Determinants of the Low SME Loan Approval Rate in Croatia: A Latent Variable Structural Equation Approach

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ABSTRACT. The paper proposes a new methodological framework for investigating consistency in loan assessment decisions and determinants of loan approval based on structural equation modelling and covariance structure analysis. We focus on a governmental SME loan programme in Croatia and investigate possible reasons for low loan approval rate that occurred in spite of interest rates subsidisation and sufficient supply of the loan funds. The novelty of the methodological approach taken is that it enables simultaneous investigation of the determinants of the loan approval and testing for consistency in the loan assessment decisions, which need not be assumed. We test several hypotheses about consistency in the loan approval decisions and lending preferences in Croatia. The empirical findings reject overall consistency of criteria but indicate a preference toward smaller loans. Among all SME loan requests, banks preferred smaller firms that requested smaller loans. The results suggest that individual banks differ in their criteria and in their loan-size preferences and that there is no positive correlation between the bank’s size and its loan-size preference.

KEY WORDS: commercial banks, credit rationing, latent variable models, loan assessment, small and medium enterprises

JEL CLASSIFICATION: C31, C51, C52, G21, H81

1. Introduction

Small and medium enterprises (SMEs) play an important role in transitional economies and have high relevance for their economic policy (Bagnasco and Sabel, 1995; Levitsky, 1996; Scase, 1997; Bateman and Lloyd-Reason, 2000; see also Tybout, 1983 for an analysis of a non-transitional developing country). In Croatia, the SMEs comprise over 96% of all business entities, thus making the SME sector a dominant part of its national economy. Nevertheless, their access to credit and loan funds is still rather limited (Boogearts et al., 2000; Barlett et al., 2002). Over the last several years, the Croatian SME sector had a mean annual employment growth of 5% while, in the same period, the employment in the large businesses sector decreased for over 30%. In addition, the SMEs currently produce over 55% of the Croatian GDP. However, the obstacles to economic development are numerous and one of the most serious is a very low SME loan approval rate in the commercial banks. Before 1998, the main obstacle to SME financing in Croatia was insufficient supply of SME credit funds (see e.g. Pissarides, 1998). By 1999, and especially in 2000, Croatian commercial banks no longer lacked funds and low loan approval rate emerged as the primary obstacle to efficient SME finance. Access to financial markets for SMEs is often problematic even in western economies (see e.g. Mullineux, 1994; Cressy et al., 1997; Assel-
bergh, 2002) and SMEs are often forced to look for alternative means of financing (e.g. Hamilton and Fox, 1998). The SME financing in Croatia is further complicated by a weak banking system and a lack of expertise in commercial banks for dealing with the SME clients (Kraft, 2000, 2002).

An appealing theoretical explanation for a low SME loan approval rate could be in “credit rationing” (Stiglitz and Weiss, 1981; see also Jaffee and Russel, 1976). In the Stiglitz and Weiss model, credit rationing might exist when some of the observationally indistinguishable loan applicants receive loans while others do not or when there are identifiable groups of potential borrowers who are unable to obtain loans at any interest rate, though with larger supply of credit they might be able to do so.

Deshmukh et al. (1983) offered an alternative theoretical explanation for a low loan approval rate in the context of optimal lending policy rules. An important special case in the Deshmukh et al. (1983) model is when the interest rate is fixed for all potential borrowers in which situation the default risk becomes the sole criterion for the lender’s decision. In this model an optimal lending policy can be expressed in terms of a critical rate of return (i.e. a credit standard), in the sense that the lender’s decision to approve a loan to a potential borrower is optimal only if the risk-adjusted rate of return from lending to the potential borrower exceeds the critical rate of return. An implication of the Deshmukh et al. (1983) model is that lending decisions based on risk-adjusted policy rules might be misinterpreted as credit rationing.

An additional element in the credit rationing theory is the role of interest rates. In a very simplified way, the Stiglitz-Weiss credit rationing model suggests that policy that decreases the interest rate and provides loan guarantees through co-financing of the loans (i.e. supply of loan funds), or provision of loan guarantees, might adversely affect credit rationing.

In an attempt to remedy the problems in SME financing, Croatian government began implementation of national SME loan schemes, starting in the year 2000 with the “Snow Ball 2000” programme (SB-2000). This scheme was designed with the purpose of co-financing the interest rate and simultaneously providing the commercial banks with additional funds for the SME loans, where eight commercial banks entered the arrangement with the purpose of providing loans to SME borrowers at a subsidised fixed interest rate. The main rationale for such loan scheme was to enable the access to loans for the SMEs who lack collateral or are in other ways unable to obtain regular commercial loans.

As the SB-2000 aimed both at increasing the supply of loan funds and at decreasing the interest rate (by subsidising it), in the context of credit rationing theory it could be expected that the access to loan funds for the SME borrowers would be improved. However, the SB-2000 programme had a loan-approval rate of below 5% at the end of the first year of administration. The programme continued through 2001, and in the end of the year about 29% of all submitted applications were approved for financing by the commercial banks. This is still too low for expecting significant growth stimuli from, otherwise available, loan funds in the SME sector and it is a possible consequence of credit rationing.

In developing and transitional countries, such as Croatia, apparent credit rationing might also be related to the low quality of business plans, or lacking expertise of the loan officers to evaluate possibly good loan applications. In this regard, even observationally distinguishable potential borrowers might be indistinguishable to the loan officers. In addition, weak banking tradition might cause suboptimal behaviour of the lenders who might consider profit-maximisation that requires administration of a larger number of smaller loans administratively too costly or simply too troublesome to deal with, and thus prefer to administer fewer larger loans, thereby displaying ‘negative attitude’ towards small lending. The term ‘negative attitudes’, first appearing in the European Commission (EC) technical assistance reports (e.g. Boogearts et al., 2000), became popular in the Croatian and EC policy circles when referring to an apparent lack of interest in small lending among the commercial banks in Croatia. This explanation, however, seems strange in an economy where 96% of all businesses are SMEs, hence the term ‘negative attitudes’ in this context implies a form of sub-optimal behaviour in the
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This difference in views has immediate policy connotations and important methodological implications for the analysis of the SME loan-approval rates. Primarily, the current policy of increased supply of loan funds and subsidising interest rates might be insufficient and it might be necessary to implement additional support services such as training programmes for the loan officers and/or for the entrepreneurs. The EC and the EBRD considered such options as part of their technical assistance programme for Croatia (CARDS), yet clear empirical evidence in support of either one of the considered alternatives was lacking.

In this paper, we investigate possible reasons for a low loan approval rate in the SB-2000 programme by analysing consistency and determinants of the commercial banks’ loan approval decisions. We collect data from the submitted loan applications under the SB-2000 programme, coding each application on a number of common variables. Such data allow us to objectively analyse the banks’ decisions by matching them with the characteristics of the loan applicants and their business projects, which has notable advantages over the self-reported data from interviews with the loan officers (such as data used e.g. by Kraft, 2002).

We propose a multivariate methodological framework based on covariance structure analysis, specifically, structural equations modelling (Jöreskog, 1973; Jöreskog et al., 2000) for analysing the consistency and determinants of bank’s loan application decisions. The logic behind using covariance structure-based analysis is simple: we assume that consistency in criteria implies that accepted applications will have similar covariance (and mean) structure, and similarly, different covariance structure from the rejected applications. Similar logic can be applied to comparative analysis across different banks.

The methodological framework we propose can be generally used in empirical research of credit rationing and loan approval rates determinants with the principal advantage of enabling testing of the consistency in loan assessment criteria, i.e., existence of optimal lending policy rules vs. randomising or credit rationing along with investigating determinants of loan approval. The traditionally employed methods in empirical
2. Research problem and hypotheses

By the mid-2001, the results of the SB-2000 programme showed that only 18.77% of the total SME credit potential (government’s funds) was transferred to the commercial banks for the SME loans. Moreover, out of the transferred amount only 12.46% was allocated to SME loans, which amounts to 2.34% of the total available credit potential for the SME finance. This alarming result was a consequence of a very low SME loan approval rate with the initial (2001) average for the SB-2000 programme of 4.71%. The continuation of the programme in 2001 resulted in additional approval of 24.29% of the applications submitted under the SB-2000 programme, thus a total of 29% of the applications submitted under SB-2000 was approved for financing, which is problematically low given the extra effort in advocating the programme throughout 2001.

The SB loan programme had two layers of loan application assessment. The first was screening by the Ministry of Crafts and SMEs and the second was the loan approval procedure in the commercial banks. The main loan-application assessment is carried out by the commercial banks on the basis of formal applications which included a description of the proposed business project (hence information about the sector, purpose, planned job openings, etc. could be extracted from the applications). Aside of the formalised two-layer assessment procedure, no other formal requirements such as interviews or site visits were made for the loan applicants, thus leaving acquisition of possible additional information at the discretion of the banks, which however, formally made loan assessment information on the basis of the submitted applications. No collateral requirement was another difference between the SB-2000 loan programme and the standard entrepreneurial loans.

Eight commercial banks participated in the programme, jointly covering all of the 21 Croatian counties, which acted as local administrative units for loan funds allocation. Aside of the counties, several towns and municipalities acted as administration units. The role of the government was in the provision of additional loan funds from the national budget and in co-financing of the interest rate in the amount of 2%. We note that out of these eight banks, two are large, namely Zagrebačka banka and Privredna banka Zagreb (PBZ).

Therefore, the main problem with the SB-2000 programme was excessively low loan approval rate despite a subsidised interest rate and sufficient (even excessive) supply of loan funds. Such low loan approval rate might be due to credit rationing in the sense of Stiglitz and Weiss (1981) theory. Credit rationing might exist when either (i) some of the, otherwise observationally indistinguishable, potential borrowers receive loans while others do not, or (ii) when specific groups of potential borrowers can be identified who are
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unable to obtain loans at any interest rate, though with larger supply of credit they might be able to do so (Stiglitz and Weiss, 1981, p. 395). Therefore, if significant determinants of loan approval are observable, than the potential borrowers who were denied loans could be observationally distinguished from those who received loans, hence the form (i) of credit rationing would not be able to explain low loan application approval rate. On the other hand, because the supply of credit funds was no doubt sufficient, the form (ii) of credit rationing did not occur. In fact, at the end of the administration of the SB-2000 programme only 1/3rd or the committed funds were used for SME lending.

Alternatively, the Deshmukh et al. (1983) model implies that an optimal lending policy can be expressed in terms of a critical rate of return (i.e. a credit standard), in the sense that the lender’s decision to approve a loan to a potential borrower is optimal only if the risk-adjusted rate of return from lending to the potential borrower exceeds the critical rate of return, which means that a low loan approval rate could be the consequence of a high critical rate of return (e.g. due to risk aversion). In such case, however, the potential borrowers who are denied loans are not observationally equivalent to those who receive loans, hence the lenders utilise the available information on the potential borrowers to form optimal risk-adjusted policy rules, which, as suggested by Deshmukh et al. (1983), might be misinterpreted as credit rationing.14

Among the Croatian and EC policymakers there were two alternative explanations for the excessively low SME loan approval rate (Boogearts et al., 2000). The first assumes that commercial banks have ‘negative attitudes’ toward small loans (‘penny-loans’) and thus prefer to invest in larger, more growth-stimulating projects. However, given the dominant SME-nature of the Croatian economy (96%) and good business results of the small enterprises (especially in comparison with the large ones) ‘negative attitudes’ towards SME lending appear strange and require more detailed clarification. In particular, what is in this context implied by ‘negative attitudes’ concerns the loan assessment criteria of the commercial banks who might be reluctant to lend to SMEs either because of the lack of appropriate training of the loan officers or because of too high perceived fixed costs incurred from administering a larger number of smaller loans.

Regardless of the underlying cause of such behaviour, an immediate implication of the tendency to over-reject otherwise qualified potential small borrowers, i.e., negative attitudes toward small lending, is that among all loan applicants, ceteris paribus, requests for larger loans will stand better chances of being approved. Therefore, among all submitted SME applications, higher chances of approval will have requests for larger loans. Therefore, the question of attitudes toward SME lending relates primarily to the banks’ preferences regarding business proposals that are smaller in the overall scope, mainly those requesting smaller amounts of money, for less ambitious business projects.15 The belief that commercial banks generally prefer larger loans, and thus have lower interest in small and micro loans, suggests that banks have ‘negative attitudes’ toward SME lending and thus over-reject potential SME borrowers, possibly due to credit rationing. If so, it follows that among a wide diversity of SME loan requests, the banks with ‘negative attitudes’ toward small loans will prefer larger, more perspective SMEs (e.g., with larger number of new job openings) and generally reject loans to the smaller ones.

The second explanation presumes that banks act rationally, evaluating loan requests on the basis of their economic merit or profitability, but that the loan applications are of poor quality. This, expectedly, is also the view promoted by the banks themselves. In the sense of Deshmukh et al. (1983), this would imply that banks have optimal lending policies with a high critical rate of return or high credit standards. Hence, low quality of loan applications (or of potential borrowers) induces that high number of potential borrowers will have the risk-adjusted rate of return bellow the bank’s critical rate of return. If this explanation is correct, the accepted applications would on average significantly differ from the rejected ones in terms of the scope of loans, sector and size of the firms, etc. Statistically, this would imply that rejected and accepted loan applications differ in terms of their covariance structures.
In a recent survey, Kraft (2002) reported the results from a series of interviews with the loan officers of the commercial banks operating in Croatia. According to the banks themselves, the main problems with the SME lending are the lack of data on past business history for SMEs, lack of client information (e.g., there is no functioning business registry), low-quality audits and inefficient court system. Consequently, the banks are reluctant to provide long-term lending to SMEs and are keener on short-term loans intended mainly for the working capital. Kraft (2002) also points out to lacking banking culture as an additional problem in most transitional countries. In these interviews, the banks’ officers generally claimed that past performance, especially past business experience, and the proposed project are the key loan-assessment criteria. Nevertheless, there are certain differences in declared emphases different banks place on the importance of the past business performance. In addition, Kraft reports that most banks wish to diversify risk by lending to a large number of smaller clients and smaller banks claim higher interest in SMEs then larger banks do. On the basis of this interview-data the second explanation above would be supported in so far as the existence of sensible and consistent criteria goes, but the preference for diversification to a larger number of smaller clients would contradict the first explanation, namely the belief that, ceteris paribus, banks prefer larger to smaller loans. However, this information is based on the claims made by the banks themselves and so far no data were collected on the actually submitted loan applications. To objectively analyse the applied criteria (or lack of it) it is necessary to look into the actual applications and compare the outcome of the loan assessment process with the characteristics of the business projects and firms that applied for loans.

The aim of the current analysis is to evaluate the applied decision criteria (i.e., their consistency) in the loan application procedure carried out by the commercial banks. Therefore, we investigate whether the banks had consistent criteria and which criteria were actually used. In particular, do banks indeed have negative attitudes towards small lending and thus credit ration small loan applicants, or do they have excessively high standards and “optimal lending policies”? Similarly, to the degree that data permits, we wish to compare the loan assessment criteria across the commercial banks that participated in the SB-2000 programme. In order to investigate these issues we formulate the following null hypotheses.

H1. The loan-assessment criteria are inconsistent, i.e., there is no significant difference between accepted and rejected applications.

H2: The banks have no specific preference regarding the size of the loans, i.e., loan applications requesting different amounts of loans have equal chances of being approved.

H3: There is no difference in loan-assessment criteria across different banks.

H4: There is no difference in the attitudes toward SME lending across different banks.

H5: There is no relationship between loan-size preference and the size of the bank.

3. Data and descriptive analysis

The primary data source comes from the loan applications submitted under the SB-2000 programme. We coded the applications on a number of variables relevant for assessing loan applicants’ business proposals. The information extracted from the individual loan requests had to be uniform across all banks and counties, which was complicated by the fact that the applications were not standardised across counties and that due to transitional situation and lacking banking tradition the data might provide only limited information. Consequently some, potentially relevant information, had to be omitted to ensure compatibility of data across all analysed banks. We, however, assume that banks had no information about potential borrowers that was systematically missing from the loan applications (but otherwise available to the loan officers).

Out of 3,919 initially submitted loan requests, 2,396 were forwarded to commercial banks by the Ministry of Crafts and SMEs. The remaining 1,423 applications were mainly incomplete or with missing documentation and were returned to the applicants, some of which re-applied with completed applications. Our data is based on the loan requests forwarder to the commercial banks.
The project and business prospects of the proposals. With the available variables, we wish to measure forms (see Table I).

Nine variables were extracted from these applications that would be financed by the entrepreneur wishes to introduce in the course of business expansion resulting from the project that would be financed by the loan funds. The amount of loan ($y_1$) refers to the total amount requested on the loan application, i.e., financial scope of the business project. Similarly, the number of new jobs ($y_2$) shows the number of planned job openings that each entrepreneur wishes to introduce in the course of business expansion resulting from the project that would be financed by the loan funds. The purpose of investment ($y_3$) is a constructed binary (dummy) indicator that takes value of one if the loan funds are requested primarily for investment purpose, i.e., business-related activities that might result in enterprise growth and business process improvement, and it is zero if the loan is requested e.g., for purchase of office furniture or facility renovation. Admittedly, coding of this variable depends on subjective judgement of the coder, however, we note that several of the banks and also local (county) loan administrators undertook coding of this variable and already classified the loan requests on the basis of their primary purpose using virtually identical criteria to those we applied. Repayment period ($y_4$) ranged from five to ten years and was requested in accordance with the provisions of the programme and total amount and purpose of the requested loan. Given the transitional nature of Croatian economy it is expected that most of the (private) SMEs are less than ten years old, as most of them were established after the fall of the communist regime in the beginning of the 1990s. However, a smaller number of SMEs in our sample existed already in the communist period. The age of the firm ($x_1$) variable measures the years of the firm’s existence, which is assumed to be of relevance in the process of credit-history assessment and past business performance. Sector ($x_2$) is another constructed dummy variable that take value of one for the cases the firm is in the production sector and zero in case of various service-type SMEs. This variable is considered important because the Croatian SME service sector is overdeveloped in comparison to its production sector; however, there is a difficulty in classifying those SMEs that are involved in both sectors simultaneously. This problem was solved by referring to the main activity as well as the purpose of the loan (i.e. whether funds are requested for production purposes or service side of the business), but we also note that only a very small fragment of SMEs encompass both production and service activities. Finally, the last tree variables ($x_3, x_4, x_5$) refer to firm’s employment size (number of employees), previously obtained loan credit in total amount (previous credit) and (gross) annual turnover of each loan applicant SME.

4. Econometric methodology

Traditional research on loan approval determinants is usually based on estimation of the loan approval probability as a function of presumably exogenous characteristics of the loan applications (i.e. characteristics of firms, projects, entrepreneurs, etc.). However, this requires a strong assumption of consistency of loan assessment criteria, namely, that the loan approval probability is a deterministic function of the characteristics of the potential borrowers, where this probability is seen as a chance that a favourable decision will be made on a loan application. Since the decision is made by the lender (i.e. loan officers), it follows that if the observable characteristics of the borrowers cause their decisions, these characteristics are exogenous and the loan approval probability is therefore endogenous. Such presumption is frequently made in the empirical literature on loan approval determinants that usually uses single equation ordinary least squares or probit/logit binary regression techniques for estimating loan approval probability given characteristics of the loan applicants. For example, Edelstein (1975) estimated a binary choice model using a...
two-stage least-squares (2SLS) method for determining the probability that a loan applicant will be a ‘good’ loan customer. Bates (1973) used discriminant analysis to study determinants of successful loan repayment and Bates (1975) applied these methods in a study of the US Small Business Administration’s minority lending in respect to incidence and causes of loan default. More recently, Munnell et al., (1996) used ordinary regression and binomial logic techniques to estimate the effects of the particular variables on the probability loan rejection using the US Home Mortgage Disclosure Act (HMDA) data.21 A similar approach was taken by Dymski and Mohanty (1999) who estimated a probit regression model of the determinants of the ethnic home-purchase loan approval in Los Angeles. Chakravarty and Scott (1999) used a logistic regression model to measure the probability of being credit rationed as a function of borrower-specific and borrower–lender relationship variables.

The main methodological problem in the loan approval research literature is in the treatment of the characteristics of loan applicants, or in the assumptions about exogeneity of loan approval determinants. It is seldom possible to a priori assume that the selection procedure appraises the applications in respect to their true merit and business prospects. When the research focus concerns lenders’ attitudes toward lending to some categories of potential borrowers and/or consistency in loan assessment criteria, it is not clear whether ‘good’ applications stand better chance of being approved than the ‘bad’ ones, regardless of how a ‘good application’ is defined. In such a case a scale for ranking applications on their relative merit (e.g. business prospects, expected profitability, etc.) might still be defined, but a variable equivalent to approval probability under positive attitudes and rationality in assessment criteria (or, similarly, full information) would be unobserved, i.e. latent. Thus, it cannot be simply assumed that the outcome (accept/reject) of the selection process is indeed linked to the characteristics of the applications; moreover, it is necessary to test such conjecture in the form of the above-defined null hypotheses.

The methodological approach we propose is to model the covariance structure of the loan application indicators (variables) using the general structural equation models with latent variables (LISREL), which is can be estimated with covariance structure analysis (CSA) methods (see Goldberger, 1972; Jöreskog, 1973; Jöreskog et al., 2000; Cziráky, 2003). CSA, in general, can be used to address the methodological issues of our research problem. To see why the CSA approach can provide insights into post hoc consistency-of-criteria analysis lets take a simple example. Suppose each loan application contains information only on the requested amount of loan and on the age of the firm, and further assume that the loan officers have no external information about the applicants. Then, consistency in the selection criteria will imply that preference is given to one of the following: (i) firms requesting smaller(larger) loans regardless of repayment period; (ii) newer(older) firms regardless of repayment period, or (iii) newer(older) firms requesting smaller(larger) amounts. If consistent criteria are applied, the covariances and means of the variables will differ between accepted and rejected applications. For the most extreme case, suppose it is found that there is no difference between accepted and rejected applications in terms of covariance between requested amount and firm’s age and also no difference in their means. This would imply random or inconsistent criteria.22

In general, analysis of the covariance structure of the variables (information) contained in the loan applications can be used to compare the relationships and various moments (e.g., means and variances) among these variables across different sub-samples such as between rejected vs. accepted applications or among different banks. In term, such analysis might uncover possible inconsistency in criteria or point out to what were the actually applied criteria.

Before proceeding with specification of a specific econometric model, it is necessary to make the assumption that the information extracted from the application forms is the key information that governed decisions of the loan officers, or that banks had no additional available information on the loan applicants that was systematically missing from the loan applications.

Assuming linear relationships among variables, we specify the model as a special case of
the general LISREL model (Jöreskog, 1973; Bol- len, 1989; Jöreskog et al., 2000; Kaplan, 2000). In matrix notation, the model can be written in three parts; the measurement model for latent exogenous variables is given by
\[ x = A_1 \xi + \delta, \] (1)
and the measurement model for latent endoge-

768 nous variables is
\[ y = A_2 \eta + \varepsilon. \] (2)

751 Finally, the structural part of the model is
\[ \eta = B \eta + \Gamma \zeta + \zeta, \] (3)
where \( A_1, A_2, B \) and \( \Gamma \) are the coefficient matrices and \( \delta, \varepsilon \) and \( \zeta \) are latent errors. Under the assumption of multivariate Gaussian distribution of the observed variables the model coefficients (given the model is identified) could be jointly estimated by minimising the (quasi) multivariate Gaussian likelihood function:
\[ F_{ML} = \ln|\Sigma| + \text{tr}\{\Sigma^{-1}\} - \ln|S| - (p + q), \] (4)
where \( S \) denotes empirical covariance matrix (computed directly from data), \( p \) and \( q \) are numbers of observed endogenous and exogenous variables, respectively, and \( \Sigma \) is the model-implied covariance matrix given by
\[ \Sigma = \begin{pmatrix} A_1(1 - B)^{-1} (\Gamma \Phi \Gamma^T + \Psi)(1 - B)^{-1} \Gamma \Phi \Gamma^T + \Theta \delta & A_1(1 - B)^{-1} \Gamma \Phi \Gamma^T + \Theta \delta \\ \end{pmatrix}. \] (5)

769 However, because our data include non-contin-
770 uous (ordinal-level) variables the Gaussianity assumption is not appropriate and the standard normal theory based on maximum likelihood estimation is not applicable (see West et al., 1995). Estimation methods for structural equation mod-
772 els with ordinal-level variables are considerably more tedious then methods for continuous multi-
774 variate-Gaussian variables (see e.g. Bartholomew and Knott, 1999), Jöreskog (2001a–d) and Jöreskog and Moustaki (2001) describe two main estimation techniques for ordinal-level variables, the underlying response variable approach, and the response function approach. The former can be divided into underlying multivariate Gaussian and bivariate Gaussian approaches. In order to estimate the postulated structural model we use asymptotic methods based on the assumption of underlying bivariate Gaussianity. This method uses weighted least squares (WLS) technique based on the polychoric correlations and their asymptotic variances. The WLS fit function minimises the criterion function \( F_{WLS} = (\hat{\rho} - \sigma(\theta))^T W^{-1}(\hat{\rho} - \sigma(\theta)) \) (see Appendix A for details).

The hypothesis of the overall equality of empirical covariance matrices, i.e., \( S_1 = S_2 = \ldots = S_k \) can be tested with the Box-M statistic, which is given by
\[ M = N \ln|S| - \sum_{i=1}^k N_i \ln|S_i|, \] (6)
where \( k \) is the number of groups. The Box-M statistic is \( \chi^2 \) distributed with degrees of freedom \( (k-1)(p+q+1)(p+q)/2 \).

780 recommend an approach based on estimation of probabilities of various response-patterns (of ordinal responses), advising that multiple ordinal variables should be modelled as a function of the latent underlying continuous variables. Jöreskog and Moustaki (2001) describe two main estimation techniques for ordinal-level variables, the underlying response variable approach, and the response function approach. The former can be divided into underlying multivariate Gaussian and bivariate Gaussian approaches. 

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where \( k \) is the number of groups. The Box-M statistic is \( \chi^2 \) distributed with degrees of freedom \( (k-1)(p+q+1)(p+q)/2 \). If overall covariance structure of the analysed matrices is found to be dissimilar across groups, we can further test for the equality of the parameters in a specific LIS- REL model. We note that testing of general invariance (Box-M) test is weaker than testing the equality of all parameters of a LISREL model (for details see Jöreskog, 1971; Kaplan, 2000).

Finally, if an acceptable model is estimated for the overall sample, and general differences among group covariance matrices are found to be relatively small it is then possible to
compute scores for the latent variables and further tests their mean differences. Alternatively, a LISREL model with means structure can be estimated and latent means can be estimated jointly with the other parameters (see Sörbom, 1978, 1981). Using the parameters of the estimated LISREL model we compute the scores for latent variables following the approach of Jöreskog (2000). This technique computes scores of the latent variables based on the estimated parameters of the LISREL model (see Appendix B for details). The latent scores approach has an advantage that once scores are computed from the full-sample model they can be used in the classical analysis of variance (ANOVA).

5. Estimation and hypotheses testing

First, we estimate the polychoric correlation matrix for the full sample, which requires estimation of threshold parameters for the ordinal-level (non-metric) variables \(y_3, y_4\) and \(x_2\). We obtained the following threshold estimates:

\[
y_4 = \begin{cases} 
5 & \Rightarrow -\infty < \tau_0 < -2.358, 6 \\
7 & \Rightarrow -1.239 < \tau_2 < -0.043, \\
8 & \Rightarrow -0.043 < \tau_3 < 1.111, \\
9 & \Rightarrow 1.111 < \tau_4 < 2.326, \\
10 & \Rightarrow 2.326 < \tau_5 < +\infty, \\
\end{cases}
\]

\[
y_3 = \begin{cases} 
0 & \Rightarrow -\infty < \tau_1 < -0.012, \\
1 & \Rightarrow \tau_1 < +\infty, \\
\end{cases}
\]

\[
x_2 = \begin{cases} 
0 & \Rightarrow -\infty < \tau_0 < -0.016, \\
1 & \Rightarrow -0.016 < \tau_1 < +\infty, \\
\end{cases}
\]

As the validity of the bivariate normality is necessary for the estimation of polychoric correlations we test this, rather then assume it. We computed two tests (results are omitted, but can be obtained upon request), the bivariate normality \(\chi^2\) test and Jöreskog’s test of close fit (Jöreskog, 2001b, appendix). The tests of close fit do not reject bivariate normality for any of the variable pairs, though more restrictive \(\chi^2\) tests do reject on several occasions. Following the advice of Jöreskog (2001c) we rely on the finding that the variables are approximately (bivariate) Gaussian and proceed with estimation of the polychoric correlations.

We estimate the polychoric correlation matrix for the full sample first (Table IV). Next we specify and estimate the structural equation model. Specification of the model is the first problem that must be solved. Strong economic theory that could guide model building for SME loan applications does not exist. Therefore, we develop our model on the grounds of some simple postulated relationships and preliminary exploratory analysis. To this end we initially perform exploratory factor analysis retaining 3 factors (for details of the procedure see Jöreskog and Sörbom, 2001). A three-factor maximum likelihood (ML) solution produced the goodness-of-fit \(\chi^2\) of 16.96 with 12 degrees of freedom, which supports the conjecture that there are only three factors in the data. The factor loadings from the unrotated (ML), verimax, and promax solutions are shown in Table II. The unrotated solution (Jöreskog, 1967) is based on the ML procedure (thus enabling the computation of a \(\chi^2\) fit statistic).
Factor analysis results (full sample, $N = 2395$)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Unrotated</th>
<th>Verimax</th>
<th>Promax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
<td>III</td>
</tr>
<tr>
<td>$y_1$</td>
<td>0.35</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>$y_2$</td>
<td>0.99</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>$y_3$</td>
<td>0.26</td>
<td>0.67</td>
<td>0.35</td>
</tr>
<tr>
<td>$x_1$</td>
<td>0.17</td>
<td>0.51</td>
<td>0.50</td>
</tr>
<tr>
<td>$x_2$</td>
<td>0.06</td>
<td>0.30</td>
<td>-0.08</td>
</tr>
<tr>
<td>$x_3$</td>
<td>0.12</td>
<td>0.94</td>
<td>-0.11</td>
</tr>
<tr>
<td>$x_4$</td>
<td>0.12</td>
<td>0.13</td>
<td>-0.04</td>
</tr>
<tr>
<td>$x_5$</td>
<td>0.06</td>
<td>0.44</td>
<td>-0.02</td>
</tr>
<tr>
<td>$x_6$</td>
<td>0.07</td>
<td>0.55</td>
<td>-0.09</td>
</tr>
</tbody>
</table>

We postulate that these three factors correspond to three latent variables: the firm’s characteristics ($\eta_1$) measured by $x_1$, $x_2$, $x_3$, $x_4$ and $x_5$, characteristics of the project ($\eta_2$) measured by $y_1$ and $y_2$, and business prospects of proposals ($\eta_3$) measured by $y_3$ and $y_4$. Therefore, in the structural model we include three latent variables, corresponding to these factors. The observed correlations among factors can be accounted for by estimation of a structural part of the model. The direction of causality, however, must follow substantive logic and cannot be empirically tested. We assume that firm’s characteristics are exogenous to the other two latent variables, and that business prospects of proposals (i.e. purpose and repayment period) affect characteristics of the project (i.e. amount requested and the number of new jobs). The model notation is defined in Table III.

We now formulate and estimate a particular (non-recursive) LISREL model comprised of measurement and structural parts. The measurement model for the firm’s characteristics (exogenous) latent variable is specified as a special case of Eq. (1), namely

$$
\begin{bmatrix}
X_1 \\
X_2 \\
X_3 \\
X_4 \\
X_5 
\end{bmatrix} =
\begin{bmatrix}
\lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{14} & \lambda_{15} \\
\lambda_{21} & \lambda_{22} & \lambda_{23} & \lambda_{24} & \lambda_{25} \\
\lambda_{31} & \lambda_{32} & \lambda_{33} & \lambda_{34} & \lambda_{35} \\
\lambda_{41} & \lambda_{42} & \lambda_{43} & \lambda_{44} & \lambda_{45} \\
\lambda_{51} & \lambda_{52} & \lambda_{53} & \lambda_{54} & \lambda_{55}
\end{bmatrix}
\begin{bmatrix}
\xi_1 \\
\xi_2 \\
\xi_3 \\
\xi_4 \\
\xi_5
\end{bmatrix} +
\begin{bmatrix}
\delta_1 \\
\delta_2 \\
\delta_3 \\
\delta_4 \\
\delta_5
\end{bmatrix},
$$

(8)
and similarly, the measurement model for the two latent endogenous variables (characteristics of business project and business prospects of proposal) is specified as a special case of Eq. (2) as follows:

\[
\begin{pmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & \gamma_{11} & \gamma_{13} \\
0 & 1 & \gamma_{22} & \gamma_{24} \\
0 & 0 & \gamma_{31} & \gamma_{34} \\
0 & 0 & 0 & \gamma_{41}
\end{pmatrix}
\begin{pmatrix}
\eta_1 \\
\eta_2 \\
\xi_1 \\
\xi_2
\end{pmatrix} +
\begin{pmatrix}
\epsilon_1 \\
\epsilon_2 \\
\epsilon_3 \\
\epsilon_4
\end{pmatrix}.
\]  

(9)

Finally, the structural part of the model is specified as a special case of the Eq. (3) as follows:

\[
\begin{pmatrix}
\eta_1 \\
\eta_2
\end{pmatrix} =
\begin{pmatrix}
0 & \beta_{12} \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
\eta_1 \\
\eta_2
\end{pmatrix} +
\begin{pmatrix}
\gamma_{11} & \gamma_{13} \\
\gamma_{21} & \gamma_{23}
\end{pmatrix}
\begin{pmatrix}
\zeta_1 \\
\zeta_2
\end{pmatrix}.
\]  

(10)

Full coefficient matrices corresponding to Eqs. (8)–(10) in the LISREL notation are specified as follows:

\[
\Lambda_x =
\begin{pmatrix}
1 & \lambda_{x1} & \lambda_{x3} & \lambda_{x4} \\
\lambda_{x1} & 1 & \lambda_{x2} & \lambda_{x4} \\
\lambda_{x3} & \lambda_{x2} & 1 & \lambda_{x3} \\
\lambda_{x4} & \lambda_{x4} & \lambda_{x3} & 1
\end{pmatrix},
\]  

\[
\Lambda_y =
\begin{pmatrix}
1 & \lambda_{y1} & \lambda_{y3} & \lambda_{y4} \\
\lambda_{y1} & 1 & \lambda_{y2} & \lambda_{y4} \\
\lambda_{y3} & \lambda_{y2} & 1 & \lambda_{y3} \\
\lambda_{y4} & \lambda_{y4} & \lambda_{y3} & 1
\end{pmatrix}.
\]  

(8)

and the residual covariance matrices are specified as

\[
\Theta_x =
\begin{pmatrix}
\delta_1 & 0 & 0 & 0 \\
0 & \delta_2 & 0 & 0 \\
0 & 0 & \delta_3 & 0 \\
0 & 0 & 0 & \delta_4 \\
0 & 0 & 0 & 0 & \delta_5
\end{pmatrix},
\]  

\[
\Theta_y =
\begin{pmatrix}
\xi_1 & 0 & 0 & 0 \\
0 & \xi_2 & 0 & 0 \\
0 & 0 & \xi_3 & 0 \\
0 & 0 & 0 & \xi_4 \\
0 & 0 & 0 & 0 & \xi_5
\end{pmatrix}.
\]  

(11)

Estimation of the model with the WLS technique produced an overall fit \( \chi^2 \) statistic of 53.66 (\( p = 0.001 \)), which is not a perfect fit; however empirically based model modifications\textsuperscript{28} did not achieve significant improvement in the fit. Alternative fit measures indicate approximately good fit of the model with normed fit index (NFI) = 0.98; non-normed fit index (NNFI) = 0.98; relative fit index (RFI) = 0.98; and the adjusted fit index (AFI) = 0.99 (see Jöreskog, et al. 2000 for details on these indices). The standardised root mean square residual of the model is 0.019, which is also indicative of relatively good fit.

The WLS parameter estimates (\( N = 2395 \), standard errors are in parentheses) are obtained as

\[
\Phi = \phi_{11} \quad \text{and} \quad \Psi = \begin{pmatrix}
\xi_1 & 0 \\
0 & \xi_2
\end{pmatrix}.
\]  

(12)

\[
\begin{pmatrix}
1 & 0 \\
2.28(0.42) & 0 \\
0 & 1 \\
0 & 0.70(0.09)
\end{pmatrix},
\]  

(13)

\[
\begin{pmatrix}
1 & 3.08(0.32) & 0.48(0.11) & 1.44(0.17) & 1.78(0.18)
\end{pmatrix},
\]  

(14)

\[
E(\eta_1^T) =
\begin{pmatrix}
0.15 & 0.11 & 0.80 \\
0.02 & 0.20 & 0.10
\end{pmatrix},
\]  

(15)

\[
\begin{pmatrix}
0.13(0.09) & 0 \\
0 & 0.37(0.10)
\end{pmatrix},
\]  

(16)

\[
\begin{pmatrix}
-0.26(0.10) & 2.10(0.19)
\end{pmatrix},
\]  

(17)

\[
\begin{pmatrix}
0 & 0.20(0.08) \\
0 & 0
\end{pmatrix}.
\]  

(18)
The estimated coefficients are generally well determined and statistically significant. The estimate of $\gamma_{11}$ is negative, which is unexpected, though its significance is marginal. Thus, it appears that firm’s characteristics do not have strong effect on the latent variable measured by the amount of loan and number of new jobs ($\eta_1$). The estimated model seems to be capable of explaining the observed covariances among the modelled variables reasonably well. Therefore, it can serve as a reference model for testing the group differences.

The first multi-group model we estimate compares the accepted and rejected applications, jointly for all banks together. The sub-samples (rejected and accepted) polychoric correlation matrices are given in Table IV.

The Box-M-test (6) for general equality of the correlation matrices of accepted vs. rejected applications is 84.49 with 45 degrees of freedom, which, taking into account that polychoric correlation matrices were used for estimation is not large enough to conclude that the two matrices differ significantly.

The multigroup estimation of the specific LISREL model (8)–(10) with WLS using polychoric correlation matrices from Table IV produced a $\chi^2$ of 138.34 (df = 69). This result was obtained by treating all parameters fixed across both groups, which is equivalent to testing that jointly

$B_{x}^{[A]} = B_{x}^{[R]}, \quad \Gamma_{x}^{[A]} = \Gamma_{x}^{[R]}, \quad \Lambda_{x}^{[A]} = \Lambda_{x}^{[R]},$

$\Phi^{[A]} = \Phi^{[R]}, \quad \Theta_{x}^{[A]} = \Theta_{x}^{[R]}, \quad \text{and} \quad \Theta_{x}^{[A]} = \Theta_{x}^{[R]}$

Relaxing the equality of error variances, i.e. $\Theta_{x}^{[A]} = \Theta_{x}^{[R]}, \quad \text{and} \quad \Theta_{x}^{[A]} = \Theta_{x}^{[R]}$, decreased the $\chi^2$ to 83.62 (df = 57), which is no longer highly significant. We conclude that the two groups of applications differ mainly in the error variances, while the structural parameters, which are of primary importance for our hypotheses, do not appear to be different. Based on these results we do not reject the hypothesis that subsamples of accepted and rejected applications have similar covariance structure (H1). This finding also contradicts the information extracted from interview data reported by Kraft (2002), i.e., that banks evaluate loan requests based on their economic merit and profitability potential (i.e. banks have ‘optimal lending policy’) because in such case far greater difference should exist between covariance structures of rejected and accepted applications.

Estimation of the scores for latent variables produced three new variables corresponding to the latent variables $\eta_1$, $\eta_2$, and $\epsilon_1$. Analysis of variance $F$-test (Table 5) suggests that the mean difference between accepted and rejected applications for $\eta_1$ is highly significant. The rejected applications score significantly higher on latent characteristics of business project ($\eta_1$), which is measured by the requested amount of loan and number of the planned new jobs. This indicates that banks, on average, preferred smaller to larger projects in terms of the size of loan and number of new jobs. This contradicts the conjecture that banks have ‘negative attitudes’ toward small lending and thus rejects the claim that banks prefer larger loans. Therefore, we reject hypothesis H2 and conclude that, ceteris paribus, preference was given, on average, to smaller loan requests. This finding agrees with the conclusion that Kraft (2002) has drawn from interview data (that
banks might be diversifying risk by lending to a
large number of smaller clients.

A similar preference to smaller loans in the
US was pointed out by Edelstein (1975) who
finds that loan size is extremely important; smal-
ner loan requests are more likely to be approved
than larger ones, while it has been demonstrated
that larger approved loans have superior repay-
ment records.

In the present study, we are interested in what
might be the reason for this observed preference
towards smaller loans in the Croatian SB-2000
loan programme? Specifically, we might consider
a possibility that smaller loans are also shorter-
term loans and hence preferred due to risk aver-
sion of the banks. In this context, risk aversion in
the form of ‘filtering out’ the ‘risky’ category of
smaller loans would imply that while the lenders
are unable to assess riskiness of the small loans,
they nevertheless should be able to classify poten-
tial borrowers into those who belong to the risky
category and those who belong to the less risky
types. The (smaller) amount of loan cannot be
the only classifier because we cannot exclude the
possibility that potential borrowers belonging to
‘less risky’ categories might also apply for smaller
loans. Therefore, the risk-filtering explanation
implies that classification is possible on the
grounds of the applicant’s observable characteris-
tics, although risk assessment of their loan
requests might be hindered by lacking informa-
tion. This has immediate empirical consequences,
which are to some degree testable. Namely, in the
context of the covariance structure analysis, risk-
filtering would imply different covariance struc-
tures between smaller and larger loans, and in
addition a link between duration of loans and
their size would be supportive of this explanation.

### TABLE IV
Polychoric correlation matrices

<table>
<thead>
<tr>
<th></th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_3$</th>
<th>$y_4$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_5$</th>
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<td>0.11</td>
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<tr>
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<td>$x_4$</td>
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<td>0.04</td>
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<td>0.55</td>
<td>0.10</td>
<td>0.23</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Therefore, a finding that smaller loans are also short-term and that these loans are generally requested by an observationally distinguishable category of potential borrowers (which itself might be perceived as too risky) would be indicative of credit rationing due to risk-filtering. In principle, this can be tested in our present methodological framework by grouping the sample by size or duration of the loans. However, such an analysis, while potentially informative, might suffer from the sample-selection bias because we would need to use arbitrary selection criteria, and thus violate one of the key assumptions behind the multigroup comparison analysis.

We first note that the correlation between loan size \( y_1 \) and the repayment period \( y_2 \) in the full sample is apparently low (0.09), which does not suggest that smaller loans are also shorter in duration; although the correlation is positive, but the strength of the relationship is not very high.

The situation is similar also in the subsamples of accepted and rejected applications (see Table IV), where this correlation is 0.07 and 0.11, respectively. A grouping into ‘larger’ and ‘smaller’ loans using a somewhat arbitrary criterion of being above and below the mean, respectively, we calculate the Box \( M \)-test for the general equality of the correlation matrices is 72.42 (df = 45), which does not provide strong evidence of significant difference between the two matrices. Similar analysis for the groups with different repayment period (above and below the mean) produced the \( M \)-test of 145.55 (df = 45), which on the other hand indicates significant differences. Proceeding with the multigroup LISREL estimation we find that the hypothesis of equality of factor structural parameters (allowing different error variances) was rejected with a \( \chi^2 \) of 185.76 (df = 69); equality of factor loadings and structural parameters (allowing different factor loadings and error variances) was rejected with \( \chi^2 \) of 167.61 (df = 57), and finally, equality of factor structural parameters (allowing different factor loadings and error variances) was rejected with \( \chi^2 \) of 132.91 (df = 51). This might indicate that the repayment period is related to the riskiness of the loans and hence it might be an indicator used for classification into ‘riskier’ and ‘less risky’ categories. If so, the ‘riskier’ category would be credit rationed, hence there should be significant difference between accepted and rejected applications in terms of the repayment period. However, as indicated by the ANOVA tests in Table V, such difference does not exist \( (p = 0.99) \), in fact, there appear to be virtually no difference in the repayment period between accepted and rejected applications. Therefore, the finding of no significant difference between accepted and rejected applications in terms of the repayment period, together with the result that smaller loan applications are not observationally distinguishable from the larger ones, does not support the risk-filtering explanation for the preference towards the smaller loans.

Table V reports ANOVA results for the observed variables, which further supports the results based on the latent scores. Namely, both indicators of \( \eta_1 \) are individually significantly different between accepted and rejected applications, both being greater for rejected applications.

For testing hypotheses H3 and H4 we first compute correlation matrices for individual banks (accepted applications), which are shown in Table VI. Testing the null of overall equality of correlation matrices (Box-\( M \)-test) produced a \( \chi^2 \) of 821.15 (df = 315), which suggests these matrices are significantly different.

Testing joint equality of all parameters of the estimated LISREL model, i.e., \( B_{ij}^{(A)} = B_{ij}^{(R)} \), \( \Gamma_{ij}^{(A)} = \Gamma_{ij}^{(R)} \), \( A_{ij}^{(A)} = A_{ij}^{(R)} \), \( \Phi^{(A)} = \Phi^{(R)} \), \( \Theta_{ij}^{(A)} = \Theta_{ij}^{(R)} \), and \( \Theta_{ij}^{(A)} = \Theta_{ij}^{(R)} \), resulted with a \( \chi^2 \) of 855.03 (df = 339) which suggests that model parameters significantly differ across subsamples. Relaxing the constraints \( \Theta_{ij}^{(A)} = \Theta_{ij}^{(R)} \) and \( \Theta_{ij}^{(A)} = \Theta_{ij}^{(R)} \) also reduced the \( \chi^2 \) to 660.69 (df = 255). It follows that accepted applications differed in structure across different banks, thus we infer that the applied assessment criteria were not equal. Therefore, we can reject hypotheses H3.

In addition to covariance structure, we also test for the difference in means across banks. Significant difference in means of latent variables would bring in question hypothesis H4, i.e., it would
imply that bank’s preferences, e.g., in terms of size of loans, differ across banks and thus that their preference toward small lending differ as well.

Using the scores computed above we perform ANOVA on accepted applications across banks (Table V) finding significant difference only in $\eta_1$. 

### TABLE V
ANOVA for differences across banks

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<tr>
<th>Variable</th>
<th>Variance</th>
<th>Sum of squares</th>
<th>df</th>
<th>Mean square</th>
<th>F-Test</th>
<th>p-Value</th>
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</table>
Therefore, we reject hypothesis H4 concluding that all banks did not have equal preferences toward small lending.

The issue of comparing banks in respect to their “attitudes” toward small lending is complicated by the fact that means of all submitted applications were not equal across banks, thus it is not appropriate to compare the absolute amounts of accepted or rejected applications among banks and on this basis draw conclusions about banks’ lending preferences. To overcome this problem we define a coefficient $\lambda$ as an indicator of bank’s preference (or attitude) toward lending. We are interested here in the average

<table>
<thead>
<tr>
<th>TABLE VI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Within-group correlation matrices for accepted applications</strong></td>
</tr>
</tbody>
</table>

| Zabrebačka banka (1), $N = 492$ | Privredna banka Zagreb (2), $N = 304$ |
|-----------------------------------------------|
| $y_1$ 1.00 | 1.00 |
| $y_2$ -0.40 1.00 | -0.03 1.00 |
| $y_3$ 0.09 0.34 1.00 | 0.17 0.46 1.00 |
| $y_4$ 0.05 0.04 0.59 1.00 | 0.04 0.40 0.58 1.00 |
| $x_1$ -0.18 0.08 -0.01 -0.03 1.00 | 0.10 0.20 0.11 0.20 1.00 |
| $x_2$ 0.12 0.08 0.55 0.44 0.16 1.00 | -0.26 0.31 0.77 0.61 0.35 1.00 |
| $x_3$ -0.06 0.12 0.31 0.13 0.07 0.12 1.00 | 0.27 0.15 -0.13 0.02 0.01 -0.36 1.00 |
| $x_4$ 0.06 -0.02 0.35 0.21 0.19 0.57 -0.04 1.00 | 0.09 0.38 0.48 0.20 0.25 0.40 -0.18 1.00 |
| $x_5$ 0.16 0.04 0.39 0.21 0.22 0.65 0.27 0.38 1.00 | -0.14 0.21 0.49 0.44 0.10 0.69 0.28 0.32 1.00 |

| Varaždinska banka (3), $N = 67$ | Podravska banka (4), $N = 50$ |
|-----------------------------------------------|
| $y_1$ 1.00 | 1.00 |
| $y_2$ -0.04 1.00 | -0.03 1.00 |
| $y_3$ 0.09 0.34 1.00 | 0.17 0.46 1.00 |
| $y_4$ 0.05 0.04 0.59 1.00 | 0.04 0.40 0.58 1.00 |
| $x_1$ -0.18 0.08 -0.01 -0.03 1.00 | 0.10 0.20 0.11 0.20 1.00 |
| $x_2$ 0.12 0.08 0.55 0.44 0.16 1.00 | -0.26 0.31 0.77 0.61 0.35 1.00 |
| $x_3$ -0.06 0.12 0.31 0.13 0.07 0.12 1.00 | 0.27 0.15 -0.13 0.02 0.01 -0.36 1.00 |
| $x_4$ 0.06 -0.02 0.35 0.21 0.19 0.57 -0.04 1.00 | 0.09 0.38 0.48 0.20 0.25 0.40 -0.18 1.00 |
| $x_5$ 0.16 0.04 0.39 0.21 0.22 0.65 0.27 0.38 1.00 | -0.14 0.21 0.49 0.44 0.10 0.69 0.28 0.32 1.00 |

| Erste (5), $N = 44$ | Poečeška banka (6), $N = 57$ |
|-----------------------------------------------|
| $y_1$ 1.00 | 1.00 |
| $y_2$ 0.30 1.00 | 0.30 1.00 |
| $y_3$ 0.31 0.56 1.00 | 0.31 0.56 1.00 |
| $y_4$ 0.01 0.57 0.64 1.00 | 0.01 0.57 0.64 1.00 |
| $y_5$ 0.01 0.06 0.20 0.16 1.00 | 0.12 0.27 0.51 0.35 1.00 |
| $x_1$ -0.61 0.73 0.63 0.47 0.48 1.00 | 0.02 0.30 0.60 0.54 0.48 1.00 |
| $x_2$ -0.09 0.12 0.24 0.12 0.23 0.38 1.00 | -0.22 0.13 -0.04 0.15 0.00 0.14 1.00 |
| $x_3$ -0.18 0.12 0.24 0.12 0.23 0.38 1.00 | -0.15 0.10 0.21 0.24 0.21 0.49 0.00 1.00 |
| $x_4$ 0.04 -0.10 0.34 0.06 0.24 0.38 0.00 0.00 1.00 | -0.02 0.41 0.49 0.55 0.27 0.43 0.11 0.15 1.00 |

| Dubravačka banka (7), $N = 62$ | Raiffeisen (8), $N = 76$ |
|-----------------------------------------------|
| $y_1$ 1.00 | 1.00 |
| $y_2$ -0.04 1.00 | -0.04 1.00 |
| $y_3$ 0.07 0.40 1.00 | 0.09 0.36 1.00 |
| $y_4$ -0.02 0.30 0.58 1.00 | 0.18 0.43 0.61 1.00 |
| $y_5$ 0.09 -0.08 0.32 0.16 1.00 | 0.07 0.01 0.39 0.19 1.00 |
| $x_1$ -0.22 0.34 0.73 0.50 0.44 1.00 | 0.10 0.25 0.77 0.58 0.31 1.00 |
| $x_2$ 0.22 0.34 0.73 0.50 0.44 1.00 | 0.10 0.25 0.77 0.58 0.31 1.00 |
| $x_3$ -0.04 0.09 0.25 -0.03 0.02 0.08 1.00 | 0.13 0.13 0.04 0.05 0.13 0.23 1.00 |
| $x_4$ -0.19 0.35 0.41 0.06 0.18 0.57 0.04 1.00 | 0.10 0.02 0.30 0.13 0.10 0.42 0.14 1.00 |
| $x_5$ 0.14 0.16 0.49 0.34 -0.09 0.64 0.06 0.16 1.00 | 0.04 -0.09 0.20 -0.16 0.05 0.47 0.12 0.15 1.00 |
sizes of particular latent quantities and wish to compare their means in sub-samples of accepted and rejected applications. We define \( \lambda \) as

\[
\lambda = \exp \left( \frac{\bar{x}_A}{\sigma_A} - \frac{\bar{x}_B}{\sigma_B} \right),
\]

(11)

where \( \bar{x}_A \) and \( \bar{x}_B \) are means of the accepted and rejected applications, respectively, and \( \sigma_A \) and \( \sigma_B \) are their standard deviations. The \( \lambda \) coefficient is computed for the latent variables, i.e., their estimated scores. We compute \( \lambda \) for each of the three latent variables (Table VII), although of primary interest is \( \lambda \) for \( \eta_1 \) (characteristics of business project) because it intends to measure banks’ preferences toward loan size (note that \( \eta_1 \) is measured by positively correlated amount of loan and number of new jobs).

The \( \lambda \)'s for other two latent variables also have meaningful interpretation due to specific nature of the covariance structure of their observed indicators. Namely, both sets of indicators, for \( \eta_2 \) and for \( \xi_1 \) are positively correlated (see Table IV) and each of them in some way measures the “size” factor of the underlying latent concepts. Specifically, larger values of the latent scores of \( \eta_2 \) indicate proposals that are more oriented toward production and have higher repayment period (i.e., longer-term loans); similarly, larger latent scores for \( \xi_1 \) indicate firms that are, on average, older, more likely to be in the production sector, have higher number of employees, larger previous credit, and greater annual turnover. Therefore, comparison of means of the latent scores across banks, so some degree, provides information

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<td>( x )</td>
<td>( s )</td>
<td>( N )</td>
<td>( x )</td>
<td>( s )</td>
<td>( \lambda )</td>
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\( \lambda \) = \text{sample mean}; s = \text{standard deviation.}
about the preferences and attitudes of particular banks for lending to certain categories of firms and business projects (larger vs. smaller, in particular). The idea behind the $\lambda$ coefficient is to measure the difference in means between approved and rejected applications by assigning higher values to larger positive differences, thus giving higher score to those banks that favour larger loans over the smaller ones ($\eta_1$), more production oriented with longer repayment period ($\eta_2$), and given to larger firms ($\zeta_1$). Figure 1 plots the $\lambda$ coefficients for all eight commercial banks, calculated for each of the three latent variables.

The highest overall value for all three $\lambda$ coefficients has Varaždinska banka, which also has the highest absolute values on $\lambda$ coefficients for $\eta_1$, and $\eta_2$, followed by the Raiffeisen bank, which also has high overall score. Požeška banka and Zagrebačka banka score low on $\lambda$ for $\eta_1$, which indicates a tendency to approve smaller loans, though apparently requested by larger firms ($\zeta_1$). This type of behaviour seems consistent with the assumption of high risk aversion and strongly contradicts the hypothesis H2, i.e., smaller loans are, in fact, strongly preferred.

The values of $\lambda$ for $\eta_1$ allow the comparison of different lending patterns in terms of loan size (i.e., scope of the loan measured by its monetary value and the number of newly opened jobs) in relation to the size of the commercial banks. We find no support to the survey results of Kraft (2002) where a link between bank’s size and its lending preference was claimed. Namely, larger banks did not show higher preference for larger loans, in fact, the $\lambda$ coefficient for $\eta_1$ is second lowest for Zagrebačka banka, the largest bank in the SB-2000 programme, while on the other hand, some smaller banks (e.g., Varaždinska banka and Raiffeisen Bank Austria) have very high $\lambda$’s. Therefore, we cannot reject hypothesis H5; there is no evidence of a positive correlation between the bank size and its loan-size preferences. Such finding differs from the US results reported by Berger et al. (1995: 89-92) who find that most of the small lending is done by smaller banks, and that large banks make very few small loans. The U.S. results are consistent with the literature on borrower-lender relationship where such relationship is considered to increase the probability of receiving a loan (see e.g., James, 1987; Lummer and McConnell, 1989; Hoshi et al., 1991; Slovin et al., 1993; Peterson and Rajan, 1994; Billett et al., 1995; Berger and Udell, 1995; Blackwell and Winters, 1997; Cole, 1998). Since smaller banks generally tend to have closer and longer relationships with smaller clients than the large banks do, a strong positive link between bank size and loan size is expectable. On the other hand, finding of a weak link indicates possible lack of banking tradition which is plausible in transitional countries with young and still under-developed banking system.
6. Discussion

This paper proposed a new multivariate methodological framework based on structural equation modelling and covariance structure analysis for analysing consistency and determinants of the loan approval process. We analysed Croatian SB-2000 programme using data from submitted loan applications and investigate consistency and determinants of the commercial banks' loan assessment criteria. Modelling the covariance structure of loan applications allowed comparison of accepted and rejected applications and testing for their difference. We investigated whether the accepted applications consistently differed from the rejected ones. In addition, we extended multi-group analysis to testing for differences across banks.

The results indicated that, on average, commercial banks lacked consistency in the loan approval criteria; hence the low loan approval rate was likely a consequence of credit rationing due to lack of loan assessment skills among the loan officers. Hence, the alternative explanation of high lending standards and optimal lending policy could not be sustained in light of the empirical evidence. In particular, both accepted and rejected applications appear to have similar covariance structures and similar coefficients in the estimated structural model. On the other hand, the results showed that banks, on average, preferred smaller loans and smaller firms.

This finding, however, might not be indicative of their understanding and support for SMEs, rather it might be a sign of high risk aversion or lack of relevant business and market research data needed for evaluation of the SME business projects. In particular, smaller loans might also be shorter-term loans and hence preferred due to risk aversion of the banks that might be 'filtering out' the risky category of smaller borrowers. This would imply that while the banks might not be able to assess the small lending risk, they nevertheless should be able to classify potential borrowers into those who belong to the risky category and those who belong to the less risky categories.

We tested this implication in the context of covariance structure analysis where risk-filtering would imply different covariance structures between smaller and larger loan applicants, and an additional link between duration of loans and their size would be supportive of this explanation. The results, however, indicate a relatively small correlation coefficient between loan size and repayment period, which hence does not support the assumption that smaller loans are on average also shorted in duration. A multigroup analysis of differences between 'larger' and 'smaller' applicants' groups did not find significant evidence of group differences. Multigroup analysis with subsamples of loans with different repayment period, on the other hand, found significant differences, namely, the overall equality; equality of factor loadings; and equality of structural parameters was rejected. However this finding has no direct relevance for lending decisions because approved and rejected loan applications did not differ significantly in terms of the repayment period, hence, apparently, these differences were not utilised by the banks in the loan assessment process.

Differences among the eight banks that participated in the SB-2000 programme were also found. Comparison of covariance structures of accepted applications across banks revealed significant differences, and similar differences were also found in the means of estimated latent variables, most notably in the average amount of the approved loans. Based on a simple measure we defined with the purpose of capturing individual bank's preferences toward lending scope, we conclude that banks differed in their preferences toward small-lending. However, we found no relationship between bank's size and average loan size, thus no evidence was found that smaller banks prefer smaller loans and vice versa, which is contrary to the situation in developed countries (e.g. in the US) where it often the case that smaller banks tend to lend to smaller borrowers more than the larger banks do. We interpret this finding as a consequence of lacking banking tradition and a young banking system where long-term relationships between banks and borrowers were not yet formed, hence the relationship-based higher loan approval rates to small borrowers made by smaller lenders is lacking in Croatia.

Given the empirical results from the SB-2000 programme, some broader policy conclusions could be drawn. First, it seems unlikely that increased supply of loan guarantee funds and/or establishment of new loan-guarantee agencies...
would itself remedy the SME lending problem. Namely, if the problem is in inadequate loan assessment skills in the banks, increased supply of guarantee funds runs a risk of higher loan default rate. This follows from inconsistency in loan assessment decisions and hence lacking ability of the banks to assess potential lending risk. Therefore, along with the supply of loan guarantees and credit funds, the government should support training schemes for loan officers and possibly also for the staff of the local business centres. The EBRD, for example, is considering technical assistance programmes for the banking officers in Croatia. This is an important initiative because the domestic institutions (e.g. Croatian Banking Association) evidently lack capacity to implement such training schemes alone. Furthermore, an improvement in the application procedures aiming at disclosing more of the lending-risk related information could decrease information asymmetry and make the loan assessment procedure more efficient.

Acknowledgments

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Notes

1 The relationship between banks and SME clients is an important issue requiring special practical and theoretical considerations (see e.g. Bornheim and Herbeck, 1998).

2 In the Jaffe and Russell (1976) model credit could be profitably rationed, hence credit rationing could be rational (i.e. profit-maximising) lending policy. Hess (1984) showed that this conclusion is flawed due to a confusion of competitive supply curves with zero profit curves in the Jaffe and Russell model (see also Jaffe and Russell, 1984 for a reply).

3 The empirical literature on credit rationing is rather extensive. Recent research on credit rationing is generally focused on particular categories of small borrowers such as SMEs (e.g. Berger and Udell, 1992, 1995), households (e.g. Chakravarty and Scott, 1999), or ethnic minorities (e.g. Dyminski and Mohanty, 1999) or on particular categories of lending such as mortgages (see inter alia Goering and Glennon, 1996; Munnell et al., 1996; Ladd, 1998). Credit rationing is also widely investigated in the context of the borrower-lender relationship where the empirical focus is on investigating the effects of such relationships on the firm’s value or on its strength (James, 1987; Lummer and McConnell, 1989; Hoshi et al., 1991; Slovin et al., 1993; Peterson and Rajan, 1994; Billett et al., 1995; Berger and Udell, 1995; Blackwell and Winters, 1997; Cole, 1998, etc.).

4 Specifically, Stiglitz and Weiss (1981) show that asymmetric information between borrowers and banks might lead to refusal of loans to some of the observationally identical borrowers. Therefore, it is possible that the observational equivalence, and thus credit rationing, is due to the lack of information that banks have on the potential borrowers.

5 These issues might have important policy implications. If credit rationing occurs as a consequence of insufficient loan funds, a policy aiming at an increased supply of credit funds could be an effective measure. The case when observationally identical potential borrowers are being credit rationed has more complex policy connotations. The problem here might be a consequence of asymmetric information between borrowers and lenders who can reject loan applications of otherwise qualified borrowers. This implies that it might be possible to overcome credit rationing by producing or collecting information about the borrower and using it in the loan assessment decisions. The information about borrowers is closely related to the risk of default, hence measures aimed at providing more information about the borrowers could reduce riskiness of the loans and thus diminish credit rationing.

6 While a novelty in transitional countries, loan programmes such as Croatian SB-2000, exist in the western countries, particularly in the US, for considerable time. Some of the best known US examples include the “Project OWN” established by the US federal government in 1968 with the purpose of fostering growth and supporting minority owned businesses, which included direct government loans and indirect assistance through commercial bank loans that were insured against default risk by the Small Business Administration (see Bates, 1975). Another US example, very similar to the SB-2000, is the “Philadelphia’s eight-bank minority loan
program", administered through the Job Loan and
Urban Venture Corporation of Philadelphia, which
functioned as an intermediary between the banks and
their potential borrowers doing pre-loan screening, and
included a loan guarantee programme in which eight
US banks participated on a proportionate basis (see
Edelstein, 1975).

The international policy issues concern primarily the
European Commission and the EBRD and the allocation
of the EU technical assistance funds (i.e. CARDS) for
SME development. The World Bank is also supporting
the SMEs in Croatia, mainly through structural adjust-
ment loans.

Note that in the Stiglitz and Weiss model the ‘dis-
tinguishable group’ type of credit rationing, by defini-
tion, could be remedied through increased supply of
credit funds, and because the supply of credit is suffi-
cient (even excessive) in Croatia, such form of credit
rationing seems implausible in this case. Furthermore,
because we are specifically analysing an SME loan
scheme, it follows that we cannot compare the loan
approval rates of SMEs with those of the large compa-
nies, as the later were not eligible for the SB-2000 lend-
ing, hence only variation in size within the SMEs is
relevant here.

The main policy issue relates to design and implemen-
tation of various training programmes for loan officers
and training programmes for local business centres,
entrepreneurs and local SME consultants. Naturally, this
implies a policy priority but not necessarily an exclusive
choice between the two approaches. It is also important
to add that a third approach, namely design and imple-
mentation of additional loan funds for SMEs was
abounded by both the European Commission and the
Croatian Government due to sufficient liquidity of the
commercial banks.

Generally, the alternative approaches in the loan
approval determinants literature investigate which charac-
teristics of the potential borrowers are statistically signifi-
cant loan approval determinants, under an (implicit or
explicit) assumption that the applied loan assessment
criteria are consistent. Alternatively, an
exclusion of the bank, which might gain access to informa-
tion from alternative sources is left at the discre-
tion of the bank, which might gain access to informa-
tion not contained in the application forms. (hence there are no specific requirements of attending
an interview or for arranging site visits, as usual with
standard commercial loans), acquisition of additional
information on credit histories, debt burdens, loan-to-
value ratios, and other factors considered in making
mortgage decisions (see Munnell et al., 1996). For exam-
ple, while the SB-2000 programme has a formalised
application procedure based on the applications alone
(hence there are no specific requirements of attending
an interview or for arranging site visits, as usual with
standard commercial loans), acquisition of additional
information from alternative sources is left at the discre-
tion of the bank, which might gain access to informa-
tion not contained in the application forms.

The HMDA data includes US loan application data on
over 12 million loan applicants from over 3000 lender,
making it the most comprehensive loan application
data set available for the research on loan approval
determinants and discrimination in lending.

Note that this example implicitly assumes that all rel-
levant information is contained in correlations, thus
excluding the possibility that some complex non-linear
criteria were applied consistently. Alternatively, an
assumption of (underlying) multivariate Gaussianity can
justify linear specification.

The bivariate Gaussian method is based on limited
information maximum likelihood estimation (LIML) of
the underlying continuous variables, while the multivariate
approach requires full information maximum likelihood
(FIML). Jöreskog and Moustaki point out that bivariate
LIML approach have greater flexibility and ability to handle larger number of latent and observed variables.  

24 We do not pursue this approach primarily because we wish to use the estimated latent scores in secondary analysis (ANOVA). In addition, the results obtained with latent scores approach are asymptotically equivalent to latent means estimates from the means-structure model (for further discussion see Kaplan, 2000).

25 LISREL 8.54 computer programme was used for estimation (see Czirký, 2003).

26 The factor analysis was performed on the estimated polychoric correlation matrix for the full sample (Table VIII) thus the use of maximum likelihood $\chi^2$-fit statistic is correct. Note, however, that performing the same analysis on the raw data would not be appropriate due to the presence of ordinal variables.

27 Czirký et al. (2002a, b) estimate a similar model with three latent variables; such "triangular" non-recursive models are often found more stable and better performing than the more complex alternatives (see also Czirký et al., 2003).

28 We compute model-modification indices proposed by Sörbom (1989).

29 Note that we report ANOVA results for all observed variables for convenience, though ANOVA is strictly inapplicable to ordinal variables ($y_3$, $y_4$ and $x_3$). The ANOVA results relating to metric variables ($y_1$, $y_2$, $z_1$, $y_3$, $y_4$ and $x_3$) are, on the other hand, appropriate.

30 The multi-group model across banks was estimated without the use of asymptotic covariance matrices, which could not be computed for samples of this size. Therefore, the reported $\chi^2$ statistics should be interpreted more conservatively.

31 The exponential is taken to make all values positive.

### APPENDIX

#### A. Weighted least-squares estimation

The method of weighted least-squares (WLS) is based on polychoric correlations and their asymptotic variances. The WLS fit function is given by

$$ F_{WLS} = [\hat{\rho} - \sigma(\theta)]^T W^{-1} [\hat{\rho} - \sigma(\theta)], $$

where $\hat{\rho} = \text{vech}(S)$, $\rho \in R^{(q(q+1)/2)}$, $\sigma(\theta) = \text{vech}(\Sigma)$ and $W \in R^{(q(q+1)/2) \times (q(q+1)/2)}$ is a positive definite weight matrix. A typical element of a suitable matrix $W$ is given by

$$ W_{mn} = \frac{1}{N} \sum_{k=1}^{N} (x_{km} - \bar{x}_m)(x_{kn} - \bar{x}_n)(x_{ji} - \bar{x}_i) \times (x_{kj} - \bar{x}_j) - S_{mnsij}, $$

(A.2)

where $x_k$ are sample observations and $s_{ij}$ are bivariate sample correlations. This method requires computation of polychoric correlations, which are based on the assumption of underlying (unobserved) continuous Gaussian variables. Polychoric correlation is a correlation between two ordinal variables. A correlation between an ordinal-level and a continuous variable is called polyserial and, a correlation between two binary (dummy) variables is usually termed tetrachoric. We refer to correlations among all estimated ordinal-level variables as "polychoric" for simplicity, though we note the correlation matrices in Table IV contain Pearson, polyserial, and tetrachoric correlations (depending on the types of variable pairs).

Joreskog (2001a–d) describes an approach based on estimation of thresholds for the unobserved variables. For an observed ordinal variable with $m$ discrete levels $z = 1, 2, \ldots, m$, a true (unobserved) value of $z$, i.e., $z^*$, is assumed to be in the interval $\tau_{i-1} < z^* < \tau_i$ where $l = 1, 2, \ldots, m$ and $-\infty = \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_{m-1} < \tau_m = +\infty$ are threshold parameters. First, the probability of a response in category $i$ is given by

$$ p_i = \Pr(z = i) = \Pr(\tau_{i-1} < z^* < \tau_i) = \int_{\tau_{i-1}}^{\tau_i} \Phi(u)du = \Phi(\tau_i) - \Phi(\tau_{i-1}), $$

(A.3)

where $\Phi(*)$ is the Gaussian distribution function. It follows that thresholds can be estimated by inverting $\Phi(*)$, i.e.,

$$ \tau_i = \Phi^{-1}\left(\frac{\sum i p_i}{m}\right), \quad i = 1, 2, \ldots, m - 1. $$

(A.4)

Note that $p_i$ can be consistently estimated by $p_{i0}$, the sample percentage of responses in category $i$, i.e., $p_i = p_{i0}$. Finally, the polychoric correlation coefficient $\rho$ between a variable pair $(1, 2)$ can be estimated by maximising the bivariate Gaussian likelihood function

$$ F_{PC} = m \sum_{i=1}^{m} \sum_{j=1}^{m} p_{ij} \ln \int_{\tau_{i-1}}^{\tau_i} \int_{\tau_{j-1}}^{\tau_j} \frac{1}{2\pi \sqrt{1-\rho}} \exp \left[ -\frac{1}{2(1-\rho^2)} (u^2 - 2\rho uv + v^2) \right] du dv, $$

(A.5)
where $m_1$ and $m_2$ are the numbers of response categories in variables 1 and 2, respectively; $z_1^{(1)}, z_2^{(1)}, \ldots, z_{m_1-1}$ are thresholds for the variables $z_1$ and $z_2$, respectively. Letting $p_{ij} = n_{ij}/N$ be the sample proportions, maximising (A5) is equivalent to minimising the following fit function

$$
\hat{F}_{PC} = \sum_{i=1}^{m_1} \sum_{j=1}^{m_2} p_{ij} \left\{ \ln p_{ij} - \ln \int_{t_1}^{t_2} \int_{t_2}^{t_3} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp \left[ -\frac{1}{2(1-\rho^2)} (u^2 - 2\rho uv + v^2) \right] du dv \right\}.
$$

For multi-group comparisons an appropriate method is the Jöreskog’s (1971) procedure for evaluating group differences in respect to group covariance matrices and group-specific model estimates (see inter alia Sörbom, 1981; Bollen, 1989; Kaplan, 2000). Specifically, we can test our hypotheses by comparing $B_{(A)} = B_{(R)}$, $\Gamma_{(A)} = \Gamma_{(R)}$, $\Lambda_{(A)} = \Lambda_{(R)}$, $\Psi_{(A)} = \Phi_{(R)}$, $\Theta_{(A)} = \Theta_{(R)}$, and $\Theta_{(A)} = \Theta_{(R)}$, where ‘A’ and ‘R’ stand for accepted and rejected, respectively, and $B_{(A)}$, $\Gamma_{(A)}$, $\Lambda_{(A)}$, $\Psi_{(A)}$, $\Phi_{(A)}$, $\Theta_{(A)}$, $\Theta_{(R)}$ are LISREL coefficient matrices.

**B. Computing latent scores**

The factor scores technique of Lawley and Maxwell (1971) and Jöreskog (2000) computes scores of the latent variables based on the estimated parameters of the Eqs. (1)–(3). Writing Eqs. (1) and (2) in a system

$$
\begin{pmatrix}
  y \\
  x
\end{pmatrix} =
\begin{pmatrix}
  \Lambda \\
  0
\end{pmatrix}
\begin{pmatrix}
  \xi \\
  \eta
\end{pmatrix} +
\begin{pmatrix}
  \epsilon \\
  \delta
\end{pmatrix},
$$

and using the following notation

$$
\begin{align*}
\Lambda &\equiv \begin{pmatrix}
  \Lambda_y \\
  0
\end{pmatrix}, \\
\xi &\equiv \begin{pmatrix}
  \eta \\
  \xi
\end{pmatrix}, \\
\delta &\equiv \begin{pmatrix}
  \epsilon \\
  \delta
\end{pmatrix}, \\
x &\equiv \begin{pmatrix}
  y \\
  x
\end{pmatrix},
\end{align*}
$$

the scores for the latent variables of a general LISREL model can be computed using the formula

$$
\xi_i = U D^{1/2} V L^{-1/2} V D^{1/2} U T \Psi_{(A)}^{-1} x_i,
$$

where $UDV^T$ is the singular value decomposition of $\Phi_{(A)} = E(\xi_i, \eta_i \delta_i)$, and $V D^{1/2} U T \psi_{(A)}^{-1}$ is the singular value decomposition of the matrix $D^{1/2} U^T B D^{1/2}$, while $\Theta_{(A)}$ is the error covariance matrix of the observed variables (for details on derivation of the Eq. (A9) see Cziráky et al., 2002c). The latent scores $\xi_{ij}$ can be computed for each observation $x_i$ in the $(9 \times N)$ sample matrix $X = \begin{pmatrix} x_1, x_2, \ldots, x_N \end{pmatrix}$ whose rows are observations on each of our 9 observed variables and $N$ is the sample size, i.e.,

$$
\begin{pmatrix}
  x_{11} \\
  x_{21} \\
  \vdots \\
  x_{n1}
\end{pmatrix}
= \begin{pmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_N
\end{pmatrix}.
$$

**References**


