Incomplete Markets, The Substitutability of Time and Money Investments, and Early Childhood Interventions.

Preliminary and Incomplete.

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Abstract. I investigate efficacy of policy interventions in the childhood skill formation process when parents face uninsurable income risk. I first provide novel analytical results demonstrating how the dynamics of parental investments are distorted by incomplete markets frictions. Because of dynamic complementarity in the skill production function, realized returns to investing in children are dependent on future investments, and therefore correlated with realized future stochastic discount factors. If investments are purely monetary this correlation is negative, implying that corrective policy distortions should decrease in child age. If investments are purely time the correlation is ambiguous. When investments are both time and money, the correlation is negative if the elasticity of substitution between them is small enough. Using data from the PSID Child Development Supplement, I estimate a production function for skills in which time and money investments combine through a nested CES aggregator. The estimated elasticity of substitution between parental investments of time and money is about 1.2. I embed this production function in a lifecycle model and conduct several policy simulations. Correcting investment distortions created by incomplete markets on an individual basis is self financing in aggregate provided that the elasticity of wages with respect to ability is larger than about 0.13. When policies only depend on age, the optimal sequence of subsidies decreases in the age of the child. This explains why early interventions have been found to be the most efficacious in many recent studies, even if parents are fully rational and optimizing.

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1 Introduction

Since the seminal work of Cunha and Heckman (2007) a large body of literature has explored issues relating to the dynamic process of skill formation in children. One aspect of this research has been to estimate the skill production function, while additional research has focused on the policy implications associated with these estimates.[1] A common approach to policy analysis has been to extend the standard lifecycle incomplete markets framework to include multiple periods of parental investment in children's human capital. A common finding in these analyses is that policy interventions targeted to younger children, i.e. preschool aged, are the most effective.[2]

A very important result regarding skill production functions is that the investments parents make at different stages in their child's development are complements. This dynamic complementarity means that human capital investments made early in a child's life positively affect the productivity of later investments. While this result would support early childhood policy interventions when parents are myopic, the argument may not follow when parents are fully rational. In the incomplete markets framework parents are such rational optimizing agents, and their decisions take dynamic complementarity fully into account. As such, it is not clear why early policy interventions are preferred to general interventions in the incomplete markets framework. This is the main question addressed in this paper.

Uninsurable idiosyncratic income risk is a core feature of the standard incomplete markets model. This risk gives rise to valuations of assets that depend on the correlation between realized dividends and stochastic discount factors. For parents, a child's ability is a quasi-asset that delivers dividends in the form of altruistic utility. In this paper I demonstrate that these payoffs are correlated with realized stochastic discount factors in a way that leads to under-investment

[1] Some of the more influential papers include Todd and Wolpin (2003), Cunha, Heckman, and Schennach (2010), Del Boca, Flinn, and Wiswall (2013) and Agostinelli and Wiswall (2016a). As this is a burgeoning literature I review many additional relevant papers in section 2.

[2] Examples of policy analysis within such a framework include Cunha (2013), Lee and Seshadri (2018) and Daruich (2018).

in younger children. The intuition is that with dynamic complementarity the payoff to early investment depends on later investment, and the resources available for such later investments is correlated with later parental consumption. Increased early investment in children also increases the dependence of parents' future utility on their realized earnings shocks, which leads them to favor safe assets more than they would in an economy with full insurance. I provide novel analytical results in this paper showing that a decreasing sequence of investment subsidies is required to close the wedge between parental choices under incomplete markets and under full insurance.

When human capital investments in children are entirely in the form of monetary expenditures, there will be a strong association between investments and parental consumption because both are financed from the same pool of resources. Adverse earnings shocks are likely to cause reductions in both consumption and investment, particularly if shocks are permanent. Because realized future consumption is inversely related to the realized stochastic discount factor, future investments will also be inversely related to the realized stochastic discount factor. In the presence of dynamic complementarity the return to early investments is positively affected by later investments. Therefore, the realized return to early human capital investments is also negatively correlated with realized stochastic discount factors. I derive analytical results for the dynamics of optimal human capital investments in children, and show that this negative correlation results in early investments being too small compared to later investments (relative to first-best optimal dynamics). A decreasing sequence of investment subsidies is required to close the wedge between chosen investment dynamics and what parents would choose under full-insurance.

When human capital investments are not purely monetary, but rather also include an element of parental time (possibly entirely time), the result that early investments are too small relative to later ones becomes ambiguous. In the case that investments are purely time, the sign of the wedge in the analytical investment dynamics equation depends on the correlation between realized stochastic discount factors and the product of realized wage growth with future investments. However, I also show that when both time and money investments are salient, being combined through a CES aggregator, under-investment at early stages is more likely as the elasticity of substitution falls. At the extreme Leontief case early investments are unambiguously too small compared to later investments (again relative to first-best optimal dynamics). To understand the empirically relevant case we first need estimates of the time-money CES investment aggregator, and then the dynamics need to be simulated in a numerical model.

Using data from the Panel Study of Income Dynamics Child Development Supplement (CDS) data I estimate the structural parameters of a time-money CES investment aggregator, and then also estimate a dynamic skill production function where the aggregated investment index is an input. I use within-household variation combined with an instrumental variables approach to address the endogenous relationship between the productivity of parental time investments and parental wages in the labor market. When estimating the dynamic skill production function I follow the method of Agostinelli and Wiswall 2016a; 2016b in order to make use of several noisy measures of unobserved latent skills. I estimate that the elasticity of substitution between time and money investments is about 1.2, that monetary investments become relatively more productive as children get older, and that there is substantial permanent heterogeneity in the relative productivity of parental time.

I nest the estimates described in the previous paragraph within a lifecycle incomplete markets model with multiple periods of parental investment. Simulations show that early investments are smaller than what would be chosen under full insurance. As a first pass at understanding the magnitude of the effect of uninsurable risk, I simulate a counterfactual experiment in which a planner compels parents to make the same investments they would have in a full-insurance model, and the planner compensates parents directly for the difference. I find that this increases the average ability of children by 3.51% when they reach the age of majority, and the cost of reimbursement is only 0.48% of aggregate earnings. This implies that correcting the effects of incomplete markets on parental investments would be self-financing as long as the elasticity of wages with respect to ability exceeds 0.136, which is considerably smaller than conventional estimates.[3] I next consider more realistic policies that are not individualized by allowing a planner to choose a sequence of child-age specific monetary investment subsidies. Given an aggregate budget constraint of $500 per child, the most efficient use of funds for producing ability (at the age of majority) is a subsidy of about 10% for the youngest children declining to about 0% at age 16.

2 Literature Review

TBC.

3 Child Development Model

For the initial sections of the paper the entire lifecycle model does not yet need to be specified. Therefore, for brevity I present only a part of that model here, and employ it in analysis of investment dynamics and skill production estimation. After those sections are complete I briefly fill in the remained of the lifecycle model, which is relatively standard.

Consider a household i with a child of age t that gains utility from consumption c_{it} and disutility from non-leisure time $\ell_{it} + h_{it}$, where ℓ_{it} is labor supply and h_{it} is time invested in their child. Periodic utility is $u(c_{it}, \ell_{it} + h_{it})$ where $u_c > 0$ and $u_\ell = u_h < 0$. Labor supply earns a wage w_{it} per unit.

Time invested in the child combines with money investments in the child, denoted x_{it},

[3] For example, Agostinelli (2017) estimates that the elasticity of a woman's skills with respect to her ability is 0.44.

according to the following CES aggregator:

$$I_{it} = \left(\alpha x_{it}^{\frac{\delta-1}{\delta}} + (1-\alpha)\zeta_{it} h_{it}^{\frac{\delta-1}{\delta}}\right)^{\frac{\delta}{\delta-1}}. \tag{1}$$

The parameter δ is the elasticity of substitution between time and monetary investments. The CES weight α is the deterministic part of the relative productivity of money versus time investment, while ζ_{it} represents time-varying heterogeneity in the relative productivity of time investment. The parameters δ and α could vary with the age of the child or other deterministic features of the household (I test this below), but subscripts on these parameters are repressed to keep the notation light.

The household's decision problem is dynamic. The state variables are the household's assets, a_{it}, the current skills of the child, θ_{it}, the current wage offer, w_{it}, and the current time-investment productivity shock, ζ_{it}. The purpose of investing in the child is to improve their future skills, which enter the continuation value of the household. Skills evolve according to the following production function:

$$\theta_{it+1} = \gamma_0 \theta_{it}^{\gamma_1} I_{it}^{\gamma_2} \eta_{it+1}. \tag{2}$$

Note that both the location (γ_0) and scale ($\gamma_1 + \gamma_2$) are free parameters in this model, and will be estimated following Agostinelli and Wiswall (2016a) (AW hereafter). The timing of shocks η_{it+1} is such that they are orthogonal to current investments, which is standard as in AW.[4]

[4] It is noteworthy that I estimate a Cobb-Douglas production function, while AW estimate a trans-log. The reason is, I have a missing data problem that can be solved by imposing weak-separability. The PSID-CDS data I am using provide measures of skills and investments only every five years, whereas the CNLSY data utilized by AW are much more frequent. It seems far fetched to assume that realized skills five years in the future are a function of only current investments and orthogonal shocks; therefore, I directly address estimating an annual production function using quinquennial data. This is fleshed out in detail below.

The recursive decision problem of household is

$$\begin{aligned}
V_t(a_{it},\theta_{it},w_{it},\zeta_{it}) &= \max_{\{a_{it+1},\ell_{it},x_{it},h_{it}\}} \{u(c_{it},\ell_{it}+h_{it})+\beta\mathbb{E}\left[V_{t+1}(a_{it+1},\theta_{it+1},w_{it+1},\zeta_{it+1})\right] \\
& s.t. \qquad (3) \\
c_{it} &= (1+r)a_{it}-a_{it+1}+w_{it}\ell_{it}-x_{it} \\
\theta_{it+1} &= \gamma_0\theta_{it}^{\gamma_1}I_{it}^{\gamma_2}\eta_{it+1} \\
I_{it} &= \left(\alpha x_{it}^{\frac{\delta-1}{\delta}}+(1-\alpha)\zeta_{it}h_{it}^{\frac{\delta-1}{\delta}}\right)^{\frac{\delta}{\delta-1}} \\
a_{it+1} &\geq \underline{a} \\
w_{it+1} &\sim \mathit{\Gamma}_w(w_{it}) \\
\zeta_{it+1} &\sim \mathit{\Gamma}_\zeta(\zeta_{it}) \\
\eta_{it+1} &\sim \mathit{\Gamma}_\eta,
\end{aligned}$$

where $\mathit{\Gamma}_w(w_{it})$ is the conditional distribution of future wages, $\mathit{\Gamma}_\zeta(\zeta_{it})$ is the conditional distribution of future time-investment shocks, and $\mathit{\Gamma}_\eta$ is the distribution of development shocks. β is the time discount factor, r is the real interest rate, and $\underline{a}$ is the household's borrowing limit.

4 Policy Implications of Incomplete Markets

In a first-best economy households would be insured against consumption risk, and therefore stochastic discount factors would reduce to the risk-free discount factor. As I demonstrate in this section, the dynamics of investments in children are also systematically different when markets are incomplete as opposed to a world with full insurance. This result has important policy implications that depend crucially upon the parameters of the investment aggregator. The key quantities are the covariances between investments and stochastic discount factors, which the investment aggregator parameters help determine.

At the corners of the parameter space the implications are clearest. When $\alpha = 1$ only monetary investments matter, and they should be subsidized more heavily in young children than older children. When $\alpha = 0$ there might still be value in subsidizing investments in children, but these subsidies should increase in the age of the child, not decrease. For intermediate ranges of α, the elasticity of substitution between time and goods investments becomes important. When time and money investments are perfect substitutes, aggregated investment is quite well insured due to the fact that parents can switch between modes of investment in response to wage shocks. However, when time a money investments are complementary, e.g. the Cobb-Douglas special case, there is greater value in subsidies that decline with the age of the child.

4.1 Illustrative Examples

Monetary Investments Only ($\alpha = 1$): Assuming that parental preferences are such that $\theta_{it} > 0$ is always chosen, an intertemporal optimality condition for investments always holds. If credit constraints bind these conditions do not break down, rather the stochastic discount factors that enter them simply become smaller. For the case $\alpha = 1$ currently under consideration, the dynamics of investments satisfy

$$x_{it} = \gamma_1 \mathbb{E}\left[\beta \frac{u_c(c_{it+1}, \ell_{it+1})}{u_c(c_{it}, \ell_{it})} x_{it+1}\right]. \tag{4}$$

As is usual with a Cobb-Douglas production function, the optimal input proportions depend on relative prices and productivity. However, the relative price of x_{it+1} is unknown because it depends on the realized stochastic discount factor.

If uncertainty due to incomplete markets were eliminated, as would occur in a first-best equilibrium, then equation 7 would simplify to

$$x_{it} = \gamma_1 R x_{it+1}, \tag{5}$$

where R is the risk-free discount factor. Although equations 7 and 5 are similar, one can show that they differ by a systematic wedge. Taking an expansion of the expectation in equation 7 results in

$$x_{it} = \gamma_1 R x_{it+1} + \gamma_1 \beta \mathbb{COV}\left[\frac{u_c(c_{it+1}, \ell_{it+1})}{u_c(c_{it}, \ell_{it})}, x_{it+1}\right]. \tag{6}$$

The covariance term is negative in this example with $\alpha = 1$ because marginal utilities from consumption rise as expenditure falls, and vice-versa.

The above derivations lead to the conclusion that x_{it} will be smaller relative to x_{it+1} under incomplete markets than in a first-best environment. In other words, the dynamics of investments will be such that parents delay investments in their children relative to what would occur in a first-best world. One way to implement a first-best pattern of investments over the course of childhood is to subsidize investments at a rate that decreases with the age of the child. Under such a scheme equation 7 would become

$$x_{it} = \gamma_1 \frac{1 - S_{t+1}}{1 - S_t} \mathbb{E}\left[\beta \frac{u_c(c_{it+1}, \ell_{it+1})}{u_c(c_{it}, \ell_{it})} x_{it+1}\right], \tag{7}$$

where S_t is the age-t specific subsidy rate. The policy wedge $(1 - S_{t+1})/(1 - S_t)$ exceeds unity, and thereby tilts the age-profile of investments towards younger ages.

Time Investments Only ($\alpha = 0$): When only time investments are productive for child development, a similar equation for optimal investment dynamics arises:

$$h_{it} = \gamma_1 \mathbb{E}\left[\beta \frac{u_c(c_{it+1}, \ell_{it+1})}{u_c(c_{it}, \ell_{it})} \frac{w_{it+1}}{w_{it}} h_{it+1}\right]. \tag{8}$$

By similar logic to the $\alpha = 1$ case, the wedge between these dynamics and first-best dynamics is

$$\gamma_1 \beta \mathbb{COV}\left[\frac{u_c(c_{it+1}, \ell_{it+1})}{u_c(c_{it}, \ell_{it})}, \frac{w_{it+1}}{w_{it}} h_{it+1}\right]. \tag{9}$$

Unlike the case with monetary investments only, the sign of this covariance is ambiguous. The product of wage growth and time investment could be positively correlated with stochastic discount factors if most of the variation arises from time-investment movements; however, if wage growth dominates the variation the covariance could be negative. Therefore, the policy implications of market incompleteness require knowledge of this covariance when investments are entirely time related.

Time and Money as Perfect Substitutes: When considering a case where both time and money investments may occur, i.e. $\alpha \in (0, 1)$, the dynamics of the aggregate investment quantity I_{it} are of policy interest. One example where a closed-form expression for these dynamics is derivable is the of time and money investments as perfect substitutes. In this case the dynamics chosen by parents would be:

$$I_{it} = \gamma_1 \alpha \mathbb{E}\left[\beta \frac{u_c(c_{it+1}, \ell_{it+1})}{u_c(c_{it}, \ell_{it})} x_{it+1}\right] + \gamma_1 (1-\alpha) \zeta_{it} \mathbb{E}\left[\beta \frac{u_c(c_{it+1}, \ell_{it+1})}{u_c(c_{it}, \ell_{it})} \frac{w_{it+1}}{w_{it}} h_{it+1}\right]. \tag{10}$$

The covariances from both equations 6 and 9 are policy relevant in this example. As a consequence, the wedge between incomplete markets dynamics and first-best dynamics will include the sum of a negative and a positive covariance. The parameter α will determine whether parental investments tend to occur too early or too late relative to the first-best. The takeaway from these derivations is that when time and money investments are substitutable the policy implications are ambiguous, and may be limited.

Time and Money as Perfect Complements: At the opposite extreme time and money are perfect complements, and the investment aggregator has a Leontief form ($\delta \to 0$). In this case the dynamics of investments can be expressed with current aggregate investment related to future aggregate investment as follows:

$$I_{it} = \gamma_1 \mathbb{E}\left[\beta \frac{u_c(c_{it+1}, \ell_{it+1})}{u_c(c_{it}, \ell_{it})} I_{it+1}\right]. \tag{11}$$

Following the same procedure as the $\alpha = 1$ case above, one can show that the wedge between investment dynamics in first-best and incomplete markets cases depends on the covariance between stochastic discount factor realizations and future investments. However, the relevant covariance can also be expressed as

$$\mathbb{COV}\left[\beta \frac{u_c(c_{it+1}, \ell_{it+1})}{u_c(c_{it}, \ell_{it})}, \frac{x_{it+1}}{\alpha}\right]. \tag{12}$$

To the extent that negative resource shocks reduce both consumption and monetary investments, this covariance will be negative. This implies that investments in children grow more quickly with their age than would occur in a first-best equilibrium.

Taking Stock: The examples above are intended to illustrate that in many cases parents under-invest in young children relative to what would occur in the first-best. In a model with only monetary investments, e.g. that of Cunha (2013), parents unambiguously under-invest in young children relative to the first-best. The negative correlation between realizations of stochastic discount factor risk and future investments in children is the source of such under-investment. However, if time investments are allowed to make up for reductions in monetary investments when bad shocks occur, this under-investment might disappear entirely. This possibility requires time and monetary investments to be substitutes. With enough complementarity between time and goods investments, we again observe the result that parents of young children under-invest relative to the first-best. To understand this policy question more fully, estimates of δ and α are required.

5 Identification and Estimation

Estimating the substitution elasticity δ: The first-order condition for the optimal (interior solution) combination of time and money investments can be written

$$\ln\left(\frac{x_{it}}{h_{it}}\right) = \delta \ln\left(\frac{\alpha}{1-\alpha}\right) + \delta \ln(w_{it}) - \delta[\ln(\overline{\zeta}_i) + \ln(\tilde{\zeta}_{it})], \tag{13}$$

where I have assumed that $\ln(\zeta_{it})$ is the sum of a permanent fixed-effect, $\ln(\overline{\zeta}_i)$, and a transitory shock, $\ln(\tilde{\zeta}_{it})$.

The first concern for recovering δ is that this equation is not always observed, and that selection out of the sample (e.g. labor force non-participation) might correlated with ζ_{it}. I assume that such selection is based on persistent characteristics, and therefore uncorrelated with the time-vary component $\tilde{\zeta}_{it}$. Put differently, I assume in first-differences form

$$\Delta \ln\left(\frac{x_{it}}{h_{it}}\right) = \delta \Delta \ln\left(\frac{\alpha}{1-\alpha}\right) + \delta \Delta \ln(w_{it}) - \delta \Delta \ln(\tilde{\zeta}_{it}), \tag{14}$$

the error term is orthogonal to factors driving sample selection.

The crucial question is what instrument to use to overcome the problem that wage innovations are likely correlated with the shocks to time-investment productivity. I construct an instrument based on wages. Specifically, the instrument I propose is the average of $\Delta \ln(w_{it})$ within the state of i's residence, where the average is computed excluding the data of i themself. To the extent that wage growth has a state-component there will be a strong first-stage (indeed the F-statistic exceeds 14 in the first-stage). Furthermore, the instrument is uncorrelated with idiosyncratic household level shocks by construction.

Estimating δ by simple IV using the proposed instrument generates $\hat{\delta} = 1.31$ (s.e. = 0.64). In an effort to improve precision, I also estimate a specification adding the square of the instrument in the first-stage, which yields $\hat{\delta} = 1.42$ (s.e. = 0.56). Further addition of higher-order terms does not help precision much. The data do not support the hypotheses that δ varies with parental education or the age of the child.

Estimating α, ζ_{it} and I_{it}: I recover α using

$$\ln\left(\frac{\alpha}{1-\alpha}\right) = \mathbb{E}\left[(1/\delta)\ln\left(\frac{x_{it}}{h_{it}}\right) - \ln(w_{it})\right]. \tag{15}$$

Hyopthesis testing supports a model where α is larger for older children, but does not support α varying with parental education. Modelling it as $\alpha = \alpha_0 + \alpha_{(age\leq 9)} \times \mathbf{1}_{age\leq 9}$ yields $\hat{\alpha}_{(age\leq 9)} = -.0390$ (s.e. = 0.0227), relative to a base estimate $\hat{\alpha}_0 = 0.244$.

I think of the deviations e_{it} from the mean in equation 15 as a combination of shocks and measurement error: $e^{\zeta}_{it} = \overline{\zeta}_i + \tilde{\zeta}_{it} + \epsilon^{\zeta}_{it}$, where ϵ^{ζ}_{it} is the measurement error. With two periods of data $E[e^{\zeta}_{it}|e^{\zeta}_{it-1}] = \overline{\zeta}_i$, so estimates of the permanent component are easily recoverable. However, separating the time-varying component from measurement error is difficult. Therefore, I introduce some approximation by including only the estimated permanent component in the calculations that follow. The permanent component is just under half of the total variation in e^{ζ}_{it}. Using this approximation estimates of I_{it} can be recovered. Below I take into account the fact these are estimates of investments, rather than actual investments. In doing so I also take into account measurement error induced by approximation of $\hat{\zeta}_{it}$.

Estimating the production function parameters γ_0, γ_1 and γ_2: Here I heavily utilize the identification results of AW to estimate a production function with unknown location and scale. However, I have an additional complication that measures of ability and investments are not available every year, rather only every five years. Using the assumed weakly separable

Cobb-Douglas production function, I can iterate equation 2 forward and write:

$$\ln(\theta_{it+5}) = g_0 + g_1 \ln(\theta_{it}) + g_2 \ln(I_{it}) + \varepsilon(I_{it+1}, I_{it+2}, I_{it+3}, I_{it+4}, .), \tag{16}$$

where the coefficients g_0, g_1 and g_2 are known deterministic functions of the structural parameters γ_0, γ_1 and γ_2. The error term ε is composed of future investments and shocks. If g_0, g_1 and g_2 could be estimated consistently then estimates of γ_0, γ_1 and γ_2 could be recovered, but the error term is clearly correlated with the RHS variables through future investments.

My proposed solution to the above identification problem is to continue to follow AW by also estimating a reduced form model of the dynamics of parental investments:

$$\ln I_{it} = b_0 + b_1 \ln \theta_{it} + b_2 \ln I_{it-1} + \nu_{it}. \tag{17}$$

Like AW emphasize, this equation captures the endogeneity of investments and current skills. This equation differs from AW in that 'permanent' features of the equation, such as mother's education, are captured through autocorrelation of investments. More subtly, the permanent component of household income is captured through investment autocorrelation, while income shocks enter ν_{it}. (The estimated investment dynamics equation will also be used to discipline parameters when calibrating a quantitative model later on.)

I choose to write equation 17 this way because I can then iterate it forward, generating expressions that relate unobserved investments, e.g. I_{it+1}, to current investments and skills. Where unobserved future skills enter these equations, the production function is used substitute them out. In this way, future investments can be substituted out of the error term of equation 16 where they are replaced by linear functions of current investments and skills, of which measures are available. As a result, I have the following two equations that can potentially be estimated:

$$\ln(\theta_{it+5}) = G_0 + G_1 \ln(\theta_{it}) + G_2 \ln(I_{it}) + \mathcal{E}^{\theta} \tag{18}$$

$$\ln(I_{it+5}) = B_0 + B_1 \ln(\theta_{it}) + B_2 \ln(I_{it}) + \mathcal{E}^{I}. \tag{19}$$

The error terms $\mathcal{E}^{\theta}$ and $\mathcal{E}^{I}$ in these equations consist of orthogonal shocks only. The six coefficients G_0, G_1, G_2, B_0, B_1 and B_2 are all known deterministic functions of the deep parameters γ_0, γ_1, γ_2, b_0, b_1 and b_2; therefore, we have six equations in six unknowns, and the deep parameters are identified if the reduced forms can be consistently estimated. Measurement issues must be addressed in order to estimate the reduced forms.

Again following AW closely, I first select three age-invariant measures $Z_{m,it}$ of θ_{it}, and follow AW's approach to recovering estimates of the parameters of the measurement system. Each measure is assumed to relate to latent ability as follows:

$$Z_{m,it} = \mu_m + \lambda_m \ln \theta_{it} + \epsilon_{m,it}. \tag{20}$$

μ_m and λ_m are measurement parameters, and $\epsilon_{m,it}$ is measurement error. Also define "residual" skill measures $\tilde{Z}_{m,it} = (Z_{m,it} - \mu_m)/\lambda_m$ and $\tilde{\epsilon}_{m,it} = \epsilon_{m,it}/\lambda_m$, such that $\ln(\theta_{it}) = \tilde{Z}_{m,it} - \tilde{\epsilon}_{m,it}$. The three measures are Letter-Word, Applied Problems and Paragraph Comprehension raw scores. I normalize the Letter-Word loading λ_{LW} at age six to unity, and recover the loadings of the other two measures, as well as the intercepts, using sample means and covariances of the measurements as in AW. I substitute residual Letter-Word scores $\tilde{Z}_{LW,it}$ and $\tilde{Z}_{LW,it+4}$ into equations 18 and 19. I also assume that the estimation error in investments is such that $\ln \hat{I}_{it} = \ln I_{it} + \epsilon_{I,it}$, where $\epsilon_{I,it}$ is orthogonal estimation/measurement error. This transforms the equations into

$$\tilde{Z}_{LW,it+5} = G_0 + G_1\tilde{Z}_{LW,it} + G_2 \ln(\hat{I}_{it}) + (\mathcal{E}^{\theta} - \epsilon_{I,it} - G_1\tilde{\epsilon}_{LW,it} + \tilde{\epsilon}_{LW,it+4}) \quad (21)$$

$$\ln(\hat{I}_{it+5}) = B_0 + B_1\tilde{Z}_{LW,it} + B_2 \ln(\hat{I}_{it}) + (\mathcal{E}^{I} - \epsilon_{I,it} - G_1\tilde{\epsilon}_{LW,it} + \epsilon^{I}_{it+4}), \quad (22)$$

where measurement errors have been collected into the error terms.

Both $\tilde{Z}_{LW,it}$ and $\ln(\hat{I}_{it})$ are correlated with their respective measurement errors. I instrument for $\tilde{Z}_{LW,it}$ with residual Applied Problems scores $\tilde{Z}_{AP,it}$, exactly as AW suggest. A suitable instrument for $\ln(\hat{I}_{it})$ is less straightforward because alternative contemporaneous measures are unavailable. Instead, I use lagged time-investment measured in the 1997 wave of the PSID-CDS. The logic is that the permanent part of time-investment productivity $\overline{\zeta}_i$ should positively influence variation in both current investment and the time component of past investments, but past time investments are uncorrelated with current measurement error.

The covariance between an alternative measure of future skills, i.e. $\tilde{Z}_{AP,it+5}$, and the production function error term can be used to recover an estimate of $Var(\mathcal{E}^{\theta})$ (again this is AW's suggested approach). Assuming that η_{it+1} is iid, I recover its variance as $Var(\eta) = Var(\mathcal{E}^{\theta})/5$.

PSID-CDS Investment Data: The Child Development Supplement of the Panel Study of Income Dynamics (PSID-CDS) began collecting data in 1997 with the goal "to provide researchers with a comprehensive, nationally representative, and longitudinal database of children and their families with which to study the dynamic process of early human capital formation." The initial sample included 3563 children under 13 years old whose parents were included in the main PSID sample. Second and third waves of data collection were carried out in 2002 and 2007. These additional waves re-interviewed 2907 and 1506 eligible children under the age of 19. Of these, a sample of 683 observations are complete in all of the required investment, wage, ability, demographic and location data.[5]

The advantage of the PSID-CDS is the time-use survey, which provides comprehensive information on the activities of children and the inclusion of parents in those activities. The time diaries include 24 hours of information for two representative days, one a weekday and the other a weekend day. For each activity a child does during the survey day information is collected on the duration of the activity, who was participating with the child, who was around but not participating, and where the activity took place. The time inputs of parents are defined as the total duration of activities in which they are actively participating with the child. An estimate of weekly totals is based on five times the weekday time allocation plus two times the weekend time allocation.

Observed child-specific expenditures were limited in 1997 and appear unreasonably low; however, in the 2002 and 2007 waves many additional expenditure items were added resulting in a more reasonable total expenditure variable. Items include tuition costs, tutoring expenses, lessons (e.g. music lessons), sports, community/religious groups, toys/books, vacations, school supplies, clothing, transportation and daycare expenses. The 2002 wave also includes spending on food specifically for the child, but this item was not included in 2007 and so it is excluded in 2002 as well for consistency.

On top of reported expenditure, I also consider the robustness of results to the inclusion of implicit spending on children through neighborhood choice. To the extent that parents pay for the expense of living in a good neighborhood, this could be considered a monetary investment in their children. However, it is difficult to map the value of a home directly to a child investment amount. For robustness purposes only, I consider a measure that takes one-tenth of either the observed or imputed annual rent as an investment in the child.[6] Results

[5]The vast majority of missing data is in either time or monetary investments. Of the 1506 possible observations, 654 are missing some element of the investment data. Of the other 199 dropped observations, 128 are missing wage data.

[6]Imputed rent is 6% of the property value. The 10% assumption is based on Kane, Riegg, and Staiger (2006) who find that the causal effect of an increase in elementary school quality from the 25th to 75th percentile is a

using the alternative measure of monetary investments that combines these implicit payments with the observed payments can be found in Appendix A.

Estimation Results Table 1 presents the main estimation results. Note that Table 2 in the appendix presents a robustness version of these results where part of parental housing costs are treated as investment.

The point estimates of the elasticity of substitution between time and monetary investments are slightly larger than unity. The degree of complementarity between these modes of investment is close to that of a Cobb-Douglas aggregator. Estimates of the CES weight parameter α are such that the importance of monetary investment increases with the age of the child. This parameter exhibits some sensitivity to the first-stage specification, but the age-pattern holds across specifications. These parameters are well within the interior of the parameter space, and simulations are required to determine the nature of any wedge between laissez-faire and first-best investment dynamics.

On the side of the skill production function, the estimates indicate mildly decreasing returns to scale. This is in contrast to AW, who find substantially increasing returns to scale. The auxiliary model of investment dynamics indicates considerable persistence in parental investments, and little tendency of parents to increase investments when abilities are higher (over and above what lagged investment captures).

10% increase in the home's value. This estimate is of something different than the proportion of a homes value that should be considered child-investment, but clearly is related.

	Linear First Stage		Quadratic First Stage	
Parameter	**Estimate**	**Bootstrapped s.e.**	**Estimate**	**Bootstrapped s.e.**
δ	1.162	0.562	1.232	0.497
α_0	0.378	0.246	0.244	0.114
$\alpha_{(age \leq 9)}$	-0.056	0.022	-0.039	0.023
σ^2_ζ	0.349	0.019	0.307	0.017
γ_0	0.656	0.380	0.946	0.254
γ_1	0.588	0.068	0.600	0.058
γ_2	0.259	0.100	0.259	0.095
σ^2_η	0.125	0.036	0.115	0.028
b_0	0.173	0.380	0.126	0.231
b_1	0.964	0.117	0.966	0.099
b_2	0.019	0.088	0.018	0.067

Table 1: Estimates of Production Function and Investment Aggregator Parameters. Bootstrapped standard errors based on 1000 replications. First-stage refers to regression of changes in wages on the instrument (state-level average wage changes). The F-statistic in the linear first-stage is 14.16, and in the quadratic first-stage is 9.33.

6 Quantitative Analysis

6.1 Full Lifecycle Model

To understand the implications of the above estimates for the dynamics of investments in children and efficacy of early childhood interventions, I embed the estimated structure within a fully specified lifecycle model. This is a steady-state model with overlapping unit mass cohorts aged $j = 0, ..., J$. From ages $j = 0$ to $j = 16$ an individual is a child cared for by their parents. From age $j = 17$ to $j = 30$ and individual is a worker who solves a standard consumption-savings problem, with the exception that the continuation value includes the expectation of becoming a parent at age $j = 31$. From $j = 31$ to $j = 47$ the individual is a parent who invests in the human capital of their own child. At age $j = 48$ a parent earns utility associated with their child's final skill level, and then solves a standard consumption savings problem until age $j = 65$. From age $j = 65$ to $j = J = 85$ the individual is retired and consumes out of private savings and social security income.

Periodic utility functions are $u(c, \ell) = \frac{1}{1-\sigma}(c^{\nu}\ell^{1-\nu})^{1-\sigma}$. Utility from a child's skills when the child comes of age is $\kappa \ln(\theta^c_{17})$, where $\theta^c_{j^c}$ is the child's skill level when they are age j^c. All children are born with the same initial skill level $\overline{\theta}_0$. An individual's wages are given by $\ln w_j = \lambda \ln(\theta) + z_j + \overline{w}$, where λ is the elasticity of wages with respect to ability and z_j follows an $AR(1)$ process. The constant $\overline{w}$ ensures that the average wage reflects the estimation data, because otherwise the estimates of α would be inappropriate. Social security pensions are approximated by annual receipts of $\$30,000$ for all retired households. Only the permanent component of parenting time productivity $\overline{\zeta}$ is included in the quantitative model as this includes substantial variation on its own, and more importantly the time-varying component cannot be separated from measurement error. Table XYZ reports the calibrated parameters of the model.

6.2 Correction of Investment Wedges

Under the intermediate degree of time-money investment complementarity estimated in the previous section, it is not possible to derive closed form analytical expressions for the wedges between investment choices under incomplete markets and under full-insurance. Instead, I compute counterfactual human capital investment decision rules that assume away future consumption risk (i.e. set $c_{t+j} = E_t[c_{t+j}]$. If full-insurance of consumption risk were introduced in this economy these decision rules would coincide with actual household decisions. The wedge (difference) between the actual and counterfactual decision rules provides a measure of how much investments are affected by uninsurable risk. The average value of these wedges is about 2.3% of what parents would choose on their own under uncertainty. If the a planner were able to enforce this policy change, i.e. top-up parental investments on a case-by-case basis without causing crowding-out, an increase in the average ability level of 3.51% would be achieved, while the cost of doing this would be 0.48% of aggregate labor income. Such a program would be self-financing provided that the elasticity of wages with respect to ability exceeds 0.137. Most estimates in the literature would support a sufficiently high elasticity.

6.3 Dynamic Subsidies

In equation 7 we saw that a decreasing sequence of subsidies could also correct the incomplete market wedges, at least in the case that investments are purely monetary. Here I consider what the linear sequence of subsidies on expenditure investments would be, subject to a $500 per child budget constraint. All households will face the same subsidy rate here. I restrict the policy space to three possibilities, all of which meet the financing constraint: (i) a constant sequence, (ii) an increasing sequence that starts at zero at age 0 and has a constant slope, (iii) a decreasing sequence that ends at zero at age 16 and has a constant slope. For the constant subsidy case the budget constraint is met at a subsidy rate of 0.86%, and the gain in average ability is 0.76%. For the increasing sequence of subsidies the slope at which the budget constraint is met is 0.1% per year, and the gain in average ability is 0.38%. For the decreasing sequence of subsidies the initial subsidy at which the constraint is met is 1.8%, and the gain in average ability is 1.27%. This indicates that even with the complicated aggregation of time and monetary investments, the effect of incomplete markets on parental decisions is that they backload human capital investments to some degree. A decreasing sequence of subsidies corrects this by effectively increasing early childhood investments.

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A Housing Costs as Monetary Investments

	Linear First Stage		Quadratic First Stage	
Parameter	**Estimate**	**Bootstrapped s.e.**	**Estimate**	**Bootstrapped s.e.**
δ	1.326	0.514	1.294	0.438
α_0	0.199	0.403	0.242	0.393
$\alpha_{(age \leq 9)}$	-0.026	0.018	-0.031	0.018
σ^2_ζ	0.128	0.008	0.136	0.008
γ_0	0.600	0.349	0.478	0.395
γ_1	0.615	0.048	0.611	0.052
γ_2	0.283	0.097	0.288	0.101
σ^2_η	0.109	0.019	0.110	0.019
b_0	0.344	0.262	0.375	0.299
b_1	0.937	0.083	0.935	0.087
b_2	0.017	0.048	0.019	0.051

Table 2: Estimates of Production Function and Investment Aggregator Parameters. Bootstrapped standard errors based on 1000 replications. First-stage refers to regression of changes in wages on the instrument (state-level average wage changes). The F-statistic in the linear first-stage is 14.16, and in the quadratic first-stage is 9.33.