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TO THE MEMORY OF MY FATHER
A TRUE BELIEVER
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Preamble

This work presents a theoretical study on some aspects of the connection between immigration and social security. The work consists of two parts: The two chapters of Part 1 study an open economy environment, while the third chapter in Part 2 studies a closed economy.

Chapter 1 (in Part I), entitled ‘Immigration Control and Intergenerational Conflict’ (CERGE-EI Working Paper 453), concentrates on the intergenerational conflict that selective immigration causes in a host economy. The chapter shows that the policy-setting generation prefers a selective migration policy (i.e. allowing only high-skilled migrants) though, under rather permissive assumptions, the successive generations and the overall economy suffer welfare losses from the policy. The reverse also holds as the results are driven by the skill-fertility trade-off: Thus a non-selective policy, that is welfare depriving for the initial generation, guarantees higher welfare for the subsequent cohorts.

Chapter 2 (in Part I), entitled ‘Can Social Security Survive a Non-Selective Immigration?’, continues on the theme of Chapter 1 and shows that the policy-setting initial population may prefer to abandon the existing unfunded social security system in order to redistribute the gains of non-selective migration from the future cohorts towards themselves. In essence, the chapter provides a new answer to an old policy question. However, as opposed to the customary reasoning of wealth redistribution towards migrants, this chapter derives the result in an environment where intra-generational redistribution is impossible and only inter-generational redistribution is allowed.

Chapter 3 (in Part II), entitled ‘The Migration Challenge for PAYG’ (forthcoming in the Journal of Population Economics), shows that increased migration, usually proposed as a tool for keeping social security solvent in an ageing economy, may actually decrease the pension benefits in the post-migration economy. The result is mostly due to the capital dilution that lower-skilled migrants cause. However, the results significantly depend on the ‘Bismarckian Factor’ - the measure of actuarial fairness of the social security system: Should there be no intra-generational redistribution, as in the previous chapter, the post-migration pensions will increase.
Shrnutí

Tato práce je teoretickou studií některých aspektů ukazujících spojitost mezi imigrací a sociálním zabezpečením. Práce se skládá ze dvou částí. První část (Part I) má dvě kapitoly a obě pojednávají o otevřené ekonomice. Třetí kapitola tvoří druhou část (Part II) studie, tato část se zabývá uzavřenou ekonomikou.

První kapitola první části, nazvaná "Immigration Control and Intergenerational Conflict" (Imigranční kontrola a mezigenerační konflikt", CERGE-EI Working Paper 453) se soustředí na mezigenerační konflikty vzniklé v hostitelské ekonomice, které jsou způsobeny selektivně-restriktivními migračními podmínkami. Tato kapitola ukazuje, že generace, určující podmínky imigrace, dává přednost selektivní imigraci (např. povoluje imigraci pouze vysoce kvalifikovaným migrantům), ovšem plnění poměrně benevolentních podmínek způsobuje úbytek blahobytu budoucí generace obyvatelstva hostitelské země. A platí to i naopak, jelikož u méně kvalifikovaných imigrantů je nižší kvalifikace vyvážena vyšší plodností. Tudíž neselektivní politika imigrace, která nepřináší blahobyt generaci současné, zaručuje lepší životní podmínky generacím budoucím.

Druhá kapitola první části, nazvaná "Can Social Security Survive a Non-Selective Immigration?" ("Je možné i při neselektivní imigraci zachovat sociální zabezpečení?"), rozvíjí koncept předchozí kapitoly a ukazuje, že generace určující pravidla imigrace může upřednostnit přerozdělení budoucích výnosů převážně z neselektivní imigrace ve vlastní prospěch, bez ohledu na současné zdroje sociálního zabezpečení. V podstatě tato kapitola přináší novou odpověď na starou otázku ohledně nejlepší politiky. Nicméně v protikladu s obvyklými tvrzeními o přerozdělování bohatství směrem k migrantům, tato kapitola přináší výsledek z prostředí, kde redistribuce uvnitř současné generace není možná, přičemž je povolena pouze redistribuce mezigenerační.

Part I

Immigration and Social Security in Open Economy
Preface

This section includes two inter-related studies that inquire into the nature of the connection between immigration and social security policies. While utilising the same framework (being part of the same model), each chapter answers a different question. The first chapter develops the question of whether selective (skill-favouring) migration policies are beneficial for an open economy, and the second chapter studies whether unfunded social security can withstand non-selective immigration.

A small open economy environment is used in order to mirror standard European welfare states that are currently (re-)designing their immigration policies and are facing the problem of an ageing population, which challenges their social security systems. Such economies face a problem of sustainability. Thus, either parametric reforms should be performed, i.e. the pension benefits should decrease or the tax rates should go up, or appropriate demographic policies (increased fertility, higher human capital, or migration) should be designed. This work focuses on the last item on the list – increased immigration.

Immigration itself is a complicated phenomenon: Migrants differ both from the native population and among themselves – language, religion, education, skill level, fertility and the like. This chapter concentrates on two of those characteristics – skill level and fertility rates – as these directly affect the tax base. In general, this part studies the direct (first-order) effect of immigration on public finances. To assess the effect of migration, a welfare measure is developed based on the changes that the fiscal policies (social security system) cause for the life-time income of the population.

The first chapter thus evolves along the issues of selective (skill-favouring) migration policies. Recently many countries (and the European Union as a whole) have adopted migration policies that clearly favour the skilled. Indeed, skilled migrants do increase the tax base more (vis-à-vis the
unskilled migrants), however, they later claim larger pension benefits\(^1\) and have lower fertility rates. Thus the more skilled the migrants are in the first period the more pensions are claimed and the smaller is the contribution base in the next period. Hence both factors decrease the welfare of the next cohort – creating an intergenerational conflict. Moreover, in the steady state, the growth rate of the population translates into the internal rate of return of the social security system, and thus the future cohorts prefer high-fertility unskilled migrants over skilled migrants. Hence the first chapter concludes with the observation that a selective migration policy is preferred by the policy-setting generation but is welfare-depriving for all the subsequent population.

Further, in order to find the average effect of the selective migration on the economy, a debt-financed social security system is assumed, *i.e.* as opposed to the case of unfunded social security, in which at any time the contributions of the working population finance the pensions of the retired population, in the case of a debt-financed system the government is allowed to externally borrow and lend while maintaining a sustainable social security system. Though this system does not correspond to the usual definition of an unfunded social security system, it still shows the effect of selective immigration on the economy overall. Further, this environment can be interpreted as an optimal tax-setting solution of a social planner, which gives equal weight to each cohort and uses appropriate discounting. In this environment selective migration is shown to be welfare-depriving under rather permissive conditions (and is confirmed by real data).

In the second chapter, the focus shifts from migration policy design to social security reforms. Given the popularity of social security reforms and the general inefficiency of selective migration policies (EU free labour movement, tied family migration, quality of foreign education and the like), the chapter claims that (unskilled) migration may cause or facilitate a social security reform. As opposed to the usual claim that unskilled migrants are a drain on the social security system and thus threaten the existence of the system, the second chapter of the present work claims that unskilled migrants threaten the unfunded social security system even when it is purely earnings-related with no intra-generational redistribution.

The chapter is based on the public choice logic, *i.e.* the policy that guarantees higher welfare

\(^1\)This part considers only earnings-related (actuarially fair, or Bismarckian) social security where each retiree receives a pension relative to the size of own contribution. Though the pure type of this system is not employed in any country, any social security system contains some (usually large) part of this system. More about this can be found in the third chapter where the ‘Bismarckian factor’ is discussed.
for the majority of the current population prevails. However, the welfare of the initial retired population is assumed to be constant (as they receive the promised pensions and the interest on the savings is unchanged in the open economy environment). Hence, the results are based on the comparison of the welfare of the initial young. Two stylised environments are considered - an environment with unfunded pay-as-you-go (PAYG) social security and an environment with no pension system (in which the claims of the initial old are satisfied and the generated debt is evenly distributed over all the generations). In essence, the second chapter builds on the intergenerational conflict result of the first chapter to claim that the policy-setting cohort may prefer to abandon the social security in order to redistribute the gains from unskilled migration towards themselves.

\[1\]

The two chapters use slightly different welfare measures: Since, in the first chapter, the pension benefit rate (the 'replacement rate') is fixed, the welfare measure is based solely on the tax rate solely, while in the second chapter the welfare measure accounts for both the tax rate as well as the pension replacement rate. Hence, the welfare measure of the first chapter is just a particular case of the welfare measure of the second chapter, viz. the results are directly transferable between the two chapters.
Chapter 1

Immigration Control and Intergenerational Conflict

ABSTRACT  The study assesses the effects of immigration control on the welfare of the current and future population of a host economy. A theoretical model of a small open economy populated with overlapping generations of heterogeneous agents is used to show that skill-favouring immigration policies are, under rather permissive conditions, welfare-depriving for the overall population. However, the policy-setting generation is shown to benefit from immigration control, thus decreasing the welfare for the future population.

Introduction

Many economies are designing (or redesigning) their national immigration policies as circumstances in the international labour markets rapidly change: While most of the developed world suffers from low fertility (which in tandem with increased longevity heavily burdens national budgets), the labour markets in many developing economies are not able to absorb their continuously increasing populations, creating an abundant supply of potential immigrant labour. However, whereas the bulk of the immigrant supply is unskilled, newly-designed migration policies mostly favour skilled immigration, e.g. the European Blue Card Programme (Djajic, Michael & Vinogradova, 2011).

This paper examines preferential immigration policies, i.e. policies which favour a high share of skilled immigrants. The effect of different immigration policies on the current and future popu-
lation is studied, as immigration potentially may cause many unwanted changes in the economy, including intra- as well as inter-generational redistribution of welfare (Auerbach & Lee, 2011). While there is a vast literature on the impact of immigrants on the host economy (Borjas, 1999; Okkerse, 2008; Kerr & Kerr, 2011; Gaston & Nelson, 2013; and Nathan, 2013, provide an excellent survey of the literature), the great majority of that work is concentrated on the immediate and short-term effects of those policies, mostly ignoring the welfare of the future population.

Much of the literature has evolved along the lines of the Borjas-Card debate on the effects of migrants on native population. Despite the remarkable volume of the literature no consensus has yet been reached and the debate continues through methodological improvements. Moreover, the studies are concentrated on the instantaneous and short-run effects and according to the meta-surveys of Okkerse (2008) and Kerr & Kerr (2011) immigration has very little (or no) effect on the employment or wages of the native population. Nathan (2013) surveys the emerging (and also still inconclusive) literature on the ‘wider effects’ of immigration. The other major strand of the literature, mostly using computational general equilibrium models, studies the effect of immigration on public finances.

Thus, Storesletten (2000), discussing the problem of the sustainability of the pension system with an ageing ‘baby-boom generation’, studies the impact of immigrants on the overall economy and advocates for a preferential immigration policy (further favouring advanced-age skilled immigrants). The results are driven by the fact that skilled immigrants with few remaining working years will contribute to the public budget the most while being least likely to claim benefits or produce a ‘costly’ child. Lee & Miller (2000) in a similar model, but with a more developed demographic side (and less developed economic side), conclude that the preferential policies are better for social security when including the costs and benefits from the immigrants’ direct descendants into the calculations. Storesletten (2003) confirmed the previous results in an open-economy environment (to simulate a typical European welfare state), vouching for the benefits of preferential immigration.

In more recent work Chojnicki, Docquier & Ragot (2011), in line with the literature, claim that the welfare of the generations concerned would be higher if the United States had had a more selective immigration policy. Again, their study summarises the short-term effect on the population and does not look into future welfare. Lancomba & Lagos (2010) study the welfare effects
of different generations in more detail in order to identify the political equilibrium in the host
country and conclude that the population will vote for (even) unskilled immigrants as almost
every group is better off with this policy. However, Lancomba & Lagos do not consider voting
for skilled vs. unskilled immigration.

Razin & Sadka (1999, 2000) are pioneers in the literature who highlight the possible benefit of
unskilled immigrants to the economy (while also shifting the analysis to the long-run effects of
migration). They claim that, with an infinite horizon view, even unskilled immigrants Pareto
improve the welfare of a heterogeneous population (while individually burdening the welfare
state). In Razin & Sadka’s framework immigration can be understood as borrowing from the
future through the pay-as-you-go (PAYG) pension system, and with the infinite horizon there
is effectively no terminal point and thus none of the current or future generations will feel the
burden of the migrants.

Similar to Razin & Sadka (1999, 2000), the current paper also studies an infinite-horizon environ-
ment, assuming that an economy never ends. However, as opposed to Razin & Sadka immigra-
tion is not assumed to be a one-time policy, but rather a continuous one, i.e. once the economy is
open to immigrants, they will also come in later periods. Further, as the unfunded PAYG system
requires the balancing of the budget at all times, each cohort is affected by migration-induced
demographic changes differently and, thus, the welfare for each cohort is studied separately.

Though this paper does not explicitly consider political equilibrium, it is still assumed that public
choice favours a policy that enhances welfare for two cohabiting generations. Thus, while the
initial retired population have no welfare changes, the preferences of the initial young will define
the hypothetical voting outcomes (as no intergenerational altruism is modelled, the effect of the
immigrants on the particular period will be decisive\(^1\)). For instance, if the initial young benefit
from the preferential migration policy, then that policy will be assumed to be the outcome of the
political equilibrium.

However, while the initial young population uses the immigrants to support themselves in fi-
nancing the (fixed) pension benefits of the initial old, all the subsequent cohorts have to pay
the migrants their pensions (which, given the earnings-related PAYG system, is higher with a
higher skill level of the migrants) and thus might prefer unskilled migrants over skilled ones.

\(^1\)Fuster, Imrohoroglu & Imrohoroglu (2007) show the importance of altruism in the studies involving social security.
Furthermore, skilled migrants have lower fertility rates and thus in the next period there are fewer immigrant descendants to support the PAYG system. Thus, the immigration policy causes an intergenerational conflict in the economy.

Further, to understand the average effect of the preferential migration policy an alternative economy is studied, where the unfunded PAYG system is replaced with a debt-financed PAYG system. Under this system the government adjusts the debt level to keep a fixed tax rate. Though the usual definition of PAYG assumes that the social security budget should be balanced at all times, in reality many governments do employ soft budgets for the general sustainability of the social security system (this has led some researchers to use the modified, debt-financed PAYG in their studies of social security policies, e.g. McGrattan & Prescott, 2013). Moreover, the debt-funded PAYG with a perpetually fixed tax can be interpreted, in line with Phelan (2006), as an optimal solution to a social-planner problem which (while using appropriate discounting) gives equal weights to all cohorts, viz. the fixed-tax-rate debt-funded PAYG environment can be treated as an average effect of the migration policy on the overall economy.

In this paper, following Razin & Sadka (1999) and Storesletten (2003), a small-open-economy model with a population of overlapping generations of heterogeneous agents is constructed (to simulate a typical European economy). The model is abstracted from detailed labour and other price-setting markets (as the effects of immigration on those are still to be identified in the empirical literature) in order not to bias the results. Hence, in the current work the effect of immigration on the welfare of the population is channelled through public finances.

The demographic side of the model is amply developed: Aside from the usual characteristics

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2 In theory, results depend on whether capital, skilled labour, and unskilled labour are used as substitutes or complements in production as well as how unemployment is introduced into the country. Thus, Razin & Sadka (2000) repeated their (1999) experiment in a closed-economy framework, and reported some welfare redistribution in the host economy due to the distortions in the capital-labour ratio. Schou (2006) discusses a semi-open economy, where the labour market was only closed, and the immigrants and natives are perfect substitutes. Schou also reports unwanted welfare redistribution. Kemnitz (2003) works out the theoretical counterpart of Razin & Sadka (1999) with unemployment, and again claims some unwanted welfare redistribution if immigrants disturb the employment rates of the native population. Alas, the empirical literature is inconclusive on the particulars of the effect of unskilled immigration, and no firm consensus has yet been reached. Thus, Card (2005) argues that evidence for immigrants harming unskilled natives is scant. In a survey article Okkerse (2008) concludes that the literature has failed to establish a firm relationship between immigration and unemployment, and that new labourers can be absorbed into an economy without damaging the labour market position of residents. Brücker & Jahn (2011) in a computational general-equilibrium model replicated the German economy and found very minor effect of migration on native wages or employment possibilities. Furthermore, some recent empirical literature reveals positive indirect effects of immigration on the local population. Cortes (2008) shows the benefit of unskilled immigration through instantaneous effect on prices. Furtado & Hock (2010) highlighted the role of unskilled immigrants on the work-fertility trade-offs for local skilled females. Bauer, Flake & Sinning (2013) claim increased employment prospects for skilled natives from immigration.
The Economic Environment

A small-open-economy environment is used. The model implicitly assumes the existence of a firm that locally hires all the available labour. No financial institution is modelled: Savings and borrowings are made based on constant (world) prices. The model is explicitly populated by heterogeneous agents and a government that manages the social security budget.

Population

The population differ in age (young, $i = 0$, and retired, $i = 1$), in skill level (skilled, $s = 1$, and unskilled, $s = 0$) and number of previous generations in the economy. Immigrants, $m$, are introduced to the economy while young and are considered the first generation in the economy, $g = 1$. Descendants of immigrant dynasties can be of any generation, $g = 2, 3, \ldots$, and the natives, $n$, belong to dynasties that are present in the economy at time $t = 0$.

While the share of skilled among immigrants, $\lambda \in (0, 1)$, is a ‘choice variable’ for the government,

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3The assumptions of small-open economy and, especially, perfect substitutability between skilled and unskilled labour limits the analysis to particular cases only. More realistic modelling of the matter may potentially bring more insights to the topic.
the locals, (i, descendants of native and immigrant dynasties) are born skilled with some probability: The share of skilled among descendants of native dynasties is \( \theta \in (0, 1) \), and the share of skilled among immigrant dynasties is either \( \lambda \) or \( \theta \) depending on the assimilation process.\(^4\) (For notation \( \gamma (s, g) \) will also be used as the probability for generation \( g \) agent having skill level \( s \).)

In the labour market the skill level directly translates into efficiency level, \( \varepsilon (s) \). The skill level also co-determines, together with generational background, the fertility rate \( \varphi (s, g) \) of the agents. Natives, for the sake of simplicity, are assumed to reproduce with unit fertility.\(^5\) Thus, if \( \mu_t (i, s, g) \) is defined as a measure on type \( (i, s, g) \) agents, the introduction of the new generation of \( (i, s) \) natives can be presented as:

\[
\mu_{t+1} (0, s, 0) = \gamma (s, 0) \sum_{s'} \mu_t (0, s', 0)
\]

and the introduction of the immigrant dynasty descendants as:

\[
\mu_{t+1} (0, s, g + 1) = \gamma (s, g + 1) \sum_{s'} \mu_t (0, s', g) \cdot \varphi (s', g)
\]

The size and quality of the first generation immigrants is a government policy:

\[
\mu_t (0, s, 1) = \gamma (s, 1) \cdot \psi \sum_{g \neq 1} \sum_{s'} \mu_t (0, s', g)
\]

where \( \psi \) is the size of the immigrant population compared to the local-born population. The government chooses not only the size of the group but also the share of skilled among the immigrants: \( \lambda = \gamma (1, 1) \).

Each agent stays in the model for two periods (except the initial retired population, \( \mu_0 (1, s, n) \), that are present only for the second period of their lives), \( \text{viz.} \) individual ageing is deterministic:

\[
\mu_{t+1} (1, s, g) = \mu_t (0, s, g)
\]

\(^4\)The immigrant dynasties will either assimilate fully and have the skill distribution of the natives, \( \theta \), or fully inherit the ancestral skill distribution (with \( \lambda \) as the share of skilled and \( 1 - \lambda \) as the share of unskilled). The skill distribution is commonly believed (e.g. Card \& Rothstein, 2007; Heath, Rothon \& Kilpi, 2008) to be between the two, but for the sake of analytic simplicity only the extreme cases are considered.

\(^5\)The constant population, though very optimistic for ageing societies, already makes unfunded pension systems costly for the participating population. Principally, welfare losses arise once the sum of the growth rates for the real wage and population is less than the real interest rate (Aaron, 1966).
\(i.e.,\) everybody ages, stays retired for one period and leaves the model afterwards (also no return migration is allowed).

The size of the effective labour force depends on the absolute size of the population and their average efficiency:

\[
N_t = \sum_{s,g} \epsilon(s) \mu_t(0, s, g) \tag{1.5}
\]

At the start of the economy half of the native population is young (age \(i = 0\)), and the other half is retired (age \(i = 1\)). Thus the effective labour force at time \(t = 0\) is

\[
N_0 = L (\theta \varepsilon_s + (1 - \theta) \varepsilon_u) = LE_0 \tag{1.6}
\]

where \(L\) is exactly half of the total population in the country, and \(E_0\) is the average efficiency of a native worker.

**Government**

The government regulates immigration (as presented in eq. (1.3)) and implements the fiscal constitution: The fiscal constitution includes taxation, pension benefits and sustainability of public debt (if it exists), and can be presented as:

\[
\sum_{s,g} \rho_t w \varepsilon(s) \mu_t(1, s, g) + B_t (1 + r) = \sum_{s,g} \tau_t w \varepsilon(s) \mu_t(0, s, g) + B_{t+1} \tag{1.7}
\]

and

\[
\lim_{t \to \infty} B_t \cdot (1 + r)^{-t} = 0 \tag{1.8}
\]

where \(\rho_t\) is the pension replacement rate\(^6\), \(\tau_t\) is the tax rate, and \(B_t\) is the debt at year \(t\) (a borrowing from time \(t - 1\) due at time \(t\)). Equation (1.7) is a general dynamic budget constraint that allows for unfunded PAYG, debt-financed PAYG, as well as a social security reform when PAYG is terminated.

\(^6\)This specification follows Bismarckian (earnings-related) social security system that allows intergenerational redistribution and excludes intragenerational redistribution.
For the traditional definition of the PAYG system the debt generation should be impossible, i.e. $B_t = 0$ for any $t$. Then (1.8) is satisfied in all cases. Further, for the sake of comparability there will be no initial debt under any type of government budget discussed below, i.e. $B_0 = 0$.

In the case when debt-financed PAYG is considered, the economy will start with no initial debt, however, the government is allowed to borrow or save, i.e. non-zero public debt is allowed $B_t \neq 0$ for some $t$. Here equation (1.7) needs to be augmented by the condition presented in (1.8) for sustainability. Though this definition of PAYG is not customary (Uebelmesser, 2004), some authors (e.g. Attanasio, Kitao & Violante, 2007; Nishiyama & Smetters, 2007) use ‘debt-financed’ PAYG based on the fact that the policy makers habitually smooth the PAYG budget by borrowing. In the following text, the debt-financed PAYG will be used only with a constant tax rate that will distribute any debt equally over the population. The debt-financed PAYG will be used to illustrate the average effect of the immigration on the economy over time.

The debt-financed government budget will also be used in Chapter 2, while analysing social security reforms i.e. the case when the government terminates the PAYG system, converts the implicit debt of the unfunded PAYG pension (the pension claims of the initial retired population) into explicit debt, and uses taxes to service the debt. In this case the initial pension replacement rate will remain at the usual level, i.e. $\rho_1 = \rho$, as the claims of the initial retired are being honoured, while for all the other periods the population will not receive any pensions, i.e. $\rho_t = 0$ for any $t > 1$.

These reforms are usually politically non-feasible (Yamada, 2011) as the policy reforming generation would have to finance the reform, i.e. contribute to the PAYG while saving for their own retirement, and thus create a double-paying generation. In order to avoid the double-paying generation problem, similar to the case of debt-financed PAYG (again, at least for the sake of comparability), a single tax will be considered that will evenly distribute the debt over the generations and the sustainability condition (1.8) will be utilised to avoid perpetual debt-generating situations.
Households and Welfare

Each household is represented by an individual agent that maximizes lifetime utility, which is derived from consumption in both periods:

\[
U(c_t^i, c_{t+1}^i) = v(c_t^i) + \beta v(c_{t+1}^i)
\]

(1.9)

where \( c_t^i \) is the consumption of an agent born at time \( i \) during time \( j \); \( U(\cdot, \cdot) \) is a time-separable utility function with \( \beta \in (0, 1) \) being the time-discount coefficient and \( v(\cdot) \) being a continuous, twice continuously differentiable, strictly increasing, strictly concave function that satisfies the Inada conditions.\(^7\) To finance consumption, an agent uses labor income net of taxes and savings in the first period, while in the second period savings and social security benefits are used. Thus, at time \( t \) a \((0, s, g)\) type agent faces the following budget constraints:

\[
c_t^i + a_t \leq w \varepsilon (s) (1 - \tau_t)
\]

(1.10)

\[
c_{t+1}^i \leq \rho_t w \varepsilon (s) + a_t (1 + r)
\]

(1.11)

where \( a_t \) is the savings. The agent’s efficiency is \( \varepsilon_s = \varepsilon (1) \) and \( \varepsilon_u = \varepsilon (0) \).

Further, as there are no borrowing constraints, (1.10) and (1.11) can be combined into one intertemporal budget constraint:

\[
c_t^i + \frac{c_{t+1}^i}{1 + r} \leq w \varepsilon (s) \left(1 - \tau_t + \frac{\rho_t}{1 + r}\right)
\]

(1.12)

Essentially the intertemporal budget constraint (1.12) shows that only the present discounted value of the lifetime after-tax income, i.e. the expression on the right-hand side of (1.12), matters for the consumption choice of an agent type \( (s) \). This observation is the basis of the following lemma:

\(^7\)Leisure is not considered in the utility function for notational simplicity: All of the following results hold with the conventional time-separable, CRRA utility function:

\[
U(c_t^i, c_{t+1}^i, n_t, n_{t+1}) = \sum \beta^t \frac{\left(\frac{c_t^i}{n_t}\right)^{\alpha} \left(\frac{n_{t+1}}{c_{t+1}^i}\right)^{\gamma}}{1 - \delta}
\]

and the budget constraint (1.10)-(1.11): The agent’s decision on leisure depends only on parameters and interest rates rather than on own or government policy variables.
Chapter 1

The Status Quo Economy

**Lemma 1** Let $U_s(\tau_t)$ be the lifetime utility as a function of the size of the social security system:

$$U_s(\tau_t) = \max U \left( c_t^s(s), c_{t+1}^s(s) \right)$$  \hspace{1cm} (1.13)

and also denote

$$W_t \equiv \frac{\rho}{\tau_t}$$  \hspace{1cm} (1.14)

then $U_s(\tau_t)$ is strictly increasing in $W_t$.

**Proof.** The first order conditions of the optimisation problem (1.9) subject to (1.12) give implicit functions of consumptions in the both periods depending only on $W_t$ defined in (1.14): $c_t^s = c_t^s(W_t)$ and $c_{t+1}^s = c_{t+1}^s(W_t)$. On the other hand $U \left( c_t^s(s), c_{t+1}^s(s) \right)$ is strictly increasing in both arguments and thus it is strictly increasing in $W_t$. Then from the definition, $U_s(\tau_t)$ is also strictly increasing in $W_t$. ■

Further, all type $s$ agents of the same generation face an identical optimisation problem (1.9)-(1.12), viz. $U_s(\tau_t)$ is independent of the agents’ generation in the country $g$ and represents the utility of all the type $s$ agents of generation $t$. Moreover, according to the lemma 1 $U_s(\tau_t)$ for $s = 0, 1$ are both strictly growing in $W_t$ at the same time. However $W_t$ is independent of agents’ skill type $s$ and thus is a valid measure of welfare for the entire generation under different government policies. Thence, based on Lemma 1, the product of the pension replacement rate and the reciprocal of the tax rate, $W_t$, shall be considered the **measure of welfare** and will be used to compare the welfare of the agents under different policies. Still, the welfare of the initial retired population is invariable: Independent of the policy changes, the retired population consumes its own savings and pension benefits.

The **Status Quo Economy**

The Status Quo economy starts with an established social security system and no immigration and those policies are maintained without changes. Thus the population dynamics (1.1)-(1.6), in
combination with zero-immigration policy

$$\psi = 0,$$  \hspace{1cm} (1.15)

takes the following form:

$$N_{t+1} = N_t = N_0.$$  \hspace{1cm} (1.16)

The government also maintains balanced PAYG system at all times, i.e. runs (1.7)-(1.8) with no borrowings:

$$B_t = B_0 = 0.$$  \hspace{1cm} (1.17)

The initial retired population, or the generation $t = 0$ at period $t = 1$, did contribute to the PAYG system in the previous period and thus anticipates social security benefits with replacement rate $\rho$ during the period $t = 1$. The subsequent generations will contribute with a tax rate $\tau_t$ and claim social security benefits with replacement rate $\rho$ during the next period.

**Definition 1** Given the (world) prices for labour and capital, $w$ and $r$, the replacement rate for public pension $\rho$, the subjective time discount factor $\beta$, skill and fertility rates $\varepsilon_s$ and $\varphi_{g,s}$, skill distribution $\gamma_{s,g}$, the initial value of public debt, $B_0$, and the size of the immigrant population compared to the native population $\psi$, the Status Quo equilibrium in the economy is an allocation

$$\left\{ c_{t0}^1; \{ c_t^i, c_{t+1}^i, a_t, \mu_{t} (i, s, g), \tau_t, B_t \}^{i=0,1}_{s=0,1, g=1,2,\ldots} \right\}$$

such that the initial (time $t = 0$) retired agents consume their savings and pension benefits, households optimise (1.9)-(1.11), the government budget (1.7)-(1.8) is balanced according to (1.17), and the population evolves according to (1.16).

**Lemma 2** In the Status Quo economy the tax rate is constant

$$\tau_t = \rho$$  \hspace{1cm} (1.18)

and the Welfare is unity

$$W^{SQ} = 1$$  \hspace{1cm} (1.19)
for all generations.

**Proof.** Using (1.17) the government budget (1.7)-(1.8) can be re-written as

\[ \rho N_t = \tau_t N_{t+1} \]  \hspace{1cm} (1.20)

that in combination with (1.16) results in (1.18). Combination of (1.18) and (1.14) results in (1.19). 

The equal welfare for all agents is a result of a balanced budget and constant population growth rate. Moreover, the unity welfare for the *Status Quo* economy makes it an ideal *numéraire* — a benchmark for comparison.

### The Economy with Immigration

In an economy with immigration the government sets a tax rate, based on the size of the effective labour force (the base for the contributions and the next period pension claims), to balance the budget (1.7)-(1.8).

\[ \rho w N_{t+1} + B_t (1 + r) = \bar{\tau} w N_t + B_{t+1} \]  \hspace{1cm} (1.21)

and

\[ \lim_{t \to \infty} B_t \cdot (1 + r)^{-t} = 0 \]  \hspace{1cm} (1.22)

In period \( t = 1 \) the government allows migrants to enter. Migrants immediately start participating in the local labour market and the social security system (by contributions first and then pension claims one period later). The government chooses the size of the immigrant group (relative to the local-born population, \( \psi \)) and the share of the skilled among migrants, \( \lambda \), so that the average efficiency of the immigrant population is

\[ E_\lambda = \lambda \varepsilon_s + (1 - \lambda) \varepsilon_u \]  \hspace{1cm} (1.23)
and the working age population at time $t = 1$ is

$$N_1 = \mathcal{L}(E_\theta + \psi E_\lambda) \quad (1.24)$$

From the second period on, the effective population $N_t$ grows based on (1.1)-(1.5), with a working assumption that the population growth should not exceed the market interest rate at any period.

**Definition 2** Given the (world) prices for labour and capital, $w$ and $r$, the replacement rate for public pension $\rho$, the subjective time discount factor $\beta$, skill and fertility rates $\varepsilon_s$ and $\varphi_{g,s}$, skill distribution $\gamma_{s,g}$, the initial value of public debt, $B_0$, and the size of the immigrant population compared to the native population $\psi$, the equilibrium in the economy with immigration is an allocation

$$\left\{c^t_0, \left\{c^t_t, c^{t+1}_t, a_t, \mu_t(i, s, g), B_t\right\}_{s=0,1, g=1,2, \ldots} \right\}$$

such that the initial (time $t = 0$) retired agents consume their savings and pension benefits, households optimise (1.9)-(1.11), the government budget sets a tax rate $\tilde{\tau}$ to balance the budget (1.21)-(1.22), and the population evolves according to (1.1)-(1.5).

The equilibrium defined above suggests that there is one level of tax rate for all the cohorts in the economy. Incontestably, the equilibrium does not fit into the customary definition of the PAYG pension system (e.g., Uebelmesser, 2004). However, smoothing of fiscal spikes is a habitual practice worldwide (that has encouraged many researchers to consider a ‘single-tax’ equilibrium for modelling social security budgets), and this debt-financed PAYG equilibrium can be used as an illustrative case for studying the ‘average’ effect of immigration policy on the entire economy.

**Lemma 3** In the Economy with Immigration the Welfare is constant, $W_t = \overline{W}$ for all cohorts $t$ and is equal to

$$\overline{W} = (1 + r) \left(1 + \frac{N_o}{\sum_{t=1}^{\infty} N_t (1+r)^t} \right)^{-1}. \quad (1.25)$$

**Proof.** Dividing (1.21) by $(1 + r)^t$ and summing over all periods results in

$$\tilde{\tau} w \sum_{t=1}^{\infty} \frac{N_t}{(1+r)^t} = \rho w \sum_{t=1}^{\infty} \frac{N_{t-1}}{(1+r)^t} + \sum_{t=1}^{\infty} B_t - B_{t-1}(1 + r) \quad (1.26)$$
which with conditions (1.22) and \( B_0 = 0 \) solves for \( \tilde{\tau} \). Plugging \( \tilde{\tau} \) into into the definition of the welfare measure (1.14) will give (1.25).

**Proposition 1** The welfare in the Economy with Immigration is larger than in the case of Status Quo Economy

\[
\tilde{W} > W^{SQ}
\]

in the case of any non-zero immigration, \( \psi > 0 \)

**Proof.** With positive immigration \( \psi > 0 \), according to (1.1)-(1.5), \( N_t \) grows compared to \( N_0 \), that results in \( \tilde{W} > 1 \), hence \( \tilde{W} > W^{SQ} \).

Virtually, the proposition shows that increased immigration is a Pareto-improving policy independent of the numbers and skill distribution of the migrants and their descendants. In other words, increased immigration does not have any channel to decrease the welfare of the locals. On the contrary, immigrants and their descendants increase the rate of return of the PAYG system and thus the welfare of the agents. As there are no other channels involved, the immigration influences only the public finances and even the lowest skilled immigrants do not burden the economy. There are two main reasons for this: First, as in Razin and Sadka (1999), in this model the immigrants participate in an unfunded pension system and shift the burden of increased pension claims to an infinitely later period (where still another cohort of immigrants will contribute to the system). Second, as suggested by Sinn (2001) the purely earnings-related (Bismarckian) social security system guarantees the positive contribution of the immigrants.

**Dynastic Assimilation**

As immigrant dynasties may follow various assimilation paths, it is necessary to discuss them separately. Here three main cases are studied: full assimilation, when the immigrant descendants adopt fertility rates and skill levels of the natives; partial assimilation, when immigrant descendants inherit fertility rates but adopt the skill levels; and no assimilation, when skill and fertility levels are both inherited.  

\(^8\)Certainly, dynastic assimilation is a multidimensional and multifarious process that involves many direct and hidden costs. However, this work does not discuss the environment or the costs associated with the process and thus potentially faces a risk of biasing the results (Krieger, 2004).
Dynastic Assimilation

Chapter 1

Full Assimilation: Uninherited Fertility and Uninherited Skills

In the case of full assimilation, the second period starts with the natives creating young natives equal to themselves in number and average efficiency, while the immigrants father, with their respective fertility rates $\varphi_s = \varphi(s, 1)$, second generation immigrants that have the skill distribution of the natives. Next, the government allows a new cohort of young immigrants to enter the country according to (1.3), so that in the second period the effective labour force is

$$N_2 = L (E_\theta + \psi E_\lambda) (1 + \psi \Phi_\lambda)$$

(1.28)

where $\Phi_\lambda$ is the average fertility of the immigrants:

$$\Phi_\lambda = \lambda \varphi_1 + (1 - \lambda) \varphi_o$$

(1.29)

From the second period on the natives reproduce, the immigrant descendants behave identically to the natives, and the immigrants enter according to (1.3), so that the population at time $t > 1$ is

$$N_t^{fa} = L (E_\theta + \psi E_\lambda) (1 + \psi \Phi_\lambda)^{t-1}$$

(1.30)

i.e. increasing proportionally with the weighted sum of the local and immigrant fertility rates.

**Lemma 4** In the case of full assimilation the welfare measure reads:

$$\tilde{W}^{fa} = \frac{E_\theta + E_\lambda \psi}{E_\lambda \psi + E_\theta (1 + r) - \Phi_\lambda E_\theta \psi} (1 + r)$$

(1.31)

**Proof.** Follows directly from rearranging equations (1.6), (1.24), (1.28), (1.30) and (1.25). \[\blacksquare\]

**Proposition 2** Welfare, in the case of full assimilation, decreases with an increase in the share of skilled among immigrants if

$$\frac{\varepsilon_s - \varepsilon_u}{\varphi_u - \varphi_s} < \frac{E_\theta + \psi E_\lambda}{r - \psi \Phi_\lambda}$$

(1.32)

and increases if the inequality holds with the opposite sign.
**Proof.** The derivative of (1.31) is

\[
\frac{\partial \tilde{W}^{fa}}{\partial \lambda} = P_{fa} \cdot \left( \frac{\partial E_{\lambda}}{\partial \lambda} r + \frac{\partial \Phi_{\lambda}}{\partial \lambda} E_{\theta} - \psi \left[ \frac{\partial E_{\lambda}}{\partial \lambda} \Phi_{\lambda} - \frac{\partial \Phi_{\lambda}}{\partial \lambda} E_{\lambda} \right] \right) \tag{1.33}
\]

where \( P_{fa} \) is a strictly positive function of model parameters, \( \frac{\partial E_{\lambda}}{\partial \lambda} = \varepsilon_s - \varepsilon_u > 0 \) and \( \frac{\partial \Phi_{\lambda}}{\partial \lambda} = \varphi_s - \varphi_u < 0 \). Hence, (1.33) is negative if (1.32) holds.

The proposition claims that under rather permissive assumptions that the difference in efficiency rate is not disproportionately large compared to the difference in fertility rates, unskilled immigration is preferred. The intuition behind condition (1.32) is that if the losses (size of the pension system losses \((r - \psi \Phi_{\lambda})\) multiplied by the losses in efficiency \((\varepsilon_s - \varepsilon_u)\)) due to unskilled immigration are smaller than the gains (average efficiency \((E_{\theta} + \psi E_{\lambda})\) multiplied by the gain in fertility \((\varphi_u - \varphi_s)\)) due to unskilled immigration, then unskilled immigration is preferred.

Thus there is a trade-off between skill and fertility levels: If there were no or were very little fertility differences then the inequality (1.32) would easily collapse and the welfare would increase with the share of skilled immigrants. However, the difference in efficiency and difference in fertility are, in general, rather similar, while \((r - \psi \Phi_{\lambda})\) has a small positive value and \(E_{\theta} > 1\) by construction, so the inequality is virtually always satisfied. For instance, Akin (2012) using German socio-economic panel data, calibrated the fertility rates for the two immigrant skill groups, \(\varphi_s = 0.84\) and \(\varphi_u = 1.14\) (found in Akin, Table 6). Based on Akin, Table 4, assuming the unskilled as a numéraire, the efficiency levels can be computed, \(\varepsilon_s = 1.46\) and \(\varepsilon_u = 1\). Similarly, the share of skilled among natives can be computed from Akin, Table 7, \(\theta = 0.31\).

To calculate the size of immigration, the current German annual level of 0.1 per cent of the entire population can be used for a 30-year period. Thus \(\psi = (1 + 0.002)^{30} - 1 = 0.06\). To obtain the real interest rate, Eurostat data on annual government bond yields on ten-year maturities over five pre-crisis years (2003-2007) is used to calculate the average 3.848 per cent. As the model does not account for economic growth, the average growth rate of Germany over the same years, 2.978, is subtracted and the remaining 0.87 per cent is used as a base for compound interest calculation \(r = (1 + 0.0087)^{30} = 0.297\). Thus, using these data the inequality (1.32) can be re-written

\[
\frac{0.46}{0.3} < \frac{1.2 + 0.03 \lambda}{0.23 + 0.02 \lambda}
\]
As $\lambda \in (0, 1)$, the right-hand side of the above inequality falls into the interval $(4.92, 5.22)$. Hence the inequality holds for any $\lambda$. That is, while the welfare increases with the size of the immigrant population, it decreases with the share of skilled immigrants.

Partial Assimilation: Inherited Fertility and Uninherited Skills

In the case of partial assimilation, the labour force dynamics are identical to the previous case up to the second period and can be presented by (1.6)-(1.28). However, from the beginning of the third period (while the natives reproduce, the immigrants produce new generation with fertility rates of, $\Phi_\lambda$, and the government allows young immigrants into the economy according to policy rule (1.3)), the second generation immigrants, having inherited the fertility level of their ancestors, produce third-generation immigrants with the skill-dependent fertility rate $\varphi_s$ (as the first-generation immigrants do).

The effective population at time $t > 1$ is

$$N_t^{\text{pa}} = L \left( (E_\theta + \psi E_\lambda) \left( \frac{\psi \Phi_\lambda}{\Phi_\theta + \psi \Phi_\lambda - 1} \left( \Phi_\theta + \psi \Phi_\lambda \right)^{t-1} + \frac{\Phi_\theta - 1}{\Phi_\theta + \psi \Phi_\lambda - 1} \right) \right)$$

(1.34)

where

$$\Phi_\theta = \theta \varphi_1 + (1 - \theta) \varphi_o$$

(1.35)

is the average fertility level of the immigrant descendants.

Hence, the immigrant generations differ in their average fertility both from the natives (unit fertility) and from the immigrants (on average $\Phi_\lambda$). Furthermore, when the share of skilled among the immigrant population is higher than among the native population, the average fertility rate of the descendants is higher than that of the immigrants. Similar to the previous case of full assimilation, the labour force growth over time is driven by the weighted sum of the fertility rates, $\Phi_\theta + \psi \Phi_\lambda$; however, the labour force growth rate reaches that level only in the new steady state.

Lemma 5  In the case of partial assimilation the welfare measure reads:

$$\bar{W}^{\text{pa}} = \frac{E_\theta + E_\lambda \psi}{E_\lambda \psi + E_\theta (1 + r) - \Phi_\lambda E_\theta \psi (1 + r - \varphi_s)} (1 + r)$$

(1.36)
**Proof.** Follows directly from rearranging equations (1.6), (1.24), (1.28), (1.34) and (1.25). ■

**Proposition 3** In the case of partial assimilation welfare decreases with increase in the share of skilled among immigrants if

\[
\frac{\varepsilon_s - \varepsilon_u}{\varphi_u - \varphi_s} < \frac{E_\theta + \psi E_\lambda}{r - \psi \Phi_\lambda + 1 - \Phi_\theta}
\]  

(1.37)

and increases if the reverse inequality holds.

**Proof.** The derivative of (1.36) is

\[
\frac{\partial \tilde{W}_{pa}}{\partial \lambda} = P_{pa} \cdot \left[ \frac{\partial E_\lambda}{\partial \lambda} \left( 1 + r - (\Phi_\theta + \psi \Phi_\lambda) \right) + \frac{\partial \Phi_\lambda}{\partial \lambda} (E_\theta + \psi E_\lambda) \right]
\]  

(1.38)

(where \(P_{pa}\) is positive) and is negative if (1.37) holds. ■

The economic intuition from the previous case still holds. Further, a similar numerical exercise results in an inequality (1.37) to be re-written

\[
\frac{0.46}{0.3} < \frac{1.2 + 0.03\lambda}{0.18 + 0.02\lambda}
\]

As \(\lambda \in (0, 1)\), the right-hand side of the above inequality falls into the interval (6.15, 6.67). Thus unskilled immigration is again preferred.

**No Assimilation: Inherited Fertility and Inherited Skills**

In the extreme case of immigrant descendants being identical (in labour-market and reproductive qualities) to their ancestors, i.e. in the case of no assimilation, the size of the population is again the same as the previous two cases for the first two periods. However, while in the first period the labour force is again identical (and is given by (1.6)-(1.24)), the immigrant descendants are different in their skill distribution: The effective labour force in the second period in case of no assimilation is:

\[
N_2^{\alpha} = L \left( E_\theta + \psi E_\lambda \Phi_\lambda + \psi E_\lambda (1 + \psi \Phi_\lambda) \right)
\]

(1.39)
which further changes both the population and the labour force size:

\[ N_t^{na} = L \left( E_\theta + \psi E_\lambda \frac{\Phi_\lambda^t (1 + \psi)^t - 1}{\Phi_\lambda (1 + \psi) - 1} \right) \]  

(1.40)

for \( t > 0 \). Again the weighted sum of the immigrant dynasty average fertility, \( \Phi_\lambda (1 + \psi) \), is the driving force for the labour force growth and is also the limit value that the labour force (and population) growth rate approaches over time.

**Lemma 6**  
In the case of no assimilation the welfare measure reads:

\[ \tilde{W}^{na} = \frac{E_\lambda \psi r}{E_\lambda \psi + E_\theta (1 + r) - \Phi_\lambda E_\theta (1 + \psi)} + 1 \]  

(1.41)

**Proof.** Follows directly from rearranging equations (1.6), (1.24), (1.39)-(1.40), and (1.25). ■

**Proposition 4**  
In the case of no assimilation welfare decreases with increase in the share of skilled among immigrants if

\[ \frac{\epsilon_s - \epsilon_u}{\varphi_u - \varphi_s} < \frac{1 + \psi}{1 + r - \Phi_\lambda (1 + \psi)} \]  

(1.42)

and increases if the reverse inequality holds.

**Proof.** The derivative of (1.41) is

\[ \frac{\partial \tilde{W}^{na}}{\partial \lambda} = P_{na} \cdot \left[ \frac{\partial E_\lambda}{\partial \lambda} (1 + r - \Phi_\lambda (1 + \psi)) + \frac{\partial \Phi_\lambda}{\partial \lambda} (1 + \psi) \right] \]  

(1.43)

(\( P_{na} \) is positive) and is negative if (1.42) holds. ■

As in the previous cases, the economic interpretation of the results and the numerical exercise direct towards similar conclusions. Thus the inequality (1.42), based on the same data, takes the value

\[ \frac{0.46}{0.3} < \frac{1.06}{0.318 \lambda + 0.089} \]

As \( \lambda \in (0, 1) \), the right-hand side of the above inequality falls into the interval (2.6, 11.9), viz. the unskilled migrants are preferred.
Chapter 1  The PAYG Economy and Immigration

The PAYG Economy and Immigration

In the PAYG economy, as opposed to the Status Quo Economy, non-zero immigration is allowed. The initial retired population receives their pension benefits, and all the other cohorts fully participate in the unfunded pension system. The government starts with zero initial debt and continues the zero-debt policy, i.e. the taxes are set to balance the budget (1.7)-(1.8) each period while maintaining
\[ B_t = 0 \]  
for any time \( t \geq 0 \).

**Definition 3** Given the (world) prices for labour and capital, \( w \) and \( r \), the replacement rate for public pension \( \rho \), the subjective time discount factor \( \beta \), skill and fertility rates \( \varepsilon_s \) and \( \varphi_{g,s} \), skill distribution \( \gamma_{s,g} \), the initial value of public debt, \( B_0 \), and the size of the immigrant population compared to the native population \( \psi \), the equilibrium in the unfunded PAYG economy is an allocation
\[
\left\{ c_0^1, \{ c_t^i, \hat{c}_{t+1}, a_t, \mu_t (i,s,g), \hat{\tau}_t, B_t \}_{i=0,1,t=1,2...} \right\}
\]
such that the initial (time \( t = 0 \)) retired agents consume their savings and pension benefits, households optimise (1.9)-(1.11), the government budget (1.7)-(1.8) is balanced according to (1.44), and the population evolves according to (1.16).

Though the definition of the equilibrium is identical in the current and the Status Quo economies, the behaviour of the variables differs significantly. Each period will be characterised by a unique tax level as the effective population size will be changing each time.

**Lemma 7** In the unfunded PAYG economy with immigration the equilibrium welfare measure changes each period and is given as
\[
\hat{W}_t = \frac{N_t}{N_{t-1}}. \tag{1.45}
\]

**Proof.** Using (1.17) the government budget (1.7)-(1.8) can be re-written as
\[
\rho N_t = \hat{\tau}_{t+1}^P N_{t+1} \tag{1.46}
\]
that solves for
\[ \hat{\tau}_t = \rho N_{t-1}/N_t \] (1.47)

Substituting for (1.47) in (1.14) results in (1.45).

The lemma suggests that the welfare measure in the unfunded PAYG economy with immigration collapses to simple effective population change, and the welfare dynamics virtually follows the dynamics of the effective population.

**Lemma 8** The young of the first period (irrespective of the assimilation scenario) have the welfare:
\[ \hat{W}_{1a}^{fa} = \hat{W}_{1a}^{pa} = \hat{W}_{1a}^{na} = E_\theta + E_\lambda \psi E_\theta = 1 + \frac{E_\lambda \psi}{E_\theta} \] (1.48)

**Proof.** Follows directly from (1.6), (1.24), and (1.45).

**Proposition 5** In the unfunded PAYG economy the equilibrium welfare of the first-period young (i.e. the policy-setting cohort) increases with the share of the skilled among immigrants.

**Proof.** The derivative of the right-hand side of (1.48) with respect to the share of skilled among immigrants, \( \lambda \), is
\[ \frac{\psi}{E_\theta} (\varepsilon_s - \varepsilon_u) > 0. \]

The economics behind the result of the proposition is rather intuitive: The initial retired population claims a fixed amount of benefits that the initial young population pays through taxes. As the skills among immigrants increase the size of the effective labour force, and thus the tax base, the tax rate decreases. Further, the presence of immigrants does not change the pension benefits that the initial young will claim once they retire (due to the design of the PAYG system with fixed pension replacement rate); that burden goes to the young of the next period.

Based on the proposition 5, a conjecture on ‘voting equilibrium’ can be made: As the welfare of the retired group is unaltered (by definition) and the young are strictly better off (from proposition 5) any type of voting will result in a selective immigration policy where only skilled immigrants are allowed in. Thus in the PAYG economy with immigration (and without dynastic altruism), the policy-setting population will prefer a policy that is welfare impairing for the entire economy (as in the case of debt-financed social security system).
In order to follow the changes in welfare of different cohorts based on the share of skilled immigrants, each assimilation scenario should be studied separately.

**Lemma 9** If the immigrant descendants fully assimilate, the welfare at period \( t \geq 2 \) is:

\[
W^{fa}_{t \geq 2} = 1 + \psi \Phi \lambda
\]  

(1.49)

**Proof.** Follows directly from (1.24), (1.30), and (1.45).

**Proposition 6** If the immigrant descendants fully assimilate, the welfare of \( t \geq 2 \) (i.e. everyone, except the policy-setting cohort) decreases with an increase in the share of the skilled among immigrants.

**Proof.** The derivative of (1.49) with respect to the share of skilled immigrants \( \lambda \) is

\[\psi (\varphi_s - \varphi_u) < 0.\]

Hence, in the case of full assimilation of the immigrant descendants the economy appears in a new steady state from the second period on (as the effective population change is constant over the periods). At the same time, the skilled immigrants produce fewer children and thus decrease the relative tax base, which decreases the welfare level.

Propositions 5 and 6 suggest that there is a strong conflict of interest between the first and subsequent cohorts of the population: While the first cohort strictly prefers skilled immigrants, all the other cohorts enjoy higher welfare if the policy does not favour the skilled. The lower the share of skilled immigrants, the higher the welfare of later cohorts is. However, it is worth noting that, if the share of skilled immigrants is very small, the initial cohort may have lower welfare than their descendants.

**Proposition 7** In the case of full assimilation, if

\[E_\lambda < E_\theta \Phi \lambda\]

(1.50)

then the welfare of the first (policy-setting) cohort is lower than the welfare of all the other cohorts.

**Proof.** Follows directly from (1.48) and (1.49).
That is, if the effective size of the immigrant population is less than the effective size of the immigrant descendant population (average efficiency multiplied by size) then the first cohort is worse off compared to every other cohort. The reverse also holds, i.e. if the share of skilled immigrants is high, then the first generation has higher welfare than the subsequent cohorts.

The inequality (1.50) can be rewritten in simpler form, i.e. in terms of the share of skilled among immigrants

\[
\lambda < \frac{E \varphi_u - \varepsilon_u}{E \varphi - \varphi_s + \varepsilon_s - \varepsilon_u}
\]  

(1.51)

Using the numbers from previous calculations, \( E \varphi = 1 + 0.31 \cdot 0.46 = 1.1426 \), the nominator becomes \( 1.14 \cdot 1.1426 - 1 = 0.3025 \), and the denominator is \( 0.46 + 1.1426 \cdot 0.3 = 0.8028 \). Hence, when \( \lambda < 0.3025/0.8028 = 0.377 \) the first (policy-setting) cohort has lower welfare than later cohorts. It is worth noting that the calculated value for the share of skilled immigrants that makes the first cohort better off, \( \lambda > 0.377 \), is well above the calculated share of skilled natives \( \varphi = 0.31 \).

**Lemma 10** If the immigrant descendants partially assimilate, the welfare of the agents in the second period is

\[
\tilde{W}_2^{f_a} = 1 + \psi \Phi_{\lambda}
\]

(1.52)

then grows over time according to

\[
\tilde{W}_t^{f_a} = \frac{\Phi_{\theta} - 1 + \psi \Phi_{\lambda} (\Phi_{\theta} + \psi \Phi_{\lambda})^t}{\Phi_{\theta} - 1 + \psi \Phi_{\lambda} (\Phi_{\theta} + \psi \Phi_{\lambda})^{t-1}}
\]

(1.53)

and approaches the level of

\[
\tilde{W}_\infty^{f_a} = \Phi_{\theta} + \psi \Phi_{\lambda}
\]

(1.54)

in the limit.

**Proof.** (1.52) and (1.53) follow directly from (1.24), (1.34), and (1.45). The derivative of (1.53) with respect to time \( t \):

\[
\frac{\partial}{\partial t} \tilde{W}_t^{f_a} = \frac{\Phi_{\lambda} \psi (\Phi_{\theta} - 1) (\Phi_{\lambda} \psi + \Phi_{\theta} - 1) (\Phi_{\lambda} \psi + \Phi_{\theta})^{t+1}}{\left( (\Phi_{\lambda} \psi + \Phi_{\theta}) (\Phi_{\theta} - 1) + \Phi_{\lambda} \psi (\Phi_{\lambda} \psi + \Phi_{\theta}) \right)^2} \ln (\Phi_{\lambda} \psi + \Phi_{\theta})
\]

(1.55)
is positive given that $\Phi_\theta > 1$ by construction. Also, it is straightforward to note that

$$
\lim_{t \to \infty} \frac{\Phi_\theta - 1 + \psi \Phi_\lambda (\Phi_\theta + \psi \Phi_\lambda)^t}{\Phi_\theta - 1 + \psi \Phi_\lambda (\Phi_\theta + \psi \Phi_\lambda)^{t-1}} = \Phi_\theta + \psi \Phi_\lambda
$$

(1.56)

thus proving the last part of the lemma. ■

**Proposition 8** If the immigrants’ descendants partially assimilate, the welfare of $t \geq 2$ (i.e. everyone, except the policy-setting cohort) decreases with an increase in the share of the skilled immigrants.

**Proof.** The derivative of (1.53) with respect to the share of skilled immigrants $\lambda$ is

$$
\frac{\partial}{\partial t} \hat{W}_{p\alpha}^{t \geq 2} = P_{t}^{p\alpha} \cdot (\varphi_s - \varphi_u)
$$

(1.57)

which is negative as $(\varphi_s - \varphi_u) < 0$ while $P_{t}^{p\alpha} > 0$. ■

As in the case of full assimilation, the results unequivocally indicate the reverse effect of skilled immigration on the welfare of the first (policy-setting) cohort and all the subsequent cohorts: While the first cohort is better off with skilled immigrants, future cohorts prefer the unskilled. However, as opposed to the case of full assimilation, in the case of partial assimilation there is a possibility that the first cohort will have higher welfare than the cohorts immediately following, but in future there will be cohorts that will enjoy welfare higher than that of the first generation.

**Proposition 9** In the case of partial assimilation

(a) the first cohort has lower welfare compared to all the other cohorts, if

$$
E_\lambda < E_\theta \Phi_\lambda
$$

(1.58)

(b) the first cohort has higher welfare compared to all the other cohorts, if

$$
E_\lambda > E_\theta \left( \Phi_\lambda + \frac{\Phi_\theta - 1}{\psi} \right)
$$

(1.59)

(c) the first cohort has higher welfare compared to some immediately following cohorts $t \in (2, t)$, but
lower than the welfare of the cohorts \( t > \bar{t} \), if

\[
E_\theta \Phi_\lambda < E_\lambda < E_\theta \left( \Phi_\lambda + \frac{\Phi_\lambda - 1}{\psi} \right) \tag{1.60}
\]

**Proof.** Appropriate comparison of (1.48), (1.52) and (1.54) with slight reshuffling result in (1.58)-(1.60). ■

Thus, the dynamics in the case of partial assimilation are richer, allowing different welfare ranking, but again a large share of skilled immigrants allows the first cohort to have the highest welfare (and the higher the share the higher the welfare is), while a small number of skilled immigrants gives the first cohort the lowest welfare.

**Lemma 11** If the immigrant descendants do not assimilate, the welfare of cohorts \( t \geq 2 \) is

\[
\widehat{W}_{t \geq 2}^{na} = \frac{(F_{\lambda,\psi} (1 + \psi) - 1) E_\theta + \left( \Phi_{\lambda} (1 + \psi)^{t+1} - 1 \right) E_\lambda \psi}{(F_{\lambda,\psi} (1 + \psi) - 1) E_\theta + \left( \Phi_{\lambda} (1 + \psi)^t - 1 \right) E_\lambda \psi} \tag{1.61}
\]

that reaches the limit value of \( \widehat{W}_\infty^{na} = \Phi_\lambda (1 + \psi) \), while growing if

\[
\Phi_\lambda (1 + \psi) > 1 + \frac{E_\lambda}{E_\theta} \psi \tag{1.62}
\]

and decreasing otherwise.

**Proof.** From (1.40) and (1.45) directly follows (1.61). The limit value of (1.61) is also easily obtainable. The derivative of (1.61) with respect to time is

\[
\frac{\partial}{\partial t} \widehat{W}_{t \geq 2}^{na} = \frac{F_{\lambda,\psi} E_\lambda \psi (F_{\lambda,\psi} + 1)^t \ln (F_{\lambda,\psi} + 1)}{(F_{\lambda,\psi} E_\theta + \left( (F_{\lambda,\psi} + 1)^t - 1 \right) E_\lambda \psi)^2} \left( F_{\lambda,\psi} E_\theta - E_\lambda \psi \right) \tag{1.63}
\]

where

\[
F_{\lambda,\psi} = \Phi_\lambda (1 + \psi) - 1 \tag{1.64}
\]

The expression in (1.63) is positive if the second multiplier in the product is positive. The (1.62) presents the required condition. ■
Proposition 10 In the case of no assimilation, if condition (1.62) holds, i.e.

$$\lambda < \frac{E_\theta \phi_u - \epsilon_u}{E_\theta (\phi_u - \phi_s) + \epsilon_s - \epsilon_u}$$

(1.65)

then the first cohort (i.e. the policy-setting cohort) has the lowest welfare, which sequentially increases for subsequent cohorts.

If condition (1.62), or (1.65), holds with the opposite sign, the first cohort has the highest welfare, while welfare sequentially decreases for the subsequent cohorts.

If the expressions on both sides of (1.62), or (1.65), are equal, then the welfare measure for all cohorts $t \geq 1$ are the same.

Proof. The welfare of the second cohort is

$$\hat{W}^{na}_2 = \frac{(\Phi_\lambda (1 + \psi) - 1) E_\theta + (\Phi_\lambda^2 (1 + \psi)^3 - 1) E_\lambda \psi}{(\Phi_\lambda (1 + \psi) - 1) E_\theta + (\Phi_\lambda^2 (1 + \psi)^2 - 1) E_\lambda \psi}$$

(1.66)

which is more than the welfare of the first cohort $(W^{na}_2 > W^{na}_1)$ if

$$1 + \frac{\Phi_\lambda^2 (1 + \psi)^2 E_\lambda \psi}{E_\theta + (\Phi_\lambda (1 + \psi) + 1) E_\lambda \psi} > 1 + \frac{E_\lambda}{E_\theta}$$

(1.67)

The expression simplifies into (1.62), or further into (1.65). Under the same condition (1.62), according to Lemma 11, the welfare measure for all subsequent cohorts grows. This concludes the proof for the first part of the proposition. The other two cases are proved similarly.

Again it is clear that the first cohort benefits from an immigration policy that favours the high-skilled, and that later cohorts would prefer the share of skilled immigrants to be smaller initially. A highly-skilled immigrant group leaves the first (policy-setting) cohort with high welfare, while the welfare of all subsequent cohorts decreases rapidly.

Note that the model is dynamically inconsistent and any cohort will prefer to increase the share of skilled immigrants in their own period, thus increasing its own welfare and decreasing the welfare of the subsequent cohorts.
Conclusion

International labour mobility has reached new heights in recent years. Additionally, over the last decades the pattern of immigration has reversed: Many traditionally source economies are now recipients. Thus, many of these economies (European Union countries in particular) are currently redesigning their immigration policies. The destination countries seem inclined to follow the established policies of more experienced host countries, such as the United Kingdom or Australia, where a so-called point system is employed. The point system is a mechanism of screening the skills of immigrants and allowing entry only to the skilled (versus the unskilled). An illustrative example is the Blue Card currently in design in the EU.

The current paper studied the effect of skilled immigration on the welfare of the host country. A small open economy was considered with overlapping generations of heterogeneous agents. The population differ in various characteristics, such as age, skill level, and the generation in the host economy. The agents were also modelled as different regarding fertility levels. A minimalistic approach was taken for modelling purposes (to eschew possible theoretical uncertainties and biases).

Three alternative economies were studied: A Status Quo economy continues the established PAYG system and does not allow immigrants in the economy (this serves as a benchmark). The second economy allows immigrants into the economy and uses a dynamic social security budget that has one tax rate for all the generations as a smoothing mechanism (this serves as an indicator of the average effect of the immigration on the economy). The third economy allows immigration and retains the unfunded PAYG system. For the last two economies, three different cases were discussed depending on the assimilation of the descendants of the migrant population: Among the infinitely many possibilities of assimilation, two extremes was studied (full assimilation and no assimilation) and an illustrative intermediate case of partial assimilation, when over generations the immigrant descendants inherit the fertility level of their ancestors but adopt the skill level of the native population. All other cases were assumed to be between the extremes.

The results show that a skill-favouring immigration policy most often decreases welfare in an economy with a unified tax rate. That is, the overall effect of the selective immigration is negative for the host economy under the condition that the efficiency gain is not disproportionately large.
compared to the fertility losses when choosing skilled immigrants. Once the model is calibrated to fit an actual economy, the required condition is satisfied for any skill level.

Moreover, in the balanced PAYG equilibrium it is apparent that the first cohort, i.e. the policy designers, benefits from a policy of skilled immigration, while all the others are worse off, when compared to an immigration policy that allows more unskilled immigrants into the economy. Hence in a hypothetical political equilibrium, where only the (non-altruistic) first cohort votes, a skill-favouring immigration policy will be chosen, thus decreasing the welfare of the future cohorts. An intergenerational conflict is present.

The results thus suggest that the decision-making policy-setting initial generation has an incentive to choose an average-welfare-depriving policy of favouring the skilled. However, the model does not suggest any optimal policy to follow: Decreasing the share of skilled immigrants increases the welfare of the average agent, yet decreases the welfare of the decision makers up to the point that the policy-setting first cohort may have the lowest welfare of all. Moreover, each subsequent cohort has an incentive to increase the share of skilled immigrants in their own period (if policy alterations were allowed). Hence, while point-system skill-favouring immigration policies are welfare depriving for the overall economy, unskilled immigration is a dynamically inconsistent policy.
Bibliography


Chapter 2

Can Social Security Survive Non-Selective Immigration?

ABSTRACT  The current study assesses the effects of immigration control on the social security system of a host economy. A theoretical model of a small open economy populated with overlapping generations of heterogeneous agents is used to show that if the immigrant population is sufficiently unskilled the native population will prefer to switch from an unfunded social security system to debt-funded or fully funded systems.

Introduction

Population ageing – joint decline in fertility and mortality rates – increases social security expenses and burdens public finances. An expanded number of retirees and a shrunken working age population results in more beneficiaries per contributor. With a smaller contribution base and increasing number of beneficiaries, governments face problems in financing their pension systems. Those systems have already become a burden for most developed countries: Social expenditures have been increasing at least for the last 20 years in tandem with the share of the old-age population. Already in 2007, public pension revenues covered only 88 per cent of the expenditures on average in OECD countries. By 2060, the revenues are likely to constitute only 64 per cent of the expenditures, and in some countries the projected gap is to reach above 10 per cent of GDP (OECD, 2012).

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Many possible ways of dealing with this problem are proposed: As the pension system is defined by the tax and benefit rates, parametric reforms (changes in those rates) should be able to restore balance in the system. However, reduction of social benefits (including any reduction in the duration of eligibility) contradicts the basic idea of the social welfare states of Europe. Also, an increase in taxes will have harmful consequences in Europe as it may result in ‘voting with feet’ (Uebelmesser, 2004; Krieger, 2005), when the young working generation prefers to emigrate because of the onerous social policy.

Alternatively, a pension system can undergo structural reforms - moving from an unfunded to a funded system. The fully funded system ensures neutrality of the social security system on the government budget. After the Pensiña reforms in Chile, pension reforms gained popularity. The transition from pay-as-you-go (PAYG) to a fully funded scheme seemingly releases the governments from the problem of financing social security.

Further, Aaron (1966) established the fact that the PAYG system is preferable to fully funded system until the sum of population and the real wage growth rates exceed the real interest rate in the economy. Hence, in the developed world, full funding is supposedly the preferred scheme as a result of the current and expected negative growth rate of the population: The fully funded system will guarantee higher pensions for retirees and will eradicate the problem of sustainability for the government.¹

However, a fully funded system cannot be introduced at once, as the PAYG system already exists. When the PAYG system was introduced, the initial generation received benefits without contributing to the system, and from then on all subsequent generations pay to cover the generated implicit debt. Should there be no terminal-generation, the initial-debt-financing problem will never emerge. Alas, the establishment of the funded system terminates the unfunded system, whereupon the implicit debt becomes an ‘explicit’ government debt that requires financing. This makes the political feasibility of the transition questionable as the taxpayers are still under the burden of the legacy of the PAYG system.² Lindbeck and Persson (2003) argue that a cut in

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benefits and increases in contribution rates suffice to sustain the current system, if not the redistributional concerns. It has been shown in the literature (e.g. Breyer, 1989; Conesa & Garriga, 2007) that such a transition *per se* does not generate an efficiency gain as the burden of the introductory gift (the first benefits distributed to the population who did not contribute) will exist in both cases.

Whilst traditional economic policies have become ineffective in sustaining social security in ageing societies, new policies have been proposed: A logical alternative to the parametric and structural reforms of social security is demographic reform. A popular reform discussed in the literature is connected to immigration\(^3\) (Storesletten, 2000; Chen & Fang, 2013), *i.e.* a foreign born population may cover the shortage of the local working population.\(^4\) The literature is mostly favourable towards this theory of ‘replacement migration’ (an excellent survey of the literature is provided by Kerr & Kerr, 2011). That is, migration is seen as free importation of production factor that can be taxed to cover the budgetary shortages connected with social expenses (e.g. Razin & Sadka, 2000; Fehr, Jokish & Kotlikoff, 2004).

Meanwhile, the ‘welfare magnet’ literature claims that low skilled immigrants are actually attracted by the redistributive welfare systems of the host economies and thus they are a burden on the public finances (Borjas, 1999; Barret & Maître, 2013). Based on that idea, Cohen, Razin & Sadka (2009) and Razin, Sadka & Suwakiri (2011) show that the population will choose to abandon\(^5\) the (redistributive) social welfare programmes in the political equilibrium. However, recent empirical findings do not confirm the existence of those welfare magnet effects. Thus, Giulietti, Guzi, Kahanec & Zimmerman (2013) based on the OECD data claim that migrant skill distribution in the economy cannot be explained by the generosity of the welfare system (at least not by the unemployment benefits). Belot & Hatton (2012) show that cultural similarities and colonial legacies are the important factors that determine the direction of migration, especially in Europe.

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\(^3\) Currently immigration policy formulation has high priority among other institution building activities in the EU (Vikhrov, 2013). A solid proof for that is the ‘Green Paper’: It pre-defines an EU approach to managing economic migration.\(^4\) The Green Paper aims to design an immigration policy that would fulfill the Lisbon Strategy (of building a knowledge-based social economy). Among other policies directly aimed at fulfilling the Lisbon Strategy (e.g. attracting students and researchers), the Paper highlights the possibility of using migration policy to secure fiscal sustainability of the social scheme.

\(^4\) While some countries are deficient in working age population, some other countries have an excess supply of it. For instance, Fargues (2005) notes the example of Egypt where, annually, an additional half a million workplaces are needed to absorb the new generation entering the labour market.

\(^5\) In their model social security is cut to absolute minimum, which is still positive, as they have assumed pensions-in-the-utility function.
However, the previous chapter showed that immigration brings intergenerational welfare conflicts even in the case of a non-redistributive PAYG system: Further, it has been shown that the less skilled the immigrants are, the more benefits go to the future population under a restricted PAYG system. Therefore, given that migration control is not always possible, most of the welfare gains from immigration may go to future cohorts. Thus, this chapter shows that lacking other intergenerational welfare smoothing mechanisms, the current population may prefer to abandon the PAYG system in favour of fully funded pension systems. Additionally, as opposed to the previous literature (i.e. Cohen, Razin & Sadka, 2009; Razin, Sadka & Suwakiri, 2011) the need to reform the social security arises not because the immigrants are a drain on the system but rather from fact that immigration delays the benefits into the future and social security reform is a mechanism for redistribution of those gains.

This chapter heavily borrows methods from the previous chapter, that is, a small open economy environment is again used, where an actuarially fair (intra-generationally non-redistributive) PAYG system already exists. However, in this chapter an economy with a structural reform is studied and it is conjectured that immigration reforms may lead to social security reforms once the initial population generates welfare gains from the reform. Further, this chapter also allows the study of joint determination of social security and immigration policies, i.e. simultaneous reforms in both policies if immigration control is possible.

**The Economic Environment**

This chapter utilises the same economic environment discussed in the previous chapter: There are young and retired, skilled and unskilled populations. Those also differ by origin, being immigrants and local-born. The local-born population can be either descendants of a native or immigrant family. While the natives and immigrants differ in their skill distribution and fertility rates, the descendants of immigrants can undergo some assimilation. Again, three main assimilation patterns will be discussed: Full assimilation (adopting both skill and fertility rates), partial

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6For instance, Vikhrov (2013) discusses the EU free labour movement policy; Belot & Hatton (2012) emphasise the often-ineffective selective policies related to the refugees and tied family migration, Razin & Wahba (2011) highlight the inability of the selective policies to control for the quality of education.

7An alternative to abandoning the balanced PAYG system in favour of funded system can be retaining PAYG in combination with borrowing, i.e. moving to the debt-financed PAYG economy. While these alternatives yield identical welfare (according to the Breyer equivalence result), still the new system cannot be considered an unfunded any more.
assimilation (adopting the skill levels but inheriting fertility rates) and no assimilation (inher-
iting the skill and fertility rates from the immigrant ancestors). Thus, the equations (1.1)-(1.6),
(1.23)-(1.24), (1.28)-(1.30), (1.34), (1.39)-(1.40) still hold.

**Government**

The Government again is responsible for maintaining immigration policy and fiscal constitution. However, in this case while the government may control the size of immigration, it is assumed that selective immigration is absolutely impossible or, alternatively, that the characteristics of the immigrants are exogenous. The fiscal constitution is also different from the previous case. In addition to the PAYG equilibrium discussed in the previous chapter, here an equilibrium without a pension system is allowed:

$$\sum_s \rho w \varepsilon(s) \mu_1(1, s, n) + B_0(1 + r) = \sum_{s, g} \bar{\tau} w \varepsilon(s) \mu_1(0, s, g) + B_1$$  \hspace{1cm} (2.1)$$

(for all the other cohorts)  \hspace{1cm} B_{t-1}(1 + r) = \sum_{s, g} \bar{\tau} w \varepsilon(s) \mu_t(0, s, g) + B_t$$  \hspace{1cm} (2.2)

and

$$\lim_{t \to \infty} B_t \cdot (1 + r)^{-t} = 0$$  \hspace{1cm} (2.3)

The unfunded social security can be abandoned, while honouring the initial retired population’s claims to benefits, and the government can levy a tax to finance the new debt (the implicit debt of the social security system - retirees’ claims to benefits - converted into an explicit debt). As before, the initial level of debt can be assumed to be zero $B_0 = 0$.

The constant tax rate $\bar{\tau}_{LF}$ is one of infinitely many solutions that would balance the government budget (2.1)-(2.3). However, there is no other tax policy which is Pareto-superior to this policy: Once the PAYG system is terminated, the government generates debt by servicing the pension claims of the initial retired population and levies taxes to service the debt. The fixed tax rate distributed the debt equally over the generations. Thus if the rate is lowered for any one generation, then the other generations have to pay higher taxes in order to finance the government debt.
Households and Welfare

Each household is again populated by a single agent, which optimises the problem (1.9)-(1.11). However, in this section a new welfare measure will be used to in order to accommodate the possibility of social security reforms. As in the case of PAYG termination, no one except the initial retired population receives pension benefits, so comparison based only on the reciprocal of the tax base will not be fully meaningful. Thus,

**Lemma 12** The lifetime utility function

\[
U_s(\tau_t, \rho_t) = \max U \left( c^t_s(s), c^{t+1}_s(s) \right)
\]

is strictly increasing in

\[
W_t \equiv 1 - \tau_t + \frac{\rho_t}{1 + r}.
\]

**Proof.** Similar to the proof of Lemma 1. \(\blacksquare\)

In essence, the welfare measure of the current chapter, \(W_t\) as given in equation (2.5) and the welfare measure \(W_t\) from equation (1.14) developed in Lemma 1 of the previous chapter can be translated into each other

\[
W_t = 1 + \rho \left( \frac{1}{1 + r} - \frac{1}{W_t} \right)
\]

However, as opposed to the welfare measure (1.14) from the previous chapter, the welfare measure given in equation (2.5) explicitly accounts for the fact that under different equilibria, the households may or may not receive pension benefits, and thus makes the equilibria comparable in welfare terms. In addition, the lemma shows that the utility is proportional to the specific form of the effect of the government policies on the lifetime income of the agent, \(W_t\). This result is due to the assumption of a small open economy where the prices of capital and labour are given, and provides a possibility to study the first order effects of the government policy on welfare.
The Laissez Faire Economy

In the Laissez Faire economy, the government allows in immigrants with exogenously given skill levels, cancels the PAYG social security system, finances the initial retired population’s pension claims through borrowing, and sets a tax rate so that the government budget is balanced over an infinite-time horizon.

Definition 4 Given the (world) prices for labour and capital, $w$ and $r$, the replacement rate for public pension $\rho$, the subjective time discount factor $\beta$, skill and fertility rates $\varepsilon_s$ and $\varphi_{g,s}$, skill distribution $\gamma_{s,g}$, the initial value of public debt, $B_0$, and the relative size of the immigrant population $\psi$, the equilibrium in the Laissez Faire economy is an allocation

$$\begin{align*}
\{ & c_{t0}^1, \{ c_{t1}^1, c_{t+1}^1, a_t, \mu_t (i, s, g), B_t \}_{t=0,1}^{i=0,1; t=1,2} \}
\end{align*}$$

such that the initial (time $t = 0$) retired agents consume their savings and pension benefits, households optimise (1.9)-(1.11), the government budget sets a tax rate, $\bar{\tau}$ to balance the budget (2.1)-(2.3), and the population evolves according to (1.1)-(1.5).

The equilibrium defined above starts from a point with an existing restricted (defined benefit) PAYG social security system and no-migration policy, i.e. the start in the Status Quo economy. The definition suggests that there is one perpetually fixed level of tax, thus making the reform a mechanism for intergenerational welfare redistribution (smoothing). Also, one tax rate makes for convenient comparison between Laissez Faire, Status Quo and debt-financed PAYG economies: While the comparison between the Status Quo and debt-financed PAYG economies shows the effect of immigration, the comparison of Laissez Faire and debt-financed PAYG economies shows the effect of the social security reforms. Similarly, comparison of the Laissez Faire with zero-migration, i.e. $\psi = 0$, and Status Quo also describes the welfare effects of social security reforms:

Proposition 11 In equilibrium Laissez Faire and Debt-financed PAYG economies yield equal welfare:

$$\bar{W} = \tilde{W}$$

(2.7)

where $\bar{W} = W_t$ in the Laissez Faire economy, and $\tilde{W} = W_t$ in the economy with debt-financed PAYG for
The government budget constraint in the debt-financed PAYG economy (1.21), divided on both sides by \((1 + r)^t\) and summed over all periods, can be rewritten as

\[
\tilde{\tau} w \sum_{t=1}^{\infty} \frac{N_t}{(1 + r)^t} = \rho w \sum_{t=1}^{\infty} \frac{N_{t-1}}{(1 + r)^t} + \sum_{t=1}^{\infty} \frac{B_t - B_{t-1}(1 + r)}{(1 + r)^t}
\] (2.8)

that, using (1.22) and \(B_0 = 0\), solves for

\[
\tilde{\tau} = \frac{\rho N_0}{1 + r} \left( \sum_{t=1}^{\infty} \frac{N_t}{(1 + r)^t} \right)^{-1} + \frac{\rho}{1 + r} \] (2.9)

Plugging \(\tilde{\tau}\) into the definition of the welfare measure (2.5)

\[
\tilde{W} \equiv 1 - \rho \sum_{t=1}^{\infty} \frac{N_t}{(1 + r)^t} \] (2.10)

The government budget constraint in \textit{Laissez Faire} economy (2.1)-(2.2), divided on both sides by \((1 + r)^t\) for each period and summed over all periods, can be rewritten as

\[
\bar{\tau} w \sum_{t=1}^{\infty} \frac{N_t}{(1 + r)^t} = \rho w \frac{N_0}{1 + r} + \sum_{t=1}^{\infty} \frac{B_t - B_{t-1}(1 + r)}{(1 + r)^t}
\] (2.11)

that, using (1.22) and \(B_0 = 0\), solves for

\[
\bar{\tau} = \frac{\rho N_0}{1 + r} \left( \sum_{t=1}^{\infty} \frac{N_t}{(1 + r)^t} \right)^{-1} \] (2.12)

Plugging \(\bar{\tau}\) into the definition of the welfare measure (2.5) and using the fact that \(\rho_t = 0\)

\[
\bar{W} \equiv 1 - \rho \sum_{t=1}^{\infty} \frac{N_t}{(1 + r)^t} \] (2.13)

Direct comparison of (2.10) and (2.13) yields the results. ■

The proposition is based on the fact that once a social security system was introduced, one generation received benefits without contributing to the system. This implicit debt stays within the PAYG system and requires financing through contributions. This part of the contribution is what
Nishiyama & Smetters (2007) refer to as the effective tax of the system - that is the difference between the tax contribution and pension benefit under the (debt-financed) PAYG system where all generations contribute to cover the internal debt of the system. In the case of a Laissez Faire economy, the internal debt is converted into an external debt and is financed through taxation and thus is equal to the effective tax of the system. Further, as the effective population growth rate is assumed to be less than the interest rate throughout this work, no welfare gains are possible under either system: should there be extra borrowing in the Laissez Faire economy, future cohorts will face higher taxes for financing the debt (as the interest on the debt will be more than the population growth rate). Similarly, the redistribution through the PAYG system generates losses.

In essence, Proposition 11 is an extended version of the Breyer (1989) equivalence result, which states that PAYG is (Pareto-)efficient or that social security reforms do not generate welfare gains themselves. Similar equivalence results in the framework of social security reforms are presented in Fenge (1995), Lindbeck & Persson (2003) and Conesa & Garriga (2008). They claim that the government can conduct Pareto-neutral reforms using appropriate debt financing. Accordingly, Proposition 11 extends the Breyer equivalence result to show that Pareto-neutral reforms are possible while there are demographic changes in the number and skill level of the population, i.e. where the heterogeneity and migration are incorporated into the model.

**Corollary 1** Increased immigration policy is welfare enhancing independent of social security policy:

\[ \tilde{W} = \bar{W} > W^{SQ} \] (2.14)

**Proof.** Follows directly from the Propositions 1 and 11.

As Corollary 1 shows, the increased immigration policy is Pareto-superior independent of social security policy. Again, the result is primarily based on the assumption of an open economy with a fully actuarial social security system (as the sole channel for immigrants to affect the welfare of the locals).
Welfare Analysis

As the population dynamics is different under each version of the dynastic assimilation, each case needs to be studied separately. Thus,

**Lemma 13** In Laissez Faire economy the welfare is

- **in the case of full assimilation**

  \[ \bar{W}^{fa} = 1 - \frac{\rho}{1 + r} \frac{E_\theta (r - \psi \Phi)}{(E_\theta + \psi E_\lambda)(1 + r - \Phi)} \]  

  \[ (2.15) \]

- **in the case of partial assimilation**

  \[ \bar{W}^{pa} = 1 - \frac{\rho}{1 + r} \frac{E_\theta r (1 + r - \Phi - \psi \Phi)}{(E_\theta + \psi E_\lambda)(1 + r - \Phi)} \]  

  \[ (2.16) \]

- **in the case of no assimilation**

  \[ \bar{W}^{na} = 1 - \frac{\rho}{1 + r} \frac{E_\theta r (1 + r - \Phi (1 + \psi))}{(E_\theta + \psi E_\lambda)(1 + r) - \Phi E_\theta (1 + \psi)} \]  

  \[ (2.17) \]

**Proof.** Follows from (2.13) and (1.30), (1.34) and (1.40).

The Case of Full Assimilation

In the case of full assimilation of the immigrant dynasties, an unfunded PAYG economy reaches a new steady state after the first post-migration period. Once the first cohort immigrants produce children (that are identical to the natives) and a new cohort of immigrants are allowed to enter, the per-period change in the effective labour stabilises and yields constant welfare for everyone. Thus, the initial \( t = 1 \) young population has a welfare level different from all the other cohorts:

**Proposition 12** In the case of full assimilation of the immigrant descendants, the welfare

\[ \hat{W}_1^{fa} < \bar{W}^{fa} < \hat{W}_{t>1}^{fa} \]  

\[ (2.18) \]
if, and only if,

$$\lambda < \frac{E_\theta \varphi_u - \varepsilon_u}{E_\theta (\varphi_u - \varphi_s) + \varepsilon_s - \varepsilon_u}$$

(2.19)

and \( \hat{W}_{fa}^a > W_{fa}^a > \hat{W}_{fa}^a_{>1} \) if the inequality (2.19) holds with the opposite sign. If the inequality (2.19) holds as equality, the welfare for all cohorts in the unfunded PAYG economy and Laissez Faire economy is identical.

**Proof.** Direct comparison of the welfare yields:

$$W_{fa}^a - \hat{W}_{fa}^a = -\frac{\rho \psi}{1 + r \psi E_\lambda + E_\theta} (E_\lambda - E_\theta \Phi_\lambda)$$

(2.20)

which is positive if, and only if,

$$E_\theta \Phi_\lambda > E_\lambda,$$

(2.21)

negative if \( E_\lambda > E_\theta \Phi_\lambda \), and zero if \( E_\lambda = E_\theta \Phi_\lambda \). Similarly,

$$W_{fa}^{a_{LP}} - W_{fa}^{a_{>1}} = \frac{\rho \psi}{1 + r (\psi E_\lambda + E_\theta) (1 + \psi \Phi_\lambda)} (E_\lambda - E_\theta \Phi_\lambda)$$

(2.22)

the sign of which also depends on the sign of \( (E_\lambda - E_\theta \Phi_\lambda) \). Further, substituting \( E_\lambda \) and \( \Phi_\lambda \) from (1.23) and (1.29) in (2.21) yields (2.19). \( \blacksquare \)

The proposition can be understood in terms of the Breyer equivalence result, i.e. the welfare in Laissez Faire economy is the ‘weighted average’ of the welfare under unfunded PAYG across cohorts. The proof of the proposition also suggests that the difference between the welfare levels of the initial young population and the Laissez Faire is larger than difference between the welfare levels of farther cohorts and the Laissez Faire, i.e. the gains or losses from terminating the PAYG is larger for the initial young population than any other agent in later periods.

Moreover, the proposition claims that if the skill distribution of the immigrants is skewed towards the lower end, the initial young population will have higher welfare under the Laissez Faire economy, i.e. the initial young population will initiate social security reforms (in order to redistribute the welfare gains from immigration for their own benefit).

Using the data from the previous chapter (based on Akin, 2012) the inequality (2.19) can be quantified. Thus, \( E_\theta = 1 + 0.31 \cdot 0.46 = 1.1426 \), and the nominator is \( 1.14 \cdot 1.1426 - 1 = 0.3025 \), while the
denominator is \(0.46 + 1.1426 \cdot 0.3 = 0.8028\). Hence, when \(\lambda < 0.3025 / 0.8028 = 0.377\), the initial young, i.e. the first (policy-setting) cohort, has lower welfare than all the others. Furthermore, the calculated value for share of skilled immigrants that makes the first cohort better off, \(\lambda > 0.377\), is well above the calculated share of skilled natives from the data \(\theta = 0.31\) and the share of the skilled immigrants \(\hat{\lambda} = 0.06\) (thus suggesting that according to the model the current German population is in favour of full termination of the PAYG system).

**The Case of Partial Assimilation**

In the case when the immigrant dynasties partially assimilate, i.e. the local born population adopt the skill distribution of the native population while inheriting the fertility levels of their ancestors, the effective labour force changes its composition each period causing intergenerational welfare imbalance in an unfunded PAYG economy. This welfare disbalance generates a possibility for welfare redistribution if the PAYG is terminated, i.e. the Laissez Faire economy is installed.

**Proposition 13**  
In the case of partial assimilation of the immigrant descendants, the welfare

\[
\hat{W}^{\text{pa}}_1 < W^{\text{pa}}
\]  

if, and only if,

\[
\lambda < \frac{E_\theta \varphi_u - \varepsilon_u + \varepsilon_s (\Phi_\lambda - 1)}{E_\theta (\varphi_u - \varphi_s) + (\varepsilon_s - \varepsilon_u)} - \frac{(\varepsilon_s - \varepsilon_u)(\Phi_\theta - 1)}{E_\theta (1 + r - \Phi_\theta)}
\]  

(2.24)

and \(\hat{W}^{\text{pa}}_1 > W^{\text{pa}}\) if the inequality (2.24) holds with the opposite sign. If the inequality (2.24) holds as equality, the welfare for the initial young in the unfunded PAYG economy and Laissez Faire economy is identical.

**Proof.** Direct comparison of the welfare yields:

\[
\hat{W}^{\text{pa}} - W^{\text{pa}} = \rho \frac{\psi}{1 + r} \frac{E_\theta \Phi_\lambda r - E_\lambda (1 + r - \Phi_\theta)}{1 + r - \Phi_\theta}
\]  

(2.25)

the sign of which equals the sign of the nominator of the third ratio on the right-hand side of the eq (2.25).
Thus, similar to the case with full assimilation, there is a possibility that the welfare of the initial young population will be higher in the Laissez Faire economy vis-à-vis unfunded PAYG economy, viz. the initial young cohort will prefer to terminate the PAYG system and shift some of the burden of the initial retired population’s pension benefit claims to be ‘financed’ by the immigration-generated gains for the future cohorts.

Further, in the case of partial assimilation, the share of skilled immigrants needs to be higher than in the case of full assimilation, in order for the unfunded PAYG (vis-à-vis the Laissez Faire economy) to bring higher welfare to the initial young population. As can be seen in the previous chapter, the level of welfare in the case of partial assimilation is identical to the welfare in the case of full assimilation for the first two periods. However, according to Lemma (10) the welfare under partial assimilation grows after the second period towards a new steady state level. Thus, the welfare in the Laissez Faire economy, as the ‘weighted average’ of the unfunded PAYG levels, guarantees higher welfare. The same relationship can be seen by comparing (2.19) and (2.24): The ratio on the right-hand side of the inequality (2.24) has an additional positive term added at the nominator and a negative term at the denominator, thus yielding a larger number. Again, the data from the previous chapter can be used to quantify the inequality (2.24): $\lambda < 0.63$. As with the case of full assimilation, the share of skilled immigrants $\lambda$ – which makes unfunded PAYG preferable for the initial young – is much higher than the current level of $\hat{\lambda} = 0.06$. Hence, according to the model and the German data, any new immigration will cause the initial young population to prefer social security reform in the case of partial assimilation as well.

**Proposition 14** In the case of partial assimilation of the immigrant descendants, the welfare

$$\bar{W}_{I \rightarrow \infty}^{pa} > W^{pa}$$

if, and only if,

$$\lambda < \frac{E_0 \varphi_u - \varepsilon_u + \varepsilon_u (\Phi_u - 1) / r + E_n (1+r)(\Phi_u - 1)}{E_0 (\varphi_u - \varphi_s) + (\varepsilon_s - \varepsilon_u) - (\varepsilon_s - \varepsilon_u)(\Phi_u - 1)/r}$$

(2.27)

and $\bar{W}_{I \rightarrow \infty}^{pa} < W^{pa}$ if the inequality (2.27) holds with the opposite sign. If the inequality (2.27) holds as equality, the welfare for the initial young in the unfunded PAYG economy and Laissez Faire economy is identical.
Proof. Direct comparison of the welfare yields:

\[
\tilde{W}_{t \to \infty}^{pa} - \tilde{W}^{pa} = \eta \cdot [(E_{\theta} \Phi r - E_{\lambda} (1 + r - \Phi_{\theta})) \psi + E_{\theta} (1 + r) (\Phi_{\theta} - 1)]
\]  
(2.28)

where \(\eta = \frac{\rho(1+r-\Phi_{\theta} - \Phi_{\lambda} \psi)(1+r)(E_{\theta} \psi + E_{\lambda} + E_{\theta})}{(1+r)(E_{\theta} \psi + E_{\lambda} + E_{\theta})} > 0\). Thus, the sign of (2.28) is determined by the sign of the second term, which is positive if the inequality (2.27) holds, negative if the inequality has the opposite sign and equal when both sides of the inequality are equal. \(\blacksquare\)

In effect, the proposition claims that most cohorts receive higher welfare in the unfunded PAYG economy as opposed to the Laissez Faire economy if the share of the skilled migrant population is not very large. However, comparison of the inequalities (2.24) and (2.27) shows the threshold value of the share is higher than that in Proposition 13, viz. there is a possibility that the share of skilled immigrants is between the two values, so that the initial young and the distant future cohorts are better off in the unfunded PAYG economy at the ‘expense’ of the intermediate cohorts. Correspondingly, there are values of \(\lambda\) that cause initial young and the immediate future cohorts to prefer the Laissez Faire economy, i.e. prefer to terminate the existing PAYG system.

Proposition 15 In the case of partial assimilation

(a) the initial young have the highest welfare in the unfunded PAYG economy compared to all the other cohorts and to the Laissez Faire economy, while all the other cohorts have lower welfare compared to the Laissez Faire economy:

\[
\tilde{W}_{t}^{pa} > \tilde{W}^{pa} > \tilde{W}_{t \to \infty}^{pa}
\]  
(2.29)

if, and only if,

\[
\lambda > \frac{E_{\theta} \varphi_{u} - \varepsilon_{u} + \varepsilon_{s}(\Phi_{\theta} - 1)}{E_{\theta} (\varphi_{u} - \varphi_{s}) + (\varepsilon_{s} - \varepsilon_{u}) - \frac{(\varepsilon_{s} - \varepsilon_{u})(\Phi_{\theta} - 1)}{\psi}}
\]  
(2.30)

(b) the initial young have the highest welfare in the unfunded PAYG economy compared to all the other cohorts and to the Laissez Faire economy, while some immediately following cohorts have lower welfare compared to the Laissez Faire economy

\[
\tilde{W}_{t}^{pa} > \tilde{W}_{t \to \infty}^{pa} > W^{pa}
\]  
(2.31)
if, and only if,

\[
\frac{E_\theta \varphi_u - \varepsilon_u + \frac{E_\theta (\Phi_{\theta} - 1)}{\psi_{\theta}}}{E_\theta (\varphi_u - \varphi_s) + (\varepsilon_s - \varepsilon_u)} < \lambda < \frac{E_\theta \varphi_u - \varepsilon_u + \frac{E_\theta (\Phi_{\theta} - 1)}{\psi_{\theta}}}{E_\theta (\varphi_u - \varphi_s) + (\varepsilon_s - \varepsilon_u)}
\]

(2.32)

(c) the initial young have the higher welfare in the unfunded PAYG economy compared to the Laissez Faire economy, while some future cohorts have higher welfare compared to the initial young and to the Laissez Faire economy

\[
\hat{W}_{pa}^{\infty} > \hat{W}_1^{pa} > \bar{W}_1^{pa}
\]

(2.33)

if, and only if,

\[
\frac{E_\theta \varphi_u - \varepsilon_u + \frac{E_\theta (\Phi_{\theta} - 1)}{\psi_{\theta}}}{E_\theta (\varphi_u - \varphi_s) + (\varepsilon_s - \varepsilon_u)} < \lambda < \frac{E_\theta \varphi_u - \varepsilon_u + \frac{E_\theta (\Phi_{\theta} - 1)}{\psi_{\theta}}}{E_\theta (\varphi_u - \varphi_s) + (\varepsilon_s - \varepsilon_u)}
\]

(2.34)

(d) the initial young have lower welfare in the unfunded PAYG economy compared to the Laissez Faire economy, while some future cohorts have higher welfare compared to the initial young and to the Laissez Faire economy

\[
\hat{W}_{pa}^{\infty} > \hat{W}_1^{pa} > \bar{W}_1^{pa}
\]

if, and only if,

\[
\lambda < \frac{E_\theta \varphi_u - \varepsilon_u + \frac{E_\theta (\Phi_{\theta} - 1)}{\psi_{\theta}}}{E_\theta (\varphi_u - \varphi_s) + (\varepsilon_s - \varepsilon_u)}
\]

Proof. Follows directly from the Propositions 9, 13, and 14.

Proposition 15 cases (a) - (c) jointly characterise those cases when the initial young have higher welfare under unfunded PAYG compared to Laissez Faire equilibrium, when the share of immigrants is higher than the threshold level given by inequality (2.24). This case is divided into three subcases. In the case of (a), when the share of skilled in the migrant population is large and the inequality (2.30) holds, the initial young population prefers the unfunded PAYG economy while all the other cohorts have a lower level of welfare compared to the Laissez Faire economy. In the cases of (b) and (c) there are some future cohorts who also benefit from being in the PAYG economy.
Figures 2.1 - 2.7 illustrate different cases that may arise in the case of partial assimilation. The vertical axis shows the level of welfare that the measure (2.5) takes in each case. The horizontal surface axes show the time (or the post-migration period, i.e. the cohort starting from the initial young) and the share of skilled migrant population, $\lambda$. That is, in each time period the welfare of agents is shown in the unfunded PAYG economy and the *Laissez Faire* economy. In the case of *Laissez Faire* economy, the welfare does not change over time and thus is represented with the white chequered flat surface: Each cross-point on the surface shows the level of welfare for generation $t$ (invariable) when the share of skilled migrant population is $\lambda$. The coloured surface shows the time-variant (cohort-dependent) welfare in the *Laissez Faire* economy. The darker shades represent lower levels of welfare.

Figure 2.1 depicts the case of the data that has been used in this and the previous chapter (i.e. based on Akin (2012), an assumption that the immigrant population is equal to 5 per cent in each period, and pension replacement rate of 22 per cent calculated based on Eichhorst *et al.*, 2011). The results of Proposition 3, *i.e.* that with debt-financed PAYG (or *Laissez Faire*) welfare decreases with an increase in the share of skilled immigrant population, can be seen in the angle of the white surface: The higher the share of skilled immigrants, the lower the welfare is. A similar observation for the period 1 (the first post-migration period) in the unfunded PAYG economy illustrates the results of Proposition 5, *i.e.* the welfare of the initial young increases with the share of skilled immigrant population. For all the other periods on the unfunded PAYG surface, selective immigration corresponds to lower welfare, as suggested by Proposition 8. Further, the results of Proposition 1 (any immigration is Pareto-improving in the given environment) can be seen by comparing the welfare levels to the welfare levels to 0.9496 of the *Status Quo* economy.

In figure 2.1, however, only the cases (c) and (d) of Proposition 15 can be observed, i.e. there are always some later cohorts that have higher welfare than the initial young population. Furthermore, for some low values of $\lambda$, the welfare of the initial young population is lower than that of any other cohort in the unfunded PAYG economy and that of the own welfare in the *Laissez Faire* economy. (The share of the skilled in the immigrant population from the German Socio-Economic Panel data, as calculated in Akin (2012), is 0.06 or 6 per cent of the entire immigrant population in Germany demonstrates the case when the initial young have the lowest welfare.) Again, it is expected that when the welfare of the initial young is higher in the *Laissez Faire* econ-
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Omy compared to the unfunded PAYG, that is in any case when the share of the skilled migrant population is less than 63 per cent of the total, the initial young will initiate a termination of the unfunded PAYG system.

Figure 2.2 and 2.3 illustrate the same results with higher or lower levels of immigration. In figure 2.2, the share of migrants in the total population is assumed to be one per cent in each period, while figure 2.3 illustrates the case of 15 per cent. Thus, when the amount of immigration is low, the welfare gains are also small (in line with Proposition 1). Moreover, the magnitude of the effect of the increase in the share of the skilled immigrant population is smaller in the case of a smaller share of the immigrant population in the total. Further, as can be seen from figure 2.3, even with large (15 per cent) migration, the PAYG system generates welfare losses for the population and only for the very late cohorts does the welfare measure reach 0.99, i.e. the system still generates a welfare loss of about 1 per cent. Also, with a larger share of immigrant population and a larger share of skilled immigrant population the welfare of the initial young population is larger than that of any other cohort in the unfunded PAYG or Laissez Faire economy (in line with case (a) in proposition 15), however, the initial young population still prefers a Laissez Faire economy over unfunded PAYG if the share of skilled migrant population is lower than 63 per cent (as predicted by Proposition 13).

Figure 2.4 and 2.6 illustrate the effects of the skill and fertility rate premiums. Thus, figure 2.4 corresponds to the case when the skill level does not translate into an efficiency level, but only defines the share of population with high and low fertility rates. In this case the welfare of the initial young population becomes immune to the share of skilled immigrants (as they do not affect the tax base of the first post-migration period, and in the second period when the initial young retire they receive the promised pensions regardless) and thus makes the Laissez Faire economy more attractive (in this case the share of skilled migrants should be higher than the previously calculated 63 per cent, and actually with the current data, the Laissez Faire economy guarantees a higher level of welfare to the initial young population with any level of \( \lambda \).

Figure 2.5 is based on an assumption that there is no difference in fertility levels between immigrants, i.e. skilled and unskilled immigrants have equally high fertility rates (replacement rate 1.4 per person based on Akin, 2012). In this case the initial young population benefits from a higher share of immigrants in the unfunded PAYG economy, but the welfare for all the future
cohorts does not change with the share of skilled migrant population. As opposed to the other cases discussed above, the welfare in the Laissez Faire economy now increases with the share of skilled migrant population as the inequality (1.37) does not hold in the Proposition 3. By the Breyer-equivalence-result logic, if the welfare of the initial young population increases while the welfare of all the other cohorts does not change with the increase in the share of skilled migrant population, then the average welfare (the welfare in the Laissez Faire economy) will increase. Alternatively, from the definition of the Laissez Faire economy, the new debt created from termination of PAYG is financed equally by all the agents, viz. the larger the tax base, the smaller the tax rate and the higher the welfare. Thus a more skilled immigrant population increases the tax base and therefore welfare. Furthermore, welfare in the Laissez Faire economy is higher than in the unfunded PAYG economy for the initial young cohort, again implying that social security reform will be initiated.

Figure 2.5 illustrates a similar case of no fertility rate premium. However, in this case all the agents in the economy are assumed to have the same unit fertility rate, i.e. each period the population is producing a young population of the same size and new migrants enter, to form a constant share of total population. In this case, similar to the previous equal fertility scenario of Figure 2.6, the first generation benefits from the higher share of skilled migrants, while all other cohorts are indifferent. Furthermore, the equality of fertility rates of immigrant and native population already stabilises the tax rate from the second period on. Again, the inequality (1.37) in Proposition 3 does not hold and the welfare in the Laissez Faire economy grows with the increase in the share of skilled migrant population. However, in this case the welfare in the Laissez Faire economy is lower (because of the smaller tax base) than the welfare under unfunded PAYG for the initial young population once the share of the skilled migrants is more than 30 per cent.

Finally, figure 2.7 illustrates a case where migrant fertility rates are lower than initially assumed. As opposed to previously used rates (based on Akin, 2012) of 1.14 for the unskilled immigrants a unit rate is assumed, and for the skilled immigrants previously used (data implied) a rate of 0.84 is still used. Though this is against the commonly applied assumption of immigrants having higher fertility rates than natives, the logic of the results still apply. The welfare of the initial young population increases with the increase in the share of the skilled migrant population, and the welfare for other cohorts in the unfunded PAYG economy decreases. Further, the welfare in
the Laissez Faire economy decreases with the increase in the share of skilled migrant population. In opposition to the result of the Lemma 10, here the welfare in the unfunded PAYG economy is decreasing over time to its new steady state, though (in accordance with Corollary 1) the welfare is still above the Status Quo level of 0.9496.

The Case of No Assimilation

In the case of no assimilation of the migrant descendants, i.e. when the immigrant generations inherit the skill and fertility levels of their migrant ancestors, the welfare in the unfunded PAYG economy starts at the same level as in the cases of full or partial assimilation and either increases or decreases over the cohorts to reach a new level. Meanwhile, the welfare in the Laissez Faire economy is constant over time: Thus

**Proposition 16** In the case of no assimilation of the immigrant descendants, the welfare

\[ \hat{W}_{na} < W_{na} < \bar{W}_{n.a \rightarrow \infty} \] \hspace{1cm} (2.35)

if, and only if,

\[ \lambda < \frac{E_\theta \varphi_u (1 + \psi) - \psi \varepsilon_u - E_\theta}{E_\theta (\varphi_u - \varphi_s) (1 + \psi) + \psi (\varepsilon_s - \varepsilon_u)} \] \hspace{1cm} (2.36)

and \( \hat{W}_{na} > W_{na} > \bar{W}_{n.a \rightarrow \infty} \) if the inequality (2.36) holds with the opposite sign. If the inequality (2.36) holds as equality, the welfare for all cohorts in the unfunded PAYG economy and the Laissez Faire economy is identical.

**Proof.** Direct comparison of the welfare yields:

\[ \hat{W}_{na} - \bar{W}_{na} = \frac{\rho \psi E_\lambda (1 + r) [\Phi_\lambda (1 + \psi) E_\theta - E_\theta - \psi E_\lambda]}{1 + r (\psi E_\lambda + E_\theta) ((\psi E_\lambda + E_\theta) (1 + r) - \Phi_\lambda (1 + \psi))} \] \hspace{1cm} (2.37)

which is positive if, and only if,

\[ E_\theta (\Phi_\lambda (1 + \psi) - 1) > \psi E_\lambda, \] \hspace{1cm} (2.38)
negative if \( \Phi_\lambda (1 + \psi) E_\theta - E_\theta - \psi E_\lambda < 0 \), and zero if \( \Phi_\lambda (1 + \psi) E_\theta - E_\theta = \psi E_\lambda \). Similarly,

\[
\bar{W}_{t \to \infty}^{na} - \hat{W}^{na} = \frac{\rho}{1 + r} \frac{\Phi_\lambda (1 + \psi) E_\theta - E_\theta - \psi E_\lambda}{(\psi E_\lambda + E_\theta) (1 + \psi) \Phi_\lambda}
\] (2.39)

the sign of which also depends on the sign of \( \Phi_\lambda (1 + \psi) E_\theta - E_\theta - \psi E_\lambda \). Rearranging the terms will result in the inequality (2.36).

As in the unfunded PAYG economy, the welfare of the initial young population does not depend on assimilation of the immigrant descendants; it increases with an increase in the share of skilled migrant population in the case of no assimilation, as with the cases of full and partial assimilation. Similarly, the welfare in the \textit{Laissez Faire} economy, as in the cases of full and partial assimilation, decreases with an increase in the share of skilled migrant population, however, it decreases faster than in the other two cases (as can be seen from the shape of the white surface in the figure 2.8). This also decreases the threshold level of the share of skilled migrant population that causes the welfare in the unfunded PAYG economy to be higher than the welfare in the \textit{Laissez Faire} economy for the initial young. Using the data from Akin (2012), as in the previous chapter, the quantified version of the inequality (2.36) is \( \lambda < 0.46 \). Furthermore, as the distribution of the welfare over the cohorts is smooth in the unfunded PAYG economy, the threshold value of \( \lambda \) also corresponds to the saddle point of the welfare surface, \textit{i.e.} to the line where either welfare starts on higher levels for the initial cohorts and decreases to lower levels for the future cohorts, or starts at lower levels for the initial cohorts and increases to higher welfare levels for the future cohorts (figure 2.8). From the smoothness of the surface, it also follows that once the welfare in the Laissez Faire economy is higher compared to the unfunded PAYG and the initial young population initiates social security reform, the second cohort and some subsequent cohorts will also be better off.

\section*{Joint Analysis: Selective Migration and Social Security Reform}

The political economy of reforms assumes that populations, at least the median voter, have higher welfare after a reform is initiated. The current environment assumes that the initial old (or any old) population will receive their pension benefits once they have contributed to the PAYG sys-
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tem. Hence, the initial young population contains the decisive voters. Further, it is assumed that the initial young population votes for the alternative that yields the highest welfare.

Figure 2.10 shows the welfare of the initial young population as a function of the share of skilled migrant population under different conditions: As established in Lemma 8, and observed in figure 2.9, in the unfunded PAYG economy, the welfare of the initial young does not depend on the characteristics of migrant descendants (and thus on the dynastic assimilation) as they already receive fixed pension benefits in the second post-migration period when the first local-born immigrants enter the economy. In the Laissez Faire economy, however, the entire population of the economy matters and thus the welfare of the initial young also depends on the migrant dynastic assimilation. The results of the previous chapter suggested that the initial young will prefer a larger share of immigrants (as can be seen from the upward-sloping starred line in figure 2.10) while the entire economy, as summarised by debt-financed balanced PAYG system, is worse off. In contrast, if the debt-financed system is understood as a Laissez Faire economy, the initial young population will prefer to reform social security (terminate the unfunded PAYG system) if the share of skilled migrants is relatively small.

However, in some cases the initial young population may reform both policies – social security and immigration – at the same time to achieve the highest possible welfare. Thus, if a ‘melting pot’ system prevails in the economy and the immigrant descendants fully assimilate into the native population, the initial young population will strongly prefer a selective immigration policy in combination with sustaining the PAYG system. If the ‘cultural mosaic’ system prevails and the immigrant dynasties do not fully assimilate, the initial young may prefer a non-selective immigration policy combined with social security reform (Laissez Faire economy). In the case of partial assimilation, as also can be observed from the line with the down-pointing triangle in Figure 2.10, in the Laissez Faire economy with a small share of skilled migrant population, welfare is comparable to that in the unfunded PAYG economy with a large share of skilled migrant population. Thus the result will mostly depend on the comparison of the skill and fertility premiums (and the possibility of perfect migrant selection, which is usually constrained by joint family migration, asylum seekers or the quality of education), and with the benchmark data, where the skill and

---

8Perfect commitment (Persson and Tabellini, 2000) to the PAYG system is assumed and not analysed in the current text. The Laissez Faire economy suggests that the initial retired population receives their pension benefits though the PAYG system is terminated, and the Debt-financed PAYG assumes that each cohort will receive pension benefits once they retire. In no case may the retired population face a risk of financing consumption solely by their own savings.
fertility premiums are rather comparable, the *Laissez Faire* economy with smaller share of skilled migrants will possibly prevail. In the extreme case when the descendants of migrants do not assimilate into the native population, the *Laissez Faire* economy with a smaller share of skilled migrant population will generate higher welfare for the initial young population compared to the unfunded PAYG economy with the higher share of skilled migrant population. Still as the results in the first chapter suggest, the preferred policy for the initial young population (policy setting cohort) will be welfare depriving for (at least distant) future cohorts. As can be observed from figures 2.1 and 2.8, the future cohorts and the initial young population always prefer the exact opposites.

Further, comparing the welfare under full assimilation to partial assimilation, also seen in figure 2.9 and figure 2.10, partial assimilation is preferred in the *Laissez Faire* and in the unfunded PAYG economy, as in the case of partial assimilation, the descendants of the migrants retain the fertility rates of their ancestors (assumed to be higher) and thus improve the PAYG system if in the unfunded PAYG economy, or increase the tax base if in the *Laissez Faire* economy. When the economies reach a steady state, in the unfunded PAYG economy, the welfare in the case of full assimilation is

\[
\hat{W}^f_{t \rightarrow \infty} = 1 + \frac{\rho}{1 + r} - \frac{\rho}{\psi \Phi \lambda}
\]  

(2.40)

and this level is constant from the second post-migration cohort on (as in Lemma 9), while in the case of partial assimilation the welfare grows further to reach the value

\[
\hat{W}^{pa}_{t \rightarrow \infty} = 1 + \frac{\rho}{1 + r} - \frac{\rho}{\Phi \theta + \psi \Phi \lambda}
\]  

(2.41)

which is larger than

\[
\hat{W}^f_{t \rightarrow \infty} < \hat{W}^{pa}_{t \rightarrow \infty}
\]  

(2.42)

as \( \Phi \theta > 1 \), or the fertility rates of the migrants are higher than the native fertility rates, by assumption. In 2.9 the welfare in the cases of full and partial assimilation is depicted by the two white planes (that continue from the kink for the welfare of the second post-migration cohort), and as can be observed, the partial assimilation is above the full assimilation for each cohort and \( \lambda \). Meanwhile, in the case of no assimilation, the welfare, starting from the same level as in the
other cases (from the level in 2.10), smoothly converges to the limit value

\[
\hat{W}_{n\rightarrow\infty} = 1 + \frac{\rho}{1 + r} - \frac{\rho}{(1 + \psi) \Phi_{\lambda}} \quad (2.43)
\]

which is larger than

\[
\hat{W}_{f\rightarrow\infty} < \hat{W}_{n\rightarrow\infty} \quad (2.44)
\]
as \(\Phi_\theta > 1\), or the fertility rates of the migrants are higher than the native fertility, by assumption, and is larger than

\[
\hat{W}_{p\rightarrow\infty} < \hat{W}_{n\rightarrow\infty} \quad (2.45)
\]

if \(\lambda > \theta\), and reverse if otherwise. Thus meaning that the ‘melting pot’ system is Pareto-inferior to the ‘cultural mosaic’ system (independent of the extent of the assimilation - partial or no assimilation) and full assimilation does cause welfare losses as compared to partial or no assimilation. However, in figure 2.9, the welfare in the case of full assimilation (the coloured surface) is above the one for the case of no assimilation once the share of skilled immigrants is large. This is due to the data used with low fertility rates for skilled migrant and the assumption of unit fertility in the model (while in the data, the native skilled have lower fertility rates).

**Conclusion**

Based on the results of chapter 1, which highlight the intergenerational welfare conflict that immigration imposes on the economy, and on the Breyer equivalence result, showing that the PAYG system can be privatised without gains or losses in general (long-term) welfare in the economy, this chapter shows an important connection between immigration and social security reforms.

Thus, once the government is not able to fully enforce immigration control on the skill level (for instance in the case of the EU free labour movement policy) the population will prefer to abandon the PAYG system in order to redistribute the gains that immigration brings to the future cohorts towards themselves, while if the share of skilled immigrants is high, the current population will prefer to retain the PAYG system.

The chapter further highlights the importance of the assimilation of the migrant dynasties into
the native population. For instance, in the case of complete non assimilation the initial cohort will prefer to have only low skilled migrants in combination with a reform in the social policy, *i.e.* moving towards a fully funded system. Further, the chapter claims that the fully assimilative ‘melting pot’ policy of some countries is actually inferior to the ‘cultural mosaic’ policy, where the descendants of migrant families can inherit at least some characteristics of their ancestors.
Bibliography


Partial assimilation of migrant dynasties is parameterised here based on Akin (2012) data. The share of migrant population in the total is assumed constant at 5 per cent at any period. The white plain surface represents the welfare in Laissez Faire economy. The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents.
Partial assimilation of migrant dynasties is parameterised here based on Akin (2012) data. The share of migrant population in the total is assumed constant at 1 per cent at any period. The white plain surface represents the welfare in Laissez Faire economy. The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents.
Partial assimilation of migrant dynasties is parameterised here based on Akin (2012) data. The share of migrant population in the total is assumed constant at 15 per cent at any period. The white plain surface represents the welfare in Laissez Faire economy. The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents.
Partial assimilation of migrant dynasties is parameterised here with 5 per cent migration rate and Akin (2012) data, with the exception of the efficiency level (the skill levels are assumed to translate into equal efficiencies, but different fertility rates). The white plain surface represents the welfare in Laissez Faire economy. The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents.
Partial assimilation of migrant dynasties is parameterised here with 5 per cent migration rate and Akin (2012), data with the exception of the fertility level: Immigrants have equally high fertility rates.

The white plain surface represents the welfare in Laissez Faire economy. The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents.
Partial assimilation of migrant dynasties is parameterised here with 5 per cent migration rate and Akin (2012) data, with the exception of the fertility level: All the population, native and migrant alike, have the same unit fertility.

The white plain surface represents the welfare in Laissez Faire economy. The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents.
Partial assimilation of migrant dynasties is parameterised here with 5 per cent migration rate and Akin (2012) data, with the exception of the fertility level: While the unskilled immigrants are assumed to have fertility rates equal to natives, the skilled migrants are assumed to have lower fertility rates.

The white plain surface represents the welfare in Laissez Faire economy. The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents.
Here no assimilation of migrant dynasties is parameterised based on Akin (2012) data. The share of migrant population in the total is assumed constant 5 per cent at any period. The white plain surface represents the welfare in Laissez Faire economy. The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents.
Figure 2.9: PAYG and Dynastic Assimilation

Here the welfare in PAYG economy is parameterised for different assimilation cases based on Akin (2012) data. The share of migrant population in the total is assumed constant 5 per cent at any period.

The coloured surface shows the welfare in the unfunded PAYG economy for each cohort of agents in case of no assimilation. The white plain surfaces represents the cases of full and partial assimilation (full assimilation is always below the partial assimilation).
Here the welfare for the initial young population is presented in various cases: While in the unfunded PAYG the welfare is growing with the share of skilled migrants, the welfare decreases in case of the *Laissez Faire* economy and is different for each assimilation case.
Part II

Immigration and Social Security in Closed Economy
Chapter 3

The Migration Challenge for PAYG

ABSTRACT  Immigration has been popularised in the economics literature as a tool that could be used to balance troubled PAYG pension systems. Pivotal research by Razin and Sadka shows that unskilled immigration can overcome the pension problem and, further, boost the general welfare in the host economy. However, a large strand of current economics research is engaged in studying mechanisms through which unskilled immigration, while solving the pension problem, is causing undesired shifts in general welfare. This work shows that recurring unskilled immigration will not only reduce the general welfare, but may be challenging the pension system by reducing the pension benefits themselves. Further, interpreting the actual data, it is suggested that immigration policies are designed either based on public finances only or in a political environment of gerontocracy.

Introduction

Decreased fertility, along with increased longevity, challenges the sustainability of unfunded pay-as-you-go (PAYG) pension systems in most developed economies: Decreased fertility shrinks the contribution base, while increased longevity burdens the system with more pension claims. Thus, sustainability of the system requires urgent reforms: parametric, demographic, or a combination of the two. Parametric reforms include increased pension contributions, decreased benefits, or a combination of these. Those parametric reforms, however, are welfare impairing: increased contributions harm the working-age population, while the retired suffer from decreased benefits.

Demographic reforms are the theoretical alternative to the parametric reforms. Two main lines

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of literature exist on the demographic alternatives. The first line discusses the problems within native-born populations, i.e. fertility enhancing policies (Cigno, 2010) and human capital accumulation (Cremer, Gahvari, and Pestieau, 2011), while the second discusses the possibility of bolstering native-born populations with immigrants (Razin and Sadka, 1999; Schou, 2006). Much of the literature focuses on studying the welfare effects of those immigration reforms. The current paper, however, shows that immigration reforms are mostly ‘equivalent’ to parametric reforms in the sense that they decrease pension benefits (and thus are not meaningful alternatives). The focus of the current paper is on unskilled immigration to an economy with an unfunded defined-contribution PAYG pension system.

Already, Razin and Sadka (1999) have shown that unskilled immigration can overcome the PAYG challenge: Unskilled immigrants do not merely pay the required contribution but, potentially, increase the welfare to everyone. Thus, immigrants come in and pay the necessary contributions required for the system’s sustainability. When they age, they increase the number of claims on benefits; however, their children would then contribute to the pension system (and bring back the initial equilibrium).

Since Razin and Sadka (1999), much has been written to show various channels through which increases in unskilled immigration could decrease the general welfare (hence implying that demographic reforms are not preferred to parametric reforms). Thus, Razin and Sadka (2000) claim that pension benefits are increased with increased unskilled immigration, but, in the case of closed economies, wages would decline and thus hurt those who mostly rely on labour for income.

Casarico and Devillanova (2003) noted that a wage decline caused by an influx of unskilled immigrants would change the skill distribution among the natives in the host economy. They show the changes that come with the possibility of endogenous skill upgrades can cause many inter- and intra-generational welfare re-distribution conflicts. Jinno (2011) develops the idea further, incorporating the possibility of an endogenous skill upgrade for the immigrants as well, while noting that there are some assimilation costs. The results suggest further re-distributional conflicts. Krieger (2004) claims that welfare impairment and re-distribution already occur when high immigrant fertility rates and the low skill levels of immigrant children are accounted for.
Kemnitz (2003, 2008) introduces the problem of unemployment into the analysis. He claims that unskilled immigration increases unemployment and harms the unskilled natives even though there is a boon to general welfare. In a similar manner, Muysken, Cörves and Ziesemer (2011) show that unskilled immigration increases unemployment, and if their skills are not upgraded, unskilled immigrants may impair the general welfare.

Many large-scale computational studies have been conducted in order to analyse whether immigration can make PAYG pension systems sustainable. Thus, Lee and Miller (2000) and Storesletten (2000) model the US economy and conclude that some immigration can help alleviate the problem of the ageing baby-boom generation. Schou (2006) presents a similar detailed study for Denmark and concludes that immigrants are net beneficiaries of the public pension system while their contribution to economic growth is marginal. Fehr (2009) provides a more detailed survey of this literature. More recently, Chojnicki, Docquier, and Ragot (2011) presented a retrospective study on the US economy and concluded that a higher level of immigration would have been beneficial to the general welfare of the population. However, their study was concentrated on the short- to medium-term effects of immigration and did not study possible long-term effects.

Meanwhile, there is a strand of literature claiming that only an excessively large inflow of immigrants can help sustain pension systems. For instance, Übelmesser (2004) claims that the EU cannot accommodate as many migrants as are needed for pension system sustainability and claims that parametric reforms are also necessary. Blake and Mayhew (2006) study a combination of parametric reforms with immigration, claiming that the need for immigrants into the UK will be constantly growing if the pension system is not reformed. Serrano, Eguía and Ferreiro (2011) claim that even with recent large immigration, it is not possible to sustain the current Spanish pension system, and parametric reforms are inevitable.

The current paper, claiming that under certain conditions unskilled immigration may result in lower pension benefits (when pension contributions are fixed), shows that increased immigration is not desirable. The paper utilises the idea of a multi-period immigration policy, i.e. immigrants enter the economy each period as opposed to the one-time-migrant-inflow framework of Razin and Sadka (2000). While in the Razin and Sadka model the old equilibrium is restored after a number of periods, multi-period immigration results in a new equilibrium, distinct from the initial one. This allows the study of the reaction of the economy in full, i.e. both in the new
equilibrium and on the transition path. An important channel that links unskilled immigration to pensions is studied: The paper employs the framework of several recent publications by Fanti and Gori (2010, 2012) which follow the dynamics of per-capita capital in a Diamond-type overlapping-generations model. Thus, the paper connects unskilled immigration to distortions in savings and demographics that result in lower capital per capita and thus in generally lower wages. Hence, even though pension benefits grow compared to wages (as in Razin and Sadka, 1999), wages may decrease such that benefits are less than they would be without the immigrants.

The current paper also complements the literature that studies the negative spiral of the PAYG system and demography (e.g. Cigno, 2006; Cipriani, 2014), as it shows how immigration policy designed to support the system actually burdens it further.

In line with the literature, general welfare changes are also discussed: Such a study in the heterogeneous agent framework allows the analysis of the political feasibility of immigration policy reform. Similar to Razin, Sadka and Suwankiri (2011) and Lacomba and Lagos (2010), the current paper identifies those policies that the current population might favour. Furthermore, in contrast to Lacomba and Lagos, this paper allows the population to chose immigrant skill level and follows the long-term welfare effects of the adopted policies. Numerical examples are used to support the results of the theoretical framework.

The rest of the paper is organised as follows: Section 2 introduces the economic environment. Section 3 defines the equilibrium and provides the pension system analysis under migration. Section 4 analyses the political equilibrium and transitional dynamics, and concluding remarks are in the final section.

The Economic Environment

Two overlapping generations exist in a closed economy. During the first period of their lives the agents work (for remuneration), save and consume. For the second period the agents consume their pension benefits and savings. The firm organises production by hiring labour and capital from households. The government collects pension contributions from the working-age young and distributes them within the retired population. All markets clear.
Chapter 3 The Economic Environment

Population

In each period, young migrants are allowed into the country equal to $\mu$ share of the native young. Thus, the total working-age population in the economy is:

$$T_t = N_t (1 + \mu),$$  \hspace{1cm} (3.1)

where $N_t$ is the size of the native-born population with the following dynamics.

$$N_t = N_{t-1} (1 + \mu) n,$$  \hspace{1cm} (3.2)

where it is assumed that immigrants and their descendants have the same fertility rate $(n - 1)$. However, the immigrants have only $\varepsilon$ of the skill level of the native population. Thus, the effective labour in the country at any time is:

$$L_t = N_t (1 + \mu \varepsilon)$$  \hspace{1cm} (3.3)

and is different from the total size of the population.

Households

Each household is represented by a single agent that solves a lifetime utility maximisation problem:

$$U_t = \max (\ln c_{i,t} + \beta \ln c_{i,t+1})$$  \hspace{1cm} (3.4)

subject to the budget constraints:

$$c_{i,t} + s_{i,t} = w_i e_i (1 - \tau),$$  \hspace{1cm} (3.5)

$$c_{i,t+1} = p_{t+1}^d + p_{t,t+1}^{err} + s_{i,t} (1 + r_{t+1}),$$  \hspace{1cm} (3.6)

where $i$ shows the status in the country (native born or migrant), $c_{i,t}$ and $s_{i,t}$ are, respectively, the consumption and savings of type $i$ agent at time $t$, $e_i$ shows the efficiency of the worker (which
The Economic Environment Chapter 3

is unity for native born and ε for immigrants), τ is the tax rate that has two components τ = \(\tau^d - \tau^er\), where \(\tau^d\) and \(\tau^er\) are, respectively, the pension contribution rates to the demogrant and earnings-related pension systems, \(p^d_t\) is the demogrant benefit is received equally by all retirees, while \(p^e_{i,t}\) is the earning-related pension benefits that each retired agent receives according to own contribution:

\[
p^e_{i,t} = \varphi_t \cdot w_{i,t},
\]

where \(\varphi_t\) is the pension replacement rate.

Optimal household savings take the value

\[
s_{i,t} = \frac{\beta}{1 + \beta} w_{i,t} (1 - \tau^d - \tau^er) - \frac{p^d_{i,t + 1} + \varphi^e_{i,t + 1} w^e_{i,t + 1}}{(1 + \beta) (1 + r^e_{t + 1})},
\]

where \(p^d_{i,t + 1}, \varphi^e_{i,t + 1}, w^e_{i,t + 1}\) and \(r^e_{t + 1}\) are the expected values of the pension benefits, replacement rate, wage, and interest rate respectively.

**Firms**

There is one firm that uses the Cobb-Douglas production function\(^1\) with an \(\alpha\) share of capital. Hence, the usual optimality conditions hold:

\[
w_t = (1 - \alpha) Ak_t^\alpha,
\]

\[
r_t = \alpha Ak_t^{\alpha - 1} - 1,
\]

where \(k_t\) is the capital per effective labour.

**The Pension System**

Two parallel pension systems will be accounted for: a demogrant (Beveridgean tradition) system that evenly distributes the income over the retired population and the earnings-related pension

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\(^1\)Certainly, the assumed perfect substitutability of skilled and unskilled workers causes obvious limitations in the model. However, there are two main reasons for this assumption: First, the current assumption makes the comparison of the results with the main body of the literature straightforward (as the assumption is the most common); and second, the empirical literature on the topic has not yet identified the exact relationship (e.g. Okkerse, 2008).
system (Bismarckian tradition) that is actuarially fair. Pension systems run balanced budgets at any time, *i.e.* the contributions collected are given out as benefits. Thus, the earnings-related pension system can be presented as

\[ p_{t}^{er}L_{t-1} = \tau^{er}w_{t}L_{t}. \]  

(3.11)

Using (3.2) and (3.3), the earnings-related benefit size can be calculated as

\[ p_{t}^{er} = \tau^{er}w_{t}e_{i}(1 + \mu)n, \]  

(3.12)

where \( p_{t}^{er} = p_{t}^{er}e_{i} \) is the (efficiency-weighted) pension that type \( i \) agent gets at period \( t \). Similarly, the demogrant system can be represented as

\[ p_{t}^{d}T_{t-1} = \tau^{d}w_{t}L_{t}. \]  

(3.13)

Using (3.1) and (3.3), the pension benefits can be calculated as

\[ p_{t}^{d} = \tau^{d}w_{t}(1 + \mu_{e})n \]  

(3.14)

*i.e.*, as in Razin & Sadka (1999) the first-order effect of the immigrants on pensions is strictly positive.

### The Post-Migration Equilibrium

Given the parameter values, \( \alpha, \beta, \varepsilon, \mu, \tau^{d}, \tau^{er} \), and the initial values \( K_{0}, N_{0} \), the equilibrium is an allocation \{\( c_{i,t}, s_{i,t}, p_{t} \)\} and a price vector \( (w_{t}, r_{t}) \) such that the population follows the dynamics given by (3.1) and (3.3), households optimise their problem (3.4)-(3.7), the firm optimises so that (3.9) and (3.10) hold, pension budgets (3.11) and (3.13) are balanced, and the capital market clears, *i.e.*

\[ K_{t+1} = N_{t}s_{n,t} + N_{t}\mu_{s}m_{t}. \]  

(3.15)

It is assumed that the economy starts in a steady state with no migration. Once immigration
is allowed, the economy undergoes a transition and eventually reaches a new, post-migration steady state.

**Post-Migration Capital**

Aggregate capital in the economy can be calculated based on (3.15), using the population dynamics (3.2) and the optimal savings (3.8). Thus,

\[
K_{t+1} = \left[ \frac{1 + \mu e}{1 + \beta} \left( w_t \left( 1 - \tau \right) \beta - \frac{\varphi_t w_t^e}{1 + r_t^{e+1}} \right) - \frac{1 + \mu}{1 + \beta} \frac{p_t^{e+1}}{1 + r_t^{e+1}} \right] \frac{N_{t+1}}{n (1 + \mu)}
\]

(3.16)

Using (3.3), (3.9), (3.10), and (3.14) and assuming that agents have rational expectations, the dynamics of capital per effective labour can be obtained:

\[
k_{t+1} = \frac{\alpha A (1 - \alpha) (1 - \tau) \beta}{n (1 + \mu) (\alpha (1 + \beta) + (1 - \alpha) \tau)} k_t^a
\]

(3.17)

that solves for the steady state equilibrium value:

\[
\bar{k} = \left[ \frac{\alpha A (1 - \alpha) \left( 1 - \tau^d - \tau^e r \right) \beta}{n (1 + \mu) (\alpha (1 + \beta) + (1 - \alpha) \tau)} \right]^{\frac{1}{1 + \beta}}.
\]

(3.18)

As can be immediately observed, the capital per effective labour decreases with the size of the immigrant population. However, the post-migration steady state level of capital (3.18) does not depend on the skill level of the immigrants: Though the immigrants have lower skill level, and thus earn and save less than the natives, in the steady state they are matched to the next period immigrant population that is equal in size and skill level, and thus they do not change the level of the capital per effective labour.

Meanwhile, those immigrants produce children that are not matched by any savings and thus lower the level of capital available to each effective labour unit. Hence, the capital dilution observed in the post-immigration steady state is caused not by the immigrants per se, but rather by the children of those immigrants.
Chapter 3  
The Post-Migration Equilibrium

Post-Migration Pension Benefits

As in the previous section, two pension systems will be discussed separately: The equilibrium value of the demogrant pension benefits can be obtained by substituting the steady state wage rate into (3.14):

\[
p^d(\mu) = \tau_{er} B \cdot \frac{1 + \mu \varepsilon}{(1 + \mu)^{1-\alpha}},
\]

(3.19)

where

\[
B = (1 - \alpha) \left( A \nu^{1-2\alpha} \right) \frac{1}{\gamma} \left( \frac{\alpha \beta (1 - \alpha) (1 - \tau)}{\alpha (1 + \beta) + \tau (1 - \alpha)} \right)^{\frac{\alpha}{1-\alpha}}
\]

does not change with the size or skill level of the immigrant population. Thus, to understand the effect of immigration on the demogrant pension benefits, it is sufficient to find the sign of the derivative of (3.19):

\[
\frac{d}{d\mu} p^\ast(\mu) = \frac{\tau_{er} B}{(1 - \alpha) (1 + \mu)^{1-2\alpha}} \left( \varepsilon (1 + \mu) (1 - \alpha) - \alpha (1 + \mu \varepsilon) \right).
\]

(3.20)

**Proposition 1**  Demogrant pension benefits decrease with the size of the immigrant population if the skill level of immigrants (compared to natives) is

\[
\varepsilon < \frac{\alpha}{1 - \alpha + \mu (1 - 2\alpha)}
\]

(3.21)

while \( \alpha < \frac{1+\mu}{1+2\mu} \).

**Proof.** The pension benefits decrease if the derivate (3.20) is negative. Since the first part (the ratio) of the product on the right-hand side of (3.20) is a positive constant, the sign of the derivative depends on the sign of the second part and is negative given that the conditions above are satisfied.

The proposition effectively claims that even though post-migration pensions are higher compared to wages, the immigration-caused capital dilution may be stronger and result in wages that generate lower pension benefits compared to their pre-migration levels. In more detail, the logic is as follows: In post-migration steady state, the effective population dynamics are stabilised on the level of immigration size \((1 + \mu)\), which (given the diminishing marginal returns) results in a capital dilution with the size of \((1 + \mu)^{1-\alpha}\) and a wage decrease with a magnitude
of \((1 + \mu)^{\alpha_{1}}\). Meanwhile, with population dynamics stabilised, in each period there are more contributions from the immigrants who increased the effective size of the population by \((1 + \varepsilon\mu)\). Hence, when the improved demographics of the pension system does not cover the decreased effective wage, pensions decrease. For example, if \(\alpha = 0.4\) and the immigration is under five per cent \((\mu < 0.05)\), then immigrants with even sixty-five per cent of local efficiency will already force the pension benefits to decrease, as the immigrants’ extra contribution to the pension system will not be enough to cover the wage loss caused by capital dilution.

In the case of earnings-related pensions, the behaviour of the post-migration steady state value of the benefits:

\[
\bar{p}_{e}^{cr} (\mu) = \tau_{er} B \cdot (1 + \mu)^{1-2\alpha} \tag{3.22}
\]

are obtained by substituting the equilibrium wage rate into (3.12), and can be studied similarly:

**Proposition 2** Earnings-related pension benefits increase with the size of the immigrant population when \(\alpha < 1/2\), and decrease otherwise.

**Proof.** Directly follows from (3.22). ■

Hence, in the case of an earnings-related pension system, as opposed to a demogrant pension system, the pension benefits always grow (in the economically meaningful case of \(\alpha < 1/2\)). This result is due to the fact that an actuarially fair distribution of earnings-related pensions guarantees that relative benefits increase more (a multiple of \((1 + \mu)\)) than the effective wage decreases (a multiple of \((1 + \mu)^{\alpha_{1}}\)). That is, the improvements in demographics are stronger than the capital losses.

The corollary of the two propositions is that the effect of immigration on the steady state level of total pension benefits \((\bar{p}_{d}^{cr} + \bar{p}_{e}^{cr})\) depends on the size and efficiency level of the immigrant population as well as on the Bismarckian factor \((\text{BF})^{2}\). Thus, if the BF is small and immigration control is poor, then the first proposition results will prevail and the pension benefits will decline. The situation is likely to occur in many European countries such as Denmark or the UK which

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\(^{2}\)The Bismarckian factor (Cremer & Pestieau, 1998) is a measure of the actuarial fairness of a pension systems. It takes values between 0 and 1, with 0 characterising a purely redistributive (demogrant, Beveridgean) pension system and 1 - a strictly actuarially fair (earnings related, Bismarckian) pension system. In the present stylised model, the Bismarckian factor can be presented as \(\tau_{er}/(\tau_{er} + \tau_{d})\).
have a low BF — 0 and 0.09 respectively (Krieger & Traub, 2013) — and are bound by the EU free labour movement policy\(^3\). The reverse situation (i.e. the results of proposition 2 prevail) can be expected from countries with a low BF and tight immigration control (such as Australia with a BF 0.03) or a combination of liberal immigration with a high Bismarckian factor (such as Germany and France with BF 0.55 and 0.72, respectively). In countries where there is moderate immigration control and a moderate BF (such as the USA with BF 0.46), results and the effects on pension benefits will be ambiguous.

**Post-Migration General Welfare**

Aside from the effects of immigration on pension benefits, immigration also affects other components of the general welfare – wages and return to capital. The lifetime utility of agents (3.4) can be rewritten as

$$U_t = D + (1 + \beta) \ln \left( w_t (1 + r^d_{t+1}) (1 - \tau) + p^d_{t+1} + p^r_{t+1} \right) - \ln (1 + r^d_{t+1})$$

(3.23)

where 

$$D = \ln \frac{\beta^\alpha}{(1 + \beta)^{1+\gamma}}.$$  

Capital dilution caused by immigration (3.18) will result in lower wages (3.9) and higher return to capital (3.10) that, combined with the conflicting results of the propositions 1 and 2, make the total effect of the immigration on general welfare ambiguous.

For the sake of analytical tractability, an environment without pensions is studied, i.e. \(\tau^d = \tau^d = 0\). This allows to analyse the effects of immigration on non-pension-related welfare and on pensions separately. This separation, however, is based on an assumption that the existence of pension systems does not significantly distort savings decisions (or alternatively that pension systems are not large). Hence, the results should be treated as an approximation to the general welfare.

**Proposition 3** The non-pension welfare decreases with the size of the immigrant population when \(\alpha > \frac{\beta}{1+2\beta}\).

**Proof.** The utility function (3.23) in an environment without pensions, i.e. \(\tau^d = \tau^d = 0\), will have

\(^3\)This result could be used to explain the extra mobility restrictions that the UK employs for the migrants from the new EU member states – Bulgaria, Croatia, and Romania.
as a variable part only \((\alpha - (1 - 2\alpha)\beta)\ln k\) that changes in the same direction as \(k\) when \(\alpha > \frac{\beta}{1+2\beta}\). Further, from (3.18) \(k\) decreases with an increase in the size of the immigrant population \(\mu\).

As the requirement \(\alpha > \frac{\beta}{1+2\beta}\) is satisfied in any economically meaningful situation, the proposition 3 allows to conjecture that in the post-migration steady state the general welfare decreases with increased immigration assuming that the possible gains from pensions are not excessively large (the numerical examples of the next section support this conjecture). Still, in the extreme, when actuarially fair pension systems are large the general welfare may increase with immigration.

**The Political Equilibrium**

There are numerous ways immigration policies might be initiated. Here two alternatives are studied: immigration policy set by a social planner and policy based on public choice (voting).

Following Song, Storesletten and Zilibotti (2012), a social planner maximises general welfare using some weights for each group. Usually, the farther away the group is in time, the less weight they get. Still, if the social planner does not discount the future heavily and bases the decision on the effect of immigration on the public finances (pensions in this case), the equilibrium solution will be in line with the corollary to propositions 1 and 2, i.e. small-sized and highly skilled immigration will be supported in an economy with a low BF, and larger-sized immigration in the case of a high BF. However, if the social planner bases the decision on the general welfare, the equilibrium will be in line with the above conjecture, i.e. very limited highly skilled immigration will be allowed.

Conversely, following Lacomba and Lagos (2010) and Razin et al. (2011), a public choice approach can be adopted. That is, it is assumed that agents populating the economy in the initial period vote for policy based on their economic self-interest, i.e. they vote for the policy that guarantees higher general welfare. Further, it is assumed that the policy will be intact once it is voted for. This approach requires studying the welfare of the initial young and retired population. Thus, using (3.17), the capital dilution caused by immigration in the initial two periods can
be presented as

\[ k_1 = \frac{\hat{k}}{1 + \mu \varepsilon}, \quad (3.24) \]

\[ k_2 = \frac{\hat{k}}{(1 + \mu \varepsilon)^\alpha (1 + \mu)}, \quad (3.25) \]

so, wages and interest rates will accordingly be

\[ w_1 = \frac{\hat{w}}{(1 + \mu \varepsilon)^\alpha}, \quad (3.26) \]

\[ w_2 = \frac{\hat{w}}{((1 + \mu \varepsilon)^\alpha (1 + \mu))^{1-\alpha}}, \quad (3.27) \]

and

\[ 1 + r_1 = \frac{1 + \hat{r}}{(1 + \mu \varepsilon)^{1-\alpha}}, \quad (3.28) \]

\[ 1 + r_2 = \frac{1 + \hat{r}}{((1 + \mu \varepsilon)^\alpha (1 + \mu))^{1-\alpha}}, \quad (3.29) \]

where \( \hat{k}, \hat{w}, \) and \( \hat{r} \) are the pre-migration steady state levels of the capital per effective labour, wage, and return to capital, respectively.

The initial retired population consume own savings from the previous period combined with higher returns caused by capital dilution. Furthermore,

**Proposition 17** Both demogrant and earnings-related pension benefits increase with the size of the immigrant population in the first post-migration period.

**Proof.** From (3.12), (3.14) and (3.26) follows

\[ \hat{p}_d = \tau_d n \hat{w} < \frac{\tau_d n \hat{w}}{(1 + \mu \varepsilon)^\alpha} = p_d^1 \]

\[ \hat{p}_r = \tau_r n \hat{w} < \frac{\tau_r n \hat{w}}{(1 + \mu \varepsilon)^\alpha} = p_r^1 \]

Though in the first post-migration period, the effective wages are lower compared to their pre-
vious steady state levels, pension benefits, both demogrant and earnings-related, continue to increase based on improved demographics. Thus, the initial retired population only gains from any immigration and, further, the higher the skill level of the immigrant population, the more the retirees gain.

In the post-migration environment, the initial young population face lower wages, higher return to savings, and ambiguous pension benefits. For the sake of analytical tractability the lifetime utility (3.23) will again be studied in two disjointed cases: pension benefits and utility without pensions. Thus,

\[
U_1 = \text{constant} + \ln w_1^{1+\beta} \cdot r_2^\beta
\]

\[
= \text{constant} + \ln \left( \frac{1}{1 + \mu \epsilon} \right)^{1+\beta} \left( \frac{1}{1 + \mu} \right)^\alpha \left( \frac{1}{1 + \mu} \right)^{(1-\alpha)\beta} + \ln \frac{1}{1 + \mu} \left( \frac{1}{1 + \mu \epsilon} \right)^{(1-2\alpha)\beta}
\]

(3.32)

The only non-constant part of the utility function is the last term on the right-hand side of (3.32) and it increases with the size of immigration and decreases with the efficiency level, causing the initial young population to favour low-skilled immigration. The logical interpretation of the result is as follows: Starting from the second period the native-born immigrant children cause capital dilution that results in higher return to capital (savings), and, as the size of the native-born immigrant children is equal to the size of immigrant population, the initial young population prefers high immigration. However, the initial young face those immigrants on the labour market and compete with them for the effective wages. Thus, the initial young population prefers large-scale low-skilled immigrant population.

As in the case of the initial retired population, the pre- and post-migration pension benefits for the initial young (second period retired) population can be calculated.

**Proposition 18** In the second post-migration period the earnings-related pension benefits increase with the size of immigrant population if

\[
\epsilon < \frac{(1 + \mu)^{\frac{1-\alpha}{\mu}} - 1}{\mu}
\]

(3.33)

and decrease otherwise.
Proof. Based on (3.12) and (3.27):

\[
p^e_2 = \tau^e nw \frac{1 + \mu}{(1 + \mu \varepsilon)^\alpha (1 + \mu)^\alpha}.
\]  

(3.34)

As before, the pre-migration steady state value of the earnings-related pension benefits is \( \hat{p}^e = \tau^e nw \), which is less than \( p^e_2 \) if the above condition is satisfied, while otherwise \( \hat{p}^e > p^e_2 \).

Proposition 19 In the second post-migration period the demogrant pension benefits increase with the size of immigrant population if

\[
\varepsilon > \frac{(1 + \mu)^\frac{\alpha}{\alpha - 1} - 1}{\mu},
\]  

(3.35)

and decrease otherwise.

Proof. Based on (3.14) and (3.27):

\[
p^d_2 = \tau^d n\hat{w} \frac{1 + \mu \varepsilon}{(1 + \mu \varepsilon)^\alpha (1 + \mu)^\alpha}.
\]  

(3.36)

As before, the pre-migration steady state value of the earnings-related pension benefits is \( \hat{p}^d = \tau^d n\hat{w} \), which is less than \( p^d_2 \) if the above condition is satisfied, and is otherwise \( \hat{p}^d > p^d_2 \).

As proposition (18) shows, immigration increases the level of earnings-related pension benefit, i.e. \( p^e_2 > \hat{p}^e \), under rather permissive condition. However, in order for the second post-migration period demogrant pension benefits to be higher than the pre-migration level, a rather restrictive constraint needs to be satisfied. That is, because of the redistributive nature of the demogrant pension system immigration of low-skilled migrants decreases pension benefits. Similar to the corollary to propositions 1 and 2, it can be concluded that the effect of immigration on pension benefits depends on the size and the skill level of the immigrant population as well as on the BF. A conjecture (similar to the one in the section above) can be made to claim that in the case of a high BF, the initial young population will benefit from low skilled immigration; and in the case of a low BF environment, they will benefit from the same if the pension benefit losses are not excessively large.

Hence in political equilibrium, with public voting, the initial population (both young and retired)
will vote for the positive migration of low-efficiency immigrants though in the long-term that policy is welfare impairing for future generations.\(^4\)

**Numerical Example**

This section uses examples of the United Kingdom and Germany to discuss the results and conjectures of the previous sections. The two countries have been chosen as the extremes: While the United Kingdom has implemented tight immigration policy (despite EU directives) and has exercised a fully distributive pension system (low BF), Germany has been more liberal in immigration policy (supporting EU free labour movement and Aussiedler return policies) and has exercised a more actuarially fair pension system (high BF).

For both countries there is a working assumption that \( \alpha = .4 \), and immigration \( \mu = 5 \) per cent. The tax rates are chosen so that they match the Bismarckian factors provided in Krieger \& Traub (2013) and the pension replacement rates from (Table 7 of) Eichhorst et al. (2011), i.e. \( \tau^{er} = 11.2 \) and \( \tau^{d} = 9 \) per cent for Germany and \( \tau^{er} = 1 \) and \( \tau^{d} = 18 \) per cent for the UK. Both countries reach their new steady states before the 13th cohort is born.

Figure 3.1 illustrates the above-described dynamics for welfare (the left-hand side) and pensions (the right-hand side) in the United Kingdom (above) and Germany (below). The horizontal axes show the immigrant efficiency level (compared to the natives) and the agents born during the respective period (with the initial retired population being the first cohort). In each graph, the pre-migration equilibrium levels are presented for comparison.

The results of the proposition 17, i.e. pure gains in pension benefits for the initial retirees, are clearly visible for both countries (the right-hand side of the rows). Comparison of the two graphs will illustrate the results of the propositions 1, 2, 18, 19 and their respective corollaries. As the UK pension system is characterised by a very small BF, the post-migration steady state level of pension benefits decrease with migration once the efficiency of the immigrant population, \( \varepsilon \), is less than 65 per cent of the efficiency level of the native population. In contrast in Germany the total pension benefits increase once the efficiency level of the immigrant population is more than

\(^4\)The result is surely bounded by many modelling assumptions, such as the absence of altruism towards future population, or the absence of the possibility for the future population to reverse the results (such as the repetitive voting considered in Razin et al. (2011)).
23 per cent. Moreover, given the high BF in Germany, the result of proposition 18 prevails over the result of proposition 19 and the total pension benefits for the initial young increases for any skill (efficiency) level of the immigrant population.

Further, figure 3.1 shows that the conjectures made on the general welfare hold for both cases, \textit{i.e.} post-migration welfare decreases with migration as a result of capital dilution and even the gains in pension benefits from skilled migration is offset by the losses in wages. However, in line with the theoretical results, the initial retired population only gain from migration and the gains increase with the increase in the skill level of the immigrant population. As for the initial young population, their general welfare gains are the highest in the case when immigrant population has the lowest efficiency and the welfare decreases with the increase in the efficiency levels. Again, the comparison of the UK and German cases shows the role of the BF: The threshold for the initial young population in the UK to gain from immigration is 59 per cent, while in Germany the initial young gain when the skill level of the immigrant population is as high as 68 per cent of the skill level of the native population.

The figure also presents actual results of the UK and German migration policies. Thus, table 1 in Algan, Dustmann, Glitz and Manning (2010) claims that the native hourly wage in the UK is 11.12, while immigrants earn on average 11.48, which combined with the data of a 73.1 per cent of immigrant labour force participation rate as opposed to a native 79 per cent, gives $\varepsilon = .955$. Meanwhile, the same table shows that the lowest skilled immigrant group has an hourly wage of 6.26, and only 55.7 per cent of them are employed, which produces $\varepsilon = .399$. The actual German immigrants (based on similar calculations) have an efficiency level of 85 per cent of the native born, while the lowest group has just 40 per cent.

Thus, in the environment of the current model, the public choice (voting) model interpretation of the actual data suggests gerontocracy in the UK and Germany, \textit{i.e.} the preferences of the initial old define immigration policy (and cause welfare losses for current young population as well as for future generations). The social planner model would also predict very large weights on the current retired generation (again can be interpreted that in a gerontocratic environment, the planner may be biased). However, the data fit a social planner model, which uses the effect of migration on public finances as a guide for policy-setting (such as in Razin et al., 2011): In the highly distributive UK economy (high BF), the point-system is implemented to guarantee
gains, and in the more actuarially fair German environment, immigration policy is less strict on
the skill level of the immigrant population. (This reasoning can also be used to explain the UK
restriction on immigrants from new and poorer EU member states.) Alternatively the data can be
interpreted in line with the recent findings in political science: Thus, according to Hainmueller
and Hiscox (2010), voter attitudes toward immigration are not explained by economic, but rather
by ‘ethnocentric or sociotropic’ considerations.

Conclusion

The challenge from a growing ageing population on unfunded public pensions has long been
studied in economics. Possible correctivemeasures are fewer pension benefits, larger contribu-
tions (including later retirement), fertility enhancement programmes, and replacement immigra-
tion. A vast literature now exists dealing with the last.

At the height of the discussion, Razin and Sadka (1999) introduced the idea of unskilled immigra-
tion solving the pension problem in an infinitely living economy (even with purely redistributive,
demogrant, Beveridgean pension policy). The idea is simple: Unskilled immigrant workers en-
ter the economy and, together with the native-born, working-age population, contribute to the
old-age pension system. When those immigrants age, natives have to share their pensions with
them; however, larger cohorts of the native-born (including the children of immigrants) would
contribute to the pensions of retired immigrants. Effectively, the economy borrows from the very
last generation (absurd in an infinitely living economy) and elevates the welfare of all involved
parties.

Many (including the mentioned authors themselves) rebelled at the idea of pure benefit of un-
skilled immigration and proposed various mechanisms that challenge the general welfare of the
involved parties. Prices, unemployment, child costs and the like have been proposed as pos-
sible sources of general welfare deprivation. However, the current work provides arguments
against the very idea of unskilled immigrants serving to sustain the pension system. It is shown
that while the unskilled immigration increases pension benefits compared to wages, it decreases
the capital available for each worker and, thus, also wages. As a result the demogrant pension
benefits (used in Razin and Sadka, 1999) decrease.
However, earnings-related (actuarially fair, Bismarckian) pensions, which are widely found in many countries combined with demogrant pensions, are shown to generate higher pension benefits compared to pre-immigration levels. Thus the effect of low-skilled migration on overall pension levels depends on the share of earnings-related pension benefits (the Bismarckian factor) and the level of immigrant labour efficiency.

Further, it is shown that the overall welfare of all agents (with the exception of the initial population) decreases with unskilled immigrants. Meanwhile, the initial retired population benefits from immigration (still preferring better skilled migrants) due to direct pension benefit increases, and the initial young population prefers migrants with low skill only in order to boost the interest on their savings while faced with little loss in wage rates. As a result, should a policy be set based on the preferences of the current population (i.e. if public choice prevails), an unskilled immigration policy would be established resulting in reduced pension benefits (and welfare) for the future population. The real data interpretation of the model suggests that either political gerontocracy prevails in the European states discussed or immigration policy is set solely on the basis of public finances.
Bibliography


Chapter 3

Appendix: Figures

Figure 3.1: Welfare and Pension Benefits in Closed Economy

Transition dynamics for pension benefits (right-hand side) and the general welfare (left-hand side) for the UK (top row) and Germany (bottom row)
The Dissertation deals with controlling immigration in the presence of intergenerational redistributive policies in a small open economy. It consists of three chapters. Chapter 1 focuses on the intergenerational conflict that selective immigration causes in a host economy. It analyzes how policy-setting generation, at an initial stage, devises a selective migration policy. The results are driven by the skill-fertility trade-off.

Chapter 2 analyzes whether the policy-setting initial population may prefer to abandon the existing unfunded social security system in order to redistribute the gains of nonselective migration from the future cohorts towards themselves. Chapter 3 analyses the proposition that increased migration, usually proposed as a tool for keeping social security solvent in an ageing economy, may actually decrease the pension benefits, due to the capital dilution that lower-skilled migration migrants cause.


The focus is on whether the offspring of migrants are assimilated with the native born population in terms of fertility rates and skill level. There are young and retired, skilled and unskilled populations. Those also differ by origin, being immigrants and native-born. The native-born population can be either a descendent of a native or immigrant family. While the natives and immigrants differ in their skill distribution and fertility rates, the descendants of immigrants can undergo some assimilation.
Three main assimilation patterns are analyzed: Full assimilation (adopting both skill and fertility rates), partial assimilation (adopting the skill levels but inheriting fertility rates) and no assimilation (inheriting the skill and fertility rates from the immigrant ancestors). Another issue pursued in analysis whether the social security system is Beveridgean or Bismarckian. Policy setting is done at some initial stage with no repeat of policy setting at some future date. The welfare level is assessed from the point of view of the policy-setting native-born generation.

The analysis is competent and is up to scholarly high standards. The propositions are correctly arrived at. The writing is transparent. The thesis qualifies the author to be awarded a Ph.D.

I would like to suggest that as a future extension of this fine thesis the author will reconsider the assumption that the policy setting is done once and for all at some initial stage with no repeat of policy re-setting at some future date. A more natural political economy setup in the overlapping framework is that voting about current migration and social security policies is done by current native born, with the expectations that the future generation of native born will vote on future migration and social security policies. See Razin, Sadka, and Suwankiri, Migration and the Welfare State, MIT Press, 2011, Chapters 5, and 7.

The bottom line is that I strongly recommend that based on his thesis, Gurgen Aslanyan will be awarded a Ph.D degree.

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Dear Sergey:

I will include my comments on Gurgen’s thesis in this letter. I apologize for being late; I have been traveling with MBA students and also to conferences outside of the U.S.

Gurgen Aslanyan’s thesis is a collection of three papers on how immigration policy interacts with human capital and fertility to influence the welfare of current and future generations in an economy with redistributive public policy (unfunded pension system).

The three components of the thesis correspond to three related essays. The first explores whether the nature of immigration policy (allowing skill-biased immigration or not) poses an intergenerational conflict in the sense that current versus future generations have different preferences over what type of immigration policy to implement. Since only current generations vote, this paper has a clear implication on what kind of immigration policy may be politically feasible.

The second essay specializes in the evaluation of the unfunded pension system and explores the decision to reform social security (privatization or laissez faire) depending on the type of immigration reform. The third essay extends the model to a closed economy setting and argues that unskilled immigration may be undesirable given its negative impact on pension benefits.

I have read the three essays and the preface that ties them together. Let me first state that all three essays further our understanding of the interaction between immigration and pensions significantly. Therefore, based on my evaluation, this draft certainly warrants a dissertation defense and fulfills the “thesis” requirement of a doctoral degree.

Overall, I find the topic very interesting, relevant, and economically significant. Immigration policy has had significant public policy implications in United States through their impact on the labor market and public finances. Europe has been less open than United States in terms of immigration but they still have significant populations of immigrants and this is a very timely issue for Europeans.
The country for which immigration may have an even bigger impact is Japan because Japan already has the highest (net) debt to GDP ratio among advanced economies (at 150%) and the aging is projected to require resources approximately equal to 7% of GDP. They need more workers to pay taxes. In February 2014, the Abe government announced a preliminary project of possible immigration into Japan. The initial proposal allows for both skilled and unskilled guest workers with a combined total of 200,000 per year, and eventually a stock of guest workers equal to 20 million in 20 years. This stock would represent a significant portion of the working age population in Japan in 2035. However, this proposal seems to have received significant backlash, casting serious doubts about the political feasibility of allowing guest workers in Japan.

Another interesting application of Gurgen’s modeling would be to study the internal migration in China. There is a large number of workers flooding Shanghai and cities like it due to higher wages, opportunities, schools and pensions which depend on average “local” wages. However, rural area Chinese citizens need permits to qualify for pensions and the right to send their kids to schools in large cities like Shanghai. I urge Gurgen to consider incorporating some of the features of the Chinese pension/schooling system into his framework to study the effects of internal migration on the domestic labor markets, skill accumulation, and pensions in China.

Gurgen Aslanyan provides the relevant literature survey and sufficient motivation for the topic but I wanted to provide additional context to highlight the importance of the topic for nearly all of the developed and developing economies.

I will now provide specific comments on the three essays.

**Essay 1:**

There is a small, open economy populated by two-period lived natives and immigrants. Individuals work in the first period, pay taxes to the government, and retire in the second period when get a pension and consume all their income, including their saving.

A fraction of natives are skilled; there is a different fraction of immigrants who are skilled. Fertility depends on human capital and native/immigrant background.

Effective labor is a function of the working-age population which depends on the size of the immigrant population in addition to that of the natives, and their efficiency.
The government taxes labor income; since labor is exogenous this is equivalent to a lump sum tax. Therefore any expenditures the government makes, possibly due to immigrants, will have zero distortion on labor supply.

In the Status Quo economy, there is no immigration, no debt, population is constant and the replacement rate equals the tax rate. The net return to social security is zero, which is the rate of growth of the economy.

The economy with immigration has population growth that depends on the ratio of immigrants to natives, and, the expected human capital of natives and immigrants. Immigrants enter as young workers, pay taxes and receive pensions when old. Their descendants become natives. The rate of growth of population is positive which means this economy has a higher return to pensions than the SQ economy and hence welfare is higher in the immigration economy.

This is a debt financed PAYG system and whether the immigrants inherit their own fertility and/or human capital as opposed to becoming native in either respect can change this result. The government can borrow and finance any fiscal problem associated with immigration or pensions subject to a transversality condition.

With full assimilation, a numerical example is used to see which way a particular inequality condition goes, and this suggests that welfare decreases with the share of skilled immigrants. This is counterintuitive at first; the intuition is given on page 22. I am not still clear on what the costs of immigration are on public finances and how immigration helps solve some of the problems. Also, is it possible to find another country for which the inequality goes the other way?

This is where a two-period model may not be a good measurement device to evaluate the welfare effects of immigration and a large-scale and calibrated overlapping generations model would be useful. I urge the author to consider taking his theoretical implications to more realistic models. I believe he has very interesting channels that create the possibility of allowing unskilled immigration (the U.S. model to a large extent) over other policies. It would be very interesting to quantify this in a calibrated overlapping generations model.

With government debt equal to zero, skilled immigration raises effective labor supply and the return to public pensions. Therefore, in this PAYG economy, the first period young are better off with skill-biased immigration and the first period old are indifferent between zero versus positive skilled immigration.
Different types of assimilation may overturn this result as lower fertility would reduce the return to public pensions in the future. Overall, whether skilled immigration raises welfare of the natives (that would allow this policy to be selected) seems to be a quantitative question depending on the skill heterogeneity, fertility rates among the natives and immigrants, and the degree to which immigrant “become like” natives. A large scale overlapping generations model is called for.

**Essay 2:**

This essay utilizes the same economic environment as in essay 1 but allows for eliminating pensions except for the initial old. (A very minor note is that $\beta$ is the subjective time discount factor and not the rate of time preference.)

The thought experiment in this essay is to start the economy off with a PAYG social security system by giving pensions to the initial old but allowing the possibility of immediately eliminating the public pension system. I would have liked to see an experiment where some cohorts had already paid taxes and then they would be allowed to switch from the unfunded to a private system.

In this essay, the author finds that immigration is Pareto superior; immigration allows the first young cohort to distribute resources in their favor and they dismantle the unfunded system. However, I would like to point out that two extensions are needed to convince the reader: endogenous labor with a depressing effect on local wages of increased immigration. These issues quickly will produce opposing effects and a large scale overlapping generations model will be required to numerically evaluate the welfare effects of immigration in the presence of endogenous labor with the wages of the native workers possibly depressed with a larger influx of immigrants.

In the section on joint analysis, it is assumed that the initial old always gets their pensions and therefore they do not have any influence on which immigration/pension policies are to be followed. The fact that the open economy assumption shields the return to their assets (that are exogenously brought into the economy) also makes them indifferent to any policy. Therefore, the first generation makes the call. Depending on the assimilation assumption, the first generation may open up the doors to immigrants and dismantle the unfunded system. The degree to which the descendants of the immigrants inherit the skills/fertility of their parents versus those of the natives makes a difference, and once again, calls for a large quantitative life cycle model to assess the overall appeal of immigration/privatization policies in richer and more realistic environments.
E\\text{SSAY 3:}

In this third essay, the author uses a general equilibrium version of the 2-period overlapping generations model of the previous two essays, still with exogenous labor, but now with immigration allowed such that the skills of immigrants are less than those of natives, although the fertility rates are the same.

Starting from a steady state, immigration is allowed and the new steady state is studied. One result is that the new steady state capital does not depend on the immigrants’ skill level. Again in the new steady state, if a certain parameter restriction is satisfied (essentially, if the skill level of immigrants is sufficiently low), then (lump sum) pension benefits decrease with the size of immigrant population. The earnings-related pensions, on the other hand, increase with immigration (as long as capital’s share of output is less than 50%). So, the total pensions may go up or down with immigration, once again suggesting the use of a large scale overlapping generations model to evaluate policy.

There are more results in the paper that can go either way depending on certain parameter restrictions but since this paper is based on the author’s forthcoming paper in the Journal of Population Economics, I would like to restrict my main comment to what I have repeatedly mentioned: Immigration, fertility, human capital are central themes in the design of optimal retirement and population policies. The thesis makes an important contribution to our knowledge in these areas with a simple two period model (with exogenous labor and mostly with a partial equilibrium concept). At least some of the important results on whether eliminating social security or allowing immigration can raise or lower welfare depending on certain parameter configurations. As a result, the question becomes one of quantifying the effects in larger, more realistic models in which precise demographic patterns can be incorporated. I urge the author to develop the necessary tools to take his important economic questions to carefully calibrated life cycle models as he develops his research program.

Please let me know if you have further questions.

Best regards,

Selahattin (Selo) İmrohoroğlu