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Master thesis

2021 **Bc. Arsenii Scherbov**

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Financial Connectedness of Eastern European Stock Markets

Master thesis

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Abstract

The connectedness of financial assets and markets represents an essential concept that has long-lasting consequences for the assessment of risk. Thus, it is important to correctly measure dependencies and describe their dynamics to predict future responses of markets to shocks. In this thesis, I focus on the connectedness of Eastern European stock markets and assess the relationships between returns and volatilities in these markets, accounting for the presence of cryptocurrency markets and other major developed markets. I describe conditional correlations of returns from the DCC model of Engle (2002, JBES). Using the spillover framework proposed by Diebold and Yılmaz (2009, EJ) I measure the connectedness from a static and dynamic perspective. The results indicate that Eastern European markets are tightly connected. The measures of connectedness were fluctuating over time and have risen significantly as a consequence of the recent pandemic. The magnitude of the increase for different groups of markets ranges from 35% to 100%.

Abstrakt

Propojení finančních aktiv a trhů představuje významný koncept při hodnocení rizika. Je důležité správně měřit závislosti a popsat dynamiku propojení trhů k předpovídání reakcí trhů na šoky. V této práci se zaměřuji na propojení akciových trhů východní Evropy a vyhodnocuji vztahy mezi výnosy a volatilitou na těchto trzích. Při své analýze beru v potaz existenci trhů s kryptoměnami i jiných významných trhů. Popisuji podmíněné korelace výnosů DCC modelem Engleho (2002, JBES). S použitím frameworku pro zkoumání přelévání šoků mezi trhy, který byl navržen Dieboldem a Yilmazem (2009, EJ), měřím statickou i dynamickou propojenost. Výsledky naznačují, že trhy výhodní Evropy jsou úzce propojeny. Míra propojení kolísá v čase a výražně se zvýšila v nedávné době jako důsledek pandemie COVIDu. Nárust propojení se v různých kategoriích trhů pohybuje od 35 % do 100 %.

Key words: Financial connectedness, Stock market, Spillovers.

Declaration of Authorship

I declare that I carried out this master thesis independently, and only with the cited sources, literature and other professional sources. It has not been used to obtain another or the same degree. I hereby proclaim that I wrote my master thesis on my own under the leadership of my supervisor and that the references include all resources and literature I have used.

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Prague, July 26, 2021

Signature

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Project of Master Thesis

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Academic year:	**2020/2021**

Theme: Financial Connectedness of Eastern European Stock Markets

Research question and motivation:

The interdependence between the economies of European countries generates gains due to cooperation but also poses the risk of domino effects during economic downturns (Sapir, 2010). Considering the economic slowdown at the beginning of 2020 due to Covid-19 quarantine measures, potential negative effects of economic integration seem to be especially important. Knowledge of the presence of negative effects of integration raises a question about the ability of European countries to maintain economic stability when responding to significant and geographically unequally distributed shocks. The effects of their connectedness are likely to be reflected in the behavior of European financial markets, and to cause co-movements in the prices of stock market indices of individual countries. Thus, a return or a volatility shock to one local market may propagate to connected markets, generating additional uncertainty. The magnitude of the effects of shocks may vary across time with changes in the degree of market connectedness. Time dynamics become even more important with the rise of new markets connected to the financial system, such as cryptocurrencies. It is widely accepted that uncertainty contributes to declines in the value of financial assets and decreases in returns. Therefore, it is essential to correctly estimate the degree of interdependence of European economies to construct possible recovery scenarios. There are various examples of works describing integration and spillover effects in European financial markets. Forbes Rigobon (2002) created a methodological environment for these studies by noting that the presence of heteroskedasticity requires separate modeling when estimating the degree of market interdependence. Various techniques are used to model heteroskedasticity: Bayesian quantile regressions (e.g., Caporin et al., 2018), extensions of the MGARCH model (e.g., Baele, 2005), and the VAR framework (e.g., Égert Kočenda, 2007, Diebold and Yılmaz, 2009, Demiralay Bayraci, 2015), to name a few. Among various MGARCH extensions there is a Dynamic Conditional Correlation (DCC) model proposed by Engle (2002) as an improvement on previous modeling approaches (for example, the BEKK model described in Engle Kroner, 1995). The specification of this model addresses the problem of the dimensionality of the vector of parameters and allows one to estimate time-varying conditional correlations, which motivate ubiquitous use of the DCC in empirical studies on financial connectedness. Another way to approach the problem is to estimate return or volatility spillovers. The framework proposed in Diebold and Yılmaz (2009) is based on forecast error variance decompositions from a VAR model that are used to construct various measures of the connectedness of markets. The measures allow one to investigate the magnitudes and directions of spillover effects across markets. To the best of my knowledge, no works have concentrated on changes in the integration of Eastern European stock markets during the recent pandemic, accounting for the influence of cryptocurrencies. The goal of this thesis is to fill this gap in the literature and provide new empirical evidence on the financial connectedness of Eastern European stock markets.

Contribution:

There have been many studies about spillover effects and the integration of Eastern European stock markets. However, they do not consider the influence of the Covid-19 pandemic on the degree of connectedness. There has been no study concentrating on Eastern European stock markets in isolation and describing how these markets are connected to cryptocurrencies. My thesis will contribute by applying the DCC model and Diebold and Yılmaz's (2009) spillover framework to study the integration of Eastern European stock markets.

Methodology:

I will use daily returns data on stock market indices of Eastern European and linked developed countries. I will represent the cryptocurrency markets with Bitcoin data. The quality of data is essential for the estimation of the model, and the availability of data on both stock market indices and different financial instruments may be beneficial for the comparison of evidence of connectedness from different models. Thus, the major part of the work will be devoted to gathering the necessary data on returns. I will compare the results from proposed approaches to the results of prior studies, with a special focus on the effects of the pandemic on the connectedness of markets. I will interpret the potential differences in the estimates of the magnitude of spillover effects. My conclusions will include policy suggestions, taking into account the current degree of interdependence between Eastern European stock markets and their dependence on the performance of cryptocurrency markets.

Outline:

1. Introduction

2. Literature review

3. Data description

4. Dynamic conditional correlations of returns

5. Return and volatility spillovers

6. Discussion

7. Conclusion

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Contents

1 Introduction

The concept of connectedness receives a lot of attention in the financial literature. The assumption that financial markets or assets are independent is usually rejected by the data in favor of integration (Billio et al., 2012; Diebold and Yılmaz, 2015a). Financial connectedness implies that the assessment of future outcomes, for example, risks, should take into account the influence of a surrounding environment. If a group of markets in a portfolio is strongly connected, the shock to one market may create a chain of shocks spreading across all other markets in the portfolio. Thus, the failure of one member of a group may cause the entire group to fail, raising the problem of systemic risk. The chain effect causes the magnitude of losses to exceed a loss implied by the initial shock in the case of independence of markets. The failure of the financial side of the economy may impact the real economy through spillover effects and cause declines in economic activity. Moreover, movements in the opposite direction are also a possibility. Such notions were shown to be true and significant in the financial crisis that began in 2008, and which attracted additional attention to the importance of financial integration and connectedness. Since not only assets but also financial institutions and countries may be, in some sense, connected, the study of financial integration provides relevant information for all kinds of professionals, from risk and portfolio managers to policymakers. It helps to predict and quantify the consequences of possible significant negative shocks for the dynamic evolution of connected systems. It may also reveal spillover effects from regulatory policies generated by the underlying connection structure. This information may guide policy decisions and increase their efficiency. The measurement of the degree of integration is also directly related to standard tools in risk management, including Expected Shortfall, Value at Risk (VaR), and its conditional CoVaR counterpart (Billio et al., 2012).

The important aspect of financial connectedness is its time-varying nature (Rockinger and Urga, 2001). The degree of integration may evolve dynamically and respond to new information coming to markets. The recent pandemic[1] events represent an illustrative example of a systemic shock that affected all countries and markets. The pandemic has likely changed the patterns of connectedness between financial markets, given tremendous changes in the real sector of the economy. The changes should be even more pronounced in regions where special efforts are made to increase the cooperation between member countries, including the European Union (EU). In such groups of countries, the concept of connectedness of both real and financial sectors is more relevant. Thus, the spillover effects are likely to be more significant for the countries and financial markets of the EU.

Another portion of time variation in financial connectedness may be explained by additional entities entering or leaving the connected network. Although no major changes

[1] Officially recognized by the World Health Organization on 11th of March 2020. Link: https://www.euro.who.int/en/health-topics/health-emergencies/coronavirus-covid-19/news/news/2020/3/who-announces-covid-19-outbreak-a-pandemic/

in EU structure have happened recently, the rise in importance of cryptocurrencies may represent an example of the entrance of a new market to the system. The recent study by Bouri et al. (2021) shows how different cryptocurrencies are connected to each other in terms of volatility of returns. Using Twitter feed data the authors explain the dynamic nature of the connectedness of cryptocurrencies by changes in sentiment (level of happiness) of traders. Other papers provide evidence on how cryptocurrencies are linked in terms of both return and volatility spillovers to markets for major commodities (Okorie and Lin, 2020; Bouri et al., 2021) and stock markets (Frankovic et al., 2021). It is possible that the pandemic increased connectedness between EU stock markets and cryptocurrencies through the sentiment channel. The above-mentioned aspects of time-variability demonstrate that static evaluation of connectedness is likely to be misleading or incomplete and a dynamic approach should be exploited instead.

The previous literature on the relationship between European stock markets is vast. Earlier studies, for example, Syllignakis and Kouretas (2010, 2011), concentrate on the return connectedness of Central and Eastern European (CEE) stock markets and conclude that the integration of these markets with developed European markets and the US increased after the 2008 financial crisis. Demiralay and Bayraci (2015) investigate volatility spillovers for the same group of markets and conclude that similar results regarding the connectedness of CEE with other major markets hold for volatility. Moreover, they provide evidence on a moderate connectedness of CEE markets, showing that around 50 % of future volatility forecasts are formed by spillovers. These studies mostly discuss the effects of the 2008 financial crisis; evidence on the effects of the pandemic on the connectedness of European markets is scarce. Aslam et al. (2021) focus mostly on developed European stock markets in the period around the start of the pandemic. The authors use 5-minute intraday volatility data and show that almost 80 % of the variation in forecasts of future volatility is due to spillover effects between markets. The significant limitation of their work comes from the high-frequency nature of the data. The authors are able to cover only a short time period before and after the pandemic. Their model captures the connectedness of markets while they are under the effects of the pandemic and does not account for previous available information on the evolution of European markets. This may lead to an overestimation of the degree of connectedness and magnitude of spillover effects. To my best knowledge, empirical evidence on the relationship between Eastern European (EE) markets and cryptocurrencies is missing. However, the issue requires a separate investigation for the reasons already described. I aim to fill the gap in the literature by providing an empirical analysis of the financial connectedness of EE markets with themselves and other developed markets, considering the role of cryptocurrencies in this picture. I also describe how the dynamics of EE markets' connectedness responded to the pandemic events.

In this thesis, I focus on the connectedness of EE financial markets and their relations to other markets. Using daily information on prices of stock market indices I study how EE markets depend on each other, financial markets of developed countries, and the cryptocurrency market. I investigate the connectedness on return and volatility levels. Firstly, I describe the dynamic conditional correlations of returns obtained from the DCC-GARCH model of Engle (2002). Then I separately quantify spillover effects using the framework proposed by Diebold and Yılmaz (2009b, 2012, 2014) for financial returns and volatilities. Based on the results I construct connectedness measures and study their dynamic evolution, concentrating on the effects of the pandemic in the last part of the sample.

My results suggest that EE markets are tightly connected. Conditional correlations of returns of EE markets were fluctuating over time around a value close to 50 % with a noticeable spike at the beginning of the pandemic. The dynamic measure of connectedness based on spillover effects showed the same pattern and significantly increased as a consequence of the pandemic. Estimates of return and volatility spillovers allow one to conclude that EE markets were mostly receiving spillovers from other markets. However, in terms of returns, some EE markets started to generate spillover effects while in terms of volatility their behavior mostly remained unchanged. I document a weak relationship between cryptocurrency and EE markets, which departed from the state close to independence only after the pandemic.

The rest of the thesis is organized as follows: Section 2 provides a literature review; Section 3 describes data employed in the analysis; Section 4 introduces the results; Section 5 contains a discussion; and Section 6 concludes.

2 Literature review

2.1 GARCH approach

It is widely accepted in the financial literature that returns of assets may depend on each other and exhibit comovements. Poon et al. (2003) investigated the importance of time-varying volatility in studying dependencies between financial assets. The authors showed that estimates of tail dependence differ significantly once the heteroskedasticity is accounted for, suggesting that the appropriate way to analyze the dependence between financial assets must include the modeling of volatility dynamics.

A natural starting point in volatility modeling is the Autoregressive Conditional Heteroskedasticity (ARCH) model proposed by Engle (1982). Since the volatility is not observable, the idea is to explicitly define an equation that governs the dynamics of volatility of return series. The simplest example of the ARCH model of order p consists of two equations describing the mean and variance processes:

$$
\begin{aligned}
r_t &= \mu + \varepsilon_t \\
\sigma_t^2 &= \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 \\
\varepsilon_t &= \sigma_t \epsilon_t
\end{aligned} \tag{1}
$$

where ϵ_t are i.i.d disturbances with zero mean and unit variance, r_t is the return of the asset with variance σ_t^2. The model filters unobserved volatility using squares of residuals from the mean equation. It is evident that the large realizations of shocks ε_t drive the variance of subsequent returns up, generating a possibility to observe greater shocks in the future. To ensure the positiveness of variance the parameters should satisfy non-negativity constraints $\omega > 0$ and $\alpha_i \geq 0$. The existence of unconditional variance requires $\sum_{i=1}^{p} \alpha_i < 1$. The model captures the stylized fact of financial time series that volatility persists in time, forming volatility clusters. However, the generated level of persistence is usually too low to correctly describe the behavior of the return series. To improve upon the ARCH idea and increase the persistence of volatility generated by the model Bollerslev (1986) introduced the Generalized ARCH (GARCH) model, which adds lags of volatility to the variance equation. In the GARCH(p,q) model the variance equation is augmented by lagged values of volatility:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{2}$$

where again the non-negativity of the parameters is required to ensure that generated variances are positive. To achieve the finite unconditional variance the condition $\sum_{i=1}^{p} \alpha_i +$

$\sum_{i=1}^{q} \beta_i < 1$ should be satisfied. The consequences of the violation of this condition for the persistence of variance generated are discussed by Nelson (1990, 1991). The proposed Integrated GARCH (IGARCH) model allows one to work in the case when $\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i = 1$, estimating the standard GARCH model with the restrictions on the coefficients of volatility persistence. The main feature of this model is that the impact of previous volatility levels persists and never dies out for any forecasting horizon. The behavior of volatility series (the existence of unconditional variance and stationarity) depend on whether the intercept ω is equal to zero or positive.

One significant limitation of the models described above is that they do not appropriately capture the asymmetric effects of shocks on the volatility level. Among other researchers, Pagan and Schwert (1990) noted that GARCH models impose strict restrictions on the dynamics of the volatility process by stating that the effect of positive and negative shocks is identical. This feature is ensured by the use of squares of shocks in the volatility equation. In fact, the symmetric impact of shocks is usually not the case for financial time series. For example, Bekaert and Harvey (1997) demonstrate in the GARCH framework that the behavior of the majority of emerging markets exhibits signs of asymmetric reactions of volatility to previous shocks. Bad news, represented by negative shocks (unanticipated drops in the return), is likely to increase the volatility more than unanticipated increases; this typical feature is usually called the leverage effect (Hamilton, 1994) and may be modeled by the slight transformation of the volatility equation. Engel (1990) proposed to model an asymmetric effect in the GARCH(1,1) model by adding an unrestricted parameter γ to the previous shock before squaring the term:

$$\sigma_t^2 = \omega + \alpha(\epsilon_{t-1} + \gamma)^2 + \beta\sigma_{t-1}^2 \tag{3}$$

The consequences of this transformation may be summarized using the terminology proposed by Engle and Ng (1993): the News Impact Curve (NIC), which shows the impact of previous return shocks on the current volatility level, is asymmetric with respect to the sign of the shock. Indeed, fixing previous volatility levels on the unconditional mean level σ^2 the NIC is centered at $\epsilon_{t-1} = -\gamma$ as opposed to zero in the GARCH(1,1) model:

$$\sigma_t^2 = \omega + \beta\sigma^2 + \alpha(\epsilon_{t-1} + \gamma)^2 = A + \alpha(\epsilon_{t-1} + \gamma)^2 \tag{4}$$

where A is a constant. If $\gamma < 0$ the NIC is shifted to the right and the impact of negative shocks is greater compared to the impact of positive shocks close to γ.

Nelson (1991) proposed the Exponential GARCH (EGARCH) model as another approach to directly address the asymmetric reaction of volatility to previous shocks of different signs. In this model the logarithm of volatility is modeled instead of the level of volatility and the equation takes the following form:

$$ln(\sigma_t^2) = \omega + \sum_{i=1}^{p} \alpha_i[\phi\epsilon_{t-i} + \psi(|\epsilon_{t-i}| - E(|\epsilon_{t-i}|))] + \sum_{j=1}^{q} \beta_j ln(\sigma_{t-j}^2) \tag{5}$$

where ln() is a natural logarithm, $\alpha_1 = 1$ and ϵ_t are i.i.d disturbances with zero mean and unit variance. Since the volatility is log-transformed the non-negativity constraints on the parameters are no longer relevant. In the case of the normality of ϵ_t, $E(|\epsilon_t|) = (\frac{2}{\pi})^{\frac{1}{2}}$ and it is evident that the part of the slope of previous shocks without α_i depends on the sign of the shock: it is equal to $\psi + \phi$ for positive realizations and $\psi - \phi$ for negative realizations, making the NIC asymmetric. If the log transformation is not desirable one may still make the NIC asymmetric using the GJR-GARCH model of Glosten et al. (1993). The approach is close to the proposition of Engle and Ng (1993) described previously but implies a less restrictive structure in the volatility equation. For the same case of $p = q = 1$ it may be expressed in the following way:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 \boldsymbol{I_{t-1}} + \beta\sigma_{t-1}^2 \tag{6}$$

where $\boldsymbol{I_{t-1}}$ is an indicator function which is equal to 1 if $\varepsilon_{t-1} < 0$ and 0 otherwise. The NIC has its minimum at $\varepsilon_{t-1} = 0$ and the impact of the previous shock is $\alpha + \gamma$ for negative realizations and α for positive ones. It is worth noting that for the symmetric distributions of ϵ_t the existence of unconditional variance is insured by the transformed condition $\alpha + \beta + \frac{\gamma}{2} < 1$ (Ling and McAleer, 2002). If $\gamma = 0$ and no asymmetry is present the condition reduces to the standard one for the GARCH(1,1) model. Rockinger and Urga (2001) study the financial integration of European economies and identify the presence of significant asymmetric GARCH effects with negative shocks generating additional volatility. This finding suggests that one needs to account for asymmetries when modeling the behavior of stock markets. Engle and Ng (1993) use Japanese stock return data to compare the performance of different asymmetric GARCH models. The authors identify the GJR-GARCH model as the best parametric alternative for modeling the asymmetric influence of past shocks on the current volatility. The EGARCH model is able to adequately account for asymmetry too. However, the implied variability of the predicted conditional variance is higher than in other models and exceeds the variability of squared returns. This speaks against the use of EGARCH in empirical applications.

The specification of the volatility equation is the central focus of the GARCH-type model selection process. However, the form of the conditional mean equation may also be transformed to accommodate theoretical facts and considerations about financial returns.

The specification of the conditional mean from Equation 1 assumes a constant mean of returns. However, the assets with higher variance may pay more to compensate for the risk. Since variance changes over time as postulated by the volatility equation the mean of returns should also change. Engle et al. (1987) proposed the GARCH-in-Mean (GARCH-M) model to account for the presence of the time-varying risk premium. Their proposal was to include some measure of risk into the mean equation:

$$r_t = \mu_t + \varepsilon_t = c + \delta h_t + \varepsilon_t \tag{7}$$

where h_t may be equal to σ_t^2, σ_t or $ln(\sigma_t^2)$, c is a constant term and δ is a parameter that describes the relationship between the return and risk of an asset. If the parameter δ is not equal to zero, the specification implies the existence of the serial correlation in returns, which is pronounced through the volatility equation. This serial correlation may be modeled directly by including autoregressive terms in the mean equation (lags of returns). For a stock market return the significant autocorrelation may indicate inefficiency. Rockinger and Urga (2000) test the efficiency of European stock markets using a sample from 1994 to 1999. The authors use the time-varying AR(1) model and identify that the Czech, Polish, and Hungarian markets drifted towards efficiency, which is captured by the insignificance of the autoregressive term. The Russian market remained inefficient with a slightly significant AR(1) parameter. Although the process takes time, the evidence suggests that stock markets become more efficient over time. The common practice in GARCH-type modeling is to include an autoregressive term in the mean equation. Another way is to include a set of variables to model the seasonality in mean returns or combine both approaches. However, when working with the returns of markets that are mature enough, the inclusion of these terms into the mean equation is likely to lead to insignificant results.

Another important part of the conditional mean equation is the unpredictable innovation. The choice of the distribution of the error term determines the distribution of the returns and affects the estimation. The vector of parameters $\boldsymbol{\theta}$ of GARCH-type models is usually estimated using Maximum Likelihood (ML) procedures (Hamilton, 1994). Given the distribution of i.i.d. standardized innovations $f(\epsilon_t(\boldsymbol{\theta})|I_{t-1})$ one may construct a sample log-likelihood conditioning on the first m observations[2] to ensure that necessary

[2]The unconditional distribution of first m observations is complicated and usually omitted by the virtue of the assumption that this distribution does not depend on the estimated parameter vector $\boldsymbol{\theta}$.

information I_{t-1} is available for the estimation for all lags (up to m)[3] at time t:

$$LL(\boldsymbol{\theta}) = \sum_{t=1}^{T}[ln(f(\epsilon_t(\boldsymbol{\theta})|I_{t-1})) - \frac{1}{2}ln(\sigma_t^2(\boldsymbol{\theta}))] \tag{8}$$

where the variance term arises from the standardization of innovations (recall from Equation 1 that $\epsilon_t(\boldsymbol{\theta}) = \frac{\varepsilon_t(\boldsymbol{\theta})}{\sigma_t(\boldsymbol{\theta})}$)). The maximization of $LL(\boldsymbol{\theta})$ delivers ML estimates $\hat{\boldsymbol{\theta}}$. One solves the problem numerically choosing starting values for all elements of $\boldsymbol{\theta}$ and m initial values for volatility and realizations of residuals.

The maximization of the conditional log-likelihood depends on the choice of density. For financial applications a researcher is free to choose from several common alternatives: standard normal, standardized Student‘s t, generalized error distribution (GED), and their modifications. The choice is guided by theoretical considerations with the aim of capturing important facts about the data. For example, Bollerslev (1987), in the GARCH framework, models exchange rates using standardized Student‘s t distributed errors to capture the tail fatness of the return distribution. Nelson (1991) employs a GED distribution for asset returns to capture the same property of data, arguing that the use of standardized Student‘s t distribution may deliver non-finite unconditional moments of the resulting distribution. One major problem with these distributions is that the misspecification of the density may lead to inconsistent estimates. The common practice is to use Quasi-maximum Likelihood (QML) estimators: if both conditional mean and variance equations are correctly specified one may assume Gaussian distribution of errors and receive consistent estimates of the parameters, sacrificing efficiency (Fan et al., 2014). However, both standardized Student‘s t and GED are not suitable for QML. Newey and Steigerwald (1997) discuss properties of non-Gaussian QML estimators and show how the identification condition may be satisfied when the true density is unimodal and symmetric around zero.

Although the wide range of univariate models discussed so far is relevant for the description of volatility dynamics of single assets, they are silent about the interdependence of assets. To address the central question of this thesis one needs to model the behavior of return series as a group, evaluating the strength of possible bidirectional relationships between them. The univariate GARCH-type models may serve as a building block for more general multivariate GARCH (MGARCH) models that are intended to work in the case when the number of considered assets is greater than 1. The general formulation of

[3] For example, in the GARCH(p,q) model m=max(p,q).

the setting may be summarized in the following form:

$$\begin{aligned} \boldsymbol{r_t} &= \boldsymbol{\mu_t} + \boldsymbol{\varepsilon_t} \\ \boldsymbol{\varepsilon_t} &= \boldsymbol{H_t^{1/2}} \boldsymbol{\epsilon_t} \end{aligned} \tag{9}$$

where bold $\boldsymbol{r_t}$, $\boldsymbol{\mu_t}$ are now $(n \times 1)$ vectors or returns and expected values of returns, respectively; $\boldsymbol{\epsilon_t}$ is an $(n \times 1)$ i.i.d. vector of innovations with zero mean and unity covariance matrix $\boldsymbol{I_n}$; $\boldsymbol{H_t}$ is an $(n \times n)$ conditional covariance matrix of $\boldsymbol{r_t}$ that obeys some structure defined by an MGARCH model. It is important for this matrix to be positive definite and symmetric since it represents the covariance structure of the returns.

The most straightforward MGARCH model called VEC(p,q)[4] was introduced by Bollerslev et al. (1988). The authors proposed to model $\boldsymbol{H_t}$ in the following form:

$$vech(\boldsymbol{H_t}) = \boldsymbol{c} + \sum_{i=1}^{q} \boldsymbol{A_i} vech(\boldsymbol{\varepsilon_{t-i}} \boldsymbol{\varepsilon'_{t-i}}) + \sum_{i=1}^{p} \boldsymbol{B_i} vech(\boldsymbol{H_{t-i}}) \tag{10}$$

where $\boldsymbol{c}$ is an $(\frac{n(n+1)}{2} \times 1)$ vector of constant parameters; $\boldsymbol{A_i}$ and $\boldsymbol{B_i}$ are $(\frac{n(n+1)}{2} \times \frac{n(n+1)}{2})$ matrices of parameters; the operator vech() stacks the lower triangular portion of a square matrix into one column vector[5]. The structure is similar to the GARCH(p,q) model. However, the important difference is that volatilities of assets depend not only on their own lags of volatilities and shocks but also on these values of other considered assets and all their combinations. The generality and flexibility of this model comes at a cost as the number of parameters to estimate is equal to $\frac{n(n+1)}{2} + (p+q)(\frac{n(n+1)}{2})^2$ and increases significantly with the number of assets n (Bauwens et al., 2006). Moreover, the generated $\boldsymbol{H_t}$ is likely to be not positive definite at least for some periods t. To make it positive definite one needs restrictive assumptions, which are hard to justify. For example, the assumption that parameter matrices $\boldsymbol{A_i}$ and $\boldsymbol{B_i}$ are diagonal delivers a simplified diagonal VEC (DVEC) model with significantly lower number of parameters $(p+q+1)\frac{n(n+1)}{2}$ for which the conditions for positive definiteness of $\boldsymbol{H_t}$ may be obtained. However, the model is restrictive in the sense that it prohibits interactions between assets (due to its diagonal structure) that are desirable for an MGARCH model. To overcome this weakness while forcing $\boldsymbol{H_t}$ to be positive definite one may use Baba, Engle, Kraft and

[4]The name comes from the column-stacking operator and should not be confused with the Vector Error Correction model.

[5]To understand the operator consider the following example with (3×3) matrix: if X= $\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$ then vech(X)= $\begin{pmatrix} x_{11} \\ x_{21} \\ x_{31} \\ x_{22} \\ x_{32} \\ x_{33} \end{pmatrix}$ is a (6×1) vector.

Kroner (BEKK(p,q,K)) model proposed by Engle and Kroner (1995) with the following dynamics of $\boldsymbol{H_t}$:

$$\boldsymbol{H_t} = \boldsymbol{CC'} + \sum_{i=1}^{q}\sum_{k=1}^{K} \boldsymbol{A'_{ki}} \boldsymbol{\varepsilon_{t-i}} \boldsymbol{\varepsilon'_{t-i}} \boldsymbol{A_{ki}} + \sum_{i=1}^{p}\sum_{k=1}^{K} \boldsymbol{B'_{ki}} \boldsymbol{H_{t-i}} \boldsymbol{B_{ki}} \tag{11}$$

with $\boldsymbol{A_{ki}}$,$\boldsymbol{B_{ki}}$, and lower triangular $\boldsymbol{C}$ $(n \times n)$ matrices of parameters. It may be shown that every BEKK model is a restricted version of a VEC model with the parameter K guiding the degree of generality (the increase of K drives the BEKK closer to an unrestricted VEC counterpart). The decreased number of parameters equals to $(p + q)Kn^2 + \frac{n(n+1)}{2}$, which is less than in VEC but higher than in the DVEC model (Bauwens et al., 2006). Identification and estimation difficulties in practical applications force one to set K equal to 1. In this model $\boldsymbol{H_t}$ is positive definite by construction and the interactions between assets are not prohibited. This creates an ability to investigate the existence of spillover effects between assets. Using a BEKK(1,1,1) model with $n = 3$ assets Yu et al. (2020) measure volatility spillovers between oil and stock markets. The model allows authors to identify changes in the direction of spillover effects in response to major political events considered to represent structural breaks in the relationship between markets.

The direct modeling of the dynamics of $\boldsymbol{H_t}$ is not the only way to proceed in multivariate volatility modeling. A completely different class of MGARCH models that concentrates on conditional correlations was introduced by Bollerslev (1990). The Constant Conditional Correlation (CCC) model expresses the conditional covariance matrix as a product of conditional standard deviations and time-invariant conditional correlations:

$$\begin{aligned} \boldsymbol{H_t} &= \boldsymbol{D_t R D_t} = (\rho_{ij}\sqrt{\sigma_{iit}\sigma_{jjt}}) \\ \boldsymbol{D_t} &= diag(\sigma_{11t}, ..., \sigma_{nnt}) \end{aligned} \tag{12}$$

with $\boldsymbol{R}$ being a $(n \times n)$ symmetric positive definite matrix of constant correlations of asset returns ρ_{ij} with elements on the main diagonal equal to 1; $\boldsymbol{D_t}$ is a $(n \times n)$ diagonal matrix of standard deviations σ_{iit}. The attractive feature of the model is that standard deviations for each asset may be modeled separately using different univariate GARCH models. In the original paper GARCH(1,1) is employed, but the choice may cover other extensions discussed previously. $\boldsymbol{H_t}$ is ensured to be positive definite provided that conditional variances are positive and $\boldsymbol{R}$ is positive definite. Another advantage of the model over the VEC and BEKK alternatives is the significant reduction of the number of parameters to estimate. For example, with the GARCH(1,1) model for each asset this number is equal to $\frac{n(n+5)}{2}$ (Bauwens et al., 2006). Moreover, the estimation of the parameters is simplified because of the stated covariance matrix decomposition.

The most important disadvantage of the CCC is the restriction on conditional correlations to be constant over time: a property which is highly unlikely to lead to a correct description of the underlying relationships between assets. To overcome this problem Engle and Sheppard (2001) and Engle (2002) proposed a Dynamic Conditional Correlation (DCC(p,q)) model that relaxes the assumption and allows the conditional correlation matrix to be time-varying, giving rise to the following decomposition extended by an additional equation for the dynamics of the structure of conditional correlations:

$$\begin{aligned}
\boldsymbol{H_t} &= \boldsymbol{D_t R_t D_t} \\
\boldsymbol{D_t} &= diag(\sigma_{11t}, ..., \sigma_{nnt}) \\
\boldsymbol{R_t} &= \boldsymbol{Q_t^{-1*} Q_t Q_t^{-1*}} \\
\boldsymbol{Q_t} &= (1 - \sum_{i=1}^{p} \alpha_i^{DCC} - \sum_{i=1}^{q} \beta_i^{DCC})\boldsymbol{\bar{Q}} + \sum_{i=1}^{p} \alpha_i^{DCC} (\boldsymbol{\epsilon_{t-i}\epsilon'_{t-i}}) + \sum_{i=1}^{q} \beta_i^{DCC} \boldsymbol{Q_{t-i}}
\end{aligned} \tag{13}$$

where $\boldsymbol{Q_t}$ is an $(n \times n)$ symmetric positive definite conditional covariance matrix of standardized residuals $\boldsymbol{\epsilon_t}$; the main diagonal of $(n \times n)$ $\boldsymbol{Q_t^*}$ contains square roots of elements from the main diagonal of $\boldsymbol{Q_t}$ and zeroes everywhere else; $\boldsymbol{\bar{Q}}$ is an $(n \times n)$ unconditional covariance matrix of $\boldsymbol{\epsilon_t}$; α_i^{DCC} and β_i^{DCC} are positive scalar parameters. As is common with GARCH-type modeling, in empirical applications the usual choice of the specification is the most parsimonious $p = q = 1$ version of the model (Engle and Sheppard, 2001), for which the condition $\alpha^{DCC} + \beta^{DCC} < 1$ is important to ensure that $\boldsymbol{Q_t}$ is appropriately defined. Dajcman et al. (2012) employ the model in the assessment of comovements of major European stock markets. The authors show how one may describe the influence of crisis events using changes in conditional correlations. In the same framework, Asaturov et al. (2015) investigates the influence of Polish and Russian markets on other EE markets. The time variation of conditional correlations helps to describe differences in comovements of prices during tranquil and crisis periods.

It is clear that scalar parameters may represent only common dynamic patterns among all assets. A way to generalize the DCC model to capture differences in dynamics of $\boldsymbol{Q_t}$ was introduced by Cappiello et al. (2006). The Asymmetric Generalized DCC (AG-DCC) model departs from the DCC specification of $\boldsymbol{Q_t}$ dynamics by using parameter matrices instead of scalar parameters. Moreover, the equation is augmented by the part which captures asymmetry effects of past shock in the multivariate setting:

$$\boldsymbol{Q_t} = (\boldsymbol{\bar{Q}} - \boldsymbol{A'\bar{Q}A} - \boldsymbol{B'\bar{Q}B} - \boldsymbol{G'\bar{Q}^{-}G}) + \boldsymbol{A'}(\boldsymbol{\epsilon_{t-1}\epsilon'_{t-1}})\boldsymbol{A} + \boldsymbol{B'Q_{t-1}B} + \boldsymbol{G'}(\boldsymbol{\epsilon^{-}_{t-1}\epsilon^{-}_{t-1}}')\boldsymbol{G} \tag{14}$$

with $(n \times n)$ parameter matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{G}$; $\boldsymbol{\epsilon_{t-1}^{-}} = \boldsymbol{I_{t-1}} \odot \boldsymbol{\epsilon_{t-1}}$, where $\boldsymbol{I_{t-1}}$ is a vector of indicator functions with entries I_{it-1} equal to 1 if $\epsilon_{it-1} < 0$ and 0 otherwise[6]; $\boldsymbol{\bar{Q}^{-}}$ being a counterpart of unconditional covariance matrix of residuals $\boldsymbol{\bar{Q}}$ that is estimated using $\boldsymbol{\epsilon_{t-1}^{-}}$ instead of $\boldsymbol{\epsilon_{t-1}}$. The evident flexibility from using parameter matrices comes at a cost because one needs to estimate the expanded set of parameters. The AG-DCC model possesses a reduced scalar form like the DCC model in Equation 13, which may be of use in empirical applications. One may prefer this specification over scalar DCC when the modeling of leverage effects is suspected to be important for the correct representation of the multivariate distribution. For example, Gjika and Horvath (2013) use scalar version of asymmetric DCC model to describe comovements of Central European stock markets and provide evidence on mild asymmetric effects in the $\boldsymbol{Q_t}$ dynamics.

The estimation of the parameters of the DCC model requires a two-step procedure: in the first step, the parameters of the specified univariate GARCH models are estimated using QML under the assumption of Gaussian innovations to ensure the consistency of the estimates in the case of a possible misspecification of the density; the estimate of $\boldsymbol{\bar{Q}}$ is obtained from standardized residuals $\epsilon_{it} = \frac{\varepsilon_{it}}{\sigma_{it}}$; in the second step, the information from the first step is used to estimate the remaining DCC parameters via ML under the likelihood which depends on the specified multivariate distribution (Bauwens et al., 2006). As in the univariate case, the choice of the distribution is affected by the properties of the data: if one wants to capture heavy tails of the joint distribution of returns the multivariate Student‘s t distribution should be employed instead of a more simple multivariate normal alternative. In this case, the first step quasi-log-likelihood for the estimation of the set of parameters of univariate GARCH-type models θ_1 will take the following form:

$$LL_{1^{st}step}(\theta_1) = \sum_{i=1}^{n}(const - \frac{1}{2}\sum_{t=1}^{T}(ln(\sigma_{it}) + \frac{\varepsilon_{it}^2}{\sigma_{it}})) \tag{15}$$

The second step log-likelihood for the estimation of the remaining DCC parameters $\theta_2 = \{\alpha^{DCC}, \beta^{DCC}, \nu^{DCC}\}$ using the multivariate Student‘s t distribution taking $\hat{\theta_1}$ as given will reduce to:

$$\begin{aligned} LL_{2^{nd}step}(\theta_2|\theta_1) = \sum_{t=1}^{T}\Big(& ln(\Gamma(\frac{\nu+n}{2})) - ln(\Gamma(\frac{\nu}{2})) - \frac{n}{2}ln(\pi(\nu-2)) \\ & - \frac{1}{2}ln(|\boldsymbol{R_t}|) - \frac{\nu+n}{2}ln(1 + \frac{\boldsymbol{\epsilon_t' R_t^{-1} \epsilon_t}}{(\nu-2)})\Big) \end{aligned} \tag{16}$$

[6] $\odot$ is a Hadamard or element-wise product. One can illustrate the operator by a simple example: $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \odot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 * b_1 \\ a_2 * b_2 \end{pmatrix}$ that can be easily generalized for any matrices with the same dimensions.

with $\Gamma()$ representing the Gamma function and $|\boldsymbol{A}|$ denoting the determinant of $\boldsymbol{A}$.

2.2 VAR approach

Another approach to modeling the dynamic relationship between financial time series is available with the use of vector autoregressive (VAR) models. Sims (1980) introduced the model as a new atheoretical approach to study the dynamic relationship between multivariate time series. Following the notation of Hamilton (1994) the standard VAR(p) model for n time series may be represented by the following equation:

$$\boldsymbol{y_t} = \boldsymbol{c} + \sum_{i=1}^{p} \boldsymbol{\Phi_i y_{t-i}} + \boldsymbol{\varepsilon_t} \tag{17}$$

or equivalently using lag polynomials[7]:

$$\boldsymbol{\Phi}(L)\boldsymbol{y_t} = \mathbf{c} + \boldsymbol{\varepsilon_t} \tag{18}$$

where bold $\boldsymbol{y_t}$, $\boldsymbol{c}$ are $(n \times 1)$ vectors, $\boldsymbol{\Phi_i}$ are $(n \times n)$ matrices, $\boldsymbol{\Phi}(L) = \boldsymbol{I_n} - \sum_{i=1}^{p} \boldsymbol{\Phi_i L^i}$, and $(n \times 1)$ vector of innovations is i.i.d. $\boldsymbol{\varepsilon_t} \sim \mathcal{N}(0, \boldsymbol{\Omega})$. The vector $\boldsymbol{y_t}$ may contain either returns of different assets or their measures of volatility, for example range-based volatilities of assets. For this model to be covariance-stationary all roots z of the equation involving a determinant operator $|\boldsymbol{I_n} - \sum_{i=1}^{p} \boldsymbol{\Phi_i z^i}| = 0$ must lie outside the unit circle. The condition implies that effects of shocks $\boldsymbol{\varepsilon_t}$ decrease over time and eventually disappear entirely. If this is the case, then the VAR(p) model possesses a vector MA(∞) representation of the following form:

$$\boldsymbol{y_t} = \boldsymbol{\mu} + \boldsymbol{\Psi(L)\varepsilon_t} \tag{19}$$

where $\boldsymbol{\mu} = [\boldsymbol{I_n} - \sum_{i=1}^{p} \boldsymbol{\Phi}_i]^{-1}\mathbf{c}$. The elements of $\boldsymbol{\Psi_s}$ matrices for all s may be calculated by solving $\boldsymbol{\Psi}(L) = [\boldsymbol{\Phi}(L)]^{-1}$ or simulating the behavior of the system[8]. While the direct interpretation of elements of $\boldsymbol{\Phi_i}$ is not available and is complicated because of the number of parameters estimated even for small values of n and p, the MA representation provides a way to understand the relationship between series. For example, in the absence of all other innovations, an element of $\boldsymbol{\Psi_s}$ on the intersection of the ith row and jth column

[7]The lag operator produces the lagged value of a time series $Ly_t = y_{t-1}$. In general, the following property holds: $L^i y_t = y_{t-i}$ for non-negative integers i.

[8]To be more precise, the following recursive relationship holds: $\boldsymbol{\Psi_i} = \boldsymbol{\Phi_1 \Psi_{i-1}} + \boldsymbol{\Phi_2 \Psi_{i-2}} + \boldsymbol{\ldots} + \boldsymbol{\Phi_p \Psi_{i-p}}$ for all positive i, $\boldsymbol{\Psi_0} = \boldsymbol{I_n}$, and $\boldsymbol{\Psi_i} = 0$ for all negative i (Pesaran and Shin, 1998).

shows the effect of a unit increase in a jth variable's shock at time t ε_{jt} on the ith variable at period $t+s$ $y_{i,t+s}$ (Hamilton, 1994). Based on this notion, one could construct impulse response functions (IRFs) plotting the appropriate elements of $\boldsymbol{\Psi_s}$ against different time horizons s to summarize the effects of different shocks to the system on the evolution of variables of interest.

The vector MA(∞) representation and IRFs help to study the structure of the error variance for the s periods ahead forecast. The error of the forecast of $\boldsymbol{y_{t+s}}$ may be written as:

$$\boldsymbol{y_{t+s}} - \boldsymbol{\hat{y}_{t+s|t}} = \sum_{i=0}^{s-1} \boldsymbol{\Psi}_i \boldsymbol{\varepsilon_{t+s-i}} \tag{20}$$

The variance of this error may be obtained by squaring the error and taking the expectation. Since the shocks $\boldsymbol{\varepsilon_t}$ may be contemporaneously correlated according to the structure summarized by $\boldsymbol{\Omega}$, additional transformations are required to proceed with the calculation and receive the convenient simplification of the variance of the sum. The original identification proposal of Sims (1980) was to orthogonalize the shocks to the system by using the Cholesky decomposition[9] of the covariance matrix $\boldsymbol{\Omega} = \boldsymbol{PP'}$ and pre-multiply $\boldsymbol{\varepsilon_t}$ with $\boldsymbol{P}^{-1}$ to receive orthogonalized shocks $\boldsymbol{u_t} = \boldsymbol{P}^{-1}\boldsymbol{\varepsilon_t}$ with covariance matrix $\boldsymbol{I_n}$ by construction. The transformation of the initial vector MA(∞) representation leads to the following equations that contain orthogonalized innovations:

$$\boldsymbol{y_t} = \boldsymbol{\tilde{\mu}} + \boldsymbol{A(L)u_t} \tag{21}$$

$$\boldsymbol{y_{t+s}} - \boldsymbol{\hat{y}_{t+s|t}} = \sum_{i=0}^{s-1} \boldsymbol{A}_i \boldsymbol{u_{t+s-i}} \tag{22}$$

where $\boldsymbol{A(L)} = \boldsymbol{\Psi(L)P}$ with $\boldsymbol{P}$ representing a part of the Cholesky decomposition of the covariance matrix $\boldsymbol{\Omega} = \boldsymbol{PP'}$ of non–orthogonalized shocks $\boldsymbol{\varepsilon_t}$ and appropriately transformed constant term $\boldsymbol{\tilde{\mu}}$. This representation leads to the following variance share of the ith variable s-step-ahead forecast error variance attributed to shocks to jth component of $\boldsymbol{y_t}$:

$$\tilde{\theta}_{ij}^{C}(s) = \frac{\sum_{h=0}^{s-1}(\boldsymbol{e_i'\Psi_s Pe_j})^{\boldsymbol{2}}}{\sum_{h=0}^{s-1}(\boldsymbol{e_i'\Psi_s\Omega\Psi_s'e_i})} = \frac{\sum_{h=0}^{s-1}(\boldsymbol{e_i'A_s e_j})^{\boldsymbol{2}}}{\sum_{h=0}^{s-1}(\boldsymbol{e_i'A_sA_s'e_i})} \tag{23}$$

[9] Symmetric positive-definite matrix $\boldsymbol{A}$ may be represented as $\boldsymbol{A} = \boldsymbol{LL'}$, where $\boldsymbol{L}$ is a lower triangular matrix with positive entries on the main diagonal and $\boldsymbol{L'}$ is a transpose of $\boldsymbol{L}$.

with $\boldsymbol{e_i}$ being a selection vector $(n \times 1)$ that contains zeros everywhere except unity on the ith position. One potential problem of the decomposition presented is that it depends on the predetermined order of variables in $\boldsymbol{y_t}$ used in the Cholesky identification. An alternative approach, to avoid the sensitivity of impulse responses and forecast error variance decompositions to the ordering of variables in a VAR model, was proposed by Koop et al. (1996). The authors defined the Generalized Impulse Response Function (GIRF) for a horizon s as:

$$\boldsymbol{GIRF}(s, \boldsymbol{\delta}, \boldsymbol{I_{t-1}}) = E(\boldsymbol{y_{t+s}} | \boldsymbol{\varepsilon_t} = \boldsymbol{\delta}, \boldsymbol{I_{t-1}}) - E(\boldsymbol{y_{t+s}} | \boldsymbol{I_{t-1}}) \tag{24}$$

with $\boldsymbol{I_{t-1}}$ representing all available history up to time t-1 and shock to the system $\boldsymbol{\delta}$. Instead of orthogonalization of shocks the composition of $\boldsymbol{\delta}$ is chosen such that only one element of the vector (say jth variable shock) is not equal to zero. Note that in case of the VAR(p) the GIRF will be history-invariant $\boldsymbol{GIRF}(s, \boldsymbol{\delta}, \boldsymbol{I_{t-1}}) = \boldsymbol{A_s \delta}$. Then the assumption of multivariate normality of $\boldsymbol{\varepsilon}_t$ is used to integrate out the influences of correlated shocks using the historical distribution:

$$E(\boldsymbol{\varepsilon_t} | \varepsilon_{jt} = \delta_j) = \frac{\boldsymbol{\Omega} e_j \delta_j}{\sigma_{jj}} \tag{25}$$

with σ_{jj} denoting the square root of the variance of errors from the jth equation of the underlying VAR model.

Based on the GIRF approach, Pesaran and Shin (1998) showed that the variance share of the ith variable s-step-ahead forecast error variance attributed to shocks to jth component of $\boldsymbol{y_t}$ is equal to:

$$\tilde{\theta}_{ij}^{G}(s) = \frac{\sum_{h=0}^{s-1} (\boldsymbol{e_i' \Psi_s e_j})^{\boldsymbol{2}}}{\sigma_{ii} \sum_{h=0}^{s-1} (\boldsymbol{e_i' \Psi_s \Omega \Psi_s' e_i})} \tag{26}$$

It is worth noting that the proposed decomposition does not depend on the ordering of variables in $\boldsymbol{y_t}$. However, one shortcoming of this approach is that while $\sum_{j=1}^{n} \tilde{\theta}_{ij}^{C}(s) = 1$ the same result does not usually hold for $\tilde{\theta}_{ij}^{G}(s)$.

The seminal paper by Diebold and Yılmaz (2009b) (hereafter the pair of authors is referred to as DY) introduced a framework for studying the connectedness of financial assets. The authors looked separately at the return and volatility connectedness of global equity markets. The main idea of the framework is to use a VAR model for returns or volatilities of assets and respective forecast error variance decompositions to construct so-called *spillover tables*. The decomposition shows how the variance of the error of the forecast of y_{t+s} is affected by shocks from different entities of y_t. Thus, it shows the dynamic connectedness of elements of y_t. If we fix the forecast horizon s and denote $d_{ij}(s)$ to be the variance share of the s-step-ahead forecast error variance of the ith variable that is explained by the shocks to the jth variable, then the spillover table for n financial assets is going to have the structure presented in Table 1. It is immediately evident that *TO others* measures represent the sum of variance shares that come from a market to all other markets disregarding the own variance share (column sum minus the element on the main diagonal) and *FROM others* measures represent the sum of variance shares that come from all other markets to a particular market disregarding the own variance share (row sum minus the element on the main diagonal).

Table 1: General form of a spillover table.

	x_1	x_2	x_3	$\cdots$	x_n	FROM others
x_1	d_{11}	d_{12}	d_{13}	$\cdots$	d_{1n}	$\sum_{j=1,j\neq 1}^{n} d_{1j}$
x_2	d_{21}	d_{22}	d_{23}	$\cdots$	d_{2n}	$\sum_{j=1,j\neq 2}^{n} d_{2j}$
x_3	d_{31}	d_{32}	d_{33}	$\cdots$	d_{3n}	$\sum_{j=1,j\neq 3}^{n} d_{3j}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_n	d_{n1}	d_{n2}	d_{n3}	$\cdots$	d_{nn}	$\sum_{j=1,j\neq n}^{n} d_{nj}$
TO others	$\sum_{i=1,i\neq 1}^{n} d_{i1}$	$\sum_{i=1,i\neq 2}^{n} d_{i2}$	$\sum_{i=1,i\neq 3}^{n} d_{i3}$	$\cdots$	$\sum_{i=1,i\neq n}^{n} d_{in}$	$\boldsymbol{\frac{1}{n}\sum_{i=1,i\neq j}^{n} d_{ij}}$

The bold corner element in the last row of the table is called the *Total spillover index* and represents an aggregated measure of connectedness for the entire set of n assets. It shows the spillover effects from shocks across all assets to the total forecast error variance, summarizing the influence of shocks of each asset i on all other assets disregarding asset i. The total spillover index is calculated as a sum of non-diagonal elements of a spillover table (or, equivalently, as a mean of FROM measures). Given a fixed forecasting horizon s the total spillover index takes the following form (may be presented as a share or as a percentage after multiplication by 100):

$$\boldsymbol{S}(s) = \frac{1}{n} \frac{\sum_{i,j=1,i\neq j}^{n} d_{ij}(s)}{\sum_{i,j=1}^{n} d_{ij}(s)} \tag{27}$$

The identification strategy one uses to plug in for $d_{ij}(s)$ is crucially important. Initially, Diebold and Yılmaz (2009b) used the Cholesky decomposition and plugged in $\tilde{\theta}_{ij}^{C}(s)$ from Equation 23. The authors showed that the TO and FROM measures naturally depend on the ordering of variables and vary significantly from one order to the other. Thus, the directional dependence (TO and FROM measures, for example) may not be identified properly. However, the total spillover index is more robust to the ordering of variables and varies less as a result of different reorderings of variables. The results showed that the static total spillover indices for returns and volatilities of global equity markets are similar for the period from 1995 to 2007 and close to 37.5 %. However, return and volatility connectedness have significantly different dynamics. To compare them, the authors used the rolling window estimation procedure and constructed the series of dynamic total spillover indices using a 75-week rolling window as a sample for the VAR estimation and construction of forecast error variance decompositions. They noted that volatility connectedness is more sensitive to crisis events and has more time variation than return connectedness. Diebold and Yılmaz (2009a) applied the same methodology in the analysis of connectedness between American markets and provided additional evidence on the robustness of the total connectedness measure to the ordering of variables when using the Cholesky decomposition.

Diebold and Yılmaz (2012) further improved the framework by using the generalized forecast error variance decompositions described previously. This change made it possible to identify directional connectedness between assets, since generalization overcomes the sensitivity of the results to the ordering of variables. Instead of using $\tilde{\theta}^C_{ij}(s)$ from Equation 23 to plug in for $d_{ij}(s)$ the authors utilized $\tilde{\theta}^G_{ij}(s)$ from Equation 26. In order not to lose the possibility of interpreting the results from the generalized decomposition as variance shares, the additional transformation to $\theta^G_{ij}(s)$ that ensures $\sum_{j=1}^n \theta^G_{ij}(s) = 1$ for all assets i by construction was performed:

$$\theta^G_{ij}(s) = \frac{\tilde{\theta}^G_{ij}(s)}{\sum_{j=1}^n \tilde{\theta}^G_{ij}(s)} \tag{28}$$

The authors summarized how each particular asset i affects all other assets (TO others), and how all other assets affect i (FROM others) using the following directional connectedness indices based on the TO others row and FROM others column of the spillover table:

$$\begin{aligned} S_{i,TO}(s) &= \frac{\sum_{j=1, i\neq j}^n \theta^G_{ij}(s)}{\sum_{j=1}^n \theta^G_{ij}(s)} \\ S_{i,FROM}(s) &= \frac{\sum_{j=1, j\neq i}^n \theta^G_{ij}(s)}{\sum_{j=1}^n \theta^G_{ij}(s)} \end{aligned} \tag{29}$$

The difference in these measures may be interpreted as an indicator of whether the asset i is a net transmitter of shocks ($S_{i,TO}(s) - S_{i,FROM}(s) > 0$) or a net receiver of shocks ($S_{i,TO}(s) - S_{i,FROM}(s) < 0$). The switch to directional connectedness measures allows one to identify the most important assets that are responsible for the observed level of total connectedness. The results of the paper suggest that the connectedness of four major asset classes, namely Stocks, Bonds, Commodities, and FX increased significantly as a result of the 2008 financial crisis. Among these assets, stock markets were relatively more influential in terms of generation of volatility spillovers.

Diebold and Yılmaz (2014) provided an additional methodological development of the connectedness framework by noting that spillover tables based on the generalized approach to decompositions may be interpreted as a structure of a weighted directed network of assets. This interpretation allows one to identify a correspondence between the proposed connectedness measures (total spillover index, TO and FROM indices) and typically accepted connectedness measures from the network theory. For example, the total spillover index may be interpreted as the mean degree of the network, which represents the average number of connections of all nodes. The correspondence creates a bridge between the DY framework and the vast body of network literature in economics. Diebold and Yılmaz (2015a) summarize the wide applicability and flexibility of the DY framework providing evidence on the use of spillover measures outside financial markets' topics. In sum, the approach is suitable for the estimation of the financial connectedness of specific groups of markets. It may be used to investigate the changes of connectedness in time and identify the influence of crisis events on it.

One additional thing regarding the DY framework is worth discussing: across all papers mentioned above the authors remain silent about uncertainty measures of their estimates of spillover effects. Connectedness measures represent non-linear combinations of the parameters of underlying VAR models, this significantly complicates the computation of their standard errors. If asymptotic approximations are hard or even impossible to derive in a closed-form one may use bootstrap techniques as an alternative strategy (Davison and Hinkley, 1997). A *correctly* executed bootstrap may provide approximations of higher quality than the asymptotic approach. The choice of the bootstrap algorithm depends on the context and differs between cross-section and time series settings. The convenient structure of the VAR model may be exploited to perform residual bootstrap and receive approximations of variability measures Berkowitz and Kilian (2000). Buse et al. (2019) proposed the following residual bootstrap procedure for DY connectedness measures:

1. For each replication[10] construct a series of bootstrap residuals drawing randomly with replacement the demeaned residuals of the VAR model from Equation 17 estimated using the entire sample.

2. With the help of a vector MA(∞) representation from Equation 19 recursively reconstruct a series of dependent variables and estimate the VAR model using this bootstrap sample.

[10]The number of replications should be large enough, for example, 1000.

3. Use the estimates to construct Table 1. A collection of tables from each bootstrap replication represents an approximate distribution of connectedness measure which can be exploited to approximate standard errors and construct confidence intervals.

The algorithm is computationally heavy, especially in cases when the number of series in the VAR model becomes large. Moreover, the combination of this algorithm with the rolling window estimation may be highly demanding in terms of time for computation. Nevertheless, this bootstrap approximation may be considered as a valid way to assess the variability of connectedness measures in the absence of valid asymptotic approximations.

3 Data

In the proceeding analysis, I examine three major Easter European stock market indices: Hungary (BUX), the Czech Republic (PX), and Poland (WIG). I also include the Russian (RTSI) index because its relation to the EE markets is likely to be relevant, given the size of the Russian market and geographical proximity. This group of markets is the main focus of my thesis. To account for the possible connections with other developed financial markets I include the US (S&P500, hereafter SP500), France (CAC-40, hereafter CAC), Germany (DAX), and Japan (Nikkei 225, hereafter Nikkei) into the analysis. I also represent cryptocurrency markets by a single, yet the most important (for example, in terms of capitalization) representative of these markets, namely, Bitcoin (BTC). Although Liu and Tsyvinski (2021) argue that Bitcoin can not represent the entire zoo of cryptocurrencies, they provide evidence suggesting that the behavior of Bitcoin closely follows that of a more general index of coins with a sophisticated time-varying structure. The information on close, open, minimum, and maximum prices was taken from the Investing.com[11] platform.

In the first part of my investigation, I work with correlations of returns and return spillovers. I define returns as the lag difference of natural logarithms of closing prices. Since the prices of indices are observed only on weekdays while the prices of cryptocurrencies are observed anytime I recalculate the log returns for BTC such that their return on Monday includes the returns on weekends. To clarify the adjustment consider a Monday and compare returns of the BUX and BTC:

$$\begin{aligned} r^{BUX}_{Monday} &= log(P^{BUX}_{Monday}) - log(P^{BUX}_{Friday}) \\ r^{BTC}_{Monday} &= log(P^{BTC}_{Monday}) - log(P^{BTC}_{Sunday}) \end{aligned} \tag{30}$$

For the BUX the return contains information from weekends, while for the BTC the return does not contain information from full weekends; thus, the returns should not be treated equally. However, the following adjustment of BTC returns makes Monday returns comparable because after the transformation both returns contain information from the same period of time:

$$\begin{aligned} \tilde{r}^{BTC}_{Monday} =& log(P^{BTC}_{Monday}) - log(P^{BTC}_{Sunday}) + \\ & log(P^{BTC}_{Sunday}) - log(P^{BTC}_{Saturday}) + \\ & log(P^{BTC}_{Saturday}) - log(P^{BTC}_{Friday}) = \\ & log(P^{BTC}_{Monday}) - log(P^{BTC}_{Friday}) \end{aligned} \tag{31}$$

[11]Link: `https://www.investing.com/`Investing.com website.

All other missed observations due to holidays or special occasions were omitted. The final sample for the analysis of returns consists of 2007 daily observations and covers the period from the 3rd of February 2012 to the 16th of June 2021. The series of returns are presented in Figure 1. Noticeable spikes in all series at the beginning of 2020 are connected to the WHO announcement about the recognition of the COVID-19 outbreak as a pandemic. Descriptive statistics for return series are presented in Table 2. The data on returns obeys common stylized facts about financial data. All series significantly depart from normality: they are positively skewed (have a negative skewness) and leptokurtic (which implies heavier tails of the distribution). Unit root tests strongly reject the null hypothesis of the existence of a unit root for all time series. These features are important because they shape the way one should perform the analysis using this data. It is worth noting that returns of all markets fluctuate in a range from -10% to 10%, while the returns of BTC differ in this regard and demonstrate greater volatility ranging from -40% to 40%.

In the second part of the empirical analysis I turn to volatility spillovers to construct alternative connectedness measures. I abstract from volatility filtering and instead construct range-based volatility measures using all available information on prices of assets. Parkinson (1980) suggested to combine highest and lowest observed daily prices and use the following optimal estimator for daily volatility:

$$\sigma_t^2 = 0.361(h_t - l_t)^2 \tag{32}$$

with l_t and h_t representing minimum and maximum log prices in period t, respectively. However, this estimate does not use the entire set of information usually available. Garman and Klass (1980) suggested to add close and open prices and use the following daily range-based volatility measure:

$$\sigma_t^2 = 0.511(h_t - l_t)^2 - 0.019[(c_t - o_t)(h_t + l_t - 2o_t) - 2(h_t - o_t)(l_t - o_t)] - 0.383(c_t - o_t)^2 \tag{33}$$

where c_t and o_t represent close and open log prices in period t, respectively. In this thesis, I use the latter approach for the estimation of volatility spillovers. In order to align BTC with market indices, I recalculate volatility on each Monday to include volatility realized during weekends by summing over the period of three days. The final sample for the analysis of volatility spillovers consists of 1626 daily observations and covers the period from the 12th of November 2013 to the 16th of June 2021. Constructed volatility series are presented in Figure 2. Summary statistics for range-based volatilities are collected in Table 3. For all series the consistent spike of the volatility around the start of the pandemic may be indicated. I find no evidence of the existence of unit roots in constructed series. It is evident that BTC stands out from the rest of the markets: both mean and median volatility is significantly higher for the cryptocurrency.

In the construction of volatility spillovers, I use the natural logarithm of range-based volatilities, which is a common transformation for volatility spillovers studies based on the DY framework. I delete 9 trading days for which the range-based volatility is equal to zero. The transformation is applied to make the distribution of volatilities less leptokurtic and to ensure that VAR models do not produce negative variance forecasts. Series of log range-based volatilities and their summary statistics are presented in Figure 3 and Table 4, respectively. For all series the null of the existence of a unit root is strongly rejected.

Table 2: Summary statistics of daily returns.

	BTC	**BUX**	**PX**	**WIG**	**RTSI**	**SP500**	**CAC**	**DAX**	**Nikkei**
Mean $\times 10^2$	0.30	0.04	0.02	0.03	0.00	0.07	0.05	0.05	0.05
Median $\times 10^2$	0.21	0.06	0.06	0.03	0.05	0.07	0.08	0.09	0.06
Maximum	0.38	0.06	0.07	0.06	0.13	0.09	0.08	0.10	0.08
Minimum	-0.48	-0.12	-0.08	-0.14	-0.14	-0.13	-0.13	-0.13	-0.08
Std.Dev.	0.05	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.01
Skewness	-0.65	-0.81	-0.83	-1.28	-0.68	-0.83	-0.78	-0.58	-0.25
Kurtosis	14.71	12.06	12.80	19.38	12.75	26.50	14.36	14.27	7.61
ADF	-10.69	-11.21	-12.37	-11.96	-11.27	-12.28	-12.49	-12.03	-12.81
Probability	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
Observations	2007	2007	2007	2007	2007	2007	2007	2007	2007

Note: Augmented Dickey—Fuller (ADF) unit-root tests are performed. <0.01 indicates the rejection of the null of a unit root at 1% level. Sample period is 03/02/2012 – 16/06/2021.

Table 3: Summary statistics of daily range-based volatilities.

	BTC	**BUX**	**PX**	**WIG**	**RTSI**	**SP500**	**CAC**	**DAX**	**Nikkei**
Mean $\times 10^2$	0.26	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.01
Median $\times 10^2$	0.07	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00
Maximum $\times 10^2$	21.46	0.75	0.48	0.26	2.90	0.26	0.38	0.44	0.30
Minimum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Std.Dev.	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Skewness	13.83	17.75	14.67	12.34	24.70	9.49	10.68	11.77	9.87
Kurtosis	277.60	447.14	271.38	216.32	759.14	118.22	154.78	188.14	139.02
ADF	-5.50	-6.90	-8.14	-8.00	-8.66	-7.22	-7.63	-7.98	-7.11
Probability	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
Observations	1626	1626	1626	1626	1626	1626	1626	1626	1626

Note: Augmented Dickey—Fuller (ADF) unit-root tests are performed. <0.01 indicates the rejection of the null of a unit root at 1% level. Sample period is 12/11/2012 – 16/06/2021.

Figure 1: Returns.

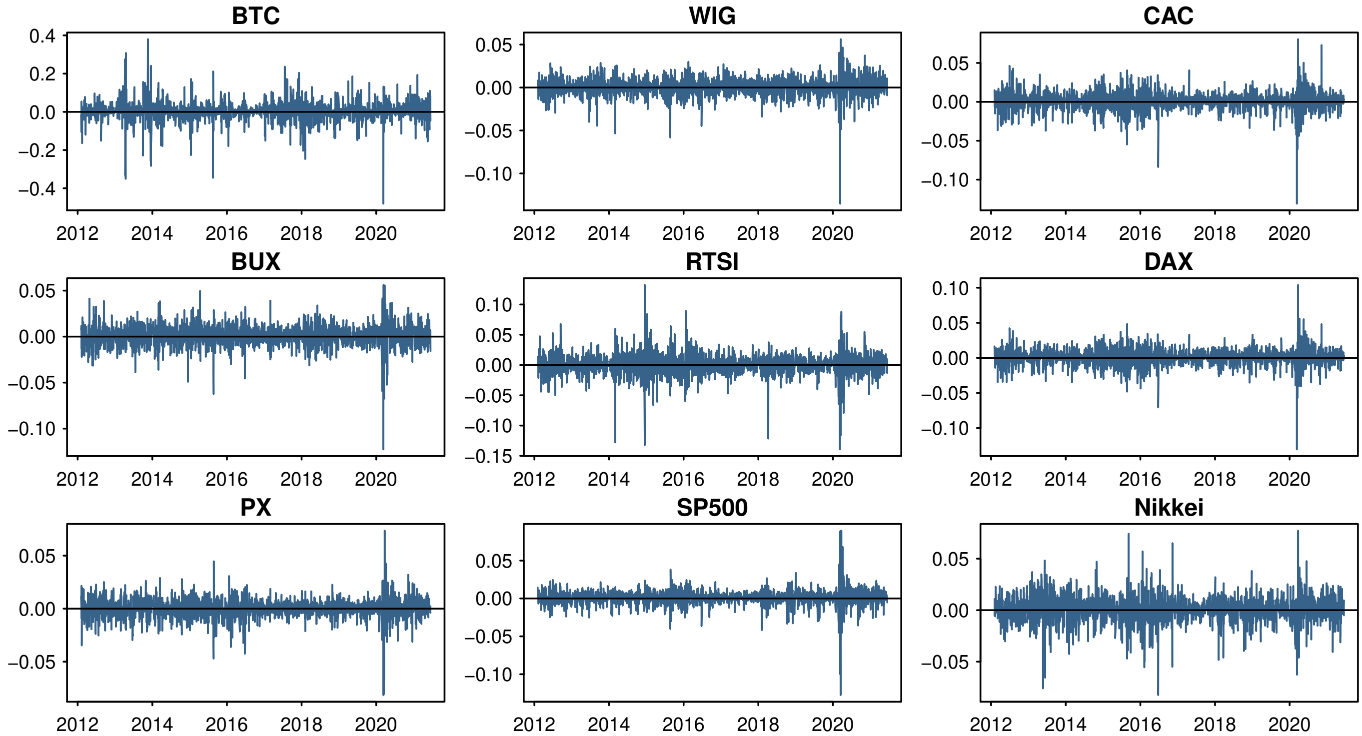


Figure 2: Range-based volatilities.

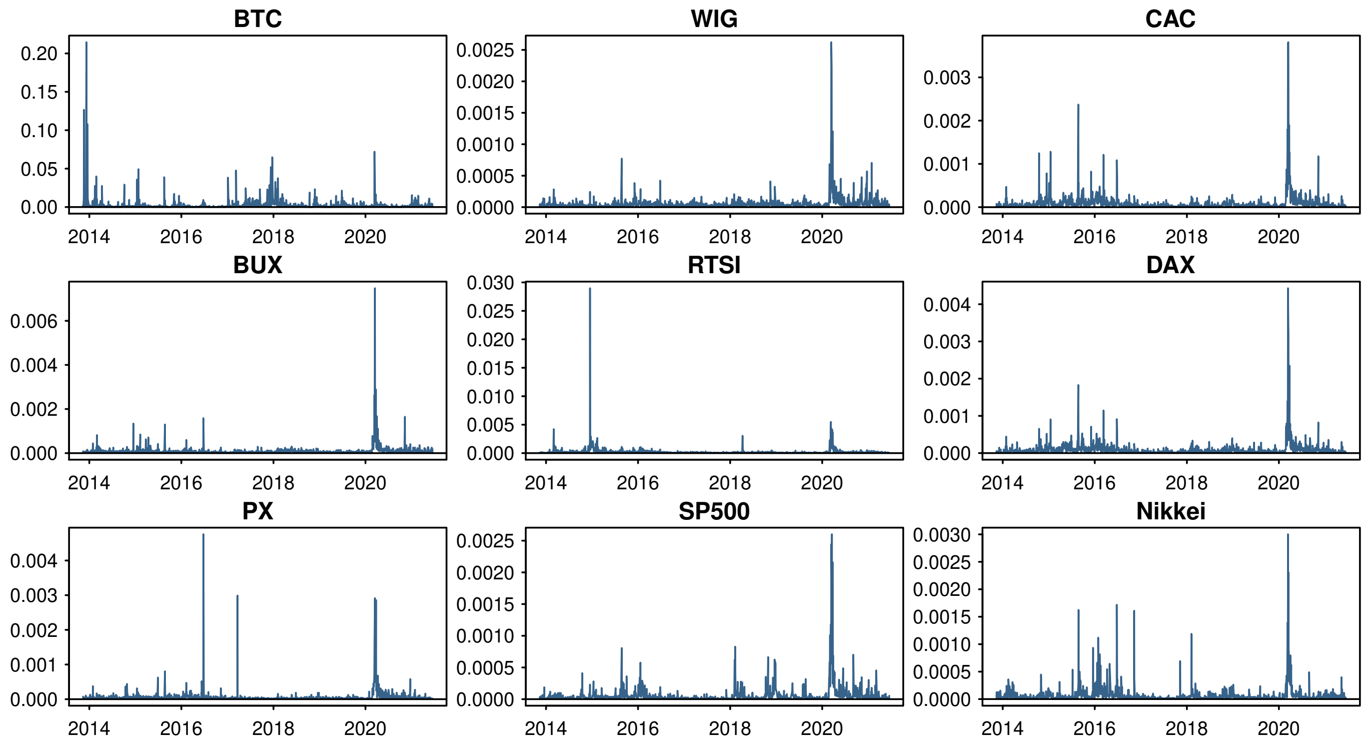
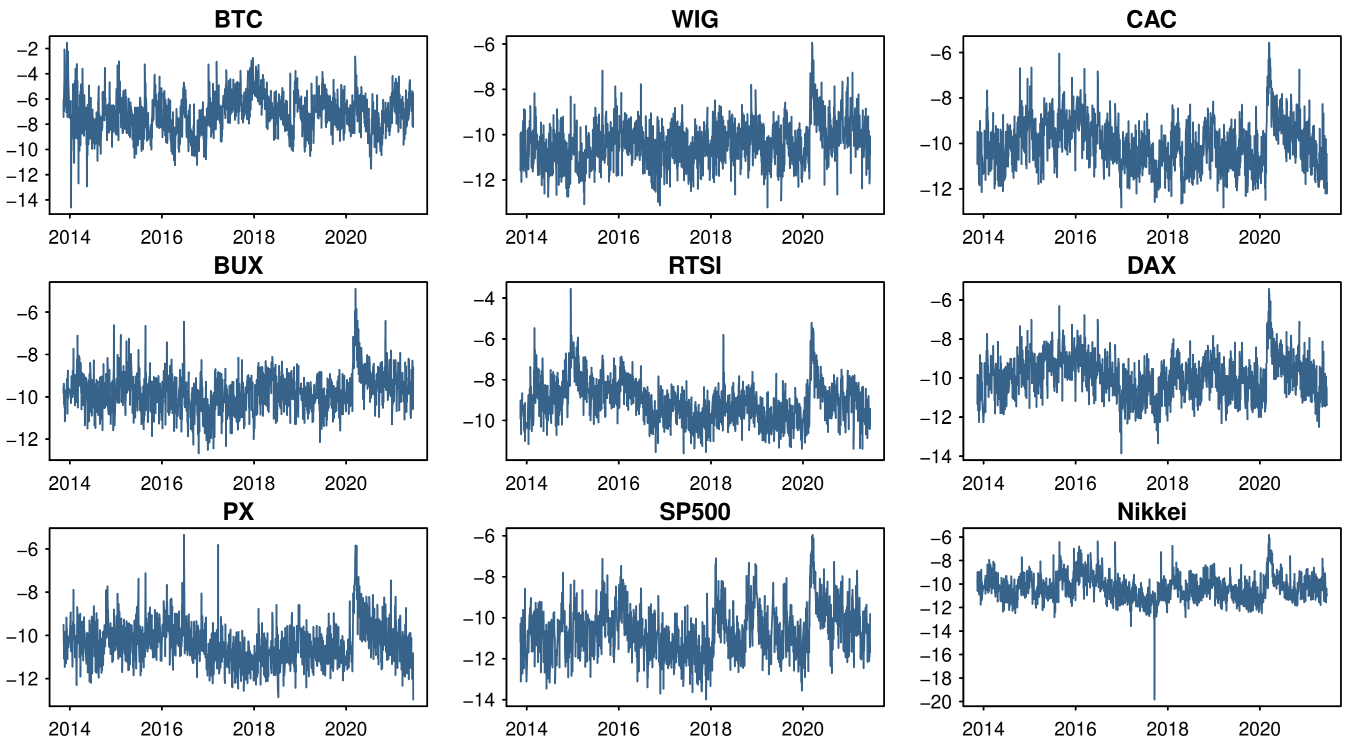


Table 4: Summary statistics of log daily range-based volatilities.

	BTC	BUX	PX	WIG	RTSI	SP500	CAC	DAX	Nikkei
Mean	-7.22	-9.77	-10.35	-10.35	-9.10	-10.64	-10.01	-9.92	-10.34
Median	-7.23	-9.80	-10.44	-10.37	-9.17	-10.71	-10.10	-9.94	-10.41
Maximum	-1.54	-4.90	-5.35	-5.94	-3.54	-5.95	-5.57	-5.42	-5.81
Minimum	-14.61	-12.68	-12.97	-13.22	-11.63	-13.99	-12.82	-13.87	-19.84
Std.Dev.	1.55	0.90	0.95	0.94	1.01	1.23	1.06	1.05	1.08
Skewness	0.03	0.50	0.81	0.39	0.66	0.44	0.41	0.27	0.13
Kurtosis	3.31	4.65	4.90	4.05	4.29	3.40	3.50	3.54	7.20
ADF	-6.34	-6.06	-5.69	-6.66	-4.93	-5.72	-5.37	-5.47	-5.26
Probability	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
Observations	1617	1617	1617	1617	1617	1617	1617	1617	1617

Note: Augmented Dickey—Fuller (ADF) unit-root tests are performed. <0.01 indicates the rejection of the null of a unit root at 1% level. Sample period is 12/11/2012 – 16/06/2021.

Figure 3: Log range-based volatilities.



4 Results

4.1 Dynamic conditional correlations of returns

To investigate how correlations of market returns evolve in time I estimate the DCC(1,1) model (Engle and Sheppard, 2001; Engle, 2002) described in Equation 13 using the two-step procedure[12]. For each asset i a univariate GJR-GARCH(1,1) of Glosten et al. (1993)

[12]To model leverage effects in the multivariate distribution I estimated scalar AG-DCC model but the parameter related to asymmetry turned out to be statistically insignificant.

from Equation 6 with AR(1) in mean and normally distributed errors is estimated using QML by maximizing the log-likelihood from Equation 15. For the SP500 and the Nikkei the GARCH(1,1) is used instead. These specifications aim to model the asymmetric influence of positive and negative shocks on future volatility when it is relevant and allow me to estimate consistently the parameters of univariate volatility models. In the second step, I use a multivariate Student‘s t distribution to match important facts about daily returns from Table 2, estimating the remaining DCC parameters by maximizing the log-likelihood from Equation 16. The results of the estimation are presented in Table 5. In the upper part of the table I report coefficient estimates and their corresponding standard errors. Each pair of rows contains information about the particular market. The lower part of the table contains DCC estimates and summarizes the results for the Q_t dynamics.

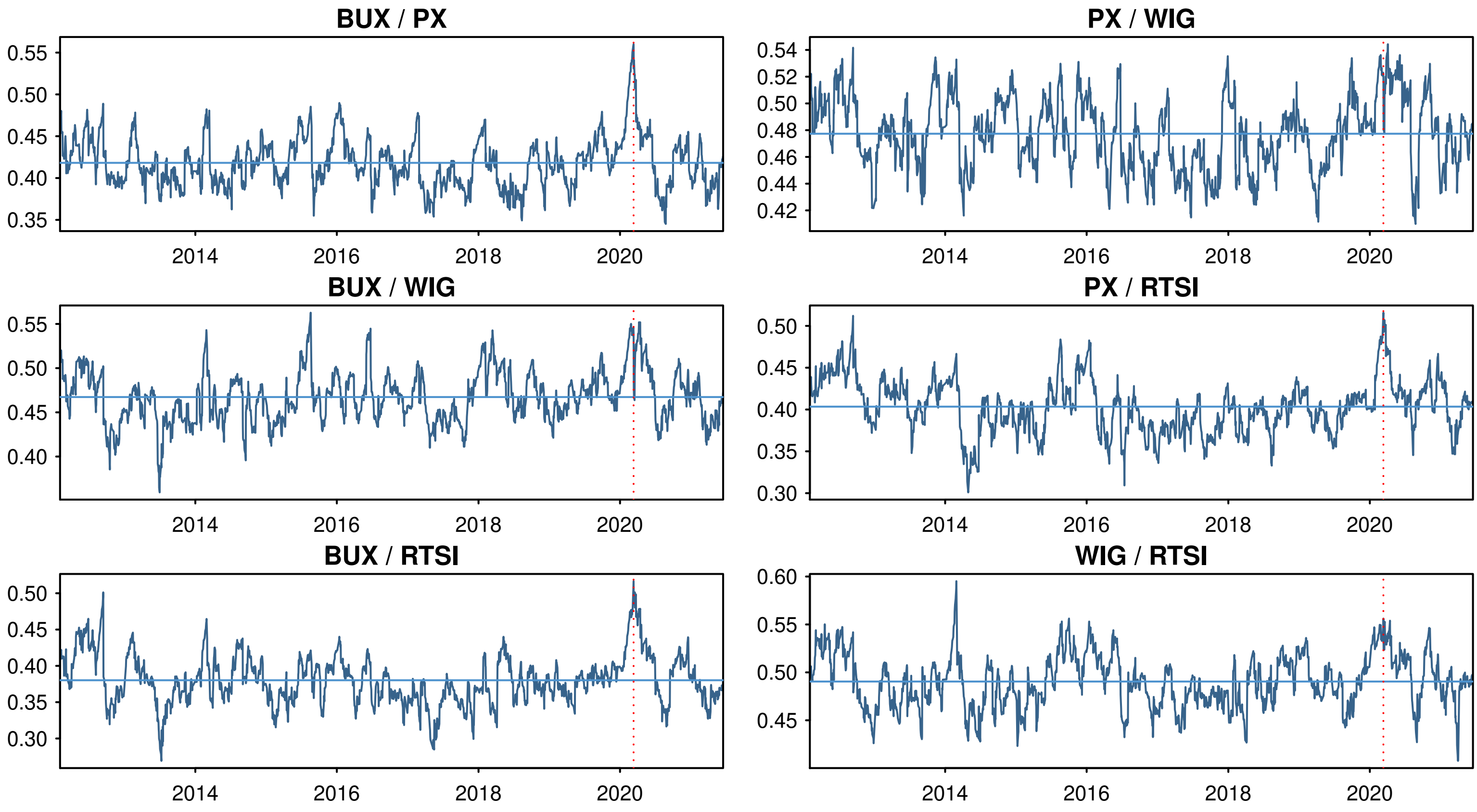
It is worth noting that for each asset the ρ coefficients in the mean equation are not statistically significant at the 1 % level. Moreover, for all assets except the BTC the γ parameter is statistically significant, suggesting the relevance of the asymmetric reactions of volatility to positive and negative shocks. Although the degrees of freedom parameter ν^{DCC} is low enough to claim that a normality assumption is not relevant for the joint distribution of these markets, it is high enough for moments to be appropriately defined. Both coefficients from the DCC equation are significantly different from zero, indicating that the assumption about dynamic and not constant conditional correlations is appropriate. I conclude that there is evidence in favor of a time-varying conditional correlation between the markets under consideration.

In all subsequent graphs the red line indicates the 11th of March 2020, when the WHO recognized the COVID-19 outbreak as a pandemic. This date is commonly used in the literature as a border between periods before and after the pandemic (e.g. Topcu and Gulal (2020); Aslam et al. (2021)). Although the pandemic started to affect different parts of the world disproportionately at different points in time, this date may approximately indicate the beginning of the pandemic, at least in an informational sense, when the majority of agents recognized the importance of a threat.

Figure 4 demonstrates pairwise dynamic conditional correlations between the Eastern European and Russian markets. The correlations between indices remain in the range from 0.25 to 0.6 and vary significantly during the period under consideration. There are no obvious positive or negative trends in the connectedness of markets. The WIG seems to be the most connected market since all pairwise correlations for it are close to 0.5 on average. The PX/WIG dynamic profile is the most volatile. For this pair, the increase of the correlation near the start of the pandemic does not lead to a level that was not observed in the past. For other pairs, the spike is more pronounced if one compares it with previously observed correlations. Although all markets become more correlated at this point, they generally return back to the unconditional average level after a short period of time. The connection of the PX with the RTSI in the 2014-2015 period dropped to the minimum level, while the responses of the BUX and WIG were less sharp. In sum, dynamic correlations indicate a significant increase in the connectedness of the EE and Russian markets during the pandemic. However, for some pairs, the effect is comparable to extreme movements in the past.

Dynamic conditional correlations of BTC with other markets are presented in Figure 5. On average, the correlations with the EE and Russian markets are positive but close to zero. The WIG and RTSI returns seem to be more correlated with BTC than the almost unrelated PX and BUX. The positive correlations with developed markets are slightly more pronounced but still quite low. Interestingly, the correlation of BTC with the Nikkei is mostly negative and was not significantly affected near the start of the pandemic. However, at the end of the sample, the correlation with the Nikkei increased and became positive. BTC did not demonstrate independence with all other markets: the correlation rapidly increased and achieved an all-time maximum but then drifted back towards zero. Judging by the dynamics of the conditional correlations, there is no persuasive evidence in favor of the connectedness of BTC with other markets.

Figure 4: Dynamic conditional correlations between EE and Russian markets.



Note: The red line indicates the 11th of March 2020 when WHO recognized COVID-19 outbreak as a pandemic. The blue horizontal line corresponds to the average correlation.

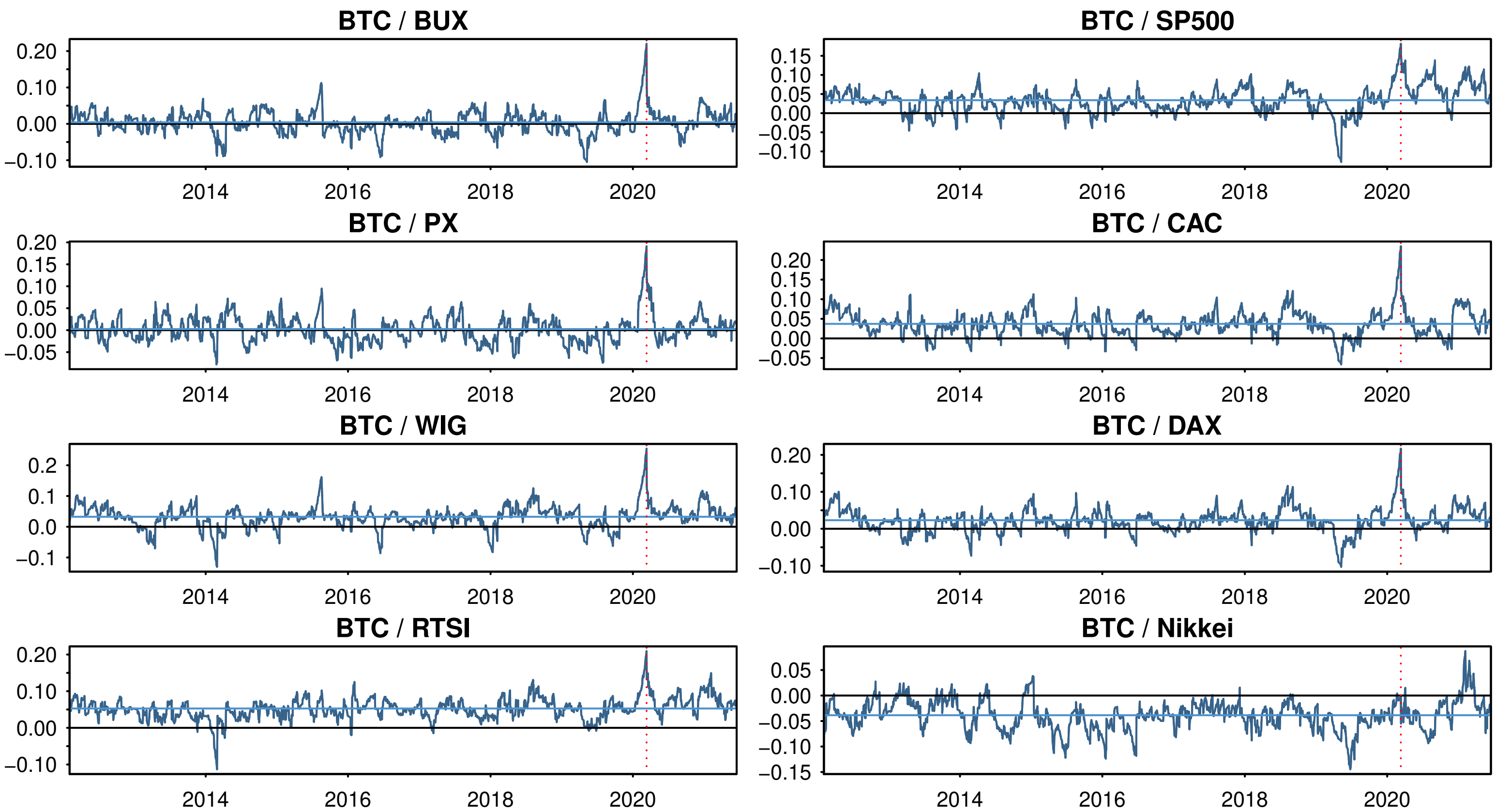
Table 5: DCC model estimates.

	Univariate GARCH estimates					
	$\mu \times 10^2$	ρ	$\omega \times 10^4$	α	β	γ
BTC	0.32	0.04	2.16	0.40	0.40	-0.17
	(0.08)	(0.03)	(0.73)	(0.10)	(0.07)	(0.10)
BUX	0.09	-0.02	0.04	0.14	0.89	-0.11
	(0.03)	(0.03)	(0.19)	(0.04)	(0.07)	(0.03)
PX	0.06	0.00	0.02	0.23	0.86	-0.20
	(0.02)	(0.02)	(0.02)	(0.04)	(0.03)	(0.04)
WIG	0.05	0.03	0.04	0.17	0.89	-0.18
	(0.02)	(0.02)	(0.00)	(0.02)	(0.01)	(0.03)
RTSI	0.08	0.03	0.06	0.18	0.89	-0.17
	(0.03)	(0.02)	(0.03)	(0.02)	(0.01)	(0.03)
SP500	0.09	-0.06	0.05	0.24	0.71	-
	(0.01)	(0.02)	(0.02)	(0.02)	(0.03)	-
CAC	0.13	0.02	0.04	0.25	0.88	-0.27
	(0.02)	(0.03)	(0.00)	(0.09)	(0.03)	(0.08)
DAX	0.11	0.01	0.03	0.18	0.91	-0.19
	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.03)
Nikkei	0.08	-0.03	0.07	0.14	0.82	-
	(0.02)	(0.03)	(0.03)	(0.02)	(0.02)	-

DCC estimates		
α^{DCC}	β^{DCC}	ν^{DCC}
0.010	0.938	11.501
(0.002)	(0.019)	(0.707)

Note: The first part of the table reports estimates and their standard errors of the univariate GJR-GARCH(1,1) model with AR(1) in mean and normally distributed errors for each asset except the SP500 and the Nikkei for which GARCH(1,1) is used instead. The second part of the table reports DCC(1,1) with multivariate Student‘s t distribution estimates and their standard errors.

Figure 5: Dynamic conditional correlations of BTC.



Note: The red line indicates the 11th of March 2020 when WHO recognized COVID-19 outbreak as a pandemic. The blue horizontal line corresponds to the average correlation.

4.2 Spillover framework

4.2.1 Return spillovers

In this section, I turn to the DY spillover framework (Diebold and Yılmaz, 2009b, 2012, 2014) and concentrate on return spillovers. Firstly, I perform static spillover analysis by estimating a VAR model using the entire sample of observations without accounting for time variability of connectedness. In other words, I present unconditional return spillover measures. Secondly, I use a rolling-window estimation to investigate how connectedness measures and spillovers change over time.

To choose the structure of the VAR model I employ information criteria for the lag selection. I use Akaike Information Criterion (AIC) for a choice decision and select the lag length of the VAR model that corresponds to the lowest AIC level. Although the AIC tends to select less parsimonious models than other information criteria, in my analysis the optimal lag length is not high enough to generate any problems with the estimation of the parameters of the model; thus, less parsimonious lag structures are feasible to estimate. It is important to use an identical sample for the construction of different models. Otherwise, the information criteria will not be comparable. For the return spillovers the optimal lag length of 3 is identified. This structure produces stable VAR models that admit vector MA(∞) representations required for the calculation of connectedness measures.

To calculate DY spillover measures that are based on forecast error variance decompositions I need to select the interval length h for h-step-ahead predictions of returns. The choice usually depends on the purpose of the calculations. For example, for daily value-at-risk calculations horizons from 1 day to 30 days (representing a month) are usually employed (Degiannakis et al., 2013; Dowd et al., 2004). In my work, for the purpose of measurement of connectedness, I set h = 12, which corresponds to half of a trading month on average after accounting for the presence of weekends. This interval is long enough to draw conclusions regarding the financial connectedness and is comparable to intervals chosen in studies based on DY's framework (Diebold and Yılmaz, 2015b; Demiralay and Bayraci, 2015). I performed robustness checks (results are not shown here) using different values 1, 10, and 30 for h and found no significant changes in the results.

Table 6 summarizes the results of the estimation of return spillovers among the markets considered. An entry of the table at the intersection of different markets is the share of forecast error variance of the row market that is generated by the return shock of the column market. For example, the value 10.98 in the upper part of the table at the intersection of BUX and WIG means that approximately 11% of the 12-days-ahead return forecast error variance of the BUX is due to the return shock of the WIG. The column FROM others (FROM o.) is the row sum of all non-diagonal elements, which identifies the share of the forecast error variance of the row market generated by spillover effects from all other markets in the system. The row TO others (TO o.) is a column sum of non-diagonal elements that represents the spillover effect generated by the market in the column to all other markets. The bold element in the lower right corner is a total spillover index. It can be calculated in many ways, for example, by taking the average of the FROM o. column. It is an aggregate measure of connectedness of markets, which represents the average share of the forecast error variance explained by shock from other markets.

A total spillover index of 55.4 indicates that more than half of the variance of returns comes from spillover effects, suggesting that the markets studied are tightly connected. The contributions of each market to the total connectedness are quite heterogeneous.

The cryptocurrency market represented by Bitcoin (BTC) is the least pronounced member of the system. It generates almost no spillovers to other markets (around 1%) and receives only 8% of the variation from return spillovers of other markets. The evidence suggests that the connectedness of BTC with other financial markets is quite weak and its influence is limited.

In contrast, the group of EE markets and Russia are significantly connected in the system. The spillover effects from other markets are the most pronounced for the Czech Republic (PX) and Poland (WIG), around 65.5% of their variances are formed by other markets. Hungary (BUX) and Russia (RTSI) are slightly less connected with FROM values close to 59%. The heterogeneity is also observed in the degree of spillover generation from these markets. The PX and WIG, on average, generate $\frac{66.4\%}{9-1} = 8.3\%$ and $\frac{64.3\%}{9-1} = 8.0\%$ of the return variation of other markets through return spillovers. The BUX and RTSI are less significant for the system in this respect with TO measures equal to 50.5 and 47.8, respectively. Interestingly, there is little heterogeneity in the connectedness of the EE and Russian indices to the developed markets. The difference between the TO and FROM measure, which helps to judge whether the market is a net transmitter or a net receiver of spillovers, is presented in Table 7. Only the Czech Republic is identified as a net transmitter among the EE and Russian stock markets, although the net measure is pretty close to zero, suggesting that in the overall system this group of markets may be considered to be receivers of return spillovers. It is possible to measure the connectedness within the group of markets by looking at the average of FROM measures in isolation[13]. The calculation of the total connectedness for this group of markets in isolation gives a value of 28.1, which is more than half of the total connectedness measure. The interpretation of this value is that, on average, 28.1% of the return forecast error variance of the EE and Russian markets is generated by return spillovers from this group of markets. This means that the significant return spillovers in the system are not solely driven by the

[13] Accounting for other members of the network (developed markets and cryptocurrency) the total connectedness within EE and Russian markets is equal to the average of adjusted FROM measures in a reduced Table 6 where only the rows and columns of the isolated group are presented.

developed markets; the relationship between the EE and Russia is of equal importance for the overall connectedness of the network.

The group of developed market indices forms a notably connected cluster. More than 30% of return forecast error variance for these markets is generated by spillovers from other developed markets. The only exception is Japan (Nikkei) for which this percentage is equal to 18%. The Nikkei is also less connected to the system; only 40% of the variation could be explained by the influence of all other markets. Germany (DAX) and France (CAC) receive more return spillovers than the US (SP500) from other markets in the network. The values are 72.1%, 72.4%, and 59.2%, respectively. Although the SP500 and Nikkei have TO measures less than those of the PX and WIG, the CAC and DAX represent important generators of return spillovers to the system in the group of developed markets with TO measures close to 90%. Table 8 identifies France and Germany as significant net transmitters of spillovers, while the SP500 is identified as a net receiver. The net connectedness measure for the Nikkei is close to zero.

One potential issue, which may undermine the static results, is the structural stability of the estimated VAR model. To check for the presence of structural breaks I use the CUSUM test (Ploberger and Krämer, 1992) for each equation of the VAR. The results indicate that the null hypothesis of the absence of structural breaks can not be rejected at 5% significance level.

Having described the static connectedness of the system I turn to the dynamic evolution of return spillovers. Time variability is accounted for with the help of the rolling window estimation of the VAR model. I set lag length to be equal to 3 (as in the static case), separately evaluate the model on each subsample of 250 trading days, and use 12-days-ahead forecast error variance decompositions to construct spillover measures similar to those presented in Table 6. The size of the window is equal to 250 because this period is equivalent to one full trading year; this number of observations is enough to estimate the model and perform subsequent calculations. I also check how the results respond to changes in the size of the window. In general, wider windows produce smoother connectedness measures, but the dynamics remain relatively similar from one size to another.

Figure 6 demonstrates the time-varying total spillover index (the bold element of Table 6). First of all, the significant time variation of the total connectedness is immediately evident from the graph. In a pre-pandemic period (to the left of the red line) the index fluctuates close to its unconditional value of 55.4. However, near the red line the total connectedness increases sharply to a value close to 75, then it levels off around this number and recovers to levels of the pre-pandemic period only at the end of the sample. The static examination of the index could not reveal the effect of the pandemic on the connectedness of markets. The increase in market connectedness implies that the part of the forecast error variance of returns due to shocks from other markets increased by approximately 20 p.p. or by almost 35% compared to the unconditional connectedness.

Table 6: Static return spillovers.

	BTC	BUX	PX	WIG	RTSI	SP500	CAC	DAX	Nikkei	FROM o.
BTC	92.26	0.61	0.76	1.10	0.35	1.73	1.11	0.78	1.31	7.74
	(2.68)	(0.43)	(0.43)	(0.60)	(0.30)	(0.62)	(0.53)	(0.44)	(0.51)	(2.68)
BUX	0.10	40.62	10.44	10.98	7.78	6.04	10.28	10.24	3.51	59.38
	(0.13)	(2.17)	(0.66)	(0.78)	(0.74)	(0.81)	(0.65)	(0.65)	(0.55)	(2.17)
PX	0.15	9.02	35.37	9.83	7.55	5.73	13.30	12.84	6.21	64.63
	(0.12)	(0.63)	(1.74)	(0.72)	(0.65)	(0.93)	(0.46)	(0.51)	(0.69)	(1.74)
WIG	0.13	9.70	10.16	35.62	9.06	7.03	11.91	12.31	4.08	64.38
	(0.17)	(0.71)	(0.64)	(1.64)	(0.65)	(0.70)	(0.49)	(0.49)	(0.55)	(1.64)
RTSI	0.32	8.03	8.92	10.73	41.27	5.62	10.76	10.01	4.33	58.73
	(0.18)	(0.72)	(0.63)	(0.68)	(2.05)	(0.76)	(0.71)	(0.77)	(0.59)	(2.05)
SP500	0.25	5.24	6.93	7.54	5.65	40.84	12.69	12.06	8.79	59.16
	(0.17)	(0.71)	(0.87)	(0.70)	(0.68)	(2.41)	(0.57)	(0.65)	(0.91)	(2.41)
CAC	0.14	6.92	10.55	9.20	7.03	9.08	27.64	23.47	5.98	72.36
	(0.12)	(0.55)	(0.48)	(0.51)	(0.61)	(0.58)	(0.90)	(0.63)	(0.60)	(0.90)
DAX	0.08	6.97	10.27	9.61	6.59	8.57	23.76	27.95	6.21	72.05
	(0.10)	(0.55)	(0.51)	(0.50)	(0.64)	(0.62)	(0.66)	(0.94)	(0.62)	(0.94)
Nikkei	0.21	3.96	8.37	5.31	3.82	3.34	7.48	7.24	60.25	39.75
	(0.19)	(0.66)	(0.84)	(0.73)	(0.61)	(0.83)	(0.92)	(0.89)	(3.79)	(3.79)
TO o.	1.38	50.45	66.40	64.29	47.83	47.15	91.29	88.95	40.42	**55.35**
	(0.85)	(3.31)	(3.12)	(3.38)	(3.13)	(3.94)	(2.12)	(2.30)	(3.52)	(1.60)

Note: The calculation is based on a VAR(3) model estimated on the full sample. 12-days-ahead forecast error variance decomposition is used. Residual bootstrap standard errors obtained with 1000 replications in parenthesis.

Table 7: Net static return spillovers.

	TO	FROM	Net	Transmitter
BTC	1.38	7.74	-6.36	**No**
	(0.85)	(2.68)	(2.38)	
BUX	50.45	59.38	-8.93	**No**
	(3.31)	(2.17)	(1.55)	
PX	66.40	64.63	1.78	**M**
	(3.12)	(1.74)	(1.71)	
WIG	64.29	64.38	-0.09	**M**
	(3.38)	(1.64)	(2.15)	
RTSI	47.83	58.73	-10.90	**No**
	(3.13)	(2.05)	(1.71)	
SP500	47.15	59.16	-12.01	**No**
	(3.94)	(2.41)	(2.92)	
CAC	91.29	72.36	18.93	**Yes**
	(2.12)	(0.90)	(1.52)	
DAX	88.95	72.05	16.90	**Yes**
	(2.30)	(0.94)	(1.64)	
Nikkei	40.42	39.75	0.68	**M**
	(3.52)	(3.79)	(3.53)	

Note: The Net column is the difference between the TO and FROM measures. M stands for marginal and indicates markets with a Net value close to 0. Residual bootstrap standard errors obtained with 1000 replications in parenthesis.

The examination of return connectedness of the EE and Russian markets in isolation, accounting for the presence of other markets in the system, reveals an even more extreme dynamic pattern and is presented in Figure 7. The increase from the unconditional 28.1 level by almost 11 p.p. is equivalent to a rise in the connectedness of the EE and Russian markets of 38%.

A close look at the time variation of net connectedness measures presented in Figure 8 partly explains the drivers of the spike in the total spillover index caused by the pandemic. While the majority of developed markets did not change their role in the system, the EE and Russian markets switched to the opposite part of the spectrum. The CAC and DAX remained net return spillover transmitters during the entire sample period; the Nikkei was always a net receiver. The SP500 generated more spillovers than it received in the pre-pandemic period, but in a post-pandemic part of the sample, this market transformed to a net receiver. The WIG and RTSI increased their influence during the pandemic, starting to generate more spillovers to other markets. The PX and BUX moved from the negative part to the region close to zero. Interestingly, in a dynamic perspective, BTC was a net transmitter of return spillovers during 2016, when the first wave of attention to cryptocurrencies developed. However, during the pandemic, BTC returned to a net receiver role. In conclusion, the EE and Russian markets increased their net connectedness measure as a consequence of the pandemic. The dynamic analysis of return spillovers showed that cryptocurrency markets are not totally unrelated to other markets under consideration.

Figure 6: Total dynamic return spillover index.

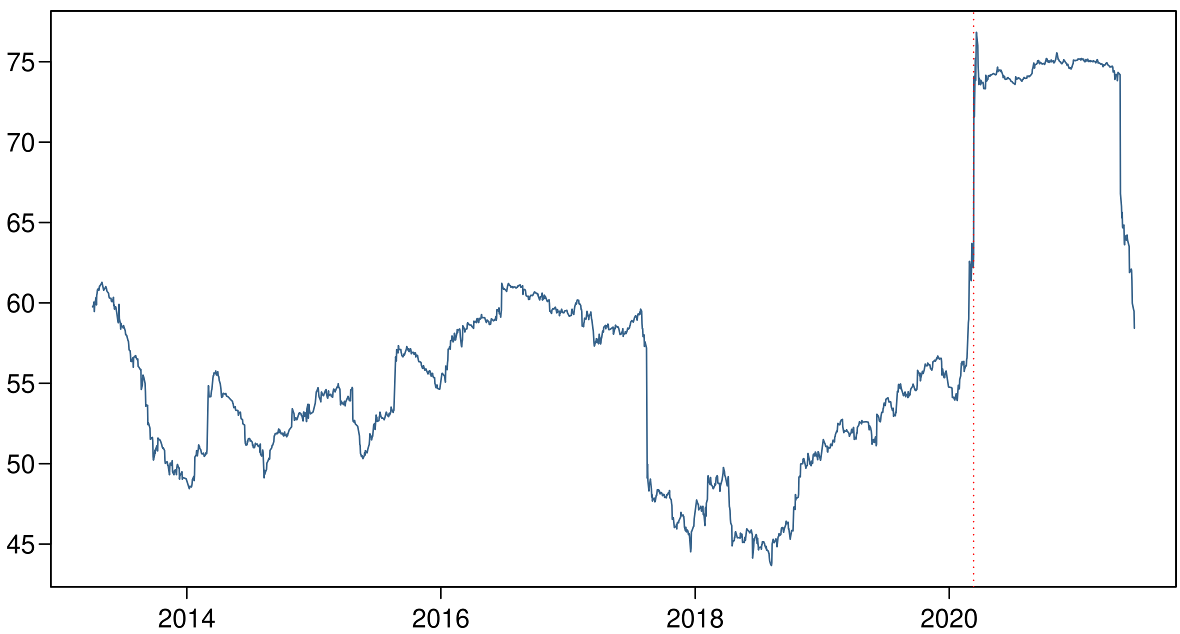
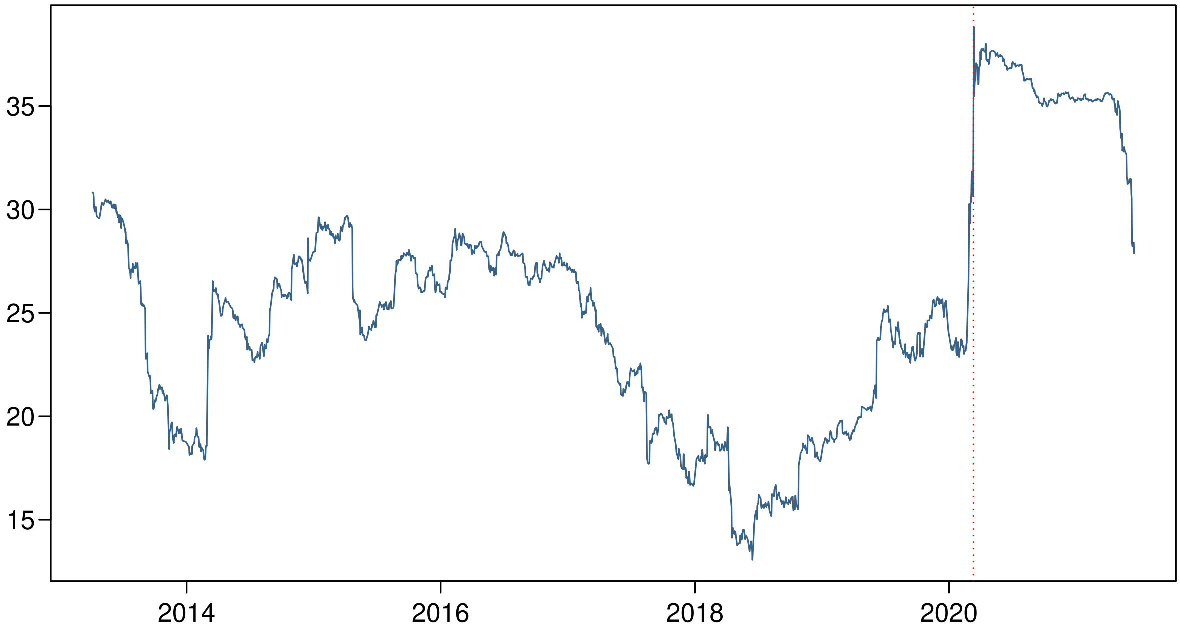


Figure 7: Total dynamic return spillover index for EE and Russian markets in isolation.



Note: The red line indicates the 11th of March 2020 when the WHO recognized the COVID-19 outbreak as a pandemic. For the estimation a 250 days rolling window is used.

4.2.2 Volatility spillovers

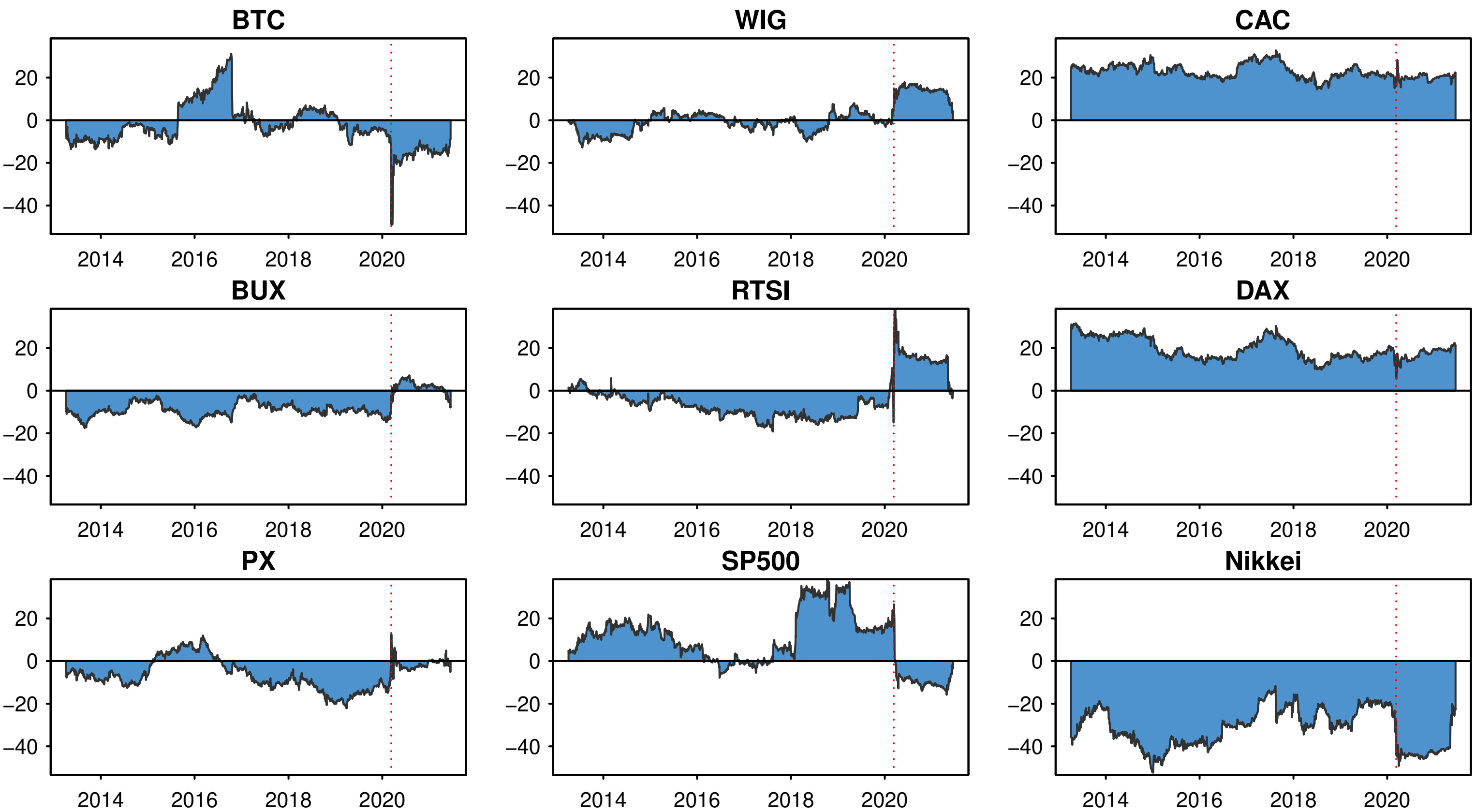
The return spillovers indicate the significant influence of the pandemic on the connectedness of markets. In this section I take a look at the same process but from a different perspective. I calculate volatility spillovers in a DY framework using range-based volatility measures instead of returns. The methodology for this part remains the same as in the return spillovers section. To ensure that the predictions of the VAR model do not deliver negative variances, I work with natural logarithms of volatilities. The AIC suggests using a lag length equal to 5 in VAR model for volatilities.

The results for static volatility spillovers are presented in Table 8. The findings on volatility connectedness differ significantly from the evidence on return connectedness. First of all, the total volatility spillover index (the bold element of Table 8) is equal to 41.6. This means that, on average, less than half of the volatility forecast error variance is driven by volatility spillovers from other markets. However, the value is still high enough to conclude that the markets we are analyzing are connected in terms of volatility.

In the static approach there is no evidence that BTC is connected to other markets. The spillover measures FROM and TO are positive but close to zero. Thus, BTC does not generate or receive any significant volatility spillovers in the system. As was seen in the analysis of return spillovers, the independence of BTC is a feature of the static approach, which does not hold after accounting for time-variability in connectedness.

The EE and Russian markets are tightly connected with other developed markets.

Figure 8: Net dynamic return spillovers.



Note: The red line indicates the 11th of March 2020 when WHO recognized COVID-19 outbreak as a pandemic. For the estimation 250 days rolling window is used.

The PX receives more volatility spillovers than others in the group. The FROM measure for this index is equal to 49.1; thus, approximately half of the variation in forecast error variance of the PX volatility is explained by spillovers from other markets. The WIG, BUX, and RTSI have this measure equal to 43.3, 39.6, and 33.3, respectively. The heterogeneity in TO measures for this group of markets is almost parallel to the structure of differences in FROM measures. The PX is a leader in volatility spillover generation with a TO measure of 44.1; the other markets generate significantly less and their TO measures are in the $29 - 34$ range. It is worth noting that the PX seems to be the most connected to developed indices among the EE and Russian markets. Net spillover measures presented in Table 9 indicate that all these markets are identified as net receivers of volatility spillovers in the system. The average TO measure for the EE and Russian markets in isolation is equal to 16.2, meaning that volatility connectedness is mostly driven by the interactions with developed markets, a result which is opposite to that observed for return spillovers. As opposed to the return spillovers evidence, the Czech Republic may be identified as a leader in connectedness with developed markets because it receives in sum the highest fraction of volatility spillovers from them. This observation suggests that volatility shocks from developed markets may propagate to the EE and Russian markets through the index of the Czech Republic.

The two leaders in connectedness in the network among the developed indices are the CAC and DAX. Around 60% of their volatility forecast error variance is due to volatility spillovers from all other markets. The CAC generates more spillovers than the DAX: on average, 10.2% of volatility forecast error variance for each market is explained by the CAC spillovers. This fraction is 9.1% for the DAX. The SP500 is less connected to the system with FROM and TO measures of 52.6 and 43.0, respectively. The Japanese market is the least connected to the network among the developed indices. Despite its size, the connectedness of the Nikkei is close to that of the RTSI. In the static case, only the CAC and DAX are identified as significant net volatility spillover transmitters. However, these static results should be considered with caution because for the BUX, WIG, and SP500 equations the CUSUM test rejects the null of the absence of structural breaks at 5% significance level. Although the breaks can not be attributed to the start of the pandemic, their existence generates additional motivation to use rolling window estimation, which accounts for the changes in parameters of the model over time.

The dynamic examination of volatility spillovers indicates a significant time variation of volatility connectedness. To construct these graphs I used a 250 day rolling window estimation and prediction horizon equal to 12 days as in the previous dynamic analysis of return spillovers. Figures 9 and 10 show how the total volatility spillover index changed over time for the entire system and for the EE and Russian markets in isolation, respectively. For the entire system the level near the start of the pandemic is 21.5 p.p. higher than the average value of 44.8. This means that the pandemic increased the volatility connectedness of the system of markets by almost 50%. For the EE and Russian markets in isolation the difference is equal to 16 p.p. (the average level is 16.5); in other words, the connectedness of these markets almost doubled. Thus, the effect of the pandemic was more pronounced for volatility than return connectedness of the markets. Moreover, the EE and Russian markets responded more sensitively than other markets. However, at the end of the sample, their isolated connectedness decreased to the full sample average value faster than the connectedness of the entire network.

In the dynamic perspective, the main spillover transmitter is the US market. The influence of the SP500 was increasing steadily before the pandemic, remained important during it, and disappeared only at the end of the sample. At the start of the pandemic the CAC, DAX, and Nikkei received more volatility spillovers than they generated. However, as opposed to the behavior of the SP500, the CAC and DAX increased their influence during the pandemic. The major switches from net transmitter to net receivers were not observed for these markets in return spillovers analysis.

BTC fluctuated around zero in the pre-pandemic period but became a strong net receiver of spillovers at the end of the sample. Thus, in terms of volatility spillovers, cryptocurrency market did not significantly influence the indices studied but rather were influenced by them.

The EE and Russian indices mostly received volatility spillovers at the pre-pandemic portion of the sample, in line with the result that was observed for return spillovers. Nevertheless, after the start of the pandemic they did not switch their role to volatility transmitters, except the WIG, as they partly did with return spillovers. For the PX a positive spike near the 11th of March 2020 is observed, which may be caused by the leadership of the Czech Republic among the EE and Russian markets in terms of connectedness to developed indices.

Table 8: Static volatility spillovers.

	BTC	BUX	PX	WIG	RTSI	SP500	CAC	DAX	Nikkei	FROM o.
BTC	95.43	2.07	0.19	0.65	0.22	0.54	0.28	0.21	0.42	4.57
	(1.90)	(1.25)	(0.39)	(0.62)	(0.36)	(0.53)	(0.40)	(0.36)	(0.45)	(1.90)
BUX	0.36	60.43	8.03	6.78	5.69	5.27	5.04	5.75	2.66	39.57
	(0.37)	(3.62)	(1.38)	(1.37)	(1.41)	(1.24)	(1.21)	(1.27)	(1.03)	(3.62)
PX	0.58	5.75	50.92	4.50	5.46	4.69	13.14	9.16	5.80	49.08
	(0.48)	(1.19)	(3.00)	(1.07)	(1.25)	(1.01)	(1.41)	(1.32)	(1.38)	(3.00)
WIG	0.37	7.34	6.45	56.70	3.26	7.60	7.91	7.15	3.22	43.30
	(0.40)	(1.44)	(1.28)	(3.18)	(0.94)	(1.33)	(1.15)	(1.12)	(1.13)	(3.18)
RTSI	0.10	4.83	5.01	1.77	66.71	2.66	8.01	6.73	4.18	33.29
	(0.31)	(1.34)	(1.40)	(0.67)	(3.91)	(1.10)	(1.63)	(1.58)	(1.43)	(3.91)
SP500	0.24	5.28	5.58	5.84	3.88	47.36	11.72	11.06	9.04	52.64
	(0.30)	(1.39)	(1.23)	(1.33)	(1.19)	(3.06)	(1.42)	(1.45)	(1.74)	(3.06)
CAC	0.29	2.44	7.97	3.79	5.25	7.64	40.77	27.35	4.51	59.23
	(0.29)	(0.73)	(1.18)	(0.81)	(1.16)	(1.11)	(1.59)	(1.17)	(1.17)	(1.59)
DAX	0.32	3.11	6.14	3.96	4.57	8.31	29.46	39.47	4.66	60.53
	(0.31)	(0.83)	(1.05)	(0.80)	(1.09)	(1.17)	(1.18)	(1.56)	(1.18)	(1.56)
Nikkei	0.11	3.21	4.73	1.75	3.95	6.32	6.32	5.79	67.81	32.19
	(0.31)	(1.25)	(1.30)	(0.94)	(1.33)	(1.44)	(1.51)	(1.53)	(4.48)	(4.48)
TO o.	2.35	34.02	44.10	29.04	32.28	43.03	81.88	73.20	34.50	**41.60**
	(1.37)	(6.16)	(6.42)	(5.34)	(6.48)	(6.07)	(6.37)	(6.49)	(7.01)	(1.97)

Note: The calculation is based on a VAR(3) model estimated on the full sample using natural logarithms of range-based volatilities. A 12-days-ahead forecast error variance decomposition is used. Residual bootstrap standard errors obtained with 1000 replications in parenthesis.

Figure 9: Total dynamic volatility spillover index.

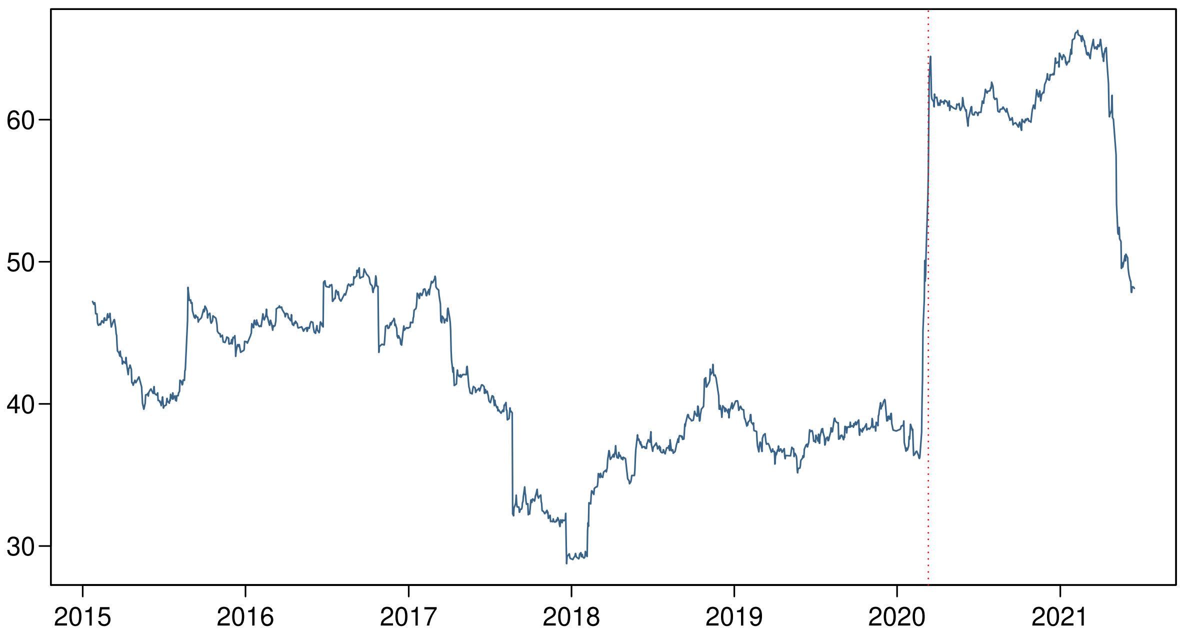
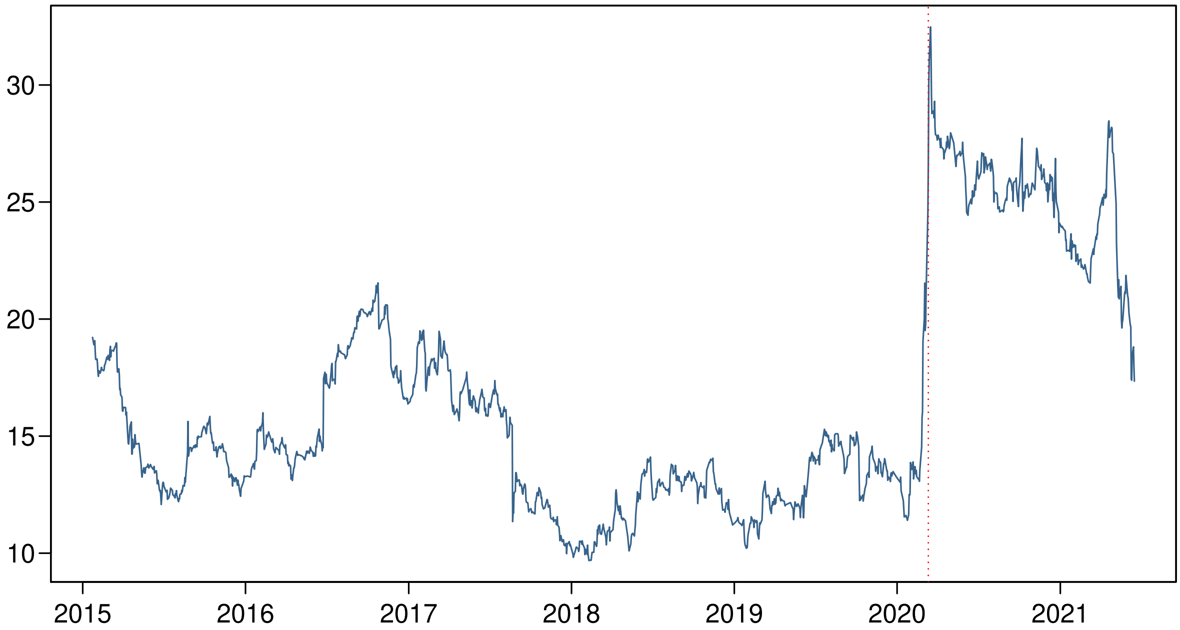


Table 9: Net static volatility spillovers.

	To	From	Net	Transmitter
BTC	2.35	4.57	-2.22	**M**
	(1.37)	(1.90)	(2.31)	
BUX	34.02	39.57	-5.55	**M**
	(6.16)	(3.62)	(6.36)	
PX	44.10	49.08	-4.98	**M**
	(6.42)	(3.00)	(6.84)	
WIG	29.04	43.30	-14.26	**No**
	(5.34)	(3.18)	(5.59)	
RTSI	32.28	33.29	-1.01	**M**
	(6.48)	(3.91)	(7.01)	
SP500	43.03	52.64	-9.61	**M**
	(6.07)	(3.06)	(6.82)	
CAC	81.88	59.23	22.64	**Yes**
	(6.37)	(1.59)	(6.64)	
DAX	73.20	60.53	12.67	**Yes**
	(6.49)	(1.56)	(6.73)	
Nikkei	34.50	32.19	2.31	**M**
	(7.01)	(4.48)	(8.13)	

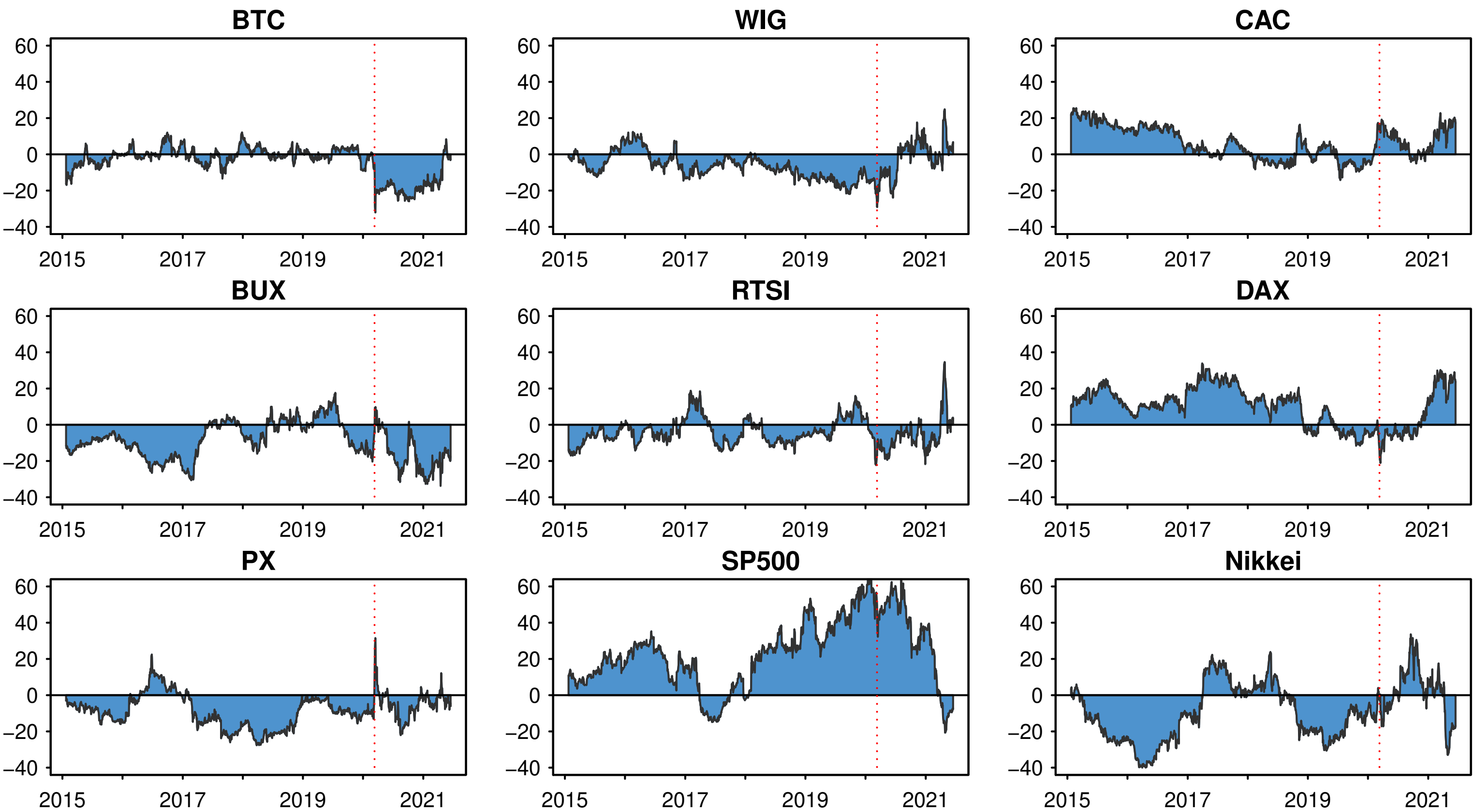
Note: The Net column is the difference between TO and FROM measures. M stands for marginal and indicates markets with Net value close to 0. Residual bootstrap standard errors obtained with 1000 replications in parenthesis.

Figure 10: Total dynamic volatility spillover index for EE and Russian markets in isolation.



Note: The red line indicates the 11th of March 2020 when the WHO recognized the COVID-19 outbreak as a pandemic. For the estimation a 250 days rolling window is used.

Figure 11: Net dynamic volatility spillovers.



Note: The red line indicates the 11th of March 2020 when the WHO recognized the COVID-19 outbreak as a pandemic. For the estimation a 250 days rolling window is used.

5 Discussion

My results may be compared to previous literature on the financial connectedness of European markets. Asaturov et al. (2015) identify the significant influence of the Russian market on Eastern European (EE) indices using the sample period 2001-2012. During the financial crisis the average conditional correlations with the BUX, PX, and WIG are approximately equal to 50 %, 55 %, and 57%, respectively. The estimates from my thesis suggest lower levels of correlations. However, the results are similar if one compares with the correlations during the beginning of the pandemic, showing that the effect of the pandemic on the connectedness of financial markets may be compared to the effect of the financial crisis of 2008. Employing data in the same period Gjika and Horvath (2013) concentrate on the connectedness of EE markets disregarding the influence of the Russian market. For this period the asymmetric DCC model suggests estimates of conditional correlations of market returns that are significantly higher than my estimates. In the second half of the sample, the correlations between the BUX, PX, and WIG exceed 60 %. The authors find significant asymmetric effects of past shocks on the level of volatility for the univariate models and only mild asymmetry in the multivariate part. Although in this thesis the asymmetric effects in the multivariate distribution turned out to be insignificant, these results are in line with my findings. One may partly explain the differences in correlation estimates by the fact that Gjika and Horvath (2013) do not consider the influence of the Russian market which proved to be important.

Demiralay and Bayraci (2015) study the EE and Russian markets using the sample period 1998-2013, which covers the 2008 financial crisis. Their static total volatility connectedness measure is equal to 57.5 and is greater than that obtained in this work (41.6). However, it is close to the total connectedness in terms of return spillovers, which is equal to 55.4. The authors show that during the 2008 financial crisis the total connectedness measure increased to levels close to 80. In this work, near the start of the pandemic, this measure was close to 65; this shows again that the effect of the pandemic on the financial connectedness of the EE and Russian markets may be compared to the effect of the financial crisis (especially, if one considers return spillovers). In general, I obtain estimates of connectedness that are uniformly less than those presented by Demiralay and Bayraci (2015). Aslam et al. (2021) study volatility spillovers in Europe using high-frequency data and a sample covering three months before and three months after the start of the pandemic. They conclude that total connectedness during this period is equal to 77.8, a result that is moderately higher than my estimates during the same period. Perhaps the overestimation of connectedness comes from the difference in return frequencies used, suggesting that the connectedness may be more pronounced in the very short term which corresponds to 5-minute interval intraday observations. Moreover, the authors use only the "stress" period in their analysis, while my work takes into account a wider pre-pandemic period that helps to understand the behavior of markets during the relatively calm periods.

The low level of connectedness of cryptocurrency and stock markets in terms of both correlations and spillover effects looks surprising. On the one hand, some sort of interdependence is expected considering the evidence on the connectedness within cryptocurrency markets (Yi et al., 2018; Ji et al., 2019) and their connectedness with commodity markets (Ji et al., 2019). On the other hand, the empirical evidence suggests that stock markets may behave differently in this regard. Tiwari et al. (2019) find low conditional correlations of six major cryptocurrencies with S&P500, the result which is close to my estimates of the relationship between BTC and the S&P500.

6 Conclusion

In this thesis, I investigate the financial connectedness of Eastern European markets. The analysis of conditional correlations of returns and spillover effects indicates tight connections between the EE markets. The significant part of the connectedness is explained by the interactions within the group of EE markets and is not driven solely by other developed markets. The relationship is not static and changes over time with different dynamics for return and volatility spillover effects. Importantly, after the start of the COVID-19 pandemic, the strength of connections increased significantly and achieved maximal levels for the 2014-2021 period. The cryptocurrency market demonstrates a limited relation to the considered markets and mostly acts as a receiver of spillovers even during crisis periods.

The results have important implications for portfolio managers. The increase in connectedness indicates limited diversification abilities for a portfolio that includes EE and Russian indices. However, a cryptocurrency may be considered as a desirable asset for risky investors that provides possibilities for diversification if one considers its‘ weak connections to other markets. Although the connection is weak, one needs to take into account that the independence may vanish during crisis periods like the recent pandemic. Based on the results of this thesis, policymakers may assess the magnitude of the response of EE financial markets to the changes in the real economy of the European Union (especially strict lockdown or travel bans). Failure to account for the described level of connectedness may lead to underestimation of possible adverse effects of strict policies and their influence on systemic risks. Moreover, one may also argue how the cryptocurrency events may affect EE markets, suggesting arguments for debates on the regulation of cryptocurrencies.

Given the increase in connectedness during the pandemic, it would be interesting to see whether the lockdown measures in different EE countries were the important drivers of the change. It seems plausible that some EE economies (for example, with significant travel and tourism orientation) could respond more sensitively to the consequences of the pandemic, forcing the financial connectedness to increase. Another avenue to improve on this work is to depart from the rolling window estimation procedure and look at dynamic connectedness using Time-Varying Parameter Vector Autoregression (TVP-VAR) model. The model allows one to maintain the assumption that parameters may change over time while using the full sample for the estimation. This can potentially increase the efficiency of the estimates or affect the results on the dynamic connectedness measures.

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