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# Attentional Role of Quota Implementation*

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**Abstract**

This paper introduces a new role of quotas, e.g., labor market quotas: the attentional role. We study the effect of quota implementation on the attention allocation strategy of a rationally inattentive (RI) agent. Our main result is that an RI agent who is forced to fulfill a quota never hires the candidates without acquiring information about them, unlike an unrestricted RI agent who in some cases bases her decision on prior belief only. We also show that in our context quotas are equivalent to other types of affirmative policies such as subsidies and blind resume policy. We show how our results can be used to set a quota level that increases the expected value of the chosen candidate and also decreases statistical discrimination and discrimination in terms of how much attention is paid to each applicant. At the same time, quota implementation could be destructive if the social planner has imperfect information about the parameters of the model.

**Keywords:** discrete choice, rational inattention, multinomial logit, quotas
**JEL classification codes:** D63, D81, D83, H23, J08

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## 1 Introduction

Labor market quotas have become a heavily-used governmental policy instrument in recent years. For example, in 2006 all publicly listed companies in Norway were required to increase female representation on their boards of directors to 40 percent. Following Norway's lead, the European Union and several countries worldwide have passed similar reforms (Bertrand et al., 2019). While there is a large body of literature that studies the effect of quota implementation on market outcomes, there is a lack of research that focuses on individual decision-making when an agent is forced to fulfill a quota. This paper introduces a new role of quotas: the attentional role.

We consider the following setup. An employer in a large firm is a rationally inattentive (RI) decision-maker. Each day she faces a group of applicants. First, she sees applicants' ethnicity and gender (or other observable characteristics), which forms her prior beliefs of the applicants' qualities and potential future productivity levels. After that, the employer can acquire additional information about the applicants – she can read resumes, ask questions, conduct tests, make comparisons of applicants and use other learning strategies. The key feature is that the information acquisition is flexible and endogenous – the employer does not have a fixed guide on how to learn about potential worker's future productivity. At the same time, the employer has cognitive (and/or time) limitations. We model these limitations as costly information acquisition. Therefore, the employer faces a trade-off between acquiring more precise information about the candidates and the cost of this information. After acquiring optimal information, the employer hires the candidate with the highest expected value for the firm.

We follow the setup introduced by Matějka and McKay (2015), in which the agent is uncertain about the value of available options. The values of the options are modeled as an unknown draws from the known distribution. The agent has an opportunity to receive additional information about the realization of the draw in the manner that is optimal given the costs, which we model using the rational inattention framework introduced by Sims (1998, 2003). We think of the labor market candidates as available options and candidates productivity levels as the values of options. Matějka and McKay (2015) show that the choice of an RI agent is typically stochastic and is characterized by the vectors of conditional and unconditional choice probabilities. In this paper we explore the effect of quota implementation on the behavior of an RI agent. We model a quota as a constraint on the unconditional choice probability of choosing a candidate from a particular group. Due to the law of large numbers, such a limitation on unconditional choice probability is essentially a limitation on the share of workers from a particular group in the overall composition of workers in the firm.

We analyze the behavior of the RI agent when quotas restrict her choice, and compare it with an unrestricted case and the situation in which a social planner subsidizes the agent's choice of certain alternatives. We find that the choice probabilities of the agent in the constrained problem have the form of a generalized multinomial logit model as in Matějka and McKay (2015) with an additional state independent component. In a choice among $N$ options with the realized values $v_i$ for $i \in \{1, ..., N\}$, our modified logit formula implies that the probability of choosing option $i$ is:

$$\mathcal{P}_i(\mathbf{v}) = \frac{q_i e^{(v_i - \varphi_i)/\lambda}}{\sum_{j=1}^{N} q_j e^{(v_j - \varphi_j)/\lambda}},$$

where $\lambda$ is the marginal cost of information, the $q_i$ terms are quotas, and $\varphi_i$ are state independent components. The form of choice probabilities shows that the agent behaves as if the value of the option is lower by $\varphi_i$. That is, the $\varphi_i$ component induces an additive utility shifter in the decision-maker's preferences. Therefore, if a choice of a particular alternative is subsidized by $-\varphi_i$ then such a subsidy has exactly the same effect as the quota, which is the result we show in Section 3.3.

These adjustments to the logit model lead to several changes in the agent's behavior. First, if the choice problem is nontrivial, the RI agent who is forced to fulfill a quota always acquires information about existing options. This feature is absent in the unconstrained problem, in which there are prior beliefs of the agent for which she decides not to acquire any additional information. Second, the amount of information she chooses to acquire depends on the level of the quota and could be less than in the unconstrained RI problem.

Further, we raise a question: which level of quotas should be considered optimal? We assume that the utilities of the agent and the social planner are partially misaligned. We consider two distinct goals of the social planner. In the first case, the social planner maximizes the expected value of the chosen options, e.g., when he does not take information costs into account. In the second case, the social planner wants to achieve fairness, i.e., that the conditional probability of being hired is independent of gender or racial identity.

In the first case, for some priors the social planner prefers not to impose a quota and the agent still does not acquire any additional information. In general, the social planner benefits by forcing the agent to fulfill quotas and, consequently, increases the expected value of the chosen option.

In the second case, the agent's choice would be based on the relative benefits from the choice of the alternative as in the standard logit model. We show that this situation is equivalent to a blind resume policy, in which the group identity is hidden from the employer. At the same time, when the social planner's goal is to eliminate any bias towards the options that are good a priori, he significantly restricts her behavior, and this leads to a decrease in the expected value of the chosen alternative.

We also study the case in which the assumption of the social planner's perfect knowledge about the distribution of the values of the alternatives is relaxed. We show that in some cases such a social planner should not intervene in the agent's decision.

Although we primarily focus on the effect of a quota in the labor market, the results of our analysis could be applied to studies of individual behavior in other areas, e.g. a quota on the proportion of safe assets that must be in the portfolio of a financial manager or a quota on the number of orders a taxi driver can reject when searching for a client using peer-to-peer ride sharing applications (such as Uber, Lyft, or Yandex). We briefly discuss these applications in Section 6.

In the next section we review the related literature. Section 3 states the formal model of the agent's behavior with quotas and subsidies. Section 4 demonstrates the implications of the model using a specific example. Section 5 discusses the optimal levels of quotas.

# 2 Literature

Our work contributes to the research on affirmative action and labor market discrimination. Affirmative action is "...any measure, beyond simple termination of a discriminatory practice, adopted to correct or compensate for past or present discrimination or to prevent discrimination from recurring in the future" (U. S. Commission on Civil Rights, 1977 p. 2). One of the most hotly debated types of affirmative action is the implementation of quotas. Coate and Loury (1993), in their famous paper, analyze a model of job assignment and show that quotas may lead to equilibria with persistent discrimination, due to feedback effects between expected job assignments and incentives to invest in human capital. Moro and Norman (2003) study the same problem in the general equilibrium setting and confirm the possibility that quotas can hurt the intended beneficiaries. These articles examine how affirmative action influences the behavior of the target group, and then its interaction with the behavior of the firm. In contrast, our study aims to investigate the individual decision-making process under quotas and consequences for policy design. A review of early studies on affirmative action can be found in Fang and Moro (2010).

To the date there is mixed empirical evidence of the effect of quotas on the quality of workers and firms's revenue. For example, Ahern and Dittmar (2012) show that firm value declined with a law mandating 40% representation of each gender on the board of public limited liability companies in Norway. In addition, the authors show that the average age and experience of the new female directors was significantly lower than that of the existing male directors and argue that this change led to a deterioration in operating performance. At the same time, Eckbo et al. (2019), using econometric adjustments and a larger data set, argue that the effect of implementation of quotas on both the value of firms and on the quality of directors was insignificant. Bertrand et al. (2019), by exploiting the same intervention, document that a quota resulted in significant improvement of the average observable qualifications of the women appointed to the boards and a decrease in the gender gap in earnings within boards, Besley et al. (2017), using Swedish data on the performance of politicians, show that a gender quota on the ballot increased the competence of male politicians. Ibanez and Riener (2018) use data from three field experiments in Colombia and show that the gains from attracting female applicants far outweigh the losses from deterring male applicants. Our paper proposes a mechanism that can possibly explain why the evidence on consequences of quotas is mixed; namely, our model demonstrates that quotas may lead to increases in labor productivity under certain conditions, while it will have negative consequences under others.

Our study fits into the rational inattention literature, which originated in studies by Sims (1998, 2003). As a benchmark, we use the modified multinomial logit model of Matějka and McKay (2015), in which agents choose among alternatives without precise information about their values, but with an opportunity to study the options for some cost.[1] We analyze this model with an additional constraint on the unconditional probabilities of the choice of a certain alternative. Lindbeck and Weibull (2017) analyze investment decisions with delegation to an RI agent. They find that optimal contracts for an agent include a high reward for good investments and punishment for bad investments. Lipnowski et al. (2019) study a model, where a principal provides information to an RI agent, but the principal does not internalize this cost. They show that if there are more than two alternatives, a principal can improve material benefits from the choice by manipulating information. We analyze a similar principal-agent problem in Section 5.1.1, but with a different mechanism. We show that, for a set of parameters, a principal can force the agent to acquire more information by defining the level of quotas on unconditional choice probabilities and, thereby, increase the expected value of the agent's choice.

Bartoš et al. (2016), in a field experiment, show that HR managers and landlords allocate their attention to job and rental applicants in line with rational inattention theory. For example, a non-European name or recent unemployment induces the HR manager to read a job application and CV in less detail, which negatively affects the probability of the applicant being invited for a job interview. The results of our study predict the attention allocation of decision-makers, such as HR managers, in the presence of quotas, i.e. whether they would blindly choose the quoted option or whether a quota would lead to greater information acquisition about the target group. Thus, the results of this study can provide a starting point for the empirical investigation of the effect of a quota on attention allocation. A detailed review of the RI literature can be found in Maćkowiak et al. (2018).[2]

[1] See also Caplin and Dean (2015) for an alternative method for characterizing solutions in a similar environment.

[2] Recent papers on strategic interactions with RI agents include Ravid (2019) and Yang (2019).

Our study also relates to discussion on whether directly administering an activity is better than fixing transfer prices and relying on utility maximization to achieve the same results in a decentralized fashion (Weitzman, 1974). We contribute to the discussion on this issue by comparing comparing an agent operating under quotas and an agent whose choices are subsidized by a social planner on the decision-making processes.

# 3 The model

This section begins with a benchmark model – we describe the standard RI problem as in Matějka and McKay (2015) and its implications. Then we state our problem involving quotas and discuss the properties of the solution. Finally, we analyze the RI problem with subsidies.

A decision-maker faces $N$ options and wants to select the option with the highest value. The state of the world is a vector $\mathbf{v} \in \mathbb{R}^N$, in which $v_i$ is the value of option $i \in \{1, ..., N\}$. The values of options differ from state to state. The decision-maker is uncertain about the realization of the state of the world. However, the decision-maker knows a distribution of possible states of the world – this prior knowledge is described by a distribution $G(\mathbf{v})$. She can refine her knowledge by processing costly information about the realization. Information processing results in a stochastic (possibly not purely stochastic) choice. The conditional probability of option $i$ being selected when the realized values are $\mathbf{v}$ is $\mathcal{P}_i(\mathbf{v})$.

## 3.1 Standard RI problem

The standard RI agent's problem is formalized as follows.

**Standard (unconstrained) RI problem.** *The agent's problem is to find a vector function of conditional choice probabilities* $\mathcal{P}^U = \{\mathcal{P}_i^U(\mathbf{v})\}_{i=1}^N$ *(the superscript "U"* *stands for "unrestricted") that maximizes expected payoff less the information cost:*

$$\max_{\{\mathcal{P}_i^U(\mathbf{v})\}_{i=1}^N} \left\{ \sum_{i=1}^N \int_{\mathbf{v}} v_i \mathcal{P}_i^U(\mathbf{v}) G(d\mathbf{v}) - \lambda \kappa(\mathcal{P}^U, G) \right\}$$

*subject to*

$$\forall i \in \{1, ..., N\}: \quad \mathcal{P}_i^U(\mathbf{v}) \geq 0 \;\; \forall \mathbf{v} \in \mathbb{R}^N, \tag{1}$$

$$\sum_{i=1}^N \mathcal{P}_i^U(\mathbf{v}) = 1 \;\; \forall \mathbf{v} \in \mathbb{R}^N, \tag{2}$$

*where unconditional choice probabilities are*

$$\mathcal{P}_i^{0,U} = \int_{\mathbf{v}} \mathcal{P}_i^U(\mathbf{v}) G(d\mathbf{v}), \quad i \in \{1, ..., N\}.$$

*The cost of information is* $\lambda\kappa(\mathcal{P}^U, G)$*, where* $\lambda > 0$ *is a given unit cost of information and* $\kappa$ *is the amount of information that the agent processes, which is measured by the expected reduction in the entropy (Shannon (1948), Cover and Thomas (2012)):*

$$\kappa(\mathcal{P}^U, G) = -\sum_{i=1}^N \mathcal{P}_i^{0,U} \log \mathcal{P}_i^{0,U} + \sum_{i=1}^N \int_{\mathbf{v}} \mathcal{P}_i^U(\mathbf{v}) \log \mathcal{P}_i^U(\mathbf{v}) G(d\mathbf{v}). \tag{3}$$

This shape of information cost is common in the literature on rational inattention. Its use has been justified both axiomatically and through links to optimal coding in information theory (see Sims,2003, Denti et al.,2019 and Matějka and McKay, 2015 for discussions).

Matějka and McKay (2015) show that, at the optimum, the conditional probabilities of choosing option $i$, $i \in \{1, ...., N\}$ follow the generalized logit form.

**Theorem** (Matějka and McKay, 2015)**.** *Conditional on the realized vector of utilities* *of options* $\mathbf{v}$, *the choice probabilities satisfy:*

$$\mathcal{P}_i^U(\mathbf{v}) = \frac{\mathcal{P}_i^{0,U} e^{v_i/\lambda}}{\sum_{j=1}^N \mathcal{P}_j^{0,U} e^{v_j/\lambda}} \qquad \textit{almost surely.}$$

*If* $\lambda = 0$, *then the agent selects the option(s) with the highest payoff with probability one.*

$\mathcal{P}_i^{0,U}$ is the probability of selecting option $i$ before the agent starts processing any information. These adjustments to the logit model reflect the fact that some options may look a priori better than others. $\mathcal{P}_i^{0,U}$ depends on the distribution $G(\mathbf{v})$ and cost of information $\lambda$.

An important property of the solution is that parameters of the model may exist for which the agent decides not to acquire any information, and instead makes her decision based solely on her prior knowledge. In this situation the agent simply chooses the option with the highest a priori expected value.[3] In terms of the labor market, this means that some categories of workers may not be given any attention and are consequently not hired. As we show in Proposition 2, the social planner can force the agent to receive at least some information about the candidates – this can be achieved via the use of quotas.

## 3.2 Quotas

We consider a departure from the standard RI problem. In the situation we are considering, the agent is not completely free in her choice. Instead, some authority limits her choice in that, for all categories, the share of the options chosen from a category $i$ should be equal to $q_i \in (0, 1)$. [4]

[3]See Caplin and Martin (2017), who analyze a discrete choice problem of an RI agent with costly information acquisition, and show that if there is a high quality default option the agent chooses zero attentional effort.

[4]We focus on the case with binding quotas for all alternatives, since (a) if the quotas have a form of weak inequality, then if it is not binding, the solution is the same as the solution to the

**RI problem with quotas.** *The agent's problem is to find a vector function of conditional choice probabilities* $\mathcal{P} = \{\mathcal{P}_i(\mathbf{v})\}_{i=1}^N$ *that maximizes expected payoff less the information cost:*

$$\max_{\{\mathcal{P}_i(\mathbf{v})\}_{i=1}^N} \left\{ \sum_{i=1}^N \int_{\mathbf{v}} v_i \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}) - \lambda \kappa(\mathcal{P}, G) \right\}$$

*subject to (1)-(3) and*

$$\forall i \in \{1, ..., N\}: \quad \mathcal{P}_i^0 = \int_{\mathbf{v}} \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}) = q_i, \quad q_i > 0, \tag{4}$$

*where* $\mathbf{q} = (q_1, ..., q_N)^T$ *is the vector of quotas and*

$$\sum_{i=1}^N q_i = 1.$$

The choice probabilities at the optimum follow:

$$\mathcal{P}_i(\mathbf{v}) = \frac{e^{(v_i + \alpha_i - \varphi_i)/\lambda}}{\sum_{j=1}^N e^{(v_j + \alpha_j - \varphi_j)/\lambda}},$$

where $\alpha_i = \lambda \log q_i$. This result is formalized in the following proposition:

**Proposition 1.** *Choice probabilities that are the solution of the RI agent problem with quotas are of generalized logit form: logit choice probabilities with an additive state-independent component.*

*Proof.* See Appendix B.1. □

The terms $\varphi_i$ are the Lagrange multipliers on the constraints on unconditional choice probabilities. In the choice probabilities they play a role of utility shifters, that is, the agent's behavior follows logit rule, but with utilities which are changed

unconstrained problem; and (b) the case with quotas only for some categories is considered in Appendix A where we show that results are similar.

by some value that depends on the marginal cost of information, prior belief and the value of a quota. In Section 3.3 we relate $\varphi$ to subsidies which are required to be paid to the agent when she hires workers from a certain category.

Proposition 1 states that the solutions to the standard RI problem and the RI problem with quotas have a similar form. However, there is a crucial difference in the information acquisition strategies of a RI agent with and without quotas. We express it in the following proposition:

**Proposition 2.** *If (i) the agent's prior is nontrivial, that is, she does not believe that some state of the world happens with certainty, and (ii) the quota does not dictate dictate that the agent must exercise some option with certainty, and (iii) the marginal cost of information $\lambda$ is finite, and (iv) $v_i - v_j$ is not constant across states $\forall i, j \in \{1, ..., N\}$ and $i \neq j$ almost surely, then the following holds: the RI agent with quotas always acquires information.*

*Proof.* See Appendix B.2. □

In a labor market context, Proposition 2 means that the employer will never blindly choose a candidate from a certain group based only on her prior beliefs, but will acquire some information.

The assumption that the cost of information is expressed as the expected reduction in entropy is not crucial for Proposition 2 to hold. For example, it also holds for the cost functions which are based on the generalized entropy (Fosgerau et al., 2018). More generally, Proposition 2 holds for any nonnegative continuously differentiable function $\kappa(P, G)$, for which it holds that $\kappa(G, G) = 0$. With such a cost function, the cost of marginal change in choice probabilities is zero at the point of quotas: $\frac{\partial \kappa(P,G)}{\partial P}\Big|_{(P,G)=(P,P)} = 0$. According to condition (iv) in Proposition 2, there is a beneficial deviation from the state-independent probabilities. Therefore, the agent can benefit from acquiring at least some information in order to improve her choice.

### 3.3 Subsidies

We are interested in understanding how an agent's attention strategy depends on the particular form of affirmative action chosen by the government. A practice exists in which a firm receives subsidies if it employs certain categories of workers (for surveys see, e.g., Card et al., 2010, 2017). We show that the agent's behavior in both situations (under quotas and with subsidies) is the same.

If the government introduces a certain level of subsidies to a category $i$, $S_i$, then the agent solves the following problem:

**RI problem with subsidies.** *The agent's problem is to find a vector function of conditional choice probabilities $\mathcal{P}^S = \{\mathcal{P}_i^S(\mathbf{v})\}_{i=1}^N$ (the superscript "S" stands for "subsidy") that maximizes expected payoff less the information cost:*

$$\max_{\{\mathcal{P}_i^S(\mathbf{v})\}_{i=1}^N} \left\{ \sum_{i=1}^N \int_{\mathbf{v}} (v_i + S_i) P_i^S(\mathbf{v}) G(d\boldsymbol{v}) - \lambda \kappa(\mathcal{P}^S, G) \right\},$$

*subject to (1)-(3) and where $S_i$ is a subsidy for choosing option $i$.*

In this case, the solution to the agent's problem follows the standard modified generalized multinomial logit formula, but with the changed value of the option $i$ by $S_i$:

$$\mathcal{P}_i^S(\mathbf{v}) = \frac{\mathcal{P}_i^{0,S} e^{(v_i+S_i)/\lambda}}{\sum_{j=1}^N \mathcal{P}_j^{0,S} e^{(v_j+S_j)/\lambda}}. \tag{5}$$

In order to compare the agent's behavior under both policies, we need to set up subsidies on the level where quotas are fulfilled:

$$\int_{\mathbf{v}} \mathcal{P}_i^S(\mathbf{v}) G(d\mathbf{v}) = q_i.$$

We refer to subsidy $S_i$ as *optimal* when unconditional probabilities of the agent's choice are equal to quotas, $\mathcal{P}_i^{0,S} = q_i$. It is important to note that the vector of optimal subsidies is not unique. Thus, if $S^*$ is a vector of optimal subsidies, then any vector which is obtained by adding any number to all components of $S^*$ is also a vector of optimal quotas.

Equations (5) and (8) provide intuition about the nature of the additive component $\varphi_i$, $i \in \{1, ..., N\}$ from the solution to the RI problem with quotas. This component can be interpreted as the government subsidies that need to be added to the values of the options to induce the RI agent to choose them with required unconditional probabilities. Therefore, the behavior of the RI agent under quotas and optimal subsidies is the same. This result is formalized in the following proposition:[5]

**Proposition 3.** *The information acquisition strategy and conditional choice probabilities of the RI agent when her choice is restricted by quotas are identical to those that exist when her choice is supported by optimal subsidies.*

# 4 Binary example with risky and safe options

In order to illustrate the logic of the model, we consider a simple example in which the agent, e.g. an employer, chooses between two alternatives, e.g. applicants. For simplicity, the type 1 applicant is the safe option that always takes the value $v_1 = R$. The type 2 applicant is the risky option that can take values $v_2 = 0$ with the probability $b$ and $v_2 = 1$ with the probability $1-b$. In a labor market context, we can assume that the share $b$ of workers from a particular category has low productivity, while the share $1 - b$ has has high productive capability. These probabilities (or shares) are the priors of the agent and she does not know what the realization of the state of the world is. The agent has an opportunity to acquire some costly information about the realization. If the realized value of the chosen risky option is

[5] In Appendix E we provide a solution for a binary example with subsidies. We show that while the behavior of the agent under quotas and subsidies is the same, the utility of the agent is different.

$v_2 = 0$ we refer to it as *bad*; if the realized value of the chosen risky option is $v_2 = 1$ we refer to it as *good*.

The agent's choice is restricted in that, on average, the share $q$ of hired applicants should be type 2 (risky) and share $1 - q$ of chosen options should be type 1 (safe). In terms of rational inattention the agent has restrictions on unconditional choice probabilities. In Appendix D we consider a case with two risky options and demonstrate that the nature of the results is similar.

To solve the problem we must find conditional probabilities $\mathcal{P}_i(\mathbf{v})$. We show in Appendix C that the solution is:

$$\mathcal{P}_1(0) = \frac{-b - q + (b + q - 1)e^{\frac{1}{\lambda}} + \sqrt{(b + q - (b + q - 1)e^{\frac{1}{\lambda}})^2 + 4q(be^{\frac{1}{\lambda}} - b)}}{2(be^{\frac{1}{\lambda}} - b)},$$

$$\mathcal{P}_1(1) = \frac{q - b\mathcal{P}_1(0)}{1 - b}.$$

For a given set of parameters, Figure 1 shows the expected reduction in entropy as a function of $b$. In the standard RI problem, when $b$ is close to 0 or 1 the agent decides not to process information and selects one of the options with certainty. However, when the agent is forced to fulfill quotas, she always acquires information and, hence, there are no non-learning areas. For example, when $b$ is close to 1, she is forced to choose a risky alternative with positive probability, and it is profitable to acquire information in order to choose the good risky option rather than to randomly choose a risky alternative.

At the same time, under quotas, the agent can acquire less information than in the standard RI problem (Figure 1). Accordingly, the effect of the quota on the amount of acquired information is ambiguous.

We now explore how the quota affects the expected value of the chosen option. In terms of the labor market this question could be restated in the following way:

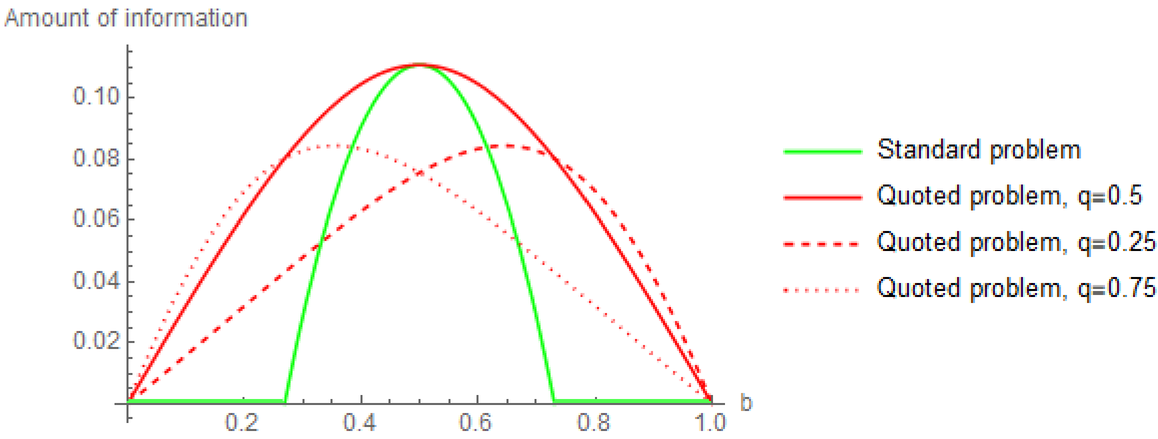


Figure 1: Amount of information as a function of $b$ and $\lambda = 0.5$, $R = 0.5$. The green curve is for the standard RI problem and the red curves are for the quoted RI problem: the solid curve is for $q = 0.5$, the dotted curve for $q = 0.75$ and the dashed curve for $q = 0.25$.

does quota necessarily mean that the expected value (or productivity) of the hired workers will fall? The definition of the expected value of the chosen risky option can be found below. The value of the safe option is always $R$.

**Definition.** *The expected value of the chosen risky option is* $\frac{(1-b)\mathcal{P}_1(1)}{\mathcal{P}_1^0}$. *This is the ratio of the probability of the chosen risky option being good to the probability of choosing any risky option.*

Figure 2 illustrates that the expected value of the chosen risky option is higher (lower) when the quota on it is smaller (larger) than the unconditional probability of choosing it in the standard RI problem.

So far we have considered only the expected value of the chosen risky option. In the next section we discuss how quotas could increase the expected value of the chosen option and minimize statistical discrimination.

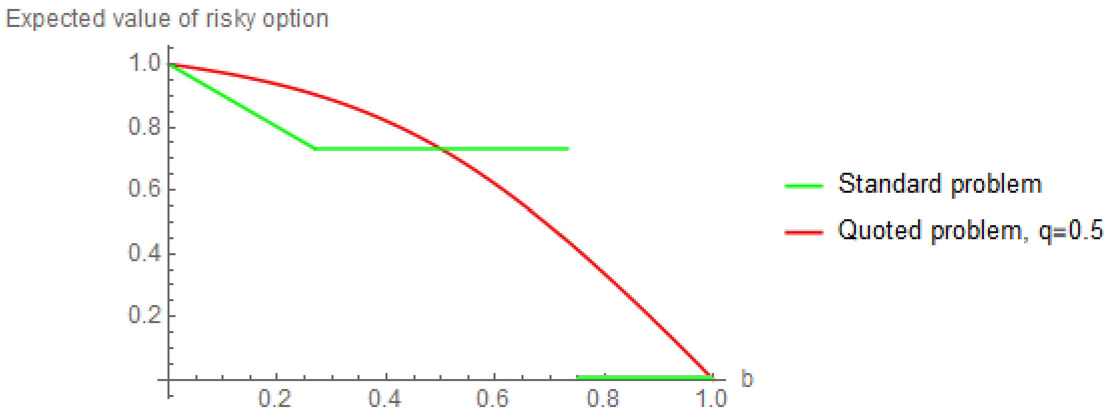


Figure 2: The expected value of the risky option conditional on being chosen as a function of $b$ and $\lambda = 0.5$, $q = 0.5$, $R = 0.5$. The green curve is for the standard RI problem and the red curve is for the quoted RI problem.

# 5 Socially optimal quota

In this section we discuss what an optimal level of quotas is. So far we have solved the agent's problem for a given level of quota. The social planner may have exogenous reasons to establish a certain level of quotas. For example, quotas may be used to compensate for under-representation of certain categories of workers, which could be important and beneficial in the long run. For instance, there is a large literature that demonstrates how diversity diversity boosts profitability (Hunt et al., 2014).

We analyze the optimal level of quotas for two different goals of the social planner. The first possible approach the social planner can take to define an optimal quota is to maximize the expected value of the chosen option without taking into account the information costs of the agent.[6]

The second approach is to minimize bias towards the options that are good a priori. Bias can lead to a situation in which the agent does not choose the option with a high realized value if the prior probability of it being good is low. In terms of the labor market, this would mean that two workers with the same productivity,

[6]We do not study the case in which the social planner maximizes total social welfare including the information cost of the agent because in this case quotas are redundant.

but from different social groups, would have different probabilities of being hired. Accordingly, the social planner wants to minimize this effect.

In this section, we define the social planner's problem for two cases and then discuss the agent's behavior under induced quotas using the binary example from Section 4.

## 5.1 Planner's objectives

### 5.1.1 Value maximizing quota

The social planner chooses optimal quota $q$ to maximize the expected value of the chosen option and does not take into account the cost of information. The maximization problem is

$$\max_q \left\{ \sum_{i=1}^{N} \int_{\mathbf{v}} v_i \mathcal{P}_i(\mathbf{v}, q) G(d\mathbf{v}) \right\},$$

where $\mathcal{P}_i(\mathbf{v}, q)$ is a solution to the RI problem with quotas and $q_i = \int_v \mathcal{P}_i(\mathbf{v}, q) G(d\mathbf{v})$.

This is reasonable, for instance, if positive production externalities exist. However, while the agent is making hiring decisions she may not take these externalities into account. Thus, the maximization problem of the agent and the social planner (organization, industry as a whole, or government) may differ. In such a case it may be beneficial for the social planner to implement quotas. We refer to the solution to this problem as a *Value maximizing quota.*

### 5.1.2 Fair quota

The social planner chooses optimal quota $q$ to minimize the Euclidian distance between the conditional probability of choosing an option in an unconstrained problem and the standard logit probability. The minimization problem is

$$\min_q \left\{ \sum_{i=1}^{N} \int_{\mathbf{v}} \left( \mathcal{P}_i(\mathbf{v}, q) - \frac{e^{v_i/\lambda}}{\sum_{j=1}^{N} e^{v_j/\lambda}} \right)^2 G(d\mathbf{v}) \right\}.$$

The solution to the standard RI maximization problem is $\mathcal{P}_i^U(\mathbf{v}) = \frac{\mathcal{P}_i^{0,U} e^{v_i/\lambda}}{\sum_{j=1}^N P_j^{0,U} e^{v_j/\lambda}}$, where $\mathcal{P}_i^{0,U}$ corresponds to the bias based on prior information about the alternative (Matějka and McKay, 2015). If the information is relatively expensive, the agent can disregard some alternatives without any information acquisition. The social planner wants the agent to choose options as if there is no bias. In this situation, the agent makes her choice according to the standard multinomial logit formula: $\mathcal{P}_i^U(\mathbf{v}) = \frac{e^{v_i/\lambda}}{\sum_{j=1}^N e^{v_j/\lambda}}$. Thus, the probability that the option will be chosen depends only on the realized value of the option and the cost of information that represents the imperfect perception of the agent. In Appendix F we show how we can find this optimal quota in terms of our model. We refer to the solution to this problem as a *Fair quota*.

## 5.2 Results

In this subsection we illustrate the consequences of optimal implementation of quotas using the example from Section 4. We depict the optimal level of quotas dependent on $b$ for two cases: when the social planner maximizes the expected value of the chosen alternatives and when he removes the effect of priors. We also depict the unrestricted unconditional choice probabilities.

Figure 3 illustrates the solution to the social planner's problem as a function of $b$, and $\lambda = 0.5$, $R = 0.5$. When the social planner maximizes the expected value of chosen alternatives, there are still non-learning areas, but they are smaller than in the standard RI problem. The reason for the presence of the non-learning areas is as follows. Consider the situation when $b$ is small, that is, the probability of the risky option being good is high. When the non-trivial quota is implemented, the agent will acquire some information in order to find out whether the risky option is good or bad, but the improvement in the expected value of chosen risky options would not compensate for the loss that comes from an abundance of good risky options.

Therefore, the social planner prefers not to constrain the agent, or, in other words, he prefers to implement the quota that would force the agent to always choose a risky option – the same action that the agent would take without any constraints. Similar logic applies in the situation when the probability of the good state is low.

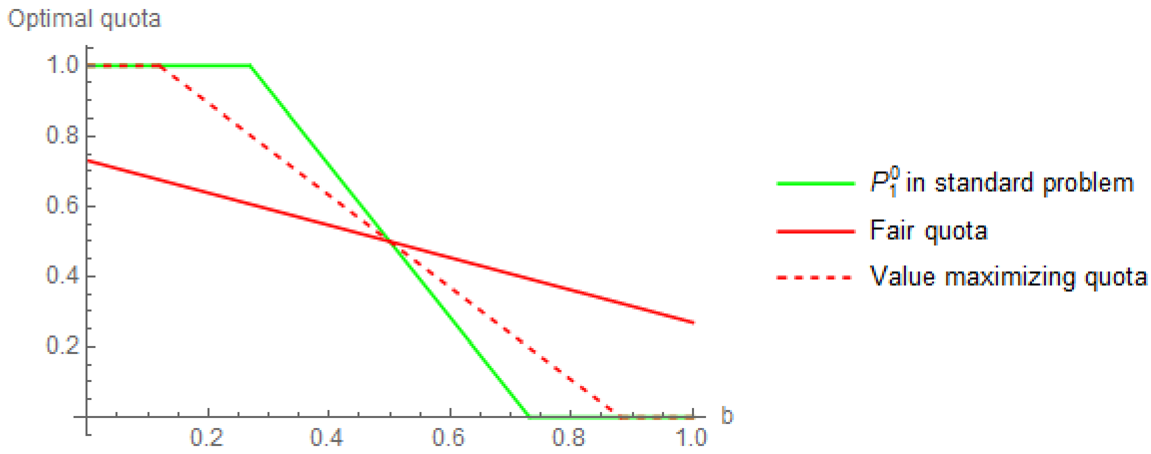


Figure 3: Optimal quota as a function of $b$ and $\lambda = 0.5$, $R = 0.5$. The red curves are for the quoted RI problem: the solid curve is for the fair quota and the dashed curve for the value maximizing quota. The green curve shows the unconditional probability of choosing a risky option for the standard RI problem.

Outside these non-learning areas, the social planner can increase the expected value of the chosen option by setting a quota (Figure 4). Thus, in this example, it is optimal to establish a quota that is higher (lower) than the unconditional probability in the standard RI problem when the state is more likely to be bad (good).

When the social planner minimizes the influence of priors and the state is likely to be bad (good), it is optimal to establish a quota that is higher (lower) than the unconditional probability in the standard RI problem and value maximizing quota. However, in this situation the social planner restricts the agent's behavior more. Thus, if we maximize the expected value of the chosen alternative, we ultimately reach a situation in which most of the good candidates are hired, and some bad alternatives are also chosen. However, when we eliminate the effect of priors, the agent is forced to select fewer good alternatives; hence, the expected value of the chosen option is lower (Figure 4).

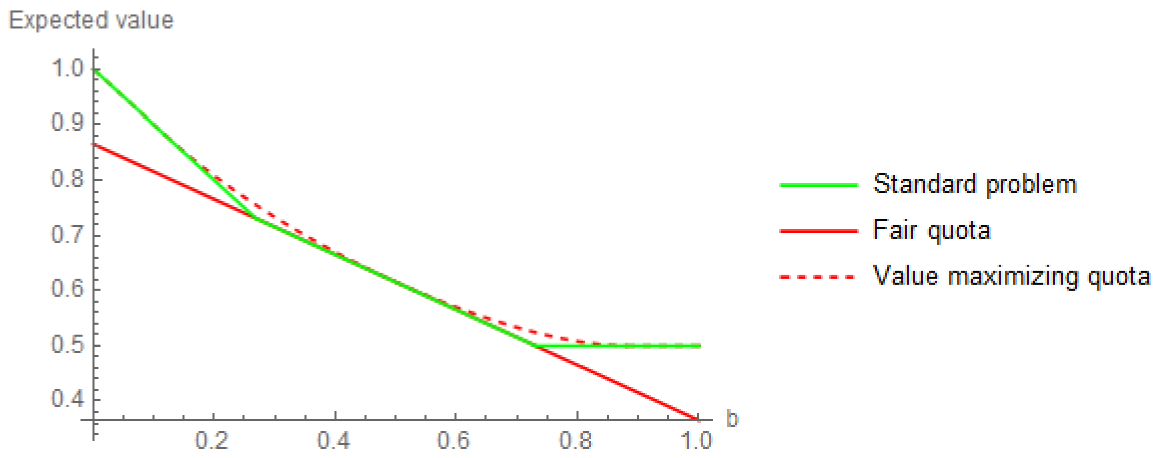


Figure 4: The expected value of the chosen alternative as a function of $b$ and $\lambda = 0.5$, $R = 0.5$. The green curve is for the standard RI problem and the red curves are for the quoted RI problem: the solid curve is for the fair quota and the dashed curve for the expected value maximizing quota.

At the same time, if the social planner does not take into account the agent's information costs, the amount of acquired information is higher than in the standard RI problem, but is lower in comparison with the situation in which the social planner eliminates the effect of priors (Figure 6). Accordingly, in the latter situation the agent pays more attention and, hence, the expected value of the risky option is increased (decreased) more when the probability of the good state is high (low) according to Proposition 4 (Figure 5).

## 5.3 Imperfect social planner

The previous section has shown that the social planner can increase the expected value of a chosen option by setting up a quota. The result was obtained under the assumption that the social planner has all relevant information to determine the optimal quota. In this section we use the example from Section 4, where the agent chooses from safe and risky alternatives. However, now we consider two situations: when the social planner does not know the actual value of the safe option, and when

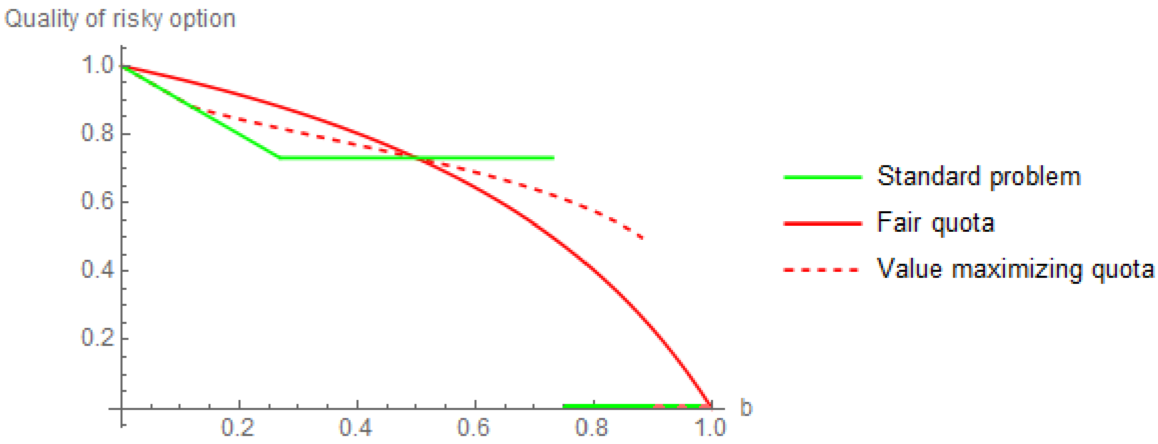


Figure 5: The expected value of the risky option conditional on being chosen as a function of $b$ and $\lambda = 0.5$, $R = 0.5$. The green curve is for the standard RI problem and the red curves are for the quoted RI problem: the solid curve is for the fair quota and the dashed curve for the value maximizing quota.

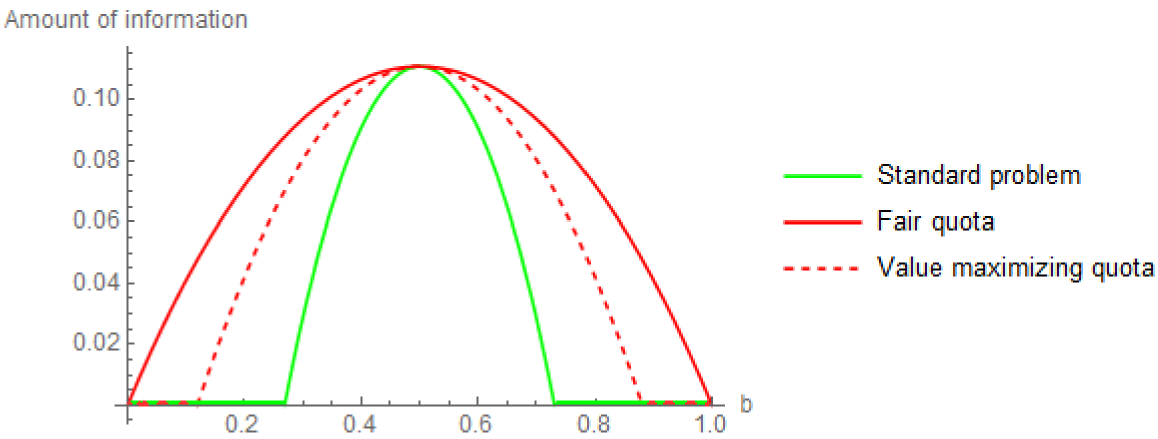


Figure 6: Amount of information as a function of $b$ and $\lambda = 0.5$, $R = 0.5$. The green curve is for the standard RI problem and the red curves are for the quoted RI problem: the solid curve is for the fair quota and the dashed curve for the value maximizing quota.

he does not know the distribution of good and bad risky options in the pool of the candidates. We demonstrate that, in these situations, setting a quota may decrease the expected value of the chosen option.

### 5.3.1 Unknown R

Let us assume that the social planner does not know $R$. He only knows that it is somewhere between 0 and the minimal threshold for the agent not to acquire information, $\underline{R}$. The social planner knows $b$ and $\lambda$; thus, he can compute $\underline{R}$. The belief of the social planner about the value of $R$ is uniform on $[0, \underline{R}]$.

Let us find $\underline{R}$. The optimal unconditional probability of choosing the risky option, $\mathcal{P}_1^{0,U}$, in the standard RI problem is

$$\mathcal{P}_1^{0,U} \in \left\{0, 1, -\frac{e^{\frac{R}{\lambda}}\left(-e^{\frac{1}{\lambda}} + e^{\frac{R}{\lambda}} - b + be^{\frac{1}{\lambda}}\right)}{\left(e^{\frac{1}{\lambda}} - e^{\frac{R}{\lambda}}\right)\left(-1 + e^{\frac{R}{\lambda}}\right)}\right\}.$$

In order to find the threshold $\underline{R}$ we solve the following equation:

$$-\frac{e^{\frac{\underline{R}}{\lambda}}\left(-e^{\frac{1}{\lambda}} + e^{\frac{\underline{R}}{\lambda}} - b + be^{\frac{1}{\lambda}}\right)}{\left(e^{\frac{1}{\lambda}} - e^{\frac{\underline{R}}{\lambda}}\right)\left(-1 + e^{\frac{\underline{R}}{\lambda}}\right)} = 1.$$

This can be rearranged to:

$$\underline{R} = \lambda \ln\left(\frac{e^{\frac{1}{\lambda}}}{1 - b + be^{\frac{1}{\lambda}}}\right).$$

The expected value of the chosen alternative from the perspective of the social planner is

$$\int_0^{\underline{R}} \left(b\Big((1 - \mathcal{P}_1(0)) \cdot R\Big) + (1 - b)\Big(\mathcal{P}_1(1) + R(1 - \mathcal{P}_1(1))\Big)\right)\frac{1}{\underline{R}}dR = U(b, q, \underline{R}),$$

where $\mathcal{P}_1(0)$ and $\mathcal{P}_1(1)$ are solutions for the quoted RI problem from Section 4.

Figure 7 illustrates the expected value of the chosen option for different $q$. The expected value of the chosen option is not higher in the presence of any quota than it would be if there is no quota.

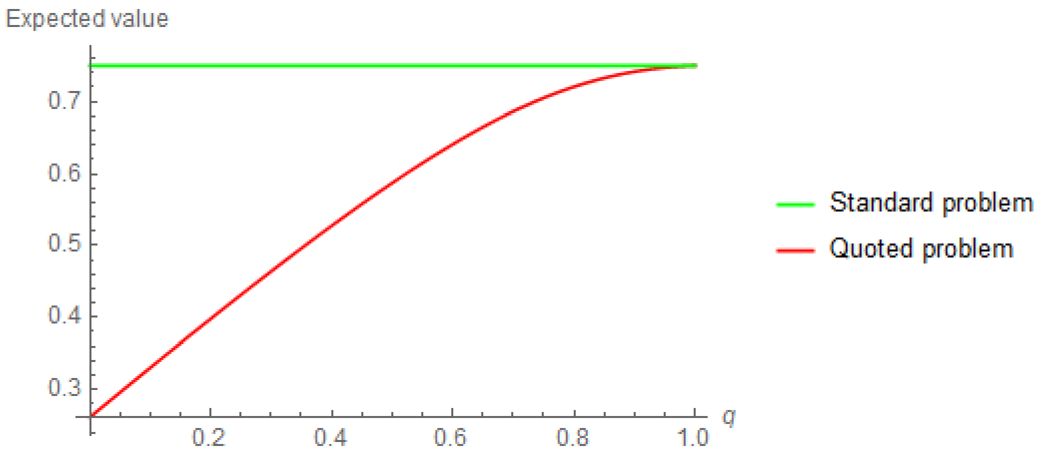


Figure 7: The expected value of the chosen alternative as a function of $q$ and $\lambda = 0.5$, $b = 0.25$. The green curve is for the standard RI problem and the red curve is for the quoted RI problem.

#### 5.3.2 Unknown b

Assume that the social planner does not know $b$. He only knows that it is somewhere between 1 and the minimal threshold for the agent not to acquire information, $\bar{b}$. The belief of the social planner is uniform on $[\bar{b}, 1]$.

Following the same procedure as in the previous subsection we find that:

$$\bar{b} = \frac{e^{\frac{1}{\lambda}} - e^{\frac{R}{\lambda}}}{e^{\frac{1}{\lambda}} - 1}.$$

The expected value of the chosen option from the perspective of the social planner is

$$\int_{\bar{b}}^{1} \left( g(1 - \mathcal{P}_1(0,b))R + (1-b)\Big(\mathcal{P}_1(1,b) + R(1 - \mathcal{P}_1(1,b))\Big)\right) \frac{1}{1-\bar{b}} db = V(\bar{b}, q, R).$$

Figure 8 illustrates the expected value of the chosen option dependent on $q$. The expected value of the chosen option is not higher in the presence of any quota than it would be if there is no quota.

This means that, when the social planner does not have perfect knowledge of the properties of the choice set, any quotas could reduce not only the utility of the agent but also the expected value of the chosen option.

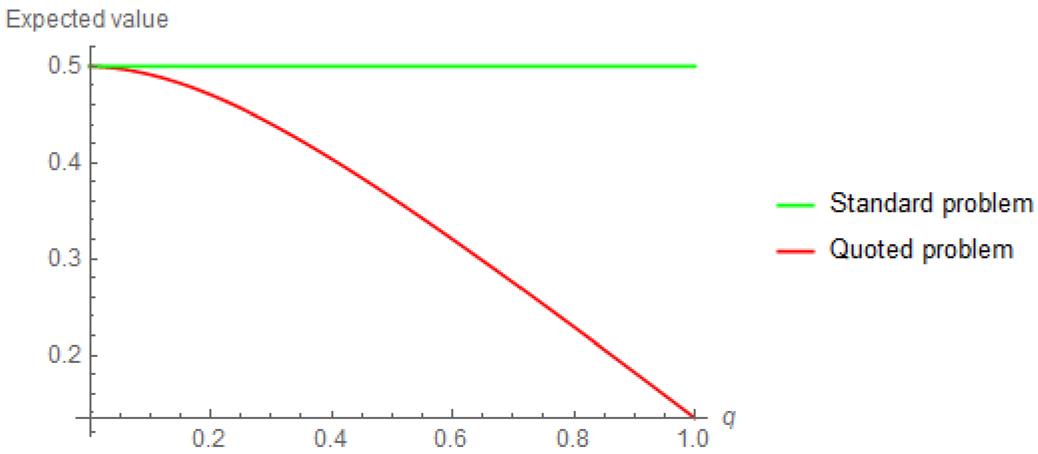


Figure 8: The expected value of the chosen alternative as a function of $q$ and $\lambda = 0.5$, $R = 0.5$. The green curve is for the standard RI problem and the red curve is for the quoted RI problem.

# 6 Conclusion

In this paper we study the optimal behavior of an RI agent who is forced to fulfill quotas when making a choice from a discrete menu. While through the paper we have used labor market settings to illustrate where quotas could influence an agents attention allocation, the proposed model could be also used to analyze the effect of quotas in other areas. For example, the model in Section 4 could be applied to analyze financial interventions. Consider a situation in which the agent manages a financial portfolio and chooses between risky and safe assets. The financial regulator wants to increase the proportion of safe assets in an agent's portfolio. Thus, the regulator imposes a quota. This intervention could lead to higher screening efforts by the agent and, hence, to the more profitable and diversified portfolio. However, as we showed in Section 5.3 such regulation should be performed only when sufficient information is available to the regulator. Otherwise, such an intervention can lead to higher risk and lower profitability.

In addition, the model could be applied to analyze the effect of quotas that are nowadays used in many mobile applications. For example, in many peer-to-peer ride sharing applications, the driver does not know some details of the order before accepting it. In addition, she faces a quota on the number of orders that she can reject. The primary goal of such quotas is to ensure that drivers accept a sufficient number of orders that could be not as profitable for her as some other orders. This restriction forces the driver to calculate the benefits and costs of accepting an order based on the distance, road condition, traffic congestion, etc. At the same time, such a policy could force the driver to switch to a competing platform. Our model could be used to find the optimal quota that will be beneficial for the platform and not too restrictive for the drivers.

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# A Details of the solution for the model with non-binding quotas

Let assume that we have $N$ alternatives and there is only one restriction on unconditional probabilities: $\mathcal{P}_1^0 = q$. Accordingly, this constraint implies that $\sum_{j=2}^{N} \mathcal{P}_j^0 = 1 - q$. Therefore, when $\lambda > 0$, then the Lagrangian of the agent's problem described in Section 3.2 is as follows:

$$\sum_{i=1}^{N} \int_{\mathbf{v}} v_i \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}) - \lambda(-\sum_{i=1}^{N} \mathcal{P}_i^0 \log \mathcal{P}_i^0 + \sum_{i=1}^{N} \int_{\mathbf{v}} \mathcal{P}_i(\mathbf{v}) \log \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}))$$
$$-\int_{\mathbf{v}} \mu(\mathbf{v})(\sum_{i=1}^{N} \mathcal{P}_i(\mathbf{v}) - 1) G(d\mathbf{v}) - \varphi_1(\int_{\mathbf{v}} \mathcal{P}_1(\mathbf{v}) G(d\mathbf{v}) - q) - \varphi_2(\sum_{j=2}^{N} \int_{\mathbf{v}} \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}) - 1 + q),$$

where $\mu(\mathbf{v})$ and $\varphi_{x \in 1,2}$ are Lagrange multipliers. The first order condition with respect to $\mathcal{P}_1(\mathbf{v})$ is:

$$v_1 - \mu(\mathbf{v}) + \lambda(\log \mathcal{P}_1^0 - \log \mathcal{P}_1(\mathbf{v})) - \varphi_1 = 0,$$

and with respect to $\mathcal{P}_j(\mathbf{v})$ is:

$$v_j - \mu(\mathbf{v}) + \lambda(\log \mathcal{P}_j^0 - \log \mathcal{P}_j(\mathbf{v})) - \varphi_2 = 0.$$

Following the same procedure described in Section 3.2 this can be rearranged to:

$$\mathcal{P}_1(\mathbf{v}) = \frac{q e^{(v_1 - \varphi_1)/\lambda}}{\sum_{j=2}^{N} \mathcal{P}_j^0 e^{(v_j - \varphi_2)/\lambda} + q e^{(v_1 - \varphi_1)/\lambda}},$$

and

$$\mathcal{P}_j(\mathbf{v}) = \frac{\mathcal{P}_j^0 e^{(v_j - \varphi_2)/\lambda}}{\sum_{j=2}^{N} \mathcal{P}_j^0 e^{(v_j - \varphi_2)/\lambda} + q e^{(v_1 - \varphi_1)/\lambda}}.$$

Therefore, the solution to the problem will be similar to that described in Section

3.2. The only difference is that now, for all alternatives for which the quota is not binding and for which $\mathcal{P}_j^0 > 0$, the additive state-independent component $\varphi_2$ is the same. This logic extends to any situation when not all quotas are binding.

# B Main proofs

## B.1 Proposition 1

When $\lambda > 0$, then the Lagrangian of the agent's problem is the following:

$$\sum_{i=1}^{N} \int_{\mathbf{v}} v_i \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}) - \lambda \left( -\sum_{i=1}^{N} \mathcal{P}_i^0 \log \mathcal{P}_i^0 + \sum_{i=1}^{N} \int_{\mathbf{v}} \mathcal{P}_i(\mathbf{v}) \log \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}) \right) +$$
$$+ \int_{\mathbf{v}} \xi_i(\mathbf{v} \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v})) - \int_{\mathbf{v}} \mu(\mathbf{v}) (\sum_{i=1}^{N} \mathcal{P}_i(\mathbf{v}) - 1) G(d\mathbf{v}) - \sum_{i=1}^{N} \varphi_i (\int_{\mathbf{v}} \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}) - q_i),$$

where $\mu(\mathbf{v}), \xi_i(\mathbf{v})$ and $\varphi$ are Lagrange multipliers. The first order condition with respect to $\mathcal{P}_i(\mathbf{v})$ is

$$v_i + \xi_i(\mathbf{v}) - \mu(\mathbf{v}) + \lambda(\log \mathcal{P}_i^0 - \log \mathcal{P}_i(\mathbf{v})) - \varphi_i = 0. \tag{6}$$

Let us note that for all $i \in \{1, ..., N\}, \mathbf{v} \in \mathbb{R}^N$ $\mathcal{P}_i(\mathbf{v}) > 0$ almost surely. That makes $\xi_i(\mathbf{v}) = 0, i \in \{1, ..., N\}$, $\mathbf{v} \in \mathbb{R}^N$ almost surely.[7] Therefore, the first order condition can be rearranged to:

$$\mathcal{P}_i(\mathbf{v}) = \mathcal{P}_i^0 e^{(v_i - \mu(\mathbf{v}) - \varphi_i)/\lambda}. \tag{7}$$

Plugging (7) into (2), we obtain:

$$e^{\mu(\mathbf{v})/\lambda} = \sum_{i=1}^{N} \mathcal{P}_i^0 e^{(v_i - \varphi_i)/\lambda},$$

[7]To see this, suppose to the contrary that $\mathcal{P}_i(\mathbf{v}) = 0$ on a set of positive measure with respect to $G$. Then $-\log \mathcal{P}_i(\mathbf{v})$ goes to infinity; in order to compensate for that in the equation (6), one of the Lagrange multipliers should be infinite, which is impossible, because they are finite scalars.

which we again use in (7) and find:

$$\mathcal{P}_i(\mathbf{v}) = \frac{\mathcal{P}_i^0 e^{(v_i - \varphi_i)/\lambda}}{\sum_{j=1}^N \mathcal{P}_j^0 e^{(v_j - \varphi_j)/\lambda}}.$$

Finally, using (4) we obtain:

$$\mathcal{P}_i(\mathbf{v}) = \frac{q_i e^{(v_i - \varphi_i)/\lambda}}{\sum_{j=1}^N q_j e^{(v_j - \varphi_j)/\lambda}}. \tag{8}$$

If we denote

$$\alpha_i = \lambda \log q_i,$$

then (8) can be written as

$$\mathcal{P}_i(\mathbf{v}) = \frac{e^{(v_i + \alpha_i - \varphi_i)/\lambda}}{\sum_{j=1}^N e^{(v_j + \alpha_j - \varphi_j)/\lambda}}.$$

## B.2 Proposition 2

Let us assume the opposite. If the agent does not acquire information, then $P_i(\mathbf{v}) = q_i$ for all $i \in 1, ..., N$. Or

$$\frac{q_i e^{\frac{v_i - \varphi_i}{\lambda}}}{\sum_{j=1}^N q_j e^{\frac{v_j - \varphi_j}{\lambda}}} = q_i.$$

We use the assumption that $q_i \neq 0$ and divide both parts of the equation above by $q_i$.

$$\frac{e^{\frac{v_i - \varphi_i}{\lambda}}}{\sum_{j=1}^N q_j e^{\frac{v_j - \varphi_j}{\lambda}}} = 1.$$

The same holds for all other options. That means that

$$v_i - \varphi_i = v_k - \varphi_k,$$

or

$$v_i - v_k = \varphi_i - \varphi_k.$$

The last equation cannot hold for all realizations of $v$. This is because the LHS of the above equation is state-dependent, while the RHS is state-independent, which contradicts assumption (iv).

# C Details of the solution for the binary example with risky and safe options

The agent's problem is:

$$\max_{\mathcal{P}=(\mathcal{P}_1(0),\mathcal{P}_1(1))} b(1-\mathcal{P}_1(0))R + (1-b)(\mathcal{P}_1(1) + (1-\mathcal{P}_1(1))R) - \lambda\kappa(\mathcal{P}, G)$$

subject to

$$\mathcal{P}_1(0), \mathcal{P}_1(1) \geq 0,$$

$$b\mathcal{P}_1(0) + (1-b)\mathcal{P}_1(1) = q,$$

and where

$$\kappa(\mathcal{P}, G) = -\sum_{i=1}^{2} q_i \log q_i + \sum_{i=1}^{2} \int_{\mathbf{v}} \mathcal{P}_i(\mathbf{v}) \log \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}).$$

The derivative with respect to $\mathcal{P}_1(0)$ of the maximand is:

$$\begin{aligned}-bR + (1-b)(-\frac{b}{1-b} + \frac{b}{1-b}R) - \lambda(b(\log\mathcal{P}_1(0) + 1 - \log(1-\mathcal{P}_1(0)) - 1) + \\ + (1-b)(-\frac{b}{1-b})(\log\mathcal{P}_1(1) + 1 - \log(1-\mathcal{P}_1(1)) - 1)) = 0.\end{aligned}$$

This is equal to:

$$-b - \lambda b(\log \mathcal{P}_1(0) - \log(1 - \mathcal{P}_1(0)) - \log \mathcal{P}_1(1) + \log(1 - \mathcal{P}_1(1))) = 0.$$

Plugging $\mathcal{P}_1(1) = \frac{q - b\mathcal{P}_1(0)}{1-b}$ into this expression we obtain:

$$-b - \lambda b(\log \mathcal{P}_1(0) - \log(1 - \mathcal{P}_1(0)) - \log(\frac{q - b\mathcal{P}_1(0)}{1-b}) + \log(1 - \frac{q - b\mathcal{P}_1(0)}{1-b})) = 0.$$

Dividing each side by $b$ and using the properties of the logarithms yields:

$$1 = \lambda \log \left( \frac{(1-b)(1 - \mathcal{P}_1(0))(q - b\mathcal{P}_1(0))}{(1-b)\mathcal{P}_1(0)(1 - b - q + b\mathcal{P}_1(0))} \right),$$

or:

$$\mathcal{P}_1(0)^2(be^{\frac{1}{\lambda}} - b) + \mathcal{P}_1(0)(b + q - (b + q - 1)e^{\frac{1}{\lambda}}) - q = 0.$$

There are two solutions to this equation:

$$\mathcal{P}_1(0) \in \{\frac{-b - q + (b + q - 1)e^{\frac{1}{\lambda}} + \sqrt{(b + q - (g + q - 1)e^{\frac{1}{\lambda}})^2 + 4q(be^{\frac{1}{\lambda}} - b)}}{2(be^{\frac{1}{\lambda}} - b)}, \\ \frac{-b - q + (b + q - 1)e^{\frac{1}{\lambda}} - \sqrt{(b + q - (b + q - 1)e^{\frac{1}{\lambda}})^2 + 4q(be^{\frac{1}{\lambda}} - b)}}{2(be^{\frac{1}{\lambda}} - b)}\}$$

The solution to the agent's problem should be positive. Only the first root is positive. This is so since the denominator $2(be^{\frac{1}{\lambda}} - b)$ is positive. For the root to be positive, the nominator should be positive. The second root is negative since $4q(be^{\frac{1}{\lambda}} - b)$ is positive, so the square root is larger than the term in front of the square root. For a similar reason, the first root is positive.

That is, the solution to the agent's problem is

$$\mathcal{P}_1(0) = \frac{-b - q + (b+q-1)e^{\frac{1}{\lambda}} + \sqrt{(b+q-(b+q-1)e^{\frac{1}{\lambda}})^2 + 4q(be^{\frac{1}{\lambda}} - b)}}{2(be^{\frac{1}{\lambda}} - b)}$$

and

$$\mathcal{P}_1(1) = \frac{q - b\mathcal{P}_1(0)}{1-b}.$$

# D Binary example with two risky options

The agent chooses between two risky alternatives.[8] The first option can take the values $v_1 = 0$ and $v_2 = 1$ with probabilities $b$ and $1-b$, correspondingly and $0.5 < b < 1$. The second option can take the values $v'_1 = 0$ and $v'_2 = 1$ with probabilities $b' = 1 - b' = 0.5$. Because of the computational difficulties, here we analyze only the area where the first option in expectation is weakly worse than the second option. The remainder is similar to the setup in Section (4).

The agent maximizes the following utility:

$$\begin{aligned}\max_{\mathcal{P}=(\mathcal{P}_1(0,0),\mathcal{P}_1(0,1),\mathcal{P}_1(1,0),\mathcal{P}_1(1,1))} \quad & b(1-b')(1-\mathcal{P}_1(0,1) + (1-b)b'\mathcal{P}_1(1,0)+ \\ & + (1-b)(1-b')(\mathcal{P}_1(1,1) + (1-\mathcal{P}_1(1,1))) - \lambda\kappa(\mathcal{P}, G)\end{aligned}$$

subject to

$$\mathcal{P}_1(0,0), \mathcal{P}_1(1,0), \mathcal{P}_1(0,1), \mathcal{P}_1(1,1) \geq 0,$$

$$bb'\mathcal{P}_1(0,0) + b(1-b')\mathcal{P}_1(0,1) + (1-b)b'\mathcal{P}_1(1,0) + (1-b)(1-b')\mathcal{P}_1(1,1) = q$$

[8]Consider a recruiter who chooses between two candidates from two different groups when she scans candidates resumes. Her beliefs about the values of candidates in these two groups could be different. However, in both groups she can potentially find candidates of high and low value.

and where

$$\kappa(\mathcal{P},G) = -\sum_{i=1}^{2} q_i \log q_i + \sum_{i=1}^{2} \int_{\mathbf{v}} \mathcal{P}_i(\mathbf{v}) \log \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}).$$

First, we derive $\mathcal{P}_1(0,0)$ and plug it back into the utility function:

$$\mathcal{P}_1(0,0) = (q - b(1-b')\mathcal{P}_1(0,1) - (1-b)b'\mathcal{P}_1(1,0) - (1-b)(1-b')\mathcal{P}_1(1,1))/bb'.$$

We take derivatives with respect to $\mathcal{P}_1(0,1)$, $\mathcal{P}_1(1,0)$, and $\mathcal{P}_1(1,1)$. We solve the resulting system of equation numerically.

For the standard RI problem we use Corollary 2 from Matějka and McKay (2015) in order to find $\mathcal{P}_1^0$:

$$\begin{aligned} bb'\frac{e^{\frac{0}{\lambda}}}{\mathcal{P}_1^0 e^{\frac{0}{\lambda}} + (1-\mathcal{P}_1^0)e^{\frac{0}{\lambda}}} + b(1-b')\frac{e^{\frac{0}{\lambda}}}{\mathcal{P}_1^0 e^{\frac{0}{\lambda}} + (1-\mathcal{P}_1^0)e^{\frac{1}{\lambda}}} + \\ + (1-b)b'\frac{e^{\frac{1}{\lambda}}}{\mathcal{P}_1^0 e^{\frac{1}{\lambda}} + (1-\mathcal{P}_1^0)e^{\frac{0}{\lambda}}} + (1-b)(1-b')\frac{e^{\frac{1}{\lambda}}}{\mathcal{P}_1^0 e^{\frac{1}{\lambda}} + (1-\mathcal{P}_1^0)e^{\frac{1}{\lambda}}} = 1. \end{aligned}$$

We solve this equation numerically and plug $\mathcal{P}_1^0$ into $\mathcal{P}_i(\mathbf{v}) = \frac{q_i e^{v_i/\lambda}}{\sum_{j=1}^{N} q_j e^{v_j/\lambda}}$ in order to obtain conditional probabilities.

Figure 9 illustrates the expected value of the first option $((1-b)\frac{\mathcal{P}_1(1,1)+\mathcal{P}_1(1,0)}{2\mathcal{P}_1^0})$ as a function of $b$. Figure 10 illustrates the expected value of the second option $(\frac{b(1-\mathcal{P}_1(0,1))+(1-b)(1-\mathcal{P}_1(1,1))}{2\mathcal{P}_2^0})$ as a function of $b$. The results are similar to the case with safe and risky options: the expected value of the chosen option is higher (lower) when the quota on it is smaller (larger) than the unconditional probability of choosing it in the standard RI problem. Accordingly, the gap in the expected values between the two options is higher when the quota implementation restricts the agent more.

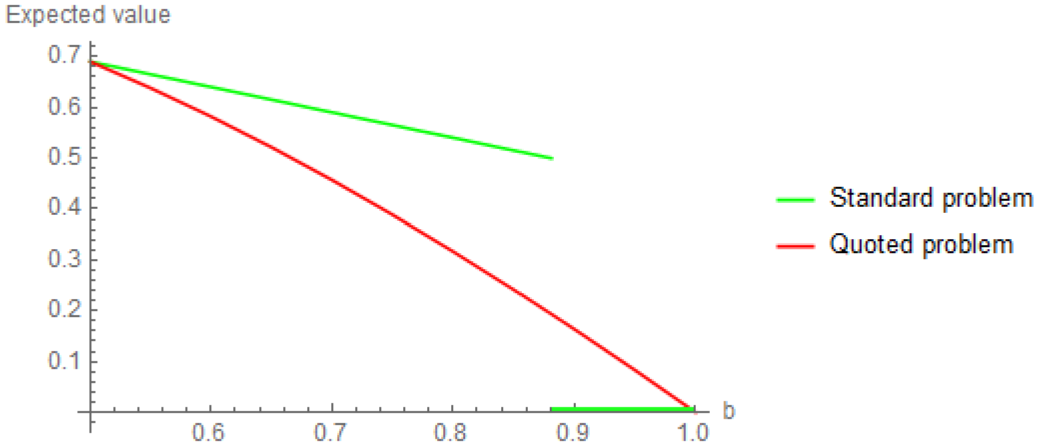


Figure 9: The expected value of the first option conditional on being chosen as a function of $b$ and $\lambda = 0.5$, $q = 0.5$. The green curve is for the standard RI problem and the red curve is for the quoted RI problem.

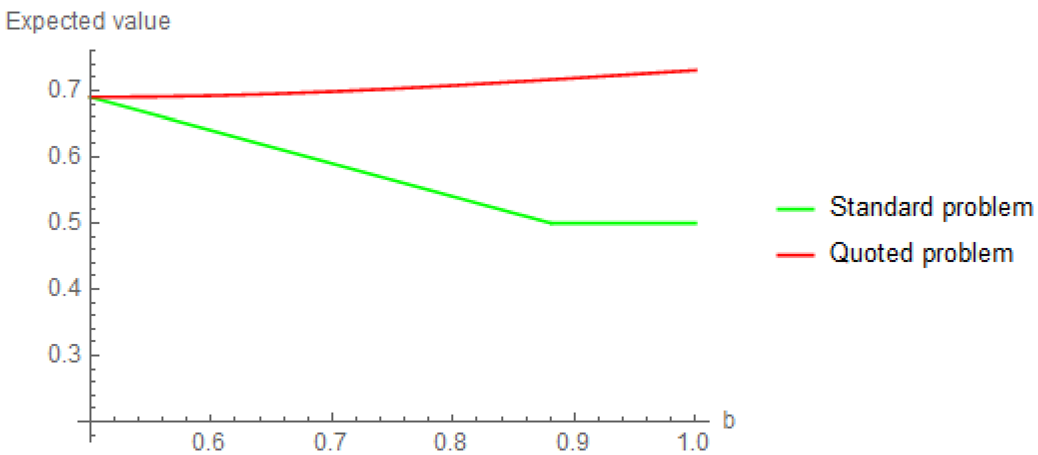


Figure 10: The expected value of the second option conditional on being chosen as a function of $b$ and $\lambda = 0.5$, $q = 0.5$. The green curve is for the standard RI problem and the red curve is for the quoted RI problem.

# E Details of the solution for the binary example: subsidies

The agent chooses between risky and safe options. The safe option always takes the value $v_1 = R$. The risky option can take the values $v_2 = 0$ with the probability $b$ and $v_2 = 1$ with the probability $1 - b$ correspondingly. These probabilities are the priors of the agent and she does not know what the realization of the state of the world is. The agent acquires costly information about the realization. The social planner sets up a subsidy for the risky option: the agent receives extra payment of $S$ if she chooses the risky option.

The maximization problem of the agent in the case of subsidy $S$ on the risky option is as follows:

$$\max_{\mathcal{P}=(\mathcal{P}_1(0),\mathcal{P}_1(1))} b(\mathcal{P}_1(0)S+(1-\mathcal{P}_1(0))R)+(1-b)(\mathcal{P}_1(1)(1+S)+(1-\mathcal{P}_1(1))R)-\lambda\kappa(\mathcal{P},G)$$

subject to

$$\mathcal{P}_1(0), \mathcal{P}_1(1) \geq 0,$$

and where

$$\kappa(\mathcal{P}, G) = -\sum_{i=1}^{2} \mathcal{P}_i^0 \log \mathcal{P}_i^0 + \sum_{i=1}^{2} \int_{\mathbf{v}} \mathcal{P}_i(\mathbf{v}) \log \mathcal{P}_i(\mathbf{v}) G(d\mathbf{v}).$$

In this case the solution has the standard modified multinomial logit form but with the value of risky option increased by $S$. Namely,

$$\mathcal{P}_1(0) = \frac{\mathcal{P}_1^0 e^{\frac{S}{\lambda}}}{\mathcal{P}_1^0 e^{\frac{S}{\lambda}} + \mathcal{P}_2^0 e^{\frac{R}{\lambda}}}$$

$$\mathcal{P}_1(1) = \frac{\mathcal{P}_1^0 e^{\frac{1+S}{\lambda}}}{\mathcal{P}_1^0 e^{\frac{1+S}{\lambda}} + \mathcal{P}_2^0 e^{\frac{R}{\lambda}}}.$$

In order to compare the agent's behavior under both policies we need to find a level of subsidies for which the risky option would be chosen by the agent with the required probability $q$:

$$(1-b)\mathcal{P}_1(1) + b\mathcal{P}_1(0) = q.$$

The unconditional probabilities in the case of the agent's problem with subsidies are as follows:

$$\mathcal{P}_1^0 = \max\{0, \min\{1, \frac{-e^{R/\lambda}(-e^{(1+S)/\lambda} + e^{R/\lambda} - be^{S/\lambda} + be^{(1+S)/\lambda})}{(e^{(1+S)/\lambda} - e^{R/\lambda})(-e^{S/\lambda} + e^{R/\lambda})}\}\}$$

$$\mathcal{P}_2^0 = 1 - \mathcal{P}_1^0.$$

Figure 11 shows the optimal subsidy that is necessary in order to equalize the unconditional probability of choosing the risky option to 0.5 as a function of $b$.

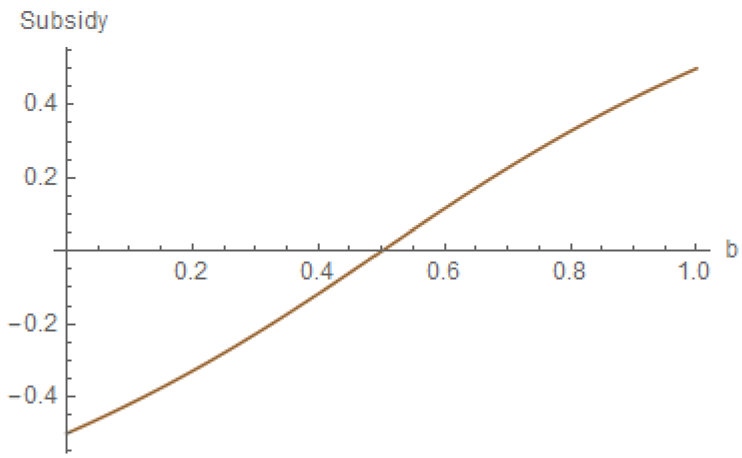


Figure 11: Optimal subsidy as a function of $b$ and $\lambda = 0.5$, $q = 0.5$, $R = 0.5$.

We see that for small $b$ the government sets a financial penalty on choosing the risky option. That is because the risky option is likely to be good and the agent would prefer to choose it more often than in half of the cases. In contrast, if $b$ is high, the government supports the choice of the risky option by establishing a positive subsidy.

When the the social planner sets the optimal subsidy, the conditional probabilities of choosing the risky option in good and bad states are the same with quotas and subsidises. Effectively, this means that if the government wants the agent to choose the risky option in some proportion, there is no difference in the agent's choice if we compare two ways of achieving the goal: through quotas or through subsidies.

At the same time, for high $b$ the utility of the firm in the case of subsidies is higher than in the case of quotas (Figure 12). Therefore, one can speculate that it is impossible to extract all subsidies from the firm afterwards and hence it is more beneficial for firms to lobby for subsidies rather than quotas.

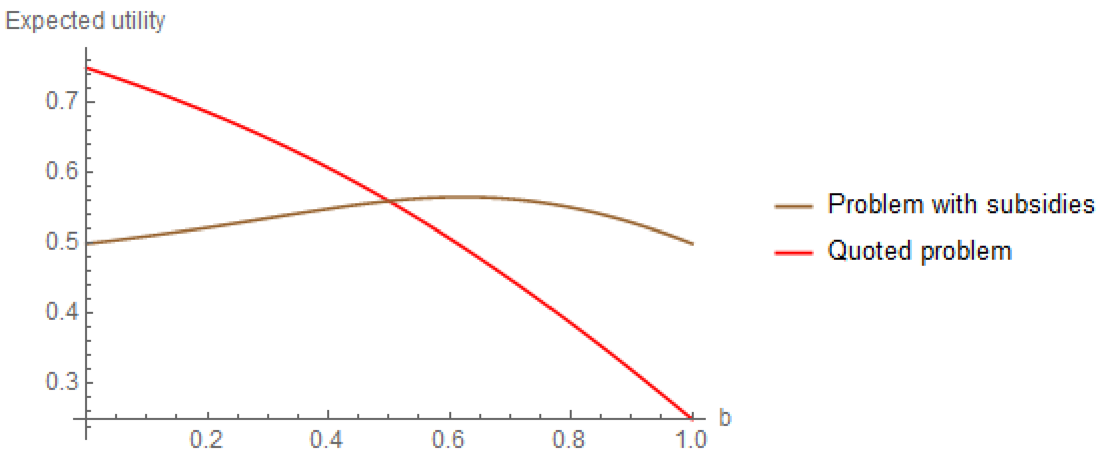


Figure 12: Utility of the agent as a function of $b$ and $\lambda = 0.5$, $q = 0.5$, $R = 0.5$. The red curve is for the quoted RI problem and the brown curve is for the RI problem with subsidies.

# F Details of the solution for the Fair quota

We can find a quota that makes conditional probabilities independent from the prior bias by solving the following equality:

$$\frac{q_i e^{(v_i-\varphi_i)/\lambda}}{\sum_{j=1}^{2} q_j e^{(v_j-\varphi_j)/\lambda}} \Big/ \frac{q_j e^{(v_j-\varphi_j)/\lambda}}{\sum_{j=1}^{2} q_j e^{(v_j-\varphi_j)/\lambda}} = \frac{e^{v_i/\lambda}}{\sum_{j=1}^{2} e^{v_j/\lambda}} \Big/ \frac{e^{v_j/\lambda}}{\sum_{j=1}^{2} e^{v_j/\lambda}},$$

or, after rearranging,

$$e^{\log q_i - \log q_j + (v_i - v_j - \varphi_i + \varphi_j)/\lambda} = e^{(v_i - v_j)/\lambda},$$

which is equal to

$$\lambda \log \frac{q_i}{q_j} - \varphi_i + \varphi_j = 0.$$

In the case when there are two options $q_2 = 1 - q$. From Section 3.3 we know that subsidies exist such that $\varphi_i$ is equal to $S_i$. We can set $\varphi_2 = 0$ and find $\varphi_1$ for any $q$. We end up with the following equation:

$$\lambda \log \frac{q}{1-q} - \varphi_1 = 0. \tag{9}$$

In Appendix E we solve the binary example with subsidies. From there we can set $\mathcal{P}_1^0 = q$ and express $S_1 = \varphi_1$ as a function of $q$:

$$\begin{aligned}\varphi_1 = \lambda \log[&\frac{-e^{R/\lambda}(e^{1/\lambda}(1-b-q)+b-q)}{e^{1/\lambda}2q} + \\ &+ \frac{\sqrt{4e^{1/\lambda+2R/\lambda}(1-q)q + e^{2R/\lambda}(e^{1/\lambda}(1-b-q)+b-q)^2)}}{e^{1/\lambda}2q}].\end{aligned}$$

Then we plug it in (9) and calculate the optimal quota $q$ as a function of $b$.

## Abstrakt

Článek studuje novou roli kvót, například kvót na pracovním trhu; a sice roli, která se týká pozornosti. Zkoumáme vliv implementace kvót na alokaci pozornosti racionálně nepozorného agenta. Naším hlavním výsledkem je zjištění, že racionálně nepozorný agent, který musí plnit kvóty, nikdy nenajme kandidáta, aniž by o něm nejdříve nezískal informace. Opačně je tomu v případě racionálně nepozorného agenta neomezeného kvótami, který může založit své rozhodnutí pouze na apriorním očekávání. Dále ukazujeme, že v našem kontextu jsou kvóty ekvivalentní ostatním typům pozitivních politik jako subvence nebo používání anonymizovaných životopisů. Prezentujeme, jak mohou být naše výsledky využity k nastavení úrovně kvót tak, aby zvýšily očekávanou hodnotu vybraného kandidáta a zároveň snížily statistickou diskriminaci i diskriminaci ve smyslu, kolik pozornosti je věnováno každému uchazeči o práci. Implementace kvót by zároveň mohla být zničující, pokud má sociální plánovač nedokonalou informaci o parametrech modelu.

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