Productivity, Taxation and Evasion:
A Quantitative Exploration of the Determinants of
the Informal Economy

VERY PRELIMINARY

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Abstract
This paper evaluates the relative importance of labor productivity vs. income
taxes and social security contributions for tax compliance in an economy with a large
degree of informality. To this end, we build a bargaining model in which matched
employer-employee pairs of heterogeneous productive capacities make decisions on
output sharing and the degree of tax evasion. The quantitative model takes as inputs
the income tax structure and the estimated aggregate productivity series. The
estimation strategy recovers the bargaining parameters and the cost function of tax
evasion in the model by matching the empirical series for the size of the informal
sector (2000-2014). The results from the performed computational experiments
point out that the most important factor is labor productivity, followed by the
corporate tax. Income tax progressivity in Bulgaria is found not to be quantitatively
relevant for tax evasion.

JEL Classifications: H24, H25, H26; C63; E62, E65

Keywords: Informal economy, progressive taxation, tax evasion, flat-tax reform

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1 Introduction and Motivation

How (and if at all) do taxes influence informality and the division of income between capital and labor? This paper contributes to this literature by quantifying the relative importance of labor productivity vs. the role of (personal and corporate) income taxes and social contributions for tax compliance in an economy with a large degree of informality.¹ Informality is a wide-spread phenomenon globally; however, the size of the grey economy is particularly high in Central and Eastern European countries;² For example, as documented in Packard et al. (2012) and Schneider and Buehn (2013), Bulgaria has the largest informal sector share relative to the official GDP (32 %) among all European countries, followed by Greece (24%) and Hungary (21%). In addition, during the last two decades Bulgaria underwent several important tax reforms: Bulgaria lowered substantially the social contributions paid by the employer, cut the corporate tax rate from 32.5% in 2000 to 10% in 2007 to attract foreign investors. Finally, Bulgaria introduced a proportional income taxation system (effective January 1, 2008), which supplanted the previously progressive tax regime in Bulgaria: Instead of facing an increasing marginal tax schedule (0-20-22-24), applied until end-2007, a uniform (flat) tax rate of 10% was used afterwards.

Therefore, Bulgaria provides an excellent ground for our research question, and the obtained results from the model in this paper could be then interpreted as an upper bound for the other EU member states (e.g. Greece, Italy, Spain, in particular). Furthermore, as a (former) transition economy, Bulgaria still carries some heritage from socialist times. For example, the shares of agriculture and manufacturing are higher than the respective average shares across the EU (El-Haj and Brada 2004). As documented in the EU Commission Report (2014), transition economies have a relatively large agricultural sector, and the degree of informality there is also high. The explanation could be in the predominant use of part-time workers, and that not all labor is registered. In addition, labor mobility within Bulgaria is quite low (NSI 2016), with migration flows mostly from

¹According to Maffezzoli (2011), Busato and Chiarini (2004, 2013) and Vasilev (2015b), the move from progressive to flat income tax systems, coupled with a decrease in effective tax rates, increases compliance and efficiency. Other studies investigating the effects of tax reform on tax evasion via micro-simulations include Gorodnichenko et al. (2009), Duncan and Peter (2010). The macro-labor literature on informal economy is represented by Albrecht et al. (2009), and Meghir et al. (2015).

²In this paper, "informal economy" refers to the unregistered production of legal goods for the purposes of income tax and social contribution evasion, and circumventing certain legal labor standards. Illegal, i.e., criminal, activity and unpaid work will be excluded from the discussion. We follow the definition of underground production by the OECD: "all legal production activities that are deliberately concealed from public authorities for the following kind of reasons: to avoid payment of income, value added or other taxes; to avoid payment of social security contributions; to avoid having to meet certain legal standards such as minimum wages, maximum hours, safety or health standards, etc."
peripheral urban areas to the capital (or abroad). Furthermore, another important factor for informality in Bulgaria could be the existence of regional monopsonies (single employers) in manufacturing. The presence of monopsonies is an important feature of the labor market in Bulgaria, and it can rationalize both the high unemployment, and the lower wages, despite the increase in productivity during the last two decades.

Last, but not least, in the rapidly growing sector of services during the transition period, another form of tax evasion became popular: As documented in the recent World Bank Enterprise survey, almost all firms in Europe are formally registered. Therefore, a stark distinction between formal and informal firms cannot be made. Instead, we should focus on this semi-formality aspect: that not all transactions are being recorded, that not all workers are on the payroll, or that at least some receive part of their wages in unmarked envelopes. In this way employers and employees evade income taxes and social security payments, which make up to 3/4 of total tax revenue in Bulgaria (NSI 2016). In service industries such as tourism, construction, entertainment the phenomenon of "envelope wages" (Williams (2008), Williams (2014b)) might also be prevalent, being three times larger than EU average, with some workers receiving up to 40% of their wage in an envelope.

Our study takes seriously into consideration both the presence of a large informal sector, and the tax reform, since, as pointed out in Schneider and Enste (2013), the growth in the shadow economy is caused by the overall tax burden (personal and corporate income taxes and social security dues combined). After all, labor income taxes (and social contributions) are likely to be quantitatively important for labor-intensive activities with low productivity, which are most of the sectors in Bulgaria. If taxes are considered to be too high and the tax system is viewed as unfair, then taxes are not paid over the full income level. The perception of unfairness can be rooted in that fact that most of the tax revenue is spent on transfers for the unemployed, or in the quality and coverage of public services. Lastly, productivity growth, coupled with salary increases in the official sector, would lead to contraction in the the size of the informal sector.

In terms of addressing the issues described above through theoretical setups, several

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3 The socialist regime produced regional monopolies, which provided most of the employment. After the change of the regime in the 1990s, many of those became private monopolies.

4 In our study we will abstract away from institutional quality considerations, as level of institutional quality, proxied by different indicators, has not changed substantially since the EU accession. Most of the gains have been already absorbed during the negotiation period. Since we focus on exogenous policies, and not on optimal government response to what is happening in the economy, the behavior of government is treated as being passive.
representative-agent models of tax policy evaluation have been constructed for Eastern European countries: Funke and Strulik (2003) explore the welfare effect of Estonia’s flat tax reform in 2003, Azacis and Gillman (2010) quantify the effect of the flat tax reform in all three Baltic countries, while recent studies by Vasilev (2015a, 2015b) did the same for Bulgaria. To the best of our knowledge, no micro-founded theoretical/structural analysis of tax policy capturing income and wealth heterogeneity has being done for any other Eastern European country. This paper aims at filling this gap and contributes to the work by Lemieux et al. (1994) for Canada, Caucutt et al. (2003) and Carroll and Young (2009) for the US. As in Guner et al. (2014), before engaging in theoretical modelling, we begin by documenting the specificity of the Bulgarian tax system, as well as providing a summary of the structure of all different types of social contributions to be made by employers and employees over the years. After documenting the stylized facts on taxes in Bulgaria, we continue to produce the corresponding income tax functions by following Bulgarian tax-, and social security legislation, and plot those functions to compare their behavior over the years, both before and after the reform. The estimated tax functions are then introduced into a model framework.

Aside from personal income taxation, including social contributions paid by both employers and employees, and corporate taxation in the model setup is crucial: since the amounts of social contributions made are very weakly related to the size of the pension to be received upon retirement in Bulgaria, those payments made during the working cycle are effectively a tax on labor, even though technically social contributions are deferred income and an important part of the social security system. Thus, when viewed as an additional burden on labor, these contributions represent the largest share of overall labor taxation, and thus can be effectively taken as an important aspect of the tax code. Similarly, corporate taxes affect profits, and thus might have effect on employment and evasion decisions.

Next, motivated by the World Bank Enterprise Survey results above, the modelling philosophy adopted in this paper is that the official and the unofficial production are identical, so all firms in the economy would be potentially "semi-formal." In other words, each firm will be formally registered, but under certain conditions not all activity will be registered. This is where the contribution of the paper lies. More specifically, the novel research direction, which has been largely ignored in earlier studies, produces new and interesting results when compared to earlier findings in the literature. We set up a model that con-

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5We are also ignoring bequests and pension considerations in the analysis to follow, and consider a closed-economy case. That is clearly unrealistic, but allows us to focus on the important issues at hand. For simplicity, government consumption in the model will be assumed to be entirely wasteful.
sists of a large number of islands, inhabited by heterogeneous firms and workers, which all differ in terms of their productivity. It will be assumed that workers are not allowed to change islands, which reflects the low labor mobility within Bulgaria (NSI 2016). Hiring and the subsequent production follow a search and matching procedure: i.e., a firm opens a single vacancy that requiring a match with a worker of a suitable ability. When the match is realized, the employer and the worker have to decide whether they will report all production and income truthfully to tax authorities, or report only a portion of it.

The main focus of the island model falls on the "report-or-not" choice margin faced by employers and employees: i.e., deciding not to report income arises as an optimal decision from a setup with information friction. As in Fortin et al. (1997), and Slemrod and Yitzhaki (2002), a technology of income hiding is introduced to impose an opportunity cost of tax evasion, which is convex in the amount of income hidden. In order to evade taxes, both sides have to agree to engage in income hiding, which takes place if both sides are better-off under evasion. Evasion is possible in the model because when the employer and the employee jointly claim that the worker is of lower productivity level, that in turn justifies the lower reported wage rate. The employer then declares a lower level of profit and hides the difference between the actual and declared level of production, net of the cost of hiding. Since the tax authority cannot directly observe individual effort level, and as long as the income evasion is consistent with the distribution of individual ability levels across the population of workers, income evasion remains unnoticed. The gain from hiding income for the employee are the savings on income taxes and social contributions. The employer also benefits by saving on the employer’s social contributions, which are a percentage of the worker’s gross wage. Thus, in order successfully to trick the government, agents will have to adopt a mimicking strategy; thus, we will investigate whether the flat tax leads to a "truth-telling" equilibrium outcome.

To understand the quantitative importance of taxes, and to isolate the effect of labor productivity, we perform several computational experiments. To discipline our theoretical model, and to make it approximate Bulgarian economy along the important for the research question dimensions, we use simulated method of moments (SMM) technique to match the estimated size of the informal sector, the average income and the level of total

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6 In what is to follow, we will use "firm owner" and "employer," and "worker" and "employee," interchangeably.

7 In Stantcheva (2015), the government also has imperfect information regarding the ability of agents. In contrast to her study, we abstract from human capital considerations. The simplification is introduced since most of the workers in Bulgaria are middle-aged individuals, most of whom do not invest in their education any longer.

8 Feldstein (1999) and Chetty (2009) interpret it as a transfer/resource cost.
factor productivity in the Bulgarian data before the reform. This allows us to recover important model parameters, such as the evasion cost parameters and the bargaining weight. Then computational experiments are performed to decompose the individual quantitative effect of the different factors driving informality. In other words, the exercise allows us to rank order the importance of different taxes and productivity. In the first exercise, we shut down labor productivity growth and leave informality to be determined only by income taxes and social security contributions. In the second, we keep taxes at their 2000 levels to study the effect of productivity on the size of the informal sector. Lastly, we assume that the progressive income taxation regime was never abolished and compare against introduction of the flat tax regime. The results from the performed quantitative experiments point out that the introduction of the flat tax regime is quantitatively irrelevant in the model. This comes in stark contrast with the findings in Vasilev (2015b) for Bulgaria vs. Duncan and Peter (2010) and Gorodnichenko et al. (2009) for Russia.\footnote{The difference could be due to the size of the economy - small (Bulgaria) vs. large (Russia).}

What matters mostly in our paper is labor productivity, which is quantitatively more important than taxes. Among the taxes considered in the model, the most important one is the corporate tax, because ultimately it is the employer who makes a decision about the surplus and how it will be split. After all, wages in the model are a small part of the total surplus, so the profit (capital) share dominates. This is consistent with the stylized behavior of labor and capital income shares in Bulgarian data. Therefore, it comes as no surprise that the tax on the aggregate surplus is the major one.

The reason why income taxation does not matter is that given the income distribution in Bulgaria, even before the adoption of the flat tax people were on average (effectively) paying 10\% of their income in the form of income tax. Thus, the flat rate introduced in 2008 was not set arbitrarily, but instead reflected the economic reality. Also, the share of revenue from personal income taxation in Bulgaria is quite low, 9-10\%. Another reason could be that productivity heterogeneity, and informational frictions matter. Another explanation can be the fact that the two technologies/sectors (official and informal) are identical. Ignoring heterogeneities and informational problems might be the important limitation of earlier studies. In addition, the representative-agent models listed above also might suffer from aggregation bias problems, while our model with heterogeneities does not. Given that all these aspects are ignored in representative-agent models with two-sectors, the results from setups with a single-member household are to be interpreted with caution.
In terms of social contributions, there is a change in the composition of employee contributions over the period, but little change in the aggregate magnitude. In contrast, there is a substantial decrease in employer’s contribution, but the decrease is done mostly in the early 2000s. In our model we abstract away from consumption taxation for several reasons: (i) income tax revenues and social contributions are much more important for government revenue than consumption taxes, and (ii) there are no important changes in the consumption tax rate. Lastly, consumption tax is a tax on demand, while the others are taxes on factors of production (inputs), or taxes on supply. As shown in Uhlig and Trabandt (2015), there is no consumption-tax Laffer curve. Moreover, we also abstract from modelling institutional quality, since it is well documented that it does not change much over the period\textsuperscript{10}, and therefore is unlikely to explain the variation in informality.

The rest of the paper is organized as follows: Section 2 presents the facts about personal income taxation, corporate taxation, and social contributions. Section 3 presents the theoretical model. Section 4 matches the model to the data\textsuperscript{11}. Section 5 contains the results of several counterfactual quantitative exercises and section 6 concludes.

\textsuperscript{10}Regarding institutional efficiency and quality of government, see evidence provided in Appendix A.4.  
\textsuperscript{11}I would say "section x discusses the estimation procedure".
2 Facts and Institutional Design

2.1 Social Security System in Bulgaria (2000-2014)

This subsection describes the specifics of the social security system in Bulgaria. We focus on the period that stared several years after the introduction of the currency board, which brought a great deal of macroeconomic stability in the economy. First, in Bulgaria both the employer and the employee make contributions towards different insurance pools, and the employee receives a payout in case one of those contingencies occurs. We discuss each type of contribution in more detail below.

2.1.1 Employee Contributions

According to the social security legislation in Bulgaria, each employee makes contributions towards unemployment, general (disease and maternity), old-age pension ("first pillar" of the pension system, which is state-managed), supplementary compulsory pension insurance ("second pillar," also state-managed; the "third pillar" consists of voluntary contributions to a private pension fund), and health insurance. These contributions generally vary during the years, but the percentage changes are not significant. (see Fig. on the next page) The sum of these contributions equals the Total Contribution Payment, which is deducted from the gross salary of an individual. If the gross salary exceeds the legislated ceiling income for contribution purpose for the year (e.g. 2600 BGN per month for 2015), the contribution payments are calculated based on that ceiling amount.

Unemployment Contributions: provide the workers with monetary benefits during temporary periods of unemployment. A worker cannot receive unemployment benefits if: (a) the worker is currently employed; (b) S/he has not been legally hired and therefore has not paid unemployment contributions; (c) S/he has not been registered as being unemployed; (d) S/he has not found another employment during the limited period for which he received the unemployment benefits. In the period 2000-2004, the unemployment contribution was stable at 1% of the gross salary. The increase to 1,05% was sustained for 2005, 2006, and 2005. In 2008, the unemployment contribution underwent a large decrease to 0,4% of the gross salary and has been unchanged since then.

General (Disease and Maternity) Contributions: They sustain a fund that provides insurance for maternity leave and temporary reduced working capacity. In 2000, the General (Disease and Maternity) contributions were set at 0,75% but underwent a series of increases - in 2005, 2006, and 2008, with the new rates being respectively 0,9%, 1,23%, and 1,4%. 
Pensions: It is a state-managed "fund for disability due to general (non-occupational) disease, old age and death" (National Tax Agency 2016). Pensions for disability are given if a person has lost fully or partly his or her working capability for an extended period or forever. A candidate for a disability pension must prove that he or she has lost 50% or more of his or her working capability. Pensions for old age are two types: (1) Social Pension - It pertains to unemployed with a minimum total insurance record of 15 years but less than the required length of service to retire under full Old-age and Insurance Record Pension. The social pension equals roughly 85% of the employment minimum full old-age and insurance pension. (2) Full Old-age and Insurance Pensions/Employment-based Pension: It is given to people who have completed a required length of service and are of the age that has been set by the government as the retirement age. (3) Death Pensions: In certain cases, once a retired person dies, a selected circle of his or her closest relatives can be eligible for receiving a part of the deceased person’s pension under certain conditions. The Pension Contributions have been reduced almost every year and are generally decreasing. Still, there are 4 years during which the contributions were increased compared to the previous year - in 2003, 2005, 2008, and 2011.

Supplementary Compulsory Pension Insurance: This is the second pillar of the pension system and it gives those born after 31.12.1959 the opportunity for additional pension and/or early retirement for those who work under the first and the second category of labor. After being set at 0,5% in 2000, this contribution underwent a series of increases

Health Insurance: The fund offers coverage of emergency care and many non-emergency procedures, checks, blood-tests, etc. For the period 2000-2005, the Health contributions were set at 1.8% of the gross salary, and has undergone three rises since then - in 2006, 2008, and 2009 with the new rates being 2.1%, 2.4%, and 3.2%.

Total Contribution Payments: Overall, the total contribution has been going up and down for the period 2000-2014 and although the lowest rate was at 10.8% in 2004, most rates are in the 12%+ range.

2.1.2 Employer Contributions

Similarly, in order to abide by the social security legislation in Bulgaria, each employer has to make the following contributions on the worker’s account: Unemployment, General (Disease and Maternity), Pensions, Supplementary Compulsory Pension Insurance, Health, Employment Accidents and Occupational Diseases, Insolvency of Employer (2000-2007)/Guaranteed Contributions (2008-2010). These contributions generally vary during the years, (see Table) but the percentage changes are not significant. The sum of these contributions equals the Total Contribution Payment, which is deducted from the gross salary of an individual and must be paid by the employer. Again, if the gross salary exceeds the legislated ceiling income for contribution purpose for the year (e.g. 2600 BGN per month for 2015), the contribution payments are calculated based on that ceiling amount.

Unemployment Contributions: The unemployment contribution rates have been steadily decreasing over the period 2000-2014. In 2000, it was set at 3% and underwent three reductions - in 2005, 2006, and 2008, with the new rates being 2.45%, 1.95%, and 0.6% respectively.

General (Disease and Maternity): In 2000, this contribution was set at 2.25% and remained fixed until 2005 when it was decreased to 2.1%. In 2006, it underwent an increase to 2.27% but in 2008 was again reduced to 2.1%.

Pension Contributions: The pension contribution has generally been reduced from 24% to 7.1% over the period 2000-2014, with the exception of two years, in which the rate was increased relative to the previous year – in 2002 and in 2010.
Compulsory Pension Insurance: This contribution had ups and downs in its rate, but comparing the values between 2000 and 2014, the supplementary compulsory pension insurance almost been doubled from 1.5% to 2.8%.

Health Contributions: In 2000, the rate for the health contribution was set at 4.2% and the decreases in 2006 and 2008 to 3.9% and 3.6% respectively were a sign of a decision to continually reduce this contribution. However, in 2009, it was increased to 4.8% and has remained fixed since then.

Employment Accidents and Occupational Diseases: This fund ensures that workers who have suffered any employment accidents and/or occupational diseases can be provided with: (a) Monetary compensations during a period of temporary reduced working capacity; (b) Monetary compensations for medical expenses and rehabilitation; (c) Disability Pension; (d) Monetary compensations for assistive technology connected to the disability. For the period 2000-2004, this contribution rate was set at 0.7%, but was reduced to 0.4% in 2005 and has been fixed at this rate since then.

Insolvency of Employer (2000-2007)/Guaranteed Contributions (2008-2010): This fund ensures that in the case of insolvency of the employer, employees are guaranteed to be paid the outstanding salaries to a certain amount. For the entire period, during which this contribution was implemented (2000-2007), the rate was set at 0.5%. Then, during 2008-
the fund was renamed to "Guaranteed Contributions" and the rate was reduced in 2009 from 0.5% to 0.1%.

Total Contribution Payments: Generally, the total contribution payment is observed to be decreased by a half from 36.15% in 2000 to 17.8% in 2014. There was only one increase in the total rate compared to the previous year - in 2003, when it was higher by 2.25% relative to the rate in 2002.

2.2 Personal Income Taxation

This tax is levied on individuals and small businesses (called DDFL or DOD in Bulgaria). The Personal Income Tax DOD/DDFL: the income tax code can be divided into two sub-groups: (i) progressive income tax system (2000-07), and (ii) proportional taxation (flat tax) - as of Jan.1, 2008. The levels and the progressivity of the two income tax schedules are depicted in Fig. 5 on the next page.

2.2.1 Progressive Income Taxation Period

Under this system, the workers gross salary net of contributions made by the employee is split in bins, called tax brackets, and the income in different sub-categories is taxed at different marginal tax rates, which increase with the bracket. The DDFL tax is calculated

\[ \text{DDFL tax} = (\text{salary \net of contributions}) \times \text{marginal tax rate} \]

In both cases the tax is applied after the employee contributions are deducted from the gross salary. Thus, it is applied on the difference between the gross salary and the total contribution payments.
on the basis of the nominal level of the worker’s salary. There is no ceiling income when it comes to the DDFL/DOD. We will treat incomes expresses as ratios to the average income in the same year. This normalization takes away any nominal effects, and allows us to make consistent comparisons of incomes and the tax burden over time. Graph of the behavior of the tax function for personal income. For simplicity, the effect of contributions has been isolated.\textsuperscript{13}

\begin{figure}
\includegraphics[width=0.5\textwidth]{income_tax_function.png}
\caption{Income tax functions: Bulgaria (2000-2014).}
\end{figure}

\textsuperscript{13}Given that the tax rate is applied to the nominal gross salary, brackets are changed over time to reflect the increase in inflation. The curvature, or the average progressivity of the income tax code, is unchanged over the years.

\subsection{Flat Tax Regime}

This is a proportional taxation system, whose rate has been fixed at 10\%. Again, it is not directly calculated from the gross salary, but rather on the difference between gross salary and total contributions made by the employee.

\subsection{Corporate Income Tax}

This is the tax that corporations pay on their nominal profits. Since 2000, the rate has been decreasing, falling from 25\% to 15\% in 2002, then increasing to 23.5\% in 2003, and then falling to 19.5\% in 2004, 15\% in 2005-06, and down to 10\% since 2007.
As seen from Table 1 above, the average effective tax rate (the first row) does not differ from 10%. To shed light on distributional effects of the tax reform, we consider the tax paid by a person whose income equals half of the mean income in the economy, a person whose income equals the mean income, and a person whose income is three times the mean income in the economy. The tax rate faced by people with lowest (taxable) incomes slightly decreases after the introduction of the flat tax. The people with the lowest income, who are subject to tax pay less as of 2008. Since then the average effective tax rate for them has increased, but it is still lower than the average rate they faced under the progressive tax regime. On the other hand, the social security contributions (being very regressive in their nature) hit lower-income population stronger. However, those workers were compensated by the increase in the minimum wage. Also the average income has increased substantially over the period (so low-income workers are paying a larger slice of an even larger pie). Lastly, since the tax rate faced by people with highest incomes (at the mean and above) decreases substantially, the drop in the rate they face is
much lower under the flat tax regime. Even when the burden of social security is added, there is not much difference across regimes.
<table>
<thead>
<tr>
<th>Year</th>
<th>Bracket</th>
<th>Tax owed</th>
</tr>
</thead>
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<tr>
<td>2000</td>
<td>From BGN 0 to BGN 80</td>
<td>0 %</td>
</tr>
<tr>
<td></td>
<td>Between BGN 80.01 to BGN 115</td>
<td>20 %</td>
</tr>
<tr>
<td></td>
<td>Between BGN 115.01 to BGN 380</td>
<td>26 % + BGN 7</td>
</tr>
<tr>
<td></td>
<td>Between BGN 380.01 to BGN 1400</td>
<td>32 % + BGN 75.9</td>
</tr>
<tr>
<td></td>
<td>Above BGN 1400</td>
<td>40 % + BGN 403.6</td>
</tr>
<tr>
<td>2001</td>
<td>From BGN 0 to BGN 100</td>
<td>0 %</td>
</tr>
<tr>
<td></td>
<td>Between BGN 100.01 to BGN 135</td>
<td>20 %</td>
</tr>
<tr>
<td></td>
<td>Between BGN 135.01 to BGN 400</td>
<td>26 % + BGN 7</td>
</tr>
<tr>
<td></td>
<td>Between BGN 400.01 to BGN 1400</td>
<td>32 % + BGN 75.9</td>
</tr>
<tr>
<td></td>
<td>Above BGN 1400</td>
<td>38 % + BGN 395.9</td>
</tr>
<tr>
<td>2002</td>
<td>From BGN 0 to BGN 110</td>
<td>0 %</td>
</tr>
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<td></td>
<td>Between BGN 110.01 to BGN 140</td>
<td>18 %</td>
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<td>Between BGN 140.01 to BGN 400</td>
<td>24 % + BGN 5.4</td>
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<td></td>
<td>Between BGN 400.01 to BGN 1000</td>
<td>28 % + BGN 67.8</td>
</tr>
<tr>
<td></td>
<td>Above BGN 1000</td>
<td>29 % + BGN 235.8</td>
</tr>
<tr>
<td>2003</td>
<td>From BGN 0 to BGN 110</td>
<td>0 %</td>
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<td></td>
<td>Between BGN 110.01 to BGN 150</td>
<td>15 %</td>
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<td></td>
<td>Between BGN 150.01 to BGN 250</td>
<td>22 % + BGN 6</td>
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<td></td>
<td>Between BGN 250.01 to BGN 600</td>
<td>26 % + BGN 28</td>
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<tr>
<td></td>
<td>Above BGN 600</td>
<td>29 % + BGN 119</td>
</tr>
<tr>
<td>2004</td>
<td>From BGN 0 to BGN 120</td>
<td>0 %</td>
</tr>
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<td>Between BGN 120.01 to BGN 150</td>
<td>12 %</td>
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<td>Between BGN 150.01 to BGN 250</td>
<td>22 % + BGN 3.6</td>
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<td>Above BGN 600</td>
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<td>24 % + BGN 99</td>
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<td>2006</td>
<td>From BGN 0 to BGN 180</td>
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<td>20 %</td>
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<td></td>
<td>Above BGN 600</td>
<td>24 % + BGN 91</td>
</tr>
<tr>
<td>2007</td>
<td>From BGN 0 to BGN 180</td>
<td>0 %</td>
</tr>
<tr>
<td></td>
<td>Between BGN 180.01 to BGN 250</td>
<td>20 %</td>
</tr>
<tr>
<td></td>
<td>Between BGN 250.01 to BGN 600</td>
<td>22 % + BGN 14</td>
</tr>
<tr>
<td></td>
<td>Above BGN 600</td>
<td>24 % + BGN 91</td>
</tr>
</tbody>
</table>
3 The Model

3.1 Introduction and Notes for Further Development

- We need a footnote that explains why we do not have a VAT in the model.
- We need a footnote explaining why social security is not an issue - why we can get away with using a static model?

3.2 Economic Environment

Consider an economy consisting of a large continuum of islands. Each island is occupied by an employer and a worker who collaborate on production of a homogeneous good. The production is undertaken according to a production function

\[ y = z\lambda, \]

where \( \lambda \) is an island-specific productivity and \( z \) is the economy-wide productivity level. The island-specific productivity is distributed according to a log-normal distribution with a mean of \(-\sigma^2/2\) and a variance of \(\sigma^2\). Thus, the average island productivity in the economy is unity and the aggregate production equals \( z \).

The productive pair, the employer and the worker, split the production outcome through a cooperative Nash-bargaining procedure with an employers bargaining power of \( \gamma \) and a worker’s bargaining power of \( 1 - \gamma \). The details of the procedure is explained in the next subsection.

The modelling is in line with Simon et al. (1982): “wages and salaries that are under-reported are generally not reported with the knowledge and aid of the employer involved. Both the employer and employee benefit from 'off the books' arrangements.” (p.8) The cost is assumed to be convex, as Simon et al. (1982), and Williams (2014a) provides evidence that workers with lowest and highest incomes hide the most (as percentage of their income) and benefit the most. Thus tax evasion affects the income distribution (Simon and Witte, 1982, p. 20). Fugazza and Jacques (2004) and Schneider and Enste (2013) provide empirical evidence from psychological studies that the evasion cost is a psychic cost, which is associated with the worker’s presence in the underground economy.

There is a government which taxes the production on all islands in the economy. In particular, the government taxes the labor income of the worker, the business income of the employer, and collects payroll taxes from both the employer and the worker. The government observes perfectly the economy-wide productivity \( z \) but does not know of the island-specific productivity levels \( \lambda \). Thus, it does not perfectly observe the island-specific production levels \( y \).
The social contributions related to a job position are paid by both the employer and the worker via payroll taxes. In particular, if the job position is associated with an earnings level of $w$, then the employer needs to contribute $s_E w$, while the worker pays $s_W w$, where $s_E$ and $s_w$ are proportional tax rates. Labor income net of payroll tax, $(1 - s_w)w$, is subject to the tax schedule $T_W(\cdot)$, which is increasing in the tax base, $T'(\cdot) > 0$, and allows for an arbitrary degree of progressivity, that is, $T''(\cdot) \geq 0$. Business income $e$ is taxed at a proportional rate $t_E$. Note that the payroll taxes $s_E w$ covered by the employer are deductible from business income $e$.

The key element of the model is the ability of the production pair to hide the scale of production they undertake on the island from the government. Hiding production is costly. In particular, if the pair coordinates on hiding $h$, they incur an output loss of $\kappa(h)$. The cost function $\kappa(\cdot)$ is increasing and convex in the hidden amount. The cost of hiding economic activity is in terms of forgone output. It reflects the resources spent on concealing the informal economic activities from the fiscal authorities.\(^{14}\) Then, the reported production is given by $\hat{y} = y - \kappa(h) - h$.

3.3 The Case of No Evasion

Suppose the employer and the worker do not hide any amount of production, i.e. $\hat{y} = y$. The outside option of both parties in this case is zero. Thus, the Nash-bargaining problem is defined as

\[
\max_{e \geq 0, w \geq 0} \left\{ [(1 - t_E)(e - s_E w)]^\gamma [(1 - s_W)w - T_W((1 - s_w)w)]^{1-\gamma} \right\} \tag{1}
\]

subject to

\[
e + w = y.
\]

The surplus of the employer is given by the amount of earned business income $e$ net of payroll taxes and corporate income taxes. The worker’s surplus equals the earnings $w$ net of payroll and labor income taxes. The solution of the above problem can be summarized by the functions which define the split of the total production,

\[
e = e^*(y) \tag{2}
\]

and

\[
w = w^*(y). \tag{3}
\]

In essence, functions (2) and (3) map the level of production to the income of the

\(^{14}\)Find some anecdotal evidence on this.
employer and the worker.

Define the after-tax income levels as

\[ c^*_E(y) = (1 - t_E)(e^*(y) - s_Ew^*(y)) \]  
\[ c^*_W(y) = (1 - s_W)w^*(y) - T_W((1 - s_W)w^*(y)) \]  

In the case of no evasion it is possible to derive some analytical results, and even closed-form solutions, for the decision rules (2)-(5). We relegated these results in Appendix (A.5).

3.4 The Case of Tax Evasion

Suppose that the production pair can hide the level of production they conduct on their island, thus evading taxation. The technology of tax evasion is associated with the cost \( \kappa(h) \) in terms of production, where \( h \) is the total hidden income.

The bargaining procedure now is slightly different. First, the threshold levels defined by the outside options of the players are not zero anymore. They are equal to the after-tax incomes \( c^*_E \) and \( c^*_W \) in the case of no evasion. Second, the employer and the worker need to decide on the reported level of production \( \hat{y} \). Third, they need to obey the publicly observed splitting rule stemming from problem (1), that is, the observed business and labor income should be given by

\[
(\hat{e}, \hat{w}) \in \arg \max_{e \geq 0, w \geq 0} \{(1 - t_E)(e - s_Ew)^\gamma \left[ (1 - s_W)w - T_W((1 - s_W)w)^{1-\gamma} \right] \}
\]
\[ \text{s.t. } e + w = y. \]

For instance, if the total agreed reported production is \( \hat{y} \), then the the reported employer’s income is consistent with the decision rule (2) and equals \( \hat{e} = e^*(\hat{y}) \). The reported worker’s income is given by \( \hat{w} = w^*(\hat{y}) \) in accord with decision rule (3). Simply put, the reported incomes of the employer and the worker sum up to the total reported production net of the cost of evasion,

\[ \hat{y} - \kappa(h) = e^*(\hat{y}) + w^*(\hat{y}). \]

Therefore, choosing the total hidden amount is equivalent to choosing the reported level of production. Then, the bargaining problem amounts to making decisions about the levels of non-reported income by each party. Denote these amounts as \( h_E \) and \( h_W \).

This motivates the following representation of the bargaining problem for an employer
and worker with a production capacity $y$,

$$\max_{h_E \geq 0, h_W \geq 0} \left\{ \left[ (1 - t_E)(\hat{e} - s_E \hat{w}) + h_E - c^*_E(y) \right] \gamma \left[ (1 - s_w)\hat{w} - T_W((1 - s_w)\hat{w}) + h_W - c^*_W(y) \right]^{1 - \gamma} \right\}$$

subject to the incentive compatibility\textsuperscript{15} constraints

$$\hat{e} = e^*(\hat{y}),$$

and

$$\hat{w} = w^*(\hat{y}),$$

where

$$\hat{y} = y - \kappa(h_E + h_W) - h_E - h_W = \hat{e} + \hat{w}.$$  

Note that reported income levels for both the employer and the employee must be non-negative, i.e. $\hat{e} \geq 0$ and $\hat{w} \geq 0$. In other words, agents are mimicking other types successfully by adopting the policy rule (truth-telling mechanism) from the non-evasion case. Only in such a way can they remain under the radar.

The solution to the bargaining problem is given by the decision rules on hidden incomes,

$$h_E = h^*_E(y)$$

and

$$h_W = h^*_W(y).$$

Then, we can redefine the reported income levels for the employer and the worker as functions of production capacity, $y$,

$$\hat{e} = e^{**}(y),$$

and

$$\hat{w} = w^{**}(y).$$

### 3.4.1 A Useful Result\textsuperscript{16}

We can also solve the tax evasion problem in two stages\textsuperscript{17} (from a numerical point of view it simplifies a lot). In the first stage, the employer and the worker determine the amount

\textsuperscript{15}Maybe we should call them "mimicking" or "no-detection" constraints.

\textsuperscript{16}Shall we move this subsection to the appendix?

\textsuperscript{17}The no tax evasion problem is left unchanged and delivers $e^*(\cdot)$ and $w^*(\cdot)$. 

20
of tax evasion $h$ in order to maximize the size of the pie, taking into account the incentive compatibility constraints. Basically they compare the marginal benefit of hiding income (less taxes) to the marginal cost (hiding income is a costly activity). In the second stage they bargain to divide the gains from evasion.

**Stage 1** Choose how much income $h$ to hide (equivalently, how much income $\hat{y}$ to report) to maximize after-tax production net of the cost of hiding income:

$$\max_{h} \{ \hat{y} + h - t_c (\hat{e} - s_e \hat{w}) - s_e \hat{w} - s_w \hat{w} - T_w ((1 - s_W) \hat{w}) \}$$

s.t.

$$\hat{y} = y - h - \kappa (h), \quad \hat{y} \geq 0, \quad \hat{e} = e^* (\hat{y}), \quad \hat{w} = w^* (\hat{y}).$$

**Stage 2.** Given optimal amount of evasion $h^*$ determined in stage 1, they solve

$$\max_{h_e, h_w} h_e^{\gamma} h_w^{1-\gamma}$$

s.t.

$$h_e + h_w = h^*$$

This delivers the following simple splitting rule:

$$h_e = \gamma h^*, \quad h_w = (1 - \gamma) h^*.$$ 

**Proposition 1** The two formulation are equivalent.

**Proof.** See Appendix (A.1). ■

### 3.5 Aggregate Statistics

The total production capacity in the economy is given by the level of the aggregate productivity shock $z$ because $E(\lambda) = 1$,

$$Y = z.$$ 

What is the size of the underground economy here? The fraction of the underground
economy production out of the total production capacity is

\[ H = \int \left[ h_E^*(z\lambda) + h_W^*(z\lambda) \right] dF(\lambda). \]

What is the aggregate efficiency loss due to underground production? This is

\[ L = \int \left[ \kappa(h_E^*(z\lambda) + h_W^*(z\lambda)) \right] dF(\lambda). \]

What is the reported production in the economy (GDP)? This is given by

\[ \hat{Y} = Y - H - L. \] (8)

What is the size of the underground economy relative to reported production? The size of the underground economy is \( H/\hat{Y} \).

What is the tax revenue raised by the government?

\[ T(T_E, T_W) = \int \left[ T_E(e^{**}(z\lambda)) + T_W(w^{**}(z\lambda)) \right] dF(\lambda). \] (9)

Note that expression (9) describes a generalized Laffer curve for this economy.

The goal of the paper is to estimate the changes in the size of the underground economy, \( H \), the efficiency loss, \( L \), reported production, \( \hat{Y} \), an tax revenue, \( T \) as functions of the individual tax schedules faced by employers and workers.
4 Quantitative Analysis

The model admits no closed form solution\textsuperscript{18}. We simulate the model for the years 2000-2014. The model period is one year.

We use non-parametric tax schedules: the personal income tax, $T_W(\cdot)$ and the corporate business tax, $T_E(\cdot)$. Finally we feed into the model the social security contributions $S_E(\cdot)$ and $S_W(\cdot)$ for the employer and the employee, respectively.

Functional Forms

We choose a parsimonious functional form for the cost of evasion, with only two parameters: $\beta$ which governs the level of the cost, and $\theta$ which controls the curvature

$$\kappa(h) = \beta \exp(\theta h)$$

For the distribution of the island-specific productivity shock we choose a lognormal distribution:

$$\log \lambda \sim N\left(-\frac{\sigma^2}{2}, \sigma^2\right)$$

This implies that $E(\lambda) = 1$

Parameters to be Estimated

We need to estimate the two parameters related to the cost of hiding: $\beta, \theta$. These two parameters are identified in particular by the time series on the informal economy size for the 2000-2014 period. We can identify the bargaining parameter $\gamma$ (more precisely, the employer’s bargaining power) by matching the level and the evolution of aggregate wages over time (2000-2014). We pin down the variance $\sigma^2$ of the idiosyncratic shock by matching the income inequality that we observe in the data. In this case we ask the model to match the 2000-2014 average only (not the time series) since our model cannot capture by construction many features that affected inequality in the data (globalization, skill-biased technological change, trade shocks, etc). Unobserved labor productivity: $\{z_t\}_{t=2000}^{2014}$. We use this to match the observed labor productivity in the data between 2000 and 2014.

Data Targets

In this subsection we discuss the moments that we attempt to match. We need to estimate 19 parameters; to do so we choose to match 46 Targets. The data targets are as follows:

- Informal economy: 15 moments

\textsuperscript{18}See however the appendix with analytical results.
- Observed average wages: 15 moments
- Output per worker: 15 moments
- Gini coefficient: 1 moment (2000-2014 average)

It is well understood that all the model parameters affect all of the above moments. We can, nonetheless, explain how the identification strategy works: in other words, which parameter matters most for which target. Heuristically, a target is informative about an unknown parameter if that target is sensitive to changes in the parameter. In order to pin down the parameters related to the cost of hiding ($\beta$ and $\theta$) we attempt to match the time series evolution of the size of the informal economy. The mean of the informal sector size is roughly informative about the level of the cost of evasion, captured by the parameter $\beta$ (trivially, a higher $\beta$ shifts down the informal sector size for all the years), whereas the slope of the decrease of tax evasion over time is affected more by $\theta$.

The evolution of wages over time helps identify the employer’s bargaining power. The wage inequality observed in the data helps identify the variance of the island-specific shock $\sigma^2_\lambda$. Finally our last target is the evolution of labor productivity (observed) between 2000 and 2015. This last set of moments is informative about the true production capacity of the economy, before the resource cost from evasion and the hidden income are taken into account. This is interesting because we can use the model to estimate a moment that cannot be observed in the data.

4.1 Estimation Strategy

The parameters to be estimated are summarized in the following vector:

$$\Theta = \{\beta, \theta, \gamma, \sigma^2_\lambda, \{z_t\}_{t=2000}^{2014}\}$$

Let $m$ represent the vector of 46 targets taken from the data. Let $\hat m(\Theta)$ be the vector of analogous moments obtained from simulating the model; of course the moments from the model, $\hat m_i(\Theta)$, are a function of the parameter vector $\Theta$. Moreover we define the difference between the data targets and the corresponding model moments as follows:

$$g_i(\Theta) = m_i - \hat m_i(\Theta)$$

for $i = 1, \ldots, 46$.

The simulated method of moments chooses the optimal parameters in order to bring the model as close as possible to the data: more precisely, it picks $\Theta$ to minimize a weighted sum of the squared deviations between the data and the model:
\[ \hat{\Theta} = \min_{\Theta} g(\Theta)' W g(\Theta) \]

where \( W \) is any positive semidefinite matrix. For simplicity we choose the identity matrix, i.e. \( W = I \). Notice that the estimator \( \hat{\Theta} \) is consistent\(^{19}\) for any positive semidefinite matrix (what is the optimal weighting matrix in our setup?). The variance-covariance matrix of the estimator \( \hat{\Theta} \) is given by \( J(\hat{\Theta}) \)

\[
Var(\hat{\Theta}) = \frac{1}{n} \left[ J(\hat{\Theta})' W J(\hat{\Theta}) \right]^{-1} J(\hat{\Theta})' W \Sigma W J(\hat{\Theta}) \left[ J(\hat{\Theta})' W J(\hat{\Theta}) \right]^{-1} \quad (10)
\]

where \( n \) is the number of observations and

\[
J(\hat{\Theta}) = \frac{\partial \hat{m}(\Theta)}{\partial \Theta}
\]

measures the sensitivity of the model moments to the parameters and \( \Sigma \) is the var-cov matrix of data moments. Notice that if the weighting matrix \( W \) is set equal to \( \Sigma^{-1} \) (i.e. if \( W \) is the optimal weighting matrix), then formula (10) collapses to

\[
Var(\hat{\Theta}) = \frac{1}{n} \left[ J(\hat{\Theta})' W J(\hat{\Theta}) \right]^{-1}
\]

### 4.2 Estimation Results

The parameter estimates are summarized in Table 3. The third column of the table reports the point estimates and the last column reportes the associated standard errors [to be completed!].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Cost of evasion parameter</td>
<td>5.58158 \cdot 10^{-7}</td>
<td>TBC</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Cost of evasion parameter</td>
<td>0.00579</td>
<td>TBC</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Employer bargaining power</td>
<td>0.97</td>
<td>TBC</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>Variance of idiosyncratic productivity</td>
<td>0.47</td>
<td>TBC</td>
</tr>
<tr>
<td>( {z_t}_{t=2000}^{2014} )</td>
<td>Output per worker</td>
<td>see figure</td>
<td>TBC</td>
</tr>
</tbody>
</table>

Table 3: Parameter Estimates

\(^{19}\)While the SMM estimator is consistent for arbitrary weighting matrix (for example, \( W = I \)), it is not necessarily efficient (Lee and Ingram [1991]).
The estimates for the cost of evasion parameters, $\beta$ and $\theta$, do not have an immediate economic interpretation. It is interesting to focus on the estimate for the employer’s bargaining power, $\gamma$. The ability of our bargaining model to match the data hinges on a very high value for $\gamma$, close to 1 and definitely higher than 0.9, meaning that the employer is able to get most of the income generated by production. This result suggests that taxes affecting the employer’s profits are going to play a major role in determining the decision to hide income. Taxes levied on the workers’ income, on the contrary, do not have a significant role precisely because wages are only a tiny fraction of total production.

The estimated value of variance of idiosyncratic productivity...

Figures (6), (7) and (8) will help the reader evaluate the goodness of fit implied by the model. The model does a fairly good job in matching the evolution of the informal economy over time in Bulgaria. It slightly overemphasizes the importance of the shadow economy at the beginning of the sample and slightly underpredicts it for the last two years, but overall the fit is quite satisfactory. It is however evident from figures (7) and (8) that the model has some difficulty mimicking the time series evolution of observed labor productivity and wages\textsuperscript{20}.

<table>
<thead>
<tr>
<th>Averages (2000-2014)</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Informal sector</td>
<td>0.337</td>
<td>0.338</td>
</tr>
<tr>
<td>Observed wages</td>
<td>202.78</td>
<td>193.39</td>
</tr>
<tr>
<td>Output per worker</td>
<td>8165.25</td>
<td>8791.58</td>
</tr>
<tr>
<td>Gini</td>
<td>0.332</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Table 4: Aggregate Statistics

Finally, in order to assess the goodness of fit on the aggregate dimension, we compute the average over time of the selected targets and we compare the actual moments with those from our simulated model. As Table (4) shows, the model has no problem matching most of the aggregate targets, but for the Gini coefficient on labor income\textsuperscript{21}. The model overpredicts inequality by a factor of 1.5. This is not surprising, however, since our model cannot capture by construction many features that affect inequality in the data (globalization, skill-biased technological change, trade shocks, etc).

We conclude this (sub)section by discussing two more interesting issues. First, our heterogeneous agents model delivers some predictions regarding the cross-section of tax

\textsuperscript{20}Should we stress that the current version of the paper is still a work in progress, etc.?

\textsuperscript{21}Is it total income or labor income?? Not clear from the data, in the model we computed both and here we reported the gini on total income.
Figure 6: informal Sector Size: Model Fit
Figure 7: Observed Labor Productivity: Model Fit
Figure 8: Observed Labor Productivity: Model Fit
evasion. While we do not have micro data at the individual/establishment level to contrast such prediction, we do find useful to discuss them in light of some anecdotal evidence. Figure (9) plots the share of income hidden from the government as a function of production capacity. It is worth noticing that at the bottom of the productivity distribution, employers and workers decide to hide everything and stay under the radar, while at the top evasion is almost negligible. Moreover, tax evasion (i.e. the fraction of income hidden over total income) declines monotonically with idiosyncratic productivity. This findings seem to be consistent with the evidence provided by the World Bank Enterprise Survey 2007. This survey analyses the prevalence of the undeclared economy by firm size and finds that small and medium-size enterprises are more likely to under-report their earnings. The Eurobarometer Survey on Undeclared Work in the EU (European Commission, 2014) also confirms that employees from small and medium-size companies are more likely to receive part of their remuneration in cash.

![Figure 9: Tax Evasion and Individual Productivity](image)

As we mentioned in the previous (sub)section, the estimated model allow us to retrieve the unobserved production capacity for the economy. We compare the unobserved vs the
Figure 10: Observed vs Unobserved
observed labor productivity in Figure (10). Both series increase over time (with the exception of the 2008-2009); the true production capacity averages around 12000 euros\textsuperscript{22} whereas the observed one averages around 8800 euros. Tax evasion implies therefore a loss of production of roughly 1/4, which is a sizeable quantity. The distance between the two lines captures both tax evasion and the deadweight loss due to tax evasion\textsuperscript{23}.

5 Experiments

Now the model economy is ready to be simulated for the counterfactual experiments. We perform a series of counterfactual experiments in order to quantify the relative effects of taxes and labor productivity on informality. In the first exercise we shut down productivity growth and we solve the model by feeding only the tax structures. The result is shown in Figure (11): the red line represents the evolution of the informal sector size in the baseline simulation (which is meant to match the data, by allowing both taxes and productivity to change), whereas the blue line represents the informal sector size that we would have observed if productivity had been constant at the 2000 level. In this way the blue line can be interpreted as the contribution of changes in taxation (both income taxes and social security contributions) to informality. Several things are worth mentioning. First of all, in the baseline scenario informality drops by around 8 percent points (from 38 to 30\% of GDP), whereas it decreases by only 3.67\% when the only exogenous variation is taxes. Hence changes in taxes can explain a bit less than half of the change in informality. Moreover, almost all of the variation in informality induced by taxes takes place in the first half of the time sample, between 2000 and 2007. This is not surprising, since the major tax reforms regarding employers were implemented before 2008, as we documented in section (2). The fact that between 2007 and 2008 taxes did not affect much informality points out that the role of the flat tax reform (which affected worker’s income) was trivially small.

In the second counterfactual exercise, we keep taxes at their 2000 levels to isolate the effect of productivity growth on the size of the informal sector (see Figure (12)).

We now turn our attention on decomposing the total effect of taxes among the corporate business tax, the personal income tax and the social security contributions.

Did the 2008 flat tax reform really matter for tax evasion? As already anticipated, not much. To quantify its effect more precisely, we solve the model by feeding into only the variation in the personal income tax. Figure (13) compares this counterfactual with the

\textsuperscript{22}Real euros 2005.

\textsuperscript{23}Reported labor productivity is computed as $\hat{Y} = Y - H - L$, see equation (8) in section (3).
Figure 11: No Productivity Growth: The role of Taxes
Figure 12: Keep Taxes at 2000 Levels: The Role of Productivity
Figure 13: The (Un)Importance of 2008 Flat Reform
Figure 14: The Importance of Corporate Income Tax
simulation where all taxes are considered. It is clear that the variation in personal income tax did not play any relevant effect on the size of the informal economy. Neither the flat tax in 2008 nor the previous reductions in the effective marginal tax rates impacted on tax evasion.

As we already mentioned, among all taxes we consider, the most important one is the corporate business tax. Figure [14] makes it clear that the change in $T_E$ alone accounted for most of the effect of taxes on informality. As we already explained in the introduction, most of the surplus from production accrues to the employer as profits. It is therefore the employer who makes the decision about how much income to hide and the taxes on profits are the relevant margin. Since wages are only a small part of the surplus from production, even the payroll tax levied on the employer does not change significantly the incentives to report income to the tax authority.

**Counterfactual Experiment - Results**

Finally, we summarize the results of the decomposition between taxes and productivity in Table [5]. The first row reports the variation of the informal economy that we observed in the Bulgarian data for the 2000-2014 period, whereas the second row reports the corresponding variation in the model. The model in its current version overpredicts the reduction of informality observed in the data.

The variation in taxes alone can account for 45% of the variation in informal sector size implied by the model (this comes from our first counterfactual exercise, see Figure[11]). When productivity is the only driving force, the model generates a variation that is equal to around 60% of the reduction in the baseline scenario.

<table>
<thead>
<tr>
<th></th>
<th>2000</th>
<th>2014</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td>36.90</td>
<td>31.00</td>
<td>-5.90</td>
</tr>
<tr>
<td><strong>Model</strong></td>
<td>38.34</td>
<td>30.24</td>
<td>-8.10</td>
</tr>
<tr>
<td>Change taxes only</td>
<td>38.34</td>
<td>34.67</td>
<td>-3.67 (45 %)</td>
</tr>
<tr>
<td>Change productivity only</td>
<td>38.34</td>
<td>33.49</td>
<td>-4.85 (60 %)</td>
</tr>
</tbody>
</table>

Table 5: Decomposing the change in Informality: Taxes vs Productivity
6 Conclusions

In this paper we evaluated the relative importance of labor productivity vs. income taxes and social security contributions for tax compliance in an economy with a large degree of informality. To this end, we used a bargaining model in which matched employer-employee pairs of heterogeneous productive capacities make decisions on output sharing and the degree of tax evasion. Our quantitative model takes as inputs the income tax structure and the estimated aggregate productivity series. The estimation strategy recovered the bargaining parameters and the cost function of tax evasion in the model by matching the empirical series for the size of the informal sector (2000-2014). The results from the performed computational experiments pointed out that the most important factor is labor productivity, followed by the corporate tax. Income tax progressivity in Bulgaria is found not to be quantitatively relevant for tax evasion.

There are several factors that we left out from this paper: pension benefits, etc. We leave this for future research.

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A Appendix

A.1 Proof of proposition 1 [equivalence result]

Problem (6) with incentive-compatibility constraint can be viewed as a dynamic game (which we solve by backward induction). In our case, we have Nash bargaining problem, which results in an efficient bargaining solution. (The split $\gamma$ vs. $1 - \gamma$ share ensures cooperative outcome. It essentially tunes the original non-cooperative game into a cooperative one) So the equilibrium allocations are on the Pareto frontier, i.e the solution is Pareto efficient. But then problem (6) is equivalent to simply maximizing social welfare, which is setup (7). It also gives a Pareto optimal solution (equilibrium).


A.2 How we computed the tax functions

Throughout this section we assume that tax liabilities are given by the function $T(y; \lambda, \tau)$ where $y$ denotes taxable personal income and $\lambda, \tau$ are parameters indexing the level of taxes and their progressivity, respectively. To derive some of the results we assume the following functional form (see Benabou (2002) and Heathcote et al. (2014)):

$$T(y; \lambda, \tau) = y - \lambda y^{1-\tau}$$

- The average tax rate (ATR) is
  $$\tau(y) \equiv \frac{T(y)}{y} = 1 - \lambda y^{-\tau}$$

- Marginal tax rate:
  $$T'(y) = 1 - \lambda (1 - \tau) y^{-\tau}$$

- Progressivity is measured by $\tau$. Indeed we have
  $$\frac{\partial}{\partial \tau} \left( \frac{T(y)}{y} \right) = \lambda y^{-\tau} \log y = \begin{cases} > 0 & \text{if } y > 1 \\ < 0 & \text{if } y < 1 \end{cases}$$
Hence a higher $\tau$ implies a higher ATR on individuals whose income is above the average.

- Recall that in a progressive tax system the ATR is increasing with income i.e. the marginal tax rate is always greater than the average. Indeed

$$\frac{1 - T'(y)}{1 - \frac{T(y)}{y}} = 1 - \tau$$

If $\tau = 0$ than we have a flat tax (average and marginal tax rate are equal). If $\tau > 0$ then the tax system is progressive.

- Another useful property of this tax function is that it implies a loglinear relation between after-tax and pre-tax income:

$$\log (y - T(y)) = \log \lambda + (1 - \tau) \log y$$

useful for the estimation!

- How does the marginal tax rate vary with $\tau$

$$\frac{\partial}{\partial \tau} (T'(y)) = \lambda y^{-\tau} [1 + (1 - \tau) \log y] > 0$$

as long as $y > \tilde{y}$ where $\tilde{y} \in (0, 1)$.

To get an idea about the properties of this function, please look at the figures below.
HSV Tax function

Income (relative to the mean)
A.3 How the informal economy is measured

A.4 Institutional Efficiency

![Graph showing changes in government regulatory quality from 2003 to 2014. The graph indicates a trend with a peak around 2008.](image)

Figure 15: The (Un)Importance of changes in the quality of institutions

A.5 Analytical results

The Nash bargaining problem in the case of NO evasion is the following\(^\text{24}\):

\[
\max_{e \geq 0, w \geq 0} \left[ (1 - t_E)(e - s_E w) \right]^{\gamma} \left[ (1 - s_W) w - T_W (w - s_W w) \right]^{1-\gamma}
\]

s.t.

\[
e + w = y
\]

where \(\gamma \in (0, 1)\) is the bargaining power of the employer. The progressive/proportional income tax schedule is denoted by \(T_W\); \(T_E(y) = t_E y\) is business income tax, \(S_E(w) = s_E w\)

\(^{24}\)In this and in the next section I follow the notation of the March 2016 draft.
are payroll taxes paid by the employer, $S_W(w) = s_Ww$ are payroll taxes paid by the worker. Notice that the profit tax is paid on profits net of payroll taxes, personal income tax is paid on wage net of payroll taxes.

- The objective function (i.e. the Nash product) is strictly concave in $(e, w)$, hence the FOCs are both necessary and sufficient for the interior maximum $(e^*, w^*)$. No corner solutions.

- Let $(w^{FB}, e^{FB})$ denote the solution in the case without taxes. Then

(i) $w^{FB} = (1 - \gamma) y$, $e^{FB} = \gamma y$

(ii) $w^* \leq w^{FB}$ and $e^* \geq e^{FB}$

Substituting the constraint (11) into the objective function to eliminate $e$ we get:

$$\max_{0 \leq w \leq y} \left[(1 - t_E)(y - w(1 + s_E))\right]^\gamma \left[(1 - s_W)w - T_W((1 - s_W)w)\right]^{1-\gamma} \quad (12)$$

For simplicity I also substitute the tax function specification of HSV yielding:

$$\max_{0 \leq w \leq y} \left[(1 - t_E)(y - w(1 + s_E))\right]^\gamma \left[\lambda ((1 - s_W)w)^{1-\tau}\right]^{1-\gamma}$$

The above problem is equivalent to

$$\max_{0 \leq w \leq y} \{ \gamma \log (y - w(1 + s_E)) + (1 - \gamma)(1 - \tau) \log w \}$$

The FOC wrt $w$ is:

$$\frac{\gamma (1 + s_E)}{y - w(1 + s_E)} = \frac{(1 - \gamma)(1 - \tau)}{w} \quad (13)$$

Solving wrt $w$ we get:

$$w^*(y) = \left[\frac{(1 - \gamma)(1 - \tau)}{(1 + s_E)(1 - \tau(1 - \gamma))}\right] y = \alpha_w y < (1 - \gamma) y$$

and of course (from the constraint):

$$e^*(y) = [1 - \alpha_w] y = \alpha_e y > \gamma y$$

It is easy to see that

$$\gamma = 0 \implies w^* = \left[\frac{1}{1 + s_E}\right] y, \quad e^* = \left[\frac{s_E}{1 + s_E}\right] y$$

$$\gamma = 1 \implies w^* = 0, \quad e^* = y$$

More formally, $(w^{FB}, e^{FB}) = \arg\max_{w, e} e^\gamma w^{1-\gamma}$ s.t. $e + w = y$. 

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Please notice that $s_E > 0$ introduces an interesting asymmetry in the splitting rule. When the employer has all the bargaining power (i.e. $\gamma = 1$) he can get the whole pie; when the worker has all the bargaining power (i.e. $\gamma = 0$), however, the wage is lower than $y$.

**Proposition.** Let $(w^*, e^*)$ denote the solution to problem (12). We have the following comparative statics results:

(i) $\frac{\partial w^*}{\partial \tau} < 0$ the higher the tax progressivity, the lower is the pre-tax wage (and the higher is the pre-tax profit)

(ii) $\frac{\partial w^*}{\partial \lambda} = 0$

(iii) $\frac{\partial w^*}{\partial s_E} < 0$

(iv) $\frac{\partial w^*}{\partial t} = 0$

(v) $\frac{\partial w^*}{\partial s_W} = 0$

**Proof.** (i) We have to show that

$$\frac{\partial w^*}{\partial \tau} < 0$$

i.e.

$$\frac{\partial w^*}{\partial \tau} = \frac{-(1 - \gamma)(1 - \tau(1 - \gamma)) - (1 - \gamma)^2(1 - \tau)}{(1 + s_E)(1 - \tau(1 - \gamma))} < 0$$

which holds true for any $\gamma \in (0, 1)$. Results (ii)-(v) are trivial.

**Comments.** The level of the personal income tax (governed by parameter $\lambda$), the business income tax and the payroll tax paid by the worker do not affect the optimal split of the surplus between the employer and the worker. Only the payroll tax on the employer affects the marginal return of increasing the wage.

The figure below should help visualize the comparative statics (baseline parametrization, see matlab file).
Figure 16: No Evasion: Comparative Statics

Pre-tax wage and profit:

- Progressivity $\tau_A$
- Business income tax $t_B$
- Payroll tax $s_e$
- Payroll tax $s_w$