) + \delta] f'(\overline{\pi} - r) + (A - 1) [f(\pi) - f(\overline{\pi} - r)]}, \tag{A-15}
\]

which exceeds 1 because of Assumption 1 and because \( A > 1 \). Likewise, using equation (3.14), it follows that,

\[
\frac{\partial \overline{\pi}}{\partial r} = \frac{\frac{\partial^2 V}{\partial r \partial \overline{\pi}}}{\frac{\partial^2 V}{\partial \overline{\pi}^2}} = -\frac{((A - 1) r + \delta) f'(\pi + r) - [r(A - 1) + \delta] f'(\pi + r) + (A - 1) [f(\pi + r) - f(\pi)]}{[r(A - 1) + \delta] f'(\pi + r) + (A - 1) [f(\pi + r) - f(\pi)]}, \tag{A-16}
\]

which is less than \(-1\) because of Assumption 1 and because \( A > 1 \). Q.E.D

**Proof of proposition 7:** Using equation (3.12) and the envelope theorem it follows that:

\[
\frac{\partial V}{\partial t} = (r(A - 1) + \delta) [f(\overline{\pi} - r) + f(\pi + r)] \frac{\partial r}{\partial t} > 0, \tag{A-17}
\]

where the inequality follows because \( A > 1 \) and because by Proposition 1, \( \frac{\partial r}{\partial t} > 0 \). Q.E.D

**Proof of Lemma 3:** (i) Suppose the exchange rate reaches the upper bound of the band. Given \( x \), a dependable policymaker will defend the upper bound of the band if
and only if $\alpha < \alpha^*(x)$, where $\alpha^*(x)$ is given by equation (A-1). Hence, the net payoff from attacking the upper bound of the band is:

$$
v^\beta(x, \alpha) = \begin{cases} 
(x - \pi) e_{-1} - t, & \text{if } \alpha \geq \alpha^*(x), \\
(1 - \beta)(x - \pi) e_{-1} - t, & \text{if } \alpha < \alpha^*(x).
\end{cases}
$$

Since $v^\beta(x, \alpha)$ has the same properties as $v(x, \alpha)$ in equation (A-2), the equilibrium analysis here is exactly as in the proof of Lemma 1. Hence, once again we have a unique equilibrium in which speculators attack the upper bound of the band if and only if they observe a signal above a unique threshold, $\overline{\theta}_\beta$.

Now, we turn to characterize the behavior of the dependable policymaker in equilibrium. Since in equilibrium, $\alpha(x)$ is increasing in $x$ ($\alpha(x)$ is given by (3.2), where $\overline{\theta}_\beta$ replaces $\overline{\theta}$), and since $C(x, \alpha(x))$ is increasing in both $x$ and $\alpha(x)$, then the dependable policymaker will exit the band if and only if $x$ is above some threshold level: $\overline{\pi}_\beta(\overline{\theta}_\beta)$.

In order to establish that $\overline{\pi}_\beta(\overline{\theta}_\beta)$ and $\overline{\theta}_\beta$ converge to each other as $\varepsilon$ approaches 0, we now show that $\overline{\pi}_\beta(\overline{\theta}_\beta)$ must be in the interval $[\overline{\theta}_\beta - \varepsilon, \overline{\theta}_\beta + \varepsilon]$. In order to see this, suppose by way of negation that $\overline{\pi}_\beta(\overline{\theta}_\beta) > \overline{\theta}_\beta + \varepsilon$. Then, speculators who observe $\overline{\theta}_\beta$ know that a dependable policymaker will defend the band. Thus, the payoff they expect to get from attacking the band is lower than $(1 - \beta) \left( \overline{\theta}_\beta + \varepsilon - \pi \right) e_{-1} - t$. By equilibrium conditions and continuity, we know that speculators who observe $\overline{\theta}_\beta$ must be indifferent between attacking the band and not attacking it. This means that $(1 - \beta) \left( \overline{\theta}_\beta + \varepsilon - \pi \right) e_{-1} - t > 0$.

However, using Assumption 3, this condition will hold only if $\overline{\theta}_\beta + \varepsilon > \pi + \delta$. Since by assumption, $\overline{\theta}_\beta(\overline{\theta}_\beta) > \overline{\theta}_\beta + \varepsilon$, this implies in turn that $\overline{\pi}_\beta(\overline{\theta}_\beta) > \pi + \delta$, thereby contradicting the fact that a dependable policymaker always exits the band when $x > \pi + \delta$. Thus, $\overline{\pi}_\beta(\overline{\theta}_\beta)$ cannot be above $\overline{\theta}_\beta + \varepsilon$. Next, suppose by way of negation that $\overline{\pi}_\beta(\overline{\theta}_\beta) < \overline{\theta}_\beta - \varepsilon$. Then, at $\overline{\theta}_\beta(\overline{\theta}_\beta)$, a dependable policymaker knows that no speculator attacks the band. By equilibrium conditions and continuity, at $\overline{\theta}_\beta(\overline{\theta}_\beta)$, a dependable policymaker must be indifferent between existing the band and maintaining it, that is, $\overline{\pi}_\beta(\overline{\theta}_\beta) = \pi + \delta$. Since by assumption, $\overline{\pi}_\beta(\overline{\theta}_\beta) < \overline{\theta}_\beta - \varepsilon$, this means that $\overline{\theta}_\beta > \pi + \delta + \varepsilon$, which contradicts the fact that speculators have a dominant strategy to attack the band when they observe signals above $\pi + \delta + \varepsilon$. Thus, $\overline{\pi}_\beta(\overline{\theta}_\beta)$ cannot be below $\overline{\theta}_\beta - \varepsilon$.

Given the fact that $\overline{\pi}_\beta(\overline{\theta}_\beta)$ is in the interval $[\overline{\theta}_\beta - \varepsilon, \overline{\theta}_\beta + \varepsilon]$ and using Equation (3.2), it follows that:
\[ \bar{\pi}(\theta^* \beta) = \frac{\varepsilon(2\pi + 2\delta - 1) + \theta^*}{2\varepsilon + 1}. \]

Since a speculator that observes \( \theta^* \beta \) is indifferent between attacking the band and not attacking it, the equation that defines \( \bar{\theta} \) is given by:

\[
\beta \int_{\pi\varepsilon(\bar{\theta} \beta)}^{\pi\varepsilon(\bar{\theta} \beta)} (x - \pi)e^{-1}f(x \mid \bar{\theta} \beta)dx + (1 - \beta) \int_{\pi\varepsilon(\bar{\theta} \beta)}^{\pi\varepsilon(\bar{\theta} - \varepsilon)} (x - \pi)e^{-1}f(x \mid \bar{\theta} \beta)dx = t, \quad (A-18)
\]

where \( f(x \mid \bar{\theta} \beta) \) is defined by equation (2.4). This equation coincides with equation (3.4) if \( \beta = 1 \). Substituting from equation (2.4) for \( f(x \mid \bar{\theta} \beta) \) into (A-18), and taking the limit as \( \varepsilon \) goes to \( 0 \), yields:

\[
\lim_{\varepsilon \to 0} \frac{\beta \int_{\pi\varepsilon(\bar{\theta} \beta)}^{\pi\varepsilon(\bar{\theta} \beta)} (x - \pi)e^{-1}f(x \mid \bar{\theta} \beta)dx + (1 - \beta) \int_{\pi\varepsilon(\bar{\theta} \beta)}^{\pi\varepsilon(\bar{\theta} - \varepsilon)} (x - \pi)e^{-1}f(x \mid \bar{\theta} \beta)dx}{F(\bar{\theta} \beta + \varepsilon) - F(\bar{\theta} \beta - \varepsilon)} = t. \quad (A-19)
\]

Using L'Hôpital's rule, and the expression for \( \bar{\pi}(\bar{\theta} \beta) \), and recalling that as \( \varepsilon \) goes to \( 0 \), \( \bar{\pi}(\bar{\theta} \beta) \) goes to \( \bar{\theta} \), we obtain:

\[
(\bar{\theta} \beta - \pi)(1 - \beta \delta + \beta(\bar{\theta} \beta - \pi))e^{-1} = t. \quad (A-20)
\]

Solving this equation for \( \theta^* \beta \) reveals that \( \bar{\theta} \beta = \pi + r^\beta \), where \( r^\beta \) is defined by equation (4.1). Using similar arguments, it follows that \( \theta^* \beta = \pi - r^\beta \).

\text{(ii)} Differentiating \( r^\beta \) with respect to \( \beta \) and we obtain:

\[
\frac{\partial r^\beta}{\partial \beta} = \frac{1}{2\beta^2} \left[ 1 - \frac{t}{e^{-1}} - \frac{\delta - \frac{1}{\beta}}{2} \right]. \quad (A-21)
\]

This derivative is positive iff the expression inside the brackets is positive. This is the case, in turn, iff

\[
\frac{t}{e^{-1}} + \frac{(\delta - \frac{1}{\beta})^2}{4} > \left[ \frac{t}{e^{-1}} - \frac{\delta - \frac{1}{\beta}}{2} \right]^2. \quad (A-22)
\]

Further rearrangement of the last inequality shows that it is equivalent to Assumption 2. Hence, \( r^\beta \) increases with \( \beta \). \quad \text{Q.E.D.}
Proof of Proposition 8: The proofs of parts (i)-(iii) of the proposition follow the corresponding proofs in Proposition 2 with $r$ replaced by $r^\beta$.

(iiv) Differentiating $\frac{\partial V^\beta}{\partial x}$ with respect to $\beta$ and $\pi$, and using the implicit function theorem, yields:

$$\frac{\partial \pi}{\partial \beta} = \frac{A \int_0^{\pi - r^\beta} [f(x) - f(\pi - r^\beta)] \, dx + [r^\beta(A\beta - 1) + \delta] f'(\pi - r^\beta) \frac{\partial r^\beta}{\partial \pi}}{-\frac{\partial^2 V^\beta}{\partial \pi^2}}.$$  (A-23)

Assumption 1 ensures that the integral term in the numerator is positive and it also ensures that $f'(\pi - r^\beta) > 0$. Since by Lemma 3, $\frac{\partial r^\beta}{\partial \pi} > 0$, it follows that the numerator is positive. By the second order conditions for maximization, the denominator is also positive so $\pi$ increases towards 0 as $\beta$ increases towards 1. Similarly, it can be shown that as $\beta$ increases towards 1, $\bar{\pi}$ decreases towards 0. Hence, an increase in $\beta$ leads to a tighter band. Q.E.D.
9 References


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