WHY FAIRNESS MATTERS:
A NEW LOOK AT THE LAFFER CURVE
AND THE DISPLACEMENT LOSS
FROM TAX EVASION

Filip PALDA

Discussion Paper No. 2001 – 65
May 2001
Why Fairness Matters: A New Look at the Laffer Curve and the Displacement Loss from Tax Evasion *

Filip Palda

May, 2001

Abstract

In the presence of the underground economy taxes give rise to a deadweight loss from displacement of efficient producers by inefficient producers. I consider an economy in which a producer faces two types of costs: the cost of production, and taxes. If the ability to evade taxes is inversely proportional to the ability to keep production costs down, high tax rates may cause inefficient producers to crowd out efficient producers. The deadweight loss due to this crowding out can be several times as large as the triangle deadweight losses from discouraged consumption. The Laffer curve also loses its concavity, and the maximum government revenues possible under tax evasion may be larger than when no one evades taxes. Keywords: Underground economy; social cost of public funds; taxation. J.E.L. classification: H26, H43, K42, O17

*Palda is a Professor at Ecole Nationale d’Administration Publique, 4750 Ave. Henri-Julien, (#4040), Montreal, Quebec, H2L 4Z1, Canada. Email:Filip.Palda@enap.uquebec.ca.I thank Thomas Borcherding, Art Denzau, Jacek Kugler, Tom Willet, and Paul Zak for their excellent comments on an earlier draft of this paper. A copy of the Displacement Engine, the Maple V software code used to generate the results in this paper, will be emailed to interested readers on request.
Shell Brasil, the Brazilian subsidiary of the Anglo-Dutch oil group, is to sell 285 service stations and six fuel deposits to Agip do Brasil, the local subsidiary of Eni, the Italian group. Shell said the move was part of efforts to concentrate on the most profitable parts of its business in Brazil, but it is understood to have sold the stations, in remote central and western regions of the country, after failing to compete with smaller distributors undercutting bigger companies by evading taxes. Financial Times of London, February 25, 2000, page 18

Introduction

Shell Brasil would not have left the retailing business in remote parts of Brasil had it been as able and willing to evade taxes as its smaller competitors. Examples of Shell’s plight abound. Officials from the Quebec ministry of finance explained to me that large hotels in the province were in a turmoil because bed-and-breakfast inns were dodging taxes and undercutting the major chains. A 1996 publication of the Quebec Ministry of Finance explains that ”Businesses that pay their taxes in full are also seriously affected by unreported work and tax evasion. They face unfair competition on the part of businesses that offer goods and services at lower prices because these businesses did not pay or collect income tax or other taxes or they do not comply with the regulations in force (page 24).” In the October 23, 1999 edition of The Economist, the Moscow correspondent wrote that ”The prime cause of all this waste is that Russian business competes on the basis of political connections rather than costs, quality and price. The distortions embedded in the system—tax breaks, access to cheap land or energy and freedom from bureaucratic harassment—mean that, though competition is often intense, the least productive companies can come out winners.”

Snippets of concern from the media and government about the uneven enforcement of taxes are common. Few I have spoken to in government have raised an eyebrow in disbelief at the suggestion that uneven enforcement of taxes creates an uneven playing field on which inefficient producers with a willingness and ability to evade taxes oust honest, efficient producers from the market. The difference between the costs of the
surviving evaders and what costs would have been without evasion is the "displacement" loss from tax evasion. I put the term displacement in quotation marks because it is a term new to economics. Public finance theorists have ignored displacement loss, or have hurried past it on sprinkling a few words of warning. Vito Tanzi (1982, p.88) is one of the few economists to have noticed that "untaxed underground activities will compete with taxed, legal ones and will succeed in attracting resources even though these activities may be less productive...There will of course be significant welfare losses associated with this transfer." Jonathan Kesselman (1997, p.300) made a related point: "If pure tax evasion is concentrated in particular industries or sectors it will raise net returns from activities in those sectors, and this will in turn tend to expand those sectors and their products as against the efficient pattern arising with uniform compliance.” Those seeking further enlightenment about displacement losses will search in vain. Public finance thinkers are like the figure in a Greek tragedy who is the last to learn that his world is not as he thought it. While the public finance industry whirrs through theorems on optimal taxation and market failure, excise officials worry about fairness in the tax system and grope for a formal display that fairness is efficient. This is the theme of the present paper.

I examine the circumstances under which a displacement loss from uneven enforcement of taxes arises. The amount of loss depends on how closely tied are a firm’s productive efficiency and evasive ability. If efficient producers are honest tax payers and inefficient producers are dishonest, then a rise in taxes creates a climate that favors the survival of tax evaders above the survival of firms with low production costs. The less related are productive efficiency and honesty, the lower is this cost. When productive efficiency and honesty go hand-in-hand, displacement losses tend to be high. Using a simple model of profit maximizing firms I show how displacement losses from the tax tend to rise as the correlation between honesty and efficiency rises. My model hints that under very general assumptions about demand and supply, displacement loss may come to as much as 10% of the value of an industry’s output. Such a figure rivals the traditional Harberger triangle loss.

I find these results using a simple economic model which I pass through numerical simulations under a wide variety of parameter values. This paper presents no analytical results, but rather follows the research agenda for economics set out in Judd’s (1998)
One of the advantages of the simulation approach to economic models is that unexpected results may fall out of the simulations. The most surprising, though perhaps not the most useful, result of the present paper is that if firms with low production costs are bad at evading taxes, and firms with high production costs are good at evading taxes, government may make more revenue from a proportional tax than if no one evaded. A positive relation between productivity and honesty converts a proportional tax into a progressive tax based on ability to pay. Without any knowledge of firm costs, government can act as a price discriminating monopolist. Firms who are not good at keeping production costs low may be able or willing to take the risk of evading part of the tax. Their evasion allows them to survive and pay taxes. Partial participation in the underground economy keeps money flowing into government coffers by boosting the survival rates of firms that would collapse under complete enforcement of taxes. This is how tax evasion may broaden a government’s tax base beyond what it would be without evasion.

The plan of the paper is first to present a model of firm survival under the rigid assumption that firms produce a fixed amount or produce nothing at all. Firm efficiency and honesty are drawn from a joint normal distribution. The simulation results all focus on the consequences of changing the correlation between efficiency and honesty. The second part of the paper assumes that firms can vary their output decisions. The results of the first section do not change under this more general assumption about firm behavior. Unequal enforcement of tax laws tilt the playing field in favor of those best able to avoid the laws, and this may give rise to a displacement deadweight loss. Fairness, defined as even enforcement of the law, is efficient.

1. Costs under fixed firm output

I begin with the simplistic case of an industry in which firms are atomistic and produce each an identical and fixed quantity of output. I follow Telser (1978) in working first with this case, as a step towards understanding more complex cost structures in which firms can vary their outputs in response to changes in market parameters.

Potential producers are infinite in number, indexed by $A$. $A$ is a productivity parameter
that differs from firm to firm. Nature grants each firm its $A$ by drawing from a truncated normal distribution $f(A)$ along the interval $[0, 1]$ with mean $\mu_A$ and standard deviation $\sigma_A$. Implicitly I assign the set of producers a measure of one, though I could have assigned them an explicitly weight of say $N$. To keep notation simple I avoid making the measure explicit and assume that firms that finds it profitable to produce are constrained to producing the identical infinitesimal output $dq$ and that the sum of these outputs cannot exceed one. The costs a firm perceives depend on its particular efficiency index $A$ and taxes $T$ in the following manner: $(1 + T) dq / A$. Costs, as perceived by the firm, fall as productivity rises and rise with taxes.

This cost function is not drawn out of thin air, but may be arrived at by assuming a Cobb-Douglas technology which makes a firm’s output $q$ a function of its Labor $L$ and capital $K$ inputs as well as productivity parameters $A$, $\alpha$, $\beta$:

$$q = AL^\alpha K^\beta$$

(1)

Firms share the same $\alpha$ and $\beta$ but differ in the parameter $A$. Firms pay $w$ for a unit of Labor and $r$ for a unit of capital. Government levies a tax of $T$ on the value of each unit of capital and labor the firm employs ($T$ may be thought of as unemployment insurance and a capital tax). The costs the firm perceives of hiring labor are $w(1 + T)$ and its costs of capital are $r(1 + T)$. If we set $\alpha = \beta = .5$ (i.e. constant returns to scale) and $w = r = .5$ then the firm’s cost function can be shown to be

$$c = \frac{1 + T}{A}q$$

(2)

If we assume each firm which decides to produce is constrained to producing an infinitesimal quantity $dq$ then the above becomes the cost function mentioned earlier.

I assume that firms either produce or do not produce depending on whether their costs are lower than price, and that all are restricted to producing the same amount so that I may be certain that the results on industry cost falling out of simulations emerge from some firms taking the place of other firms as taxes change. I could have chosen many other types of cost function and taxes. The most obvious tax to have chosen would have been a tax on revenues from sales (price multiplied by output). I avoided this type of tax because it gives rise to messy, complicated bounds of integration when
figuring out the cumulative joint distribution function of productive and evasive talents. No important insights are lost by examining the case of taxes on inputs, and this tack considerably simplifies the modeling of displacement deadweight loss. The only downside to this simplification is that I have to be explicit about where my cost structure came from. The cost structure I examine in the present section can be considered a special case of the cost function I will examine section 3. In the Cobb-Douglas world without capacity constraints of section 3, everyone produces, and displacement takes place at the intensive rather than the extensive margin. In both cost-structures emerges the stark result that taxes may increase the average production cost of an industry and lead to displacement deadweight losses. I have also chosen to model two cost structures because each calls for modeling techniques that I believe span most of the problems a researcher would encounter in studying displacement losses from taxation.

A firm’s decision to produce depends on whether its costs \((1 + T)/A\) are less than the market price \(P\) which is paid in terms a numeraire (produced in some other industry without distortions). Supply \(Q^s\) is the proportion of firms who satisfy this condition:

\[
Q^s = Pr\left(\frac{1 + T}{A} \leq P\right)
\]

\[
= \int_{\frac{1}{1+T}}^{P} f(A)dA
\]  

Equation (4) traces a logistic shaped supply function with a maximum output of 1 attained as \(P\) tends to infinity and sweeps even the highest cost firms (those with very low \(A\)) into the market. Figure 1 illustrates the simple logic of the supply curve. I assume a linear demand curve of the form \(Q^d = a + bP\) where \(a > 0\) and \(b < 0\). Despite the simple layout, there is no analytical solution to equilibrium owing to the indeterminacy of the integral of the normal distribution and due to the fact that price \(P\) appears in the limits of integration. Finding an equilibrium in this case calls for numerical methods which search over a broad range of possible prices to see which price equates supply and demand. To narrow the range of possible equilibrium prices searched I have used the bisection method described by Judd (1998) and also to be found in Burden and Faires (1997). Details of the algorithm are available in Maple format from the author upon email request. Figure 2 shows the supply curve and demand curve under the assumption that \(a = 2, b = -.5,\)
\(\mu_A = .5, \sigma_A = .1\) I have chosen these demand parameters so that demand and supply intersect roughly at their halfway marks. To ensure the area under the normal curve comes to unity I have renormalized the truncated normal between zero and one using the procedure Madalla (1983) prescribes for truncated normal distributions.

As taxes rise, average firm production costs should fall. This is the standard public finance result. In a high tax environment only firms with the lowest costs will remain standing. To see this more formally recall that a firm’s production costs depends on its productivity parameter \(A\) as follows: \(1/A\). This differs from total costs \((1 + T)/A\) as perceived by the firm. Production costs of the industry are

\[
\int_{1+T}^{1} f(A) \frac{1}{A} dA
\]

Simulations (not shown here) show the expected result that as taxes rise, the average industry costs (total production costs divided by total output, equation 5 divided by equation 4) fall.

The thesis of this paper is that some firms who are less able or willing to evade taxes than others will find themselves pushed out of the market by less efficient producers with a greater zest for tax evasion. We can allow for the possibility of evasion by assuming that nature endows each firm with a particular propensity to pay a proportion \(i\) of its taxes. In this case, a firm perceives its costs to be \((1 + iT)/A\). A high \(i\) signifies the firm pays most of its taxes. The variable \(i\) is the subject of a subfield of public finance on tax evasion begun by Allingham and Sandmo (1972). Researchers in this field ask how preference parameters for risk, distribution functions of the risk of apprehension, and the structure of fines interplay to give a level of tax evasion. I do not look into these details but rather treat the field as a black box from which emerges the prediction that some people will evade more than others. I summarize the field with the parameter \(i\) which I refer to as evasive ability. The lower is \(i\) the greater is a firm’s evasive ability, and the more it evades taxes.

We can model the notion that each firm has its own particular productivity and evasive ability by assuming that each firm draws its productivity \(A\) and its evasive ability \(i\) from a truncated joint normal distribution. Both \(A\) and \(i\) range between zero and one. Their correlation coefficient is \(\rho_{A,i}\). The marginal distributions of \(A\) and \(i\) have means and
standard deviations of \((\mu_A, \mu_i, \sigma_A, \sigma_i)\). Formally, the untruncated bivariate normal density looks as follows:

\[
g(A, i) = \frac{1}{2\pi \sigma_A \sigma_i \sqrt{1 - \rho_{A,i}^2}} \times e^{-\frac{1}{2(1 - \rho_{A,i}^2)} \left[ \left( \frac{A - \mu_A}{\sigma_A} \right)^2 - 2\rho_{A,i} \left( \frac{A - \mu_A}{\sigma_A} \right) \left( \frac{i - \mu_i}{\sigma_i} \right) + \left( \frac{i - \mu_i}{\sigma_i} \right)^2 \right]} \tag{6}
\]

The truncated normal equation I use throughout the paper \(f(A, i)\), takes into account that \(A\) and \(i\) are restricted to between zero and one:

\[
f(A, i) = \frac{g(A, i)}{\int_0^1 \int_0^1 g(A, i) dAdi} \tag{7}
\]

I could have chosen other distributions than the bivariate normal. Using methods described in McFadden (1971) one may take two independently, uniformly distributed random variables and by arbitrarily restricting their range construct a joint distribution with some non-zero correlation between the two. I prefer using the normal because of its fame, and the ease with which one can visualize changes in the correlation parameter, means, and variances. The downside of using the bivariate normal is that its non-integrability forces us to rely on numerical approximations.

Supply falls out of an infinite number of firms, each asking itself whether the sum of its tax and production cost \((1 + iT)/A\) is below the market price \(P\). The sum of answers to these many questions is summarized by the supply function:

\[
Q^s(P) = \int \int_{\{(A,i):(1+iT)/A\leq P\}} f(A, i) dAdi \tag{8}
\]

The term in curly brackets beneath the double integrals is the area of integration which captures the criterion each firm uses in deciding whether to produce. The precise bounds of integration can be gleaned from Figure 3. If \((1 + T)/P \geq 1\) then firms with the combinations of productivity parameter \(A\) and evasion parameter \(i\) in the shaded area 1, are those firms who produce. If \((1 + T)/P \leq 1\), firms in the area 1 and 2 produce. Integrating the density function \(f(A, i)\) over the appropriate bound gives the weight of firms producing. How supply is determined can also be seen in Figure 4, the three dimensional representation of Figure 3. Those firms north of the ”wall” described by \((1 + iT)/A\) decide to produce. How many of these firms there are depends on how much
of the density function falls on this north side of the wall. The density function I have graphed has \( \rho_{A,i} = .9 \), which means that there is a strong tendency for firms with high productivity to pay most of their taxes. I have set the means of both \( A \) and \( i \) to be 0.5 and their standard deviations to be 0.1. Figure 3 indicates that the supply function comes in two parts. If \((1 + T)/P \leq 1\), then

\[
Q^s = \int_0^1 \int_{1+iT/P}^1 f(A,i) dAdi
\]

and when \((1 + T)/P \geq 1\),

\[
Q^s = \int_0^{P-1} \int_{1+iT/P}^1 f(A,i) dAdi
\]

Figure 5 shows the industry supply function for a range of prices and taxes in the cases where \( \rho_{A,i} = (-0.9, 0.0, 0.9) \). The supply curves have the familiar logistic shape, and tend to one as price rises and zero as tax rises, holding all else constant. It is only at higher tax levels that a difference emerges between the three supply curves. At around \( T = 5 \), the curve with the highest supply is the one generated by \( \rho_{A,i} = 0.9 \) the lowest supply curve is the one with \( \rho_{A,i} = -0.9 \). This is the first hint of a result to emerge when we look at tax revenues. A positive correlation between productive ability and honesty (or similarly, ineptitude in evading) transforms a tax rate which officially is the same for all, into a tax based on ability to pay. Some weak firms are able to hang on at higher tax rates that would not have been able to hang on at these rates under a lower correlation between \( A \) and \( i \).

How do average (also the same as unit), industry production costs vary with the tax? In the case of no evasion, unit costs unambiguously fell with the tax. The same might not hold true in the presence of tax evasion if rising taxes allow firms skilled at evasion but unskilled at production to oust from the market firms with great productive skills but poor evasive skills. Intuition suggests that when \( A \) and \( i \) are positively correlated (high productivity accompanies high tax-paying) average cost may rise with the tax level. A firm’s production costs are \( 1/A \) (recall that the costs a firm perceives \( (1 + iT)/A \) differ from its actual costs of production \( 1/A \)). Total industry costs are the sum of individual firm production costs. Figure 3 indicates that total industry costs in the case of tax
evasion again come in two parts. If $(1 + T)/P \leq 1$, then

$$C = \int_0^1 \int_{1+iT}^{1} f(A, i) \frac{1}{A} dAdi$$

(11)

and when $(1 + T)/P \geq 1$,

$$C = \int_0^{P-1} \int_{1+iT}^{1} f(A, i) \frac{1}{A} dAdi$$

(12)

Unit costs are the above industry production costs divided by total industry output as given by equations (9) and (10).

We now have all the structure we need to see whether average costs rise with the tax and whether the rise is larger for higher levels of $\rho_{A,i}$. Figure 6 shows equilibrium average industry costs for a range of taxes and correlations. To generate this graph my computer program took a value of $T$ and $\rho_{A,i}$. This nailed down some of the parameters in the supply function (see equations 7 and 8). The program then searched for that price which would equalize supply and demand. I then took this equilibrium price and plugged it into the cost equations 9 and 10. This gave me industry costs. To get average industry costs at a particular $(T, \rho_{A,i})$ combination I plugged equilibrium price into the supply function (equations 9 and 10) and divided total industry costs by total industry output. One repeats this process for a wide range of taxes and correlations between productive and evasive abilities.

To isolate the production side I want to minimize interaction between the demand and supply curves. To do so I make demand infinitely inelastic. Equilibrium output is fixed at $Q_{fixed}$. As supply parameters vary, equilibrium price varies. Output stays the same, but the identity of the producers changes. If my notions about displacement are correct then as taxes rise, higher cost producers displace lower cost producers when good evaders tend to be inefficient producers ($\rho_{A,i} > 0$). In Figure 6 I have set fixed demand at $Q_{fixed} = .5$ (recall that maximum supply is one). Figure 6 shows that for positive levels of correlation between $A$ and $i$ (inefficient producers are good evaders) average industry costs rise with taxes.

Figure 6 shows that when efficiency and evasive ability are uncorrelated, costs can still rise with taxes. This result may strike one as odd. If production costs bear no systematic
relation to evasive ability, should not evasive ability simply be a "noise" through which emerges the well-acquainted opposite flux between taxes and average industry costs? I uncovered that average costs may rise with the tax in my earlier work (Palda 1998, 2000a, 2000b, 2001) which assumed that evasive ability and productive efficiency were independently and uniformly distributed. In Palda (1998) I explained that "even in such a world, some firms with poor productive ability but a verve for tax evasion will manage to survive and displace more productive, but less wily rivals (p.1136)." In that paper I was unable to explore whether the standard result that taxes drive down average industry costs arose the instant the correlation between $A$ and $i$ passed from zero, to less than zero because the assumption of a uniform distribution does not lend itself easily to the construction of different correlations between $A$ and $i$. The structure of the present paper allows me to vary this correlation with ease. Remarkably, even as we move into an industry where $\rho_{A,i} < 0$, average industry costs continue to rise with the tax. It seems that even when good producers tend to be good tax evaders, some poor producers will be even better tax evaders and will displace from the market some more efficient competitors. What accords nicely with intuition in Figure 6 is that as $\rho_{A,i}$ falls, so does the average industry cost, except at the highest range of correlations where the tendency reverses and average costs fall with correlation. The reason for this switch is that past a certain point the cost-raising effects of displacement, due to increasing correlation between $A$ and $i$ are overcome by the non-linear depressing influence on costs of a rising $A$. To see this, note in Figure 7, that the costs a firm perceives fall non-linearly with $A$ but only linearly with $i$. Correlation is a measure of linear association between $A$ and $i$ so that a rise in correlation will tend to increase linearly the group of low-productivity, high-evasion firms who push high-productivity, low- evasion firms out of the market. As correlation rises, high productivity firms and low productivity firms receive increasing weight in aggregate supply. Because $A$ depresses costs in a non-linear fashion, the added linear weight to the high productivity firms who remain in the market begins to outweigh the cost-increasing influence of the displacement of high-productivity firms by low productivity firms. Readers familiar with numerical modeling will forgive this tedious description of the effect of correlation on unit costs by recognizing that pretty graphs seldom reveal all secrets about a complex model.

Displacement loss is the difference between actual costs and minimum possible costs.
I calculate the minimum possible costs of producing an output $Q_{fixed}^*$ by solving the following equation for price:

$$Q_{fixed}^* = \int_{P_{minimum}^{*}}^{1} f(A) dA$$

The right hand side of this equation is the supply curve when no one evades the tax and when the tax level is zero. The price $P_{minimum}^*$ I have solved out for here is the price that when plugged into the industry cost equation will sweep under the cost integral the most efficient firms who could produce the amount $Q_{fixed}^*$. The price $P_{minimum}^*$ can be plugged into the cost equation when there is no tax evasion to give the minimum cost of producing the level of output $Q_{fixed}^*$. This minimum cost is

$$\int_{P_{minimum}^{*}}^{1} f(A) \frac{1}{A} dA$$

We can set the tax level to zero in the above calculations for convenience because when everyone pays their full tax, there is no displacement loss, so that the cost to the industry of producing a certain quantity with a tax is the same as producing it without a tax.

The minimum average industry cost of producing an output $Q_{fixed}^* = .5$ is 1.7421 units of the numeraire and does not vary either with tax or correlation between productive and evasive talents. I have represented this minimum cost as the flat plane in Figure 6. At all tax and correlation levels the evasion unit cost is above the no evasion unit cost of producing the same quantity. Displacement loss is the difference between the two planes in Figure 6. Figure 8 shows displacement loss as a percentage of the value of industry output. Figure 8 has much the same shape as Figure 6. The value of Figure 8 is that it shows that displacement loss can be a significant percentage of the value of industry output.

So far I have kept demand insensitive to price. I did this to isolate the change of the identity of suppliers due to the displacement effects of a tax. We can go to the opposite conceptual extreme and make the demand curve infinitely elastic, so that equilibrium price is a parameter of the demand curve. Figures 9(a)-(c) show how the results previously derived carry over to this opposite extreme. When good evaders are inefficient producers ($\rho_{A,i} > 0$), Figure 9(a) shows that there is a range of prices for which average industry
costs rise with the tax. This result still holds for a zero correlation between \( A \) and \( i \) but disappears (Figure 9c) when the correlation becomes negative. Not being able to find a positive relation between average industry costs and taxes does not mean that displacement deadweight losses disappear as \( \rho_{A,i} \) drops below zero. Even when \( \rho_{A,i} < 0 \) exceptions can be found to the tendency that efficient firms are good evaders. These exceptions will displace more efficient, less evasively talented firms and in displacing them force industry costs to be higher than if there were no displacement. Not finding a positive relation between taxes and average industry costs in the simulations to which I refer above may simply mean that the displacement loss from a tax is not prominent enough to show itself in a graph of taxes and average industry costs. To show the existence of a displacement we need to compare average industry costs under evasion to the costs of producing an identical output at the minimum possible industry cost, over a broad range of \( \rho_{A,i} \). In simulations not shown here, displacement loss was positive at all tax and correlation levels.

Finally, we should ask how my results change when the demand curve has a finite elasticity. In simulations not shown here, the main difference once again is that unit costs may not rise with the tax, but that displacement loss is ever-present.

2. **Laffer curve under fixed firm output**

While experimenting with my numerical model I discovered by chance that maximal government revenues under tax evasion may be larger than without evasion. Recall that I assumed a firm pays tax on the value of labor and capital it employs, so that the firm’s tax bill is \( w(1 + iT)L + r(1 + iT)K \). With the Cobb-Douglas technology, and the \( w = r = .5 \) and \( \alpha = \beta = .5 \) it is simple to show that under evasion the revenue a firm with evasive ability \( i \) and productive ability \( A \) pays to government is \( T \times (i/A) \). The intuition for \( i \) is obvious. The less evasively gifted a firm is (high \( i \)) the more tax it pays. Because each firm that decides to produce, produces the same amount, a more efficient firm (high \( A \)) will use less labor and capital than a less efficient firm and so pay less tax.

Under tax evasion, if \((1 + T)/P \leq 1\), then government revenues \( R \) can be shown to be

\[
R(P, T, \rho A, i, \mu_A, \mu_i, \sigma_A, \sigma_i) = \int_0^1 \int_{\mu_A+\sigma_A}^{\mu_i+\sigma_i} \frac{f(A, i)}{A} \times i \times T A dA di
\]  

(15)
and when \((1 + T)/P \geq 1\),

\[
R(P, T, \rho A, i, \mu_A, \mu_i, \sigma_A, \sigma_i) = \int_{0}^{1} \int_{\frac{1-T}{1+T}}^{1} \frac{f(A, i)}{A} \times i \times TdAdi
\]  

(16)

Following the same logic as above, without tax evasion government revenues are

\[
R(P, T, \mu_A, \sigma_A) = \int_{\frac{1-T}{1+T}}^{1} \frac{f(A)}{A} TdA
\]  

(17)

To calculate revenue at any given tax rate \(T\) we must calculate equilibrium price \(P\) at that tax and then plug this price into the appropriate bounds of the above integral. This is done once again by setting the supply equation with no evasion (4) equal to a demand curve \(Q^d = a + bP\) and applying a bisection algorithm to solve for the price that equilibrates supply and demand. In this section I have abandoned the notion of an infinitely elastic demand curve in order to be able to generate a Laffer (1981) curve with the familiar U-shape. The parameters values for demand I have chosen are \(a = 2\) and \(b = -0.5\).

Figure 10 shows the Laffer curves under evasion and without evasion. Under evasion and without evasion government revenues at first rise with the tax rate, and then fall. Along the correlation axis government revenues are constant in the no evasion case, because no matter what is the correlation between \(A\) and \(i\) government does not allow \(i\) to express itself. With evasion, government revenues have a varied relation to \(\rho_{A,i}\). For low tax rates, government revenues rise with \(\rho_{A,i}\). As those who are inefficient producers are increasingly able to evade the tax, the uniform tax rate \(T\) becomes a tax according to producer’s ability to pay. Government grows increasingly able to act as a tax discriminating monopolist. The revenue generating effects of a high \(\rho_{A,i}\) may be so great that the maximum possible government revenue under tax evasion exceeds that without tax evasion. Figure 9 shows that more government revenue are possible under tax evasion than without it.

A high correlation between productive ability and honesty does not guarantee high tax revenues. Figure 10 shows that for high tax levels, government receipts fall as \(\rho_{A,i}\) rises. To understand this relation suppose an economy is at \(\rho_{A,i} = 1\) and \(T = 3\). This is the
trough of the Laffer Curve, where, in spite of the perfect correlation between productivity and honesty, government revenues are zero. Revenues are low because taxes have pushed consumers into a very elastic section of the demand curve. Now lower $\rho_{A,i}$. This makes it easier than before for low cost producers to survive. They pass these savings on through price and bring consumers back to part of the demand curve Laffer curve that is less elastic. Government revenues rise.

The superior revenue generating powers of tax evasion under high $\rho_{A,i}$ should not be taken to mean that tax evasion is good for society, nor should any easy prescriptions for public policy be inferred. Tax evasion carries with it a displacement loss. Research into optimal taxation must calculate the marginal social cost of public funds for any given $\rho_{A,i}$ and equate this to the marginal benefit of a social welfare function in order to judge where on the $(\rho_{A,i}, T)$ grid government should place itself.

3. Displacement under variable firm output

The results of the previous section were stark. Tax revenue can be higher under tax evasion than without evasion. Evasion allows some inefficient firms to displace efficient firms so that as the tax rises, average industry costs rise. How much of these findings are due to constant cost structure I assumed and due to my assumption that each firm that decides to produces the same amount as all other firms? In this section I give firms once again a Cobb-Douglas production function but lift the output constraints I had imposed in the previous sections of this paper and allow for decreasing returns to scale. I find that under a Cobb-Douglas production function with diminishing returns to scale, displacement deadweight loss gives results that are even more pointed than those of the previous section.

The Cobb-Douglas technology I assume makes a firm’s output $q$ a function of its labor $L$ and capital $K$ inputs as well as productivity parameters $A, \alpha, \beta$:

$$q = AL^{\alpha}K^{\beta}$$

(18)

As before, firms share the same $\alpha$ and $\beta$ but differ in the parameter $A$. Firms pay $w$ for a unit of labor and $r$ for a unit of capital. Government levies a tax of $T$ on the value of each unit of capital and labor the firm employs ($T$ may be thought of as unemployment
insurance and a capital tax) and a firm pays only $i$ percent of this tax, and avoids the rest. My main reason for studying this particular type of tax is that it considerably simplifies the cost structure of firms and makes modeling easy. The parameter $i$ is specific to each firm. The costs the firm perceives of hiring labor are $Lw(1 + iT)$ and its costs of capital are $Kr(1 + iT)$. If we set $\alpha = \beta = 0.25$ (i.e. increasing costs) and $w = r = 0.5$ then the firm’s cost function is

$$c = \frac{1 + iT}{A^2} - q^2$$  \hspace{1cm} (19)

The above is tedious but straightforward to derive from the firm’s cost-minimizing problem and the interested reader may consult Varian (1984, p. 28) for details. If the price of the good which the firm produces is $P$, a firm’s profit maximizing calculations will lead it to produce:

$$q = \frac{A^2}{2} \frac{P}{1 + iT}$$  \hspace{1cm} (20)

The above supply function looks like familiar terrain, but a short tour around its landmarks will help us to make sense of the results that follow on displacement deadweight loss. Equation (18) says that under diminishing returns to scale a firm with $\alpha = 0.25$ and $\beta = 0.25$ increases its output linearly with price. Under these circumstances marginal costs rise at a constant rate so that the supply curve is positively sloped and linear. If $A$ were to double, a quick glance at the production function (18) shows that provided $\alpha + \beta < 1$ labor and capital could fall by more than one half and while keeping output constant. Another way of putting this is that output rises exponentially with the productivity parameter $A$, as equation (20) shows. Output falls as the tax $T$ rises, and falls as the fraction of this tax that the firm pays $i$ rises. Taxes have a non-linear effect on output because I assumed that taxes work their way through labor and capital inputs. A rise in the tax is like an equal rise in wages and interest. At the original level of output this would simply raise costs linearly because the similar change in price of labor and capital does not warrant their recombination in order to soften the effect of the tax on costs. The linear increase in costs forces the firm to cut back output. As its output falls the firm sees its production costs fall at an increasing rate (we are going backwards along a decreasing returns to scale production function). Further increases in taxes will have less and less effect on firm output as the firm retreats into the high productivity segment of
its production function. This explains why taxes have a non-linear and diminishing effect on firm output.

There are no complicated bounds of integration to worry about in deriving the industry supply function under a Cobb-Douglas production function with increasing costs, as there were in the cost structure I examined in the previous section. Each firm produces something. If each firm produces, the entire range of productive abilities \( A \), (zero to one), and evasive abilities \( i \), (zero to one) express themselves in aggregate supply. Aggregate supply is

\[
Q^s = \int_0^1 \int_0^1 f(A, i)q(A, i, P, T)dAdi
\]

\[
= \frac{P}{2} \int_0^1 \int_0^1 f(A, i)\frac{A^2}{1+iT}dAdi
\]

Aggregate supply is linear in price. This was not the case in the previous section where supply bore an S-shaped relation to price. In the present case all firms produce. Price does not sweep across a normal distribution of costs. All firms in the distribution are included in supply from the start. Price works its effects through each individual firm’s supply function (20) in a linear manner. The sum of linear price effects on firm output shows up in a linear relation between industry output and price.

The industry supply curve (22) indicates that taxes interact with the distribution of productive talents \( A \) because evasive ability \( i \) transforms the general tax \( T \) into a particular firm tax \( iT \). When the tax and the distribution of talents interact, we might expect that the effect of taxes on output would depend in part on the correlation between evasive and productive talents, as the degree of this correlation affects the weight that different combinations of \( A \) and \( i \) receive in calculating industry supply. Contrary to this expectation and unlike in the fixed firm output case, Figure 11 shows that changes in the correlation between evasive and productive abilities have no effect on industry supply. It seems that under the parameters of the Cobb-Douglas function I have chose, the increased output of high-cost, high-evasive-ability firms balances the decreased output of low-cost low-evasive-ability firms, as low-efficiency producers become increasingly adept at evading taxes.

Equilibrium price occurs where supply (22) equals demand, which as before we take to
be $Q^d = a + bP$. Equating (22) to the demand equation gives an equilibrium price under tax evasion of

$$P_{\text{evasion}} = \frac{1}{2} \int_0^1 \int_0^1 f(A, i) \frac{A^2}{1 + iT} dAdi - b$$

(23)

No bisection algorithm is needed here to search for equilibrium price. $P$ appears explicitly in $Q^s$ and not in the bounds of integration as it did in the previous formulation of supply (equation Y). This allows us to isolate $P$ directly instead of needing to integrate over $f(A, i)$ (an impossible task in section 1) to isolate $P$.

A firm’s production costs are not the same as the costs it perceives. The firm perceives costs of $(1 + iT)q^2/A^2$. The resource costs of producing the amount $q$ are $q^2/A^2$. Substituting the expression (20) for $q$ in production costs of a particular firm gives overall industry costs in equilibrium as

$$COSTS_{\text{evasion}} = \frac{P_{\text{evasion}}^2}{2} \int_0^1 \int_0^1 f(A, i) \left( \frac{A}{1 + iT} \right)^2 dAdi$$

(24)

The displacement loss from tax $T$ is the above cost of producing equilibrium industry output when all producers evade to greater or lesser degrees, less what it would cost to produce this output if no one evaded the tax. In an honest world the industry costs of producing any given level of output will fall below the costs of producing this output under tax evasion. A firm’s production costs without evasion and at zero tax are

$$c = \left( \frac{q}{A} \right)^2 = \left( \frac{AP}{2} \right)^2$$

(25)

(26)

where in the second line I have made use of the fact that firm supply $q$ is $(A^2P)/2$. Summing (26) over all firms gives industry costs of

$$\frac{P^2}{4} \int_0^1 A^2 f(A) dA$$

(27)

We cannot simply subtract the above minimum cost from actual costs as given by $Y$ to get the deadweight loss from displacement. We must find that price without evasion (call it $P_{\text{honesty}}^*$) in the above equation that would lead to the same industry output as under evasion at a particular tax $T$. Without tax evasion a firm’s output is $q = A^2P/[2(1 + T)]$. 
Setting \( T = 0 \) simplifies this expression. As explained throughout this paper, whether tax is positive or zero makes no difference to the calculation of minimum cost in a world without tax evasion. Taking firm output as \( q = A^2P/2 \) gives industry supply under honesty of

\[
Q_{\text{honesty}}^* = \frac{P_{\text{honesty}}}{2} \int_0^1 f(A)A^2dA
\]

The equilibrium price under honesty \( P_{\text{honesty}}^* \) at which equilibrium industry output under evasion \( Q_{\text{evasion}}^* \) is equal to output under honesty \( Q_{\text{honesty}}^* \) as given by equation (28) comes from equating \( Q_{\text{honesty}}^* \) to \( Q_{\text{evasion}}^* \) and solving out for \( P_{\text{honesty}}^* \). This gives

\[
P_{\text{honesty}}^* = \frac{2Q_{\text{evasion}}^*}{\int_0^1 f(A)A^2dA}
\]

Substituting this price into industry cost under honesty as given by (27) gives what would be the least cost way of producing that output which an industry with evaders produces at tax \( T \). The difference between actual costs as given by equation (24) and minimum costs as given by equation (27) is the displacement loss from the tax.

Once again I assume demand to be completely inelastic so that I can focus on how different producers displace each other in conditions where demand forces them to compete for a fixed pie. Figure 12 shows how the unit costs of production under tax evasion vary with the tax and the correlation between productive ability \( A \) and evasive ability \( i \). I calculate equilibrium price under evasion using equation (23) and plug this into equation (24) to calculate industry costs. I then divided industry costs by equilibrium output to get average production costs for the industry. In the previous section I showed how unit costs could increase with tax. Standard public finance theory says that average industry production costs fall as taxes increase. Figure 12 shows that, as in the fixed output case, average industry costs rise with the tax. The figure differs in that average industry costs tend to fall uniformly as the correlation between \( A \) and \( i \) increases. In the fixed output case of section 2 I explained that average costs are under two opposing stresses. As correlation rises, displacement rises linearly, and so pushes up average costs, but as correlation rises the efficient firms who remain standing have non-linearly decreasing costs. In the present Cobb-Douglas case with variable output the second effect seems to dominate as we increase the value of the correlation coefficient.
Nothing in the arguments I have developed insists that unit costs must rise with the tax for a deadweight loss form displacement to exist. Rising unit costs are an unmistakable sign of displacement loss, but subtler signs can be found. A displacement loss exists if unit costs are greater under tax evasion than they would be without tax evasion. We need this insight when examining cases where the demand curve is not perfectly inelastic. When I set demand parameters to \( a = 2 \) and \( b = -0.5 \) a graph of unit costs (not show here) no longer shows a negative relation between taxes and average industry costs. This does not mean displacement does not exist.

4. Laffer curves under variable firm output

So far the results of the previous section have carried over to the case of a Cobb-Douglas production function. What about the finding that tax revenue may be larger with evaders about than in a world where all pay government its due? With tax evasion, taxes collected from each firm are \( wL \times iT + rK \times iT \). If, as before, \( w = r = 0.5 \), and \( \alpha = \beta = 0.25 \), it is simple to show that labor and capital are employed in equal quantities \( L = K = (q/A)^2 \). The taxes a firm pays reduce to \((q/A)^2 \times iT\). Substituting the complete expression for \( q \) as given in equation (20) into revenues gives government revenue paid by any particular firm as

\[
\frac{TP^2}{4} \frac{A^2i}{(1 + iT)^2}
\]

Total government revenues under evasion \( R_{\text{evasion}} \) are the sum of the above over all firms:

\[
R_{\text{evasion}} = \frac{TP^2}{4} \int_0^1 \int_0^1 f(A, i) \frac{A^2i}{(1 + iT)^2} dAdi
\]

The tax government collects when there is no evasion can be easily show to be as

\[
\frac{A^2}{2} \frac{P^2}{1 + T}
\]

Summing the above across all firms gives government revenues under honesty as

\[
R_{\text{honesty}} = \frac{TP^2}{4(1 + T)^2} \int_0^1 f(A) A^2 dA
\]
Figure 13 superimposes revenue under evasion and revenue under honesty in the case where the demand parameters are $a = 2$ and $b = -.5$. What comes through once again is that tax evasion may allow government to raise more revenue than if everyone paid their allotted tax. Government revenues under evasion are highest for very high correlations between $A$ and $i$. The explanation for this is the same as in the previous section: when efficient producers are poor evaders, tax evasion turns an equal tax on all into a tax based on ability to pay. By chance, government benefits from the distribution of productive and evasive talents to discriminate between taxpayers and raise revenues to heights not possible in a world where none cheat.

5. Extensions and testing

Taxes are not the only form of government intervention that produce displacement losses. As I showed in Palda (2000a), evasion of the minimum wage can force honest, high-productivity producers who comply with the minimum wage to retreat before dishonest, inefficient producers who evade the minimum wage. In Palda (2000b) I showed how firms that are good at lobbying government for favors can oust from the market firms that are less talented at lobbying but who are more efficient at producing. The notion that a person may have two talents that help him survive in the market goes back at least to the work of Roy (1951) and in the 1980’s labor economists such as Willig (1986) (in a masterful survey) took up the theme of how the joint distribution of multiple-talents tugs a man into one occupation instead of to another. The question of displacement loss never arose in those models because the multiplicity of talents they considered were productive talents. My model explicitly considers how a productive talent bundled Januslike with an unproductive talent can make a firm a menace to economic welfare.

Like vultures circling, deadweight losses signal a crippled market. Externalities, adverse selection, tax wedges between cost and reservation demand price, are the big game of welfare economics, not just for their economic interest, but also because of their interest to politics. According to McNeill (1975), Becker (1983), and many others, political systems may evolve to dampen deadweight losses. This is why deadweight loss is not the exclusive plaything of the welfare theorist, but also of those who study the formation of government
and of other institutions geared to minimizing waste. Displacement deadweight losses are easy to banish in circumstances where a firm can separate its productive from its unproductive abilities. The institution of transferable fishing quotas, and transferable emissions quotas are intended to allow inefficient firms who win the lottery for the right to produce to transfer their rights to more efficient firms. Such an institution wipes out displacement losses at the cost of organizing the markets for the quotas. Societies which do not create the conditions for such markets to flourish may labor under displacement losses. How do institutions deal with less easily separable talents such as productive ability and the ability to evade taxes? Perhaps complexity crept into tax codes in order that those with an knack for tax evasion could put on the cloak of accountants, lawyers, and lobbyists and sell their services to the highest bidders. Those who would bid highest are the most productive, least evasively gifted producers. What Public Choice economists deride as the "rent-seeking" costs from lobbying may be echoes of a larger loss from displacement that lobbyists mitigate by helping to separate evasive from productive abilities.

The insights of my model of displacement may extend to sociology. Most of us have worked with officemates whom we know to be incompetents, but who have a knack for bringing their meager accomplishments to the attention of the department head or the shop-floor supervisor. Viennese psychoanalyst Erich Fromm called this type the "marketing personality." Social norms that limit boasting may be an attempt to limit the deadweight losses from the displacement of talented workers by their mundane colleagues afflicted with the gift of the gab.

Are there any testable implications in my model or is it just another shack in the shantytown of metaphysics that now sprawls over economics? A prediction of my theory is that we might find that rising taxes lead to increased industry costs. This is only half the story. Science is about connecting causes to effects. To do that you need measurement. We may be able to measure industry costs, but how do we measure evasive talents? If this challenge could be met, then my theory could be subject to testing. I do not wish to bow my head too low in humility on this point. No one has ever seen a Harberger triangle, those three-sided sprites of the welfare-economics forest, but this never stopped the masters of computational general equilibrium models from practicing their insightful alchemy. Even when they are not directly testable, numerical simulations tell us what
happens over a range of possible parameters. If my model is not directly testable, then the challenge to making it interesting is to find out how evasive and productive abilities are related so that we may form some idea of the parameters of their joint distribution, and calibrate our simulations with these benchmarks.

6. Conclusion

When a productive talent such as the ability to make a good efficiently combines in a firm with a talent for tax evasion, low-productivity firms with good evasive abilities may oust from the market efficient firms with poor evasive abilities. The difference between the cost of production of those inefficient firms who remain standing in the market and the lower costs of the efficient firms they ousted is the displacement deadweight loss from tax evasion. This deadweight loss generally increases with the correlation between efficiency and honesty in paying taxes. When efficient firms are poor tax evaders, and inefficient firms are good tax evaders a uniform tax on firm inputs becomes a tax according to ability to pay. This transformed tax allows the government to act as would a price discriminating monopolist, so that we may find that government revenues may be greater under tax evasion than without tax evasion.

My analysis casts a negative light on the underground economy. Tax evasion is "unfair" in the sense that it allows inefficient producers to push their more efficient more honest rivals to the fringes of the market. The difference in the costs of the actual producers and those who would have survived had government enforced taxes evenly is a social loss. Unfairness is inefficient.

BIBLIOGRAPHY


When $P$ is less than $K$ but greater than $T$, the proportion of producing is $(P/K)$ less the area $B$.

\[
\frac{1+T}{P}
\]

\[
\frac{P-1}{T}
\]
production cost under evasion
production cost under honesty
Figure 7

production plus tax cost
displacement as a percentage of industry output value
Figure 10

evasion Laffer curve

honesty Laffer curve
Figure 11: Two faces of the supply curve
Figure 12: Average firm costs under evasion

Figure 13: Laffer curves under evasion and honesty