Informative Advertising in a Monopoly with Network Externalities

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Abstract

This paper studies the incentives for a monopolistic firm producing a good with network externalities to advertise when consumers face imperfect information and therefore must search to realize their actual willingness to pay for the good. A firm may disclose market information through advertising if it finds it beneficial. The results suggest that advertising is more likely in the case of a negative network effect and less likely with a positive network effect. When a monopolist faces a strong network externality, it chooses to support the maximum possible network and charge a price equal to the value of the externality. Finally, depending on the value of the search cost and type of network externality, a monopolist may use different advertising content: no information, price information only, product characteristics, or both price and product characteristics. Specifically, if all consumers have the same search cost, as the search cost grows the firm must include more information in the advertising content, while as the network externality changes from negative to positive, the firm reduces the content. In contrast, if consumers differ in their search costs, the firm tends to provide more information as the externality changes from negative to positive.

JEL codes: D42, D83, D85, L12

Keywords: Advertising, Search, Network Effects, Consumption Externality, Bandwagon, Snob Effect, Monopoly, Industrial Organization

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1 Introduction

In some markets, the individual buying decision of a consumer may depend on the number of other consumers who own or buy the same good. In particular, the telecommunication, luxury products, books, gyms, swimming pools, software and fashion. Markets are characterized by strong network effects (also known as network externalities). These externalities may be positive or negative depending on how they affect consumers’ willingness to pay. A network externality is positive when a consumer’s utility increases with the number of consumers using the same good, i.e. consumers benefit from the greater clientele. One can observe this effect in, among others, the software, books, fashion, music markets. When the network effect is negative, a consumer’s willingness to pay is decreasing in the number of consumers who buy the same good. No one likes overcrowded beaches or swimming pools, and some people who desire uniqueness and exclusivity enjoy goods with limited editions such as status and luxury goods.

Network effects are divided into two groups depending on the origin of the effect. The first group is technology side network effects, which are explained by the supply side of the market, specifically originating from technology, and include telecommunication, software, and hardware. They are characterized by a positive externality and the most important research questions are technology adoption and compatibility problems of competing brands. The second group is demand side effects (or network externalities in consumption), which usually originate from consumer preferences for social-economic attributes of goods found in the markets of status goods, fashion, music, books, and sub-cultures. In the economic literature, a positive consumption effect is called conformity or bandwagon effect and a negative consumption externality is called vanity, snob effect or snobbism. The body of the literature on network externalities in consumption is small and mainly represented by signalling models and taxation of positional goods.

The research goal of this paper is to combine network externalities and a disclosure game to study the incentives of a monopoly to reveal any market information. In markets
with network externalities, consumers make their buying decision before they realize the actual volumes of sales, and therefore they must form expectations based on the available market information. However, this information is not easy to obtain and therefore firms may disclose it themselves (at least partly) if needed. Surprisingly, related studies have not yet considered the problem of information frictions in these markets.

Literature on the effects of consumption network externalities on market functioning consists of several articles considering an oligopolistic setting where consumers rationally anticipate a market outcome with fulfilled expectations (Navon et al., 1995; Grilo et al., 2001; Griva and Vettas, 2001). Specifically, these studies assume that consumers are rational, perfectly informed, aware of market prices, and able to foresee the actual clientele size. Moreover, they do not consider any commitment problems related to prices. In reality, consumers face imperfect information, limited abilities to rationally foresee the market outcome and they may not also easily observe prices if firms have not advertised them. In this case consumers cannot correctly form their expectations about clientele sizes and realize their actual willingness to pay for a good. For this reason, many producers of goods with network effects deliver some market information in the form of price advertising, announcement of total supply or product characteristics. This information is used by consumers to correctly foresee the market outcome. Additionally, advertising also works as a commitment device to ensure that firms adhere to their publicly announced prices or output.

Advertising is widely used in search models as a means of information disclosure. When consumers are *ex ante* poorly informed about charged prices or valuations for the good (product characteristics), they may search and learn necessary information by incurring some time or monetary costs. Otherwise, firms may disclose this information themselves in the form of advertising. In the latter case, all disclosed information becomes public knowledge, and as a result consumers are able to optimally make their buying decisions.

This paper considers a model in which consumers are prone to consumption external-
ities but face a need to search because of incomplete information. Specifically, consumers are assumed to be *ex ante* unaware of prices and their actual valuation for the good. There are two ways to obtain necessary information: a costly search by consumers or advertising by firm. If consumers need to search, they compare their expected benefits of a purchase with the cost of the search that is assumed to be either homogeneous or heterogenous. If the monopolist advertises, it chooses how much information to disclose. The model considered in this paper serves to explain how consumers decide on a search, what price internalizes a consumption externality and what conditions influence the choice of the advertising content. In particular, the central research question is how the network externality affects the information disclosure decision of the firm.

The results suggest that when search cost is homogeneous, the firm needs to advertise for a negative network effect since the expected benefits of search decrease in the externality and thus consumers search less. As the network externality moves from negative to positive, the firm reduces the advertised content if search costs are not large. When search cost is heterogenous, the firm advertises less information for a negative network effect and advertises more for a positive network effect. This occurs due to a more sensitive demand, since the probability of buying does not only depend on the consumer’s match alone but also if her search cost is low enough. Moreover, for a negative network effect all consumers prefer a small clientele, and therefore providing little information reduces visits and thus restricts demand. Conversely, for a positive network effect all consumers benefit from a larger clientele, and thus providing more information increases visits and expands demand.

This paper is organized as follows. Section 2 is a review of the related literature. Section 3 describes the search decision of consumers, price-settings of the firm and an advertising game. Section 4 presents results and concluding remarks.
2 Literature Review

There are three groups of literature closely related to this study. The first is a set of papers devoted to the social attributes of consumption. Network externalities in consumption was initially discussed by Veblen (1899) and then formalized by Leibenstein (1950) who coined the terms bandwagon effect, snob effect and Veblen effect. These effects are the key terms used in studies associated with consumption externalities. Further literature on the topic is a set of signalling models and a theory of conformity explaining behavioral reasons as to why individuals are sensitive to a bandwagon or snobbism.

The second group of literature is related to network economics. A detailed review of network economics is found in Shy (2011), the author determines a network effect as a special kind of externality when consumer’s utility or firm’s profits are directly or indirectly affected by the number of adopters of the same buying decision or technology. Economides and Himmelberg (1995) analyze the equilibrium size of networks under different market structures and conclude that monopoly provides the smallest network, prefect competition results in the largest network, and oligopoly has a moderate network. Navon et al. (1995), Grilo et al. (2001), and Vettas and Griva (2011) study network externalities in oligopoly with product differentiation. These papers conclude that a negative network effect softens price competition, while a positive network effect leads to lower prices and stronger competition. Moreover, with a strong bandwagon effect a firm with a locational advantage may even capture a whole market. These studies shed light on how consumption externalities influence price competition in oligopoly. The core

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1Veblen effect describes a situation in which demand positively reacts to a higher price of the good. Buying an expensive good (usually status goods or positional goods) shows a high social-economic status of the buyer. A higher price of a Veblen good serves as a signal of the status. It is important to distinguish between snob effect and Veblen effect. Snob effect is a demand-reducing effect associated with the total clientele size. With snob effect price is not importnat. Consumers only care how many other individuals own the same good. Snob effect can only decrease price elasticity but cannot contradict the law of demand. Veblen effect, in turn, changes the direction of the price effect from negative to positive.

2For instance, Bagwell & Bernheim (1996) and Corneo & Jeanne (1997) assume that buying a conspicuous good signals the social-economic status of consumers.

3Bernheim (1994) explains why people with heterogeneous preferences over behavioral patterns sometimes conform to a single conduct.
limitation of the studies is an assumption that consumers are able to perfectly foresee the market outcome, i.e. the authors consider equilibria with fulfilled expectations. This assumption has to be relaxed because in reality consumers face bounded rationality and incomplete information. Nevertheless, research in network economics has contributed to the building of bridges between the technological nature of networks and behavioral aspects of consumption.

The third group of related literature is devoted to search theory. This theory implies that with incomplete market information consumers need to incur some costs (e.g. time, effort, money) to obtain necessary information. In other words, they are engaged in a costly search. This market friction complicates a buying decision and reduces demand for firms. Anderson and Renault (2006) show that by advertising relevant information such as prices and valuations for the good, a firm can secure profits in the presence of search costs. Konishi and Sandfort (2002) consider an advertising game in monopoly and duopoly. In their paper, price advertising expands firms’ demand and therefore firms may find it profitable to incur advertising costs in order to increase revenues. Depending on the values of advertising and search costs, firms choose between staying silent and advertising.

This paper studies how the network externalities in consumption influence the advertising decision of a monopolist if consumers face a problem of incomplete information. Section 3 presents a model in which a monopolist decides whether to disclose any market information or make consumers search for this information themselves.
3 Model

This section presents a monopoly model of advertising in a market of a good with network effects when consumers are not able to correctly form their expectations about the potential clientele size, because they are poorly informed. Consumers may learn market information by searching or through the firm’s advertising. If consumers search, they incur some search cost which is simply a cost of visiting the store. Otherwise, a monopolist may disclose some market information using advertising. Once consumers have learnt the information they are able to correctly anticipate future sales, form their willingness to pay and, make a buying decision. In this sense, the good is a search good.

A continuum of consumers is independently and uniformly distributed on a unit interval $[0,1]$. Each consumer has a valuation for the commodity $\theta$ which belongs to this interval. However, consumers have ex ante identical tastes, because in the beginning they are not informed about how much they value the product of the monopolist (e.g., they do not know product characteristics, their matches to the product). To learn both $\theta$ and a price, each consumer needs to visit the store and pay a search cost $c$. Search cost is public knowledge.

Every consumer has a utility function $U = \theta + \gamma d^e - p$, where $p$ stands for the market price and $d^e$ is the expected clientele size (future sales). The measure of the network externality is reflected by $\gamma$. If $\gamma > 0$, there is a bandwagon effect (a positive network effect) and if $\gamma < 0$, there is a snob effect (a negative network effect). Without perfect information about market price and $\theta$, the consumer is not able to correctly foresee $d^e$.

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4Nelson (1970, 1974) introduces two types of market goods: search goods and experience goods. A search good is a good with easily verified consumption characteristics, consumers are able to realize their willingness to pay (utility gain) after a search (a visit to the store) but before the purchase. With an experience good consumers can realize their actual utility gain only upon consumption, because product characteristics cannot be observed in advance.

5As discussed before, when consumers decide to buy a conspicuous good (or any good with a network effect), they base their decision on how many other consumers will own this good. Therefore, their willingness to pay is dependent on the clientele size (actual sales). If the price of this good is public knowledge, everyone is able to correctly anticipate actual sales. However, in reality due to bounded rationality and imperfect market information, consumers are not able to perfectly foresee this and thus must spend some time, effort, and money to fix the problem. This situation can be, for instance, resolved with a search.
and consequently she cannot realize her actual benefits of the purchase.

A monopolistic firm produces a good at zero marginal cost, decides on the price and whether it wants to disclose any information with advertising. Advertising is costless. The model also assumes "truth-in-advertising law", whereby it is illegal to announce false information. A monopolist commits to its announcements with advertising. The game considered in this model has the following timing:

1. In the beginning of the game, the firm decides whether to advertise or not. Consumers do not know their valuations and the market price.
   - Case A: There is no advertising.
   - Case B: Only the price is advertised.
   - Case C: Only horizontal matches $\theta$ are advertised.
   - Case D: Both the price and $\theta$ are advertised. Consumers have no information problem but still need to pay $c$ as a visiting cost.

2. Observing the advertising decision of the firm, consumers choose whether to search or not. If a consumer searches, she incurs a search cost $c$.

3. If there was no advertising, each consumer who decides to search realizes her match $\theta$ which is randomly drawn from the interval $[0,1]$

4. Once consumers have learnt both $\theta$ and the price, they make their buying decision.

In this section, two types of search costs are considered. The first case deals with a homogenous search cost, i.e. when all consumers have the same search cost $c$. In the second case, it is assumed that consumers are heterogenous in search costs and each consumer $i$ has her own $c_i$. This search cost does not depend on $\theta_i$.
3.1 Case A. No advertising

Let us start with the problem of a representative consumer who observes no advertising from the firm. In this case she does not know her $\theta$ and the charged price, and thus she must search incurring some sunk visit cost $c$. A consumer $i$ will buy the good if her surplus is not negative: $\theta_i + \gamma d^v - p \geq 0$, which means that the share of consumers with non-negative surplus is $(1 - \hat{\theta})$, with $\hat{\theta} = p - \gamma d^v$.

The firm cannot influence the search decision of consumers without advertising and thus takes the number of searching consumers as given$^6$. Let us denote the share of searching consumers as $s$. In this case, the profit function of the monopolist is as follows:

$$\pi^n(p_n) = p_n \cdot s \cdot (1 - p_n + \gamma d^v)$$

Taking $s$ and $d^v$ as given, the FOC gives the monopoly price $p_n = \frac{1 + \gamma d^v}{2}$. As expected, this price increases in $\gamma$. If there is a bandwagon effect, a greater clientele size increases the consumer’s valuation for the product and thus increases the price. In contrast, with a snob effect, product valuation decreases with a larger volume of sales and thus it reduces the price.

Consumers anticipate this price and decide to search only if their benefits of the search exceed the search cost $c$. The expected benefit of a visit is the expected consumer surplus and therefore the search condition is as follows:

$$E(CS) = \int_{\hat{\theta}}^{1} (\theta_i + \gamma d^v - p) \, d\theta \geq c$$

This search rule implies that a consumer decides to visit the store if the expected benefits of search $E(CS) \geq c$, and remains inactive otherwise$^7$.

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$^6$Since a firm cannot influence the number of consumers who search, it also cannot influence the expectations of consumers, i.e. $d^v$.

$^7$When the expected benefits of a search are equal to the visiting cost, $E(CS) = c$, two types of equilibrium may exist: full participation in which all consumers decide to search, and partial participation in which consumers randomize between visiting and being inactive.
When consumers visit the store, they learn all information and thus in equilibrium a market clearance condition must satisfy: 

\[ d^e = s(1 - p + \gamma d^e) \]

In other words, rational consumers must foresee that their expectations about actual sales \( d^e \) are exactly what is produced by the firm. In turn, this means that 

\[ d^e = s \frac{(1 - p)}{1 - \gamma s} \]

If we solve this condition for the monopoly price \( p_n = \frac{1 + \gamma d^e}{2} \), then 

\[ d^e = \frac{s}{2 - s \gamma} \]

The corresponding monopoly price is therefore \( p_n = \frac{1}{2 - s \gamma} \).

Therefore, the expected benefits of a search can be computed as follows:

\[
E(CS) = \int_{\frac{1}{2 - s \gamma}}^{1} \left( \theta_i + \gamma \frac{s}{2 - s \gamma} - \frac{1}{2 - s \gamma} \right) d\theta = \frac{1}{2(2 - s \gamma)^2}
\]

### 3.1.1 Homogenous visiting costs

If a visiting cost is the same for everyone, then the search condition is identical for each consumer and the search decisions of all consumers coincide. This implies that a share of consumers who decide to visit, \( s \), is either 1 or 0. If \( s = 0 \), no one is active and there is no market. If \( s = 1 \), then everyone searches and the corresponding equilibrium is defined by 

\[ p_n = \frac{1}{2 - \gamma}, \quad \hat{\theta} = \frac{1 - \gamma}{2 - \gamma}, \quad d^e = \frac{1}{2 - \gamma} \text{ and } \pi^n = \frac{1}{(2 - \gamma)^2}. \]

It is important to note that two different equilibria are possible, depending on the value of \( \gamma \). In particular, the equilibrium described above is only possible for \( \gamma < 1 \). However, with a strong bandwagon effect \( \gamma \geq 1 \), the equilibrium demand function \( d^e = \frac{(1 - p)}{1 - \gamma} \) is upward slopping and thus the pricing rule changes. Let us start with the case in which \( \gamma < 1 \).

The corresponding search condition is described by the following inequality:

\[
E(CS) = \frac{1}{2(2 - \gamma)^2} \geq c
\]

To avoid randomization, it is assumed that consumers prefer buying to having nothing and therefore they decide to search in any case. This assumption applies to the rest of the paper as well. An equilibrium with partial participation is considered in Appendix A.
Let us denote the threshold cost where this condition holds as a strict equality as $\bar{c}$.

If we investigate how this threshold cost changes with the measure of the externality $\gamma$, we will obtain the following result:

$$\frac{d\bar{c}}{d\gamma} = (2 - \gamma)^{-3} > 0$$

This shows that as $\gamma$ grows, the threshold search cost increases as depicted in Figure 1 and implies that the set of search costs for which consumers decide to search expands with $\gamma$. In other words, consumers are more likely to search for a positive $\gamma$ and more likely to stay inactive for a negative $\gamma$. This conclusion is summarized in Lemma 1.

**Lemma 1.** If a monopolist does not advertise prices and $\gamma < 1$, consumers tend to search more for a product with a bandwagon effect and tend to search less for a product with a snob effect. This implies that advertising is more effective in the case of a snob effect.

With a bandwagon effect (i.e. a positive network effect) greater $\gamma$ increases the expected consumer surplus, which in turn increases search intensity. With a snob effect (i.e. a negative network effect) greater expected sales reduce consumer surplus and thus the benefits of a search decrease.

The second option is that consumer preferences are characterized by a strong positive consumption externality, $\gamma \geq 1$. In this setting, the equilibrium demand function increases in price $d^e = \frac{(1-p)}{1-\gamma}$ and due to this functional form higher sales of the monopolist are always associated with a higher price. When there is a strong positive network effect, it can dominate the negative effect of price on demand and thus the only way the firm may have a positive market share is to charge a higher price. The only equilibrium compatible in this setting is when everyone searches, everybody buys, and the monopolist charges

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8 Appendix B presents a detailed explanation on how equilibrium demand with fulfilled expectations is formed for network goods.
the maximum possible price that supports this equilibrium. This price can be found from two conditions: \( d^e = \frac{(1-p)}{1-\gamma} \) and \( d^e = 1 \). Thus, the only price that satisfies the conditions is \( p = \gamma \). Consumers rationally anticipate this price and compute their expected surplus as:

\[
E(CS) = \int_0^1 (\theta_i + \gamma - \gamma) d\theta = \frac{1}{2}
\]

Therefore, consumers search for \( c < \frac{1}{2} \) and the resulting price is equal to \( \gamma \). The corresponding profit is also \( \gamma \).

**Lemma 2.** If a monopolist does not advertise and \( \gamma \geq 1 \), the only equilibrium is when everyone searches, the monopolist serves all consumers and charges a price equal to the value of the network externality \( \gamma \).

This result is intuitive: when the network effect is strong, the utility gain of consumers
approaches its maximum at any price $p < \gamma$, since everyone is willing to buy the good. Thus, the monopolist charges the highest possible price that induces the full participation of consumers. In this case, both price and profit increase in the network effect: greater $\gamma$ allows the monopolist to charge a higher price and obtain a higher profit.

To sum up, the firm can remain silent with a homogenous search cost in two cases: when $\gamma < 1$ and $c \leq \frac{1}{2(2-\gamma)^2}$ it charges $p_n = \frac{1}{2-\gamma}$ serving $\frac{1}{2-\gamma}$ share of consumers; when $\gamma > 1$ and $c < \frac{1}{2}$ the firm charges $p_n = \gamma$ selling to everyone. Otherwise, there is no market because no one searches. Therefore, the only way to make consumers visit the store is to provide information in the form of advertising.

### 3.1.2 Heterogenous visiting costs

The case with heterogenous search costs means that the costs are different for every consumer. This difference may be explained by different abilities for a search, a different distance to the store, or a different value of time, etc. However, the key issue is that consumers do not have the same search costs. This implies that the share of visiting consumers $s$ can take any value from 0 to 1.

Let us assume that each consumer $i$ has a visit cost $c_i$ which is uniformly distributed on $[0,1]$ and is independent of $\theta$. The problem of the firm is the same as before and the expected consumer surplus is as follows:

$$E(CS) = \int_{\frac{1-s\gamma}{2-s\gamma}}^{1} \left( \theta_i + \gamma \frac{s}{2-s\gamma} - \frac{1}{2-s\gamma} \right) d\theta = \frac{1}{2(2-s\gamma)^2}$$

A consumer who decides to search must have a search cost no larger than $\frac{1}{2(2-s\gamma)^2}$, and given the uniform distribution of the visiting costs, $s = \frac{1}{2(2-s\gamma)^2}$ \footnote{Indeed, since $c_i$ is uniformly distributed on $[0,1]$, a share of consumers with $c_i < \frac{1}{2(2-s\gamma)^2}$ is a share of $\frac{1}{2(2-s\gamma)^2}$. These are the consumers who decide to search, i.e. $s$.}. This condition can be transformed into an implicit function $F(s,y) = 0$ which indirectly expresses $s$ via...
It is of interest to see how the share of searching consumers depends on the network effect $\gamma$. This can be done using the implicit function theorem:

$$\frac{ds}{d\gamma} = -\frac{F_\gamma'}{F_s'} = \frac{(2 - s\gamma)^3}{s}$$

Figure 2 shows that the share of searching consumers is higher for a bandwagon effect and lower for a snob effect even with heterogenous search costs. The same explanation as before is applicable to this result: a greater clientele size increases consumer surplus for a positive network effect and decreases the surplus for a negative effect. This influences the search decision of consumers and, correspondingly, the advertising policy of the firm which is reflected in Lemma 1.

When the monopolist does not advertise and consumers have heterogenous search costs, the resulting equilibrium is described by $p_a = \frac{1}{2 - s\gamma}$, $\hat{\theta} = \frac{1 - s\gamma}{2 - s\gamma}$, $d^c = \frac{s}{2 - s\gamma}$ and $\pi^a = \frac{s}{(2 - s\gamma)^2}$. As one can see, heterogenous search costs bring lower price, sales, and profit in comparison with the homogenous costs case, because heterogenous costs reduce the share of potential buyers even more. Figure 3 shows the curves of equilibrium $s$, $d$, $p$ in the space of $\gamma$ (horizontal axis). All three increase in the externality. Larger $\gamma$ enhances the expected consumer surplus and thus stimulates a search and sales, and increases price.

As in the previous case, two options are possible: $s\gamma < 1$ and $s\gamma \geq 1$. By the same reasoning, if the network effect is high enough, there can be an equilibrium when a monopolist serves all consumers. This possibility occurs when the expected consumer surplus exceeds $1^{10}$:

$$E(CS) = \int_0^1 (\theta_i + \gamma - p) \, d\theta = \frac{1}{2} + \gamma - p \geq 1$$

This suggests a price $p = \gamma - \frac{1}{2}$ that supports an equilibrium with full participation. However, this equilibrium is only possible with a very large positive $\gamma$.

\footnote{Note that since $c_i \in [0, 1]$, a consumer with the maximum search cost searches only if the expected surplus exceeds 1.}
To summarize the results of the case when the monopolist does not advertise any information, let us state the proposition that follows:

**Proposition 1.** When a monopoly provides no information about its price and consumers’ matches, the likelihood of a visit increases in the bandwagon effect and decreases in the snob effect. Heterogeneous visiting cost has lower equilibrium sales and price compared to the case when the cost is homogeneous. A difference in the visiting costs of consumers reduces the search benefit even more and thus consumers tend to search less.

### 3.2 Case B. Only price is advertised

#### 3.2.1 Homogeneous visiting costs

Let us suppose that, at the first stage of the game, the monopolist decides to announce its price. This situation takes place when the search cost exceeds the threshold value $\tilde{c}$ and thus there will be no market for the good without advertising. The firm must
advertise at least its price to reassure consumers that visiting the store is worthwhile. By disclosing its price alone, the firm can internalize the consumption externality, but consumers still need to search because they do not know their horizontal matches, i.e. $\theta$.

As in Anderson and Renault (2006), the firm advertises a price that renders expected consumer surplus net of search cost zero. This means that, with homogeneous visiting costs, the advertised price is a critical price at which all consumers are indifferent between searching and being inactive. If the monopolist advertises some price $p$, a consumer will be indifferent between visiting the store and being inactive if:

$$E(CS) = \int_{p-\gamma d^e}^{1} (\theta_i + \gamma d^e - p) \, d\theta = \frac{(1 - p + \gamma d^e)^2}{2} = c$$

Given $d^e = \frac{1-p}{1-\gamma}$ if all consumers search, the monopolist advertises a price $p^* = 1 - \sqrt{2c(1 - \gamma)} > 0$ and sells to $\sqrt{2c}$ consumers. If $\gamma < 1$, this equilibrium is possible.
only for $\sqrt{2} < \frac{1}{\sqrt{2(1-\gamma)}}$; if $\gamma \geq 1$, the equilibrium always exists. The resulting profit is $\pi^* = \sqrt{2c}(1 - \sqrt{2c(1-\gamma)})$. It is interesting to note that actual sales do not depend on the network effect. When the firm advertises its price only, it chooses a target clientele size irrespective of the consumption externality and charges a price that captures the whole expected consumer surplus. When price is advertised alone, the firm can fully internalize the consumption externality with the announced price only. Since consumers still need to search to realize their matches, their visiting decision crucially depends on the value of $c$. Consequently, the equilibrium demand depends on the visit cost $c$ only.

Lemma 3. If a monopolist decides to advertise its price only, then the advertised price is $p^* = 1 - \sqrt{2c(1-\gamma)}$ and the share of served consumers is $\sqrt{2c}$. This equilibrium exists only for $\gamma > \frac{\sqrt{2c} - 1}{\sqrt{2c}}$.

The equilibrium price decreases in the search cost for a snob effect and a weak bandwagon, $\gamma < 1$. This result is parallel to that in Anderson and Renault (2006) in which a larger search cost makes the firm advertise a lower price to attract consumers. However, when bandwagon is strong ($\gamma \geq 1$), the price increases in $c$. This can be explained by the unusual functional form of the demand function with a strong bandwagon effect. When $\gamma \geq 1$, demand function increases in price and thus a larger market share is always associated with a higher price. Actual sales of the monopolist equal $\sqrt{2c}$ and thus a higher search cost raises the equilibrium price for a strong positive network effect. In addition, if $c \geq \frac{1}{2}$, the monopolist needs to sell to the whole market, which implies that the advertised price is the price that induces full participation, $p = \gamma$. In contrast, in the previous case with no advertising, it was shown that an equilibrium with full participation of consumers is possible only if $c < \frac{1}{2}$. Therefore, price only advertising cannot have a fully covered market.
3.2.2 Heterogenous visiting costs

When visiting costs differ across consumers, the expected benefit of the purchase shows a fraction of consumers for whom visiting costs are lower than their expected consumer surplus. Therefore, a share of searching consumers, \( s \), is equal to \( E(CS) = \frac{(1-p+\gamma d^e)^2}{2} \).

When the firm advertises its price, it can influence the search decision of consumers with the announced price and thus its profit function is as follows:

\[
\pi^p = p \cdot s \cdot (1 - p + \gamma d^e) = p \cdot \frac{(1 - p + \gamma d^e)^2}{2} \cdot (1 - p + \gamma d^e) = p \cdot \frac{(1 - p + \gamma d^e)^3}{2}
\]

Both fulfilled expectations and market clearing conditions imply that consumers correctly anticipate what the firm will sell in the market: \( d^e = \frac{(1-p+\gamma d^e)^3}{2} \). This condition is an implicit equilibrium demand function.

The firm chooses to announce the price that maximizes its profit. FOC with respect to price is:

\[
\pi^p = \frac{3(1-p+\gamma d^e)^2}{3\gamma(1-p+\gamma d^e)^2 - 2} - p \cdot \frac{(1-p+\gamma d^e)^3}{2} = 0
\]

The corresponding equilibrium is defined by the system of three equations which implicitly express market price \( p \), share of visiting consumers \( s \), and equilibrium volume of sales \( d \):

\[
\begin{align*}
\begin{cases}
  s = \frac{(1-p+\gamma d)^2}{2} \\
  d = s \sqrt{2s} \\
  p = \sqrt{2s(1-3\gamma)}
\end{cases}
\end{align*}
\]

The corresponding curves of equilibrium \( s, d, p \) are shown in Figure 4. The horizontal axis is a space of \( \gamma \). Both sales and the share of visiting consumers increases in \( \gamma \) as expected, while price decreases in the externality. When the firm advertises its price, it
Figure 4: Equilibrium $s$, $d$, $p$ when price is advertised only
can influence the expectations of consumers and thus it uses a price announcement to support a particular expectation about the clientele size. Specifically, if \( \gamma \) is negative, the firm must set a sufficiently high price to have a small clientele since a smaller clientele implies a higher valuation for the good. However, when \( \gamma \) approaches the bandwagon effect, the firm must charge a low price to attract more consumers since a higher clientele enhances consumers’ willingness to pay. The firm can use this price advertising only for \( \gamma < 0.36 \) (a condition on positive values of \( s, d \) and \( p \)).

**Proposition 2.** If a monopolist advertises its price alone and does not provide any match information, then with a homogeneous visiting cost it chooses a fixed target volume of sales and internalizes the network externality with price only. This price increases in the network effect. In contrast, with a heterogeneous visiting cost both sales and the share of visiting consumers increase in the network effect while price positively reacts to the externality. The firm commits to its price with advertising and thus it can positively affect consumers’ expectations with a higher price and negatively affect their expectations with a lower price.

### 3.3 Case C. Advertising of \( \theta \) only

#### 3.3.1 Homogeneous visiting costs

When the monopolist advertises only \( \theta \), consumers learn their matches (which differ across consumers). This type of advertising leads to a hold-up problem and consequently to the Diamond paradox where no one wants to visit the store. Thus, a monopolist never advertises \( \theta \) only.

To explain why consumers never visit the store when they are informed only about their valuations for the good, let us consider the reasoning as follows. When consumers know their \( \theta \) and no price is advertised, they rationally expect some realization of the price \( p \) charged by the firm and the associated sales \( d^e \). If any consumer visits the
firm, then this consumer has a willingness to pay that exceeds the sum of the price and the consumer’s search cost: \( \theta + \gamma d^e > p + c \). Although the firm takes it into account, it cannot influence the expectations of the consumer \( d^e \) with price (it simply cannot commit to price) and therefore tends to increase the price until the consumer’s surplus is fully taken by the firm. This reasoning leads to the Diamond paradox, in which no price exists below the upper price limit and thus there will be no visits of consumers. This result is similar to that in Anderson and Renault (2006).

### 3.3.2 Heterogeneous visiting costs

The introduction of heterogeneous costs allows us to avoid the Diamond paradox. As discussed in Anderson and Renault (2006), with heterogeneous search costs equilibrium prices may be less than the monopoly price and tend smoothly toward marginal cost as the search cost distribution puts more weight in the neighborhood of zero.

When the firm discloses horizontal matches to consumers, a particular consumer \( i \) expects some price \( p^* \) and visits the store if her \( \theta_i > p^* + c_i - \gamma d^e \). Therefore, the firm knows that for each \( c_i \) a share of visiting consumers is equal to \( \int_{p+c_i-\gamma d^e}^{p+c_i} d\theta = 1 - p - c_i + \gamma d^e \).

An integration over all \( c_i \) gives a demand function as follows:

\[
D = \int_0^1 (1 - p - c + \gamma d^e) \, dc = \frac{1}{2} - p + \gamma d^e
\]

Since the firm advertises matches only, it cannot influence the expectations of consumers with its price, hence \( d^e \) is taken by the firm as given. The corresponding profit function is \( \pi^m = p \left( \frac{1}{2} - p + \gamma d^e \right) \) and FOC with respect to price gives \( p = \frac{1 + 2 \gamma d^e}{4} \). Consumers rationally anticipate this price and, given that consumers’ expectations are fulfilled in equilibrium, the firm’s sales are \( d = \frac{1}{4(2-\gamma)} \). The corresponding equilibrium price is \( p = \frac{1}{2(2-\gamma)} \) and \( \pi^m = \frac{1}{4(2-\gamma)^2} \). Match advertising is only possible for \( \gamma < 1.5 \), since \( d \in (0, 1) \).
Both price and sales increase in the consumption externality $\gamma$. Derivatives of both are positive: $d'_\gamma = p'_\gamma = \frac{1}{2(2-\gamma)^2} > 0$. Larger $\gamma$ enhances consumers’ valuation for the good and therefore increases both sales and the price.

**Proposition 3.** A monopolist can use match advertising only if visiting costs are heterogeneous due to the Diamond paradox. Consumers learn their horizontal matches and thus different types of consumers have different searching rules: higher $\theta$ has a greater share of visits. Both demand and price increase in the network effect, since the externality positively affects the expected benefit of a purchase.

### 3.4 Case D. Advertising of both price and $\theta$

#### 3.4.1 Homogeneous visiting costs

Let us suppose that the monopolist at the first stage of the game decides to reveal both $\theta$ and the price. By disclosing them, the firm can fully internalize the consumption externality and consumers can correctly form their expectations. With this advertising all information is public, so consumers do not search but still need to pay visiting costs. A consumer is willing to buy if her $\theta \geq p + c - \gamma d(p)$. In equilibrium actual sales must be equal to the production of the firm: $d(p) = 1 + \gamma d(p) - p - c$. This gives a demand function $d(p) = \frac{1 - p - c}{1 - \gamma}$.

As before, two cases are possible: $\gamma < 1$ and $\gamma \geq 1$. In the latter case, the demand function increase in price and thus the pricing rule of the firm differs. Let us start with the case when $\gamma < 1$.

The monopolist’s profit function is as follows:

$$\pi^a(p_a) = p_a \frac{1 - c - p_a}{1 - \gamma}$$

The profit maximization problem results in $p_a = \frac{1 - c}{2}$, $d_a = \frac{1 - c}{2(1 - \gamma)}$ and $\pi^a = \frac{(1 - c)^2}{4(1 - \gamma)}$. 

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As one can see, the firm can charge the monopoly price $p_a = \frac{1-c}{2}$ to a greater share of the market\textsuperscript{11}. This implies that with full disclosure the firm can internalize the consumption externality and charge the monopoly price. Specifically, this price is independent of $\gamma$, which in turn allows the firm to charge a high price even in the presence of the negative network effect\textsuperscript{12}. This result supports Lemma 1, in which a monopolist would prefer to advertise in the case of negative $\gamma$. With full disclosure the firm can perfectly influence the expectations of consumers and consequently the search decision. Therefore, it can internalize the externality with the volume of equilibrium sales while charging a regular monopoly price.

It is important to note that the equilibrium described above is only possible for $\gamma \leq \frac{1+c}{2}$, because for any $\gamma \in \left(\frac{1+c}{2}; 1\right)$ the monopolist obtains all consumers at the price equal to $\frac{1-c}{2}$. In turn, this means that the firm can charge $p = \gamma - c$ and still sell to all consumers. Larger $\gamma$ benefits the firm because it can charge a higher price and consequently receive greater profits.

When $\gamma \geq 1$, a positive consumption externality compensates the negative effect of price and thus the demand function positively reacts to the price increase: $d(p) = \frac{1-p-c}{1-\gamma}$. As in the previous case, the only equilibrium that survives is the one where the firm sells to everyone and charges a price equal to the size of the network effect net of $c$: $p = \gamma - c$. Using the same reasoning as before, with a very strong bandwagon effect, the demand function positively depends on price and thus the firm is willing to sell to all consumers. The maximum possible price that supports full participation of consumers is found from: $d(p) = \frac{1-p-c}{1-\gamma} = 1$. It is equal to $p = \gamma - c$ and the corresponding profit is $\pi^a = \gamma - c$.

\textbf{Lemma 4.} If a monopolist chooses to advertise both price and $\theta$, then for a product with a snob effect or a weak bandwagon effect ($\gamma < \frac{1+c}{2}$), a regular monopoly price is

\textsuperscript{11}A regular monopoly without the network effect would charge $p^m = \arg\max_p [p(1-p-c)] = \frac{1-c}{2}$ and sell the good to $\frac{1-c}{2}$ share of the market.

\textsuperscript{12}Without advertising, the price was $p_n = \frac{1}{2-\gamma}$; with advertising, it is $p_a = \frac{1-c}{2}$, which is larger for $\gamma < \frac{2c}{2-\gamma} < 0$. 

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charged $p^m = \frac{1+c}{2}$ that does not depend on $\gamma$. In the case of a strong bandwagon effect with $\gamma \geq \frac{1+c}{2}$, a monopolist sells to all consumers and charges a price equal to the size of the consumption externality net of $c$, $p = \gamma - c$.

### 3.4.2 Heterogeneous visiting costs

When both price and matches are public information, each consumer $i$ observes the advertised price $p^*$ and visits the store if her $\theta_i > p^* + c_i - \gamma d^e$. Moreover, since the firm advertises its price it can perfectly influence the expectations of consumers, and consumers use the advertised price to calculate the actual sales. Therefore, the firm’s demand function can be found from the equation as follows:

$$d = \int_0^1 \int_{p+c-d}^{1} d\theta dc = \frac{1}{2} - p + \gamma d$$

Rearranging the terms brings $d = \frac{1-2p}{2(1-\gamma)}$ and the resulting profit function is $\pi^b = p \frac{(1-2p)}{2(1-\gamma)}$. FOC with respect to price gives $p = \frac{1}{4}$ and $d = \frac{1}{4(1-\gamma)}$. As in the case with homogeneous visiting costs, the equilibrium price does not depend on $\gamma$. Moreover, this price is a regular monopoly price (in the model with heterogeneous visiting costs). Thus, the firm can charge a monopoly price to a greater share of the market while sales are adjusted to the consumption externality. This implies that when the firm advertises both matches and price, it internalizes the externality by means of sales only. This equilibrium exists for $\frac{1}{4(1-\gamma)} \leq 1$ (or $\gamma \leq 0.75$).

When $\gamma$ exceeds 1, the firm faces a strong bandwagon effect and the only equilibrium is where the firm sells to everyone. The lowest type consumer receives a surplus $CS = 0 - p + \gamma - 1$ and thus the price supporting the equilibrium with a corner solution is $p = \gamma - 1$.

**Proposition 4.** When a monopoly fully discloses market information, it commits to its announced price and all consumers realize their matches. Therefore, the share
of visiting consumers is equal to the actual volume of sales. The firm is able to set a monopoly price and fully internalizes the network externality with its output only. When the firm faces a strong bandwagon effect, it serves all consumers and charges a price equal to the value of the externality net of the maximum visiting cost.

To sum up, we have considered four strategies of the firm. In the first scenario, the firm stays silent and does not advertise any information, and thus consumers must search to obtain necessary market information. In the second case, the firm advertises its price only. The third scenario is never used because the advertising of θ only leads to the Diamond paradox and zero sales if search costs are homogeneous. Finally, the firm may disclose full information and thus consumers make their buying decisions without any search frictions.
3.5 Advertising decision

3.5.1 Homogeneous visiting costs

Let us now consider the very beginning of the game when the monopolist chooses whether it is beneficial to advertise and which information to disclose. To know whether it is beneficial, the firm should compare its profits: \( \pi^a, \pi^* \) and \( \pi^a \). If a monopolist does not advertise, only two equilibria exist: either when \( \gamma < 1 \) and \( c \leq \frac{1}{2(2-\gamma)^2} \), the firm charges \( p_n = \frac{1}{2-\gamma} \) serving \( \frac{1}{2-\gamma} \) share of consumers; or when \( \gamma \geq 1 \) and \( c < \frac{1}{2} \), the firm charges \( p_n = \gamma \) selling to everyone. If a monopolist decides to advertise price only, the equilibrium price is \( p^* = 1 - \sqrt{2c(1-\gamma)} \) and sales are \( \sqrt{2c} \). This equilibrium exists for \( c < \frac{1}{2} \) and \( 1 - \sqrt{2c(1-\gamma)} > 0 \). Only \( \theta \) advertising is never chosen because of the Diamond paradox.

Finally, if a firm chooses to advertise both price and \( \theta \), then for \( \gamma < \frac{1+c}{2} \), a regular monopoly price \( p^m = \frac{1-c}{2} \) is charged to \( \frac{1-c}{2(1-\gamma)} \) share of consumers, and for \( \gamma \geq \frac{1+c}{2} \), the firm sells to all consumers and charges a price \( p = \gamma - c \).

Depending on the values of \( \gamma \) and \( c \) the firm chooses under which conditions a particular advertising brings higher profits (or any positive profit if staying silent means no market). In particular, we are interested in finding the regions where the firm considers information disclosure a dominant strategy. In other words, the goal is to determine where \( \pi^* \) or \( \pi^a \) exceed \( \pi^a \).

There are four threshold values of \( \gamma : 1 - \sqrt{\frac{1}{2c}}, 2 - \sqrt{\frac{1}{2c}}, \frac{1+c}{2} \) and 1. First, only price advertising may exist only for \( 1 - \sqrt{2c(1-\gamma)} > 0 \), which is identical to \( \gamma > 1 - \sqrt{\frac{1}{2c}} \). Second, the firm can stay silent only if \( c \leq \frac{1}{2(2-\gamma)^2} \) which implies that \( \gamma \in [2 - \sqrt{\frac{1}{2c}}, 2 + \sqrt{\frac{1}{2c}}] \). Third, with advertising of both price and \( \theta \), the firm changes its pricing policy at \( \gamma \geq \frac{1+c}{2} \). Fourth, without advertising the firm faces a strong network effect at \( \gamma > 1 \) and thus also changes its pricing policy\(^\text{13}\). At the same time, a threshold \( \gamma = 2 - \sqrt{\frac{1}{2c}} \) may have three different locations: \( 2 - \sqrt{\frac{1}{2c}} < \frac{1+c}{2} \) for \( c < \frac{(1-\sqrt{3})^2}{2} \); \( \frac{1+c}{2} \leq 2 - \sqrt{\frac{1}{2c}} \leq 1 \)

\(^{13}\)Since \( 2 + \sqrt{\frac{1}{2c}} > 1 \), a threshold value of \( \gamma = 2 + \sqrt{\frac{1}{2c}} \) does not have any specific meaning in the analysis.
\[ c \in \left[ \frac{(1-\sqrt{3})^2}{2}, \frac{1}{2} \right]; \quad 2 - \sqrt{\frac{1}{2c}} > 1 \text{ for } c > \frac{1}{2}. \] These possible locations of threshold \( \gamma \) define three regions in the space of the search cost \( c \): low search costs when \( c < \frac{(1-\sqrt{3})^2}{2} \); moderate search costs with \( c \in \left[ \frac{(1-\sqrt{3})^2}{2}, \frac{1}{2} \right] \); high search costs with \( c > \frac{1}{2} \). Let us investigate each case separately.

a) Low search costs

Consumers face low search costs if \( c < \frac{(1-\sqrt{3})^2}{2} \). There are five regions in \( \gamma \)-space as it is depicted in Figure 5:

- when \( \gamma < 1 - \sqrt{\frac{1}{2c}} \), the firm must advertise both price and \( \theta \) because without this advertising a search cost exceeds the expected consumer surplus and consumers do not visit the store at all. The only way to make it work is to advertise a price \( p^m = \frac{1-c}{2} \) and sell to \( \frac{1-c}{2(1-\gamma)} \) share of consumers. Since \( \frac{1}{2(1-\gamma)} < 1 \), some consumers do not buy and the market is uncovered\(^{14}\);

- when \( \gamma \in \left[ 1 - \sqrt{\frac{1}{2c}}; 2 - \sqrt{\frac{1}{2c}} \right] \), the firm needs to advertise to make consumers visit the store. It may use only price advertising if \( \pi^s > \pi^a \) or disclose full information if \( \pi^a > \pi^s \);

- when \( \gamma \in \left[ 2 - \sqrt{\frac{1}{2c}}; \frac{1+c}{2} \right] \), the firm can choose between advertising or not. It compares \( \pi^n \), \( \pi^s \), \( \pi^a \) and chooses a strategy that brings higher payoffs;

- when \( \frac{1+c}{2} < \gamma < 1 \), the firm also faces a choice whether to advertise or stay silent. However, only price advertising is always dominated by staying silent in this region, and thus it is never used. If the firm does not advertise, it obtains \( \pi^n = \frac{1}{(2-\gamma)^2} \). If the firm advertises both price and \( \theta \), it faces a strong positive network effect, and thus it charges \( p = \gamma - c \), sells to all consumers, and receives \( \pi^a = \gamma - c \);

- when \( \gamma \geq 1 \), the firm faces a strong positive network effect. Since \( \pi^s < \gamma - c < \gamma \), the firm chooses not to advertise at all. It charges a price equal to \( \gamma \), consumers expect this price, and all choose to search since search costs are low\(^{15}\).

\(^{14}\)If some consumers are not served in equilibrium, the market is uncovered. If all consumers participate and buy, the market is fully covered.

\(^{15}\)Note that when \( \gamma \geq 1 \), without advertising consumers search if \( c \leq \frac{1}{2} \).
### Figure 5: Low search cost, \( c < \frac{(1-\sqrt{3})^2}{2} \)

<table>
<thead>
<tr>
<th>Advertising both ( \theta ) and ( p )</th>
<th>Advertising of ( p ) or both ( \theta ) and ( p )</th>
<th>Any advertising content: no info, ( p ) only or both ( \theta ) and ( p )</th>
<th>Advertising both ( \theta ) and ( p )</th>
<th>No advertising</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncovered market</td>
<td>Uncovered market</td>
<td>Uncovered market</td>
<td>Covered market</td>
<td>Covered market</td>
</tr>
</tbody>
</table>

- \( 1 - \sqrt{\frac{1}{2}c} \)
- \( 2 - \sqrt{\frac{1}{2}c} \)
- \( \frac{1 + c}{2} \)
- \( 1 \)

\( \gamma \)

b) **Moderate search costs**

Consumers face moderate search costs if \( c \in \left[ \frac{(1-\sqrt{3})^2}{2}; \frac{1}{2} \right] \). There are five regions in \( \gamma \)-space as depicted in Figure 6:

- when \( \gamma < 1 - \sqrt{\frac{1}{2c}} \), the firm needs to advertise both price and \( \theta \). The advertised price is \( p^m = \frac{1-c}{2} \) and sales are \( \frac{1-c}{2(1-\gamma)} \). A search is not affordable;

- when \( \gamma \in \left( 1 - \sqrt{\frac{1}{2c}}; \frac{1+c}{2} \right) \), the firm also needs to advertise, but in this region only price advertising is also possible, and thus the firm compares advertising payoffs and chooses the best advertising option;

- when \( \gamma \in \left[ \frac{1+c}{2}; 2 - \sqrt{\frac{1}{2c}} \right) \), only price advertising brings higher profits than full information disclosure. Therefore, the firm advertises its price only;

- when \( \gamma \in \left( 2 - \sqrt{\frac{1}{2c}}; 1 \right) \), the firm prefers to stay silent because search is possible and both types of advertising result in lower profits: \( \pi^s > \pi^f > \gamma - c \). Since \( \gamma < 1 \) the market is uncovered;

- when \( \gamma \geq 1 \), the firm faces a strong positive network effect. Since search is possible and \( \pi^s \) exceeds both \( \pi^f \) and \( \gamma - c \), the firm chooses not to advertise at all and charges \( p = \gamma \). Consumers expect this price and choose to search because search costs are low enough. Since \( \gamma \geq 1 \), the market is fully covered.

Figures 5 and 6 demonstrate that a negative network effect makes the firm provide
as much information as possible (advertising of both θ and price). Meanwhile, a strong positive externality brings higher profits when the firm is silent because the expected benefit of a search is positively related to the network externality. A negative network effect decreases the expected consumer surplus and thus consumers search less. The only way to make consumers visit the store is to provide necessary information in the form of advertising. As Anderson and Renault (2006) and Renault (2016) show, price is never advertised alone if the firm can reveal match information partially. However if match information must be fully informative, then the firm chooses to advertise price alone for intermediate visit costs.

c) High search costs

Consumers face high search costs if \( c > \frac{1}{2} \). There are only two regions in \( \gamma \)-space as depicted in Figure 7:

- when \( \gamma < \frac{1+c}{2} \), the firm needs to advertise both θ and a price \( p^m = \frac{1−c}{2} \). A search is not affordable because the cost is high. Since sales are equal to \( \frac{1-c}{2(1-\gamma)} \), the market is uncovered;

- when \( \gamma \geq \frac{1+c}{2} \), the firm also needs to advertise both price and θ. However, in this region, it faces a strong positive network effect, and therefore it charges \( p = \gamma - c \) and sells to all consumers. Moreover, the firm must advertise its price even for \( \gamma \geq 1 \),
because with $c \geq \frac{1}{2}$ a search is not affordable. Thus, no consumer searches even for a strong bandwagon effect due to a high cost of a search. No market exists if there is no advertising.

**Proposition 5.** When all consumers have identical visiting costs, the firm chooses its optimal advertising strategy depending on the network externality and the value of the visiting cost. The firm tends to disclose more information as the network externality moves from positive to negative when the cost of a visit is low or moderate, while a higher visiting cost always induces full disclosure.

Unlike in the previous literature, an advertising decision of a monopolist does not only depend on $c$ (as in Konishi and Sandfort, 2002; Anderson and Renault, 2006), but it also depends on the network effect $\gamma$. In particular, a relatively weak bandwagon effect and a regular snob effect require advertising if consumers are poorly informed. As Anderson and Renault (2006) and Renault (2016) show, as the visit cost grows, the optimal advertising strategy of the firm changes from no advertising to full disclosure. In our case, the optimal advertising strategy changes from no advertising to full disclosure as $\gamma$ decreases. Moreover, market coverage positively reacts to the increase in $\gamma$, reaching its maximum when the externality is strong.
3.5.2 Heterogeneous visiting costs

Considering the advertising decision of the firm when consumers are different in their visiting costs, this decision depends on the type of the consumption externality and its size. Since the firm prefers advertising content which gives the highest profit, it compares the profits under different advertising policies. When the firm provides no information, it receives

$$\pi^m = \frac{s}{(2-s\gamma)^2},$$

where $$s = \frac{1}{2(2-s\gamma)^2}$$. If the firm chooses to advertise its price alone, it obtains

$$\pi^p = \frac{2s^2(1-3s\gamma)}{3},$$

where $$s = \frac{3+\sqrt{2s(6s\gamma-1)}}{18}$$, but this advertising policy is possible only for $$\gamma < 0.36$$. Only match advertising takes place for $$\gamma < 1.5$$ and brings

$$\pi^m = \frac{1}{4(2-\gamma)^2}.$$

Finally, advertising of both price and match results in profits

$$\pi^b = \frac{1}{16(1-\gamma)}$$

if $$\gamma \leq 0.75$$ and $$\pi^b = \gamma - 1$$ if $$\gamma \geq 1$$.

Since $$\pi^b$$ is always greater than $$\pi^m$$ for $$\gamma \leq 0.75$$, the firm prefers full disclosure to "match only" advertising. However, "both price and match" advertising is not achievable for $$\gamma \in (0.75; 1)$$, and therefore the firm uses match advertising in this region. Moreover, match advertising is also implemented for $$\gamma \in [1; 1.5]$$ because $$\pi^m > \pi^b$$ for these values. When $$\gamma > 1.5$$, the firm cannot use match advertising and thus it fully discloses both match information and price. Further analysis of the advertising policies and their comparison are shown in the figures that follow.

Figures 8 and 9 show profit curves for different advertising policies of the firm depending on the value of $$\gamma$$. The vertical axis is the value of profit and the horizontal axis is a space of $$\gamma$$. Figure 8 shows four profit curves for $$\gamma < 0.36$$, since equilibrium with "only price" advertising does not exist for $$\gamma < 0.36$$.

Clearly, "only price" advertising gives higher profits for $$\gamma < -0.95$$. After that point "both price and match" advertising dominates any other advertising decision. "Match only" and zero advertising is never chosen, since they result in lower profits. When the snob effect is strong, it is more profitable to influence the expectations of consumers with price only. If consumers benefit substantially from a small clientele, the firm advertises a high price to commit to a small sales in equilibrium. Indeed, since the snob effect makes
Figure 8: Profit curves under different advertising policies, $\gamma < 0.36$
demand less elastic, the equilibrium price with "price only" advertising decreases in $\gamma$ as shown in Figure 4. With a strong snob effect the firm prefers to advertise a high price to support smaller sales, since demand is inelastic. Therefore, "price only" advertising brings the highest profits to the firm\textsuperscript{16}.

However, when $\gamma > -0.95$, full disclosure brings higher levels of profit. Demand becomes less inelastic and hence it is more profitable to have a higher clientele and lower price. Since the "both price and match" option gives the highest sales among all four possible advertising policies, the firm benefits from disclosing both $\theta$ and price.

Figure 9 shows profit curves for three advertising policies when $\gamma > 0.36$. "Both price and match" advertising dominates all other regimes for $\gamma \in (-0.95; 0.75)$ for the same reason as before: higher $\gamma$ implies a more elastic demand and thus it is profitable to have larger clientele, advertising a lower price helps the firm commit to larger sales in equilibrium, and disclosing matches increases the probability of visits. Any other advertising policy results in lower equilibrium sales.

However, "both price and match" advertising cannot be used for $\gamma \in (0.75; 1)$. Therefore, "match only" advertising takes place for the given interval of $\gamma$. Moreover, for $\gamma > 1$ the firm serves the whole market and charges $p = \gamma - 1$ under full disclosure, which gives lower profits when $\gamma \in [1; 1.5]$. Therefore, the firm also uses "match only" advertising for $\gamma \in [1; 1.5]$. For $\gamma > 1.5$ the firm advertises both price and matches, and sells to all consumers at price $p = \gamma - 1$, which obviously increases in the externality, since larger $\gamma$ implies greater benefits from the bandwagon effect.

To sum up, when consumers are different in their visiting costs, the optimal advertising policy depends on the consumption externality. Specifically, if $\gamma < -0.95$ the firm benefits more from "price only" advertising; if $\gamma \in (-0.95; 0.75)$ the firm advertises both price and matches, and has not fully covered market; if $\gamma \in [0.75; 1.5]$ the firm discloses horizontal matches only; if $\gamma > 1.5$ the firm announces both matches and price, and does\textsuperscript{16}When the firm announces both price and match, demand becomes perfectly elastic, since the firm charges a fixed monopoly price $p = \frac{1}{4}$.

\textsuperscript{16}
not have a fully covered market.

**Proposition 6.** When consumers have heterogenous visiting costs, a monopoly never remains silent and thus at least the price is advertised. As the network effect changes from negative to positive, the firm includes more information in its advertising content.

The most important result is that with heterogeneous visiting costs, as \( \gamma \) changes from negative to positive values, the firm includes more information in its advertising. This result is opposite the case with homogeneous cost. This occurs due to a more sensitive demand, since the probability of buying does not only depend on the consumer’s match alone, but also if her search cost is low enough. Moreover, for a negative network effect all consumers prefer a small clientele, because consumers’ willingness to pay increases as clientele decreases. Therefore, providing little information reduces visits and thus restricts demand, while for a positive network effect all consumers benefit from a larger clientele, because consumers’ willingness to pay increases as clientele increases. Hence, providing
more information increases visits and expands demand. When the visit cost is the same for everyone, the firm only cares if the expected benefit of a search exceeds a given threshold, while the benefit decreases in $\gamma$.

4 Results and concluding remarks

The preceding section considers the incentives for a monopoly to disclose market information. In contrast to the previous literature, the model presented in this paper combines network externalities and an information disclosure game. Network effects in consumption are considered using a model of a market where the decision to buy a product depends on the total sales of the good. Disclosure game uses a framework of search and advertising. This implies that if the firm remains silent, consumers must search to obtain necessary market information; if the firm decides to reveal the information itself in the form of advertising, the information becomes public knowledge. This setting better describes the functioning of the markets with network goods, because the existing literature on the topic does not consider search frictions and price commitment problem as the main obstacles for consumers when they face network externalities. First, with network goods consumers make a buying decision based on their expectations about the actual sales (clientele size). This can be easily done if consumers are able to correctly foresee the market outcome. However, due to bounded rationality or a lack of necessary market information (e.g. price) forming the correct expectation is complicated. This explains why sellers of conspicuous goods usually reveal some information to help consumers to form correct expectations about possible clientele size. This information is usually transmitted via announcements of the total supply (or limited editions), product characteristics or price advertising. Second, the announcements and price advertising work as a commitment device, since any public announcement in the form of the official advertising obliges the firm to fulfil what it announced. Therefore, consumers are assured that the firm will not deviate and break promises.
Advertising as a disclosure method is widely used in search models to show that information disclosure may expand demand and secure higher profits in the presence of search costs (e.g. Anderson and Renault, 2006; Konishi and Sandfort, 2002). In network economics, consumers are assumed to rationally anticipate prices and actual sales (e.g. Grilo et al., 2001; Griva and Vettas, 2011). However, this is only possible with no information problems. Thus, this assumption has to be relaxed because in real markets information is not perfect and therefore the formation of consumers’ expectations is complicated. The model considered in this paper describes how consumers make their search and buying decisions, and what explains a firm’s decision regarding what advertising content to use. This decision making process is a three-stage game.

In the beginning of the game a monopolist has an option to remain silent and keep consumers uninformed or to advertise and reveal either price only, match only or both price and consumers’ matches which are *ex ante* unknown. When the firm chooses to stay silent consumers search if their search cost exceeds the expected consumer surplus. If the firm decides to advertise any information, consumers use this information to compute the expected clientele and decide on buying. The advertising strategy of the firm depends on two parameters: the size of the search cost and the measure of the network effect.

First, advertising is more likely for a negative network effect and less likely for a positive network effect. In other words, the benefits of search increase in the network effect, because a greater clientele size increases the expected consumer surplus. Search benefits are small in the case of a negative network effect and therefore the only way to secure profits is by advertising.

Second, a strong positive network effect can eliminate a negative price effect. With a strong positive externality the demand function increases in price because a greater clientele increases a consumer’s willingness to pay more than a reduction due to price increase. Therefore, a monopolist prefers to sell to all consumers and charges a price equal to the value of the network externality.

Third, the previous two results hold for any type of search cost: both homogenous
and heterogenous. Homogenous search costs mean that all consumers face the same value of the cost. With heterogenous search costs, consumers differ in the costs due to their different value of time, different search abilities or locations.

Finally, when visiting costs are homogeneous, the advertising decision of the firm also depends on the costs of a search. When search costs are low or moderate, the firm must advertise when the search costs exceed the expected benefit of the search or when price advertising gives higher profits. It is important to note that a monopolist needs to provide as much information as possible for a negative network effect, while a strong positive network effect brings higher profits when the firm remains silent. Depending on the value of the network externality, the market can be either partially served (uncovered market) or with the full participation of consumers (covered market). When search costs are high, the only way to sell is to provide consumers with full information about prices and product characteristics.

When visiting costs are heterogenous, zero advertising content is never chosen by the firm; at least price or matches should be disclosed. The firm advertises its price alone for a strong negative consumption externality, because the consumer’s valuations for the good increase with a smaller clientele. Advertising a higher price and undisclosed matches reduce visits and consequently prevent large equilibrium sales. When demand becomes more elastic (increase in $\gamma$), it is more profitable to charge a lower price and facilitate visits. This can be done by disclosing as much information as possible. Thus, the firm prefers to advertise both price and matches whenever it is possible for a positive network externality.

Compared to the network literature, these results show that market frictions that complicate a consumer’s ability to form correct expectations significantly affect the decision making process of consumers and therefore the market outcome. Moreover, the addition of network externalities to the advertising game in the search model enrich the conclusions of the search literature, because information disclosure decision becomes dependent not only on the costs of a search, but also on consumption externalities.
Appendix A

As shown in Janssen et al. (2005), when every search is costly, two types of equilibrium are possible. In particular, when the expected surplus of a purchase $E(CS)$ is equal to the visiting cost, either all consumers may decide to visit the store (this equilibrium is considered in the paper) or consumers may randomize between visiting the store and being inactive. The latter equilibrium implies that the probability of a visit is equal to the share of visiting consumers, $s$. In equilibrium where consumers randomize, $s$ becomes endogenous and is determined in equilibrium.

Consumers may choose to randomize when $E(CS) = \frac{1}{2(2-s)^2} = c$. Therefore, the equilibrium share of visiting consumers or the probability of a visit, $s^*$, is a solution to the equation $\frac{1}{2(2-s)^2} - c = 0$. The corresponding price is $p = \frac{1}{2-\gamma s^*}$ and equilibrium sales are $d = \frac{s^*}{2-\gamma s^*}$. Let us investigate the properties of the equilibrium with partial participation.

First, to have positive price and sales, the condition $(2 - \gamma s) > 0$ must hold. This gives the equilibrium $s^* = \frac{2\sqrt{c}-1}{\gamma}$.

Hence, $p = \sqrt{2c}$ and $d = \frac{2\sqrt{c}-1}{\gamma}$. Since $0 < d < 1$ and $0 < s < 1$, this equilibrium exists for

\[
\begin{cases}
    c > 0.125 \\
    \gamma > 0 \\
    \gamma > 2\sqrt{2c} - 1 \\
    \sqrt{2c(2 - \gamma)} < 1
\end{cases}
\quad\text{and}\quad
\begin{cases}
    c < 0.125 \\
    \gamma < 0 \\
    \gamma < 2\sqrt{2c} - 1 \\
    \sqrt{2c(2 - \gamma)} > 1
\end{cases}
\]

Figure 10 presents the curves of $s = \frac{2\sqrt{c}-1}{\gamma\sqrt{c}}$ for different values of $c$. The vertical axis is $s$, the horizontal axis is $\gamma$. When $\gamma$ is negative, the probability of a visit decreases in the snob effect and in the costs of search as expected because both negatively influence the consumer surplus. When $\gamma > 0$, the probability of a visit decreases in $\gamma$ and increases in $c$. If $\gamma$ grows and $c$ is fixed, this would increase $E(CS)$, but to keep $E(CS) = \frac{1}{2(2-s)^2} = c$, the equilibrium $s$ should decrease. If $c$ grows and $\gamma$ is fixed, the equilibrium $s$ should decrease to keep the condition for randomization unchanged.

As shown, for a particular set of parameters $\gamma$ and $c$ an equilibrium with randomizing consumers may exist when $E(CS) = c$. However, in the main analysis it is assumed that all consumers prefer buying to being inactive.
Figure 10: Locus of equilibrium $s$ as a function of $\gamma$ for different $c$
Following the analysis in Leibenstein (1950), market demand is a function of consumers’ expectations about the actual sales of the good with a network effect. It is therefore possible to treat expectations as a parameter and see how market demand changes with different expectations. Let the market demand $D_j$ indicate the quantities demanded at alternate prices if all consumers expect that total sales are equal to $d_j$. Thus an increase in $d_j$ shifts the demand curve $D_j$ outwards. Considering a graphical analysis of snob and bandwagon effects, assume that alternative consumers’ expectations of the sales are $d_A < d_B < d_C < \ldots < d_N$ and corresponding demand curves are $D_A$, $D_B$, $D_C$, ..., $D_N$ as shown in Figures 11, 12 and 13.

**Snob Effect.** Figure 11 demonstrates a snob effect. As shown, a higher expected clientele corresponds to lower levels of demand. If we assume that consumers are rational

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**Appendix B**

Following the analysis in Leibenstein (1950), market demand is a function of consumers’ expectations about the actual sales of the good with a network effect. It is therefore possible to treat expectations as a parameter and see how market demand changes with different expectations. Let the market demand $D_j$ indicate the quantities demanded at alternate prices if all consumers expect that total sales are equal to $d_j$. Thus an increase in $d_j$ shifts the demand curve $D_j$ outwards. Considering a graphical analysis of snob and bandwagon effects, assume that alternative consumers’ expectations of the sales are $d_A < d_B < d_C < \ldots < d_N$ and corresponding demand curves are $D_A$, $D_B$, $D_C$, ..., $D_N$ as shown in Figures 11, 12 and 13.
and they can correctly foresee the total sales at every market price, then only one point on any of the curves $D_A, D_B, D_C, ..., D_N$ could be on the equilibrium demand curve. The points on each curve $D_A, D_B, D_C, ..., D_N$ represent the amounts that consumers expect to be the total sales. In these equilibrium points A, B, C, ..., N market demand at market price is equal to consumers’ expectations. The locus of these points $\tilde{D}$ is therefore the actual demand curve for the conspicuous commodity. $\tilde{D}$ is less elastic compared to the demand curves $D_A, D_B, D_C, ..., D_N$ which treat consumers’ expectations as parameters. The snob effect reduces the price sensitivity of demand.

Let us consider a price increase leading to a transition from equilibrium C to equilibrium A. Total decrease in the demanded quantities is $d_A d_C$, but only a part of this change is the price effect. To measure the price effect we go along the demand curve $D_C$ to a new price level, which tells us the quantity that would be demanded at the new price if all consumers did not adjust their expectations. This transitional point is denoted as X. Therefore, the price effect is $x d_C$. The snob effect is $d_A x$, and shows that some consumers will enter the market due to the decreased expected clientele in new equilibrium A, because lower clientele increases a valuation for the good. Although price effect dominates the snob effect, market demand is now less elastic since the price effect and snob effect are of the opposite direction. Reduced demand elasticity allows the firm to charge a higher price.

**Bandwagon Effect.** Figure 12 demonstrates a bandwagon effect. As shown, a higher expected clientele corresponds to higher levels of demand. The rest of the analysis of the bandwagon effect is parallel to the snob effect. The locus $\tilde{D}$ is the actual demand curve for the conspicuous commodity. $\tilde{D}$ is more elastic compared to the demand curves $D_A, D_B, D_C, ..., D_N$ which treat consumers’ expectations as parameters. This enhanced price sensitivity is explained by the bandwagon effect. Let us consider a price increase leading to a transition from equilibrium C to equilibrium A. Total decrease in the demanded quantities is $d_A d_C$, but only a part of this change is the price effect. To measure the price effect we go along the demand curve $D_C$ to a new price level, which tells us the
Figure 12: Bandwagon Effect
quantity that would be demanded at the new price if all consumers did not adjust their expectations. This transitional point is denoted as X. Therefore, the price effect is $xd_C$.

The bandwagon effect is $d_A x$, and represents an additional reduction in the number of consumers who left the market due to the decreased expected clientele in new equilibrium A. Therefore, the bandwagon effect enhances the price elasticity of market demand and thus it tends to lower prices. The price effect and bandwagon effect are of the same direction.

**Strong Bandwagon Effect.** A different analysis takes place with a strong bandwagon effect when a higher price is always associated with larger equilibrium sales. Actual market demand with fulfilled expectations is upward slopping now, as shown in Figure 13. Let us assume that the initial market state was at point A. There was a change in consumers’ expectations about the actual sales from $d_A$ to $d_C$ and a new equilibrium with
fulfilled expectations is at point C. Higher clientele enhances consumers’ valuations for the network good and therefore there is a higher price in equilibrium C. To decompose the total change in the demanded quantities $\Delta d$, let us measure both price and bandwagon effects as shown in Figure 13. We go along the demand curve $D_A$ to the new price level, which tells us the quantity that would be demanded at the new price if all consumers keep their expectations fixed. The corresponding price effect is $xd_A$, which is negative. The bandwagon effect is $d_Cx$, which is positive. In the case of a strong bandwagon, a negative price effect is dominated by a positive effect of the externality. Therefore, the total effect is positive and actual market demand is upward slopping. With a strong bandwagon effect, an enhanced consumers’ valuation for the good dominates a loss in utility due to the increase in price. Hence, the actual demand with fulfilled expectations has a positive slope.
References


Corneo, G., & Jeanne, O. (1997). Conspicuous consumption, snobbism and con-

application to the US fax market. Discussion Paper No. EC-95-11, Stern School of
Business, New York University.

trial organization*, 14(6), 673-699.


University Press.


Department of Economics University of North Carolina.

is characterized by conformity or vanity. *Journal of Public Economics*, 80(3), 385-408.


Hopkins, E., & Kornienko, T. (2004). Running to keep in the same place: Consumer

Ireland, N. J. (2001). Optimal income tax in the presence of status effects. *Journal
of Public Economics*, 81(2), 193-212.

sequential search and oligopolistic pricing. *International Journal of Industrial Organiza-
tion*, 23(5), 451-466.

Konishi, H., & Sandfort, M. T. (2002). Expanding demand through price advertise-


Abstrakt

Tento článek se zabývá reklamními aktivitami monopolistické firmy, která vyrábí produkt vykazující charakteristiky síťové externality, v situaci, kdy spotřebitelé nemají úplné informace, což je vede k hledání jejich rezervační ceny. Firma může, pokud uzná za vhodné, pomocí reklamy odhalit určité tržní informace. Výsledky naznačují, že reklama je pravděpodobnější v situaci negativní síťové externality a méně pravděpodobná v situaci pozitivní síťové externality. Pokud monopolista čelí silné síťové externalitě, pak podporuje maximální možnou síť a požaduje cenu, která odpovídá hodnotě této externality. Monopolista si může zvolit obsah své reklamy na základě typu síťové externality a nákladů spotřebitelů na hledání rezervační ceny. Obsah reklamy tak může obsahovat pouze informace o ceně produktu nebo pouze o vlastnostech produktu a/nebo může obsahovat informace jak o ceně, tak o vlastnostech produktu. Pokud mají všichni spotřebitelé stejné náklady na hledání rezervační ceny, pak s růstem těchto nákladů musí firma do reklamy zahrnout více informací. Pokud dojde ke změně síťové externality z negativní na pozitivní, pak firma sníží množství poskytovaných informací v reklamě. Pokud ovšem spotřebitelé mají různé náklady na hledání rezervační ceny, pak při změně externality z negativní na pozitivní, má firma sklon publikovat reklamu, jež obsahuje více informací.
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