Does CPI Approximate Cost-of-Living?: Evidence from the Czech Republic

Jiri Podpiera
DOES CPI APPROXIMATE COST-OF-LIVING? EVIDENCE FROM THE CZECH REPUBLIC

JIRÍ PODPIERA∗

Abstract. An original method based on an n-markets simultaneous partial equilibrium model is designed to evaluate the second order bias in the Consumer Price Index's (CPI) approximation of the Cost-of-Living (COL) and to test the statistical significance of its mean value over a variety of horizons. In the empirical application of the model I consider nine goods markets that correspond to the first strata level breakdown of the CPI in the Czech Republic for the period 1994-2000. Having evaluated the substitution bias, i.e., the difference between the growth rate of the CPI and the growth rate of the COL, I find that on a yearly basis it ranges from -0.83 to 0.51 p.p. In addition, I find that, on average, the bias statistically vanishes on the time horizon of five quarters. Different levels of inflation such as moderate (10%) and lower (2%) characterized the sample period and therefore I conclude that the derived result is robust up to moderate inflation.

Abstrakt. V článku je představena originální metoda založená na modelu částečné rovnováhy, která vytváří odchylku indexu spotřebitelských cen (CPI) od životních nákladů (COL) a testuje dočasnost této odchylky. V empirické aplikaci byla použita data za Českou Republiku v období 1994-2000 na prvním strata souboru CPI. Kvantiﬁkovaná odchylka se pohybuje mezi -0.83 a 0.51 p.b. v ročních průměrech. Avšak v klozavých průměrech pěti čtvrtletí je odchylka statisticky nevýznamná. Tento výsledek je robustní vůči změně inflace v řádu procentních bodů.

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1. Introduction

Correctly measuring the growth in the Cost-of-Living (COL) is of very high importance since it is among the most closely watched information that enters into decisions about monetary policy setting, wage adjustment, and operations of capital and money markets, for instance. In recent years, the traditional Consumer Price Index (CPI) as an approximation of the COL Index has been reconsidered and two sources of bias in the CPI have been identified and analyzed. Hausman (2002) recognizes two categories of biases: first and second order biases. First order bias arises from the mismeasurement of price indices generated by the introduction of new goods, quality changes in existing goods, and by a shift in shopping patterns to lower-priced stores. The second order bias, called substitution bias, is caused by an insensitivity of the CPI to substitution effects induced by relative price changes.

In the 1990s, a great portion of literature was devoted to the evaluation of the second order bias. More recently, two approaches to evaluating the second order bias in the CPI have been employed. The first approach is based on a comparison of the CPI and the COL that is approximated by the Fisher Superlative Index \(^1\) (see for instance Hanousek and Filer 2001). The second approach employs a single good demand system to evaluate the CPI bias directly (Hamilton 2001; Costa 2001).

The former approach is legitimate to use only in the case of homothetic consumer behavior, because the Laspeyres and the Paasche Indices represent upper and lower bounds for the true cost of living only under homothetic preferences (see Diewert 1976). Nevertheless, such a situation is rarely observable in the real world. Pollak and Wales (1992) and Bankes et al. (1997), for example, showed that the Engel curves are quadratic and hence proved that consumer behavior is non-homothetic. The non-homothetic consumer behavior introduces into superlative indices a so-called “income bias”. Omitting the non-homotheticity leads to a functional dependence of measured COL on the level of real income. Dumagan and Mount (1997) have shown, using simulations, that income bias can be even greater than substitution bias in the Laspeyres Index \(^2\) and hence an application of the Fisher Superlative Index is conditional on the Paasche Index without income bias. \(^3\)

The latter approach focuses on the demand system estimation and allows for non-homothetic consumer behavior and hence provides more realistic results. The disadvantage of the demand system approach in Hamilton (2001) or Costa (2001) is in high data intensity and narrow focus of only one or two goods.

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\(^1\)The Fisher Superlative Index is computed as the geometric average over Paasche and Laspeyres Price Indices and is, under the assumption of homothetic consumer behavior, equal to the Cost of Living Index.

\(^2\)The fact that the COL can exceed the Laspeyres index implies that the homothetic assumption introduces another bias that might be even more severe.

\(^3\)This argument applies to all non-homothetic indices.
Therefore, I design an original method that combines the supply-demand system with a direct computation of the unbiased COL. In particular, I develop a supply-demand partial equilibrium model consisting of a system of demands for $n$ goods, the Almost Ideal Demand System derived by Deaton and Muellbauer (1980), and a system of corresponding marginal cost-driven supply functions as advocated for instance by Sbordone (2001). Under the assumption that the development of sub-price indices of the CPI is collinear, I use the fact that the growth rate of the Stone Price Index without income bias is equal to the growth rate of the COL. The computation of the COL is performed by simulation in the estimated partial equilibrium model under constant real income. By fixing the real income I eliminate the impact of the variation in real income on budget shares and thus on the COL. Having obtained the unbiased COL I test whether the COL and the CPI growth rates differ significantly in their mean values and set the minimal horizon in which the CPI approximates in mean value the COL or settles at a certain fixed difference. In the empirical application, I work with quarterly data on the first CPI strata level in the Czech Republic during the period 1994-2000. The Czech data application is chosen for the reason that the Czech Republic in this period underwent economic transition and real income exhibited high volatility. Hence, income bias is likely to play a significant role in the true substitution bias evaluation. Moreover, the data sample allows me to investigate the second order bias both during the period of relatively higher CPI inflation (around 10% in 1995-97) and the period of lower inflation (around 2% in 1999-2000).

Recent empirical findings by Gordon (1995), Boskin et. al. (1996) or Dievert (1995), on a U.S. data, stipulated that the CPI overstates the COL by more than one percentage point per year. All these studies have focused on the evaluation of various biases connected with quality adjustments, homothetic substitution bias, new goods introduction, and the like. Their results differ only in magnitude; all find a positive CPI bias. Only most recently, a new approach evaluating the true bias in the CPI has appeared. Costa (2001) and Hamilton (2001) have shown, using a non-homothetic demand system, that the CPI bias in the U.S. between 1888-1994 ranged from -0.1 to 2.7 p.p. To my knowledge, there is no similar study using non-homothetic consumer behavior for the Czech Republic. Research studies using data for the Czech Republic, for instance in Hanousek and Filer (2001), focus on the evaluation of an absolute homothetic substitution bias by comparing the Fisher Superlative Index to the Laspeyres Index at various strata levels in 1991-1999. Whereas Hanousek and Filer (2001) find that the bias

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4A sub-price index denotes a lower level of aggregation of price index following the structure of the Consumer Price Index.
5Stone Price Index only after de-logarithmic adjustment. See Deaton and Muellbauer (1980) and Section 2.4 for details.
6Some studies, for instance Wynne and Sigalla (1994) or Pakes (1995) concluded that the CPI bias is lower than one percentage point.
on the fourth strata level ranged between -0.48 p.p. (in 1992) and 4.1 p.p. (in 1991), on the first strata level the extreme values were -0.25 p.p. (in 1992) and 0.75 p.p. (in 1991). My findings from the empirical application of the model on the first strata level Czech data in 1994-2000 show that the yearly CPI bias ranged from -0.83 p.p. (in 1995) to 0.51 p.p. (in 1999). On the overlapping period in both studies 1994-1999, the bias found by Hanousek and Filer (2001) on the first strata level was -0.05 p.p. and 0.1 p.p. as compared to my results -0.81 p.p. and 0.51 p.p. As a result, the CPI substitution bias accounting for non-homotheticity is more volatile than the bias evaluated under the assumption of homotheticity. In addition, I verified that the five quarters is the minimal horizon in which the CPI approximates the COL on average.

Hanousek and Filer (2001) considered four strata levels and found that the substitution bias is greater when descending the strata levels. This would mean that the minimum horizon of the approximation of the COL by the CPI might possibly be longer than five quarters. Nevertheless, to assess the magnitude of substitution bias in lower strata levels would necessitate more detailed data in a longer time series.

The rest of the paper is organized as follows. In Section 2 I present the theoretical framework for the estimation. In Section 3 I describe the estimation techniques and data used together with results. Section 4 concludes by summarizing the main findings.

2. The Partial Equilibrium Model

The partial equilibrium model consists of \( n \)-demand and supply systems, following the breakdown of the CPI that are simultaneously in equilibrium. A detailed description of the demand (Section 2.1) and supply (Section 2.2) introduce the set-up of the partial equilibrium model. Section 2.3 derives the partial equilibrium model solution under constant real income. In Section 2.4 is presented the test for the minimum time horizon of the CPI’s approximation of the COL.

2.1. Demand. The demand side of the partial equilibrium model is represented by a demand system on \( n \) markets. In particular, I model the optimal decision of an average consumer who has preferences that satisfy the price independent generalized linearity condition in logarithmic form\(^7\) (PIGLOG). The PIGLOG class of preferences guarantees that the investigated economic relations (namely symmetry and separability restrictions) are valid for a particular single consumer as well as for an average consumer. An average consumer maximizes his utility by deciding about the allocation of his income among \( n \) groups of products.\(^8\)

\(^7\)See Muellbauer (1975) for details.

\(^8\)Note that the aggregation over groups of products is easier since it is solved by the introduction of the separability condition in indirect utility function; for details see Muellbauer (1975) p. 525.
The demand system specification follows the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980). The AIDS satisfies all restrictive properties of a demand system automatically i.e., it is derived from the consumer maximization problem using PIGLOG class of preferences. In addition, it is an arbitrary first order approximation to any demand system and it aggregates perfectly over consumers. Equations (2.1)-(2.6) have been directly taken from Deaton and Muellbauer (1980). The demand functions expressed in average budget shares \( \bar{w} \) as a function of price vector \( p \) and average real income takes the following form:\(^9\)

\[
\bar{w}_{it} = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \log p_{jt} + \beta_i \log \frac{x_t}{P_t}
\]

where \( \bar{w}_{it} \) denotes the average consumer budget share for good \( i \), and \( \alpha_i, \gamma_{ijt}, \) and \( \beta_{it} \) are parameters, \( p_{jt} \) is the \( j \)-th product group price index, \( x_t \) is the average nominal expenditure, and \( P_t \) is the true price index of the following form:

\[
\log P_t = \alpha_0 + \sum_k \alpha_k \log p_{kt} + \frac{1}{2} \sum_j \sum_k \gamma_{kj} \log p_k \log p_j
\]

which, under the assumption of collinear development of price indices, can be approximated by the Stone Price Index:

\[
\log P_t \approx \log \phi + \sum_k \bar{w}_{kt} \log p_{kt}
\]

where \( \bar{w}_{kt} \) denotes the average consumer budget share for a good \( k \). The following restrictions result from the definition of the demand system (2.1):

\[
\sum_{i=1}^{n} \alpha_{ki} = 1, \text{ for all } k, \sum_{i=1}^{n} \gamma_{ij} = 0, \sum_{i=1}^{n} \beta_{i} = 0, \text{ and } \sum_{j} \gamma_{ij} = 0, \text{ and } \gamma_{ij} = \gamma_{ji}
\]

In particular, restrictions (2.4) and (2.5) represent homogeneity of degree zero, Slutsky symmetry and the adding up of marginal expenditures to zero.

Using approximation (2.3) we can rewrite the demand system in log-linear form useful for direct estimation which is given by:

\[
\bar{w}_{it} = \alpha_i^* + \sum_{j=1}^{n} \gamma_{ij} \log p_{jt} + \beta_i \log \left( \frac{x_t}{P_t} \right) + \varepsilon_{it}
\]

\(^9\)For the details of a more general form of PIGLOG demand \( w_i(y, p) = \log yA_i(p) + B_i(p) \) and expenditure functions \( y = H(p)^{\alpha}B(p) \), see Muellbauer (1975) p. 526.
where $\alpha^*_k = \alpha_k - \beta_i \log \phi$ and $\log P^*_t = \sum_k \pi_{kt} \log p_{kt}$.

2.2. Supply. The absence of a perfectly elastic supply is an important factor of price determination on the market. The importance of modeling supply function as possibly less elastic becomes a relevant feature of the model when applied to economies in transition rather than established western market economies, where the assumption of a perfectly elastic supply is likely to hold. One of the specific features of the supply function in transition economies is its lower price elasticity as the economy undergoes structural changes and reforms. For this reason, a change in demand causes a change in price as well and this in turn has an effect on the optimal allocation of the household’s budget.

An average firm’s price setting in a specified market is usually assumed to be backward looking and is modeled by marginal cost determinants. In theory, there are multiple approaches to modeling supply function that differ mainly in the degree to which forward- and backward-looking factors are included. Roberts (1997) for instance argues that the forward-looking complement incorporated in the New Keynesian Phillips Curve should be discounted because it is extremely sensitive to the type of survey measures of inflation expectations. I employ a fully backward looking version of supply function. Additionally, in a small open economy an exchange rate channel might have a significant direct impact on supply in certain markets. The following relation can generally describe the supply in each category of products:

\[
\log p_{it} = \delta_i \log p_{i(t-1)} + \rho_i \bar{w}_{it} + \psi_i \log PPI_t + \phi_i R_t + \xi_i \log w_{it} + \theta_i \log \text{REER}_t + \nu_{it}
\]

for $i = 1, ..., N$. $p_{i(t-1)}$ represents a lagged price index of good $i$, $\bar{w}_{it}$ stands for the average budget share devoted to good $i$, $PPI_t$ denotes the producer price index, $R_t$ is the nominal interest rate, $w_{it}$ is the increase in the wage index in a good’s category $i$, and $\text{REER}_t$ represents the real effective exchange rate deflated by the PPI.

The development on the demand side, represented in this model through budget shares, together with the changes in price competitiveness with respect to the price of imported goods and services, represented by the exchange rate, are significant determinants of the supply function. Since the price setting in the current period is to a certain extent derived from the price in the previous period, I assume continuity in price setting and consider the first order autoregressive process in the specification of market supply. Remaining variables in supply equation (2.7) stand for the marginal cost factors such as wages, interest rate and the production price index representing the cost of intermediate goods. In specifications (2.7) by including the $\text{REER}_t$, representing the real effective exchange rate deflated by the $PPI_t$, together with the $PPI_t$ we can analyze which of these two variables
i.e., the nominal effective exchange rate or the $PPI_t$ has a more significant impact on the price setting.

2.3. Partial Equilibrium under Constant Real Income. The partial equilibrium model under constant real income can be derived using the estimated parameters of the supply-demand model described above. Since I observe a sequence of equilibrium decisions of budget share allocation and price indices, I can identify a simultaneous equilibrium model and estimate the parameters of the model (equations (2.1) and (2.7)). Using the estimated parameters I simulate the partial equilibrium model under constant real income.

The following equation states that the real income stays at the same level as of the initial (base year) level i.e., for an initial set of prices and the nominal income I derive the initial level of real income as follows.

\[
\log(x_t) - \left( \sum_{j=1}^{n} \bar{w}_it \log p_{jt} \right) = \log(x_0) - \left( \sum_{j=1}^{n} \bar{w}_{i0} \log p_{j0} \right),
\]

Incorporating equation (2.8) into systems (2.1) and (2.7), I define the partial equilibrium system under the constant real income of the year that coincides with the base year for the CPI.

Denoting $x_0$ as the nominal income in the base year, $\bar{w}_{it}$ as the observed budget shares, $x_t$ as the observed nominal income level, $x_t^*$ as the nominal income level under constant real income and the optimal budget shares as $\bar{w}_t^*$ under a constant real income, then the whole demand system after incorporation of relation (2.8) can be written in the following matrix form:

\[
B_t^{D*} W_t^{*} = A_t^{D*}
\]

where

\[
B_t^{D*} = \begin{pmatrix}
1 + \hat{\beta}_1 \log p_{1t} & \hat{\beta}_1 \log p_{2t} & \ldots & \hat{\beta}_1 \log p_{nt} & -\hat{\beta}_1 \\
\hat{\beta}_2 \log p_{1t} & 1 + \hat{\beta}_2 \log p_{2t} & \ldots & \hat{\beta}_2 \log p_{nt} & -\hat{\beta}_2 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\hat{\beta}_n \log p_{1t} & \hat{\beta}_n \log p_{2t} & \ldots & 1 + \hat{\beta}_n \log p_{nt} & -\hat{\beta}_n \\
-\log p_{1t} & -\log p_{2t} & \ldots & -\log p_{nt} & 1
\end{pmatrix},
\]

\[
A_t^{D*} = \begin{pmatrix}
\hat{\alpha}_1^* + \sum_{j=1}^{n} \hat{\gamma}_{1j} \log p_{jt} \\
\hat{\alpha}_2^* + \sum_{j=1}^{n} \hat{\gamma}_{2j} \log p_{jt} \\
\vdots \\
\hat{\alpha}_n^* + \sum_{j=1}^{n} \hat{\gamma}_{nj} \log p_{jt} \\
\log x_0 - \sum_{j=1}^{n} \bar{w}_{i0} \log p_{j0}
\end{pmatrix}.
\]
The solution for the optimal budget shares $x_t^*$ and the nominal income $x_t^*$ under constant real income depending on estimated parameters, initial price indices and the nominal income in the base year can be computed using the following relation:

$$W_t^* = B_t^D x_t^* A_t^{-1}$$

The computed optimal budget shares corresponding to the initial price vector and the constant real income I further use for the derivation of the set of the optimal price vector which I obtain from the supply function. The supply system written in matrix form is given by the following equation:

$$B_t^{S*} \Pi_t^* = A_t^{S*}$$

where

$$B_t^{S*} = \begin{pmatrix}
1 & \ldots & \ldots & 0 \\
0 & 1 & \ldots & 0 \\
\vdots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & 0 & 1
\end{pmatrix},$$

$$A_t^{S*} = \begin{pmatrix}
\delta_1 \log p_{1t-1} + \rho_1 \omega_{1t} + \psi_1 \log PPI_t + \phi_1 R_t + \\
+ \xi_1 \log w_{1t} + \theta_1 \log REER_t \\
\delta_n \log p_{nt-1} + \rho_n \omega_{nt} + \psi_n \log PPI_t + \phi_n R_t + \\
+ \xi_n \log w_{nt} + \theta_n \log REER_t
\end{pmatrix}$$

and

$$\Pi_t^* = [\log p_{1t}^*, \log p_{2t}^*, \ldots, \log p_{nt}^*].$$

where $p_{it}^*$ denotes the optimal price of good $i$ at time $t$. The solution for the set of the optimal prices depending on estimated parameters and budget shares can be computed as follows:

$$\Pi_t^{**} = B_t^{S**} A_t^{S**}$$

By iterating between system of supplies (2.12) and demands (2.10) I determine the equilibrium vectors of budget shares and prices. The $x_{k0}^*$ and $p_{k0}^*$ define the initial condition, i.e., such budget shares that have been used for the computation of the CPI in a base year and prices that correspond to it are the same as prices observed at that time. Since the real income is fixed at the level of time zero (time zero is identical to the CPI base year), the COL and the CPI are normalized to be equal at this point in time. The normalization is based on the argument that the Laspeyres, Paasche, Stone and Fisher Price Indices are equal in the base year. Hence, the COL is equal to the CPI.

2.4. CPI Bias Evaluation and Testing. The direct computation of the growth rate in the COL, approximated by the Stone Price Index adjusted for the income bias in budget shares and price indices, is given by:
DOES CPI APPROXIMATE COST-OF-LIVING?

\[ PCOL_{GR} = \frac{P^*_t}{P^*_{t-1}} - 1 = \frac{\phi 10^k \sum \omega^*_k \log p^*_k}{\phi 10^k \sum \omega^*_{k-1} \log p^*_{k-1}} - 1 \]

where \( P^*_t \) denotes the Stone Price Index without the income bias, \( \omega^*_k \) and \( p^*_k \) represent the equilibrium budget shares and prices under constant real income, and \( \phi \) stands for the unknown collinear multiple which cancels out and need not be evaluated.

Further, by denoting the CPI growth rate as

\[ PCPI_{GR} = \frac{CPI_t}{CPI_{t-1}} - 1 \]

I can define the CPI bias as the difference between the growth rate of the CPI and the growth rate of the COL. Formally,

\[ BIAS = PCPI_{GR} - PCOL_{GR} \]

**Definition 1.** The expected value of the function of the n-dimensional discrete random variable \( (X_1, ..., X_n) \) denoted as \( E[g(X_1, ..., X_n)] \) is defined as 

\[ E[g(X_1, ..., X_n)] = \sum g(x_1, ..., x_n) f_{X_1,...,X_n}(x_1, ..., x_n), \]

where the function \( g(, ..., .) \) is a function of the n-dimensional random variable. (for details see Mood et al. 1974)

**Theorem 1.** For two discrete linearly related random variables \( X_1 \) and \( X_2 \) holds the following relation \( E[X_1 - X_2] = E[X_1] - E[X_2], \) where \( E[.] \) denotes the expectation.

**Proof.**

\[ E[X_1 - X_2] = \sum_i \sum_j (x_{1,i} - x_{2,j}) f_{X_1, X_2}(x_{1,i}, x_{2,j}) = \sum_i (x_{1,i}) \sum_j f_{X_1, X_2}(x_{1,i}, x_{2,j}) = \sum_j (x_{2,j}) \sum_i f_{X_1, X_2}(x_{1,i}, x_{2,j}) = E[X_1] - E[X_2]. \]

Applying Theorem 1, it must hold, under the assumption that the growth rates are distributed joint normally, that

\[ E_{t:a}(BIAS) = E_{t:a}(PCPI_{GR}) - E_{t:a}(PCOL_{GR}) \]

where index \( t = 1, ..., T - a \) denotes the different starting time of expected values (moving average) and parameter \( a \) specifies the time horizon over which we take the average. Assuming that \( E_{t:a \to \infty}(BIAS) \sim N(\mu, \sigma^2) \) we can test whether the expected value of the CPI bias is statistically different from zero on a certain horizon \( a \).

By evaluating the CPI bias at the different horizon \( a \) from any possible time \( t \) (moving average), I can statistically test when the expected value of the growth rate of the CPI and the growth rate of the COL coincide \( (E_{t,a}(BIAS) = 0) \) or stabilize at certain fixed difference \( m \), \( (E_{t,a}(BIAS) = m) \). To test the expected value I use \( t - distribution \), as it is for a small sample an appropriate approximation for the normal distribution.
3. Empirical Application

The empirical application of the model described in Section 2 to data from the Czech Republic breaks down into three subsections: Data, Estimation and Results.

3.1. Data. The data used in this analysis was obtained from the Czech Statistical Office and the Czech National Bank. The sample consists of quarterly data series of household budget shares, expenditure level and price indices classified by purpose of expenditure from the 1st quarter of 1994 to the 4th quarter of 2000. The Consumer Price Index (CPI) on the first strata level data consists of 12 groups classified by purpose of expenditure. However, I design my own classification of nine groups of goods in order to account for the change in methodology of goods classification by the Czech Statistical Office in 1999. Even though the comparability of classifications reported prior to and after 1999 has affected mainly the lower strata levels and not the first strata level, which remained fairly consistent, in order to assure maximum comparability between methodologies I have made the following adjustments.\footnote{In parenthesis is given the degree of comparability in the Czech Statistical Office’s product group consistency scale before and after 1998: (1) full consistency, (2) satisfactory consistency and (3) poor consistency. The consistency denotes the degree of comparability after the transfer of indicators among groups had taken place. By merging the groups within which the transfers have taken place we significantly improve the comparability.}

Beverages consist of non-alcoholic beverages (3), alcoholic beverages (3) and tobacco (1). The consistency of all three subcategories is, however, improved by merging the non-alcoholic and alcoholic beverages since the lower comparability was caused almost exclusively by non-alcoholic beer, which had been moved from the first category to the second one. Thus, the overall consistency can instead be considered (2).

In the case of rent for dwelling and household equipment, merging them into one category yields a 50% improvement in the comparability from the worst classification (3). Likewise, merging culture (3), entertainment (3) and education (3), earns a comparability improvement of 30%, yet this still remains the worst consistent group of products. The final categories that I redefine and that cover the structure of the CPI are summarized in Table 1.

Although the adjustment effected by merging groups was necessary to improving the methodological consistency of the time series, after the adjustment some of these groups became quite broadly defined and, consequently, the investigated price elasticities might be determined by a stronger tendency in one of the merged subgroups of goods. For instance, if the share of rent for dwelling and household equipment increases, it is impossible to distinguish in which relation of these two categories the growth has appeared and this complicates the inference about single product group. Prior to estimation, the data has been seasonally adjusted using the standard method Census X11-additive in order to eliminate the seasonal effects that might
have an influence on the final horizon of the CPI’s approximation of the COL.

### Table 1: Summary of Good Classification

<table>
<thead>
<tr>
<th>Category</th>
<th>Content</th>
<th>Comparability</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Food</strong></td>
<td>food (01)</td>
<td>(1)</td>
</tr>
<tr>
<td>Beverages</td>
<td>alcoholic (02), non-alcoholic (01)</td>
<td>(2)</td>
</tr>
<tr>
<td><strong>Cloth</strong></td>
<td>clothing (03) and footwear (03)</td>
<td>(1)</td>
</tr>
<tr>
<td>Heating</td>
<td>heating (04) and lighting (04)</td>
<td>(1)</td>
</tr>
<tr>
<td>Household</td>
<td>rent for dwelling (04) and household equipment (05)</td>
<td>(2)</td>
</tr>
<tr>
<td><strong>Personal</strong></td>
<td>personal (12) and medical care (06)</td>
<td>(1)</td>
</tr>
<tr>
<td>Culture</td>
<td>culture (09), education (10), entertainment (09) and recreation (11)</td>
<td>(2-3)</td>
</tr>
<tr>
<td>Transport</td>
<td>transport (07) and communications (08)</td>
<td>(2)</td>
</tr>
<tr>
<td><strong>Miscellaneous</strong></td>
<td>miscellaneous goods and services (12)</td>
<td>(1)</td>
</tr>
</tbody>
</table>

Note: 1) See footnote 10.

The data source for the wage indices in equivalent classification to the nine groups of goods classification was the Czech National Bank database. The production prices index (PPI) and the real effective exchange rate (deflated by PPI) have been obtained from the Czech National Bank database as well. All data series are presented in Figures 1-4.

3.2. Estimation. I have performed an estimation of the simultaneous system of supplies (defined in Section 2.2) and demands (defined in Section 2.1) on nine goods markets. The markets’ classifications correspond to the first CPI strata level data, i.e., market for Food, Beverages, Clothing, Heating, Household, Personal, Culture, Transport and Miscellaneous (as defined in Section 3.1).

The empirical application of the model remains limited to the substitution bias in the first CPI strata level since longer time series for the Czech Republic which would grant an increase in the number of degrees of freedom are unavailable. Thus, any interpretation of the results needs to take into account that the investigated second order bias, i.e. the substitution bias, is not fully evaluated as I expect that the substitution effect will be greater along the lower strata levels. Nevertheless, the contribution of the application is in the evaluation of the true substitution bias in the CPI on the first strata level data.

In applying the supply function to the groups of goods, I have taken into account the product group’s specific character. The Food and Beverages final product markets, as well as Transport together with Miscellaneous goods and services, are assumed to be influenced by interest rate only indirectly.
These markets are representative of intermediary supply in contrast to "outlets", factory or branch shops such as clothing or household equipment markets. In the case of Transport and Miscellaneous goods and services, I have made ex ante exclusion of the direct effect of the nominal effective exchange on price setting since these goods and services are non-tradables. The simultaneous system estimation of nine demands and nine supplies has been estimated using 3SLS, and the results for the demand and supply are presented in Table 2 and Table 3.

### Table 2: Consumer Demand, By Categories of Goods

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>$\alpha_i^*$</td>
<td>1.9</td>
<td>0.23</td>
<td>0.74</td>
<td>-0.6</td>
<td>-1.1</td>
<td>-0.08</td>
<td>0.73</td>
<td>-0.61</td>
<td>-0.5</td>
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<td></td>
<td>(6.7)</td>
<td>(1.4)</td>
<td>(3.0)</td>
<td>(-3.0)</td>
<td>(-2.2)</td>
<td>(-0.6)</td>
<td>(2.6)</td>
<td>(-1.2)</td>
<td>(-0.5)</td>
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<tr>
<td>$\gamma_{i1}$ Food</td>
<td>0.1</td>
<td>-0.12</td>
<td>-0.14</td>
<td>0.26</td>
<td>0.16</td>
<td>0.03</td>
<td>0.05</td>
<td>-0.02</td>
<td>-0.7</td>
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<tr>
<td></td>
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<td>(-1.5)</td>
<td>(3.5)</td>
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<td>(0.7)</td>
<td>(0.6)</td>
<td>(-0.1)</td>
<td>(-2.3)</td>
</tr>
<tr>
<td>$\gamma_{i2}$ Bev.</td>
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<td>0.14</td>
<td>-0.1</td>
<td>-0.1</td>
<td>-0.5</td>
<td>0.04</td>
<td>-0.04</td>
<td>-0.2</td>
<td>-0.13</td>
</tr>
<tr>
<td></td>
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<td>(2.4)</td>
<td>(-0.9)</td>
<td>(-1.1)</td>
<td>(-2.3)</td>
<td>(0.8)</td>
<td>(-0.3)</td>
<td>(-0.7)</td>
<td>(-0.3)</td>
</tr>
<tr>
<td>$\gamma_{i3}$ Cloth</td>
<td>0.6</td>
<td>0.05</td>
<td>0.25</td>
<td>0.02</td>
<td>-0.4</td>
<td>0.02</td>
<td>-0.13</td>
<td>0.05</td>
<td>0.7</td>
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<tr>
<td></td>
<td>(4.6)</td>
<td>(0.7)</td>
<td>(2.2)</td>
<td>(0.2)</td>
<td>(-3.7)</td>
<td>(0.6)</td>
<td>(-2.2)</td>
<td>(0.5)</td>
<td>(3.6)</td>
</tr>
<tr>
<td>$\gamma_{i4}$ Heat.</td>
<td>-0.1</td>
<td>-0.02</td>
<td>-0.18</td>
<td>0.07</td>
<td>0.21</td>
<td>-0.14</td>
<td>0.2</td>
<td>-0.3</td>
<td>-1.5</td>
</tr>
<tr>
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<td>(-1.8)</td>
<td>(-0.5)</td>
<td>(-3.4)</td>
<td>(1.5)</td>
<td>(0.96)</td>
<td>(-1.7)</td>
<td>(0.9)</td>
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<tr>
<td>$\gamma_{i5}$ Hous.</td>
<td>0.3</td>
<td>0.03</td>
<td>0.23</td>
<td>0.05</td>
<td>1.12</td>
<td>-0.03</td>
<td>-0.6</td>
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<td>(1.5)</td>
<td>(0.27)</td>
<td>(1.5)</td>
<td>(0.35)</td>
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<td>(-2.2)</td>
<td>(1.1)</td>
<td>(0.05)</td>
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<tr>
<td>$\gamma_{i6}$ Pers.</td>
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<td>-0.09</td>
<td>-0.54</td>
<td>0.66</td>
<td>1.0</td>
<td>0.17</td>
<td>-0.5</td>
<td>-0.24</td>
<td>0.7</td>
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<td>(-4.1)</td>
<td>(-0.6)</td>
<td>(-2.3)</td>
<td>(3.3)</td>
<td>(2.1)</td>
<td>(1.5)</td>
<td>(-3.7)</td>
<td>(-0.7)</td>
<td>(1.1)</td>
</tr>
<tr>
<td>$\gamma_{i7}$ Cult.</td>
<td>-0.7</td>
<td>-0.01</td>
<td>0.42</td>
<td>-0.5</td>
<td>-0.47</td>
<td>-0.01</td>
<td>0.9</td>
<td>-0.1</td>
<td>0.23</td>
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<tr>
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<td>(2.1)</td>
<td>(-3.1)</td>
<td>(-1.3)</td>
<td>(-0.1)</td>
<td>(4.5)</td>
<td>(-0.4)</td>
<td>(0.9)</td>
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<tr>
<td>$\gamma_{i8}$ Trans.</td>
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<td>-0.01</td>
<td>-0.03</td>
<td>-0.09</td>
<td>-0.44</td>
<td>0.03</td>
<td>-0.1</td>
<td>0.3</td>
<td>0.45</td>
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<td>(-0.5)</td>
<td>(-1.5)</td>
<td>(-3.2)</td>
<td>(0.89)</td>
<td>(-1.4)</td>
<td>(1.8)</td>
<td>(1.1)</td>
</tr>
<tr>
<td>$\gamma_{i9}$ Misc.</td>
<td>0.02</td>
<td>-0.02</td>
<td>-0.06</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
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<td>(-2.4)</td>
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<td>(-0.1)</td>
<td>(-1.1)</td>
<td>(-0.8)</td>
<td>(1.5)</td>
<td>(-0.01)</td>
</tr>
<tr>
<td>$\beta_i$ Income el.</td>
<td>-0.53</td>
<td>-0.07</td>
<td>-0.22</td>
<td>-0.01</td>
<td>-0.07</td>
<td>-0.06</td>
<td>-0.08</td>
<td>0.38</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>(-7.8)</td>
<td>(-1.8)</td>
<td>(-3.6)</td>
<td>(-0.2)</td>
<td>(-0.6)</td>
<td>(-2.1)</td>
<td>(-1.3)</td>
<td>(3.4)</td>
<td>(3.4)</td>
</tr>
</tbody>
</table>

$\sum \gamma_{ij}$

$\sum_{j} \gamma_{ij}$

S.S.E. ($10^{-3}$)

R$^2$

D.W.

<table>
<thead>
<tr>
<th>\text{Comm.}</th>
<th>\text{Food}</th>
<th>\text{Bev.}</th>
<th>\text{Cloth}</th>
<th>\text{Heat}</th>
<th>\text{Hous.}</th>
<th>\text{Pers.}</th>
<th>\text{Cult.}</th>
<th>\text{Trans.}</th>
<th>\text{Misc.}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_i^*$</td>
<td>1.9</td>
<td>0.23</td>
<td>0.74</td>
<td>-0.6</td>
<td>-1.1</td>
<td>-0.08</td>
<td>0.73</td>
<td>-0.61</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

$\gamma_{i1}$ Food | 0.1 | -0.12 | -0.14 | 0.26 | 0.16 | 0.03 | 0.05 | -0.02 | -0.7 |

$\gamma_{i2}$ Bev. | 0.2 | 0.14 | -0.1 | -0.1 | -0.5 | 0.04 | -0.04 | -0.2 | -0.13 |

$\gamma_{i3}$ Cloth | 0.6 | 0.05 | 0.25 | 0.02 | -0.4 | 0.02 | -0.13 | 0.05 | 0.7 |

$\gamma_{i4}$ Heat. | -0.1 | -0.02 | -0.18 | 0.07 | 0.21 | -0.14 | 0.2 | -0.3 | -1.5 |

$\gamma_{i5}$ Hous. | 0.3 | 0.03 | 0.23 | 0.05 | 1.12 | -0.03 | -0.6 | 0.5 | 0.05 |

$\gamma_{i6}$ Pers. | -1.1 | -0.09 | -0.54 | 0.66 | 1.0 | 0.17 | -0.5 | -0.24 | 0.7 |

$\gamma_{i7}$ Cult. | -0.7 | -0.01 | 0.42 | -0.5 | -0.47 | -0.01 | 0.9 | -0.1 | 0.23 |

$\gamma_{i8}$ Trans. | 0.2 | -0.01 | -0.03 | -0.09 | -0.44 | 0.03 | -0.1 | 0.3 | 0.45 |

$\gamma_{i9}$ Misc. | 0.02 | -0.02 | -0.06 | -0.01 | -0.01 | -0.01 | -0.02 | 0.07 | -0.01 |

$\beta_i$ Income el. | -0.53 | -0.07 | -0.22 | -0.01 | -0.07 | -0.06 | -0.08 | 0.38 | 0.67 |

| $\sum \gamma_{ij}$ | -0.4 | -0.1 | -0.1 | 0.36 | 0.67 | 0.1 | -0.2 | 0.1 | -0.2 |

S.S.E. ($10^{-3}$)

R$^2$

D.W.

Note: t-statistics are presented in parentheses.

As can be seen from Table 2, demands for the majority of goods have been estimated with a high coefficient of determination, on average 0.85. However, such a good fit could be due to a small sample because there is a limited number of degrees of freedom. A more appropriate measure of the model’s...
performance is the ratio of significant coefficients to the total, which is equal to 0.43. When compared to the result 0.33 in Deaton and Muellbauer (1980), who estimated the demand system with a similar data set for the U.K., the model can be considered satisfactorily well-estimated. Regarding the classification of goods into necessity and luxury, Food, Beverages, Clothing and Personal turn out to be necessities, as their income elasticity is negative (coefficients $\beta_i$ in Table 1), which is in correspondence with intuition. By the same reasoning, Transport and Miscellaneous goods prove to be luxuries. Household, Heating and Culture are rather indeterminate with respect to statistical insignificance.

Restriction (2.4), that the sum of the coefficients $\beta_i$ is equal to zero, is satisfied\footnote{This has to be satisfied because the data used was such that the budget shares sum up to one (as in the CPI).} as $\sum_i \beta_i = 0.015$. Restrictions on cross and own price elasticities $\gamma_{ij}$ are presented in Table 2 in the row denoted as $\sum_i \gamma_{ij}$. Their values are close to zero as implied by restriction (2.5).

The estimates of the marginal effects (elasticities) of real income on budget shares are in general significantly different from zero, prompting the belief that real income is one of the determinants of the consumption decision and hence that the consumer’s preferences are non-homothetic.

All estimated price elasticities are lower than one in absolute values except for the demand for rent for dwelling and household equipment, where a unitary increase in price causes an increase in the quantity purchased. This feature might be a transition specific one, possibly due to a persistently unsaturated demand for household equipment and further due to the administratively regulated housing market, in which a slowly increasing availability of apartments for rent (through slow liberalization of the housing market) is accompanied by increasing rents.\footnote{This is a specific feature of transition in the Czech Republic since apartments that are under regulated rents remain regulated until there is a change in tenantship (See Act 176/1993 about the rents of flats; Act 526/1990 about price regulation). There is no limit on the contracts with new tenants, which are determined on a competitive basis. This causes an increase in availability of apartments for rent accompanied by an increase in the charged rent, which might seem at first glance counterintuitive.}

The results of estimation of the supply system are presented in the Table 3. Supplies turn out to be relatively elastic but not perfectly elastic (see positive coefficients $\rho_i$ in Table 3). The least elastic supply is that for Heating: a percentage change in budget share causes an increase in price by 2.2 %. On the other side, Clothing and Culture represent highly elastic supplies since a percentage increase in budget share causes an 0.06 % increase in price in the case of Clothing and zero increase in the case of Culture (perfectly elastic supply).\footnote{Taking into account the t-test of the estimated coefficient for Culture, which shows that the coefficient is not different from zero at 10% significance level.} The elasticity of supply of Food (0.51) Beverages (0.61), and
Miscellaneous (0.56) are less elastic than that of Household (0.33), Personal (0.48) and Transport (0.28).

Wage increase in respective sector (coefficients $\xi_i$), real effective exchange rate (coefficient $\theta_i$), interest rate (coefficient $\phi_i$) as well as producer prices (coefficient $\psi_i$) prove to be significant cost factors influencing the price setting (see the t-statistics in Table 3).

Table 3: Supplies By Categories of Goods

<table>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{i,t-1}(\delta_i)$</td>
<td>0.76</td>
<td>0.72</td>
<td>0.96</td>
<td>0.59</td>
<td>0.79</td>
<td>0.92</td>
<td>0.79</td>
<td>0.94</td>
<td>0.82</td>
</tr>
<tr>
<td>(19.3)</td>
<td>(10.3)</td>
<td>(5.6)</td>
<td>(8.1)</td>
<td>(15.1)</td>
<td>(24.4)</td>
<td>(20.7)</td>
<td>(18.6)</td>
<td>(9.3)</td>
<td></td>
</tr>
<tr>
<td>$\varpi_i (\rho_i)$</td>
<td>0.51</td>
<td>0.61</td>
<td>0.06</td>
<td>2.2</td>
<td>0.33</td>
<td>0.48</td>
<td>0.11</td>
<td>0.28</td>
<td>0.56</td>
</tr>
<tr>
<td>(9.3)</td>
<td>(2.2)</td>
<td>(2.08)</td>
<td>(4.9)</td>
<td>(3.0)</td>
<td>(2.8)</td>
<td>(0.93)</td>
<td>(3.0)</td>
<td>(1.46)</td>
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<tr>
<td>wage ($\zeta_i$)</td>
<td>0.04</td>
<td>0.05</td>
<td>0.25</td>
<td>0.13</td>
<td>0.006</td>
<td>0.003</td>
<td>0.004</td>
<td>0.02</td>
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<tr>
<td>(1.8)</td>
<td>(1.77)</td>
<td>(3.4)</td>
<td>(3.6)</td>
<td>(0.84)</td>
<td>(0.14)</td>
<td>(0.21)</td>
<td>(0.11)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R (\phi_i)$</td>
<td>-</td>
<td>-</td>
<td>0.31</td>
<td>0.34</td>
<td>0.12</td>
<td>0.06</td>
<td>0.3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(5.6)</td>
<td>(2.3)</td>
<td>(1.8)</td>
<td>(1.5)</td>
<td>(5.2)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPI ($\psi_i$)</td>
<td>0.39</td>
<td>0.28</td>
<td>0.0006</td>
<td>0.002</td>
<td>0.0007</td>
<td>0.0004</td>
<td>0.0005</td>
<td>0.05</td>
<td>0.17</td>
</tr>
<tr>
<td>(6.1)</td>
<td>(3.1)</td>
<td>(5.4)</td>
<td>(3.0)</td>
<td>(2.6)</td>
<td>(5.18)</td>
<td>(2.3)</td>
<td>(1.3)</td>
<td>(1.13)</td>
<td></td>
</tr>
<tr>
<td>REER ($\theta_i$)</td>
<td>-0.23</td>
<td>-0.06</td>
<td>-0.03</td>
<td>-0.26</td>
<td>-0.06</td>
<td>-0.004</td>
<td>-0.08</td>
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<tr>
<td>(-4.9)</td>
<td>(-1.3)</td>
<td>(-1.5)</td>
<td>(-2.4)</td>
<td>(-1.4)</td>
<td>(-0.24)</td>
<td>(-2.8)</td>
<td></td>
<td></td>
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<tr>
<td>S.S.E. ($10^{-3}$)</td>
<td>0.4</td>
<td>0.5</td>
<td>0.07</td>
<td>4.3</td>
<td>0.6</td>
<td>0.05</td>
<td>0.23</td>
<td>0.5</td>
<td>26.2</td>
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<tr>
<td>$R^2$</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.93</td>
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<tr>
<td>D.W.</td>
<td>1.63</td>
<td>1.87</td>
<td>1.7</td>
<td>2.6</td>
<td>2.6</td>
<td>1.11</td>
<td>2.5</td>
<td>2.29</td>
<td>2.9</td>
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</tbody>
</table>

Note: t-statistics are presented in parentheses.

In addition, as can be seen from the positive estimate of the coefficient $\psi_i$ in front of the PPI, the PPI creates a higher pressure on price setting than does the nominal effective exchange rate. The nominal exchange rate appreciation influences supply prices negatively, while the interest rate, wages as well as producer prices have a positive impact on price setting. The signs of the estimated coefficients are in line with expectations. The ratio of significant coefficients to all coefficients is 0.72, which together with the almost perfect fit measured by coefficient of determination (0.99 on average) makes the model reasonably well estimated.

3.3. Results. A simulation of the partial equilibrium model under constant real income has been carried out following the derivation in Section 2.3 with the estimated coefficients from the nine-market equilibrium model in Section 3.2. In particular, I simulated the prices and budget shares of each category of goods under a scheme of inflation compensation that keeps real income at a constant level for the year 1994, when the CPI and the COL coincide. The results of the simulated time series of price indices and budget shares
DOES CPI APPROXIMATE COST-OF-LIVING?

DOES CPI APPROXIMATE COST-OF-LIVING?

15

together with the original data for all indicators can be found in the Figures 1-4. In the figures, simulated values are labeled as implied values and original series are included for comparison. The substitution bias can be found in Figure 1.4.

Using the computed COL and CPI growth rates I have evaluated the CPI substitution bias on a yearly basis. As can be seen in Table 4, the absolute CPI substitution bias on the yearly frequency ranges from -0.83 to 0.51 p.p. From the point of view of the level of the CPI inflation, in periods when the inflation was around 2%, the absolute bias tends to stabilize at a level of 0.5 p.p. in absolute value. On the contrary, the value of the relative CPI substitution bias tends to increase in absolute value as the inflation level decreases.

<table>
<thead>
<tr>
<th>Table 4: Yearly CPI Substitution Bias</th>
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<tbody>
<tr>
<td>------</td>
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<tr>
<td>Absolute Bias CRI (p.p.)(^2)</td>
</tr>
<tr>
<td>Relative Bias CRI (%)(^3)</td>
</tr>
<tr>
<td>Absolute Bias (p.p.)(^4)</td>
</tr>
</tbody>
</table>

Note: 1) The substitution bias in the year 2000 has been computed using the data for the first three quarters; 2) The CRI denotes constant real income; 3) Relative bias has been computed as the ratio of the respective bias and the growth rate of the COL; 4) The Absolute Bias (homothetic) on the first strata level data (Laspeyres Index minus Fisher Superlative Index) has been taken from Hanousek and Filer (2001).

From the comparison of the substitution bias computed under constant real income (the first row in Table 4) and homothetic substitution bias (the third row in Table 4), I infer that the non-homothetic bias is more volatile than the homothetic.

In addition, based on the relations expressed in equations (2.13) - (2.16), I express the expected value of the BIAS for a variety of horizons a as follows:

\[
E_{t,a}(BIAS) = E_{t,a} \left[ \frac{CPI_t}{CPI_{t-1}} - \left( \frac{\phi \sum \bar{w}_k \log p_{kt}}{\phi \sum \bar{w}_{kt-1} \log p_{kt-1}} \right) \right]
\]

where \(\bar{w}_k\) and \(p_k\) represent the equilibrium budget shares and prices, and \(\phi\) stands for the unknown collinear multiple which cancels out. Index \(t = 1, ..., T - a\), denotes the different starting time of expected values (moving averages) and parameter \(a\) specifies the time horizon over which I take the expected value.

I have computed the expected value of the BIAS with a different horizon of averaging \(a\) (moving average). In particular, using relation (3.1) I have fixed a certain level of \(a\) and performed the computation of the \(E_{t,a}(BIAS)\) starting at time \(t = 1\) and finishing at the time \(T - a\). The set of derived expected values for time \(t = 1, ..., T - a\), I subjected to a statistical test, using...
t-distribution, such that each single average is different from zero. Table 5 presents the average p-value of not rejecting the zero hypothesis: probability that the averages are different from each other. Following this procedure, I have determined the minimal horizon in which the COL growth rate is approximated by the growth rate of the CPI. The horizon $a = 5$ quarters.

<table>
<thead>
<tr>
<th>Horizon $a$ (quarters)</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average p-value$^{1)}$</td>
<td>0.381</td>
<td>0.287</td>
<td>0.023$^{**}$</td>
<td>0.022$^{**}$</td>
<td>0.02$^{**}$</td>
</tr>
<tr>
<td>$\sigma^2$ of p-values</td>
<td>0.0701</td>
<td>0.0408</td>
<td>0.0002</td>
<td>0.0001</td>
<td>0.00009</td>
</tr>
</tbody>
</table>

Note: $^{***}$ denotes significance level 1%, $^{**}$ significance level 5%, and $^*$ 10%.  $^{1)}$ The p-value represents the probability of not rejecting the zero hypothesis that the averages are different from each other.

As can be seen from Table 5, the probability of not rejecting the zero hypothesis, which at the same time means that the average growth rate of the COL and the average growth rate of the CPI are not different from each other, is decreasing with increasing horizon $a$ (number of quarters).

The variability of the $p-values$, presented in Table 5, decreases to a very low value at the horizon of five and more quarters. The five quarters horizon of averages shows that the probability of unequal averages of CPI and COL is 2.3%. A higher time span $a > 5$ quarters only intensifies the result obtained at the five quarters as can be seen from Table 5. Thus, I conclude that the horizon of five quarters is the minimum horizon for eliminating the CPI substitution bias on the first strata level data in the Czech Republic for the period 1994-2000.

4. Conclusion

The CPI mismeasurement of the COL analyzed in this paper has focused on the true substitution bias in the CPI. An original method consisting of an n-markets supply-demand partial equilibrium model has been designed. The system of demands builds up on the Almost Ideal Demand System by Deaton and Muellbauer (1980) and the system of supplies is based on marginal cost driven supply functions that correspond to the structure of demands. Under the assumption of collinear development of sub-CPI indices, I used the fact that the growth rate of the delogarithmic Stone Price Index without the income bias is equal to the growth rate of the COL. In order to eliminate the income bias in the Stone Price Index I performed a simulation in the estimated partial equilibrium model under constant real income of the period in which the CPI has its base. For the obtained COL (delogarithmic Stone Price Index without the income bias), I designed a simple test to verify whether the growth rate of the COL differs from the growth rate of the CPI significantly in their mean values over a variety of horizons of moving averages.
The developed theoretical framework, which builds upon the non-homothetic behavior of consumers, has been applied to quarterly data for the Czech Republic for 1994-2000 at the first CPI strata level; the substitution bias has been evaluated.

The yearly bias ranges from -0.83 to 0.51 p.p. In comparison with the bias found in Hanousek and Filer (2001), who performed the computation under the assumption of homothetic behavior of consumers, the bias on the first strata level ranges from -0.05 to 0.1 p.p.

As I have confirmed by empirical application, a change in the preferences due to a change in real income can cause the true substitution effect to take both positive and negative values. This is due to the fact that besides the intuitive substitution effect towards cheaper goods (positive bias) there is the income effect stemming from the change in real income causing a change in preferences, which can go either way and cause a positive or a negative substitution bias. Especially in the periods of economic transition such as the period of 1994-2000 in the Czech Republic, there were significant changes in real income and thus the income bias proved to be a significant factor in the true substitution bias evolution.

However, despite its higher volatility, the evaluated non-homothetic substitution bias does not prove to be, statistically, significantly different from zero on the average of five quarters on the first strata level data. Nevertheless, on the fourth quarter horizon, the statistically significant difference of the bias from zero has shown that the concerns of the bias in a yearly horizon are still valid.

That the horizon of five quarters on average was determined by evaluating moving averages over the whole investigated period and that the data has been quarterly seasonally adjusted excludes the possibility of an impact of seasonal effects on the determination of the minimal horizon. Moreover, since the sample consisting of quarterly data for the Czech Republic from 1994-2000 contained both a period of relatively higher (moderate) CPI inflation (around 10\% in 1995-97) and period of lower (stable) inflation (around 2\% in 1999-2000), the determined horizon of five quarters remains robust even under such CPI inflation variability.

However, this conclusion drawn from the empirical application applies only to the substitution bias on the first CPI strata level. At the lower strata levels we would expect to find a higher substitution bias and consequently the minimal horizon would probably be longer.
REFERENCES


Figure 1.1: Stone Price Index

Figure 1.2: Consumer Price Index

Figure 1.3: Inflation

Figure 1.4: Substitution Bias

Figure 1.5: Nominal Expenditures

Figure 1.6: Real Expenditures

Figure 1.7: Price Index: Food

Figure 1.8: Price Index: Beverages
Figure 2

DOES CPI APPROXIMATE COST-OF-LIVING?
Figure 3: Budget Share: Beverages

Figure 3.1

Figure 3.2: Budget Share: Clothing

Figure 3.3: Budget Share: Heating

Figure 3.4: Budget Share: Household

Figure 3.5: Budget Share: Personal

Figure 3.6: Budget Share: Culture

Figure 3.7: Budget Share: Transport

Figure 3.8: Budget Share: Miscellaneous

Figure 3
Figure 4