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Essays on Implications of Rational Inattention to Discrete Choices

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Abstract

In the first chapter we study fundamental links between two popular approaches to consumer choice: the multinomial logit model of individual discrete choice and the CES utility function, which describes a multiple choice of a representative consumer. We base our analysis on the rational inattention (RI) model and show that the demand system of RI agents, each of whom chooses a single option, coincides with the demand system of a fictitious representative agent with a CES utility function. Thus, the diversified choice of the representative agent may be explained by the heterogeneity in signals received by the RI agents. We obtain a new interpretation for the elasticity of substitution and the weighting coefficients of the CES utility function. Specifically, we provide a correspondence between parameters of the CES utility function, prior knowledge and marginal cost of information.

In the second paper we investigate the role of a value of a known policy with certain payoff on agents' information acquisition and belief polarization. We model agents to be rationally inattentive: some information about the new policy can be acquired before a choice is made, but doing so is costly. We show that even small changes in the agents' perception of the status quo can lead to polarization of opinions. Such behavior is caused by agents not learning about the states separately, but by endogenously pooling them into groups and acquiring only the information necessary to understand which group the realized state is from. As a consequence, the agents might update their expected belief about the value of the new policy wrongly, away from the true payoff.

In the third chapter we introduce a new role of quotas: the attentional role. We study the effect of quota implementation on the attention allocation strategy of a RI agent. First, we find that a RI agent who is forced to fulfill a quota always acquires information about existing options, unlike an unrestricted RI agent who can decide not to acquire any information. Second, we show that the same behavior could be achieved by subsidizing certain alternatives. Finally, we analyze optimal quotas from the social planner's point of view under two scenarios: when the social planner takes into account production externalities and when he eliminates the influence of priors on the agent's choice.

Abstrakt

První kapitola dizertace se zabývá propojením dvou populárních přístupů k teorii spotřebitelského výběru: Modelu Multinomického Logitu diskrétního výběru a CES užitkové funkce, která slouží k modelování výběru reprezentativního spotřebitele. Analýzou modelu racionální nepozornosti ukážeme, že systém poptávek racionálně nepozorných spotřebitelů, z nichž každý vybírá jednu možnost, splývá se systémem fiktivního reprezentativního spotřebitele s CES užitkovou funkcí. Výběr různých možností tak může být vysvětlen jako důsledek heterogeneity v signálech obdržených racionálně nepozornými spotřebiteli. Nabízí se tak nová interpretace elasticity substituce a vážicích koeficientů CES užitkové funkce. Konkrétně předkládáme vztah mezi parametry CES užitkové funkce, předchozími znalostmi (prior), a mezními náklady na informaci.

V druhé kapitole dizertace se zaměříme na dopady nového opatření s neznámými dopady na zisk informací a polarizaci přesvědčení aktérů. Modelujeme aktéry pomocí racionální nepozornosti: některé informace ohledně nového opatřeni je možné získat před rozhodnutím, ale zisk informací je nákladný. Ukážeme, že i malé změny ve vnímání aktérů opatření, které je status-quo, mohou vést k polarizaci přesvědčení. Takové chování je způsobeno aktéry, kteří informace nezískávají o jednotlivých stavech světa, ale vytváří endogenní balíčky stavů světa a následně získávají pouze informace, které umožňují rozliäit mezi těmito balíčky. V důsledku aktéři mohou získat informace, které vedou k chybám v přesvědčení o dopadech opatření.

V třetí kapitole předkládáme nové dopady kvót skrze alokaci pozornosti. Zaměříme se na dopady implementace kvót na výběr strategie alokace pozornosti v modelu s racionálně nepozornými aktéry. Nejprve ukážeme, že racionálně nepozorní aktéři, kteří jsou nuceni splnit kvótu, vždy, na rozdíl od kvótou neomezených racionálně nepozorných aktérů, získávají informace o dostupných možnostech. Poté ukážeme, že totožné chování je možné docílit dotováním určitých možností. Závěrem předkládáme analýzu optimálních kvót z pohledu společenského plánování v dvou případech: když společenské plánování zahrnuje externality z výroby a když společenské plánování dokáže odstranit dopad předchozích znalostí (prioru) na volby aktérů.

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All errors remaining in this text are my responsibility.

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Introduction

People's attention is a scarce resource. The theory of Rational Inattention, which was introduced in the seminal works of Sims (1998, 2003) studies the optimal allocation of attention. The key components of the model are that people choose what information to attend to and that the choice of information structure is unrestricted. Such an assumption seems appropriate for the modern world: with all the recent technological developments people are overwhelmed with information and are also free to choose from many information sources. In this dissertation I study several implications of Rational Inattention to discrete choices.

In the first paper, "Logit, CES, and Rational Inattention", we explore the connection between the CES demand system of a representative consumer and the discrete choice of an individual consumer who is rationally inattentive. More specifically, we show that the CES-type behavior can be generated by aggregating the choices of the decision makers who are rationally inattentive. In particular, if we consider a population of consumers who (i) are endowed with a budget y, (ii) must select which of N goods to spend their endowment on, each of which gives utility $\ln q$ if consumed in quantity q, and (iii) is rationally inattentive to prices with costs based on Shannon mutual information, then the resulting expected demand, conditional on realized prices, is the same as that of a consumer maximizing CES preferences. Hence, the parameters of the CES utility function can be related to the primitives of the Rational Inattention problem.

In the second paper, "The Status Quo and Belief Polarization of Inattentive Agents" (joint work with Vladimír Novák), we explore the role of the status quo on the attention allocation strategy and demonstrate the possibility of opinion polarization for rationally inattentive agents. We study the evolution of belief of a rationally inattentive agent who is choosing between two options, risky (new policy) and safe (status quo), and characterize situations in which the belief would be updated, on average, in the wrong direction. We find that the key determinant of the direction of belief updating is the perception of the status quo. The position of the status quo determines the information acquisition strategy. In our interpretation, the agent splits states of the world into categories and learns, to some extent, about these categories, not distinct states. This type of learning might lead to updating of belief in the wrong direction. The division into categories is determined exactly by the perception of the status quo. If the two agents have different perceptions of the status quo, they might diverge in their opinions after information acquisition. Interestingly, the difference in their opinions can become greater if the information becomes cheaper to acquire.

The last paper of this dissertation, "Attentional Role of Quota Implementation" (joint work with Sergei Mikhalishchev), is devoted to studying the optimal behavior of a RI agent who is forced to fulfill quotas when making a choice from a discrete menu. In this situation, the agent always (for any non-trivial quota and any non-trivial prior belief) acquires information. We show that a social planner using quotas could force an agent to make a better choice (for the economy) and reduce the attentional discrimination, which can take place because of costly attention. At the same time, it is important to note that quotas restrict the agent, and the effect of quota implementation could be negative.

Chapter 1

Logit, CES, and Rational Inattention

1.1 Introduction

People choose different products for various reasons, and, perhaps, the two most important are variation in preferences and in information. Correspondingly, there are models of individual choice based either on heterogeneous idiosyncratic preferences or on variation in information received by agents. Both types of models have become workhorses in microeconomics, decision making and related topics. However, for the analysis of behavior of a set of consumers, rather than a single consumer, an "as if" model of a fictitious representative consumer with aggregate utility function is commonly used, often having the shape of a constant elasticity of substitution (CES). The existing microfoundation of the CES utility function is based exclusively on preference heterogeneity, and thus any change in its parameters is interpreted as a change in the idiosyncratic preferences of underlying agents, while possible informational reasons are ignored.

In this paper we broaden the approach to the microfoundation of the CES utility function and show that this functional form might be obtained by aggregation of choices of rationally inattentive (RI) consumers who make a discrete choice with costly information acquisition. Our approach explains the origins of both the weighting coefficients (which have previously been interpreted as a consumer's preferences for separate goods) and of the elasticity of substitution of the CES utility function.

The new microfoundation is important since it has different features on the individual level and different comparative statics and testable implications. The parameters (elasticity of substitution, weighting coefficients) are endogenous and thus the "as if" CES changes if the environment does. It allows the expansion of understanding of the concept of a representative agent and opens a way to study the role of the informational environment (for shaping CES utility) in many models of macroeconomics, international trade and economic geography that are based on the CES utility function of the representative agent.

The multinomial logit model and the CES utility function are among the most popular tools for dealing with consumer choice problems. Despite the fact that these models use quite different assumptions (discrete individual choice and multiple choice of a representative consumer, correspondingly), there is a deep and illuminating link between them.

The existing literature that relates the CES utility function to the multinomial logit model of discrete choice is based on a random utility model (Anderson, De Palma, and Thisse (1987, 1988)). Hence, the elasticity of substitution of the aggregate utility is determined by an exogenous parameter of a specific (extreme value Gumbel) cumulative distribution function of taste dispersion. Since this parameter reflects idiosyncratic differences in preferences, it is difficult to forecast its changes under economic shocks.

This paper, in contrast, uses rational inattention (RI) (Sims (1998, 2003)) as a microfoundation, and reveals the link between the parameters of the RI model, the multinomial logit model, and the elasticity of substitution and weighting coefficients of the CES utility function.

We model a situation in which a consumer is facing a discrete choice problem: she possesses some income and spends it to purchase only one kind of several divisible goods. We assume that, despite the goods having certain prices, the consumer is not able to observe the prices perfectly. Limitations in consumers' attention to prices are confirmed empirically (e.g. Zeithaml (1988), Rosa-Díaz (2004)).

The assumption of uncertain prices is not crucial for our analysis. Instead we could assume that the consumer does not observe purchasable quantities perfectly. The uncertainty appears either because of prices or quantities, and we stick to uncertainty of prices for the sake of definiteness.

The RI consumer observes signals about the prices, but the structure of the signals (any joint distribution of signal and state) is itself chosen by the consumer. As is usually assumed in RI models, the information is costly, and the cost of information is proportional to entropy-based reduction in uncertainty between the prior and the posterior distributions.

We explore the demand structure of RI consumers with logarithmic utility and the marginal cost of information λ. We show that this demand structure is the same as the one generated by the CES utility function that belongs to an aggregate representative consumer who possesses perfect information, for which the elasticity of substitution is $\sigma = 1/\lambda + 1$. That is, the higher the cost of information, the smaller the response of market demand to changes in prices. We show that the weighting coefficients of the CES utility function are defined by the prior knowledge of the RI consumers and the marginal cost of information.

Our model leads to new implications. Firstly, in our model the weighting coefficients of the resulting aggregate CES utility function are endogenous. That means that if the environment changes,[1] then the parameters of the information acquisition and decision problem of the RI agent change; thus the representative CES utility changes and the magnitude of consumers reactions would be different from that implied by the CES function with exogenous coefficients. A further implication of our model concerns the reaction of demand to change in the marginal cost of information, λ. We show that the weighting coefficients of the CES utility depend on it; that is, the marginal cost of information enters the model deeper than the parameter of taste heterogeneity, μ, in the model of Anderson, De Palma, and Thisse (1987, 1988). We can even see that information can become so expensive that some goods are never chosen (the weighting coefficient becomes 0).

Our paper is related to several strands of literature. The microfoundation of the utility function of the representative consumer is still an open question (see Kirman (1992), Sheu (2014), Tito (2016)). It is especially important to microfound the CES shape because the CES function is used in many models of macroeconomics, international trade, economic geography and industrial organization (see, e.g., Atkin, Faber, and Gonzalez-Navarro (2018), Mrázová and Neary (2014), Sheu (2014)). It is notable that one of the reasons for the critique of the welfare analysis based on models with CES utility (see Kirman (1992), Tito (2016)) is that the relation between the fictitious representative consumer and real consumers is not clear and the welfare of the representative consumer seems not to be informative.

The relation between the logit model of discrete choice and the CES utility function of a representative consumer was first explored by Anderson, De Palma, and Thisse

[1]For example, because of the introduction of a new trade barrier on a foreign good.

(1987,1988). They use a random utility model as a foundation for logit and show that the demand system derived from a nested logit model is also generated by the CES utility function. In particular, they show that the elasticity of substitution σ of the CES utility function is determined by a positive constant (Gumbel distribution parameter), which serves as a scale parameter of the random term in the definition of stochastic utility. More general results on the connection between multinomial logit and demand systems are presented, in the same vein, by Thisse and Ushchev (2016) and by Tito (2016).

The model of RI, first introduced by Sims (1998, 2003), was applied to consumer behavior by Caplin and Dean (2015), Joo (2019), Martin (2017), Matějka and McKay (2012), Matějka (2015) and Tutino (2013). Matějka and McKay (2015) proposed a foundation for the multinomial logit model based on RI, and we use their model in this paper.

The structure of the rest of the paper is as follows. In Section 1.2 we describe the RI model of consumer choice and derive from it the CES utility function of a representative consumer. In Section 1.3 we consider the cases of homogeneous and heterogeneous price distributions and study price elasticity and elasticity of substitution. Section 1.4 concludes.

1.2 The model

There are N types of goods that are perfect substitutes for the individual consumer. The consumer is endowed with budget y, which she spends entirely on one type of good[2]. The consumer would like to purchase the cheapest type of good to have as large a quantity of it as possible; however, at the moment of choice of the good she does not observe prices perfectly[3]. For example, there are various packages which have various prices, as well as some discounts or taxes which are not so obvious at first sight. The true payoffs related to the chosen good are revealed only after the choice is made. One can think of the following interpretation of the model: different goods are sold by different vendors, who are located in different places. The consumer learns the price when she arrives at the location of the vendor and it is too late to change the vendor.

[2] We could also assume that there are N types of goods that are perfect substitutes and one more good with a known price $p = 1$ (a numeraire good). The consumer has an opportunity to spend part of her budget on the numeraire good. Following Anderson, De Palma, and Thisse (1988), we could assume that the utility of consumption good i and good 0 is $v_i = \ln q_i + \alpha \ln q_0$. The resulting utility function of the representative consumer would have a shape of CES utility function multiplied by q_0^{α}. This function is similar to the utility function that was considered in the original Dixit and Stiglitz (1977) paper.

[3] Alternatively, we could assume uncertain quantities instead of prices.

Following Anderson, De Palma, and Thisse (1987), we assume that the utility of consumption of good i by the individual is

$$v_i = \ln q_i, \quad i = 1, ..., N,$$

where q_i is the quantity. If the individual chooses good i to purchase, then, obviously, the consumed quantity is

$$q_i^* = \frac{y}{p_i}, \tag{1.1}$$

where p_i is the price, and the indirect utility is

$$V(y, p_i) = \ln\left(\frac{y}{p_i}\right). \tag{1.2}$$

We assume that the consumer exhausts her budget entirely. For example, that can be achieved in the following way. The buyer hands over her budget to the seller, e.g., \$10, and gets in return the amount of the good that the budget is sufficient for.

1.2.1 Choice of the good

Following Matějka and McKay (2015), the agent is rationally inattentive and chooses from N products characterized by utility values considered by the agent as a random vector $v = (v_1, ..., v_N)$ with distribution $G(v) \in \Delta(\mathbb{R}^N)$, where $\Delta(\mathbb{R}^N)$ is the set of all probability distributions on $\mathbb{R}^N$. More precisely, the price vector $p = (p_1, ..., p_n)$ is random, which makes $v_i = V(y, p_i)$, $(i = 1, ..., N)$ random variables. The belief about v, i.e. $G(v)$, is given exogenously by the agent's prior knowledge of prices.

The agent is able in principle to obtain precise information about the realization of the random price vector $p = (p_1, ..., p_N)$ (and, correspondingly, about the realization of the random vector of utilities $v = (v_1, ..., v_N)$). However, for the agent the information about the realization is costly. She constructs her information-action strategy in advance by solving a problem of maximization of the expected utility less the information cost.

The information-action strategy includes the choice of information (signal) about the realization and the choice of action (selected product) conditional on the signal. The second choice is standard: the agent simply chooses the option providing the highest expected value. The first choice is the hallmark of rational inattention.

It is assumed that, to reduce the uncertainty, the agent has to pay a cost $\lambda\kappa$, where

$\lambda > 0$ is the marginal cost of information, and $\kappa > 0$ is the amount of information processed. The latter is the expected entropy[4] reduction between the agent's prior and posterior beliefs about v.

According to Lemma 1 from Matějka and McKay (2015) the state-contingent choice behavior of the RI consumer can be found as the solution to a simpler maximization problem that does not make reference to signals or posterior beliefs. That is, each information-action strategy may be characterized by a vector function $(P_1(v), ..., P_N(v))$, where $P_i(v)$ is a conditional probability that product i will be chosen under the realization v. The probabilities reflect the agent's choice under incomplete information, when she receives a noisy signal but does not know the realization of v precisely.

Formally, the consumer's problem is described in the following way.

Consumer's Problem. *The consumer's problem is to find an information processing strategy maximizing expected utility less the information cost:*

$$\max_{(P_1(v),P_2(v),...,P_N(v))} \{\sum_{i=1}^{N} \int_v v_i P_i(v) G(dv) - \lambda\kappa(P,G)\}, \tag{1.3}$$

where

$$\kappa(P,G) = -\sum_{i=1}^{N} P_i^0 \ln P_i^0 + \int_v \left(\sum_{i=1}^{N} P_i(v) \ln P_i(v) \right) G(dv),$$

$P_i(v)$ is the conditional on the realized vector v probability of choosing good i, and P_i^0 is the unconditional probability that the product of type i will be chosen,

$$P_i^0 = \int_v P_i(v) G(dv), \quad i = 1, ..., N.$$

Probabilities P_i^0 are obtained as a solution of the problem (1.3); they reflect prior knowledge $G(v)$ and do not depend on the realization of p. However, they may depend on the marginal cost of information λ.

It is shown by Matějka and McKay (2015) that the solution, $P_i(v)$, follows the modified logit formula:

$$P_i(v) = \frac{P_i^0 e^{\frac{v_i}{\lambda}}}{\sum_{j=1}^{N} P_j^0 e^{\frac{v_j}{\lambda}}}, \quad i = 1, ..., N. \tag{1.4}$$

By plugging (1.2) into (1.4) we obtain for the probability of choosing product i as a

[4] The entropy of a continuous random variable X with probability density function $f(x)$ with respect to a probability measure m is $H(X) = -\int f(x) \log f(x) m(dx)$.

function of price vector and prior beliefs:

$$P_i\left(v(p)\right) = \frac{P_i^0 p_i^{-\frac{1}{\lambda}}}{\sum_{j=1}^N P_j^0 p_j^{-\frac{1}{\lambda}}}, \quad i = 1, ..., N. \tag{1.5}$$

The conditional expected demand for good i is $D_i = P_i(v(p))q_i^*$. Equations (1.1) and (1.5) imply the following.

Lemma 1. *The conditional expected demand for good i, $D_i = P_i(v(p))q_i^*$, is*

$$D_i = \frac{P_i^0 p_i^{-\frac{1}{\lambda}-1}}{\sum_{j=1}^N P_j^0 p_j^{-\frac{1}{\lambda}}} y, \quad i = 1, ..., N. \tag{1.6}$$

Thus, the market share of the good i is

$$M_i = \frac{p_i D_i}{y} = P_i^0 \left(\frac{p_i}{\mathbb{P}}\right)^{-\frac{1}{\lambda}},$$

where $\mathbb{P}$ is a price index,

$$\mathbb{P} = \left(\sum_{j=1}^N P_j^0 p_j^{-\frac{1}{\lambda}}\right)^{-\lambda}.$$

1.2.2 The link between rational inattention and the CES utility function

In the following proposition we show that the demand of the aggregate of RI agents is the same as if there was a fictitious representative consumer maximizing the CES utility function under full information.

Proposition 1 (The CES demand structure of rationally inattentive agents)**.** *The demand structure (1.6) representing the rational inattention model of discrete choice with logarithmic preferences is generated by the CES utility function*

$$U = \left(\sum_{j=1}^N \beta_j q_j^\rho\right)^{\frac{1}{\rho}},$$

which is maximized by the representative consumer subject to the budget constraint

$$\sum_{j=1}^N p_j q_j \leq y,$$

where the elasticity of substitution is

$$\sigma = \frac{1}{1-\rho} = \frac{1}{\lambda} + 1, \tag{1.7}$$

and the "weighting" coefficients are

$$\beta_i = \gamma \left(P_i^0\right)^{1-\rho} = \gamma \left(P_i^0\right)^{\frac{\lambda}{1+\lambda}}, \quad i = 1, ..., N, \tag{1.8}$$

where γ is any positive coefficient.

Proof: see the Appendix.

Thus, the goods seem as if they are not perfect substitutes for the representative consumer, despite being perfect substitutes for each of the underlying RI consumers.

From (1.7) we see that the elasticity of substitution σ is higher than 1 and depends negatively on the marginal cost of information λ. If the cost of information λ increases, then the behavior of the representative (aggregate) consumer is the same as if the elasticity of substitution went down. The reason is that the individual consumer inspects prices less, and consequently she is more likely to make errors and thus react less to changes in prices.

The weighting coefficients β_i depend positively on the corresponding unconditional probabilities P_i^0.

Corollary 1. *The indirect utility function of the representative consumer is*

$$\mathcal{V}(y, p_1, ..., p_N) = \gamma^{\frac{1}{\rho}} \frac{y}{\mathbb{P}},$$

where $\mathbb{P}$ is the price index,

$$\mathbb{P} = \left(\sum_{j=1}^{N} P_j^0 p_j^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}} = \left(\sum_{j=1}^{N} P_j^0 p_j^{1-\sigma} \right)^{\frac{1}{1-\sigma}} = \left(\sum_{j=1}^{N} P_j^0 p_j^{-\frac{1}{\lambda}} \right)^{-\lambda}.$$

1.3 Implications

1.3.1 The case of a priori homogeneous options

The important case is when all the options enter the prior G symmetrically, i.e. the individual does not distinguish between them before she starts processing information.

Such options are referred as *a priori homogeneous.*

In this case unconditional probabilities are $P_1^0 = ... = P_N^0 = 1/N$ and conditional probabilities of choice of the goods,

$$P_i(v) = \frac{e^{\frac{v_i}{\lambda}}}{\sum_{j=1}^N e^{\frac{v_j}{\lambda}}}, \quad i = 1, ..., N,$$

do not depend on prior belief. This is the *multinomial logit* formula. Correspondingly,

$$P_i(v(p)) = \frac{p_i^{-\frac{1}{\lambda}}}{\sum_{j=1}^N p_j^{-\frac{1}{\lambda}}}, \quad i = 1, ..., N$$

and expected demands are

$$D_i = \frac{p_i^{-\frac{1}{\lambda}-1}}{\sum_{j=1}^N p_j^{-\frac{1}{\lambda}}} y, \quad i = 1, ..., N.$$

In the case of a priori homogeneity, as in the general case, the choice of the CES function is not determined in a unique way, but up to a constant multiplier. Natural candidates for such a CES function are two "standard" functions:

$$\overline{U} = \left(\sum_{i=1}^N q_i^\rho \right)^{\frac{1}{\rho}} \tag{1.9}$$

and

$$\tilde{U} = \left(\sum_{i=1}^N \frac{1}{N} q_i^\rho \right)^{\frac{1}{\rho}}. \tag{1.10}$$

Function (1.9) corresponds to $\gamma = N^{1-\rho}$ in the formula (1.8), and function (1.10) corresponds to $\gamma = N^{-\rho}$. These two functions can explain the same consumer choices based on the same market data; however, they possess different properties. In particular, function (1.10) at the limit as $\lambda \to \infty$ converges to the Cobb-Douglas function. The function (1.9), in its turn, goes to infinity as $\lambda \to \infty$, which is somewhat intractable.

Moreover, it is easy to show that function (1.10) is decreasing in the marginal cost of information, while function (1.9) is increasing. That is, for the representative consumer with utility function (1.10) an increase of the cost of information is "bad news", while for the consumer with function (1.9) it is "good news". This is an example of how implications do change from a singular agent level (where lower cost of information clearly leads to higher welfare) to an aggregate representative agent level. This affirms that one should be careful when using aggregate models in policy analysis.

1.3.2 Simple example regarding a priori heterogeneous options

For the case of homogeneous options the main implications of RI foundation of the CES utility function are rather similar to the implications of the Random Utility foundation. In this section we show in a simple example how, in the case of asymmetric distribution of prices the CES utility function (namely, its weighting coefficients) of a representative agent changes with respect to a change in parameters of the RI model – marginal cost of information and prior knowledge. One implication of such a change is the possibility of a zero weighting coefficient for some good – the information becomes too costly to acquire or the good is a priori too expensive – the consumer does not consider it and never buys.

Let us assume that a RI consumer chooses one of two goods. The first good is sold at a fixed price. The second good, in turn, is sometimes sold with a discount and sometimes has a higher price. How will such pricing affect the demand of the representative agent? How does the demand change in response to a change of the environment?

More precisely, there are two goods and two states of the world. Different goods are optimal in different states, but it is costly to identify the realization of the state of the world. The first good always has price 1. The second good costs 0.5 in the first state and 1.5 in the second state. The agent possesses prior knowledge on the probability distribution of the state of the world: g_1 and $g_2 = 1 - g_1$ are probabilities of state 1 and state 2, correspondingly. As part of her information strategy the agent obtains unconditional probabilities of choosing good 1 and good 2, P_1^0, $P_2^0 = 1 - P_1^0$, correspondingly. These probabilities depend on her prior knowledge and marginal cost of information. As formula (1.8) demonstrates, these unconditional probabilities together with the parameter of information cost determine the weighting coefficients of the CES function of the representative agent. The exact formulas and the way they are obtained can be found in the Appendix.

In Figure 1.1 we can see how exactly coefficients β_1 and β_2 change with respect to the information cost parameter λ under the fixed prior $g_1 = g_2 = 0.5$. In Figure 1.2 the marginal cost of information is fixed ($\lambda = 0.5$) and we vary the prior (g_1, g_2).

In Figure 1.1 the states of the world are equiprobable: $g_1 = g_2 = 0.5$. The marginal cost of information changes from 0 to 1; the blue (solid) line depicts the weighting coefficient for good 1, the red (dashed) line – for good 2.

Figure 1.1: Coefficients β_1 (blue solid line) and β_2 (red dashed line) dependent on λ when $g_1 = g_2 = 0.5$

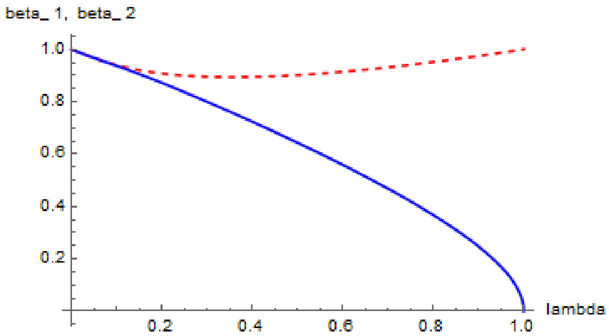
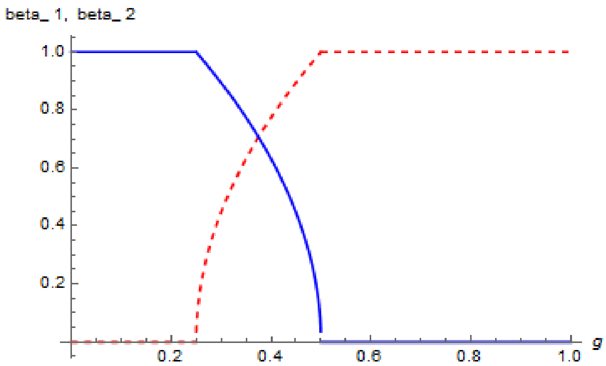


Figure 1.2: Coefficients β_1 (blue solid line) and β_2 (red dashed line) dependent on g_1.



The standard point of view on the CES utility function is that its weighting coefficients reflect consumer's preferences (Eaton and Kortum). We can see in Figure 1.1 that for all values of λ the weighting coefficient for the first good is lower than for the second one. It might look as if for the consumers the second good is intrinsically more preferable (recalling the common view on the weights in the CES utility function of the representative consumer). But this is not the case – the higher weighting coefficient is explained by the information. Indeed, when we look at Figure 1.2, we see that for a fixed value of λ the weighting coefficients vary with the prior knowledge of the consumer (the probability of the first state of the world g_1). Idiosyncratic tastes do not change but the prior does change.

This example demonstrates that the approach offered in the present paper not only provides an alternative foundation for the CES utility but also helps to understand the nature of the weighting coefficients of the CES function. We have shown that the weighting coefficients change with respect to prior knowledge and marginal cost of information.

1.3.3 The effect of change in belief or in price on elasticity and elasticity of substitution

There are two sources of demand change in our model: 1) change in price and 2) change in belief (change in price distribution). The first effect does not change the demand structure, while the second one changes the utility function of the representative consumer. Correspondingly, the price elasticity and the elasticity of substitution of the expected demand in our model differ from those for the representative agent with the CES utility function.

The price elasticity with respect to change in price realization is (all derivations are in the Appendix)

$$\frac{dD_i}{dp_i}\frac{p_i}{D_i} = (-\frac{1}{\lambda} - 1) + \frac{1}{\lambda} \cdot \frac{P_i^0 p_i^{-\frac{1}{\lambda}}}{\sum_{j=1}^N P_j^0 p_j^{-\frac{1}{\lambda}}}.$$

The price elasticity with respect to change in price distribution is

$$\frac{dD_i}{dp_i} \cdot \frac{p_i}{D_i} = \frac{dP_i^0}{dp_i}\frac{p_i}{P_i^0} - \frac{\sum_{j=1}^N \frac{dP_j^0}{dp_i} p_j^{-\frac{1}{\lambda}} p_i}{\sum_{j=1}^N P_j^0 p_j^{-\frac{1}{\lambda}}}.$$

Changes in beliefs lead to the new demand structure (different CES function), which corresponds to different price elasticities for the demand of RI agents in comparison with the usual price elasticities for the CES utility function. This fact, for the example with two goods, is expressed in the following proposition.

Proposition 2 (Distinction between price elasticity of rationally inattentive agents and price elasticity for the CES utility function)**.** *Let $N = 2$ and initially (before the change in price distribution) the feasible unconditional probability P_1^0 is unique, and the change in price distribution is such that in all states of the world price p_1 increases and price p_2 decreases ; the probabilities of the states of the world themselves do not change. Then the price elasticity of expected demand is lower in comparison with the price elasticity for the CES utility function.*

Proof. See the Appendix 1.D. □

The demand structure of RI agents, generally speaking, does not have a constant elasticity of substitution property. The elasticity of substitution is

$$\varepsilon_{ji} = \frac{d\left(\frac{D_j}{D_i}\right)}{d\left(\frac{p_i}{p_j}\right)} \cdot \frac{\frac{p_i}{p_j}}{\frac{D_j}{D_i}}.$$

The elasticity of substitution of the representative agent, if RI agents are unaware of changes in price distribution (the prior does not change), is $\frac{1}{\lambda} + 1$.

However, when the agents are rationally inattentive, the unconditional probabilities might change with the elasticity of substitution

$$E_{ji} = \frac{d\left(\frac{P_j^0}{P_i^0}\right)}{d\left(\frac{p_i}{p_j}\right)} \cdot \frac{\frac{p_i}{p_j}}{\frac{P_j^0}{P_i^0}}.$$

That is, elasticity of substitution of the expected demand of RI agents is

$$\varepsilon_{ji} = E_{ji} + \frac{1}{\lambda} + 1.$$

1.4 Conclusion

It is often assumed that changes in the aggregate consumer's demand are due to changes in idiosyncratic preferences of individual consumers[5]. We propose an alternative story: the demand shifts for particular goods might sometimes be better explained by a change in information about goods when the consumers are rationally inattentive.

According to our model, the demand structure changes due to shifts in information costs and the structure of prior knowledge of consumers, not in the idiosyncratic preferences. In many markets there was a reduction in the costs of information (due to the appearance of websites with information on products, such as google.com/shopping, special search engines to compare the prices of airline tickets, hotels, restaurants, etc.). All this directly affects the information costs and consumer's prior beliefs. The information coming from different countries or regions and making their products salient might also change a consumer's priors. Accordingly, we can anticipate changes in the structure of the CES utility function and the aggregate consumer behavior. Thus, our model extends the understanding of why changes in demand, which are usually interpreted as a change in preferences, often occur after certain events (shocks) in the economy, such as crises,

[5]Of course, this is not the only explanation for shifts in the demand that has been considered in the economic literature. Papers on consumer search are a good example. A distinctive feature of our approach is that the consumer's information acquisition is unconstrained.

opening of new markets, and changes in the advertising policy of certain firms.

We show that the demand system generated by the CES utility function is equivalent to a model of rational inattention to discrete choice. That is, we endogenize (microfound) the CES utility function with the RI model. We show that the elasticity of substitution and "weighting" coefficients of the CES function are determined by the parameters of the RI model, namely marginal cost of information and prior beliefs. Such a link helps us to connect the intensively developing RI theory with neoclassical economic models.

The results of this paper may help to find estimates for the cost of information. In the literature there are estimations of elasticities of substitution for the CES function (e.g. Bergstrand, Egger, and Larch (2013), Coloma (2009), Redding and Weinstein (2016)). Based on such estimations and using formula (1.7), which connects elasticity of substitution, σ, and marginal cost of information, λ, it is now possible to obtain the estimates for the parameter of cost of information.

1.A Proof of Proposition 1

Proof. Indeed, for the problem

$$\max\left(\sum_{j=1}^{N}\beta_j q_j^{\rho}\right)^{\frac{1}{\rho}}$$

s.t.

$$\sum_{j=1}^{N} p_j q_j = y, \tag{1.11}$$

the F.O.C. is

$$\frac{\beta_1 q_1^{\rho-1}}{p_1} = ... = \frac{\beta_N q_N^{\rho-1}}{p_N}. \tag{1.12}$$

From (1.11) and (1.12) it follows that

$$q_i = \frac{\beta_i^{\frac{1}{1-\rho}} p_i^{\frac{1}{\rho-1}}}{\sum_{j=1}^{N}\beta_j^{\frac{1}{1-\rho}} p_j^{\frac{\rho}{\rho-1}}} y. \tag{1.13}$$

By comparing (1.6) and (1.13) we see that the elasticity of substitution between goods in the CES utility function is

$$\sigma = \frac{1}{1-\rho} = \frac{1}{\lambda} + 1.$$

Correspondingly,

$$\rho = \frac{1}{\lambda+1}.$$

Each coefficient β_i of the CES function is defined by the corresponding unconditional probability and the marginal cost of information in the following way:

$$\beta_i = \gamma\left(P_i^0\right)^{1-\rho} = \gamma\left(P_i^0\right)^{\frac{\lambda}{1+\lambda}}, \quad i = 1, ..., N,$$

where γ is a positive coefficient. □

1.B Derivation of weighting coefficients in the example

We find the unconditional probabilities P_i^0, $i = 1, 2$ using the Corollary 2 from (Matějka, McKay, 2015). They should satisfy the equality:

$$\sum_{i=1}^{2} \frac{e^{\frac{v_i}{\lambda}}}{\sum_{j=1}^{2} P_j^0 e^{\frac{v_j}{\lambda}}} g_i = 1.$$

After computing the unconditional probabilities, we plug them into equation (1.8) and obtain the weighting coefficients of the corresponding CES function.

In our particular example under $\gamma = 1$:

$$\beta_1 = \left(P_1^0\right)^{\frac{\lambda}{1+\lambda}} = \left(\frac{g_1 2^{\frac{1}{\lambda}} + \left(\frac{2}{3}\right)^{\frac{1}{\lambda}} - g_1 \left(\frac{2}{3}\right)^{\frac{1}{\lambda}} - 1}{-1 + \left(\frac{2}{3}\right)^{\frac{1}{\lambda}} - \left(\frac{4}{3}\right)^{\frac{1}{\lambda}} + 2^{\frac{1}{\lambda}}} \right)^{\frac{\lambda}{1+\lambda}},$$

and

$$\beta_2 = \left(P_2^0\right)^{\frac{\lambda}{1+\lambda}} = \left(1 - \frac{g_1 2^{\frac{1}{\lambda}} + \left(\frac{2}{3}\right)^{\frac{1}{\lambda}} - g_1 \left(\frac{2}{3}\right)^{\frac{1}{\lambda}} - 1}{-1 + \left(\frac{2}{3}\right)^{\frac{1}{\lambda}} - \left(\frac{4}{3}\right)^{\frac{1}{\lambda}} + 2^{\frac{1}{\lambda}}} \right)^{\frac{\lambda}{1+\lambda}}.$$

1.C Price elasticity of expected demand

The price elasticity of demand is

$$\frac{dD_i}{dp_i}\frac{p_i}{D_i} = \frac{(-\frac{1}{\lambda} - 1)p_i^{-\frac{1}{\lambda}-2} \sum_{j=1}^{N} P_j^0 p_j^{-\frac{1}{\lambda}} - p_i^{-\frac{1}{\lambda}-1} P_i^0 (-\frac{1}{\lambda} p_i^{-\frac{1}{\lambda}-1})}{(\sum_{j=1}^{N} P_j^0 p_j^{-\frac{1}{\lambda}})^2} \cdot \frac{p_i \sum_{j=1}^{N} P_j^0 p_j^{-\frac{1}{\lambda}}}{p_i^{-\frac{1}{\lambda}-1}}$$

$$= (-\frac{1}{\lambda} - 1) + \frac{1}{\lambda} \cdot \frac{P_i^0 p_i^{-\frac{1}{\lambda}}}{\sum_{j=1}^{N} P_j^0 p_j^{-\frac{1}{\lambda}}}.$$

If the agent knows that the distribution of prices changed (unconditional probabilities P_i^0 can change), then the elasticity of expected demand in the states in which p_i changes is:

$$\frac{dD_i}{dp_i} \cdot \frac{p_i}{D_i} = (\left(\frac{dP_i^0}{dp_i} p_i^{-\frac{1}{\lambda}-1} + P_i^0 (-\frac{1}{\lambda} - 1) p_i^{-\frac{1}{\lambda}-2} \right) \cdot (\sum_{j=1}^{N} P_j^0 p_j^{-\frac{1}{\lambda}}) -$$

$$-\left(\sum_{j=1}^{N}\frac{dP_j^0}{dp_i}p_j^{-\frac{1}{\lambda}}-\frac{1}{\lambda}P_i^0 p_i^{-\frac{1}{\lambda}-1}\right)P_i^0 p_i^{-\frac{1}{\lambda}-1})\cdot\left(\sum_{j=1}^{N}P_j^0 p_j^{-\frac{1}{\lambda}}\right)^{-2}\cdot\frac{p_i\left(\sum_{j=1}^{N}P_j^0 p_j^{-\frac{1}{\lambda}}\right)}{P_i^0 p_i^{-\frac{1}{\lambda}-1}}=$$

$$=(-\frac{1}{\lambda}-1)+\frac{1}{\lambda}\cdot\frac{P_i^0 p_i^{-\frac{1}{\lambda}}}{\sum_{j=1}^{N}P_j^0 p_j^{-\frac{1}{\lambda}}}+\frac{dP_i^0}{dp_i}\frac{p_i}{P_i^0}-\frac{\sum_{j=1}^{N}\frac{dP_j^0}{dp_i}p_j^{-\frac{1}{\lambda}}p_i}{\sum_{j=1}^{N}P_j^0 p_j^{-\frac{1}{\lambda}}}.$$

The term

$$\frac{\sum_{j=1}^{N}\frac{dP_j^0}{dp_i}p_j^{-\frac{1}{\lambda}}p_i}{\sum_{j=1}^{N}P_j^0 p_j^{-\frac{1}{\lambda}}}$$

is a weighted sum of elasticities

$$\sum_{j=1}^{N}\delta_j\frac{dP_j^0}{dp_i}\frac{p_i}{P_j^0},$$

where $0<\delta_j<1$, $\sum_{j=1}^{N}\delta_j=1$ and

$$\delta_j=\frac{P_j^0 p_j^{-\frac{1}{\lambda}}}{\sum_{k=1}^{N}P_k^0 p_k^{-\frac{1}{\lambda}}}=P_i(v).$$

1.D Proof of Proposition 2

Using Proposition 3 from (Matějka, McKay 2015):

$$\frac{dP_1^0}{dp_1}\frac{p_1}{P_1^0}<0$$

and

$$\frac{dP_2^0}{dp_1}\frac{p_1}{P_2^0}>0.$$

Using the formula for price elasticity for RI and comparing it with the formula for CES, the difference between the price elasticities of expected demand of RI consumer and CES-generated demand is

$$(1-\delta_1)\frac{dP_1}{dp_1}\frac{p_1}{P_1^0}-\delta_2\frac{dP_2^0}{dp_1}\frac{p_1}{P_2^0}<0.$$

Chapter 2

The Status Quo and Belief Polarization of Inattentive Agents

Co-authored with Vladimír Novák (CERGE-EI).

2.1 Introduction

In recent years people have tended to disagree more and extreme opinions are becoming more prevalent in the public discourse. Voters in many countries have experienced elections that can almost be interpreted as a referendum between so called mainstream and populist candidates. Examples include the series of 2016-2017 democratic votes in Europe: the Brexit referendum, presidential elections between Macron and Le Pen in France, parliamentary elections in the Netherlands with Geert Wilders as an expected front runner and many others since. The recent increase in partisanship is well documented through the measurement of congressional speeches by Gentzkow, Shapiro, and Taddy (2016). Importantly, people also seem to be moving farther away from each other in their beliefs regarding the issues which require almost a purely scientific approach of benefits and costs assessments. Many researchers have connected this rise of populism first with the aftermath of the financial crisis, and later with the immigration crisis and the backlash against globalization.[1] What all these situations have in common is that they created winners and losers. Therefore, individuals perceive the favorability of implemented policies differently. This is also documented by studies suggesting not only

[1] See, for instance, Rodrik (2018), Pastor and Veronesi (2018), Inglehart and Norris (2016).

that U.S. citizens disagree on the issues, but they disagree on what the issues actually are.[2] This raises several theoretical questions. Can such polarization be explained by endogenous information acquisition? What is the role of an unprecedented increase in information availability? Is it a result of rational behavior or is it necessary for individuals to have biased reasoning? How should we take into account the fact that people hold on to their opinions of the established policies?

We present a discrete choice problem where the agent can choose between a safe known option (status quo) and a risky option (new policy) with an unknown payoff. Before choosing, she has an opportunity to receive information about the realized state, but doing so is costly. In order to account for endogenous information acquisition, without imposing any exogenously given biases, we model the decision maker to be rationally inattentive, e.g., Sims (1998, 2003). Our theoretical model reveals that inattentiveness can lead to belief polarization.

A key role in our analyses belongs to the status quo and we model it as a safe option with a known payoff. The value of the status quo influences what information the agent acquires. We show that it leads to a *state pooling effect*. In particular, the agent is not trying to find information about which particular state of the world is going to occur and what the exact payoff from the risky option will be, i.e. a new policy; but she pools the states into two categories. One category consists of the states perceived a priori as more favorable than the status quo, and other states form the second group. We assume that there exist at least three states of the world, otherwise the state pooling effect would not occur. Because of the limited attention constraint and costly information acquisition, the decision maker decides to learn which category of states contains the realized state. As a consequence the agent might be confused about which particular state from a category has occured and updates her belief about the particular state incorrectly.

Specifically, if we take the position of an observer who knows which state of the world is happening we will see the following behavior: in the extreme states the agent updates her belief about the expected value of the new policy in the direction of the realized value of the risky policy. However, in the moderate states the decision maker can update her

[2] Pew Research Center: 2018 Midterm Voters: Issues and Political Values, available online: http://www.people-press.org/2018/10/04/2018-midterm-voters-issues-and-political-values/

belief about the expected value of the new policy away from the realized payoff of the new policy. As we show in section 2.3.4 this effect is determined by the relative position of prior expected value of the new policy and the status quo. Belief polarization consequently arises for the categories of agents with a different prior belief and/or perception about the status quo.

The importance of this result lies in highlighting that two agents with the same prior belief can become polarized if they perceive the favorability of the already implemented policy differently enough. The omission of the status quo in the empirical studies that focus on polarization might lead to biased results or at least leaves one of the important polarization channels uninvestigated.

One of the consequences of the updating in the opposite direction from the realized value of the new policy that we describe above is that the agents who are pessimistic about the new policy become over-pessimistic, and the optimistic ones become over-optimistic. Moreover, in our setting, even the agents who choose the same option after information acquisition can become more distant in their beliefs. The prior expected payoff from the new policy and status quo can work similarly in our model and changes in both of them might cause polarization. However, valuation of the status quo is crucial for creation of state categories. This emphasizes the importance of understanding how the citizens value the current situation for polarization mitigation in comparison with just communicating the more probable outcomes of the proposed policies.

Our model also gives clear predictions of when the agents update toward the true value and when heterogeneous agents would converge in their opinions. Particularly, a conservative approach, in the sense that the agent expects a very similar outcome from the new policy as from the currently implemented policy, ensures the agents' truthful learning independently of their partisanship. We further show how the difference between the prior and posterior expected values from the new policy depends on the realized state of the world. We also characterize the set of states in which the agents update in the opposite direction from the realized value. The elements of this set are only the moderate states and never the extreme states. As a result, agents tend to become more polarized in neither excellent nor disastrous times, but when everything seems just fine. This finding explains why disagreement in society is rising even though most of the developed western economies are in a decent condition.

We also investigate the impact of the cost of information on polarization. As expected, more expensive information enlarges the areas of the prior beliefs for which the agent chooses not to acquire any information. Similarly, agents update more and, thus, would disagree more when the information is cheaper. Importantly, the updating away from the realized value occurs for every positive parameter of the marginal cost of information, only the magnitude of such updating changes.

The rest of the paper is organized as follows. The next section describes related literature, in Section 2.2 we present the general model for n states and two actions and present our main theoretical results. Section 2.3 illustrates the results for the example with three states and thus provides crucial insights. In Section 2.4 we focus on understanding what the implications of a different cost of information are on agent disagreement.

2.2 Related literature

This paper provides a general framework for studying belief polarization with endogenous information acquisition. A theoretical framework builds upon the results from recent literature on rational inattention, which was introduced by Sims (1998, 2003) and further developed and implemented in macroeconomics, e.g., Maćkowiak and Wiederholt (2009, 2015), Woodford (2009), Afrouzi and Yang (2019), finance, e.g., Van Niewerburgh and Veldkamp (2010) and other fields.[3] A survey of this literature is provided in Matějka, Maćkowiak, and Wiederholt (2018). RI theory relies on the central premise that information is plentiful, but attention is a scarce resource. Thus it allows the agent to choose an optimal signal, which she wants to obtain given the information constraint. The main finding of the present paper is demonstrated using the discrete choice model with rationally inattentive agents (Matějka and McKay 2015), which uses a Shannon cost function.

We contribute to the literature on belief polarization. The classicar result is that the beliefs of unconstrained and rational Bayesian agents converge over time and that they will almost surely assign probability 1 to a true state (Savage (1954), Blackwell and Du-

[3]See also Caplin and Dean (2015), Caplin, Dean, and Leahy (2019).

bins (1962)). Relaxing the rationality assumption started the stream of literature which focuses on finding the conditions when persistent disagreement may occur. The common method for obtaining such results is to assume that the agent is facing a misspecified model (Berk 1966) or that the agents have some form of biased perception or learning. A survey of this literature is provided by Gerber and Green (1999). Dixit and Weibull (2007) present a model in which political polarization is driven by the bimodality of preferences. In this paper, we diverge from any exogenous behavioral biases by considering a rationally inattentive decision maker who can select the optimal signal given her information capacity constraint and updates her belief in the Bayesian fashion; thus any documented biases are endogenously created.

The question of how an agent's inattentiveness leads to persistent disagreement and how it can explain confirmation bias has been the focus of several studies. Su (2015) demonstrates that a belief divergence may occur if an agent's learning is rationally inattentive. However, this is only for a setting with utility that has a quadratic loss form in which the agent can only choose the variance in the Gaussian signal model as a true value plus noise, and also with an ad hoc assumption of the attention cost being proportional to the so called observation window. The most prominent attempt to study persistent disagreement for inattentive agents was presented by Nimark and Sundaresan (2019)[4], which is the paper with the setting closest to ours. Their main objective is to investigate the persistence of belief polarization of inattentive agents in a two actions, two states setting. They also discuss in greater detail different implications of the two different approaches for the measurement of information costs. In this paper we emphasize a different channel that influences information acquisition and thus belief polarization that is the effect of the status quo. We believe that this is an important aspect of decision-making which deserves greater attention. Moreover, we study a setting with n states and two actions, which after the endogenously obtained state pooling effect might collapse to a two states, two action setting. This provides a clear connection with the majority of papers studying belief polarization.

More generally, we contribute to the literature of reference-dependent preferences. Kőszegi and Rabin (2006, 2007) build a model of reference point formation and study shifts in risk attitudes, but they cannot account for the fixed reference points. In our

[4]An earlier version of the paper is known as Sundaresan and Turban (2014).

model we keep the status quo as exogenously given and fixed for the agent. This makes us closer to the approach of Bordalo, Gennaioli, and Shleifer (2012), who present a theory where the agent is drawn to salient payoffs and thus provides a context dependent representation of lotteries. An endogenously arising state pooling effect connects us to the categorical thinking literature, represented mainly by Mullainathan (2002). In this area of research it is assumed that people make inferences using coarse categories. As a consequence people do not update continuously, but change categories only when they see enough data to suggest that a different category better fits the data. Our approach provides a theory of endogenous categories creation. However, the main difference is that our agent does not make their inference through categories, but acquires only the information necessary for a resolution between categories. Nonetheless, the common ground for both theories is that due to the possibility that different states can collapse to the same category, the decision maker cannot sufficiently distinguish between types of states.

We provide a general framework that can be further used in applications answering specific questions connected with polarization of political partizanship, see, e.g., Gentzkow, Shapiro, and Taddy (2016), Gentzkow (2016), Boxell, Gentzkow, and Shapiro (2017), polarization on cultural views (Krasa and Polborn 2014) and linkages with inequality and conflict (Esteban and Ray 2011). The behavior presented in this paper may, for the reader, resemble the effects described in papers about overconfidence (Ortoleva and Snowberg 2015), limited memory (Wilson 2014) and several others. In this paper we solve the static problem, but thanks to the state pooling effect the multiple states are collapsed into two categories, which gives us the possibility to gain intuition about the behavior in dynamics from papers focusing on a two state, two action case in the dynamic setting. For instance, Che and Mierendorff (2017) study optimal sequential decision making with limited attention in a two states, two action environment.

Our paper also makes several predictions relevant to the empirical literature on information preferences. For instance, Charness, Oprea, and Yuksel (2018) study whether people choose optimal or biased information sources. Their results suggest that a confirmatory seeking rule is the most common one, but they assume only a two states, two actions setup. Important insights for our predictions arise from Ambuehl and Li (2018), who show that people disproportionally value information that yields certainty. Our model may provide a rational explanation for such a valuation. In future research it might also be interesting to explore, using referendum survey data, whether voters use similar voting cues and information sources as predicted by our model.

2.3 The model

In this section we describe the general agent's decision problem, introduce a methodology for beliefs evolution assessment, and present the main theoretical results.

2.3.1 Description of the setup

A single agent faces a discrete choice problem between two options. The first option, which we refer to as *a new policy*, provides a payoff $v_s \in \mathbb{R}$ that depends on an unobservable state of the world $s \in S = \{1, \ldots, n\}$, where $n \in \mathbb{N}$. The states are labeled in such a way that $v_1 < v_2 < \ldots < v_n$. The second option yields a known fixed payoff $R \in \mathbb{R}$, $v_k \leq R \leq v_{k+1}$ for some $k \in \{1, 2, ..., n-1\}$. The agent is risk-neutral. We assume that $v_1 < R < v_n$ in order to exclude trivial cases[5]. One can imagine a situation where the agent has to choose between a currently implemented policy with a known payoff, i.e. preservation of *the status quo*[6], or selects a new policy that might possibly lead to several different outcomes.

The decision maker is uncertain which state of the world s is going to be realized and we denote her prior beliefs as a vector of probabilities $\mathbf{g} = [g_1 \quad g_2 \quad \ldots \quad g_n]^T$, such that $\sum_{s=1}^{n} g_s = 1$ and $g_s \geq 0$, $\forall s \in S$. We model the agent to be rationally inattentive in the fashion of Sims (2003, 2006). The agent wishes to select the option with the highest payoff. Prior to making the decision, the agent has the possibility to acquire some information about the actual value of the new policy, which is modeled as receiving a signal $x \in \mathbb{R}$. The distribution of the signals, $f(x, s) \in P(\mathbb{R} \times S)$, where $P(\mathbb{R} \times S)$ is the set of all probability distributions on $\mathbb{R} \times S$, is subject to the agent's choice. The joint distribution of signals and states which can be chosen by the agent is restricted to be consistent with the agent's prior belief, that is, the unconditional expectation of her

[5]If $R \leq v_1$, the safe option is weakly dominated by the risky option, and if $R \geq v_n$, the risky option is weakly dominated by the safe option. In both of these cases the agent does not have incentives to acquire information on realization of the state of the world.

[6]The status quo option can be perceived just as an imposed reference point, thus we denote it as R.

posterior belief should be equal to her prior belief, $\int_x f(dx,s) = g_s \quad \forall s \in S$. However, this information is costly and we assume the cost to be proportional to the expected reduction in entropy[7]. For detailed treatment of this measure, see, for example, Cover and Thomas (2012). Upon receiving a signal, the agent chooses an action, and her choice rule is modelled as $\sigma(x) : x \to \{1,2\}$. Given the updated belief, the agent chooses the action with the highest expected payoff. The timing of the decision problem is depicted in figure 2.1.

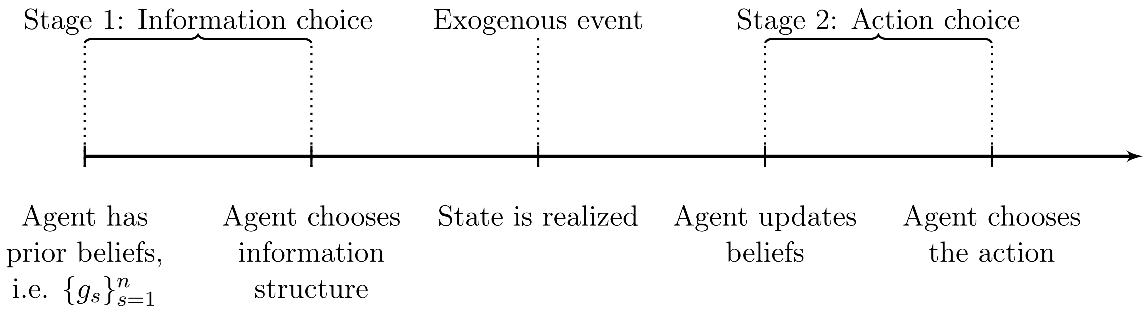


Figure 2.1: Timing of the events in the problem. The decision problem consists of two stages: the information strategy selection stage and the standard choice under uncertainty stage.

2.3.2 Agent's problem

The agent's problem is to find an information strategy maximizing the expected utility less the information cost. According to Lemma 1 from Matějka and McKay (2015) the state-contingent choice behavior of the rationally inattentive consumer can be found as the solution to a simpler maximization problem that does not make reference to signals or posterior beliefs. The information strategy is characterized by the collection of conditional probabilities of choosing option i in state of the world s : $\mathcal{P} = \{\mathcal{P}(i|s) \big| \, i = 1,2; \; s \in S\}$, where $i \in \{\text{new policy, status quo}\} = \{1,2\}$ denotes the option and s is the state. The agent solves:

$$\max_{\mathcal{P}=\{\mathcal{P}(i|s)|i=1,2;s=1,\ldots,n\}} \left\{ \sum_{s=1}^{n} \left(v_s \mathcal{P}(i=1|s) + R\mathcal{P}(i=2|s)\right) g_s - \lambda\kappa \right\}, \tag{2.1}$$

subject to

[7]The entropy $H(Z)$ of a discrete random variable Z with support $\mathcal{Z}$ and probability mass function $\mathcal{P}(z) = Pr\{Z = z\}, z \in \mathcal{Z}$ is defined by $H(Z) = -\sum_{z\in\mathcal{Z}} p(z) \log p(z)$.

$$\forall i: \ \mathcal{P}(i|s) \geq 0 \qquad \forall s \in S\ , \tag{2.2}$$

$$\sum_{i=1}^{2} \mathcal{P}(i|s) = 1 \qquad \forall s \in S\ , \tag{2.3}$$

and

$$\kappa = \underbrace{-\sum_{i=1}^{2} \mathcal{P}(i) \log \mathcal{P}(i)}_{\text{prior uncertainty}} - \underbrace{\left(-\sum_{s=1}^{n} \left(\sum_{i=1}^{2} \mathcal{P}(i|s) \log \mathcal{P}(i|s)\right) g_s\right)}_{\text{posterior uncertainty}} . \tag{2.4}$$

where κ denotes the expected reduction in entropy, $\lambda \geq 0$ is the unit cost of information and thus $\lambda\kappa$ reflects the cost of generating signals with different precision. $\mathcal{P}(i)$ is the unconditional probability that the option i will be chosen and is defined as

$$\mathcal{P}(i) = \sum_{s=1}^{n} \mathcal{P}(i|s) g_s, \qquad i = 1, 2.$$

Matějka and McKay (2015) study a general case of the static problem described above, and show that the agent's choice of action is in line with the modified multinomial logit formula.[8] This result translates into our setting in the following way:

Lemma 2. *Conditional on the realized state of the world s^*, the probability of choosing a new policy, i.e. option 1 for $\lambda > 0$ is*

$$\mathcal{P}(i = 1|s^*) = \frac{\mathcal{P}(i = 1) e^{\frac{v_{s^*}}{\lambda}}}{\mathcal{P}(i = 1) e^{\frac{v_{s^*}}{\lambda}} + (1 - \mathcal{P}(i = 1)) e^{\frac{R}{\lambda}}}, \tag{2.5}$$

the probability of choosing the status quo is

$$\mathcal{P}(i = 2|s^*) = \frac{(1 - \mathcal{P}(i = 1)) e^{\frac{R}{\lambda}}}{\mathcal{P}(i = 1) e^{\frac{v_{s^*}}{\lambda}} + (1 - \mathcal{P}(i = 1)) e^{\frac{R}{\lambda}}}, \tag{2.6}$$

where $\mathcal{P}(i = 1)$ is the unconditional probability of choosing a new policy.

[8] For the dynamic version of this result see (Steiner, Stewart, and Matějka (2017)).

When $\lambda = 0$, the agent chooses from two available options: the new policy or the status quo, the option with the highest value with probability one.

Proof. Lemma 2 is a direct consequence of the Theorem 1 from Matějka and McKay (2015).

□

Lemma 2 tells us that the optimal information-processing strategy of the decision maker leads her to choose in line with a biased logit model. Specifically, the agent's prior knowledge and information strategy are reflected by the unconditional probability of choosing the new policy. The appearance of this probability and the marginal cost of information λ in the choice rule uncovers two forces that can shift the choice probabilities. However, it is important to state that choice probabilities are not fully biased because they still depend on the true payoffs of both options.

An important feature of the solution to the agent's problem is that for the given vector of payoffs of the new policy $(v_1, ..., v_n)$, the value of status quo R and the marginal cost of information λ there exist prior beliefs of the agent for which she decides not to acquire any information. In this case we say that the agent is in a *non-learning area.* Once the agent is in a non-learning area, she makes her decion only based on her prior beliefs. That is, when the agent is in a non-learning area, if $\mathbb{E}v = \sum_{s=1}^{n} v_s g_s > R$, then the agent chooses the new policy with certainty and $\mathcal{P}(i = 1) = 1$, if $\mathbb{E}v < R$, then the agent chooses the status quo with certainty and $\mathcal{P}(i = 1) = 0$, or if $\mathbb{E}v = R$, then the agent is indifferent between the two policies and the agent can have any $\mathcal{P}(i = 1) \in [0, 1]$. Let us assume, without loss of generality, that in the latter case the agent would decide to keep the status quo, that is, $\mathcal{P}(i = 1) = 0$. Given this assumption, the unconditional choice probabilities of the agent who is in a non-learning area, are either 0 or 1. If the agent's prior is such that she decides to acquire at least some information, we say that the agent is in a *learning area.* For such prior beliefs the unconditional choice probabilities lie in the open interval $(0, 1)$.

2.3.3 Description of belief evolution

The uncertainty in this model is about the realized state of the world and thus about the actual payoff of the new policy. Without the information acquisition stage of the problem the agent would choose the option based on the comparison of the status quo payoff R with the agent's prior expected payoff from the new policy being

$$\mathbb{E}v = \sum_{s=1}^{n} v_s g_s.$$

In order to judge how this expected payoff from the new policy changes after the signal is received and the option is chosen, we take the position of an external observer. The observer knows that a realized state of the world is s^* and is interested in the agent's posterior belief about the payoff of the new policy v_s given the realized state s^*. Note that the agent's posterior belief is given by the signal she receives and thus the observer not only wants to know what the expected posterior belief is for a given signal, but is interested in the expected posterior belief about the new policy on average across all possible signals the agent may choose. Since there is a one to one mapping between the selected information structure and consequently chosen action, the posterior expected belief of interest is

$$\mathbb{E}_i[\mathbb{E}(v_s|i)|s^*] = \sum_{i=1}^{2} \left(\sum_{s=1}^{n} v_s \mathcal{P}(s|i) \right) \mathcal{P}(i|s^*),$$

where option $i \in \{1,2\} = \{\text{new policy}, \text{status quo}\}$. For the rationally inattentive agent it can be further formalized as:

Proposition 3. *The expected posterior value of the new policy given the state s^* for the rationally inattentive decision maker with a marginal cost of information λ and for $i \in \{$new policy, status quo$\} = \{1,2\}$ is*

$$\mathbb{E}_i[\mathbb{E}(v|i)|s^*] = \sum_{s=1}^{n} v_s g_s \frac{\mathcal{P}(i=1|s^*) e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1|s^*)) e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1) e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1)) e^{\frac{R}{\lambda}}}. \tag{2.7}$$

Proof. The proof is presented in Appendix 2.A. □

The main indicator for the expected belief evolution that we consider can be defined as $\Delta(s^*)\mathbb{E}_i[\mathbb{E}(v|i)|s^*] - \mathbb{E}v$. In particular, we are interested in the sign of $\Delta(s^*)$, which informs us whether the posterior expected belief is moving from $\mathbb{E}v$ towards v_1 or v_n or stays equal to $\mathbb{E}v$.

Proposition 4. *If the agent is in a learning area, the sign of the change in the mean of beliefs in state s^* $\Delta(s^*)$ is the same as the sign of $(v_{s^*} - R)$.*

Proof. The proof is presented in Appendix 2.B

□

Proposition 4 significantly simplifies the considerations of the beliefs evolution. At the same time, it also demonstrates the important link between the value of the status quo R and updating process. Namely, it demonstrates that the sign of $\Delta(s^*)$ is affected only by the the relative location of R and v_{s^*}, no matter what $\mathbb{E}v$ is.

We can note the following corollary, which specifies conditions for which there is no change in the mean of beliefs.

Corollary 2. $\Delta(s^*) = 0$ *holds if at least one of the following conditions is satisfied:*

(a) $v_{s^*} = R$

(b) the agent is not acquiring any information, i.e. $\exists\, i : \mathcal{P}(i) = 1$.

2.3.4 Updating in the opposite direction from the realized value and beliefs polarization

This section describes the impact of the value of the status quo on the opinion polarization of inattentive agents. For the rest of this section we assume that the agent is in the learning area, i.e. $0 < \mathcal{P}(i = 1) < 1$ and we denote the set of states $S' = S \setminus \{1, n\}$ as the set of the *intermediate states.*

Definition 1. The agent is updating in the *opposite direction from the realized value* v_{s^*} in the state $s^* \in S'$, if the condition $(\mathbb{E}v - v_{s^*}) \cdot \Delta(s^*) > 0$ is satisfied.

In the following theorem we provide conditions for the presence of the states in which the agent is updating in the opposite direction from the realized value of the risky option.

Theorem 1. *If there exists a state $s^* \in S'$ for which $(v_{s^*} - R)(\mathbb{E}v - v_{s^*}) > 0$, then, in this state of the world, the agent is updating in the opposite direction from the realized value v_{s^*}.*

Proof. The theorem immediately follows from Proposition 4. □

Dichotomy of the set of the states of the world and intuition for the result. Due to costly information acquisition the rationally inattentive decision maker chooses only the necessary information in order to disentangle whether to select the status quo or the new policy. This leads to the **state pooling effect**, when the agent divides the states into two categories. Namely, as Proposition 4 states, for all the states s in which $v_s > R$ the expected posterior belief about the value of the risky option is higher than the prior belief, which unites or pools all such states into one category. Similarly, all the states s for which $v_s < R$ are pooled into another category. For all the states from one category the direction of updating of the expected belief about the value of the risky option is the same. It is important to notice that the agent's expected posterior beliefs are not the same for the states from one category. We discuss the magnitude of updating for different realized states $s^* \in S$ in subsection 2.3.5.

The state pooling effect induces updating in the opposite direction from the realized value, when for the realized state s^* holds that $\mathbb{E}v < v_{s^*} < R$ or $R < v_{s^*} < \mathbb{E}v$. The updating in the opposite direction from the realized value can cause agents with different perception of the status quo and/or with different prior beliefs to become polarized.

Let us consider a situation with two agents $j = 1, 2$ who have (i) different preferences about the status quo policy R^j and, (ii), different prior beliefs about the value of the new policy $\mathbb{E}^j v$. The expected posterior belief of the agent j about the value of the new policy, conditional on the realized state s^*, is denoted by $\mathbb{E}_i^j[\mathbb{E}(v|i)|s^*]$. The difference between the expected posterior beliefs of the agent j in the state s^* and the prior beliefs of the agent j is denoted by $\Delta_j(s^*)$, $\Delta_j(s^*) = \mathbb{E}_i^j[\mathbb{E}(v|i)|s^*] - \mathbb{E}^j v$.

Definition 2. We say that two agents $j = 1, 2$ that are characterized by the pair $(R^j, \mathbb{E}^j v)$ become *polarized in the state* s^* when the following two conditions are satisfied

1. $|\mathbb{E}_i^1[\mathbb{E}(v|i)|s^*] - \mathbb{E}_i^2[\mathbb{E}(v|i)|s^*]| > |\mathbb{E}^1 v - \mathbb{E}^2 v|$
2. $\Delta_1(s^*) \cdot \Delta_2(s^*) < 0$

The first condition secures that the expected posterior beliefs in the state s^* of two agents are further apart, whereas the second ensures that they update in opposite directions in the state s^*. In the following theorem we provide conditions for the presence of the states of the world in which the agents become polarized.

Theorem 2. *Let us assume that there are two agents $j = 1, 2$ that are characterized by the pair $(R^j, \mathbb{E}^j v)$. If in state of the world $s^* \in S'$ the conditions $(\mathbb{E}^1 v - \mathbb{E}^2 v)(v_{s^*} - R^1) > 0$*

and $(\mathbb{E}^1 v - \mathbb{E}^2 v)(v_{s^*} - R^2) < 0$ *hold, then the two agents become polarized in this state of the world.*

Proof. Without loss of generality, let us assume that $\mathbb{E}^1 v > \mathbb{E}^2 v$. For the condition $(\mathbb{E}^1 v - \mathbb{E}^2 v)(v_{s^*} - R^1) > 0$ to be satisfied, it is necessary that $v_{s^*} > R^1$. Proposition 4 states that in this case $\Delta_1(s^*) > 0$. Using similar reasoning, the expected posterior belief is lower than the prior belief for agent 2. That is, they update in different directions and the expected posterior beliefs are farther away from each other than the priors are. Both conditions from the definition are satisfied and the agents, indeed, become polarized in state s^*. □

2.3.5 Monotonicity of $\Delta(s^*)$

So far we have shown that updating in the opposite direction from the realized value can occur and it might lead to belief polarization of agents. A natural question arises: How is the difference between the prior and the posterior expected payoff from the new policy $\Delta(s^*)$ influenced by the realized true state s^*? The answer to this question is provided by the following proposition.

Proposition 5. *Change in mean of beliefs* $\Delta(s^*)$ *is an increasing function of* s^*.

Proof. The proof is presented in Appendix 2.C. □

This finding, together with the fact that $\Delta(s^* = 1) < 0$ and $\Delta(s^* = n) > 0$, implies that $\Delta(s^*)$ reaches its minimum in state 1, its maximum in state n and $\Delta(s^*) = 0$ may occur in between. We remind the reader, at this point, that we have defined state k such that $v_k \leq R < v_{k+1}$. Note that inside the learning area $\Delta(s^*) = 0$ when $v_{s^*} = R$. Thus, either $v_k = R$ and $\Delta(s^* = k) = 0$, or such a state does not exist, but state k is the highest state where $\Delta(s)$ is negative. Using definition 1, we know that in states for which the condition $(\mathbb{E}v - v_{s^*}) \cdot \Delta > 0$ is satisfied, the agent updates in the opposite direction from the realized value of the risky option. Let us assume that the agent's prior expected value of the new policy is $\mathbb{E}v > R$. Then one can see that the agent is updating correctly for all states where $\Delta(s)$ is negative. However, updating in the opposite direction from the realized value occurs for all states that have payoffs smaller than $\mathbb{E}v$ and at the same time higher than R (see figure 2.2).

If we assume that $\mathbb{E}v < R$ then updating in the opposite direction from the realized value would happen in all states s^* for which it holds that $R > v_s^* > \mathbb{E}v$. When we

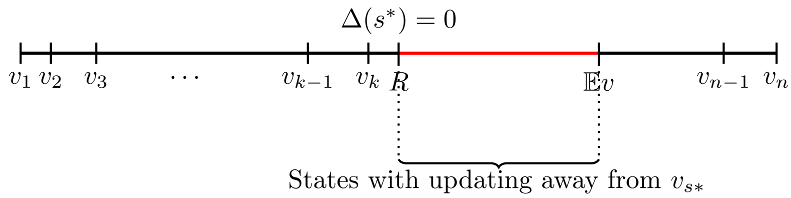


Figure 2.2: Set of states where the agent updates in the opposite direction from the realized value

denote by W the set of states where the agent updates in the opposite direction from the realized value we can write that

$$W = \begin{cases} \{s \mid R < v_s < \mathbb{E}v\}, & \text{if } \mathbb{E}v > R, \\ \{s \mid \mathbb{E}v < v_s < R\}, & \text{otherwise.} \end{cases}$$

It is worth noticing that the set of states where the agent updates in the opposite direction from the realized value are those where payoffs are neither very good nor very bad. That has significant implications for predictions when the inattentive decision makers become polarized. We can also see that the number of states in the set W is determined by the status quo payoff R and by the prior expected value of the new policy. Before we study what influences the magnitude of $\Delta(s^*)$, we first study the example with three states in the following section.

2.4 Over-optimism and polarization: intuition and implications

In order to understand in detail the previously stated results and their implications, we focus now on the case with three states. We assume that a rationally inattentive agent is choosing between the new policy that takes values $v_1 < v_2 < v_3$ in the states of the world $s = 1, 2, 3$, correspondingly; and keeping the status quo that has a payoff $v_1 < R < v_3$, independently of the realized state of the world. The decision maker has a prior expectation of the value of the new policy $\mathbb{E}v = v_1 g_1 + v_2 g_2 + v_3 g_3$.

In Proposition 4 we have shown that for fixed state s^*, the sign of $\mathbb{E}_i[\mathbb{E}(v|i)|s^*] - \mathbb{E}v$ is determined by the sign of $(v_{s^*} - R)$. When we consider the true realization of the state to be $s^* = 1$ (the worst payoff of the new policy), the agent on average shifts her belief about the value of the new policy down ($\Delta < 0$ because $v_1 - R < 0$), for any $\mathbb{E}v$ and R that are inside the interval (v_1, v_3). There is no surprise here: the value of the new policy is the lowest possible v_1 and the agent on average shifts her expectation of this option's payoff down, towards the true value. Similarly, when $s^* = 3$, implementing the new policy would lead towards the highest possible value v_3 and the agent correctly shifts the expected posterior belief closer to v_3 (because $\Delta > 0$).

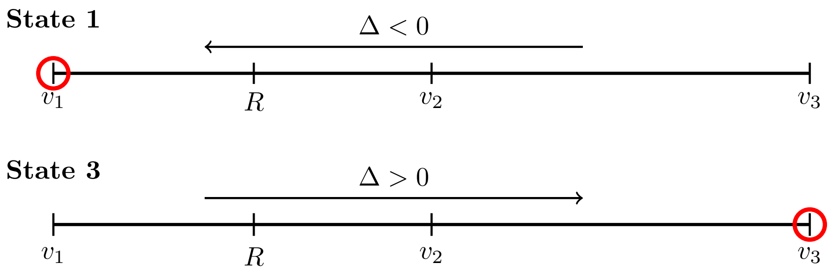


Figure 2.3: Updating when extreme states are realized. The true state is highlighted by the red circle.

Updating in the opposite direction from the realized value and state pooling effect

A more interesting situation occurs when intermediate states are realized. In this example, it is only the case when $s^* = 2$. First, without loss of generality, we assume that prior beliefs of the city residence are such that $\mathbb{E}v < v_2$ and we fix them. We consider different valuations of the status quo R. For $R \leq \mathbb{E}v$ and $\mathbb{E}v < R < v_2$ holds that $\Delta > 0$, the agent updates her average posterior belief about the payoff of the new policy towards the true realized payoff v_2. However, when $R > v_2$ then $\Delta < 0$ meaning that the agent updates her expected belief to the left, i.e. away from the true payoff of the new policy.[9] All these three cases are depicted in figure 2.4. In all three scenarios, the decision maker is rather pessimistic about the new policy, i.e. $\mathbb{E}v < v_2$. In the first two cases, when $R < v_2$, the agent on average understands that the impact of the new policy is beneficial. The reason is that she knows that keeping the status quo would lead to a relatively bad outcome and thus when the realized value of the risky option is relatively high, she correctly increases her expected belief about the probabilities that the new policy can lead to better outcomes (v_2 and v_3).

[9]Note that this is not possible with Bayesian updating and exogenous Gaussian signals.

When $R > v_2$ the agent shifts her expectation of the new policy down, closer to the outcome v_1, away from the true payoff v_2, i.e. she updates in the *opposite direction from the realized value*. One could expect that this result is just a consequence of confirmatory learning. However, we would like to emphasize that in the problem described in this paper, the wrong updating is a consequence of a different mechanism. Specifically, the agent chooses between the new policy and preserving the status quo. Initially she expects the new policy not to be very good and at the same time she perceives the status quo as quite good. The decision maker would prefer to choose the status quo in the realizations of the state $s^* = 1, 2$. Hence, to some extent, she acquires information that would allow her to disentangle whether state 3 is realized. She learns it (with some noise), and on average she understands that the realization is indeed not $s^* = 3$, but since, to some extent, she does not care which one of the other two it is exactly, both her posterior probabilities of states 1 and 2 rise. This is the *state pooling effect* mentioned in section 2.3.4, i.e. the agent endogenously pools states into categories. In this example, one category is composed from states 1 and 2; and the second category from state 3. Consequently, the direction of updating of the expected belief about the value of the risky option depends on the category to which the realized state belongs. This may result in the presense of updating in the opposite direction from the realized value of the risky option.

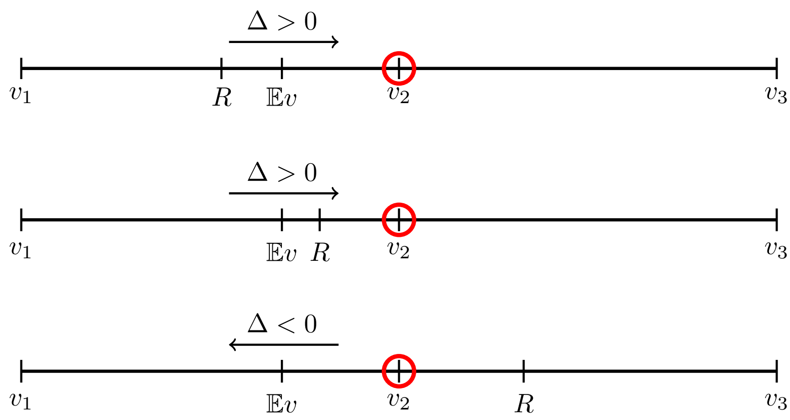


Figure 2.4: Updating for the changing status quo when state $s^* = 2$ is realized.

Symmetry, over-optimism and over-pessimism

Note that the whole effect works symmetrically, that is, in the previously discussed example with $s^* = 2$ and updating in the opposite direction from the realized value we can exchange the R and $\mathbb{E}v$. The analyses could also be done for the fixed R and changing the prior expected value of the new policy $\mathbb{E}v$, but we emphasize the role of the status quo, which is not usually considered in the papers studying polarization. Nevertheless, consider two situations where $s^* = 2$: the first with $\mathbb{E}v < v_2 < R$ and the second with $R < v_2 < \mathbb{E}v$. In the first situation the decision maker has a low prior expected value from the new policy and then updates towards v_1. In the second situation the prior expected value is quite high and then it is updated upwards, towards v_3. Stated differently, in the first situation the agent is pessimistic about the new policy and consequently becomes even more pessimistic. In the second case, the opposite is true. The agent is optimistic and becomes over-optimistic about the outcome of the new policy. This provides new insights for studies investigating whether people prefer positively or negatively skewed information, e.g., (Masatlioglu, Orhun, and Raymond 2019). In particular, how the change in the environment can shape the preferences for the skewed information.

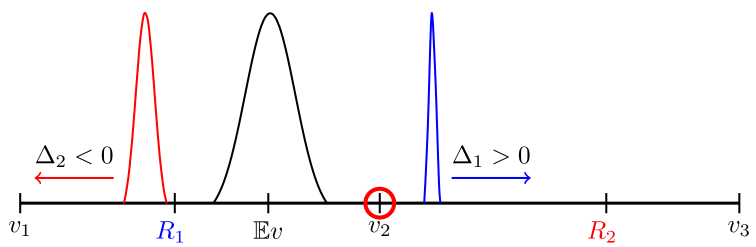


Figure 2.5: Polarization of the two groups with different status quo, illustrated using density functions when $s^* = 2$.

Polarization

Suppose two types of agents that differ only in how they perceive the current situation $R_i,\ i = 1, 2$. This is often the situation that different people do not necessarily need to have different expectations about future policy, but they disagree about the favourability of the current policies. This is especially common for disputes connected with globalization. It might be possible to explain that another policy would bring only a low outcome, but those who currently benefit from the current situation and those who, for instance, lost their jobs due to globalization would have a totally different opinion about the payoff of the current set of policies. Let us assume that the first group (blue in figure 2.5) benefits highly from the current situation and the second group opposes the current policies, i.e. $R_1 > v_2 > R_2$. Based on the result formulated in Proposition 4, group 1 would on average update their belief up ($\Delta_1 > 0$) and group 2 would on average update down ($\Delta_2 < 0$). This situation is depicted in figure 2.5, showing the posterior expected belief for group 1 (in blue) and group 2 (in red), with accompanying figurative probability density functions. The prior expected belief is the same for both groups (in black). We can observe that posterior expected beliefs for these two groups move further apart from each other, which documents the polarization situation created only by the difference in evaluation of the current policy.

Convergence

In order to draw the whole picture, our framework also has clear predictions for the case when the beliefs of two agents converge and at the same time are closer to the true value. Such a situation occurs when two agents have different prior expectations of the new policy and they also perceive the status quo differently. Moreover, their $\mathbb{E}v$ and R are close to each other. We can label this situation as both agents being conservative, in the sense that they expected the new policy not to differ greatly from the currently implemented policy. This situation is depicted in figure 2.6.

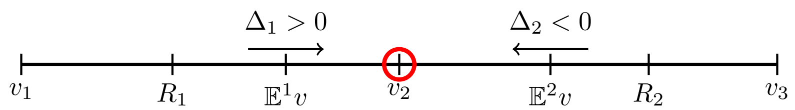


Figure 2.6: Illustration of the situation when the agents' beliefs converge

Non-learning areas

In all our results we assume that the agent is in the learning area. Here we try to shed more light on the non-learning area. We know that the agent is not acquiring any information when $P(i = 1) = 0$ or $P(i = 1) = 1$. Because the decision maker can choose only from two options, she would be in the non-learning area in the two following cases. First, when her perception of the status quo is in close proximity either to the lowest possible payoff v_1 or to the highest possible outcome v_3. Similarly, the second possibility corresponds to the situation when her prior belief is close to v_1 and v_3 and the plot of $P(i = 1)$ for varying $\mathbb{E}v$ as depicted in figure 2.7. This behavior is expectable. For instance, when the agent a priori believes that the new policy is extremely good, while acquiring the information is costly, she would choose not to obtain any information.

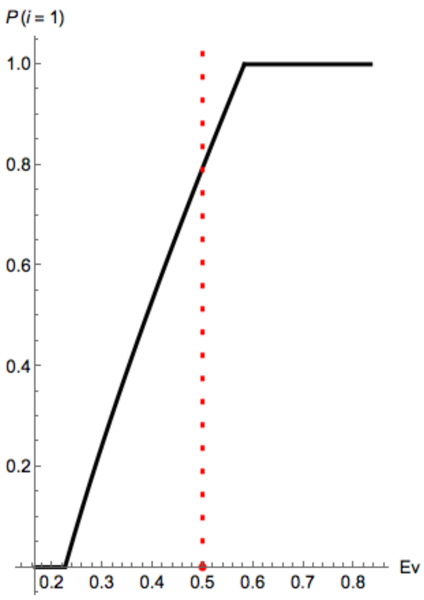


Figure 2.7: $\mathcal{P}(i = 1)$ as function of $\mathbb{E}v$, $\lambda = \frac{1}{4}$, $R = \frac{3}{8}$.

2.5 Comparative statics

Section 2.4 investigated conditions when inattentive agents are becoming polarized, i.e. in what direction they are updating. In this section we explore the magnitude of their updating in more detail. How much does the expected posterior belief about the new policy differ from the prior expected value? What is the role of the cost of information? Does the model predict that the agents become more polarized in a situation with a higher cost of information? Does the actual perception of the status quo have an influence on the value of $\Delta(s^*)$ or does it have an influence only on whether the agent is updating correctly or incorrectly? All these questions are immensely hard to answer analytically for the general case. Therefore, we again take advantage of the example with the three states and two actions. This problem is a simple benchmark that exhibits the basic features of most solutions to the problem.

Problem of the decision maker

The decision maker chooses between two options $i = \{1, 2\} = \{\text{new policy}, \text{status quo}\}$ and can acquire information with marginal cost λ. The payoff of the new policy takes the value v_s with corresponding probability g_s, where $s = 1, 2, 3$. The status quo carries the payoff $R \in (0, 1)$ with certainty. In the following scenarios we use several different values of R and λ. All the parameter values are summarized in table 2.1.

v_1	v_2	v_3	g_1	g_2	g_3	R_1	R_2	λ_1	λ_2	λ_3	λ_4
0	$\frac{1}{2}$	1	$g \in (0, \frac{2}{3})$	$\frac{1}{3}$	$\frac{2}{3} - g$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	1

Table 2.1: Parameters used in this section

Note that keeping g_2 fixed, we can vary g only between $(0, \frac{2}{3})$. Also $\mathbb{E}v$ can vary only from $\frac{1}{6}$ to $\frac{5}{6}$. To solve this problem it is necessary to find the unconditional probabilities $\mathcal{P}(i = 1)$ and $\mathcal{P}(i = 0)$. First, for a given set of parameters, the unconditional probability $\mathcal{P}(i = 1)$ as a function of $\mathbb{E}v$ for different values of λ is shown in figure 2.8.

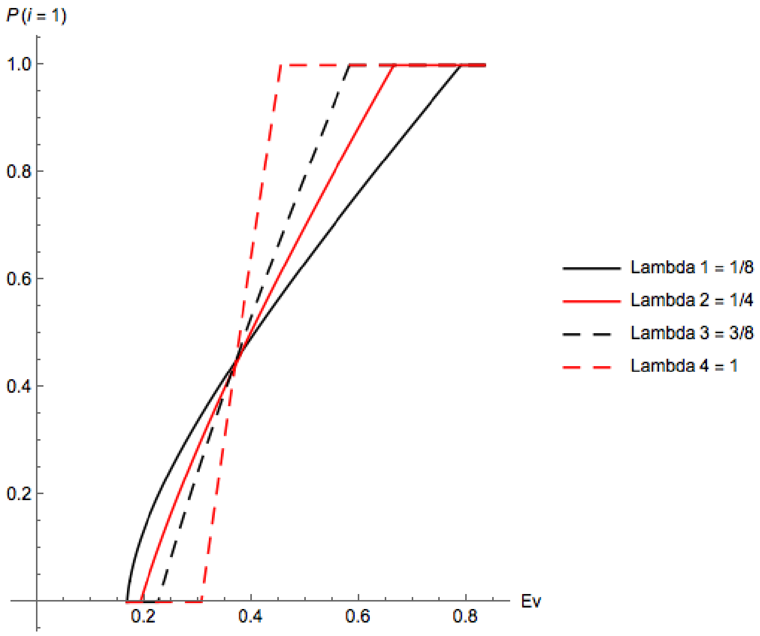


Figure 2.8: $\mathcal{P}(i = 1)$ as function of $\mathbb{E}v$ for different λ, R_1

For $\mathbb{E}v$ close to $\frac{1}{6}$ and $\frac{5}{6}$, the agent does not process any information and chooses with certainty the status quo and the new policy, respectively. With increasing marginal cost of information, the area in which she chooses with certainty grows. In the middle area, the agent acquires information and the unconditional probability of selecting the new policy is an increasing function of the prior expected value from the new policy. With an increase marginal cost of information λ the small changes in $\mathbb{E}v$ translate into bigger changes in the $\mathcal{P}(i = 1)$.

In order to draw a full picture, see figure 2.9 that depicts $\mathbb{E}_i[\mathbb{E}(v|i)|s^*]$ as a function of

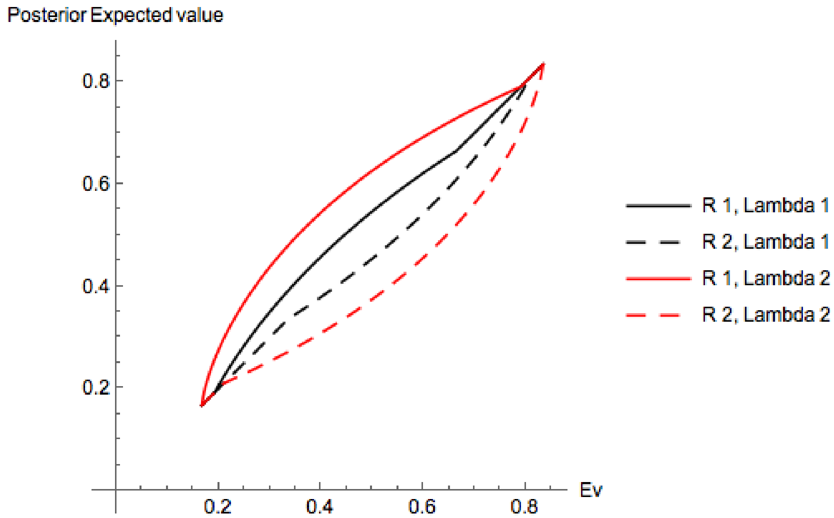


Figure 2.9: $\mathbb{E}_i[\mathbb{E}(v|i)|s^*]$ as a function of $\mathbb{E}v$ for different levels of R and λ. The solid lines are the case with R_1 and dashed with R_2. Black corresponds to cases with λ_1 and red is used for λ_2.

$\mathbb{E}v$ for different levels of R and λ. Similarly to the previous figures, a different R changes the direction of updating and also the convexity of the line. The role of the marginal cost of information is clear from this figure. The cheaper the information $\lambda_2 < \lambda_1$, the further away the prior expected values are from the posterior expected value of the new policy. This is also manifested by the fact that the decision maker is learning even for the prior beliefs, where she was not acquiring information for λ_1. Therefore, in our example, when the cost of information is smaller the polarization of agents is more severe.

Can agents diverge in their opinions but move beliefs in the same direction?
Until this point we have considered only the polarization when two agents are updating in opposite directions. However, in the situation when both agents share the same valuation of the status quo, but one group is very optimistic about the new policy ($\mathbb{E}v$ is high) and the second group is pessimistic about the new policy ($\mathbb{E}v$ is small), they might diverge in opinions, in the sense that their posterior expected values are further away from each other than their prior expected values (see figure 2.10).

This result indicates behavior that resembles the behavior of confirmatory learning agents. Those who a priori prefer the new policy would move their posterior further to the right. But can this result occur?

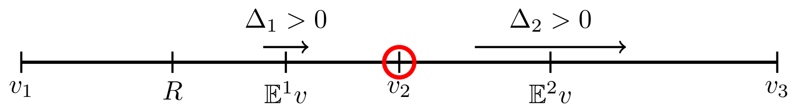


Figure 2.10: Illustration of the situation when the agents who are a priori optimistic about the new policy become more polarized

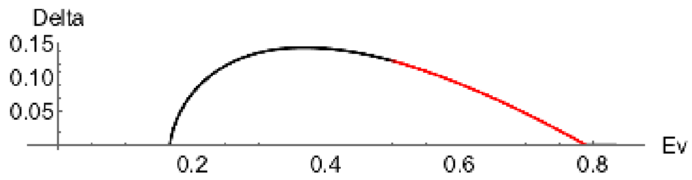


Figure 2.11: $\Delta(s^* = 2)$ as a function of $\mathbb{E}v$ for R_1 and λ_2. The red area depicts the area of updating in the opposite direction from the realized value.

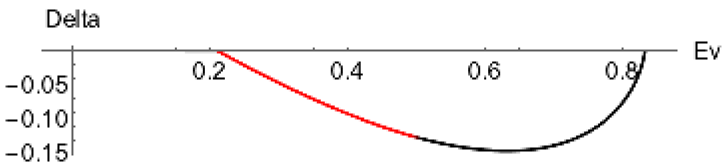


Figure 2.12: $\Delta(s^* = 2)$ as a function of $\mathbb{E}v$ for R_2 and λ_2. The red area depicts the area of updating in the opposite direction from the realized value.

Figures 2.11 and 2.12 show the dependence of $\Delta(s^* = 2)$ on $\mathbb{E}v$ for two values of R, correspondingly. The former figure corresponds to the situation when the agent is updating to the right, towards v_3, and the latter – to updating to the left, towards v_1. In both cases the red area indicates the area where the decision maker is updating away from the true state v_2. We can notice several interesting implications. First, two agents updating in the same direction with the same perception of the status quo might diverge in their opinions only when they are updating correctly. Particularly, their $\mathbb{E}v$ in the case for R_1 has to be inside interval $(\frac{1}{6}, \frac{3}{8})$. As a consequence, two agents that have the same valuation of the status quo and the same marginal cost of information, but one is optimistic and the second is pessimistic about the new policy, cannot diverge in their opinions by updating in the same direction. However, if the two agents have different R or λ this situation might happen.

A second interesting insight is that since the maximal value of $\Delta(s* = 2)$ is achieved for prior beliefs, which are close to the payoff associated with true state 2, it suggests that someone who is updating correctly can move her belief from something which is lower than v_2 to something which is higher than v_2. Moreover, we observe that the more the agent is optimistic about the new policy the less he updates, but that is not surprising in this example due to keeping the g_2 fixed and $\sum_{s=1}^{3} g_s = 1$.

2.6 Conclusion

People's opinions about the proposed policies and pertinent issues often become polarized. The literature provides several explanations of the phenomena: preference for information which confirms existing beliefs, imperfect memory, interpretation of ambiguous evidence as confirming existing beliefs, etc. In this paper we explore a new source of belief polarization, which arises as a consequence of the state-pooling effect if the information is costly to acquire.

We study the evolution of the beliefs of a rationally inattentive agent who chooses between two options, risky (new policy) and safe (status quo), and characterize situations in which the beliefs would be updated on average in the opposite direction from the realized value of the risky option. We find that the key determinant of the direction of belief updating is the perception of the status quo. The position of the status quo determines the information acquisition strategy. In our interpretation, the agent splits the states of the world into categories and learns, to some extent, about these categories, not distinct states. This type of learning might lead to the updating of beliefs in the opposite direction from the realized value. The division into categories is determined exactly by the perception of the status quo. If the two agents have different perceptions of the status quo, they might diverge in their opinions after information acquisition. Interestingly, the difference in their opinions can become greater if the information becomes cheaper to acquire.

Our paper sheds new light on the problem of opinion polarization in society that is taking place currently. It provides a crisp explanation of why polarization can become more severe when information is cheaper to obtain. Extensions of the model for multiple actions could possibly create several endogenous categories and thus provide more insights into the connection with the models of categorical thinking. Another interesting extension could be to add a voting layer on top of the model presented. We also encourage future research testing the implications of our model on actual referendum data.

2.A Proof of Proposition 3

The agent's posterior belief about the payoff of the new policy v given the fixed state s^* for option $i \in \{\text{status quo }, \text{new policy}\} = \{1, 2\}$ is

$$\mathbb{E}_i[\mathbb{E}(v|i)|s^*] = \mathcal{P}(i = 1|s^*)\mathbb{E}(v|i = 1) + \mathcal{P}(i = 2|s^*)\mathbb{E}(v|i = 2).$$

This, after substituting for the conditional probabilities $\mathcal{P}(i|s^*)$ $\forall i$ according to lemma 2 and applying the Bayes rule can be rewritten as

$$\mathbb{E}_i[\mathbb{E}(v|i)|s^*] = \frac{\mathcal{P}(i=1)e^{\frac{v_{s^*}}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_{s^*}}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}} \cdot \sum_{s=1}^{n} v_s g_s \frac{e^{\frac{v_s}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}} +$$
$$+ \frac{(1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_{s^*}}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}} \cdot \sum_{s=1}^{n} v_s g_s \frac{e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}}.$$

Lemma 2 shows that

$$\mathcal{P}(i = 1|s^*) = \frac{\mathcal{P}(i=1)e^{\frac{v_{s^*}}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_{s^*}}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}}.$$

Thus,

$$\mathbb{E}_i[\mathbb{E}(v|i)|s^*] = \sum_{s=1}^{n} v_s g_s \frac{\mathcal{P}(i=1|s^*)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1|s^*))e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}((i=1))e^{\frac{R}{\lambda}}}.$$

2.B Proof of Proposition 4

First we prove the following lemma that we further use for proving Proposition 4.

Lemma 3. *Relations* $\mathcal{P}(i = 1|s^*) \gtreqless P(i = 1)$ *for* $0 < \mathcal{P}(i = 1) < 1$ *are equivalent to* $v_{s^*} \gtreqless R$.

Proof. After substitution for the conditional probabilities, the conditions $\mathcal{P}(i = 1|s^*) \gtreqless P(i = 1)$ can be rewritten as

$$\frac{\mathcal{P}(i=1)e^{\frac{v_{s*}}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_{s*}}{\lambda}}+(1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}} \gtrless \mathcal{P}(i=1),$$

these are equivalent to

$$\left(\mathcal{P}(i=1)-\mathcal{P}^2(i=1)\right)\left(e^{\frac{v_{s*}}{\lambda}}-e^{\frac{R}{\lambda}}\right) \gtrless 0.$$

For $0 < \mathcal{P}(i=1) < 1$ the term in the first parenthesis is always positive. Therefore, the left hand side of the inequality is positive when $v_{s^*} > R$ and negative for $v_{s^*} < R$. □

Now we can continue with the proof of Proposition 4.

Proof. In order to solve the agent's problem given by equations 2.1 - 2.4 we need to find $\mathcal{P}(i=1)$ and $\mathcal{P}(i=2)$ that is defined as $\mathcal{P}(i=2) = 1 - \mathcal{P}(i=1)$. These probabilities have to be internally consistent, i.e. $\mathcal{P}(i) = \sum_{s=1}^{n} \mathcal{P}(i|s)g_s$. After dividing both sides of these conditions by $P(i)$ we obtain the following conditions

$$1 = \sum_{s=1}^{n} \frac{e^{\frac{v_s}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}}+\mathcal{P}(i=2)e^{\frac{R}{\lambda}}} g_s, \quad \text{if } \mathcal{P}(i=1) > 0,$$

$$1 = \sum_{s=1}^{n} \frac{e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}}+\mathcal{P}(i=2)e^{\frac{R}{\lambda}}} g_s, \quad \text{if } \mathcal{P}(i=2) > 0.$$

The difference of these two equations is

$$\sum_{s=1}^{n} \frac{e^{\frac{v_s}{\lambda}}-e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}}+\mathcal{P}(i=2)e^{\frac{R}{\lambda}}} g_s = 0.$$

For k for which holds that $v_k \leq R \leq v_{k+1}$ we can further write the above equation as

$$\frac{e^{\frac{v_k}{\lambda}}-e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_k}{\lambda}}+\mathcal{P}(i=2)e^{\frac{R}{\lambda}}} v_k g_k = -\sum_{s\neq k} \frac{e^{\frac{v_s}{\lambda}}-e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}}+\mathcal{P}(i=2)e^{\frac{R}{\lambda}}} v_k g_s. \tag{2.8}$$

We will use the last equation for determining the sign of $\Delta(s^*)$ that can be written as

$$\Delta(s^*) = \sum_{s=1}^{n} v_s g_s \frac{\mathcal{P}(i=1|s^*)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1|s^*))e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}} - \sum_{i=1}^{n} v_s g_s,$$

$$\Delta(s^*) = \sum_{s=1}^{n} v_s g_s \frac{\mathcal{P}(i=1|s^*)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1|s^*))e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}} - \sum_{i=1}^{n} v_s g_s \frac{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}},$$

$$\Delta(s^*) = \sum_{i=1}^{n} v_s g_s \frac{(\mathcal{P}(i=1|s^*) - \mathcal{P}(i=1))(e^{\frac{v_s}{\lambda}} - e^{\frac{R}{\lambda}})}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}},$$

$$\Delta(s^*) = (\mathcal{P}(i=1|s^*) - \mathcal{P}(i=1)) \cdot \sum_{s=1}^{n} v_s g_s \frac{e^{\frac{v_s}{\lambda}} - e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}}.$$

Substituting the equation (2.8) into the sum in the last equation we obtain

$$\Delta(s^*) = (\mathcal{P}(i=1|s^*) - \mathcal{P}(i=1)) \left[\sum_{s \neq k} (v_s - v_k) g_s \frac{e^{\frac{v_s}{\lambda}} - e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}} \right].$$

The expression in the square brackets is positive, because for the afore defined k the sign of $(v_s - v_k)$ and the sign of $e^{\frac{v_s}{\lambda}} - e^{\frac{R}{\lambda}}$ are the same. Hence $\Delta(s^*)$ has the same sign as $(\mathcal{P}(i=1|s^*) - \mathcal{P}(i=1))$ that further, by lemma 3, has the same sign as $(v_{s^*} - R)$.

□

2.C Proof of Proposition 5

Proof. We are interested in the monotonicity of $\Delta(s^*)$ when the true state of the world s^* is changing. In appendix 2.B we derive that

$$\Delta(s^*) = (\mathcal{P}(i=1|s^*) - \mathcal{P}(i=1)) \left[\sum_{s \neq k} (v_s - v_k) g_s \frac{e^{\frac{v_s}{\lambda}} - e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}} \right].$$

Let us consider two states of the world s_1^* and s_2^*, such that $s_1^* > s_2^*$. Demonstrating that $\Delta(s_1^*) - \Delta(s_2^*) \geq 0$ would prove the monotonicity of $\Delta(s^*)$.

$$\Delta(s_1^*) - \Delta(s_2^*) = \left[\sum_{s\neq k}(v_s - v_k)g_s \frac{e^{\frac{v_s}{\lambda}} - e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}}\right] \cdot$$
$$\cdot \left(\mathcal{P}(i=1|s_1^*) - \mathcal{P}(i=1) - \mathcal{P}(i=1|s_2^*) + \mathcal{P}(i=1)\right) =$$
$$= \left[\sum_{s\neq k}(v_s - v_k)g_s \frac{e^{\frac{v_s}{\lambda}} - e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}}\right] \cdot$$
$$\cdot \left(\mathcal{P}(i=1|s_1^*) - \mathcal{P}(i=1|s_2^*)\right) =$$
$$= \left[\sum_{s\neq k}(v_s - v_k)g_s \frac{e^{\frac{v_s}{\lambda}} - e^{\frac{R}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_s}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}}\right] \cdot$$
$$\cdot \left(\frac{\mathcal{P}(i=1)e^{\frac{v_{s_1^*}}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_{s_1^*}}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}} - \frac{\mathcal{P}(i=1)e^{\frac{v_{s_2^*}}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_{s_2^*}}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}}\right)$$

The term in the square brackets is positive, so the sign of $\Delta(s_1^*) - \Delta(s_2^*)$ is determined by the sign of the term in the round brackets.

Let us show that

$$\frac{\mathcal{P}(i=1)e^{\frac{v_{s_1^*}}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_{s_1^*}}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}} - \frac{\mathcal{P}(i=1)e^{\frac{v_{s_2^*}}{\lambda}}}{\mathcal{P}(i=1)e^{\frac{v_{s_2^*}}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}} > 0.$$

The last inequality is equivalent to

$$\mathcal{P}(i=1)e^{\frac{v_{s_1^*}}{\lambda}}\left(\mathcal{P}(i=1)e^{\frac{v_{s_2^*}}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}\right) -$$
$$- \mathcal{P}(i=1)e^{\frac{v_{s_2^*}}{\lambda}}\left(\mathcal{P}(i=1)e^{\frac{v_{s_1^*}}{\lambda}} + (1-\mathcal{P}(i=1))e^{\frac{R}{\lambda}}\right) > 0,$$

$$(1-\mathcal{P}(i=1))\, e^{\frac{v_{s_1^*}}{\lambda}} e^{\frac{R}{\lambda}} - (1-\mathcal{P}(i=1))\, e^{\frac{v_{s_2^*}}{\lambda}} e^{\frac{R}{\lambda}} > 0,$$

which, in turn, is equivalent to

$$e^{\frac{v_{s_1^*}}{\lambda}} > e^{\frac{v_{s_2^*}}{\lambda}}.$$

The last inequality holds, so $\Delta(s^*)$ is an increasing function.

$\square$

Chapter 3

Attentional Role of Quota Implementation

Co-authored with Sergei Mikhalishchev (CERGE-EI).

3.1 Introduction

Labor market quotas have become a heavily-used governmental policy instrument in recent years. For example, in 2006 all publicly listed companies in Norway were required to increase female representation on their boards of directors to 40 percent. Following Norway's lead, the European Union and other countries worldwide have passed similar reforms (Bertrand et al. 2019). While there is a large body of literature that studies the effect of quota implementation on market outcomes, there is a lack of research that focuses on individual decision-making when an agent is forced to fulfill a quota. This paper introduces a new role of quotas: the attentional role. Although we primarily focus on the effect of quotas in the labor market, the results of our analysis could be applied to studying individual behavior in other areas, e.g. a quota on the number of orders a taxi driver could reject when searching for a client using peer-to-peer ride sharing applications (such as Uber, Lyft, or Yandex).

In this paper we explore the effect of quota implementation on the behavior of a rationally inattentive (RI) agent facing a discrete choice. We follow the setup introduced by Matějka and McKay (2015), in which the agent's choice in the unconstrained problem is characterized by the set of conditional and unconditional choice probabilities. Such an agent has prior beliefs about the values of the available options. The values of the options are modeled as an unknown draw from the known distribution. The agent has an opportunity to receive additional information about the realization of the draw in the manner that is optimal given the costs, which we model using the rational inattention framework introduced by Sims (1998, 2003).

The attention allocation strategy of the RI agent will change if her choice would be constrained by the quotas, which we model as a constraint on unconditional choice probabilities. We analyze the behavior of the RI agent when quotas restrict her choice and compare it with an unrestricted case. We also compare it with the situation in which a social planner subsidizes the agent's choice of a certain alternative. Specifically, we explore the change in the attention strategy and, consequently, changes in choice probabilities, the average quality of chosen options, and social welfare.

We find that the choice probabilities of the agent in the constrained problem have the form of a generalized multinomial logit model as in Matějka and McKay (2015) with an additional state independent bias. In a choice among N options with values v_i for $i \in \{1, ..., N\}$, our modified logit formula implies that the probability of choosing option i is:

$$\mathcal{P}_i(\mathbf{v}) = \frac{q_i e^{(v_i - \varphi_i)/\lambda}}{\sum_{j=1}^{N} q_j e^{(v_j - \varphi_j)/\lambda}},$$

where λ is the marginal cost of information, the q_i terms are quotas, and φ_i is a state independent bias. The interpretation of this bias is as follows: it is the subsidy that the social planner should add to options if he wants to induce the choice of a certain alternative with some probability.

These adjustments to the logit model lead to several changes in the agent's behavior. First, if the choice problem is non-trivial, the RI agent who is forced to fulfill a quota always acquires information about existing options. This feature is absent in the unconstrained problem, in which there are prior beliefs of the agent for which she decides not to acquire any additional information. Second, the overall amount of the acquired information depends on the level of the quota and could be less than in the unconstrained RI problem.

We further investigate what the optimal quota is. We assume that the utilities of the agent and the social planner are partially misaligned. We consider two distinct goals of the social planner. In the first case the social planner maximizes the expected value of the chosen option, e.g. when he does not take into account information costs. In the second case the social planner wants to achieve fairness, i.e., he wants to eliminate the influence of priors on the agent's choice.

In the first case, for some priors the social planner prefers not to impose a quota and the agent still does not acquire any information. In general, the social planner benefits by forcing the agent to fulfill quotas and, consequently, increases the overall quality of the choice.

In the second case the agent's choice would be based on the relative benefits from the choice of the alternative as in the standard logit model. At the same time, when the social planner's goal is to eliminate the influence of the agent's priors, he significantly restricts her behavior, and this leads to a decrease in the overall quality of the choice. We also study the case in which the assumption of the social planner's perfect knowledge is relaxed. We show that in some cases such a social planner should not intervene in the agent's decision.

Our work brings new arguments to the ongoing discussion regarding the benefits and costs of different kinds of affirmative action. Thus, when we discuss the effects of quota implementation, we need to take into account the nature of the agent's behavior and define the main purposes of such a policy intervention. In addition, future empirical work could build upon our results to study the effect of quotas and subsidies on individual behavior.

In the next section we review the related literature. Section 3.3 states the formal model of the agent's behavior with quotas and subsidies. Section 3.4 demonstrates the implications of the model using a specific example. Section 3.5 discusses the optimal level of quotas. Finally, Section 3.6 concludes.

3.2 Literature

Our work contributes to the research on affirmative action and labor market discrimination. Affirmative action is "...any measure, beyond simple termination of a discriminatory practice, adopted to correct or compensate for past or present discrimination or to prevent discrimination from recurring in the future." (US Commission on Civil Rights, 1977 p.2). One of the most hotly debated types of affirmative action is quota implementation. Coate and Loury (1993), in their famous paper, analyze a model of job assignment and show that introduction of quotas may lead to equilibria with persistent discrimination, due to feedback effects between expected job assignments and incentives to invest in human capital. Moro and Norman (2003) study the same problem in the general equilibrium setting and confirm the possibility that quotas could hurt the intended beneficiaries.

These articles examine how affirmative action influences the behavior of the target group and then its interaction with the behavior of the firm. In contrast, our study aims to investigate the individual decision-making process under quotas and its consequences for policy design.

At the same time, a number of empirical studies find an overall positive effect of quota implementation (for example, Ibanez and Riener (2018), Niederle, Segal, and Vesterlund (2013)). Besley et al. (2017), using Swedish data on the performance of politicians, show that a gender quota on the ballot increased the competence of male politicians. Bertrand et al. (2019) document that a law mandating 40% representation of each gender on the board of public limited liability companies in Norway resulted in an overall improvement of labor quality and a decrease in the gender gap in earnings within boards. Our paper proposes a mechanism that could partially explain this evidence; namely, our model demonstrates that implementation of quotas may lead to an increase in labor productivity. A review of early studies on affirmative action can be found in Fang and Moro (2010).

Our study fits into the rational inattention literature, which originated in studies by Sims (1998, 2003). As a benchmark, we use the modified multinomial logit model of Matějka and McKay (2015), in which agents choose among discrete alternatives without precise information about their values, but with an opportunity to study the options for some cost. We solve this model with an additional constraint on the unconditional probabilities of the choice of a certain alternative. Lindbeck and Weibull (2017) analyze investment decisions with delegation to a RI agent. They find that optimal contracts for an agent include a high reward for good investments and punishment for bad investments. We analyze a similar principal-agent problem, but with a different mechanism; we assume that a principal could influence an agent's behavior by defining the level of quotas on unconditional choice probabilities.

Bartoš et al. (2016), in a field experiment, show that HR managers and landlords allocate their attention to job and rental applicants in line with rational inattention theory. For example, a non-European name or recent unemployment induces the HR manager to read a job application and a CV in less detail, consequently affecting the probability of the applicant being invited for a job interview. The results of our study could predict the attention allocation of decision-makers, such as HR managers, in the event of quota implementation, i.e. whether they would blindly choose the quoted option or whether quota implementation would lead to higher information acquisition about the target group. Thus, the results of this study could provide a starting point for the empirical investigation of the effect of quota implementation on attention allocation.

Our study also relates to the discussion about whether directly administering the activity is better than fixing transfer prices and relying on utility maximization to achieve the same results in a decentralized fashion (Weitzman 1974). We contribute to the discussion on this issue by comparing the agent with quotas in the decision-making process and the agent with the choice subsidized by a social planner.

3.3 The Model

In this section we first describe the standard RI problem and its implications as a benchmark model. Then we solve the problem with quotas and discuss its properties. Finally, we analyze the RI problem with subsidies.

We consider a model of discrete choice with costly information acquisition as in Matějka and McKay (2015). The agent faces the menu of N options and wants to select the option with the highest value. The state of the world is a vector $\mathbf{v} \in \mathbb{R}^N$, where v_i is the value of option $i \in \{1, ..., N\}$. Thus, the values of options differ from state to state. The agent has imperfect information about the state of the world, and so is unsure of the payoff from the choice. The agent's prior knowledge is described by a joint distribution $G(\mathbf{v})$. She can refine her knowledge by processing information about the options. This information processing is costly. The conditional probability of option i being selected when the realized values are $\mathbf{v}$ is $\mathcal{P}_i(\mathbf{v})$.

3.3.1 Standard RI problem

The standard RI agent's problem is formalized as follows.

Standard (unconstrained) RI problem. *The agent's problem is to find a vector function of conditional choice probabilities* $\mathcal{P}^U = \{\mathcal{P}_i^U(\mathbf{v})\}_{i=1}^N$ *(the superscript "U" stands for "unrestricted") that maximizes expected payoff less the information cost:*

$$\max_{\{\mathcal{P}_i^U(\mathbf{v})\}_{i=1}^N} \left\{ \sum_{i=1}^N \int_{\mathbf{v}} v_i \mathcal{P}_i^U(\mathbf{v}) G(d\mathbf{v}) - \lambda \kappa(\mathcal{P}^U, G) \right\}$$

subject to

$$\forall i \in \{1, ..., N\}: \quad \mathcal{P}_i^U(\mathbf{v}) \geq 0 \quad \forall \mathbf{v} \in \mathbb{R}^N, \tag{3.1}$$

$$\sum_{i=1}^{N} \mathcal{P}_i^U(\mathbf{v}) = 1 \quad \forall \mathbf{v} \in \mathbb{R}^N, \tag{3.2}$$

where unconditional choice probabilities are

$$\mathcal{P}_i^{0,U} = \int_{\mathbf{v}} \mathcal{P}_i^U(\mathbf{v}) G(d\mathbf{v}), \quad i \in \{1, ..., N\}.$$

The cost of information is $\lambda\kappa(\mathcal{P}^U, G)$, where $\lambda > 0$ is a given unit cost of information and κ is the amount of information that the agent processes, which is measured by the expected reduction in the entropy (Shannon (1948), Cover and Thomas (2012)):

$$\kappa(\mathcal{P}^U, G) = -\sum_{i=1}^{N} \mathcal{P}_i^{0,U} \log \mathcal{P}_i^0 + \sum_{i=1}^{N} \int_{\mathbf{v}} \mathcal{P}_i^U(\mathbf{v}) \log \mathcal{P}_i^U(\mathbf{v}) G(d\mathbf{v}). \tag{3.3}$$

This shape of information costs is common in the literature on rational inattention. Its usage has been justified both axiomatically and through links to optimal coding in information theory (see Sims (2003) and Matějka and McKay (2015) for discussions).

It is shown by Matějka and McKay (2015) that at the optimum the conditional probabilities of choosing option i, $i \in 1, ..., N$ follow the generalized logit form.

Theorem 3 (Matějka and McKay (2015))**.** *Conditional on the realized vector of utilities of options* $\mathbf{v}$, *the choice probabilities satisfy:*

$$\mathcal{P}_i^U(\mathbf{v}) = \frac{\mathcal{P}_i^{0,U} e^{v_i/\lambda}}{\sum_{j=1}^{N} \mathcal{P}_j^{0,U} e^{v_j/\lambda}} \qquad \text{almost surely.}$$

If $\lambda = 0$, *then the agent selects the action(s) with the highest payoff with probability one.*

$\mathcal{P}_i^{0,U}$ is the marginal probability of selecting action i before the agent starts processing any information. The vector of these probabilities reflects the fact that some options might look a priori better than others. $\mathcal{P}_i^{0,U}$ depends on the prior knowledge of the probabilities $G(\mathbf{v})$ and cost of information λ.

The important property of the solution is that there may exist such priors for which the agent decides not to acquire any information and makes her decision purely based on her prior knowledge. In this situation the agent just picks an option with the highest a priori expected value.

3.3.2 Quotas

We consider a departure from the standard RI problem. In the situation which we are considering, the agent is not free to choose options as often as she wants. Instead, some authority limits her choice in that the share of the chosen options from a particular category i should equal $q_i \in (0,1)$. We focus on the case with binding quotas for all alternatives, since when quotas are not binding for any i this condition is redundant and the solution to the maximization problem is the same as in the standard RI. In appendix 3.A we show that results are similar in situation when quotas are binding for one or more alternatives, but not for all of them.

RI problem with quotas. *The agent's problem is to find a vector function of conditional choice probabilities $\mathcal{P} = \{\mathcal{P}_i(\mathbf{v})\}_{i=1}^N$ that maximizes expected payoff less the information cost:*

$$\max_{\{\mathcal{P}_i(\mathbf{v})\}_{i=1}^N} \left\{ \sum_{i=1}^N \int_{\mathbf{v}} v_i \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}) - \lambda\kappa(\mathcal{P}, G) \right\} \tag{3.4}$$

subject to

$$\forall i \in \{1, ..., N\}: \quad \mathcal{P}_i(\mathbf{v}) \geq 0 \quad \forall \mathbf{v} \in \mathbb{R}^N, \tag{3.5}$$

$$\sum_{i=1}^N \mathcal{P}_i(\mathbf{v}) = 1 \quad \forall \mathbf{v} \in \mathbb{R}^N \tag{3.6}$$

and

$$\forall i \in \{1, ..., N\}: \quad \mathcal{P}_i^0 = \int_{\mathbf{v}} \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}) = q_i, \quad q_i > 0, \tag{3.7}$$

where $\mathbf{q} = (q_1, ..., q_N)^T$ is the vector of quotas and

$$\sum_{i=1}^N q_i = 1.$$

The cost of information $\lambda\kappa(\mathcal{P}, G)$ is defined according to

$$\kappa(\mathcal{P}, G) = -\sum_{i=1}^N \mathcal{P}_i^0 \log \mathcal{P}_i^0 + \sum_{i=1}^N \int_{\mathbf{v}} \mathcal{P}_i(\mathbf{v}) \log \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}). \tag{3.8}$$

When $\lambda > 0$, then the Lagrangian of the agent's problem is the following:

$$\sum_{i=1}^N \int_{\mathbf{v}} v_i \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}) - \lambda \left(-\sum_{i=1}^N \mathcal{P}_i^0 \log \mathcal{P}_i^0 + \sum_{i=1}^N \int_{\mathbf{v}} \mathcal{P}_i(\mathbf{v}) \log \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}) \right) +$$

$$+\int_{\mathbf{v}}\xi_i(\mathbf{v}\mathcal{P}_i(\mathbf{v})G(d\mathbf{v}))-\int_{\mathbf{v}}\mu(\mathbf{v})(\sum_{i=1}^{N}\mathcal{P}_i(\mathbf{v})-1)G(d\mathbf{v})-\sum_{i=1}^{N}\varphi_i(\int_{\mathbf{v}}\mathcal{P}_i(\mathbf{v})G(d\mathbf{v})-q_i),$$

where $\mu(\mathbf{v}), \xi_i(\mathbf{v})$ and φ are Lagrange multipliers. The first order condition with respect to $\mathcal{P}_i(\mathbf{v})$ is

$$v_i+\xi_i(\mathbf{v})-\mu(\mathbf{v})+\lambda(\log\mathcal{P}_i^0-\log\mathcal{P}_i(\mathbf{v}))-\varphi_i=0. \tag{3.9}$$

Let us note that for all $i\in\{1,...,N\}, \mathbf{v}\in\mathbb{R}^N$ $\mathcal{P}_i(\mathbf{v})>0$ almost surely. That makes $\xi_i(\mathbf{v})=0, i\in\{1,...,N\}$, $\mathbf{v}\in\mathbb{R}^N$ almost surely. To see this, suppose to the contrary that $\mathcal{P}_i(\mathbf{v})=0$ on a set of positive measure with respect to G. Then $-\log\mathcal{P}_i(\mathbf{v})$ goes to infinity; in order to compensate that in the equation (3.9), one of the Lagrange multipliers should be infinite, which is impossible, because they are finite scalars.

What is left of the first order condition can be rearranged to:

$$\mathcal{P}_i(\mathbf{v})=\mathcal{P}_i^0e^{(v_i-\mu(\mathbf{v})-\varphi_i)/\lambda}. \tag{3.10}$$

Plugging (3.10) into (3.2), we obtain:

$$e^{\mu(\mathbf{v})/\lambda}=\sum_{i=1}^{N}\mathcal{P}_i^0e^{(v_i-\varphi_i)/\lambda},$$

which we again use in (3.10) and find:

$$\mathcal{P}_i(\mathbf{v})=\frac{\mathcal{P}_i^0e^{(v_i-\varphi_i)/\lambda}}{\sum_{j=1}^{N}\mathcal{P}_j^0e^{(v_j-\varphi_j)/\lambda}}.$$

Finally, using (3.7) we obtain:

$$\mathcal{P}_i(\mathbf{v})=\frac{q_ie^{(v_i-\varphi_i)/\lambda}}{\sum_{j=1}^{N}q_je^{(v_j-\varphi_j)/\lambda}}. \tag{3.11}$$

If we denote

$$\alpha_i=\lambda\log q_i,$$

then (3.11) can be written as

$$\mathcal{P}_i(\mathbf{v})=\frac{e^{(v_i+\alpha_i-\varphi_i)/\lambda}}{\sum_{j=1}^{N}e^{(v_j+\alpha_j-\varphi_j)/\lambda}}.$$

This result is formalized in the following proposition:

Proposition 6. *Choice probabilities that are the solution of the RI agent problem with quotas are of generalized logit form: logit choice probabilities with additive state-independent bias.*

Proposition 6 states that the solutions to the standard RI problem and the RI problem with quotas have a similar form. However, there is a crucial difference in the information acquisition strategies of a RI agent with and without quotas. We express it in the following proposition:

Proposition 7. *If (i) the agent's prior is nontrivial, that is, she does not believe that some state of the world happens with certainty, and (ii) the quota does not dictate the agent to take some option with certainty, and (iii) the marginal cost of information λ is finite, and (iv) the matrix function $A(\mathbf{v})$ with elements $a_{ij} = v_i - v_j$ is state-dependent, then the following holds: the RI agent with quotas always acquires information.*

Proof. Let us assume the opposite. If the agent would not acquire information, then $P_i(\mathbf{v}) = q_i$ for all $i \in 1, ..., N$. Or

$$\frac{q_i e^{\frac{v_i - \varphi_i}{\lambda}}}{\sum_{j=1}^{N} q_j e^{\frac{v_j - \varphi_j}{\lambda}}} = q_i.$$

We use the assumption that $q_i \neq 0$ and divide both parts of the equation above by q_i.

$$\frac{e^{\frac{v_i - \varphi_i}{\lambda}}}{\sum_{j=1}^{N} q_j e^{\frac{v_j - \varphi_j}{\lambda}}} = 1.$$

The same holds for all other options. That means that

$$v_i - \varphi_i = v_k - \varphi_k,$$

or

$$v_i - v_k = \varphi_i - \varphi_k.$$

The last equation cannot hold for all realizations of v. That is so since the LHS of the above equation is state-dependent, while the RHS is state-independent, which contradicts assumption (iv). □

Proposition 7 tells us that in the context of the labor market, an introduction of quotas would not lead to a situation in which HR managers are hiring certain categories of workers without information acquisition.

3.3.3 Subsidies

There exists a practice when a firm receives subsidies if it employs certain categories of workers (for surveys see, e.g., Card, Kluve, and Weber (2010, 2017)). We are interested in understanding how the agent's attention strategy depends on the particular form of affirmative action that was chosen by the government. We show that the agent's behavior in both situations (under quotas and with subsidies) is the same.

If the government introduces a certain level of subsidies, S_i, the agent solves the following problem:

RI problem with subsidies. *The agent's problem is to find a vector function of conditional choice probabilities* $\mathcal{P}^S = \{\mathcal{P}_i^S(\mathbf{v})\}_{i=1}^N$ *(the superscript "S" stands for "subsidy") that maximizes expected payoff less the information cost:*

$$\max_{\{\mathcal{P}_i^S(\mathbf{v})\}_{i=1}^N} \left\{ \sum_{i=1}^N \int_{\mathbf{v}} (v_i + S_i) P_i^S(\mathbf{v}) G(d\boldsymbol{v}) - \lambda\kappa(\mathcal{P}^S, G) \right\},$$

subject to

$$\forall i \in \{1, ..., N\}: \quad \mathcal{P}_i^S(\mathbf{v}) \geq 0 \quad \forall \mathbf{v} \in \mathbb{R}^N, \tag{3.12}$$

$$\sum_{i=1}^N \mathcal{P}_i^S(\mathbf{v}) = 1 \quad \forall \mathbf{v} \in \mathbb{R}^N, \tag{3.13}$$

where S_i *is a subsidy for choosing option* i. *The cost of information* $\lambda\kappa(\mathcal{P}^S, G)$ *is defined according to*

$$\kappa(\mathcal{P}^S, G) = -\sum_{i=1}^N \mathcal{P}_i^{0,S} \log \mathcal{P}_i^0 + \sum_{i=1}^N \int_{\mathbf{v}} \mathcal{P}_i^S(\mathbf{v}) \log \mathcal{P}_i^S(\mathbf{v}) G(d\mathbf{v}). \tag{3.14}$$

In this case the solution to the agent's problem follows the standard modified generalized multinomial logit formula, but with the changed value of the option i by S_i:

$$\mathcal{P}_i^S(\mathbf{v}) = \frac{\mathcal{P}_i^{0,S} e^{(v_i+S_i)/\lambda}}{\sum_{j=1}^N \mathcal{P}_j^{0,S} e^{(v_j+S_j)/\lambda}} \tag{3.15}$$

In order to compare the agent's behavior under both policies we need to set up subsidies on the level where quotas are fulfilled:

$$\int_{\mathbf{v}} \mathcal{P}_i^S(\mathbf{v})G(d\mathbf{v}) = q_i.$$

We refer to subsidy S_i as *optimal* when unconditional probabilities of the agent's choice are equal to quotas, $\mathcal{P}_i^{0,S} = q_i$. It is important to note that the vector of optimal subsidies is not unique. We formalize it in the following Lemma:

Lemma 4. *If S^* is a vector of optimal subsidies, then any vector which is obtained by adding any number to all components of S^* is also a vector of optimal quotas.*

Proof. Assume that the optimal vector of quotas is $\mathbf{S} = \{S_1, ..., S_N\}$. Let's add any α to all subsidies and denote this vector of quotas $\mathbf{S}^* = \{S_1^*, ..., S_N^*\}$. Then we can rewrite equation (3.15) as follows:

$$\mathcal{P}_i^{S^*}(\mathbf{v}) = \frac{\mathcal{P}_i^{0,S^*} e^{(v_i+S_i^*)/\lambda}}{\sum_{j=1}^N \mathcal{P}_j^{0,S^*} e^{(v_j+S_j^*)/\lambda}} = \frac{\mathcal{P}_i^{0,S^*} e^{(v_i+S_i+\alpha)/\lambda}}{\sum_{j=1}^N \mathcal{P}_j^{0,S^*} e^{(v_j+S_j+\alpha)/\lambda}} = \frac{e^{\alpha}\mathcal{P}_i^{0,S^*} e^{(v_i+S_i)/\lambda}}{e^{\alpha}\sum_{j=1}^N \mathcal{P}_j^{0,S^*} e^{(v_j+S_j)/\lambda}}$$

Therefore we can reduce e^{α} and since $\mathcal{P}_i^{0,S} = \int_{\mathbf{v}} \mathcal{P}_i^S(\mathbf{v})G(d\mathbf{v})$ we obtain equation (3.15). □

Equations (3.15) and (3.11) provide us with intuition about the nature of the additive bias φ_i, $i \in \{1, ..., N\}$ from the solution to the RI problem with quotas. This bias can be interpreted as the subsidies from the government that are needed to be added to the values of the options in order to make the RI agent choose them with required unconditional probabilities. Therefore, the behavior of the RI agent with quotas and with optimal subsidies is the same. This result is formalized in the following proposition[1]:

Proposition 8. *The information acquisition strategy and conditional choice probabilities of the RI agent when her choice is restricted by quotas are identical to the situation when her choice is supported by optimal subsidies.*

[1]In Appendix 3.D we solve a binary example with subsidies. We show that while the behavior of the agent under quotas and subsidies is the same, the utility of the agent is different for two cases, which is a reason why quotas could be more appropriate.

3.4 Binary example with risky and safe options

In order to illustrate the logic of the model let us consider a simple example where the agent chooses between risky and safe alternatives. The safe option always takes the value $v_1 = R$. The risky option can take values $v_2 = 0$ with the probability b and $v_2 = 1$ with the probability $1 - b$. These probabilities are the priors of the agent and she does not know what the realization of the state of the world is. The agent has an opportunity to acquire some costly information about the realization. If the realized value of the chosen risky option is $v_2 = 0$ we refer to such an alternative as bad; and if the realized value of the chosen risky option is $v_2 = 1$ we refer to it as good.

Let us add a small remark which shows that any RI decision problem with two alternatives can be formulated as a binary choice problem between risky and safe options. The idea is that we do not change the behavior of the agent by substracting a constant from the maximization problem. It means that we can shift the values of the options in such a way that the maximization problem would be equivalent to the initial one, but one of the options would be safe in the new decision problem. Indeed, if the initial maximization problem is

$$\max\{\int_{\mathbf{v}} \sum_{j=1}^{2} (v_j \mathcal{P}_j(\mathbf{v}))\, G(d\mathbf{v}) - \lambda\kappa\}.$$

Then we can extract a number from it and the optimal choice probabilities will not change. Let us extract $\int_{\mathbf{v}} \left(\sum_{j=1}^{N} v_i\right) G(d\mathbf{v})$. Then the initial maximization problem is equivalent to

$$\max\{\int_{\mathbf{v}} \sum_{j=1}^{2} ((v_j - v_i)\mathcal{P}_j(\mathbf{v}))\, G(d\mathbf{v}) - \lambda\kappa\}.$$

The last problem is a problem of choice between 2 alternatives, one of which has a constant value, and thus can be perceived as a safe option. In the context of the labor market this means that in order to study the consequences of quotas' implementation, we can consider the choice problem in which one option is safe (the productivity of workers in one group is constant). This binary example can also serve as an illustration of the financial market situation, when the agent chooses between a safe asset and a risky asset while facing restriction from the regulator on the ratio of riskiness of her portfolio.

Let us get back to the example. The agent's choice is restricted in that on average the share q of chosen options should be risky and share $1 - q$ of chosen options should be safe. In terms of rational inattention the agent has restriction on unconditional choice probabilities. In Appendix 3.C we consider a case with two risky options and demonstrate that the nature of the results is similar.

To solve the problem we must find conditional probabilities $\mathcal{P}_i(\mathbf{v})$. We show in Appendix 3.B that the solution is:

$$\mathcal{P}_1(0) = \frac{-b - q + (b+q-1)e^{\frac{1}{\lambda}} + \sqrt{(b+q-(b+q-1)e^{\frac{1}{\lambda}})^2 + 4q(be^{\frac{1}{\lambda}} - b)}}{2(be^{\frac{1}{\lambda}} - b)},$$

$$\mathcal{P}_1(1) = \frac{q - b\mathcal{P}_1(0)}{1-b}.$$

For a given set of parameters, Figure 3.1 shows reduction in entropy as a function of b. In the standard RI problem, when b is close to 0 or 1 the agent decides not to process information and selects one of the options with certainty. However, when the agent is forced to fulfill quotas she always acquires information and, hence, there are no non-learning areas. For example, when b is close to 1, she is forced to choose a risky alternative with positive probability, and it is profitable to acquire information in order to choose the good risky option rather than to make a random choice of a risky alternative.

At the same time, under quotas the agent could prefer to acquire less information than in the standard RI problem (Figure 3.1). Accordingly, the effect of the quota implementation on the amount of acquired information is ambiguous.

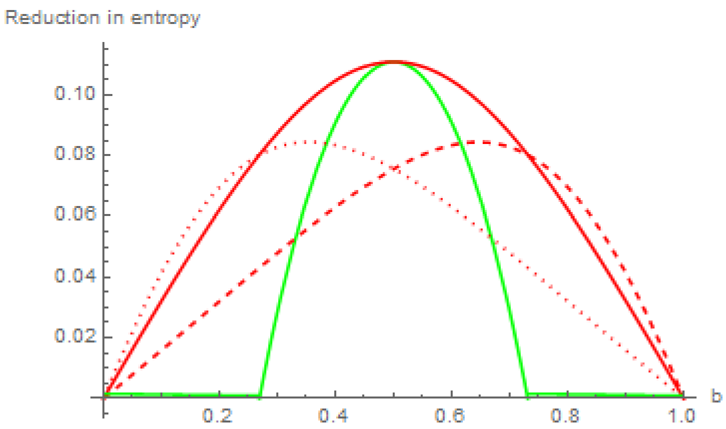


Figure 3.1: Reduction in entropy as a function of b and $\lambda = 0.5$, $R = 0.5$. The green line is for the standard RI problem and the red lines are for the quoted RI problem: the solid line is for $q = 0.5$, the dotted line for $q = 0.75$ and the dashed line for $q = 0.25$.

We now explore how the quota implementation affects the quality of the chosen options. In terms of the labor market this question can be restated in the following way:

does quota implementation necessarily mean that the quality (or productivity) of the hired workers will fall? The definition of the quality of the chosen risky option can be found below. The quality of the safe option is always R.

Definition 3. The quality of the chosen risky option is $\frac{(1-b)\mathcal{P}_1(1)}{\mathcal{P}_1^0}$. This is the ratio of the probability of the chosen risky option being good to the probability of choosing any risky option.

Figure 3.2 illustrates the quality of the chosen risky option. Accordingly, the quality of the chosen risky option is higher (lower) when the quota on it is smaller (larger) than the unconditional probability of choosing it in the standard RI problem.

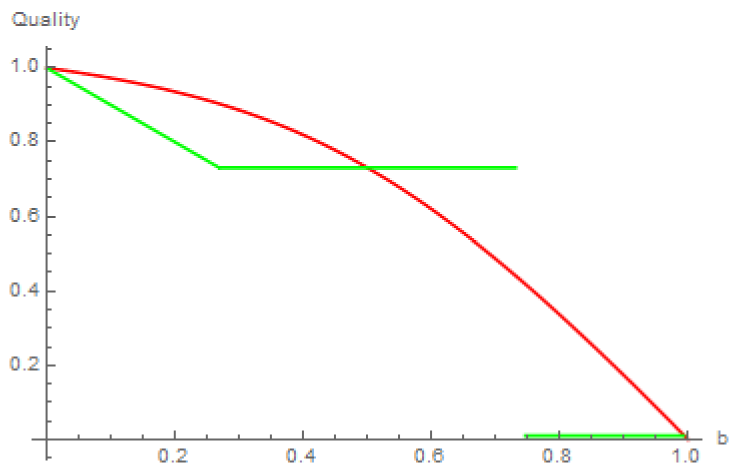


Figure 3.2: Quality of the risky option conditional on being chosen as a function of b and $\lambda = 0.5$, $q = 0.5$, $R = 0.5$. The green line is for the standard RI problem and the red line is for the quoted RI problem.

So far we have considered only the quality of the chosen risky option. In the next section we discuss how quotas can increase the overall quality of the choice and minimize the statistical discrimination.

3.5 Optimal action of the social planner

In this section we discus what an optimal level of quotas is. So far we have solved the agent's problem for a given level of quota. The social planner may have exogenous reasons why he wants to establish a certain level of quotas rather than the quality of the alternatives. For example, quotas might be used in order to compensate for under-representation of certain categories of workers that could be important and beneficial in the long run. For instance, there is a large literature that demonstrates how diversity brings a boost to profitability (Hunt, Layton, and Prince 2014).

We analyze the optimal level of quotas for two different goals of the social planner. The first possible approach the social planner could take to define an optimal quota is to maximize a function that is similar to the agent's utility, but which also accounts for productivity externalities.

The second approach is to minimize a bias towards the options that are good a priori. Such bias could lead to the situation when the agent does not choose the option with a high realized value if the prior of it being good is low. In terms of the labor market it would mean that it could be that two workers with the same productivity, but from different social groups, would have different probabilities of being hired. Accordingly, the social planner could be willing to minimize this effect.

In Appendix 3.E we discuss the case in which the social planner faces a tradeoff between the two objectives.

In this section, we define the social planner's problem for two cases and then discuss the agent's behavior under induced quotas using the binary example from Section 3.4.

3.5.1 Externalities

If we think of our model as a labor market model, then it is natural to assume that production externalities take place. However, while the agent is making hiring decisions she might not take these externalities into account. Thus, the maximization problem of the agent and the social planner (organization, industry as a whole, or government) might differ. In such a case it might be beneficial for the social planner to implement quotas.

The social planner takes into account production externalities. That is, for the social planner the values of the options are multiplied by a number $\alpha > 1$.

$$\max_q \left\{ \alpha \sum_{i=1}^{N} \int_{\mathbf{v}} v_i \mathcal{P}_i(\mathbf{v}, q) G(d\mathbf{v}) - \lambda \kappa(q, G) \right\},$$

where $\mathcal{P}_i(\mathbf{v}, q)$ is a solution to the RI problem with quotas and $q_i = \int_v \mathcal{P}_i(\mathbf{v}, q) G(d\mathbf{v})$.

If we denote $\beta = \frac{1}{\alpha}$, then, since $\alpha > 1$ and $0 < \beta < 1$, the maximization problem is equivalent to

$$\max_q \left\{ \sum_{i=1}^{N} \int_{\mathbf{v}} v_i \mathcal{P}_i(\mathbf{v}, q) G(d\mathbf{v}) - \beta \lambda \kappa(q, G) \right\}.$$

For simplicity of exposition we consider the case $\beta \to 0$. In this case the maximization problem is

$$\max_q \left\{ \sum_{i=1}^N \int_{\mathbf{v}} v_i \mathcal{P}_i(\mathbf{v}, q) G(d\mathbf{v}) \right\}.$$

This is the case in which the social planner is interested in maximizing the expected value of the chosen option and does not take into account the cost of information. We refer to the solution of this problem as a *Quality maximizing quota*.

3.5.2 Influence of priors

Another possible goal of quota implementation is to decrease the influence of priors. The social planner could be willing to minimize the influence of priors on the agent's strategy of acquiring information and final choice. In the standard RI model the unconditional probabilities are defined before the agent starts processing any information (Matějka and McKay 2015). Therefore, if the information is relatively expensive the agent could disregard some alternatives without any information acquisition. As a result, the probability of an alternative being chosen could be lower than its relative utility. An example of a policy that also aims to reduce the influence of prior knowledge on the agent's decision is blind resume practices. We show how the social planner can accomplish the same results by the means of quota implementation.

The solution to the standard RI maximization problem is $\mathcal{P}_i^U(\mathbf{v}) = \frac{\mathcal{P}_i^{0,U} e^{v_i/\lambda}}{\sum_{j=1}^N P_j^{0,U} e^{v_j/\lambda}}$, where $\mathcal{P}_i^{0,U}$ corresponds to the effect of priors. The social planner wants the agent to choose options as if $\mathcal{P}_i^{0,U} = \mathcal{P}^{0,U} = \frac{1}{N}\ \forall i \in \{1, ..., N\}$. Consequently, the agent makes her choice according to the standard multinomial logit formula: $\mathcal{P}_i^U(\mathbf{v}) = \frac{e^{v_i/\lambda}}{\sum_{j=1}^N e^{v_j/\lambda}}$.

In terms of our model, we could find a quota that makes conditional probabilities independent from the prior bias by solving the following equality:

$$\frac{q_i e^{(v_i - \varphi_i)/\lambda}}{\sum_{j=1}^2 q_j e^{(v_j - \varphi_j)/\lambda}} \Big/ \frac{q_j e^{(v_j - \varphi_j)/\lambda}}{\sum_{j=1}^2 q_j e^{(v_j - \varphi_j)/\lambda}} = \frac{e^{v_i/\lambda}}{\sum_{j=1}^2 e^{v_j/\lambda}} \Big/ \frac{e^{v_j/\lambda}}{\sum_{j=1}^2 e^{v_j/\lambda}},$$

or, after rearranging,

$$e^{\log q_i - \log q_j + (v_i - v_j - \varphi_i + \varphi_j)/\lambda} = e^{(v_i - v_j)/\lambda},$$

which is equal to

$$\lambda \log \frac{q_i}{q_j} - \varphi_i + \varphi_j = 0.$$

In the case when there are two options $q_2 = 1 - q$. From the Section 3.3.3 we know that there exist such subsidies that φ_i is equal to S_i. Therefore, using Lemma 4 we could set $\varphi_2 = 0$ and find φ_1 for any q. We end up with the following equation:

$$\lambda \log \frac{q}{1-q} - \varphi_1 = 0.$$

In Appendix 3.D we solve the binary example with subsidies. From there we can set $\mathcal{P}_1^0 = q$ and express $S_1 = \varphi_1$ as a function of q:

$$\varphi_1 = \lambda \log[\frac{-e^{R/\lambda}(e^{1/\lambda}(1-b-q)+b-q)+\sqrt{4e^{1/\lambda+2R/\lambda}(1-q)q+e^{2R/\lambda}(e^{1/\lambda}(1-b-q)+b-q)^2)}}{e^{1/\lambda}2q}].$$

Then we plug it into the expression above and calculate the optimal quota q as a function of b. We refer to the solution of this problem as a *Fair quota*.

3.5.3 Results

In this subsection we illustrate the consequences of optimal implementation of quotas by solving the example from Section 3.4. We depict the optimal level of quotas dependent on b for two cases: when the social planner maximizes the expected value of the chosen alternatives and when he minimizes the effect of priors. We also depict the unrestricted unconditional choice probabilities. Figure 3.3 illustrates the solution to the social planner's problem as a function of b, and $\lambda = 0.5$, $R = 0.5$. When the social planner maximizes the expected value of chosen alternatives, there are still non-learning areas, but they are smaller than in the standard RI problem. The reason for the presence of the non-learning areas is as follows. Let us consider the situation when b is small, that is the probability of the risky option being good is high. When the non-trivial quota is implemented, the agent will acquire some information in order to find out whether the risky option is good or bad, but the improvement in the quality of chosen risky options would not compensate for the loss that comes from an abundance of good risky options. Therefore, the social planner prefers not to constrain the agent, or, in other words, he prefers to implement the quota that would force the agent to always pick a risky option – the same action that the agent would take without any constraints. Similar logic applies for the situation when the probability of the good state is low.

Outside these non-learning areas the social planner could increase the overall quality of the chosen options by setting a quota (Figure 3.4). Thus, in this example, it is optimal

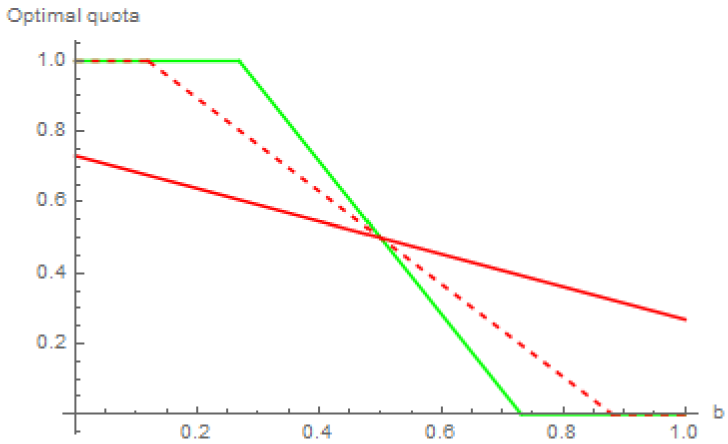


Figure 3.3: Optimal quota as a function of b and $\lambda = 0.5$, $R = 0.5$ The red lines are for the quoted RI problem: the solid line is for the fair quota and the dashed line for the quality maximizing quota. The green line shows the unconditional probability of choosing a risky option for the standard RI problem.

to establish a quota that is higher (lower) than the unconditional probability in the standard RI problem when the state is more likely to be bad (good).

When the social planner minimizes the influence of priors, it is optimal to establish a quota that is higher (lower) than the unconditional probability in the standard RI problem, as well as quotas in the former situation when the state is likely to be bad (good). However, in this situation the social planner restricts the agent's behavior more. Thus, if we maximize the overall quality of the chosen alternatives we end up in a situation where most of the good candidates are hired, while some bad alternatives were also chosen. However, when we eliminate the effect of priors the agent is forced to select less good alternatives than exist; hence, the overall quality of the chosen option is lower (Figure 3.4).

At the same time, if the social planner doesn't take into account the agent's information costs the overall information acquisition is higher than in the standard RI problem, but lower in comparison to the situation when he eliminates the effect of priors (Figure 3.6). Accordingly, in the latter situation the agent pays more attention and, hence, the quality of the risky option is increased (decreased) more when the probability of the good state is high (low) according to Proposition 4 (Figure 3.5).

It is important to notice that while the optimal quota here is state dependent, the information acquisition and the quality of the chosen option are the same as in the situation with a constant quota, discussed in Section 3.4. At the same time, the utility of the agent in the former situation is going to be higher, as well as overall welfare: the

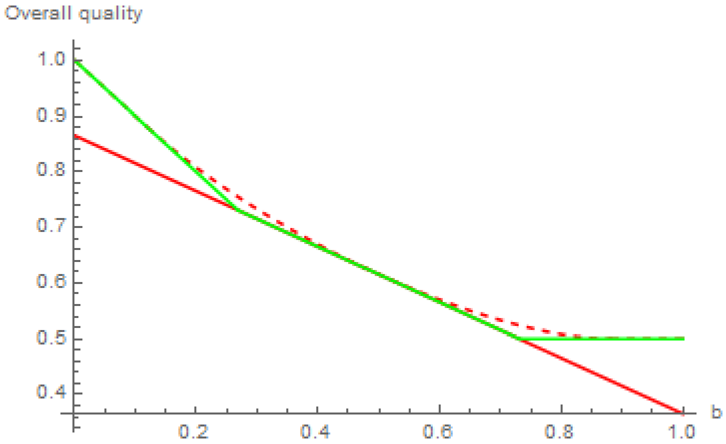


Figure 3.4: Overall quality of chosen alternatives as a function of b and $\lambda = 0.5$, $R = 0.5$. The green line is for the standard RI problem and the red lines are for the quoted RI problem: the solid line is for the fair quota and the dashed line for the quality maximizing quota.

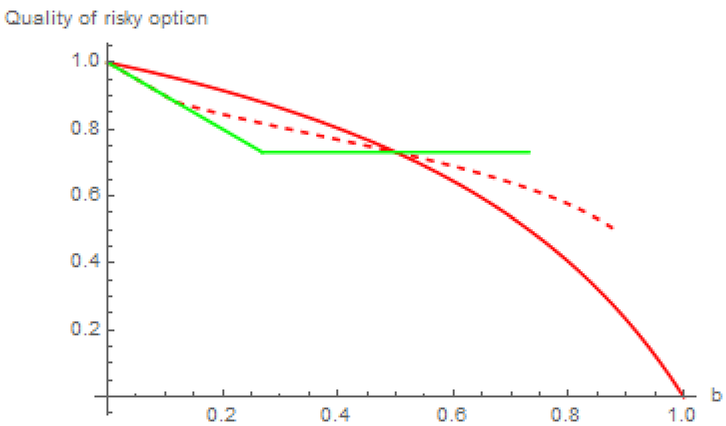


Figure 3.5: Quality of risky option conditional on being chosen as a function of b and $\lambda = 0.5$, $R = 0.5$. The green line is for the standard RI problem and the red lines are for the quoted RI problem: the solid line is for the fair quota and the dashed line for the quality maximizing quota.

agent chooses more good options and fewer bad options.

Figure 3.7 shows that if a risky option is likely to be bad, the social planner, who does not take into account the agent's information costs, establishes a quota that is lower than the probability of the risky option being good. If the risky option is likely to be good, the social planner establishes a quota that is higher than the probability of the risky option being good. The point at which the optimal quota schedule crosses the probability that the risky option is good depends on the parameters. At the same time, if the social planner's goal is to eliminate the effect of priors, the opposite is true: the quota is higher (lower) than the probability of the risky option being good if the risky option is likely to be bad (good).

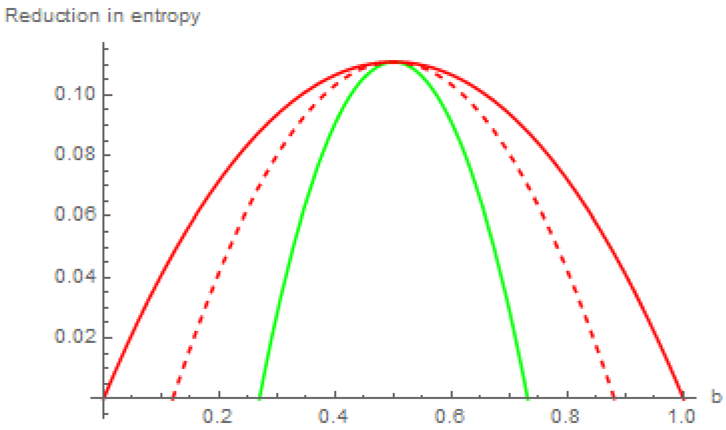


Figure 3.6: Reduction in entropy as a function of b and $\lambda = 0.5$, $R = 0.5$. The green line is for the standard RI problem and the red lines are for the quoted RI problem: the solid line is for the fair quota and the dashed line for the quality maximizing quota.

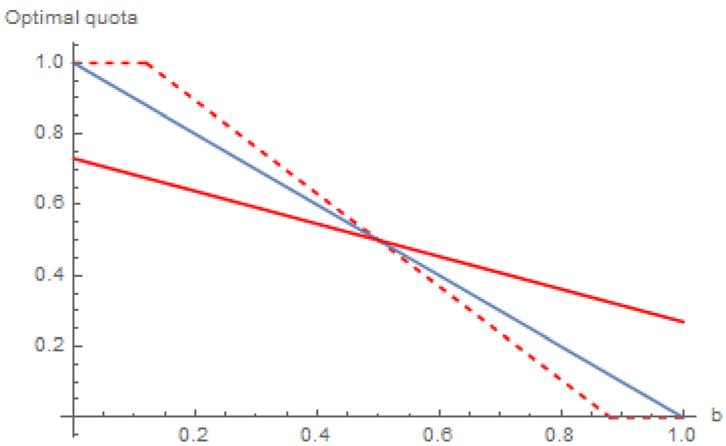


Figure 3.7: Optimal quota as a function of b and $\lambda = 0.5$, $R = 0.4$. The red lines are for the quoted RI problem: the solid line is for the fair quota, the dashed line for the quality maximizing quota). The blue line is the probability that the risky option is good.

3.5.4 Imperfect social planner

We have shown in the previous section that the social planner can increase the overall quality of choice by setting up some quota. The result was obtained under the assumption that the social planner has all relevant information for setting up the quota. In this section we demonstrate that if the social planner does not have perfect information about the parameters of the model, then setting up the quotas might hurt the overall quality.

We use the example from Section 3.4, where the agent chooses from safe and risky alternatives. However now we consider two situations: when the social planner does not know the actual value of the safe option, or when he does not know the distribution of good and bad risky options in the pool of the candidates.

Unknown R

Let us assume that the social planner does not know R. He only knows that it is somewhere between 0 and the minimal threshold for the agent not to acquire information, $\underline{R}$. The social planner knows b and λ; thus, he can compute $\underline{R}$. The belief of the social planner about the value of R is uniform on $[0, \underline{R}]$.

Let us find $\underline{R}$. The optimal unconditional probability of choosing risky option, $\mathcal{P}_1^{0,U}$, in the standard RI problem is

$$\mathcal{P}_1^{0,U} \in \left\{0, 1, -\frac{e^{\frac{R}{\lambda}}\left(-e^{\frac{1}{\lambda}} + e^{\frac{R}{\lambda}} - b + be^{\frac{1}{\lambda}}\right)}{\left(e^{\frac{1}{\lambda}} - e^{\frac{R}{\lambda}}\right)\left(-1 + e^{\frac{R}{\lambda}}\right)}\right\}.$$

In order to find the threshold $\underline{R}$ we solve the following equation:

$$-\frac{e^{\frac{R}{\lambda}}\left(-e^{\frac{1}{\lambda}} + e^{\frac{R}{\lambda}} - b + be^{\frac{1}{\lambda}}\right)}{\left(e^{\frac{1}{\lambda}} - e^{\frac{R}{\lambda}}\right)\left(-1 + e^{\frac{R}{\lambda}}\right)} = 1.$$

This can be rearranged to:

$$\underline{R} = \lambda \ln\left(\frac{e^{\frac{1}{\lambda}}}{1 - b + be^{\frac{1}{\lambda}}}\right).$$

The expected overall quality of the chosen alternatives from the perspective of the social planner is

$$\int_0^{\underline{R}} \left(b\Big((1-\mathcal{P}_1(0)) \cdot R \Big) + (1-b)\Big(\mathcal{P}_1(1) + R(1-\mathcal{P}_1(1)) \Big) \right) \frac{1}{\underline{R}} dR = U(b, q, \underline{R}),$$

where $\mathcal{P}_1(0)$ and $\mathcal{P}_1(1)$ are solutions for the quoted RI problem from Section 3.4.

Figure 3.8 illustrates the overall quality of choice for different q and for the case when there is no quotas. The expected quality of the chosen option is not higher for any quota than in the case with no quotas.

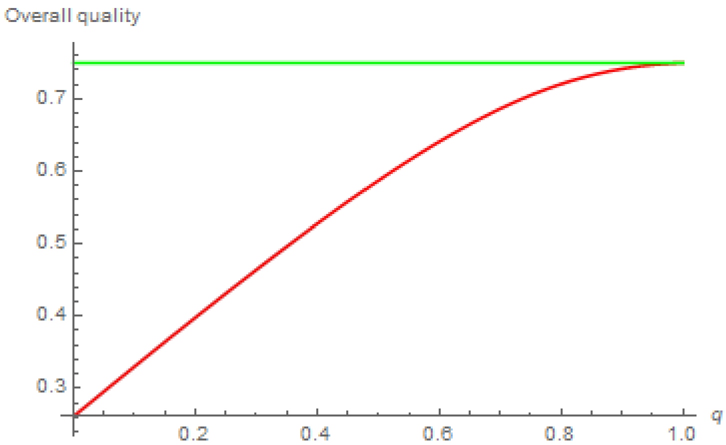


Figure 3.8: Overall quality of chosen alternatives as a function of q and $\lambda = 0.5$, $b = 0.25$. The green line is for the standard RI problem and the red line is for the quoted RI problem.

Unknown b

Let us assume that the social planner does not know b. He only knows that it is somewhere between 1 and the minimal threshold for the agent not to acquire information, $\bar{b}$. The belief of the social planner is uniform on $[\bar{b}, 1]$.

Following the same procedure as in the previous subsection we find that:

$$\bar{b} = \frac{e^{\frac{1}{\lambda}} - e^{\frac{R}{\lambda}}}{e^{\frac{1}{\lambda}} - 1}$$

The expected overall quality of the chosen options from the perspective of the social planner is

$$\int_{\bar{b}}^{1} \left(g(1-\mathcal{P}_1(0,b))R + (1-b)\Big(\mathcal{P}_1(1,b) + R(1-\mathcal{P}_1(1,b)) \Big) \right) \frac{1}{1-\bar{b}} db = V(\bar{b}, q, R).$$

Figure 3.9 illustrates the overall quality of choice for different q and for the case when there is no quotas. The expected quality of the chosen option is not higher for any quota than in the case with no quotas.

This means that when the social planner does not perfectly know the properties of the choice set any quotas could reduce not just the utility of the agent but also the overall quality of the choice.

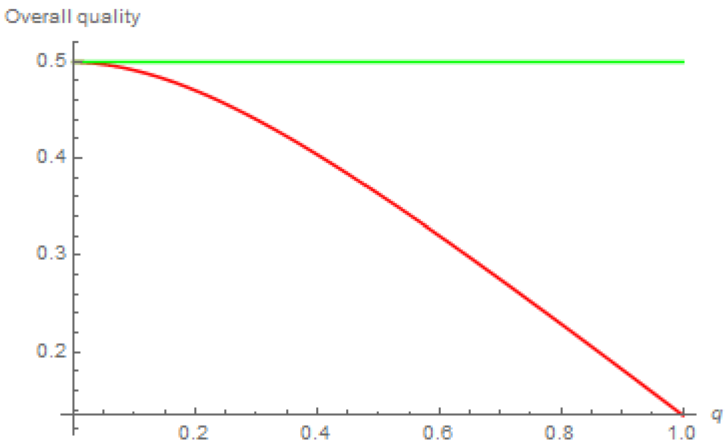


Figure 3.9: Overall quality of chosen alternatives as a function of q and $\lambda = 0.5$, $R = 0.5$. The green line is for the standard RI problem and the red line is for the quoted RI problem.

3.6 Conclusion

In this paper we study the optimal behavior of a RI agent who is forced to fulfill quotas when making a choice from a discrete menu. In this situation, the agent always acquires information. We show that a social planner using quotas could force an agent to make a better choice and reduce the attentional discrimination, which can take place because of costly attention. At the same time, it is important to note that quotas restrict the agent, and the effect of quota implementation could be negative.

This study contributes to the discussion concerning how affirmative action can influence individual economic behavior. In addition, the results of the model provide a testable prediction that could be exploited in future empirical work in order to test the rational inattention theory.

3.A

Details of the solution for the model with non-binding quotas

Let assume that we have N alternatives and there is only one restriction on unconditional probabilities: $\mathcal{P}_1^0 = q$. Accordingly, this constraint implies that $\sum_{j=2}^{N} \mathcal{P}_j^0 = 1 - q$. Therefore, when $\lambda > 0$, then the Lagrangian of the agent's problem described in Section 3.3.2 is as follows:

$$\sum_{i=1}^{N} \int_{\mathbf{v}} v_i \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}) - \lambda(-\sum_{i=1}^{N} \mathcal{P}_i^0 \log \mathcal{P}_i^0 + \sum_{i=1}^{N} \int_{\mathbf{v}} \mathcal{P}_i(\mathbf{v}) \log \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}))$$
$$- \int_{\mathbf{v}} \mu(\mathbf{v})(\sum_{i=1}^{N} \mathcal{P}_i(\mathbf{v}) - 1) G(d\mathbf{v}) - \varphi_1(\int_{\mathbf{v}} \mathcal{P}_1(\mathbf{v}) G(d\mathbf{v}) - q) - \varphi_2(\sum_{j=2}^{N} \int_{\mathbf{v}} \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}) - 1 + q),$$

where $\mu(\mathbf{v})$ and $\varphi_{x \in 1,2}$ are Lagrange multipliers. The first order condition with respect to $\mathcal{P}_1(\mathbf{v})$ is:

$$v_1 - \mu(\mathbf{v}) + \lambda(\log \mathcal{P}_1^0 - \log \mathcal{P}_1(\mathbf{v})) - \varphi_1 = 0,$$

and with respect to $\mathcal{P}_j(\mathbf{v})$ is:

$$v_j - \mu(\mathbf{v}) + \lambda(\log \mathcal{P}_j^0 - \log \mathcal{P}_j(\mathbf{v})) - \varphi_2 = 0.$$

Following the same procedure described in Section 3.3.2 this can be rearranged to:

$$\mathcal{P}_1(\mathbf{v}) = \frac{q e^{(v_1 - \varphi_1)/\lambda}}{\sum_{j=2}^{N} \mathcal{P}_j^0 e^{(v_j - \varphi_2)/\lambda} + q e^{(v_1 - \varphi_1)/\lambda}},$$

and

$$\mathcal{P}_j(\mathbf{v}) = \frac{\mathcal{P}_j^0 e^{(v_j - \varphi_2)/\lambda}}{\sum_{j=2}^{N} \mathcal{P}_j^0 e^{(v_j - \varphi_2)/\lambda} + q e^{(v_1 - \varphi_1)/\lambda}}.$$

Therefore, the solution to the problem is going to be identical to that described in Section 3.3.2. The only difference is that now, for all alternatives for which the quota is not binding and for which $\mathcal{P}_j^0 > 0$, the additive state-independent biases φ_2 are the same. This logic extends to any situation when not all quotas are binding.

3.B

Details of the solution for the binary example with risky and safe options

The agent's problem is:

$$\max_{\mathcal{P}=(\mathcal{P}_1(0),\mathcal{P}_1(1))} b(1-\mathcal{P}_1(0))R+(1-b)(\mathcal{P}_1(1)+(1-\mathcal{P}_1(1))R)-\lambda\kappa(\mathcal{P},G)$$

subject to

$$\mathcal{P}_1(0),\mathcal{P}_1(1)\geq 0,$$

$$b\mathcal{P}_1(0)+(1-b)\mathcal{P}_1(1)=q,$$

and where

$$\kappa(\mathcal{P},G)=-\sum_{i=1}^{2} q_i\log q_i+\sum_{i=1}^{2}\int_{\mathbf{v}}\mathcal{P}_i(\mathbf{v})\log\mathcal{P}_i(\mathbf{v})G(d\mathbf{v}).$$

The derivative with respect to $\mathcal{P}_1(0)$ of the maximand is:

$$-bR+(1-b)(-\frac{b}{1-b}+\frac{b}{1-b}R)-\lambda(b(\log\mathcal{P}_1(0)+1-\log(1-\mathcal{P}_1(0))-1)+$$
$$+(1-b)(-\frac{b}{1-b})(\log\mathcal{P}_1(1)+1-\log(1-\mathcal{P}_1(1))-1))=0.$$

This is equal to:

$$-b-\lambda b(\log\mathcal{P}_1(0)-\log(1-\mathcal{P}_1(0))-\log\mathcal{P}_1(1)+\log(1-\mathcal{P}_1(1)))=0.$$

Plugging $\mathcal{P}_1(1)=\frac{q-b\mathcal{P}_1(0)}{1-b}$ into this expression we obtain:

$$-b-\lambda b(\log\mathcal{P}_1(0)-\log(1-\mathcal{P}_1(0))-\log(\frac{q-b\mathcal{P}_1(0)}{1-b})+\log(1-\frac{q-b\mathcal{P}_1(0)}{1-b}))=0.$$

Dividing each side by b and using the properties of the logarithms yields:

$$1=\lambda\log\left(\frac{(1-b)(1-\mathcal{P}_1(0))(q-b\mathcal{P}_1(0))}{(1-b)\mathcal{P}_1(0)(1-b-q+b\mathcal{P}_1(0))}\right),$$

or:

$$\mathcal{P}_1(0)^2(be^{\frac{1}{\lambda}}-b)+\mathcal{P}_1(0)(b+q-(b+q-1)e^{\frac{1}{\lambda}})-q=0.$$

There are two solutions to this equation:

$$\mathcal{P}_1(0) \in \{\frac{-b-q+(b+q-1)e^{\frac{1}{\lambda}}+\sqrt{(b+q-(g+q-1)e^{\frac{1}{\lambda}})^2+4q(be^{\frac{1}{\lambda}}-b)}}{2(be^{\frac{1}{\lambda}}-b)},$$
$$\frac{-b-q+(b+q-1)e^{\frac{1}{\lambda}}-\sqrt{(b+q-(b+q-1)e^{\frac{1}{\lambda}})^2+4q(be^{\frac{1}{\lambda}}-b)}}{2(be^{\frac{1}{\lambda}}-b)}\}$$

The solution to the agent's problem should be positive. Only the first root is positive. This is so since the denominator $2(be^{\frac{1}{\lambda}}-b)$ is positive. For the root to be positive, the nominator should be positive. The second root is negative since $4q(be^{\frac{1}{\lambda}}-b)$ is positive, so the square root is bigger than the term in front of the square root. For a similar reason, the first root is positive.

That is, the solution to the agent's problem is

$$\mathcal{P}_1(0) = \frac{-b-q+(b+q-1)e^{\frac{1}{\lambda}}+\sqrt{(b+q-(b+q-1)e^{\frac{1}{\lambda}})^2+4q(be^{\frac{1}{\lambda}}-b)}}{2(be^{\frac{1}{\lambda}}-b)}$$

and

$$\mathcal{P}_1(1) = \frac{q-b\mathcal{P}_1(0)}{1-b}.$$

3.C

Binary example with two risky options

The agent chooses between two risky alternatives[2]. The first option can take the values $v_1 = 0$ and $v_2 = 1$ with probabilities b and $1-b$, correspondingly and $0.5 < b < 1$. The second option can take the values $v_1' = 0$ and $v_2' = 1$ with probabilities $b' = 1 - b' = 0.5$. Because of the computational difficulties, here we analyze only the area where the first option in expectation is weakly worse than the second option. The remainder is similar to the setup in Section (3.4).

The agent maximizes the following utility:

$$\max_{\mathcal{P}=(\mathcal{P}_1(0,0),\mathcal{P}_1(0,1),\mathcal{P}_1(1,0),\mathcal{P}_1(1,1))} b(1-b')(1-\mathcal{P}_1(0,1)+(1-b)b'\mathcal{P}_1(1,0)+$$

[2]Consider the recruiter who chooses between two candidates from two different groups when she scans candidates resumes. Her beliefs about the quality of candidates in these two groups could be different. However, in both groups she could potentially find candidates of high and low quality.

$$+ (1-b)(1-b')(\mathcal{P}_1(1,1) + (1-\mathcal{P}_1(1,1))) - \lambda\kappa(\mathcal{P}, G)$$

subject to

$$\mathcal{P}_1(0,0), \mathcal{P}_1(1,0), \mathcal{P}_1(0,1), \mathcal{P}_1(1,1) \geq 0,$$

$$bb'\mathcal{P}_1(0,0) + b(1-b')\mathcal{P}_1(0,1) + (1-b)b'\mathcal{P}_1(1,0) + (1-b)(1-b')\mathcal{P}_1(1,1) = q$$

and where

$$\kappa(\mathcal{P}, G) = -\sum_{i=1}^{2} q_i \log q_i + \sum_{i=1}^{2} \int_{\mathbf{v}} \mathcal{P}_i(\mathbf{v}) \log \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}).$$

First, we derive $\mathcal{P}_1(0,0)$ and plug it back into the utility function:

$$\mathcal{P}_1(0,0) = (q - b(1-b')\mathcal{P}_1(0,1) - (1-b)b'\mathcal{P}_1(1,0) - (1-b)(1-b')\mathcal{P}_1(1,1))/bb'.$$

We take derivatives with respect to $\mathcal{P}_1(0,1)$, $\mathcal{P}_1(1,0)$, and $\mathcal{P}_1(1,1)$. We solve the resulting system of equation numerically.

For the standard RI problem we use Corollary 2 from (Matějka and McKay 2015) in order to find $\mathcal{P}_1^0$:

$$bb' \frac{e^{\frac{0}{\lambda}}}{\mathcal{P}_1^0 e^{\frac{0}{\lambda}} + (1-\mathcal{P}_1^0)e^{\frac{0}{\lambda}}} + b(1-b')\frac{e^{\frac{0}{\lambda}}}{\mathcal{P}_1^0 e^{\frac{0}{\lambda}} + (1-\mathcal{P}_1^0)e^{\frac{1}{\lambda}}} +$$
$$+ (1-b)b' \frac{e^{\frac{1}{\lambda}}}{\mathcal{P}_1^0 e^{\frac{1}{\lambda}} + (1-\mathcal{P}_1^0)e^{\frac{0}{\lambda}}} + (1-b)(1-b')\frac{e^{\frac{1}{\lambda}}}{\mathcal{P}_1^0 e^{\frac{1}{\lambda}} + (1-\mathcal{P}_1^0)e^{\frac{1}{\lambda}}} = 1.$$

We solve this equation numerically and plug $\mathcal{P}_1^0$ into $\mathcal{P}_i(\mathbf{v}) = \frac{q_i e^{v_i/\lambda}}{\sum_{j=1}^{N} q_j e^{v_j/\lambda}}$ in order to obtain conditional probabilities.

Figure 3.10 illustrates the quality of the first option ($(1-b)\frac{\mathcal{P}_1(1,1)+\mathcal{P}_1(1,0)}{2\mathcal{P}_1^0}$) as a function of b. Figure 3.11 illustrates the quality of the second option ($\frac{b(1-\mathcal{P}_1(0,1))+(1-b)(1-\mathcal{P}_1(1,1))}{2\mathcal{P}_2^0}$) as a function of b. The results are similar to the case with safe and risky options: the quality of the chosen option is higher (lower) when the quota on it is smaller (larger) than the unconditional probability of choosing it in the standard RI problem. Accordingly, the gap in the quality between two options is higher when the quota implementation restricts the agent more.

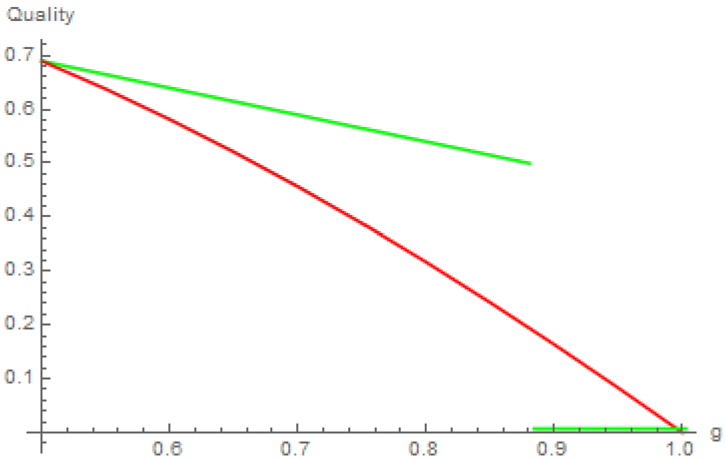


Figure 3.10: Quality of the first option conditional on being chosen as a function of b and $\lambda = 0.5$, $q = 0.5$. The green line is for the standard RI problem and the red line is for the quoted RI problem.

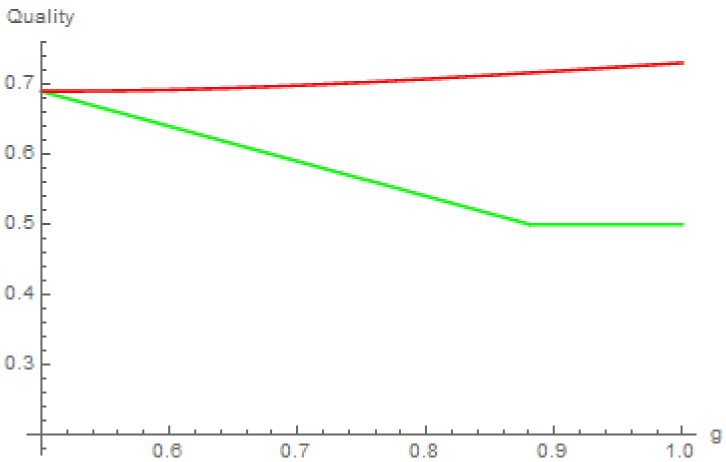


Figure 3.11: Quality of the second option conditional on being chosen as a function of b and $\lambda = 0.5$, $q = 0.5$. The green line is for the standard RI problem and the red line is for the quoted RI problem.

3.D

Details of the solution for the binary example: subsidies

The agent chooses between risky and safe options. The safe option always takes the value $v_1 = R$. The risky option can take the values $v_2 = 0$ with the probability b and $v_2 = 1$ with the probability $1 - b$ correspondingly. These probabilities are the priors of the agent and she does not know what the realization of the state of the world is. The agent acquires costly information about the realization. The social planner sets up a subsidy for the risky option: the agent receives extra payment of S if she chooses the risky option.

The maximization problem of the agent in the case of subsidy S on the risky option is as follows:

$$\max_{\mathcal{P}=(\mathcal{P}_1(0),\mathcal{P}_1(1))} b(\mathcal{P}_1(0)S+(1-\mathcal{P}_1(0))R)+(1-b)(\mathcal{P}_1(1)(1+S)+(1-\mathcal{P}_1(1))R)-\lambda\kappa(\mathcal{P},G)$$

subject to

$$\mathcal{P}_1(0), \mathcal{P}_1(1) \geq 0,$$

and where

$$\kappa(\mathcal{P}, G) = -\sum_{i=1}^{2} \mathcal{P}_i^0 \log \mathcal{P}_i^0 + \sum_{i=1}^{2} \int_{\mathbf{v}} \mathcal{P}_i(\mathbf{v}) \log \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}).$$

In this case the solution has the standard modified multinomial logit form but with the value of risky option increased by S. Namely,

$$\mathcal{P}_1(0) = \frac{\mathcal{P}_1^0 e^{\frac{S}{\lambda}}}{\mathcal{P}_1^0 e^{\frac{S}{\lambda}} + \mathcal{P}_2^0 e^{\frac{R}{\lambda}}}$$

$$\mathcal{P}_1(1) = \frac{\mathcal{P}_1^0 e^{\frac{1+S}{\lambda}}}{\mathcal{P}_1^0 e^{\frac{1+S}{\lambda}} + \mathcal{P}_2^0 e^{\frac{R}{\lambda}}}.$$

In order to compare the agent's behavior under both policies we need to find a level of subsidies for which the risky option would be chosen by the agent with the required probability q:

$$(1 - b)\mathcal{P}_1(1) + b\mathcal{P}_1(0) = q.$$

The unconditional probabilities in the case of the agent's problem with subsidies are as follows:

$$\mathcal{P}_1^0 = \max\{0, \min\{1, \frac{-e^{R/\lambda}(-e^{(1+S)/\lambda} + e^{R/\lambda} - be^{S/\lambda} + be^{(1+S)/\lambda})}{(e^{(1+S)/\lambda} - e^{R/\lambda})(-e^{S/\lambda} + e^{R/\lambda})}\}\}$$

$$\mathcal{P}_2^0 = 1 - \mathcal{P}_1^0.$$

Figure 3.12 shows the optimal subsidy that is necessary in order to equalize the unconditional probability of choosing the risky option to 0.5 as a function of b.

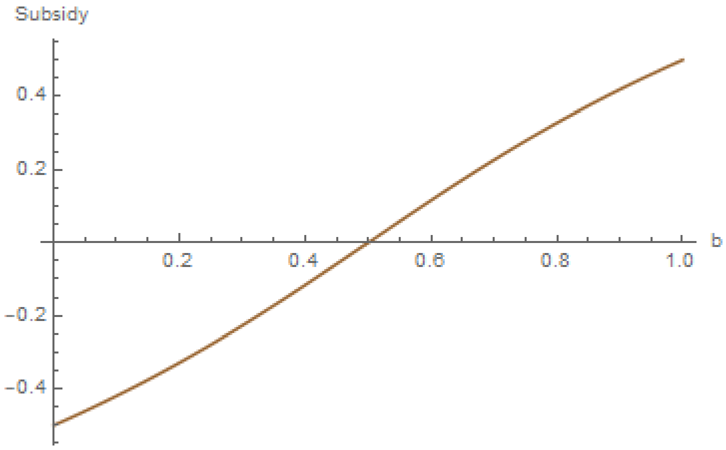


Figure 3.12: Optimal subsidy as a function of b and $\lambda = 0.5$, $q = 0.5$, $R = 0.5$.

We see that for small b the government sets a financial penalty on choosing the risky option. That is because the risky option is likely to be good and the agent would like to choose it more often than in half of the cases. In contrast, if b is high, the government supports the choice of risky option by establishing a positive subsidy.

When the the social planner sets the optimal subsidy, the conditional probabilities of choosing the risky option in good and bad states are the same in the cases with quotas and subsidises. Effectively, it means that if the government wants the agent to choose the risky option in some proportion, there is no difference in the agent's choice if we compare two ways of achieving the goal: through quotas or through subsidies.

At the same time, for high b the utility of the firm in the case of subsidies is higher than in the case of quotas (Figure 3.13). Therefore, one could speculate that it is impossible to extract all subsidies from the firm afterwards and hence it is more beneficial for firms to lobby for subsidies rather than quotas.

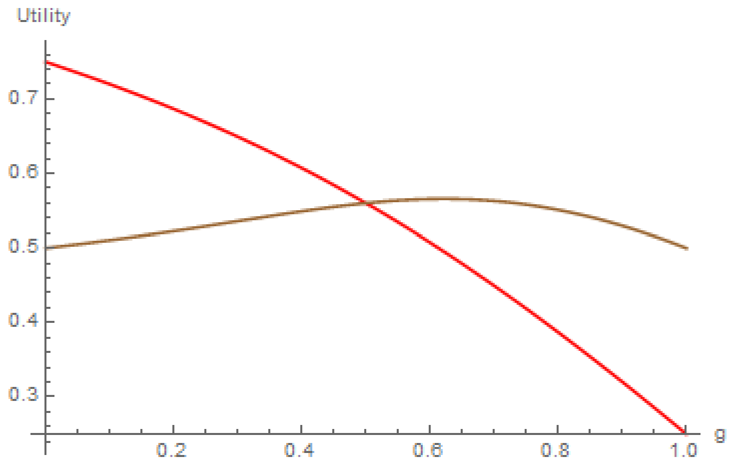


Figure 3.13: Utility of the agent as a function of b and $\lambda = 0.5$, $q = 0.5$, $R = 0.5$. The red line is for the quoted RI problem and the brown line is for the RI problem with subsidies.

3.E

Tradeoff situation

We demonstrate the situation when the social planner faces a tradeoff between maximizing the overall quality of the chosen alternatives and minimizing the influence of priors. We model the latter by Kullback-Leibler divergence: a measure of how the chosen probability distribution is different from a fair quota which is defined in subsection 3.5.2.

$$\max_{q_i}\{\alpha \sum_{i=1}^{N} \int_{\mathbf{v}} v_i \mathcal{P}_i(\mathbf{v}, q) G(d\mathbf{v}) - (1-\alpha) \sum_{i=1}^{N} q_i^f \log \frac{q_i^f}{q_i}\},$$

where $\mathcal{P}_i(\mathbf{v}, q)$ is a solution to the RI problem with quotas and q_i^f is fair quota.

Figure 3.14 illustrates the solution to the social planner's problem as a function of b, and $\lambda = 0.5$, $R = 0.5$.

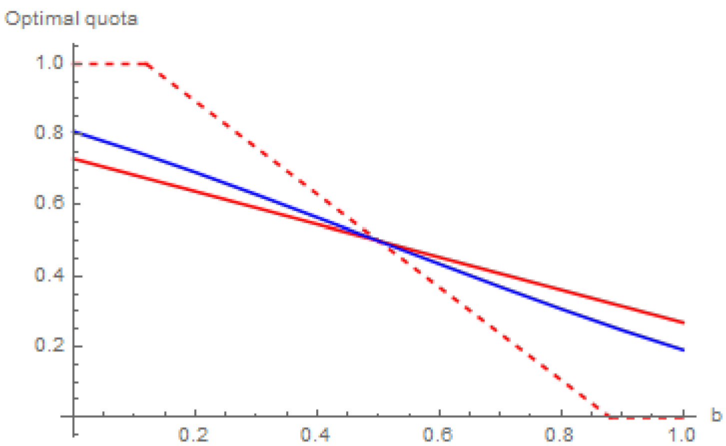


Figure 3.14: Optimal quota as a function of b and $\lambda = 0.5$, $R = 0.5$. The red solid line is for $\alpha = 0$ (fair quota), the red dashed line is for $\alpha = 1$ (quality maximizing quota); the blue line is for $\alpha = 0.5$.

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