Three Essays on Monetary Policy

Jakub Matějů

Dissertation

Prague, August 2018
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To my family and friends.
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First essay suggests that the non-fundamental component of asset prices is one of the drivers of the credit cycle. The proposed model builds on the financial accelerator literature by including a stock market where leveraged investors trade stocks of productive firms with stochastic returns. Investors borrow funds from the banking sector and can default. Their limited liability induces a moral hazard problem which shifts demand for risk and drives prices of risky assets above their fundamental value. Embedding the contracting problem in a New Keynesian general equilibrium framework, the model shows that expansionary monetary policy leads to a rise in both the fundamental and non-fundamental components of stock prices. A positive shock to the non-fundamental component triggers a credit cycle: collateral value rises, and lending and default rates decrease. The credit boom lasts only while stock market growth maintains sufficient momentum. However, monetary policy does not reduce the volatility of inflation and the output gap by reacting to asset prices.

Second essay introduces an "updating channel" of monetary policy, which can play a significant role if a central bank is deemed to possess superior information compared to general public. Assuming an information advantage of a central bank, an unexpected change in monetary policy interest rates signals the state and the outlook of the economy to outside agents. The subsequent update of their expectations goes in an adverse direction, counteracting the conventional transmission from interest rates to inflation and output. We develop a simple model laying down a theoretical basis for the updating channel. We further detect the presence of the updating channel in private forecasts of inflation in a cross-country sample of selected OECD countries.

Third essay illustrates the non-linear reaction of small (satellite) currencies to increased financial stress in the large (core) economy. We suggest that the safe haven status of a satellite currency may hold in calm periods, but breaks down when risk aversion is elevated. A stylized model of portfolio allocation between assets denominated in euro and the satellite currency suggests the presence of two regimes characterized by different reactions of the exchange rate to an increased stress in the euro area. In the “diversification” regime, the satellite currency appreciates in reaction to an increase in the expected return variance in the core economy, while in the “flight to safety” regime, the satellite currency depreciates in response to increased expected volatility. Using the Bayesian Markov-switching VAR model, the presence of these regimes is identified in the case of the Czech koruna, the Hungarian forint and the Polish zloty.
Abstrakt

První esej přichází s myšlenkou, že nefundamentální část ceny aktiv je jedním ze spouštěcích úvěrového cyklu. Navrhovaný model staví na literatuře finančního akcelerátoru a přidává k ní akciový trh, kde investoři provádějí pákové obchody s akcemi produktivních firem se stochastickými výnosy. Tito investoři využívají úvěry od bankovního sektoru a mohou zbankrotovat. Jejich omezené ručení vede k morálnímu hazardu, který zvyšuje poptávku po riziku a náhuku cenu rizikových aktiv nad jejich fundamentální úroven. Tento kontrakt zapojuje do nové keynesiánského modelu obecné rovnováhy a ukazuje, že uvolněná měnová politika vede k růstu jak fundamentální, tak i nefundamentální části cen aktiv. Cenový šok do nefundamentální části cen aktiv spouští úvěrový cyklus: růst hodnoty závěrů umožňuje výšší úvěrování při nižších úrokových sazbách. Úvěrová expanze však trvá jen dokud je růst aktiv dostatečně rychlý. Neukazuje se ale, že by měnová politika zmenšila výkyvy v mezicí výstupu či inflaci pokud by explicitně reagovala na ceny aktiv.

Druhý esej představuje nový informační kanál transmise měnové politiky, který může být významný v případech kdy má centrální banka k dispozici lepší informace než veřejnost. Při informační výhodě centrální banky může její politika signalizovat tyto informace vnějším agentům. Jejich následné přehodnocení situace pak působí nezamýšleným směrem, a působí proti standardnímu působení úrokových sazeb na inflaci a mezicí výstupu. Představujeme jednoduchý model pokládající teoretické základy tohoto mechinismu, a nacházíme pro něj empirickou ilustraci v dynamice prognóz inflace ve vzorku vybraných zemí OECD.

Třetí esej ukazuje nelineární reakci malých, satelitních měn na změnu finančního stresu ve velké, jádrové ekonomice. Navrhujeme, že statut měny jako "bezpečného přístavu" může platit v klidných dobách, to se ale mění se zvyšující se averzí vůči riziku. Stylizovaný model alokace portfolia složeného z eurových aktiv a aktiv denominovaných v satelitní měně ilustruje, že reakce na zvýšený stres může být v různých režimích odlišná. V režimu "diferzifikace" satelitní měna v reakci na zvýšenou nejistotu výnosů v eurozóně zhodnocuje, naopak v režimu "útěk do bezpečí" satelitní měna reaguje na zvýšenou nejistotu znelodnocením. Pomocí bayesovského Markov-switching VAR modelu ukazujeme, že v těchto režimech se pohybuje česká koruna, maďarský forint i polský zlotý.
I am indebted to Michal Kejak who offered guidance, motivation, numerous insights and ideas in helping to develop my research. I also thank the remaining members of my dissertation committee for valuable discussions and comments. I am grateful to my co-authors Jan Filáček, Tomáš Adam and Soňa Benecká for venturing with me into some uncharted territories. I wouldn’t be able to complete the work without the patience and support from my wife Jana and occasional tolerance on the side of my son Cyril.

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Introduction

I would argue that policy today must rely less on the old normal as a guidepost and instead be sensitive to the contours that shape today’s "new normal."

Federal Reserve Governor Lael Brainard
September 12, 2016, at the Chicago Council on Global Affairs

After years of monetary policies operating on the zero lower bound for nominal interest rates, the global macroeconomic cycle entered an upward phase with a firm development in demand, output and inflation gradually returning towards the targets of central banks. This situation warrants normalization of monetary policies, with tapering of unconventional tools and a return to its standard instrument of short term nominal interest rate. Meanwhile, the monetary and financial world has changed. Increasingly complex financial intermediation has become a dominant source of economic fluctuations. Information frictions and asymmetries have been shown to explain a substantial portions of market inefficiencies. Financial market sentiments appear to have larger effects on capital flows and asset prices than previously thought. Therefore, it is crucial for the monetary policy makers to understand and evaluate the impact of their interest rate policies in this "new normal."

This dissertation contributes to this understanding by exploring monetary policy issues under various forms of financial and informational inefficiencies, using both theoretical structural models and empirical assessment. It is organized in three chapters (essays).
The first essay offers an answer to questions about the impact of monetary policy on asset price bubbles and explores the implication of bubbles on financial and macroeconomic variables. The essay proposes a general equilibrium model of the interaction of monetary policy and endogenously arising asset price bubbles. The bubble originates from an agency problem between financial intermediaries (banks) and investors, where the investors are leveraged and operate under limited liability with a possibility to default. This raises both their risk appetite and the demand for risky assets, as well as, consequently, risky asset prices. Expansionary central bank policy is shown to amplify the non-fundamental component of asset prices. The bubble also has expansionary effects on real variables through the easing of collateral constraint. This channel further strengthens when I assume that the overpriced assets can remain on investors’ balance sheets for longer periods of time. However, adding asset prices or the non-fundamental component to the monetary policy reaction function does not help to stabilize inflation or output beyond what can be achieved with a standard monetary policy rule.

Informational frictions may interfere with monetary policy transmission particularly when there is high uncertainty about future economic developments. The second essay suggests that the information asymmetry between the central bank and outside agents may give rise to an "updating channel", which works in an adverse way against the intended monetary policy action. Assuming that the central bank possesses information unavailable to outside agents, monetary policy actions issue informative signals to private agents about the current and future state of the economy. We set up a simple model where part of the economy is populated by boundedly rational agents who observe actions of the central bank and update their expectations of future economic developments. Following a forward-looking Taylor rule, the central bank, by conducting its monetary policy, signals its forecast of the inflation and output gap. Observing the short term interest rate, the partially rational agents update their expected inflation and output and adjust their behaviour accordingly, which may counteract the intended monetary policy adjustment. The updating channel thus explains the "price puzzle" - an empirical observation of inflation increase after monetary policy tightening.

We also measure how an unexpected change in monetary policy rate affects private expectations of inflation and output. Although the standard understanding of monetary policy transmission would suggest that inflation expectations fall after a restrictive monetary policy shock, the data show the opposite reaction, i.e. an increase in inflation expectations. Using different specifications of the monetary policy surprise (both as a
deviation from an expected interest rate change, and from a forward- and backward-looking Taylor rule) we show that the positive correlation between unanticipated interest rate changes and the adjustments of inflation expectations is robust and stable across all specifications.

The monetary policy in small open economies in particular is strongly affected by spill-overs of global events, manifested often through exchange rates of domestic currencies. The third essay investigates the reaction of small currencies to global financial stress. Using a stylized model of portfolio risk management strategies we show that there can be several regimes of reaction depending on the attitude towards stress. In a "diversification" regime, the small currency appreciates in response to increased uncertainty in the core economy as international investors use assets denominated in the small currency to diversify their portfolio. In a "flight to safety" regime the small currency depreciates in reaction to increased uncertainty as the investors enter the "risk off" mode. We use a Bayesian estimated Markov-switching VAR model to identify the regimes on the data from the Czech Republic, Poland and Hungary and to estimate the exchange rate responses in each regime.
Abstract

This essay suggests that the dynamics of the non-fundamental component of asset prices are one of the drivers of the credit cycle. The model presented builds on the financial accelerator literature by including a stock market where investors with limited liability trade stocks of productive firms with stochastic productivities. Investors borrow funds from the banking sector and can go bankrupt. Their limited liability induces a moral hazard problem, which shifts demand for risk and drives prices of risky assets above their fundamental value. Embedding the contracting problem in a New Keynesian general equilibrium framework, the model shows that expansionary monetary policy induces loose credit conditions and leads to a rise in both the fundamental and non-fundamental components of stock prices. A positive shock to the non-fundamental component triggers a credit cycle: collateral value rises, and lending and default rates decrease. These effects reverse after several quarters, inducing a credit crunch. The credit boom lasts only while stock market growth maintains sufficient momentum. However, monetary policy does not reduce the volatility of inflation and the output gap by reacting to asset prices.

Keywords: credit cycle, endogenous asset price bubbles, monetary policy

JEL Codes: E32, E44, E52, G10
1.1 Introduction

Asset price dynamics are one of the major drivers of the credit cycle. As asset prices enter the balance sheets of both financial and non-financial companies, they determine the collateral constraints of borrowers and contribute to how much banks are willing to lend at a given risk premium. An asset price may consist of both fundamental and non-fundamental components. In this paper, the fundamental value represents the part of the asset price which can be justified by discounted future dividend income under efficient market allocation. The remaining part of the asset price is labeled as non-fundamental and can be interpreted as a bubble. Empirical observations suggest that the non-fundamental component is more volatile than the fundamental one, which tends to be rather stable and is related to the discounted sum of expected future dividends. Consequently, the non-fundamental (or bubble) component may be an important driver of the credit cycle.

This paper builds a general equilibrium model capable of monetary policy simulations, based on the financial accelerator literature. It extends the standard model by including an asset (stock) market where assets are endogenously priced above their fundamental values. The assets traded are shares in productive firms, which consist of claims on future returns on capital. These returns (dividends) are stochastic and subject to idiosyncratic productivity shocks. There is information asymmetry between lenders (commercial banks) and borrowers (stock market investors), which induces the costly-state-verification problem, and gives rise to debt contracts where the leveraged stock market investors have limited liability for the outcomes of their investment decisions. The limited liability induces excessive risk-taking by the investors and leads to overpricing on the market for risky assets. As the overpriced assets affect the collateral constraints on borrowers, the value of the non-fundamental (as well as fundamental) component has implications for the real economy: it affects the amount of lending, investment, and output. Moreover, the asset price dynamics also affect the loan default rate. Expansionary monetary policy boosts both the fundamental and non-fundamental components. Through the collateral constraints of stock market investors, the higher stock prices induce lower borrowing rates and higher investment. A positive shock to the non-fundamental component of the asset price eases the collateral constraint and temporarily decreases the lending rate. Although the default rate immediately declines with lower interest rates, it picks up later with higher lending rates as the asset price shock fades out, again tightening the col-
lateral constraint. Despite the suggested importance of asset prices for fluctuations in macroeconomic and financial variables, the estimated monetary policy efficiency frontiers show that the central bank achieves the lowest combinations of inflation and output gap volatilities by not reacting to either asset prices or their non-fundamental component.

The paper is organized as follows. Section 1.2 presents the empirical motivation and Section 1.3 relates the paper to other literature. Section 1.4 describes financial intermediation in a partial equilibrium setup, describing the interactions between banks, investors with limited liability, and firms, and shows that this setup leads to inflated prices on the asset market. Section 1.5 describes an extension where the overpriced assets have longer maturities and can be used as collateral in the periods before maturity. Section 1.6 extends the model to general equilibrium by describing relations to the remaining sectors of the economy, and Section 1.7 presents the responses of model variables to shocks and discusses the implications for efficient monetary policy.

1.2 Empirical Motivation

This paper claims that the credit cycle is, to a large extent, driven by asset price developments, where the non-fundamental component of asset prices plays a prominent role. This is indeed confirmed by the observed data. Figure 1.1 shows the normalized log-deviations (from the HP trend) of the Dow Jones Industrial Average index and the total value of credit market instruments (liabilities) since 1990, both taken from the FRED database of the St. Louis Fed. It is apparent that in this period, the Dow Jones index, used as a proxy for asset prices, was a leading factor of the amount of provided credit. The credit boom in the late 1990s was preceded by steady growth in asset prices. The asset price bust of the dot-com bubble in 2001 was followed by a negative credit gap, which began to close only after asset prices rebounded in 2003. In 2007, it was again asset prices (both stock and real-estate) which preceded the credit crunch associated with the global financial crisis in 2007–2009. Lending began to pick up only after asset prices returned to growth after 2010.

As another illustrative stylized fact, simple Granger causality tests (Table 1.1) on a quarterly sample of US data (taken from the FRED database), ranging from 1949Q1 to 2014Q4, suggest that it is the dynamics of asset prices that lead the credit cycle rather than vice versa (all variables in percentage changes). Assuming that the fundamental value of an asset price should reflect dividend income, a proxy for the fundamental com-
ponent was obtained by regressing the log of dividend income (including dividend income lags and leads of up to the 4th order) on the log of asset prices. The residual of this regression, i.e., the component of prices which cannot be explained by dividend flows, is a naïve estimate of the non-fundamental component of asset prices. The Granger causality tests show that this non-fundamental component (a bubble), rather than the fundamental value, leads the credit cycle.¹

1.3 Related Literature

The global financial and European debt crises have pointed out the importance of the financial sector in transmitting and amplifying economic shocks. The literature has reacted to this increased interest by building on the general equilibrium models with financial frictions of the late 1990s. The most important contributions involve works by Carlstrom and Fuerst (1997) and the subsequent synthesis of Bernanke, Gertler, and Gilchrist (1999),

¹As a side note, the amount of lending, in turn, predicts the fundamental returns with marginal significance, possibly because credit-financed investment was followed by higher profits and dividends. It is also important to keep in mind the limitations of both the Granger causality test and the estimation of the fundamental component, and these computations should only be viewed as an illustrative stylized fact.
who integrate the previous models of financial frictions with New Keynesian rigidities and are therefore able to analyze the role of monetary policy. Another stream of literature builds on Kiyotaki and Moore (1997), which establishes a link between collateral value and the business cycle, but does not explicitly model loan defaults. In recent years, enormous work has been done to incorporate other aspects of financial intermediation, such as the role of collateral constraints in the housing market (Iacoviello and Neri 2010) or the role of unconventional tools of monetary policy (Gertler and Karadi 2011; Cúrdia and Woodford 2011; Gertler and Karadi 2013). A canonical model for the analysis of financial frictions and policy responses during the crisis was established by Gertler and Kiyotaki (2010). The role of liquidity constraints for the possibility of bank runs was analyzed by Gertler and Kiyotaki (2013). The effects of the recently widely adopted tools of macro-prudential regulation have been explored by many, e.g. Kashyap, Tsomocos, and Vardoulakis (2014). Similarly to this paper, Farhi and Tirole (2012) concluded that a form of limited liability gives rise to increased risk preference.

This paper presents a general equilibrium model capable of monetary policy simulations, which captures the characteristics of the credit cycle. It is inspired by a broadly held view of how large financial crises develop and spread. The idea of Adrian and Shin (2010) and the model of Allen and Gale (2000) are possibly closest to our view. While Allen and Gale (2000) provide a rational explanation (albeit only in a partial equilibrium setup) for the overpricing of risky assets when the assets are priced by investors with limited liability, Adrian and Shin (2010) show how changes in the value of assets

| Table 1.1: Granger Causality between the Asset Prices the Credit Cycle |
|---------------------------|--------|-----------|--------|
| Pairwise Granger Causality Tests |        |           |        |
| Sample: 1949Q1 2014Q4 |        |           |        |
| Lags: 4 |        |           |        |
| Null Hypothesis: | Obs | F-Statistic | Prob. |
| %\(\Delta\text{(D.J.I.A.)}\) does not Granger Cause %\(\Delta\text{(Credit)}\) | 246 | 4.86702 | 0.0009 |
| %\(\Delta\text{(Credit)}\) does not Granger Cause %\(\Delta\text{(D.J.I.A.)}\) | 1.72503 | 0.1451 |
| %\(\Delta\text{(Non-fund.)}\) does not Granger Cause %\(\Delta\text{(Credit)}\) | 243 | 2.48795 | 0.0442 |
| %\(\Delta\text{(Credit)}\) does not Granger Cause %\(\Delta\text{(Non-fund.)}\) | 1.42583 | 0.2261 |
| %\(\Delta\text{(Fund.comp.)}\) does not Granger Cause %\(\Delta\text{(Credit)}\) | 243 | 1.28622 | 0.2761 |
| %\(\Delta\text{(Credit)}\) does not Granger Cause %\(\Delta\text{(Fund.comp.)}\) | 1.80739 | 0.1281 |
used as collateral can immediately lead to large swings in balance sheet sizes (causing a contraction in credit and real activity) when the market is highly leveraged.

This paper suggests that asset price and credit booms may be explained and described using a combination of these two effects. Because the incentive structures faced by the managers of certain types of financial intermediaries (such as investment banks) induce excessive risk-taking, there may be over-investment in risky assets, leading to endogenously inflated prices. If these overpriced assets are allowed to serve as collateral for new loans, the shocks to asset prices may have pronounced and non-trivial effects.

The main contribution of this paper is that it constructs a full New Keynesian general equilibrium model with the feedback between asset prices, credit conditions, investment, and capital returns. The model also illustrates how the monetary policy of an inflation-targeting central bank interferes with this mechanism. The simulations suggest that expansionary monetary policy leads to an increase in both the fundamental and non-fundamental element of asset prices and consequently increases collateral value, reduces lending rates, and boosts economic activity. A non-fundamental shock to asset prices can also trigger a credit cycle: shortly after a positive price shock the lending rate falls and the default rate decreases as collateral constraints ease. After several quarters, however, lending rates and the default rate start to rise as investors' wealth shrinks with a stock market slowdown.

Our paper contains a number of other results (contributions). In particular, it finds that the non-fundamental component of asset prices may work as a shock absorber. Similarly to Martín and Ventura (2014), the existence of the non-fundamental component (or a bubble) can have some beneficial aspects. As the model presented implies, the bubble can partially absorb the effects of exogenous shocks by affecting the wedge between the risk-free rate and capital returns in a counter-cyclical manner. This is most obvious in the case of a monetary policy shock, where the impact of an interest rate shock on capital returns and investment is much smaller compared to the similarly parametrized benchmark of Bernanke, Gertler, and Gilchrist (1999).

This paper also shows that if the growth of asset prices and traded volumes maintains a sufficient (precisely defined) momentum, borrowing constraints are relaxed and the dynamics of real economic variables exceed the benchmark of Bernanke, Gertler, and Gilchrist (1999). If the growth of the stock market slows below the defined threshold, the dynamics fall below the benchmark model.² The paper also shows that under limited

²Although this paper studies the effects of overpriced assets on the economy, the model does not
liability, leveraged investors prefer risky investment over diversified portfolios.

1.4 Financial Intermediation Structure and Asset Pricing

Inspired by the model of asset bubbles presented by Allen and Gale (2000), and building on the model of the financial accelerator in the New Keynesian framework of Bernanke, Gertler, and Gilchrist (1999), this paper constructs a partial equilibrium model which generates incentives-based overpricing in asset prices. The limited liability of investors (i.e., they do not suffer the full cost in the event of default) induces them to prefer risky assets and price them above their fundamental value. The fundamental value is defined as the price at which the investors would invest their own resources, which is consistent with pricing based on the present value of future dividend income. In later sections, this contracting problem is embedded in the general equilibrium framework of Bernanke, Gertler, and Gilchrist (1999), and we show that non-fundamental overpricing of risky assets can emerge within this widely used model framework. This allows us to conduct monetary policy experiments to study the impact of monetary policy shocks on the size of the non-fundamental component of asset prices. The latter has real implications: it affects credit availability and consequently the amount of lending, investment, and economic activity.

The model can, however, be interpreted more broadly, as limited liability is widely present in the whole economy as an inherent result of the principal-agent problem. A similar incentive structure applies to the setting where an investment fund is managed by a fund manager with limited liability, whose salary is dependent on the fund’s performance. When the return on the managed portfolio is sufficiently high, the manager’s income increases with the returns. When the returns become negative, the manager can be fired, but does not directly bear the cost of the portfolio loss. By analogy to the model presented below, the limited liability leads to increased demand for risky assets. Further, corporate management is typically partially rewarded in company stock options. In case of stock price growth, managers execute their options, while in case of stock price fall they do not bear the losses. This limited liability leads to more risky projects being undertaken feature an explicit mechanism for the burst of the endogenous asset bubble. The bubble is interpreted as a structural, long-term feature of the financial market, where limited liability leads to mispricing of risky assets even in the steady state of the model.
in comparison to the first-best setting (which we will call fundamental), where managers would decide about the investment of funds that they themselves own.

By examining the impacts of limited liability on asset pricing and real activity, this paper suggests that limited liability is at the heart of increased risk-taking in the financial sector.\footnote{The assets used in this model are claims on the future capital returns of firms and can be interpreted as stocks. However, the same mechanism would apply for other types of assets, where returns are uncertain and investors have limited liability, such as real estate. The model would imply mispricing of real estate in a similar manner, which could trigger a financial cycle according to Borio (2014) and others. However, the macroeconomic implications may be slightly different (one of the most notable differences being the large exposure of households to housing assets in comparison to stocks), and are beyond the scope of this paper.}

1.4.1 Contract Timing and Payoffs

This model builds on Bernanke, Gertler, and Gilchrist (1999), and extends it by incorporating a stock market where the shares of productive firms are traded by investors with limited liability. Figure 1.2 depicts the agents in the contracting problem, the flow of funds, and the respective interest rates. Risk-neutral investors can invest in risky shares $S_{t+1}$ of productive firms, but have only limited own wealth $N_{t+1}$. These shares of productive firms are traded at an endogenously determined price $P_{t+1}$ and yield a stochastic return equal to the return on the firm’s installed capital $\omega R_{t+1}^{K}$, where $\omega$ is an idiosyncratic productivity element, which is i.i.d. with $E[\omega] = 1$. $R_{t+1}^{K}$ is aggregate capital productivity. The realization of idiosyncratic return $\omega$ is not known at the time of the investment decision. It is revealed to the investor ex-post, but it cannot be contracted on, which gives rise to the costly-state-verification problem. The investor needs to borrow $B_{t+1} = P_{t} S_{t+1} - N_{t+1}$. We initially assume that ownership of shares entitles the holder to capital returns in one period, but later we will relax this assumption to allow for multi-period asset holdings, which enter the collateral constraints and lead to more pronounced effects of asset prices on other financial and macroeconomic variables.

The risk-neutral financial intermediaries (banks) are willing to lend $B_{t+1}$ at a contractual rate $Z_{t+1}$ as soon as their expected payoff from the contract exceeds the opportunity cost of investing in a risk-free asset with certain return $R_{t+1}$ (the banking sector is competitive). Lending to investors is generally risky, as an investor can default on the loan whenever the realized return on the portfolio is low enough, such that he is not able to repay the borrowed amount at the contractual rate $Z_{t+1} B_{t+1}$. Let us also assume that investors with limited liability are the only agents in the economy who are capable of...
in investing in risky assets. In general, an investor can also invest positive amounts in a risk-free asset yielding $R_{t+1}$. However, he will invest a zero amount in this risk-free asset, as his external financing costs (the contractual rate) will generally be higher than the risk-free return ($Z_{t+1} > R_{t+1}$), because the contractual rate $Z_{t+1}$ would need to compensate the bank for any default risk. The riskiness of an investment is not ex-ante observable by the bank, which can only monitor the returns ex-post by paying agency costs (as in the standard costly-state-verification problem). Whenever the realized return on the risky asset is below a certain threshold, where the investor is unable to pay the loan back, he declares bankruptcy (Figure 1.3). The threshold for default is the break-even idiosyncratic return $\bar{\omega}_{t+1}$ on the risky asset, defined by the following constraint:

$$Z_{t+1}B_{t+1} = \bar{\omega}_{t+1}P_{t+1}^{K}S_{t+1}$$

The default-threshold idiosyncratic return $\bar{\omega}_{t+1}$ is such that the borrower will just be able to repay the borrowing $B_{t+1}$ times the contractual rate $Z_{t+1}$. In the event of default, the borrower (bank) pays a fraction $\mu$ as auditing costs, to collect whatever remained from
the project. The contractual rate has to satisfy the participation constraint for the bank:

\[(1 - F(\bar{\omega}_{t+1}))Z_{t+1}B_{t+1} + (1 - \mu) \int_{0}^{\bar{\omega}_{t+1}} \omega R_{t+1}^{K} S_{t+1} dF(\omega) \geq R_{t+1}B_{t+1} \quad (1.2)\]

where the bank’s expected payoff from lending to the investors must be higher than the opportunity cost of investing in risk-free bonds. To avoid facing idiosyncratic risk, the banks diversify their loan portfolio among many ex-ante identical investors, charging a flat rate \(Z_{t+1}\). The timing of the financial intermediation contract for a representative investor-bank pair is the following:

1. The bank lends \(B_{t+1}\) to the investor at a flat rate \(Z_{t+1}\), which compensates the bank for the ex-ante symmetric risk of default by the investor.

2. The stock market opens and the investor may sell and buy risky assets \(S_{t+1}\) for an endogenously determined price \(P_{t}\).

3. The idiosyncratic risk \(\omega\) is realized and the assets \(S_{t+1}\) yield \(\omega R_{t+1}^{K}\) to the investor.

4. The investor either repays \(Z_{t+1}B_{t+1}\) to the bank or defaults. In the case of default, the bank pays auditing costs \(\mu\) and collects the residual value of the investment.

This is a variant of the standard costly-state-verification problem of Townsend (1979), and the described risky debt contract (including the true reporting of default) was shown to be Pareto-optimal by Gale and Hellwig (1985). The contract payoffs are depicted in Figure 1.4.

In the case of default, the investor’s payoff is zero. In the case of success, the investor is the residual claimant, after satisfying the participation constraint of the bank. The bank itself does not face any risk on aggregate, as its assets are perfectly diversified among loans to individual investors, and the loss from the fraction of defaulting loans is ex-ante covered by higher contractual rates. Therefore, households’ savings are not subject to any risk either.
1.4.2 The Investor’s Problem and Demand Pricing of the Risky Asset

The investor chooses the amount of risky investment \( S_{t+1} \) and the default threshold \( \bar{\omega}_{t+1} \) to maximize

\[
\max_{S_{t+1}, \bar{\omega}_{t+1}} \left[ \int_{\omega_{t+1}}^{\infty} \omega R^{K}_{t+1} S_{t+1} dF(\omega) - (1 - F(\bar{\omega}_{t+1})) Z_{t+1} B_{t+1} \right]
\]

subject to the bank’s participation constraint (1.2). In other words, the investor only takes into account the “optimistic” part of the return distribution where he has positive profit, and he does not internalize the full cost of the losses. Because of the limited liability, the investor’s payoff is zero in the case of default. This causes the investor’s subjective return distribution to be more optimistic than the true fundamental return distribution. Next, we show that limited liability increases the investor’s appetite for investing in the risky asset, and raises stock market prices.\(^4\)

\(^4\)Several conceptual points are worth noting at this point. First, we can think about the idiosyncratic \( \omega \) realizations as shocks to distinct sectors of the economy. We conjecture that investors endogenously prefer to fully face the idiosyncratic risk and not to diversify their asset holdings among sectors. The idiosyncratic risk is preferred by investors because the limited liability makes the non-diversified risky investment more attractive. Appendix 1.B shows in more detail that investors with limited liability prefer not to diversify their asset holdings.

Second, let us also assume a continuum of sectors of measure 1, so that the probability of default represents the fraction of defaulting investors. There are infinitely many firms in each sector, so that firms do not have any bargaining power vis-à-vis investors, who become residual claimants. To ensure ex-ante symmetry among the wealth of investors at the beginning of each period, we assume that investors gather in households of investors, and each investor’s household has one investor covering each sector.
Using $B_{t+1} = P_t S_{t+1} - N_{t+1}$, the investor’s problem becomes linear in $S_{t+1}$. That implies that there is a price of a risky asset $P_t$ above which the demand is infinite, and below which the investors would like to short-sell the asset in an infinite amount. As we will show later, the non-degenerate supply ensures a unique and stable finite equilibrium. The no-profit condition for investors requires:

$$\int_{\omega_{t+1}}^{\infty} \omega R_{t+1}^K dF(\omega) - (1 - F(\bar{\omega}_{t+1})) Z_{t+1} P_t = 0$$

This equality will be achieved because of the competitive investors sector and the convex costs of creating investment opportunities (see below). We assume that there are many ex-ante identical investors and they take the universally charged rate $Z_{t+1}$ as exogenous. In other words, the contractual rate $Z$ is not conditioned on the individual characteristics of investors, such as the individual amount of shares invested. The investors’ wealth is equalized across households of investors. From condition (1.4), we can express the price of the risky asset as observed on the stock market:

$$P_t = \frac{1}{Z_{t+1}} \int_{\omega_{t+1}}^{\infty} \omega R_{t+1}^K dF(\omega)$$

The price reflects the investors’ valuation of the risky asset, where the expected realization from a truncated return distribution is discounted by the contractual rate multiplied by the probability it will be paid. We claim that this price of risky shares observed on the stock market is overpriced in comparison to the fundamental value as a result of the limited liability of investors. Following Allen and Gale (2000), we define the fundamental value as the price the investor would pay if he invested his own funds without leverage. Such a “fundamental” investor would solve

$$\max_{S_{t+1}} \left[ \int_0^{\infty} \left( \omega R_{t+1}^K S_{t+1} + R_{t+1}(N_{t+1} - P_t S_{t+1}) \right) dF(\omega) \right]$$

where the investor’s own wealth $N_{t+1}$ is sufficient to cover all desired spending on the

---

The investors in each sector operate individually (importantly, they cannot repay debt using returns from other sectors), coming “home” together and pooling their wealth among household members only after the uncertainty is realized and returns are paid. Also, the “creative investors” described below hand their profits over to the household pool. Therefore, from the point of view of households of investors, the idiosyncratic risk of individual investment is pooled and vanishes at the household level.

Finally, to prevent arbitrage conducted by banks or households driving the price down to the fundamental level, the model assumes that trading on the stock market is restricted to investors with limited liability, who possess the unique property of controlling and operating the firms which they own.
risky asset and the rest \((N_{t+1} - P_tS_{t+1})\) is left for investing in the risk-free asset. The no-profit condition of fundamental investors is:

\[
P_t^F = \frac{1}{R_{t+1}} (E[\omega] R^K_{t+1}) = \frac{P^K_{t+1}}{R_{t+1}}
\]  

(1.7)

The fundamental price is no higher than the present value of the one-period-ahead claim on expected returns on capital installed. Comparing the fundamental price with the price with limited liability, we show that

\[
P_t \geq P_t^F
\]  

(1.8)

The full proof can be found in Appendix 1.A. Interestingly, because investors with limited liability are willing to pay a higher price, they effectively drive away any fundamental investors (whose fundamental valuation is lower) from the market.

We have shown that under certain reasonable assumptions (most notably that investors enjoy limited liability) the prices which are observed on the stock market are endogenously inflated compared to their fundamental values. The non-fundamental component is the difference between the observed price of the risky asset and its fundamental value.

### 1.4.3 Supply of the Risky Asset

To complete the model of the stock market, we assume that in each household there are investors who “create” investment opportunities (and sell them to other investors on the stock market). The profit from creating an investment opportunity, and the creative investors’ objective function, is

\[
\max_{S_{t+1}} (P_tS_{t+1} - c(S_{t+1}))
\]  

(1.9)

where \(c(S_{t+1})\) is an increasing convex cost function which links the costs of creating an investment opportunity to the number of investment assets created. To ensure interior equilibrium, assume that \(c'(.) > 0, c''(.) > 0\). The amount of stocks created \(S_t\) is then sold on the stock market at price \(P_t\), which the competitive creative investors take as exogenous and which potentially exceeds their average costs and creates economic profit. The creative investors are evenly distributed among households of investors, so that any profit from trading on the stock market stays in the investors’ sector and is equalized.
across households to prevent heterogeneous paths of wealth. The first-order condition of the creative investors’ profit maximization problem is

\[ P_t = c'(S_{t+1}) \]  

(1.10)

This equation describes the supply side of the market for the risky asset.

### 1.4.4 Value of the Investment and Wealth Accumulation

Using the substitution for \( Z_{t+1}B_{t+1} \) from eq. (1.1), the investors’ objective (1.3) can be rewritten as

\[
\max_{S_{t+1}, \omega_{t+1}} \left[ \int_{\omega_{t+1}}^{\infty} \omega R^K_{t+1} S_{t+1} dF(\omega) - (1 - F(\omega_{t+1}))(\omega_{t+1} R^K_{t+1} S_{t+1}) \right]
\]

(1.11)

Substituting the last term from the banks’ participation constraint (1.2), we arrive at the following expression for the value of investment:

\[
R^K_t S_t - \left( R_t + \frac{\mu \int_{0}^{\omega_{t+1}} \omega R^K_{t+1} S_{t+1} dF(\omega)}{P_{t-1} S_{t} - N_{t}} \right) (P_{t-1} S_{t} - N_{t})
\]

(1.12)

As noted above, some of the investors are “creative” and they actively generate investment opportunities, establishing contact with firms. They are able to generate shares of firms, which they will be able to sell for \( P_t \), by paying convex costs \( c(S_{t+1}) \). Because we have not yet established any particular functional form for \( c() \), let us normalize it such that \( c(S_{t+1}) \approx 1 \) and that the deviations around this value resulting from the positive derivative are of second-order importance for investors’ wealth. Then the investors’ net profit can be expressed as \( S_{t+1}(P_t - 1) \), which they store on a risk-free account for the rest of the period. We assume that these creative investors are uniformly distributed across households of investors and pool their gain in the wealth of a representative household of investors. Further imposing the market-clearing condition that \( S_t = Q_{t-1} K_t \) (i.e., stocks...
of firms entitle their holders to a share of firms’ installed capital), we get

\[ V_t = R^K_t Q_{t-1} K_t - \left( R_t + \frac{\int_0^{\omega_t+1} \omega R^K_t Q_{t-1} K_t \, df(\omega)}{P_{t-1} Q_{t-1} K_t - N_t} \right) \left( P_{t-1} Q_{t-1} K_t - N_t \right) + R_t Q_{t-1} K_t (P_{t-1} - 1) \]

\[ = R^K_t Q_{t-1} K_t - \left( R_t + \frac{\int_0^{\omega_t+1} \omega R^K_t Q_{t-1} K_t \, df(\omega)}{d} \right) \left( P_{t-1} Q_{t-1} K_t - N_t \right) \left( Q_{t-1} K_t - N_t \right) \]

\[ (1.13) \]

which is identical to the wealth accumulation equation of Bernanke, Gertler, and Gilchrist (1999) (although here it results from a different financial market structure). In other words, we consider a sector of limited liability investors between firms and banks, and we show that the prices of assets traded on the financial market are inflated. Because of a different incentive structure leading to different investment decisions, this leads to a different allocation of real resources in comparison to Bernanke, Gertler, and Gilchrist (1999).

However, if the inflated assets are held for more than one period and can be used as collateral for further loans, the overpricing will have an even stronger impact on credit availability, lending, investment, and real activity. In the next section we explain how the model economy behaves when the assets (with inflated prices as a result of the mechanism described above) are held for two periods before they mature.

1.5 Extension: Multi-Period Assets and the Collateral Constraint

In this section we consider an extension of the model presented, where the agents hold and trade the assets for multiple periods until they mature. In that case, the prices of assets held for more than one period will affect the evolution of investors’ wealth, which is used as collateral. If the prices of these assets are inflated similarly to the single-period case described above, the investors’ wealth (which serves as collateral) is inflated as well. Most importantly, the results suggest that if the asset prices maintain a sufficient (precisely defined) growth momentum, investors’ wealth is higher than in the Bernanke, Gertler, and Gilchrist (1999) benchmark. If the asset price growth slows down, the wealth decreases below the Bernanke, Gertler, and Gilchrist (1999) benchmark.
1.5.1 Financial Intermediation Contract with Two-Period Assets

Assume there is an asset which is purchased in the first period, held in the second, and transforms into a claim on productive capital in the third. As a result, in every period \( t \) two types of asset are traded: the old ones \( S_{t+1}^{old} \) (issued in the previous period \( t-1 \) and maturing in \( t+1 \)) with price \( P_{t}^{old} \) and the newly issued ones \( S_{t+1}^{new} \) with price \( P_{t}^{new} \). While \( S_{t+1}^{old} \) yield \( R_{t+1}^{K} \) at the beginning of the next period, \( S_{t+1}^{new} \) yield nothing in period \( t+1 \), but can be traded as \( S_{t+2}^{old} \), which in turn yield \( R_{t+2}^{K} \) at the beginning of period \( t+2 \).

The cash-flow constraint, i.e., the relationship between the contractual rate \( Z_{t+1} \) and the threshold idiosyncratic productivity \( \bar{\omega}_{t+1} \) (formerly eq. (1.1)), in this case becomes

\[
Z_{t+1}B_{t+1} = \bar{\omega}_{t+1}R_{t+1}^{K}S_{t+1}^{old} + P_{t+1}^{old}S_{t+1}^{new} \tag{1.14}
\]

because in addition to capital returns on maturing assets, the investors will in \( t+1 \) own previously purchased assets maturing in the following period. The bank participation constraint (1.2) changes to

\[
(1-F(\bar{\omega}_{t+1}))Z_{t+1}B_{t+1} + (1-\mu) \int_{0}^{\bar{\omega}_{t+1}} \left( \omega R_{t+1}^{K}S_{t+1}^{old} + P_{t+1}^{old}S_{t+1}^{new} \right) dF(\omega) \geq R_{t+1}B_{t+1} \tag{1.15}
\]

1.5.2 Investors’ Objective and Asset Pricing

The investors’ objective (previously eq.(1.3)) is to maximize expected profit, which, in the presence of two-period assets, is defined as

\[
\max_{S_{t+1}^{new}, S_{t+1}^{old}, \bar{\omega}_{t+1}} \left[ \int_{\bar{\omega}_{t+1}}^{\infty} \left( \omega R_{t+1}^{K}S_{t+1}^{old} + P_{t+1}^{old}S_{t+1}^{new} \right) dF(\omega) - (1-F(\bar{\omega}_{t+1}))Z_{t+1}B_{t+1} \right] \tag{1.16}
\]

The amount of borrowing is given by \( B_{t+1} = P_{t}^{new}S_{t+1}^{new} + P_{t}^{old}S_{t+1}^{old} - N_{t+1} \). When substituted into (1.16), the no-profit conditions of investors can be derived, which lead to the following (demand) pricing equations:

\[
P_{t}^{old} = \frac{1}{Z_{t+1}} \frac{\int_{\bar{\omega}_{t+1}}^{\infty} \omega R_{t+1}^{K} dF(\omega)}{1-F(\omega)} \tag{1.17}
\]

\[
P_{t}^{new} = \frac{P_{t+1}^{old}}{Z_{t+1}} = \frac{1}{Z_{t+1}Z_{t+2}} \frac{\int_{\bar{\omega}_{t+1}}^{\infty} \omega R_{t+2}^{K} dF(\omega)}{1-F(\omega)} \tag{1.18}
\]
Both of these can be shown to be higher with respect to their corresponding fundamental values defined as if there were no information asymmetry (analogously to the case of single-period assets).

### 1.5.3 Investors’ Wealth Accumulation Under Two-Period Assets

Combining the investors’ objective (1.16) with the bank participation constraint (1.15), substituting for $Z_{t+1} B_{t+1}$ using (1.14), and imposing the market-clearing condition that $S^\text{old}_t = Q_{t-1} K_t$, one can obtain the evolution of the aggregate value of investors’ assets.

$$V_t = R^K_t Q_{t-1} K_t + P^\text{old}_t S^\text{new}_t$$

$$- \left( R_t + \frac{\mu \int_0^{\bar{\omega}_{t+1}} (\omega R^K_t Q_{t-1} K_t + P^\text{old}_t S^\text{new}_t) dF(\omega)}{P^\text{new}_t S^\text{new}_t + P^\text{old}_t Q_{t-1} K_t - N_t} \right) (P^\text{new}_t S^\text{new}_t + P^\text{old}_t Q_{t-1} K_t - N_t)$$

(1.19)

The value of investment is now different in comparison to single-period assets and to the Bernanke, Gertler, and Gilchrist (1999) benchmark. This is because (overpriced) assets can be used as collateral in the periods before maturity. However, the purchase of overpriced assets also constitutes extra costs with respect to the Bernanke, Gertler, and Gilchrist (1999) benchmark. Comparing the wealth accumulation equation with the benchmark, we are able to establish the conditions under which the mispriced risky assets have boosting effects on the economy, and when their effect is restrictive.

The question is whether the difference between investors’ wealth in Bernanke, Gertler, and Gilchrist (1999) and that in the present model is positive or negative:

$$V_t^{\text{INF}} - V_t^{\text{BGG}} = P^\text{old}_t S^\text{new}_t (1 - \mu F(\bar{\omega}_{t+1})) - R_t (P^\text{old}_t Q_{t-1} K_t) \leq 0$$

(1.20)

This inequality translates (using the fact that new assets will become old in the next period, $S^\text{new}_t = S^\text{old}_{t+1}$) into the question of whether the nominal growth of the stock market has sufficient momentum:

$$\frac{P^\text{old}_t S^\text{old}_{t+1}}{P^\text{old}_t S^\text{old}_t} \leq \frac{R_t}{1 - \mu F(\bar{\omega}_{t+1})}$$

(1.21)

If the growth of asset prices and/or the volume of assets traded remains above this threshold, the investors’ wealth and the amount of borrowing, investment, and economic activity
exceeds the Bernanke, Gertler, and Gilchrist (1999) benchmark. When the growth of the asset market loses momentum, the effects of non-fundamental prices on the real economy become negative. The threshold implies that for the non-fundamental pricing to have an expansionary effect on the economy, the growth of the volume of the risky asset market, multiplied by the fraction which is not lost to auditing costs needs to cover the risk-free interest rate.

1.6 Other Sectors of the New Keynesian General Equilibrium Model

Now we embed the contracting problem described above in a general equilibrium model. We follow the framework of Bernanke, Gertler, and Gilchrist (1999) closely. In addition to investors and firms, there are retailers, households, the central bank, and the government.

1.6.1 Investors and Banks

The households of investors (as distinct from the ordinary households described below) are risk-neutral, but leave the system at a rate of \( \gamma \). This ensures that investors always demand credit and do not accumulate enough wealth to be eventually fully self-financing. After departure, they consume the remaining part of their wealth. Investors accumulate wealth according to

\[
N_{t+1} = \gamma V_t + W_t^i
\]  

(1.22)

where \( V_t \) is the value of investment in firms’ shares as defined above by (1.13). When a household of investors dies, it consumes all its wealth and departs the scene. This process creates the investors’ consumption \( C_t^e \). \( W_t^i \) is investors’ wages.

The demand price of assets \( P_t \) was defined by (1.5), and the supply side by (1.10). Dividing the banks’ participation constraint (1.2) by \( B_{t+1} \) and using (1.1) for substitution of the term inside the integral of after-default asset recovery, we can express the risk premium (the difference between \( Z_{t+1} \) and \( R_{t+1} \)) as a function of the default threshold \( \bar{\omega}_{t+1} \):

\[
\frac{R_{t+1}}{Z_{t+1}} = 1 - F(\bar{\omega}_{t+1}) + \frac{1 - \mu}{\bar{\omega}_{t+1}} \int_{0}^{\bar{\omega}_{t+1}} \omega dF(\omega) \equiv \Psi(\bar{\omega}_{t+1})
\]  

(1.23)

where \( \frac{\partial \Psi(\omega_{t+1})}{\partial \omega_{t+1}} < 0 \). Similarly, the demand price of risky assets is a function of the aggregate capital returns \( R_{t+1}^{K} \), the contractual rate \( Z_{t+1} \), and the default threshold \( \bar{\omega}_{t+1} \)
eq. (1.5), and can be transformed such that

\[
P_t Z_{t+1} \frac{R^K_{t+1}}{R_{t+1}} = \int_{\bar{\omega}_{t+1}}^{\infty} \omega dF(\omega) \left(1 - F(\bar{\omega}_{t+1})\right) = \Theta(\bar{\omega}_{t+1})
\]

(1.24)

where \(\frac{\partial \Theta(\bar{\omega}_{t+1})}{\partial \bar{\omega}_{t+1}} > 0\), i.e., the price of the risky asset increases with the risk, which is a result of investors’ elevated risk preference induced by the limited liability. Combining equations (1.23) and (1.24), one can see that the wedge between the risk-free rate \(R_{t+1}\) and capital returns \(R^K_{t+1}\) can be expressed as a function of \(\bar{\omega}_{t+1}\) and the risky asset price \(P_t\):

\[
\frac{R^K_{t+1}}{R_{t+1}} = \frac{P_t}{\Psi(\bar{\omega}_{t+1})\Theta(\bar{\omega}_{t+1})}
\]

(1.25)

where \(P_t\) is in turn determined by the increasing supply-side marginal costs, which link it to \(S_{t+1}\) (1.10).

Finally, the market-clearing condition links the financial sector to the production sector:

\[
S_t = Q_t K_t
\]

(1.26)

### 1.6.2 Firms

A representative firm produces output \(Y_t\) using the production function with capital \(K_t\) and aggregate labor \(L_t\) as inputs.

\[
Y_t = A_t K_t^\alpha L_t^{1-\alpha}
\]

(1.27)

where \(A_t\) is stochastic total factor productivity following an autoregressive process. The capital share is denoted by \(\alpha\). Labor consists of workers’ labor \(H_t\) and investors’ labor \(H^i_t\) (consider venture capitalists):

\[
L_t = H_t^\Omega (H^i_t)^{1-\Omega}
\]

(1.28)

where \(\Omega\) is the share of workers’ labor. New capital creation involves installment costs, while old capital depreciates at rate \(\delta\).

\[
K_{t+1} = \Phi \left(\frac{I_t}{K_t}\right) - (1 - \delta)K_t
\]

(1.29)
The installation costs can be thought of as a competitive sector of capital producers who purchase investment and rent capital stock to produce new capital using the production function $\Phi \left( \frac{I_t}{K_t} \right)$ to sell it at price $Q_t$. The FOC of their problem determines the “replacement cost” component of the price of capital: $Q_t = \left[ \Phi' \left( \frac{I_t}{K_t} \right) \right]^{-1}$. 

Firms produce wholesale goods, which are sold to monopolistically competitive retailers at a relative price $\frac{1}{X_t}$. The firms sector is assumed to be competitive. Return on capital $R^K$ is equal to the marginal product of capital multiplied by the price of wholesale goods produced, augmented by the change in the value of capital (consisting of the change in $Q_t$ and depreciation). In expectation terms:

$$E[R^K_{t+1}] = E \left[ \frac{\frac{1}{X_{t+1}} \frac{\alpha Y_{t+1}}{K_{t+1}} + Q_{t+1}(1 - \delta)}{Q_t} \right]$$

(1.30)

Wages in the workers and investors sectors of the labor market are competitive and follow the marginal products of labor:

$$W_t = (1 - \alpha)\Omega \frac{1}{X_t} \frac{Y_t}{H_t}$$

(1.31)

$$W^w_t = (1 - \alpha)(1 - \Omega) \frac{1}{X_t} \frac{Y_t}{H^w_t}$$

(1.32)

1.6.3 Households and Retailers

In addition to the agents involved in the contracting problem, the model features a standard New Keynesian general equilibrium setup. Households derive utility from consumption $C_t$, leisure $1 - H_t$, and real money holdings $\frac{M_t}{P_t}$. This gives rise to demand for consumption (the Euler equation), labor supply, and demand for real money balances. Household savings are deposited ($D_t$) at a risk-free rate $R_t$ in banks, which use them to lend to investors (market clearing implies $D_t = B_t$). The expected utility

$$E_t \sum_{k=0}^{\infty} \beta^k \left[ \ln(C_{t+k}) + \zeta \ln\left( \frac{M_{t+k}}{P^C_{t+k}} \right) + \xi \ln(1 - H_{t+k}) \right]$$

(1.33)

where $\zeta$ is a preference parameter of real money holdings and $\xi$ is a preference parameter of leisure, is maximized subject to the budget constraint

$$C_t = W_t H_t - T_t + \Pi_t R_t D_t - D_{t+1} + \frac{(M_{t-1} - M_t)}{P^C_t}$$

(1.34)
where $T_t$ are taxes and $\Pi_t = P_t^C / P_{t-1}^C$ is consumer price inflation. The first-order conditions of this problem form the Euler equation, labor supply, and money demand:

\[
\frac{1}{C_t} = E_t \{ \beta \frac{1}{C_{t+1}} R_{t+1} \} \tag{1.35}
\]

\[
\frac{W_t}{C_t} = \xi \frac{1}{1 - H_t} \tag{1.36}
\]

\[
\frac{M_t}{P_t^C} = \zeta C_t \left( \frac{R_{t+1}^n - 1}{R_{t+1}^n} \right) \tag{1.37}
\]

Monopolistically competitive retailers buy wholesale goods from the producers and costlessly diversify products to establish market power. Retailers set prices according to Calvo pricing, where only a fraction of retailers change prices each period. The final product is sold to households. Monopolistically competitive retailers face the Dixit-Stiglitz demand functions for the final product varieties

\[
Y_t(z) = \left( \frac{P_t^C(z)}{P_t^C} \right)^\epsilon \tag{1.38}
\]

where $Y_t(z)$ is the quantity demanded, $P_t^C(z)$ is the price of consumption good $z$, and $\epsilon$ is the elasticity of substitution. The consumption goods are aggregated into final consumption bundles using

\[
Y_t = \left[ \int_0^1 Y_t(z)^{(\epsilon-1)/\epsilon} dz \right]^{\epsilon/(\epsilon-1)} \tag{1.39}
\]

and the consumption price index is

\[
P_t^C = \left[ \int_0^1 P_t^C(z)^{(1-\epsilon)} dz \right]^{1/(1-\epsilon)} \tag{1.40}
\]

In each period, only a fraction $\theta$ of retailers choose prices $P_t^*$ to maximize expected profits until the next expected price change. Retailers transfer profits back to workers’ households.

\[
P_t^C = [\theta P_{t-1}^{1-\epsilon} + (1 - \theta)(P_t^*)^{1-\epsilon}]^{1/(1-\epsilon)} \tag{1.41}
\]

The optimal price-setting of monopolistically competitive retailers leads to a New Keynesian Phillips curve. In log-linear form, where lower-case letters define log-deviations
from the steady state, this is expressed as:

$$\pi_t = E_{t-1}[\kappa(-x_t) + \beta \pi_{t+1}]$$

(1.42)

where $\beta$ is the discount factor for workers’ households and $\kappa = (1 - \theta)(1 - \theta \beta)/\theta$.

### 1.6.4 Government Policies and the Resource Constraint

The government consumes a fraction of output, financing it by taxes collected and seigniorage received.

$$G_t = \frac{M_t - M_{t-1}}{P_t} + T_t$$

(1.43)

The central bank sets the nominal interest rate according to an inflation-targeting monetary policy rule.

$$R^n_{t+1} = (R^n_t)^\rho \Pi_t \varepsilon^n_{t+1}$$

(1.44)

where $\varepsilon^n_{t+1}$ is a monetary policy shock.

The resource constraint is the national accounts identity in a closed-economy setting

$$Y_t = C_t + I_t + G_t + C^e_t + \phi^y_t$$

(1.45)

where $\phi^y_t$ represents the resources devoted to monitoring costs, which are lost. The complete log-linearized model can be found in Appendix 1.C, which pays special attention to describing the financial sector block.

### 1.7 Model Simulations

#### 1.7.1 Calibration

The parameters are calibrated similarly to Bernanke, Gertler, and Gilchrist (1999). The values are summarized in Table 1.2.

The idiosyncratic risk $\omega$ is log-normally distributed with $\text{var}[\omega] = 0.28$ and $E[\omega] = 1$. With an assumption of a 3% default rate, this implies a default threshold of 0.19 and the p.d.f. at this point is approximately equal to 0.1.
Figure 1.5: Impulse Responses of Macro Variables to a Restrictive MP Shock

Note: Impulse responses to a monetary policy shock (to the nominal interest rate) of size 0.1. The responses are in percentage deviations from the respective steady states. The red line represents the responses from the Bernanke, Gertler, and Gilchrist (1999) benchmark model. The darkest blue line represents the present model with single-period assets; lighter colors represent models with longer asset maturities (2 to 5 periods).
Table 1.2: Calibrated Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Note</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>quarterly discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\eta$</td>
<td>labor supply elasticity</td>
<td>3</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>capital share</td>
<td>0.35</td>
</tr>
<tr>
<td>$\Omega(1-\alpha)$</td>
<td>workers’ labor share</td>
<td>0.64</td>
</tr>
<tr>
<td>$(1-\Omega)(1-\alpha)$</td>
<td>investors’ labor share</td>
<td>0.01</td>
</tr>
<tr>
<td>$\delta$</td>
<td>quarterly capital depreciation</td>
<td>0.025</td>
</tr>
<tr>
<td>$G/Y$</td>
<td>s.s. share of government expenditures</td>
<td>0.2</td>
</tr>
<tr>
<td>$\phi$</td>
<td>elasticity of $Q$ to capital-to-investment</td>
<td>0.25</td>
</tr>
<tr>
<td>$K/N$</td>
<td>capital-to-worth ratio</td>
<td>2</td>
</tr>
<tr>
<td>$\Xi$</td>
<td>s.s. default threshold (3% default rate)</td>
<td>0.19</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>investors’ quarterly departure rate</td>
<td>0.0272</td>
</tr>
<tr>
<td>$\mu$</td>
<td>asset recovery (auditing) costs</td>
<td>0.12</td>
</tr>
<tr>
<td>$\theta$</td>
<td>quarterly Calvo parameter</td>
<td>0.75</td>
</tr>
<tr>
<td>$\psi$</td>
<td>monetary policy sensitivity to inflation</td>
<td>0.11</td>
</tr>
<tr>
<td>$\rho$</td>
<td>monetary policy shock smoother</td>
<td>0.9</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>autocorrelation of government expenditure shock</td>
<td>0.95</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>autocorrelation of technology shock</td>
<td>0.999</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>autocorrelation of asset price shock</td>
<td>0.9</td>
</tr>
</tbody>
</table>

1.7.2 Policy Simulations

Figures 1.5 and 1.6 show the impulse responses of the model variables (expressed in log-deviations from their steady-state values) to a 0.1 percentage point increase in the nominal monetary policy rate. In all the graphs, the red line shows the responses of the benchmark Bernanke, Gertler, and Gilchrist (1999) model, the darker blue line shows the responses of the single-period assets version of the present model, and the lighter blue lines relate to longer maturities in the multiple-period assets version of the model. Because of the standard New Keynesian features of the model (monopoly power of retailers, price rigidities), the nominal interest rate hike transfers to an increase in the real interest rate. Consumption falls as households’ optimal allocation shifts towards savings. Further, output additionally falls and inflation drops as marginal costs decrease. Investment also falls, but in a much smaller magnitude in comparison to the Bernanke, Gertler, and Gilchrist (1999) benchmark. The reason is that a large part of the shock is absorbed by the financial sector, most notably by the prices of risky assets, while capital returns and investment are affected much less. In a sense, the financial sector in this model works as an immediate shock absorber, as the non-fundamental price reacts to the shocks in a counter-cyclical manner by narrowing the wedge between capital returns and the risk-free
Figure 1.6: Impulse Responses of Financial Variables to a Restrictive MP Shock

Note: Impulse responses to a monetary policy shock (to the nominal interest rate) of size 0.1. The responses are in percentage deviations from the respective steady states. The red line represents the responses from the Bernanke, Gertler, and Gilchrist (1999) benchmark model. The darkest blue line represents the present model with single-period assets; lighter colors represent models with longer asset maturities (2 to 5 periods).

With a nominal interest rate hike, the financing costs of loans rise, inducing an elevated default threshold. Asset prices, including their non-fundamental component, fall (as they are discounted by the lending rate). As the cost of borrowing increases, investors’ wealth falls. Unlike in the benchmark model, these reactions are weaker on impact but more persistent, giving rise to momentum of the credit cycle. In the present model, borrowing reacts with an increase after several quarters, as the investors’ wealth falls and the remaining funds are borrowed, partially absorbing the shock and smoothing the investment cycle. This effect occurs because the elevated idiosyncratic risk of default makes investors demand more risky assets (which are claims on installed capital) and thus mitigates the fall of investment observed in the benchmark model.

Figures 1.7 and 1.8 show the responses of the model variables to a shock to the
Figure 1.7: Impulse Responses of Macro Variables to a Shock to the Non-Fundamental Part of the Asset Price

Note: Impulse responses to an asset price shock of size 0.1. The responses are in percentage deviations from the respective steady states. The darkest blue line represents the model with single-period assets; lighter colors represent models with longer asset maturities (2 to 5 periods).
Figure 1.8: Impulse Responses of Financial Variables to a Shock to the Non-Fundamental Part of the Asset Price

Note: Impulse responses to an asset price shock of size 0.1. The responses are in percentage deviations from the respective steady states. The darkest blue line represents the model with single-period assets; lighter colors represent models with longer asset maturities (2 to 5 periods).
Figure 1.9: Monetary Policy Efficiency with a Backward-Looking Rule

Note: The vertical and horizontal lines show the s.d. of the output gap and inflation, respectively, both in percentage deviations from the steady state. The reaction parameters are in the range (0,1) except for the low values of the inflation reaction parameter, which do not ensure determinacy. Based on 1,000 simulations.

non-fundamental component of asset prices. This shock is just an autoregressive term added to the log-linearized pricing equation (1.5). As asset prices rise, investors’ wealth increases, inducing more investment. Consumption temporarily falls, as it is optimal to postpone consumption and invest. Because of higher marginal costs, inflation rises too. The responses of the lending rate, the amount of borrowing, and the default threshold heavily depend on whether asset prices are treated as collateral as a part of investors’ wealth and on how long the inflated prices stay in the portfolio. When portfolio turnover is fast and assets mature quickly, wealth increases only temporarily and expenditure on more expensive risky assets is financed by borrowing, which increases the default threshold and lending rate. When longer maturities dominate, wealth rises more and the lending rate falls, as do the amount of borrowing and the default threshold.
1.7.3 Should Monetary Policy React to Asset Prices?

For the purposes of this exercise, the assumption of a strict inflation-targeting monetary authority was relaxed, allowing the central bank also to react to the output gap and asset prices, in both the backward-looking and forward-looking policy rules. A grid search among various combinations of reaction parameters to inflation, the output gap, and asset prices was conducted to locate the combinations of the lowest implied standard deviations of the two target variables, inflation and the output gap, under shocks to technology, government spending, asset prices, and the monetary policy rate. The combinations of minimized standard deviations define the monetary policy efficiency frontier from which the central bank can choose its optimal reaction function based on its preferences (the relative disutility from output and inflation fluctuations). Figure 1.9 illustrates the monetary policy frontier when reacting to inflation and the output gap only (in red) in comparison to the outcome when monetary policy can also react to asset prices (in blue). The version with 2-period asset maturity is used for this exercise, so that asset prices do affect the collateral constraints. The simulations suggest that there is virtually no gain from reacting to asset prices, as the lower envelope of the minimized combinations of the standard deviations of inflation and the output gap are achieved when the backward-looking central bank does not react to asset prices. The reaction to the non-fundamental component of asset prices was also tested, with a similar result: a monetary policy reaction to the non-fundamental component does not lead to lower volatility of output and inflation.

The analysis for the forward-looking monetary policy rule shows the same results: efficient combinations of the standard errors of inflation and output on the monetary policy frontier are achieved when forward-looking monetary policy does not explicitly react to current asset prices. The same holds for the reaction to the non-fundamental component of asset prices. Still, this analysis does not imply that monetary policy should not react to asset prices at all. More precisely, it suggests that the reaction should not go beyond the effects which asset prices have on the inflation and output forecast. This result is broadly in line with previous research, such as (Bernanke and Gertler 1999). Figure 1.10 shows the simulation in which all shocks except the shock to the non-fundamental component of asset prices were switched off. Even in this case, where asset prices should be most informative about the future evolution of the target variables, the numerical grid search was not able to find a parametrization under which adding asset prices would
Figure 1.10: Monetary Policy Efficiency with a Shock to the Non-Fundamental Component of Asset Prices Only

Note: The vertical and horizontal lines show the s.d. of the output gap and inflation, respectively, both in percentage deviations from the steady state. The reaction parameters are in the range (0,1), except for the low values of the inflation reaction parameter, which do not ensure determinacy. Based on 1,000 simulations.
dominate the standard monetary policy reaction function.

The explanation for this may be that asset prices, and most importantly their non-fundamental component, work as a shock absorber that cushions exogenous shocks (including monetary policy). Interfering in the endogenous absorber mechanism then does not lead to lower volatilities of the target variables.

1.8 Concluding Remarks

This paper analyzed the role of asset prices and their non-fundamental component in the dynamics of the credit cycle. We presented a model of financial intermediation where the limited liability of investors gives rise to overpricing on the market for risky assets such as shares in productive firms. The model is based on the established framework of Bernanke, Gertler, and Gilchrist (1999), but is altered substantially to include a stock market where stocks of firms with stochastic idiosyncratic returns are traded by investors with limited liability. We show a number of results. First, the prices of assets on this market exceed their fundamental values (which are defined as in the case of the absence of the principal-agent problem, i.e., if investors did not borrow but only used their own funds for trading). Second, investors prefer to face idiosyncratic, sector-specific risk rather than diversify their portfolios, because of their limited liability. Third, we show that when the nominal amount of assets traded (i.e., the product of the asset price and the number of assets traded) maintains a sufficient growth momentum, there is also a boom in investors’ wealth, credit, investment, and output. When the growth of asset prices (or traded volumes) slows down, wealth, credit, investment, and output fall below the benchmark allocations – a credit crunch occurs.

The financial sector can therefore be a source of real economic fluctuations. A positive shock to asset prices gives rise to a credit cycle. First, the lending rate and default rate decrease and wealth increases. After several quarters (depending on the maturity structure of the portfolios) the lending rate and the default rate rise as the asset price falls and shrinks collateral value. At the same time, however, the financial sector also functions as a shock absorber, because in periods of elevated risk the non-fundamental component of asset prices rises as investors prefer higher risk. This, in turn, stabilizes investors’ wealth and encourages funding for capital investment.

The model illustrates that an expansionary monetary policy shock temporarily boosts the financial market by reducing the lending rate and the fraction of defaulting investors,
and increasing asset prices (including the non-fundamental component), which in turn inflates investors’ wealth.

Finally, our estimates of monetary policy efficiency frontiers suggest that reacting to asset prices or the non-fundamental component does not help achieve more favorable combinations of inflation and output gap volatilities.

1.A Proof of Overpricing of Risky Assets

Proof. We need to show that

\[
P_t = \frac{1}{Z_{t+1}} \int_{\bar{\omega}_{t+1}}^{\infty} \omega R^K_{t+1} dF(\omega) \geq \frac{p_{t+1}^K}{r_{t+1}} = P^F_t
\]

(1.46)

We define \( \tilde{Z}_{t+1} \) and \( \tilde{P}_t \) such that

\[
\tilde{P}_t = \frac{1}{\tilde{Z}_{t+1}} \int_{\bar{\omega}_{t+1}}^{\infty} \omega R^K_{t+1} dF(\omega) = \frac{p_{t+1}^K}{r_{t+1}} = P^F_t
\]

(1.47)

By showing that \( \tilde{Z}_{t+1} > Z_{t+1} \) we will prove that \( P_t > P^F_t \). Using \( \tilde{P}_t = P^F_t = \frac{r^K_{t+1}}{r_{t+1}} \):

\[
\tilde{Z}_{t+1}(1 - F(\bar{\omega}_{t+1})) = \frac{r_{t+1}}{r^K_{t+1}} \int_{\bar{\omega}_{t+1}}^{\infty} \omega R^K_{t+1} dF(\omega)
\]

(1.48)

Eliminating \( R^K_{t+1} \) and multiplying by \( B_{t+1} \):

\[
\tilde{Z}_{t+1}B_{t+1}(1 - F(\bar{\omega}_{t+1})) = R_{t+1}B_{t+1} \int_{\bar{\omega}_{t+1}}^{\infty} \omega dF(\omega) > R_{t+1}B_{t+1}
\]

(1.49)

Now using the banks’ participation constraint (1.2):

\[
Z_{t+1}B_{t+1}(1 - F(\bar{\omega}_{t+1})) = R_{t+1}B_{t+1} - (1 - \mu) \int_{0}^{\bar{\omega}_{t+1}} \omega R^K_{t+1} S_{t+1} dF(\omega) < R_{t+1}B_{t+1}
\]

(1.50)

therefore \( \tilde{Z}_{t+1} > Z_{t+1} \) and \( P_t > P^F_t \).

Therefore, the risky asset is overpriced in comparison to the fundamental price.
1.B Investors Prefer Not to Diversify

We show that everything else being equal, investors prefer non-diversified over diversified portfolios in the presence of limited liability. Therefore, investors do not have incentives to deviate from the no-diversification equilibrium and the equilibrium is stable.

The expected return on shares net of financing costs under full diversification (no risk) equals

$$R^K_{t+1} - Z_{t+1}B_{t+1} = \int_0^\infty \omega R^K_{t+1} - Z_{t+1}B_{t+1}dF(\omega) \quad (1.51)$$

The expected return on shares net of expected financing costs when there is no diversification and the investor with limited liability fully faces the idiosyncratic risk is

$$E[R^K_{t+1} - Z_{t+1}B_{t+1}|\omega > \bar{\omega}_{t+1}] = \int_{\bar{\omega}_{t+1}}^\infty \omega R^K_{t+1} - Z_{t+1}B_{t+1}dF(\omega) \quad (1.52)$$

Now it is obvious that $E[R^K_{t+1} - Z_{t+1}B_{t+1}|\omega > \bar{\omega}_{t+1}] > R^K_{t+1} - Z_{t+1}B_{t+1}$ as

$$\int_0^\infty \omega R^K_{t+1} - Z_{t+1}B_{t+1}dF(\omega) = \int_{\bar{\omega}_{t+1}}^{\omega_{t+1}} \omega R^K_{t+1} - Z_{t+1}B_{t+1}dF(\omega) + \int_{\bar{\omega}_{t+1}}^\infty \omega R^K_{t+1} - Z_{t+1}B_{t+1}dF(\omega) < 0$$

Therefore, the investor prefers idiosyncratic risk over diversification. Because of the limited liability, the investor enjoys profits in the case of success, while he does not internalize the costs in the case of default. Limited liability thus shifts investors’ demand for risk.

1.C Log-Linearized Model

This section represents the log-linearized model including the extension of two-period assets as it enters the simulations. The financial sector block is discussed in greater detail than other parts, which come directly from Bernanke, Gertler, and Gilchrist (1999). The lowercase letters denote log-deviations from steady-state values. The ratios of capital letters denote the steady-state values of the respective ratios. Further, we define the nominal interest rate $r^n_{t+1} = r_{t+1} + E[\pi_{t+1}]$. 

37
\textbf{1.C.1 Firms}

\[ y_t = a_t + \alpha k_t + (1 - \alpha)\Omega h_t \]  
\[ k_{t+1} = \delta i_t + (1 - \delta)k_t \]  
\[ r^k_{t+1} = (1 - \epsilon)(y_{t+1} - k_{t+1} - x_{t+1}) + \epsilon q_{t+1} - q_t \]  
\[ q_t = \phi(i_t - k_t) \]  

\textbf{1.C.2 Retailers and Households}

\[ \pi_t = E_t \left[ \kappa(-x_t) + \beta \pi_{t+1} \right] \]  
\[ c_t = -r_{t+1} + E_t[c_{t+1}] \]  
\[ c^e_t = n_{t+1} + \psi^e_t \]  
\[ y_t - h_t - x_t - c_t = \eta^{-1}h_t \]  

\textbf{1.C.3 Financial Intermediation}

Although some of the log-linearized equations and variables could be eliminated in this block, the full set of equations is presented and the dynamics of all corresponding variables are shown, as the financial sector is a key part of the general equilibrium model. In addition to the main text, we introduce a shock to the (non-fundamental) asset price, \( \nu_t \), with a standard deviation of 0.1, corresponding to the empirical moment of the log-deviation of the quarterly Dow Jones index from its HP trend. \( \Xi \) stands for the steady-state value and \( \omega \) for the log-deviation from the steady state of the default threshold \( \bar{\omega}_{t+1} \), while \( f(\Xi) \) is the probability density function of the idiosyncratic return distribution evaluated at the steady-state default threshold.

\[ z_{t+1} - r_{t+1} = \frac{1}{2}(1 - \mu f(\Xi))\Xi \bar{\omega}_{t+1}; \]  
\[ p_t = E \left[ r^k_{t+1} \right] - z_{t+1} + f(\Xi)\Xi E \left[ \bar{\omega}_{t+1} \right] + \nu_t; \]  
\[ p^f_t = p_t - (r^k_{t+1} - r_{t+1}) \]  
\[ \bar{\omega}_t + r^k_t + k_t = z_t + b_t \]  
\[ p_t + q_t + k_{t+1} = \frac{N}{QK}n_{t+1} + (1 - \frac{N}{QK})b_{t+1} \]
Equation (1.62) is the bank participation constraint, linking the risk premium to the default threshold. Equation (1.63) is the log-linear version of the demand for risky assets, which, together with the previous equations, defines the wedge between the risk-free rate and capital returns as a function of the default threshold and asset prices. When we examine the determinacy conditions of the model, it turns out that the term involving the expectation of the default threshold in equation (1.63) has to be lower than the right-hand side of equation (1.62) for the model to be determined, i.e., the default has to be a sufficiently improbable (tail) event. Equation (1.64) defines the fundamental price, equation (1.65) defines the default threshold, and equation (1.66) defines borrowing. Equation (1.67) is the law of motion of investors’ wealth. In the case where assets are held for multiple periods, this equation changes to

\[ n_{t+1} = \frac{\gamma RK}{N} (r_t^k - r_t) + r_t + n_t \]

where the asset price dynamics enter the investors’ wealth. The cost of previously purchased assets \( p_{cost_{t-1}} \) equals \( p_{t-1} \) in the case of two-period assets, but can include longer lags of asset purchasing costs to create the desired mix of asset maturities in the portfolio. Similarly to Bernanke, Gertler, and Gilchrist (1999), we assume that the composition of portfolios is stable over time and the shares of assets held (including different maturities) stay close to their steady-state values. In the plots presented in the main text, maturities from one to four quarters ahead are combined in the respective portfolios with equal weights. Finally, equation (1.68) describes the supply side of the investment asset market, linking its price to the quantity of investment opportunities produced via increasing marginal costs.

**1.C.4 Monetary Policy, the Resource Constraint, and Shocks**

\[ r_t^n = \rho r_{t-1}^n + \psi \pi_{t-1} + \xi_t^n \]  

\[ y_t = \frac{C}{Y} c_t + \frac{I}{Y} i_t + \frac{G}{Y} g_t + \frac{C^e}{Y} c_t^e + \psi^y_t \]  

\[ g_t = \rho_y g_{t-1} + \xi_t^g \]
\[ a_t = \rho_a a_{t-1} + \varepsilon_a^t \]  \hspace{1cm} (1.73)

\[ v_t = \rho_p v_{t-1} + \varepsilon_p^t \]  \hspace{1cm} (1.74)
Chapter 2
Adverse Effects of Monetary Policy Signalling: The Updating Channel

Abstract
Assuming an information advantage of a central bank, an unexpected change in monetary policy interest rates signals the state and the outlook of the economy to outside agents. The subsequent update of their expectations moves in an adverse direction, counteracting the conventional transmission from interest rates to inflation and output. We develop a simple model laying down a theoretical basis for the updating channel. We further detect the presence of the updating channel in private forecasts of inflation in a cross-country sample of selected OECD countries.

Keywords: monetary transmission, monetary policy signalling, updating channel

JEL Codes: E17, E43, E58

2.1 Introduction
In this paper we explore how the actions of a central bank can influence the expectations of private agents in an adverse, perhaps unintended way. The main idea is that central bank’s actions are based on information about the state of the economy which may not be available to some of the outside agents. Information asymmetry between the central bank and outside agents can arise for several reasons. First, central banks may have access
to more detailed, but not publicly available, data (Peek, Rosengren, and Tootell 2003). Second, central banks typically assign more resources to producing reliable forecasts than private forecasters. Finally, central bank forecasts are to some extent self-fulfilling, as policymakers typically conduct monetary policy in accordance with the forecast in order to meet their policy targets. Assuming this type of information asymmetry, monetary policy actions send out informative signals to outside agents about the current and future state of the economy. These signals may counteract the conventional effects of monetary policy.

To illustrate the potential of non-standard effects of monetary policy signalling, consider the example of a surprising interest rate cut by the central bank. By monetary policy easing, a forward-looking central bank reveals that it has information suggesting anti-inflationary economic developments in the future. If the central bank’s forecasts, which are based on this information, are perceived to be reliable, outside agents may want to revise their assessment of future developments downwards and adjust their consumption/investment decisions accordingly, to reflect the central bank’s more pessimistic perception of the future state of the economy. As some authors use the term "signalling channel" in a more general sense, also including the forward guidance about future policy changes, we use the term "updating channel" to label the non-standard effects of monetary policy signalling explored in this paper. This reflects the fact that the information contained in an unexpected policy rate change leads outside agents to update their expectations about future economic developments.

The updating channel can act against the intended effects of monetary policy measures. In an extreme case, inflation expectations and inflation itself might decline after an interest rate cut, in sharp contrast to the common understanding of interest rate transmission. Indeed, a negative reaction of inflation to an interest rate cut and vice versa (the "price puzzle") has been observed in estimated VAR models (Sims 1992; Eichenbaum 1992). This anomaly casts serious doubts on the ability of central banks to control inflation in the short run.

In this paper we first suggest a simple New Keynesian general equilibrium model with a mix of fully and boundedly rational agents, where interest rate decisions by a forward-looking central bank issue signals about future inflation and output. The boundedly rational agents deviate from full rationality by their inability to form rational expectations. Instead, they update their expectations upon observing the actions of the central bank, which leads to distortions in the functioning of the monetary transmission mech-
anism. For example, if boundedly rational agents observe an interest rate hike, they interpret it as a reaction of the central bank to expected higher inflation and output and shift their inflation expectations, weakening the standard transmission of restrictive monetary policy. Under reasonable parameter values, this effect even leads to a reversed transmission of monetary policy shocks at early horizons, explaining the price puzzle. Further, we test for this updating mechanism on a panel of selected OECD countries, while previous studies (Melosi 2012; Tang 2013) focused on U.S. data. Indeed, we find that monetary policy surprises have non-standard effects on inflation expectations, based on a survey among professional forecasters (Consensus Forecasts) in accordance with the updating channel. The structure of the paper is as follows. In Section 2 we review the existing literature. In Section 3 we set up a simple New Keynesian model with boundedly rational agents, to illustrate the functioning of the updating channel and motivate the empirical part of the paper. In Section 4 we test for the existence and significance of the updating channel on a sample of OECD countries. Section 5 concludes the paper.

### 2.2 Related literature on the effects of monetary policy signalling

The theoretical foundations of the non-standard effects of monetary policy signalling have been established recently, as progress has been made in the literature on the role of information frictions in monetary policy (Sims 2010; Angeletos and La’O 2011; Mackowiak and Wiederholt 2011; Paciello and Wiederholt 2014; Adam 2007). For the updating channel of monetary policy signalling to function, the crucial assumption is that the central bank knows more than the public. Romer and Romer (2000) show that the Federal Reserve (Fed) has substantially richer knowledge about future inflation and that monetary policy actions provide signals to commercial forecasters, who substantially revise their forecasts in response to these signals. Peek, Rosengren, and Tootell (2003) also find that the Fed has an information advantage over the public in the form of confidential supervisory knowledge (for example about non-traded companies in default) which could stay undisclosed for a prolonged period of time. A number of papers (Cukierman and Meltzer 1986; Faust and Svensson 2001; M. 2005) use the information asymmetry between the public and the central bank to explain inflation bias.

Morris and Shin (2002) establish that the central bank forecast works as a coordination
device for private agents. In their model, the central bank sends signals to private agents, which have dispersed information about the state of the economy. They show that private agents tend to put more weight on the public signal than is justified by the level of its precision. Svensson (2006) shows that in the Morris-Shin model, public signals enhance welfare within a reasonable range of model parameters.

Campbell et al. (2012) distinguish between Odyssean and Delphic forward guidance. While the former involves a public commitment of the central bank to a given policy path, the latter is merely a macroeconomic forecast and a hint on possible policy reaction, often with a lot of conditioning. In our view, the updating channel may operate under both types of communication, but might be more pronounced under Delphic guidance, as this does not involve an explicit commitment to any policy response to the disclosed surprise. Supporting this view, Campbell et al. (2017) show that the puzzling reaction of private sector forecasts on Fed FOMC announcement days documented by Gürkaynak, Sack, and Swanson (2005) can be attributed to Delphic-type guidance. In this paper, we further explore this puzzling reaction, showing that it also holds for the private sector expectations of inflation itself, and proposing an explanation along the lines of the updating channel.

In relation to modelling uncertainty, Lorenzoni (2009) defines "noise shocks". In his model, households are hit by heterogeneous productivity shocks. They observe their own productivity and a noisy public signal regarding aggregate productivity. The public signal is obscured by "noise shocks", which resemble aggregate demand shocks; they increase output, employment and inflation in the short run and have no effects in the long run. Rousakis (2013) extends the model to include heterogeneity among producers and shows that noise shocks can resemble both demand and supply shocks. Another stream of literature studies the effects of expectational shocks on an economy (Beaudry and Portier 2006; Barsky and Sims 2011; Blanchard, L’Huillier, and Lorenzoni 2013).

With an idea very similar to the updating channel, Melosi (2012) proposed a general equilibrium model with dispersed information, where the central bank observes several shocks hitting the economy (technology, demand and monetary policy shocks) and signals these shocks to private agents via setting the policy rate. The model is fitted to the U.S. data, using inflation expectations from the Survey of Professional Forecasters (SPF). His results suggest that the central bank’s signals improve the effectiveness of monetary policy stabilisation in the face of demand shocks, whereas no such effect is found in the case of technology shocks.

Optimal discretionary policy with an updating channel in place leads to more emphasis on inflation. If the signal-update effect is strong, optimal policy under discretion converges to optimal policy under commitment. Tang uses SPF probability distributions to calculate a measure of subjective uncertainty and shows that when uncertainty about future inflation is high, the responses of inflation forecasts to policy rate surprises are strongly positive.

Finally, Nakamura and Steinsson (2013) show that high-frequency responses of output expectations is counter-intuitive after Fed FOMC announcements - further support for the updating channel. We extend the analysis to a panel of countries (sacrificing the high frequency) and show that these effects are much more wide-spread and can be observed in a number of inflation targeting economies.

2.3 Model

In this section, we set up a model where the signals issued by a central bank’s monetary policy may have non-standard, perhaps unintended effects. Following a forward-looking Taylor rule, the central bank signals its forecast of inflation and the output gap. Observing the short-term interest rate, the boundedly rational agents update their otherwise static expectations about future inflation and output and adjust their behaviour accordingly, which may counteract the conventional transmission of monetary policy. We show that a restrictive monetary policy shock makes the boundedly rational agents in the economy believe that the output gap and inflation will be higher than expected, which reduces the strength of the transmission of monetary policy. In some model specifications this adverse effect might even lead to reversed transmission of monetary policy, where inflation and the output gap rise with a monetary policy contraction (and fall with a restriction).

2.3.1 Model setup

We extend the standard New Keynesian setup by considering two sectors in the economy, one consisting of fully rational agents, who know the model and are able to form rational expectations, and the other consisting of boundedly rational agents. We assume a specific form of bounded rationality, under which the relevant agents are not able to form rational expectations based on the rational expectations solution of the model. In other words, the boundedly rational agents are not able (or willing) to solve the model, but otherwise behave rationally and maximize their expected utility in respect to their constraints.
Table 2.1: Notation of variables in the two sectors

<table>
<thead>
<tr>
<th>Produced by..</th>
<th>Consumed by..</th>
<th>fully rational</th>
<th>boundedly rational</th>
</tr>
</thead>
<tbody>
<tr>
<td>fully rational</td>
<td>$C_{F,t}^F, Y_{F,t}^F, \ldots$</td>
<td>$C_{F,t}^B, Y_{F,t}^B, \ldots$</td>
<td></td>
</tr>
<tr>
<td>boundedly rational</td>
<td>$C_{B,t}^F, Y_{B,t}^F, \ldots$</td>
<td>$C_{B,t}^B, Y_{B,t}^B, \ldots$</td>
<td></td>
</tr>
</tbody>
</table>

However, the boundedly rational agents still observe the actions of the central bank and can update their static expectations by filtering the information contained in interest rate decisions. Fully rational agents are aware of the bounded rationality of the agents in the boundedly rational sector and take it fully into account.

One of the key assumptions of our model is that the expectations central bank forecast are also formed rationally, in a similar manner to the expectations formed by fully rational agents. The assumption that central banks to some extent have an informational advantage is supported by empirical studies, such as Romer and Romer (2000), and is also used by Melosi (2012) and others.

As a starting point we use a standard open economy model of Galí and Monacelli (2005), which we alter to feature a common monetary policy and trivial exchange rate. We further assume that expectations in one of the economies (sectors) is formed adaptively, with learning from the monetary policy signals of the common central bank. The model consists of two sectors. We denote the variables relating to the fully rational sector of the economy by the superscript $F$ and the boundedly rational sector by the superscript $B$. The fully rational sector populates the continuum $[0, n)$, and the boundedly rational sector is populated on $[n, 1]$. For further notation, define $\omega^F \equiv \omega$ and $\omega^B \equiv 1 - \omega$. Each sector consists of consumers, producers and monopolistically competitive price-setting retailers. Subscripts denote the production side (origin) of the goods, while superscript denotes their consumption side (destination), as summarized in Table 2.1.

### 2.3.2 Consumers

Consumers in a sector $i \in \{F, B\}$ maximize their infinite lifetime expected utility derived from consumption (including habit formation) $C_{i,t}^{H,i}$ and amount of supplied labor $N_{i,t}$. 

discounted by the factor $\beta$.

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U \left( C_{t-1}^{H,i}, N_t^i \right) \quad (2.1)$$

Consumption involves habit formation $C_{t-1}^{H,i} = C_t^i - \chi C_{t-1}^i$ where $\chi$ is the external habit persistence parameter.\(^\dag\) The final consumption index $C_t^i$ consists of consumption bundles produced in the same sector $i$ and goods produced in the other sector $j$.

$$C_t^i = \left[ (1 - \alpha)^{\frac{1}{\eta}} C_{t-1}^{i} \frac{\eta - 1}{\eta} + \alpha^{\frac{1}{\eta}} C_{t-1}^{j} \frac{\eta - 1}{\eta} \right]^{\frac{\eta}{\eta - 1}} \quad i, j \in \{F,B\}; j \neq i \quad (2.2)$$

Here $\alpha$ is a parameter of relative preference for the bundles from the other sector. We assume symmetric preferences of consumers in both sectors. The consumption bundle consists of a consumption variety, giving a monopoly power to the price-setting retail sector. The consumption bundles are defined as follows.

$$C_{F,t}^F = \left[ \left( \frac{1}{\omega} \right)^{\frac{1}{\eta}} \int_0^{\omega} C_{F,t}^F(k) \frac{\xi - 1}{\xi} \, dk \right]^{\frac{1}{\xi - 1}} \quad C_{B,t}^F = \left[ \left( \frac{1}{1 - \omega} \right)^{\frac{1}{\eta}} \int_{\omega}^{1} C_{B,t}^F(k) \frac{\xi - 1}{\xi} \, dk \right]^{\frac{1}{\xi - 1}} \quad (2.3)$$

$$C_{B,t}^B = \left[ \left( \frac{1}{1 - \omega} \right)^{\frac{1}{\eta}} \int_{\omega}^{1} C_{B,t}^B(k) \frac{\xi - 1}{\xi} \, dk \right]^{\frac{1}{\xi - 1}} \quad C_{F,t}^B = \left[ \left( \frac{1}{\omega} \right)^{\frac{1}{\eta}} \int_0^{\omega} C_{F,t}^B(k) \frac{\xi - 1}{\xi} \, dk \right]^{\frac{1}{\xi - 1}} \quad (2.4)$$

Utility is maximized subject to the budget constraint. The disposable income consists of nominal payoff $D_t^i$ of a portfolio held at the end of the last period, wage income $W_t^i N_t^i$ and net transfers $T_t^i$. The expenditure side consists of consumption of goods produced in the same and the other sector, and portfolio purchases ($Q_t^i$ is a price for a bond paying off one unit of currency in the next period, and also stands for the expected stochastic discount factor). The law of one price ensures that goods produced in one sector are priced equally, regardless of where they are consumed (i.e. $P_{i,t}$ has no superscript).

$$\int_0^{\omega} P_{F,t}(k) C_{F,t}^F(k) \, dk + \int_{\omega}^{1} P_{B,t}(k) C_{B,t}^F(k) \, dk + Q_t^F D_{t+1}^F \leq D_t^F + W_t^F N_t^F + T_t^F \quad (2.5)$$

\(^\dag\)To better capture the persistence of consumption and the hump-shape of impulse response functions, we use a simplified linear version of external habit persistence as in Smets and Wouters (2003) and Dennis (2009), i.e. we assume that the representative consumer does not affect $\overline{C}_t^i$. In the following period, $\overline{C}_{t-1}^i = C_{t-1}^i$. 
\[
\int_\omega^1 P_{B,t}(k) C_{B,t}^B(k) \, dk + \int_0^\omega P_{F,t}(k) C_{F,t}^B(k) \, dk + Q_t^B D_{t+1}^B \leq D_t^B + W_t^B N_t^B + T_t^B \quad (2.6)
\]

Optimal allocation gives demand functions within each category of goods.

\[
C_{i,t}^i(k) = \frac{1}{\omega^i} \left( \frac{P_{i,t}(k)}{P_i^t} \right)^{-\varepsilon} C_{i,t}^i \\
C_{j,t}^i(k) = \frac{1}{1-\omega^i} \left( \frac{P_{j,t}(k)}{P_j^t} \right)^{-\varepsilon} C_{j,t}^i \quad (2.7)
\]

Here \(\omega^i\) is the size of sector \(i \in \{F, B\}\) and \(P_{F,t} \equiv \left( \frac{1}{\omega} \int_0^\omega P_{F,t}(k)^{1-\varepsilon} \, dk \right)^{\frac{1}{1-\varepsilon}}\), \(P_{B,t} \equiv \left( \frac{1}{1-\omega} \int_\omega^1 P_{B,t}(k)^{1-\varepsilon} \, dk \right)^{\frac{1}{1-\varepsilon}}\) are defined as price indices of goods produced in each sector. The optimal demand allocation between goods produced in the two sectors is then given by:

\[
C_{i,t}^i = (1-\alpha^i) \left( \frac{P_{i,t}}{P_i^t} \right)^{-\eta} C_i^i \\
C_{j,t}^i = \alpha^i \left( \frac{P_{j,t}}{P_j^t} \right)^{-\eta} C_i^i \quad i, j \in \{F, B\}; j \neq i \quad (2.8)
\]

The solution of the inter-temporal problem of the consumer with a CES utility

\[
U(C_{t}^i, N_{t}^i) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \]

yields the Euler equation:

\[
Q_t^i = \beta E_t \left[ \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right] \quad (2.9)
\]

Here \(P_t = [(1-\alpha) (P_{F,t})^{1-\eta} + \alpha (P_{B,t})^{1-\eta}]^{\frac{1}{1-\eta}}\) is the consumption price index. Note that the total consumption price index is the same for both sectors, given symmetric preference weight \(\alpha\). However, the inflation expectations, as well as the prices of goods produced by each sector can differ. Defining the consumption price inflation as \(\Pi_t = \frac{P_t}{P_{t-1}}\) (in log-linear terms \(\pi_t = p_t - p_{t-1}\)), the Euler equation can be log-linearized as:

\[
c_t^i = \frac{1}{1+\chi} E_t^i \left[ c_{t+1}^i \right] + \frac{\chi}{1+\chi} c_{t-1}^i - \frac{1-\chi}{\sigma (1+\chi)} \left( r_t - E_t^i [\pi_{t+1}] - \rho \right) + \varepsilon_t^D \quad (2.10)
\]

Where \(r_t \equiv -\log Q_t^i\) is the common nominal interest rate determined by the central bank, (implying also the same expected stochastic discount factors). \(\rho \equiv -\log \beta\) is the discount rate and \(\varepsilon_t^D\) is a demand shock. Finally, the optimality condition for the decision between consumption and leisure yields the inverse labour demand, \(\frac{W_{i,t}}{P_t} = C_t^{i\sigma} N_t^{i\phi}\), in log-linear terms \(w_{i,t} - p_t = \sigma c_t^i + \phi n_t\).
2.3.3 Price setting

The behaviour of producers and price-setters closely follows Galí and Monacelli (2005). The price-setting follows the standard Calvo pricing assumption, i.e. each period a randomly selected share $1 - \theta$ of firms is allowed to reoptimize prices. The optimal price-setting therefore follows

$$p_{i,t}^* = \mu + (1 - \beta \theta) \sum_{l=0}^{\infty} (\beta \theta)^l E_{i,t} \{ mc_{i,t+l} + p_{i,t+l} \}$$

(2.11)

where $p_{i,t}^*$ are the reoptimized prices and $\mu \equiv \log \frac{\varepsilon}{\varepsilon - 1}$ is the log of the steady-state markup. Combining this with Calvo aggregate price dynamics $\pi_{i,t} = (1 - \theta) (p_{i,t}^* - p_{i,t-1})$ and relating price changes to expected marginal cost deviations yields the aggregate supply equation (Phillips curve):

$$\pi_{i,t} = E_{i,t} [\pi_{i,t+1}] + \lambda mc_{i,t}$$

(2.12)

2.3.4 Production, marginal costs and relative prices

The production function for a differentiated product of a firm $k$ is linear in labor input, scaled by productivity.

$$Y_{i,t}(k) = A_t N_{i,t}(k)$$

(2.13)

Therefore the marginal cost is defined, in log terms, as wages less output prices and the productivity of labour; $mc_{i,t} = w_{i,t} - p_{i,t} - a_t$. Note that both sectors have access to identical technology.

Analogous to the open economy\(^2\) model, terms of trade between the two sectors can be defined as $S_{i,t}^j = \frac{P_{j,t}}{P_{i,t}}$, in log-linear form $s_{i,t} = p_{j,t} - p_{i,t}$. Note that from the log-linearized definition of price level,

$$p_{i,t} = (1 - \alpha^i) p_{i,t} + \alpha^i p_{j,t} = p_{i,t} + \alpha s_{i,t}$$

(2.14)

Combining this with the definition of marginal cost and the inverse labour demand:

$$mc_{i,t} = \sigma c_{i,t} + \phi n_{i,t} + \alpha s_{i,t} - a_t$$

(2.15)

\(^2\)The nominal exchange rate can be thought of as equal to unity, and because of symmetry of preferences in both sectors and similarity of consumption price levels, the real exchange rate between the two sectors is also trivially equal to unity.
2.3.5 Market clearing and aggregate output

Market clearing for the consumption goods requires that an output of an individual firm in each sector is either consumed domestically, or exported to the other sector:

\[ Y_{i,t}(k) = C_{i,t} + C_{j,t} = \left( \frac{P_{i,t}(k)}{P_t} \right)^{-\varepsilon} \left[ \frac{1 - \alpha}{\omega^i} \left( \frac{P_{i,t}}{P_t} \right)^{-\eta} C_t^i + \frac{\alpha}{1 - \omega^i} \left( \frac{P_{i,t}}{P_t} \right)^{-\eta} C_t^j \right] \] (2.16)

The second equality makes use of the demand functions. The aggregate output in each sector can be defined as

\[ Y^F_{t} = \left( \frac{1}{\omega} \int_0^{\omega} Y^F_{F,t}(k)^{1-\frac{1}{\omega}} dk \right)^{\frac{1}{1-\frac{1}{\omega}}}, \quad Y^B_{t} = \left( \frac{1}{1-\omega} \int_0^{1-\omega} Y^B_{B,t}(k)^{1-\frac{1}{\omega}} dk \right)^{\frac{1}{1-\frac{1}{\omega}}} \]

Using the definition of production price levels, output can be written as

\[ Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\eta} \left[ \frac{1 - \alpha}{\omega^i} C_t^i + \frac{\alpha}{1 - \omega^i} C_t^j \right] \] (2.17)

In log-linear terms, this equation describes the relationship between domestic output and consumption; \( y_{i,t} = \eta \alpha s_{i,t} + c_t^i + \theta \left( c_t^j - c_t^i \right) \) where \( \theta = \frac{\omega^i}{(1-\omega)(1-\omega^i)} \).

Aggregate output of the whole economy is then defined as \( Y_t = Y^F_t + Y^B_t \), in log-linear terms \( y_t = \omega y^F_{t} + (1 - \omega) y^B_{t} \). The aggregate consumer price inflation is same as consumer price inflation in each sector, resulting from the symmetric relative preference parameter \( \alpha \).

2.3.6 Monetary policy and expectations formation

To close the model, we consider a forward-looking central bank, which is able to form rational expectations similar to agents in the fully rational sector. The central bank sets nominal interest rate common to both sectors:

\[ r_t = \rho + \phi^\pi E_t^F [\pi_{t+1}] + \phi^\gamma E_t^F [y_{t+1}] + \varepsilon_t^{MP} \] (2.18)

The central bank decides on the setting of the interest rate \( r_t \) based on the natural rate of interest \( \rho \) and its fully rational forecasts of aggregate consumer price inflation \( E_t^F [\pi_{t+1}] \) and the aggregate output \( E_t^F [y_{t+1}] \) (both expressed in log-deviations from their steady states) for the following period \( (t+1) \). Sensitivity of central bank decisions to inflation and output forecasts is captured by parameters \( \phi^\pi \) and \( \phi^\gamma \), respectively. By setting the monetary policy interest rate according to the rule, the forward-looking central bank signals the expected state of the economy to the boundedly rational agents.
The core assumption of our model is that boundedly rational agents are unable to form rational expectations by solving the model of the economy. Otherwise, the agents in the boundedly rational sector behave optimally given their expectations. These expectations are not rational, but are partly backward-looking and partly updated using the information contained in the monetary policy decisions of the central bank. Particularly, boundedly rational agents observe the choice of the short-term interest rate \( r_t \) and are able to invert the monetary policy rule to update their inflation and output expectations \( E^B[\cdot] \) about the following period:

\[
E^B_t[\pi_{t+1}] = \gamma^\pi r_t - \rho - \phi^\pi E^B_t[y_{t+1}] + (1 - \gamma^\pi) \pi_t \tag{2.19}
\]

\[
E^B_t[y_{t+1}] = \gamma^y r_t - \rho - \phi^y E^B_t[\pi_{t+1}] + (1 - \gamma^y) y_t \tag{2.20}
\]

where \( \gamma^\pi \) and \( \gamma^y \) are the parameters driving the gain with which boundedly rational agents update their static expectations of inflation and output, using the information contained in the interest rate decisions. We assume that the boundedly rational agents expect the sector-specific inflation and output to be the same as their expectations of aggregates, \( E^B_t[y_{i,t+1}] = E^B_t[y_{t+1}] \) and \( E^B_t[\pi_{i,t+1}] = E^B_t[\pi_{t+1}] \), and further that they have equal consumption and income expectations \( E^B_t[c_{i,t+1}] = E^B_t[y_{i,t+1}] \). \(^3\)

### 2.3.7 Shock processes

We introduce technology, demand and monetary policy shocks to the model, and assume that each of them follows an AR(1) process. Note that all shocks are common to both sectors.

\[
a_t = \rho^a a_{t-1} + \nu^a_t \tag{2.21}
\]

\[
\epsilon^D_t = \rho^D \epsilon^D_{t-1} + \nu^D_t \tag{2.22}
\]

\(^3\)We have experimented with a deeper structural expectation formation of boundedly rational agents, recognizing that their expected inflation and output may differ across sectors and that expected consumption may differ from expected output. However, this led to more complex mutual dependencies in expectation formation, which we considered inconsistent with our relatively simple definition of boundedly rational agents. Moreover, we often encountered indeterminacies caused by the attempts to infer 4-6 expectation terms from a single policy rate.
ε_t^{MP} = ρ^{MP} e_{t-1}^{MP} + ν_t^{MP} \quad (2.23)

The error terms \( ν_t^A, ν_t^D, ν_t^{CP} \) and \( ν_t^{MP} \) are all serially and mutually independent and identically distributed.

### 2.4 Model implications

The distinctive features and key parameters of our model are the share of fully rational agents \( ω \) and the learning parameters \( γ^x \) and \( γ^y \) of boundedly rational agents. In the limiting case of \( ω = 1 \) the model collapses to the simple New Keynesian model with fully rational agents only. In the opposite case when boundedly rational agents have fully static expectations \( γ^x = γ^y = 0 \), they do not infer any information from the interest rate setting and no signalling effects emerge.

To illustrate the updating channel, we start by assuming that 70% of the agents are fully rational \( (ω = 0.7) \) and that boundedly rational agents derive their expectations equally from past observations and from learning from the policy rate, \( γ^x = γ^y = 0.5 \). We also set the relative preferences for goods produced in both sectors to be equal, \( α = 0.5 \), i.e. there is no "home bias". The habit persistence parameter is set to \( χ = 0.9 \), in line with Fuhrer (2000). The other model parameters are calibrated in line with Clarida, Gali, and Gertler (1999).

#### 2.4.1 Basic dynamics of the updating channel

We report the impulse response functions to all three types of shocks in the model: demand shock, technology shock and monetary policy shock. The size of all shocks is 1 percentage point. Note that we assume common shocks to both sectors. The responses represent log-deviations from the respective steady state values. We compare the results from our model featuring the updating channel against a baseline model with the same specification under full rationality.

The impulse response functions show that the technology shock \( ν_t^a \) has standard effects on all the variables of the model (Figure 2.1). It raises output and reduces inflation in both sectors and in the aggregate. The central bank reacts to the shock by cutting interest rates. However, the boundedly rational agents see the cut as a signal of future slowdown, so they increase output by a smaller amount compared to the baseline model.
Figure 2.1: Responses of model variables to technology shock

Figure 2.2: Responses of model variables to demand shock
The responses to demand shock \( \nu^D_t \) also show standard behaviour (Figure 2.2). Higher aggregate output leads to higher inflation in both the fully rational and boundedly rational sectors. The central bank reacts by raising interest rates, which gradually pushes aggregate output and inflation back towards the steady state. The weak response of the output of boundedly rational agents is the result of high persistence of their output expectations which makes the output reaction (largely driven by consumption smoothing according to the Euler equation) very weak, and further restrained by expected higher inflation and restrictive monetary policy reaction.

The monetary policy shock \( \nu^M_P_t \) is of key interest to us (Figure 2.3). The boundedly rational agents interpret the change in interest rates as a standard monetary policy reaction to expected inflation and output. Therefore, in the case of a contractionary monetary policy shock, the boundedly rational agents expect higher inflation which also spills over to the fully rational sector and creates a price puzzle. This reinforces the early restrictive effects of the interest rate hike on output, especially in the boundedly rational sector. The fully rational sector partially compensates for this, leading to a hump-shaped response, peaking after around 8-10 quarters in both output and inflation - a very sluggish response for a structural model, being closer to empirical estimates from vector autoregressive models.
2.4.2 Sensitivity to key parameter values

The effects of the monetary policy shock on aggregate inflation and output depends heavily on some of the parameters of the model. In the following text we explore the sensitivity of model dynamics to the share of boundedly rational agents, their learning gain and habit persistence. In each of the exercises, all the remaining parameter values are held constant and identical to the benchmark case described above.

The key parameter of our model is the share of boundedly rational agents. When their share is high ($\omega = 0.1$), responses of inflation to a restrictive monetary policy shock are conventionally negative, but have a pronounced hump-shape peaking around 4 quarters after the impact, the lag being caused by the slow learning of boundedly rational agents. In the opposite case, with most agents being fully rational (also our main specification model with $\omega = 0.7$), the initial reaction of inflation is positive as discussed above, offering an explanation of the empirically observed "price puzzle" based on the updating channel. Further, the lag of the peak response gets even longer.

Another important parameter for the strength of the updating channel are the learning parameters of boundedly rational agents (in the simulation we set $\gamma = \gamma^\pi = \gamma^y$). When learning is weak ($\gamma = 0.1$), the response of inflation and output to a restrictive monetary policy shock is conventionally negative and hump-shaped, peaking earlier (4 quarters) for inflation and later (6 quarters) for output. However, when the boundedly rational agents react more strongly to the policy signal ($\gamma = 0.9$), the updating channel is stronger; both inflation and output rise early after the impact of the shock (the price puzzle) and then continue with a pronounced hump-shape, with longer lags of the peak response (Figure 2.5).

Finally, the impulse responses of aggregate inflation and output to the monetary
policy shock also depend on the habit persistence parameter $\chi$. In the model without habit persistence ($\chi = 0$), both inflation and output decline in response to the monetary policy shock. However, with strong habit formation (close to 0.9), the impulse responses of inflation become positive on the early horizons as shown in Figure 2.6. Therefore a sufficient degree of habit formation is needed to observe sizable effects of the updating channel.

Overall, the simulation results suggest that our model is sensitive to the calibration of its key parameters: the learning parameter of boundedly rational agents, their share in the economy and habit persistence. A slight shift in the values of the key parameters can change the direction of the impulse response reactions to a monetary policy shock. The updating channel emerges with strong learning from monetary policy signals, low share of boundedly rational agents and high habit persistence.
2.5 Empirical assessment of the updating channel

In this section we evaluate the significance and magnitude of the updating channel of monetary policy on a cross-country sample of OECD countries, using the Consensus Forecasts (CF) as a proxy for inflation expectations.

2.5.1 Empirical methodology

The question to be tested is whether an unexpected interest rate decision triggers an update of the inflation expectations of outside agents in a non-standard direction. In particular, the model presented in the previous section suggests that an unexpected hike in the policy rate can signal to the boundedly rational agents that the central bank expects high inflation, leading to an update of inflation expectations in an adverse direction, counteracting the effects of the policy restriction. To test for the presence and the strength of the updating channel, we conduct an empirical estimation of how interest rate surprises influence inflation expectations in a panel of selected OECD countries.

We estimate the following basic specification:

\[
\Delta E_{t,i}^{PUB} [\pi_{t+h}] = \delta_i + \lambda^{PUB} \Delta E_{t-1,i}^{PUB} [\pi_{t+h,i}] + \lambda^{CB} \Delta E_{t-1,i}^{CB} [\pi_{t+h,i}] + \phi_{t-1,i}^{SURP} + \\
+ \tau^{TI} I_{t-1,i}^{SURP} + \tau^{E} T_{t-1,i}^{E} \Delta E_{t-1,i}^{CB} [\pi_{t+h,i}] + \tau T_{t-1,i} + \beta \Delta X_{t,i} + \mu C E_t + \nu_{t,i} \quad (2.24)
\]

In this equation, we examine whether the change in the inflation forecast of the outside public \( E_{t,i}^{PUB} [\pi_{t+h}] \) for horizon \( h \) depends on the lagged change of the central bank’s forecast \( E_{t-1,i}^{CB} [\pi_{t+h,i}] \) and the lagged policy interest rate surprise \( I_{t-1,i}^{SURP} \), while controlling for the effects of contemporaneous macroeconomic news contained in the vector \( X_{t,i} \) and the calendar effect \( C E_t \) (see below for explanation). The country fixed effects \( \delta_i \) control for time-invariant country-specific endogeneity (such as overall more sluggish reaction of expectations in a given economy).

Although in the model presented above the boundedly rational agents update their inflation and output gap forecasts by filtering from the changes in the interest rate, it should be noted that many central banks publish forecasts to guide expectations, which may interfere with the information content of interest rate changes. Therefore, we control for published central bank forecasts in the estimated equations. We suppose that if there was any new information value in a published central bank forecast, the outside public would use it to update their forecasts. This effect is captured by the estimated parameter...
We expect that there might be an additional piece of information contained in the interest rate decision itself. This is either because the published forecast may be biased to guide expectations in a desired way, or because the interest rate decision may contain more judgment-based and/or more recent information than the published forecast. The differences between the views of the staff preparing the forecast and the views of the decision-making committee may also carry a valuable signal. We estimate equation 2.24 to examine whether the interest rate surprises affect agents’ expectations about inflation, and if so, in which direction and to what degree. This effect is captured by the parameter \( \phi \). Non-zero values of \( \phi \) would imply that the monetary policy surprise conveys extra information in addition to its published forecasts and other publicly available information. The standard understanding of the transmission of monetary policy shocks would imply negative \( \phi \); an unexpected monetary policy tightening would drive down expected inflation, as higher interest rates are understood to reduce inflation in the medium term. However, if the updating channel works as hypothesised in the previous section, we can observe positive \( \phi \); an unexpected interest rate hike increases the inflation expectations of outside agents. This may be because the agents update their prior information on future inflation by filtering the signal of perceived inflationary pressures, which is revealed by the central bank hiking its rates.

Finally, we include the central bank transparency index \( TI_t \) and the interaction terms of \( r_{t-1,i}^{SURP} \) and \( E_{t-1,i}^{CB} [\pi_{t+h,i}] \) with the transparency index. If the central bank reveals detailed information about future macroeconomic developments, an interest rate surprise might be understood as an unexpectedly timed expected policy. However, when the central bank does not publish its forecast, an interest rate surprise might convey much more information, and private agents are more likely to change their expectations about the fundamental trends in the economy. On the other hand, central bank transparency and credibility may strengthen the value of the information contained in monetary policy actions. As we show later, the empirical results for the effects of central bank transparency are as inconclusive as these theoretical considerations.

2.5.2 Data

The explained variable in the main regression equations, the change in agents’ inflation expectations \( \Delta E_{t,i}^{PUB} [\pi_{t+h}] \), is computed from the Consensus Forecasts professional sur-
vey for the countries concerned. Although Consensus Forecasts are published monthly, the surveys only collect forecasts referring to current and next calendar years. This causes problems in January, when the current year and next year reference switches to a different calendar year. We mitigate the problem by constructing the change of private expectations in January as the difference between the January forecast for the current year and the previous month (December) forecast for the next year. Further, using lagged values of forecast changes forces us to leave out the January observation in every year. Due to the use of calendar years, a jump in the forecast horizon occurs at the beginning of each year, with a horizon of one year ahead in December changing to a horizon of one-to-two years ahead in February. We account for this effect by adding a month index $CE_t$ which starts from 1 in January and increases to 12 in December each year. This calendar effect captures the fact that the updates of expectations might have different strength with changing forecast horizons.

For the construction of the central bank forecast change we gathered vintage data of central banks’ inflation forecasts from their inflation reports and websites. Here again, the published forecast figures usually refer to calendar years, which makes them consistent with the Consensus Forecast data. In some cases in earlier periods, the projections were reported for the following 12 months. Where these one-year-ahead forecasts roughly correspond to a given calendar year, we include these observations in the sample. Further, central bank forecasts are generally not issued every month. We assume, for example, that a forecast issued in March is still valid in April and May, until the new forecast is released in June.

Another key explanatory variable is the interest rate surprise. As the main hypothesis of this paper concerns the effects of this particular variable, we construct three alternative measures of the interest rate surprise to see how robust the results are. First, we construct the interest rate surprise as the difference between Consensus Forecasts expected change in the 3-month money market rate and the realized change in the monetary policy rate

$$r_{t,i}^{SURP} = [r_{t,i}^{CB} - r_{t-1,i}^{CB}] - [E_{t-1,i}^{PUB}[r_{t+3,i}^{3M}] - r_{t-1,i}^{3M}]$$

(2.25)

However, we were concerned that this definition of the policy rate change might be prone to endogeneity, because news arriving between the most recent Consensus Forecast release (and thus affecting the change in the forecast) and the interest rate decision does affect both the interest rate surprise and the change in Consensus Forecasts expectations.
To reduce the endogeneity problem, we lag the interest rate surprise by one month and use it as an instrumental variable for the current surprise.

Furthermore, we construct an alternative measure of surprise as the deviation from the monetary policy rate implied by an estimated Taylor rule. We use both a backward-looking (based on current observations) and a forward-looking Taylor rule (based on central banks’ forecasts), both of which are estimated for each country separately. The endogeneity bias should be smaller as the new information is already embedded in the forecasts entering the Taylor rule.

We interact the policy rate surprise and central bank forecast variables with a measure of central bank transparency. As proxy for central bank transparency we use Siklos’ (2010) transparency index database, which we update for improvements in central banks’ transparency after 2009. As further macroeconomic control variables we include the change in the inflation rate (we use one lag because inflation data are typically published with a one-month lag) and the real-time most recently observed change in real GDP growth (we use the OECD database of real-time data to get the vintages of GDP revisions). We also include the change in the nominal effective exchange rate as a control variable. Inclusion of these variables should further reduce the endogeneity problem.

We use data from 12 economies, including both inflation-targeting countries (Canada, the Czech Republic, Hungary, Norway, Poland, Sweden, Turkey, Japan and the United Kingdom) and economies where inflation targeting is not explicit (the euro area, Switzerland and the United States). The selection of countries was guided by data availability and structural similarity (developed economies, inflation targeters). The availability of both central bank forecasts and Consensus Forecasts was another criterion. The sample covers the period between January 2001 and March 2013. We compiled more than 1000 effective observations on a monthly frequency.

Table 2.2 shows the summary statistics of the key variables entering the regressions. Notably, all the measures of the policy interest rate surprise are centered around mean values which are not statistically different from zero (the standard deviation comes from the pooled sample). The same holds for the mean revisions in the central bank forecasts and Consensus Forecasts. Interestingly, the standard deviations of the surprises are marginally lower when the deviations from Taylor rules are used as the definition, suggesting that Taylor rules may be better predictors for interest rate changes than surveys among forecasters and analysts. The summaries for individual countries (available upon request) show that the forecast revisions and policy rate surprises were generally higher
Table 2.2: Summary statistics of key variables

<table>
<thead>
<tr>
<th></th>
<th>count</th>
<th>mean</th>
<th>sd</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change of CPI forecast, CF</td>
<td>1452</td>
<td>-0.0092975</td>
<td>0.302191</td>
<td>-3.1</td>
<td>2.5</td>
</tr>
<tr>
<td>Change of CPI forecast, CB</td>
<td>1393</td>
<td>0.0027563</td>
<td>0.3507938</td>
<td>-1.8</td>
<td>3.7</td>
</tr>
<tr>
<td>Policy rate surprise, CF</td>
<td>1464</td>
<td>0.0174932</td>
<td>0.54146</td>
<td>-2.8</td>
<td>9.9</td>
</tr>
<tr>
<td>Policy rate surprise, Taylor BW</td>
<td>1657</td>
<td>-1.67e-09</td>
<td>0.3828077</td>
<td>-3.666882</td>
<td>5.534725</td>
</tr>
<tr>
<td>Policy rate surprise, Taylor FW</td>
<td>1212</td>
<td>2.95e-09</td>
<td>0.2434574</td>
<td>-2.052365</td>
<td>3.05042</td>
</tr>
</tbody>
</table>

in converging economies such as the Czech Republic, Hungary and Turkey than in the other economies in the sample.

We ran both fixed effects and random effects estimators when estimating equation 2.24. Although the fixed effects estimator is more likely to be consistent as it controls for time-invariant endogeneity, the Hausman test suggests that there is no significant difference between random and fixed effects estimates. We present only the results of the random effects regressions because of their higher efficiency. We also attempt to control for further sources of endogeneity in estimating the key parameters. We control for the endogeneity resulting from employing a lagged dependent variable in a panel regression by GMM-style instrumenting for the lagged dependent variable in the spirit of Arellano and Bond (1991).

As already noted above, further endogeneity might arise from the fact that some important macroeconomic news (not immediately observed in macroeconomic data, such as the extreme case of the fall of Lehman Brothers in October 2008) might have arrived between the time when the Consensus Forecast was released and the date of the monetary decision. To eliminate this source of endogeneity we lag the interest surprise in regression 2.24 and instrument the interest rate surprise by using its own lagged values and other exogenous regressors. However, part of this type of endogeneity may still be present, as it is not clear when exactly the Consensus Forecast sources update their forecasts and what information they manage to reflect. In other words, the lag of the Consensus Forecast revision after an event may be even longer than one month.

In equation 2.24, all the coefficients except the intercept are assumed to be uniform across countries, which gives us a sufficient number of observations for the estimation. However, this assumption might be too strong and the coefficients might differ significantly between countries. To some degree, we treat this potential heterogeneity by
defining interest rate surprises as deviations from Taylor rules estimated separately for each country. The proper way to handle heterogeneity would be to extend the estimated equation to include the interaction terms of $\pi_{t-1,i}^{SURP}$ and $E_{CB,t-1,i}^{\pi}$, $E_{CB,t-1,i}^{\pi+h,i}$ with the ratio of the estimated Taylor coefficients $\phi_{\pi}/\phi_{y}$. We ran these estimations and found some of these interaction terms were statistically significant with negative signs. However, the core parameters we examine (the effects of an interest rate surprise and central bank forecasts) and the power of the estimation remained mostly unchanged. Therefore, and for the sake of clarity of the results, we present only baseline estimation results based on equation 2.24.

2.5.3 Empirical results

The estimation results for inflation expectations suggest that the effects of central banks’ signals are robustly significant for all three specifications of the interest rate surprise (Table 2.3). This is consistent with the results of other empirical studies (Romer and Romer 2000; Filáček and Saxa 2012). What is striking is the robust significance of the coefficient on the interest rate surprise, which goes in the direction of our "updating channel" hypothesis.

According to the estimation results in all specifications, a contractionary monetary policy surprise triggers an increase in private inflation expectations, which contradicts the traditional understanding of monetary policy transmission. Our hypothesis is that part of this effect can still be ascribed to endogeneity resulting from common news, but part is the adverse signalling effect of monetary policy. As agents observe an unexpected interest rate hike, they may infer that the central bank expects inflation to rise and adjust their expectations accordingly. It is important to stress that this effect is present even when we control for the effect of published central bank forecasts. The sensitivity of expectations to the interest rate surprise is roughly of the same size as the sensitivity to the central bank forecast. This suggests that the information contained in the unanticipated interest rate decision complements the information contained in the forecast.

The signalling effect seems to be independent of the degree of central bank transparency. Transparency does not appear to play a significant role in affecting private forecasts in the regression, regardless of whether we use the overall transparency index or its economic transparency sub-component. This might be explained by the ambiguous effects of central bank transparency on the strength of the updating channel. As
mentioned in the introduction, on the one hand higher transparency reduces the effects of surprising monetary policy actions on expectations, but on the other, makes surprises less likely. If a transparent central bank makes a surprising move, it might have more pronounced effects. Transparency also often goes hand in hand with credibility which can make the monetary policy signals more informative.

We do not observe significant effects of macroeconomic control variables except for observed inflation; the calendar effect is significant for all specifications.

2.6 Conclusions

In this paper we explored the possibility that an unexpected change in the monetary policy rate guides the expectations of private agents in an adverse direction. We label these effects of monetary policy signalling as an "updating channel", because the information contained in the unexpected policy rate change leads to an update of outside agents’ expectations about future economic developments.

First, we have built a simple New Keynesian general equilibrium model with boundedly rational agents to illustrate the idea of the adverse effects of monetary policy signalling and explored the sensitivity of the strength of the updating channel to deep parameters, such as the learning strength of boundedly rational agents. We illustrate that when the strength of learning from central bank decisions is high, a substantial price puzzle emerges and the transmission of monetary policy shocks to the economy can be reversed for several quarters. This may happen, for example, in the case of monetary policy changes close to the zero lower bound on nominal interest rates, where the uncertainty is high and monetary policy changes might carry valuable information content.

Second, we explore the behaviour of Consensus Forecasts (as a proxy for expectations) in response to unexpected interest rate changes. We document a significant and robust relationship between interest rate surprises and changes in expected inflation, which appears to counteract the standard monetary policy transmission. This effect, however, can be partially ascribed to the presence of endogeneity in the estimation, although we use a number of techniques to mitigate its impact.

Overall, we have illustrated the possibility of the adverse effects of monetary policy signalling in a theoretical model and documented the presence of these effects in the data. Further research is needed to explore, confirm and measure the strength of the updating channel, and also to propose a policy to mitigate or counteract its adverse effects.
Table 2.3: Effects of central bank decisions and forecasts on expected inflation

<table>
<thead>
<tr>
<th></th>
<th>Basic Surprise</th>
<th>Taylor BW Surprise</th>
<th>Taylor FW Surprise</th>
</tr>
</thead>
<tbody>
<tr>
<td>L.Change of CPI forecast, CF</td>
<td>-0.0491 (-0.81)</td>
<td>0.0273 (0.47)</td>
<td>0.233*** (2.67)</td>
</tr>
<tr>
<td>L.Policy rate surprise</td>
<td>0.0602*** (4.23)</td>
<td>0.116*** (4.68)</td>
<td>0.102*** (4.19)</td>
</tr>
<tr>
<td>L.Change of CPI forecast, CB</td>
<td>0.125*** (6.12)</td>
<td>0.108*** (5.49)</td>
<td>0.0993*** (4.34)</td>
</tr>
<tr>
<td>Transparency</td>
<td>0.000284 (0.10)</td>
<td>-0.000128 (-0.05)</td>
<td>-0.000953 (-0.38)</td>
</tr>
<tr>
<td>Transp.× P.R.surprise</td>
<td>0.00732 (0.89)</td>
<td>0.00913 (0.72)</td>
<td>0.00408 (0.32)</td>
</tr>
<tr>
<td>Transp.× Chng of CPI forec.CB</td>
<td>-0.00309 (-0.30)</td>
<td>-0.00131 (-0.13)</td>
<td>0.00155 (0.15)</td>
</tr>
<tr>
<td>LD.CPI Inflation</td>
<td>0.0829*** (8.59)</td>
<td>0.0791*** (8.33)</td>
<td>0.0716*** (6.75)</td>
</tr>
<tr>
<td>D.NEER</td>
<td>0.00183 (0.74)</td>
<td>0.000943 (0.39)</td>
<td>-0.000805 (-0.33)</td>
</tr>
<tr>
<td>D.GDP growth vintage</td>
<td>-0.291 (-0.60)</td>
<td>-0.665 (-1.38)</td>
<td>0.101 (0.19)</td>
</tr>
<tr>
<td>Calendar effect</td>
<td>-0.00392*** (-2.65)</td>
<td>-0.00471*** (-3.24)</td>
<td>-0.00528*** (-3.53)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.0275** (2.37)</td>
<td>0.0312*** (2.78)</td>
<td>0.0325*** (2.79)</td>
</tr>
<tr>
<td>Observations</td>
<td>1032</td>
<td>1036</td>
<td>838</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.122</td>
<td>0.167</td>
<td>0.190</td>
</tr>
</tbody>
</table>

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Chapter 3
Financial Stress and Its Non-Linear Impact on CEE Exchange Rates

Jakub Matějů, Tomáš Adam and Soňa Benecká

Abstract

This essay explores the reaction of selected CEE (satellite) currencies to increased financial stress in the euro area (core) and also in global financial markets. We suggest that this reaction might be non-linear; the “safe haven” status of a satellite currency may hold in calm periods, but breaks down when risk aversion is elevated. A stylized model of portfolio allocation between assets denominated in euro and the satellite currency suggests the presence of two regimes characterized by different reactions of the exchange rate to an increased stress in the euro area. In the “diversification” regime, the satellite currency appreciates in reaction to an increase in the expected variance of EUR assets, while in the “flight to safety” regime, the satellite currency depreciates in response to increased expected volatility. We suggest that the switch between regimes is related to changes in risk aversion, driven by the level of strains in the financial system as captured by financial stress indicators. Using the Bayesian Markov-switching VAR model, the presence of these regimes is identified in the case of the Czech koruna, the Hungarian forint and the Polish zloty.

Keywords: asset allocation, exchange rates, financial stress, Markov-switching

JEL Codes: E44, F31, G12, G20
3.1 Introduction

One of the financial stability challenges of central European economics (CEE) outside of the euro area stems from foreign exchange risk. As in other small economies, movements in exchange rates affect the balance sheets of private firms involved in international trade by altering the value of their exports and imports. On the financial side, corporations and households are exposed to foreign exchange risk due to the mismatch of the currency structure of their assets and liabilities. The impact of foreign exchange movements on the financial position of the real sector is further complicated in the CEE region by the ownership structure of its banks. Since the major banks, on which the private sector is dependent, are foreign owned and the parent companies are located in the euro area, investors' perception of their health can affect the exchange rates in the CEE countries. It is therefore plausible that a financial shock in the euro area countries ("core") directly unrelated to the CEE ("periphery") countries can result in the depreciation of the latter's currencies.

This paper explores the impact of financial stress in the "core" countries on exchange rates in the "periphery" countries. It suggests that the relationship can be non-linear and dependent on the degree of risk aversion. Our analysis is novel in several aspects. Previous literature has often considered the reaction of exchange rates to stress, with the focus on carry trades and has assumed that the "safe haven" status of a currency is time-invariant. Here we take a more elaborate approach, recognizing that currencies which are "safe havens" in some periods may lose this status in more turbulent times. Further, we theoretically link these distinct regimes (labelled "diversification" and "flight to safety") to the attitude of institutional investors towards risk - risk aversion in general terms. Finally, we test empirically for the presence of such regimes in CEE currencies, finding that the Czech koruna (CZK), the Polish zloty (PLN) and the Hungarian forint (HUF) have switched the regimes several times, and those switches have coincided with periods of elevated risk aversion.

There are two major difficulties in studying the link between exchange rate dynamics, financial stress and risk aversion. First, risk aversion as a behavioral characteristic of financial agents is an unobserved variable, and there are only estimates and proxies available. At the same time, risk aversion supposedly rises when the degree of financial stress itself is elevated. Therefore we are forced to reduce the three-dimensional problem into a two-dimensional one, investigating the reaction of exchange rates to financial
stress, and interpreting the non-linear responses and observed regime switches as related
to elevated risk aversion causing behavioral change. At the same time, we argue that the
regime switches do not occur at specific thresholds, as the measures of financial stress
themselves are imperfect, and a commonly measured level of stress in recent years may
exceed peaks in the previous decade, reflecting also structural changes in financial mar-
kets. Therefore, a more elaborate approach to the regime switching process is needed,
and we opt for a Bayesian Markov-switching VAR model.

Second, the existing literature has often suggested a simple link: an increase in risk
aversion causes funds to be withdrawn from emerging economies and their currencies to
depreciate. In contrast to this finding, we suggest that the reaction of emerging currencies
(CZK, PLN and HUF) to increased uncertainty depends on the level of risk aversion in the
core advanced economy (the euro area in our case). When euro area financial markets are
calm and hence risk aversion is low, the search-for-yield effect drives trades in emerging
currencies, including carry trades. This may lead to emerging currencies appreciating
in response to a mild increase in uncertainty in the euro area. On the other hand,
when advanced markets become turbulent, funds start to be withdrawn from satellite
economies. Any increase in financial stress causes the satellite currencies to depreciate.
As a result, the link can be non-linear and the relationship can operate in several regimes.

We provide both theoretical background and empirical evidence. To start with, we
present a simple model of a stylized currency portfolio where the endogenously selected
weights of assets denominated in “core” and “satellite” currencies in the optimal portfolio
respond to changes in the variance (uncertainty) of euro area assets. In this model we
identify two regimes (related to the degree of risk aversion and portfolio management
strategy), based on the different reactions of the exchange rate to increased uncertainty.
The “diversification” regime is characterized by the koruna appreciating in response to
increasing uncertainty of euro asset returns, while the “flight to safety” regime is charac-
terized by the koruna depreciating in response to increasing uncertainty of euro asset
returns. Finally, using the Markov-switching model we manage to identify different ex-
change rate reaction regimes in the case of the CZK and HUF in reaction to financial
stress captured by the CISS indicator of euro area financial stress. However, the evidence
for the PLN is more mixed. Also, we do not find different regimes in the reaction of
exchange rates to global stress captured by the VIX index, as the global stress does not
directly relate to euro area domestic uncertainty and is therefore more likely to affect
satellite currencies more uniformly in the “flight to safety” fashion.
The paper is structured as follows. First, it reviews relevant literature related to the concept of financial stress, its measures, and its impact on exchange rates. Next, based on a model of portfolio rebalancing, it provides a theoretical motivation for the link between the exchange rate and risk aversion. Finally, it estimates how the CZK, PLN and HUF have reacted to the evolution of financial stress in the various regimes identified in both the euro area and the global economy.

3.2 Financial Stress: Measurement and Potential Impact on Exchange Rates

Financial stress can be described as a situation where the normal (smooth) functioning of financial markets is severely impaired. Under these conditions, the financial system is threatened by substantial losses. Financial stress is marked by a higher degree of perceived risk (a wider distribution of probable losses) as well as uncertainty (decreased confidence in the shape of that distribution), according to Misina and Tkacz (2008). The uncertainty leads to increased volatility of asset prices, which can then alter the risk aversion of traders. As a recent study by Kandasamy et al. (2014) shows, during periods of extreme market volatility traders’ attitudes toward taking risks changes. We make use of this link when exploring the reaction of a selected currency exchange rate to increased uncertainty in foreign financial markets.

3.2.1 Measuring Financial Stress

Financial stress indicators aggregate a set of stress measures, such as volatilities and spreads, from various market segments, such as the money market, bond market, stock market, and foreign exchange market,\(^1\) into a single time series. Even early papers on financial stress, which used simply constructed stress indicators (in terms of the aggregation methods or variables used), were able to capture most stressful events as perceived by experts (Illing and Liu 2006). However, over time, more elaborate indices have been constructed. In particular, the global financial crisis gave a strong impulse to research in this field, highlighting the importance of financial stress for real economic activity.

\(^1\)Some studies additionally include macroeconomic or financial stability indicators (such as private credit) to capture the overall economic conditions. We stick to the financial markets context as we intend to assess the impact of financial stress only.
The construction of indices varies both in the stress measures included and in the methods used to aggregate them. To the best of our knowledge, there is little theoretical background for modeling financial stress, so these choices are often arbitrary. Several indicators have been constructed for individual economies (such as the U.S., the euro area, and Canada), as well as more general indicators to be used across countries (Cardarelli, Elekdag, and Lall 2009). A number of studies have shown the impact of financial stress on real or financial variables, as well as on monetary policy. However, the financial stress indicators themselves seem difficult to predict, according to Slingenberg and de Haan (2011), and their potential use in forecasting seems to be limited so far.

In the post-crisis period, the focus has shifted to the construction of financial stress indicators to capture systemic risk. The contagion effect is an important element of systemic risk, so these indices should reflect situations where stress materializes simultaneously in several interconnected markets. Brave and Butters (2012) constructs a state-space representation of the level of systemic stress. This approach takes into consideration the cross-correlations of a large number of financial variables (100 indicators), and the past development of the index, to set the weights for each sub-index. Standard portfolio theory is used by Holló, Kremer, and Lo Duca (2012), who aggregate sub-indices in a way which reflects their cross-correlation structure. This approach has been applied to Czech data (Adam and Benecká 2013) and Hungarian data (Holló 2012), but generally the attention paid to the role of financial stress in the Central European region (CEE) has been relatively limited.\(^2\)

In this paper, we build on the financial stress literature and use two financial stress indices for the euro area in our analysis. The Composite Indicator of Systemic Stress (CISS) by Holló, Kremer, and Lo Duca (2012) will be employed first as an index of financial stress in the “core” economy (euro area). Its construction not only incorporates information about conditions in individual markets, based primarily on volatility, but also captures the effect of simultaneous stress in each of them. The CISS index is updated on a weekly frequency, and is regularly used as a measure of euro area systemic stress by the ECB and by the ESRB in its Risk Dashboard, as well as being employed in academic studies (Vašíček et al. 2017; Dovern and van Roye 2014; Gelman et al. 2015). As an alternative, we will use the VIX index of implied option volatilities in the S&P 500 index, which may better capture the global stress.

\(^2\)Due to the importance of banking sectors in the CEE compared to other financial segments, the focus has been on developing banking-oriented or broader financial stability indices.


Figure 3.1: Financial stress indicators

Figure 3.1 shows the evolution of both indicators starting from the launch of the euro area in 1999.

3.2.2 Exchange Rate Dynamics and Financial Stress

A number of studies investigate the role of traditional exchange rate determinants, but financial measures such as financial stress have attracted relatively little attention from researchers. The link between exchange rate movements and risk aversion (measured by the VIX index) is investigated, for example, in De Bock and Carvalho Filho (2013). During risk-off episodes, when risk aversion dominates globally, the Japanese yen, the Swiss franc, and the U.S. dollar appreciate against other G-10 and emerging market currencies. Ranaldo and Söderlind (2010) also document the safe-haven or hedge function of selected currencies. Habib and Stracca (2012) establish fundamental determinants of currencies’ “safe haven” status, and find that net foreign asset position, low external vulnerability, large economy and past hedge status predict if a currency will function as a safe haven. At the same time, the profitability of currency carry trades is found to be procyclical in financial stress (Brunnermeier, Nagel, and Pedersen 2009; Menkhoff et al. 2012).

The determinants of CEE currencies have been also investigated by previous studies. Égert and Kočenda (2014) establish the effects of macroeconomic news and central bank communication on CEE exchange rates. Feldkircher, Horváth, and Rusnák (2014) find, on a large sample of countries and indicators, that previous price stability is a good
predictor for exchange rate reactions to global crisis. Similarly to the findings of this paper, Bubák, Kočenda, and Žíkeš (2011) suggest that the nature of the volatility spillovers from global markets to CEE exchange rates may have changed with the onset of the global crisis. Finally, it is important to note that the CEE currencies have often operated in managed float regimes with (symmetric or asymmetric) target bands. Fidrmuč and Horváth (2008) show that exchange rates tend to be more volatile if far away from target rates.

The question of how the conditions in financial markets affect portfolio decisions has been treated to some extent in the literature. According to Raddatz and Schmukler (2012), both investors and fund managers relocate their portfolios when facing a stressful event, either in the domestic country or in a foreign country where their investment is exposed. As a result, major institutional investors, and in particular their fund managers, have a substantial impact on capital flows during periods of financial stress. These capital flows then have a direct impact on the exchange rates under scrutiny. Closer to our approach, Lo Duca (2012) discusses the time-varying nature of capital flows, where push and pull factors have a different impact based on market conditions. Also Sarno, Tsiakas, and Ulloa (2016) find that over 80% of variation in bond and equity flows to satellite countries can be attributed to push factors from the US. When market tensions are elevated, investors pay attention to regional developments in emerging economies. However, when panic occurs, uncertainty and risk aversion start to drive flows and regional developments play a marginal role.

To sum up, the literature suggests that financial stress plays some role in exchange rate dynamics. In the traditional view, calm conditions in advanced countries’ financial markets stimulate carry trades to high-yielding currencies of emerging economies, which then tend to appreciate. An increased level of risk aversion leads to appreciation of safe haven currencies, while emerging markets are hit by capital outflows and their currencies depreciate vis-à-vis the U.S. dollar (with carry trade reversal too). In the following section, we show that satellite currencies can operate in several regimes – they appreciate in response to increased external financial stress in the "diversification" regime, and they depreciate in response to increased financial stress in the "flight to safety" regime, while we relate the switch from the former to the latter to the changes in risk aversion. The distinction between safe-haven and high-yielding currency status is therefore no longer time-invariant, but depends on global investors’ changing attitude to risk.

71
3.3 A Model of Portfolio Rebalancing

In this section we present a highly stylized model of portfolio allocation where investors decide about the composition of a portfolio consisting of assets denominated in “core” and “satellite” currencies. The purpose is to investigate the possibly non-linear relationship between risk aversion and the exchange rate, i.e., a different reaction of the relative exchange rate to increased uncertainty, based on the level of risk aversion and different portfolio management strategies. This relates to the findings of Cenedese et al. (2016) who acknowledge that stock market returns and exchange rate returns can be either positively or negatively correlated, and build a similar model of international currency portfolios, drawing implications for systematic profitability of carry trades. We use these ideas to exploit the effects of changes in risk aversion and of different types of risk management strategies on international capital flows and exchange rates.

We will consider two types of portfolio risk management: a simple mean-variance utility maximization and an optimization with a constraint on the maximum variance of the portfolio. We include the limit on the total portfolio return variance as an analogy to the value-at-risk indicator, which has become a widely used tool in portfolio risk management over the last few decades.\(^3\) Subsequently, two regimes (related to the degree of risk aversion and the portfolio risk management strategy) can be identified, based on the different reactions of the exchange rate to increased uncertainty. The “diversification” regime is characterized by the satellite currency appreciating in reaction to increasing uncertainty of asset returns in the core economy, while the “flight to safety” regime is characterized by the satellite currency depreciating in response to increasing uncertainty of core-currency asset returns.

Moreover, we investigate the reasons for regime switching – we suggest that regime switches may occur due to changes in investors’ behavior, notably to shifts in the degree of risk aversion, and possibly also to changes in fund managers’ objectives related to changes in the perception of risk. In particular, we will show that either an increase in risk aversion or a change from simple mean-variance optimization to value-at-risk-constrained optimization (or both at the same time) causes a switch from the “diversification” to the

\(^3\)Value-at-risk (VaR) has also been considered for currency portfolios, see e.g. Yamai and Yoshiba (2005) or Berger and Missong (2014), who also show that VaR is notoriously difficult to forecast. Note that in reality the VaR constraint may be combined with mean-variance optimization, and the moment when it becomes binding (a regime switch) might be related e.g. to perceived higher return variance after negative shocks have been observed. Therefore, it is likely that after turbulent financial market events, VaR constraints may become strictly binding for the majority of portfolios.
"flight to safety" regime.

### 3.3.1 Portfolio Composition

We model the behavior of an investor deciding about the composition of a portfolio where one class of assets is denominated in a core currency, while the other class is denominated in a satellite currency. Matching the stylized facts for the CZK, PLN and HUF, we assume that the expected return on satellite-currency assets is higher than that on core-currency assets,

\[ E[R_{sat}] \geq E[R_{core}] \quad (3.1) \]

and the variance of satellite-currency asset returns is higher than that of core-currency asset returns.

\[ \sigma_{sat} \geq \sigma_{core} \quad (3.2) \]

Assume a portfolio \( P \), composed of satellite-currency and core-currency assets. The expected return on the portfolio is

\[ E[R_P] = \lambda_{sat} E[R_{sat}] + (1 - \lambda_{sat}) E[R_{core}] \quad (3.3) \]

where \( \lambda_{sat} \) is the weight of satellite-currency assets. The variance of the portfolio returns is then

\[ \sigma_P^2 = \lambda_{sat}^2 \sigma_{sat}^2 + (1 - \lambda_{sat})^2 \sigma_{core}^2 + 2 \lambda_{sat}(1 - \lambda_{sat}) \sigma_{sat,core} \quad (3.4) \]

The aim is to study the changes in portfolio allocation (particularly the share of assets denominated in satellite currency \( \lambda_{sat} \)) in response to increased uncertainty related to the returns on core-currency assets, i.e., an increase in \( \sigma_{core} \). Figure 3.2 shows the mean-variance frontier\(^4\) of the portfolio for a particular parametrization. The preferences of investors increase toward the north-west of the diagram. It is clear that very low values of \( \lambda_{sat} \) are strictly dominated by their higher counterparts with equal variance but higher returns. The preference schedule also implies that the allocation with minimum variance (the vertex of the mean-variance parabola) is usually not the optimal one, as the investor can achieve higher expected returns with an infinitely small increase in the return variance.

\(^4\)Parameter values: \( E[R_{sat}] = 1.1, E[R_{sat}] = 1.01, \sigma_{sat} = 0.15, \sigma_{core} = 0.08 \)
We consider two types of portfolio management: mean-variance utility maximization and optimization with a constraint on the maximum variance. The latter is identical to a constraint on theoretical value-at-risk, a widely used tool for portfolio risk management.

### 3.3.2 Mean-variance Utility Maximization

First we consider an investor who maximizes her mean-variance utility derived from the portfolio return. The problem is to choose the share of satellite-currency assets $\lambda_{sat}$ to maximize

$$\max_{\lambda_{sat}} \{ E[R_P] - \frac{\gamma}{2} \sigma_P^2 \}$$

(3.5)

where $\gamma$ is the Arrow-Pratt coefficient of absolute risk aversion. The first-order conditions illustrate the optimal allocation. The investors are, in general, allowed to hold negative amounts of any of the assets (short-sell).

Figure 3.3 shows the optimal share $\lambda_{sat}$ of satellite-currency assets, with increasing uncertainty in core-currency assets, and for different values of the risk aversion parameter.

---

5 Value-at-risk, $VaR_\alpha$, is defined as the $\alpha$-quantile of the return distribution, and can be interpreted as the loss amount which will not be exceeded with probability $(1 - \alpha)$. Under the assumption of return normality, the constraint on the $\alpha$-quantile is equivalent to the constraint on the variance.

6 It can be shown that maximizing the exponential utility function $U = -e^{-\gamma R_P}$, where $\gamma$ is a coefficient of absolute risk aversion, is equivalent to maximizing the mean-variance objective $MV = E[R_P] - \frac{\gamma}{2} \sigma_P^2$.

7 Parameter values: $E[R_{sat}] = 1.1$, $E[R_{sat}] = 1.01$, $\sigma_{sat} = 0.15$, $\sigma_{corc} \in (0, 0.9\sigma_{sat})$, $\gamma \in (3, 100)$
Most importantly, for low values of the risk aversion parameter $\gamma$, the investor optimally reacts to increased core uncertainty by switching to satellite-currency assets, increasing the share $\lambda_{sat}$. For higher values of $\gamma$, however, the relationship reverses: with increased core uncertainty, the investor's optimal response is to reduce the share of satellite-currency assets. This is because satellite-currency asset returns still have relatively higher variance, and an increase in core uncertainty in the case of high risk aversion calls for a "flight to safety".

### 3.3.3 Optimization with Variance Constraint

The second type of investor behavior is motivated by the widespread use of the Value-at-Risk (VaR) indicator as a risk management tool in the last decade. When risk concerns dominate the portfolio allocation decision, it is reasonable to assume that portfolio managers are forced to pay more attention to VaR-type indicators. The major modeling difference from the previous case is that the portfolio managers have to fulfill the constraint of maximum variance. The objective is the following:

$$\max_{\lambda_{sat}} \{ E[R_P] \} \quad \text{s.t.} \quad \sigma_P^2 \leq \overline{\sigma}_P^2$$  \hspace{1cm} (3.6)
Figure 3.4: Optimal $\lambda_{sat}$ for changing $\sigma_{core}^2$, constrained $\sigma_p^2$

Because $E[R_{sat}] \geq E[R_{core}]$, the decision consists of choosing the highest $\lambda_{sat}$ such that $\sigma_p^2 = \sigma_{sat}^2$. Figure 3.4 presents the variance-constrained optimal choices of the share of satellite-currency assets in the portfolio with changing core-currency asset return variance. When the portfolio manager faces a binding constraint on the portfolio return variance, the share of satellite-currency assets decreases with higher uncertainty of core-currency returns. As the variance of core-currency asset returns rises, the manager needs to reduce the exposure to satellite-currency assets, which are still riskier.

### 3.3.4 Implications for Exchange Rate Behavior

The results are summarized in Table 3.1. Based on the risk attitude and objectives of the investors and/or portfolio managers, the share of satellite-currency assets in the model portfolio can switch between regimes, which we call “diversification” and “flight to safety.” When portfolio managers maximize their mean-variance utility and risk aversion ($\gamma$) is low, satellite-currency assets serve as a diversification tool and their share in the representative portfolio increases with increasing uncertainty of core-currency asset returns. When risk aversion rises, the attitude toward satellite-currency assets changes to “flight to safety” – the share of satellite-currency assets declines with increased core uncertainty. When the portfolio decision is made with a constraint on the portfolio variance (or VaR),
Table 3.1: Results Summary: the Response of Investors to an Increase in the Expected Variance of Core-Currency Assets

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<thead>
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<th>M-V optimization</th>
<th>VaR constraint</th>
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<tr>
<td>low risk aversion</td>
<td>diversification</td>
<td>flight to safety</td>
</tr>
<tr>
<td>high risk aversion</td>
<td>flight to safety</td>
<td>flight to safety</td>
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the “flight to safety” regime dominates.

We suggest that the changes described in investors’ attitude toward satellite-currency assets induce international capital flows, which translate into analogous behavior of exchange rates. The simple and stylized model presented above offers an explanation of the regime switches observed in the reaction of the satellite currency exchange rate to stress in the core economy. When risk aversion is low and portfolio managers operate in standard mode (mean-variance utility maximization), the satellite currency may serve for “diversification” when uncertainty in the core economy increases. On the contrary, if risk aversion rises, or portfolio managers start to operate under strictly binding constraints on the portfolio return variance (such as VaR), the regime switches to “flight to safety”, where the satellite currency reacts to increased uncertainty in the core economy by depreciation.

3.4 The Effects of Financial Stress on CEE currencies

In this section, we estimate the reaction of three CEE exchange rates to shocks to financial stress. In line with the proposed theoretical model, we believe that the reaction may be non-linear, i.e., the same shock may lead to a different reaction under different regimes. In the first, “diversification”, regime, a currency appreciates in response to elevated financial stress due to the portfolio diversification motive. However, this behavior may alter in times of financial panic, when investors resort to safe assets in advanced countries (due to increased risk aversion) and thus the currency depreciates again (the “flight to safety” regime).

We do not assume that the regimes are defined only by the level of financial stress or by its value relative to an estimated threshold. Instead, we assume that regime switching is driven endogenously by unobserved variables, such as risk aversion or credit constraints. The reason we do not associate regimes with the level of financial stress is path depen-
dency—investors may react to a rise in the level of stress in a different way when the stress has been at elevated levels for a long time than they do when it rises by the same amount in calm times, for example. As mentioned in Section 2, increased volatility of asset prices can alter the risk aversion of traders. Also, after a substantial shock to a financial system the credit constraints change. Investors are hence more sensitive to market volatility, which is an unobserved variable to be treated using endogenous regime switching.

As a result, we opt for the Markov-switching vector autoregression model, where regime switching is driven endogenously by an unobserved Markov process. This is in contrast to threshold VAR, which would define regimes based on the level of stress relative to an estimated threshold. We also assume that the transition probabilities of switching from one regime to another are constant and do not depend on the level of financial stress, due to the path dependency mentioned above.

### 3.4.1 Bayesian Markov-switching VAR Methodology

Markov-switching VAR is a non-linear variant of the VAR model, where two or more VAR models switch over time according to an endogenous unobserved Markov chain process. This model is convenient in our context, as we aim to study the response of an endogenous variable (the exchange rate) to shocks to another variable (a financial stress index) and we assume that the reaction is state contingent, with the regime depending on the state of an unobserved variable.

We estimate the model in the Bayesian setting, since we want to include our prior information regarding the nature of the exchange rate and financial stress (they should be close to a random walk). In addition, we have strong prior information that shocks to the CEE exchange rates do not affect financial stress in the euro area (or stress on the global scale), but the exchange rate itself reacts to changes in financial stress. Finally, this estimation method allows us to draw impulse response functions easily and does not suffer from convergence problems as in the case of MLE.

Let \( e_t \) be the exchange rate of a CEE currency against the euro, \( y_t \) be an \((m \times 1)\) vector of endogenous variables at time \( t \) and let \( N \) be the number of regimes. In our case, we have \( m = 2 \), \( y_t = (ind_t, e_t) \), and \( N = 2 \) or \( N = 3 \). A Markov-switching vector autoregression model can be written as follows:

\[
y_t = \mu_{s_t} + B_{1,s_t}y_{t-1} + \ldots + B_{p,s_t}y_{t-p} + (\Sigma_{s_t})^{1/2} \epsilon_t \tag{3.1}
\]
where $\epsilon_t \sim i.i.d. N(0, I_m)$. We assume that the unobserved state variable $s_t$ indicating the realization of the regime at time $t$ follows a first-order Markov chain with $N$ regimes and transition probabilities

$$p_{ij} = \text{Prob}(s_{t+1} = j | s_t = i), \quad \forall i, j \in \{1, \ldots, N\}, \quad \sum_{j=1}^{N} p_{ij} = 1 \quad (3.2)$$

This means that both the coefficients in the VAR model and the variances of shocks are governed by the same endogenous Markov process (some studies assume that the coefficients are governed by one process and the variances by another – see Krolzig (1997) for example; this reference also provides models where only the coefficients or covariance matrices are regime-dependent).

In the Bayesian setting, the parameters of the model are regarded as random variables. Let us define the vector of parameter blocks to be estimated as

$$\Theta = \{B_1, B_2, B_3, \Sigma_1, \Sigma_2, \Sigma_3, P, S_t, t = 1, \ldots, T\} \quad (3.3)$$

Were $T$ is the number of observations, $S_t$ is the state variable indicating the regime at time $t$, and $P$ is the transition matrix:

$$P = \begin{pmatrix}
    p_{11} & p_{12} & \cdots & p_{1N} \\
    p_{21} & p_{22} & \cdots & p_{2N} \\
    \vdots & \vdots & \ddots & \vdots \\
    p_{N1} & p_{N2} & \cdots & p_{NN}
\end{pmatrix} \quad (3.4)$$

The method of estimating Bayesian Markov-switching VAR models can be seen as an extension of estimating Markov-switching univariate autoregressive processes to a multivariate setting. The former, in the two- and three-regime cases, is described in Kim and Nelson (1999), for example, while the multivariate extension is described in Krolzig (1997).

In order to draw further inferences regarding the model parameters and impulse response functions, we need to choose the number of regimes to be estimated, to impose priors on the parameters, and to simulate approximations of the marginal posterior distributions of each parameter. The choice of the number of regimes was done informally.
we ran the estimation using three regimes and, if only a few periods were identified in one of the regimes, we decreased the number of regimes to two. The latter two steps – setting the priors and simulating draws from the posterior distribution by means of Gibbs sampling – are described in Appendix 3.C.

3.4.2 Data

We use weekly data since the introduction of the euro (1 January 1999) in the bi-variate Markov-switching VAR model. In each model considered, the two endogenous variables represent the exchange rate of one particular CEE currency vis-a-vis the euro and one financial stress index. Regarding the exchange rates, we consider the Czech koruna (CZK), the Polish zloty (PLN) and the Hungarian forint (HUF), i.e., the most traded currencies in the CEE region. Their exchange rates are expressed as the number of currency units per euro (e.g., CZK/EUR), which implies that an increase in exchange rates corresponds to the weaker local currency. Regarding the financial stress indices, we consider the CISS index and the VIX index. Since the CISS index is constructed on a weekly frequency, we estimate the eight models using weekly data. The CISS indicator and exchange rate time series were downloaded from the ECB Statistical Data Warehouse. The VIX index was downloaded from the Federal Reserve Economic Data (FRED) website.

The time series of the exchange rate were transformed into logarithms and the gap (from the trend extracted using the Hodrick-Prescott filter) was taken to isolate the effect of trend appreciation and the effects of fundamental variables, which are not included in our model. The levels of the resulting time series can be interpreted as percentage deviations of the level of currencies from their trend. The end of the estimation sample was set to August 31, 2012, because in the following months the Czech National Bank started to verbally intervene on the foreign exchange market and we believe that this date could coincide with a structural break in the relationship we are trying to estimate. The exchange rate time series used for the analysis can be seen in Figure 3.5.
Figure 3.5: CEE currencies vis-a-vis the euro

Data used for the analysis
exchange rates of a currency against the euro

levels
czkeur

levels
hufeur

levels
plneur

gap
czkeur

gap
hufeur

gap
plneur

00 02 04 06 08 10 12

0.10
0.05
0.00
-0.05
-0.10
00 02 04 06 08 10 12

0.15
0.10
0.05
0.00
-0.05
-0.10
00 02 04 06 08 10 12

35
30
25
20
15
10
5
0
-5
-10
-15
00 02 04 06 08 10 12
3.5 Results

Using the Markov-switching model, we identified three regimes of the reaction of exchange rates to changes in financial stress.\footnote{Initially, we estimated econometric models consistently with the theoretical model, i.e., we considered models with two regimes. This initial testing revealed that under the restriction to two regimes, the estimation uncertainty was high and impulse responses were often insignificant. Therefore we arrived at the specification with three regimes, which we consistently apply to models of all three CEE currencies.} Figure 3.6 presents estimated transition matrices, which summarise the probabilities of switching from regime $i$ to regime $j$. One can observe that transitions between regimes tend to be smooth, i.e., it is unlikely that regimes would switch from Regime 1 (low volatility\footnote{We would like to remind the reader that volatility of shocks to financial stress indices are increasing across regimes by assumption on labelling the regimes, as described in the previous section.}) to Regime 3 (high volatility) and vice versa directly. This is consistent with the observation that there are distinct periods of run-ups to crises and aftermaths of crises, with different dynamic characteristics to calm periods and periods of crises.

The transition matrices can be transformed in order to compute expected durations of remaining in a given regime, once the regime is reached (Figure 3.7). Their values indicate that durations of the calm regime tend to be longest, around 1 year for the CISS index. For the VIX index, the durations of the calm regime are also longest, albeit significantly shorter (less than or around 20 weeks), which indicates that the VIX index may be less suitable for capturing investors’ activity in the CEE region. This can reflect the fact that the VIX index captures financial stress related to the US stock market, while the CISS index captures several dimension of financial (systemic) stress in the euro area, which is clearly more relevant for the CEE currencies.

Figure 3.8 summarises the estimated probabilities of each regime, in that it indicates which regime had the highest probability in a given period.\footnote{A chart showing the time series of exact probabilities for each regime is provided in Figure 3.10 in the Appendix.} It can be observed that Regime 3 (high volatility) emerged in similar periods for all the currencies considered - around the period of financial crisis (2008 - 2009) - and in the second half of 2011 (sovereign debt crisis). Interestingly, periods when Regime 3 is identified using the VIX index are subsets of those identified using the CISS index. This can be explained by the nature of the VIX index, which considers stress in a narrow segment of financial markets (stock market in the US). The VIX index also has a more global nature, and euro area financial stress was only partially caused by global factors. Regime 2 (medium volatility) was identified in the run-up to the financial crisis and between the financial and sovereign
Figure 3.6: Transition matrices

Transition matrices

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<th>hufeur-ciss</th>
<th>plneur-ciss</th>
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Figure 3.7: Expected durations of regimes

Expected durations of regimes

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debt crisis for the Czech koruna and the Polish zloty. For the Hungarian forint, Regime 2 was identified in similar periods and also in the period between 2004 - 2006.

The lower panel of Figure 3.8 presents the average level of financial stress (captured by the CISS index) in each of the estimated regimes, separately on two sub-samples (1999 - end of 2005, 2006 - 2013). The ranking of the regimes by average financial stress coincides with the ranking by volatility, with the exception of the Polish zloty (1999-2006), for which Regime 2 was associated with higher stress than Regime 3. It is also worth mentioning that the average level of financial stress in each regime was generally lower in the first sub-sample than in the second, indicating that there is no fixed threshold for the regime switch. This is caused by the structural changes in financial markets which may have adapted to higher levels of stress, or by an imperfect measurement of financial stress by the stress indices. These two findings support our choice of the Markov switching model compared to a threshold VAR model, in that the reaction of a currency to the same shock does not depend on a fixed threshold of a stress index but on an unobserved factor (which, according to our model, may be related to risk aversion).

Regarding the reaction of exchange rate dynamics to financial stress shocks, Figure 3.9 suggests their highly non-linear nature\textsuperscript{15}. In Regime 3 (high volatility), satellite currencies depreciate in response to a shock to financial stress, with the exception of the Polish zloty, where the reaction is not significant. The depreciation in response to stress shock can be the result of safe haven flows, when investors shift their funds to safe assets due to increased risk aversion (as suggested by our model) or due to herd behaviour. In the case of the Polish zloty, the reaction is not significant, which could reflect that the Polish economy is more closed compared to the Czech and Hungarian economies, or that it has also retained some of the safe haven status during the more severe phases of past crises.

In Regime 2, the reaction of currencies in models with the CISS index is also consistent with our model and corresponds to the “diversification” regime. As a result of an increased stress in the euro area, investors diversify their portfolio and shift a part of funds to the CEE region, which leads to the appreciation of CEE currencies. On the other hand, shocks to the VIX index still lead to the depreciation of satellite currencies. Therefore, in reaction to global stress shocks, we do not observe the “diversification” regime, which is very reasonable as the uncertainty in such cases may not originate from the euro area.

Compared to the behaviour of exchange rates in Regime 3 and Regime 2, their be-

\textsuperscript{15}Impulse response functions with their confidence bands are summarised in Figure 3.11
Figure 3.8: Estimated regimes

Estimated regimes

Average level of the CISS index in the estimated regimes
Responses to 1 s.d. shock to financial stress

--- regime 1 --- regime 2 --- regime 3

Responses of exchange rates to shocks to financial stress indices (higher values indicate a weaker value of a given currency)

haviour in Regime 1 (which is characterized by low volatility and mild levels of stress) is less clear-cut. In response to a financial stress shock captured by the CISS index, the Czech koruna and the Polish zloty depreciate, while the reaction of the Hungarian forint is insignificant. This may reflect some of the business-as-usual behavior when increase in uncertainty triggers a limited rebalancing involving mild outflows from satellite markets. A further increase in financial stress may then lead to a switch to Regime 2 in a search for safe haven outside of the core (euro area) economy. At such medium levels of financial stress, CEE currencies may still qualify for the safe haven status. This does not hold anymore when the further surge in financial stress triggers the switch to Regime 3.

Overall, the results provide supportive evidence for our theoretical motivation. The emerging market currency can have different responses to increased stress based on actual financial market conditions. When the financial stress to the euro intensifies, low risk aversion leads to an increase in diversification motives and currency appreciation.

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16 In the model with the HUF, Regime 1 was observed only in period 2000 - 2001.
panic on financial markets causes risk aversion to increase, leading to capital withdrawals and depreciation of emerging market currencies.
3.6 Conclusion

This paper has analyzed how changes in financial stress and risk aversion, in the euro area and on the global level, affect the exchange rates of satellite CEE currencies. Using a highly stylized model of international currency portfolio allocation, we have illustrated that the impact of elevated stress in the core economy (euro area) on the exchange rates of satellite currencies can be non-linear and can operate in regimes: In the the well-known “flight to safety” regime, the satellite currency depreciates in response to an increase in financial stress. In the second, “diversification” regime, the CEE currencies have a safe haven status. We relate switches between the regimes to changes in risk aversion and changes in portfolio risk management strategy.

The empirical results, based on Markov-switching VAR models support the idea that the reactions of CEE currencies to financial stress operate in different regimes. Although the “flight to safety” regime prevails when financial stress indicators are peaking, in transition periods (run up to crisis or crisis aftermath) we often observe the “diversification” regime where CEE currencies respond to an increase in financial stress by appreciation. Interestingly, in calm periods when the levels of stress are low, we observe a third regime characterized by lower volatility, but also negative correlations between financial stress and satellite exchange rates which are otherwise typical for the “flight to safety” regime.

To put the results into a wider perspective, we have illustrated that with deepening financial integration, outside financial conditions and changes in investors’ sentiment can play a substantial role in exchange rate dynamics, especially in the short run when international parity based on macroeconomic fundamentals is less reliable. This paper thus also proposes a potential avenue for future research: the questions to be answered include issues of incorporating financial stress indicators into exchange rate forecasting and into practical monetary policy decision making.
3.A Estimated regime probabilities

Figure 3.10: Estimated regime probabilities
3.B Impulse responses

**Figure 3.11**: Responses to a 1 s.d. shock to financial stress indices
3.C Setting the Priors and Gibbs Sampling

3.C.1 Priors

We want to produce results that are as independent of our priors as possible, so we set most of the priors as non-informative. The only block of parameters which we set as informative is the block regarding the regression parameters, matrices $B_S$. All the priors are described subsequently.

**VAR Regression Coefficients: $B_S, \Sigma_S$**

For the coefficients in the VAR models in each regime, we specify relatively standard priors. Also, since we do not want to make any specific assumptions regarding the regimes, we assume the same priors for each regime.

We assume an independent normal-inverse-Wishart prior for the VAR coefficients. The VAR coefficients $B_S$ ($S \in \{1, \ldots, N\}$) have Minnesota prior form as described in Babu, Giannone, and Reichlin (2009), for example. Therefore, we assume the following prior $B_{S_t} \sim N(B, V_B)$:

\[
(B_k)_{ij} = \begin{cases} b_i, & \text{if } i = j, k = 1 \\ 0 & \text{otherwise} \end{cases}
\]

\[
V(B_k)_{ij} = \begin{cases} (\frac{1}{\lambda_k})^2, & \text{if } i = j \\ (\frac{\sigma_i \lambda_1 \lambda_2}{\sigma_j k \lambda_3})^2 & \text{otherwise} \end{cases}
\]

\[
\mu_S \sim N(0, c)
\]

The AR coefficients are set very close to one (the priors are estimated using univariate AR(1) regression), which is a very plausible prior for exchange rates and also stress indicators. The prior covariances between the regression parameters were set to zero, which is common practice. The variances are assumed to be distributed according to the inverse-Wishart distribution with scale matrix $S$ and prior degrees of freedom $T$ (in our case, $S^{-1} = 0$ and $T = 0$, which is a non-informative prior as described in Koop and Korobilis (2010)):

\[
\Sigma_S^{-1} \sim W(S^{-1}, T)
\]
In addition, as we have a prior belief that shocks to the Czech koruna do not affect the level of stress in the eurozone (and thus the value of stress indices), we incorporate a tight prior on the parameters reflecting the effect of the exchange rate on the stress index: $b_{ik} \sim N(0, c)$ (where $i$ is the equation for the stress index and $k$ indicates all parameters pertaining to the lagged exchange rate values), where $c$ is a very small constant.

The number of lags in the VAR model in each regime was chosen using the information criteria in the frequentist VAR model. Although a more rigorous way would be to select the number of lags using the marginal likelihood, we opted for the approach based on the information criteria due to its simplicity. In addition, changing the number of lags does not change the results dramatically.

As for the prior hyperparameters on the $B$ coefficients in each VAR model, we chose those suggested in Canova (2007) (which are very loose in our case). The prior on the constants in the VAR model is also set as very loose ($c = 10,000$).

Finally, one more prior assumption was imposed to alleviate the so-called label switching (identification) problem (Frühwirth-Schnatter 2006), which Markov-switching models suffer from when the priors are symmetric, as in our case. A possible solution is to assume a ranking of some coefficients across regimes and order the draws accordingly (as applied in Billio et al. (2013), for example). A plausible way to choose such a ranking is to order the regimes according to the variance of some shocks. This is the solution we chose. In our case, we assume that $\sigma_{ind,1} < \sigma_{ind,2} < \sigma_{ind,3}$, that is, the reduced-form shocks to financial stress indicators have the lowest volatility in Regime 1 and the highest volatility in Regime 3.

**Priors on Transition Matrix $P$**

In the case where the state variable has two states, we follow Kim and Nelson (1999) and assume a beta prior on the diagonal elements of the transition matrix (due to the adding-up property, one needs to impose only one prior in each row of the transition matrix). Specifically, $p_{ij} \sim \text{beta}(u_{ij}, \bar{u}_{ij})$. We opt for a non-informative version of the prior beta$(0.5, 0.5)$. In the case of three regimes, we assume a non-informative Dirichlet distribution prior (which is an extension of the beta distribution to multivariate random variables) for each row of the transition matrix: $p_i \sim \text{Dir}(0.5, 0.5, 0.5)$. 

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Priors on State Variable $S_t$

Similarly to the algorithm by Carter and Kohn (1994), it can be shown that due to the Markov property, all posterior distributions of $S_t$ depend on $S_0$ (Kim and Nelson 1999). Since we draw the parameters conditionally on other parameters, we assume an ergodic solution for the initial $S_t$ for a given draw of transition matrix $P$.

3.C.2 Gibbs Sampling

Since no analytical solution of the model exists, we employ the Gibbs sampling algorithm to draw samples from the joint posterior distribution of the parameters:

$$\Theta = \{ B_S, \Sigma_S, S_t, t \in \{1, \ldots, T\}, p_{ij}, i, j \in \{1, \ldots, N\}\}$$ (3.9)

We can draw from the conditional distributions of each block of parameters, which, after a sufficient number of iterations, converges to draws from the joint posterior distribution. The steps are similar to those sketched in Krolzig (1997):

1. Filtering and smoothing step: draw indicators of state (regime) $S_t$ for each $t$. This is done using multi-move sampling, which first employs the Hamilton filter to obtain the posterior distribution of the state variable at time $T$ and then samples backward states at each time $t$ given a draw at $t + 1$. This procedure is in principle very akin to the Carter and Kohn algorithm in linear state-space models (Carter and Kohn 1994).


3. Regression step: given the draws of the state variable, the whole sample can be split into $N$ sub-samples. For each sub-sample, parameter $B_s$ can be drawn from the same conditional posterior distribution (multivariate normal) as in the standard Bayesian VAR model, e.g. Koop and Korobilis (2010).

4. Similarly to the previous step, covariance matrices can be drawn for each sub-sample from its conditional posterior distribution, which is from the inverse-Wishart family.

5. Draw impulse response functions: given the draws of $B$ and $\Sigma$, we draw impulse response functions identified using recursive identification (where we assume that
the shock to stress comes first, so the shock from the koruna does not affect it contemporaneously).

We iterated this procedure 80,000 times, threw out the first 50,000 draws as a burn-in sample, and retained every 3rd draw of the remaining draws. Thus the posterior quantiles were taken from 10,000 samples from the marginal distribution functions.


