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Essays on Monetary Policy and Estimation of DSGE Models

Dissertation

Prague, August 2010

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Abstract

In the first chapter of the thesis, I assess possible risks and challenges of implementing inflation targeting strategy in more complicated, but at the same time more realistic, DSGE model economies. I focus on analysis of optimal monetary policy and welfare in a model of a small open economy with multiple domestic sectors, which have different structural characteristics. The findings suggest that openness to trade as well as sector-specific features do matter for monetary policy design thus generating important implications for optimal stabilization objectives. The ranking of simple rules indicates that flexible CPI targeting regime is able to closely replicate the optimal solution and outperform the policy of domestic inflation stabilization. The presence of sectoral asymmetries may alter the relative performance of alternative policy rules. The second part of the thesis (with S. Slobodyan) is devoted to robustness issues of Bayesian estimation of DSGE models. We investigate the consequences of relaxing the rational expectations hypothesis and contrast model fit, estimated parameters, and perceived inflation persistence for several DSGE models of the Euro area estimated under adaptive learning and rational expectations. We find that assuming adaptive expectations results in better model fit than if rational expectations are used, especially when the agents use very little information to form their beliefs. Estimated parameters and the model fit depend significantly on the information set used by the agents, which might explain widely divergent results of previous estimations under adaptive learning.

V první části práce přiřazuji možná rizika a výzvy strategii implementování cílení inflace v komplikovanějších, ale o to více realističtějších, DSGE modelových ekonomikách. Zaměřuji se na analýzu optimální monetární politiky a blahobytu v DSGE modelu malé otevřené ekonomiky s několika domácími sektory, které mají rozdílné strukturální charakteristiky. Výsledky ukazují, že otevřenost k obchodu stejně jako na sektoru závislé prvky jsou významné pro monetární politiku a tudíž jsou významné pro tvořené optimálních stabilizačních cílů. Oceňování jednoduchých pravidel naznačuje, že flexibilní cílení CPI je schopné přesně replikovat optimální řešení a předčít politiku domácí inflační stabilizace. Přítomnost sektorové asymetrie může změnit relativní výkonnost různých monetárních pravidel. Druhá část práce (s S. Slobodyanem) se věnuje otázkám robustnosti při Bayesovském odhadování v DSGE modelech. Zkoumáme dopady opuštění hypotézy racionálního rozhodování a dáváme do souvislosti fit modelu, odhad-

nuté parametry a inflační setrvačnost pro několik DSGE modelů Euro oblasti pro adaptivní učení a racionální očekávání. Zjišťujeme, že předpoklad adaptivních očekávání vede k lepšímu odhadu modelu než při racionálních očekávání, obzvláště když agenti používají velmi málo informací pro tvorbu svých odhadů. Odhadnuté parametry a fit modelu výrazně závisí na použitém informačním setu používaném agenty, což může vysvětlit velmi rozdílné výsledky odhadů pro adaptivní učení.

Introduction

The first part of the thesis is motivated by current practice of policy conduct that has been recently implemented by many monetary institutions. In particular, a new operational framework, inflation targeting, has been introduced by the most advanced central banks. At the same time, DSGE models became widely used for systematic evaluation of macroeconomic effects of a certain policy as well as for forecasting. The second motivating factor was the observation that modern economies represent complex systems composed of elements that may have very different characteristics. Empirical evidence, which exists for many European countries, indicates that the pace of productivity improvements, price and wage setting schemes, and as result inflation persistence, may vary significantly across sectors. For such economies it is not straightforward to identify whether targeting of the aggregate (domestic or consumer price) inflation would be the best approach from the welfare view point. In the first chapter of the thesis, I assess possible risks and challenges of implementing inflation targeting policy in more complicated, but at the same time more realistic, DSGE model economies. The question I address is whether it is optimal to account for sector-specific characteristics when implementing monetary plan or can welfare be maximized following simple, uniform strategies. More specifically, I analyze the stabilization objectives of optimal monetary policy and the trade-offs facing the central bank in a two-sector, small open economy model obtained as a limiting case of a two-country Dynamic Stochastic General Equilibrium framework. It is assumed that sectors have different characteristics (sector-specific shocks, preference parameters, degree of nominal rigidities). The findings suggest that openness to trade as well as sector-specific features do matter for monetary policy design thus generating important implications for optimal stabilization objectives and social welfare. The ranking of simple rules indicates that flexible CPI targeting regime is able to closely replicate the optimal solution and outperform the policy of domestic inflation stabilization. Finally, the sensitivity analysis demonstrates that the presence of sectoral asymmetries may alter the relative performance of alternative policy rules.

The second part of the thesis (with S. Slobodyan) is devoted to robustness issues of Bayesian estimation of DSGE models. This work was motivated by recent tendency to use estimated DSGE models for policy analysis and by large literature that highlights challenges in the modeling approach, and more specifically, in the ability of DSGE models to match empirically the observed features of the data. Standard RBC models with rational expectations tend to miss the observed persistence of the key macroeconomic time series such as inflation, output, investment, etc. The proposed extensions to a standard framework suggest complementing a micro-founded stylized models with a number of nominal and real rigidities such as habit formation, Calvo pricing, indexation, adjustment costs, which enable capturing the dynamic properties of real data. However, recent DSGE-VAR analysis provides evidence that even friction-augmented DSGE models remain misspecified. Various assumptions about the potential source of persistence can result in a different dynamics of the key macro variables. For example, the inclusion of "mechanical" endogenous persistence mechanism, such as habit formation and price indexation, can influence the consumption and inflation dynamics and considerably change the overall performance of the model. For that reason, the empirical literature attempts to assess the validity of alternative modeling assumptions and evaluate the ability of various DSGE models to fit macroeconomic data. In this paper, we attempt to improve DSGE model fit as well as to address the issue of possible model misspecifications by departing from the rational expectation (RE) hypothesis and incorporating more empirically realistic mechanism of public's expectation formation. More specifically, we investigate the consequences of relaxing the rational expectations hypothesis and contrast model fit, estimated parameters, and perceived inflation persistence for several DSGE models of the Euro area estimated under rational expectations and adaptive learning. We add to answering the following underlying questions: How restrictive is the RE hypothesis for estimated DSGE models? Can adaptive learning generate endogenous persistence at the same time reducing structural rigidities? In other words, we evaluate empirically the relative importance of several types of "frictions" - "mechanical" rigidities like habit formation, Calvo pricing, adjustment costs etc. versus learning. Our major contribution is that we provide the answer to these questions by offering a comprehensive analysis of the factors which could determine a diversity of the estimation results under adaptive learning. In such a way we wish to reconcile contradicting conclusions from the previous studies. We study the robustness of the estimation results in several dimensions: by varying the model size, information set available to the learning agents, and the way of forming agents' initial beliefs. We find that assuming adaptive expectations results in better model fit than if rational expectations are used, especially when the agents use very little information to form their beliefs. Estimated parameters and the model fit depend significantly on the information set used by the agents, which might explain widely divergent results of previous estimations under adaptive learning. We also find that different ways of forming the initial beliefs influence the dynamics of the model under learning.

Chapter 1

The Implications of Sectoral Heterogeneity for Monetary Policy and Welfare in a Small Open Economy: A Linear Quadratic Framework

Abstract:

Modern economies exhibit various structural and dynamic characteristics. At the same time, many central banks have implemented the similar strategy, i.e. inflation targeting, as an operational framework. Controversial normative issue - is such stabilization objective welfare maximizing for more complex models with heterogeneous elements across sectors? This article analyzes optimal monetary strategy and policy trade-offs in a DSGE model of an open economy with traded and non- traded sectors. We approximate the utility of the representative consumer to obtain a micro-founded quadratic loss function of the form extensively used for monetary policy assessment. The central bank's optimal strategy is computed and optimal and simple policy rules compared according to the derived welfare measure. We assess the role of openness, structural characteristics, and relative prices for monetary policy design. The model is calibrated to match the moments of main macroeconomics variables of Canadian economy. The findings suggest that central bank's objectives display sector-specific features thus generating important implications for optimal policy and welfare. The ranking of simple rules indicates that flexible CPI targeting regime that includes a certain degree of internal relative prices management is able to closely replicate the optimal solution and outperform the policy of domestic inflation stabilization. Finally, we conduct the sensitivity analysis and evaluate welfare implications of sectoral heterogeneity for targeting the alternative price indices.

JEL classification: E52, E58, E61, F41

Keywords: DSGE models, non-traded goods, optimal monetary policy

1.1 Introduction

In recent decades the approach to monetary policy conduct has shifted to a more systematic one. Many central banks have formulated their policy objectives explicitly and, more specifically, have announced their commitment to price stabilization as the overriding policy goal. As a result, a new operational framework, inflation targeting, has been introduced by the most advanced central banks. At the same time, important features of modern economies, such as the social and economic consequences of unemployment, uncertainties of various types, asymmetric economic structure, and interrelations with the rest of the world, have brought about efforts to widen the range of policy objectives beyond inflation (price) stability alone. Therefore, over the past several years, the attention of economists has turned to the issue of whether strict inflation targeting indeed represents the best strategy from the welfare viewpoint. Another important question is the sensitivity of the conclusions to different, more complicated model frameworks.

The important attribute of real economies is that they represent the complex systems with various structural and dynamic characteristics. Should policymakers account for structural heterogeneity across economic elements when implementing the monetary strategy, or should they assume that welfare can be maximized under the uniform specification of the policy objectives? This paper aims to contribute to the discussion of this crucial issue of monetary policy design and practical implementation.

The analysis of optimal monetary strategies has been performed in a number of studies. One thread in the literature computes optimal policy under *assumed* welfare objectives. In particular, the loss function of the central bank usually takes the quadratic form with terms such as inflation (CPI or domestic) and the output gap, with the weights in front of each target chosen ad hoc. This approach is very popular in applied research because it greatly simplifies the derivations and brings the model dynamics closer to the real data. At the same time, such an approach assumes certain policy objectives a priory. An alternative methodology analyzes optimal monetary policy on the basis of the objective function of the central bank which is derived from micro-foundations. This paper contributes to the second class of literature and adds to the analysis of optimal policy in *open* economies, where the formulation of policy targets appears to be more controversial compared to a closed economy setting. It has been shown that welfare-maximizing monetary policy in a closed economy should aim to completely stabilize CPI inflation and the output gap (Woodford, 2003). In the literature on open economies, the critical questions are whether the central bank should also target open economy variables, i.e. the exchange rate, and how the targeting of domestic variables changes under the exposure of the economy to external factors. Another topic which has attracted a great deal of attention from both researchers and practitioners is related to the determination of the appropriate inflation measure that has to be stabilized. This issue gains particular relevance for the studies of models with heterogeneous economic structure, which implies the differentiated response of domestic elements to disturbances of the same type.

A surprising conclusion drawn by several authors who have performed explicit welfare derivation for models of open economies is that exchange rate fluctuations have no direct impact on welfare. Specifically, Clarida, Gali, and Gertler (2001) find that under perfect exchange rate pass-through, the qualitative results for the closed economy carry over to the open economy. Gali and Monacelli (2005), who characterize the welfare of a small open economy for a special case of parameter values and under the balanced trade assumption, support the previous result and conclude that the small open economy problem is identical to that of a closed economy. The above results taken at face value imply optimality of complete exchange rate flexibility.

However, a number of recent studies have challenged this finding. Specifically, Corsetti and Pesenti (2005), Sutherland (2002), and Monacelli (2003) show that under incomplete pass-through, optimal policy is not purely inward looking. Benigno and Benigno (2006) analyze the gains from international monetary policy cooperation. They study the conditions under which individual countries have incentives to influence the terms of trade and thus to deviate from the socially optimal point. De Paoli (2006) finds that the simple violation of purchasing power parity (PPP), which arises from home bias in consumption, brings in a role for targeting the real exchange rate in a one-sector small open economy model. Liu and Pappa (2005) consider a two-sector, open economy model in a two-country framework. Their study provides interesting insights into the impact of an asymmetric structure between sectors on the gains from cooperation. Their results suggest that in an economy with multiple sectors, and thus multiple sources of nominal rigidities, optimal monetary policy cannot replicate a flexible price allocation creating the scope for coordination. The important limitation of their work for the analysis of optimal monetary policy is the assumption of unitary elasticity of substitution across goods and a logarithmic utility function. As a result, under this very special case, important welfare effects vanish and general conclusions concerning the optimal monetary policy cannot be derived.

In this work, we analyze the stabilization objectives of optimal monetary policy and the trade-offs facing the central bank in a two-sector, small open economy model obtained as a limiting case of a two-country Dynamic Stochastic General Equilibrium framework. We assess the role of structural asymmetries, general preferences, and multiple relative prices for monetary policy design and welfare evaluation. We contribute to the normative analysis of open economies by introducing a more complicated economic structure, namely, multiple domestic sectors combined with a variety of sector-specific and foreign shocks. In addition, we consider a general specification of preferences (the elasticity of substitution is non-unitary). These features of the model differentiate our work from the previous studies, which derived their results for the special cases of unitary elasticity of substitution across goods or, alternatively, relied on the ad hoc objective functions. By abstracting from those simplifying assumptions we are able to uncover additional welfare effects specific to the open multisectoral economy and make a methodological contribution by deriving the utility-based welfare measure and the optimal reaction function of the central bank under more generalized preferences. For this purpose we employ the linear-quadratic solution methods discussed in Benigno and Benigno (2006) and Benigno and Woodford (2005), which involve computation of a second-order approximation of the utility function and model structural equations. This approach enables us to analyze the determinants of optimal monetary policy and rank alternative monetary policy regimes on the basis of a rigorous welfare measure derived from micro foundations and approximated by a tractable quadratic form. In addition, we study how the optimal price index that has to be stabilized is affected by structural asymmetries. In particular, we evaluate the welfare benefits from targeting sector-specific versus aggregate price indices (domestic or CPI inflation) for various degrees of relative price stickiness. We calibrate the model to match the moments of variables of Canadian economy.

The results of our study suggest that the loss function of the central bank, which describes the welfare maximizing stabilization objectives, displays the features of both an open economy and multisectoral economic structure. Specifically, it is shown that social welfare is affected by variations in domestic inflation rates and output gaps (with sector-specific weights) as well as in the relative prices (including the exchange rate). We derive the optimal targeting rule, which determines the variables (targets) to which the central bank should respond in order to achieve efficient allocation of resources as well as the magnitude of such a response. Furthermore, we experiment with alternative simple rules and analyze their ability to replicate the optimal solution. We present a ranking of alternative simple rules, which indicates the costs of implementing alternative monetary strategies and can provide useful information for managing the conflicting policy objectives. Our results suggest that, in general, targeting the aggregate (domestic or CPI) inflation is not the best approximation for the optimal policy, and social welfare can be improved by accounting for sector-specific inlfaiton rates as well as other policy objectives, namely, the output gap and the relative prices.

We show that simple rules with aggregate variables (inflations) which incorporate a response to relative price changes achieve better stabilization of sector-specific volatilities, improve welfare, and thus closely approximate the optimal solution. Such a result is important because a strategy which differentiates the response between domestic sectors is difficult to design and implement in practice. Generally, the simple rules perform quite well in terms of macroeconomic stabilization (relative to the optimal rule) and can deliver reasonable welfare results. We perform a sensitivity analysis in order to study the impact of sectoral heterogeneity in the degree of price stickiness, the elasticity of substitution, and the degree of openness on the relative performance of policy rules with sector-specific and aggregate variables (inflation rates). We find that the implications of asymmetric nominal rigidities differ for closed and open economies. Welfare benefits from targeting the "core" versus broader inflation index increase as the economy becomes more open and prices in the non-traded sector relatively stickier. In addition, it is welfare improving to weigh appropriately the sectoral inflation rates if the elasticity of substitution between home and foreign goods rises. On the contrary, as goods in the non-traded sector become relatively more elastic, the benefits from targeting the measure of the core inflation gradually vanish and policy of the domestic inflation stabilization approximates the optimal strategy rather well.

The paper is organized as follows. Section 2 presents the model and section 3 describes the equilibrium dynamics. Section 4 analyzes the monetary policy problem and welfare. Section 5 describes the results of the numerical simulation. Section 6 illustrates the welfare implications of alternative simple rules. The sensitivity analysis is presented in section 7. Finally, the results of the paper are summarized in section 8.

1.2 A Two-Sector, Small Open Economy Model

The framework is represented by a two-country dynamic general equilibrium model where both sides, Home (the open economy – H) and Foreign (the rest of the world, the relatively closed economy – F), are explicitly modeled. The small open economy problem is derived as a limiting case of such a framework (as in De Paoli, 2006). Each country has two domestic sectors, which produce traded and non-traded goods; the share of non-traded goods may vary in the consumption basket of each country. A continuum of infinitively lived households consumes the final consumption good, which includes goods produced in both domestic sectors as well as imported goods. Households produce differentiated intermediate goods and receive disutility from production. We introduce monopolistic distortion and sticky prices in both sectors. These assumptions represent the standard way of introducing the role for monetary policy into such class of models. Households as consumers maximize their utility and solve the optimal price-setting problem as producers.

The model specification allows us to consider the closed economy, the open one-sector economy, and the economy with unitary elasticity of substitution as special cases of our more general analysis. We assume sector-specific productivity, fiscal, and mark-up shocks; the degree of nominal rigidities may also differ across sectors. Furthermore, we assume production subsidies in order to offset the monopolistic distortions in both sectors. The international and domestic asset markets are complete.

1.2.1 Representative Households

In our two-country framework a continuum of domestic households belong to the interval $[0, n)$, while foreign agents belong to the segment $(n, 1]$. The utility function of a representative consumer in country H or F is given by:

$$U_t^j = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} [U(C_s^j) - V(y_{s,T}(j), A_{s,T}^i) - V(y_{s,N}(j), A_{s,N}^i)] \right\},$$

where j is the index specific to the household, and i is the country index; E_t denotes the expectation operator conditional on the information set at time t, and β is the intertemporal discount factor. U(.) represents the flows of utility from consumption of a composite good and $V(.)$ stands for the flows of disutility from production of differentiated goods. Each household produces two types of differentiated goods – traded and non-traded. The home economy produces a continuum of differentiated traded goods indexed on the interval $[0, n]$, whereas the foreign economy's traded goods belong to the interval $(n, 1]$. In addition, a continuum of differentiated non-traded goods are indexed on the interval $[0, n]$ and $(n, 1]$ for the home and foreign country, respectively. A denotes a productivity shock that can be country and sector specific. The subscript T stands for the traded sector, whereas N denotes the non-traded sector.

In our analysis we assume that preferences have isoelastic functional form:

$$U(C_s^j) = \frac{(C_s^j)^{1-\rho}}{1-\rho}, \qquad V(y_{s,L}(j), A_{s,L}^i) = (A_{s,L}^i)^{-\eta}\frac{(y_{s,L}(j))^{1+\eta}}{1+\eta},$$

where $L = H, N$; $\rho > 0$ is the inverse of the intertemporal elasticity of substitution in consumption, and $\eta \geq 0$ is equivalent to the inverse of the elasticity of goods production. The composite consumption good C is a Dixit-Stiglitz aggregator of traded and non-traded goods defined as:

$$C^j = [\gamma^{\frac{1}{\omega}}(C_N^j)^{\frac{\omega-1}{\omega}} + (1-\gamma)^{\frac{1}{\omega}}(C_T^j)^{\frac{\omega-1}{\omega}}]^{\frac{\omega}{\omega-1}},$$

where C_N and C_T are the consumption sub-indices that refer to the consumption of non-traded and traded goods, respectively, $\omega > 0$ is the intratemporal elasticity of substitution, and γ is a preference parameter that measures the relative weight that individuals put on non-traded goods.

Preferences for the rest of the world are specified in a similar fashion:

$$C^{j*} = [(\gamma^*)^{\frac{1}{\omega}}(C_N^{*j})^{\frac{\omega-1}{\omega}} + (1-\gamma^*)^{\frac{1}{\omega}}(C_T^{*j})^{\frac{\omega-1}{\omega}}]^{\frac{\omega}{\omega-1}},$$

where the asterisk denotes a foreign country variable.

Traded consumption goods are the aggregators of goods produced at home and abroad and defined as:

$$\begin{aligned} C_T^j &= [\upsilon^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-\upsilon)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}}, \\ C_T^{j*} &= [(\upsilon^*)^{\frac{1}{\theta}} (C_H^*)^{\frac{\theta-1}{\theta}} + (1-\upsilon^*)^{\frac{1}{\theta}} (C_F^*)^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}}, \end{aligned}$$

where υ and υ^* are the parameters that determine the preferences of agents in countries H and F, respectively, for the consumption of goods produced at Home.

As in Sutherland (2002) and De Paoli (2006) we assume that υ^*, the share of imported goods from country H in the consumption basket of country F, increases proportionally to the relative size of the home economy n and the degree of openness $\widetilde{\upsilon}^*$. Thus we assume that $\upsilon^* = n \cdot \widetilde{\upsilon}^*$. Similarly, $(1-\upsilon) = (1-n) \cdot \widetilde{\upsilon}^*$. Such a specification allows modeling of home bias in consumption as a consequence of different country size and degree of openness.

The consumption sub-indices of non-traded, home-produced, and foreign-produced differentiated goods are defined as follows:

$$\begin{aligned} C_N &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c_N(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}, & C_N^* &= \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c_N^*(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}, \\ C_H &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c_h(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}, & C_F &= \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c_f(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}, \\ C_{H^*} &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c_h^*(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}, & C_F^* &= \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c_f^*(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}, \end{aligned}$$

where $\sigma > 1$ is the elasticity of substitution across the differentiated goods.

The corresponding consumption-based price indexes for countries H and F take the form:

$$P = [\gamma P_N^{1-\omega} + (1-\gamma) P_T^{1-\omega}]^{\frac{1}{1-\omega}} \tag{1}$$

$$P_T = [\upsilon P_H^{1-\theta} + (1-\upsilon) P_F^{1-\theta}]^{\frac{1}{1-\theta}} \tag{1a}$$

$$P^* = [(\gamma^*)(P_N^*)^{1-\omega} + (1-\gamma^*)(P_T^*)^{1-\omega}]^{\frac{1}{1-\omega}} \tag{2}$$

$$P_T^* = [(\upsilon^*)(P_H^*)^{1-\theta} + (1-\upsilon^*)(P_F^*)^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{2a}$$

The price sub-indices for home, foreign, and non-traded goods in the two economies are:

$$\begin{aligned}
P_N &= \left[\left(\frac{1}{n}\right)\int_0^n p_N(z)^{1-\sigma}d(z)\right]^{\frac{1}{1-\sigma}} & P_N^* &= \left[\left(\frac{1}{1-n}\right)\int_n^1 p_N^*(z)^{1-\sigma}d(z)\right]^{\frac{1}{1-\sigma}}, \\
P_H &= \left[\left(\frac{1}{n}\right)\int_0^n p_h(z)^{1-\sigma}d(z)\right]^{\frac{1}{1-\sigma}} & P_F &= \left[\left(\frac{1}{1-n}\right)\int_n^1 p_f(z)^{1-\sigma}d(z)\right]^{\frac{1}{1-\sigma}}, \\
P_H^* &= \left[\left(\frac{1}{n}\right)\int_0^n p_h^*(z)^{1-\sigma}d(z)\right]^{\frac{1}{1-\sigma}} & P_F^* &= \left[\left(\frac{1}{1-n}\right)\int_n^1 p_f^*(z)^{1-\sigma}d(z)\right]^{\frac{1}{1-\sigma}},
\end{aligned}$$

where $p_N(z), p_H(z)$, and $p_F(z)$ are prices in units of the domestic currency of the home-produced non-traded and traded goods, and foreign-produced goods. The law of one price holds for differentiated goods, i.e., $p_h(z) = S \cdot p_h^*(z)$ and $p_f(z) = S \cdot p_f^*(z)$, where S is the nominal exchange rate, defined as the price of the foreign currency in terms of the domestic currency. This in turn implies that $P_H = S \cdot P_H^*$ and $P_F = S \cdot P_F^*$. However, equations (1) and (2) demonstrate that the presence of non-traded goods and the home bias in consumption result in a violation of the Purchasing Power Parity (PPP), i.e., $P \neq S \cdot P^*$. Thus, the real exchange rate is not equal to one and is defined as $ER = \frac{S \cdot P^*}{P}$. The real exchange rate determinants will be more explicitly analyzed in subsection 2.5.

1.2.2 Aggregate Demand

By solving the consumer's cost minimization problem, we derive the total demand for the differentiated goods produced in countries H and F as well as the demand for the non-traded goods in both countries. The resulting demand equations for country H take the following form:

$$y_h^d(z) = \left(\frac{p_h(z)}{P_H}\right)^{-\sigma} \left[\begin{array}{l} \left(\frac{P_T}{P}\right)^{-\omega} \left(\frac{P_H}{P_T}\right)^{-\theta} \times \\ \left\{ \begin{array}{l} \upsilon(1-\gamma)C + \left(\frac{1}{ER}\right)^{-\omega} \times \\ \left[\begin{array}{c} \left(\frac{\upsilon^*}{\upsilon+(1-\upsilon)(P_{FH})^{1-\theta}}\right) + \\ \left(\frac{1-\upsilon^*}{\upsilon(P_{FH})^{\theta-1}+(1-\upsilon)}\right) \end{array} \right]^{\frac{\theta-\omega}{1-\theta}} (1-\gamma^*)\upsilon^* C^* \frac{1-n}{n} \end{array} \right\} + G_H \end{array} \right] \tag{3}$$

$$y_N^d(z) = \left(\frac{p_N(z)}{P_N}\right)^{-\sigma} \left[\left(\frac{P_N}{P}\right)^{-\omega} \gamma C + G_N \right], \tag{4}$$

and for goods produced in country F:

$$y_f^d(z) = \left(\frac{p_f(z)}{P_F}\right)^{-\sigma} \left[\begin{array}{l} \left(\frac{P_T}{P}\right)^{-\omega} \left(\frac{P_F}{P_T}\right)^{-\theta} \times \\ \left\{ \begin{array}{l} (1-\upsilon)(1-\gamma)C\frac{n}{1-n} + \left(\frac{1}{ER}\right)^{-\omega} \times \\ \left[\begin{array}{c} \left(\frac{\upsilon^*}{\upsilon+(1-\upsilon)(P_{FH})^{1-\theta}}\right) + \\ \left(\frac{1-\upsilon^*}{\upsilon(P_{FH})^{\theta-1}+(1-\upsilon)}\right) \end{array} \right]^{\frac{\theta-\omega}{1-\theta}} (1-\gamma^*)(1-\upsilon^*)C^* \end{array} \right\} + G_F^* \end{array} \right] \tag{5}$$

$$y_N^d(z) = \left(\frac{p_N^*(z)}{P_N^*}\right)^{-\sigma} \left[\left(\frac{P_N^*}{P^*}\right)^{-\omega} \gamma^* C^* + G_N^* \right], \tag{6}$$

where G and G^* are country and sector-specific government purchase shocks, $P_{FH} = \frac{P_F}{P_H}$ is the relative price of foreign to home-produced goods, i.e., the terms of trade, and ER is the real exchange rate.

In order to obtain the small open economy version of our general two-country framework, we apply the assumptions $\upsilon^* = n \cdot \widetilde{\upsilon}^*$ and $(1-\upsilon) = (1-n) \cdot \widetilde{\upsilon}^*$ and take the limit $n \to 0$ similar to De Paoli (2006). As a result, the demand equations can be simplified to:

$$y_h^d(z) = \left(\frac{p_h(z)}{P_H}\right)^{-\sigma} \left[\begin{array}{l} \left(\frac{P_T}{P}\right)^{-\omega} \left(\frac{P_H}{P_T}\right)^{-\theta} \times \\ \left\{ \upsilon(1-\gamma)C + \left(\frac{1}{ER}\right)^{-\omega} \left[\left(\frac{1}{\upsilon(P_{FH})^{\theta-1}+(1-\upsilon)}\right)\right]^{\frac{\theta-\omega}{1-\theta}} (1-\gamma^*)\widetilde{\upsilon}^* C^* \right\} + G_H \end{array} \right] \tag{7}$$

$$y_f^d(z) = \left(\frac{p_f(z)}{P_F}\right)^{-\sigma} \left[\begin{array}{l} \left(\frac{P_T}{P}\right)^{-\omega} \left(\frac{P_F}{P_T}\right)^{-\theta} \times \\ \left\{ \left(\frac{1}{ER}\right)^{-\omega} \left[\left(\frac{1}{\upsilon (P_{FH})^{\theta-1} + (1-\upsilon)} \right) \right]^{\frac{\theta-\omega}{1-\theta}} (1-\gamma^*) C^* \right\} + G_F^* \end{array} \right]. \quad (8)$$

Therefore, the demand side for our two-sector, small open economy model is represented by equations (4), (6), (7), and (8).

The demand equations illustrate the small open economy implications, the impact of the economic structure, and a more general specification of preferences. In particular, the demand for goods produced at Home depends on both domestic and foreign consumption, whereas the demand for foreign-produced goods is not affected by changes in Home consumption. Moreover, the two-sector model specification brings in the differentiated impact of the terms of trade and the real exchange rate on the total demand for tradable goods. This happens under the general assumption that $\theta \neq \omega$. The literature on open economies usually assumes that $\theta > \omega$, $\theta > 1$, and ω is small. This implies that non-traded and traded goods are complements in the consumption basket. At the same time, home and foreign-produced goods are considered as substitutes.

1.2.3 International Risk Sharing

Foreign and domestic households have access to the international financial market, where state-contingent nominal bonds are traded. Households at home and abroad make their optimal consumption-saving decisions. They maximize their utility subject to the sequence of budget constraints for $t = 0, 1, ..$:

$$P_t C_t + E_t D_{t,t+1} B_{t+1} \leq B_t + \Pi_t + T_t,$$

where B_{t+1} is the holding of a nominal state-contingent bond that pays one unit of home currency in period $t+1$, $D_{t,t+1}$ is the period t price of the bond, Π_t is the profit income from goods production, and T_t is the transfer from the government. The complete-market assumption implies that the marginal rate of substitution between consumption in the two countries is equalized:

$$\frac{U_C(C^*_{t+1})}{U_C(C^*_t)}\frac{P^*_t}{P^*_{t+1}}\frac{S_t}{S_{t+1}} = \frac{U_C(C_{t+1})}{U_C(C_t)}\frac{P_t}{P_{t+1}}. \tag{9}$$

The international risk-sharing equation presented above illustrates the equality of nominal wealth in both countries in all states and time periods. The violation of PPP implies that fluctuations in the real exchange rate may result in a divergence in consumption across countries even under optimal risk sharing.

Consumers' optimization problem implies the following Euler equation:

$$U_C(C_t) = \beta\left[U_C(C_{t+1})R_t\frac{P_t}{P_{t+1}}\right],$$

where R_t is the nominal interest rate. Log-linearization of this condition leads to the following expression:

$$\widehat{r}_t = \rho\left(\widehat{C}_{t+1} - \widehat{C}_t\right) + E\pi_{t+1}. \tag{9a}$$

1.2.4 Optimal Pricing Decisions

Each household is a monopolistic producer of one differentiated traded and one non-traded good. The domestic household sets the price $p_N(z)$ and $p_h(z)$ and takes as given P, P_N, P_H, P_F, and C. The price-setting behavior is modeled according to Calvo (1983). In countries H and F in each time period a fraction $\alpha_L \in [0,1)$ of randomly picked producers in each sector ($L = N, H$) are not allowed to change their prices. Thus the parameter α_L reflects the level of price stickiness. The remaining fraction $(1-\alpha_L)$ can choose the optimal sector-specific price by maximizing the expected discounted value of profits:

$$E_t\sum_{S=t}^{\infty}(\alpha_L\beta)^{S-t}\left[\frac{U_C(C_S)}{P_S}(1-\tau_S)\widetilde{p}_{t,L}(z)\widetilde{y}_{t,S,L}(z) - V(\widetilde{y}_{t,S,L}(z), A_{S,L})\right],$$

where after-tax revenues in each sector are evaluated using the marginal utility of nominal income, $\frac{U_C(C_S)}{P_S}$, which is identical for all households in the country under the assumption of complete markets; τ_S is the tax rate; $\widetilde{p}_{t,L}(z)$ is the price of the differentiated good z, which is produced in sector L, chosen at time t , and $\widetilde{y}_{t,S,L}(z)$ is the total demand for good z, produced in sector L, at time S, conditional on the fact that the price $\widetilde{p}_{t,L}(z)$ has not been changed. All producers who belong to the fraction $(1-\alpha_L)$ choose the same price.

The optimal price $\widetilde{p}_{t,L}(z)$, which is derived from the first-order conditions, takes the following form:

$$\widetilde{p}_{t,L}(z) = \frac{E_t \sum_{S=t}^{\infty} (\alpha_L \beta)^{S-t} V(\widetilde{y}_{t,S,L}(z), A_{S,L}) \widetilde{y}_{t,S,L}(z)}{E_t \sum_{S=t}^{\infty} (\alpha_L \beta)^{S-t} \frac{U_C(C_S)}{P_S} \frac{1}{\mu_S} \widetilde{y}_{t,S,L}(z)}, \tag{10}$$

where $\mu_{S,L} = \frac{\sigma}{(1-\tau_{S,L})(\sigma-1)}$ represents the overall degree of monopolistic distortion and leads to an inefficient gap between the marginal utility of consumption and the marginal disutility of production. Benigno and Benigno (2006) and De Paoli (2006) refer to this gap as the mark-up shock. A Calvo-type setting implies the following law of motion for the sectoral price indices:

$$P_{L,t} = [\alpha_L (P_{L,t-1})^{1-\sigma} + (1-\alpha_L) \widetilde{p}_{t,L}(z)^{1-\sigma}]^{\frac{1}{1-\sigma}}. \tag{11}$$

Similar conditions can be derived for the producers in country F.

1.2.5 Real Exchange Rate Decomposition and PPP Violation

In order to explore the structural economic factors that result in PPP violation, we consider the real exchange rate decomposition. The real exchange rate is defined as $ER = \frac{S \cdot P^*}{P}$. We use the price indexes (1), (1a), (2), and (2a) to express the real exchange rate as a function of relative prices and preference parameters. We also use the fact that the law of one price holds for tradable goods, i.e., $P_H = S \cdot P_H^*$ and

$P_F = S \cdot P_F^*$. The real exchange rate can be presented as:

$$ER = \left(\frac{\upsilon^* + (1-\upsilon^*)(P_{FH})^{1-\theta}}{\upsilon + (1-\upsilon)(P_{FH})^{1-\theta}} \right)^{\frac{1}{1-\theta}} \left(\frac{\gamma^*(P_{NT}^*)^{1-\omega} + (1-\gamma^*)}{\gamma(P_{NT})^{1-\omega} + (1-\gamma)} \right)^{\frac{1}{1-\omega}}, \qquad (12)$$

where P_{FH} is the terms of trade defined in the previous sections, and $P_{NT} = \frac{P_N}{P_T}$ and $P_{NT}^* = \frac{P_N^*}{P_T^*}$ are the relative prices of non-traded goods in the two countries. Such a decomposition enables us to analyze the different channels of PPP violation. First of all, we note that under $\upsilon \neq \upsilon^*$,the ER is affected by the terms of trade. For our small open economy model specification, given the assumptions on υ and υ^*, the difference in country size necessarily results in different shares of consumption of home-produced goods in countries H and F. This so-called home bias channel has also been analyzed by De Paoli (2006) and Sutherland (2002).

Another important component that explains the deviation of the ER from PPP is determined by the multisectoral economic structure. Specifically, different preferences for consumption of non-traded goods across countries, i.e., $\gamma \neq \gamma^*$, as well as changes in the relative price of non-traded goods determine the fluctuation in the ER. The divergence in relative prices may occur as a result of country or sector-specific productivity shocks. Moreover, the law of one price holds for traded goods only. Nothing can ensure that the same equality will hold for the goods produced in the non-traded sector. Therefore, the exchange rate in our model is a composite term of two types of relative prices. As far as the policy issues are concerned, such a distinction implies a more difficult task of exchange rate management.

1.3 Equilibrium Dynamics

1.3.1 Sticky Price Equilibrium

The equilibrium dynamics under sticky prices are characterized by the optimality conditions derived in section 2. Here, we present a log-linearized version of the model.

We define $\widehat{x_t} \equiv \ln \frac{x_t}{\overline{x}}$ as the log deviation of the equilibrium variable x_t under sticky prices from its steady state value. $\widehat{x_t}^{flex} \equiv \ln \frac{x_t^{flex}}{\overline{x}}$ represents the log deviation of the equilibrium variable x_t under flexible prices from its steady state value. Under the assumption of flexible prices, producers can re-optimize every period so that their pricing decisions are synchronized. As a result the price dispersion among the differentiated goods is zero. Therefore, the price index in each sector is equal to the price set by each producer in this sector, and the main source of domestic distortion is eliminated. We will refer to $\widehat{x_t} - \widehat{x_t}^{flex}$ as the deviation of the variable $\widehat{x_t}$ from its natural level, i.e., the gap. At the same time, Benigno and Woodford (2005) and De Paoli (2006) demonstrate that under certain conditions, the flexible price equilibrium does not represent the most efficient allocation of resources, and the desired levels of variables which the policymaker wishes to achieve in order to eliminate the loss may differ from the flexible price allocation. Specifically, in the presence of mark-up and fiscal shocks as well as the condition $\rho\theta \neq 1$, the flexible price allocation diverges from the desired targets. Therefore, in general, the optimal policy aims to stabilize of the variables relative to their *target* level. Thus, we define the welfare relevant gap as $\widehat{x_t} - \widehat{x_t}^T$, where $\widehat{x_t}^T$ is the target level of the variable $\widehat{x_t}$. Both the flexible price equilibrium and the target variables are functions of shocks that affect the economy.

Moreover, we define the price change in the traded sector as $\Pi_H = \frac{P_{H,t}}{P_{H,t-1}}$ and that in the non-traded sector as $-\Pi_N = \frac{P_{N,t}}{P_{N,t-1}}$; consequently, the producer price inflation rates in the traded and non-traded sectors are $\pi_{H,t} \equiv \ln\left(\frac{P_{H,t}}{P_{H,t-1}}\right)$ and $\pi_{N,t} \equiv \ln\left(\frac{P_{N,t}}{P_{N,t-1}}\right)$, respectively. We approximate the model around the steady state, in which producer prices do not change, i.e., $\Pi_H = \frac{P_{H,t}}{P_{H,t-1}} = 1$ and $\Pi_N = \frac{P_{N,t}}{P_{N,t-1}} = 1$ at all times. A more detailed description of the steady state is presented in the Appendix.

1.3.2 Log-Linearization of the Optimality Conditions

We log-linearize the equilibrium conditions (4), (6)–(10), and (12) and obtain the following set of log-linear equations describing the dynamics of the multisectoral small open economy:

$$\pi_{H,t} = k_H \left(\eta \widehat{Y}_{H,t} + \rho \widehat{C}_t + (1-\upsilon)\widehat{P}_{FH,t} + \gamma \widehat{P}_{NT,t} + \widehat{\mu}_{H,t} - \eta \widehat{A}_{H,t} \right) + \beta E_t \pi_{H,t+1}, \tag{13}$$

$$\pi_{N,t} = k_N \left(\eta \widehat{Y}_{N,t} + \rho \widehat{C}_t - (1-\gamma)\widehat{P}_{NT,t} + \widehat{\mu}_{N,t} - \eta \widehat{A}_{N,t} \right) + \beta E_t \pi_{N,t+1}, \tag{14}$$

$$\widehat{Y}_{H,t} = -[\theta + (\theta-\omega)\upsilon]\widehat{P}_{HT,t} + \omega\gamma \widehat{P}_{NT,t} + \upsilon \widehat{C}_t + w(1-\upsilon)\widehat{ER}_t + (1-\upsilon)\widehat{C}_t^* + \widehat{g}_{H,t}, \tag{15}$$

$$\widehat{Y}_{N,t} = \widehat{C}_t - w(1-\gamma)\widehat{P}_{NT,t} + \widehat{g}_{N,t}, \tag{16}$$

$$\widehat{C}_t = \frac{1}{\rho}\widehat{ER}_t + \widehat{C}_t^*, \tag{17}$$

$$\widehat{ER}_t = \upsilon \widehat{P}_{FH,t} - \gamma \widehat{P}_{NT,t} + \gamma^* \widehat{P}^*_{NT,t}, \tag{18}$$

$$\Delta \widehat{P}_{NT,t} = \pi_{N,t} - \pi_{H,t} - (1-\upsilon)\Delta \widehat{P}_{FH,t}. \tag{19}$$

Moreover, from the price index relation (1a) we note that:

$$\widehat{P}_{HT,t} = -(1-\upsilon)\widehat{P}_{FH,t}. \tag{19a}$$

The Phillips curve relations in the two sectors are presented by equations (13) and (14), where $k_L = \frac{(1-\alpha_L\beta)(1-\alpha_L)}{\alpha_L(1+\sigma\eta)}$ is the constant that measures the response of the sectoral inflation rates to variations in real marginal costs. The characterization of real marginal costs in the open economy setting differs from that of the closed economy due to the gap between production and consumption as well as to the impact of relative prices, which reflect the distinction between domestic and consumer prices. An improvement in the terms of trade (a decrease in $\widehat{P}_{FH}$) or a positive productivity shock results in a fall in marginal costs in the traded sector. The marginal costs in the non-traded sector are independent of direct changes in the terms of trade. However, the sectoral marginal costs are linked through the relative prices of non-traded goods. This impact is opposite in sign and symmetric in magnitude. Producers' pricing decisions are forward-looking due to price stickiness. As a result, the Phillips curve takes the expectation-augmented form. Equations (15) and (16) describe the aggregate demand for domestic goods in the two sectors. We consider $\widehat{C}_t^*$ as a term that cannot be affected by dynamics in the home country. This variable is exogenous from the small open economy perspective. Relation (17) is the log-linearized optimal risk-sharing condition. It describes variations in domestic consumption depending on fluctuations in the real exchange rate and consumption abroad. Equation (18), which is derived from (12), summarizes the determinants of the real exchange rate. Again, the relative price of non-traded goods in the foreign country is treated as exogenous. This equation illustrates the implication of the multisectoral economic structure. In particular, changes in the terms of trade do not necessarily imply a corresponding adjustment of the exchange rate, due to the impact of the relative prices of non-traded goods at home and abroad. Finally, expression (19), which is in fact an identity, is obtained from the definitions of non-traded and traded goods inflation and describes the evolution of the price indexes for both sectors. The equation that characterizes traded goods inflation is presented in the next sub-section.

1.3.3 Domestic Inflation, CPI Inflation, and Some Aggregation Results

In this sub-section, we present several useful definitions and identities, which will be used in the subsequent analysis. Log-linearization of price indexes (1) and (1a) yields :

$$\widehat{P}_t = \gamma \widehat{P}_{N,t} + (1-\gamma)\widehat{P}_{T,t} \tag{20}$$

$$\widehat{P}_{T,t} = \upsilon \widehat{P}_{H,t} + (1-\upsilon)\widehat{P}_{F,t}. \tag{21}$$

Applying the definition of inflation $\pi_t = \ln\left(\frac{P_t}{P_{t-1}}\right) = \widehat{P}_t - \widehat{P}_{t-1}$, we obtain the expressions for CPI inflation and traded inflation:

$$\pi_t = \gamma \pi_{N,t} + (1-\gamma)\pi_{T,t} \tag{22}$$

$$\pi_{T,t} = \upsilon \pi_{H,t} + (1-\upsilon)\pi_{F,t}. \tag{23}$$

Moreover, the definition of the terms of trade implies that $\pi_{F,t} = \Delta\widehat{P}_{FH,t} + \pi_{H,t}$. The combination of the equations presented above results in the following relationship between CPI and domestic inflation:

$$\pi_t = \pi_t^D + (1-\gamma)(1-\upsilon)\Delta\widehat{P}_{FH,t}, \tag{24}$$

where domestic inflation equals:

$$\pi_t^D = \gamma\pi_{N,t} + (1-\gamma)\pi_{H,t}. \tag{25}$$

Total output is given by:

$$P_t Y_t = P_{N,t}Y_{N,t} + P_{H,t}Y_{H,t}. \tag{26}$$

Log-linearization of equation (26) yields:

$$\widehat{Y}_t = \gamma\widehat{Y}_{N,t} + (1-\gamma)\widehat{Y}_{H,t} - (1-\gamma)(1-\upsilon)\widehat{P}_{FH,t}. \tag{26a}$$

This relation implies that in an open multi-sectoral economy, aggregate output is not only the weighted average of the sectoral outputs, but also a function of relative prices.

Moreover, the evolution of the nominal exchange rate is derived from the definition of the real exchange rate and takes the form:

$$\widehat{ER}_t - \widehat{ER}_{t-1} = \widehat{S}_t - \widehat{S}_{t-1} + \pi_t^* - \pi_t, \tag{27}$$

where $\widehat{S}_t$ is the nominal exchange rate, and π_t^* is CPI inflation for the foreign country. We assume that the monetary authority abroad is implementing an inflation-targeting policy, and thus, $\pi_t^* = 0$. Such an assumption is common in the small open economy literature (Gali and Monacelli, 2005).

1.4 The Monetary Policy Problem and Welfare

This section will present the formulation of the monetary policy strategy and an analysis of the competing objectives of the central bank. We will see that the model specification implies deviations of the optimal monetary policy from complete price stabilization. Specifically, we present a formal welfare analysis and derive the objective function of the central bank based on a second-order approximation of both the household's utility and the structural equilibrium conditions (13)–(19). Optimal monetary strategy involves the maximization of the quadratic social welfare function (a minimization of the loss function) subject to linear constraints. Monetary policy is able to achieve the best outcome from the welfare perspective by implementing the optimal plan. In this analysis, we focus on optimal targeting rules, which are strongly advocated by Svensson and Woodford.

1.4.1 The Objective Function of the Central Bank for an Open Economy with Multiple Domestic Sectors

In order to obtain the analytical expression for welfare in a purely quadratic form, we apply the linear-quadratic solution methods described in Woodford (2003) and Benigno and Woodford (2005). This approach is based on the idea presented in Sutherland (2002) to explore the dynamic characteristics of the model and thus to account for the impact of the second moments of the variables on their levels. The derivation of the objective function of the central bank is presented in the Mathematical Appendix. We show that the utility function of the representative household can be approximated by the following expression:

$$W_{to} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \times \tag{28}$$

$$\left[\begin{array}{c} \widehat{C}_t - (\overline{\mu}_N)^{-1}\gamma\widehat{Y}_{N,t} - (\overline{\mu}_H)^{-1}(1-\gamma)\widehat{Y}_{H,t} + \frac{1}{2}(1-\rho)\widehat{C}_t^2 \\ -\frac{1}{2}(\overline{\mu}_N)^{-1}\gamma(1+\eta)\widehat{Y}_{N,t}^2 - \frac{1}{2}(\overline{\mu}_H)^{-1}(1-\gamma)(1+\eta)\widehat{Y}_{H,t}^2 \\ +(\overline{\mu}_N)^{-1}\gamma\eta\widehat{A}_{N,t}\widehat{Y}_{N,t} + (\overline{\mu}_H)^{-1}(1-\gamma)\eta\widehat{A}_{H,t}\widehat{Y}_{H,t} \\ -\frac{1}{2}\gamma\frac{\sigma}{\overline{\mu}_N k_N}\pi_{N,t}^2 - \frac{1}{2}(1-\gamma)\frac{\sigma}{\overline{\mu}_H k_H}\pi_{H,t}^2. + t.i.p + (\left\|\xi^3\right\|) \end{array}\right].$$

We eliminate the linear terms in the objective function by using a second-order approximation of the equilibrium structural equations (13–19). As a result, we obtain an objective function that is purely quadratic. The expression takes the following form:

$$L_{to} = U_C\overline{C}E_{to}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\times \tag{29}$$

$$\left[\begin{array}{c} \frac{1}{2}W_{Y_N}(\widehat{Y}_{N,t} - \widehat{Y}_{N,t}^T)^2 + \frac{1}{2}W_{Y_H}(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^T)^2 + \frac{1}{2}W_{ER}(\widehat{ER}_t - \widehat{ER}_t^T)^2 \\ +\frac{1}{2}W_{P_{NT}}(\widehat{P}_{NT,t} - (\widehat{P}_{NT,t}^T)^2 + W_{Y_NY_H}(\widehat{Y}_{N,t} - \widehat{Y}_{N,t}^T)(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^T) \\ +W_{ER,P_{NT}}(\widehat{ER}_t - \widehat{ER}_t^T)(\widehat{P}_{NT,t} - \widehat{P}_{NT,t}^T) + \frac{1}{2}W_{\pi_N}(\pi_{N,t})^2 + \frac{1}{2}W_{\pi_H}(\pi_{H,t})^2 \end{array}\right] + t.i.p,$$

where $\widehat{Y}_{N,t}^T$, $\widehat{Y}_{H,t}^T$, $\widehat{ER}_t^T$, and $\widehat{P}_{NT,t}^T$ are welfare-relevant target variables, which are functions of stochastic shocks and, in general, may not be identical to the flexible price allocations.

Equation (29) implies that the social welfare of the two-sector, small open economy is affected by deviations in the sectoral inflation rates, output gaps, and relative prices from their target values.

In fact, the objective function reflects the impact of various economic distortions on social welfare and illustrates their relative contributions to the loss. First of all, price rigidities and monopolistic distortions in both sectors, which may not be fully offset by production subsidies, result in economic inefficiencies and introduce a role for inflation and output gap stabilization. The cross-output variable $(\widehat{Y}_{N,t} - \widehat{Y}_{N,t}^T)(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^T)$ describes the impact of co-movement in the sectoral output gaps on social welfare. When the weight in the objective function associated with the interaction term is positive, the sectoral asymmetries might be welfare improving. When this weight is negative, a co-movement of the sectoral outputs reduces welfare losses. In general, the weights next to each of the quadratic terms are represented by complicated functions of the structural parameters of the model (details are presented in the Appendix).

Furthermore, when price rigidities are present in both sectors and domestic shocks are imperfectly correlated, price changes are not synchronized following a shock. This results in inefficient output dispersion *between* sectors and introduces a role for relative prices into the monetary policy design problem. In this case, not only do the levels of inflation in both sectors matter for welfare, but so does the deviation of the relative price from its target level. The open economy formulation brings an additional, cross-country, dimension into the problem described above. Specifically, nominal rigidities may prevent prices in both countries from adjusting efficiently after exchange rate movements. In other words, the so-called relative price channel can fail to function accurately; this may result in welfare gains from exchange rate stabilization. On the other hand, in an open economy the policymaker can manipulate the terms of trade in order to increase expected consumption and decrease the expected disutility of production, i.e., to improve welfare. Those incentives are called the terms of trade externality and were analyzed by Benigno and Benigno (2006). Therefore, the weight next to the exchange rate term in the loss function balances the stability objective determined by the economic distortions (nominal rigidities) with the incentive of creating additional volatility in excess of the fundamental shocks. The cross factor $(\widehat{ER}_t - \widehat{ER}_t^T)(\widehat{P}_{NT,t} - \widehat{P}_{NT,t}^T)$ represents another "international dimension" term, which appears due to the fact that the relative price of non-traded to traded goods partially drives the evolution of the real exchange rate. This term, therefore, describes the additional welfare effects that originate from the correlation between the two relative prices.

Equation (29) indicates that the loss function derived for our model specification is not identical to the one of the closed economy or to the loss function obtained under the assumption $\rho = \theta = \omega = 1$. The general welfare representation, however, embodies these two special cases, which coincide in terms of policy objectives and imply that

$W_{Y_N Y_H} = 0$ and $W_{ER} = W_{P_{NT}} = W_{ER,P_{NT}} = 0$.

The presence of open economy terms is not the only implication of the exposure to external factors that can be observed in the objective function. The relative weights on the sectoral inflation rates and output gaps are not only affected by the structural asymmetries, like in the case of the closed economy, but also display the incentives that arise under openness to trade of one of the domestic sectors. Specifically, in an open economy, the weights in the objective function imply relatively higher stabilization of the non-traded sector compared to the traded sector variables. Figures 1.1 and 1.2 present the weights on inflation rates and output gaps as functions of the non-traded sector size derived for the closed and open economies, respectively. The weights are computed under the baseline parameterization, for illustration purpose the nominal price rigidities are assumed to be equal across sectors and are set to the value 0.66.

Figure 1.1: Sector-Specific Weights for the Closed Economy Model

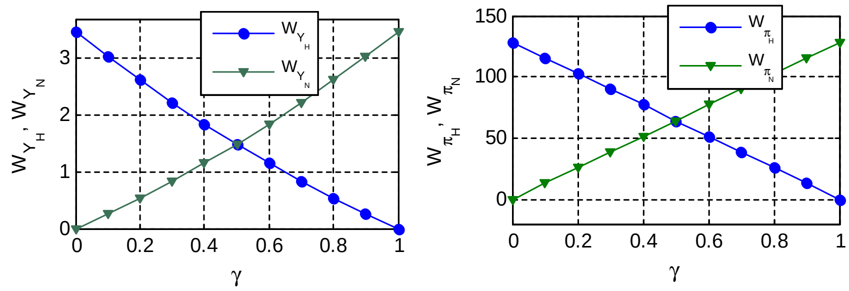
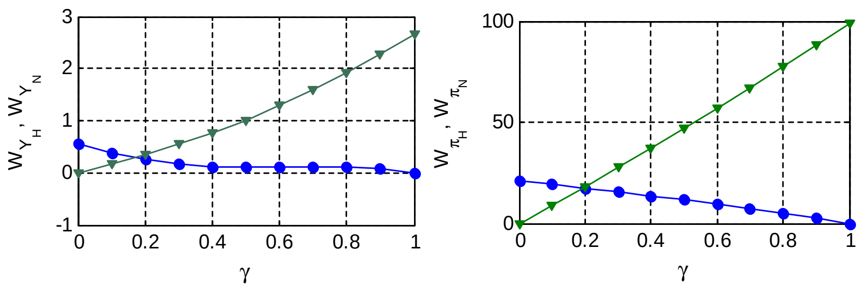


Figure 1.2: Sector-Specific Weights for the Open Economy Model



Two important results can be highlighted when analyzing Figures 1.1 and 1.2. First, these graphs indicate that both sectors are more volatile under the optimal policy when the economy is open (the weights are lower for all values of γ). Secondly, the decomposition of weights between sectors changes depending on whether the economy is subject to external factors. In particular, Figure 1.1 indicates that the weights derived for the closed economy model are symmetric and determined mainly by the parameter γ (under the equal stickiness of prices). The equal size of both sectors ($\gamma = 0.5$) implies their equal contribution to the loss function. In contrast, Figure 1.2 demonstrates that in the open economy, the stabilization "bias" is shifted toward the non-traded sector. In other words, the sector that is open to trade is allowed to adjust more at the optimum compared to the sector that produces goods only for internal consumption. Such a result can be explained by incentives that arise in the open economy. In particular, domestic households can benefit from volatility in the traded sector by varying consumption of imported goods and domestic output in response to shocks. The possibility to substitute for foreign goods in the consumption basket provides the mechanism to hedge against unfavorable economic conditions. Households are able to "divert" a part of production abroad and thus to lower the costs of the home-goods inflation and reduce the economic inefficiencies. Moreover, in the open economy, there exists the motivation to explore the terms of trade externality, i.e. to manage the relative prices (both internal and external) in a welfare improving manner. This effect is increasing in the elasticity of substitution between home and foreign traded goods θ. The similar mechanism applies to the non-traded sector. Higher elasticity of substitution between two types of domestic goods w (in the baseline calibration is assumed to be small and equal to 0.5) would involve relatively lower weights on non-tradable inflation and output in the loss function and thus more volatile dynamics of these variables under the optimal policy. Note, that in the loss function derived under the assumption of unitary elasticity of substitutions and logarithmic preferences ($\rho = \theta = \omega = 1$), the weights on inflation and output are independent on θ and w, and welfare effects describe above vanish.

1.4.2 The Relevance of the Welfare-Based Objective Function to the Current Practice of Central Banks

The loss functions widely assumed in the literature on monetary policy are typically represented by a quadratic expression that includes a weighted combination of inflation (CPI or domestic) and total output gap terms. Analyzing the micro-founded welfare objective function (29) we can see that it differs from the ad hoc forms in two important respects. First of all, it includes an open economy term and, therefore, prescribes a certain degree of exchange rate management. Secondly, it reflects the multisectoral economic structure and differentiates between sector-specific inflation rates and output gaps. Thus, the loss function derived on the basis of the economic fundamentals appears to be significantly more complex than the ad hoc policy objectives.

It is important to clarify why the objective function (29) does not explicitly display an important practical feature of current monetary policy conduct. Specifically, the majority of the central banks which have adopted inflation targeting as an operational framework have specified their monetary policy objective in terms of CPI inflation. Equation (29) indicates that the welfare loss of a small open economy depends on the *appropriate* measure of domestic inflation rates and is not explicitly affected by import price inflation. On the other hand, equation (25) illustrates the role of relative prices in movements of the foreign inflation rate. Thus, the welfare-based objective function indirectly includes all components of CPI inflation (except the lagged relative prices) but with the optimal weights.

At the same time it is possible to describe the conditions under which the explicit $\pi_{F,t}$ term can appear in the loss function. In the most general case, the loss function captures the distortions present in the domestic economy *as well as* the interrelations with the rest of the world. In particular, when countries are big enough, economic developments in the neighboring economy can affect domestic welfare and vice versa. The set of structural constraints for each country includes, in this case, both the domestic and foreign equations. Since the quadratic welfare objective function is derived from the approximation of the welfare function *and* the structural equations, the interaction between economies can bring foreign variables into the loss function of the domestic economy with country-specific weights. Such a framework is presented in Benigno and Benigno (2006), where they consider a two-country model with countries of comparable size. This paper demonstrates that despite the non-zero weight on foreign inflation in the loss function, the optimal targeting rules suggest a certain role for CPI inflation only in the case of cooperation between countries. Such a result can be explained by the fact that under the Nash regime (the non-cooperative case) the objective function is minimized only with respect to domestic variables, and the strategy of the other policymaker and the sequence of the foreign inflation rate are taken *as given*. In other words, the monetary authority does not care about the impact of domestic policy on the other country. In the cooperative case, the effects of the actions in both countries are internalized and the social planner optimizes with respect to all endogenous variables (domestic and foreign). As a result, the optimal targeting rule contains the proper measure of world inflation, which brings a role for CPI targeting.

Coming back to the model presented in this paper, the small open economy framework and, more specifically, the limiting case ($n \to 0$) imply that the domestic economy cannot influence the foreign country because of its small size, and the rest of the world can be treated as a closed economy. In this sense, countries are not directly interrelated in terms of consumption and production. The set of structural constraints for country H contains only the domestic equations and the foreign variables are treated as *exogenous* from the small open economy perspective. The implications of the foreign variables as well as other structural shocks can be observed in the targets, the deviations from which the central bank is trying to minimize. All other effects of the foreign dynamics are out of the control of the domestic policymaker and can be interpreted as unavoidable losses or as terms that are independent of policy.

1.4.3 The Optimal Monetary Policy Rules

In order to obtain the optimal targeting policy rules, we minimize the objective function (29) subject to the set of constraints, which are given by:

$$\pi_{H,t} = k_H \left[\eta(\widehat{Y}_{H,t} - \widehat{Y}^T_{H,t}) + \frac{1}{\upsilon}(\widehat{ER}_t - \widehat{ER}^T_t) + \frac{\gamma}{\upsilon}(\widehat{P}_{NT,t} - \widehat{P}^T_{NT,t}) + u^H_t \right] + \beta E_t \pi_{H,t+1}, \tag{30}$$

$$\pi_{N,t} = k_N \left[\eta(\widehat{Y}_{N,t} - \widehat{Y}^T_{N,t}) + (\widehat{ER}_t - \widehat{ER}^T_t) - (1-\gamma)(\widehat{P}_{NT,t} - \widehat{P}^T_{NT,t}) + u^N_t \right] + \beta E_t \pi_{N,t+1}, \tag{31}$$

$$(\widehat{Y}_{H,t} - \widehat{Y}^T_{H,t}) = \frac{l+1}{\rho\upsilon}(\widehat{ER}_t - \widehat{ER}^T_t) + \gamma \left[\frac{(l+1) + \upsilon^2(\rho\omega - 1)}{\rho\upsilon} \right] (\widehat{P}_{NT,t} - \widehat{P}^T_{NT,t}) + \chi^H_t, \tag{32}$$

$$(\widehat{Y}_{N,t} - \widehat{Y}^T_{N,t}) = \frac{1}{\rho}(\widehat{ER}_t - \widehat{ER}^T_t) - \omega(1-\gamma)(\widehat{P}_{NT,t} - \widehat{P}^T_{NT,t}) + \chi^N_t, \tag{33}$$

$$(1-\upsilon)\Delta(\widehat{ER}_t - \widehat{ER}^T_t) = \upsilon(\pi_{N,t} - \pi_{H,t}) - (\upsilon + \gamma(1-\upsilon))\Delta(\widehat{P}_{NT,t} - \widehat{P}^T_{NT,t}) + \varepsilon_t, \tag{34}$$

where $l = (\rho\theta - 1)(1-\upsilon)(1+\upsilon)$, and the terms $u^H_t, u^N_t, \chi^H_t, \chi^N_t$, ε_t are functions of exogenous shocks and arise when the target levels of variables and flexible price allocations diverge. The conditions (30)–(34) are obtained by combining the log-linearized equilibrium conditions (13)–(19) and expressing the relations in terms of gap variables. We assume that the central bank can commit to the policy that maximizes welfare and consider the timeless perspective approach described in Woodford (2003). The timeless perspective optimal policy assigns the particular value to the commitment to expectations prior to period 0. The constraints on the initial conditions result in the time-invariant first-order conditions and thus optimal policy. Therefore, the time inconsistency problem is eliminated. Following such a strategy, the policymaker chooses the path for endogenous variables $\pi_{H,t}$, $\pi_{N,t}$, $\widehat{Y}_{H,t}$, $\widehat{Y}_{N,t}$, $\widehat{ER}_t$, $\widehat{P}_{NT,t}$ subject to constraints (30)–(34) and given the initial conditions on π_{Ho}, π_{No}, $\widehat{Y}_{Ho}$, $\widehat{Y}_{No.}$ The Lagrange multipliers associated with the set of constraints are $\lambda_{1,t} - \lambda_{5,t}$ respectively. In addition before the optimization, we divided equation (30) by k_H, equation (31) by k_N, and equation (34) by υ. The first-order conditions to the problem are given by:

$$W_{\pi_H} k_H \pi_{H,t} = \lambda_{1,t} - \lambda_{1,t-1} + \lambda_{5,t} k_H, \tag{35}$$

$$W_{\pi_N} k_N \pi_{N,t} = \lambda_{2,t} - \lambda_{2,t-1} - \lambda_{5,t} k_N, \tag{36}$$

$$W_{Y_H}(\widehat{Y}_{H,t} - \widehat{Y}^T_{H,t}) + W_{Y_N Y_H}(\widehat{Y}_{N,t} - \widehat{Y}^T_{N,t}) = \lambda_{3,t} - \eta\lambda_{1,t}, \tag{37}$$

$$W_{Y_N}(\widehat{Y}_{N,t} - \widehat{Y}^T_{N,t}) + W_{Y_N Y_H}(\widehat{Y}_{H,t} - \widehat{Y}^T_{H,t}) = \lambda_{4,t} - \eta\lambda_{2,t}, \tag{38}$$

$$W_{ER}(\widehat{ER}_t - \widehat{ER}^T_t) + W_{ER,P_{NT}}(\widehat{P}_{NT,t} - \widehat{P}^T_{NT,t}) = \tag{39}$$
$$-\frac{1}{\upsilon}\lambda_{1,t} - \lambda_{2,t} - \frac{(l+1)}{\rho\upsilon}\lambda_{3,t} - \frac{1}{\rho}\lambda_{4,t} + \frac{1-\upsilon}{\upsilon}(\lambda_{5,t} - \beta\lambda_{5,t+1})$$

$$W_{P_{NT}}(\widehat{P}_{NT,t} - \widehat{P}^T_{NT,t}) + W_{ER,P_{NT}}(\widehat{ER}_t - \widehat{ER}^T_t) = -\frac{\gamma}{\upsilon}\lambda_{1,t} + (1-\gamma)\lambda_{2,t} - \tag{40}$$
$$-\frac{\gamma(l+1+\upsilon^2(\rho\omega-1))}{\rho\upsilon}\lambda_{3,t} + \omega(1-\gamma)\lambda_{4,t} + \left(1 + \frac{(1-\upsilon)\gamma}{\upsilon}\right)(\lambda_{5,t} - \beta\lambda_{5,t+1}).$$

Combining equations (35)–(40), we can eliminate the Lagrange multipliers and express the optimal policy rule in the following general form:

$$A^0 \Delta\widetilde{X}_t + A^1 \Delta\widetilde{X}_{t-1} + A^2 \Delta\widetilde{X}_{t+1} = 0, \tag{41}$$

where A^0, A^1, A^2 are the matrices of parameters, $\Delta\widetilde{X}_t = \widetilde{X}_t - \widetilde{X}_{t-1}$, and $\widetilde{X}_t = \widehat{X}_t - \widehat{X}^T_t$, i.e., $\widetilde{X}_t$ denotes the vector of the endogenous variables $(\pi_{H,t}, \pi_{N,t}, \widehat{Y}_{H,t}, \widehat{Y}_{N,t}, \widehat{ER}_t, \widehat{P}_{NT,t})$ in deviations from their target values. Therefore, the optimal policy rule is represented by a fairly complicated expression that prescribes the response to deviations in the sectoral inflation rates and output gaps as well as to fluctuations in relative prices. The reaction function (41) includes both backward and forward-looking endogenous variables. The matrices of the parameters A, which describe the optimal magnitude of the response, depend on the optimal weights and the structural parameters of the model.

For comparison, the optimal policy rule derived with the use of the similar methodology for the one-sector, open economy model takes the general form: $A^0\Delta\widetilde{X}_t = 0$. Therefore, the multi-sectoral model specification brings in more complex dynamics of variables under the optimal policy. Specifically, rule (41) is more persistent, i.e., it prescribes the response to the first *and* the second lag of the endogenous variables. Moreover, the rule contains forward-looking components since $A^2 \neq 0$. The characteristics of the policy rule mentioned above are determined by the persistent structure of one of the model equations (34), which describes the evolution of the sector-specific inflation rates and the two types of relative prices.

1.4.4 Policy Trade-Offs

The welfare function (29) indicates that the monetary authority is confronted with several policy objectives. In particular, the central bank has to control the sector-specific inflation rates and output gaps, as well as relative prices. In order to study the optimal plan, it is important to investigate whether the policy goals can be simultaneously attained or the central bank has to decide how to balance them appropriately. Where the objectives do not conflict with each other, the central bank can achieve the first best allocation and completely eliminate the loss. In this section, we describe the policy trade-offs that arise in a generalized model of a two-sector, small open economy.

We analyze the combination of equations (18) and (19) expressed in terms of the welfare-relevant gap variables:

$$(1-\upsilon)\Delta(\widehat{ER}_t - \widehat{ER}_t^T) = \upsilon(\pi_{N,t} - \pi_{H,t}) - (\upsilon + \gamma(1-\upsilon))\Delta(\widehat{P}_{NT,t} - \widehat{P}_{NT,t}^T) + \varepsilon_t. \quad (42)$$

The gaps depend on the target levels of the variables, which in turn are functions of the shocks and parameters and vary over time. Equation (42) indicates that it is not possible to stabilize inflation rates in each sector and to eliminate the gaps between relative prices and their target values at the same time. In fact, relative prices act as endogenous shocks that do not allow the same policy to attain zero inflation in both sectors. For example, under a productivity shock in the non-traded goods sector (Figure 4), the optimal policy implies depreciation of the nominal exchange rate. Complete stability of non-traded inflation would require an even larger increase in the exchange rate. This, however, would result in a further worsening of the terms of trade and a greater rise in home-goods inflation. A similar trade-off exists under fiscal and mark-up shocks. Moreover, the impulse-responses indicate that the magnitude of the response differs across sector-specific variables. The different sensitivity of the domestic sectors to shocks is determined not only by structural asymmetries such as sector size, elasticity of substitution, and the level of nominal rigidities, but also by the openness to trade of one of the domestic sectors. Therefore, the optimal policy cannot comply with all the sector-specific stabilization objectives simultaneously. Woodford (2003) illustrates that a corresponding trade-off also exists in the closed economy model (υ=1) if the target rate of the relative price (the natural rate) is not constant.

Furthermore, we address the question of whether complete stability of the *aggregate* variables is attainable under the given economic structure. We present the Phillips curve relations in terms of gap variables and use the definition of domestic inflation. Moreover, in this analysis we assume for simplicity that the target variables and flexible price allocations coincide and the degree of nominal rigidities is equal across sectors. We combine the constraints (30)–(33) and apply the definition of domestic inflation (25). As a result, the following relationship arises:

$$\pi_t^D = k \left[\begin{array}{c} (\eta+\rho)\left(\gamma(\widehat{Y}_{N,t} - \widehat{Y}_{N,t}^{flex}) + (1-\gamma)(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^{flex})\right) - \\ \frac{(1-\gamma)}{\upsilon} l(\widehat{ER}_t - \widehat{ER}_t^{flex}) - \frac{\gamma(1-\gamma)}{\upsilon}\widetilde{l}(\widehat{P}_{NT,t} - \widehat{P}_{NT,t}^{flex}) \end{array} \right] + \beta E_t \pi_{t+1}^D, \qquad (43)$$

where $l = (\rho\theta - 1)(1-\upsilon)(1+\upsilon)$, $\widetilde{l} = l - (\rho\omega - 1)(1-\upsilon)\upsilon$, and the flexible price allocations of the variables are functions of the exogenous shocks $\widehat{A}_{H,t}$, $\widehat{A}_{N,t}$, $\widehat{P}_{NT,t}^{*}$, C_t^{*}. Moreover, we make use of equation (26a) and provide the alternative domestic Phillips curve relation in order to analyze the impact of the aggregate output gap instead of the differentiation between the sectoral variables:

$$\pi_t^D = k\left[\begin{array}{l} (\eta+\rho)\left((\widehat{Y}_t - \widehat{Y}_t^{flex}) + (1-\gamma)(1-\upsilon)(\widehat{P}_{FH,t} - \widehat{P}_{FH,t}^{flex})\right) - \\ \frac{(1-\gamma)}{\upsilon} l(\widehat{ER}_t - \widehat{ER}_t^{flex}) - \frac{\gamma(1-\gamma)}{\upsilon}\widetilde{l}(\widehat{P}_{NT,t} - \widehat{P}_{NT,t}^{flex}) \end{array}\right] + \beta E_t \pi_{t+1}^D. \tag{44}$$

We present two special cases of our more general analysis in order to describe the role of relative prices in generating the policy trade-offs. First, we consider a two-sector, closed-economy setting, i.e., $\upsilon = 1, \gamma > 0$. In such a situation $l = \widetilde{l} = 0$. Equations (43) and (44) illustrate that the sectoral Phillips curves reduce to the classical aggregate relation, which, at the same time, describes the dynamics for the one-sector, closed economy. Therefore, there is no conflict between inflation and output gap stabilization, and optimal monetary policy is able to implement the first best, i.e., flexible price allocation. This result has been shown by Woodford (2003).

Secondly, we assume the special case of unitary elasticity of substitution and a unitary coefficient of relative risk aversion, i.e., the balanced trade model specification as in Liu and Pappa (2005). Again, we have $l = \widetilde{l} = 0$. Thus, the exchange rate and relative prices vanish from the Phillips curve relations (43) and (44). Moreover, the assumption $\rho = \theta = \omega = 1$ implies that the exchange rate does not characterize a welfare-relevant policy objective. In this situation, the terms of trade act as an endogenous "cost-push shock," which generates tension between domestic inflation and the output gap. In fact, such a trade-off can be generated in closed economy models in the presence of mark-up shocks or adjustment costs (Benigno and Woodford, 2005; Erceg and Levin, 2006).

Finally, we consider a two-sector model under general preferences. The Phillips curve (43) illustrates that the stabilization of domestic inflation and outputs in both sectors does not involve equivalent policies due to the presence of relative prices. Moreover, equation (44) indicates that there is tension between domestic inflation and relative price (internal and external,i.e. the exchange rate) stability in addition to the trade-off between domestic inflation and the aggregate output gap variability. Therefore, unless preferences are specified in the general form, the conflict between managing domestic inflation and the relative prices ceases to exist.

The fairly complex economic structure and general model specification determine the non-trivial task facing policymakers, i.e., the search for the second-best optimal policy given that the flexible price efficient allocation of resources cannot be replicated. The optimal reaction function (41), in fact, represents such a second-best solution. A similar result is obtained in the one-sector, open-economy model analyzed by De Paoli (2006). In our case, however, the definition of the real exchange rate implies a distinction between the two types of relative prices and enables us to characterize the dynamics and impact of each variable separately. Moreover, the multiple sectors imply an additional policy challenge, i.e., the proper management of the "between-sector" terms.

1.5 Impulse-Response Functions

In this section we examine the impulse-responses of key macroeconomic variables to exogenous shocks. Specifically, we compare the numerical results under the optimal plan with the outcomes achieved under the basic simple rules common in the literature, such as domestic inflation targeting (DIT), consumer price index inflation targeting (CPIT), and an exchange rate peg (PEG). We consider four types of shocks, i.e., productivity, foreign, fiscal, and mark-up shocks. For the numerical exercise we calibrate the model parameters to match the moments of Canadian data (Table 2). We assume the coefficient of relative risk aversion $\rho = 3$ and the elasticity of substitution between differentiated goods $\sigma = 6$ as in Benigno and Benigno (2006). Following Rotenberg and Woodford (1997) we set $\beta = 0.99$ and $\eta = 0.47$. The elasticity of substitution between traded home and foreign goods θ is assumed to be equal to 1.5 and the parameter that measures the substitution between non-traded and traded goods ω is set to 0.5. These assumptions are common in the open economy literature. The level of price rigidities in tradable sector is set to $\alpha_H = 0.55$ and in non-tradable sector the Calvo parameter is assumed to be somewhat higher and equal to $\alpha_N = 0.6$. The share of non-traded goods in the consumption basket γ is set to 0.5. The corresponding parameter for the foreign country $\gamma^* = 0.6$. The degree of openness $\upsilon = 0.6$, implying a 40% import share. Finally, the steady state mark-up in the traded sector $\overline{\mu}_H$ is set to the value $1/\upsilon$ as in Liu and Pappa (2005) and De Paoli (2006) in order to guarantee the optimal subsidy policy. In addition, the equal size of both domestic sectors implies that $\overline{\mu}_H = \overline{\mu}_N$. The calibration of the parameters of stochastic processes and the policy rule are based on Dib (2008) and Ortega and Rebei (2006), who performed the Bayesian estimation of multi-sectoral DSGE models of Canadian economy. The calibrated parameters are summarized in the Table 1.

Figure 1.3 represents the impulse-responses to a productivity shock in the traded sector, $\widehat{A}_H$. All regimes (except PEG) imply a reaction of the monetary authority that induces a depreciation of the nominal exchange rate. Such dynamics, together with a fall in the price of home goods, worsen the terms of trade and thus result in a real depreciation. The increase in the exchange rate is the largest under DIT, because in this case the central bank stabilizes inflation more aggressively. In fact, higher home-goods inflation stability is traded for some additional exchange rate volatility. CPI inflation rises under DIT and the optimal plan. Under PEG, the nominal exchange rate is stable and the effect of the productivity shock on CPI inflation is determined by the fall in inflation in the home-goods sector. Domestic output increases due to the real exchange rate depreciation. Domestic goods become relatively cheaper than foreign goods. However, the increase in output is not large enough to boost production above its target level and the total impact on the output gap is negative. The expenditure switching effect is the most pronounced under the DIT regime, which implies no control over the exchange rate and thus allows for greater real depreciation. As a result, the output response is the largest. On the contrary, under PEG and CPIT the expenditure switching effect is minimized and the output gap falls by more compared to the other regimes. The negative response of home-goods inflation under all the regimes is determined by the direct impact of the productivity shock, which lowers the marginal costs in this sector. However, the marginal costs in the non-tradable sector increase. Non-traded output increases and the relative price of non-traded to traded goods $\widehat{P}_{NT}$ falls under DIT and the optimal plan, due to nominal depreciation. As a result, non-traded inflation rises.

Figure 1.4 presents the impulse-response to a productivity shock in the non-traded sector, $\widehat{A}_N$. The dynamics of the variables can be described in a similar fashion. It is important to note that non-traded inflation is stabilized to a greater extent under the optimal plan compared to the alternative simple rules. The reason for such a policy reaction is that the optimal welfare function assigns the greatest weight to stabilization of non-traded inflation. At the same time, the productivity shock $\widehat{A}_N$ directly affects the price change in this sector and, hence, induces greater dynamics of this variable. In order to prevent large swings in non-traded inflation, the central bank allows greater adjustments in relative prices and output. In addition, the response of relative prices ($\widehat{P}_{NT}$ and $\widehat{ER}$) is almost two times stronger than the responses of these variables following the productivity shock $\widehat{A}_H$. Again, the reason is that instability of non-traded inflation has larger negative welfare consequences than changes in home-goods inflation. The output reaction is positive in both sectors due to the large expenditure switching effect under DIT and the optimal plan. Unlike the negative response of the output gap following the productivity shock in the home-goods sector, the $\widehat{A}_N$ shock results in an increase of output above its target level due to the more expansionary policy.

Figure 1.5 presents the responses of domestic variables to the innovation in foreign consumption, $\widehat{C}^*$. We can observe that the DIT regime is very similar to the optimal plan in terms of the direction and magnitude of the response. The foreign consumption shock risess domestic consumption through the risk-sharing condition. This, in turn, may induce an increase in domestic output. At the same time, the nominal and real exchange rates appreciate and the terms of trade fall. Domestic goods become relatively less competitive and demand shifts to foreign goods. The net effect on home output is negative under DIT and the optimal plan. The impact of the shock on the macro-variables is qualitatively different under the CPIT and PEG regimes. Specifically, the monetary authority stabilizes relative prices and the real appreciation is small. The expenditure switching effect is dominated by the positive impact of the shock on domestic consumption and demand. As a result, the outputs in both sectors as well as the output gap show a significant increase. Such a boost in production increases marginal costs, and inflation in both sectors rises.

Figure 1.6 presents the impulse-responses to a shock to foreign relative prices, $\widehat{P}^*_{NT}$. The DIT regime almost perfectly replicates the optimal response. The policy reaction following the $\widehat{P}^*_{NT}$ shock displays a sharp contrast between the responses under the CPIT and PEG regimes, whereas under the other types of shocks these two regimes induce very similar changes in economic activity. Specifically, under the CPIT regime the central bank prevents large movements in the terms of trade at the expense of additional domestic inflation volatility. The policy implies a large nominal depreciation so as to mitigate the negative impact of foreign prices on the terms of trade. The nominal depreciation under the stabilized CPI inflation results in real depreciation. This, in turn, increases domestic production and inflation in both sectors. On the contrary, the PEG regime induces a policy that is closer to the optimal plan and DIT. When foreign and home goods are substitutes, the optimal response implies a greater nominal exchange rate stabilization in order to improve the terms of trade and divert production abroad by switching to consumption of foreign goods. Such a policy is welfare improving because it enables one to take advantage of the foreign productivity shock by reducing domestic marginal costs and the inefficient output dispersion associated with price rigidities.

Figure 1.7 shows the impulse responses to a mark-up shock in the home-goods sector, $\widehat{\mu}_H$. The optimal policy diverges from complete domestic inflation stabilization and the other alternative simple rules. The positive shock leads to a rise in home-goods inflation, which returns to its initial level after several periods of deflation, and a temporary fall in the output gap. The extent to which the shock affects output versus inflation depends on the weight that the central bank places on output gap variability. Specifically, the optimal policy, unlike the alternative simple rules, implies a certain degree of output gap stability. Therefore, inflation is allowed to increase more and the output gap to fall less under the optimal plan. The response of the monetary authority to a mark-up shock implies fall in the nominal interest rate, depreciation of the exchange rate, an increase in the terms of trade, and a fall in the relative price of non-traded to traded goods. Outputs in both domestic sectors and consumption rise in response to a shock. The output gap, however, falls due to the fall in home-goods output below its target value.

The responses to a mark-up shock in the non-traded sector, $\widehat{\mu}_N$, are presented in figure 1.8. Again, the central bank has to balance conflicting policy objectives – to absorb the upward pressure on inflation in the non-traded sector by a fall in the output gap. The exchange rate appreciates and consumption and output decrease under the optimal plan. The DIT regime implies a greater economic contraction and thus the largest fall in output and consumption. CPIT and PEG represent strongly suboptimal regimes because they induce excessive stabilization of relative prices and a higher response of non-traded inflation. The comparative analysis of impulse-responses under the $\widehat{\mu}_H$ and $\widehat{\mu}_N$ shocks suggests that the optimal policy reacts more aggressively under the disturbance to a non-traded mark-up.

Figures 1.9 and 1.10 illustrate the responses to fiscal shocks in the traded and non-traded sectors, respectively. Again, the optimal policy differs significantly from the simple policy rules. The rise in government spending $\widehat{g}_H$ increases home-goods output. The central bank, which aims at domestic inflation stabilization, offsets the upward pressure on home-goods inflation by a corresponding decrease in non-traded inflation. The response induces an initial appreciation of the exchange rate, a fall in the terms of trade, and a rise in the relative price of non-traded to traded goods. As a result, consumption and non-traded output decrease. The optimal plan, on the contrary, implies an expansionary policy. The exchange rate depreciates, implying an additional stimulus to output in both domestic sectors. Such a policy prevents the initial drop in consumption. The CPI and PEG regimes imply greater stability of relative prices.

The government spending shock $\widehat{g}_N$ increases non-traded output and creates upward pressure on non-traded inflation. Therefore, unlike in the previous case, the optimal policy implies the economic contraction. The response of the central bank is the most aggressive compared to the alternative policy rules. As a result, greater non-traded inflation stability is achieved at the expense of additional volatility of inflation and output in the traded sector, as well as a larger adjustment of relative prices.

The analysis of the numerical results suggests that the type of shock and the economic structure are important determinants of the comparative performance of optimal versus simple policy rules. Specifically, the responses under the optimal policy differ the most from the simple rules under fiscal and mark-up shocks. On the contrary, the DIT regime better approximates the optimal plan under foreign and productivity shocks. In addition, the optimal and PEG regimes come closer under a foreign relative price shock. Shocks of the same type but affecting different domestic sectors may induce qualitatively distinct economic responses. This happens due to the different sensitivity of welfare-relevant economic variables to sector-specific shocks and greater stabilization of the non-traded sector under the optimal policy. In particular, the optimal policy is expansionary with respect to fiscal and mark-up shocks in the traded sector, whereas identical shocks in the non-traded sector call for an economic contraction. The DIT regime induces a more expansionary policy under a traded-sector productivity shock, whereas the policy is less active following foreign shocks. Fiscal and mark-up disturbances result in an economic contraction under DIT. Under the CPI and PEG regimes, the policy is less aggressive in response to domestic productivity shocks and it becomes more expansionary under foreign shocks.

1.6 Welfare Implications of the Alternative Simple Rules

The study of the optimal policy problem presented in the previous sections provides a useful theoretical foundation for the design of monetary strategy and offers a rigorous benchmark for comparing the performance of alternative monetary regimes. At the same time, the prescriptions of the optimal policy given by expression (41) might be too difficult for the general public to interpret and too difficult to put into practice. Therefore, the analysis of the alternative policy rules, which deliver reasonable welfare results and at the same time are simple and transparent, and the optimal rule, which has normative implications, should interact in a complementary way in order to provide beneficial economic conclusions. In this section we enhance the analysis of the optimal policy with a discussion of the alternative simple rules and present their comparative performance. Specifically, we use Dynare software in order to compute *optimal* simple rules (OSRs) of the form: $\widehat{r}_t = \rho_r \widehat{r}_{t-1} + \psi \widehat{X}_t + \varepsilon_r$, where $\widehat{X}_t$ is a vector of endogenous variables, ψ is a vector of optimized parameters, and ε_r is a policy shock with standard deviation set to 0.003.We also set the value of the parameter ρ_r to 0.75. In fact, we compute the parameters of a policy rule which maximize a linear-quadratic loss function (29) subject to constraints (30-34). As a result, we are able to analyze the performance of rules with a simple structure but with optimal coefficients.

We address two important issues. First, we consider several types of alternative simple rules classified depending on the variables entering the rules and investigate the extent to which alternative monetary regimes are able to replicate the optimal solution. Secondly, we explore the implications of the alternative simple rules for macroeconomic volatility.

The welfare ranking is performed on the basis of the value of the loss, which is computed by taking the unconditional expectations of expression (29), i.e., the second-order approximation to the utility of the representative consumer, expressed as a fraction of the steady state consumption. As a result, we present the value of the loss in terms of the variances/covariances of the sector-specific inflation rates, output gaps, and relative prices:

$$V \equiv \frac{1}{2}\frac{\beta}{1-\beta}\times\left[\begin{array}{c} W_{Y_N} var(\widetilde{Y}_{N,t}) + W_{Y_H} var(\widetilde{Y}_{H,t}) + 2W_{Y_N Y_H} covar(\widetilde{Y}_{N,t}\widetilde{Y}_{H,t}) + \\ +W_{ER} var(\widetilde{ER}_t) + W_{P_{NT}} var(\widetilde{P}_{NT,t}) + 2W_{ER,P_{NT}} covar(\widetilde{ER}_t\widetilde{P}_{NT,t}) + \\ +W_{\pi_N} var(\pi_{N,t}) + W_{\pi_H} var(\pi_{H,t}) \end{array}\right]. \tag{45}$$

Table 3 reports the welfare losses associated with various types of OSRs. Specifically, we consider simple rules which include domestic variables and rules that prescribe the response to both closed and opened economy terms. In addition, we would like to evaluate the benefits of targeting sector-specific inflation rates and outputs versus aggregate variables. This issue is practically important since central banks do not usually differentiate their policy response depending on the economic sector and consider aggregate variables, due to the problem of policy implementation and a lack of information.

Table 3 indicates that the welfare losses under the OSRs that target domestic inflation are on average 15-30% larger compared to the optimal rule. The losses associated with strict CPI inflation targeting are somewhat larger than rules that completely stabilize the domestic inflation. At the same time, certain forms of flexible CPI targeting may outperform flexible DIT rules (compare rules 4,5 and 10,11). Targeting the non-tradable inflation provides better welfare results than DIT or CPI targeting (rules 13 and 4,10). In general, the DIT regime performs worse compared to the results obtained in the previous literature. In particular, in the special case of the open economy model presented in Gali and Monacelli (2005) and the framework with ad-hoc welfare objectives as in Soto (2004), the DIT regime represents or nearly replicates the first-best. In our case, the presence of mark-up and government spending shocks determines the deviation of optimal policy from the DIT. The ranking of alternative regimes suggests that strict inflation targeting (DIT or CPI) is suboptimal compared to policies that account for other objectives, namely the interest rate smoothing, the output gap, and/or the relative prices. The rules that target the sector-specific variations in outputs and inflation rates perform significantly better compared to rules targeting the aggregate variable. Thus stabilization of the appropriately weighted average of the sectoral inflation rates (rule 12) produces better results than DIT or CPI. At the same time, augmenting the rule that responds to the aggregate inflation (domestic or CPI) and total output gap with the relative price term allows one to better account for sector-specific features of the economy. For example, rule 11 indicates that flexible CPI targeting regime that includes a certain degree of the internal relative price management can achieve a welfare result that is close to the case of targeting the sector-specific inflation rates. Furthermore, across all types of rules (4 and 5, 10 and 11, 13 and 14), the internal relative prices do better job in capturing sector-specific characteristics than external relative prices (the exchange rate). Thus the inclusion of the relative price of non-traded goods in the policy rule brings higher welfare gains. The improvement in welfare coming from the response to the change in the exchange rate is higher for the CPI targeting rules because of the excess smoothness of relative prices which this regime entails.

The values of the optimized coefficients k1, k2, k3, and k4 displayed in table 3 provide information about the relative magnitude of the policy response to deviations in key macroeconomic variables. Specifically, the OSRs indicate that the policy should respond more aggressively to variations in the non-traded sector variables (output and inflation rates).

The important criterion for evaluating the performance of the simple rules is the level of macroeconomic stability which they induce. Alternative regimes may generate comparable welfare results but, at the same time, imply different volatility of the macroeconomic variables. This issue becomes particularly important prior to entering the Eurozone, when the monetary authority has to fulfill specific and sometimes conflicting stabilization objectives. Table 4 presents the standard deviations of the key variables under different OSRs relative to the standard deviations implied by the optimal policy.

Comparing the volatility under the alternative regimes we note that the rules that strictly target aggregate variables naturally perform the worst in terms of stabilization of the particular economic sectors. Thus, under the DIT, CPI, and PEG regimes, the volatility of the sector-specific variables diverges the most from the deviations implied by the optimal rule. In particular, sectoral inflation rates are 50% over (for home inflation) and about 2 times under-stabilized (for non-traded inflation) under the strict DIT regime. At the same time, the output gaps in the home-goods and non-traded sectors are 10% and 17% respectively more volatile compared to their standard deviations under the optimal policy. In all cases of strict inflation targeting (rules 1,2,6,7,12), the fulfillment of the inflation objectives comes at the expense of somewhat higher volatility of the output gap, at sector-specific and/or aggregate levels. The comparison of DIT, CPI and the rule that targets the properly weighted domestic inflation index (rule 12) indicates that under the latter, the volatility of sector-specific inflation rates is closer to the optimal values and thus non-traded inflation is less volatile. At the same time relative prices and especially traded inflation display higher volatility. Greater stability of non-traded inflation is achieved due to the different magnitude of the optimal policy response with respect to the sectoral inflation rates expressed by the values of the parameters k1 and k2. The rules that do not differentiate the response across sectoral variables but instead incorporate the reaction to changes in the relative prices (rules 5,10,11,13,14) allow the standard deviations of the sector-specific inflation rates to be brought closer to the optimal values. Such an improvement can be achieved at the expense of increased domestic and/or CPI inflation volatility as well as the standard deviations of some of the relative prices. Moreover, regimes, which display the features of an open economy, i.e., prescribes a certain degree of exchange rate management (rules 4,16,17) bring higher stability of the CPI inflation, but may imply somewhat higher variation in output and domestic and non-tradable inflation.

The results of this section demonstrate the tension between the sector-specific inflation objectives, inflation and relative price stabilization as well as the inflation-output gap policy trade-off common in the literature. We also numerically assess the welfare benefits of differentiating the policy response depending on economic sectors compared to stabilizing aggregate variables. Moreover, we show that the welfare results achieved under the "sector-specific" targeting rules can be closely replicated by a rules with an appropriate combination of aggregate variables, namely, the CPI inflation, total output gap and the internal relative price change. Responding to the relative prices may facilitate targeting the sector-specific variables and contribute to welfare improvements when the central bank does not have enough information about domestic sectors.

The exercise presented in this section has important practical implications. In particular, it could provide policymakers with a tool for analyzing the relative importance (in terms of welfare consequences) of various monetary policy objectives and facilitate the design of strategies aimed at achieving several competing goals.

1.7 Sensitivity analysis

1.7.1 Price stickiness

In the previous sections, we analyzed the performance of various policy rules under the assumption that prices in the non-traded sector are more rigid compared to the level of nominal rigidities in the traded sector. At the same time, the estimated values of the parameters of price stickiness may vary across different studies even for the same country. In this section, we would like to provide a more general analysis of the impact of sectoral heterogeneity in the degree of price stickiness on the relative performance of policy rules with sector-specific and aggregate variables (inflation rates). In other words, we would like to check how sectoral asymmetries affect the optimal inflation index being stabilized. Moreover, we will compare the implications of asymmetric nominal rigidities for closed and open economies. Specifically, we compare our results with conclusions derived by Aoki (2001) who studied the optimal policy in a two-sector closed economy model where prices are fully flexible in one sector but sticky in the other. His main result implies that the central bank should target the core inflation rather than changes of a broader price index.

For the sensitivity analysis we evaluate 5 types of rules: optimal policy, DIT and CPI targeting with interest rate smoothing (rules 2 and 7), policy rule with sector-specific inflation rates (rule 12, which approximates the core inflation), and the rules which incorporate the response to the CPI or DIT inflations and the relative price change. We construct a measure of the sectoral asymmetries in relative price rigidities $0 \leq \chi \leq 1; \chi = \frac{\alpha_i}{\alpha_i + \alpha_j}$. It measures the level of price stickiness in a sector i relative to the overall level of nominal rigidities. In the case that $\chi = 0.5$ is chosen, $\alpha_i = \alpha_j$ i.e. sectoral prices are equally sticky. This measure allows us to vary the assumed relative stickiness of prices in the two sectors between the two extremes of complete flexibility in sector i (non-tradable, $\alpha_i = 0, \chi = 0$) and complete flexibility in sector j (tradable, $\alpha_j = 0, \chi = 1$). We compute welfare losses for values of χ from 0 to 1, for each point we consider all possible combinations of α_i and α_j and aggregate the results across all options.

The results are presented on the Figures 1.11 and 1.12. We plot the welfare losses under alternative policy regimes for various degrees of relative price rigidities. We compute optimal and DIT policy for the closed economy as a special case of an open economy, i.e. we assume that the share of imports is equal to zero (degree of openness) and open economy shocks are shut off. Figures 1.11 and 1.12 indicate that implications of equal degree of nominal rigidities across sectors ($\chi = 0.5$) differ for closed and open economies. In particular, in the closed economy model where the sectoral prices are equally sticky, the DIT policy is nearly optimal. At the same time, for values $\chi < 0.5$ or $\chi > 0.5$, targeting of aggregate price index is suboptimal. The central bank should weigh the sectoral inflation rates according to their price stickiness; the sector with higher rigidities should be more stabilized at the optimum. This result corresponds to the one shown by Aoki. The difference (in terms of welfare) between the DIT and the optimal policy is higher on the interval where nominal rigidities in sector j are greater than in sector i ($\chi < 0.5$). Such a result arises due to the assumption of asymmetric disturbances, which greater affect the sector j . In case of identical sector-specific shocks the welfare losses following the DIT policy would be symmetric on the intervals $\chi < 0.5$ and $\chi > 0.5$ because, in the closed economy, volatilities in domestic sectors equally contribute to the welfare loss function.

The results obtained for the open economy model indicate that equality of sectoral price rigidities does not imply the optimality of targeting the aggregate inflation indices (CPI or domestic). At the same time, the regimes that stabilize the measures of sector-specific and aggregate inflations produce similar welfare results on the interval $0.1 \leq \chi \leq 0.4$. In particular, for $\chi = 0.2 - 0.3$ there are almost no gains from targeting sector-specific versus domestic inflation rates. This implies that the optimal policy in the open economy may prescribe the equivalent response to changes in sectoral price indices even if sectoral nominal rigidities are asymmetric (prices in the non-tradable sector are more flexible). Such a result is obtained because, in general, the non-tradable inflation has to be more stabilized under the optimal policy relative to the tradable inflation. The gain from targeting sector-specific versus aggregate inflation rates is increasing on the interval $\chi > 0.4$, where prices in sector i (non-tradable) become stickier and the discrepancy between optimal weights assigned to sectoral inflations is increasing. Moreover, in this case flexible CPI targeting outperforms the corresponding regime of domestic inflation stabilization (see Figure 1.13, left panel). For all values of relative price stickiness, the policy rule that combines CPI with the relative price management is able to closely replicate the optimal solution and the "core" inflation (sector-specific) rule. In addition, Figure 1.11 indicates that nominal rigidities in non-tradable sector are more costly comparing to the case when prices in the tradable sector are stickier (the point where $\chi = 0$ implies lower welfare losses compared to the point $\chi = 1$). This result indicates that greater stability of non-tradable inflation comes at the expense of higher volatility of other welfare relevant variables. The stabilization of tradable inflation generates less severe volatility trade-offs.

1.7.2 Degree of openness and elasticity of substitution

In this paper we have demonstrated that openness to trade as well as the general specification of consumer preferences generate important welfare effects in a multi-sectoral small open economy model. Therefore, it is useful to understand how the results could change depending on the different values of these parameters. Specifically, we vary the parameter that determines the preferences of agents in country H for the consumption of goods produced at Home, i.e. the degree of openness υ, from 0.1 (very open economy) to 1 (completely closed economy). We perform the analogous exercise as in the previous subsection in order to evaluate welfare gains from targeting the "core" versus broader (DIT) inflation index. In addition, we compare welfare implications of flexible CPI and DIT regimes. For this analysis we consider $\rho = 1$, $\alpha_H = \alpha_N = 0.66$, and the rest of the parameters are fixed at their calibrated values. Figure 1.13 (right panel) displays the results. In particular, the gains from targeting the "core" inflation in the open economy (for all $\upsilon < 1$) are positive (relatively to both the CPI and DIT) even under the same sector size and the price stickiness, in contrast to the zero gains for the closed economy. The higher degree of openness implies the greater benefits from the stabilization of appropriately weighted inflation index compared to DIT regime. For relatively closed economies, stability of the domestic inflation delivers better welfare results than the CPI targeting, but the opposite is true for more open economies.

Figure 1.14 (left panel) displays welfare gains from targeting the alternative price indices as a function of the elasticity of substitution between home and foreign goods θ. The elasticity of substitution between traded and non-traded goods is kept at its calibrated value (0.5). The results suggest that the gain from targeting the core rather than the aggregate price index is increasing when domestic households receive a greater opportunity to substitute for foreign goods in the consumption basket. For higher values of θ , the optimal volatility of tradable inflation and output risess relatively to the volatility of non-tradable sector variables and it becomes more important to account for such an increased discrepancy between sectors. Comparing the performance of CPI versus DIT regimes we notice that for home and foreign goods that are complements the targeting of the domestic inflation strongly outperforms the CPI. As the elasticity of substitutes rises, the gains form switching to CPI targeting regime become more sizable.

Figure 1.14 (right panel) presents the similar welfare analysis depending on the preference parameter ω; θ is fixed at the calibrated value 1.5. The graph indicates that benefits from differentiating the response between sectoral inflation rates diminish with the increasing elasticity of substitution of non-tradables. This happens due to the fact that non-tradable inflation becomes less heavily weighted in the welfare objective function and the optimal volatilities in both sectors converge. Moreover, for ω higher than 0.9, the DIT regime is a better approximation of the optimal policy comparing to CPI. Note that for the special case of unitary elasticity of θ, the domestic inflation targeting regime would basically reproduce the core index and outperform the consumer price inflation stabilization for all considered values of ω. From this analysis we also notice that welfare implications of heterogeneity between the elasticity of substitution parameters (θ and ω) differ depending on which type of goods becomes more substitutes. In case when the sectoral elasticities diverge due to the increase in θ, the "core" inflation targeting does better job in replicating the optimal solution. As the structural asymmetries rise due to higher elasticity of substitution of non-tradables, the DIT regime delivers higher welfare.

1.8 Conclusions

In this paper we analyzed the stabilization objectives of optimal monetary policy in a two-sector small open economy model obtained as a limiting case of a two-country DSGE framework. We assessed the role of sectoral heterogeneity, general preferences, and multiple relative prices for monetary policy design and welfare evaluation. The stabilization objectives derived for our model specification and represented by the loss function display the features of an open economy and reflect a multisectoral economic structure. Specifically, it is shown that social welfare is affected by deviations in inflation rates and output gaps (with sector-specific weights) as well as in relative prices from their target values. Therefore, the micro-founded welfare objective function differs from the ad hoc forms widely assumed in the applied literature. The exposure of one of the domestic sectors to the external environment not only determines the presence of open economy terms in the loss function, but also affects the decomposition of weights between domestic variables. In particular, the sector that is open to trade is allowed to adjust more at the optimum compared to the sector that produces goods for internal consumption only. Such a result implies a qualitatively different policy response to deviations in sector-specific variables compared to the closed economy setting and determines the asymmetric response of the domestic sectors to various shocks. We characterized the optimal policy by the optimal targeting rule, which is a rather complex expression.

Furthermore, we experimented with alternative simple rules and analyzed their ability to replicate the optimal solution. The numerical results suggest that the type of shock is an important determinant of the comparative performance of optimal versus simple policy rules. Specifically, the optimal responses differ the most from the simple rules under fiscal and mark-up shocks. On the contrary, the DIT regime better approximates the optimal plan under foreign and productivity shocks.

An analysis of the welfare implications of alternative simple rules suggests that strict targeting of domestic and CPI inflation does not yield the best approximations for the optimal policy, and social welfare can be improved by accounting for other policy objectives, namely, the output gap and the relative prices. We presented a ranking of alternative simple rules and evaluated the welfare benefits of targeting the core versus broader inflation indices. In addition, we showed that the simple rules which incorporate a response to the relative price changes achieve more efficient stabilization of sector-specific variables. Finally, the sensitivity analysis demonstrates that implications of equal degree of price stickiness across sectors differ for closed and open economies. Unlike the policy implemented in the closed economy, the optimal strategy in the open economy may prescribe the equivalent response to changes in sector-specific price indices (and thus targeting the aggregate price index) even under the diverse values of sectoral nominal rigidities. Targeting the "core" rather than domestic inflation delivers higher welfare as prices in the non-traded sector become relatively stickier, the economy is more open, and the elasticity of substitution between home and foreign goods increases. The DIT regime becomes welfare beneficial as the possibility to substitute between non-tradable and tradable goods rises.

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Appendix

The Steady State

We approximate the model around the steady state, in which $\overline{A}_N = \overline{A}_H = 1$, $\overline{G}_H = \overline{G}_N = 0$, $\overline{\mu}_H \geq 1$, $\overline{\mu}_N \geq 1$. We assume that producer prices do not change in the steady state, i.e., $\Pi_H = \frac{P_{H,t}}{P_{H,t-1}} = 1$ and $\Pi_N = \frac{P_{N,t}}{P_{N,t-1}} = 1$ at all times. The optimal risk-sharing condition implies that $ER_t = \frac{U_C(C_t^*)}{U_C(C_t)} k_o$. Under the given functional forms, we obtain the condition for the steady state: $\overline{ER} = \left(\frac{\overline{C}}{\overline{C}^*}\right)^{\rho} k_o$. By choosing $k_o = \left(\frac{\overline{C}}{\overline{C}^*}\right)^{-\rho}$ we obtain the steady state real exchange rate equal to unity, i.e., $\overline{ER} = 1$. We normalize the price indices of traded goods at home and abroad so that $\overline{P}_H = \overline{P}_F$, as usually assumed in the literature, i.e., in the steady state the terms of trade P_{FH} are equal to unity. Moreover, from the price index equation (1a) it follows that $\overline{P}_H = \overline{P}_T$. We can write the general price index (1) as: $1 = [\gamma \overline{p}_N^{1-\omega} + (1-\gamma)\overline{p}_T^{1-\omega}]^{\frac{1}{1-\omega}}$ where $\overline{p}_N = \frac{\overline{P_N}}{\overline{P}}$, $\overline{p}_T = \frac{\overline{P_T}}{\overline{P}}$. From this relation we obtain $\overline{P}_N = \overline{P}_T = \overline{P}$. The price index equations and the fact that $\overline{ER} = 1$ imply that in the steady state prices at home and abroad are equalized. Furthermore, the price setting equations imply the following relationships in the steady state:

$$U_C(\overline{C})\frac{\overline{P_H}}{\overline{P}} = \overline{\mu}_H V_y(\overline{Y}_H), \tag{1}$$

$$U_C(\overline{C})\frac{\overline{P_N}}{\overline{P}} = \overline{\mu}_N V_y(\overline{Y}_N)\ . \tag{2}$$

From the aggregate demand equations (7) and (4) (main text) we obtain:

$$\overline{Y}_H = \left[(1-\gamma)\upsilon\overline{C} + (1-\gamma^*)\widetilde{\upsilon}^*\overline{C}^*\right], \tag{3}$$

$$\overline{Y}_N = \gamma\overline{C}. \tag{4}$$

The world aggregate resource constraint is given by: $\overline{Y}+\overline{Y}^{*}=\overline{C}+\overline{C}^{*}$. Combining this condition with (3) and (4) we obtain:

$$\frac{\overline{C}}{\overline{C}^{*}}=\frac{(1-\gamma^{*})\widetilde{\upsilon}^{*}}{(1-\gamma)(1-\upsilon)}. \tag{5}$$

This relation demonstrates that even under the complete market assumption, the structural asymmetries result in a wedge between consumption in the two countries. Finally, $k_{o}=\left(\frac{\overline{C}}{\overline{C}^{*}}\right)^{-\rho}=\left(\frac{(1-\gamma^{*})\widetilde{\upsilon}^{*}}{(1-\gamma)(1-\upsilon)}.\right)^{-\rho}$.

Second-Order Approximation of the Utility Function and Equilibrium Conditions

We apply the methodology described in Woodford (2003) and Benigno and Woodford (2005) in order to obtain the second-order approximation to the utility function of the form:

$$U_{t}^{j}=E_{t}\left\{\sum_{s=t}^{\infty}\beta^{s-t}[U(C_{s}^{j})-V(y_{s,T}(j),A_{s,T}^{i})-V(y_{s,N}(j),A_{s,N}^{i})]\right\}. \tag{6}$$

We assume that preferences have isoelastic functional form and we arrive at the following expression:

$$W_{to}=U_{C}\overline{C}E_{t_{0}}\sum_{t=t_{0}}^{\infty}\beta^{t-t_{0}}\times \tag{7}$$

$$\left[\begin{array}{c}\widehat{C}_{t}-(\overline{\mu}_{N})^{-1}\gamma\widehat{Y}_{N,t}-(\overline{\mu}_{H})^{-1}(1-\gamma)\widehat{Y}_{H,t}+\frac{1}{2}(1-\rho)\widehat{C}_{t}^{2}\\ -\frac{1}{2}(\overline{\mu}_{N})^{-1}\gamma(1+\eta)\widehat{Y}_{N,t}^{2}-\frac{1}{2}(\overline{\mu}_{H})^{-1}(1-\gamma)(1+\eta)\widehat{Y}_{H,t}^{2}\\ +(\overline{\mu}_{N})^{-1}\gamma\eta\widehat{A}_{N,t}\widehat{Y}_{N,t}+(\overline{\mu}_{H})^{-1}(1-\gamma)\eta\widehat{A}_{H,t}\widehat{Y}_{H,t}\\ -\frac{1}{2}\gamma\frac{\sigma}{\overline{\mu}_{N}k_{N}}\pi_{N,t}^{2}-\frac{1}{2}(1-\gamma)\frac{\sigma}{\overline{\mu}_{H}k_{H}}\pi_{H,t}^{2}.+t.i.p+(\left\|\xi^{3}\right\|)\end{array}\right],$$

where $t.i.p.$ denotes terms that are independent of policy and $(\|\xi^3\|)$ denotes terms that are of third order and higher. We can write (7) in a vector-matrix form as:

$$W_{to} = U_C\overline{C}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[z_x^{'}x_t - \frac{1}{2}x_t^{'}Z_x x_t - x_t^{'}Z_\xi\xi_t - \frac{1}{2}z_{\pi_H}\pi_{H,t}^2 - \frac{1}{2}z_{\pi_N}\pi_{N,t}^2\right] + t.i.p + (\|\xi^3\|), \tag{8}$$

where

$$x_t^{'} \equiv \begin{bmatrix} \widehat{Y}_{H,t} & \widehat{Y}_{N,t} & \widehat{C}_t & \widehat{P}_{HT,t} & \widehat{P}_{NT,t} & \widehat{ER}_t \end{bmatrix},$$

$$\xi_t^{'} \equiv \begin{bmatrix} \widehat{A}_{H,t} & \widehat{A}_{N,t} & \widehat{\mu}_{H,t} & \widehat{\mu}_{N,t} & \widehat{g}_{H,t} & \widehat{g}_{N,t} & \widehat{C}_t^{*} & \widehat{P}_{NT,t}^{*} \end{bmatrix},$$

$$z_x^{'} \equiv \begin{bmatrix} (-(\overline{\mu}_H)^{-1}(1-\gamma)) & (-(\overline{\mu}_N)^{-1}\gamma) & 1 & 0 & 0 & 0 \end{bmatrix},$$

$$Z_x \equiv \begin{bmatrix} (\overline{\mu}_H)^{-1}(1-\gamma)(1+\eta) & 0 & 0 & 0 & 0 & 0 \\ 0 & (\overline{\mu}_N)^{-1}\gamma(1+\eta) & 0 & 0 & 0 & 0 \\ 0 & 0 & -(1-\rho) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Z_\xi \equiv \begin{bmatrix} -(\overline{\mu}_H)^{-1}(1-\gamma)\eta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\overline{\mu}_N)^{-1}\gamma\eta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$z_{\pi_H} \equiv (1-\gamma)\frac{\sigma}{\overline{\mu}_H k_H} \qquad z_{\pi_N} \equiv \gamma\frac{\sigma}{\overline{\mu}_N k_N},$$

where $k_L = \frac{(1-\alpha_L\beta)(1-\alpha_L)}{\alpha_L(1+\sigma\eta)}$, for $L = H, N$.

We now derive the second-order approximation to the structural equilibrium conditions. Following Benigno and Woodford (2005), we approximate the optimal price-setting equation (expression (10) in the main text) for both domestic sectors as well as the law of motion for the sectoral price indices (11). We combine the corresponding expressions and, after integrating forward, obtain the following relations:

$$V_0^H = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \times \tag{9}$$

$$\left\{ \begin{array}{l} \left[\eta \widehat{Y}_{H,t} + \rho \widehat{C}_t - \widehat{P}_{HT,t} + \gamma \widehat{P}_{NT,t} + \widehat{\mu}_{H,t} - \eta \widehat{A}_{H,t}\right] + \frac{1}{2}\gamma(1-\omega)(1-\gamma)\widehat{P}_{NT,t}^2 \\ +\frac{1}{2} \left[\begin{array}{c} \eta \widehat{Y}_{H,t} + \rho \widehat{C}_t - \widehat{P}_{HT,t} + \\ \gamma \widehat{P}_{NT,t} + \widehat{\mu}_{H,t} - \eta \widehat{A}_{H,t} \end{array} \right] \times \left[\begin{array}{c} (2+\eta)\widehat{Y}_{H,t} - \rho \widehat{C}_t + \widehat{P}_{HT,t} - \\ \gamma \widehat{P}_{NT,t} + \widehat{\mu}_{H,t} - \eta \widehat{A}_{H,t} \end{array} \right] \\ +\frac{1}{2}\frac{\sigma(1+\eta)}{k_H}\pi_{H,t}^2 + s.o.t.i.p. + (\left\|\xi^3\right\|) \end{array} \right\},$$

$$V_0^N = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \times \tag{10}$$

$$\left\{ \begin{array}{l} \left[\eta \widehat{Y}_{N,t} + \rho \widehat{C}_t - (1-\gamma) \widehat{P}_{NT,t} + \widehat{\mu}_{N,t} - \eta \widehat{A}_{N,t}\right] + +\frac{1}{2}\gamma(1-\omega)(1-\gamma)\widehat{P}_{NT,t}^2 \\ +\frac{1}{2} \left[\begin{array}{c} \eta \widehat{Y}_{N,t} + \rho \widehat{C}_t - (1-\gamma)\widehat{P}_{NT,t} + \\ \widehat{\mu}_{N,t} - \eta \widehat{A}_{N,t} \end{array} \right] \times \left[\begin{array}{c} (2+\eta)\widehat{Y}_{N,t} - \rho \widehat{C}_t + (1-\gamma)\widehat{P}_{NT,t} + \\ +\widehat{\mu}_{N,t} - \eta \widehat{A}_{N,t} \end{array} \right] \\ +\frac{1}{2}\frac{\sigma(1+\eta)}{k_N}\pi_{N,t}^2 + s.o.t.i.p. + (\left\|\xi^3\right\|) \end{array} \right\},$$

where $s.o.t.i.p.$ denotes second-order terms independent of policy. We can present equations (9) and (10) in a vector-matrix form as :

$$V_0^H = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[a_x' x_t + a_\xi' \xi_t + \frac{1}{2} x_t' A_x x_t + x_t' A_\xi \xi_t + \frac{1}{2} a_{\pi_H} \pi_{H,t}^2 \right] + s.o.t.i.p. + (\left\|\xi^3\right\|), \tag{11}$$

$$V_0^N = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[b_x' x_t + b_\xi' \xi_t + \frac{1}{2} x_t' B_x x_t + x_t' B_\xi \xi_t + \frac{1}{2} b_{\pi_N} \pi_{N,t}^2 \right] + s.o.t.i.p. + (\left\|\xi^3\right\|), \tag{12}$$

where

$$a_x' \equiv \left[\begin{array}{cccccc} \eta & 0 & \rho & -1 & \gamma & 0 \end{array} \right],$$

$$a_\xi' \equiv \left[\begin{array}{cccccccc} -\eta & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{array} \right].$$

$$A_x \equiv \begin{bmatrix} \eta(2+\eta) & 0 & \rho & -1 & \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \rho & 0 & -\rho^2 & \rho & \rho\gamma & 0 \\ -1 & 0 & \rho & -1 & \gamma & 0 \\ \gamma & 0 & \rho\gamma & \gamma & -\gamma^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_\xi \equiv \begin{bmatrix} -\eta(1+\eta) & 0 & (1+\eta) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$a_{\pi_H} \equiv \frac{\sigma(1+\eta)}{k_H}.$$

$$b_x' \equiv \begin{bmatrix} 0 & \eta & \rho & 0 & -(1-\gamma) & 0 \end{bmatrix},$$

$$b_\xi' \equiv \begin{bmatrix} 0 & -\eta & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$B_x \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \eta(2+\eta) & \rho & 0 & -(1-\gamma) & 0 \\ 0 & \rho & -\rho^2 & 0 & \rho(1-\gamma) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(1-\gamma) & \rho(1-\gamma) & 0 & (1-\gamma)^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_\xi \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\eta(1+\eta) & 0 & (1+\eta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$b_{\pi_N} \equiv \frac{\sigma(1+\eta)}{k_N}.$$

The traded-goods demand equation is of the form:

$$Y_H = \left(\frac{P_T}{P}\right)^{-\omega} \left(\frac{P_H}{P_T}\right)^{-\theta} \times \tag{13}$$
$$\left\{ \upsilon(1-\gamma)C + \left(\frac{1}{ER}\right)^{-\omega} \left[\left(\frac{1}{\upsilon(P_{FH})^{\theta-1} + (1-\upsilon)}\right)\right]^{\frac{\theta-\omega}{1-\theta}} (1-\gamma^*)\widetilde{\upsilon}^* C^* \right\}.$$

We take the second-order expansion of (13) and obtain the following relation:

$$\begin{aligned} \widehat{Y}_{H,t} \quad = \quad & -\left[\theta + (\theta-\omega)\upsilon\right] \widehat{P}_{HT,t} + \omega\gamma\widehat{P}_{NT,t} + \upsilon\widehat{C}_t + \omega(1-\upsilon)\widehat{ER}_t + (1-\upsilon)\widehat{C}_t^* + \\ & +\widehat{g}_{H,t} + \frac{1}{2}\upsilon(1-\upsilon)\widehat{C}_t^2 + \frac{1}{2}\omega^2\upsilon(1-\upsilon)\widehat{ER}_t^2 + \frac{1}{2}\omega(1-\omega)\gamma(1-\gamma)\widehat{P_{NT,t}}^2 - \\ & -\frac{1}{2}\frac{\upsilon}{(1-\upsilon)}\left[(1-\theta)(\theta-\omega) - (\theta-\omega)^2\upsilon^2\right]\widehat{P}_{HT,t}^2 - (\theta-\omega)\omega\upsilon^2\widehat{ER}_t\widehat{P}_{HT,t} - \\ & -\omega\upsilon(1-\upsilon)\widehat{ER}_t\widehat{C}_t + (\theta-\omega)\upsilon^2\widehat{C}_t\widehat{P}_{HT,t} + \omega\upsilon(1-\upsilon)\widehat{ER}_t\widehat{C}_t^* - (\theta-\omega)\upsilon^2\widehat{P}_{HT,t}\widehat{C}_t^* - \\ & -\upsilon(1-\upsilon)\widehat{C}_t\widehat{C}_t^* - \omega\gamma\widehat{P}_{NT,t}\widehat{g}_{H,t} + [\theta + \upsilon(\theta-\omega)]\widehat{P}_{HT,t}\widehat{g}_{H,t} - \upsilon\widehat{C}_t\widehat{g}_{H,t} - \\ & -\omega(1-\upsilon)\widehat{ER}_t\widehat{g}_{H,t} + s.o.t.i.p. + (\left\|\xi^3\right\|). \end{aligned} \tag{14}$$

In a vector-matrix form the expression above takes the following form:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[c_x^{'} x_t + c_\xi^{'} \xi_t + \frac{1}{2} x_t^{'} C_x x_t + x_t^{'} C_\xi \xi_t\right] + s.o.t.i.p. + (\left\|\xi^3\right\|) = 0, \tag{15}$$

where

$$c_x^{'} \equiv \begin{bmatrix} -1 & 0 & \upsilon & -\left[\theta+(\theta-\omega)\upsilon\right] & \omega\gamma & \omega(1-\upsilon) \end{bmatrix},$$

$$c_\xi^{'} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & (1-\upsilon) & 0 \end{bmatrix}.$$

$$C_x \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \upsilon(1-\upsilon) & (\theta-\omega)\upsilon^2 & 0 & -\omega\upsilon(1-\upsilon) \\ 0 & 0 & (\theta-\omega)\upsilon^2 & \frac{\upsilon}{(1-\upsilon)}\begin{bmatrix} (1-\theta)(\theta-\omega)- \\ (\theta-\omega)^2\upsilon^2 \end{bmatrix} & 0 & -(\theta-\omega)\omega\upsilon^2 \\ 0 & 0 & 0 & 0 & \omega(1-\omega)\gamma(1-\gamma) & 0 \\ 0 & 0 & -\omega\upsilon(1-\upsilon) & -(\theta-\omega)\omega\upsilon^2 & 0 & \omega^2\upsilon(1-\upsilon) \end{bmatrix},$$

$$C_\xi \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\upsilon & 0 & -\upsilon(1-\upsilon) & 0 \\ 0 & 0 & 0 & 0 & \left[\theta+\upsilon(\theta-\omega)\right] & 0 & -(\theta-\omega)\upsilon^2 & 0 \\ 0 & 0 & 0 & 0 & -\omega\gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega(1-\upsilon) & 0 & \omega\upsilon(1-\upsilon) & 0 \end{bmatrix}.$$

Similarly, the demand equation for non-traded goods takes the following form:

$$Y_N = \left(\frac{P_N}{P}\right)^{-\omega}\gamma C. \tag{16}$$

The second-order approximation of this equation yields the following expressions:

$$\widehat{Y}_{N,t} = \widehat{C}_t - w(1-\gamma)\widehat{P}_{NT,t} + \widehat{g}_{N,t} + \frac{1}{2}(1-\gamma)\gamma\omega(1-\omega)\widehat{P}_{NT,t}^2 - \tag{17}$$
$$\widehat{C}_t\widehat{g}_{N,t} + \omega(1-\gamma)\widehat{P}_{NT,t}\widehat{g}_{N,t} + (\left\|\xi^3\right\|),$$

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[d_x^{'}x_t + d_\xi^{'}\xi_t + \frac{1}{2}x_t^{'}D_x x_t + x_t^{'}D_\xi\xi_t\right] + s.o.t.i.p. + (\left\|\xi^3\right\|) = 0. \tag{18}$$

$$d_x^{'} \equiv \begin{bmatrix} 0 & -1 & 1 & 0 & -w(1-\gamma) & 0 \end{bmatrix},$$

$$d_\xi^{'} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$D_x \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-\gamma)\gamma\omega(1-\omega) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$D_\xi \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega(1-\gamma) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

The second-order approximation of the risk-sharing equation (9) in the main text takes the form:

$$\widehat{C}_t = \frac{1}{\rho}\widehat{ER}_t + \widehat{C}_t^{*}. \tag{19}$$

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[e_x^{'} x_t + e_\xi^{'} \xi_t + \frac{1}{2} x_t^{'} E_x x_t + x_t^{'} E_\xi \xi_t \right] + s.o.t.i.p. + (\left\| \xi^3 \right\|) = 0. \tag{20}$$

$$e_x^{'} \equiv \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & \frac{1}{\rho} \end{bmatrix},$$

$$e_\xi^{'} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

$$E_x = 0; \qquad E_\xi = 0.$$

Finally, the real exchange rate equation (12) approximated up to the second order takes the form:

$$\begin{aligned} \upsilon\widehat{P}_{HT,t} &= -(1-\upsilon)\widehat{ER}_t - \gamma(1-\upsilon)\widehat{P}_{NT,t} + \gamma^*(1-\upsilon)\widehat{P}^*_{NT,t} - \frac{1}{2}\frac{(1-\upsilon)}{\upsilon}(1-\theta)\widehat{ER}_t^2 - \\ & -\frac{1}{2}\gamma(1-\upsilon)\left[\frac{\gamma(1-\theta)}{\upsilon} + (1-\omega)(1-\gamma)\right]\widehat{P}^2_{NT,t} - \frac{\gamma(1-\upsilon)}{\upsilon}(1-\theta)\widehat{ER}_t\widehat{P}_{NT,t} + \\ & + \frac{(1-\upsilon)}{\upsilon}(1-\theta)\gamma^*\widehat{ER}_t\widehat{P}^*_{NT,t} + \frac{(1-\upsilon)}{\upsilon}(1-\theta)\gamma\gamma^*\widehat{P}_{NT,t}\widehat{P}^*_{NT,t} + s.o.t.i.p. + (\|\xi^3\|). \end{aligned} \tag{21}$$

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[f_x^{'}x_t + f_\xi^{'}\xi_t + \frac{1}{2}x_t^{'}F_x x_t + x_t^{'}F_\xi\xi_t\right] + s.o.t.i.p. + (\|\xi^3\|) = 0. \tag{22}$$

$$f_x^{'} \equiv \begin{bmatrix} 0 & 0 & 0 & -\upsilon & -\gamma(1-\upsilon) & -(1-\upsilon) \end{bmatrix},$$

$$f_\xi^{'} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma^*(1-\upsilon) \end{bmatrix},$$

$$F_x \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\gamma(1-\upsilon)\left[\frac{\gamma(1-\theta)}{\upsilon} + (1-\omega)(1-\gamma)\right] & -\frac{\gamma(1-\upsilon)}{\upsilon}(1-\theta) \\ 0 & 0 & 0 & 0 & -\frac{\gamma(1-\upsilon)}{\upsilon}(1-\theta) & -\frac{(1-\upsilon)}{\upsilon}(1-\theta) \end{bmatrix},$$

$$F_\xi \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(1-\upsilon)}{\upsilon}(1-\theta)\gamma\gamma^* \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(1-\upsilon)}{\upsilon}(1-\theta)\gamma^* \end{bmatrix}.$$

We combine constraints (11), (12), (15), (18), (20), and (22) in order to get rid of the linear terms in the objective function (8). We collect vectors that contain the linear components of the above constraints and derive the vector λ, such that:

$$\begin{bmatrix} a_x & b_x & c_x & d_x & e_x & f_x \end{bmatrix} \times \lambda = z_x.$$

We solve the system of linear equations using the symbolic Matlab toolbox and derive values $\lambda_1 - \lambda_6$ associated with each of the constraints. After the linear terms cancel, we obtain the following expression for the loss function:

$$L_{to} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{1}{2} x_t^{'} \widetilde{Z}_x x_t + x_t^{'} \widetilde{Z}_\xi \xi_t + \frac{1}{2} \widetilde{Z}_{\pi_H} \pi_{H,t}^2 + \frac{1}{2} \widetilde{Z}_{\pi_N} \pi_{N,t}^2 \right] + K_0 + t.i.p + (\|\xi^3\|), \tag{23}$$

where

$$\widetilde{Z}_x = Z_x + \lambda_1 A_x + \lambda_2 B_x + \lambda_3 C_x + \lambda_4 D_x + \lambda_5 E_x + \lambda_6 F_x,$$

$$\widetilde{Z}_\xi = Z_\xi + \lambda_1 A_\xi + \lambda_2 B_\xi + \lambda_3 C_\xi + \lambda_4 D_\xi + \lambda_5 E_\xi + \lambda_6 F_\xi,$$

$$\widetilde{Z}_{\pi_H} = z_{\pi_H} + \lambda_1 a_{\pi_H},$$

$$\widetilde{Z}_{\pi_N} = z_{\pi_N} + \lambda_2 b_{\pi_N},$$

$$K_0 \equiv U_C \overline{C} \left[\lambda_1 V_0^H + \lambda_2 V_0^N \right].$$

Vectors $\widetilde{Z}_x$, $\widetilde{Z}_{\pi_H}, \widetilde{Z}_{\pi_N}$ represent the weights next to the endogenous variables in the objective function.

Furthermore, we would like to present the loss function (23) in terms of the variables $\widehat{Y}_{N,t}$, $\widehat{Y}_{H,t}$, $\widehat{ER}_t$, $\widehat{P}_{NT,t}$. Thus, we map the vector of all endogenous variables $x_t^{'} \equiv \begin{bmatrix} \widehat{Y}_{H,t} & \widehat{Y}_{N,t} & \widehat{C}_t & \widehat{P}_{HT,t} & \widehat{P}_{NT,t} & \widehat{ER}_t \end{bmatrix}$ into the variables of interest with the use of matrices Q and Q_ξ such that:

$$x_t = Q \begin{bmatrix} \widehat{Y}_{H,t} & \widehat{Y}_{N,t} & \widehat{P}_{NT,t} & \widehat{ER}_t \end{bmatrix}^{'} + Q_\xi \xi_t, \tag{24}$$

and

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ (1-\gamma) & \gamma & \frac{-\gamma(1-\gamma)(\widetilde{l}+1-\upsilon)}{\rho\upsilon} & \frac{-(1-\gamma)(l+1-\upsilon)}{\rho\upsilon} \\ 0 & 0 & \frac{-\gamma(1-\upsilon)}{\upsilon} & \frac{-(1-\upsilon)}{\upsilon} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$Q_\xi = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -(1-\gamma) & -\gamma & 0 & \frac{\gamma^*(1-\gamma)(\widetilde{l}+1-\upsilon)}{\rho\upsilon} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\gamma^*(1-\upsilon)}{\upsilon} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where $l = (\rho\theta - 1)(1-\upsilon)(1+\upsilon)$ and $\widetilde{l} = l - (\rho\omega - 1)(1-\upsilon)\upsilon$. Therefore, the loss function (23) can be expressed as follows:

$$L_{to} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{1}{2} X_t' W_x X_t + X_t' W_\xi \xi_t + \frac{1}{2} W_{\pi_H} \pi_{H,t}^2 + \frac{1}{2} W_{\pi_N} \pi_{N,t}^2 \right] + K_0 + t.i.p + (\left\|\xi^3\right\|), \tag{25}$$

where $X_t' = \begin{bmatrix} \widehat{Y}_{H,t} & \widehat{Y}_{N,t} & \widehat{P}_{NT,t} & \widehat{ER}_t \end{bmatrix}$, $W_x = Q' \widetilde{Z}_x Q$, $W_\xi = Q' \widetilde{Z}_x Q_\xi + Q' \widetilde{Z}_\xi$, $W_{\pi_H} = \widetilde{Z}_{\pi_H}$, $W_{\pi_N} = \widetilde{Z}_{\pi_N}$.Finally, we present the variables in the objective function in terms of the deviations from their target values. Thus, we denote the gap as $\widetilde{X}_t = (X_t - X_t^T)$. The target values are functions of the exogenous shocks and take the following general form: $X_t^T = \left(-\frac{W_\xi}{W_x} \xi_t \right)$. As a result, we are able to present the objective function in the following quadratic form:

$$\begin{aligned} L_{to} &= U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{1}{2} (X_t - X_t^T)' W_x (X_t - X_t^T) + \frac{1}{2} W_{\pi_H} \pi_{H,t}^2 + \frac{1}{2} W_{\pi_N} \pi_{N,t}^2 \right] + \\ &\quad + K_0 + t.i.p + (\left\|\xi^3\right\|). \end{aligned} \tag{26}$$

Expression (26) corresponds to formula (29) in the main text.

Tables and Figures

Table 1. Calibration of the parameters

Parameters	Definition	Value
Structural parameters		
β	discount factor	0.99
ρ	inverse of intertemporal elasticity of substitution	3
σ	intermediate goods elasticity of substitution	6
θ	elasticity of substitution between home and foreign goods	1.5
ω	elasticity of substitution between tradable and non-tradable goods	0.5
η	inverse of elasticity of goods production	0.47
γ	share of non-tradables	0.5
υ	degree of openness	0.6
γ^*	share of non-tradables in foreign country	0.5
α_H	Calvo price parameter, tradable sector	0.55
α_N	Calvo price parameter, non-tradable sector	0.6
Parameters of stochastic shocks		
ρ_{Ah}	technology autoregressive coefficient, tradable sector	0.8
σ_{Ah}	technology standard deviations, tradable sector	0.024
ρ_{An}	technology autoregressive coefficient, non-tradable sector	0.9
σ_{An}	technology standard deviations, non-tradable sector	0.008
ρ_{Gh}	autoregressive coefficient government spending shock, tradable sector	0.7
σ_{Gh}	standard deviations of government spending shock, tradable sector	0.01
ρ_{Gn}	autoregressive coefficient government spending shock, non-trad. sector	0.8
σ_{Gn}	standard deviations of government spending shock, non-trad. sector	0.008
$\rho_{\mu h}$	autoregressive coefficient mark-up shock, tradable sector	0.7
$\sigma_{\mu h}$	standard deviations of mark-up shock, tradable sector	0.02
$\rho_{\mu n}$	autoregressive coefficient mark-up shock, non-tradable sector	0.8
$\sigma_{\mu n}$	standard deviations of mark-up shock, non-tradable sector	0.014
ρ_{C^*}	autoregressive coefficient of foreign consumption shock	0.9
σ_{C^*}	standard deviations of foreign consumption shock	0.007
ρ_{pnt^*}	autoregressive coefficient of foreign relative price shock	0.7
σ_{pnt^*}	standard deviations of foreign relative price shock	0.012
Parameters of calibrated monetary policy rule		
ρ_r	smoothing coefficient	0.75
ψ_π	inflation coefficient	0.49
ψ_Y	output gap coefficient	0.038
ε_r	standard deviations of monetary policy shock	0.003

Table 2. Matching the moments: Data and the baseline model
(Canadian data, HP-filtered series, sample 1981Q1-2007Q4)

Series, std. in %	Data	Model
Total output	1.45	1.33
Output, tradable sector	3.14	3.77
Output, non-tradable sector	1.22	1.47
CPI inflation	0.47	0.48
Inflation, tradable sector	0.73	0.73
Inflation,non-tradable sector	0.44	0.40
Real exchange rate	3.48	3.60
Nominal interest rate	0.36	0.41

Definitions of used data series (data source - Statistics Canada):

- Real exchange rate: computed as nominal CDN$-US$ exchange rate deflated by Canadian and US CPI data.

- Nominal interest rate: Canadian 3-month T-bill interest rate.

- CPI inflation rate: the percentage change in the consumer price index.

- CPI inflation, tradable sector: the percentage change in the consumer price index for goods.

- CPI inflation, non-tradable sector: the percentage change in the consumer price index for services.

- Total output: GDP at 1997 constant dollars, s.a.

- Output, tradable sector: commodities and manufactured goods 1997 constant dollars, s.a.

- Output, non-tradable sector: services 1997 constant dollars, s.a.: utilities, construction, wholesale and retail trade, transportation and warehousing, information and cultural industries, finance and insurance, real estate and renting and leasing and management of companies and enterprises, professional scientific and technical services, administrative and support, waste management and remediation services, educational services, health care and social assistance, arts entertainment and recreation, accommodation and food services, other services, public administration.

Table 3. Welfare Ranking of Optimal Simple Rules

Policy Rule	Optimized coefficients				Loss to optimal
	k1	k2	k3	k4	$\frac{V^{OSR}}{V^{OPT}}$
Domestic Inflation Stabilization					
1.$\widehat{r}$=0.75$\widehat{r}_{-1}$+$\infty\pi^{D}$	-	-	-	-	1.284
2.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^{D}$	1.644	-	-	-	1.184
Flexible Domestic Inflation Targeting					
3.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^{D}$+k$_2\widetilde{Y}$	1.68	0.017	-	-	1.18
4.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^{D}$+k$_2\widetilde{Y}$+k$_3\Delta\widetilde{ER}$	1.81	0.018	0.056	-	1.178
5.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^{D}$+k$_2\widetilde{Y}$+k$_3\Delta\widetilde{P}_{NT}$	1.656	0.018	0.288	-	1.137
6.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^{D}$+k$_2\widetilde{Y}_H$+k$_3\widetilde{Y}_N$	3.44	0.044	0.601		1.144
CPI Inflation Stabilization					
7.$\widehat{r}$=0.75$\widehat{r}_{-1}$+$\infty\pi^{CPI}$	-	-	-	-	1.294
8.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^{CPI}$	1.606	-	-	-	1.198
Flexible CPI Inflation Targeting					
9.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^{CPI}$+k$_2\widetilde{Y}$	1.664	0.021	-	-	1.191
10.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^{CPI}$+k$_2\widetilde{Y}$+k$_3\Delta\widetilde{ER}$	1.771	0.0178	-0.258	-	1.148
11.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^{CPI}$+k$_2\widetilde{Y}$+k$_3\Delta\widetilde{P}_{NT}$	5.317	0.07	1.659	-	1.060
Sector-specific inflation targeting:					
12.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi_H$+k$_2\pi_N$	1.345	5.721	-	-	1.055
13.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi_N$+k$_2\Delta\widetilde{ER}$	3.935	0.183	-	-	1.123
14.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi_N$+k$_2\Delta\widetilde{P}_{NT}$	4.789	-0.685	-	-	1.072
15.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi_H$+k$_2\pi_N$+k$_3\widetilde{Y}$	1.532	6.858	0.11	-	1.047
16.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi_H$+k$_2\pi_N$+k$_3\Delta\widetilde{ER}$	1.918	8.418	0.264	-	1.075
17.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi_H$+k$_2\pi_N$+k$_3\widetilde{Y}$+k$_4\Delta\widetilde{ER}$	2.292	10.586	0.174	0.33	1.04
18. PEG	-	-	-	-	3.09

Table 4. Macroeconomic Volatility under Alternative Policy Rules (st.dev. in % relative to the optimal rule, $\frac{st.dev^{OSR}}{st.dev^{OPT}}$)

Policy Rule	Variables									
	π_H	π_N	π^D	π^{CPI}	$\widetilde{Y}_H$	$\widetilde{Y}_N$	$\widetilde{Y}$	$\widetilde{P}_{NT}$	$\widetilde{ER}$	$\widehat{r}$
Optimal, std %	0.66	0.17	0.28	0.52	4.25	1.57	1.6	3.36	3.96	0.65
Domestic Inflation Stabilization										
1.$\widehat{r}$=0.75$\widehat{r}_{-1}$+$\infty\pi^D$	0.53	2.06	0	1.08	1.10	1.17	1.09	0.94	1.04	1.02
2.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^D$	0.70	1.82	0.53	0.94	1.08	1.01	1.07	0.95	1.01	0.74
Flexible Domestic Inflation Targeting										
3.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^D$+k$_2\widetilde{Y}$	0.70	1.88	0.54	0.94	1.06	1.02	1.05	0.95	1.01	0.74
4.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^D$+k$_2\widetilde{Y}$+k$_3\Delta\widetilde{ER}$	0.70	1.90	0.56	0.86	1.06	1.03	1.06	0.94	0.99	0.69
5.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^D$+k$_2\widetilde{Y}$+k$_3\Delta\widetilde{P}_{NT}$	0.83	1.53	0.75	1.23	1.07	0.96	1.05	1.01	1.05	0.91
6.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^D$+k$_2\widetilde{Y}_H$+k$_3\widetilde{Y}_N$	0.77	1.71	0.64	0.94	1.06	0.94	1.06	0.96	1.01	0.75
CPI Inflation Stabilization										
6.$\widehat{r}$=0.75$\widehat{r}_{-1}$+$\infty\pi^{CPI}$	0.86	2	1	0	1.08	1.17	1.11	0.88	0.86	1.08
7.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^{CPI}$	0.86	1.82	0.93	0.44	1.07	1.01	1.09	0.92	0.90	0.71
Flexible CPI Inflation Targeting										
9.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^{CPI}$+k$_2\widetilde{Y}$	0.86	1.82	0.93	0.46	1.06	1.02	1.07	0.93	0.91	0.72
10.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^{CPI}$+k$_2\widetilde{Y}$+k$_3\Delta\widetilde{ER}$	0.82	1.65	0.75	0.71	1.06	1	1.06	0.95	1	0.85
11.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi^{CPI}$+k$_2\widetilde{Y}$+k$_3\Delta\widetilde{P}_{NT}$	1.03	0.94	1.07	0.83	1.07	0.88	1.07	0.99	0.99	0.65
Sector-specific inflation targeting:										
12.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi_H$+k$_2\pi_N$	0.97	0.88	0.89	1.02	1.09	0.86	1.08	1.01	1.02	0.60
13.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi_N$+k$_2\Delta\widetilde{ER}$	1.23	0.47	1.43	1.10	1.09	0.84	1.10	1.09	0.98	0.66
14.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi_N$+k$_2\Delta\widetilde{P}_{NT}$	1.07	0.76	1.14	0.79	1.08	0.89	1.09	0.99	0.97	0.69
15.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi_H$+k$_2\pi_N$+k$_3\widetilde{Y}$	0.98	0.88	0.93	1.02	1.07	0.89	1.06	1.01	1.02	0.6
16.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi_H$+k$_2\pi_N$+k$_3\Delta\widetilde{ER}$	0.98	0.88	0.93	0.90	1.09	0.88	1.09	1.01	0.99	0.62
17.$\widehat{r}$=0.75$\widehat{r}_{-1}$+k$_1\pi_H$+k$_2\pi_N$+k$_3\widetilde{Y}$+k$_4\Delta\widetilde{ER}$	0.98	0.94	0.93	0.90	1.07	0.90	1.07	1	0.99	0.63
18. PEG	1.5	4.52	2.86	1.38	1.10	1.56	1.17	0.83	0.77	0.91

Figure 1.3: Impulse Response to a Productivity Shock in the Traded Sector

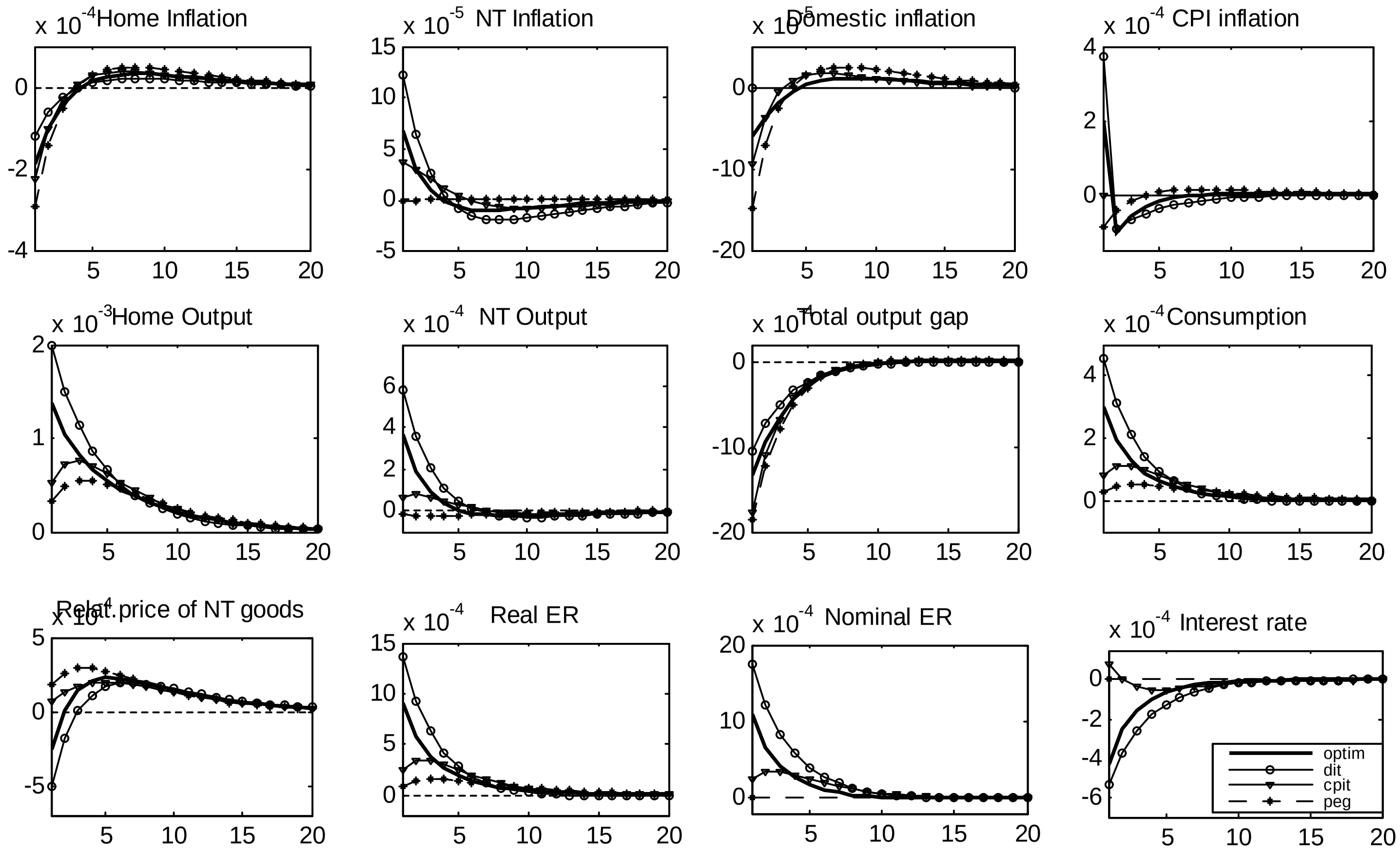


Figure 1.4: Impulse Response to a Productivity Shock in the Non-Traded Sector

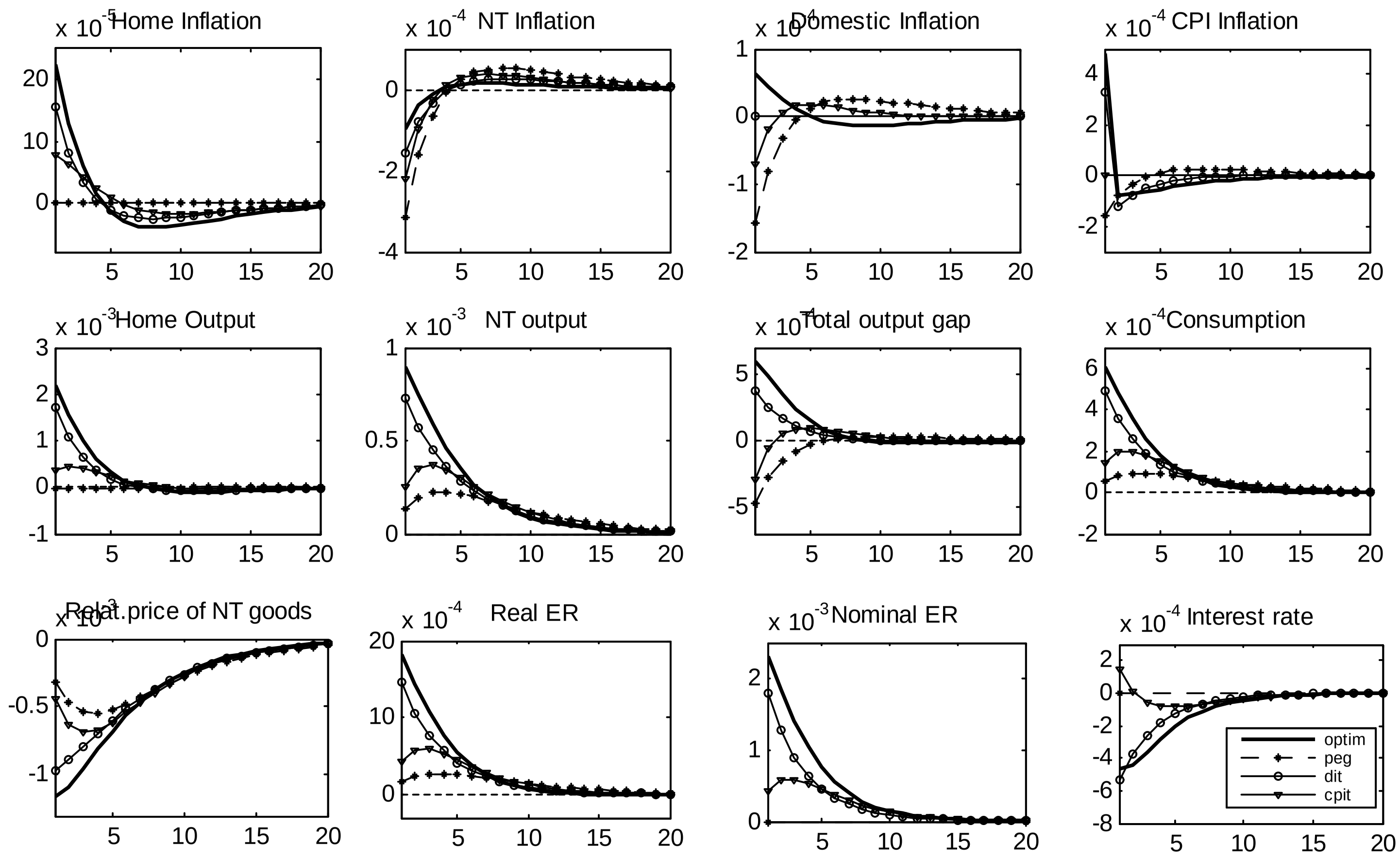


Figure 1.5: Impulse Response to a Foreign Consumption Shock

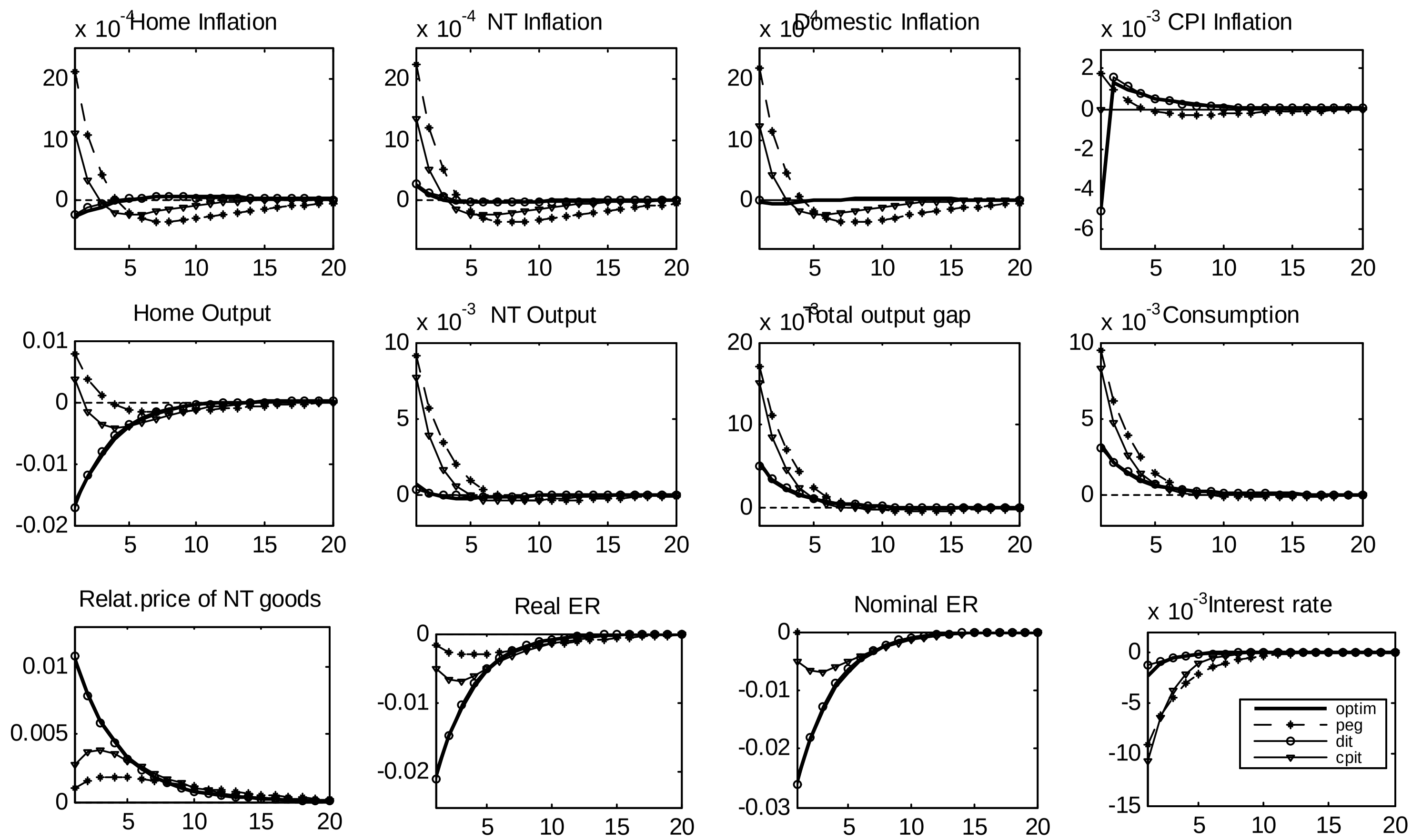


Figure 1.6: Impulse Response to a Foreign Relative Price Shock

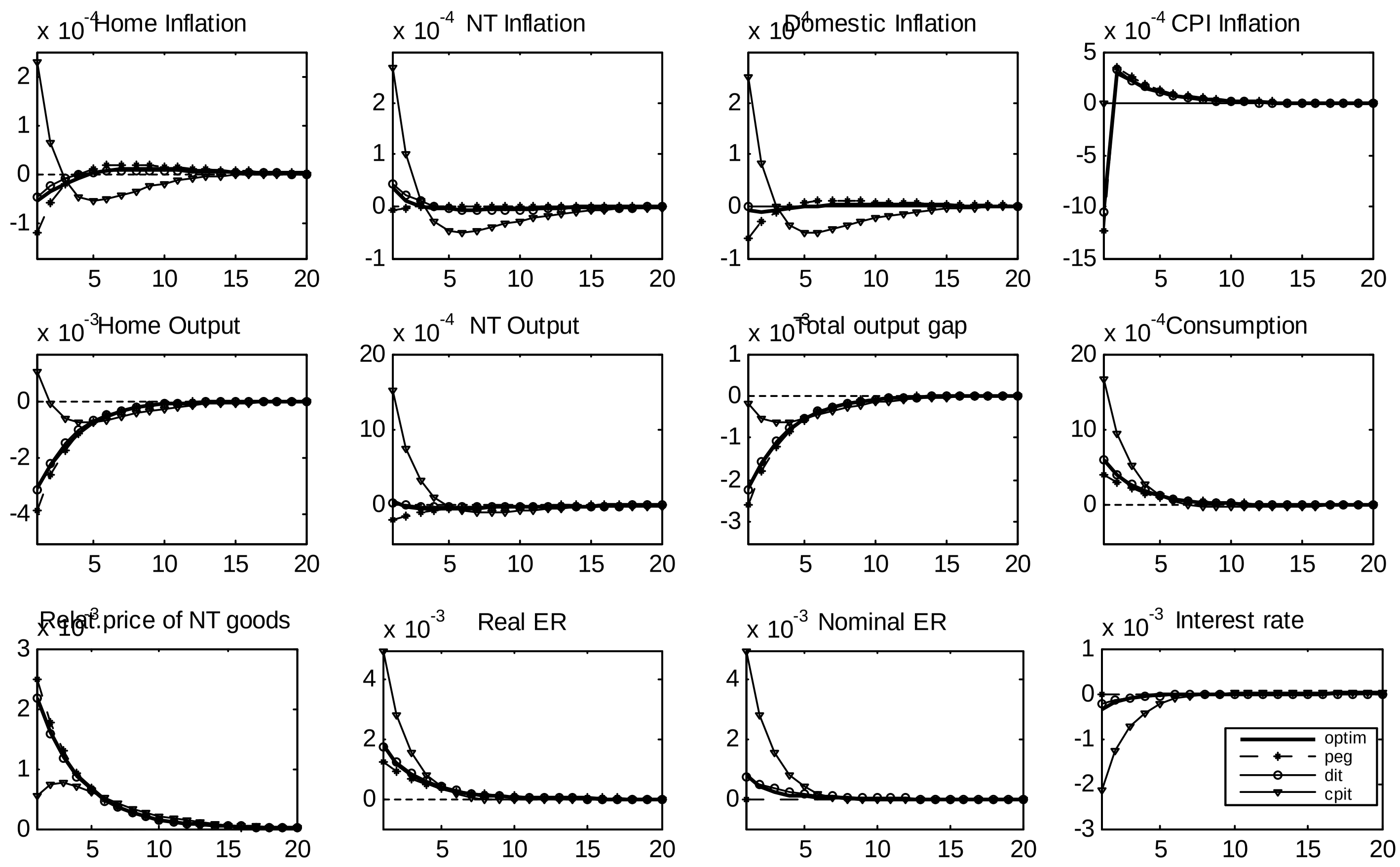


Figure 1.7: Impulse Response to a Mark-Up Shock in the Traded Sector

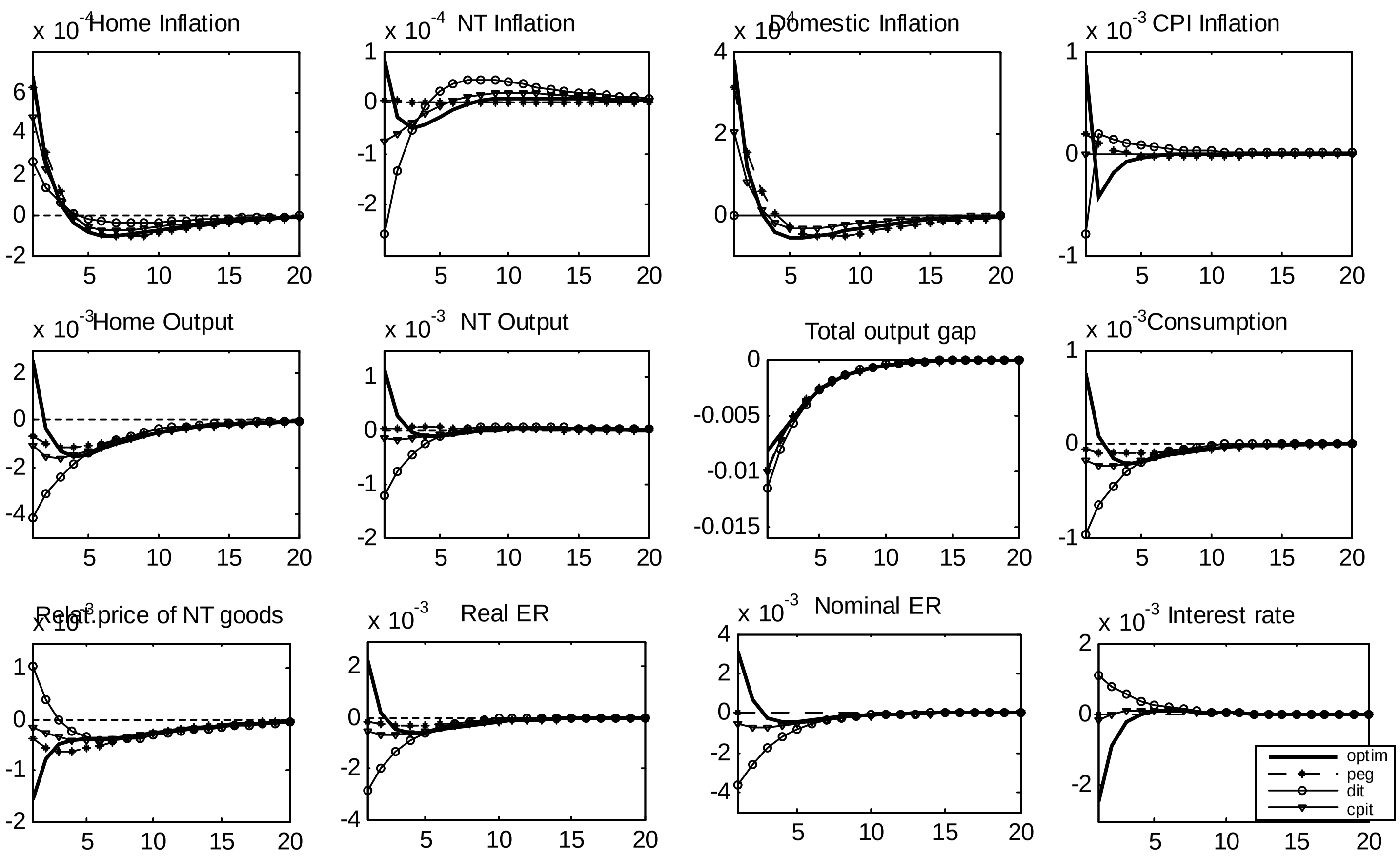


Figure 1.8: Impulse Response to a Mark-Up Shock in the Non-Traded Sector

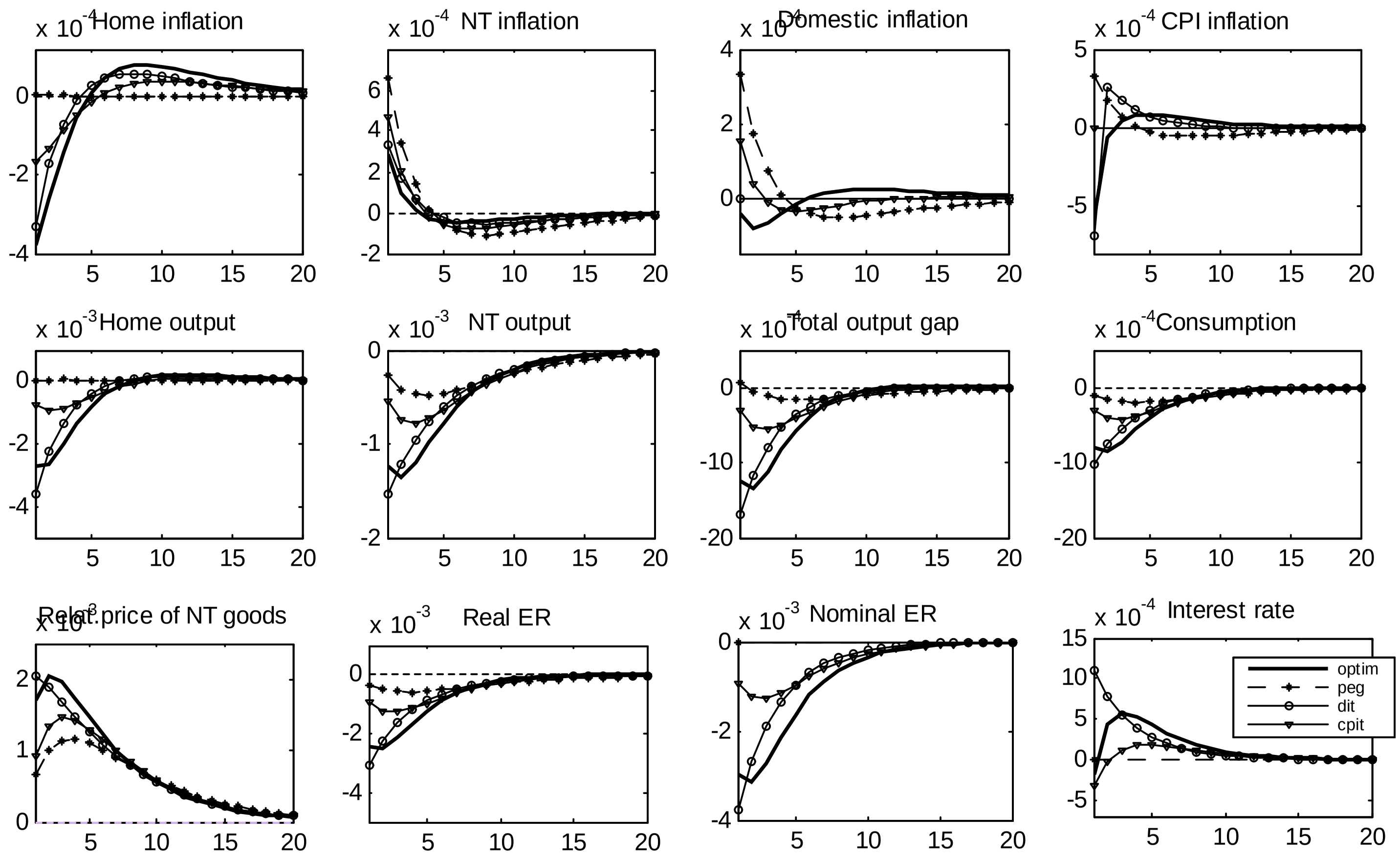


Figure 1.9: Impulse Response to a Fiscal Shock in the Traded Sector

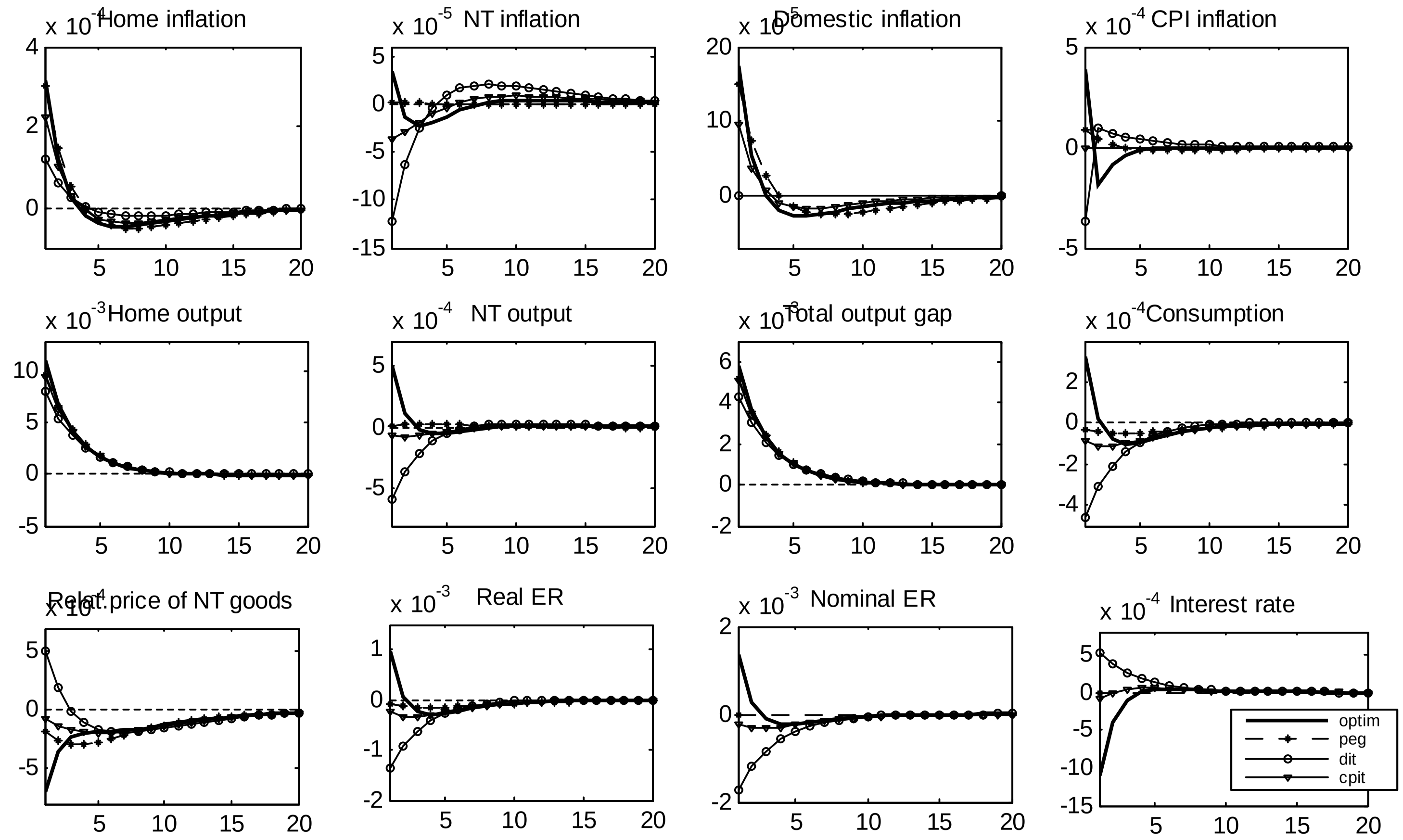


Figure 1.10: Impulse Response to a Fiscal Shock in the Non-Traded Sector

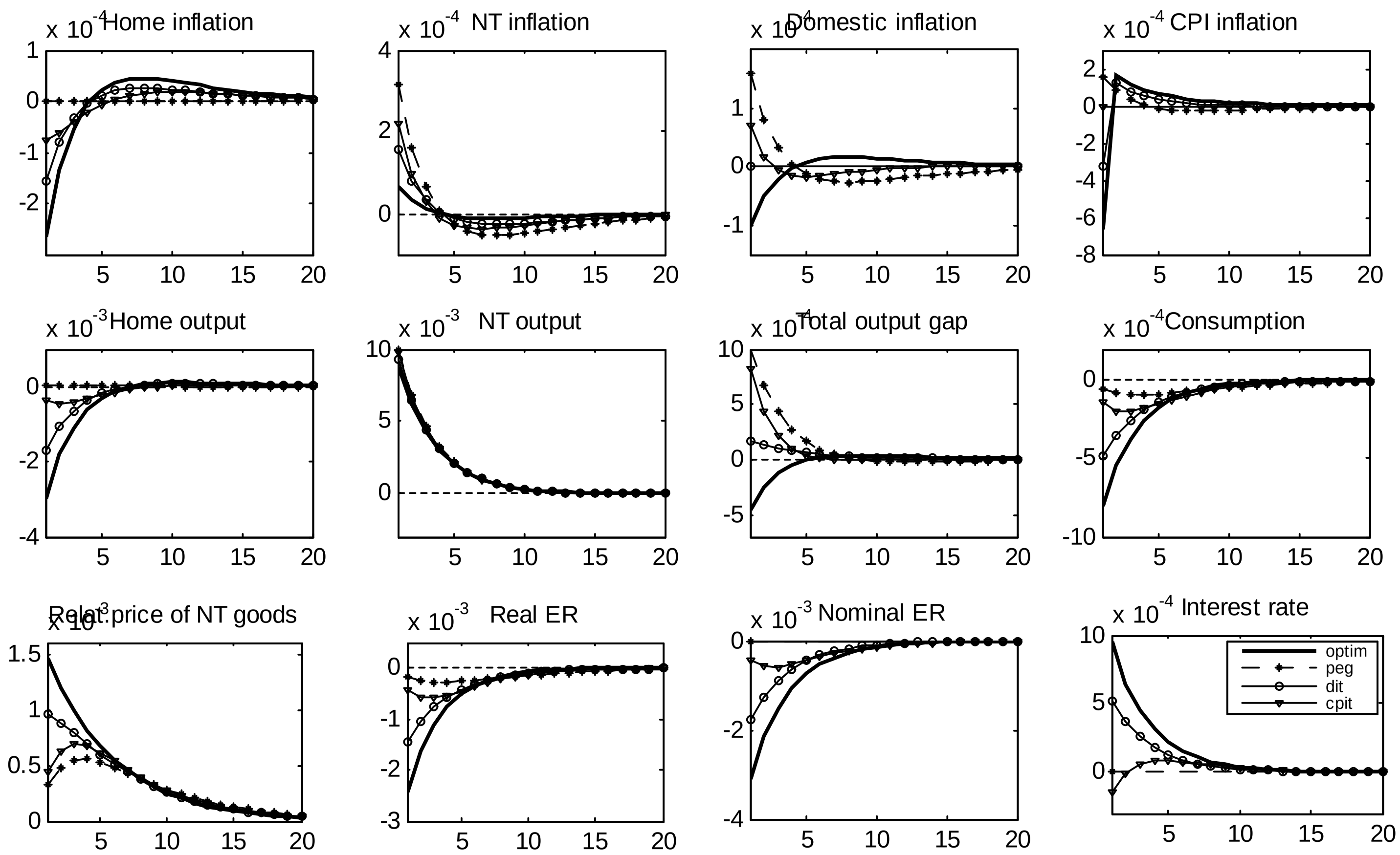


Figure 1.11: Welfare losses under alternative policies in the open economy

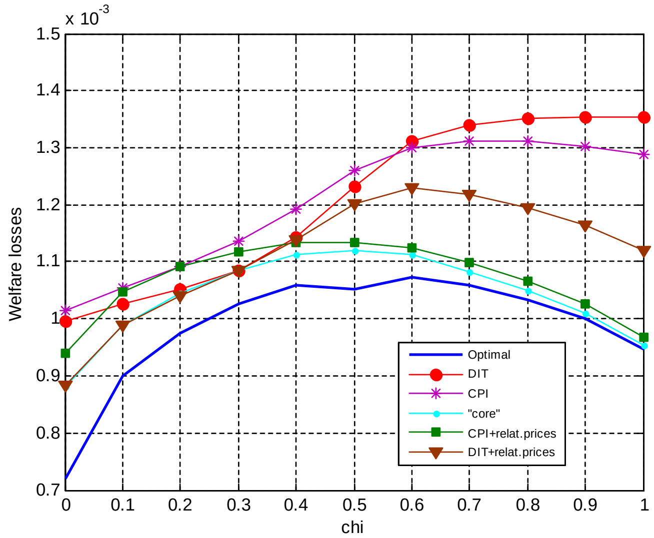


Figure 1.12: Welfare losses under alternative policies in the closed economy

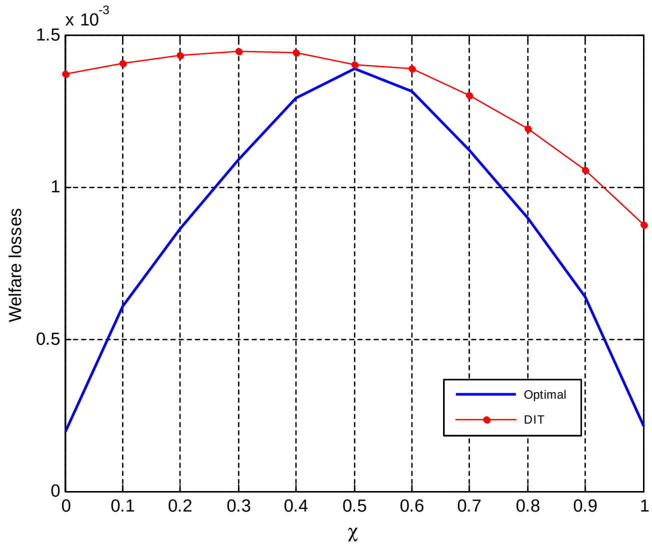


Figure 1.13: Welfare gains from targeting alternative inflation indices: price stickiness and openness

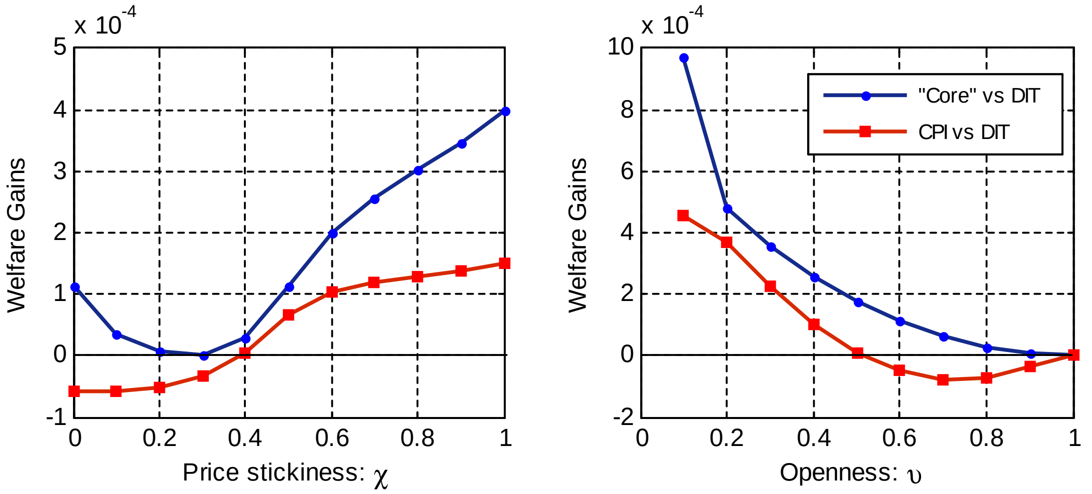
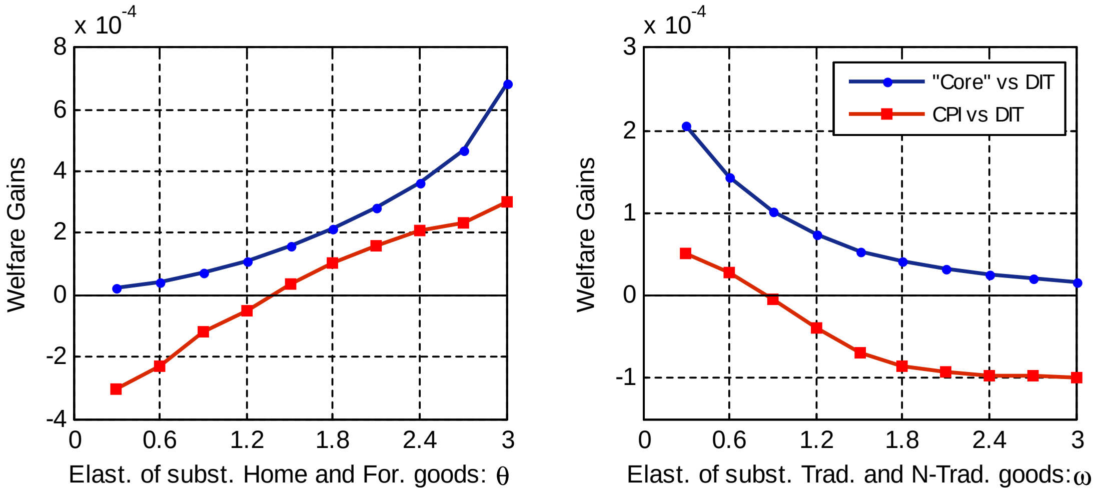


Figure 1.14: Welfare gains from targeting alternative inflation indices: elasticities of substitution



Chapter 2

Bayesian Estimation of DSGE Models under Adaptive Learning: Robustness Issues

Abstract:

We evaluate model fit, estimated parameters, and perceived persistence of inflation in several DSGE models of Euro area estimated under adaptive learning and rational expectations (RE). We systematically vary model size, information set available to the learning agents, and the way of forming agents' initial beliefs. We find that assuming adaptive expectations results in better model fit than if RE is used, especially when the agents use very little information to form their beliefs. Initial beliefs which are restricted to be consistent with the estimated RE equilibrium are found to be rather robust, as varying them significantly results in an essentially identical estimation. Pre-sample regression based initial beliefs, while more consistent with the in-sample data on average than REE-consistent initial beliefs, suffer from significant volatility and result in worse model fit. Estimated parameters and the model fit depend significantly on the information set used by the agents, which might explain widely divergent result of previous estimations under AL.

JEL classification: C11, D84, E30, E52

Keywords: DSGE models, estimation, adaptive learning

2.1 Introduction

The recent ability to implement advanced econometric techniques for systematic policy analysis has encouraged a large literature on building and estimating DSGE models. Matching empirically observed features of the data, for example persistence and hump-shaped Impulse Response Functions (IRF) of key macroeconomic variables such as inflation, output, employment, etc., necessitates inclusion of a variety of rigidities into a standard micro-founded New-Keynesian framework model. These rigidities, both real (habit formation in consumption, investment adjustment costs, variable capital utilization, fixed costs) and nominal (Calvo prices and wages, partial indexation of prices and wages to past inflation), enable models to capture the dynamic properties of observed data, see Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003, 2007). For example, the inclusion of "mechanical" endogenous persistence mechanisms, such as habit formation and price indexation, can influence the consumption and inflation dynamics and considerably change the overall performance of the model. For that reason, the empirical literature attempts to assess the validity of alternative modelling assumptions and evaluate the ability of various DSGE models to fit macroeconomic data. This issue gains further relevance when considering the growing interest in the application of micro-founded DSGE models to policy making in central banks. In particular, misspecification of the model's microfoundations may affect the welfare criteria and result in an inaccurate ranking of alternative policy regimes.

Some of the rigidities that were used in DSGE models recently, for example partial inflation indexation, have been criticized as being ad-hoc and having no theoretical foundation, see Chari, Kehoe, and McGrattan (2009). Bayesian estimation of the models requires the number of stochastic shocks driving the model to be at least equal to the number of observed variables, but certain shocks included into these models were criticized as lacking structural interpretation; assuming that such shocks are highly persistent could be questioned as well.

Even rigidities-augmented DSGE models which are driven by many shocks could remain misspecified, as evidenced, for example, by recent DSGE-VAR analysis of Del Negro and Schorfheide (2006). One hypothesis regarding the source of the residual misspecification has been the fact that the agents' expectations are rational, meaning that their subjective expectations of forward-looking variables are always consistent with the model and coincide with true mathematical expectations for given parameter values and assumed stochastic structure of the exogenous processes (shocks). Testing this hypothesis has been a major motivation behind a recent literature on estimating New Keynesian DSGE models under assumption that the agents, instead of having Rational Expectations (RE), behave as econometricians and constantly re–estimate the relation between forward–looking and other variables of the model, trying to learn the true functional form of the expectation formation mechanism.

An additional source of support for the less-than-rational beliefs hypothesis can be found in estimates of New Keynesian Phillips curves under assumption of sticky information for price settlers. As Reis (2009) summarizes, often the major source of the lack of fit for these models is an assumption that the agents are making decisions based on expectations that are rational even when based on incomplete information. Gomes (2010) attempts to develop a sticky information model where the agents who cannot act on the latest information use adaptive learning (AL) of the form described by Evans and Honkapohja (2001) to form expectations, instead of expectations constructed rationally on basis of outdated information.

In adaptive learning literature, the agents are assumed to possess imperfect knowledge about the reduced form parameters of the model when forming expectations about the future. Agents forecast future values of the lead variables with a linear function of state and exogenous variables. Agents' beliefs about the dynamics of forward looking variables are updated using the constant-gain Recursive Least Square algorithm. Learning represents an alternative source of endogenous inertia; in addition, it affects the transmission mechanism of the model and makes it time-varying through variation in agents' beliefs.

In the sticky prices DSGE literature, several recent studies made efforts to improve the model fit as well as to address the issue of possible model misspecification by departing from the RE hypothesis and incorporating adaptive learning as an expectation formation mechanism. These studies have documented dramatically different conclusions when comparing the fit and estimated parameters (especially structural rigidities) of the models under rational expectations and with learning. The strongest result in favor of integrating the assumption of bounded rationality into the DSGE models was presented by Milani (2007), who considers a very simple three equation New Keynesian model with inflation indexation, Calvo prices, and habit formation, estimated under assumption that the agents are adaptive learners using constant gain Recursive Least Squares (RLS) learning, and compares the results with estimates under RE. The models are estimated using Bayesian methods. Judged by marginal data density, the model with adaptive learning fits the US data significantly better than the RE model. Under adaptive learning certain structural rigidity parameters reduce significantly, conclusion being that the persistence observed in macroeconomic variables such as inflation might be endogenous and caused by agents' learning. Similar results are reported by Milani (2008).

Murray (2007) estimates a simple New Keynesian model augmented with firm-specific capital and constant gain RLS learning using maximum likelihood method rather than Bayesian estimation. He pays a special attention to selecting initial beliefs that the learning agents hold before the estimation period. Adaptive learning models do not fit the US data better than RE models, and Milani results on unimportance of structural rigidities in presence of adaptive learning are not confirmed. On the contrary, some structural rigidities such as capital adjustment costs become significantly more important under learning.

Vilagi (2007) considers several models, one of them very similar to that studied by Milani (2007). He estimates the models using Bayesian methods and the euro-area data. He concludes that model estimated under adaptive learning fits the data significantly better than the RE model, especially if the agents are assumed to form their expectations using only simple univariate AR(1) processes in observed variables. Some structural rigidities become less pronounced under adaptive learning, but in general Vilagi does not confirm results of Milani (2007, 2008). It is unclear whether the differences are caused by the data (US vs. EU) used for the estimation.

Slobodyan and Wouters (2009) estimate the medium size model of Smets and Wouters (2007) under adaptive learning, where the agents are using constant gain RLS. They pay particular attention to the question of forming initial beliefs of the agents, and to the information set available to the agents forecasting future values of the forward-looking variables. Several of the models with learning fit the data equally well or even better than the RE model. Specific initial beliefs contribute significantly to this result: best performing models are the ones where the initial beliefs are optimized to explain the in-sample data, consistent with previous results. Limiting the set of variables used in the forecasting equations can generate models with improved data fit. Learning models are able to generate a rapid and short lived inflation response to productivity shocks, while the response to monetary shocks is slow but very persistent. These results overcome some of the major shortcomings of the model under RE. Having forecasting equations that differ significantly from those implied by the REE is the key to this result. The additional dynamics that are introduced by the learning process do not systematically alter the estimated structural parameters of the DSGE model, contradicting claims in Milani (2007, 2008).

Slobodyan and Wouters (2010) study what happens if agents' forecasts are based on very small forecasting models, in particular on a model where expected value of a forward-looking variable depends on a constant and two lags of the variable. This forecasting model is similar to the best method of forecasting in Vilagi (2007). In contrast to other AL papers reviewed, the agents estimate simple forecasting models by Kalman filter. The results indicate that a model in which agents use a simple forecasting model to form expectations does fit the data better than the RE model. Relative to the

DSGE model under rational expectations, models with learning are estimated to have lower persistence of the exogenous shocks, especially of price and wage markup shocks; structural rigidity parameters decrease insignificantly. The results are robust to the sample period and precise specification of the forecasting model and initial beliefs.

Finally, Jaaskela and McKibbin (2010) estimate a small open economy DSGE model for Australia under constant gain RLS learning. They find that "mechanical" sources of persistence do not become unimportant under AL, and that the data marginally prefer the model with adaptive learning to the RE model.

In this paper, we contribute to the literature on estimation of DSGE models under adaptive learning. We evaluate empirically the relative importance of several types of "frictions" ("mechanical" rigidities) versus learning. Our major contribution is that we provide the answer to these questions by offering a comprehensive analysis of the factors which could determine a diversity of the estimation results under adaptive learning. In such a way we wish to reconcile contradictory conclusions from the previous studies. We perform Bayesian estimation and compare (in terms of the model fit and structural parameters) DSGE models under RE and different AL schemes and study the robustness of the estimation results in several dimensions: by modifying the model size, way of generating initial beliefs, and the set of variables included into the agents' forecasting models. All models are estimated on Euro Area data set described in Fagan et al. (2001) over the period 1970:Q1–2007:Q4 using from 3 to 7 observable macro variables. Previously, only Vilagi (2007) used this data in Bayesian estimation of a DSGE model under adaptive learning. By treating the models in a unified way, we attempt to shed some light on the general outcomes that could be expected from other DSGE model with adaptively formed expectations, and discuss possible sources of discrepant results observed so far in the literature.

The rest of the paper is organized as follows: in Section 2, we discuss the models used. Adaptive learning set-up, ways of forming initial beliefs, and selection of information sets are taken up in Section 3. Section 4 is devoted to the estimation results, and Section 5 concludes.

2.2 Models

We estimate and study the effects of AL on the three types of models, denoted in the rest of the paper M1–M3.

M1 is a simple 3 equation New Keynesian (NK) model with Calvo prices, price indexation, and habit formation in consumption. This is a model similar to that studied by Milani (2007, 2008), Vilagi's Model C, and one of the models in Murray (2007). The log-linearized version of the model consists of 3 equations.

The first equation of the model is aggregate demand equation, derived from the optimization problem of households:

$$y_t = \frac{hy_{t-1} + E_t\left[y_{t+1}\right] - \frac{1-h}{\sigma}(r_t - E_t[\pi_{t+1}])}{1+h} + e_t^b, \tag{1}$$

where y_t is real GDP, π_t the inflation rate, and r_t the nominal interest rate. The parameter h represents external habit formation, giving raise to the presence of backward-looking component in the Euler equation. σ is the measure of the elasticity of intertemporal substitution. The exogenous disturbance e_t^b is the measure of the preference shock, and follows the first-order autoregressive process,

$$e_t^b = \rho_b e_{t-1}^b + \upsilon_t^b.$$

The presence of nominal rigidities (Calvo pricing and indexation to lagged inflation) imply the following Phillips curve relation:

$$\pi_t = \frac{1}{1+\beta\iota_p} \times \left\{ \begin{array}{c} \beta E_t[\pi_{t+1}] + \iota_p\pi_{t-1} + \left(\frac{(1-\xi_p\beta)(1-\xi_p)}{\xi_p}\right) \times \\ \left[\left(\frac{\alpha}{1-\alpha} + \frac{\sigma}{(1-h)} + \frac{\eta}{1-\alpha}\right) y_t - \frac{h\sigma}{1-h} y_{t-1}\right] \end{array} \right\} + \upsilon_t^p, \tag{2}$$

where ι_p is the degree of inflation indexation, ξ_p the Calvo parameter,η the labor disutility parameter[1], and α is the capital share parameter of the Cobb-Douglas production function. The innovation $\upsilon_t^p \sim N(0, \sigma_p)$ is an *i.i.d.* process.

Finally, the policy of the central bank in setting the nominal interest rate is described by the following rule:

$$r_t = wr_{t-1} + (1-w)(\varsigma_\pi \pi_t + \varsigma_y y_t) + \upsilon_t^r, \tag{3}$$

where $\upsilon_t^r \sim N(0, \sigma_p)$ is the i.i.d. shock.

M2 is a model *M1* augmented with sticky wages, wage indexation, and sticky employment (similar to Vilagi's Model A). Our model differs from Vilagi's by introducing a shock to the labor supply e_t^l, which enables better capturing of the properties of the wage inflation process. Derivations of the model's log-linearized equations can be found in Vilagi (2007). The model consists of the following equations:

The aggregate demand equation is the same as in the model *M1* and is given by (1).

The price inflation equation is now given as

$$\pi_t = \frac{1}{1+\beta\iota_p} \times \left\{ \beta E_t[\pi_{t+1}] + \iota_p \pi_{t-1} + \left(\frac{(1-\xi_p\beta)(1-\xi_p)}{\xi_p} \right) (wr_t + \alpha l_t - e_t^a) \right\} + \upsilon_t^p, \tag{4}$$

where wr_t is real wage rate and e_t^a the first order autoregressive productivity shock,

$$e_t^a = \rho_a e_{t-1}^a + \upsilon_t^a.$$

The wage inflation equation is given by:

$$wr_t \quad = \quad \frac{1}{1+\beta} \times (\beta E_t[wr_{t+1}] + wr_{t-1} + \beta E_t[\pi_{t+1}] - (1+\beta\iota_w)\pi_t + \iota_w \pi_{t-1}) + \tag{5}$$

[1]The utility fucntion takes the form:

$$U_t = e_t^b \left[\frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - e_t^l \frac{(l_t)^{1+\eta}}{1+\eta} \right],$$

where $H_t = hC_{t-1}$ is external habit.

$$+\frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\beta)\xi_w(1+\theta_w\phi)}\left(\frac{\sigma}{(1-h)}(y_t-hy_{t-1})+\eta l_t-wr_t+e_t^l\right)+\upsilon_t^w,$$

where l_t are labor hours, ξ_w and ι_w are Calvo wage and wage indexation parameters, and θ_w is the elasticity of substitution between different types of labor. e_t^l is a labor supply shock which follows first order autoregressive process:

$$e_t^l=\rho_l e_{t-1}^l+\upsilon_t^l.$$

The innovation $\upsilon_t^w \sim N(0,\sigma_w)$ is and *i.i.d.* process.

The log-linearized firm's technology process takes the form

$$y_t=(1-\alpha)l_t+e_t^a. \tag{6}$$

Policy rule of the central bank is described by equation (3).

Finally, following Vilagi (2007) and Smets and Wouters (2003), sticky employment is modeled as follows:

$$em_t-em_{t-1}=\beta\left(E_t[em_{t+1}]-em_t\right)+\frac{(1-\xi_l\beta)(1-\xi_l)}{\xi_l}(l_t-em_t)+\upsilon_t^m, \tag{7}$$

where em_t denotes the number of people employed, ξ_l is the Calvo-type employment parameter, and $\upsilon_t^m \sim N(0,\sigma_m)$ is an *i.i.d.* shock. The inclusion of such an auxiliary equation for employment is motivated by the absence of the consistent euro area data on aggregate labor hours, whereas the employment variable is available. Since the response of employment to macroeconomic shocks is rather persistent, it is assumed that only a constant fraction ξ_l of firms can adjust employment to its desired total labor input.

The model *M3* is a Smets and Wouters (2003) model. In addition to the frictions included into *M2*, this model has investment adjustment costs and variable capital utilization. Detailed description of the model can be found in the original paper.

To summarize, the model *M1* has 3 endogenous variables (y_t, π_t, r_t), 3 exogenous shocks $(\upsilon_t^p, \upsilon_t^r, e_t^b)$, and 3 observables (y_t, π_t, r_t). The model *M2* is described by 6 model variables $(y_t, \pi_t, r_t, wr_t, l_{t,} em_t)$, 7 shocks $(\upsilon_t^p, \upsilon_t^w, \upsilon_t^m, \upsilon_t^r, e_t^l, e_t^b, e_t^a)$, and 5 observables $(y_t, \pi_t, r_t, wr_t, em_t)$. *M3* contains 11 variables, the *M2* set plus real consumption, real investment, capital stock, its rental rate, and price of capital $(c_{t,} inv_{t,} k_t, rk_t, q_t)$, and 10 exogenous shocks, the *M2* set plus investment shock, shock to government spending and the asset price shock $(e_t^{inv}, e_t^G, e_t^Q)$. Model *M3* is estimated using 7 observable variables $(y_t, c_t, inv_{t,} \pi_t, r_t, wr_t, em_t)$.

2.3 Variations of adaptive learning

2.3.1 Constant Gain RLS

We implement the adaptive learning within the DYNARE 3.064 MATLAB toolbox which is used to estimate and simulate DSGE models.[2] The models are driven by the exogenous stochastic processes w_t which are either iid random variables or univariate AR(1) processes:

$$w_t = \Gamma w_{t-1} + \Pi \epsilon_t. \tag{8}$$

Up to the first order of approximation, DYNARE represents our models in the following way:

$$A_0 \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + A_1 \begin{bmatrix} y_t \\ w_t \end{bmatrix} + A_2 E_t y_{t+1} + B_0 \epsilon_t = const, \tag{9}$$

where the vector y_t includes endogenous variables of the model. This representation is exact in our case because we work we log-linearized models. Under RE, the DYNARE solution is

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu + T \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R\epsilon_t. \tag{10}$$

[2]This is the same toolbox that was used in Slobodyan and Wouters (2009, 2010).

Deviating from the RE assumption and following Marcet and Sargent (1989) and Evans and Honkapohja (2001), we assume that the agents forecast future values of the forward-looking variables using a linear function of endogenous variables and exogenous driving processes,

$$y_t^f = \phi_{t-1}^T Z_{t-1}, \tag{11}$$

where the exact set of variables included into Z depends on the information set which the agents are assumed to use in forming their forecast. For more details on information sets, see below.[3] The agents' beliefs about reduced form coefficients ϕ are updated using a constant-gain variant of the Recursive Least Squares (RLS). The constant gain algorithm is one of many adaptive methods that allow operating in a non-stationary environment. Besides an advantage of being widely studied in the adaptive learning literature, this method has a natural interpretation as Weighted Least Squares where the weight of a data point depends geometrically on its vintage, with the most recent point getting the highest weight. The agents thus "forget" information from the distant past which might be desirable if the environment, and in particular the dependence of forward-looking variables on elements of Z, is perceived to be time-varying.

Every period, the agents are updating their beliefs in a constant gain RLS step:

$$\phi_t = \phi_{t-1} + gR_t^{-1}Z_{t-1}(y_t^f - \phi_{t-1}^T Z_{t-1})^T, \tag{12a}$$

$$R_t = R_{t-1} + g(Z_{t-1}Z_{t-1}^T - R_{t-1}). \tag{12b}$$

All endogenous model variables have zero means. Therefore, the beliefs should not include a constant. In some specifications of the learning we do not include the constant to be consistent with the theoretical solution. However, if we assume that the agents are also (implicitly) learning the values of the growth rates or inflation target, we include the constant into (12).

Given current beliefs, it is possible to derive the value of $E_t y_{t+1}^f$ as a function of

[3] In the adaptive learning literature, this equation is called the Perceived Law of Motion (PLM).

a constant, y_t, and w_t. One can then solve equation (9) for $\left(y_t^T, w_t^T\right)^T$ and derive a time–varying VAR representation of the model:

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu_t + T_t \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R_t \epsilon_t. \tag{13}$$

The values of μ_t, T_t, and R_t are then used to form expectations of the next period model variables in the Kalman filter. Thus, the estimation of a DSGE model under adaptive learning reduces to calculating a time–varying law of motion for the model and plugging it into the Kalman filter step, leaving the rest of the DYNARE toolbox largely untouched.

This procedure makes T_t a complicated function of the data, current parameters, and beliefs which could easily become unstable for one or several periods. As in common in the learning literature, we use a projection facility that skips an updating in such cases (see for instance Evans and Honkapohja 2001).[4]

2.3.2 Initial Beliefs

Equations (12) allow us to track the agents beliefs over time, if both the data and the initial beliefs are known. Following Slobodyan and Wouters (2009), we use three ways of selecting initial beliefs. In the first two ways, initial beliefs are consistent with some REE, while the third is based on regression estimates of the pre–sample data.

The first two ways of selecting the initial beliefs use equation (14) below to calculate ϕ_0 and R_0. At any REE, given for example by (10), one could derive a matrix of second moments of the model variables, Ω. These moments imply a relation between the forward-looking variables y^f and the variables used in forecasting Z, $y_t^f = \phi_0^T Z_{t-1}$, given by a projection of y_t^f onto Z_{t-1}. Initial condition for the second moments matrix R used in equations (12) is taken directly from the corresponding rows and columns of

[4]Standard projection facility is invoked when beliefs become unstable. Given that not all information sets lead to beliefs that could be described by some VAR, we have to resort to imposing projection facility when the transition matrix T_t loses stability.

Ω. The formulae for ϕ_0 and R_0 are given by

$$\phi_0 = E\left[Z_{t-1}Z_{t-1}^T\right]^{-1} \cdot E\left[Z_{t-1}\left(y_t^f\right)^T\right], \tag{14a}$$

$$R_0 = E\left[Z_{t-1}Z_{t-1}^T\right], \tag{14b}$$

where the expectations $E[]$ are derived using Ω.

Denote the parameter vector that is used to derive the model equations θ. Denote $\widetilde{\theta}$ a vector of parameters for auxiliary model which generates matrix $\Omega\left(\widetilde{\theta}\right)$ that is then used for calculations in (14). Then, in the first way of deriving initial beliefs, denoted W1, $\theta = \widetilde{\theta}$ at all times. In other words, initial beliefs are consistent with the REE produced by the estimated parameter vector θ. W1 is the closest to the pure rational expectations as only in-sample data variations could break the mapping of REE-implied relations between forward-looking variables y_t^f and predictors Z_{t-1} into the agents' perceptions of these relations; in the beginning of the sample, the two are the same. The way W1 is equivalent to the Case 2 in Murray (2007).

In the second way, W2, $\widetilde{\theta}$ is fixed while θ changes in the posterior optimization or MCMC steps. In principle, $\widetilde{\theta}$ could be selected to be any parameter vector. In this paper, we take several (usually 10 to 20) draws of $\widetilde{\theta}$ from the posterior distribution of parameters, approximated by the multivariate normal distribution, obtained after posterior maximization step under adaptive learning with W1 beliefs. W2 allows for more flexibility than W1, as the initial beliefs could now vary independently from the model itself. On the other hand, thus constructed sets of initial beliefs are consistent with RE equilibria that are rather similar to each other, because the parameter draws $\widetilde{\theta}$ are drawn from the same distribution; therefore, we consider W2 beliefs as a relatively minor disturbance that allows us to check the sensitivity of estimation results to the initial beliefs. In order to take the results of estimation under adaptive learning seriously, the estimation should pass some minimum set of requirements, such as being robust to W2 beliefs. The way W2 is close to, but not equivalent, to the Case 3 of

Murray (2007).

Our third initialization approach, W3, uses regression-based initial beliefs, obtained by running a regression of y_t^f on Z_{t-1} using pre-sample data. We pick the point estimate rather than a random point from the distribution of regression estimates, the latter being proposed by Giannitsarou and Carceles-Poveda (2007). This way represents a more serious robustness check for AL estimation that W2 for two reasons. First, correlation structure of the variables could change significantly between pre- and in-sample data, in which case pre-sample regression-based initial beliefs are of not much help to the agents in navigating in-sample environment. Second, the model could be so misspecified that even W1 beliefs consistent with REE of the pre-sample estimated model are still very far from those which could have been obtained by any regression. In both cases, our W3 beliefs are likely to be significantly different from the W1 ones, thus allowing us to observe the effect of initial beliefs on the estimation results.

2.3.3 Information Sets

Most of the theoretical results in the AL literature have been obtained for the case of Minimum State Variable (MSV) learning, where the agents form their expectations using a linear function of endogenous variables and stochastic shocks that is equivalent to the function one would derive as the REE solution of the model. In particular, the set of variables that is assumed to be available to the agents coincides with the variables that determine rational expectations of forward-looking variables. Thus, in MSV learning only the coefficients of the expectation-forming function could differ from their REE counterparts. MSV learning is one of the information sets that we use in this paper, denoted by I1. As is standard in the learning literature, we assume that the agents know exactly the law of motion (8) of the exogenous driving processes.

Assuming that the agents have access to the values of exogenous shocks is theoretically appealing as it enables convergence to the REE as an outcome of certain learning algorithms, namely, RLS with decreasing gain (a recursive analog of standard OLS regression) when the REE is E-stable, see Evans and Honkapohja (2001) for details. However, this assumption could be criticized as unrealistic. Therefore, we employ a second information set, I2, which assumes that the agents use the same endogenous model variables as the ones present in the REE solution, but not the exogenous stochastic processes.

Several papers in the small but growing literature on estimation of DSGE models under AL used an extreme informational assumption, making forecasts of macroeconomic variables depend only on own lag(s) of the variable itself and possibly a constant. Thus, the forecasting equations become a set of univariate AR(1) or AR(2) processes. Despite the fact that this approach denies the agents access to a significant amount of information available in the model, it was shown to lead to a very good model fit, see Vilagi (2007) and Slobodyan and Wouters (2010), among others. For this reason, we include an information set assumption I3 into our study, where all the forward-looking variables are believed by the agents to be simple univariate AR(1) processes.

In contrast to low–dimensional models studied by Milani (2007), Sargent, Williams, and Zha (2006), or Vilagi (2007), combination of some of the information sets and some models leads to necessity of accessing values of endogenous variables that are not observed. In such cases we use output from the Kalman filter used to construct the likelihood function for a particular combination of parameters on both sides of the updating equation (12).[5]

2.4 Estimation Results

2.4.1 Data and Priors

For our estimations, we use the data set constructed in Fagan et al (2001) over the period 1970:Q1–2007:Q4. The set of observables (varies from 3 to 7 variables depending

[5]We use only filtered estimates of endogenous variables, both of right- and left-hand sides of the forecasting equations.

on the model) includes the key macro-economic variables of the euro area. When constructing the observables, the following time series were used: real GDP (YER), GDP deflator (YED), compensation to employees (WIN), number of employees (LNN), short-term nominal interest rate (STN), real consumption expenditures (PCR), real investment (ITR). The time series of real wages is constructed as WR=(WIN/LNN)/YED. The STN time series was divided by 4 to obtain quarterly data. The natural logarithms of all variables except the STN were taken. The inflation rate is given by $\ln(YED_t) - \ln(YED_{t-1})$. Real variables are linearly detrended using a separate trend for each variable, estimated by OLS; inflation and the nominal interest rate are detrended by the same linear trend in inflation as in Smets and Wouters (2003).

We estimate all the models using Bayesian methods. The table of priors is presented in the Appendix A. We mostly followed the priors chosen in the original papers: Smets and Wouters (2003) for *M3* and Vilagi (2007) for *M1*. For the model *M2* are undertook a combined approach: prior distributions for shocks and some of the structural parameters (like habit and Calvo prices) are based on Smets and Wouters (2003), at the same time we wanted to keep priors on some of the nominal rigidities (which have most controversial empirical support , like price and wage indexation) as loose as possible. In this approach, we followed Vilagi who assumed uniform prior distributions for indexation. For the same reasons, we chose uniform prior on investment adjustment cost in model *M3*, and thus departed slightly from Smets and Wouters (2003). Overall, since the major task of this paper was to investigate the impact of AL on structural parameters, in particular nominal and real rigidities, we tried to avoid restricting such parameters by strict priors, providing instead for maximum flexibility while attempting to obtain unimodal posterior distributions under both RE and AL. Some of the rigidities such as habit formation appeared to be rather robust to the change of priors.

A number of parameters were calibrated. Similarly to Smets and Wouters (2003), we fix discount factor β at 0.99, capital share $\alpha = 0.30$, the depreciation rate $\tau = 0.025$; the share of steady state consumption in total output and the steady state investment share

are assumed to be equal to 0.6 and 0.22 respectively. The labor disutility parameter η is assumed to be fixed in model *M1* and equals 2.5.

Typically, 200,000 to 500,000 MCMC draws were performed, using two (in some cases three) MCMC chains. For more details on Bayesian estimation of DGSE models, see An and Schorfheide (2007).

2.4.2 Model Fit

The model fit of a model estimated using Bayesian methods can be ascertained using marginal data density, defined as

$$p\left(Y|\mathcal{M}\right) = \int \mathcal{L}\left(\theta|Y\right)p(\theta)d\theta,$$

where $\mathcal{L}\left(\theta|Y\right)$ is the likelihood function of the data Y given parameters of the model θ, and $p(\theta)$ is the prior density. This measure allows a straightforward comparison of two models, say $\mathcal{M}_1$ and $\mathcal{M}_2$ that are estimated on the same data. Posterior odds ratio, a measure of how much more likely a model $\mathcal{M}_1$ is when compared to the model $\mathcal{M}_2$, is given by

$$\frac{\pi\left(\mathcal{M}_1\right)}{\pi\left(\mathcal{M}_2\right)} \cdot \frac{p\left(Y|\mathcal{M}_1\right)}{p\left(Y|\mathcal{M}_2\right)},$$

where $\pi\left(\mathcal{M}_i\right)$ represents prior probability of a model $\mathcal{M}_i$. The first term in the above expression is known as prior odds, and the second as Bayes factor. Usually, the prior probabilities are taken to be equal, and thus a posterior odds ratio equals the corresponding Bayes factor. For more details on model comparison, consult An and Schorfheide (2007).

Logarithms of marginal data densities from the estimations of our models are presented in the Table 2.1. Initial beliefs are constructed using the RE-consistent method of way W1. Out major result is that the RE hypothesis is indeed restrictive. Relaxing the rationality assumption through introduction of adaptive learning improves the marginal data density of the model for essentially all learning specifications: the only case where RE and AL models have similar fit is model *M2*, information set I1 (MSV learning) with a constant. In all other cases the AL model fit is significantly better that its counterpart under rational expectations. It is hard to compare the Bayes factors across models that have different number of observable variables — three in *M1*, five in *M2*, and seven in *M3*. If the hypothesis of the rational expectations as the main source of misspecification of a DSGE model is correct, then the adaptive learning could correct some of it. One could presume that the resulting improvement in marginal data density of the model under AL relative to the RE model then reflects the degree of mis-specification which could be different in models *M1*–*M3*. Testing this conjecture is beyond the scope of the current paper.

Table 2.1: Model Comparison in Terms of MArginal Likelihood

Model specification	*M1*	*M2*	*M3*
REE	-134.96	-182.83	-468.83
AL without constant:			
I3: univariate AR(1)	-125.61	-137.66	-421.65
I2: endogenous states	-130.39	-147.71	-436.76
I1: endogenous states and shocks	-129.36	-174.27	-449.11
AL with constant:			
I3: univariate AR(1)	-119.36	-129.2	-419.6
I2: endogenous states	-131.45	-153.22	-442.19
I1: endogenous states and shocks	-123.5	-182.7	-461.68

Log marginal data densities for the three models using different information set assumptions and REE-consistent initial beliefs W1. Bayes factor — a relative probability of one model over another, equals exp of the difference between the corresponding log densities.

Another result is that the most restrictive information set I3 is indeed the best for all three models. This result has been observed previously by Vilagi (2007). Slobodyan and Wouters (2010) also suggest that endowing the agents with a minimal set of variables used in forecasting may work well in practice. The evidence on the other two information sets is more mixed: I1, the largest set which is consistent with the rational expectations MSV solution, is marginally better than the restricted MSV set I2 for the smallest model *M1* but is significantly worse in the larger models *M2* and *M3*. Overall, though, we can observe a clear ranking I3–I2–I1, making a very strong case for the statement that the more restrictive is the information set available to the agents for forecasting forward-looking variables, the better is the model fit.

Comparing the AL estimations with and without the constant, we observe a clear separation between the best set of variables I3 and the worse group of I2 and I1. For the very economical forecasting equations implied by assuming I3 (just own lag in every forecasting equation), including the constant improves the model's marginal data density significantly, especially for the smaller models *M1* and *M2*. For sets I1 and I2, including the constant worsens the marginal data density, sometimes by so much that the overall model fit is essentially the same as under the RE (information set I1 with constant, model *M2*). The only exception to this rule is model *M1*, set I1. We believe that large sets of regressors I1 and I2, when used in forecasting equations on the real data over the estimation period, might lead to overfitting. In this case, adding an extra variable — a constant — makes the overfitting problem worse. In the model *M1*, the overfitting problem is not as severe because the total number of right-hand side variables is small (three endogenous variables and one shock). Notice that for the intermediate information set I2, worsening of the fit after including the constant is minimal in model *M1* and moderate in model *M2*, which is consistent with the overfitting of the forecasting equations explanation.

Finally, we analyze the relative fit of alternative model specifications as a function of time. Specifically, we would like to find out whether the gain in the model fit observed under the information set I3 comes from a specific (short-lasted) time period, or whether the superior performance of the model based on univariate forecasting rule holds for the longer time span. Figure 2.1 shows the cumulative likelihood for model specifications I2 (dashed line) and I3 (dotted line) relative to I1. If a line is trending up in this graph on some time interval, this means that on average the likelihood on this interval is better relative to the I1 model. Figure 2.1 indicates that I3 model does better than I1 almost all the time, except for 1985-1988 and 1990-1993. Before 1980 and after 1993 we observe a persistent positive trend in the I3 relative cumulative likelihood. This means that for most of the sample the model with I3 set is more appropriate for describing the data generating process than the specification implied by the information set I1; the better model fit is broadly based rather than being due to a particularly favorable performance at a specific time. On another hand, the gain in the model fit under I2 relative to I1 is relatively modest, especially in the second half of the sample. In the first half of 1970es the model with I2 information set performs worse than the one with the full set of variables and shocks I1.

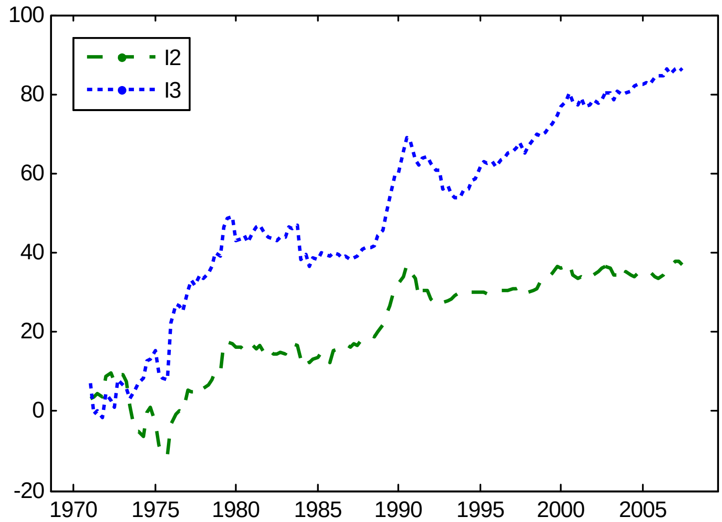


Figure 2.1: Cumulative likelihood for estimated model specifications I2 and I3 relative to I1.

2.4.3 Estimated Parameters

Given a large number of treatments in the paper, we compare the effect of AL assumption for the estimates of structural rigidity parameters and persistence of exogenous shocks only for the best information set identified in the previous section, namely I3 with a constant. We will treat this specification as a baseline and consider the outcomes with other sets of variables as a form of sensitivity analysis.

Table 2.2 presents an overview of the main results of our estimation. As is obvious from the Table, under AL some estimated structural rigidities and persistence of the shocks fall, sometimes significantly. Among the parameters present in all three models, habit persistence parameter h presents the clearest picture: its estimate is lower under AL than under RE. The drop is quite significant: in *M1* and *M2*, posterior mean under adaptive learning lies outside of the 95% Highest Probability Density (HPD) interval of RE estimation, for *M3* it is less expressed because the HPD under RE is very wide. Estimated Calvo pricing and inflation indexation parameters fall in *M1* but stay unchanged or even increase marginally in larger models *M2* and *M3*. Among parameters present in *M2* and *M3* only, Calvo wages parameter falls marginally, wage indexation remains at essentially zero level as under RE, and Calvo parameter for employment falls. Finally, the largest (*M3*) model-specific parameters — elasticities of investment adjustment costs and of capital utilization — both fall, with adjustment costs exhibiting the most dramatic decline among all the parameters studied (from 9.44 under RE to 3.21 under AL).

These results taken in the whole signal that there is indeed an overall drop in structural rigidity parameters when the RE assumption is replaced by the AL one. The parameters that are estimated to be somewhat extreme under the RE fall the most (habit persistence in consumption h, investment adjustment costs φ, Calvo prices ξ_p, Calvo employment ξ_e), with Calvo wage parameter ξ_w being somewhat exceptional. An increase in importance of "mechanical" sources of rigidities is very seldom observed — basically, only price indexation parameter in the model *M2* inches up marginally under AL from a very low level of 0.20 estimated under RE.

We also observe that the overall decrease in importance of "mechanical" frictions is most pronounced in a small model *M1* where there are few rigidities. This outcome is consistent with the view that adaptive learning to a significant degree serves as a tool of remedying misspecifications. For example, both Calvo pricing and price indexation are estimated to be extremely high under the RE in model *M1* ($\xi_p = 0.97$, $\iota_p = 0.71$). These parameters drop significantly under the AL. Larger model *M2* adds wage rigidities which probably relax the misspecification present in *M1*. As a result, there is not much movement in this group of parameters (Calvo prices and wages, indexation of prices and wages) between RE and AL estimations in *M2*. This result indicates that the ability of adaptive learning to substitute for real and nominal rigidities can be overestimated if one uses very small model. Therefore, Milani's conclusions who obtained his results in the estimated three equation NK model cannot be expected to apply in more complicated models to the same extent.[6] We conclude that learning can substitute for "mechanical" source of rigidities only partially; some of the structural frictions remain quite strong.

Turning attention to the persistence of exogenous processes, we see that there is no clear pattern: productivity shock can become less persistent while employment equation shock persistence goes up. Demand shock becomes less persistent and become more precisely estimated in *M1* and *M3* but remains well within Rational Expectations HDP for this parameter that is very wide in both cases.

Comparing out results to others in the literature, we do not observe significant decline of consumption habits and price indexation parameters to zero in as in Milani (2007) in a simple model *M1*, while in more complex models *M2* and *M3* price indexation is low already under the RE. Murray (2007), comparing Cases 1 and 3 (Case 3 is similar to the information set I2, not presented in the Table 2.2), observes an essentially unchanged habit formation parameter (0.11 under RE to 0.12), and a dramatic *increase* in capital adjustment friction, a parameter playing a role that is similar to our φ, which is found to drop significantly under the AL in *M3*, the only model where this friction is present. Murray also finds a significant decrease in price indexation from 0.36 to 0.0, which is somewhat consistent with our estimates. Jääskelä and McKibbin (2010) estimate an open economy model of Australia with initial beliefs constructed by a method rather close to our W1 and find that both habit persistence and share

[6] We have to note that similarly to Murray (2007) and Vilagi (2007), we do not confirm Milani results who found some of the structural rigidity parameters to become insignificant undel learning. Whether this discrepancy is due to the estimation method used (Murray (2007) used Maximum Likelihood estimation) or the data (European in Vilagi (2007) and here vs. US in Milani 2007, 2008) is a subject of further study.

of rule-of-thumb consumers (this parameter plays a role somewhat akin to the price indexation parameter) *increase* under AL. Slobodyan and Wouters (2008, 2009) report that in a large model (very similar to *M3*) investment adjustment cost elasticity φ typically drops significantly while habit persistence can grow (in Slobodyan and Wouters 2009; note, however, that they report habits increase from 0.70 under RE to 0.74÷0.79 under AL, while here habits *drop* to 0.73, *i.e.*, to approximately the same value) or decrease slightly (in Slobodyan and Wouters 2008; from 0.77 to 0.68). Finally, Vilagi (2007) finds that habit persistence could either decrease from 0.99 to 0.79 (in a small model equivalent to *M1*) or stay unchanged at 0.99 in a model similar to *M2*. Price indexation drops significantly in both a small model similar to *M1* (from 0.63 to 0.15) and a larger *M2*-like model (from 0.44 to 0.29). Wage indexation also reduces from 0.75 to 0.02. Insignificance of indexation parameters under AL is confirmed by us. Vilagi also finds an unambiguous although small in magnitude drops in Calvo parameters in price- (both *M1*-like and *M2*-like models) and wage-setting (*M2*-like model), which are confirmed by us only for the *M1* model case.

Thus, putting aside results of Milani, one consistent finding of the literature is that capital adjustment cost parameter falls under AL. Another, more tentative, conclusion is that adaptive learning estimation leads to habit persistence parameter's increase if under RE it is less than 0.75, but it decreases if already high under RE as in this paper or Slobodyan and Wouters (2008). It is difficult to establish a common pattern among the parameters describing nominal rigidities because of differences in this modelling block and estimated parameters; overall, there seem to be no uniform movement in nominal rigidities in one or another direction under AL in the papers surveyed, with Vilagi (2007) being an important exception. Our results partially confirm Vilagi (2007) but for the fact that indexation parameters are already extremely low in larger models.

Table 2.2: Model Comparison in Terms of Estimated Parameters

	h	φ	ι_p	ι_w	ξ_p	ξ_w	ξ_e	ψ	ρ_a	ρ_b	ρ_g	ρ_l	ρ_i	g
M1:RE	0.89		0.71		0.97					0.58				
AL	0.79		0.61		0.91					0.50				0.101
M2:RE	0.90		0.20	0.05	0.91	0.80	0.86		0.96	0.50		0.92		
AL	0.78		0.24	0.04	0.91	0.77	0.75		0.93	0.51		0.97		0.063
M3:RE	0.88	9.44	0.23	0.05	0.91	0.79	0.75	0.21	0.99	0.42	0.93	0.89	0.93	
AL	0.73	3.21	0.24	0.04	0.90	0.73	0.58	0.14	0.97	0.34	0.94	0.97	0.89	0.046

In every cell, the top number is the posterior mean of the estimated parameter distribution under RE, and the bottom one is the result of estimation under the baseline AL specification (only own lag and a constant used to forecast every forward-looking variable). Only structural rigidity parameters and persistence of exogenous shocks are presented.

2.4.4 Information Sets

As stated previously, information set which the agents use in forming their expectations affects the model fit to a large degree. To access whether a similar effect is observed with respect to the estimated parameters, in the Table 2.3 below we present the estimates for the middle-of-the-road model *M2* under adaptive learning for all information sets. The most striking result is that parameter values are affected to a significant degree by the information set. For example, some structural rigidity parameters actually increase, rather than decrease, under AL estimation when the agents are allowed access to the MSV solution consistent information set I1. Calvo wages and employment and price indexation are the lowest in the baseline I3 estimation and the highest in MSV I1 estimation, with I2 set being right in the middle. With I1, these parameters are actually higher than under RE. Overall, the REE-consistent set I1 delivers the worst outcome in terms of "mechanical" sources of rigidities, as all parameters either stay the same as under RE or slightly increase, with ρ_l, persistence of the employment shock, being the only exception.

Another interesting feature to note is that information sets with and without the constant deliver the most consistent results for the smallest set I3. The only exception is the estimated gain parameter which is rather high in the baseline estimation with a constant (0.063) but is much lower without it (0.024). As described in the following subsection, high gain implies beliefs that change rather fast in reaction to the data. On the other hand, all other estimations deliver much lower value of the gain in the region of $0.02 \div 0.03$ (and even 0.007 for MSV solution consistent set I1 with constant). If the beliefs are close to being unstable at some point during the estimated sample, the estimation procedure could select a lower gain in which case probability of invoking the projection facility declines.[7] We discuss this issue in more detail in the next subsection where we describe the beliefs about inflation process implied by different information sets.

Considering the whole set of estimations for different information sets, we can say that in accordance with the model fit results, the set of variables that the agents are assumed to be using for forming expectations influences the estimation significantly. Therefore, any comparison of the results of estimation under adaptive learning across different models should take into account the variables used by the agents; in case the assumed information sets differ significantly, the results could not be compared.

Table 2.3: Information Sets Comparison in Terms of Estimated Parameters

Model M2	h	ι_p	ι_w	ξ_p	ξ_w	ξ_e	ρ_a	ρ_b	ρ_l	g
Rational Expectations	0.90	0.20	0.05	0.91	0.80	0.86	0.96	0.50	0.92	
Adaptive Learning, no constant										
I3: univariate AR(1)	0.80	0.20	0.05	0.90	0.75	0.74	0.94	0.46	0.95	0.024
I2: endogenous states	0.79	0.41	0.18	0.90	0.82	0.78	0.92	0.50	0.92	0.022
I1: endogenous states and shocks	0.92	0.68	0.08	0.89	0.91	0.88	0.97	0.56	0.51	0.028
Adaptive Learning, with constant										
I3: univariate AR(1)	0.78	0.24	0.04	0.91	0.77	0.75	0.93	0.51	0.97	0.063
I2: endogenous states	0.78	0.51	0.14	0.90	0.81	0.79	0.91	0.53	0.92	0.022
I1: endogenous states and shocks	0.88	0.58	0.06	0.90	0.86	0.96	0.96	0.58	0.68	0.007

2.4.5 Initial Beliefs and Sensitivity

As mentioned previously, different ways of initializing initial beliefs could be considered as a sensitivity analysis for the AL estimation. Estimation under adaptive learning

[7]Estimations with frequent projection facilities tend to fit the data poorly.

naturally introduces some free parameters. In W1 way of initializing the learning — REE-consistent initial beliefs — there is only one extra parameter, the gain. However, if the estimation depends too sensitively on the initial beliefs, one could criticize AL estimation with W1 initial beliefs as using in-sample data for the belief optimization.[8] If, on the other hand, we could establish that the estimation results remain relatively stable when initial beliefs are selected from a distribution centered on the W1 beliefs, we could claim that the estimation is robust to small errors that the agents could make in forming the beliefs. In the latter case, the estimation results are a function of the time variability in expectation-forming function and of changes in the transmission mechanism that are associated with a particular information set, not the initial beliefs that could be somewhat arbitrary.

To generate W2 initial beliefs, we take the multivariate normal distribution of parameters implied by the posterior maximization of a model under adaptive learning with W1 initial beliefs, and select $10 \div 20$ draws from this distribution.[9] For every parameter draw, the corresponding REE is then constructed and initial beliefs are derived that are consistent with the REE in accordance with (14). These initial beliefs are then fixed for the duration of estimation using MCMC. As we argued before, W2 beliefs allow for some flexibility by disentangling the initial beliefs' REE (and, thus, transmission mechanism) and the REE implied by the current parameter draw in the MCMC. On the other hand, W2 beliefs stay constant during the MCMC, which represents a constraint on the estimation. In case the estimation results are very sensitive to the

[8] Several papers using estimation under AL have used optimized initial beliefs, cf. some specifications in Milani (2007) and Sargent, Williams, and Zha (2005), effectively using the in-sample data twice. This procedure is hard to justify if one takes the story of agents as econometricians seriously.

On the other hand, REE-consistent W1 initial beliefs in this paper correspond to the agents who know the probability distribution that would be obtained under particular parameter values and use is as a starting point, but allow for non-stationarity and/or structural breaks when forming expectations in real time.

[9] One could also use the MCMC output directly and randomly select points from there, thus drawing from true posterior distribution of parameters, not its multivariate normal approximation that could be significantly incorrect. Beside simplicity, our procedure has an advantage of allowing to draw from scaled up or down distribution easily. We had to restrict our distribution to 50% of the approximate multivariate normal in order to guarantee that most draws for initial beliefs result in a point that satisfies inequality restrictions imposed on parameters during the estimation.

initial beliefs, one would expect either a significant divergence of estimation results in terms of the model fit or estimated parameters, or at least an increase in confidence intervals of the estimated posterior distribution of parameters. Neither of these effects is expected when the estimation is insensitive to the initialization.

Another consequence of W2 initial beliefs is as follows. Consider for a moment posterior probability maximization step under W1 initial beliefs. In the mode, the REE that is used to construct the initial beliefs (defined by the parameter vector $\widetilde{\theta}$) and the REE implied by the model parameter vector θ are constrained to be equal, $\theta = \widetilde{\theta}$. Denote posterior mode as $\theta_0 = \widetilde{\theta}_0$. Now, fix the initial beliefs' parameter vector at $\widetilde{\theta}_0$ and re-optimize the posterior with respect to θ. It is clear that at the resulting posterior mode, θ_1, we should have a higher value of the posterior, because it is always possible to set $\theta_1 = \widetilde{\theta}_0$ and get the W1 posterior value.[10] Normally, higher posterior is translated into better marginal data density, and thus we expect that W2 estimation with the initial beliefs' parameter vector $\widetilde{\theta}$ fixed at W1 posterior mode value will fit the data better (will have higher marginal data density). If, on the other hand, W2 beliefs parameters $\widetilde{\theta}$ are fixed at values that are close to, but not equivalent, to the W1 parameter vector θ, this "partial optimization" effect is counterbalanced by the fact that the initial point in this procedure is likely to have lower posterior that the W1 estimation. If the W2 initial beliefs are close to W1 ones, we would expect the "partial optimization" effect to be stronger, and thus the model fit to be improved.

We conduct analysis with W2 beliefs for the baseline information set I3 and all models. The results across models are qualitatively similar, and thus we concentrate on reporting middle-of-the-road model *M2* estimations only.

Consider first the model fit. Marginal data density for the baseline AL estimation for *M2* equals -129.2 (see Table 2.1). In 17 W2 estimations for the same model with baseline information set, the marginal data density varies between -129.5 and -127.3,

[10] This estimation could then be considered as a first step in the joint optimization of model parameters θ and belief parameters $\widetilde{\theta}$, the next step being fixing the model paramters at θ_1 and optimizing with respect to the belief parameters $\widetilde{\theta}$ to get $\widetilde{\theta}_1$, etc.

with the mean of -128.0. In other words, the changes are tiny, but on average we observe a slight improvement, consistent with the "partial optimization" logic given above.

Now, let turn attention to the estimated parameters. We select three W2 estimations, those with the highest, the lowest, and the median marginal data density among the 17 W2 runs that we have conducted. Table 2.4 presents, for the parameters that reflect nominal and real rigidities, the posterior mean and the confidence interval for the selected estimation runs. Clearly, there is little or no difference between the three estimations. Only for the gain parameter we do observe somewhat larger changes, but they are well within the estimated HPD intervals. Comparing the parameter estimates with the corresponding row in the Table 2.3, we see that the parameter estimates under W2 are extremely to those under W1 but sometimes slightly biased, which is consistent with the marginal likelihood being higher on average with W2 initial beliefs than W1 beliefs.

A very similar picture can be observed for other models under baseline AL estimation. Thus, we can conclude this part of the sensitivity analysis by stating that at least for the best information set (only own lag and a constant are used to form expectations), estimation under adaptive learning with REE-consistent initial beliefs is extremely robust to small disturbances in the beliefs.

Table 2.4: Sensitivity of Estimated Parameters to Initial Beliefs

Marg.lik	h	ι_p	ι_w	ξ_p	ξ_w	ξ_e	ρ_a	ρ_b	ρ_l	g
Lowest	0.79	0.25	0.04	0.91	0.77	0.75	0.93	0.50	0.96	0.058
	0.68 0.91	0.14 0.34	0.0 0.09	0.89 0.93	0.72 0.82	0.72 0.78	0.88 0.97	0.39 0.62	0.92 0.99	0.04 0.07
Median	0.79	0.25	0.04	0.91	0.77	0.75	0.93	0.51	0.95	0.066
	0.68 0.91	0.15 0.36	0.0 0.09	0.89 0.93	0.71 0.82	0.72 0.78	0.88 0.97	0.36 0.62	0.92 0.99	0.05 0.08
Largest	0.78	0.25	0.04	0.91	0.76	0.75	0.93	0.51	0.95	0.064
	0.67 0.90	0.15 0.36	0.0 0.09	0.88 0.93	0.71 0.82	0.72 0.78	0.88 0.98	0.40 0.62	0.93 0.99	0.05 0.08

Model *M2* estimated under the baseline AL specification (only own lag and a constant used to forecast every forward-looking variable), W2 beliefs. Only 3 (out of 17) estimations are presented, with lowest, median, and highest marginal likelihood. In every cell, the top number is the posterior mean of the estimated parameter distribution under AL, and the bottom one is the corresponding confidence interval. Only structural rigidity parameters and persistence of exogenous shocks are presented.

We next turn to another robustness exercise, where initial beliefs are derived from OLS regression using pre-sample data — W3 beliefs. We use 20 quarters of the data to form initial beliefs, and estimate the model using the rest of the sample. Again, we present the results only for the Model *M2*, baseline information set I3 with a constant. Given that the estimated sample is now different, we re-estimate the model using this shorter sample under W1 beliefs as well.

Regarding the model fit, W3 beliefs are doing worse than our baseline adaptive learning specification: marginal likelihood is just -54.8 *vs.* -43.2 for W1 beliefs. Both specifications, though, fit the data significantly better than Rational Expectations estimation at -96.0. The reason is probably that pre-sample based initial beliefs generate forecasting functions that are largely inconsistent with the in-sample data. We discuss the issue in more detail in the following subsection.

Comparison of the estimated parameters shows that estimated parameters differ surprisingly little between the two AL specifications: the only parameter that differs noticeably is price indexation, which increases from 0.25 in the baseline specification to 0.46 under pre-sample beliefs. The gain also differs. As the gain parameter is related to the speed with which the beliefs are updated, this implies that the biggest difference between the beliefs could be their volatility and the speed with which expectation formation function adjusts to the new data. We return to this question again in the next subsection. Finally, for this shorter sample the difference between estimations under RE and baseline AL with W1 beliefs is similar to that presented in the Table 2.2.

Table 2.5: Initial Beliefs Comparison in Terms of Estimated Parameters

	h	ι_p	ι_w	ξ_p	ξ_w	ξ_e	ρ_a	ρ_b	ρ_l	g
RE	0.87	0.25	0.11	0.88	0.77	0.83	0.934	0.41	0.92	
W1 beliefs	0.81	0.25	0.11	0.83	0.70	0.70	0.96	0.47	0.91	0.058
W3 beliefs	0.79	0.46	0.06	0.81	0.72	0.71	0.95	0.45	0.89	0.025

Posterior means of the estimated parameters under RE, AL with REE-consistent initial beliefs, and AL with pre-sample based beliefs. Baseline AL specification (own lag and a constant), model *M2*. Only structural rigidity parameters and persistence of exogenous shocks are presented.

2.4.6 Beliefs About Inflation and Transmission Mechanism

Adaptive learning could affect model fit in several ways. First, time variation of beliefs allows the model itself to become time varying, cf. (13). This could improve model fit if the process that generates time series of observed variables is itself time-varying. On the other hand, if the beliefs updating process is too volatile, parameter uncertainty could lead to deterioration of the fit. Another channel through which adaptive learning operates is through change in the transmission mechanism. Even when the beliefs are consistent with a REE and are not time varying, if the information set used by the agents to form expectations differs from the MSV set then the transmission mechanism differs from that under the Rational Expectations. The fact that information set affects estimations to a larger degree than initial beliefs (compare the results presented in Tables 2.1 and 2.3 with those of Tables 2.4-2.5) informs us that the transmission mechanism effect could be more pronounced than that of time variation.

To illustrate the effect of the transmission mechanism, observe Fig. 2.2 which shows the coefficients in the agents' forecasting function for inflation (also called the Perceived Law of Motion, or PLM),

$$\pi_t = a + \rho\pi_{t-1}.$$

We present values of a and ρ for W1 and W3 beliefs, I3 information set, model *M2*, *i.e.*, the same adaptive learning specifications that were described in the Table 2.5. As beliefs evolve over time, a and ρ are time varying. One difference between the initial beliefs is immediate: pre-sample regression based initial beliefs W3 (blue lines) are much more volatile, despite the fact that estimated gain g for W3 beliefs is much lower than under W1 beliefs (0.025 *vs.* 0.058). According to the updating equations (12), innovations to beliefs are given as $gR_t^{-1} \cdot Z_{t-1}\epsilon_t^T$ where ϵ_t is the forecasting error at time t. Amount of update is then proportional to the effective gain gR_t^{-1}. Under pre-sample beliefs, effective gain is much larger despite the gain g being lower, which implies that R_t, the second moments matrix (essentially, variance of inflation), is much

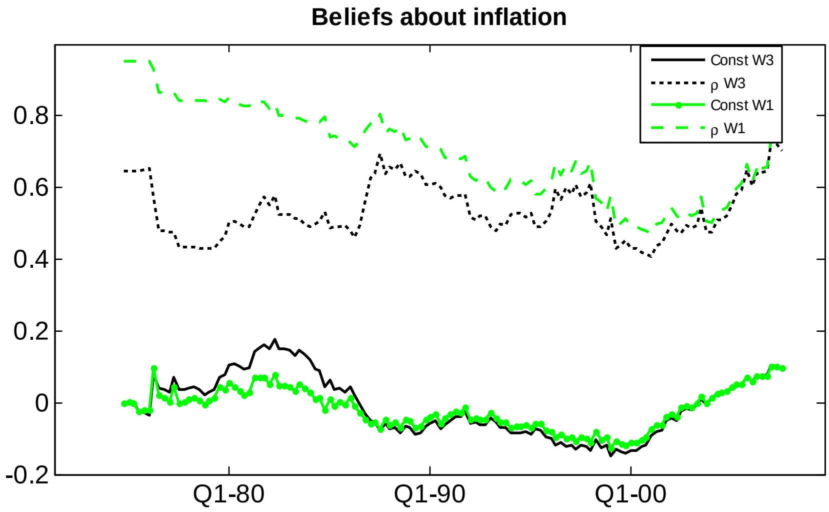


Figure 2.2: PLM beliefs about inflation (constant and persistence) under REE-consistent initial beliefs (W1) and pre-sample based initial beliefs (W3). The agents perceive the following inflation process:

$$\pi_t = a + \rho\pi_{t-1}.$$

smaller in the pre-sample than the value implied by the REE with which W1 beliefs are consistent. Despite the fact that W3 beliefs seem to be more 'correct' (perceived inflation persistence ρ, blue dotted line, is essentially constant over the estimated time interval, while a systematic decline is observed for W1 beliefs), their volatility leads to a significantly worse model fit.

'Initial beliefs' in the Fig. 2.2 are given as the very first point of the graph. In the Rational Expectations Equilibrium implied by the model parameters in W1 estimation, the agents believe in a very persistent inflation. This feature is shared by the models *M2* and *M3* but, significantly, not *M1* (initial inflation persistence in model *M1* is closer to 0.5). Notice that if the initial persistence is close to unity, the effective gain parameter is restricted to low numbers; otherwise, large forecasting errors could result in update of persistence to values above one, which invokes projection facility. Estimation with large numbers of projection facilities tend to result in a very bad model fit.

Thus, there is a basic tension in initial beliefs that are REE-consistent. The beliefs contain point estimates of the parameters in the forecasting functions (ϕ in Eq. 12), and perceived volatility of variables used to forecast forward-looking variables. If the perceived volatility is inconsistent with the data-generating process of observed variables, the estimation procedure whould attempt to adjust the gain parameter to counteract effect of 'wrong' R_0. However, the gain parameter that is too large could then lead to frequent projection facility hits and very volatile beliefs ϕ, with deteriorating model fit as a consequence. A way of overcoming this problem might be to introduce two gain parameters, one for updating point forecasting function parameters ϕ and another for updating the second moments matrix R, or adding a scale parameter for the matrix R that could be either estimated or calibrated.

To illustrate the effect of the information set used for forecasting on the transmission mechanism, we perform the following. We take a model *M2* estimated under Rational Expectations and fix the parameters at their posterior mode values. Then, taking these parameter values as given, we construct initial beliefs under the information sets I1, I2, and I3 that are compatible with this REE.[11] Finally, using thus derived transmission mechanism (matrices T and R in Eq. 10), we calculate impulse response function of inflation, interest rate, and output to productivity, price mark-up, consumption preference, and monetary policy shocks. The results are presented in Fig. 2.3. The solid, dashed, and dotted lines correspond to impulse responses under I1, I2, and I3 information set, respectively. As we see from the figure, even in cases when the impact effect of a particular shock on a variable is similar under the three information sets, the impulse responses show very disparate transitions towards the long-run steady state (*i.e.*, the response of nominal interest rate to consumption preference shock). On another hand, there are impulse responses that diverge already on impact. Some of them also evolve in different directions during the adjustment (for example, the response of output to price mark-up shock). Therefore, even for the same model parameters, the three alternative information sets used by learning agents for

[11] At the REE, the constants in forecasting functions must equal zero, therefore, information sets with and without the constant generate equivalent initial beliefs.

forecasting imply different transmission mechanisms. Our results also indicate that there is a tendency for impulse responses with I3 information set to be more persistent but less pronounced in magnitude, at least at the REE consistent initial beliefs.

Another exercise useful for studying the properties of the transmission mechanism under alternative learning schemes is the analysis of inflation persistence implied by different information sets. In order to perform this task, we take estimation results under learning for the model *M2* with three information sets. In every case, we fix the parameters at the corresponding posterior mode values and compute inflation persistence implied by the Actual Law of Motion, given by (13).[12] The results are presented in Fig.2.4. In accordance with the previous analysis of Impulse Responses, we see that initial I3 REE-consistent beliefs imply very persistent inflation. Another important observation is that implied inflation persistence under I1 and I2 information sets is rather stable over time, whereas it exhibits dramatic changes under I3. This means that if "true" inflation persistence based on real data was changing fast, the I3 learning model would be better positioned to capture this data generating process. Given that I3 learning rule outperforms I2 and I1 in terms of the model fit (see Table 2.1), we suggest that the data generating process was indeed changing with the time, and I3 model was successful in capturing such a dynamic adjustment. We believe that our results are in line with some of the previous studies on inflation persistence (especially those that estimated the persistence with time-variation in the mean of inflation or over short time sample), which suggest that inflation in the euro area might have been only moderately persistent in 1995-2002. For a detailed survey of such results see Table 3.1 in Altissimo et al (2006).

To provide somewhat more intuition about the dynamics of implied inflation persistence, we consider the components that contribute to the update of the beliefs coefficients. Equation (12a) illustrates that, given the same forecasting error, the update

[12] Note that we cannot compare the beliefs about inflation directly as in the Fig. 2, because the forecasting equations are different under the information sets I1-I3. In terms of the adaptive learning literature, Fig. 4 presents the Actual Law of Motion, or ALM, for inflation.

of belief coefficients ϕ will depend on the value of the "effective" gain gR^{-1}, where R^{-1} is inverse of the matrix of the second moments. Figure 2.5 shows one of the terms of the second moments matrix corresponding to expected squared inflation for 3 models I1, I2, and I3. The graph indicates that squared inflation is expected to be very high under I1 information set, thus making the effective gain very small and resulting in relatively minor updates to the belief coefficients in response to the forecasting errors. This, in turn, leads to very smooth time series for implied inflation persistence. In a model with I2 information set, agents believe that the second moment of inflation is much smaller and rather stable, which implies that beliefs are updated stronger than in I1 case, but the implied inflation persistence still does not vary much. Finally, agents using I3 information set believe that, initially, expected squared inflation is rather high but drops fast, which is consistent with a noticeable fall in implied inflation persistence. This downward adjustment of perceived second moment of inflation combined with high estimated gain coefficient (about 2 times higher than for I2 or I1) result in a high value of "effective" gain. High gain means significant updates of belief coefficients which is reflected in a significant fall of implied inflation persistence under I3 beliefs before 2000.[13]

2.5 Conclusions

As far as the sensitivity of the estimation results to the chosen learning rule is concerned, we find that the more restrictive information set available to the agents is, the better is the model fit. In particular, the greatest improvement in marginal data density and the most significant change in the estimated parameters relative to the RE estimation are observed when the forecasts are made using univariate AR(1) processes. In general, the estimated parameters associated with different information sets used in forecasting rules vary significantly. We demonstrate that for the full information set

[13] Subsequent increase in implied persistence is explained by the fact that during this period the simple AR1 forecasting model was mostly underpredicting inflation, which led the agents to update perceived persistence of inflation upwards.

consisting of all endogenous states and shocks — the same set of variables as under rational expectations, the estimated structural rigidity parameters as well as model marginal data density are closest to those obtained under RE. This conclusion is in line with some of the earlier research which documented little difference between the estimation results under RE and adaptive learning. Very importantly, we have established that this conclusion holds independently of the model complexity: In all three models considered here, ranging from a three equation New Keynesian model to a Smets and Wouters type model, forecasting forward-looking variables using univariate AR(1) processes brings the best results.

We believe that the reason for significant differences among the estimation results with alternative information sets is the effect of the set on transmission mechanism of the model: even for identical parameter values, the impulse responses implied by the REE-consistent initial beliefs (i.e., before any belief updating has taken place) show very disparate dynamic behavior of macroeconomic variables. Magnitude and persistence of the responses depend on the assumed information set to a significant degree, leading to divergence in the overall model fit and estimated parameters.

We also find that different ways of forming the initial beliefs influence the dynamics of the model under learning. REE-consistent initial beliefs produce more persistent and less volatile evolution of inflation expectations than pre-sample regression based initial beliefs. High volatility of pre-sample beliefs is a probable reason for the fact that they generally lead to a worse model fit than REE-consistent initial beliefs in our estimation. On the other hand, pre-sample beliefs could be very sensitive to short-term but pronounced developments in the data, which might lead to the agents' forecasting functions changing dramatically over the in-sample. This sensitivity is likely to lead to a large variability of estimated results, making them less credible.

On the contrary, the REE-consistent initial beliefs are found to be very robust in adaptive learning estimations, in the sense that moderate deviations in the REE employed to construct such initial beliefs lead to essentially identical marginal data density and estimated parameters. The fact that REE-consistent initial beliefs produce rather robust outcomes means that the estimation results are mainly driven by the time-varying transmission mechanism introduced by adaptive learning rather than by the specific initial conditions. The robustness of the REE-consistent initial beliefs can also motivate the preference for such "model-driven" approach to form the initial conditions when estimating the DSGE models for policy analysis and forecasting.

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Appendix. Tables and Figures

Table of priors

Parameter description		Prior distribution, M1			Prior distribution, M2			Prior distribution, M3		
	Notation	Type	Mean	St.err	Type	Mean	St.err	Type	Mean	St.err
Stand. errors of shocks:										
preference cons.	σ_b	*I.Gam.*	0.1	2	*I.Gam*	0.2	2	*I.Gam*	0.2	2
policy rule	σ_r	*I.Gam.*	0.1	2	*I.Gam*	0.1	2	*I.Gam*	0.1	2
price markup	σ_p	*I.Gam.*	0.1	2	*I.Gam*	0.15	2	*I.Gam*	0.15	2
wage markup	σ_w				*I.Gam*	0.25	2	*I.Gam*	0.25	2
productivity	σ_a				*I.Gam*	0.4	2	*I.Gam*	0.4	2
labor supply	σ_l				*I.Gam*	0.1	2	*I.Gam*	0.1	2
employment	σ_m				*I.Gam*	0.1	2	*I.Gam*	0.1	2
investment	σ_{inv}							*I.Gam*	0.1	2
price of capital	σ_q							*I.Gam*	0.4	2
governm.spend.	σ_g							*I.Gam*	0.3	2
Autoreg. coefficient:										
preference consump.	ρ_b	$U(0,1)$	0.5	0.29	*Beta*	0.85	0.1	*Beta*	0.85	0.1
productivity	ρ_a				*Beta*	0.85	0.1	*Beta*	0.85	0.1
labor supply	ρ_l				*Beta*	0.85	0.1	*Beta*	0.85	0.1
investment	ρ_{inv}							*Beta*	0.85	0.1
govern. spending	ρ_g							*Beta*	0.85	0.1
Structural parameters:										
Utility function consumption	σ	*Norm.*	1	0.375	*Norm.*	1	0.375	*Norm.*	1	0.375
Utility function labor	η				*Norm.*	2.5	0.25	*Norm.*	2	0.5
Habit	h	*Beta*	0.7	0.1	*Beta*	0.7	0.1	*Beta*	0.7	0.1
Indexation prices	ι_p	$U(0,1)$	0.5	0.29	$U(0,1)$	0.5	0.29	$U(0,1)$	0.5	0.29
Indexation wages	ι_w				$U(0,1)$	0.5	0.29	$U(0,1)$	0.5	0.29
Calo prices	ξ_p	*Beta*	0.75	0.1	*Beta*	0.75	0.05	*Beta*	0.75	0.05
Calvo wages	ξ_w				*Beta*	0.75	0.05	*Beta*	0.75	0.05
Calvo employment	ξ_m				*Beta*	0.75	0.15	*Beta*	0.5	0.15
Inter.rate smoothing	w	$U(0,1)$	0.5	0.29	$U(0,1)$	0.5	0.29	*Beta*	0.8	0.1
policy rule, inflation	ζ_π	*Norm.*	1.5	0.1	*Norm.*	1.5	0.1	*Norm.*	1.7	0.1
policy rule, output term	ζ_y	*Norm.*	0.5	0.1	*Norm.*	0.5	0.1	*Norm.*	0.125	0.05
Investment ajustment cost	φ							$U(1,11)$	6	2.5
Var. capital utilization	ψ							*Norm.*	0.2	0.075
Gain (learning param.)	g	*Gam.*	0.035	0.03	*Gam.*	0.035	0.03	*Gam.*	0.035	0.03

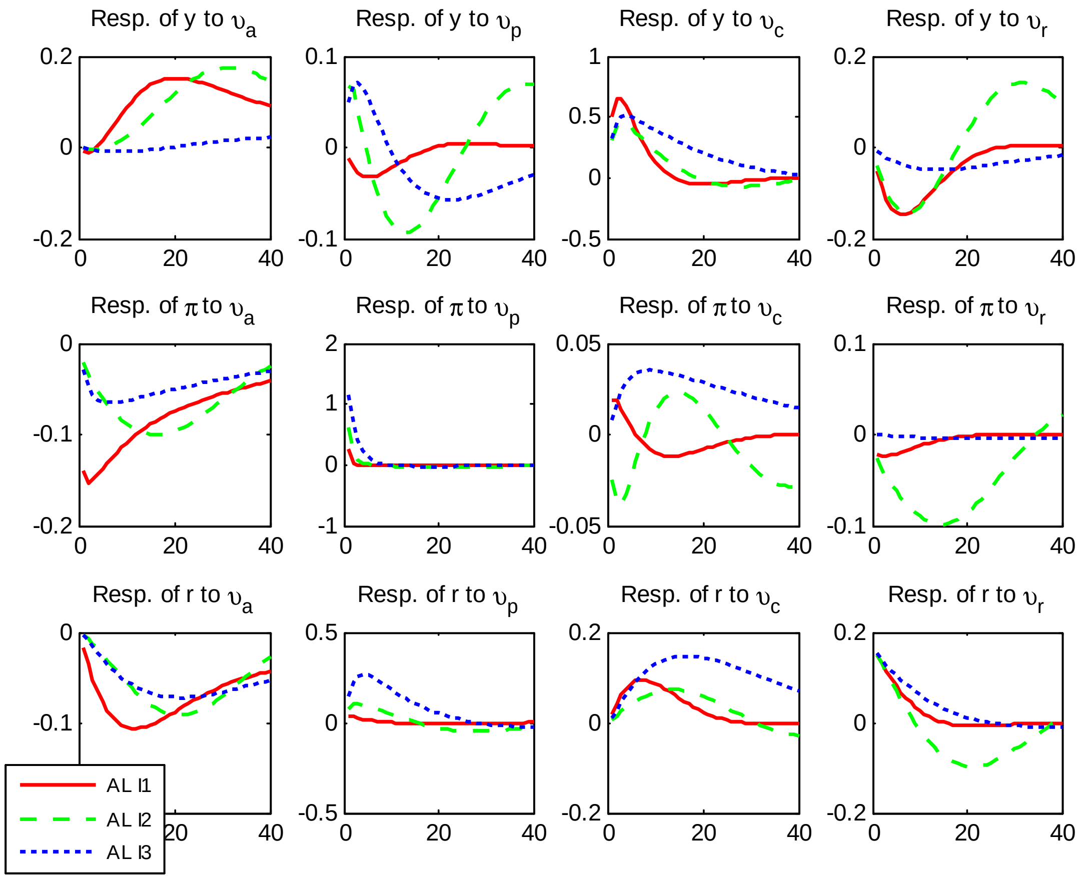

Figure 2.3: Impulse response functions under different information sets

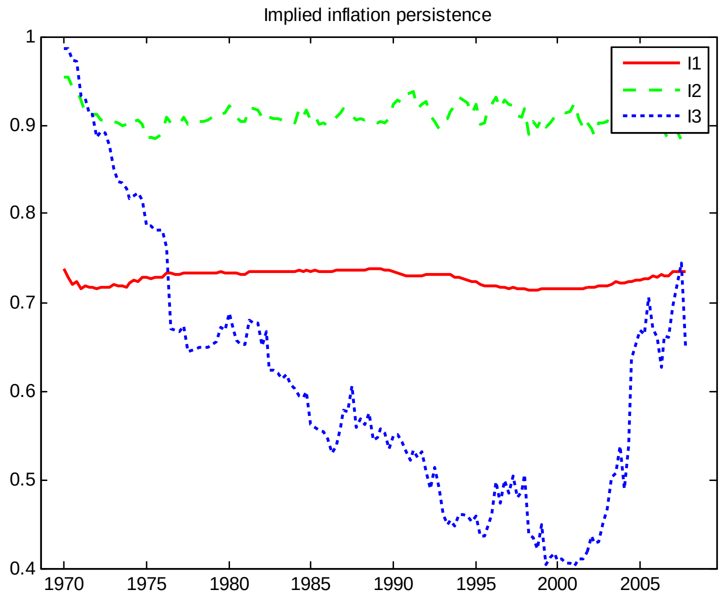

Figure 2.4: Implied inflation persistence under different information sets

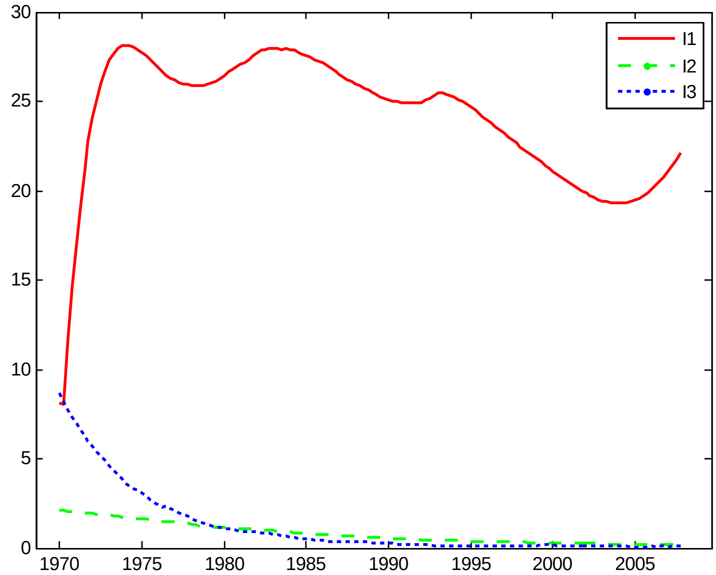

Figure 2.5: Implied second moments of inflation under different information sets