CERGE
Center for Economic Research and Graduate Education
Charles University

Essays on Macroeconomic Policies and Family Economics

Vera Tolstova

Dissertation

Prague, May 2021

Dissertation Committee:

Sergey Slobodyan (CERGE-EI, Chair)

Radim Bohacek (CERGE-EI)
Byeongju Jeong (CERGE-EI)
Marek Kapicka (CERGE-EI)
Filip Matejka (CERGE-EI)
Ctirad Slavik (CERGE-EI)

Referees:

Christopher Herrington (Virginia Commonwealth University)
Georgi Kocharkov (Deutsche Bundesbank)

Contents

Acknowledgements

I would like to thank my supervisor Sergey Slobodyan for his enormous support and guidance throughout my doctoral studies. I am very grateful to my dissertation committee members Radim Bohacek, Marek Kapicka, Filip Matejka, Byeongju Jeong and Ctirad Slavik for very helpful suggestions and discussions.

I would like to express special thanks to Mattias Doepke for his excellent supervision and support during my stay at Northwestern university. I am also very grateful to Deborah Novakova and Andrea Downing for their excellent editing assistance throughout my doctoral studies, the CERGE-EI SAO for administrative assistance and my CERGE-EI schoolmates for providing continuing support and inspiration.

I acknowledge financial support from the Czech Science Foundation project No. P402/12/ G097 (DYME Dynamic Models in Economics) and institutional support RVO 67985998 from the Czech Academy of Sciences.

Any remaining errors are my own.

Prague, Czech Republic　　　　Vera Tolstova

May, 2021

Abstract

In a real economy, decisions on investments in the human capital of children are made by families rather than by atomistic parents as is typically assumed in the literature. The first chapter of this dissertation incorporates family formation into an otherwise standard dynastic framework with human capital accumulation. The study finds that accounting for differences in taxation and education policies between the U.S. and 10 OECD countries is sufficient to replicate cross-country variations in the degree of assortative matching and its positive correlation with the intergenerational persistence of earnings. Positive assortative matching is crucial to a model's ability to generate realistic levels of the intergenerational earnings correlations observed in the data.

In the second chapter, I develop a simple dynastic model in the style of Barro and Becker (1989), with endogenous fertility and human capital accumulation, to quantify the optimal progressivity of higher education subsidies. I find that the optimal policy is characterised by a higher degree of progressivity than current U.S. education subsidies. Additionally, the relations between the degree of progressivity of education policies and welfare/ population growth are hump-/ U-shaped respectively. While an assumption of endogenous fertility is quantitatively important, heterogeneity in fertilities is sufficient to generate these results. This is because welfare gains from more progressive subsidies are driven not only by decreases in the fertility rates of low income individuals, but also because their children frequently relocate to states associated with higher incomes and relatively lower fertility rates.

In the third chapter, I study a politico-economic dynamic general equilibrium model to quantify the importance of endogenous fertility in explaining the generosity of redistribution and education policies in the U.S. Policies are endogenised as outcomes of majority voting. I find that accounting for endogenous fertility is essential for strong performance of the model in matching the levels of both transfers and education subsidies in the U.S. economy. The predictions of the model for a cross-section of U.S. states are used to verify the plausibility of the finding that fertility decisions respond to policies.

Abstrakt

V reálných rozhodnutích o investování do lidského kapitálu dětí je rozhodnutí častěji činěno rodinami nežli atomárními rodiči, jak se obvykle předpokládá v literatuře. Tento článek zahrnuje formování rodiny do jinak standardního rodového frameworku s akumulací lidského kapitálu. Studie zjišťuje, že započtení rozdílů ve zdanění a politikách vzdělávání mezi USA a 10 zeměmi OECD je postačující k replikování variability asortativní shody a positivní korelace s persistencí mezigeneračních výdělků napříč zeměmi. Positivní asortativní shody jsou klíčové pro schopnost modelu generovat realistické úrovně korelace výnosů napříč generacemi, které jsou pozorovány v datech.

V tomto článku studuji jednoduchý dynastický model s endogenní plodností a akumulací lidského kapitálu, podobný Barro a Becker modelům, za účelem kvantifikace optimální progresivity dotací vyššího vzdělání. Zjišťuji, že optimální dotace jsou progresivnější než současném dotace v USA. Zároveň ukazuji, že růst blahobytu s růstem progresivity dotací nejdříve roste a pak klesá, zatímco růst populace s růstem progresivity dotací nejdříve klesá a pak roste. Přestože předpoklad endogenní plodnosti je kvantitativně důležitý, různorodost plodnosti je dostatečným předpokladem k dosažení těchto výsledků. Je to dáno skutečností, že růst blahobytu v důsledku progresivnějších dotací je způsoben nejen poklesem plodnosti nízkopříjmových jednotlivců, ale také sociální mobilitou směrem k vyšším příjmům a tedy k nižší plodnost.

Tento článek využívá politicko-ekonomický dynamický model obecné tržní rovnováhy ke kvantifikaci důležitosti endogenní fertility při vysvětlení štědrosti přerozdě- lování a podob vzdělávacích politik v USA. Vzdělávací politiky jsou určeny vnitřně jako výsledek hlasování většiny. Zjišťuji, že započítání endogenní fertility je nezbytné pro schopnost modelu dobře popsat úrovně transferů i dotování vzdělání v USA. Predikce modelu ohledně průřezu napříč státy USA jsou použity k ověření možných reakcí fertility na politiky vzdělávání a v konečném důsledku i kredibility mých výsledku.

Introduction

Consideration of decision-making within families has become an important branch of macroeconomics and can provide novel insights into classic macroeconomic topics such as investments in child human capital, optimal public policies, and cross-country differences in the intergenerational persistence of earnings.

In the first chapter, I analyse marital sorting. I incorporate marital decisions into an otherwise standard dynastic framework with human capital accumulation to study the importance of marital sorting in explaining cross-country differences in the intergenerational persistence of earnings. I find that considerations of cross-country differences in taxation and education policies explains positive relationships between the degree of assortative matching and intergenerational persistence of earnings observed in the data. Moreover, positive assortative matching is crucial to a model's ability to replicate realistic levels of intergenerational correlation of earnings in OECD countries. Accounting for differences in taxation and education policies between the U.S. and 10 OECD countries, the model with a marriage market has a superior fit to a counterpart framework that does not consider a marriage market. Therefore, differences in public policies and degrees of assortative matching are not stand-alone factors, but are interconnected in explaining cross-country variations in the intergenerational persistence of earnings.

In the second two chapters, I abstract from marital decisions, but introduce endogenous fertility. Parents decide on the number of their children and on their investments in the human capital of each child. In the second chapter, I introduce endogenous fertility into a simple dynastic model in the style of Barro and Becker (1989), with human capital accumulation, to quantify the optimal progressivity of subsidies for higher education. I find that a welfare-maximising policy is characterised by a higher degree of progressivity than current U.S. education subsidies. The main driver behind this result is the assumption of endogenous fertility. When education subsidies become more progressive, the share of low-productivity individuals decreases, not only because this category of agents invest more in the education of their children, but also because they reduce their fertility rates and their children often relocate to states associated with higher incomes and relatively lower fertility rates. In contrast, the counterpart model with exogenous fertility predicts that more progressive subsidies for higher education do not result in material welfare gains.

In the third chapter, I add an assumption of endogenous fertility to a politico-economic dynamic general equilibrium model. Redistribution and education policies are endogenised as outcomes of majority-voting equilibrium. I find that accounting for endogenous fertility is essential for strong performance of the model in matching the levels of both transfers and education subsidies in the U.S. economy. This is because consideration of endogenous fertility makes transfers and education subsidies more costly from the perspective of financing policies via tax revenues. The implications of the model for a cross-section of U.S. states are used to validate the framework and to circumvent the lack of empirical evidence on the elasticities of fertilities with respect to public policies. Taking redistribution policies as given, the model quite closely replicates the variations in and levels of education subsidies, average numbers of children in families, and fertility differentials across U.S. states. This verifies the plausibility of the idea that fertility decisions respond to policies.

Chapter 1

Marital Sorting and Cross-Country Differences in Intergenerational Earnings Persistence

Published as CERGE-EI Working Paper Series No. 680

1.1 Introduction

Cross-country variations in intergenerational persistence of earnings is a widely documented phenomenon in empirical literature. Among Western economies, Scandinavian countries and Canada have the lowest intergenerational correlation of earnings, while the U.S., U.K. and Southern Europe have the highest earnings persistence (Corak, 2006; Holter, 2015). Understanding the reasons for these differences may shed light on the underlying factors behind intergenerational earnings persistence and optimal policies to promote social mobility. Consequently, a wide range of theoretical and quantitative studies investigating this phenomenon has appeared. The key factors affecting intergenerational persistence of earnings found by the literature include taxation, public education financing, intergenerational correlation of abilities, parental investments in human capital, and borrowing constraints. By employing dynastic life-cycle frameworks incorporating these factors, existing quantitative studies have succeed in explaining at most half of the gap in the intergenerational earnings elasticity between the countries with the highest and the lowest intergenerational persistence of earnings. For instance, Holter (2015) explains from 21 to 54 % of the gap between three Scandinavian countries (Denmark, Norway, Finland) and the U.S. by accounting for cross-country differences in education and taxation policies. Herrington (2015) explains 8.7% of the gap between Norway and the U.S. by accounting for differences in tax policies and regional redistribution of compulsory education subsides. Blankenau & Youderian (2015) explain 8.5% of the gap between the U.S. and Denmark and Norway through accounting for differences in early education subsidies.

In a real economy, decisions on investments in child human capital are made by families rather than by atomistic parents as is typically assumed in existing studies. Moreover, empirical literature demonstrates variations in the degree of assortative matching and its positive connection to the persistence of intergenerational earnings in a number of developed countries (Chadwick and Solon, 2002; Ermisch et al., 2004; Eika et al., 2019). This paper incorporates marital sorting into an otherwise standard dynastic framework with a human capital accumulation process. The study finds that consideration of cross-country differences in taxation and education policies explains positive relationships between the degree of assortative matching and intergenerational persistence of earnings observed in the data. Moreover, positive assortative matching is crucial to a model's ability to replicate realistic levels of intergenerational correlation of earnings in OECD countries. Accounting for differences in taxation and education policies between the U.S. and 10 OECD countries, the benchmark with a marriage market calibrated into the U.S. economy has a superior fit, as opposed to Holter (2015), and the exact counterpart framework that does not consider a marriage market. Therefore, differences in public policies and degrees of marital sorting are not stand alone factors, but are interconnected in explaining cross-country variations in intergenerational earnings persistence.

The remainder of this paper is organised as follows. Section 1.2 presents an overview of existing literature on cross-country differences in intergenerational persistence of earnings and marital sorting. Section 1.3 documents a positive correlation between the degree of assortative matching and intergenerational persistence of earnings, and a negative correlation between the degree of assortative matching and tax progressivity. Section 1.4 studies impacts of taxation and education policies on the degree of assortative matching and parental investments in the human capital of their children in a simplified theoretical setup. Sections 1.5 and 1.6 present a quantitative model and its calibration to the U.S. economy. Section 1.7 concludes with the results.

1.2 Related Literature

This paper builds on the quantitative literature analysing the role of public policies in explaining cross-country differences in intergenerational earnings elasticity and literature on marital sorting in quantitative macroeconomic models.

The phenomenon of intergenerational persistence of earnings has been studied for several decades, generating a large number of empirical and theoretical studies. Bjorklund & Jantti (1997), Solon (2002), Corak (2006), Jantti at al. (2006), Blanden (2013) and Landerso & Heckman (2017), among others, document substantial cross-country differences in intergenerational persistence of earnings. Moreover, they find that intergenerational earnings elasticity is lower in Nordic countries than in the U.S. and U.K.

Theoretical literature pioneered by Becker and Tomes (1979, 1986) has sought explanations for this phenomenon. Becker and Tomes introduced a model of human capital formation through both imperfect transmission of abilities and investments in human capital made by parents and government. Parents are altruistic towards their children and care about their utilities. Therefore, in this framework, parents invest in the human capital of their children in order to increase their utilities through their future productivity. Relying on this basic assumption, theoretical and quantitative literature has proposed several explanations for the phenomenon of intergenerational persistence of earnings and its variations across countries.

One frequently analysed hypothesis is cross-country differences in taxation and education policies. Existing literature documents a negative correlation between tax progressivity and intergenerational persistence of earnings; see Holter (2015). The study finds that accounting for cross-country differences in tax progressivity explains around 50 % of the variation in intergenerational earnings persistence between the U.S. and 10 OECD countries. Herrington (2015) finds that more progressive taxes in Norway contribute to less intergenerational earnings persistence than in the U.S.

Moreover, the literature finds connections between intergenerational persistence of earnings and public spending on education. Holter (2015) demonstrates that countries with higher social mobility tend to have more generous public education investments at state and federal levels, especially at the tertiary level. This empirical finding is in line with Solon's (2004) analytical result that less generous public investments into education lead to higher intergenerational persistence of earnings. However, in Holter's quantitative model, the estimated impact of education policies on intergenerational persistence of earnings turns out to be moderate. A quantitative study by Blankenau & Youderian (2015) finds that accounting for cross-country differences in public expenditures on early childhood education explains around 10% of the gap in intergenerational persistence of earnings between the U.S. and Denmark and Norway.

Overall, most existing quantitative studies correctly predict the directions of tax and education policies impacts on intergenerational persistence of earnings (see Holter, 2015; Blankenau & Youderian, 2015; Herrington, 2015). However, on average, around 50 % of the gap in intergeneration earnings elasticity between the U.S. and other OECD countries remains unexplained. This study finds that accounting for marital sorting improves the performance of an otherwise standard dynastic life-cycle model of human capital formation in explaining cross-country differences in intergenerational mobility. Following the standard approach of modelling tax and education policies in quantitative studies, this paper finds that the sum of squared errors of model predictions versus data is reduced by around 33 % as opposed to a counterpart model without a marriage market.

This paper is connected to the literature on the marriage market and its role in explanating intergenerational earnings mobility and inequality. Alm & Whittington's (1999) and Wiik et al.'s (2010) empirical papers show the importance of economic factors including taxation for marital decisions. A broad empirical literature represented by Atkinson et al. (1983), Chadwick and Solon (2002) and Ermisch et al. (2004), among others, demonstrates positive connection between assortative matching and intergenerational earnings persistence in the U.K., U.S. and Germany, respectively. Greenwood et al. (2014) and Eika et al. (2019) show that positive assortative matching has a non-negligibly positive impact on income inequality in the U.S. and in several European countries, respectively. Relying on a search model of marital decisions pioneered by Mortensen (1988), Fernandez et al. (2005) explain a positive correlation between wage inequality and the degree of assortative matching via multiple equilibria phenomenon. The empirical part of their paper demonstrates substantial cross-country differences in the degree of assortative matching and a positive correlation between the degree of assortative matching and wage inequality.

This paper demonstrates that variations in tax and education policies provides an alternative explanation for cross-country differences in marital sorting and its connection to intergenerational earning persistence and income inequality. This relates the current paper to quantitative literature on marriage markets and decision making within couples. Early studies include Aiyagari et al. (2000), Regalia & Rios-Rull (2001), Fernandez & Rogerson (2001), Greenwood et al. (2003) and others. Those papers develop quantitative models with marital sorting to analyse factors affecting intergenerational mobility, increases in single motherhood, income inequality and welfare consequences of child support policies, respectively. The models are calibrated to the U.S. economy. Similarly, this paper develops a dynastic life-cycle model in which altruistic agents decide on their marital status and investments in the human capital of their children. Assumptions including family decision making through Nash bargaining, fertility choices and divorces are excluded from the model. This simplification allows for the introduction of larger numbers of search rounds for realistic modelling of the assortative matching.

In more recent literature, Guner & Knowles (2007), Greenwood & Guner (2008) and Greenwood at. al. (2016) develop dynamic search models of marriage and divorce. These studies mainly focus on the role of household sector progress, increases in skill premiums, declines in marriage rates, growing degrees of marital sorting, and income inequality observed in the U.S. over recent decades. Similarly to those studies, this paper develops a dynamic search model of marriage. I introduce multiple periods of matching rounds and parental human capital investments to analyse the role of public policies in explaining cross-country differences in the degree of marital sorting and its connection to intergenerational earnings persistence.

This paper also relates to recent studies by Chakraborty et al. (2015) and Bick & Fuchs-Schundeln (2018) analysing the roles of taxation and marital patterns in labor supply decisions of both men and women. Chakraborty et al. (2015) show that divorce rates and taxation differences could explain around half of variations in labor supply between the U.S. and European countries. Bick & Fuchs-Schundeln (2018) demonstrate that tax treatment of married couples is crucial for explaining gender differences and aggregate variations in labor supply across European countries. In contrast to these studies, this paper abstracts from labor supply decisions but endogenises marital sorting and human capital formation.

1.3 Stylised Facts

This section presents empirical evidence on the relationships between intergenerational persistence of earnings, degrees of assortative matching, tax policy parameters and income inequality. Existing literature has analysed the connections between some of these indicators. The roles of public policies and assortative matching have been analysed as separate hypotheses explaining cross-country differences in intergenerational persistence of earnings and income inequality. Holter (2015) documents a negative correlation between intergenerational persistence of earnings, tax progressivity, tax levels and public expenditures on tertiary education across 11 OECD countries. Atkinson et al. (1983), Chadwick and Solon (2002) and Ermisch et al. (2004) show a positive connection between assortative matching and intergenerational persistence of earnings across several European countries. Fernandez at al. (2005) find a positive link between the degree of assortative matching and the wage differential between high and low skilled workers across a wide range of countries.

To the best of my knowledge, there is no existing study analysing connections between all of these indicators together in a cross-country setting and showing that taxation policies and assortative matching are interconnected rather than functioning as separate factors that affect cross-country differences in intergenerational persistence of earnings. The remainder of this section presents assessment and data sources for each of the indicators analysed and the results of the correlations.

Intergenerational persistence of earnings. Following the literature, intergenerational persistence of earnings or intergenerational earnings elasticity (IEE) is measured in a standard way using a coefficient β in the regression of the logarithm of sons' earnings $\ln(y_{son})$ versus the logarithm of fathers' earnings $\ln(y_{father})$: $\ln(y_{son}) = \alpha + \beta \ln(y_{father}) + \epsilon$. The estimates of intergenerational persistence of earnings are based on Corak (2006), who provides an overview of the most recent cross-country results. The estimates for Italy and Spain are from Piraino (2007) and Pla (2009). The numbers illustrate a well-known pattern: high persistence of earnings in the U.S and U.K. and low persistence in Scandinavian countries and Canada; see Appendix A in 1.9, table 1.9.1, column "IEE".

Assortative matching. Following Fernandez at al. (2005), I estimate the degree of assortative matching as the correlation between the partners' years of schooling based on the Luxembourg Income Study (LIS) household level survey data covering 1995 and 2000.[1] The sample for estimation includes couples between 25 and 65 years of age[2] for whom information on years of education is available for both partners. Cohabiting couples are treated as married. The estimates are presented in the Appendix A in 1.9, table 1.9.1, column "Assortative matching". Alternative approaches to estimation of the degree of assortative matching may involve dividing agents into low, medium and highly educated groups, and measuring the correlation between education groups of partners, or calculating a ratio of the probability an agent will marry a partner with a particular education level versus the likelihood of random matching disregarding education, as in Eika et al. (2019). These measures are less preferable than the approach applied in this paper, because human capital is assumed be a continuous variable rather than a discrete education group, to facilitate comparisons with Holter's counterpart study, making an analogous assumption. In real life, earnings variation is indeed

[1] These dates are similar to the timespan employed in the literature for intergenerational correlation of earnings and public policies parameters estimation.

[2] Corresponds to the age of the adult population in the model.

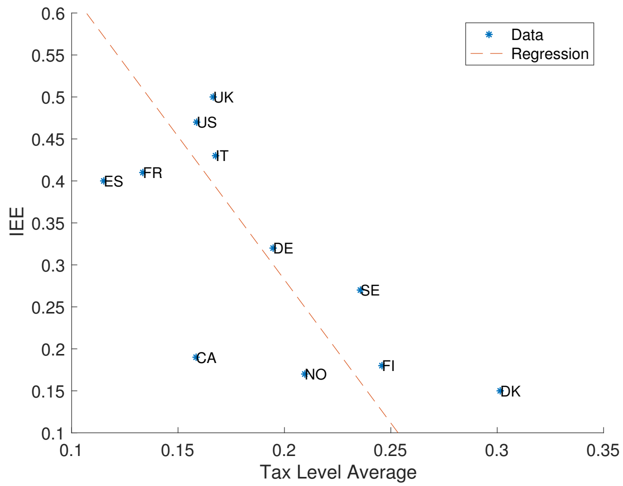
much broader than several income levels corresponding to several education groups. The number of years of schooling can be seen as the closest proxy for modelling human capital available in the data.

Taxes, education subsidies and income inequality. The tax progressivity wedge is defined in a standard way as $1 - \frac{1-T'(y_2)}{1-T'(y_1)}$ where $T'(y_2)$ and $T'(y_1)$ are marginal taxes at income levels y_1 and y_2; see Guvenen et al. (2014). Following Benabou (2002), I employ a standard assumption of $T'(y) = 1 - \theta_0 y^{-\theta_1}$. Given this functional form specification, the tax progressivity wedge becomes $1 - (\frac{y_2}{y_1})^{-\theta_1}$. The estimates of θ_1 are provided by Holter et al. (2019), while $y_2 = 2AW$ and $y_1 = 0.5AW$, as in Holter (2015), where AW is an average wage. The tax level is captured by the tax rate corresponding to average earnings based on OECD tax and benefit calculator data.[3] The estimates of tax policy parameters appear in the columns "Tax Progressivity Wedge" and "Average Tax". Income inequality is measured as the logarithm of 90th to 10th gross earnings percentiles ratio, from OECD Statistics; column "Log P90/P10".

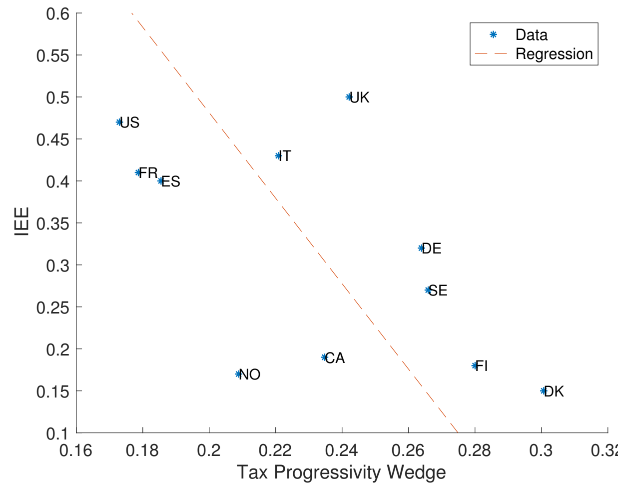
The results for 11 OECD countries analysed in this paper demonstrate that countries with more progressive taxes have lower intergenerational persistence of earnings and income inequality. This is in line with Holter's (2015) findings. Similarly to Fernandez at al. (2005), who demonstrate a positive correlation between assortative matching and skill premiums, the degree of assortative matching is positively correlated with income inequality measured by the logarithm of P90/P10 ratio; see table 1.9.2, Appendix A in 1.9. This paper uncovers new relationships which are beyond the scope of existing literature. Countries with more progressive taxation and higher average levels of taxes have lower degrees of assortative matching. Moreover, the degree of assortative matching is positively correlated with intergenerational persistence of earnings and income inequality. The scatter plots and corresponding regression lines are depicted in figure 1.3.1 below.

[3] https://www.oecd.org/els/soc/benefitsandwagestax-benefitcalculator.htm. The tax was estimated as an average tax rate of a 40 year old head of household with average earnings, 2 children and a partner who makes 39% (OECD average) of his income.

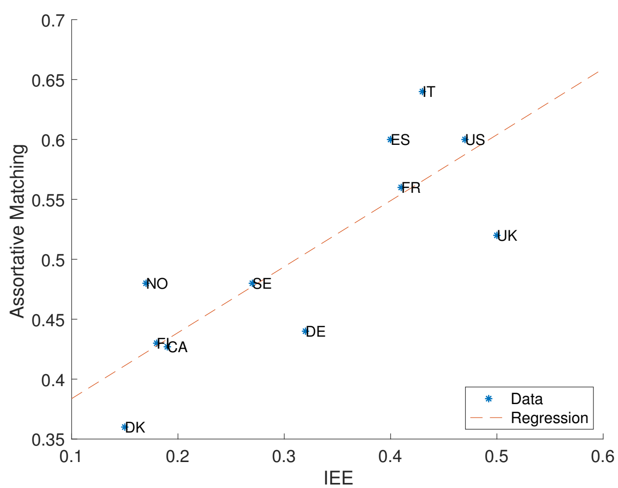
Figure 1.3.1: Differences in assortative matching, earnings persistence and tax policies across OECD countries



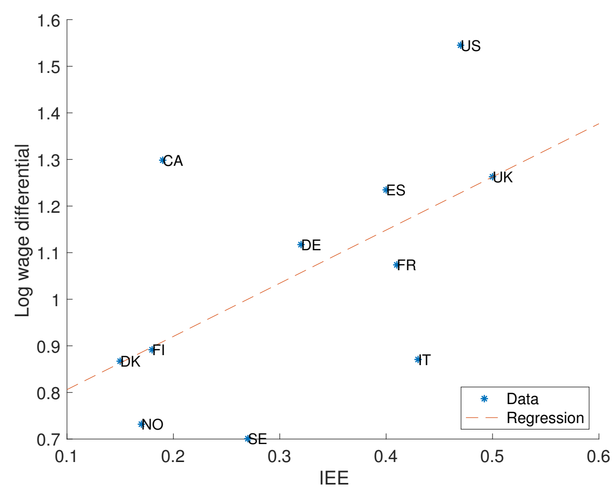
A: Tax level average vs. IEE



B: Tax progressivity wedge vs. IEE



C: Assortative matching vs. IEE



D: Log wage differential vs. IEE

1.4 A Simple Theoretical Example

To illustrate the qualitative connections between assortative matching, public policies, and investments in the human capital of children, I construct a simple, analytically solvable model based on Solon (2004) and Fernandez et al. (2005).

I analyse a 2 period economy populated by male and female agents.[4] The distribution of productivities is exogenous. For the sake of tractability, assume that one half of the population of each gender has low productivity y_l, while the other agents of both genders have productivities y_h, $y_h > y_l$.[5] There is no gender gap in pre- and after tax incomes between individuals of an identical productivity type. Following Fernandez et al. (2005), I assume that there are two rounds of matching in the marriage market. In the first period of their lives agents meet each other randomly and decide whether to marry or not depending on the observed productivity of the potential partner and match-specific quality shock $b_1 \geq 0$, where b_1 is a random variable with CDF F_b. For simplicity, there is no divorce in the economy. If they are married in the first period, a couple decides how much to consume and how much to invest in the human capital of their children. In the second period, agents who did not form a household in the first period are randomly matched with potential partners and draw a random realisation of matching shock $b_2 \geq 0$, where b_2 is a random variable with CDF F_b. In the first period, agents pay net taxes t_h and t_l depending on productivity types. Taxes finance education subsidies for children.

A couple married in the first period shares a common utility function, which is defined as follows:

$$u^{HM_1}(i, j, c_1, c_2, b_1) = log(c_1) + \alpha log(h_c) + b_1 + \beta log(c_2)$$

where i and j are the productivity types of agent i and his or her partner j respectively (due to gender symmetry, there is no need for male and female subscripts), c_t is the consumption of a couple in the periods $t = 1, 2$, h_c is the human capital of each child, b_1 is a match quality shock drawn in period 1 when a couple meets, α is an altruism factor, β is a time discount coefficient. I do not explicitly model the connection between chid human capital and productivity type, but assume that they are positively related, so that parental utility is increasing with human capital of each child. The human capital of each child is determined by parental investments in education e according to the following technology:

$$h_c = A(e + g)^{\psi}$$

where $A > 0$, $\psi \in (0, 1)$, $g > 0$ is a government education subsidy.

If married in the second period, agents do not have children and make only consumption decisions. Therefore, the utility of a couple married in the second period depends on consumption and matching

[4] In a dynastic multi-period setup, the utility of parents would depend on the utilities of their children, which in turn would depend on their consumption and marital decisions. This property makes an analytical solution of the dynamic model non-feasible and motivates the 2 period simplification.

[5] Arbitrary distribution of productivity types makes implications of taxation and education policies for marriage patterns and intergenerational income mobility substantially less tractable.

quality only:

$$u^{HM_2}(i,j,c_2,b_2) = log(c_2) + b_2,$$

$$c_2 \leq y_i + y_j.$$

The assumption of $b_2 \geq 0$ guarantees that all single agents marry in the second period.

1.4.1 Matching assumptions

To make obtaining analytically-tractable results feasible, I make the following assumptions.

Assumption 1. Both high and low productive individuals always accept the same productivity partners in the first period, while a highly productive individual may reject or accept a low productive candidate in the first period depending on the realisation of matching quality shock b_1. This assumption holds under the following restrictions on utilities:

$$u^{*HM_1}(h,h,b_1=0) > log(y_h - t_h) + \beta\big(V(h) + E(b_2)\big)$$

$$u^{*HM_1}(l,l,b_1=0) \geq log(y_l - t_l) + \beta\big(V(l) + E(b_2)\big)$$

$$u^{*HM_1}(h,l,b_1=0) < log(y_h - t_h) + \beta\big(V(h) + E(b_2)\big)$$

where $V(i)$ is expected utility depending on the household consumption in the second period for a single agent of type $i \in \{h,l\}$, u^{*HM_1} is an optimal utility value defined below.

Denote $b = b_1 - \beta E(b_2)$. Under the conditions above, there is a threshold level b^*, so that for all $b \geq b^*$ high productive individual marries a low productive partner in the first period. By assumption 1, high and low productive individuals who meet partners with the same productivity in the first period match assortatively. Therefore, the degree of assortative matching in the first period is determined by b^* only.[6] The levels of expected second period utilities that sustain this type of equilibrium correspond to:

$$V(i) = \frac{1}{2}u^{HM_2}(i,h,E(b_2)) + \frac{1}{2}u^{HM_2}(i,l,E(b_2))$$

since there are equal shares of high and low productive individuals who remain single at the beginning of the second period, $i \in \{h,l\}$.

1.4.2 A household problem formed in a first period

A couple married in the first period solves the following problem:

$$u^{HM_1}(i,j,c_1,c_2,b_1) = log(c_1) + \alpha log(h_c) + b_1 + \beta log(c_2)$$

[6] Fernandez et al. consider a similar case, but to support this type of equilibrium, they assume that in the second period, single agents are exogenously matched with partners of the same productivity.

$$c_1 + en \leq y_i - t_i + y_j - t_j,$$

$$c_2 \leq y_i + y_j,$$

where $i, j \in \{h, l\}$.

After solving the married couple problem, the optimal level of investments in education e^* and utility u^{*HM_1} become

$$e^* = \frac{\alpha\psi(y_i - t_i + y_j - t_j) - gn}{n(1 + \alpha\psi)},$$

$$u^{*HM_1}(i, j, b_1) = log\left(\frac{y_i - t_i + y_j - t_j + gn}{1 + \alpha\psi}\right) + \alpha log\left(A\left(\frac{\alpha\psi(y_i - t_i + y_j - t_j + gn)}{n(1 + \alpha\psi)}\right)^{\psi}\right)$$

$$+ b_1 + \beta log\left(y_i + y_j\right).$$

1.4.3 The problem of an individual who remains single in the first period

If an agent with productivity type i decides to remain single in the first period, in a second period he or she is matched randomly with a potential partner, draws a match quality shock realisation b_2, does not have children and decides on his or her consumption maximising the following utility function:

$$u^{HS_1}(i, c_1, c_2, E(b_2)) = log(c_1) + \beta\big(V(i) + E(b_2)\big)$$

given budget constraint

$$c_1 \leq y_i - t_i$$

and expected utility $V(i)$ depending on the agent's productivity type, $i \in \{h, l\}$. After solving the single agent problem, optimal utility corresponds to

$$u^{*HS_1}(i, E(b_2)) = log(y_i - t_i) + \beta\big(V(i) + E(b_2)\big).$$

1.4.4 Propositions

The condition under which a high productive individual would marry a low productive individual is defined by the following inequality:

$$u^{*HM_1}(h, l, b_1) \geq u^{*HS_1}\big(h, E(b_2)\big) \ (*)$$

Since for all $b \geq b^*$ a high productive individual would marry a low productive agent, the proportion of married couples formed by different productivity types in first period would constitute $\frac{1}{2}\big(1 - F(b^*)\big)$. Therefore, the lower the b^*, the higher $1 - F(b^*)$, or proportion of "mixed" households, resulting in a lesser degree of assortative matching.

The government budget constraint in the first period corresponds to:

$$\big(1 - \frac{1}{2}F(b^*)\big)gn \leq t_l + t_h$$

Consider two different reforms similar to the quantitative model analysis in the next section. First, assume that education subsidy g is fixed, while tax progressivity is increasing due to higher net transfer $-t_l$ for low productive individuals.

Proposition 1. The degree of assortative matching is decreasing with net transfers to low productive individuals ($-t_l$). For proof, see Appendix B in 1.10.

Optimal parental investments in education e^* is an increasing linear function of net parental income. Since declines in the net tax of low productive individuals is exactly offset by increases in the net tax of his or her high productive partner, the income of mixed couples remains unchanged. In contrast, the net income of couples consisting of matched high/low productive agents and, consequently, their investments in the education of their children decline/increase, respectively. Therefore, intergenerational earnings persistence declines.

Now assume that tax progressivity is fixed, while education subsidy g increases. Denote $t_h = at$, $t_l = t$, $a > 1$, $t > 0$. The fixed tax progressivity assumption is captured by constant parameter a. Education subsidy g can be adjusted through parameter t corresponding to the average taxation level.

Proposition 2. The degree of assortative matching is decreasing with public spendings on education g. The same property applies if a more generous education subsidy is financed by an increase in the tax for high productive individuals instead of an increase in the average taxation level. For proof, see Appendix B in 1.10.

Optimal parental investments in education e^* is a decreasing function of g. Therefore, higher public expenditures lead to lower parental or private investments in the education of children and, consequently, lower intergenerational correlation of earnings.[7]

The results demonstrate that in the current simplified framework, more generous education subsidies and higher net transfers for low productive individuals would always lead to a lower degree of assortative matching and lower intergenerational persistence of earnings.

In a rich life-cycle dynastic framework, connections between public policies, marital sorting and intergenerational persistence of earnings are more complicated, because parents consider the expected future utilities of their children when making decisions. Those utilities in turn depend on the children's marriage market prospects. In the remainder of this paper, I demonstrate that a rich quantitative model correctly replicates the positive correlation between the degree of assortative matching and intergenerational persistence of earnings, when cross-country differences in public policies are accounted for. Moreover, the sum of squared errors in intergenerational earnings elasticity predictions reduces by around one third as opposed to those produced by an analogous model without marital decisions.

[7] This result is analogous to Holter's (2015) finding based on a simplified theoretical example and driven by an assumption of substitutability between consumption and child human capital in the utility function of married couples.

1.5 Model

1.5.1 Setup

The economy is populated by an equal number of males m and females f. A period in the model corresponds to 5 years. An agent's life duration is deterministic and corresponds to 13 periods or 65 years. The first 5 periods are equivalent to the 0-24 years individuals spend in the parental household and do not make economic decisions. By the age of 25, agents start their independent life. There are 8 periods of adult life, starting at 25 - 29 years of age. Each individual receives labor income determined by her or his productivity. Single agents decide whether to get married or wait till next matching round. Marriage occurs only when both individuals agree. If individuals get married before the age of 45 (the 10th lifetime period) they give birth to an exogenously defined number of children and incur time and education costs of raising them. Time costs are exogenous, while expenditures on education are chosen by parents, who are altruistic towards their children. There are no divorces in the economy. Additionally, there are no births outside of wedlock.

The simplifying assumptions of an absence of divorces and single-parent families dramatically reduce the computational burden of the model. This is because introduction of divorce and single-parenthood options would lead to substantial expansion of the model dimension related to inclusion of child birth timing as a choice and child human capital and abilities as extra state variables. Moreover, according to the U.S. Census Bureau, Survey of Income and Program Participation (SIPP), 2008 Panel, the share of currently divorced individuals in the overall population is relatively low, at 9.1% for males and 11.3% for females in the U.S. in 2009.[8] According to the 1996 and 2000 waves of LIS data used for model calibration, the share of divorced individuals is about 13% of the 25-65 years old population. The estimate is calculated as the proportion divorced or separated individuals in the sample. Other possible statuses include "married", "never married/not in union", where "married" or "union" refers not only to de jure but also de facto situations, and "widowed" (about 1.5%).

On the one hand, the proportion of single parents in the data is also relatively low in the years close to the model calibration period. Using PSID data, Regalia and Rios-Rull (2001) finds that, the share of single mothers is 13.7% in 2001. According to the OECD Family Database 2011 data, the proportion of single-parent households is around 11% in the U.S. An average estimate for OECD countries is around 8%.[9] On the other hand, according to Iacovou & Skew (2010), the percentage of children living in single-parent households constitutes 25.8% in the U.S. and 14.9% in OECD countries in 2007. Nevertheless, the focus of this paper is on the role of assortative matching and its interaction with taxation and education policies, rather than

[8] https://www.census.gov/prod/2011pubs/p70-125.pdf, the average proportion of individuals who have ever been divorced among the adult population is 22.4% for females and 20.5% for males.

[9] http://www.oecd.org/els/family/database.htm, only couple and single parent or single adult households are considered; "other" household types are excluded from the calculations.

on the role of single parenthood in explaining cross-country differences in intergenerational persistence of earnings. Omitting divorce and remarraige but allowing the possibility of never-married single parenthood would be computationally feasible. However, according to the U.S. Census, the share of children born to never-married women is 7.1%.[10] Therefore, assigning around 25% of children to never-married single-mother families may be an over-assumption. Overall, consideration of single-parent households could amplify the gap in intergenerational earnings correlations between the U.S. and European countries.

For simplicity, retirement age is not modelled in the current framework, because marital decisions as well as investments in human capital of the children are typically made before retirement age. Only same age individuals can get married. Given that the duration of each period is 5 years, this is a realistic assumption.

1.5.2 Households of married couples

At each period, single individuals meet each other on the marriage market. Marriage can start at any period $i = 1, .., 8$ of adult life. Once met, a couple draws a match-specific bliss shock $b(i) \in N\big(\bar{b}(i), \sigma_b^2\big)$ where $\bar{b}(i)$ is a mean, and σ_b is a standard deviation of the corresponding bliss shock distribution. The mean $\bar{b}(i)$ depends on life period i when potential marriage may start.

$$\bar{b}(i) = \begin{cases} b_y, & \text{if } 1 \leq i \leq 4, \\ b_o, & \text{if } 5 \leq i \leq 8. \end{cases}$$

An assumption $b_y \geq 0$ guarantees that getting married young and having children may be desirable given the negative sign of the expected utility of children due to $\sigma = 2$ determining intertemporal elasticity of substitution in the CRRA utility function.[11] The bliss shock is constant during the marriage. Parameter b_o is normalised to 0. As demonstrated in the calibration section, once the proportion of individuals who got married young (under 45) is matched by calibrating b_y along with other parameters and setting $b_o = 0$, the overall proportion of married agents - potential calibration target for $b_o = 0$ is very close to the U.S. data. The standard deviation of σ_b affects the degree of assortative matching. Higher values of the standard deviation would lead to a lower degree of assortative matching due to higher importance of non-income factors for the marital decision.

Given the realisation of a bliss shock, each partner's characteristics including ability a_g and productivity p_g, $g \in \{f, m\}$, and life period i when individuals meet, a couple solve the following problem if marriage takes place.

[10] The percentage is calculated for the cohort of women aged 45-50 in 2018 which roughly corresponds to parents in a model economy: see Table 2 at https://www.census.gov/data/tables/2018/demo/fertility/ women-fertility.html.

[11] This assumption is analogous to Regalia and Rios-Rull's (2001) assumption of direct utility function Ω added to the expected value function of children when agents grow older.

1.5.2.1 Agents who marry in life periods 1-4 (ages 25 - 44)

Agents who get married in periods 1-4, have children. For simplicity, fertility is not modelled in the current framework,[12] and, there is no option to be childless. All married couples give birth to n children during the first period of their marriage. The first 4 periods of married life are devoted to raising and educating children. If agents marry in the life time period i, then the duration of married life corresponds to $9-i$ periods or 25 - 40 years.

First period of marriage

Agents incur time costs of raising 0-4 year old children. No investments in education are made in this period. Utility V^{HM} of a household consisting of a married couple and their children is defined as follows:

$$V^{HM}\left(i, j, p_m, a_m, p_f, a_f, b(i)\right) = \max_{c} \quad \left\{u(c, b(i)) + \beta E_{a_c}\left[V^{HM}(i, j', a_c, p_m, p_f, b(i)|a_m, a_f)\right]\right\}$$

$$\text{s.t.} \quad c = \xi(i,j)\left[wgp_f(1-\omega)(1-\tau[wgp_f(1-\omega)]) + wp_m(1-\tau[wp_m]) + 2tr\right]$$

where $i = 1, .., 4$ is an adult life period when marriage starts, $j = 1$ is a period of marriage, $j' = j+1$, p_m, a_m, p_f, a_f are male and female productivity and ability shocks realisations respectively, c is consumption of the household, g is the gender wage gap, ω is the time costs of raising n children, $\tau[y]$ is a tax function depending on family type (married couple with or without children or a single person) and an agent's labor income y, tr is a transfer, a_c is the ability of children, the utility of the household formed by a married couple is defined as:

$$u(c, b(i)) = \frac{c^{1-\sigma}}{1-\sigma} + b(i)$$

where $\sigma > 0$, $b(i)$ is a matching bliss shock specified above, $\xi(i,j)$ is an adult equivalence scale parameter which depends on the life period when marriage takes place (determined by both i and j). This parameter captures economies of scale in household consumption and takes the following functional form, as in Greenwood et al. (2003):

$$\xi(i,j) = \begin{cases} \left(\frac{1}{2+qn}\right)^{\chi}, & \text{if } 1 \le i \le 4 \text{ and } j \le 5, \\ \left(\frac{1}{2}\right)^{\chi}, & \text{if } 5 \le i \le 8 \text{ or } j > 5. \end{cases}$$

In the case of standard parameters values $q = 0.3$, $\chi = 0.5$ and number of children per family $n > 0$, equivalence scale parameter $\xi > 0.5$ provides the economic motive for marriage.

Child abilities are correlated with the abilities of both parents and follow a standard assumption of an autoregressive process:

$$\log(a_c) = \rho_a \log(a_p) + \epsilon_a, \;\; \epsilon_a \in N(0, \sigma_a)$$

[12] Introduction of endogenous fertility would substantially increase the computational burden of the model. Moreover, there is a lack of data on fertility-income profiles for OECD countries (except for the U.S.) needed for evaluation of the model is performance.

where ρ_a is a parameter responsible for intergenerational correlation of abilities, a_p is average parental abilities corresponding to $\frac{a_m+a_f}{2}$, ϵ_a is a normally distributed random component.

Second - fourth periods of marriage

Agents make decisions on investments in primary and secondary education of their children.

$$V^{HM}(i,j,a_c,p_m,p_f,b(i)) = \max_{c,e_{j-1}} \quad \{u(c,b(i)) + \beta V^{HM}(i,j'=j+1,a_c,p_m,p_f,b(i),h_{c,j-1})\}$$

$$\begin{aligned} \text{s.t.} \quad & c = \xi(i,j)\big[wgp_f(1-\tau[wgp_f]) + wp_m(1-\tau[wp_m]) + 2tr - ne_1\big] \\ & h_{c,j-1} = h_{c,j-2} + a_c\big[h_{c,j-2}(e_{j-1}+\tilde{g}(j))\big]^{\psi_0} \end{aligned}$$

where $i = 1,..,4$, $j = 2,3,4$, e_1 are parental investments in primary education, $\{e_{j-1}\}_{j=3}^{4}$ are parental investments in the first and second periods of secondary education, n is the number of children, $h_{c,0}$ is a minimum level of human capital, $0 < \psi_0 < 1$ is a parameter determining the curvature of the human capital production function in the period of primary and secondary education, $\tilde{g}(j=2) = g_1$, is government subsidies for primary education, $\tilde{g}(j=2,3) = g_{2,3}$, is public spendings on secondary education.

As in Restuccia & Urrutia (2004) and Holter (2015), this paper treats private (parental) investments in child human capital as a locally funded education expenditure. Central and regional components contribute to "pure" public expenditures on education. Public and private funding are perfect substitutes in this model. This is a realistic assumption, as education spendings are interpreted as monetary investments. Parental time investments in child human capital are not explicitly modelled in this paper and are assumed to be a part of the abilities correlated across generations.

Fifth period of marriage

Parents make decisions on whether to send their children to college and, if yes, how much to spend on their eduction.

$$\begin{aligned} V^{HM}(i,j,a_c,p_m,p_f,b(i),h_{c,j-2}) = \max_{c,e_{j-1}} \quad & u(c,b(i)) + \gamma\big[(1-\delta(a_c,h_{c,3}))\{0.5 \sum_{k\in m,f} E_{z_c}V_k(p_c^0,a_c)\} \\ & + \delta(a_c,h_{c,3})\{0.5 \sum_{k\in m,f} E_{z_c}V_k(p_c^1,a_c)\}\big] \\ & + I(i<4)\beta V^{HM}(i,j'=j+1,p_m,p_f) \end{aligned}$$

$$\begin{aligned} \text{s.t.} \quad & c = \xi(i,j)\big[wgp_f(1-\tau[wgp_f]) + wp_m(1-\tau[wp_m]) + 2tr - ne_4\big] \\ & p_c^1 = z_c h_{c,4}^1, \; p_c^0 = z_c h_{c,4}^0 \\ & h_{c,4}^1 = h_{c,3} + a_c\big[h_{c,3}(e_4+g_4)\big]^{\psi_1}, h_{c,4}^0 = h_{c,3} \\ & \delta(a_c,h_{c,3}) = 1 - exp(\theta a_c h_{c,3}) \end{aligned} \tag{1.1}$$

where $i = 1,..,4$, $j = 5$, $I(i<4) = 1$ if $i < 4$ and 0 otherwise, so that for agents who got married in life period 4, the 5th period of marriage is the last life period, and also the last period of marriage, $\gamma > 0$ is parental altruism, $\delta(a_c,h_{c,3})$ is the probability of college completion which increases with a child's abilities a_c

and human capital accumulated by the time of high school completion $h_{c,3}$, $\theta < 0$, V_k, $k \in \{m, f\}$, are value functions of male m and female f individuals at the time they enter adult life, defined below. For simplicity it is assumed that parents are not aware of their children's gender and have equal proportions of male and female children. Therefore, the probability of a child being female or male equals $\frac{1}{2}$.[13] By the time of leaving the parental household, children draw labor market luck shock z which determines their productivity p in their adult lives along with human capital $h_{c,4}$.

Periods six to eight of marriage

The duration of marriage for agents who got married in the period $i = 1, 2, 3$ of their adult lives corresponds to 8, 7 and 6 periods respectively. After the 5th period, children leave the household so their parents do not make any economic decisions and their utility depends only on consumption as defined below:

$$\begin{aligned} V^{HM}(i, j, p_m, p_f, b(i)) = \quad & u(c, b(i)) + \beta V^{HM}(i, j' = j + 1, p_m, p_f) \\ \text{s.t.} \quad & c = \xi(i, j)\big[wgp_f(1 - \tau[wgp_f]) + wp_m(1 - \tau[wp_m]) + 2tr\big] \\ & 5 < j < J(i) \end{aligned}$$

$$\begin{aligned} V^{HM}(i, j, p_m, p_f, b(i)) = \quad & u(c, b(i)) \\ \text{s.t.} \quad & c = \xi(i, j)\big[wgp_f(1 - \tau[wgp_f]) + wp_m(1 - \tau[wp_m]) + 2tr\big] \\ & j = J(i) \end{aligned}$$

where $i = 1, 2, 3,\ J(i) = 9 - i$ is the last period of marriage.

1.5.2.2 Agents who merry in life periods 5-8 (ages 45 - 64)

Agents who get married in life periods 5-8, corresponding to ages 45 - 64, do not have children. This assumption is realistic since typically women complete their fertility decisions by the age of 45. Consequently, no economic decisions are made and for each marriage period utility is defined as follows.

$$\begin{aligned} V^{HM}(i, j, p_m, p_f, b(i)) = \quad & u(c, b(i)) + \beta V^{HM}(i, j' = j + 1, p_m, p_f) \\ \text{s.t.} \quad & c = \xi(i, j)\big[wgp_f(1 - \tau[wgp_f]) + wp_m(1 - \tau[wp_m]) + 2tr\big] \\ & j < J(i) \end{aligned}$$

$$\begin{aligned} V^{HM}(i, j, p_m, p_f, b(i)) = \quad & u(c, b(i)) \\ \text{s.t.} \quad & c = \xi(i, j)\big[wgp_f(1 - \tau[wgp_f]) + wp_m(1 - \tau[wp_m]) + 2tr\big] \\ & j = J(i) \end{aligned}$$

where $i = 5, .., 8,\ J(i) = 9 - i$ is the last period of marriage.

[13] This simplifying assumption is analogous to one from Regalia & Rios-Rull's (2001) paper. Due to the presence of a gender wage gap, returns on investments in education are different for male and female children. Consequently, the dimensionality of the problem would grow substantially if parents were aware of their children's gender and deciding on investments in education of female and male children separately.

1.5.3 Single adult households

Every period a single adult household meets another single male $k = m$ or female $k = f$ individual characterised by productivity p' and abilities a' with probability $\frac{\Omega_k(i+1,p',a')}{\int_{x'} d\Omega_k(i+1,p',a')}$, $x' = \{p', a'\}$, $k \in \{f, m\}$ where $\Omega_k(i+1, p', a')$ is a non-normalised distribution of single individuals across productivity and ability types in the period $i + 1$.

For a male individual with productivity p and ability a the utility of being single in period i is defined as follows:

$$\begin{aligned} V_m^{HS}(i,p,a) = \quad & u^{HS}(c) + \beta \int_{b'} \int_{x'} \Big[i_m(i+1,p,a,p',a',b') V^{HM}(i+1,1,p,a,p',a',b') + \\ & + [1 - i_m(i+1,p,a,p',a',b')] V_m^{HS}(i+1,p,a) \Big] \frac{d\Omega_f(i+1,p',a')}{\int_{x'} d\Omega_f(i+1,p',a')} dF_i(b') \\ \text{s.t.} \quad & c = wp(1 - \tau[wp]) + tr \end{aligned}$$

For a female individual with productivity p and ability a the utility of being single in period i is defined as follows:

$$\begin{aligned} V_f^{HS}(i,p,a) = \quad & u^{HS}(c) + \beta \int_{b'} \int_{x'} \Big[i_m(i+1,p',a',p,a,b') V^{HM}(i+1,1,p',a',p,a,b') + \\ & + [1 - i_m(i+1,p',a',p,a,b')] V_f^{HS}(i+1,p,a) \Big] \frac{d\Omega_m(i+1,p',a')}{\int_{x'} d\Omega_k(i+1,p',a')} dF_i(b') \\ \text{s.t.} \quad & c = wgp(1 - \tau[wgp]) + tr \end{aligned}$$

where $i_m(i+1, p, a, p', a', b)$ indicates a positive marital decision between a male with productivity p and abilities a and a female with productivity p' and abilities a' in life period $i+1$, b' is a realisation of matching bliss shock, $F_i(b')$ is corresponding CDF that depends on the period i when a potential marriage may start, $i = 1, .., 8$.

$$i_m(i+1,p,a,p',a',b') = \begin{cases} 1, & \text{if a male with } p, a \text{ marries a female with } p', a' \text{ given } b', \\ 0, & \text{otherwise.} \end{cases}$$

The one-period utility function $u^{HS}(c)$ of single households is defined as:

$$u^{HS}(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where $\sigma < 1$.

In the last life period $i = 8$ for both genders, utility is specified as follows.

$$\begin{aligned} V_k^{HS}(i,p_k,a_k) = \quad & u^{HS}(c) \\ \text{s.t.} \quad & c = \begin{cases} wp_m(1 - \tau[wp_m]) + tr, & \text{if } k = m, \\ wgp_f(1 - \tau[wgp_f]) + tr, & \text{if } k = f. \end{cases} \end{aligned}$$

1.5.4 Value functions of agents at the beginning of the first adult life period

Since agents are altruistic towards their children, the utility of a household consisting of a married couple depends on the expected value functions of their children at the moment when they enter adult life. The value function of a young individual of gender $k \in \{m, f\}$ with productivity p and ability a at the beginning of the first period of adult life is defined as:

$$
\begin{aligned}
V_m(p,a) = \quad & \int_b \int_x [i_m(1,p,a,p',a',b)V^{HM}(1,1,p,a,p',a',b)+ \\
& [1 - i_m(1,p,a,p',a',b)]V_m^{HS}(1,p,a)] \\
& \frac{d\Omega_f(1,p',a')}{\int_{x'} d\Omega_f(1,p',a') = 1} dF_1(b) \\
V_f(p,a) = \quad & \int_b \int_x [i_m(1,p',a',p,a,b)V^{HM}(1,1,p',a',p,a,b)+ \\
& [1 - i_m(1,p',a',p,a,b)]V_f^{HS}(1,p,a)] \\
& \frac{d\Omega_m(1,p',a')}{\int_{x'} d\Omega_m(1,p',a') = 1} dF_1(b)
\end{aligned}
$$

1.5.5 Marital decisions

Given the realisation of a matching bliss shock b, a male with productivity p and ability a marries a female with productivity p' and ability a' if the utility of being single is less than or equal to the utility of being married for both individuals:

$$
\begin{aligned}
& V_m^{HS}(i,p,a) \leq V^m(i,j=1,p,a,p',a',b) \\
& V_f^{HS}(i,p',a') \leq V^m(i,j=1,p,a,p',a',b)
\end{aligned} \tag{*}
$$

$$
i_m(i,p,a,p',a',b) = \begin{cases} 1, & \text{if (*) is satisfied,} \\ 0, & \text{otherwise.} \end{cases}
$$

1.5.6 Stationary distributions

The non-normalised distribution of single male and female individuals across productivity and ability types in the period $i = 1$ is determined by the stationary distribution of young agents across human capital $h_{c,4}$ levels acquired by the moment of entering adult life and realisation of market luck z and abilities a shocks. The productivity is defined as $p = zh_{c,4}$.

Denote $X = A \times P$, where A and P are sets of possible values of abilities and productivities respectively, $x = \{a, p\} \in X$, $x' = \{a', p'\} \in X'$, $X' = X$. The non-normalised distribution of single male and female individuals across productivity and ability types in the periods $2 \leq i \leq 8$ is given by $\Omega_m(i,p,a)$ and $\Omega_f(i,p,a)$ respectively.

$$
\Omega_m(i,x) = \int_b \int_X^x \int_{X'} [1 - i_m(i-1,x,x',b)] \frac{d\Omega_f(i-1,x')}{\int_X d\Omega_f(i-1,x')} d\Omega_m(i-1,x) dP(b)
$$

$$\Omega_f(i,x) = \int_b \int_X^x \int_{X'} [1 - i_m(i-1,x',x,b)] \frac{d\Omega_m(i-1,x')}{\int_X d\Omega_m(i-1,x')} d\Omega_f(i-1,x)dP(b)$$

Similarly, non-normalised distribution of couples of males of type $\bar{x} = \{\bar{p}, \bar{a}\}$ and females of type $\bar{x}' = \{\bar{p}', \bar{a}'\}$ who draw realisation b of matching bliss shock and who got married in the period i is given by $G(i+1, \bar{x}, \bar{x}', b)$.

$$G(i+1, \bar{x}, \bar{x}', b) = \int_b \int_X^{\bar{x}'} \int_X^{\bar{x}} i_m(i,x,x',b) d\Omega_m(i,x) d\Omega_f(i,x') dP(b).$$

Normalise the size of female and male cohorts who enter adult lives in each period of the model economy to 1.[14] Since there is no random mortality in the economy, the size of the cohort remains stable till the end of the life-time period, and the sum of female or male individuals who got married and those who remained single in each life period $i = 1,..,8$ is equal to 1:

$$\int_b \int_X \int_X G(i,x,x',b)dxdx'db + \int_X \Omega_k(i,x)dx = 1; k \in \{f,m\}$$

1.5.7 Government

The government collects income taxes to finance uniform transfers tr, lump-sum education subsidies $g_i, i = 1; \{2,3\}; 4$ and exogenous government expenditures G. Taxes are defined according to the following formula from Guvenen et al. (2011). For an individual with labor income y, the tax rate τ is specified by the following equation:

$$\tau(y) = \tau_1 \left(\frac{y}{\bar{y}}\right)^{0.2} + \tau_2 \left(\frac{y}{\bar{y}}\right)^{0.4} + \tau_3 \left(\frac{y}{\bar{y}}\right)^{0.6} + \tau_4 \left(\frac{y}{\bar{y}}\right)^{0.8}$$

The estimates of parameters τ_1,.., τ_4 for 11 OECD countries and different family types are from Holter (2015): singles without children, married couples without children, and married couples with one child corresponding to families with children in this model; see table 1.11.2 in Appendix C in 1.11.

Following Holter (2015), I assume lump-sum educational subsidies. Subsidies for primary education are denoted as g_1, for the first and second periods of secondary education, subsidies are assumed to be identical and are denoted as $g_{2,3}$, for tertiary education, subsidies are denoted as g_4. This functional form choice facilitates comparability of the results in this paper with those obtained by Holter. The education subsides are estimated based on UNESCO data[15] on government expenditures per student as % of GDP per capita, and OECD "Education at a Glance"[16] data on the sources of public educational funds; see results in table 1.11.4, Appendix C in 1.11.[17]

[14] Population growth determined by fertility rates is taken into considerations for calculations of aggregate values.

[15] http://data.uis.unesco.org/

[16] https://www.oecd.org/education/education-at-a-glance/

[17] Holter (2015) evaluates education subsidies based on UNESCO and OECD data from 2000-2005. However, there are no estimates provided for Canada, Sweden and Germany due to unavailability of the data. In this paper, I update the estimates and fill in the gaps for Canada and Germany based on the latest data from 2006 - 2010.

Exogenous government expenditures G are set equal to 16.1 % of GDP. This estimate is obtained by subtracting total government spendings on education (2.9%[18] of GDP) from 19% corresponding to the estimate of total government spending (including expenditures on education) provided by Krusell & Rios-Rull (1999).

1.5.8 Stationary equilibrium

A stationary equilibrium consists of t_0 population size N_0, cohort growth rate ζ, average earnings $\bar{y}$, government policies including parameters determining labor income taxes τ, education subsidies $g_i, i = 1; \{2,3\}; 4$ and transfer tr defined in subsection 1.5.7, equilibrium wage $w = 1$, a set of value functions $V^{HM}(i, j = 1, p_m, a_m, p_f, a_f, b(i))$, $V^{HM}(i, j = 2, ..5, a_c, h_{c,j-2}, p_m, p_f, b(i))$, $V^{HM}(i, j = 6, ..J(i), p_m, p_f, b(i))$ for couples who get married in periods $i = 1, ..4$; $V^{HM}(i, j = 1, ..J(i), p_m, p_f, b(i))$ for agents who get married in periods $i = 5, ..8$, $J(i) = 9 - i$; $V_k^{HS}(i, p, a)$ for single individuals; $V_k(p, a)$ for young individuals entering adult lives, $k \in \{f, m\}$; marital decision rules $i_m(i, p_m, a_m, p_f, a_f, b(i))$; parental decision rules regarding investments in the human capital of their children; non-normalised stationary distributions of married and single individuals $G(i, p_m, a_m, p_f, a_f, b(i))$ and $\Omega_k(i, p, a)$, $k \in \{f, m\}$ for $i = 1, .., 8$ such that:

a) value functions V^{HM} and parental decision rules regarding investments in the human capital of their children solve the problem of a household consisting of married couples specified in the subsection 1.5.2 given the value functions of young individuals entering their adult lives V_k, $k \in \{f, m\}$, education subsidies $g_i, i = 1; \{2,3\}; 4$, taxes τ, and transfer tr;

b) value functions V_k^{HS}, $k \in \{f, m\}$ solve the problem of households consisting of single individuals specified in subsection 1.5.3 given marital decision rules i_m, value functions V^{HM} of the households consisting of married couples and non-normalised stationary distributions of single individuals Ω_k, $k \in \{f, m\}$, taxes τ, and transfer tr;

c) value functions of young individuals entering their adult lives V_k, $k \in \{f, m\}$ is defined as specified in subsection 1.5.4 taken V^{HM}, V_k^{HS}, $k \in \{f, m\}$, marital decision rules i_m and non-normalised stationary distributions of single individuals Ω_k, $k \in \{f, m\}$ as given;

d) marital decision rules i_m are defined as in subsection 1.5.5 given V^{HM}, V_k^{HS}, $k \in \{f, m\}$;

e) non-normalised stationary distributions of single individuals Ω_k, $k \in \{f, m\}$, and married couples G follow recursive rules defined in subsections 1.5.6 and 1.5.7, given marital decision rules i_m and the distribution of young individuals across productivity and ability types;

f) stationary distribution of young individuals entering their adult lives across productivity and ability types is determined by parental decision rules on investing in the human capital of their children and the stochastic process of intergenerational transmission of abilities a and stationary distribution of labor market luck shock z;

[18] Education at a Glance, 2005.

g) the government budget is balanced and the growth rate of the cohort is determined by individuals' marital decisions.

1.6 Calibration

The parameters of the model can be divided into two groups. The first group includes parameters that are set exogenously, taking standard values or values estimated by the results of empirical studies. The second group consists of parameters that are calibrated jointly, so that the model matches key moments in the U.S. economy. The parameters are summarised in table 1.6.1 below.

Table 1.6.1: Exogenously set parameters from external sources

Parameter	Value	Description	Source
g_1 (primary)	0.101	Public expenditures on education	UNESCO & OECD
$g_{2,3}$ (secondary)	0.114		2000-2005
g_4 (tertiary)	0.209		
ω	1	Time cost of children	De la Croix & Doepke (2003)
g	0.21	Gender earnings gap	OECD, https://data.oecd.org
n	2.87	Number of children per married couple	Greenwood et al. (2003)
χ	0.5	Consumption equivalence parameter	Greenwood et al. (2003)
q	0.3		Greenwood et al. (2003)
t_c^1	0.8	Time costs, college	Restuccia & Urrutia (2004)
t_c^0	0.4	Time costs, college, dropouts	Restuccia & Urrutia (2004)

Public expenditures on education are evaluated based on government expenditures per student expressed as a percentage of GDP per capita provided by the World Bank, World Development Indicators, 2000 - 2005. The information on initial sources of public educational funds and final purchasers of educational resources by level of government for primary, secondary and tertiary (tables B4.3a and b, Education at a Glance, 2004) is utilised to evaluate central and regional expenditures treated as public spending on education and local expenditures treated as private spending, as in Restuccia & Urrutia (2004).

The number of children per family is from Greenwood et al. (2003), who estimate fertility rates for married women based on PSID data. The data covers 1983-1990 and provides information on the number of children born. The estimation period is similar to that used to evaluate intergenerational persistence of earnings by Solon (2004). Since a non-negligible proportion of females in the sample were still of fertile age, the regression was used to predict the number of children borne by women by age 44. The estimated number of children ever borne by married women is utilised as an estimate of the number of children per family, since no out-of-wedlock births are allowed in the model.[19]

Parameters χ and q translate household income into consumption per adult family member, taking scale effects into consideration. Following Greenwood et al. (2003), this study assumes intermediate values of those estimates reported by the literature.

De la Croix & Doepke (2003) employ the estimate of time costs of raising children as equal to 2.25 years per child. Given that the number of children born is set to 2.87 for each family as in Greenwood et al. (2003), this would imply total costs equal to 6.46 years in the model economy. Assigning non-zero time costs of children to the second period of marriage (years 5-9) would non-negligibly increase the computational burden of the model because, instead of applying the overall income of a couple as a state variable in the household problem for the second period of marriage, one would need to keep track of male and female productivities separately. Therefore, for the sake of simplicity the fraction of time spent on children is set equal to 1 in the first period of marriage, which corresponds to 5 years.

Following Greenwood et al. (2015), I assume that females earn a fraction of the salary of a male with equivalent productivity. This fraction corresponds to 1 minus the gender wage gap g. The OECD provides an estimate of the gender wage gap, defined as the difference between median earnings of women relative to the median earnings of men (see https://data.oecd.org/earnwage/gender-wage-gap.htm). The data refers to full-time employees.

Time costs of attending college t_c are set equal to 0.8, which implies 4 years corresponding to the standard duration of tertiary education for students who complete college. For students who drop out, time costs of college correspond to 0.4, which is equivalent to 2 years. This assumption is in line with the estimate employed by Restuccia & Urrutia (2004).

The remaining set of parameters presented in table 1.6.2 below is calibrated jointly so that the model replicates key characteristics of the U.S. economy. I minimise the sum of squared deviations of the statistics (corresponding to the targets in the table below) predicted by the model from corresponding data moments. I denote the parameter vector as $\Psi = (h_{min}, \phi_0, \phi_1, \gamma, b_y, \sigma_b, \theta, \sigma_a, \rho_a, \sigma_z)$, x_i as statistics simulated in the model, $\bar{x_i}$ as its empirical analog, $i = 1, .., 10$, the calibration procedure can be formulated as follows:

$$\Psi^* = \arg\min_\Psi \sum_{i=1}^{10} \left(\frac{x_i(\Psi) - \bar{x_i}}{\bar{x_i}} \right)^2 .$$

Standard errors for the parameters obtained are not provided due to unavailability of estimates for variances of empirical moments.

As the model is highly nonlinear, there is no one-to-one correspondence between model parameters and statistical targets. Therefore, parameters are assigned to calibration targets based on the principle of sensitivity. Parameters of the human capital production function and of parental altruism γ affect parental

[19] Given the estimated share of married individuals at 69.5% and assuming no out-of-wedlock births, the average number of ever-born children per woman in the model is equal to 2.87*0.695 + 0*0.305 = 1.995. According to the World Bank data, available at https://data.worldbank.org/, the total number of births per woman fluctuates between 1.97 and 2.08, with an average of 2.02. Therefore, the current framework does not inflate the average fertility rate.

incentives to invest into the education of their children. Parameters ϕ_0, ϕ_1 and h_{min} influence returns on investments in education and, therefore, correspond to the shares of private spending on pre-university and university education, and university attendance ratios, respectively. Parameter γ determines the importance of the expected utility of children for parents. Because child utility is positively affected by parental investments into children's education, this parameter is assigned to the university premium. Parameter θ determining the probability a child will fail at university corresponds to the university drop out rate.

Parameters affecting ability and labor market luck shocks influence inequality and intergenerational persistence of earnings. The standard deviation of the ability shock σ_a affects the variance of the ability shock and, consequently, the contribution of human capital to earnings variations. Therefore, this parameter is assigned to the share of human capital in the earnings variance. Parameter ρ_a is responsible for intergenerational correlation of abilities and positively affects intergenerational persistence of earnings and, therefore, is assigned to this calibration target. The standard deviation of the labor market luck shock σ_z affects inequality and, consequently, corresponds to log 90 to 10 ratio calibration target.

Parameters b_y and σ_b determine agents' incentives to marry and, consequently, are responsible for marriage market statistics. Parameter b_y positively affects individuals' incentives to marry at a young age and, therefore, is assigned to the share of young married couples. The standard deviation of the match quality shock σ_b influences a random component in agents' marital decisions. Consequently, this parameter is assigned to the degree of assortative matching.

Table 1.6.2: Endogenously calibrated parameters and benchmark model fit

Name	Value	Description	Target	Data	Model
ϕ_0	0.585	Human capital, before university	Private spending, prior-university	0.493	0.485
ϕ_1	0.495	Human capital, in university	Private spending, university	0.631	0.600
h_{min}	1.700	Minimum level of human capital	University attendance ratio	0.530	0.535
γ	0.365	Parental altruism	University premium	1.75	1.771
θ	-0.155	Parameter affecting university failure	University drop out rate	0.321	0.319
σ_a	0.388	Std. of ability shock	Share of h.c. in earnings variance	0.615	0.621
ρ_a	0.125	Autocorrelation, ability shock	IEE	0.470	0.484
σ_z	0.360	Std. deviation, labor market shock	Log 90 to 10 ratio	1.545	1.535
b_y	0.057	Additional value of marriage, young	Share of young married agents	0.695	0.698
σ_b	0.005	Std. deviation of match quality	Degree of assortative matching	0.605	0.602

The benchmark model fit for endogenously calibrated parameters is presented in 1.6.2 above[20]. The results demonstrate that the model captures the U.S. economy quite accurately. Moreover, the marriage

[20] Table 1.11.1 in Appendix C in 1.11 presents the datasources for estimation of U.S. statistics. A slight imprecision in the matching of certain calibration targets may be explained by the discrete nature of the model. Productivity, human capital, labor market luck shock and ability grids sizes are set at the levels that allow for avoidance of grid-dependence of the model solution, and at the same time guarantee reasonable computational time.

market parameters including the degree of assortative matching and the share of married individuals below the age of 45 is matched relatively precisely. Nevertheless, even though parameter b_o (determining the mean of the marriage bliss shock for older individuals) is normalised to 0, the model captures the overall share of married individuals adequately: 0.801 in the model versus 0.772 in the data.

1.7 Results

Employing the framework of marital sorting and human capital formation presented above, I evaluate whether accounting for marital sorting and public policies may improve the performance of the otherwise standard life-cycle model in explaining cross-country differences in the intergenerational persistence of earnings. I find that accounting for differences in taxation and education policies between the U.S. and 10 OECD countries is sufficient to replicate a positive relationship between the degree of assortative matching and intergenerational persistence of earnings. Moreover, the model provides more accurate predictions of intergeneration earnings correlation in 11 OECD countries. The sum of squared errors reduces by about one third compared to Holter's model, which does not account for the marriage market. The results of the decomposition analysis in sections 1.7.1 and 1.7.2 show that improvement in the model performance may be partially driven by a simplifying assumption of an absence of inter vivo transfers and savings in the economy. Nevertheless, a positive degree of assortative matching is crucial for the model's ability to produce realistic levels of intergenerational correlation of earnings observed in OECD countries.

1.7.1 Explaining cross-country differences in assortative matching and intergenerational earnings persistence

I evaluate the model's performance in explaining cross-country differences in the degree of assortative matching and intergenerational persistence of earnings if differences in national public policies are accounted for. As in Holter (2015), 10 OECD countries are analysed. I start by introducing taxation and education policies of the 10 countries into the model, while keeping the rest of the parameters as in the benchmark (U.S.) economy: see policies parameters in tables 1.11.2 and 1.11.4, Appendix C in 1.11. Then I discuss the model's performance in replicating key stylised facts discussed in the section 1.3. To disentangle the effects of different policies, I repeat a similar exercise by replacing only taxes or education subsidies with corresponding country specific policies.

Taxation and education subsidies

The model correctly replicates the relationships between intergenerational persistence of earnings, income inequality, tax progressivity and average tax levels. The model predicts a strong negative correlation between intergenerational persistence of earnings and average tax levels (-0.68 in the model vs. -0.7 in the data); intergenerational persistence of earnings and the tax progressivity wedge (-0.64 in the model vs. -0.6 in the data): see figure 1.7.1 below, subfigures A and B, and table 1.11.3 with correlations in Appendix C in 1.11. As in Holter's model, higher tax levels and progressivity discourage parents from investing in the human capital of their children. Consequently, the share of private contributions to overall investments in human capital decreases and weakens the correlation between parental and child earnings, leading to lower intergenerational persistence of earnings. Moreover, since distribution of private spending on the education of children is compressing, income inequality also decreases. The model correctly predicts a strong negative correlation between income inequality and average tax levels (-0.49 in the model vs. -0.62 in the data) and inequality and the tax progressivity wedge (-0.502 in the model vs. -0.498 in the data). However, the model overestimates the levels of inequality for the countries with the lowest earnings persistence (see figure 1.7.1, subfigure D).

A key novel assumption introduced in this paper is the presence of a marriage market. The model correctly replicates a positive link between intergenerational persistence of earnings and the degree of assortative matching in the data (the correlation in the model is 0.81 vs. 0.82 in the data: see figure 1.7.1, subfigure C).

On the one hand, higher average tax levels, tax progressivity and education subsidisation have negative impacts on the degree of assortative matching. A more generous taxation system typically implies higher levels of transfers, lower after-tax income of more productive individuals and higher after-tax income of less productive agents. Higher after-tax income of low productive individuals may increase the relative benefits of marriage with a low income partner. More generous transfers and education subsidisation[21] may discourage parents from spending on the education of their children and, consequently, reduce the costs or increase the benefits of marriage. Therefore, for a given realisation of matching quality shock, a high productive agent is more likely to marry a low productive individual than in an economy with a less generous tax system and less education subsidisation. Consequently, the degree of assortative matching declines. This directly contributes to lower intergenerational correlation of earnings, through weakening of the income correlation within a couple, and indirectly, through reducing marriage market returns on investments in the human capital of children. This intuition is similar to propositions 1 and 2 for the simple model example in section 1.4.

On the other hand, higher average tax levels, tax progressivity, and education subsidisation may have positive impacts on the degree of assortative matching. As demonstrated in section 1.4, even in a very simple framework, the impact of taxes and education subsidies on the degree of assortative matching can be ambigious. Lower net taxes may increase the relative benefits of single life. More generous education subsidisation and redistribution compress the distribution of agents across productivity levels, implying lower

[21] The degree of education subsidisation is difficult to capture with a single measure such as, for instance, tax progressivity, since public spending on primary, secondary and tertiary education may vary within a given country. For instance, in Scandinavian countries, public expenditures (coming from federal and regional sources) on primary and secondary education are quite moderate, while public expenditures on university education are the highest of the 11 OECD countries. In contrast, in Italy and Spain, the order of these two types of education spending is the opposite. Nevertheless, employing the same data on education subsidies as in this paper, Holter (2015) demonstrates that countries with higher average tax levels and tax progressivity tend to have more generous public expenditures on tertiary education.

pre- and after-tax incomes for the most productive individuals. Lower after-tax income of highly productive agents may increase the marginal benefits of marrying an individual with relatively high productivity. Those factors positively affect the degree of assortative matching.

Nevertheless, the results obtained in this model of marital sorting and human capital formation demonstrate that the resulting impacts of higher average tax levels, tax progressivity and more generous education subsidies on the degree of assortative matching is negative.

Moreover, the model correctly captures the positive correlation between the degree of assortative matching and inequality measured by log P90/P10. This finding is similar to Ferdandez et al. (2005), who demonstrate a positive connection between the degree of marital sorting (measured as the correlation of partners' years of schooling, as in this paper) and the skill premium equivalent to the inequality measure in their model. In contrast to Fernandez at. al. (2005), who explain this pattern though multiple equilibria in a theoretical model, this paper demonstrates that accounting for cross-country differences in taxes and education subsidies is sufficient to generate a positive link between the degree of marital sorting and inequality.

Only taxation

I assume that tax functions are country-specific, while education subsidies expressed as percentage of GDP per capita are fixed at the U.S. level. The model that accounts only for differences in taxation policies, replicates key stylised facts nearly as precisely as the model that accounts for variation in both taxation and education policies (benchmark); see figure 1.11.1 in Appendix C in 1.11. Moreover, the model provides a somewhat more accurate prediction of cross-country differences in intergenerational persistence of earnings than the benchmark model. This happens due to the fact that countries with high persistence of earnings such as Italy, Spain, and France have relatively high proportion of federal and state financing of primary and secondary education, which has negative impacts on the persistence of earnings in the model. If education subsidies are fixed at the U.S. level, the model predicts more precise higher values of intergenerational earnings correlation for those countries.

Only education subsidisation

Now assume that education subsidies expressed as percentage of GDP per capita take country-specific values, while the tax function corresponds to the benchmark U.S. economy. The model in which only differences in education policies are considered has substantially inferior performance compared to the frameworks that account for country-specific taxes. The model overestimates intergenerational persistence of earnings and Log P90/P10 ratios for most countries, confirming the dominant role of cross-country differences in taxation systems in explaining intergenerational correlation of earnings and income inequality patterns. Additionally, the model underestimates the degree of assortative matching across countries, demonstrating the resulting negative impact of more generous education subsidisation on the degree of marital sorting; see figure 1.11.2 in Appendix C in 1.11. In most countries, except the U.K., either primary and secondary or tertiary education subsidisation levels are higher than in the U.S., leading to lower degrees of marital sorting than in the benchmark model.

The results demonstrate that consideration of cross-country differences in taxation is more important for explanation of the key stylised facts than education policies. In subsequent subsections I analyse the importance of marriage markets and positive assortative matching assumptions for model performance.

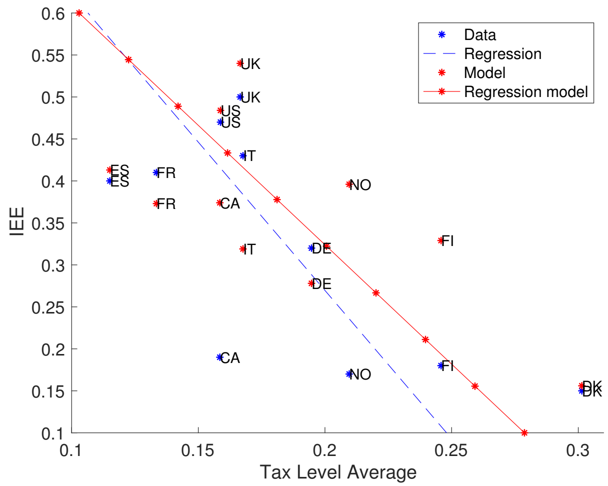
1.7.2 The role of the marriage market assumption

In this paper, for the sake of computational feasibility of the model, labor is the only production factor and investments in human capital of children is the only form of intergenerational transfers. In contrast to Holter's (2015) framework, agents are not able to accumulate savings in the form of capital or to make inter vivos transfers to their children.

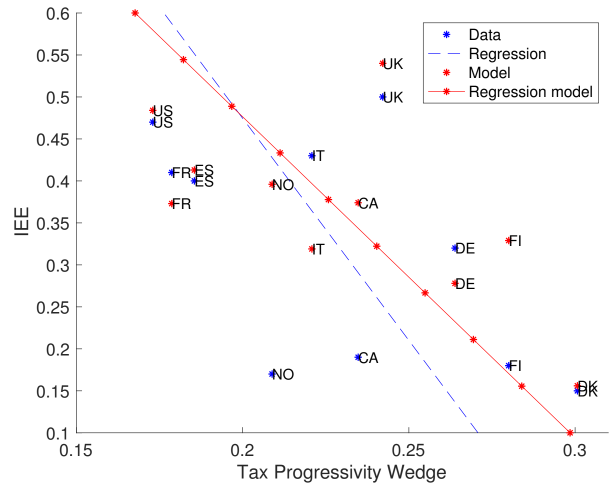
On the one hand, the intuition of the results in this paper would be preserved. If agents could enter adult life with financial transfers from their parents and borrow to finance their education as in Holter's model, individuals would still prefer to marry assortatively due to intergenerational persistence of savings. High-skilled individuals are more likely to enter their adult lives with higher transfers from their parents and, consequently, more resources to finance university education than their lower skilled peers. If redistributive and education policies are less generous, this increases the attractiveness of high-skilled agents on the marriage market even further and amplifies the impact of policies on the degree of assortative matching. Consequently, the degree of assortative matching would be higher in economies with less generous redistributive and education policies.

On the other hand, due to an absence of savings in the model, comparison of this paper's results with Holter's (2015) framework might be misleading. To mitigate this limitation and introduce an "intermediate step" between this model and Holter's setup, I construct an exact counterpart of the benchmark model in this paper, but without marital sorting. The counterpart model assumes an analogous human capital accumulation process and abstracts from the presence of savings in the economy. A household consists of a single agent who enters adult life at the age of 25 and retires at 65. An exogenous number of children arrive in the first period of adult life. The number of children per household is estimated as $\bar{n}/2 = 1$, where $\bar{n}$ is the average number of children per women in a benchmark model. Investments in the human capital of children are made in the 2nd-5th periods of the agent's life.

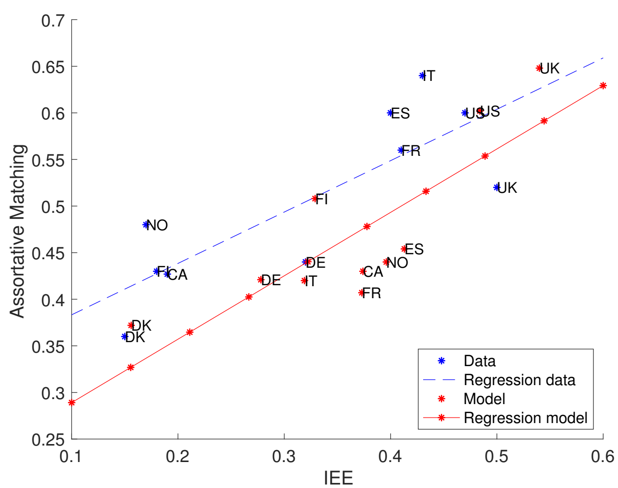
Figure 1.7.1: Model performance in explaining stylized facts. Taxes and education subsidies



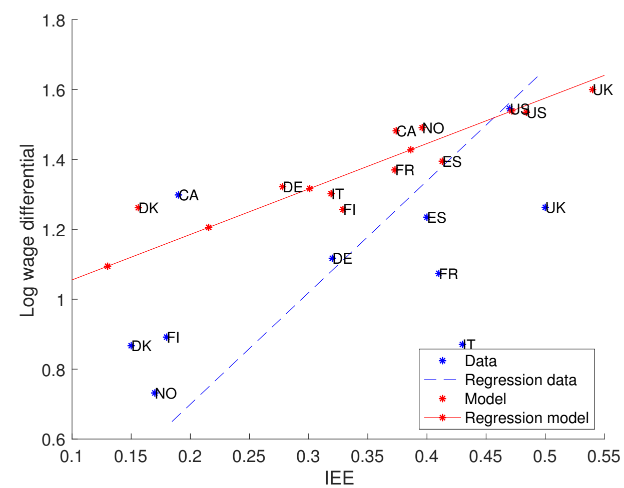
A: Tax level average vs. IEE



B: Tax progressivity wedge vs. IEE



C: Assortative matching vs. IEE



D: Log wage differential vs. IEE

The parameters including h_{min}, ϕ_0, ϕ_1, γ, b_y, σ_b, θ, σ_a, ρ_a are jointly calibrated so the model replicates key statistics of the U.S. economy. The calibrated parameters and model fit are presented in Appendix C in 1.11, tables 1.11.5 and 1.11.6.

Given a calibrated model, I repeat same exercise as in the previous subsection by considering differences of taxes and education subsidies, as well as only taxes or education subsidies. The results and comparison with the benchmark model are presented in table 1.7.1. The sum of squared errors "SSE all" is calculated

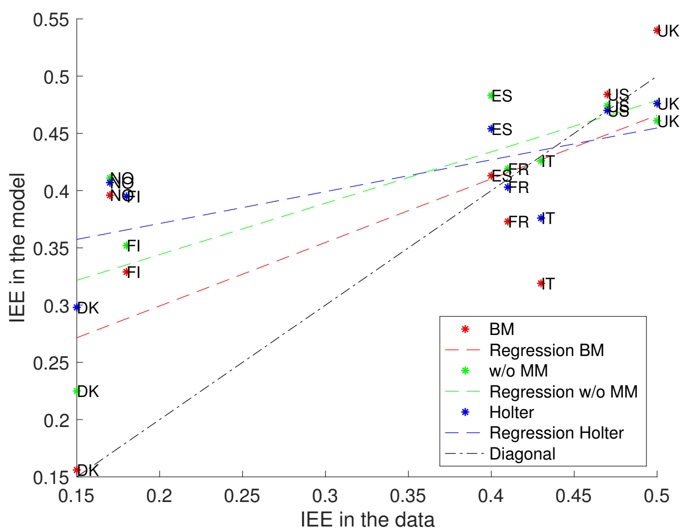
Table 1.7.1: Cross-country differences in public policies and IEE. Role of marriage market

Country	Data	Taxes & Education			Taxes			Education		
		BM	w/o MM	H	BM	w/o MM	H	BM	w/o MM	H
Denmark	0.15	0.156	0.225	0.298	0.139	0.186	0.299	0.356	0.423	0.439
Norway	0.17	0.396	0.411	0.407	0.352	0.392	0.404	0.373	0.474	0.458
Finland	0.18	0.329	0.352	0.395	0.297	0.324	0.375	0.458	0.487	0.468
Canada	0.19	0.374	0.485	-	0.479	0.472	0.463	0.416	0.499	-
Sweden	0.27	-	-	-	0.31	0.382	0.382	-	-	-
Germany	0.32	0.278	0.372	-	0.297	0.384	0.384	0.391	0.461	-
Spain	0.4	0.413	0.483	0.454	0.472	0.481	0.481	0.408	0.478	0.439
France	0.41	0.373	0.419	0.403	0.449	0.443	0.443	0.379	0.475	0.432
Italy	0.43	0.319	0.426	0.376	0.419	0.438	0.438	0.335	0.486	0.425
U.S.	0.47	0.484	0.474	0.47	0.484	0.47	0.47	0.484	0.474	0.47
U.K.	0.5	0.54	0.461	0.476	0.469	0.467	0.467	0.543	0.491	0.477
SSE all	-	0.125	0.192	-	0.141	0.172	0.215	0.229	0.39	-
SSE H	-	0.089	0.102	0.131	0.141	0.172	0.215	0.173	0.275	0.252

for all countries for which both taxation and education parameters are estimated in this paper, while "SSE H" are calculated for countries with both types of parameters estimated by Holter (2015). The benchmark model ("BM") demonstrates a superior fit versus the counterpart model without a marriage market ("w/o MM"). The sum of squared errors of model IEE predictions for all countries decreases by 35/18/41 %, if cross-country differences in both taxation and education policies, taxes only or education subsidies only are accounted for; see columns "BM" versus "w/o MM", line "SSE all". If only differences in education subsidies are considered, the model fit depreciates in the case of both frameworks.

Additionally, the benchmark model demonstrates superior performance compared to both the counterpart model without a marriage market and Holter's framework ("H") for the identical set of countries as in Holter (2015). However, the difference in the models' performance is minor. The sum of squared deviations of fitted versus actual values of IEE decreases by about 12.7/32.1 % if both policies are factored in, 18/34.4 % if taxes only, 37.1/31.3 % if education subsidies only are accounted for; line "SSE H", see also figure 1.7.2 with actual vs. fitted IEE values for different models. Consequently, consideration of a marriage market may amplify the impact of taxation and education policies on intergenerational earnings persistence and improve performance of an otherwise standard life-cycle dynastic model of human capital formation. On the one hand, the degree of assortative matching may have a direct impact on intergenerational persistence of earnings through weakening or strengthening correlations between fathers' and sons' earnings. On the other hand, when deciding on their children's education, parents consider not only labor market but also marriage market returns on human capital investments. More generous taxation and education subsidies decrease the income gap between the most and least productive individuals. Therefore, the benefits of marrying a relatively high productive partner decline and discourage parents from investing in their child's human capital and, consequently, contribute to further decreases in intergenerational earnings persistence.

Figure 1.7.2: Actual vs. model IEE. Taxes and education subsidies, countries as in Holter (2015)



1.7.3 The role of positive assortative matching

To evaluate the importance of positive assortative matching for explaining cross-country differences in intergenerational persistence of earnings, I compare the results in the benchmark model versus the counterpart model with random matching of agents ("RM"). This assumption is equivalent to the marital decision matrix i_m having all elements equal to 1, so that agents accept an offer from any candidate they meet. As table 1.7.2 demonstrates, the intergenerational earnings persistence predicted by the model drops dramatically for all countries. Countries with the highest intergenerational persistence of earnings, the U.S. and U.K., become similar to Denmark and Norway in a real economy, while for model Scandinavian countries, intergenerational earnings correlations fall below 10 %. These results demonstrate the high importance of positive assortative matching for explaining the observed levels of intergenerational earnings mobility in OECD countries.

Table 1.7.2: Impact of random matching on IEE

Country	**Data**	**Taxes & Education**	
		BM	**RM**
Denmark	0.15	0.156	0.039
Norway	0.17	0.396	0.094
Finland	0.18	0.329	0.067
Canada	0.19	0.374	0.192
Sweden	0.27	-	-
Germany	0.32	0.278	0.047
Spain	0.4	0.413	0.205
France	0.41	0.373	0.118
Italy	0.43	0.319	0.077
U.S.	0.47	0.484	0.168
U.K.	0.5	0.54	0.185

1.8 Conclusion

I develop a life-cycle dynastic model of marital sorting and human capital formation to study whether consideration of a marriage market together with public policies may improve model performance in explaining cross-country differences in intergenerational earnings persistence. I find that accounting for differences in taxation and education policies between the U.S. and 10 OECD countries replicates a positive relationship between the degree of assortative matching and intergenerational persistence of earnings. This demonstrates that cross-country differences in public policies and the degree of marital sorting are interconnected rather than separate factors in determining variations in intergenerational earnings elasticity. The model with marital sorting reduces the sum of squared errors of intergenerational earnings persistence predictions by nearly one third, as opposed to Holter's (2015) model, which does not consider a marriage market. This improvement may be partially driven by the simplifying assumption of an absence of inter vivo transfers and savings in the model. Nevertheless, the study shows that positive assortative matching is crucial to the model's ability to replicate reasonable levels of intergenerational correlation of earnings in OECD countries.

Future research may consider introduction of savings and the possibility of divorces and single parenthood into the economy for more realistic modelling of intergenerational transfers and family formation. Availability of country data on regional distribution of public education expenditures as in the case of Norway, analysed by Herrington (2015), and consideration of imperfect substitution between private and public investments in child human capital could make analysis of education subsidisation more comprehensive. Additionally, this study assumes that cross-country differences in public policies are exogenous. Endogenizing taxes and subsidies as an outcome of political processes or a social planner problem might be an interesting extension.

1.9 Appendix A

This section presents details of key stylised facts.

Table 1.9.1: Stylised facts for 11 OECD countries

Country	IEE	Assortative Matching	Log P90/P10	Tax Progr. Wedge	Average Tax
Denmark	0.15	0.36	0.87	0.3	0.3
Norway	0.17	0.48	0.73	0.21	0.21
Finland	0.18	0.43	0.89	0.28	0.25
Canada	0.19	0.43	1.3	0.23	0.16
Sweden	0.27	0.48	0.7	0.27	0.24
Germany	0.32	0.44	1.12	0.26	0.19
Spain	0.4	0.6	1.23	0.19	0.12
France	0.41	0.56	1.07	0.18	0.13
Italy	0.43	0.64	0.87	0.22	0.17
U.S.	0.47	0.61	1.55	0.17	0.16
U.K.	0.5	0.52	1.26	0.24	0.17

Table 1.9.2: Stylised facts. Correlation matrix

	IEE	Assort. Matching	Log P90/P10	Tax Progressivity	Avg. Tax
IEE	1				
Assort. Matching	0.82	1			
Log P90/P10	0.563	0.334	1		
Tax Progressivity	-0.599	-0.806	-0.498	1	
Avg. Tax	-0.703	-0.754	-0.624	0.846	1

1.10 Appendix B

This section includes proofs of propositions from section 1.4 on a simple theoretical example.

Proof of Proposition 1. Denote b_{t-1} as a level of b determined at the iteration $t-1$ and taken as given by agents at the beginning of iteration t. The level of b_{t-1} determines government spending on education subsidies through the number of married couples. Then by plugging in the government budget constraint, we can rewrite equality (*) as a difference equation of the following form

$$b_t = f(b_{t-1}) - \beta log(y_h + y_l) + \beta V(h),$$

$f(b_{t-1}) = log\big(y_h + t_l - [1 - \frac{1}{2}F(b_{t-1})]gn\big) - log\big(\frac{y_l + y_h + \frac{1}{2}F(b_{t-1})gn}{1+\alpha\psi}\big) - \alpha log\big(A\big(\alpha\psi\frac{y_h + y_l + \frac{1}{2}F(b_{t-1})gn}{n(1+\alpha\psi)}\big)^{\psi}\big).$
A fixed point solution b^* of a given equation is stable if $|f_b'(b^*)| < 1$. Applying an implicit differentiation rule obtain:

$$\frac{db^*}{d(-t_l)} = \frac{-1}{\big(1 - f_b'(b^*)\big)\big(y_h + y_l + \frac{1}{2}F(b^*)gn\big)} < 0.$$

Given fixed point stability conditions, $1 - f_b'(b^*) > 0$. Therefore, the RHS of the equation above is negative and a higher net transfer to low productive individuals leads to lower b^*.

Proof of Proposition 2. Assume tax progressivity captured by the parameter a is fixed, $t_l = t$, $t_h = at$. As in the proof of proposition 1 above, applying implicit differentiation obtains:

$$\frac{db^*}{dg} = \frac{-1}{1 - f_b'(b^*)}\Big(\frac{(1+\alpha\psi)nF(b^*)}{2(y_h + y_l + \frac{1}{2}F(b^*)gn)} + \frac{a(1 - \frac{1}{2}F(b^*))n}{(1+a)y_h - a(1 - \frac{1}{2}F(b^*))gn}\Big) < 0.$$

Due to fixed point stability conditions, $1 - f_b'(b^*) > 0$. Therefore, the RHS of the equation above is negative, and higher levels of education subsidies leads to lower b^*.

If, instead, more generous education subsidies are financed by higher levels of the tax for high productive individuals, so that tax progressivity increases, then the equation above modifies to:

$$\frac{db^*}{dg} = \frac{-1}{1 - f_b'(b^*)}\Big(\frac{(1+\alpha\psi)nF(b^*)}{2(y_h + y_l + \frac{1}{2}F(b^*)gn)} + \frac{(1 - \frac{1}{2}F(b^*))n}{y_h + t_l - (1 - \frac{1}{2}F(b^*))gn}\Big) < 0.$$

Therefore, a negative relationship between education subsidies and b^* is preserved.

1.11 Appendix C

This section presents details of the model calibration and results.

Table 1.11.1: Data sources for endogenously calibrated parameters targets

Target	Source
University attendance ratio; university drop out rate	Census Bureau, 2000-2005
Private spending, prior-university and university	UNESCO & OECD, 2000-2005
University premium; log 90 to 10 ratio	OECD, Education at a Glance
Share of young married agents; degree of assortative matching	LIS 96-00
Share of h.c. in earnings variance; IEE	Hugget et al. (2011); Corak (2006)

Table 1.11.2: Country tax functions, from Holter (2015)

	Married, 2 children				**Married, no children**				**Single**			
	τ_1	τ_2	τ_3	τ_4	τ_1	τ_2	τ_3	τ_4	τ_1	τ_2	τ_3	τ_4
DK	-2.737	6.483	-4.329	0.932	-0.834	2.654	-1.782	0.373	-1.852	4.995	-3.492	0.775
NO	-0.915	2.357	-1.436	0.277	-0.915	2.357	-1.436	0.277	-0.919	2.453	-1.512	0.289
FI	-0.834	2.654	-1.782	0.373	-0.834	2.654	-1.782	0.373	-2.315	5.579	-3.776	0.827
CA	-3.044	6.513	-4.211	0.893	-1.005	2.384	-1.468	0.294	-0.306	0.806	-0.255	-0.015
SE	-1.899	4.382	-2.787	0.573	0.044	0.256	0.117	-0.102	-0.862	2.485	-1.602	0.322
DE	-2.832	6.707	-4.575	1.004	-2.342	6.033	-4.321	0.989	-1.279	3.924	-2.909	0.672
ES	-0.854	1.800	-0.944	0.156	-0.695	1.483	-0.725	0.105	-0.746	1.710	-0.925	0.156
FR	0.145	-0.226	0.483	-0.174	-0.066	0.523	-0.244	0.037	-0.640	1.996	-1.385	0.315
IT	-2.973	6.547	-4.289	0.916	-2.199	5.172	-3.501	0.771	-2.339	5.629	-3.884	0.867
US.	-1.513	3.474	-2.235	0.470	-0.595	1.637	-1.008	0.197	-1.183	3.181	-2.253	0.513
UK.	-3.387	7.400	-4.917	1.067	-1.752	4.313	-3.017	0.684	-1.816	4.587	-3.269	0.752

Table 1.11.3: Model results. Correlation matrix

	IEE	Assort. Matching	Log P90/P10	Tax Progressivity	Avg. Tax
IEE	1				
Assort. Matching	0.814	1			
Log P90/P10	0.862	0.687	1		
Tax Progressivity	-0.64	-0.205	-0.502	1	
Avg. Tax	-0.68	-0.242	-0.488	0.846	1

Table 1.11.4: Country federal and state expenditures per student, % of GDP per capita

Country	Primary	Secondary	Tertiary
Denmark	9.94	13.86	55.8
Norway	10.49	14.46	44.18
Finland	7.75	12.15	34.84
Canada	13.3	13.84	38.34
Sweden	-	-	-
Germany	13.92	18.7	36.75
Spain	17.71	21.58	23.78
France	15.63	24.29	34.89
Italy	19.33	21.38	23.87
U.S.	10.1	11.4	20.9
U.K.	5.34	6.53	23.3

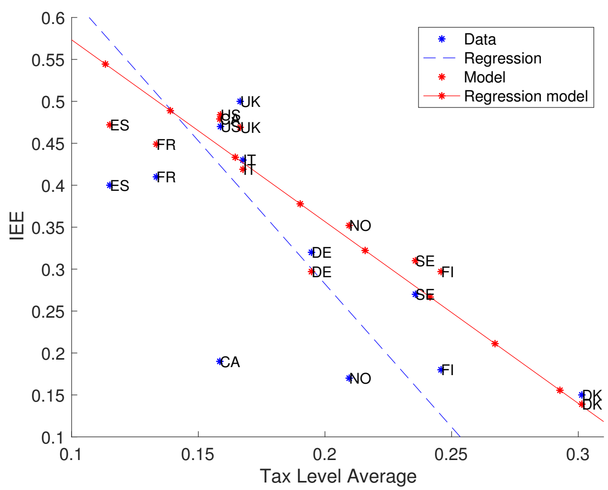
Table 1.11.5: Model without a marriage market. Endogenously calibrated parameters

Parameter	Value	Description	Target
h_{min}	1.55	Minimum level of human capital	University attendance ratio
ϕ_0	0.535	Human capital production, before university	Private spending, prior-university
ϕ_1	0.6	Human capital production, in university	University premium
γ	0.219	Parental altruism	Private spending, university
θ	-0.199	Parameter affecting university failure	University drop out rate
σ_a	0.315	Std. of ability shock	Share of h.c. in earnings variance
ρ_a	0.185	Autocorrelation parameter of ability shock	Intergenerational earnings persistence
σ_z	0.389	Std. deviation of labor market luck shock	Log 90 to 10 ratio

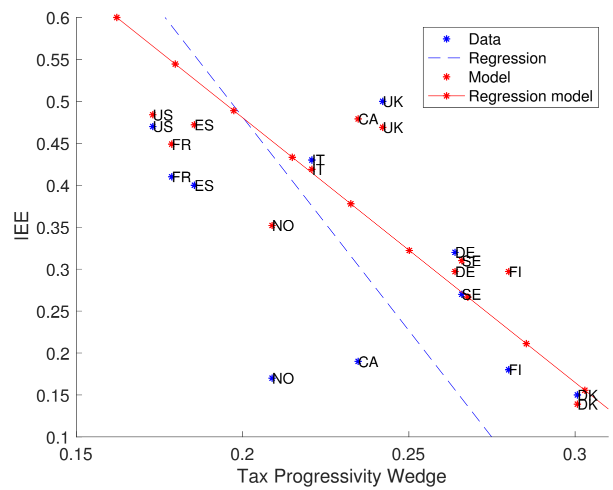
Table 1.11.6: Model without a marriage market. Calibrated parameters and fit

Target	U.S.	Model w/o marriage market
University attendance ratio	0.530	0.536
Share of private spending, primary and secondary education	0.493	0.491
University premium	1.75	1.749
Share of private spending, university education	0.631	0.616
University drop out rate	0.321	0.327
Share of h.c. in earnings variance	0.615	0.616
Earnings persistence	0.470	0.475
Log 90 to 10 ratio	1.545	1.552

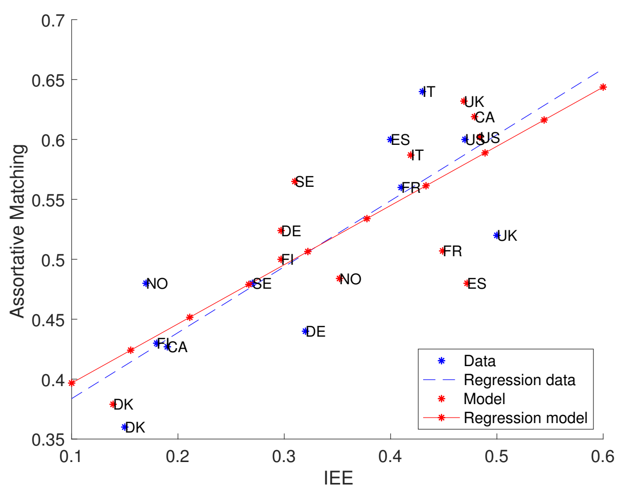
Figure 1.11.1: Model performance explaining stylized facts. Only taxes



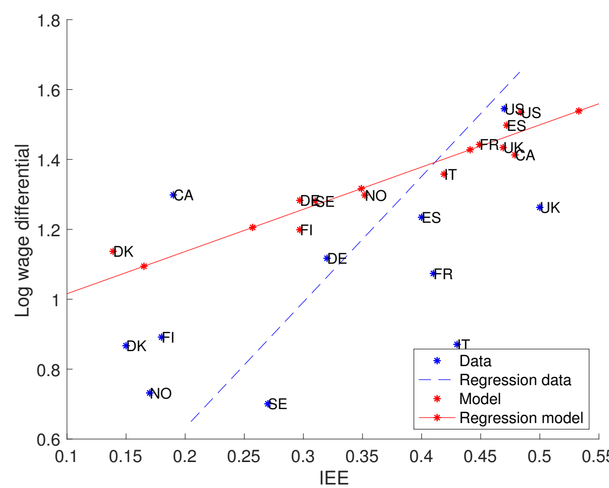
A: Tax level average vs. IEE



B: Tax progressivity wedge vs. IEE

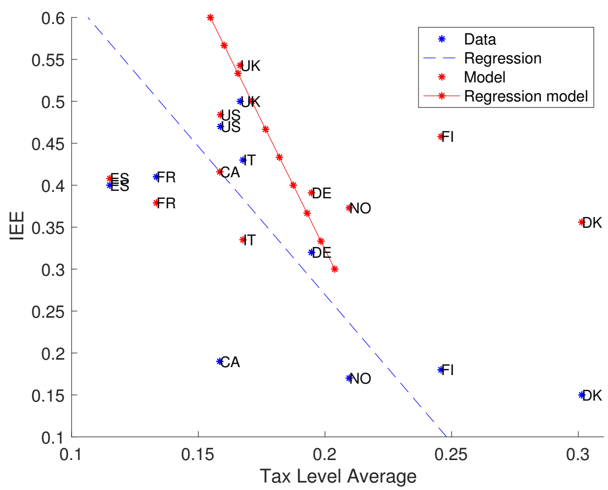


C: Assortative matching vs. IEE

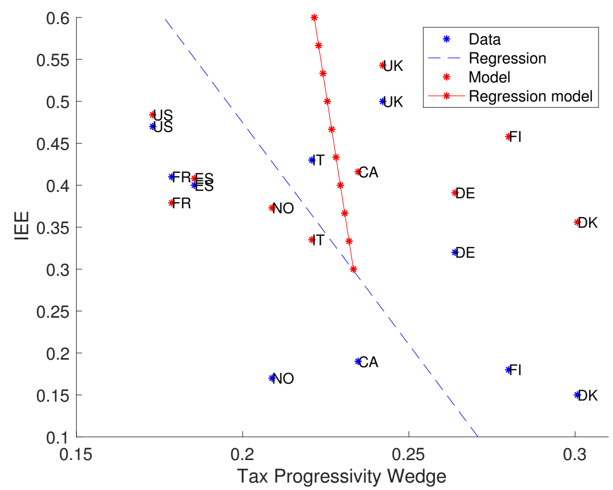


D: Log wage differential vs. IEE

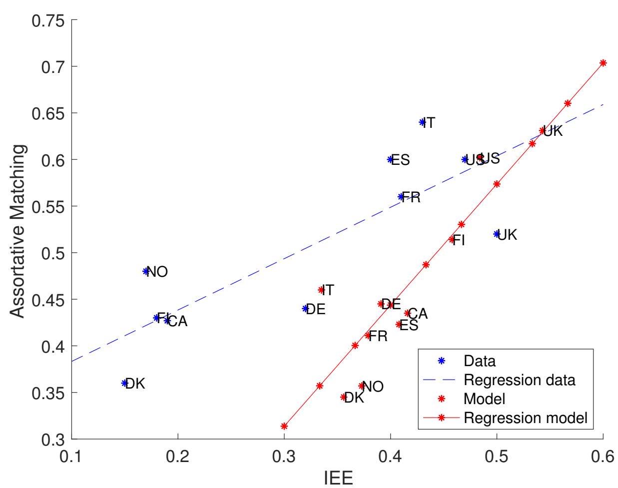
Figure 1.11.2: Model performance explaining stylized facts. Only subsidies



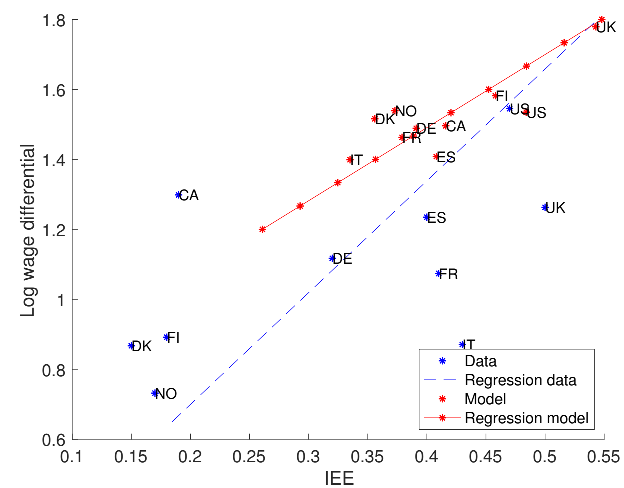
A: Tax level average vs. IEE



B: Tax progressivity wedge vs. IEE



C: Assortative matching vs. IEE



D: Log wage differential vs. IEE

Chapter 2

On the Optimal Progressivity of Higher Education Subsidies: the Role of Endogenous Fertility

Published as CERGE-EI Working Paper Series No. 613

2.1 Introduction

In this paper I quantify the optimal degree of progressivity of higher education subsidies in a dynamic dynastic model with market luck shock, intergenerational transmission of human capital in the form of parental investments in college education, and endogenous fertility. The assumption of endogenous fertility is novel for this kind of economic framework, and therefore distinguishes this study from the existing literature. Progressive higher education subsidies provide insurance against negative ability shock and serve as a policy instrument for redistribution of income across ex-ante heterogeneous agents.[1] However, this policy might disincentivise high-ability individuals from investing in higher education for their children, who are also likely to be highly-able. This creates a standard equity-efficiency tradeoff for utilitarian social planner.

This study modifies the social planner's problem by considering endogenous fertility, and finds that a welfare-maximising higher education subsidisation policy is characterised by a higher degree of progressivity than current U.S. policy.[2] The main driver behind this result is the assumption of endogenous fertility. When education subsidies become more progressive, the share of low-productivity individuals decreases, not only because this category of agents invest more in the education of their children but also because they

[1] Progressive subsidies imply that low-income individuals are subsidised at higher rates compared to high-income individuals.

[2] In this paper I employ a Millian social welfare criterion equal to the expected welfare of a new born individual and follow the tradition of the Ramsey taxation problem, implying full information and simple functional forms of taxes and subsidies.

cut their fertility rates and their children transit to the states corresponding to higher productivity, and consequently, relatively lower fertilities. In contrast, the counterpart model with exogenous fertility predicts that the introduction of more progressive higher education subsidies does not lead to any material welfare gains.

This study combines two strands of literature on education subsidisation and endogenous fertility, which are discussed in more detail below. Studies analysing optimal education policies typically do not account for endogenous fertility (see Loury, 1981; Benabou, 2002; Caucatt & Kumar, 2003; Bohacek & Kapicka, 2012; Abbot et al., 2013). Studies which analyse both education policies and endogenous fertility do not consider heterogeneity of agents nor, in the main, the implications of individuals' fertility decisions for the distribution of productivity types (see Doepke & De la Croix, 2004; Moav, 2005; Baudin, 2011). This paper fills this gap and finds that consideration of fertility decisions and their implications for the distribution of productivity types is quantitatively important for welfare.

The remainder of the paper is organised as follows. Section 2.2 presents a review of related literature. Section 2.3 introduces the model. The calibration is described in section 2.4. Section 2.5 begins with an assessment of the benchmark model fit and then presents the main results and decomposes the effects of endogenous fertility and adjustments in factor prices and distribution of productivities.

2.2 Related Literature

Analysis of optimal education policy places this paper in relation to a large strand of literature focusing on optimal education subsidisation. The most influential analytical studies in this area are those of Loury (1981), Benabou (2002) and Bovenberg & Jacobs (2005). Loury (1981) finds that public education provision increases mean incomes and decreases variation of incomes in a framework with liquidity constraints and heterogeneous agents. Benabou (2002) shows that education policy is superior to redistribution policy from the perspective of income growth, but inferior from the perspective of insurance against negative income shock. Bovenberg & Jacobs (2005) demonstrate that subsidisation of education is an essential component of optimal fiscal policy, as this policy instrument allows for the mitigation of the distortive impact of progressive labor income taxes on human capital accumulation. The main contribution of this paper is that it investigates optimal education policies from a novel perspective - in a model with endogenous fertility.

Similarly to the analytical papers listed above, the existing quantitative studies on higher education policies abstract from endogenous fertility. I follow Caucutt & Kumar (2003) employing a relatively simple general equilibrium model, although a substantial number of studies in this area rely on rich general equilibrium frameworks which allow for detailed modelling of education policy (see Akyol & Athreya, 2005; Garriga & Knightly, 2007; Bohacek & Kapicka, 2012; Abbott et al., 2013; Kruger & Ludwig, 2013; Colas et al., 2021).

The results of these studies are mixed. While some papers find that alternative education subsidisation policies may be welfare-improving compared to the current U.S. policy, others do not find any material welfare gains in deviation from the current U.S. status quo. Specifically, Bohacek & Kapicka (2012) find that increases in higher education subsidies in the U.S. to European levels could lead to 1.5 % growth of welfare if reform is financed by higher tax rates. Akyol & Athreya (2005) find that more generous subsidisation of college education compared to the current U.S. policy may be welfare-improving, since subsidies decrease the risk of college completion failure. Analogously, Kruger & Ludwig (2013) demonstrate that the optimal policy would require more generous subsidisation of college education, since it allows for mitigation of the distortive impact of progressive labor income taxation on human capital accumulation.

Similarly to this paper, Colas et al. (2021) find that, in a structural life-cycle model, optimal need-based college subsidies are characterised by a higher degree of progressivity than the current U.S. policy. However, the intuition behind this finding is different and relates to the relatively low costs of subsidising children from low income families rather than endogenous fertility considerations. Interestingly, Colas et al.'s result does not rely on redistribution concerns. A social planner aiming to maximise tax revenue would calculate optimal subsidies characterised by a similar degree of progressivity if utilitarian welfare maximisation were a target.

In contrast, Caucutt & Kumar (2003) and Abbott et al. (2013) do not find any material welfare gains arising from the deviation from current U.S. policy. Additionally, these two papers analyse the optimal degree of progressivity of education subsidies. Specifically, Caucutt & Kumar (2003) find that college subsidies which are more progressive than the current U.S. policy do not lead to any material welfare gains. Similarly, Abbott et al. (2013), employing a rich life-cycle framework, find that introduction of more progressive education subsidisation would lead to relatively small welfare gains, equivalent to a 0.2% increase in life-time consumption. In contrast, this paper finds an almost 1% welfare gain and shows that the assumption of endogenous fertility is essential for this result.

The second strand of literature related to the current study is devoted to endogenous fertility. The studies in this area typically focus on the explanation of inequality patterns (see Moav, 2005; De la Croix and Doepke, 2003; Kremer & Chen, 2002) or implications of different education financing systems and fertility differentials for economic growth (De la Croix and Doepke, 2004).

To the best of my knowledge, the only study in this area which analyses optimal education subsidies in the economy with endogenous fertility is the paper by Baudin (2011). Employing a representative agent framework with endogenous child quality and quantity, Baudin shows that it is optimal to subsidise education and tax the number of children, due to the presence of human capital externalities. In contrast, when the social welfare function allows for preferences for population growth, education might be either subsidised or taxed, depending on the sign of population growth. The fundamental difference of this paper from Baudin's study is the assumption of heterogeneity of agents across productivity types. This assumption leads to a novel type of discrepancy between individual and social planner preferences that is absent in Baudin's paper. Specifically, while deciding on the number and education of their children, individuals do not take into consideration that their decisions affect the distribution of productivity types across agents and, consequently, social welfare.

Methodologically the current paper builds on a seminal study by Becker & Barro (1989) who formalise the child quality-quantity trade off, and Alvarez (1999), who generalise this basic model by the introduction of stochastic ability shocks and endogenous human capital formation, as well as Knowles (1999a), who further extends this setting to a general equilibrium model. The current paper can be seen as one of the first attempts to model parental investments in the college education of their children and analyse optimal higher education subsidisation in this type of economic environment.

2.3 The Model

I consider an overlapping-generations model populated by a continuum of three-period lived individuals. A period in the model is equal to a 25 year interval, so that the model periods correspond to actual life ages 0-25, 26-50 and 51-75 respectively. Given that a mother's average age at first birth is 26 (for 2011) and estimated life expectancy is 78 for both sexes (for the period 2005-2010) in the U.S., timing in the model is consistent with a real economy.[3] Agents are referred to as children in the first period of their lives, young adults or parents in the second period, and old adults in the third period.

All decisions are taken by young adults. At the beginning of adulthood individuals are characterised by state vector $x = (z, s)$ where z is market luck shock, and s is investments in college education. Adult individuals choose the number of their children and investments in the higher education of each child. Higher education is subsidised by government. For simplicity I assume that there is no social security system in the economy. Therefore, savings are the only source agents can rely on to finance their consumption after retirement.

2.3.1 Human capital production

In the model, an individual's human capital h corresponds to his labor market productivity and depends on both market luck shock z and investment in college education s, according to the following human capital production technology:

$$h = z(\kappa + s)^{\eta}$$

where $0 < \eta < 1$, $\kappa > 0$. Clearly, this functional form satisfies the standard properties including $\frac{\partial h}{\partial s} > 0$ and $\frac{\partial^2 h}{\partial^2 s} < 0$. Additionally, marginal return in the case of $s = 0$ is finite: $\frac{\partial h}{\partial s}|_{s=0} < \infty$. The latter assumption makes the model consistent with the data, suggesting that there are no infinite returns on education, especially at the college stage (see Psacharopoulos & Patrinos, 2004). The functional form of the human capital production function is close to that employed in De la Croix & Doepke (2003) and Erosa & Koreshkova (2007).

[3] Sources: Central Intelligence Agency: the World Factbook; World Population Prospects, 2012.

I assume that market luck shock is correlated across generations. The assumption of correlation of z across generations allows us to capture the importance of factors including the effect of networks and neighbourhoods. Individuals who are successful on the labor market are more likely to form a network with other top earners and live in richer neighbourhoods than those who are less lucky on the labor market. These factors are very likely to improve the labor market prospects of their children. Individuals' human capital is also affected by the investment in college education chosen by their parents. Since the focus of this paper is higher education subsidisation, I do not model primary and secondary schooling. Instead I assume that all children are endowed with an exogenous positive level of human capital, captured by parameter κ and interpreted as compulsory primary and secondary education. College education is financed by parents and subsidised by government.[4] The assumption of parental choice of their children's education is supported by the findings of Belley & Lochner (2007) who, employing NLSY97 data, demonstrate that family income is an important determinant of college attendance and quality of college. Additionally, individuals are eligible for student loans. For simplicity, the share of college expenditures covered by loans is assumed to be exogenous.[5]

One of the key assumptions of this paper is the modelling of investments in college education of children as a continuous choice problem, as in Herrington (2015). In other words, parents can choose any non-negative investment in college education of their children $s' \geq 0$. In contrast to the modelling of college investments as a binomial choice problem, typically employed in the literature, this assumption allows us to capture heterogeneity in the costs of different colleges and the amount of time individuals spend on acquisition of higher education.

2.3.2 Decision problem

In the current model all decisions are taken by young adults characterised by state vector $x = (z, s)$ consisting of ability shock z and investments in college education s, determined by their parents. Young individuals choose consumption c, savings b to finance consumption at old age c_o, the number of their children n and investments in higher education of each child s'. For simplicity, the ability of children z' is assumed to be unobservable for parents when they are making decisions. Therefore, one should think about an individual in a model economy as an "average" agent characterised by state $x = (z, s)$. The assumption of an unobserved shock to children's ability may be interpreted as an uncertainty about their ability to graduate from college. In real life, this uncertainty remains even after the high school performance of children has been observed. Old adults exogenously supply $\epsilon < 1$ time units of labor, which allows for the incorporation of a retirement period into the model, and consume their savings $b(1 + r)$.

Young individuals are endowed with one unit of time allocated between labor market activities and child rearing. Raising one child takes fraction ϕ of parental time. Children are also costly in terms of goods,

[4] Given that primary and secondary education in the U.S. is compulsory and publicly provided, while higher education is not compulsory and only partially subsidised, this assumption provides a relatively accurate approximation of the U.S. economy.

[5] Exclusion of student loans from the model would make individuals more financially constrained. Therefore, fertility responses to education subsidies and, consequently, the welfare effect might be inflated.

including investment in college education and the exogenous cost of children, interpreted as necessities consisting of expenditures on housing, clothing and food. Government compensates fraction θ of costs of higher education per child. I assume that θ depends on relative parental productivity.[6] The remaining part of college expenditures is paid by parents and their children. Specifically, parents contribute $(1-\theta(y)-\theta_l)s'$ per child, while children repay $\theta_l s'(1+r_l)$ as a student loan together with interest r_l when they enter adulthood. The student loan ratio θ_l is exogenous and identical for all individuals (discussed in more detail in Section 4).

A young adult's objective is to maximise utility from consumption and the expected utility of each of their children, weighted by altruism discount factor χ and by an increasing and concave function of family size n^{ξ}. Individuals' preferences are modelled in the style of Barro and Becker (1989). The individual decision problem is formulated recursively as follows:

$$V(z,s) = \max_{c,b,n,s'} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta \left(\frac{c_o^{1-\sigma}}{1-\sigma} + \chi n^{\xi} E_t \left[V(z',s') | z \right] \right) \right\}$$

subject to

$$c + b + \left[(1-\theta(y)-\theta_l)\, s' + g(y,h) \right] n \leq wh(1-\phi n)(1-\tau) - \theta_l s(1+r_l)$$

$$c_o \leq b(1+r) + wh\epsilon$$

$$\theta_l + \theta(y) \leq 1$$

$$h = z(\kappa+s)^{\eta}$$

where β is the time discount factor, $g(y,h)$ is the cost of children in terms of goods, which is exogenous and defined in Section 4, τ is a proportional labor income tax rate to finance education subsidies, w is wage, and r is interest rate.

The dependence of discount factor χn^{ξ} on the number of children, which is endogenous, together with non-convexities of a budget constraint makes the dynamic programming problem above non-standard. However, as Alvarez & Stokey (1998) show, due to the homogeneity of the utility function, this class of dynamic programming problems can be analysed by similar tools to those used in the standard case.

2.3.3 Production

The production sector in the economy consists of a large number of firms renting physical capital and effective labor from households in a competitive market. The aggregate production is defined by a standard Cobb-Douglas production function:

$$Y = K^{\alpha} L^{1-\alpha}$$

where K is aggregate capital determined by savings made by the previous generation, and L is aggregate effective labor supply.

[6] Relative parental productivity is defined as human capital h related to average human capital in the economy.

2.3.4 Distribution of agents and population dynamics

As mentioned above, agents are heterogeneous with respect to ability shock z and investments in college education s. Therefore, a formal means of description of heterogeneity in the model is needed. I follow the standard approach employed in the literature. Denote the individual state vector as $x = (z, s) \in X$, where $X = \mathcal{Z} \times \mathcal{S}$, $\mathcal{Z} = [\underline{z}, \bar{z}]$, $\mathcal{S} = [\underline{s}, \bar{s}]$. Then the distribution of individual states across agents is described by probability measure ψ defined on subsets of a state space. Denote the probability space as $(X, B(X), \psi)$, where X is the state space, and $B(X)$ is the Borel σ-algebra on X. Therefore, for each set $B \in B(X)$, $\psi(B)$ shows the fraction of agents whose individual states lie in B as a share of all adult agents. Then for all $B \in B(X)$, the law of motion for the probability measure ψ is given by:

$$\psi'(B) = \frac{\int_X P(x, B) n(x) d\psi(x)}{\gamma}$$

where $P(x, B)$ is a transition function equal to the probability that children of a parent with individual state x will transit to the set B next period, and depends on the transition probability $p(z'|z)$ determined by the stochastic process for market luck shock, $n(x)$ is the number of children chosen by a parent with state x, γ is average fertility, corresponding to the size of the current generation of children relative to that of the current young adults:

$$P(x, B) = \begin{cases} p(z'|z), & \text{if } (z', s(x)) \in B, \\ 0, & \text{otherwise;} \end{cases}$$

$$\gamma = \int_X n(x) d\psi(x).$$

2.3.5 Stationary equilibrium

The analysis focuses on the concept of stationary equilibrium, according to which factor prices, average fertility, capital, and labor per adult population are constant over time.

Definition. A stationary competitive equilibrium is a set of decision rules $c^*(x)$, $c_o^*(x)$, $n^*(x)$, $s'^*(x)$, $b^*(x)$, factor prices w^* and r^*, interest rate on loans r_l^*, average fertility γ^*, per capita K^*, L^*, average productivity y^* and stationary distribution $\psi^*(x)$ such that:

1) decision rules constitute the solution of the individual's problem with the individual taking factor prices and population growth as given;

2) factor prices w^* and r^* are determined by optimal behaviour of the representative firm:
$w^* = (\frac{K^*}{L^*})^\alpha$, $r^* = (\frac{K^*}{L^*})^{\alpha-1} - \delta$.

3) markets clear:

a) market of goods: $Y^* + (1-\delta)K^* = \int_X \left[c^*(x) + \frac{1}{\gamma^*} c_o^*(x) + \gamma^* K^* + g(y, x) n^*(x) + \theta(x) s'^*(x) n^*(x)\right] d\psi^*(x)$
where relative parental productivity $y = \frac{h}{\int_X h d\psi^*(x)}$;

b) market of capital: $K^* = \int_X \frac{1}{\gamma^*} b^*(x) d\psi^*(x)$;

c) labor market: $L^* = \int_X \left[h(1-\phi n^*(x)) + \frac{1}{\gamma^*}h\epsilon\right]d\psi^*(x)$;

4) government budget is balanced: $\tau w^* L^* = \int_X \theta(x)s'^*(x)n^*(x)d\psi^*(x)$[7];

5) loans: government finances loans to the current generation of children from the repayments of loans by the current generation of adults so that $\theta_l \int_X s'^*(x)n^*(x)d\psi^*(x) = (1+r_l)\theta_l \int_X s d\psi^*(x)$, since the economy is in a steady state $1 + r_l^* = \gamma^*$;

6) a stationary distribution $\psi^*(x)$ is consistent with households' decision rules and stochastic process for z.

In general it is difficult to guarantee the existence and uniqueness of equilibrium in heterogeneous-agents models. I follow intuition from Knowles (1999a), who analyses a heterogeneous-agents model with endogenous fertility. The presence of market luck shock guarantees that the child of the parent with the lowest realisation of market luck shock may draw the highest realisation of market luck shock and vice versa. In other words, both the American dream and the American nightmare are possible. Together with the assumption of minimal human capital of children whose parents choose not to invest in their college education (due to $\kappa > 0$) this guarantees the absence of poverty traps. In other words, an individual with no college education may draw high realisation of market luck shock and become rich. Moreover, due to mean-reverting properties of the market luck shock and decreasing marginal return on human capital, extremely high productivity is also not an absorbing state.

2.4 Calibration

The benchmark model presented above is calibrated to match salient characteristics of the U.S. economy. For all statistics, except for individuals' fertility rates, I use the U.S. Census data and *National Postsecondary Student Aid Study* (NPSAS) 2011-2012 data. In order to estimate average fertilities for different parental income quintiles, I use data from the *National Longitudinal Survey of Youth* 1979.

2.4.1 Higher education subsidies and loans

The U.S. government subsidises higher education through financing of public colleges and through direct financial aid to students. Additionally, students may receive financial support from higher education institutions. I model the higher education subsidy as total financial aid in the form of grants and scholarships received from both sources: directly from government (federal, state grants and scholarships) and from institutions (institutional grants and scholarships). As in Herrington (2015), I assume that the higher education subsidy is proportional to the total price of college s. The coefficient of proportionality $\theta(y)$ is a linear function of relative parental productivity y:

$$\theta(y) = \max(a_\theta + b_\theta y, 0).$$

[7] All aggregates are per total number of adults in the economy. Similarly to the representative agent models, the total number of individuals is irrelevant for equilibrium and can be normalised to any positive number.

Parameters a_θ, b_θ are estimated from NPSAS data on student financial aid and parental income for undergraduates, 2011 - 2012. For estimation I restrict the sample to the individuals whose parental income is less than 200 thousands dollars (95 % of all observations in the sample fall into this category, additionally, nearly 96 % of all households in the U.S. belong to this income group). The NPSAS data can be analysed solely by online tools. Since neither evaluation of $\theta(y)$ for each individual nor non-linear regression analysis is available via these tools, estimation of a_θ and b_θ based on individual data is unfeasible.

To overcome these difficulties I proceed as follows. First, the whole sample is divided into 24 income groups so that the income interval is equal to 5 thousand dollars for the bottom and middle parts of the income distribution and 50 thousand dollars for individuals with high incomes. For each group I estimate the ratio of average financial aid to average price of college within a group. Second, I regress these group by group ratios of financial aid to price of college on group by group relative parental incomes to obtain the estimates of a_θ and b_θ. The results including standard errors and R^2 are presented in table 2.4.1 (the regression fit is depicted in figure 2.7.1 in Appendix in 2.7).

Table 2.4.1: Estimates of parameters for $\theta(y)$

Parameter	Value
a_θ	0.51
	(0.014)
b_θ	-0.145
	(0.013)
R^2	0.83

For simplicity it is assumed that student loans to finance college expenditures are exogenous and the ratio of the loan to the price of college is homogenous across individuals. According to the NPSAS data 2011-2012, the ratio of loan to the price of college does not vary much across parental income quintiles (from 0.141 to 0.173). Consequently, parameter θ_l is set equal to the average ratio of the loan to the price of college for all individuals (0.15).

2.4.2 Cost of children in terms of goods

The model assumes that along with time cost of children and expenditures on college, there is a cost of children in terms of goods. This type of cost is interpreted as expenditures on necessities including housing, clothing and food and, therefore, assumed to be exogenous in the model.

According to the *Expenditures on children by families* 2013 report published by the U.S. Department of Agriculture, cost per child is larger in absolute value for families with higher pre tax annual income. However, the cost per child related to parental pre-tax income is lower for richer individuals. In order to capture the empirical properties of this type of cost I employ the following approximation. I assume that the annual cost per child in terms of goods is proportional to the parental effective wage wh, while the coefficient of proportionality decreases with the relative parental effective wage, which is essentially equal to relative parental productivity:

$$g(y,h) = \tilde{g}(y)wh, \text{ where } \tilde{g}(y) = a_g exp(-b_g y), a_g > 0, 0 < b_g < 1.$$

Effective wage is chosen as a scale since the data is provided for annual parental income, which corresponds to effective wage in the model. This assumption is in line with Knowles (1999a) who assumes the exogenous cost of children to be proportional to parental income. However, the current study generalises the approach of Knowles by the assumption of a more general functional form and disciplining the model based on the data.

The data from the *Expenditures on children by families* 2013 report is available only for 3 income groups: lowest, middle and highest tertiles, see table 2.4.2. I use estimated cost per child related to life-time parental income proxied by annual income times 25 years (length of young adulthood) for the lowest and highest tertiles, to solve for a_g and b_g.

Table 2.4.2: Cost of children in terms of goods as a share of life-time parental income

Income group	Average relative parental income	Average costs per child
Lowest tertile	0.4	0.16
Middle tertile	0.85	0.1
Highest tertile	1.91	0.075

The resulting estimates are presented in the table 2.4.3 below. As was assumed, $0 < b_g < 1$ and, therefore, costs of children in terms of goods increase with parental income.

Table 2.4.3: Parameters for $\tilde{g}(y)$

Parameter	Value
a_g	0.18
b_g	0.47

Plugging values for the middle income group which were not used for estimation, one would obtain $\tilde{g}(0.85) = 0.12$, which is quite close to the data.

2.4.3 Other parameters

In this subsection I discuss the estimation of the remaining parameters of the model, including the characteristics of ability shock, preferences and the human capital production function. The following parameters take values which are standard for macroeconomic literature: $\alpha = 0.34$, capital depreciation rate $\delta = 1 - (1 - 0.1)^{25} = 0.928$. I set fraction ϕ of parental time spent per child, equal to 0.09. Given that a model period is equal to 25 years, this corresponds to 2.25 years per child, as in De la Croix and Doepke (2003). The parameter determining labor supply in old adulthood, is set equal to 0.5, since retirement starts at the age 65 in the U.S. This roughly corresponds to the middle of the old adulthood period (51-75 years).

The rest of the parameters, including ρ, σ_z, β, σ, ξ, η, χ, κ are jointly calibrated to minimise the quadratic loss function (sum of squared deviations of statistics predicted by the model from statistics in the data) so that the model replicates salient features of the U.S. economy. The target statistics, and estimated values of the parameters in the data and in the model are presented in the table 2.5.1 below.

There is no one-to-one link between parameters of the model and corresponding targeted statistics. The parameters are assigned to the stylised facts based on the principle of sensitivity. Specifically, while intergenerational persistence of earnings is affected by all parameters in the model, this statistic is the most sensitive to persistence ρ in the AR(1) process for z. Similarly, standard deviation σ_z is assigned to the variance of log earnings. The value of β positively affects savings. Therefore, this parameter is assigned to annual capital to GDP ratio. Parameter σ plays an important role in the model with endogenous fertility since it affects the quality-quantity tradeoff through the curvature of the utility function (Knowles, 1999a).[8] Higher values of σ imply lower curvature of the utility function. Consequently, as Knowles suggests, individuals would choose the lower number of children. Since σ affects fertilties, I assign this parameter to average fertility rates in the economy. Parameter ξ affects utility from the number of children, and is, therefore, assigned to a fertility differential between the bottom and top income quintiles.

The remaining parameters η and κ characterise human capital production technology, and parameter χ corresponds to a parental altruism factor. These parameters affect parental incentives to invest in the education of their children. Parameter η determines marginal return on investments in education. Therefore, the targeted statistic for η is the college wage premium. Parameter κ also affects marginal returns on investments in education and corresponds to the share of individuals without higher education. The remaining parameter χ influences the contribution of the expected value of children to overall parental utility and, consequently, affects the importance of child quality for parents. Since education subsidies are proportional to total investments in college education chosen by parents, public expenditures on college are influenced by χ as well. Therefore, this parameter is assigned to the share of public expenditures on higher education in GDP.

[8] Additionally, in a Barro & Becker type of model $\sigma \in (0, 1)$. This guarantees that utility increases with the number of children.

2.5 Results

2.5.1 Benchmark model fit

As table 2.5.1 demonstrates, the benchmark model predicts targeted statistics quite well: nearly all stylised facts are almost perfectly matched. Small discrepancies in the cases of the share of individuals with no higher education, wage premium and average fertilities could be explained by the fact that the model is highly non-linear. Consequently, there is no guarantee of the existence of parameters delivering a precise match.

Additionally, the model also replicates certain important properties of the U.S. economy which were not targeted in the calibration exercise. Specifically, the number of children decreases with parental income, while investments in college education per child increase with parental income in the U.S. (see figure 2.5.1 below). These observations are consistent with the results of empirical studies including Jones & Tertilt (2008), Hanushek (1992) and the empirical part of De la Croix & Doepke's 2009 study. The model replicates these properties of the data quite closely.

Table 2.5.1: Estimates of jointly calibrated parameters

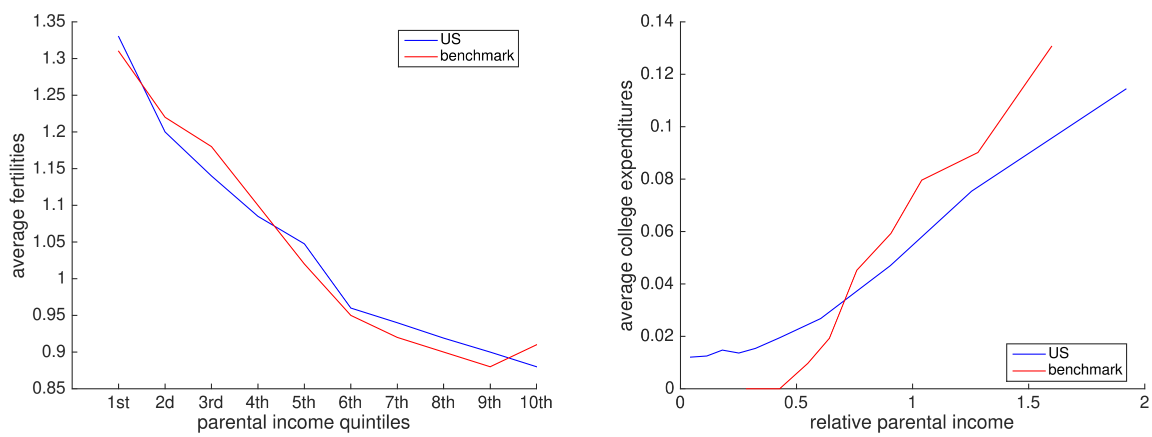
Description	Parameter	Value	Target	Data	Model
Persistence in AR(1) process for z	ρ	0.31	Intergenerational persistence of log earnings	0.47	0.473
Std dev of noise in AR(1) process for z	σ_z	0.52	Variance of log earnings	0.36	0.363
Time discount factor	β	0.61	Annual capital to GDP ratio	3	2.99
Risk aversion parameter	σ	0.6	Average fertility rate	1.04	1.036
Curvature of altruism discount factor	ξ	0.21	Fertility differential	1.41	1.42
Elasticity of HC output w.r.t. input	η	0.36	Wage premium of higher education	1.61	1.62
Altruism discount factor	χ	0.55	Public expenditures on higher education, share of GDP	0.95%	0.946%
Minimal human capital level parameter	κ	0.0065	Share of individuals, no higher education	42%	40%

Notes. The fertility differential is defined as a ratio of average fertility rates of the bottom quintile of income distribution to the top quintile. The estimated values of intergenerational persistence and variance of log earnings are taken from Corak (2012) and Mulligan (1997) respectively. The remaining stylised facts are author's calculations. Average fertility rates for different parental income quintiles are estimated based on NLSY79. Wage premium is evaluated as a ratio of average monthly earnings of full-time workers with at least some college to those of workers with no college based on Survey of Income and Program Participation, 2008, US Census Bureau. The share of public expenditures on higher education in GDP (for the year 2011) is taken from Education at a Glance 2014.

Interestingly, as Alvarez (1999) demonstrates, the Barro-Becker model does not guarantee the replication of these stylised facts in general. Using a dynastic formulation of the current model, I show that investment in education per child s' increases with parental education s (see Appendix in 2.7). Additionally, as figure 2.5.1 demonstrates, investments in the education of children predicted by the model lie in a reasonable range. In contrast, the negative relation between fertilities and parental income cannot be guaranteed in this model. However, the assumption of time cost of children, which implies that opportunity costs are higher for high income parents, and investment in education of children increases with parental income, deliver a quite accurate match of the fertility-income profile.

The results presented above suggest that the model fits the relevant characteristics of the U.S. economy quite well, including those that were not directly targeted by calibration. Therefore, the model may serve as a proper laboratory for quantitative analysis of higher education subsidisation policies.

Figure 2.5.1: Incomes, fertilities and investments in education



A: Fertilities

B: Investments in education

Notes. The fertility-income profile is estimated based on NLSY 1979 data. The data on total investment in higher education (accounting for number of years of schooling) by parental income is not available. To overcome these restrictions, the following approximation is employed. For each parental income group, average years of schooling is estimated based on the data on the highest level of education earned by total family income, provided by the NPSAS. Then average years of schooling is multiplied by average student budget from NPSAS data. Finally, resulting expenditures on education are normalised by average life-time income.

2.5.2 Policy experiments

In this subsection I solve for welfare-maximising progressivity of higher education subsidies. The analysis focuses on a so-called small reform in the style of Piketty & Saez (2012, 2013) implying that total spending on higher education is fixed at the absolute current U.S. level. I focus on the class of linear policies:

$$\theta(y) = \begin{cases} 1-\theta_l, & \text{if } a_l + b_l y \geq 1-\theta_l; \\ a_l + b_l y, & \text{if } 0 < a_l + b_l y < 1-\theta_l; \\ 0, & \text{otherwise.} \end{cases}$$

Parameter a_l is a free parameter, which can take either positive or negative values and is interpreted as progressivity of education subsidies. Given a_l, parameter b_l is chosen so that the absolute level of public expenditures on higher education is the same as in the U.S. economy. Higher values of a_l implies that b_l is negative and the function $\theta(y)$ decreases with y. Therefore, subsidies are progressive. In contrast, if a_l becomes sufficiently low or negative, the corresponding values of b_l are likely to be positive. This implies that the education subsidy $\theta(y)$ becomes regressive.[9]

The current study follows the Millian social welfare criterion, which ranks allocations of different population size by comparing the expected utility of a newborn individual:

$$W(a_l) = \int_X V(x, a_l) d\psi(x, a_l).$$

On the one hand, more progressive subsidies could allow for better insurance against negative ability shock. On the other hand, since market luck shock is correlated across generations, more progressive subsidisation may result in efficiency lost. This is a standard equity-efficiency trade off. However, the assumption of endogenous fertility modifies this classical social planner's problem, due to its non-trivial effect on the distribution of productivity types across agents. Consequently, in the model with endogenous fertility more progressive subsidisation of higher education may be desirable from the welfare maximisation perspective, because this policy stimulates low income individuals to not only invest more in the education of their children, but also cut their fertility rates. Therefore, the share of low productivity individuals declines and welfare increases.

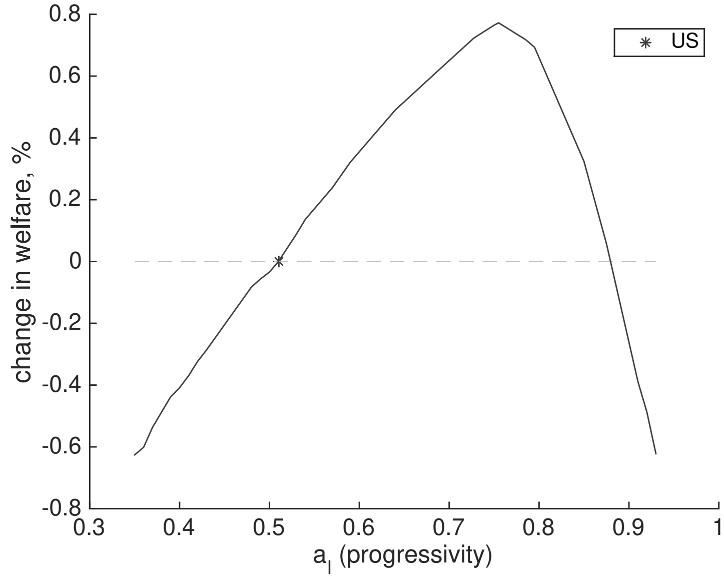
2.5.2.1 Welfare and population growth

Figures 2.5.2 and 2.5.3 below depict average per capita welfare and fertility rates in a steady state equilibrium as functions of a_l (given that parameter b_l is adjusted to match education budget as in the U.S. economy). Figure 2.5.4 depicts per capita welfare as a function of average fertility rates for different education policies. The number markers on the figure 2.5.4 correspond to the values of progressivity a_l.

Table 2.5.2 below presents a comparison of aggregate characteristics in the benchmark case (U.S.) and in the case of welfare-maximising policy. All aggregate variables are normalised to the amount of the working population. Variables including C, Y, K are translated to annual terms and normalised to the GDP in the benchmark economy.

[9] Alternatively, one could fix a_l and vary parameter b_l. This might seem like a more natural parameter choice to reflect the progressivity of education subsidies. However, the results of this exercise would be identical to the setup proposed in this paper.

Figure 2.5.2: Welfare

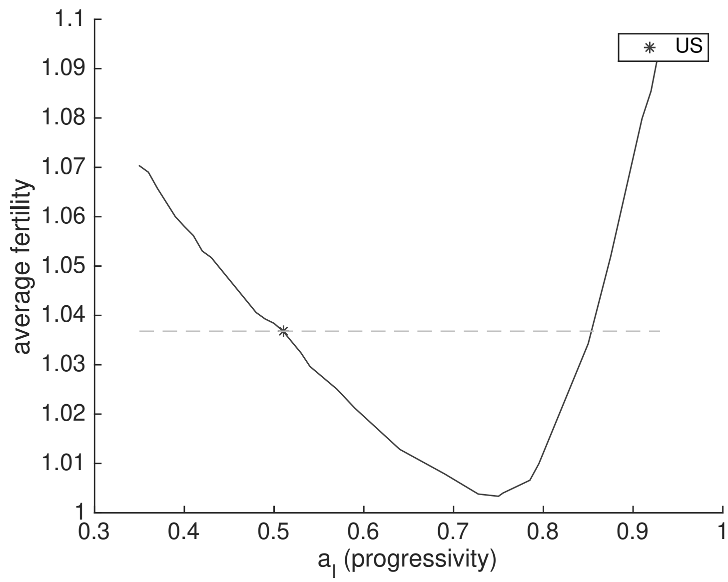


As mentioned above, parameter a_l characterises the degree of progressivity of education subsidies. Higher values of a_l correspond to lower (negative) values of slope parameter b_l. As the results demonstrate, the dependance between a_l and welfare is hump-shaped, while dependance between a_l and average fertilities is U-shaped (see figures 1 and 2 above). In order to provide the intuition for this result, I use examples of policies corresponding to $a_l = 0.35, a_l = 0.76$ and $a_l = 0.85$. I label these policies as slightly progressive, optimum and highly progressive respectively. The first policy is slightly progressive since the corresponding slope parameter $b_l = -0.03$, so that subsides are almost flat. Both the second and the third policies are more progressive compared to the benchmark economy, since the corresponding slopes b$_l$ are equal to -0.42, -0.57 respectively versus $b_l = -0.14$ in the benchmark. While the second policy induces the highest per capita welfare and lowest average fertility rates among all linear policies, the third policy leads to lower welfare, whereas average fertility rates are the same as in the U.S. status quo.

First, it is worth mentioning that since education subsidies are proportional, increases in subsidisation rates unambiguously lead to higher total expenditures on children's college education. However, since the share of college expenditures contributed by parents $1 - \theta(y)$ declines, the adjustment in college spending paid by parents has an ambiguous sign. The results suggest the effect of total expenditures on college education dominates and, consequently, college spending paid by parents is positively related to education subsidisation rates.

Now let us return to the example. Assume a less regressive higher education policy corresponding to $a_l = 0.35$ is introduced. Since individuals with low incomes are offered lower education subsidies than in the status quo, this category of agents invest less in the education of their children and consequently, since the cost of children declines, increase their fertility rates (see subfigures A and B, figure , in Appendix in 2.7).

Figure 2.5.3: Average fertilities



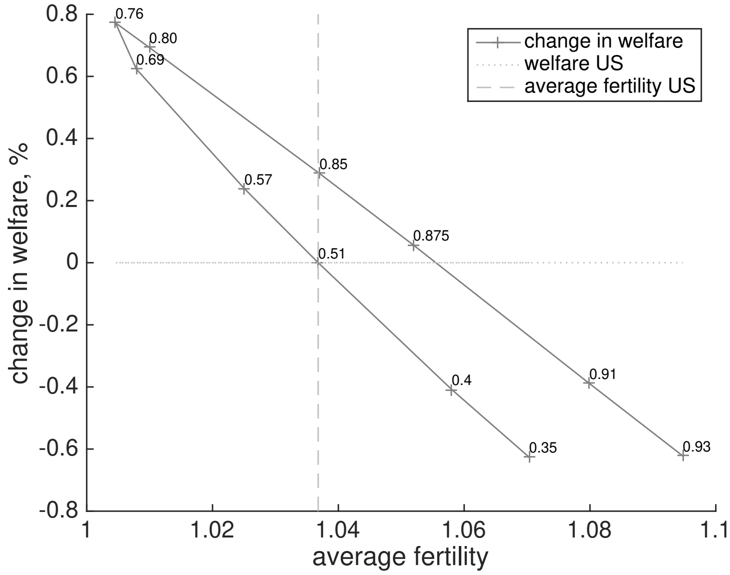
Therefore, the share of low income individuals increases. Due to decline in education subsidies agents with low and middle incomes are worse off. High income agents are worse off as well, since increases in fertility rates lead to a decline in the capital-to-labor ratio and, consequently, to a decline in equilibrium wages (see subfigures C, figure , in Appendix in 2.7).[10]

In contrast, when the policy becomes more progressive (see the case of $a_l = 0.76$) than a status quo policy, poor individuals increase investments in education of their children and decrease their fertility rates. Therefore, average productivity increases. Low income individuals are better off. High income individuals are better off as well, since the decline in population growth leads to increases in the capital-to-labor ratio and, consequently, to higher equilibrium wages. Consequently, welfare is higher than in the case of a status quo policy. However, if education policy becomes even more progressive (see the case $a_l = 0.85$), agents with middle and high incomes substantially decrease investments in the education of their children, and increase their fertility rates. Increases in higher education subsidies make low income individuals better off. However, individuals with middle and high incomes are worse off, due to a decrease in education subsidies and lower equilibrium wages. Consequently, welfare declines.

The relationship between progressivity of subsidies and average fertility is U-shaped. As discussed above, increases in progressivity of education subsidies lead to declines in fertilities of low income parents. This effect quantitatively dominates increases in fertilities of high income individuals. Therefore, average fertility declines with progressivity. However, when policy becomes substantially progressive, the fertility rates of a

[10]The capital-to-labor ratio is affected by fertility rates since savings are made by the previous generation. Therefore, an increase in population growth leads to a decline in capital per working population and, consequently, contributes to a decline in the capital-to-labor ratio.

Figure 2.5.4: Welfare and average fertilities



sizeable part of the population, including middle and high income individuals, increase. This effect starts to dominate the decline in fertilities of low income parents. Consequently, when the progressivity of education subsidies increases substantially, average fertility starts to increase.

As can be seen in figure 2.5.4, education subsidisation policies associated with higher welfare induce lower average fertility rates than a status quo policy. Interestingly, comparison of the policies that deliver the same average fertility rates demonstrates that more progressive higher education subsidies are superior in terms of welfare compared to their less progressive counterparts (see figure 2.5.4). This result suggests that an increase in welfare driven by decline in fertility differentials quantitatively dominates a decline in welfare driven by lower productivity of individuals with high market luck realisations.

The results presented in this section rely on the assumption that fertility decision responses to education subsidies are plausible in the model. In general, an empirical evaluation of fertility decision responses to government policies is a non-trivial exercise due to the long-term nature of lifetime fertility choices. Nevertheless, the subsection below demonstrates that, even in the absence of fertility decision adjustments with respect to education subsidies, the main results of the paper are preserved as long as the number of children is heterogenous across agents and decreasing with parental income as observed in the U.S. data.

2.5.2.2 The role of distributional effects and heterogeneous fertilities

As discussed above, in an economy with endogenous fertility it might be relatively "easy" to achieve higher welfare by the introduction of policies leading to lower population growth than in the status quo. However, lower fertility rates are not the main factor leading to welfare improvement. Positive welfare gains associated with more progressive education subsidies are mostly driven by distributional effects rather than

Table 2.5.2: Key economic variables. Benchmark and optimum

Variable	Description	U.S.	Optimum
S	Investments in higher education, share of U.S. GDP	3.34%	3.28%
C	Total consumption, share of U.S. GDP	80%	81%
Y	Output	100%	100.1%
K	Capital	2.99	3.03
w	Wage	0.2	0.222
r	Interest rate, annual	4.37%	4.3%
τ	Tax	1.14%	1.14%
γ	Average fertility	1.036	1.004
n_d	Fertility differential	1.42	1.17
$\underline{\psi}$	Share of agents, no higher education	0.40	0.04

by a decline in average fertility.

In order to see this, let us evaluate per capita welfare assuming that distribution is fixed, as in the benchmark case. Specifically, first, equilibrium utilities are evaluated in new equilibria corresponding to education subsidies with different degrees of progressivity. Then welfare is calculated by integrating equilibrium utilities over distribution corresponding to the status quo. As subfigure A, figure 2.7.3, in Appendix in 2.7 demonstrates, more progressive subsidisation policies do not lead to any material welfare gains. Therefore, adjustments in distribution are essential.

Another important result is that the assumption of heterogenous (exogenous) fertility rates is sufficient to generate the main results of this paper. Assumption of heterogenous fertilities implies that fertility rates are assigned to individuals exogenously, based on the benchmark decision rule, and do not change in response to education policies. That is, fertility rates negatively depend on parental productivity. In order to demonstrate the role of heterogenous fertility, I run the same policy experiments as in subsection 5.2.1 (TBA), but assume that the number of children n exogenously depends on parental state variables z and s in the same way as in the benchmark economy.

The results show that similarly to the endogenous fertility (or full adjustment) case analysed above, the dependence between the degree of progressivity of education subsidies and per capita welfare/population growth is hump-shaped/U-shaped (see subfigures B, C, figure 2.7.3, in Appendix in 2.7). Additionally, the policy maximising per capita welfare is also very similar to the endogenous fertility case ($a_l = 0.75$, $b_l = -0.39$). The individuals' ability to adjust their fertility rates in response to education subsidies amplifies absolute changes in welfare and population growth; but the direction of these changes is the same. The intuition is as follows. When education subsidies become more progressive, low income individuals increase investments in the education of their children. Consequently, their children transit to the states associated with higher human capital and lower fertility rates. On the other hand, individuals with middle and high incomes would invest less in the education of their children. Therefore, their children transit to the states characterised by a lower level of human capital and higher fertility rates than in the benchmark. This leads to higher per capita welfare and lower fertility rates.

2.5.2.3 The role of endogenous fertility

The key assumption of this paper is endogenous fertility. In this subsection I check the sensitivity of the main results to this assumption.

Assume uniform (exogenous) fertility implying that the number of children is identical across individuals and equal to average fertility in the benchmark economy (1.036). The replication of the same exercise as in subsection 5.2.1, assuming that model parameters are the same as in the benchmark economy, shows that an increase in progressivity of education subsidies does not lead to any material welfare improvement compared to the status quo (see subfigure A, figure 2.7.4, in Appendix in 2.7).

However, the model with assumption of exogenous fertility and benchmark parameters generates an unrealistically low share of individuals with no higher education: 18 % versus 40 % in the data. The prediction of other statistics is also imprecise. To address this caveat, I calibrate the model with exogenous fertility to match target statistics in the U.S. data. I assume that parameter ξ is the same as in the benchmark model, since the fertility differential is always equal to 1 in this model. Additionally, parameter σ is also set at the same level as in the benchmark economy, since average fertility is exogenous. The estimates of other parameters and model fit are provided in table 2.7.1 in Appendix in 2.7. Similar to the case of benchmark model parameters, this model also predicts the absence of any material gains of more progressive policies (see subfigure B, figure 2.7.4, in Appendix in 2.7). This result demonstrates that assumption of endogenous fertility is crucial for the main findings of the current paper.

2.6 Conclusion

In this paper I have developed a dynastic general equilibrium model for analysis of higher education subsidies. The key assumption of the model which distinguishes it from the existing literature is endogenous fertility. Employing this model calibrated to the U.S. economy, I quantify the optimal degree of progressivity of education subsidies. The analysis focuses on linear policies. I find that the optimal policy is characterised by a higher degree of progressivity than current U.S. policy. Additionally, the relation between progressivity of subsidies and welfare/average fertility is hump- and U-shaped respectively.

While the assumption of endogenous fertility is quantitatively important, heterogeneity in fertilities across individuals is sufficient to generate these results. This is because welfare gains of more progressive subsidies are driven not only by declines in fertility rates of low income individuals, but also by the fact that their children transit to the states associated with relatively low fertilities.

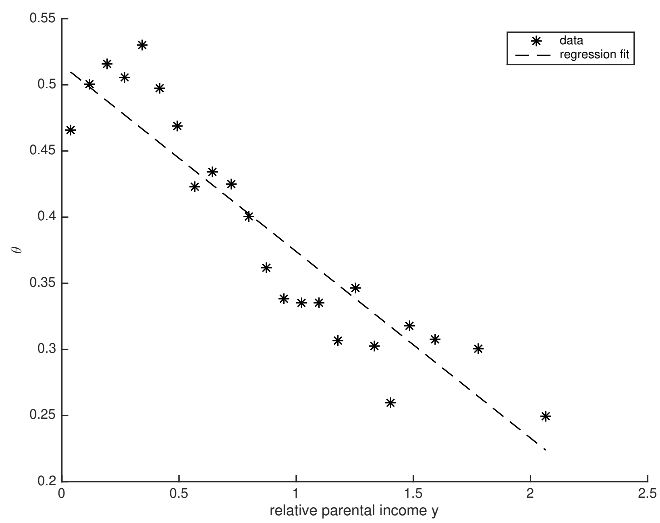
Future research may focus on relaxing the assumption of unobservable children's ability shock z' which is a current model simplification. On the one hand, if agents could observe the abilities of their children, variations in parental education spending and, consequently, earnings inequality would increase, because abilities are positively correlated across generations. Therefore, model recalibration might be needed to match U.S. stylised facts. On the other hand, the intuition of the main results would be preserved, because in the case of observed children's abilities, low/high income parents would still decrease/increase their fertility rates in response to more progressive higher education subsidies.

Another interesting extension of this paper could be an assessment of the plausibility of fertility decision responses to education subsidies based on a cross-country case study. Policies related to higher education observed in various developed countries could be plugged into the model to evaluate whether resulting equilibrium fertility rates are plausible. Last but not least, future research may focus on introducing student loans, which are exogenous in this paper, as an additional education policy parameter, and on solving for an optimal combination of financial aid and student loans.

2.7 Appendix

This section presents the plot of the financial aid to the price of college ratio (proxy for $\theta(y)$) against relative parental income together with regression fit, provides analytical proof of certain properties of the model and depicts the results.

Figure 2.7.1: Regression fit for $\theta(y)$



It can be analytically verified that the model predicts positive dependence between investments in education per child s' and parental education s. Additionally, keeping s, s' positively depends on education subsidy θ.

In order to prove this result, first, write down the dynastic formulation of the individual's problem without higher education subsidies. Using optimality conditions, express c_o as a function of c:

$$c_o = c\left(\beta\left(1+r\right)\right)^{\frac{1}{\sigma}}.$$

Therefore, the problem can be rewritten as:

$$V(z,\bar{h}) = \max_{c,n,\bar{h}'} \{\omega_u \frac{c^{1-\sigma}}{1-\sigma} + \beta\chi n^{\xi} E_t\left[V(z',\bar{h}')|z\right]\}$$

$$\omega c + \left(s' + g(\bar{h},z)\right)n \leq wz\bar{h}(1-\phi n)(1-\tau) + \frac{wz\bar{h}\epsilon(1-\tau)}{1+r}$$

$$\bar{h} = (\kappa + s)^{\eta}$$

$$s' \geq 0$$

$$\omega_u = 1 + \frac{\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}}{(1-\sigma)^{1-\sigma}}$$

$$\omega = 1 + \frac{(\beta(1+r))^{\frac{1}{\sigma}}}{1+r}$$

Now using the human capital production function express s' as a function of child human capital $\bar{h}'$: $s' = \max(\bar{h}'^{\frac{1}{\eta}} - \kappa, 0)$. Denote $\bar{h}'^{\frac{1}{\eta}} = e(\bar{h}')$. Following Alvarez (1999), premultiply the utility function by N^{ξ}, where N is a total size of dynasty with current level of productivity equal to $z\bar{h}$. Then rewrite the problem in a so-called dynastic form:

$$V(\bar{H}, N, z) = \max_{C, \bar{H}', N'} U(C, N) + \beta\chi E_t \left[V(\bar{H}', N', z') | z \right]$$

$$\omega_\lambda C + \left(e\left(\frac{\bar{H}'}{N'}\right) - \kappa + g\left(\frac{\bar{H}}{N}, z\right)\right) N' \leq wz\bar{H}(1-\tau) - wz\frac{\bar{H}}{N}(1-\tau)\phi N' + \frac{wz\bar{H}\epsilon(1-\tau)}{1+r}$$

$$\frac{\bar{H}'}{N'} \geq \kappa^{\eta}$$

where N is a total number of individuals in a dynasty, $C = cN$, $\bar{H} = hN$, $\bar{H}' = h'N'$, $U(C, N) = w_u \frac{(\frac{C}{N})^{1-\sigma}}{1-\sigma} N^{\xi}$, $V(\bar{H}, N, z) = N^{\xi} v(\frac{\bar{H}}{N}, z)$.

The utility function is homogenous of degree ξ with respect to C and N by construction. Additionally, the budget set is a cone. According to Alvarez & Stokey (1998), dynamic programming problems where the objective function is homogenous and the budget set is a cone can be analysed by similar tools as those for standard bounded dynamic programming problems.

Now fix consumption C at some level $\bar{C}$ and write first order conditions with respect to $\bar{H}'$ and N' for interior solution:

$$E_t \left[V_1'(\bar{H}', N', z') | z \right] + \lambda \left(-e_1' \left(\frac{\bar{H}'}{N'} \right) \right) = 0$$

$$E_t \left[V_2'(\bar{H}', N', z') | z \right] + \lambda \left(-e + \kappa + e_1' \left(\frac{\bar{H}'}{N'} \right) \left(\frac{\bar{H}'}{N'^2} \right) N' - g\left(\frac{\bar{H}}{N}, z\right) - (1-\tau)\phi w \frac{\bar{H}}{N} \right) = 0$$

Dividing the first equation above by the second one we get:

$$\frac{E_t \left[V_1'(\bar{H}', N', z') | z \right]}{E_t \left[V_2'(\bar{H}', N', z') | z \right]} = \frac{e_1'\left(\frac{\bar{H}'}{N'}\right)}{e - \kappa - e_1'\left(\frac{\bar{H}'}{N'}\right)\frac{\bar{H}'}{N'} + g\left(\frac{\bar{H}}{N}, z\right) + (1-\tau)\phi wz\frac{\bar{H}}{N}}.$$

Denote $\frac{E_t\left[V_1'(\bar{H}', N', z')|z\right]}{E_t\left[V_2'(\bar{H}', N', z')|z\right]} = \nu(\bar{H}', N', z)$. Since value function $V(\bar{H}, N, z) = N^{\xi} v(\frac{\bar{H}}{N}, z)$ is homogenous by degree $\xi < 1$, the ratio of its derivatives is homogenous of degree zero with respect to $\bar{H}'$ and N'. This implies that ν can be rewritten as a function of $\frac{\bar{H}'}{N'} = \bar{h}'$ and z. Since the value function is concave, ν is decreasing: $\frac{\partial \nu}{\partial \bar{h}'} < 0$. Plugging the functional form for e, we obtain:

$$\nu(\bar{h}', z) = \frac{e_1'\left(\bar{h}'\right)}{\left(1 - \frac{1}{\eta}\right) e\left(\bar{h}'\right) - \kappa + g\left(\bar{h}, z\right) + (1-\tau)\phi wz\bar{h}}.$$

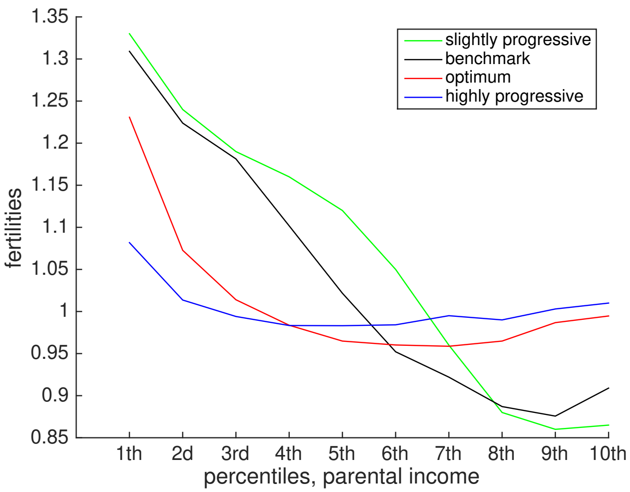
Denote the right hand side of the equation above as $\mu(\bar{h}, \bar{h}^{'})$. It is straightforward to show that $\frac{\partial\mu}{\partial\bar{h}'} > 0$. Since $\mu(\bar{h}, 0) = 0$, $\nu(\bar{h}^{'}, z) > 0$ and $\frac{\partial\nu}{\partial\bar{h}'} < 0$, this implies that solution $\bar{h}^{'}(\bar{h})$ exists for any $\bar{h} > 0$. Denote the interception point of $\nu(\bar{h}^{'}, z)$ and $\mu(\bar{h}, \bar{h}^{'})$ as $\bar{h}^{*}$. Therefore, if $\bar{h}^{*} < \kappa^{\eta}$, then $\bar{h}^{'}(\bar{h}) = \kappa^{\eta}$, otherwise $\bar{h}^{'}(\bar{h}) = \bar{h}^{*}$. Since $g_1^{'}(\bar{h}, z) > 0$, $\frac{\partial\mu}{\partial\bar{h}} < 0$, therefore, $\bar{h}^{'}$ is a strictly increasing function of h when the solution is interior.

Education subsidies. If one introduces education subsidies into the model, then the equation determining $\bar{h}^{'}(\bar{h})$ modifies as follows:

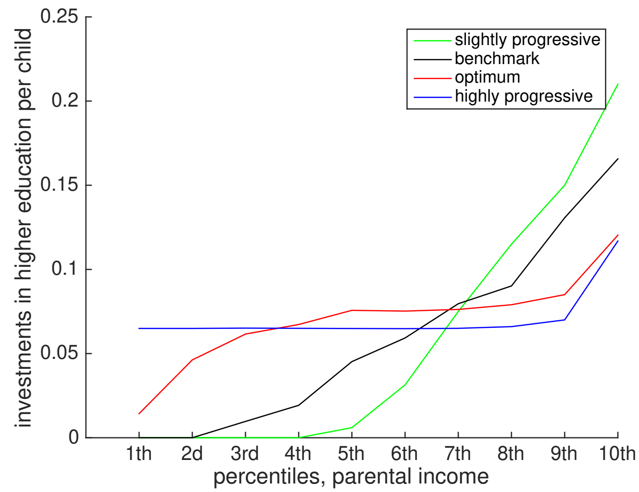
$$\nu(\bar{h}^{'}, z) = \frac{e_1^{'}(\bar{h}^{'})}{\left(1 - \frac{1}{\eta}\right) e\left(\bar{h}^{'}\right) - \kappa + \frac{g\left(\bar{h}, z\right) + (1-\tau)\phi w z \bar{h}}{1-\theta\left(\bar{h}, z\right)}}.$$

Given that $\theta(\bar{h})$ is a function of $\bar{h}$, dependence between $\bar{h}^{'}$ and $\bar{h}$ can turn to negative (if education subsidies are highly progressive). However, an increase in the level of $\theta(\bar{h}, z)$ for given $\bar{h}$ leads to the decrease of the right hand side of the equation above. Consequently, higher subsidies positively affect child human capital $\bar{h}^{'}$.

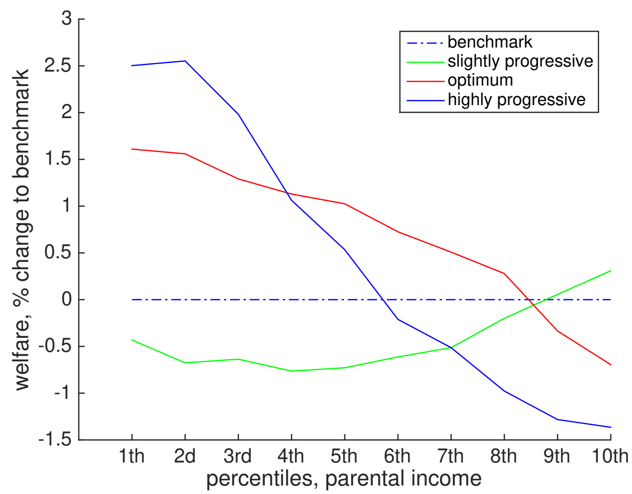
Figure 2.7.2: Fertilities, investments in education of children and welfare



A: Fertility

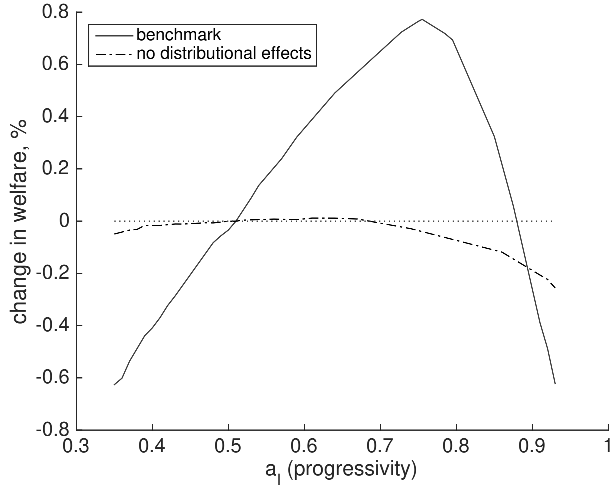


B: Investments in education per child

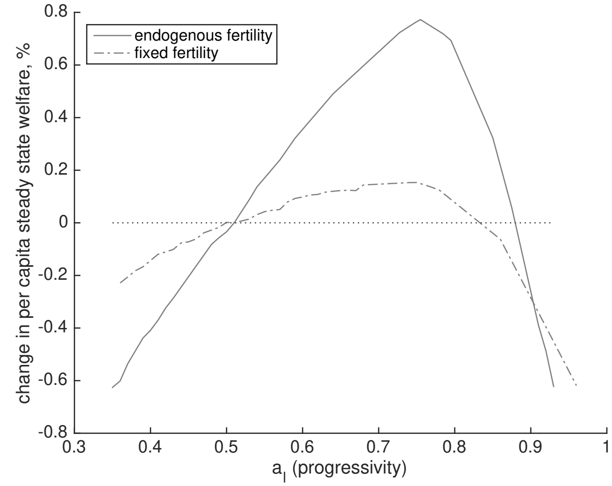


C: Welfare change

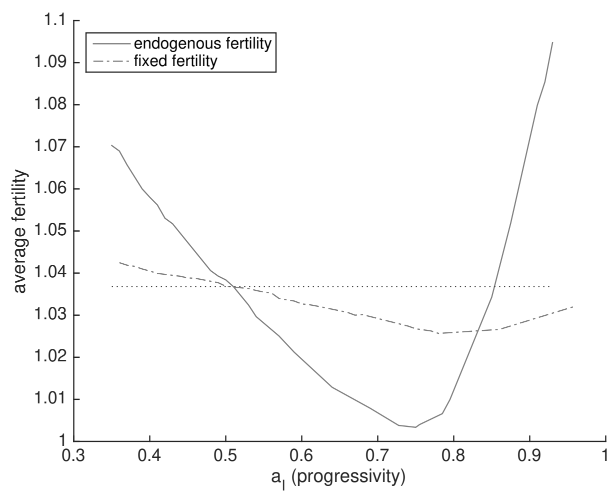
Figure 2.7.3: Decomposition of results I



A: Welfare. Elimination of distributional effects.



B: Welfare. Heterogenous (exogenous) fertility

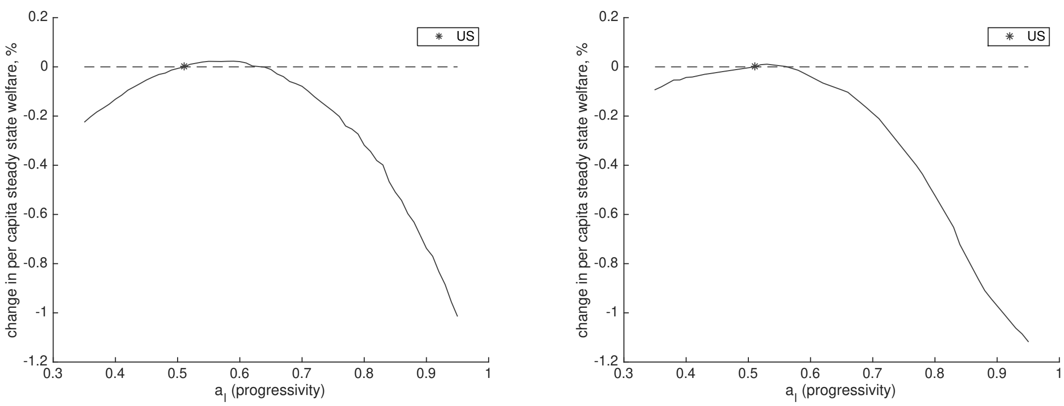


C: Fertility. Heterogenous (exogenous) fertility

Table 2.7.1: Estimates of jointly calibrated parameters. Uniform (exogenous) fertility

Target	Parameter	Value	Data	Model
Intergenerational persistence of log earnings	ρ	0.32	0.47	0.468
Variance of log earnings	σ_z	0.52	0.36	0.35
Annual capital to GDP ratio	β	0.63	3	3.05
Wage premium of higher education	η	0.55	1.61	1.61
Share of public expenditures on higher education in GDP	χ	0.65	0.95%	0.957%
Share of individuals, no higher education	κ	0.005	42%	40%

Figure 2.7.4: Decomposition of results II



A: Welfare. Uniform (exogenous) fertility. Benchmark parameters

B: Welfare. Uniform (exogenous) fertility. Calibrated parameters

Chapter 3

Voting on Education and Redistribution Policies in the U.S: Does Endogenous Fertility Matter?

Published as CERGE-EI Working Paper Series No. 681

3.1 Introduction

In modern societies, including the U.S, in which birth control is widely used, the number of children becomes an individual choice, which is very likely to be affected by public policies. Even though empirical and theoretical studies demonstrate that fertility decisions indeed interact with redistribution and education policies in non-negligible ways, the existing literature on determinants of public policies apparently abstracts from endogenous fertility.[1]

This paper fills this gap by demonstrating that endogenous fertility is an important determinant of redistribution and education policies and is one of few studies performing a quantitative analysis of public policies.[2] Specifically, relying on a novel politico-economic extension of a standard framework in the style of Barro & Becker (1989), this paper finds that majority-voting equilibrium transfers and education subsidies predicted by this model are quite close to the U.S. data (5.5% and 2.9% of GDP in the model versus 5.4% and 2.5% of GDP in the data). However, when endogenous fertility is eliminated from the model, equilibrium transfers and education subsidies increase dramatically - to 8.3% and 4.7% of GDP, respectively.

[1] A wide range of empirical studies finds that expansion of transfers leads to non-negligible increases in birth rates in the U.S. (Georgellis & Wall, 1992; Whittington et al., 1990) and other developed countries (Bjorklund, 2006, Sweden; Ermisch, 1998, UK). Additionally, De la Croix & Doepke (2009) demonstrate theoretically that voting on public education interacts with fertility decisions. The predictions of their theory are consistent with the data on U.S. states and cross-country evidence.

[2] One of the most celebrated studies in this area is Krusell & Rios-Rull's (1999) paper.

Endogenous fertility is important because it makes transfers and education subsidies more costly for two reasons. First, both types of policies positively affect fertility differentials between low and high income individuals, because income and substitution effects act in the same direction for high income as opposed to low income parents. Since both transfers and education subsidies imply redistribution of resources from rich to poor agents, the income effect of policies on fertilities is positive for low income parents and negative for high income parents. In contrast, since both policies lead to declines in the opportunity costs of children and investments in education per child, the substitution effect increases the incentives to have children for both types of parents. Therefore, transfers and education subsidies positively affect fertility differentials. Increases in fertility differentials lead to higher shares of low productive individuals and, consequently, to less resources available to finance public policies. Consequently, equilibrium tax rates increase.

Second, both policies positively affect the average number of children. The substitution effect quantitatively dominates the negative income effect for high income parents, and fertility rates of both productivity types increase. Increases in the average number of children lead to declines in aggregate labor supply and growth of total expenditures on education subsidies. Both factors contribute to increases in equilibrium tax rates. Therefore, a median voter would choose less generous transfers and education subsidies, which turn out to be much closer to the U.S. data than in the absence of endogenous fertility.

The implications of the model regarding a cross-section of U.S. states are used to validate the framework and circumvent the lack of empirical evidence on the elasticities of fertilities with respect to public policies. Taken redistribution policies as given, the model replicates the variations and levels of education subsidies, average numbers of children, and fertility differentials across U.S. states quite closely.[3] This confirms the plausibility of the model and, consequently, the credibility of the main result, which highlights the importance of endogenous fertility for high performance of the model in explaining the levels of public expenditures in the U.S.

The remainder of this study is organised as follows. Section 3.2 discusses related literature, section 3.3 presents the model and section 3.4 introduces calibration. Section 3.5 describes the results including evaluation of the model's ability to replicate the levels of public expenditures observed in the U.S. data, and presents model validation based on data from a cross-section of U.S. states.

3.2 Related Literature

The contribution of this paper is twofold. First, it demonstrates that a politico-economic extension of a standard model with endogenous fertility in the style of Barro & Becker (1989), in which redistribution and education subsidisation policies are determined by majority voting, goes a long way to explain the levels of both types of public expenditures in the U.S. Notably, the assumption of endogenous fertility is

[3] Since the greatest part of transfers are financed from federal budget, redistribution policy is assumed to be exogenous at the state level.

quantitatively important for this result. Second, the study builds a quantitative theory connecting transfers, public education provision, intergenerational persistence of earnings, and endogenous fertility in the U.S.

The first contribution makes the current study close to theoretical and quantitative political economy literature. There is a wide range of theoretical studies analysing determinants of redistribution and education policies. As opposed to this paper, most theoretical studies focus on factors related to individuals' perceptions and preferences, which are typically very difficult to quantify. For instance, Piketty (1995) shows that preferences for redistribution depend on individual histories of productivities. Benabou & Ok (2001) suggest that demand for redistribution is influenced by individuals' perceptions of upward mobility. Alesina & Angeletos (2005) demonstrate that individual choices of redistribution policies might be also affected by individuals' ability to distinguish between the "luck" and "effort" components of income. Additionally, the choice of public provision of education may depend on community income (Fernandez & Rogerson, 1998). Agents may also support public provision of education due to positive externalities related to accumulation of human capital (Benabou, 1996). The studies listed above endogenise either redistribution or education policies. Bernasconi & Profeta (2012) and Ono (2016) are among the few papers integrating both types of policies. The quantitative analysis presented in this study, which also integrates both policies, may be seen as complimentary to these theoretical papers.

One of the most influential quantitative studies in this area is Krusell & Rios-Rull (1999). Similarly to their work, this paper analyses a dynamic framework and investigates whether the level of redistribution observed in U.S. data could be rationalised as an outcome of politico-economic equilibrium. However, beyond endogenous fertility, the fundamental assumption of this paper, which distinguishes it from Krusell & Rios-Rull, is uncertainty driven by ability shocks. Consequently, the costs-benefit comparison motive of demand for redistribution in the style of Meltzer & Richard (1981) is augmented by parental willingness to insure their children against negative ability shocks.

The main contribution of this work to the politico-economic literature discussed above is its consideration of endogenous fertility. The literature in this area is scarce. One example is a study by De la Croix & Doepke (2009), who connect private education and voting on public funding for schooling with endogenous fertility. This paper abstracts from the choices between public and private schools, but extends the setup analysed in their study to a dynamic dynastic framework in which both education and redistribution policies are endogenous and determined through majority voting.

The second contribution relates this paper to a wide range of studies devoted to theoretical and quantitative modelling of redistribution and education policies and their interaction with intergenerational correlation of earnings and income inequality. Restuccia & Urrutia (2004), Sephardi & Yuki (2004), Erosa & Koreshkova (2007) and Krueger & Ludwig (2013) are examples of papers in this area. Most of these studies evaluate the roles of redistribution and education policies separately, while this study integrates both policies, similarly to Krueger & Ludwig. Additionally, this paper contributes to this strand of literature by endogenising fertility. The literature in this area is scarce: De la Croix & Doepke (2003,2004), Moav (2005) and Fan & Zhang

(2013) are among the few examples of theoretical studies in this domain. This paper extends frameworks developed in these papers to a dynamic dynastic setting with stochastic abilities.

The example of quantitative study in this area is Knowles (1999a,1999b). Similarly to this paper, Knowles develops a general equilibrium model in which both decisions on the human capital of children and fertilities are endogenous. This study extends the work of Knowles by considering public provision of education and endogenising education and redistribution policies through majority voting.

3.3 Model

This section begins with the description of the economic environment and a definition of equilibrium assuming that education and redistribution policies are exogenous. Then the concept of politico-economic equilibrium is formulated.

The methodology in this work builds on Barro & Becker (1989) and Alvarez (1999), who introduce endogenous fertility and human capital formation into a dynamic dynastic model and Knowles (1999a), who extends this basic framework to a general equilibrium setting. This paper further extends the methodology developed in these studies by introducing education policy and endogenising both redistribution and education policies as outcomes of politico-economic equilibrium.

3.3.1 The economic environment. Exogenous policies

Consider a two-period overlapping-generation dynastic model populated by an infinite number of individuals. Agents live for two periods corresponding to 0-25 and 26-50 years in a real economy. All decisions in the model are taken by parents who decide on the number of their children, their investments in the education of each child, and consumption. There is a government in the economy that proportionally taxes incomes and uses tax revenue to finance transfers, education subsidies and exogenous government expenditures. The distribution of transfers across income groups is exogenous and replicates the combination of direct and means-tested benefits in the U.S. economy.

3.3.1.1 Education and human capital

This study focuses on a primary education (corresponding to K-12 in the U.S.) and for simplicity abstracts from college education. This simplification is motivated by the fact that expenditures on primary education are likely to be more important for fertility decisions than expenditures on college, which take place later after child birth. Additionally, as demonstrated by Keane & Wolpin (1997), around 90% of the variance in lifetime utility is determined by the heterogeneity of skills acquired by the age of 16, prior to entering college. The human capital production technology takes the following form:

$$h' = z'[h^{\kappa}(\theta + e)^{1-\kappa}]^{\eta}$$

where $0 < \kappa, \eta < 1$, h' denotes the human capital of the child, z' is iid ability shock for the child, h is parental human capital, θ is public provision of primary education, e is private spending on primary education.

Following Restuccia & Urrutia (2004), I assume that public and private funding are perfect substitutes. Public provision of education θ corresponds to state and federal funding of public schools, while private expenditures are interpreted as local funding of primary education financed by local taxes. Consequently, each productivity type should be interpreted as a community of homogeneous agents who choose the amount of local funding of public schools along with number of children and consumption.

3.3.1.2 Individual problem

Individuals are referred to as children and adults in first and second periods of their lives. All decisions are made by adults. At the beginning of the second period, individuals enter adulthood and decide on the number of their children, investments in the education of each child, and consumption. The ability shock of child is revealed after decisions on the number of children and investments in the education of each child is made. Preferences are in the style of Barro & Becker (1989) with parental utility depending on current period consumption and the expected utility of each child, weighted by an increasing concave function of the number of children. Individual earnings are determined by human capital or productivity and time devoted to paid work.

For now, education subsidies θ_t and transfers T_t are exogenous. I keep time indexes for variables because public policies and, consequently, individual decisions may change over time. The dynamic programming formulation of individual problem is as follows:

$$V_t(h) = \max_{c_t, n_t, e_t} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} + \beta n_t^{\xi} E_{z'}[V_{t+1}(h')] \right\}$$

subject to

$$\begin{aligned} c_t + [g_t(i_t) + e_t]n_t &\leq w_t h(1 - \phi n_t)(1 - \tau_t) + T_t(i_t/\bar{i}_t), \\ T_t(i_t/\bar{i}_t) &= \lambda_{0t}\lambda(i_t/\bar{i}_t), \\ g_t(i_t) &= \bar{g}(i_t), \\ h' &= z'[h^{\kappa}(\theta_t + e_t)^{1-\kappa}]^{\eta} \end{aligned}$$

where h is human capital or productivity, w_t is wage, ϕ is the share of parental time spent per child, n_t is the number of children, $i_t = w_t h(1 - \phi n_t)$ is income, $\bar{i}_t$ is average income, τ_t is a labor income tax, T_t is transfers which depend on income according to parameter λ_{0t} and function $\lambda(i_t/\bar{i}_t)$ discussed in the calibration section below, z is iid ability shock, $\log(z) \in N(0, \sigma_z^2)$, c_t is the consumption of an adult individual, g_t is the cost of children in terms of goods, which depends on parental income according to function $\bar{g}(i_t)$ discussed in the calibration section below.

3.3.1.3 Production

A large number of firms rent effective labor from households in competitive markets. The production technology is determined by linear function: $Y_t = L_t$, where L_t is an aggregate effective labor supply.

3.3.2 Economic equilibrium

Let $\psi_t(h)$ denote the share of agents at time t with human capital $h \in H = [\underline{h}, \bar{h}]$.

The definition of recursive competitive equilibrium assumes that public policies are exogenous. Given an initial measure ψ_0 of individuals, a competitive equilibrium is a sequence of value functions and policy rules $\{V_t, c_t, e_t, n_t\}_{t=0}^{\infty}$, production plans $\{Y_t, L_t\}_{t=0}^{\infty}$, sequence of transfers, education subsidies and labor income taxes $\{T_t, \theta_t, \tau_t\}_{t=0}^{\infty}$, sequence of prices $\{w_t\}_{t=0}^{\infty}$ and sequence of measures $\{\psi_t\}_{t=1}^{\infty}$ such that:

1) given prices and government policies, $\{V_t, c_t, e_t, n_t\}_{t=0}^{\infty}$ solves the individual problem specified above;

2) prices $\{w_t\}_{t=0}^{\infty}$ are determined by the optimal behaviour of the representative firm;

3) the government budget is balanced in all periods t:

$$\int [T_t(i_t/\bar{i}_t) + \theta_t n_t] d\psi_t + E_t = \tau_t w_t L_t,$$

where $E_t = \delta Y_t$ are exogenous government expenditures, $0 < \delta < 1$;

4) markets clear in all periods t:

a) market of goods: $Y_t = \int \big[c_t + [g_t(i_t) + e_t + \theta_t]n_t\big] d\psi_t + E_t$;

b) labor market: $L_t = \int \big[h(1 - \phi n_t)\big] d\psi_t$;

5) $\psi_{t+1} = G_t^{\psi}(\psi_t)$, where G_t^{ψ} is a law of motion for measures of individuals, which is determined by households' decision rules and a stochastic process for abilities z. The explicit definition of the law of motion for measures is as follows. For all subsets $B \in H$ the Markov transition function at time t is defined as

$$P_t(h, B) = \begin{cases} p(z'), & \text{if } h'(z', h, e_t) \in B, \\ 0, & \text{otherwise.} \end{cases}$$

In other words, the probability of going from state h to a subset of states B is zero if that set does not include the next period child human capital h', which is determined according to the given human capital production technology $h'(z', h, e_t) = z'[h^{\kappa}(\theta_t + e_t)^{1-\kappa}]^{\eta}$. If B includes h', then transition probability is fully determined by the stochastic process for abilities.

Given the definition of the Markov transition function, the next period's measures of individuals are given by:

$$\psi_{t+1}(B) = \frac{\int P_t(h, B) n_t d\psi_t}{\gamma}$$

where γ is equal to average fertility and shows the relative size of the next period generation to the current generation:

$$\gamma = \int n_t d\psi_t.$$

3.3.3 Politico-Economic equilibrium

Now assume that transfers T and education subsidies θ are endogenous and determined as outcomes of majority voting. That is, similar to Meltzer & Richard (1981) and Krusell & Rios-Rull (1999), agents vote for the levels of transfers and education subsidies that maximise their equilibrium utility, and equilibrium policies coincide with those preferred by a median voter.

In order to derive individuals' preferences over transfers and education subsidies, I introduce the following assumptions. First, agents anticipate that the policies chosen in the current period will be in place forever. This assumption is quite realistic given that agents are likely to be myopic.[4] Second, as in Meltzer & Richard (1981) and Krusell & Rios-Rull (1999), agents correctly predict the general equilibrium effects of redistribution and education policies and calculate their utilities accordingly.

Given these assumptions, individuals' preferences over transfers and education subsidies are defined in the following way. Assume that given levels of redistribution and education policies (T, θ) were run forever so that the economy is in a steady state and at the beginning of the current (zero) period there is an occasional opportunity to re-vote on policies. The utility of an individual with human capital h associated with introduction of an alternative level of transfers T' and education subsidies θ' is given by:

$$W(h, \theta, T, \theta', T') = V_0(h) \text{ so that } T_{-1} = T,\ \theta_{-1} = \theta \text{ and } T_t = T',\ \theta_t = \theta'\ \forall\ t \geq 0.$$

$V_0(h)$ is the utility of an individual with productivity h, T_{-1}, θ_{-1} are policies at the beginning of the current period, T', θ' are alternative policies introduced in the current period and run forever.[5] Utility $W(h, \theta, T, \theta', T')$ depends on status quo policies (T, θ) since these policies determine current period distribution of individuals ψ_0 and, consequently, affect subsequent distributions $\{\psi_t\}_{t=0}^{\infty}$, which, in turn, influence individual utility through equilibrium tax rates.

Since policy space is bi-dimensional, Nash equilibrium may not necessarily exist in a majority voting game. Therefore, following Conde-Ruiz & Galasso (2003) and Ono (2016), I use the so-called issue-by-issue voting concept formalised by Shepsle (1979). Under this concept, a sufficient condition for 2-tuple (T^*, θ^*) to constitute a politico-economic equilibrium of voting game is that T^* constitutes an majority-voting equilibrium, given that the other policy θ is fixed at the level θ^* and vice versa. Additionally, preferences must be single-peaked in each dimension of the policy space. In this study, I quantitatively verify that these conditions are satisfied.

According to the issue-by-issue voting concept, in this model majority-voting equilibrium is defined as follows. If a median voter prefers not to deviate from the current policy T, then T constitutes majority-voting

[4] The current approach might be criticised from the position of the dynamic voting paradigm of Krusell & Rios-Rull's (1999) and subsequent literature. In this study, implementation of dynamic voting is not feasible due to uncertainty.

[5] When calculating their utilities, individuals take into consideration the transition path from the current steady state to the new stationary equilibrium under alternative policies (T', θ').

equilibrium transfers given education subsidies θ:

$$T = T(T, \theta) = \underset{T'}{\operatorname{argmax}}\ W(h_m(\theta, T), \theta, T, \theta' = \theta, T')$$

where $h_m(\theta, T)$ is the human capital of a median voter.

Similarly, if agents are voting on education subsidies, and a median voter prefers not to deviate from the current policy θ, then θ constitutes a majority-voting equilibrium education subsidisation policy given transfers T:

$$\theta = \theta(T, \theta) = \underset{\theta'}{\operatorname{argmax}}\ W(h_m(\theta, T), \theta, T, \theta', T' = T)$$

Consequently, the fixed-point condition determining issue-by-issue voting equilibrium (T^*, θ_0^*) is determined by solving a two equation system:

$$T^* = T(\theta^*, T^*);\ \theta^* = \theta(\theta^*, T^*).$$

3.4 Calibration

This section describes estimation of the model parameters. The model is calibrated assuming that policies are set exogenously at the corresponding U.S. levels and the economy is in the steady state.

The following parameters take values which are standard for macroeconomic literature: $\beta = 0.366 = 0.99^{100}$ so that the quarter discount rate is 0.99, as in Aiyagari (1994). The share of parental time spent on children ϕ is equal to 0.09 as in De la Croix and Doepke (2003), who set an analogous time cost parameter equal to 0.075 in a model with a period of 30 years. Because in this current model, the period is 25 years, the parameter is adjusted to $0.09 = 0.075 * 30/25$. A government expenditure-to-GDP ratio of 0.165 is calculated as 0.19 (Krusell & Rios-Rull, 1999) less 0.025 corresponding to the ratio of expenditures on public primary education to GDP, which is endogenous in the current model.

The remaining parameters are jointly calibrated so that, given the redistribution and education policies set exogenously at the levels corresponding to the U.S. data, the steady state equilibrium replicates the relevant statistics of the U.S. economy described below.

3.4.1 Data

I begin by discussing the timing of the data used to evaluate characteristics of the U.S. economy. This study focuses on 1992-2002, because I use a cross-section of U.S. states to evaluate model performance. 1992 is the earliest date that Census data on state government finances is available, and 2002 is close to the year when the 1980-1982 cohort entered adulthood. This cohort is analysed in Chetty et al. (2014), which is apparently the only data source on intergenerational correlation of earnings across U.S. states.

3.4.1.1 Transfers

The distribution of transfers across individuals is exogenous and governed by the function $\lambda(i_t/\bar{i}_t)$. This study focuses on money transfers and does not consider population-based services and public goods. Money transfers in the U.S. consist of direct and means-tested benefits. Direct transfers include expenditures on Social Security, Medicare, Unemployment Insurance and Worker's Compensation. Payments for retirees are excluded from transfers, because old age is not analysed in the model. While direct transfers are not conditional on the income of recipients, means-tested transfers mostly benefit low income individuals. The largest means-tested programs are Medicaid, Earned Income Tax Credit (EITC), Food Stamps and Supplemental Security Income (SSI).

As in Krusell & Rios-Rull (1999), total expenditures on direct benefits are set at 1.7% of GDP (spending on pensions account for 5.1% GDP, according to the OECD Social Expenditure Statistics 1995). Means-tested transfers are set at 3.7% of GDP including payments from the federal budget (1.1% of GDP) and spendings from state and local budgets (2.6% of GDP).[6] Using estimates of aggregate expenditures on transfers and distribution of transfers across income groups from *The Redistributive State: The Allocation of Government Benefits, Services, and Taxes in the United States* report provided by the Heritage Foundation, I evaluate the distribution of total transfers across income quintiles (see table 3.4.1 below).[7]

Table 3.4.1: Distribution of transfers

Income quintile	First	Second	Third	Fourth	Fifth
Direct transfers	0.24	0.26	0.2	0.15	0.14
Means-tested transfers	0.46	0.29	0.15	0.08	0.04
Total transfers estimated	0.4	0.28	0.16	0.1	0.07

Notes. The first two rows are based on data from the Heritage Foundation. The last row shows the author's calculations.

As seen in table 3.4.1, the distribution of total benefits is skewed to the bottom of the income distribution. In order to replicate this property of distribution of transfers, I employ the following functional form:

$$\lambda(i/\bar{i}) = (i/\bar{i} + \lambda_1)^{-\lambda_2}$$

where $\lambda_1 > 0$ guarantees that transfers received by individuals with zero income is finite. Since the absolute level of λ is irrelevant, λ_1 could be normalised to 1 without loss of generality. Parameter $\lambda_2 > 0$ jointly calibrated with other parameters ensures that distribution of transfers is skewed to the bottom of the income distribution.[8]

[6] Krusell & Rios-Rull use 1995 as an example year to estimate expenditures on transfers, while this study focuses on 1992-2002. However, the calculations based on the data from the *Statistical abstract of the United States* for the latter period are very close to those provided in Krusell & Rios-Rull's paper. Therefore, I use the estimates from this influential study to make the results of this paper more comparable to existing literature.

[7] http://www.heritage.org.

[8] Knowles (1999a), who also analyses a framework with endogenous fertility, approximates transfers by a second order

3.4.1.2 Cost of children in terms of goods

This type of cost includes necessary expenditures on children of housing, food and clothing. According to data from *Expenditures on Children by Families* 1996 provided by the U.S. Department of Agriculture, expenditures on children in terms of goods are increasing with parental income. However, the share constituted by these costs to life-time parental income is decreasing with parental income. This finding is illustrated in table 3.4.2 below.

Table 3.4.2: Expenditures on children by husband-wife families

Income tertile	First	Second	Third
Before-tax mean annual income, in thousands of dollars	34.7	46.1	87.3
Expenditures on children, in thousands of dollars	66.75	86.4	126.96
Expenditures on children as a share of life-time income	0.24	0.15	0.12

In order to capture these properties of the costs of children in terms of goods, the following functional form is assumed:

$$g_t = g_1 {i_t}^{g_2}$$

where $i_t = wh(1 - \phi n_t)$, $g_1 > 0$, $0 < g_2 < 1$ are parameters to estimate. I choose income shares of expenditures on children for the first and second tertiles as target statistics. This is because, for individuals with relatively low incomes, the costs of children are more likely to be interpreted as necessities than for families from the top of income distribution.

3.4.1.3 Education

The data on public and local investments in education are from the *Annual Survey of School System Finances* provided by the U.S. Census Bureau. For 1992-2002, public provision of primary education in the U.S. constituted 2.5% of GDP. Total expenditures on primary education from both public and local sources accounted for 4.5% of GDP.

3.4.1.4 Variance and intergenerational correlation of earnings

Variance of log earnings takes a value equal to 0.36, which is standard for macroeconomic literature (Mulligan, 1997); the estimate of intergenerational correlation of earnings is non-standard and taken from Chetty et al. (2014). Their study demonstrates that standard estimates based on intergenerational correlation of log earnings provided by existing literature including Solon (1992) and Corak (2006) may be biased due to non-linearities of intergenerational earnings elasticity.

polynomial. However, I do not follow his approach, because transfers may become non-monotone when the distribution of productivity types changes.

In order to avoid this bias, Chetty et al. use an alternative approach based on the correlation between the rank of child income in the income distribution of children and the rank of parental income in the income distribution of parents. This type of estimate has been shown to be substantially more robust than intergenerational earnings elasticity estimates. Additionally, this study follows Chetty et al. because, to the best of my knowledge, this is the only data source for intergenerational correlation of earnings across U.S. states. Based on Chetty et al., the estimated national level of intergenerational earnings correlation is 0.33.[9]

3.4.1.5 Demographics

To estimate average fertility rates and fertility-income profiles, I use data on the number of Children Ever Born from the 1990 U.S. Census. In the case of women aged 40-49, this variable may serve as an appropriate proxy for life-time fertility, since women are very likely to complete their child birth processes by that age. For the purpose of comparability of estimates of different statistics, I use 1990 data and restrict the sample to women aged 40-45, because this cohort is closest to the cohort of mothers of children born in 1980-1982.[10]

Household income is proxied by total family income per adult family member. Since incomes of individuals stabilise after the age of roughly 40, the annual income of individuals in that age group may serve as an appropriate proxy of life-time incomes. Jones & Tertilt (2008) offer an alternative proxy based on an occupational income score.[11]

I evaluate fertility differentials using both total annual total family income and occupational income scores. Fertility differentials are measured as a ratio of the average fertilities of the bottom income quintile over average fertilities of the top quintile. Average fertility rates and fertility-income profiles for both methodologies are presented in table 3.4.3 below. Because there are only minor differences between the two, family income methodology is used for the calibration exercise, due to its larger sample size.

Table 3.4.3: Fertility-Income profiles

		Income quintiles				
Methodology	Average	First	Second	Third	Fourth	Fifth
Income	1.13	1.31	1.16	1.11	1.06	0.99
Occupation score	1.11	1.29	1.14	1.10	1.04	0.97

[9] This estimate is below standard evaluations equal to 0.4 (Solon, 1992) and 0.47 (Corak, 2006). However, as the sensitivity analysis presented in Appendix A in demonstrates, the main results of this study are robust to consideration of standard estimates.

[10] As in Jones & Tertilt (2008), I restrict the sample to married women.

[11] According to the U.S. Census definition, the occupation income score is a constructed variable that assigns a measure of the median earned income for each occupation. This study uses a variable based on the 1990 occupational classification scheme.

3.4.2 Jointly calibrated parameters

The parameters $\xi, \sigma, \theta_0, \eta, \kappa, \sigma_z, \lambda_0, \lambda_2, g_1, g_2$ are calibrated to match the statistics of the U.S. economy described above. While there is no one-to-one correspondence between parameters and target statistics, parameters are assigned based on the principle of sensitivity. Higher values of σ increase the curvature of the utility function. Consequently, as Knowles (1999a) suggests, number of children increases. Therefore, σ is assigned to average fertility.[12] Parameter ξ affects the marginal utility of an additional child. I assign ξ to the fertility differential between low and high income individuals. Investments in education are governed by the parameters of the human capital production function. Parameter θ denotes government subsidisation of primary education as a share of GDP. Parameter η reflects returns on education and, therefore, corresponds to total investments in education. I assign κ to the intergenerational correlation of earnings because this parameter influences the relative importance of parental human capital for the future human capital of a child. The variance of ability shock σ_z is assigned to the variance of log earnings.

The parameter of transfer function λ_0 affects an aggregate level of transfers as a share of GDP. Parameter λ_2 influences the curvature of the transfer function and, therefore, corresponding target statistics is a share of transfers paid to middle quintile of the income distribution. Clearly, one parameter is not enough to match the whole distribution of transfers. However, as I demonstrate later in the text, the resulting distribution of transfers in the equilibrium is very close to that in the data. Parameters g_1 and g_2 of the function $\bar{g}$ are responsible for expenditures on children in terms of goods for the bottom and middle tertiles of the income distribution. The estimates of calibrated parameters are presented in table 3.4.4 below. Columns "U.S." and "Model" demonstrate that the model replicates target statistics of the U.S. economy quite well.

Table 3.4.4: Estimates of jointly calibrated parameters

Description	Parameter	Value	Target	U.S.	Model
Risk aversion parameter	σ	0.41	Average fertility rate	1.13	1.128
Curvature of altruism factor	ξ	0.42	Fertility differential	1.31	1.31
Std dev of noise in z	σ_z	0.56	Variance of log earnings	0.36	0.362
Education subsidy	θ	0.009	Public education, % GDP	2.5	2.5
Elasticity of HC w.r.t. inputs	η	0.39	Total education, % GDP	4.5	4.6
Exponent on parental HC	κ	0.54	Persistence of earnings	0.33	0.33
Generosity of transfers	λ_0	0.27	Total transfers, % GDP	5.4	5.4
Slope of $\lambda(i/\bar{i})$	λ_2	2.9	Transfers, middle tertile	0.16	0.16
Generosity of child cost	g_1	0.07	Child cost, bottom tertile	0.24	0.237
Slope of child cost	g_2	0.32	Child cost, middle tertile	0.15	0.148

[12] Note that, due to endogenous fertility, $1 > \sigma > 0$.

3.4.3 Benchmark model fit

In this subsection I evaluate the performance of the calibrated model in replicating characteristics of the U.S. economy, which are important for the results but not directly targeted in the calibration.

The distribution of transfers across income groups is important because it affects demand for redistribution. Table 3.4.5 below demonstrates that the distribution of transfers generated by the model is very close to the data. Therefore, the choice of the functional form of λ is reasonable. The distribution of earnings is

Table 3.4.5: Distribution of transfers

Income quintile		First	Second	Third	Fourth	Fifth
Distribution of transfers	Benchmark	0.41	0.26	0.16	0.11	0.06
	U.S.	0.40	0.27	0.16	0.10	0.07
Distribution of earnings	Benchmark	0.37	0.64	0.84	1.13	2.01
	U.S.	0.36	0.66	0.85	1.11	2.01

another important characteristic of the economy because it affects the median voter's demand for insurance. In the calibration exercise, the target parameter is the variance of log earnings. However, as the table above shows, the calibrated model is capable of replicating the whole distribution of earnings. The relative position of the median voter determined by the median-to-mean income ratio is matched very closely: 0.837 versus 0.835 in the data.[13]

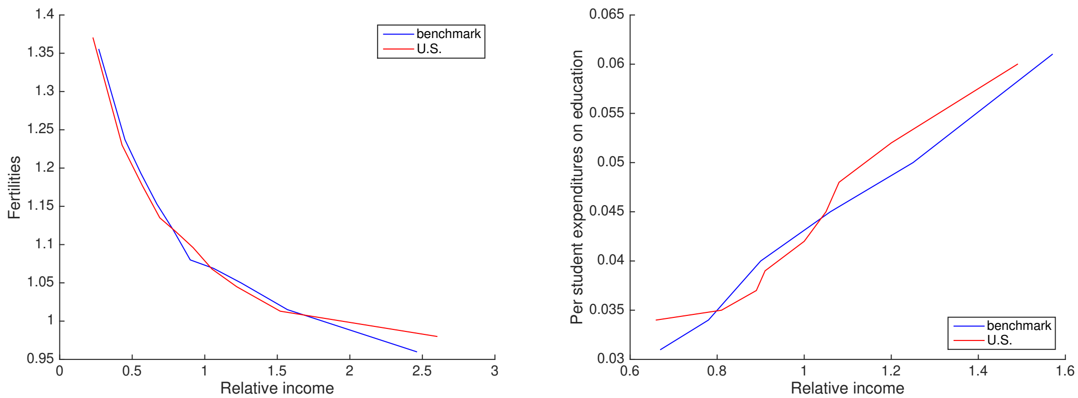
Additionally, the model replicates negative/positive relations between the number of children/expenditures on education per child and parental incomes (see 3.4.1 below)[14]. Although, in general, the Barro-Becker model does not guarantee reproduction of these properties.

The results presented above demonstrate that the model is able to account for the salient features of the U.S. economy and, therefore, may serve as a proper laboratory for quantitative investigation of the role of endogenous fertility in explaining redistribution and education policies in the U.S.

[13]Data source: U.S. Census 1990.

[14]The fertility-income profile is estimated from U.S. 1990 Census data. The relation between investments in education per child and parental income is evaluated based on data on per student expenditures on primary education provided by the Census's *Annual Survey of School System Finances* report, and the data on local incomes provided by the Bureau of Economic Analysis.

Figure 3.4.1: Fertility-income and education-income profiles



A: Fertilities B: Investments in education

3.5 Results

3.5.1 Politico-economic equilibrium: education and redistribution policies in the U.S.

In this section I analyse whether distribution and education policies observed in the U.S. can be rationalised as outcomes of the politico-economic equilibrium defined in the section 3.3 above. For this purpose, relying on estimates of parameters σ, ξ, η, κ, σ_z, g_1, g_2, λ_2, I solve numerically for politico-economic equilibrium redistribution and education policies.[15]

In order to quantify the role of endogenous fertility in determining redistribution and education policies, I compare the benchmark model (with endogenous fertility) with the model in which fertility is exogenous. The latter model assumes that fertilities are homogeneous across individuals and are set equal to the average number of children in the benchmark economy.[16]

Since this study employs the concept of issue-by-issue voting, this implies that, practically, the politico-economic equilibrium can be found by first solving for the majority-voting equilibrium level of transfers given education subsidies $T(\theta)$ and second, by solving for equilibrium education subsidies given transfers $\theta(T)$. Generally, there is no guarantee of the existence and uniqueness of a politico-economic equilibrium in the model. However, it can be quantitatively verified that functions $T(\theta)$ and $\theta(T)$ are well behaved and that the resulting politico-economic equilibrium is unique. Before turning to the main results of the paper, I

[15] The procedure of calibrating the model assuming that policies are exogenous and then employing a calibrated model to analyse whether policies observed in the data can be rationalised as outcomes of politico-economic equilibrium is similar to that employed by Krusell & Rios-Rull (1999).

[16] The elimination of endogenous fertility from the model does not substantially change its ability to replicate the key characteristics of the U.S. economy; see table 3.7.1 in Appendix A in 3.7.

discuss the complementarity between redistribution and education policies demonstrated by the model, and differences in the slopes of response functions $T(\theta)$ and $\theta(T)$.

3.5.1.1 Complementarity between policies

As can be seen from figure 3.5.1 below, both $T(\theta)$ and $\theta(T)$ are increasing functions of their arguments. Note that assumption of endogenous fertility is not essential for complementarity between policies.

The intuition for this result is as follows. A redistribution policy impedes individuals' incentives to invest in educating their children. Education subsidies may mitigate this adverse effect of transfers by improving average productivity. Therefore, the given level of transfers may be financed by lower equilibrium tax rates. Consequently, when transfers become more generous, a median voter would support more generous education subsidies, and vice versa. In other words, the policies are complements. This result is in line with the findings of existing literature including Bovenberg & Jacobs (2005) and Krueger & Ludwig (2013), although, these studies analyse policies from the social planner's perspective as opposed to the political economy paradigm employed in this paper.

Additionally, response function $\theta(T)$ corresponding to equilibrium education subsidies is steeper than response function $T(\theta)$ corresponding to equilibrium transfers. Similarly to the property of complementarity, an assumption of endogenous fertility is not crucial for this result. To see the intuition, consider the cases of voting on each of two policies separately. If redistribution policy is a given and individuals vote on education policy, more generous transfers discourage parents from investing in educating their children. Consequently, in an economy with higher levels of transfers, the median voter is poorer compared to a mean-income individual and supports more generous redistribution through education subsidies. This median-voter effect amplifies the effect of complementarity discussed above.

In contrast, when education policy is a given and agents vote on redistribution, more generous education subsidies improve productivity and reduce inequality due to crowding-out of private investments in education. Consequently, in an economy with higher education subsidisation, the median voter is richer than the mean-income individual and prefers less generous redistribution through transfers.In this case, the median-voter effect acts in the opposite direction to the complementarity effect.[17] Therefore, the slope of the response function $T(\theta)$ is lower than the slope of $\theta(T)$.

3.5.1.2 Equilibrium policies and endogenous fertility

The benchmark model predicts equilibrium transfers and education subsidies equal to 5.5% and 2.9% of GDP respectively (see column "Benchmark", table 3.5.1). This is close to the U.S. data (column "Calibrated model"). Given that equilibrium policies are quite close to the U.S. data, the statistics of the U.S. economy predicted by the benchmark model are close to the data as well (see table 3.7.1, Appendix A in 3.7).

[17] However, the equilibrium level of transfers is increasing with education subsidisation, because the complementarity effect is quantitatively more important than the median-voter effect.

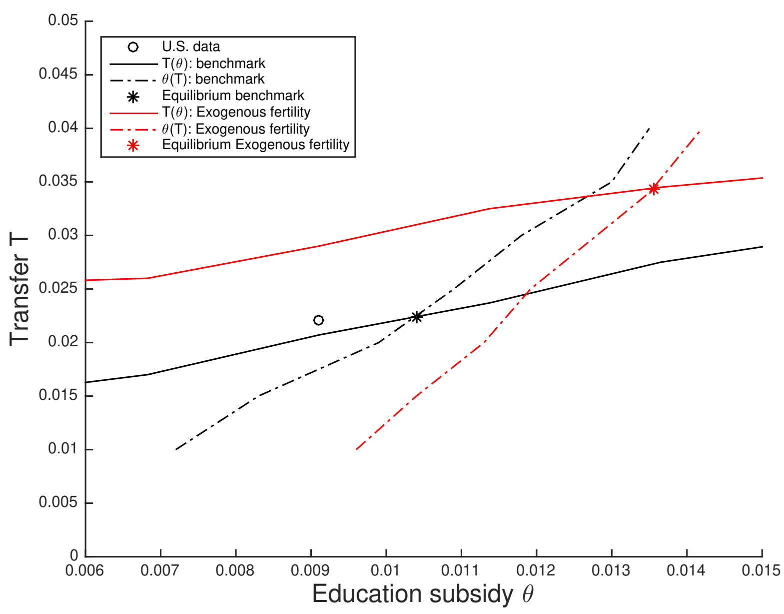
This study demonstrates that assumption of endogenous fertility is important for the ability of the model to perform well in replicating U.S. data. Once endogenous fertility is eliminated from the framework, equilibrium levels of both transfers and education subsidies increase substantially. Transfers increase by 53%, while education subsidies increase by 31% compared to the benchmark (see column "Exogenous fertility").

Table 3.5.1: Politico-economic equilibria

Statistics	Calibrated model	Benchmark	Exogenous fertility
Education subsidisation per child θ	0.0091	0.0104	0.0136
Education subsidisation, % of GDP	2.5	2.9	4.7
Transfers per capita T	0.0221	0.0224	0.0343
Transfers, % of GDP	5.4	5.5	8.3

Endogenous fertility is important because it increases the costs of transfers and education subsidies, since both policies positively affect fertility differentials between low and high income parents and the average number of children.

Figure 3.5.1: Politico-economic equilibrium



Notes. Black and red colours correspond to endogenous fertility (benchmark) and exogenous fertility models, respectively. Solid lines correspond to equilibrium transfers $T(\theta)$ given an education policy. Dashed lines correspond to equilibrium education subsidies $\theta(T)$ given transfers. The interceptions of response functions $T(\theta)$ and $\theta(T)$ correspond to politico-economic equilibrium in each case.

In contrast to low income individuals, for high income parents, income and substitution effects act in the opposite directions. Since both transfers and education subsidies imply redistribution of resources from rich to poor agents, the income effect of policies on fertilities is positive for low income parents and negative for high income parents. In contrast, since both policies lead to declines in the opportunity costs of children and investments in education per child, the substitution effect increases the incentives to have children for both types of parents. Therefore, transfers and education subsidies positively affect fertility differentials.[18] Increases in fertility differentials, in turn, lead to a higher share of low productive individuals and declines in average productivity. Consequently, one would need a higher budget-balancing tax rate to finance the given levels of public policies.

Similarly to fertility differentials, the average number of children is positively affected by transfers and education subsidies, because the substitution effect quantitatively dominates income effect for high income parents and fertilities of both productivity types increase (see the example in figure 3.7.1, subfigure C, in Appendix A in 3.7). Increases in the average number of children positively affect equilibrium tax rates as well, due to their negative impact on aggregate labor supply and positive impact on aggregate expenditures on education subsidies.

Consequently, due to higher equilibrium tax rates, the costs of transfers are more substantial when fertility is endogenous and the median voter would support lower levels of transfers and education subsidies, which are much close to the U.S. data than in the absence of endogenous fertility (see 3.5.1 above).

Additionally, a concern that may arise is whether the results presented above are driven by the impact of fertility decisions on the labor supply (due to time costs of raising children) or by the impact of fertility and child education decisions on the distribution of productivity types. In order to shed some light on this question, I compare equilibria in the benchmark and exogenous fertility models with an Exogenous Fertility - Benchmark Labor (ExF-BL) model (see subfigure D, figure 3.7.1, in Appendix A in 3.7). In the latter framework, fertilities are exogenous, and the labor supply is fixed and corresponds to that in the benchmark model with endogenous fertility. This setup preserves labor supply responses as in the model with endogenous fertility, but eliminates the impact of fertility decisions on the distribution of productivities from the analysis, because individuals are not choosing the number of their children.

As subfigure D, figure 3.7.1, in Appendix A in 3.7 demonstrates, the equilibrium in the ExF-BL model is very close to that in the model with exogenous fertility. Therefore, the impact of fertility and child education decisions on the distribution of productivities is quantitatively more important than the impact of fertility decisions on the labor supply. Consequently, the assumptions of both endogenous fertility and human capital introduce a significant element into the model which is unlikely to be mimicked by assumptions of endogenous labor supply and human capital.

Finally, the main results of this paper are robust to small variations in the parameters of the model (see table 3.7.2 in Appendix A in 3.7).

[18] See examples in subfigures A and B, figure 3.7.1, in Appendix A in 3.7, illustrating adjustments in fertility differentials on the transition path from the equilibrium corresponding to the current U.S. policies to equilibria under alternative policies.

3.5.2 Model validation based on a cross-section of U.S. states

The credibility of the results presented above crucially depends on the plausibility of fertility elasticities with respect to redistribution and education policies predicted by the model. There are a number of empirical studies including Georgellis & Wall (1992), and Whittington et al. (1990) confirming positive effects of transfers on fertilities in the U.S. However, the estimates obtained in these studies cannot be used to discipline the current model due to a lack of information regarding the elasticities of completed fertilities. In addition, apparently there is no empirical evidence on the impact of expansion of public schooling subsidies on fertilities in the U.S.

In order to circumvent these difficulties, I validate the model based on a cross-section of U.S. states. This setting can serve as an excellent case study, because all states operate within a similar political system while exhibiting substantial variations in the levels of transfers, tax rates and public subsidies for schooling. Transfers are financed mostly from the federal budget. According to data from the *Federal Expenditures by State* and *Census Annual State Government Finance* reports provided by the 1990 U.S. Census, only 20% of total expenditures on transfers are financed by state and local governments.[19] Therefore, this variable is treated as exogenous at the state level. In contrast, expenditures on primarily education are determined mostly at the state and local levels. According to the 1990 U.S. *Census Annual Survey of School System Finances* report, only 7% of expenditures are financed from the federal budget, while the rest is financed from state and local budgets.[20] Therefore, education policy is treated as endogenous at the state level and determined through majority voting.

Federal expenditures on transfers are divided into two groups. The first is direct payments from the federal budget to individuals in the form of direct benefits and means-tested transfers. The second is federal payments to state and local governments in the form of aid for financing means-tested transfers. Both types of federal payments vary across states for various reasons.

First, since U.S. states form a federal fiscal union, the federal government may transfer relatively more resources in the form of transfers to states with relatively low incomes. According to the 1999 *Congressional Research Service* report, federal financing of certain types of transfers is an inverse function of per capita state income. One example is the Federal Medical Assistance Percentage Program used to finance Medicaid, which accounts for near half of all spending on means-tested transfers. Second, political preferences vary across states. Third, as pointed out by Serrato & Wingender (2016), allocation of a wide range of federal spending programs depends on local population measurements. Discrepancies in methodologies used to assess population in different years (Census and non-Census) lead to measurement errors and, consequently, to substantial variations in the allocation of federal expenditures on transfers across states.

Finally, though the greatest portion of funding comes from the federal budget, state and local governments

[19] In the states with the highest levels of funding from their own resources (Minnesota and New Hampshire) contributions from state and local governments account for at most 36% of total expenditures.

[20] The highest level of federal budget contribution is 13% among all states.

have a certain degree of freedom to decide on eligibility requirements and benefits amounts for various welfare programs. Medicaid and TANF (Temporary Aid to Needy Families) are among these programs.[21] For example, in 2000, Medicaid expenditures per enrollee varied between \$ 3043 and \$ 7825, and TANF (Temporary Aid to Needy Families) varied between \$ 3879 and \$ 11877 for families of three with no income.[22]

3.5.2.1 Empirical evidence

I start with a description of the data sources on public policies, fertilities and intergenerational correlation of earnings across U.S. states. The 1992-2002 time period considered for a cross-section of U.S. states is the same as for the "national" model discussed above.

Transfers. I use data on direct payments to individuals financed by the federal budget from the *Federal Expenditures by State* reports provided by the 1999 Census. The data on state and local expenditures on transfers including aid from the federal budget is from the 1999 *Census Annual State Government Finance*. In order to eliminate variations in the size of transfers due to differences in demographic and population characteristics across states, I adjust expenditures on transfers by the number of recipients using the data on the number of beneficiaries for major programs including Unemployment Insurance, Supplemental Security Income, TANF and Food Stamps programs provided by the 1997 Census.[23]

Taxes. Following Armenter & Ortega (2007), I use federal and state personal current taxes as empirical counterparts for income tax rates in the model. The data is provided by the *Bureau of Economic Research*, regional accounts. The tax rates are evaluated as an average ratio of total personal current tax revenue over total personal income for 1992-2002.

It is important to account for tax rates variation across states, because expenditures on transfers are budget-balanced in the model. In contrast, in the U.S. economy expenditures on transfers are not balanced by tax revenue, since the greatest part of transfers is financed from federal budget. Therefore, in order to assess the credibility of the assumption of budget-balancing transfers, it is useful to check whether more generous expenditures on transfers are associated with higher tax rates. Below I demonstrate that this is indeed the case.

Other variables. The sources of the data on primary education, intergenerational mobility and demographic variables are the same as in the calibration section above.

Table 3.5.2 presents the correlations between key economic variables. All correlations are significant at least at a 10 per cent level (the notations of the variables are provided in table 3.8.1, Appendix B in 3.8). States with more generous transfers have more generous public subsidies for primary schooling, higher tax

[21] Additionally, states may receive bonuses from the federal government for high performance in meeting program goals. One example is TANF (Temporary Aid to Needy Families).

[22] Sources: U.S. Department of Health and Human Services and the *Kaiser Family Foundation*, www.kff.org.

[23] Specifically, first I calculate transfers (financed from both federal, state and local sources) per recipient, which I then premultiply by the average share of transfer recipients in a population across states. The data on recipients can be found via https://www.census.gov/prod/99pubs/99statab/sec12.pdf.

rates and lower intergenerational correlation of earnings. Additionally, these states are characterised by higher fertilities and fertility differentials.

Notably, positive correlation between transfers and tax rates cannot be explained by more generous public subsidies for primary education. In order to demonstrate this, I introduce a so-called net tax measure equal to the tax rate less the ratio of public subsidies for primary schooling to aggregate state incomes. As can be seen from table 3.5.2, the correlation between net tax and transfers is positive. The relationships between variables are preserved in the case of alternative measures of fertility differentials, transfers adjusted by the costs of living and transfers and education subsidies measured as a share of state GDP (see table 3.8.2, Appendix B in 3.8). Additionally, the results are robust to controlling for a number of factors which are beyond the scope of the current model (see table 3.8.3, Appendix B in 3.8).

Table 3.5.2: Correlations of key economic variables

Variable	TR	TAX	NETTAX	ED	RM	FERT	FERTDIFF
TR	1						
TAX	0.62***	1					
	(0.000)						
NETTAX	0.57***	0.91***	1				
	(0.000)	(0.000)					
ED	0.46**	0.57***	0.28*	1			
	(0.001)	(0.000)	(0.04)				
RM	-0.35*	-0.46**	-0.34*	-0.38*	1		
	(0.01)	(0.001)	(0.01)	(0.07)			
FERT	0.32*	0.46**	0.24	0.27	-0.4**	1	
	(0.02)	(0.001)	(0.1)	(0.06)	(0.004)		
FERTDIFF	0.41***	0.48*	0.27*	0.45**	-0.26*	0.32*	1
	(0.003)	(0.001)	(0.06)	(0.001)	(0.07)	(0.02)	

*Note *p<.05, **p<.01, ***p<.001

3.5.2.2 Policy experiments

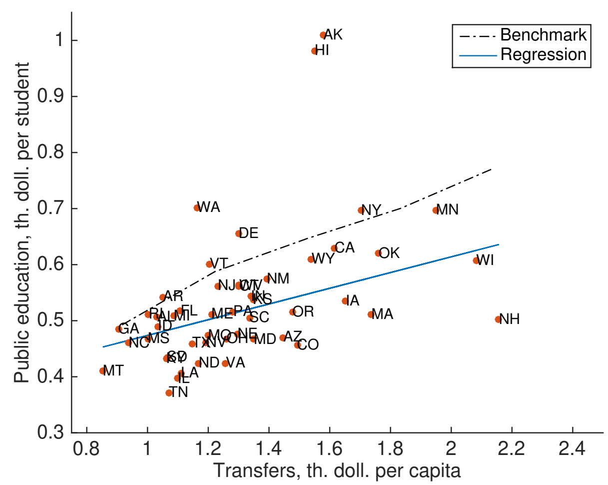
In this subsection I evaluate the accuracy of the model's predictions for a cross-section of U.S. states. I solve for majority-voting equilibrium education subsidies given transfers. The resulting equilibrium corresponds to $\theta(T)$ response function in the benchmark. Additionally, I solve for corresponding fertilities, fertility differentials and intergenerational correlation of earnings.

The results are depicted in figure 3.5.2. As subfigure A, figure 3.5.2, shows, the model predicts a positive relationship between the levels of transfers and public subsidies for education as in the data (though the model slightly overestimates the generosity of education subsidies). Additionally, the model predicts that states with more generous transfers will have higher fertilities and fertility differentials (see subfigures B and C, figure 3.5.2). While the relations between transfers and the average number of children is a bit steeper than that in the data, the fertility differentials predicted by the model are remarkably close to the data.

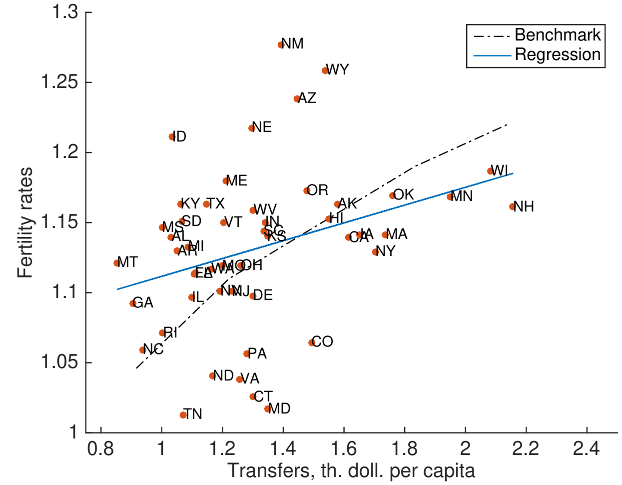
Moreover, the model correctly predicts lower intergenerational correlation of earnings in states with more generous redistribution policies (see subfigure D, figure 3.5.2). The mechanism driving this result is as follows. More generous transfers discourage parents from investing in educating their children. Therefore, education becomes less correlated with parental income and intergenerational correlation of earnings declines. Additionally, more generous subsidies for public education crowd out parental investments in children's education and, consequently, contributes to further decreases in the intergenerational correlation of earnings.

Finally, the model delivers a relatively accurate prediction of the data in the case of transfer and education subsidies measured as a share of state GDP (see figure 3.5.3). The results above demonstrate that, given redistribution policy, the model delivers relatively accurate predictions of politico-economic equilibrium education policies and corresponding fertilities, fertility differentials and intergenerational correlation of earnings. This confirms that the model can serve as a reliable tool for quantitative analysis of the role of endogenous fertility in explaining the outcomes of generosity of both redistribution and education policies at the U.S. national level presented in the subsection 3.5.2.

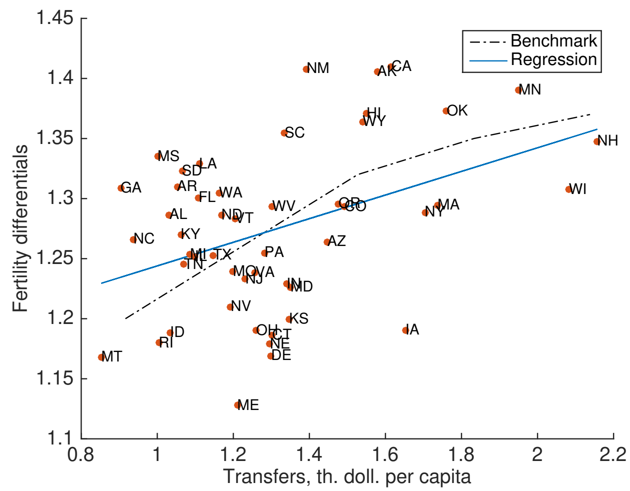
Figure 3.5.2: Model predictions for a cross-section of U.S. states: transfers and education subsidies are measured in per capita/ per student terms



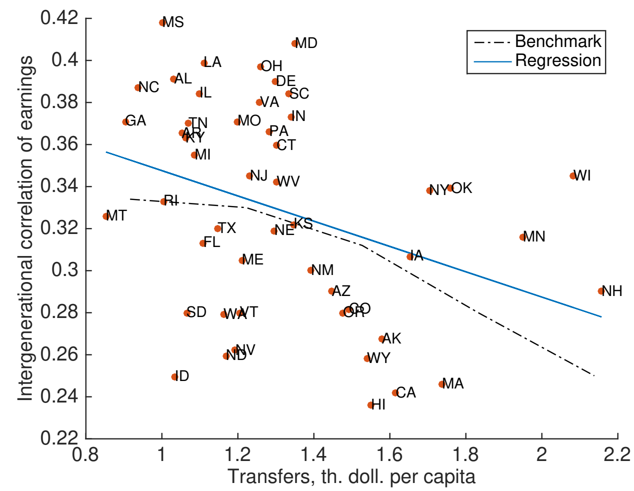
A: Education subsidies



B: Fertilities

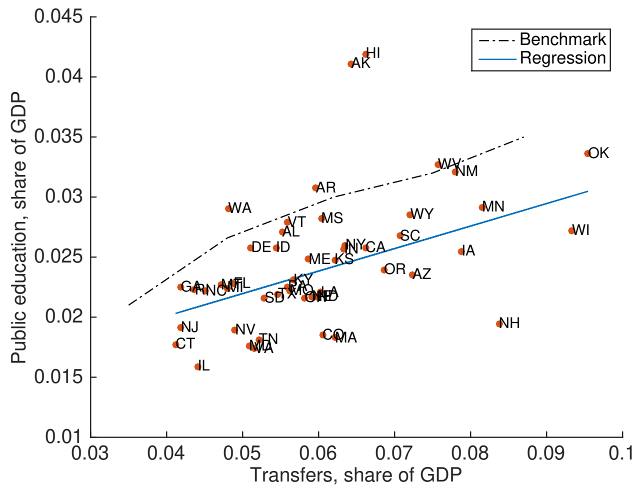


C: Fertility differentials

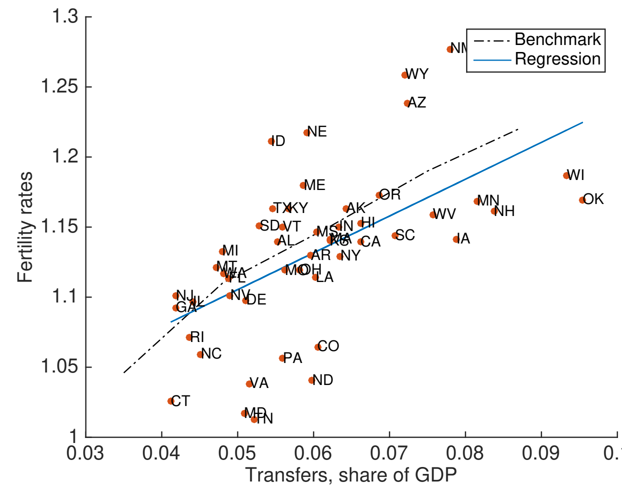


D: Intergenerational correlation of earnings

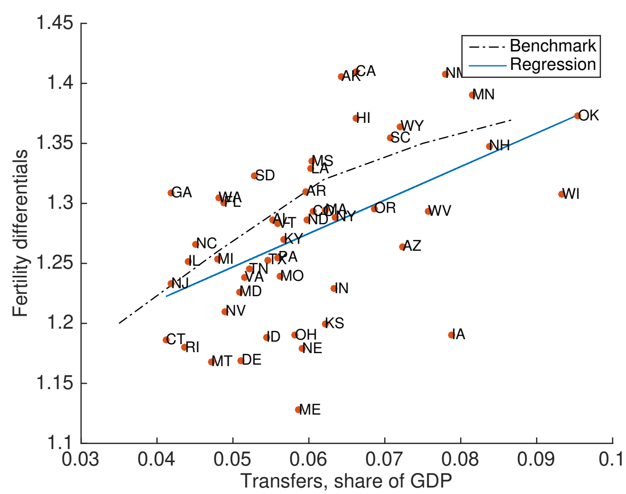
Figure 3.5.3: Model predictions for a cross-section of U.S. states: transfers and education subsidies are measured as a share of state GDP



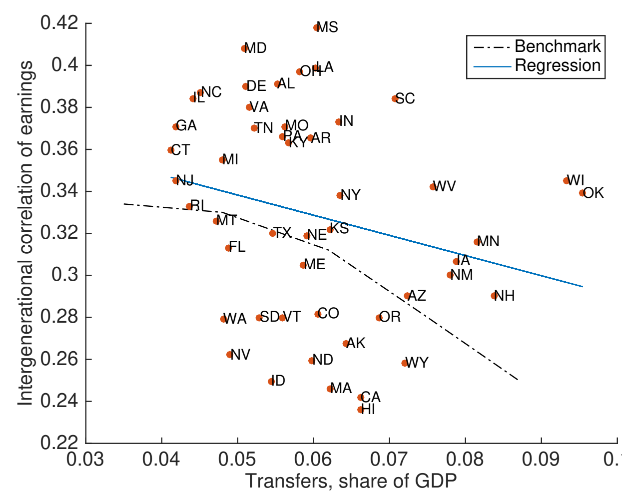
A: Education subsidies



B: Fertilities



C: Fertility differentials



D: Intergenerational correlation of earnings

3.6 Conclusion

This study demonstrates that assumption of endogenous fertility is quantitatively important for explaining the levels of redistribution and education policies in the U.S. The analysis builds on a political economy extension of a dynamic general equilibrium model in the style of Barro & Becker (1989). The model is calibrated to the U.S. economy. Redistribution and education policies are endogenised as outcomes of majority voting.

The study demonstrates that the model with endogenous fertility predicts political equilibrium levels of redistribution and education policies that are quite close to the U.S. data. However, elimination of endogenous fertility from the analysis leads to substantially higher equilibrium levels of both redistribution and education policies. This is because an assumption of endogenous fertility adds costs of transfers and education subsidies, since both policies positively affect fertility differentials and the average number of children. The validation of the model based on a cross-section of U.S. states demonstrates the plausibility of fertility decisions responses and, consequently, the credibility of the main result of this study.

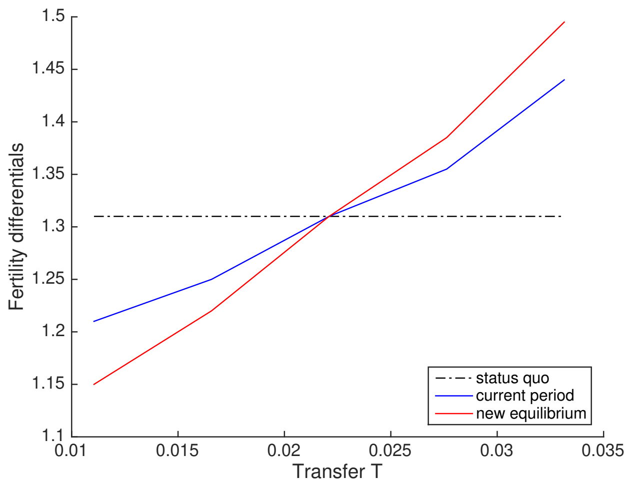
3.7 Appendix A

This section presents key statistics predicted by different settings, depicts fertilities and fertility differentials on a transition path including initial stationary equilibrium (U.S. status quo), a current period in which alternative policies are introduced, and a new stationary equilibrium. This section concludes by evaluating the sensitivity of the main results with respect to small changes in parameter values.

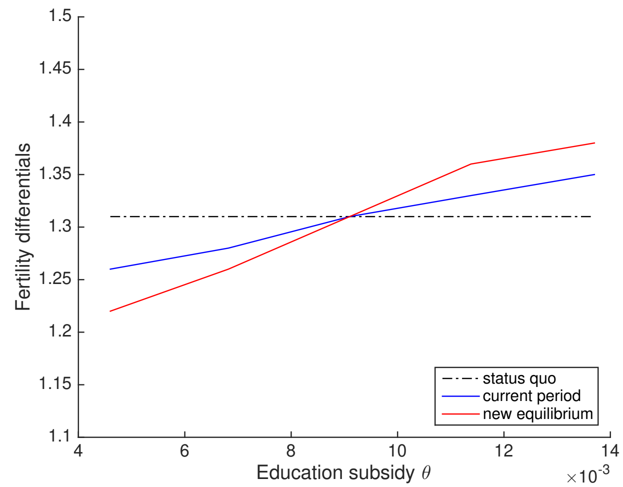
Table 3.7.1: Replication of the key statistics by different models

Statistics	U.S.	Calibrated model	Benchmark	Exogenous fertility
Average fertility rate	1.13	1.128	1.148	1.13
Fertility differential	1.31	1.31	1.34	1
Variance of log earnings	0.36	0.362	0.358	0.35
Public education, % GDP	2.5	2.5	2.9	2.47
Total education, % GDP	4.5	4.6	4.7	4.6
Intergenerational correlation of earnings	0.33	0.33	0.32	0.31
Total transfers, % GDP	5.4	5.4	5.5	5.3
Transfers, middle tertile	0.16	0.16	0.162	0.161
Cost of children, bottom tertile	0.24	0.237	0.236	0.233
Cost of children, middle tertile	0.15	0.148	0.148	0.147

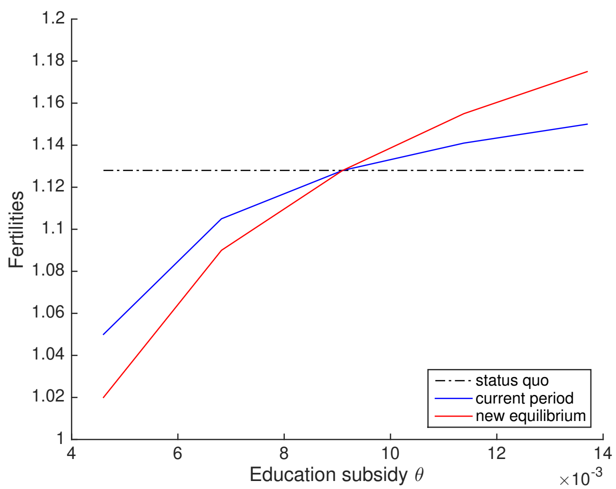
Figure 3.7.1: Fertilities, fertility differentials and decomposition of results



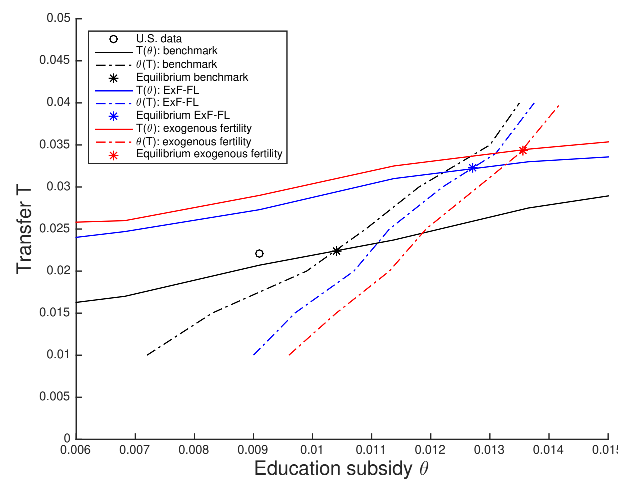
A: Fertility differentials. Education = U.S. level



B: Fertility differentials. Transfers = U.S. level



C: Fertilities. Transfers = U.S. level



D: Decomposition of results

Table 3.7.2: Sensitivity analysis

Parameters	Endogenous fertility		Exogenous fertility	
	Transfers	Education	Transfers	Education
Benchmark calibration	5.49	2.91	8.30	4.70
5-percent increase in σ	5.43	2.85	8.15	4.62
5-percent increase in ξ	5.41	2.83	8.08	4.65
5-percent increase in η	5.45	2.85	8.09	4.63
IEE = 0.4	5.38	2.79	8.65	4.95
IEE = 0.47	5.29	2.71	8.81	5.00
5-percent increase in λ_2	5.41	2.95	7.98	4.81
5-percent increase in σ_z	5.57	2.93	8.03	4.84

Notes. In the U.S. data transfers and education subsidies account for 5.4% and 2.5 % of GDP respectively. As the results in table 3.4.2 demonstrate, the model with endogenous fertility performs much better in replicating the levels of public policies observed in the U.S. data than the counterpart model with exogenous fertility for various alternative parameter values. In the case of IEE (intergenerational earnings elasticity) the model is recalibrated so that it matches alternative estimates of intergenerational earnings correlation, which are standard in macroeconomic literature (0.4 Solon, 1992; 0.47 Corak, 2006). As table 3.4.2 shows, higher intergenerational correlation of earnings makes the assumption of endogenous fertility even more important. Consequently, differences in equilibrium levels of policies predicted by the models with and without endogenous fertility become more substantial than in the case of benchmark calibration implying IEE equal to 0.33.

3.8 Appendix B

This section introduces notations, presents a robustness check of the relationships between variables in table 3.5.2.

Table 3.8.1: Key notations

Variable	Notation
Transfers	TR
Transfers adjusted by costs of living	TRCL
Labor income tax %	TAX
Labor income tax less state expenditures on public education	NETTAX
Expenditures on public education	ED
Intergenerational correlation of earnings	RM
Average fertility rates	FERT
Fertility differentials between bottom and top quintiles of income distribution	FERTDIFF
Fertility differentials between below- and above-median income groups	FERTDIFF50
Fertility differentials between no-college and college individuals	FERTDIFFED
Fertility differentials between bottom and top quintiles of occupation scores distribution	FERTDIFFSC
State government ideology	IDEOL
Student enrollment	SHARESTUD
Percentage of Black	BLACK
Percentage of Hispanic or Latino	HISP
Income Gini	GINI
Percentage of children in single-parent household	FRACCHILD
Percentage of individuals who commute less than 15 minutes to work	FRACCOMMUT
Social capital index	SOCIALCAP
Test scores adjusted by parental income	TESTSC
High school dropout rates	HSDROP

Table 3.8.2: Robustness check

Variable	FERTDIFF50	FERTDIFFED	FERTDIFFSC	TRCL	TRSH	EDSH
TR	0.35*	0.31*	0.39**	0.95***	0.79***	0.34*
	(0.01)	(0.02)	(0.005)	(0.000)	(0.000)	(0.016)
TAX	0.4**	0.43**	0.5***	0.64***	0.42**	0.37**
	(0.004)	(0.002)	(0.000)	(0.000)	(0.002)	(0.009)
NETTAX	0.26*	0.27*	0.36*	0.64***	0.33*	0.3*
	(0.06)	(0.05)	(0.01)	(0.000)	(0.018)	(0.029)
ED	0.35*	0.46***	0.50***	0.52***	0.3*	0.8***
	(0.01)	(0.000)	(0.000)	(0.000)	(0.03)	(0.000)
RM	-0.32*	-0.35*	-0.34*	-0.37**	-0.24	-0.27
	(0.02)	(0.01)	(0.01)	(0.009)	(0.09)	(0.06)
FERT	0.26	0.23	0.27	0.28*	0.58***	0.51***
	(0.06)	(0.1)	(0.05)	(0.04)	(0.000)	(0.000)
FERTDIFF	0.67***	0.58***	0.73***	0.33*	0.51***	0.53***
	(0.000)	(0.000)	(0.000)	(0.019)	(0.000)	(0.000)

*Note *p<.05, **p<.01, ***p<.001

Notes. The correlations are evaluated in the case of alternative measures of fertility differentials. Additionally, the robustness of correlations is checked in the case of transfers adjusted by costs of living (TRCL) as well as transfers (TRSH) and education subsidies (EDSH) measured as a share of state GDP. Costs of living are proxied using the index estimated in Berry et al. (2000).

Table 3.8.3: Robustness check of correlations

Dependant variable	ED		FERT		FERTDIFF		RM	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
TR	1.21***	1.29***	0.063*	0.040	0.099**	0.114***	-0.06**	-0.06**
	(0.33)	(0.30)	(0.027)	(0.028)	(0.031)	(0.033)		(0.019)
IDEOL		0.014***						
		(0.004)						
SHARESTUD		0.166*						
		(0.063)						
BLACK				-0.001		0.001		
				(0.001)		(0.002)		
HISP				0.002		0.001		
				(0.001)		(0.002)		
GINI				-0.177		0.265		0.015
				(0.237)		(0.282)		(0.16)
FRACCHILD								0.93
								(0.25)
FRACCOMMUT								-0.27
								(0.08)
SOCIALCAP								0.03
								(0.012)
TESTSC								0.0007
								(0.001)
HSDROP								0.19
								(0.5)

Notes. The columns correspond to dependant variables. The rows correspond to regressors. The regression coefficients presented above serve solely for evaluation of conditional correlations and should not be interpreted as estimates of causal links between variables.

Table 3.8.3. Comments. As table 3.8.2 shows, correlations are robust to controlling for a number of factors.

Transfers and education subsidies. Positive relationships between levels of transfers, public schooling and taxes might be explained by the fact that the size of government is likely to be positively correlated across different dimensions. Therefore, more liberal states may support more generous transfers and education subsidies. Below I add a measure of state government ideology (I chose 1996 as an example year) provided by Berry et al.'s (1998) study as a control variable. Additionally, the positive link between per student expenditures on public schooling might be explained by the relatively low share of school-age children as a proportion of the population. The results demonstrate that positive relationships between levels of transfers and public schooling is preserved and highly significant after controlling for political preferences and the share of school-age individuals in a population (see columns "ED 1" and

"ED 2").

Transfers, fertilities and fertility differentials. The variation of fertilities and fertility differentials across states may be explained by variations in income inequality and the racial composition of population. In the case of fertilities and transfers, introduction of the Gini coefficient and percentage of Black and Hispanic populations to the regression leads to insignificance of the level of transfers, although, the proportion of variance in fertilities explained by inequality and racial composition is low as well. This indicates that positive connections between the generosity of transfers and fertility rates is moderate. However, positive relationships between transfers and fertility differentials remain highly significant after controlling for inequality and racial compositions across states (see columns "FERT 1, 2" and "FERTDIFF 1, 2").

Transfers and intergenerational correlation of earnings. Finally, I evaluate whether negative correlation between transfers and intergenerational correlation of earnings is robust to controlling for factors which are found to be important in Chetty et al. (2014). These factors include the percentage of the Black population, income inequality, spatial segregation, school quality proxied by test scores and high school dropout rates, social capital and the percentage of children living in single-parent households. Correlation between intergenerational correlation of earnings and transfers remains negative and highly significant after controlling for these factors (see columns "RM 1, 2").

Bibliography

Abbott, B., Gallipoli, G., Meghir, C., & Violante, G. L. (2013). Education policy and intergenerational transfers in equilibrium (No. w18782). National Bureau of Economic Research.

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. The Quarterly Journal of Economics, 109(3), 659-684.

Aiyagari, S. R., Greenwood, J., & Guner, N. (2000). On the State of the Union. Journal of Political Economy, 108(2), 213-244.

Akyol, A., & Athreya, K. (2005). Risky higher education and subsidies. Journal of Economic Dynamics and Control, 29(6), 979-1023.

Alesina, A., & G.M. Angeletos (2005). Fairness and Redistribution: US vs. Europe. American Economic Review, 95: 913-35.

Alm, J., & Whittington, L. A. (1999). For love or money? The impact of income taxes on marriage. Economica, 66(263), 297-316.

Alvarez, F. (1999). Social mobility: the Barro-Becker children meet the Laitner-Loury dynasties. Review of Economic Dynamics, 2(1), 65-103.

Alvarez, F., & Stokey, N. L. (1998). Dynamic programming with homogeneous functions. Journal of economic theory, 82(1), 167-189.

Atkinson, A. B., Maynard, A. K., & Trinder, C. G. (1983). Parents and Children: Incomes in two generations. SSRC. DHSS Studies in Deprivation and Disadvantage, 10.

Barro, R. J., & Becker, G. S. (1989). Fertility choice in a model of economic growth. Econometrica: journal of the Econometric Society, 481-501.

Baudin, T. (2011). Family policies: what does the standard endogenous fertility model tell us? Journal of Public Economic Theory, 13(4), 555-593.

Becker, Gary S., and N. Tomes, "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility," Journal of Political Economy, LXXXVII (1979), 1153 - 1189.

Becker, Gary S., and N. Tomes, "Human Capital and the Rise and Fall of Families," Journal of Labor Economics, IV (1986), S1 - S39.

Belley, P., & Lochner, L. (2007). The changing role of family income and ability in determining educational achievement (No. w13527). National Bureau of Economic Research.

Benabou, R. (1996). Equity and efficiency in human capital investment: the local connection. The Review of Economic Studies, 63(2), 237-264.

Benabou, R. (2002). Tax and education policy in a heterogeneous-agent economy: What levels of redistribution maximize growth and efficiency? Econometrica, 70(2), 481-517.

Benabou, R., & Ok, E. A. (2001). Social mobility and the demand for redistribution: the POUM hypothesis. The Quarterly Journal of Economics, 116(2), 447-487.

Bernasconi, M., & Profeta, P. (2012). Public education and redistribution when talents are mismatched. European Economic Review, 56(1), 84-96.

Berry, W. D., Fording, R. C., & Hanson, R. L. (2000). An Annual Cost of Living Index for the American States, 1960 - 1995. Journal of Politics, 62(2), 550-567.

Berry, W. D., Ringquist, E. J., Fording, R. C., & Hanson, R. L. (1998). Measuring citizen and government ideology in the American states, 1960-93. American Journal of Political Science, 327-348.

Bick, A., & Fuchs-Schundeln, N. (2018). Taxation and labour supply of married couples across countries: A macroeconomic analysis. The Review of Economic Studies, 85(3), 1543-1576.

Bjorklund, A. (2006). Does family policy affect fertility? Journal of population economics, 19(1), 3-24.

Bjorklund, A., & Jantti, M. (1997). Intergenerational income mobility in Sweden compared to the United States. The American Economic Review, 87(5), 1009-1018.

Blanden, J. (2013). Cross-country rankings in intergenerational mobility: a comparison of approaches from economics and sociology. Journal of Economic Surveys, 27(1), 38-73.

Blankenau, W., & Youderian, X. (2015). Early childhood education expenditures and the intergenerational persistence of income. Review of Economic Dynamics, 18(2), 334-349.

Bohacek, R., & Kapicka, M. (2012). A quantitative analysis of educational reforms in a dynastic framework. Manuscript, University of California Santa Barbara.

Bovenberg, A. L., & Jacobs, B. (2005). Redistribution and education subsidies are Siamese twins. Journal of Public Economics, 89(11).

Caucutt, E. M., & Kumar, K. B. (2003). Higher education subsidies and heterogeneity: A dynamic analysis. Journal of Economic Dynamics and Control,27(8), 1459-1502.

Colas, M., Findeisen, S., & Sachs, D. (2021). Optimal need-based financial aid. Journal of Political Economy, 129(2), 492-533.

Chadwick, L., & Solon, G. (2002). Intergenerational income mobility among daughters. American Economic Review, 92(1), 335-344.

Chakraborty, I., Holter, H. A., & Stepanchuk, S. (2015). Marriage stability, taxation and aggregate labor supply in the U.S. vs. Europe. Journal of Monetary Economics, 72, 1-20.

Chetty, R., Hendren, N., Kline, P., & Saez, E. (2014). Where is the land of opportunity? The geography of intergenerational mobility in the United States. The Quarterly Journal of Economics, 129(4), 1553-1623.

Corak, M. (2006). Do poor children become poor adults? Lessons from a cross-country comparison of generational earnings mobility. Research on economic inequality, 13(1), 143-188.

Corak, M. (2012). Inequality from generation to generation: the United States in comparison.

Conde-Ruiz, J. I., & Galasso, V. (2003). Early retirement. Review of Economic Dynamics, 6(1), 12-36.

De La Croix, D., & Doepke, M. (2003). Inequality and growth: why differential fertility matters. American Economic Review, 93(4), 1091-1113.

De la Croix, D., & Doepke, M. (2004). Public versus private education when differential fertility matters. Journal of Development Economics, 73(2), 607-629.

De la Croix, D., & Doepke, M. (2009). To segregate or to integrate: education politics and democracy. The Review of Economic Studies, 76(2), 597-628.

Eika, L., Mogstad, M., & Zafar, B. (2019). Educational assortative mating and household income inequality. Journal of Political Economy, 127(6), 2795-2835.

Ermisch, J. (1988). Econometric analysis of birth rate dynamics in Britain. The Journal of Human Resources, 23(4), 563-576.

Ermisch, J., Francesconi, M., & Siedler, T. (2005). Intergenerational economic mobility and assortative mating.

Erosa, A., & Koreshkova, T. (2007). Progressive taxation in a dynastic model of human capital. Journal of Monetary Economics, 54(3), 667-685.

Fan, C. S., & Zhang, J. (2013). Differential fertility and intergenerational mobility under private versus public education. Journal of Population Economics, 26(3), 907-941.

Fernandez, R., Guner, N., & Knowles, J. (2005). Love and money: A theoretical and empirical analysis of household sorting and inequality. The Quarterly Journal of Economics, 120(1), 273-344.

Fernandez, R., & Rogerson, R. (2001). Sorting and long-run inequality. The Quarterly Journal of Economics, 116(4), 1305-1341.

Georgellis, Y., & Wall, H. J. (1992). The fertility effect of dependent tax exemptions: Estimates for the United States. Applied Economics, 24(10), 1139-1145.

Greenwood, J., & Guner, N. (2008). Marriage and divorce since World War II: aing the role of technological progress on the formation of households. NBER Macroeconomics Annual, 23(1), 231-276.

Greenwood, J., Guner, N., Kocharkov, G., & Santos, C. (2014). Marry your like: Assortative mating and income inequality. American Economic Review, 104(5), 348-53.

Greenwood, J., Guner, N., Kocharkov, G., & Santos, C. (2016). Technology and the changing family: A unified model of marriage, divorce, educational attainment, and married female labor-force participation. American Economic Journal: Macroeconomics, 8(1), 1-41.

Greenwood, J., Guner, N., & Knowles, J. A. (2003). More on marriage, fertility, and the distribution of income. International Economic Review, 44(3), 827-862.

Guner, N., & Knowles, J. (2007). Marital Instability and the Distribution of Wealth. Unpublished manuscript.

Guvenen, F., Kuruscu, B., & Ozkan, S. (2014). Taxation of human capital and wage inequality: A cross-country analysis. Review of Economic Studies, 81(2), 818-850.

Hazan, M., & Zoabi, H. (2015). Do highly educated women choose smaller families? The Economic Journal, 125(587), 1191-1226.

Hanushek, E. A. (1992). The trade-off between child quantity and quality. Journal of political economy, 84-117.

Herrington, C. M. (2015). Public education financing, earnings inequality, and intergenerational mobility. Review of Economic Dynamics, 18(4), 822-842.

Holter, H. A. (2015). Accounting for cross-country differences in intergenerational earnings persistence: The impact of taxation and public education expenditure. Quantitative Economics, 6(2), 385-428.

Holter, H. A., Krueger, D., & Stepanchuk, S. (2019). How do tax progressivity and household heterogeneity affect Laffer curves? Quantitative Economics, 10(4), 1317-1356.

Huggett, M., Ventura, G., & Yaron, A. (2011). Sources of lifetime inequality. American Economic Review, 101(7), 2923-54.

Iacovou, M., & Skew, A. (2010). Household structure in the EU (No. 2010-10). ISER working paper series.

Jacobs, B., & Bovenberg, A. L. (2010). Human capital and optimal positive taxation of capital income. International Tax and Public Finance, 17(5), 451-478.

Jantti, M., Bratsberg, B., Roed, K., Raaum, O., Naylor, R., Osterbacka, E., ... Eriksson, T. (2006). American exceptionalism in a new light: a comparison of intergenerational earnings mobility in the Nordic countries, the United Kingdom and the United States.

Jones, L. E., & Tertilt, M. (2008). Chapter 5 An Economic History of Fertility in the United States: 1826 - 1960. In Frontiers of family economics (pp. 165-230).

Keane, M. P., & Wolpin, K. I. (1997). The career decisions of young men. Journal of political Economy, 105(3), 473-522.

Knowles, J. (1999a). Social Policy, Equilibrium Poverty, and Investment in Children. Manuscript. Philadelphia, Pennsylvania: University of Pennsylvania, Department of Economics.

Knowles, J. (1999b). Can Parental Decisions Explain US Wealth Inequality? Dissertation.

Kremer, M., & Chen, D. L. (2002). Income distribution dynamics with endogenous fertility. Journal of Economic growth, 7(3), 227-258.

Krueger, D., & Ludwig, A. (2013). Optimal progressive labor income taxation and education subsidies when education decisions and intergenerational transfers are endogenous. The American Economic Review, 103(3), 496-501.

Krusell, P., & Rios-Rull, J. V. (1999). On the size of US government: political economy in the neoclassical growth model. American Economic Review, 89(5), 1156-1181.

Landerso, R., & Heckman, J. J. (2017). The Scandinavian fantasy: The sources of intergenerational mobility in Denmark and the US. The Scandinavian journal of economics, 119(1), 178-230.

Loury, G. C. (1981). Intergenerational transfers and the distribution of earnings. Econometrica: Journal of the Econometric Society, 843-867.

Meltzer, A. H., & Richard, S. F. (1981). A rational theory of the size of government. Journal of political Economy, 89(5), 914-927.

Moav, O. (2005). Cheap children and the persistence of poverty. The economic journal, 115(500), 88-110.

Mortensen, D. T. (1988). Matching: Finding a partner for life or otherwise. American Journal of Sociology, 94, S215-S240.

Mulligan, C. B. (1997). Parental Priorities and Economic Inequality (Chicago: University of Chicago Press).

Ono, T. (2016). Inequality and the politics of redistribution. International Tax and Public Finance, 23(2), 191-217.

Piketty, T. (1995). Social mobility and redistributive politics. The Quarterly journal of economics, 110(3), 551-584.

Piketty, T., & Saez, E. (2012). Optimal labor income taxation (No. w18521). National Bureau of Economic Research.

Piketty, T., & Saez, E. (2013). A theory of optimal inheritance taxation. Econometrica, 81(5), 1851-1886.

Piraino, P. (2007). Comparable estimates of intergenerational income mobility in Italy. The BE Journal of Economic Analysis & Policy, 7(2).

Pla, M. C. (2009). Measuring intergenerational earnings mobility in Spain: A selection-bias-free approach.

Psacharopoulos, G., & Patrinos, H. A. (2004). Returns to investment in education: a further update. Education economics, 12(2), 111-134.

Regalia, F., & Rios-Rull, J. V. (2001). What accounts for the increase in the number of single households? University of Pennsylvania, mimeo.

Restuccia, D., & Urrutia, C. (2004). Intergenerational persistence of earnings: The role of early and college education. The American Economic Review, 94(5), 1354-1378.

Shepsle, K. A. (1979). Institutional arrangements and equilibrium in multidimensional voting models. American Journal of Political Science, 27-59.

Solon, G. (1992). Intergenerational income mobility in the United States. The American Economic Review, 393-408.

Solon, G. (2002). Cross-country differences in intergenerational earnings mobility. Journal of Economic Perspectives, 16(3), 59-66.

Solon, G. (2004). A model of intergenerational mobility variation over time and place. Generational income mobility in North America and Europe, 2, 38-47.

Wiik, K. A., Bernhardt, E., & Noack, T. (2010). Love or money? Marriage intentions among young cohabitors in Norway and Sweden. Acta Sociologica, 53(3), 269-287.

Whittington, L. A., Alm, J., & Peters, H. E. (1990). Fertility and the personal exemption: implicit pronatalist policy in the United States. The American Economic Review, 80(3), 545-556.