Change Detection in Nonstationary Time Series in Linear Regression Framework \(^1\)
Software Support and Applications

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Chapter 1

Introduction

This Technical Report represents the second phase of the grant entitled "Change Detection in Nonstationary Time Series in Linear Regression Framework", received in 2002 from CERGE-EI Foundation under a program of Global Development Network.

The main goal of this project was to give a unified framework for the design and performance evaluation of some algorithms and methods for solving change detection problem in time series with application in econometrics. The following objectives have been taken into account:

1. To establish a methodological approach to deal with change detection in time series with application in the field of economics.

2. To evaluate the performances of some algorithms and methods for change detection in time series, presented in the literature, and to develop new methods and algorithms.

3. To design an integrated software support, implementing the best methods and algorithms for change detection in time series.

4. To prove the implemented methods and algorithms on case studies in the field of economics.

The Technical Report representing the first phase of the grand (Popescu 2002) had as subject some of the best algorithms and methods for change detection in nonstationary time series. Also, some numerical results for the discussed methods have been studied by simulation, to rank the methods under consideration.

The second phase of the grant is dedicated to a software support implementing the best methods and algorithms for change detection in nonstationary time series. Performance evaluation of some methods, by simulation, to investigate their robustness, constitutes a distinct part of this phase. Also, some case studies in the fields of economics, using real data are reported.

The report includes three main chapters, Chapter 2 has as objective to give an overview on the CHANGE program package for detection of changes, implemented on personal computers compatible IBM/PC. Chapter 3 is devoted to some experimental results, obtained via simulation, for the test statistics, when the change detection methods based on quadratic forms are used. Also, the robustness of the methods, as to the assumption of autoregressive data and to the model structure is discussed. Chapter 4 presents the results obtained in segmentation of some nonstationary financial and economic time series using the algorithms based on "distance" measure, quadratic forms and Kitagawa-Akaike method.
Chapter 2

Software support for change detection

The objective of this chapter is to give an overview on the CHANGE program package for detection of changes, implemented on personal computers compatible IBM/PC. The capabilities of CHANGE package make it a powerful, user-friendly and computationally efficient software package. Computational algorithms have been selected so that the user can have complete confidence in the results of its use. All CHANGE data processing is controlled through a fully integrated menu-driven environment. The package is file oriented; it enables the user to create a large collection of numerical data and descriptive information which can be easily maintained, modified, copied and stored. It is compatible with the IDPACK/PC program package (Popescu, 1991) for system identification and time series analysis. So these packages can import and export data between them and sometimes they are used together in solving of change detection in different applications.

2.1 Software structure

CHANGE software package includes three main components:

- **MES** - Multilevel Exploitation Subsystem
- **DMS** - Data Management Subsystem
- **APS** - Application Programs Subsystem

that function together to provide a full range of capabilities for handling, processing of data and presenting the results.

**MES** offers the frame in which may be loaded and executed data management and application programs, with a menu-driven environment that eliminates the need to memorize or look up a series of special commands for each program or function the user want to perform.

One of the most important aspects of the package is its file orientation. All the application programs that require data read it, generally, from data files created by **DMS**. The data may be listed, checked and, if necessary edited prior to use. Also, the data can be analysed by several programs and new files can be created by partitioning and merging existing data files. These options are given in **Database Manager Menu**.

**APS** contains a comprehensive collection of procedures for change detection in signals and systems, in time and frequency domain, but and other programs for preliminary analysis and investigation of the signals and systems (data filtering, spectral analysis, parameter estimation, simulation, etc.). All these functions are given in **CHANGE Master Menu**.
2.2 CHANGE functions

In the following we present the main functions of the package CHANGE by the options implemented in the subsystems MES, DMS and APS.

2.2.1 Multilevel Exploitation Subsystem

The MES offers the user the following facilities:

- data base and application programs starting and selection.
- advancing to a new menu.
- returning to the previous menu.
- displaying a "Help" file associated to the current menu.
- exit from the current menu and returning in the operating system.

Thus, starting from a Master Menu, the user has the possibility, specifying a figure or a letter to execute any data base or application program, to obtain information about system or program functions, or to return in any point of the menu tree.

<table>
<thead>
<tr>
<th>CHANGE Master Menu</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Database Manager</td>
</tr>
<tr>
<td>3. Discrete Simulation</td>
</tr>
<tr>
<td>5. Spectral Analysis</td>
</tr>
<tr>
<td>7. Change Detection in Data</td>
</tr>
<tr>
<td>9. Graphics</td>
</tr>
</tbody>
</table>

H - Help          Ctrl/C - Exit
R - Return to previous menu

Choice —>

The system can be used by following instructions and prompts displayed on the monitor, having the feature of a user friendly software package.

2.2.2 Data Management Subsystem

CHANGE was designed to be a file oriented program package. DMS creates all the data files for subsequent use in the application programs. Each data file, created by DMS, produces two data files. The first is called header file and contains information about the number of variables and cases, variable names, a file label and the name of the associated file containing numerical data. The second file is a standard random access file that contains the actual
numeric data. A numeric data file can be thought as a table or matrix of numbers. Each column is defined as a variable and each row represent one case, or observation, expressed in some unit or measurement. The menu for DMS is presented below:

**Database Master Menu**

<table>
<thead>
<tr>
<th>1. Enter Data</th>
<th>2. List Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. Edit Data</td>
<td>4. Edit File Header</td>
</tr>
<tr>
<td>7. Delete Samples</td>
<td>8. Vertical Augment</td>
</tr>
</tbody>
</table>

H - Help Ctrl/C - Exit  
R - Return to previous menu

Choice —>

The options mentioned above have the following functions:

- **Enter Data** permits the user to create a new file or to add data to an existing file.

- **List Data** is to list all variables or just a subset of the variables, all of cases or a subset of cases.

- **Edit Data** allows changes to be made in individual cases.

- **Edit File Header** serves two purposes: first, it allows display of the file header (i.e. number of variables, number of cases, variables names and file label); second, the program may be used to change the variable names and/or the file label.

- **Delete Files** is simply a convenience feature in that it allows files to be destroyed while in the program mode.

- **Move/Merge/Transform** performs three different functions. **Move** simply refers to transferring data from one file to another. **Merge** is the same as **Move** except that the variables are selected from two different files. New variables may be created by performing **Transform** function on variables from input file(s).

- **Vertical Augment** allows some cases from one file to be appended to another file and to create a new file.

All the programs included in CHANGE read the header file first, when a data file is need, and display its content.
2.2.3 Application Programs Subsystem

This subsystem of the package consists of about 40 main programs which implements various procedures for data generation, discrete simulation, data filtering, spectral analysis, parameter estimation, change detection in data and systems (more than 20 programs). These modules are grouped in 7 options of the CHANGE Master Menu, positions 2-8. For each such option correspond one or more programs, so that by selecting a certain option another menu will be displayed and the desired application program can be chosen.

All programs and subprograms are written in standard Fortran 77 language. A great attention was paid to obtain efficient and reliable codes and to implement the best known numerical algorithms.

CHANGE programs permit the user to select the variables/or cases and confirm the selection made. The output is normally listed on the terminal, but the user can require to obtain for many programs the results on the printer. A title for each problem solved may by optionally entered. The programs present to user lists of options. Generally, the options are indicated by a figure. Default are provided.

Data generation

This function of the package assures the generation of different data to be used in simulation:

- Deterministic data (pulse, step, ramp, frequential, etc.)
- Stochastic data (pseudo-random binary sequence, univariate gaussian data with given mean and variance, univariate data with given spectrum).

Discrete simulation

It is performed simulation of ARX and ARMAX models, for MISO systems, in deterministic or stochastic conditions.

Data filtering

This function includes options for lowpass, highpass and bandpass data sets filtering, after designing of a corresponding Butterworth sinus filter (Othnes and Enochson, 1978).

Spectral analysis

These functions are used for dynamic properties evaluation of data and systems. Functional dependence between input-output data can be determined once the input-output data properties are known. Options are provided for power and cross spectrum computation using parametric models (Akaike, 1978).

Parameter estimation

This option implements the on-line parameter estimation for the following types of models: AR, ARMA, ARX and ARMAX, investigated for change detection in their behaviour. The methods used are least squares method and prediction error method, in recursive form (Tertisco, Stoica and Popescu, 1987). These programs can be used in the preliminary analysis of data, to obtain a priori information, before the change detection analysis.
Change detection in data

The menu associated to this option is presented below:

<table>
<thead>
<tr>
<th>Change Detection in Data Menu</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Change Detection in Mean</td>
</tr>
<tr>
<td>2. Change Detection in Amplitude</td>
</tr>
<tr>
<td>3. Change Detection in Spectrum</td>
</tr>
<tr>
<td>4. Change Detection with Cepstrum Distance</td>
</tr>
<tr>
<td>5. Change Detection with Cusum Tests</td>
</tr>
<tr>
<td>6. Change Detection with Quadratic Forms</td>
</tr>
<tr>
<td>7. Change Detection in AR Parameters</td>
</tr>
</tbody>
</table>

H - Help
Ctrl/C - Exit
R - Return to previous menu

Choice —>

All these options are implemented in the sequential detection mode (on-line), and for the great part of these are implemented all the approaches, described in the section dedicated to the implementation aspects (Popescu, 2002). The off-line detection of changes can be direct applied as a particular case of the on-line detection. All the test statistics evaluated are graphically displayed.

The option Change Detection in Mean performs the change detection in the mean value of a signal using Hinkley test (Hinkley, 1971; Basseville and Benveniste, 1983).

The second option, for Change Detection in Amplitude, implements three detection techniques based on Kullback information, Kullback divergence and Bhattacharyya distance (Ishii, Iwata and Suzumura, 1979).

Change Detection in Spectrum performs discrimination of two spectral densities and uses Kullback information. This is evaluated by the correlations between the regressive coefficients of AR model and data sequences analysed (Ishii, Iwata and Suzumura, 1979).

Change Detection with Cepstrum Distance implements change detection in frequency contents of a signal using cepstral distance (Gray and Markel, 1976; Markel and Gray, 1977); three approaches concerning data selection for reference and current model are implemented.

Change Detection with Cusum Tests performs implementation of three methods for change detection using one model and two models (Basseville and Benveniste, 1983). The last two methods, based on two models, make use of logarithmic likelihood ratio and of mutual entropy between the conditional probability laws. The decision concerning change occurring is taken on the Hinkley test. Only the A3 approach is implemented for all three methods.

Change Detection with Quadratic Forms makes use of quadratic forms of some stochastic Gaussian variables (model parameters, serial and partial correlations of residuals, etc.), which have a $\chi^2$ distribution in the absence of a change (Stoica, 1990); three approaches concerning data selection for reference and current model are implemented.

The option Change Detection in AR Parameters performs change detection in these parameters of an ARMA model with MA coefficients unknown and strong nonstationary (Basseville, Benveniste and Moustakides, 1984)
Change detection in systems

The following menu is associated to this option:

<table>
<thead>
<tr>
<th>Change Detection in Systems Menu</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Change Detection in Time</td>
</tr>
</tbody>
</table>

H - Help
R - Return to previous menu

Choice —>

The both implemented options work in on-line mode, A1 approach. The off-line version can be obtained as a particular case of the on-line operation mode.

Change Detection in Time performs change detection in the dynamics of a system SISO, described by an ARX model (Carlsson, 1988; Popescu 1995). After the estimation of parameter and covariance matrix of these, using input-output data, three test variables \((T_1, T_2, T_3)\) are computed and used in decision concerning the presence or absence of a change. The evolutions of these statistics are graphically displayed, the decision concerning the presence of a change being made using \(T_3\) statistics.

Change Detection in Frequency implements the statistical tests \((T_5, T_6, T_7, T_8)\) suggested by Wahlberg (1989), based on the previous results obtained by Ljung (1987). These test variables are more robust to the experimental conditions than the test variables in time domain, depending also on the spectral density functions of the input and noise. The model used is an ARMAX model for a SISO system. The evaluation of the spectral density functions for the input and for the noise is based on a parametric method (Akaike, 1978). As in the case of change detection in time domain, the evolutions of the computed test variables are displayed, the decision concerning the presence of a change being made using \(T_5\) statistics.

Graphics

The package offers, also, some graphic facilities for the data and for the results, presented in the following menu:
### Graphics Menu

<table>
<thead>
<tr>
<th>1. Data Plot (2D)</th>
<th>2. Data Plot (3D)</th>
</tr>
</thead>
</table>

H - Help
R - Return to previous menu

Choice —>

These functions are performed using DISPLAY graphic processor (Netoiu, 1990).
Chapter 3

Performance evaluation of some change detection methods

This section is devoted to some experimental results, obtained via simulation, for the test statistics when the change detection methods based on quadratic forms (Popescu, 2002), described in Section II. Also, the robustness of the methods, as to the assumption of autoregressive data and to the model structure is discussed.

The methods have been applied to the cases shown in Table 3.1. In each case it was generated one realization of \( \{y_t^{(1)}\} \) and 100 independent realizations of \( \{y_t^{(2)}\} \), of 500 sample points each. Using the multiple simulation runs, we can evaluate the probability of accepting \( H_1 \) under \( H_0 \) (first type of risk), which is also called “false alarm”, and the probability of accepting \( H_0 \) under \( H_1 \) (second type of risk) for the testing methods under consideration. Note that the studied cases are grouped into two classes: for the first 3 cases in Table 3.1, the assumption concerning the autoregressive data are satisfied, while for the last 3 cases are not.

In all cases, in the beginning, only the filter which identifies the model AR\( _1 \) is activated, and after 200 sample points the second filter (sliding block) and the test are activated. If the size of the window used for identifying model AR\( _1 \) is too small, false alarms may occur due to poor estimation of AR coefficients. For this reason the window size has been chosen of 200 samples. Because the number of sample points used for the second filter is 200, it results that two successive changes which occur within less than 200 sample points could not be detected by the investigated methods. For all the methods, the critical probability value \( \alpha \) was set to \( \alpha = 0.05 \).

3.1 Test statistics comparison

The results obtained for C1, C2 and C3 are given in Table 3.2. It can be noted that the combination MIII-A3 has no sense. The model order used was: \( p = 1 \) for C1, \( p = 2 \) for C2 and \( p = 4 \) for C3.

Remark 1. It can be noted that the first type of risk for MI is greater (for A1 and A2 approaches) than that of MIII and MIII. At the same time MI leads to the smallest second type of risk in all cases considered.

Remark 2. Initially, the data window for the reference model will contain only data from \( \{y_t^{(1)}\} \). When the data window used for the current model includes enough data from \( \{y_t^{(2)}\} \), the change is detected. Afterwards, the data window for the reference model will contain data
from \( \{y_{t}^{(1)}\} \) and \( \{y_{t}^{(2)}\} \) and the data window for current model will include only data from \( \{y_{t}^{(2)}\} \). Sometimes, in this case a second change is detected. This depends on the number of data samples from \( \{y_{t}^{(1)}\} \) for which the reference model is computed. Thus, the real change instant will be included between two successive change detection instants. Table 3.3 presents for C1, the number of cases with a single and double change, in the analysed realizations. It can be noted that the number of double change detections reduces for A3 approach, in comparison with the A1 and A2 approaches, for MI and MII. It results that for MI-A3 and MII-A3 the change detection instant will be very close to the real change instant.

**Remark 3.** MII and MIII are not sensitive to a scaling of data. More exactly, MIII is completely insensitive to scaling (it is based on correlations that are not affected by scaling) and MII is only slightly sensitive (due to a slight modification of the AR model fitted to the concatenated set \( \{y_{t}^{(1)}, y_{t}^{(2)}\} \), produced by a ”reasonable” scaling of \( \{y_{t}^{(2)}\} \)).

**Remark 4.** Concerning the computational burden involved, MI is comparable to MIII.

### 3.2 Assumption of autoregressive data

For the C4, C5 and C6 the assumption of autoregressive data is not satisfied. The results obtained in these cases are given in Table 4, in the same manner as for the cases C1, C2 and C3. The model order was chosen in all cases, \( p = 3 \).

**Remark 5.** The results obtained for C4, where \( \{y_{t}^{(2)}\} \) data are generated by an AR process, are similar to the previous results. For C5, where there appears only a small change (the angular frequency jumps from \( 0.2\pi \) to \( 0.23\pi \)) all the methods and approaches indicate a great second type of risk. In C5, where this insignificant change is accompanied by an increase of variance, the second type of risk will decrease, especially for MI.

### Table 3.1: The cases considered in simulation

<table>
<thead>
<tr>
<th>Case</th>
<th>Generation of ( {y_{t}^{(1)}} ) and ( {y_{t}^{(2)}} )</th>
</tr>
</thead>
</table>
| C1   | \( y_{t}^{(1)} = 0.6y_{t-1}^{(1)} + \epsilon_{t}; \sigma^{2} = 1. \)  
\( y_{t}^{(2)} = 0.1y_{t-1}^{(2)} + \epsilon_{t}; \sigma^{2} = 1. \) |
| C2   | \( y_{t}^{(1)} = 0.3y_{t-1}^{(1)} + 0.5y_{t-2}^{(1)} + \epsilon_{t}; \sigma^{2} = 4. \)  
\( y_{t}^{(2)} = 0.3y_{t-1}^{(2)} + 0.5y_{t-2}^{(2)} + \epsilon_{t}; \sigma^{2} = 0.25 \) |
| C3   | \( y_{t}^{(1)} = 0.3y_{t-1}^{(1)} + 0.5y_{t-2}^{(1)} + \epsilon_{t}; \sigma^{2} = 0.09 \)  
\( y_{t}^{(2)} = 0.5y_{t-1}^{(2)} - 0.3y_{t-2}^{(2)} + 0.6y_{t-3}^{(2)} - 0.5y_{t-4}^{(2)} + \epsilon_{t}; \sigma^{2} = 0.16 \) |
| C4   | \( y_{t}^{(1)} = \sqrt{2} \sin (0.2\pi t) + \epsilon_{t}; \sigma^{2} = 0.64 \)  
\( y_{t}^{(2)} = 0.7y_{t-1}^{(2)} + 0.5y_{t-2}^{(2)} - 0.56y_{t-3}^{(2)} + \epsilon_{t}; \sigma^{2} = 1.0 \) |
| C5   | \( y_{t}^{(1)} = \sqrt{2} \sin (0.2\pi t) + \epsilon_{t}; \sigma^{2} = 0.64 \)  
\( y_{t}^{(2)} = \sqrt{2} \sin (0.23\pi t) + \epsilon_{t}; \sigma^{2} = 0.64 \) |
| C6   | \( y_{t}^{(1)} = \sqrt{2} \sin (0.2\pi t) + \epsilon_{t}; \sigma^{2} = 0.64 \)  
\( y_{t}^{(2)} = \sqrt{2} \sin (0.23\pi t) + \epsilon_{t}; \sigma^{2} = 1.0 \) |
<table>
<thead>
<tr>
<th>Case</th>
<th>Testing method</th>
<th>Estim. first type of risk</th>
<th>Estim. second type of risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>MI-A1</td>
<td>0.10</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A2</td>
<td>0.08</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A3</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MII-A1</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MII-A2</td>
<td>0.04</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MII-A3</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MIII-A1</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MIII-A2</td>
<td>0.05</td>
<td>0.00</td>
</tr>
<tr>
<td>C2</td>
<td>MI-A1</td>
<td>0.06</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A2</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A3</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MII-A1</td>
<td>0.00</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>MII-A2</td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>MII-A3</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MIII-A1</td>
<td>0.02</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>MIII-A2</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>C3</td>
<td>MI-A1</td>
<td>0.22</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A2</td>
<td>0.23</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A3</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MII-A1</td>
<td>0.10</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MII-A2</td>
<td>0.15</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MII-A3</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MIII-A1</td>
<td>0.10</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MIII-A2</td>
<td>0.14</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 3.2: Results for C1, C2, C3 cases

<table>
<thead>
<tr>
<th>Testing method</th>
<th>No. cases with single change</th>
<th>No. of cases with double change</th>
</tr>
</thead>
<tbody>
<tr>
<td>MI-A1</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>MI-A2</td>
<td>3</td>
<td>97</td>
</tr>
<tr>
<td>MI-A3</td>
<td>83</td>
<td>17</td>
</tr>
<tr>
<td>MII-A1</td>
<td>3</td>
<td>97</td>
</tr>
<tr>
<td>MII-A2</td>
<td>7</td>
<td>93</td>
</tr>
<tr>
<td>MII-A3</td>
<td>88</td>
<td>12</td>
</tr>
<tr>
<td>MIII-A1</td>
<td>15</td>
<td>85</td>
</tr>
<tr>
<td>MIII-A2</td>
<td>10</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 3.3: No. of cases with single and double change for C1
<table>
<thead>
<tr>
<th>Case</th>
<th>Testing method</th>
<th>Estim. first type of risk</th>
<th>Estim. second type of risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>C4</td>
<td>MI-A1</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A2</td>
<td>0.13</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A3</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A1</td>
<td>0.11</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A2</td>
<td>0.12</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A3</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A1</td>
<td>0.14</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A2</td>
<td>0.18</td>
<td>0.00</td>
</tr>
</tbody>
</table>

| C5   | MI-A1          | 0.00                      | 0.87                      |
|      | MI-A2          | 0.00                      | 0.40                      |
|      | MI-A3          | 0.00                      | 0.67                      |
|      | MI-A1          | 0.00                      | 0.53                      |
|      | MI-A2          | 0.00                      | 0.49                      |
|      | MI-A3          | 0.00                      | 0.97                      |
|      | MI-A1          | 0.00                      | 0.54                      |
|      | MI-A2          | 0.00                      | 0.45                      |

| C6   | MI-A1          | 0.00                      | 0.10                      |
|      | MI-A2          | 0.00                      | 0.08                      |
|      | MI-A3          | 0.00                      | 0.68                      |
|      | MI-A1          | 0.00                      | 0.14                      |
|      | MI-A2          | 0.00                      | 0.22                      |
|      | MI-A3          | 0.00                      | 0.92                      |
|      | MI-A1          | 0.00                      | 0.29                      |
|      | MI-A2          | 0.00                      | 0.33                      |

Table 3.4: Results for C4, C5, C6 cases

### 3.3 Importance of model order

In the cases where the order of the AR model is not known, for the investigated methods, the underestimation of this order can cause poor detection. The results obtained for C3 case with a filter of order 3 and respective 2, instead of real order 4, are given in Table 3.5.

*Remark 6.* It can be noted that the behaviour of the detector, especially for the second type of risk, is not affected by the underestimation of the model order. It seems that the practice of identifying AR filters in lattice form may prevent this fact (see A2, A3 for all methods). The first type of risk will be affected by an underestimation of the order.

*Remark 7.* A strong improvement of the change detection for the second type risk can be noted for the C5 (non autoregressive data), when the model order increases from $p = 3$ to $p = 5$ and respectively $p = 10$. The results are given in Table 3.6. This improvement is accompanied by a slight increase of the first type of risk.
<table>
<thead>
<tr>
<th>p</th>
<th>Testing method</th>
<th>Estim. first type of risk</th>
<th>Estim. second type of risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>MI-A1</td>
<td>0.25</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A2</td>
<td>0.37</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A3</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MIII-A1</td>
<td>0.34</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MIII-A2</td>
<td>0.39</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MIII-A3</td>
<td>0.05</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MIII-A1</td>
<td>0.32</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MIII-A2</td>
<td>0.35</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 3.5: Results for C3 case: $p = 3, p = 2$

<table>
<thead>
<tr>
<th>p</th>
<th>Testing method</th>
<th>Estim. first type of risk</th>
<th>Estim. second type of risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>MI-A1</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>MI-A2</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A3</td>
<td>0.00</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>MIII-A1</td>
<td>0.03</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>MIII-A2</td>
<td>0.07</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>MIII-A3</td>
<td>0.01</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>MIII-A1</td>
<td>0.06</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>MIII-A2</td>
<td>0.21</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>p</th>
<th>Testing method</th>
<th>Estim. first type of risk</th>
<th>Estim. second type of risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>MI-A1</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A2</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MI-A3</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MIII-A1</td>
<td>0.02</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>MIII-A2</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>MIII-A3</td>
<td>0.00</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>MIII-A1</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MIII-A2</td>
<td>0.02</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 3.6: Results for C5 case: $p = 5, p = 10$
3.4 Conclusions

The performance evaluation problem of some methods for sequential detection of changes in non-stationary time series has been addressed. The detection algorithms, considered in the paper, are based on quadratic forms of a Gaussian random variable (estimated AR parameters, estimated residual variance and sample serial and partial residual correlations). The robustness of these algorithms is also investigated. The final conclusion is that, of the methods studied in the paper, MI and the approaches A2 and A3, should be preferred in most practical applications.
Chapter 4

Experimental results

The objective of this section is to compare the results obtained in segmentation of some non-stationary financial and economic time series using the algorithms based on "distance" measure, quadratic forms and Kitagawa-Akaike method (Popescu 2002). The following data have been used:

- US bond yield daily 1 April - 29 December 1989.
- UK bond yield daily 1 April - 29 December 1989.
- West Germany bond yield daily 1 April - 29 December 1989.
- Japan bond yield daily 1 April - 29 December 1989.
- 1 month - tbill monthly 30.01.1926 - 30.12.1996.

The source of the first four data sets is the book "The Econometrics Modelling of Financial Time Series" Terence C. Mills, Cambridge University Press, 1993, and the source of the last two data sets is US Federal Reserve web site at the St. Louis Fed. FRED.

4.1 US bond yield daily 1 April - 29 December 1989

The results obtained for this data set are represented in figure 4.1: BONDUS and statistics U, U1, U2 (Popescu 2002) when cusalem tests were applied, figure 4.2: BONDUS and statistics X, X1 and X2 for method I, II and III when quadratic form based tests were applied (Popescu, 2002), and in figure 4.3 under the form of the BONDUS and resulting segmentation (vertical lines) when three AR models and evaluation of Akaike Information Criterion (AIC) - Kitagawa - Akaike method was applied.

For the statistics used in figure 4.1 it was used A1 approach (Popescu, 2002) and Hinkley test with drift parameter \( \nu = 1 \), and the threshold \( h = 1 \). The sliding window size was \( L = 50 \) and the model order \( p = 3 \). It can be noted strong similarities when cusalem tests, based on "distance" measures were used.

For the statistics represented in figure 4.2 it was used A1 approach and \( \chi^2_{0.05}(4) \) and \( \chi^2_{0.05}(6) \) as thresholds for the statistics X, X1 and X2 when method I and respectively methods II and III were used. The sliding window size was \( L = 50 \) and the model order \( p = 3 \). It can be
Figure 4.1: US bond yield daily 1 April - 29 December 1989 - Cusum Tests

Figure 4.2: US bond yield daily 1 April - 29 December 1989 - Quadratic Form Tests
noted that the methods II and III are less sensitive to possible changes in data dynamics in the interval 150 - 600. For the rest of the data the results are similar.

The results represented in figure 4.3 were obtained for a sliding window size of $L = 50$. The model maximum order was $p = 10$. The detailed results obtained in this case are given in Appendix A.

4.2 UK bond yield daily 1 April - 29 December 1989

The results obtained for this data set are represented in figure 4.4: BONDUK and statistics U, U1, U2 when cusum tests were applied, figure 4.5: BONDUK and statistics X, X1 and X2 for method I, II and III when quadratic form tests were applied, and in figure 4.6 under the form of the BONDUK and resulting segmentation (vertical lines) when three AR models and evaluation of Akaike Information Criterion (AIC) - Kitagawa - Akaike method was applied.

For the statistics used in figure 4.4 it was used A1 approach (Popescu, 2002) and Hinkley test with drift parameter $\nu = 1$, and the threshold $h = 1$. The sliding window size was $L = 50$ and the model order $p = 3$. It can be noted strong similarities when cusum tests, based on "distance" measures were used.

For the statistics represented in figure 4.5 it was used A1 approach and $\chi^2_{0.05}(4)$ and $\chi^2_{0.05}(6)$ as thresholds for the statistics X, X1 and X2 when method I and respectively methods II and III were used. The sliding window size was $L = 50$ and the model order $p = 3$. It can be noted that the methods II and III are less sensitive to possible changes in data dynamics in the interval 300 - 900. For the rest of the data the results are similar.

The results represented in figure 4.6 were obtained for a sliding window size of $L = 50$. 

Figure 4.3: US bond yield daily 1 April - 29 December 1989 - Kitagawa - Akaike Method
Figure 4.4: UK bond yield daily 1 April - 29 December 1989 - Cusum Tests

Figure 4.5: UK bond yield daily 1 April - 29 December 1989 - Quadratic Form Tests
Figure 4.6: UK bond yield daily 1 April - 29 December 1989 - Kitagawa - Akaike Method

The model maximum order was \( p = 10 \). The detailed results obtained in this case are given in Appendix A.

### 4.3 West Germany bond yield daily 1 April - 29 December 1989

The results obtained for this data set are represented in figure 4.7: BONDWG and statistics U, U1, U2, when cusum tests were applied, figure 4.8: BONDWG and statistics X, X1 and X2 for method I, II and III when quadratic form tests were applied, and in figure 4.9 under the form of the BONDWG and resulting segmentation (vertical lines) when three AR models and evaluation of Akaike Information Criterion (AIC) - Kitagawa - Akaike method was applied.

For the statistics used in figure 4.7 it was used A1 approach (Popescu, 2002) and Hinkley test with drift parameter \( \nu = 1 \), and the threshold \( h = 1 \). The sliding window size was \( L = 50 \) and the model order \( p = 3 \). It can be noted strong similarities when cusum tests, based on "distance" measures were used.

For the statistics represented in figure 4.8 it was used A1 approach and \( \chi^2_{0.05}(4) \) and \( \chi^2_{0.05}(6) \) as thresholds for the statistics X, X1 and X2 when method I and respectively methods II and III were used. The sliding window size was \( L = 50 \) and the model order \( p = 3 \).

The results represented in figure 4.9 were obtained for a sliding window size of \( L = 50 \). The model maximum order was \( p = 10 \). The detailed results obtained in this case are given in Appendix A.
Figure 4.7: West Germany bond yield daily 1 April - 29 December 1989 - Cusum Tests

Figure 4.8: West Germany bond yield daily 1 April - 29 December 1989 - Quadratic Form Tests
4.4 Japan bond yield daily 1 April - 29 December 1989

The results obtained for this data set are represented in figure 4.10: BONDJP and statistics U, U1, U2, when cusum tests were applied, figure 4.11: BONDJP and statistics X, X1 and X2 for method I, II and III when quadratic form tests were applied, and in figure 4.12 under the form of the BONDJP and resulting segmentation (vertical lines) when three AR models and evaluation of Akaiake Information Criterion (AIC) - Kitagawa - Akaiake method was applied.

For the statistics used in figure 4.10 it was used A1 approach (Popescu, 2002) and Hinkley test with drift parameter $\nu = 1$, and the threshold $h = 1$. The sliding window size was $L = 50$ and the model order $p = 3$. It can be noted strong similarities when cusum tests, based on "distance" measures were used.

For the statistics represented in figure 4.11 it was used A1 approach and $\chi^2_{0.05}(4)$ and $\chi^2_{0.05}(6)$ as thresholds for the statistics X, X1 and X2 when method I and respectively methods II and III were used. The sliding window size was $L = 50$ and the model order $p = 3$. It can be noted that the methods II and III are less sensitive to possible changes in data dynamics in the interval 300 - 750. For the rest of the data the results are similar.

The results represented in figure 4.12 were obtained for a sliding window size of $L = 50$. The model maximum order was $p = 10$. The detailed results obtained in this case are given in Appendix A.
Figure 4.10: Japan bond yield daily 1 April - 29 December 1989 - Cusum Tests

Figure 4.11: Japan bond yield daily 1 April - 29 December 1989 - Quadratic Form Tests
**4.5 1 month - tbill monthly 30.01.1926 - 30.12.1996.**

The results obtained for this data set are represented in figure 4.13: TBILL and statistics U, U1, U2, when cusum tests were applied, figure 4.14: TBILL and statistics X, X1 and X2 for method I, II and III when quadratic form tests were applied, and in figure 4.15 under the form of the TBILL and resulting segmentation (vertical lines) when three AR models and evaluation of Akaike Information Criterion (AIC) - Kitagawa - Akaike method was applied.

For the statistics used in figure 4.13 it was used A1 approach (Popescu, 2002) and Hinkley test with drift parameter \( \nu = 1 \), and the threshold \( h = 1 \). The sliding window size was \( L = 50 \) and the model order \( p = 3 \). It can be noted strong similarities when cusum tests, based on "distance" measures were used.

For the statistics represented in figure 4.14 it was used A1 approach and \( \chi^2_{0.05}(4) \) and \( \chi^2_{0.05}(6) \) as thresholds for the statistics X, X1 and X2 when method I and respectively methods II and III were used. The sliding window size was \( L = 50 \) and the model order \( p = 3 \).

The results represented in figure 4.15 were obtained for a sliding window size of \( L = 50 \). The model maximum order was \( p = 10 \). The detailed results obtained in this case are given in Appendix A.

**4.6 US treasury bill 2nd market - middle rate, daily 11.06.1986 - 1.12.1995**

The results obtained for this data set are represented in figure 4.16: US-TR and statistics U, U1, U2, when cusum tests were applied, figure 4.17: US-TR and statistics X, X1 and X2 for
Figure 4.13: 1 month - tbill monthly 30.01.1926 - 30.12.1996 - Cusum Tests

Figure 4.14: 1 month - tbill monthly 30.01.1926 - 30.12.1996 - Quadratic Form Tests
method I, II and III when quadratic form tests were applied, and in figure 4.18 under the form of the US-TR and resulting segmentation (vertical lines) when three AR models and evaluation of Akaike Information Criterion (AIC) - Kitagawa - Akaike method was applied.

For the statistics used in figure 4.16 it was used A1 approach (Popescu, 2002) and Hinkley test with drift parameter $\nu = 1$, and the threshold $h = 1$. The sliding window size was $L = 50$ and the model order $p = 3$. It can be noted strong similarities when cusum tests, based on "distance" measures were used.

For the statistics represented in figure 4.17 it was used A1 approach and $\chi^2_{0.05}(4)$ and $\chi^2_{0.05}(6)$ as thresholds for the statistics X, X1 and X2 when method I and respectively methods II and III were used. The sliding window size was $L = 50$ and the model order $p = 3$. It can be noted that the methods II and III are less sensitive to possible changes in data dynamics.

The results represented in figure 4.18 were obtained for a sliding window size of $L = 50$. The model maximum order was $p = 10$. The detailed results obtained in this case are given in Appendix A.
Figure 4.16: US treasury bill 2nd market - middle rate, daily 11.06.1986 - 1.12.1995 - Cusum Tests

Figure 4.17: US treasury bill 2nd market - middle rate, daily 11.06.1986 - 1.12.1995 - Quadratic Form Tests
Figure 4.18: US treasury bill 2nd market - middle rate, daily 11.06.1986 - 1.12.1995 - Kitagawa - Akaike Method
Chapter 5

Conclusions

The main goal of this project was to give a unified framework for the design and performance evaluation of some algorithms for solving change detection problem in time series with application in econometrics. The following objectives have been taken into account:

1. To establish a methodological approach to deal with change detection in time series with application in the field of economics.

2. To evaluate the performances of some algorithms and methods for change detection in time series, presented in the literature, and to develop new methods and algorithms.

3. To design an integrated software support, implementing the best methods and algorithms for change detection in time series.

4. To prove the implemented methods and algorithms on case studies in the field of economics.

The proposed problem in this project assumes off-line or back-wise data processing, although the solution in data and an on-line data processing can be used. The segmentation model is the simplest possible extension of linear regression models to series with abruptly changing properties, or piece-wise linearizations of non-linear models. It is assumed that the time series can be described by one linear regression within each segment with distinct parameter vector and noise variance.

The significance of the research can be considered from two points of view:

From methodological point of view:

• To establish a unified and integrated approach for change detection in time series to be used in economics.

• To promote advances solutions (methods and algorithms) to problems in the field of analysis of economical processes.

From practical point of view:

• To propose a set of recommendations, based on the performance evaluation of the methods and on the case studies.
• To build an integrated software, implementing the best methods and algorithms for change
detection problem solving in nonstationary time series analysis.

In conclusion, concerning the problem making the object of this grant, we can mention the
following remarks:

• Although the problem of change detection reached the maturity, there is a gap between
theory and practice.

• The effort is now directed to robust change detection and diagnosis methods using reduced
order models and adequate distance measures.

• These methods can not be reduced to repeated identification. Our purpose isn’t to deter-
mine a good model, we use the model only like a tool in change detection schemes. Good
and precise models offer high performance in change detection schemes, but also biased
parametric models can be used for change detection and isolation. This bias decreases,
but does not annihilate the performance of the detection procedure.

In our opinion, a coherent methodology is now available to the designer, together with the
corresponding set of tools, which enables him to solve a large variety of change detection problem
in dynamical systems. The general opinion of the scientific community with preoccupations in
this field is that there is a gap between theory and practice and that the model based methods
have many more possibilities in the real practical problems than they so far have proved to have.
The topics of change detection are of increasing practical importance and therefore theoretical
as well as applied research is a challenge for the future.
References


APPENDIX - Analysis results for Kitagawa - Akaike method

US bond yield daily 1 April - 29 December 1989 Series

Title ---> BONDUS

No. of data used ---> 960
Maximum order of AR model ---> 10
Length of basic local span ---> 50
Parameter KSW ---> 0

Basic Autoregressive Model

\[ X(I) = A(1)*X(I-1) + A(2)*X(I-2) + \ldots + A(M)*X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

Initial local model: NS = 50 MS = 1 SDS = .120222D-01 AICS = -217.051

Current Model

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Innovation Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I A(I) SD = .1202195852D-01</td>
<td></td>
</tr>
<tr>
<td>1 .9647011268</td>
<td></td>
</tr>
</tbody>
</table>

This model was fitted to the data \( X(\ 11), \ldots ,X(\ 60) \).

Basic Autoregressive Model

\[ X(I) = A(1)*X(I-1) + A(2)*X(I-2) + \ldots + A(M)*X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
The following two models are compared

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .4355D-02 AICS = -484.869
Constant model: (NP = 100) MP = 1 SDP = .8534D-02 AICP = -472.374

O******************************
*****
***** NEW MODEL ADOPTED *****
*****
******************************

Current Model

Coefficients      Innovation Variance

I     A(I)     SD = .4355342029D-02
1     1.0120927316

This model was fitted to the data (X( 61), ... , X( 110)).

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

The following two models are compared

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .5413D-02 AICS = -524.765
Constant model: (NP = 100) MP = 1 SDP = .4974D-02 AICP = -526.359
O***** Constant model adopted *****
Current Model

Coefficients  Innovation Variance
I  A(I)  SD = 0.4973728178D-02
1  1.0034315481

This model was fitted to the data (X(61), ..., X(160))

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

The following two models are compared ---

OMoving model: (NF = 100, NS = 50) MS = 1 SDS = 0.2220D-02 AICS = -827.874
Constant model: (NP = 150) MP = 1 SDP = 0.4062D-02 AICP = -821.927

NEW MODEL ADOPTED

Current Model

Coefficients  Innovation Variance
I  A(I)  SD = 0.2219868363D-02
1  0.9997866692

This model was fitted to the data (X(161), ..., X(210))

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .3907D-02 AICS = -572.763
Constant model: (NP = 100) MP = 2 SDP = .3081D-02 AICP = -572.254

O******************************************
*****          *****
*****  NEW MODEL ADOPTED       *****
*****          *****
O******************************************

0 ............................. Current Model .............................

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.                              .
.
.                          Coefficients  Innovation Variance
.                          I   A(I)  SD =       .3907163636D-02
.                          1   1.2818947339
.                          2  -.2907320183
.
.
. This model was fitted to the data ( X( 211), ... ,X( 260) ).

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .9121D-02 AICS = -502.107
Constant model: (NP = 100) MP = 1 SDP = .6883D-02 AICP = -493.865

35
******************************************************************************

******          ******
****** NEW MODEL ADOPTED  ******
******          ******
******************************************************************************

0 ................................ Current Model .................................
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.
Coefficients  Innovation Variance
.
  I     A(I)       SD = .9120822507D-02
.
  1     .9143075460
.
.
This model was fitted to the data ( X( 261), ..., X( 310) ).
******************************************************************************

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 3 SDS = .4429D-02 AICS = -493.841
Constant model: (NP = 100) MP = 1 SDP = .7222D-02 AICP = -489.057

******************************************************************************

******          ******
****** NEW MODEL ADOPTED  ******
******          ******
******************************************************************************
0 ........................ Current Model ........................

  .
  .
  .
  .
  .
  .
  .

Coefficients  Innovation Variance

I A(I) SD = 0.4428769764D-02
1 0.9948522896
2 -0.5377818677
3 0.5066074625

This model was fitted to the data (X(311), ..., X(360)).

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = 0.2201D-01 AICS = -449.804
Constant model: (NP = 100) MP = 4 SDP = 0.1229D-01 AICP = -429.889

0*****************************0*****************************
*****                         *****
*****  NEW MODEL ADOPTED      *****
*****                         *****
0*****************************0*****************************
Basic Autoregressive Model

\[ X(I) = A(1) \times X(I-1) + A(2) \times X(I-2) + \ldots + A(M) \times X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .4510D-02 AICS = -452.893
Constant model: (NP = 100) MP = 4 SDP = .1213D-01 AICP = -431.187

O******************************

*****

***** NEW MODEL ADOPTED *****

*****

O******************************

0 ......... Current Model ..............

\[ \begin{array}{ccc}
\text{Coefficients} & \text{Innovation Variance} \\
I & A(I) & SD \\
1 & .9276658174 \\
\end{array} \]

This model was fitted to the data (X(411), ..., X(460))

Basic Autoregressive Model

\[ X(I) = A(1) \times X(I-1) + A(2) \times X(I-2) + \ldots + A(M) \times X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .3415D-02 AICS = -546.046
Constant model: (NP = 100) MP = 1 SDP = .4013D-02 AICP = -547.810

38
0**** Constant model adopted ****

0 .......................... Current Model ............................
  .
  .
  .
  .
  Coefficients         Innovation Variance .
  .  I   A(I)           SD = .4013470944E-02 .
  .  1   .9856813910 .
  .
  . This model was fitted to the data ( X( 411), ... ,X( 510) ) .

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + ... + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 1 SDS = .31500-02 AICS = -831.822
Constant model: (NP = 150) MP = 1 SDP = .37261-02 AICP = -834.858
0**** Constant model adopted ****

0 .......................... Current Model ............................
  .
  .
  .
  .
  Coefficients         Innovation Variance .
  .  I   A(I)           SD = .3726129391E-02 .
  .  1   .9847138189 .
  .
  . This model was fitted to the data ( X( 411), ... ,X( 560) ) .

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + ... + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
The following two models are compared

Moving model: (NF = 150, NS = 50) MS = 6 SDS = .2206D-02 AICS = -1126.695
Constant model: (NP = 200) MP = 4 SDP = .3421D-02 AICP = -1125.586

************
***** NEW MODEL ADOPTED *****
************

Current Model

<table>
<thead>
<tr>
<th>I</th>
<th>A(I)</th>
<th>Innovation Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8675696810</td>
<td>.2205635344D-02</td>
</tr>
<tr>
<td>2</td>
<td>0.3558243791</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.0454105580</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-0.3029665955</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-1.064942638</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.2607045555</td>
<td></td>
</tr>
</tbody>
</table>

This model was fitted to the data (X(561), ..., X(610)).

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

The following two models are compared

Moving model: (NF = 50, NS = 50) MS = 1 SDS = .2219D-02 AICS = -593.375
Constant model: (NP = 100) MP = 1 SDP = .2658D-02 AICP = -589.027
O*************************************************************
****            ****
**** NEW MODEL ADOPTED         ****
****            ****
*************************************************************

O  ......................  Current Model  .........................
    .
    .
    .
    .
    .  Coefficients  Innovation Variance  .
    .  I  A(I)  SD =  .2218880803D-02  .
    .  1  .9767738533  .
    .
    . This model was fitted to the data  ( X(611), ..., X(660) )  .

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M:  Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model:  (NF = 50, NS = 50)  MS = 1  SDS = .4677D-02  AICS = -565.791
Constant model: (NP = 100)  MP = 1  SDP = .3634D-02  AICP = -557.749

O*************************************************************
****            ****
**** NEW MODEL ADOPTED         ****
****            ****
*************************************************************
0 ................................. Current Model .................................

Coefficients          Innovation Variance
I         A(I)        SD =  .467716497D-02
1         1.011546157

This model was fitted to the data ( X(  661), ... ,X(  710) )

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 3 SDS = .3305D-02 AICS = -541.862
Constant model: (NP = 100) MP = 3 SDP = .3975D-02 AICP = -544.777
0***** Constant model adopted *****

0 ................................. Current Model .................................

Coefficients          Innovation Variance
I         A(I)        SD =  .3974842718D-02
1         1.0945675756
2         -.3202662468
3         .2367242031

This model was fitted to the data ( X(  661), ... ,X(  760) )

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
The following two models are compared

**Moving model:** \((NF = 100, \ NS = 50)\)  \(MS = 5\)  \(SDS = .6751D-02\)  \(AICS = -782.677\)

**Constant model:** \((NP = 150)\)  \(MP = 1\)  \(SDP = .5881D-02\)  \(AICP = -766.416\)

**NEW MODEL ADOPTED**

This model was fitted to the data \((X(761), \ldots, X(810))\).

---

**Basic Autoregressive Model**

\[ X(I) = A(1)*X(I-1) + A(2)*X(I-2) + \ldots + A(M)*X(I-M) + E(I) \]

Where

- **M:** Order of the model
- **E(I):** Gaussian white noise with 0 mean and SD(M) variance

---

The following two models are compared

**Moving model:** \((NF = 50, \ NS = 50)\)  \(MS = 1\)  \(SDS = .7967D-02\)  \(AICS = -475.521\)

**Constant model:** \((NP = 100)\)  \(MP = 1\)  \(SDP = .8541D-02\)  \(AICP = -472.291\)
O**********************************************************************
*****       *****
*****   NEW MODEL ADOPTED   *****
*****       *****
************************************************************************

0 ................................ Current Model ............................
.
.
.
.
.        Coefficients     Innovation Variance  .
.        I       A(I)     SD = .7967155081D-02  .
.        1        .9107487291  .
.
. This model was fitted to the data ( X( 811), ... ,X( 860) )  .

--------------------------------------------------------------------------
Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M:   Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model:  (NF = 50, NS = 50)  MS = 10  SDS = .5032D-02  AICS = -480.213
Constant model: (NP = 100)       MP = 1  SDP = .7868D-02  AICP = -480.498
O***** Constant model adopted *****

O ................................ Current Model ............................
.
.
.
.
.        Coefficients     Innovation Variance  .
.        I       A(I)     SD = .7867754870D-02  .
.        1        .9415110814  .
.
. This model was fitted to the data ( X( 811), ... ,X( 910) )  .

--------------------------------------------------------------------------
Basic Autoregressive Model

44
\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

\( M \): Order of the model

\( E(I) \): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---

OMoving model: (NF = 100, NS = 50) MS = 3 SDS = .2447D-02 AICS = -773.152

Constant model: (NP = 150) MP = 1 SDP = .6179D-02 AICP = -758.990

*********************************************************

***** NEW MODEL ADOPTED *****

*********************************************************

Current Model

. Coefficients Innovation Variance .
. I  A(I)  SD = .244656769D-02 .
. 1  1.0806702410 .
. 2  -4640445237 .
. 3  .2478232619 .

This model was fitted to the data ( X( 911), \ldots ,X( 960) ) .
UK bond yield daily 1 April - 29 December 1989 Series

Title --> BONDUK

No. of data used --> 960
Maximum order of AR model --> 10
Length of basic local span --> 50
Parameter KSW --> 0

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0Initial local model: NS = 50 MS = 1 SDS = .67356D-02 AICS = -246.017

0 ................................ Current Model .................................
..............................................................
..............................................................
..............................................................
..............................................................

Coefficients Innovation Variance

\[ \begin{array}{lll}
I & A(I) & SD = .6735632579D-02 \\
1 & .9980234744 & \\
\end{array} \]

..............................................................

This model was fitted to the data (X(11), ..., X(60)).

..............................................................

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .4241D-02 AICS = -515.160
Constant model: (NP = 100) MP = 1 SDP = .5555D-02 AICP = -515.299
O***** Constant model adopted *****

0 .............................. Current Model ..............................

. ............................
. ............................
. ............................
. Coefficients           Innovation Variance
. I          A(I)      SD = .5555377255D-02
. 1          .9939921151

. This model was fitted to the data ( X(11), ... , X(110) ).

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared --- OMoving model:
(NF = 100, NS = 50) MS = 5 SDS = .1282D-01 AICS = -721.117
Constant model: (NP = 150) MP = 8 SDP = .7760D-02 AICP = -710.825

O**************************
*****                     *****
***** NEW MODEL ADOPTED    *****
*****                     *****
************************************************************************
0 ........................ Current Model ..............................

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Innovation Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I A(I)</td>
<td>SD = .1282494656D-01</td>
</tr>
<tr>
<td>1 1.1133009149</td>
<td></td>
</tr>
<tr>
<td>2 -.1739047004</td>
<td></td>
</tr>
<tr>
<td>3 .2372752394</td>
<td></td>
</tr>
<tr>
<td>4 .1271409361</td>
<td></td>
</tr>
<tr>
<td>5 -.3135832495</td>
<td></td>
</tr>
</tbody>
</table>

This model was fitted to the data ( X( 111), ... ,X( 160) ).

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .4476D-02 AICS = -472.270
Constant model: (NP = 100) MP = 6 SDP = .8674D-02 AICP = -460.743

0******************************
*****
***** NEW MODEL ADOPTED *****
*****
******************************

0 ........................ Current Model ..............................

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Innovation Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I A(I)</td>
<td>SD = .4475980942D-02</td>
</tr>
<tr>
<td>1 0.9766990573</td>
<td></td>
</tr>
</tbody>
</table>

This model was fitted to the data ( X( 161), ... ,X( 210) ).

Basic Autoregressive Model
\[
X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I)
\]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 10 SDS = 0.6558D-02 AICS = -495.807
Constant model: (NP = 100) MP = 3 SDP = 0.7019D-02 AICP = -487.920

*NEW MODEL ADOPTED*

<table>
<thead>
<tr>
<th>I</th>
<th>A(I)</th>
<th>Innovation Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3520613156</td>
<td>0.6557776025D-02</td>
</tr>
<tr>
<td>2</td>
<td>-0.3910405942</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.3685640625</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-0.7185330099</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.7007781741</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-0.6916839127</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-0.7582497334</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-0.6190429380</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-0.5253373025</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-0.2968507686</td>
<td></td>
</tr>
</tbody>
</table>

This model was fitted to the data (X(211), \ldots ,X(260)).

Basic Autoregressive Model

\[
X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I)
\]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
The following two models are compared

OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .8541D-02 AICS = -461.497
Constant model: (NP = 100) MP = 2 SDP = .9107D-02 AICP = -463.867

Constant model adopted

-----------------------

Current Model

Coefficients Innovation Variance
1 A(1) SD = .9107415758D-02
1.1966703563
2 -1.1975618333

This model was fitted to the data (X(211), ..., X(310))

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

The following two models are compared

OMoving model: (NF = 100, NS = 50) MS = 1 SDS = .1425D-01 AICS = -672.421
Constant model: (NP = 150) MP = 2 SDP = .1100D-01 AICP = -670.423

NEW MODEL ADOPTED
Current Model

Coefficients    Innovation Variance
I    A(I)    SD = .1424875424D-01
1    .9710230618

This model was fitted to the data ( X( 311), ... ,X( 360) ).

Basic Autoregressive Model

\[ X(I) = A(1) \times X(I-1) + A(2) \times X(I-2) + \ldots + A(M) \times X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .2702D-01 AICS = -385.117
Constant model: (NP = 100) MP = 1 SDP = .2082D-01 AICP = -383.170

NEW MODEL ADOPTED

Current Model

Coefficients    Innovation Variance
I    A(I)    SD = .2701767717D-01
1    1.0112190602

This model was fitted to the data ( X( 361), ... ,X( 410) ).

Basic Autoregressive Model
\[ X(I) = A(1) \times X(I-1) + A(2) \times X(I-2) + \ldots + A(M) \times X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .6710D-02 AICS = -422.768
Constant model: (NP = 100) \[ MP = 6 \quad SDP = .1505D-01 \quad AICP = -405.620 \]

--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 3 SDS = .5485D-02 AICS = -498.488
Constant model: (NP = 100) \[ MP = 1 \quad SDP = .6456D-02 \quad AICP = -500.272 \]

--- Constant model adopted ---
Basic Autoregressive Model

\[ X(I) = A(1) \times X(I-1) + A(2) \times X(I-2) + \ldots + A(M) \times X(I-M) + E(I) \]

Where

- \( M \): Order of the model
- \( E(I) \): Gaussian white noise with 0 mean and SD(M) variance

---

The following two models are compared ---

OMoving model: (NF = 100, NS = 50) MS = 2 SDS = .6440D-02 AICS = -746.532
Constant model: (NP = 150) MP = 1 SDP = .6549D-02 AICP = -750.264

**** Constant model adopted *****

Basic Autoregressive Model

\[ X(I) = A(1) \times X(I-1) + A(2) \times X(I-2) + \ldots + A(M) \times X(I-M) + E(I) \]

Where

- \( M \): Order of the model
- \( E(I) \): Gaussian white noise with 0 mean and SD(M) variance
The following two models are compared

OMoving model: (NF = 150, NS = 50) MS = 1 SDS = .8217D-02 AICS = -986.339
Constant model: (NP = 200) MP = 1 SDP = .7230D-02 AICP = -981.903

*******************************
***** NEW MODEL ADOPTED *****
*******************************

Current Model

... This model was fitted to the data (X(561), ..., X(610))

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

The following two models are compared

OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .5091D-02 AICS = -494.088
Constant model: (NP = 100) MP = 1 SDP = .7011D-02 AICP = -492.022

*******************************
***** NEW MODEL ADOPTED *****
*******************************
0 ........................ Current Model ........................

.  
.  
.  
.  
.  Coefficients  Innovation Variance 
.  I  A(I)  SD  =  .5091147733D-02 
.  1 1.2067549224  
.  2 -.2359478897  
.  
.  This model was fitted to the data ( X( 611), ... ,X( 660) ) . 

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I) 

Where  
M:  Order of the model  
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .7747D-02 AICS = -495.034 
Constant model: (NP = 100) MP = 2 SDP = .6488D-02 AICP = -497.783
0**** Constant model adopted ****

0 ........................ Current Model ........................

.  
.  
.  
.  
.  Coefficients  Innovation Variance 
.  I  A(I)  SD  =  .6487807469D-02 
.  1 1.2748744410  
.  2 -.2803526764  
.  
.  This model was fitted to the data ( X( 611), ... ,X( 710) ) . 

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I) 

Where  
M:  Order of the model  
E(I): Gaussian white noise with 0 mean and SD(M) variance
The following two models are compared

OMoving model: \((NF = 100, \ NS = 50)\)  \(MS = 1\)  \(SDS = .6117D-02\)  \(AICS = -748.615\)

Constant model: \((NP = 150)\)  \(MP = 2\)  \(SDP = .6419D-02\)  \(AICP = -751.285\)

Constant model adopted

Current Model

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Innovation Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I (A(I))</td>
<td>SD = .6418515869D-02</td>
</tr>
<tr>
<td>1 1.2126915027</td>
<td></td>
</tr>
<tr>
<td>2 -2.203504876</td>
<td></td>
</tr>
</tbody>
</table>

This model was fitted to the data \((X(\ 611), \ldots, X(\ 760)\) .

Basic Autoregressive Model

\[X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I)\]

Where

- \(M\): Order of the model
- \(E(I)\): Gaussian white noise with 0 mean and SD(M) variance

The following two models are compared

OMoving model: \((NF = 150, \ NS = 50)\)  \(MS = 1\)  \(SDS = .7963D-02\)  \(AICS = -988.931\)

Constant model: \((NP = 200)\)  \(MP = 2\)  \(SDP = .6861D-02\)  \(AICP = -990.383\)

Constant model adopted

Current Model

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Innovation Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I (A(I))</td>
<td>SD = .6860916360D-02</td>
</tr>
<tr>
<td>1 1.1813645389</td>
<td></td>
</tr>
<tr>
<td>2 -1.811811690</td>
<td></td>
</tr>
</tbody>
</table>

This model was fitted to the data \((X(\ 611), \ldots, X(\ 810)\) .

Basic Autoregressive Model
\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

---

The following two models are compared

OMoving model: (NF = 200, NS = 50) MS = 3 SDS = .7045D-02 AICS =-1230.155
Constant model: (NP = 250) MP = 2 SDP = .7121D-02 AICP =-1230.170

Constant model adopted

---

Current Model

<table>
<thead>
<tr>
<th>I</th>
<th>A(I)</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.183615083</td>
<td>.7121182891D-02</td>
</tr>
<tr>
<td>2</td>
<td>-.1882209690</td>
<td></td>
</tr>
</tbody>
</table>

This model was fitted to the data (X(611), \ldots, X(860))

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

---

The following two models are compared

OMoving model: (NF = 250, NS = 50) MS = 1 SDS = .9314D-02 AICS =-1459.981
Constant model: (NP = 300) MP = 2 SDP = .7582D-02 AICP =-1458.583

NEW MODEL ADOPTED

---

57
Basic Autoregressive Model

\[ X(I) = A(1) \times X(I-1) + A(2) \times X(I-2) + \ldots + A(M) \times X(I-M) + E(I) \]

Where
- \( M \): Order of the model
- \( E(I) \): Gaussian white noise with 0 mean and \( SD(M) \) variance

--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .5741D-02 AICS = -483.817
Constant model: (NP = 100) MP = 1 SDP = .7591D-02 AICP = -484.085

O**** Constant model adopted ****

---
West Germany bond yield daily 1 April - 29 December 1989 Series

Title ---> BONDWG

No. of data used ---> 960
Maximum order of AR model ---> 10
Length of basic local span ---> 50
Parameter KSW ---> 0
Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

Initial local model: \( NS = 50 \ MS = 1 \ SDS = .11843D-02 \ AICS = -332.931 \)

Current Model

<table>
<thead>
<tr>
<th>I</th>
<th>A(I)</th>
<th>Innovation Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9834050874</td>
<td>.1184275481D-02</td>
</tr>
</tbody>
</table>

This model was fitted to the data \( X(11), \ldots, X(60) \).

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
The following two models are compared
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .2463D-03 AICS = -744.373
Constant model: (NP = 100) MP = 7 SDP = .6443D-03 AICP = -718.733

O***********************
*****
***** NEW MODEL ADOPTED *****
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********************************

O .......................... Current Model ..............................

. .
. .
. .
. I   A(I)             SD = .2463315733D-03 .
. 1   1.0144169954

. This model was fitted to the data ( X( 61), ... ,X( 110) ) .
************************************************************

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

The following two models are compared
OMoving model: (NF = 50, NS = 50) MS = 3 SDS = .5847D-03 AICS = -775.662
Constant model: (NP = 100) MP = 6 SDP = .4818D-03 AICP = -749.802

O***********************
*****
***** NEW MODEL ADOPTED *****
*****
********************************

60
Current Model

Coefficients  Innovation Variance
I  A(I)  SD = .5847050198D-03
1  .6287258746
2  -.1058545691
3  .4519342190

This model was fitted to the data (X(111), ..., X(160)).

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50)  MS = 2  SDS = .8770D-03  AICS = -710.172
Constant model: (NP = 100)  MP = 3  SDP = .8448D-03  AICP = -699.640

*************************************************************
*****  NEW MODEL ADOPTED  *****
*************************************************************

Current Model

Coefficients  Innovation Variance
I  A(I)  SD = .8769738156D-03
1  .7166081950
2  .3005653564

This model was fitted to the data (X(161), ..., X(210)).

Basic Autoregressive Model
\( X(I) = A(1) \cdot X(I-1) + A(2) \cdot X(I-2) + \ldots + A(M) \cdot X(I-M) + E(I) \)

Where

\( M \): Order of the model

\( E(I) \): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .6172D-03 AICS = -709.469
Constant model: (NP = 100) MP = 1 SDP = .8175D-03 AICP = -706.924

O************

*****

***** NEW MODEL ADOPTED *****

*****

O************

O .......................... Current Model ..........................

\begin{tabular}{|c|c|}
\hline
\textbf{I} & \textbf{A(I)} & \textbf{Innovation Variance} \\
\hline
1 & 1.2877193008 & \\
2 & -.283314395 & \\
\hline
\end{tabular}

This model was fitted to the data \( (X(211), \ldots, X(260)) \).

Basic Autoregressive Model

\( X(I) = A(1) \cdot X(I-1) + A(2) \cdot X(I-2) + \ldots + A(M) \cdot X(I-M) + E(I) \)

Where

\( M \): Order of the model

\( E(I) \): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 3 SDS = .6848D-03 AICS = -719.838
Constant model: (NP = 100) MP = 3 SDP = .6746D-03 AICP = -722.142
O***** Constant model adopted *****

62
0 ........................ Current Model ........................

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. Coefficients         Innovation Variance  

. I     A(I)         SD = .6745806098D-03  

. 1    1.3551472794         .  

. 2    -.4967597080         .  

. 3    .1430579047         .  

.  

This model was fitted to the data (X( 211), ... ,X( 310)) .

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

OMoving model: (NF = 100, NS = 50) MS = 2 SDS = .4591D-03 AICS =-1100.451
Constant model: (NP = 150) MP = 2 SDP = .6234D-03 AICP =-1101.042
O***** Constant model adopted *****

0 ........................ Current Model ........................

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. Coefficients         Innovation Variance  

. I     A(I)         SD = .6234261030D-03  

. 1    1.3435457615         .  

. 2    -.3448985578         .  

.  

This model was fitted to the data (X( 211), ... ,X( 360)) .

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model: (NF = 150, NS = 50) MS = 10 SDS = .2315D-02 AICS =-1382.457
Constant model: (NP = 200) MP = 10 SDP = .1246D-02 AICP =-1315.514

O******************************************************************************
*****                             *****
*****      NEW MODEL ADOPTED      *****
*****                             *****
******************************************************************************

O ........................ Current Model .................................

                        Coefficients      Innovation Variance
                        I      A(I)         SD = .2315107491D-02

   1  1.4643241027
   2 -.7576640551
   3 .4993084323
   4 -.2293942647
   5 .3238658328
   6 -.3702638391
   7 .0984628629
   8 .1550519331
   9 -.7996350070
  10 .579883548

This model was fitted to the data ( X( 361), ... ,X( 410) ).

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
The following two models are compared:

**OMoving model:** \( NF = 50, \ NS = 50 \) \( MS = 2 \ SD = .7614D-03 \ AICS = -634.435 \)

**Constant model:** \( NP = 100 \) \( MP = 10 \ SDP = .1852D-02 \ AICP = -607.173 \)

NEW MODEL ADOPTED

---

Current Model

.................

Coefficients Innovation Variance

1  1.3437469987

2  -.3406983869

This model was fitted to the data \( X(411), \ldots, X(460) \).

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

\( M \): Order of the model

\( E(I) \): Gaussian white noise with 0 mean and SD(M) variance

---

The following two models are compared:

**OMoving model:** \( NF = 50, \ NS = 50 \) \( MS = 1 \ SD = .6149D-03 \ AICS = -718.725 \)

**Constant model:** \( NP = 100 \) \( MP = 1 \ SDP = .7374D-03 \ AICP = -717.234 \)

NEW MODEL ADOPTED

---
Current Model

Coefficients | Innovation Variance
---|---
I A(I) SD = \( .6148780173D-03 \)
1 1.0032922526

This model was fitted to the data \( X(461), \ldots, X(510) \).

Basic Autoregressive Model

\[
X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I)
\]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and \( SD(M) \) variance

--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1623D-02 AICS = -682.889
Constant model: (NP = 100) MP = 1 SDP = .1182D-02 AICP = -670.034

NEW MODEL ADOPTED

Current Model

Coefficients | Innovation Variance
---|---
I A(I) SD = \( .1622664722D-02 \)
1 .9841365294

This model was fitted to the data \( X(511), \ldots, X(560) \).

Basic Autoregressive Model

\[
X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I)
\]

66
Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 3 SDS = .1916D-02 AICS = -622.069
Constant model: (NP = 100) MP = 1 SDP = .1853D-02 AICP = -625.117
O***** Constant model adopted *****

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0 .......................... Current Model ..........................

Coefficients             Innovation Variance

I  A(I)  SD = .1100013699D-02
1 .9443502949

This model was fitted to the data ( X( 611), ... ,X( 660) )

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50)  MS = 2  SDS = .9120D-03  AICS = -680.615
Constant model: (NP = 100)         MP = 2  SDP = .1028D-02  AICP = -682.053
O***** Constant model adopted *****

0 .......................... Current Model ..........................

Coefficients             Innovation Variance

I  A(I)  SD = .1027602730D-02
1 1.2904473838
2 -.3071816898

This model was fitted to the data ( X( 611), ... ,X( 710) )

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 8 SDS = .1503D-02 AICS = -989.072
Constant model: (NP = 150) MP = 8 SDP = .1281D-02 AICP = -981.031

O*******************************
    *****
    ***** NEW MODEL ADOPTED  *****
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O ........................... Current Model ...........................

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   Coefficients               Innovation Variance
   .
   I   A(I)          SD = .1502847926D-02
   .
   1  1.6207701926
   .
   2 -.7285185127
   .
   3 .1314636948
   .
   4 -.5241078468
   .
   5  1.1058045429
   .
   6 -.9397992070
   .
   7  .6064463859
   .
   8 -.2704506142
   .

   This model was fitted to the data ( X( 711), ... ,X( 760) )

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .6615D-03 AICS = -669.072
Constant model: (NP = 100) MP = 6 SDP = .1303D-02 AICP = -650.282
************
*****  NEW MODEL ADOPTED  *****
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O ........................ Current Model ........................

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Coefficients         Innovation Variance
.
  I     A(I)     SD = .6614708593D-03
  1     1.0036801016
.
.
This model was fitted to the data ( X( 761), ... ,X( 810) )

----------------------------------------------

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M:    Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .4668D-03 AICS = -741.533
  Constant model: (NP = 100)   MP = 4  SDP = .5311D-03 AICP = -744.052
O***** Constant model adopted *****

O ........................ Current Model ........................

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Coefficients         Innovation Variance
.
  I     A(I)     SD = .5311195171D-03
  1     1.0797625809
  2     .0493674008
  3     .1413924465
  4     -.2713690251
.
.
This model was fitted to the data ( X( 761), ... ,X( 860) )

----------------------------------------------

Basic Autoregressive Model

70
\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model:  (NF = 100, NS = 50)  MS = 8  SDS = .8387D-03  AICS = -1080.238
Constant model: (NP = 150)  MP = 4  SDP = .8349D-03  AICP = -1053.227

O**************************************
*****
*****  NEW MODEL ADOPTED  *****
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O*******************************

0 ...............................  Current Model  ..........................
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Coefficients  Innovation Variance
.
1  .6224985571
2  .2060069357
3  -.0983751496
4  .1166707613
5  -.3172579043
6  -.0550672288
7  .2350280058
8  .3244465187
.
.
This model was fitted to the data  \( X(861), \ldots, X(910) \).

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

71
The following two models are compared

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1475D-02 AICS = -658.149
Constant model: (NP = 100) MP = 1 SDP = .1412D-02 AICP = -652.248

***********************************************************
*****      *****
***** NEW MODEL ADOPTED      *****
*****      *****
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............... Current Model .........................

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............... 

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............... 

............... Coefficients Innovation Variance 

............... 

............... I A(I) SD = .1474723240D-02 

............... 

............... 1 1.0021982427 

............... 

............... This model was fitted to the data ( X( 911), ... ,X( 960) ) 

.........................................................
Japan bond yield daily 1 April - 29 December 1989 Series

Title ---> BONDJP

No. of data used ---> 960
Maximum order of AR model ---> 10
Length of basic local span ---> 50
Parameter KSW ---> 0

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0Initial local model:   NS = 50   MS = 2   SDS = .72540D-03   AICS = -355.439

0  ..............................  Current Model  ..............................

.  ...........................................

.  ...........................................

.  Coefficients  Innovation Variance

.  I  A(I)  SD = .7254000779D-03

.  1  1.4898167733

.  2  -.4886664694

.  This model was fitted to the data \(X(11), \ldots, X(60)\).

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
The following two models are compared

OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .7592D-03 AICS = -708.601
Constant model: (NP = 100) MP = 3 SDP = .8707D-03 AICP = -696.625

******************************************************
*****          NEW MODEL ADOPTED          ******
**************************************************************

0  ................. Current Model  .........................

.  
.  
.  
.  Coefficients  Innovation Variance  
.  I  A(I)  SD = .7592170885D-03  
.  1  .6753691505  
.  2  .3025671994  
.  
.  This model was fitted to the data (X(61), ..., X(110)).

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

The following two models are compared

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1724D-02 AICS = -667.315
Constant model: (NP = 100) MP = 2 SDP = .1228D-02 AICP = -664.205

******************************************************
*****          NEW MODEL ADOPTED          ******
**************************************************************

74
Current Model

Coefficients  Innovation Variance
I  A(I)  SD = .1724037498D-02
1  .9792729084

This model was fitted to the data (X(111), ..., X(160))

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

The following two models are compared

OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .3255D-03 AICS = -709.661
Constant model: (NP = 100) MP = 1 SDP = .1132D-02 AICP = -674.371

NEW MODEL ADOPTED

Current Model

Coefficients  Innovation Variance
I  A(I)  SD = .3255050749D-03
1  1.5096599871
2  -.4825622027

This model was fitted to the data (X(161), ..., X(210))

Basic Autoregressive Model
X(I) = A(1)X(I-1) + A(2)X(I-2) + ... + A(M)X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 5 SDS = .1298D-02 AICS = -715.842
Constant model: (NP = 100) MP = 2 SDP = .1056D-02 AICP = -679.344

0*****************************************************************************
***** ****
***** NEW MODEL ADOPTED  *****
***** ****
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0 ............................ Current Model .................................

. 
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. 
. Coefficients Innovation Variance .
. I A(I) SD = .1298280204D-02 .
. 1 .5425737917 .
. 2 .2986217667 .
. 3 -.1395631686 .
. 4 .0538871910 .
. 5 .283992318 .
.
.
. This model was fitted to the data ( X( 211), ... ,X( 260) ) .

Basic Autoregressive Model

X(I) = A(1)X(I-1) + A(2)X(I-2) + ... + A(M)X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model:  (NF = 50, NS = 50)  MS = 2  SDS = .1170D-02  AICS = -651.893
Constant model: (NP = 100)  MP = 1  SDP = .1571D-02  AICP = -641.612

O******************************************************************************
*****                    *****
*****   NEW MODEL ADOPTED     *****
*****                    *****
******************************************************************************

0                      Current Model                      0
                        .
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                        .
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                        .
                        .
                        .
Coefficients  Innovation Variance
   I      A(I)     SD = .1169530509D-02
   1     1.2859301480
   2    -.2891679558

This model was fitted to the data ( X( 261), ... ,X( 310) ) .

-------------------------------------------------------------------
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M:  Order of the model
E(I):  Gaussian white noise with 0 mean and SD(M) variance

O---  The following two models are compared  ---
OMoving model:  (NF = 50, NS = 50)  MS = 1  SDS = .2244D-02  AICS = -632.534
Constant model: (NP = 100)  MP = 3  SDP = .1761D-02  AICP = -626.167

O******************************************************************************
*****                    *****
*****   NEW MODEL ADOPTED     *****
*****                    *****
******************************************************************************
0 .......................... Current Model ..........................

. ..........................

. ..........................

. Coefficients  Innovation Variance .
. I  A(I)  SD =  .2243929721D-02 .
. 1  .9579703050 .

. This model was fitted to the data ( X( 311), ... ,X( 360) ) .

---------------------------------------------------------------
Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared --
OMoving model: (NF = 50, NS = 50)  MS = 8  SDS = .6083D-02  AICS = -538.086
Constant model: (NP = 100)  MP = 4  SDP = .5253D-02  AICP = -514.901

O************************************************************
***** NEW MODEL ADOPTED *****
***** ************************************************************

O .......................... Current Model ..........................

. ..........................

. ..........................

. Coefficients  Innovation Variance .
. I  A(I)  SD =  .6083408498D-02 .
. 1  .9272812516 .
. 2  .4219626062 .
. 3  -.1921564440 .
. 4  -.4313933505 .
. 5  .1544657128 .
. 6  .1599322419 .
. 7  .5455013413 .
. 8  -.6428171162 .

. This model was fitted to the data ( X( 361), ... ,X( 410) ) .

---------------------------------------------------------------
Basic Autoregressive Model

78
\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

\( M \): Order of the model

\( E(I) \): Gaussian white noise with 0 mean and SD(M) variance

---

***The following two models are compared***

**Moving model:** (NF = 50, NS = 50) MS = 8 SDS = .1934D-03 AICS = -646.656

**Constant model:** (NP = 100) MP = 8 SDP = .3775D-02 AICP = -539.929

---

***NEW MODEL ADOPTED***

---

Current Model

\[
\begin{array}{ccc}
I & A(I) & \text{Innovation Variance} \\
1 & 1.1224340817 & .1933661209D-03 \\
2 & -.3124072805 & \\
3 & .0477740883 & \\
4 & .1070076141 & \\
5 & .0222366888 & \\
6 & -.0351555439 & \\
7 & -.0126970020 & \\
8 & .0825312850 & \\
\end{array}
\]

This model was fitted to the data \((X(411), \ldots, X(460))\).

---

**Basic Autoregressive Model**

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

\( M \): Order of the model

\( E(I) \): Gaussian white noise with 0 mean and SD(M) variance
The following two models are compared:

**OMoving model:** (NF = 50, NS = 50)  MS = 1  SDS = 0.1678D-02  AICS = -725.048
Constant model: (NP = 100)  MP = 1  SDP = 0.9871D-03  AICP = -688.072

************
***** NEW MODEL ADOPTED  *****
************

0 ...........................  Current Model  ...........................

. . .
. . .
. .
. Coefficients  Innovation Variance .
. 1  A(I)  SD  =  0.1678213507D-02 .
. 1  1.0108015295 .
. .
.
. This model was fitted to the data ( X(461), ..., X(510) ).

Basic Autoregressive Model

\[ X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I) \]

Where
- **M:** Order of the model
- **E(I):** Gaussian white noise with 0 mean and SD(M) variance

The following two models are compared:

**OMoving model:** (NF = 50, NS = 50)  MS = 2  SDS = 0.2839D-03  AICS = -717.851
Constant model: (NP = 100)  MP = 2  SDP = 0.1037D-02  AICP = -681.174

************
***** NEW MODEL ADOPTED  *****
************

80
0 .......................... Current Model ..........................
   .
   .
   .
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   .
   .

   Coefficients      Innovation Variance
   .
   I    A(I)      SD =  .2838687468D-03
   .
   1    1.5001070959
   .
   2    -.5054635193
   .

   This model was fitted to the data ( X( 511), ... ,X( 560) ).

---------------------------------------------------------------

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model: (NP = 50, NS = 50) MS = 1 SDS = .1825D-02 AICS = -713.654
Constant model: (NP = 100) MP = 1 SDP = .1206D-02 AICP = -668.048

O*************************************************************************

*****  *****
***** NEW MODEL ADOPTED  *****
*****  *****
*************************************************************************

0 .......................... Current Model ..........................
   .
   .
   .
   .
   .
   .

   Coefficients      Innovation Variance
   .
   I    A(I)      SD =  .1825154463D-02
   .
   1    .9117581309
   .

   This model was fitted to the data ( X( 561), ... ,X( 610) ).

---------------------------------------------------------------

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

81
Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1404D-02 AICS = -635.736
Constant model: (NP = 100) MP = 1 SDP = .1707D-02 AICP = -633.284

***** NEW MODEL ADOPTED *****

0 ........................................ Current Model ........................................
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Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .3192D-03 AICS = -722.918
Constant model: (NP = 100) MP = 1 SDP = .8632D-03 AICP = -701.492
O****************************************************************************************
*****     *****
*****     NEW MODEL ADOPTED     *****
*****     *****
****************************************************************************************

O                      Current Model                      ...........
                        .
                        .
                        .
                      Coefficients  Innovation Variance .
                      I     A(I)  SD =  .3191888529D-03 .
                      1     1.0037259143 .
                     .
                      This model was fitted to the data ( X(  661), ... ,X(  710) ) .
---------------------------------------------------------------------------------------------------------------------

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

    Where

    M:   Order of the model
    E(I): Gaussian white noise with 0 mean and SD(M) variance


O---  The following two models are compared  ---

OMoving model: (NF = 50, NS = 50) MS = 3  SDS = .8431D-03  AICS = -744.406
Constant model: (NP = 100)        MP = 6  SDP = .5684D-03  AICP = -733.273

O****************************************************************************************
*****     *****
*****     NEW MODEL ADOPTED     *****
*****     *****
****************************************************************************************

83
0 ........................... Current Model .................................
  
  Coefficients               Innovation Variance
  I       A(I)            SD = .8431351199D-03
  .
  1       1.0935788532 .
  .
  2       .2799304014 .
  .
  3       -.3895814826 .
  .
  This model was fitted to the data ( X( 711), ... ,X( 760) ) .
  
Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared --- OMoving model:
(NF = 50, NS = 50) MS = 6 SDS = .1399D-03 AICS = -775.652
Constant model: (NP = 100) MP = 6 SDP = .5033D-03 AICP = -745.436

0***********************
     *****
     ***** NEW MODEL ADOPTED     *****
     *****
***********************

0 ........................... Current Model .................................
  
  Coefficients               Innovation Variance
  I       A(I)            SD = .1398907373D-03
  .
  1       .9640741939 .
  .
  2       -.0376125072 .
  .
  3       .1257255023 .
  .
  4       .7668361222 .
  .
  5       -.4581815884 .
  .
  6       -.3533310149 .
  .
  This model was fitted to the data ( X( 761), ... ,X( 810) ) .
  
Basic Autoregressive Model
\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 3 SDS = .1786D-03 AICS = -853.256
Constant model: (NP = 100) MP = 3 SDP = .2186D-03 AICP = -834.843

NEW MODEL ADOPTED
---

Current Model

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Innovation Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>A(I)</td>
</tr>
<tr>
<td>1</td>
<td>1.2292336061</td>
</tr>
<tr>
<td>2</td>
<td>0.971009491</td>
</tr>
<tr>
<td>3</td>
<td>-.3282494335</td>
</tr>
</tbody>
</table>

This model was fitted to the data (X(811), \ldots, X(860)).

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .5480D-03 AICS = -792.982
Constant model: (NP = 100) MP = 3 SDP = .3760D-03 AICP = -780.593

85
0 ........................ Current Model ........................

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.  Coefficients  Innovation Variance  
.     I     A(I)  SD    =      .5480348919D-03  
.     1      1.3060049544  
.     2     -.2968285390  

.  This model was fitted to the data ( X(  861), ... ,X(  910) )  

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model:  (NF =  50, NS =  50)  MS =  2  SDS = .4132D-03  AICS = -753.037
Constant model:  (NP =  100)  MP =  2  SDP = .4878D-03  AICP = -756.563

0**** Constant model adopted ****

0 ........................ Current Model ........................

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.  Coefficients  Innovation Variance  
.     I     A(I)  SD    =      .4877893866D-03  
.     1      1.3915793141  
.     2     -.3865144088  

.  This model was fitted to the data ( X(  861), ... ,X(  960) )  

..........................................................................................
1 month - tbill monthly 30.01.1926 - 30.12.1996 Series

Title ---> TBILL

No. of data used ---> 2000
Maximum order of AR model ---> 10
Length of basic local span ---> 50
Parameter KSW ---> 0

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0
Initial local model: NS = 50 MS = 1 SDS = .42011D-02 AICS = -269.621

0 .......................... Current Model ..............................
   .
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   .
   .

This model was fitted to the data \( X(11), \ldots , X(60) \).

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

88
OMoving model:  (NF = 50, NS = 50) MS = 1 SDS = .2603D-02 AICS = -563.179
Constant model: (NP = 100) MP = 1 SDP = .3426D-02 AICP = -563.634
O***** Constant model adopted *****

O .......................... Current Model ..........................
  .
  .
  .
  .
  . Coefficients  Innovation Variance .
  .  I  A(I)  SD = .342608054D-02 .
  .  1  .9804021510 .
  .
  . This model was fitted to the data ( X( 11), ... ,X( 110) ) .
  ........................................................................
Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 1 SDS = .1600D-02 AICS = -881.532
Constant model: (NP = 150) MP = 1 SDP = .2819D-02 AICP = -876.699

O******************************
  *****  *****
  ***** NEW MODEL ADOPTED  *****
  *****  *****
  *****************************************

O .......................... Current Model ..........................
  .
  .
  .
  .
  . Coefficients  Innovation Variance .
  .  I  A(I)  SD = .159965572D-02 .
  .  1  .9926502236 .
  .
  . This model was fitted to the data ( X( 111), ... ,X( 160) ) .
  ........................................................................
Basic Autoregressive Model
\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .2965D-02 AICS = -604.947
Constant model: (NP = 100) MP = 1 SDP = .2451D-02 AICP = -597.128

*******************************************************************************
****** NEW MODEL ADOPTED ******
****** ******
*******************************************************************************

          Current Model

                Coefficients       Innovation Variance
                .                .
                .  .
                .  .
                I  A(I)          SD =  .2964711313D-02  .
                1  .8177201100  .

This model was fitted to the data (X( 161), ..., X( 210)).

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 6 SDS = .7829D-02 AICS = -515.546
Constant model: (NP = 100) MP = 2 SDP = .6707D-02 AICP = -494.461
0....................... Current Model ......................

.  .  .
.  .  .  
.  .  .
.
.
. Coefficients      Innovation Variance .
. I  A(I)   SD = .7828794350D-02 .
.  1  .7303433506 .
.  2  -.2006145382 .
.  3  -.3283505685 .
.  4  .1161868098 .
.  5  .0041268626 .
.  6  -.325631923 .
.
. This model was fitted to the data ( X( 211), ... ,X( 260) ).

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M:   Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .4265D-02 AICS = -497.357
Constant model: (NP = 100)         MP = 2 SDP = .8060D-02 AICP = -476.079
0 ..........................  Current Model ..........................
  .
  .
  .
  .
  .
  Coefficients  Innovation Variance  .
  . I   A(I) SD = .4265490095D-02 .
  . 1   .9726952078  .
  .
  .
  .
  This model was fitted to the data ( X( 261), ... ,X( 310) ).

Basic Autoregressive Model

\[ X(I) = A(1)\times X(I-1) + A(2)\times X(I-2) + \ldots + A(M)\times X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .1647D-01 AICS = -468.170
Constant model: (NP = 100) MP = 4 SDP = .9897D-02 AICP = -451.557

0************************************************************
*****  *****
***** NEW MODEL ADOPTED  *****
*****  *****
************************************************************

0 ..........................  Current Model ..........................
  .
  .
  .
  .
  .
  Coefficients  Innovation Variance  .
  . I   A(I) SD = .1647020230D-01 .
  . 1   1.4914838828  .
  . 2   -.5181329165  .
  .
  .
  .
  This model was fitted to the data ( X( 311), ... ,X( 360) ).

Basic Autoregressive Model
\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
\( M \): Order of the model
\( E(I) \): Gaussian white noise with 0 mean and SD(M) variance

---

### The following two models are compared

<table>
<thead>
<tr>
<th>Model Type</th>
<th>( NF )</th>
<th>( NS )</th>
<th>( MS )</th>
<th>SDS</th>
<th>AICS</th>
<th>AICP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moving</td>
<td>50</td>
<td>50</td>
<td>1</td>
<td>0.1636D-01</td>
<td>-400.966</td>
<td>-393.027</td>
</tr>
<tr>
<td>Constant</td>
<td>100</td>
<td></td>
<td>2</td>
<td>0.1849D-01</td>
<td>-393.027</td>
<td>-393.027</td>
</tr>
</tbody>
</table>

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************

***** NEW MODEL ADOPTED *****

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### Current Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
\( M \): Order of the model
\( E(I) \): Gaussian white noise with 0 mean and SD(M) variance

---

### The following two models are compared

<table>
<thead>
<tr>
<th>Model Type</th>
<th>( NF )</th>
<th>( NS )</th>
<th>( MS )</th>
<th>SDS</th>
<th>AICS</th>
<th>AICP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moving</td>
<td>50</td>
<td>50</td>
<td>2</td>
<td>0.2820D-02</td>
<td>-489.204</td>
<td>-459.716</td>
</tr>
<tr>
<td>Constant</td>
<td>100</td>
<td></td>
<td>1</td>
<td>0.9685D-02</td>
<td>-459.716</td>
<td>-459.716</td>
</tr>
</tbody>
</table>
O**********************************
  *****                        *****
  *****         NEW MODEL ADOPTED  *****
  *****                        *****
**********************************

O                      Current Model                      ..............
  .                      .                                  ...
  .                      .                                  ...
  .                      .                                  ...
  .                      Coefficients                  Innovation Variance
  .                      I       A(I)                  SD =   0.2820141750D-02
  .                      1       0.6200490247
  .                      2       0.2326659984
  .                      .                      
  .                      This model was fitted to the data ( X( 411) , ... , X( 460) ) .
-----------------------------------------------------------------------------------------------

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = 0.4185D-02 AICS = -557.355
Constant model: (NP = 100) MP = 1 SDP = 0.3828D-02 AICP = -552.550

O**********************************
  *****                        *****
  *****         NEW MODEL ADOPTED  *****
  *****                        *****
**********************************
Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 4 SDS = .2494D-02 AICS = -559.491
Constant model: (NP = 100) MP = 1 SDP = .3503D-02 AICP = -561.421
O**** Constant model adopted ****

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
The following two models are compared

Moving model: (NF = 100, NS = 50) MS = 3 SDS = .2553D-02 AICS = -851.946
Constant model: (NP = 150) MP = 8 SDP = .2981D-02 AICP = -854.305

Constant model adopted

Current Model

<table>
<thead>
<tr>
<th>I</th>
<th>A(I)</th>
<th>Innovation Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.9854575446</td>
<td>.2981379768D-02</td>
</tr>
<tr>
<td>2</td>
<td>1.608721015</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-.3275966153</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>.1333461090</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>.0956518662</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-.2188706840</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>.0443924516</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>.1372545462</td>
<td></td>
</tr>
</tbody>
</table>

This model was fitted to the data (X(461), ..., X(610))

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

The following two models are compared

Moving model: (NF = 150, NS = 50) MS = 3 SDS = .5432D-02 AICS =-1107.078
Constant model: (NP = 200) MP = 1 SDP = .3952D-02 AICP =-1102.726

NEW MODEL ADOPTED

**************
0 .......................... Current Model ..........................

... Coefficients  Innovation Variance ...
... I  A(I)  SD = .5431931617D-02 ...
... 1  .8912814546 ...
... 2  -.1587836332 ...
... 3  .2787328738 ...

... This model was fitted to the data ( X( 611), ... ,X( 660) ) ... 

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50)  MS = 1  SDS = .2692D-02  AICS = -544.654
Constant model: (NP = 100)  MP = 3  SDP = .4110D-02  AICP = -541.439

********* NEW MODEL ADOPTED *********

0 .......................... Current Model ..........................

... Coefficients  Innovation Variance ...
... I  A(I)  SD = .2691598426D-02 ...
... 1  1.0039010099 ...

... This model was fitted to the data ( X( 661), ... ,X( 710) ) ... 

Basic Autoregressive Model
\[ X(I) = A(1)*X(I-1) + A(2)*X(I-2) + \ldots + A(M)*X(I-M) + E(I) \]

Where
- \( M \): Order of the model
- \( E(I) \): Gaussian white noise with 0 mean and SD(M) variance

---

The following two models are compared

OMoving model: (\( NF = 50, \ NS = 50 \)) \( MS = 1 \) SDS = .5011D-02 AICS = -552.691
Constant model: (\( NP = 100 \)) \( MP = 1 \) SDP = .3913D-02 AICP = -550.353

**********

***** NEW MODEL ADOPTED *****

**********

---

Current Model

- Coefficients
- Innovation Variance

<table>
<thead>
<tr>
<th>I</th>
<th>A(I)</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.9984327540</td>
<td>.5010561222D-02</td>
</tr>
</tbody>
</table>

This model was fitted to the data \( (X(711), \ldots, X(760)) \).

Basic Autoregressive Model

\[ X(I) = A(1)*X(I-1) + A(2)*X(I-2) + \ldots + A(M)*X(I-M) + E(I) \]

Where
- \( M \): Order of the model
- \( E(I) \): Gaussian white noise with 0 mean and SD(M) variance

---

The following two models are compared

OMoving model: (\( NF = 50, \ NS = 50 \)) \( MS = 1 \) SDS = .5663D-02 AICS = -515.503
Constant model: (\( NP = 100 \)) \( MP = 4 \) SDP = .5032D-02 AICP = -519.199

***** Constant model adopted *****
Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---

OMoving model: (NF = 100, NS = 50) MS = 2 SDS = 0.5065D-02 AICS = -777.472

Constant model: (NP = 150) MP = 8 SDP = 0.4735D-02 AICP = -784.928

Constant model adopted *****

Basic Autoregressive Model
\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

---

The following two models are compared

Moving model: (NF = 150, NS = 50) MS = 4 SDS = 0.3972D-02 AICS = -1051.358
Constant model: (NP = 200) MP = 6 SDP = 0.4659D-02 AICP = -1059.800

Constant model adopted

---------------------------------------------

Current Model

<table>
<thead>
<tr>
<th>I</th>
<th>A(I)</th>
<th>Innovation Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.1647501844</td>
<td>0.4658797625D-02</td>
</tr>
<tr>
<td>2</td>
<td>-1.719389254</td>
<td></td>
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<tr>
<td>3</td>
<td>-1.775309544</td>
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<tr>
<td>4</td>
<td>1.056589687</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.2465816368</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-1.700998212</td>
<td></td>
</tr>
</tbody>
</table>

This model was fitted to the data (X(711), \ldots, X(910)).

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

---

The following two models are compared

Moving model: (NF = 200, NS = 50) MS = 1 SDS = 3.3392D-02 AICS = -1340.119
Constant model: (NP = 250) MP = 6 SDP = 0.4356D-02 AICP = -1345.036

Constant model adopted

100
Current Model

Coefficients

I  A(I)  SD =  .4356248907D-02
1  1.1495396983
2  -.1456618724
3  -.2040601471
4  .1209593453
5  .2381570361
6  -.1612078556

This model was fitted to the data (X(711), ..., X(960)).

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

The following two models are compared:
OMoving model: (NF = 250, NS = 50) MS = 3 SDS = .1666D-02 AICS = -1656.909
Constant model: (NP = 300) MP = 6 SDP = .4015D-02 AICP = -1641.318

NEW MODEL ADOPTED

This model was fitted to the data (X(961), ..., X(1010)).

Basic Autoregressive Model
\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

- **M**: Order of the model
- **E(I)**: Gaussian white noise with 0 mean and SD(M) variance

---

The following two models are compared

**Model**
- **OMoving model**: (NF = 50, NS = 50) MS = 1 SDS = .1809D-02 AICS = -623.612
- **Constant model**: (NP = 100) MP = 3 SDP = .1757D-02 AICP = -626.417

**Constant model adopted**

---

Current Model

<table>
<thead>
<tr>
<th>I</th>
<th>A(I)</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.9415292936</td>
<td>.1756965423D-02</td>
</tr>
<tr>
<td>2</td>
<td>-.1295319572</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>.1883656762</td>
<td></td>
</tr>
</tbody>
</table>

This model was fitted to the data ( \( X(961), \ldots ,X(1060) \) )

---

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

- **M**: Order of the model
- **E(I)**: Gaussian white noise with 0 mean and SD(M) variance

---

The following two models are compared

**Model**
- **OMoving model**: (NF = 100, NS = 50) MS = 1 SDS = .1917D-02 AICS = -935.269
- **Constant model**: (NP = 150) MP = 1 SDP = .1874D-02 AICP = -937.954

**Constant model adopted**
Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

\[ M: \quad \text{Order of the model} \]
\[ E(I): \quad \text{Gaussian white noise with 0 mean and SD(M) variance} \]

--- The following two models are compared ---

OMoving model:  (NF = 150, NS = 50)  MS = 4  SDS = .3342D-02  AICS = -1213.010
Constant model:  (NP = 200)  MP = 4  SDP = .2281D-02  AICP = -1206.615

*********************************************************
*****
***** NEW MODEL ADOPTED *****
*****
*********************************************************

Current Model

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Innovation Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I: A(I)</td>
<td>SD = .3342246862D-02</td>
</tr>
<tr>
<td>1</td>
<td>.7593613394</td>
</tr>
<tr>
<td>2</td>
<td>-.001399676</td>
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<tr>
<td>3</td>
<td>-.0014993162</td>
</tr>
<tr>
<td>4</td>
<td>.2358494097</td>
</tr>
</tbody>
</table>

This model was fitted to the data ( X( 961), ..., X( 1110) )

*********************************************************
Basic Autoregressive Model

\[ X(I) = A(1)\times X(I-1) + A(2)\times X(I-2) + \ldots + A(M)\times X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .2658D-02 AICS = -567.571
Constant model: (NP = 100) MP = 1 SDP = .3284D-02 AICP = -567.876

Constant model adopted

--- Current Model ---

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Innovation Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>A(I)</td>
</tr>
<tr>
<td>1</td>
<td>.9907699211</td>
</tr>
</tbody>
</table>

This model was fitted to the data (X(1111), \ldots ,X(1210)).

Basic Autoregressive Model

\[ X(I) = A(1)\times X(I-1) + A(2)\times X(I-2) + \ldots + A(M)\times X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---

OMoving model: (NF = 100, NS = 50) MS = 2 SDS = .1787D-02 AICS = -878.250
Constant model: (NP = 150) MP = 1 SDP = .2868D-02 AICP = -874.108

NEW MODEL ADOPTED

104
0 .................................. Current Model ..................................


'This model was fitted to the data ( X( 1211), ... ,X( 1260) ) .

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1786525166D-02 AICS = -622.889
Constant model: (NP = 100) MP = 5 SDP = .1671D-02 AICP = -627.459
0***** Constant model adopted *****

0 .................................. Current Model ..................................


'This model was fitted to the data ( X( 1211), ... ,X( 1310) ) .

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 1 SDS = .1890D-02 AICS = -937.008
Constant model: (NP = 150) MP = 2 SDP = .1846D-02 AICP = -938.194
0**** Constant model adopted *****

0 ......................... Current Model .................................

.  
.  
.  
.  
. Coefficients  Innovation Variance  .
. I A(I) SD = .1846198112D-02  .
. 1 1.0858834220  .
. 2 -.1226891946  .
.  
. This model was fitted to the data ( X( 1211), ... ,X( 1360) ).

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model: (NF = 150, NS = 50) MS = 1 SDS = .8895D-03 AICS =-1285.434
Constant model: (NP = 200) MP = 2 SDP = .1681D-02 AICP =-1271.635

0*********************
*****  *****
***** NEW MODEL ADOPTED  *****
*****  *****
******************************************************************************
0 ........................ Current Model ........................
  
  .
  .
  .
  .
    Coefficients                 Innovation Variance
  .
    I          A(I)             SD =  .3046558348D-02
  .
    1          1.0108262528     
  .
  .  This model was fitted to the data ( X( 1411), ... ,X( 1460) )
  
-----------------------------------------------
Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model:  (NF = 50, NS = 50)  MS = 1  SDS = .3047D-02  AICS = -632.928
Constant model: (NP = 100)         MP = 1  SDP = .1982D-02  AICP = -618.350

0*****************************************************************************
  *****
  *****   NEW MODEL ADOPTED   *****
  *****
/*****************************************************************************

0 ........................ Current Model ........................
  
  .
  .
  .
  .
    Coefficients                 Innovation Variance
  .
    I          A(I)             SD =  .3046558348D-02
  .
    1          1.0108262528     
  .
  .  This model was fitted to the data ( X( 1411), ... ,X( 1460) )
  
-----------------------------------------------
Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .7202D-03 AICS = -643.488
Constant model: (NP = 100) MP = 1 SDP = .2000D-02 AICP = -617.453

O*******************************************************************************
*****  *****
***** NEW MODEL ADOPTED  *****
*****  *****
*******************************************************************************

O ........................ Current Model .................................
  .
  .
  .
  . Coefficients Innovation Variance .
  .  I    A(I)   SD =   .7201706675D-03 .
  .  1    .9973465840 .
  .
  . This model was fitted to the data ( X( 1461), ... ,X( 1510) ).
*******************************************************************************
Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1916D-02 AICS = -666.690
Constant model: (NP = 100) MP = 3 SDP = .1281D-02 AICP = -657.984
O********************
*****    *****
***** NEW MODEL ADOPTED    *****
*****    *****
********************

0 ........................ Current Model ........................
  ......................................................
  ......................................................
  ......................................................
  ......................................................
  ......................................................
  ......................................................
  ......................................................
  ......................................................
  ......................................................
  ......................................................

Coeficients   Innovation Variance
  I    A(I)        SD = .1915503690D-02
  1    1.0026858287

This model was fitted to the data ( X( 1511), ... ,X( 1560) )

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .2100D-02 AICS = -613.177
Constant model: (NP = 100)      MP = 1 SDP = .2014D-02 AICP = -616.778
O***** Constant model adopted *****

0 ........................ Current Model ........................
  ......................................................
  ......................................................
  ......................................................
  ......................................................
  ......................................................
  ......................................................
  ......................................................
  ......................................................
  ......................................................

Coeficients   Innovation Variance
  I    A(I)        SD = .2013698405D-02
  1    1.0040522764

This model was fitted to the data ( X( 1511), ... ,X( 1610) )

Basic Autoregressive Model
\[ X(I) = A(1) \cdot X(I-1) + A(2) \cdot X(I-2) + \ldots + A(M) \cdot X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

---

The following two models are compared

OMoving model:  (NF = 100, NS = 50)  MS = 1  SDS = 0.2327D-02  AICS = -915.932
Constant model:  (NP = 150)  MP = 1  SDP = 0.2134D-02  AICP = -918.467

Constant model adopted

---

Current Model

Coefficients  Innovation Variance
I  A(I)  SD  =  0.2133945333D-02
1  1.0024936061

This model was fitted to the data (X(1511), ..., X(1660)).

Basic Autoregressive Model

\[ X(I) = A(1) \cdot X(I-1) + A(2) \cdot X(I-2) + \ldots + A(M) \cdot X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

---

The following two models are compared

OMoving model:  (NF = 150, NS = 50)  MS = 2  SDS = 0.9173D-03  AICS = -1262.172
Constant model:  (NP = 200)  MP = 1  SDP = 0.1858D-02  AICP = -1253.644

NEW MODEL ADOPTED

---

110
0 ......................... Current Model ..............................
  .
  .
  .
  .
  .

  Coefficients  Innovation Variance  
  . I A(I) SD = .9172940519D-03 
  . 1 .7978437591 
  . 2 .2002976118 
  .

  This model was fitted to the data ( X(1661), ... ,X(1710) ) .

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .8150D-03 AICS = -695.318
Constant model: (NP = 100) MP = 2 SDP = .8813D-03 AICP = -697.413
0**** Constant model adopted ****

0 ......................... Current Model ..............................
  .
  .
  .
  .
  .

  Coefficients  Innovation Variance  
  . I A(I) SD = .8812835049D-03 
  . 1 .8541138389 
  . 2 .1459299097 
  .

  This model was fitted to the data ( X(1661), ... ,X(1760) ) .

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 1 SDS = .6877D-03 AICS = -1057.518
Constant model: (NP = 150) MP = 1 SDP = .8300D-03 AICP = -1060.114
**** Constant model adopted ****

--- The following two models are compared ---
OMoving model: (NF = 150, NS = 50) MS = 1 SDS = .7622D-03 AICS = -1415.078
Constant model: (NP = 200) MP = 1 SDP = .8137D-03 AICP = -1418.784
**** Constant model adopted ****

Current Model

Coefficients Innovation Variance .
   I    A(I)     SD = .8299915492D-03 .
   1    .9999655304 .

This model was fitted to the data (X(1661), ..., X(1810)).

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

Current Model

Coefficients Innovation Variance .
   I    A(I)     SD = .8136991383D-03 .
   1    .9997913478 .

This model was fitted to the data (X(1661), ..., X(1860)).

Basic Autoregressive Model
\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---

OMoving model: \( (NF = 200, NS = 50) \) MS = 3 SDS = .4064D-03 AICS = -1801.190
Constant model: \( (NP = 250) \) MP = 1 SDP = .7432D-03 AICP = -1797.138

*************************
*****
***** NEW MODEL ADOPTED *****
*****
*************************

Current Model

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Innovation Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I 1 1.1522670490</td>
<td></td>
</tr>
<tr>
<td>2 -.5031986610</td>
<td></td>
</tr>
<tr>
<td>3 .3519184456</td>
<td></td>
</tr>
</tbody>
</table>

This model was fitted to the data \( (X(1861), \ldots ,X(1910)) \).

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---

OMoving model: \( (NF = 50, NS = 50) \) MS = 8 SDS = .2833D-03 AICS = -772.853
Constant model: (NP = 100) \[ \text{MP} = 3 \quad \text{SDP} = .4002D-03 \quad \text{AICP} = -774.361 \]

O***** Constant model adopted *****

0 ............................. Current Model ............................

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................. Coefficients ................... Innovation Variance ........

..........................

1 1.0743705113

2 -.3876565459

3 .3135270338

..........................

This model was fitted to the data (X(1861), ..., X(1960)).

--------------------------------------------------------------------------

Basic Autoregressive Model

\[ X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I) \]

Where

\( M \): Order of the model

\( E(I) \): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared --- OMOVING model:

(NF = 100, NS = 40) \[ \text{MS} = 2 \quad \text{SDS} = .6870D-03 \quad \text{AICS} = -1059.686 \]

Constant model: (NP = 140) \[ \text{MP} = 2 \quad \text{SDP} = .5396D-03 \quad \text{AICP} = -1047.461 \]

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***** NEW MODEL ADOPTED *****

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O ............................. Current Model ............................

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................. Coefficients ................... Innovation Variance ........

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1 1.4502860384

2 -.4515822692

..........................

This model was fitted to the data (X(1961), ..., X(2000)).

--------------------------------------------------------------------------

114
US treasury bill 2nd market - middle rate, daily 11.06.1986 - 1.12.1995

Title ---> USTR

No. of data used ---> 852
Maximum order of AR model ---> 10
Length of basic local span ---> 50
Parameter KSW ---> 0

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

Initial local model: NS = 50 MS =10 SDS = .10739D+00 AICS = -89.564

0 ....................... Current Model ........................

Coefficients Innovation Variance
. I A(I) SD = .1073918978D+00
. 1 .8142619274 .
. 2 -.0243569474 .
. 3 .1628427160 .
. 4 .0673123556 .
. 5 -.0842926555 .
. 6 .1585937023 .
. 7 .0551659674 .
. 8 -.1093448476 .
. 9 .2547753917 .
. 10 -.5130654729 .
. This model was fitted to the data (X( 11), \ldots ,X( 60))

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]
Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .1274D+00 AICS = -186.601
Constant model: (NP = 100) MP = 2 SDP = .1422D+00 AICP = -189.028
O***** Constant model adopted *****

O ......................... Current Model .........................

Coefficients        Innovation Variance
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This model was fitted to the data (X(11), ..., X(110))

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 2 SDS = .7303D-02 AICS = -429.002
Constant model: (NP = 150) MP = 2 SDP = .9727D-01 AICP = -343.535

O*****************************
*****
***** NEW MODEL ADOPTED *****
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*****************************
0 ........................ Current Model ........................

........... Coefficients  Innovation Variance ...........
  I  A(I)  SD =  .7302964645D-02 
  1  .7751441340 
  2  .2257251631 

This model was fitted to the data ( X( 111), ... ,X( 160 ) )

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared --- OMoving model:
(NF = 50, NS = 50) MS = 4 SDS = .3381D-02 AICS = -514.448
Constant model: (NP = 100) MP = 4 SDP = .5386D-02 AICP = -512.389

O*********NEW MODEL ADOPTED*********

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]
Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

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0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 8 SDS = .2710D-03 AICS = -667.147
Constant model: (NP = 100) MP = 10 SDP = .1872D-02 AICP = -606.065

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**** NEW MODEL ADOPTED ****
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0 ............................ Current Model ............................


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Coefficients  Innovation Variance
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I  A(I)  SD = .2709854070D-03
.
1  -.0634244337
.
2  .1858700878
.
3  .5131659912
.
4  .2876769606
.
5  .2986671268
.
6  .2396089525
.
7  -.2492092033
.
8  -.2125375850
.
.
.
This model was fitted to the data ( X( 211), ... ,X( 260) )

---------------------------------------------------------------------

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

---------------------------------------------------------------------

118
The following two models are compared

OMoving model: (NF = 50, NS = 50) MS = 2 SDS = 0.1437D-01 AICS = -598.806
Constant model: (NP = 100) MP = 3 SDP = 0.7683D-02 AICP = -478.873

**************************************************************
****** ************ NEW MODEL ADOPTED ************
****** ************
**************************************************************

Current Model

Coefficients                  Innovation Variance
I   A(I)           SD      = 0.1436915400D-01
1   0.6257819454
2   0.3607041739

This model was fitted to the data (X(261), ..., X(310)).

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

The following two models are compared

OMoving model: (NF = 50, NS = 50) MS = 6 SDS = 0.3711D-01 AICS = -356.826
Constant model: (NP = 100) MP = 6 SDP = 0.2798D-01 AICP = -343.618

**************************************************************
****** ************ NEW MODEL ADOPTED ************
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................. Current Model ......................

................. Coefficients Innovation Variance .................

................. This model was fitted to the data ( X( 311), ... ,X( 360) ) .

Basic Autoregressive Model

\[ X(I) = A(1) \times X(I-1) + A(2) \times X(I-2) + \ldots + A(M) \times X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 8 SDS = .1242D+00 AICS = -237.000
Constant model: (NP = 100) MP = 8 SDP = .9025D-01 AICP = -222.518

O********************************************************************************************************

*****
***** NEW MODEL ADOPTED *****

********************************************************************************************************
Current Model

Coefficients

<table>
<thead>
<tr>
<th>I</th>
<th>A(I)</th>
<th>Innovation Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.7221252841</td>
<td>SD = 0.1241631874D+00</td>
</tr>
<tr>
<td>2</td>
<td>0.2530333371</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.1503961570</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.2738013687</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.3532092769</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-0.2063205405</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-0.3516248412</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.4651069241</td>
<td></td>
</tr>
</tbody>
</table>

This model was fitted to the data (X(361), ..., X(410)).

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where

\( M \): Order of the model
\( E(I) \): Gaussian white noise with 0 mean and SD(M) variance

--- Moving model: ---

\( NF = 50, NS = 50 \)
\( MS = 2 \)
\( SDS = 0.9775D-01 \)
\( AICS = -196.573 \)

Constant model: \( MP = 8 \)
\( SDP = 0.1159D+00 \)
\( AICP = -197.501 \)

Constant model adopted

This model was fitted to the data (X(361), ..., X(460)).

Basic Autoregressive Model
\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) \( MS = 1 \) SDS = .6236D-01 AICS = -332.243
Constant model: (NP = 150) \( MP = 8 \) SDP = .1028D+00 AICP = -323.191

*****************************************************
***** NEW MODEL ADOPTED *****
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Current Model

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. .
. . Coefficients Innovation Variance .
. I A(I) SD = .6236033434D-01 .
. 1 .9927226049 .
. This model was fitted to the data \( X(\ 461), \ldots ,X(\ 510) \) .

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) \( MS = 1 \) SDS = .2433D+00 AICS = -201.408
Constant model: (NP = 100) \( MP = 1 \) SDP = .1531D+00 AICP = -183.680
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******     NEW MODEL ADOPTED     ******

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O  ......................  Current Model  ......................

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Coefficients     Innovation Variance

  I     A(I)     SD   .2433325292D+00

  1     .9615332505

. This model was fitted to the data ( X( 511), ... ,X( 560) ).

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where

M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared --- OMoving model:
(NF = 50, NS = 50) MS = 1 SDS = .4979D+00  AICS = -97.532
Constant model: (NP = 100)      MP =  3  SDP = .3559D+00  AICP = -95.319

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******     NEW MODEL ADOPTED     ******

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O  ......................  Current Model  ......................

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Coefficients     Innovation Variance

  I     A(I)     SD   .4979180959D+00

  1     .9699178392

. This model was fitted to the data ( X( 561), ... ,X( 610) ).

Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 8 SDS = .1176D+01 AICS = -4.748
Constant model: (NP = 100) MP = 9 SDP = .9200D+00 AICP = 11.666

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***** NEW MODEL ADOPTED *****
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0--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1124D+01 AICS = 35.943
Constant model: (NP = 100) MP = 9 SDP = .1171D+01 AICP = 35.776

O***** Constant model adopted *****

0 ........................ Current Model .................................

...

Coefficients  Innovation Variance

I    A(I)       SD = .1170888238D+01

1    1.1070490875
2    -.2285442687
3    -.0466392163
4    .0189499919
5    .1121471468
6    .0724526218
7    -.3378385263
8    .5124858283
9    -.2186610934

This model was fitted to the data (X( 611), ..., X( 710)).

-----------------------------------------------
Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

OMoving model: (NF = 100, NS = 50) MS = 4 SDS = .3420D+00 AICS = -7.875
Constant model: (NP = 150) MP = 9 SDP = .9406D+00 AICP = 10.809

O******************************

*****                        *****

*****  NEW MODEL ADOPTED  *****

 *****                        *****

**************************************************************************
0 .......................... Current Model ..........................
  .
  .
  .
  .
  .  Coefficients  Innovation Variance .
  .  I      A(I)  SD =  .3419755844D+00 .
  .  1    .8519317821 .
  .  2    .1205712707 .
  .  3    -.3326560785 .
  .  4    .3591311411 .
  .
  .  This model was fitted to the data ( X( 711), ... ,X( 760) ) .
  .................................................................

Basic Autoregressive Model

\[ X(I) = A(1)X(I-1) + A(2)X(I-2) + \ldots + A(M)X(I-M) + E(I) \]

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared --- OMoving model:
(NF = 50, NS = 50) MS = 1 SDS = .1405D+00 AICS = -137.790
Constant model: (NP = 100) MP = 4 SDP = .2454D+00 AICP = -130.497

O*******************************

***** ****

***** NEW MODEL ADOPTED *****

***** ****

*******************************

0 .......................... Current Model ..........................
  .
  .
  .
  .
  .  Coefficients  Innovation Variance .
  .  I      A(I)  SD =  .1404674898D+00 .
  .  1    .9561232626 .
  .
  .
  .  This model was fitted to the data ( X( 761), ... ,X( 810) ) .
  .................................................................

Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where
M: Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance

--- The following two models are compared ---
OMoving model: (NF = 50, NS = 42) MS = 5 SDS = .5965D-01 AICS = -200.551
Constant model: (NP = 92) MP = 2 SDP = .1116D+00 AICP = -195.778

NEW MODEL ADOPTED

Current Model

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Innovation Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I A(I)</td>
<td>SD = .5964624286D-01</td>
</tr>
<tr>
<td>1 .7530995438</td>
<td></td>
</tr>
<tr>
<td>2 .3741982381</td>
<td></td>
</tr>
<tr>
<td>3 -.0830894482</td>
<td></td>
</tr>
<tr>
<td>4 .3396315855</td>
<td></td>
</tr>
<tr>
<td>5 -.4103577267</td>
<td></td>
</tr>
</tbody>
</table>

This model was fitted to the data (X(811),...,X(852))