# Auctioning off the Agenda: Bargaining in Legislatures with Endogenous Scheduling 

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#### Abstract

An ubiquitous feature of all legislatures is that plenary time is scarce. However, no models of legislative bargaining explicitly include this scarcity and the resulting competition for floor time it induces. In this paper, we develop a general model of legislative scheduling with scarcity that we call agenda auctions. As the name suggests, the intuition for our model comes out of framing the problem as a special type of a multi-good auction in which a single agent is charged with choosing which set of bills will be considered for votes. The "goods" to be auctioned off are the limited number of slots for floor votes, and this scarcity induces competition for these limited slots. We show how this agenda competition affects legislative bargaining both for the case of redistributive and non-redistributive (i.e., spatial) politics. We also compare legislative bargaining with agenda auctions to that with content-neutral scheduling, where the probability a bill is voted on in independent of its content.


## 1. INTRODUCTION

One of the ubiquitous features of all legislatures, and indeed most voting bodies, is that plenary time - the time consumed in plenary session when important bills are considered by formally stated motions that must be voted upon - is scarce (Cox 2006). ${ }^{1}$ In fact, this scarcity has been argued to lead to many of the structures we see in modern legislatures (Cox 2006, see also Polsby, Gallaher, Rundquist 1969). Yet, no models of legislative bargaining explicitly include this scarcity. The standard models assume what we will call content-neutral scheduling. That is, the likelihood that a bill is considered by the full chamber is independent of its content and the other bills proposed. For example, in Shepsle (1979) every committee is a monopoly supplier of proposals in their given jurisdiction with guaranteed access to a floor vote for their bill. Baron and Ferejohn (1989), hereafter BF, allow that some agents may not necessarily be able to make proposals, but the likelihood that a particular agent can make one does not depend on her actual proposal. The BF model has been extended to allow for endogenous proposal probabilities, but these probabilities are determined ex ante, given a member's seniority (McKelvey and Reisman 1992). Yet, no modern legislature that we know of works this way. For example, in the U.S. House of Representatives

[^0]scheduling matters are handled by the Rules Committee, arguably the most powerful committee in the chamber. The Rules Committee is well aware of a bill's content, as well as any proposed amendments, before it allows it to be scheduled for a floor vote. ${ }^{2}$

In this paper we develop a general model of legislative scheduling with scarcity that we call agenda auctions. As the name suggests, the intuition for our model comes out of framing the problem as a special type of a multi-good auction in which a single agent is charged with choosing which set of bills will be considered for votes. The "goods" to be auctioned off are the limited number of slots for floor votes, and this scarcity induces competition for these limited slots.

The simplest form of agenda auctions are for the case of pure redistribution. In these models the voting decision in the legislature does not affect the amount of surplus in the economy, but only how to allocate it. Legislative bargaining typically creates externalities because the final outcome of the legislative process affects all agents' payoffs. However, in the case of pure redistribution, it is possible to use side-payments to compensate individual agents for these externalities and, therefore, to ensure the existence of equilibria. After developing our general model of agenda auctions, we show how agenda competition affects outcomes in a static version of the legislative redistribution model of Baron and Ferejohn (1989). The agenda competition will allow the scheduling agent to possibly extract considerable rents, depending on the minimal share of the budget that must be allocated to obtain a majority of votes in favor of the proposal. Thus, the cheaper it is to garner a majority of votes, the higher the rents to the scheduling agent.

Agenda auctions become more complicated in case of non-redistributive policy making, such as the common spatial model of politics. In the non-redistributive situation, if another agent's proposal is chosen, it will change the set of final enacted policies and, therefore, all agents' payoffs. Side payments may not be allowed in some political settings and, therefore, proposers can then only provide inducements to the scheduling agent in terms of policy proposals and they must all "consume" the enacted policies. ${ }^{3}$ That is, there are built-in externalities within the model. Given that in most democracies contracting on side payments with politicians is illegal, it will be difficult to internalize and price these externalities. Even with this difficulty, we show that equilibria generically exist in agenda auctions.

While our general model provides a very flexible framework that can be extended in many directions, we add some details in order to analyze environments of non-redistributive policy making. We do so in what we think of as the simplest case of complete-information legislative bargaining where the political problem is split along different jurisdictions, and each proposer may only propose to move the policy along her jurisdiction. We thus use our framework to generalize Shepsle's (1979) model. In our model of legislative bargaining, competition for access to the floor induces a final policy to move towards the policy preferences of the scheduling agent.

In Shepsle's (1979), which has dominated the literature, committees are monopoly suppliers of policy proposals, there are serious agency problems within the legislature because the committees can use their agenda power to extract all the surplus from the majority (see also Denzau and Mackay 1983). ${ }^{4}$ The conventional wisdom was that the only real way to ameliorate this agency loss was by screening. This lead to a voluminous empirical literature on whether or not legislative committees are preference outliers (see, for example, Cox and McCubbins 1993, Londregan and

[^1]Snyder 1994, Poole and Rosenthal 1997). Our model, however, suggests a new way to think about this agency problem. When we include competition for plenary time in the model, this competition can prevent the agency loss if the scheduling agent is chosen correctly: when committees are outliers the outcome may be moderate if the scheduling agent is a moderate. Nevertheless, this may create another agency problem in the choice of the scheduler. ${ }^{5}$

We show that when the scheduling agent's preferences are closer to the status quo than the preferences of committees, the outcome will also be closer to the status quo due to the competition between the committees to get their proposals onto the agenda. On the other hand, when the scheduling agent is an outlier relative to the status quo the outcome might still not be entirely extreme even in the absence of constraints imposed by majority voting: a committee might prefer the status quo to a very radical outcome in his jurisdiction, which limits the competition between the committees through the outside option given by the status quo. Thus in either case, there may be a bias for the status quo in equilibrium. Fernandez and Rodrik (1991) show that such status-quo bias can be a consequence of incomplete information. We show that such bias might in the absence of incomplete information be due to the legislative structure.

As noted above, the legislative scheduling problem has not really been studied. The most directly related work is by Cox and McCubbins (1993), who model the scheduling process under incomplete information as a multi-armed bandit problem. However, their model is non-strategic and they do not examine the impact of competition on proposal behavior. They instead examine the order the scheduling agent would have proposals voted on. McKelvey and Riezman (1992) take a totally different approach whereby the recognition probability of a member is determined by a seniority rule in order to endogenously generate an incumbency advantage for the members of the legislature, but they assume independent scheduling. There have also been a number of papers on endogenous agenda formation, such as Banks and Gasmi (1987) and Penn (2005), but these are best characterized as determining the amendment tree of a single bill and again assume independent scheduling of the final votes. ${ }^{6}$

The rest of this paper is organized as follows. In the next section, we consider a very simple example to generate some intuition for agenda competition in legislative bargaining. In Section 3 we give the general framework for agenda auctions and prove the existence of equilibria for them. We then turn to the case of distributive politics in Section 4. In Section 5 we consider agenda auctions in the case of non-redistributive. We then compare agenda auctions to the more typical content-neutral scheduling used in the literature. The final sections considers ways to extend agenda auctions to dynamic bargaining situations or ones with incomplete information.

## 2. SIMPLE EXAMPLE OF AN AGENDA AUCTION

The following example serves as a very simple illustration of the model. A legislature must make a decision whether to move the policy from some status quo to either of two different alternatives, $x_{1}$ and $x_{2}$, e.g., status quo is "do nothing", $x_{1}$ is military invasion of some rogue state, and $x_{2}$ is financing health care for the poor. There are two committees, the defense committee, and the health-care committee. Each committee can propose a policy change in its respective jurisdiction and one proposal can be chosen - thi can be either due to the scarcity of floor time or the scarcity

[^2]of resources needed to implement either of the two decisions. Which proposal is chosen is decided by an agenda setter, player 0 . The agenda setter has some preferences over the three outcomes, and so do both committees (think of committee's preferences as the preferences of some representative member of the committee). In a simplified version of the model once the agenda setter chooses a proposal it is implemented - in our general model and most of the examples, there are some additional legislators who vote between the status quo and the chosen proposal (when the chosen proposal is status quo then this vote is irrelevant). So suppose that the committee 1's preferences are $x_{1} \succ_{1} x_{s q} \succ_{1} x_{2}$, committee 2's preferences are $x_{2} \succ_{2} x_{s q} \succ_{2} x_{2}$, and the agenda setter's preferences are $x_{s q} \succ_{0} x_{1} \succ_{0} x_{2}$. When committees simultaneously make their proposals, they must take into account the agenda setter's preferences. This competition to make it onto the agenda here induces the final outcome of any subgame-perfect Nash equilibrium to be $x_{s q}$. If there were no legislator with agenda-selection power and the slots were instead assigned randomly, then the outcome would be either $x_{1}$ or $x_{2}$, depending on which committee obtained the slot. In the general model, equilibrium outcome must also be majority preferred to the status quo.

The previous example most of the elements of our model and it outlines the kind of incentives that different actors will face when making their choices. We emphasize that in our model preferences and the structure of decision making are known to everyone. First, there are proposers, who simultaneously make their proposals, which can only be made within the proposers' jurisdicitons. Next, the agenda setter selects some of these proposals, where the number of proposals he can select is restricted by an exogenous constraint. Finally, there are voters who must approve the chosen selection against the status quo. Our formal model is specified in a continuous policy space, and we do not explicitly consider such discrete examples; any such discrete example could easily be described in our general framework. ${ }^{7}$ As in this example, in more general applications of our model competition induces the outcomes of such legislative process to be more responsive to the agenda-setter's preferences than to the preferences of the committees.

## 3. THE MODEL

We consider $n+1$ legislators, or voters, and the set of voters is denoted by $N=\{0,1, \ldots, n\}$. In our model of a legislature, some of these voters, possibly all of them, have the power to make proposals. These voters can be thought of as committees, ${ }^{8}$ we call them proposers, and we denote the set of proposers by $N p=\{1,2, \ldots, n p\} \subset N \backslash\{0\}$, where $n p \leq n$. Voter 0 is the agenda setter, and he has a special role, which we shall describe in a moment.

In our model, the set of all possible policy outcomes is fairly general and it is given by $R^{K}$, where $K \geq 1$ is the dimension of the policy space. For instance, when considering the problem of distributing a finite pie or a budget amongst the legislators, $K=n+1$, where a dimension $k$ denotes the amount of resources that goes to the legislator $i=k$; such setting is usually called an environment of redistributive politcs. In a different example one dimension might describe economic policies, and the other social policies, $K=2$; that would be an example of a non-redistributive politics environment. We consider a setting where there is a default policy, or a status quo policy, which is denoted by $x_{s q} \in R^{K}$.

An additional structure in our model is given by the proposers' jurisdictions, These are subsets

[^3]of the policy space, such that each proposer may propose moves of the policy away from the status quo only in her jurisdiction. For example, in a setup of redistributive politics, all proposers might have the same jurisdiction, given by the set of all possible splits of the pie (i.e., the budget set). In a setting of non-redistributive politics positive observation seems to suggest considering jurisdictions which restrict proposals. For example, a proposer with the jurisdiction over defence, i.e., the defence committe, may propose changes in defence policies, while a proposer with the jurisdiction over health may propose changes in health policies. Formally, the jurisdiction of proposer $i \in N p$ is denoted by $M_{i} \subset R^{K}$, and we assume that each $M_{i}$ is compact.

In our model each proposer makes one proposal. We denote a proposal of proposer $i$ by $m_{i} \in M_{i}$, so that $m_{i}$ is interpreted as a move from 0 by the vector $m_{i}$, resulting in a policy $0+m_{i}=m_{i} \in R^{K}$. We denote the vector of proposals of all the proposers by $m=\left(m_{1}, m_{2}, \ldots, m_{n p}\right) \in \times_{i \in N_{p}} M_{i} \subset$ $R^{n p \times K}$.

The final element of our model is an exogenous constraint on the number of proposals that actually make it to the floor. One interpretation of this is that the floor time in a legislature is limited. ${ }^{9}$ We denote this constraint by $T$, and we assume that $0<T \leq n p$. We assume that if there are more than $T$ proposals, then exactly $T$ proposals make it to the floor, and if there are $T$ proposals, then all of them do. ${ }^{10}$ In the example of distributive politics, only one of the proposals to split the budget is ultimately implemented, so that there, $T=1$.

We call the set of proposals that end up being considered an agenda. Since each proposer makes one proposal, we can represent such an agenda by the indices of proposers whose proposals are on the agenda, and we denote such agenda by $A, A \subset N p,|A|=T$. Given the constraint $T$, we denote by $\mathcal{A}_{T}$ the set of all possible agendas, so that $\mathcal{A}_{T}=\{A \subset N p,|A|=T\}$. Given a vector of proposals $m$ and given an agenda $A \in \mathcal{A}_{T}$, we assume that the resulting policy is given by simply adding up all the proposed moves on the agenda $A$. Denote this policy by $x_{A}(m) \in R^{K}$, so that, $x_{A}(m)=\sum_{i \in A} m_{i} .{ }^{11}$

In our general model, voters have quasi-concave preferences over the outcome space. In our examples we consider a particular case of this where voters have linear preferences. That is, each voter $i \in N$ has an ideal point $y_{i} \in R^{K}$, so that $u_{i}(x)=-\sum_{k=1}^{K} \omega_{i, k}\left|x_{k}-y_{i, k}\right|$, where $\omega_{i}=$ $\left(\omega_{i, 1}, \ldots, \omega_{i, K}\right)$ is the vector of weights describing how much $i$ cares about the different dimensions. For example, when all voters care equally about all dimensions, $\omega_{i, j}=1, \forall i \in N, \forall j \in K$. This is the simplest case of single-peaked preferences in a multi-dimensional environment.

There are several advantages of this linear environment. It is sufficiently flexible to fit all our examples, and it makes for simple algebra. It also allows for different rates of substitution across the different policy dimensions. To account for the example of distributive politics, with the size of the pie equal to 1 , and when voters haver purely selfish preferences, we can set $\omega_{i, i}=1, \forall i \in N$, and $\omega_{i, i}=1, \forall i \in N, \forall j \in K$, with $y_{i, i}>1$, i.e., the ideal point of each voter being greater than 1 on dimension $i$. Dimension $i$ then represents the share of the pie that goes to legislator $i$. In

[^4]non-distributive examples, the weights that a legislator puts on different dimensions describe his relative preferences for the moves along these different dimensions.

### 3.1. The agenda auction

We call our model of the procedure by which the agenda is chosen, and consequently voted upon, an agenda auction. In the agenda auction the proposers first simultaneously make their proposals $m$. The agenda setter (player 0 ) then chooses $T$ of the proposals that have been made, and thus puts together an agenda $A$ from the proposals in $m$. Finally, the whole legislative body, i.e., all voters in $N$, simultenously vote between the status quo, and the whole agenda. That is, the final vote is an up or down vote between $x_{A}(m)$ and $x_{s q}$.

This simplification allows us to abstract from issues arising from sequential proposals or sequential voting. For example, when voting on a bill providing for free education for poor people, it might be necessary to first vote on a bill providing food subsidies. Viewing the agenda as a reduced-form outcome of a legislative session thus has its obvious limitations. Our approach can also be viewed as the simplest modeling choice. Moreover, under the assumption of voter spohistication, any equilibrium outcome of a one-shot model where voting is done proposal by proposal will also be an outcome of the single-vote model; any equilibrium outcome of a model where the votes are announced sequentially rather than simultaneously will similarly be an equilibrium outcome of the simultaneous-vote model.

Formally, the agenda auction is a one-shot extensive-form game in three stages.
Stage 1. Proposers simultaneously submit their proposals $m_{i} \in M_{i}, i \in N p$. Thus, $m_{i}$ is the action of proposer $i$ in stage 1 .
Stage 2. The agenda setter chooses an agenda $A \in \mathcal{A}_{T}$. Given $m \in M$, denote by $A(m)$ the agenda setter's action in the second stage, i.e., the agenda setter's history-contingent agenda choice, so that,

$$
A(.): \times_{i \in N p} M_{i} \rightarrow \mathcal{A}_{T}
$$

Stage 3. All voters $i \in N$ simultaneously submit their votes between $x_{s q}$ and $x_{A}$. Given $m \in M$ and $A \in \mathcal{A}_{T}$, denote by $d_{i}(m, A)$ the action of voter $i \in N$ in the third stage, i.e., $i$ 's historycontingent vote, so that,

$$
d_{i}(., .): \times_{i \in N p} M_{i} \times \mathcal{A}_{T} \rightarrow\{\text { aye }, \text { nay }\} ;
$$

We denote $d=\left(d_{i}\right)_{i \in N}$.
The final outcome of the agenda auction is given by the $q$ - majority voting correspondence, where $q$ is an exogenous parameter, $q \geq \frac{1}{2}$. Let $C_{q}\left(x, x^{\prime}, d\right)$ denote the $q$-majority voting correspondence, between policies $x$ and $x^{\prime}$, and given voting decisions $d$. Hence, the outcome of the agenda auction is given by $C_{q}\left(x_{s q}, x_{A(m)}(m), d(m, A(m))\right)$. In most common situations, in order to pass a bill, either a simple majority is needed, so that $q=\frac{1}{2}$, or a $\frac{2}{3}$-majority might be required, so that $q=\frac{2}{3}$. In all our examples, we take $q=\frac{1}{2}$. We denote by $O_{m a j}\left(x_{s q}\right)$ the set of policies, which are (weakly) majority preferred to the status quo. In all our examples, equilibria have the property that whenever a voter weakly prefers $x_{A}$, he votes for $x_{A}$, so that $x_{A}$ is the outcome as long as $x_{A} \in O_{m a j}\left(x_{s q}\right)$. We will then use a simplified notation and denote the outcome of the agenda auction by $x_{a a}(m, A)$.

In this extensive-form game, the strategy of each player is defined in the usual way by specifying his history-contingent actions at all points where he is to move. A strategy of a player $i$ is denoted
by $s_{i}$ : For each proposer, $s_{i}=\left(m_{i}, d_{i}(.,).\right)$; For the agenda setter, $s_{i}=\left(A(),. d_{0}(.,).\right)$; And for all other voters $i \notin N p \cup\{0\}, s_{i}=d_{i}(.,$.$) . We denote a mixed strategy by \sigma_{i}$, by $s=\left(s_{i}\right)_{i \in N}$ a profile of strategies, and by $\sigma=\left(\sigma_{i}\right)_{i \in N}$ a profile of mixed strategies. We denote by $x_{a a}(\sigma)$ the outcome of agenda auction when players follow a strategy profile $\sigma$.

A profile of strategies is a subgame-perfect Nash equilibrium SPNE if, at every history where he is to move, each player chooses a plan of action to maximize his utility function, given the strategies of the other players. Thus, in the agenda auction at stages 1 and 2, players apply backward induction in order to evaluate the consequences of their actions on the final outcome. ${ }^{12}$

As in any voting model, $x_{s q}$ is always supported as an equilibrium outcome. The reason is that if all voters vote against the proposed move, then if one of the voters changed his mind and voted in favor of the move that would make no difference as the majority would still vote in favor of $x_{s q}$. On the flip side, whenever there exists a group of voters comprising a majority who prefer the move, voting "nay" is weakly dominated for any voter in this group. To avoid such outcomes where nobody votes for the move because nobody else votes for the move, we focus on equilibria in strategies which are weakly undominated at the voting stage. Since voting is only done once, this assumption is equivalent to assuming truthful voting at the voting stage whenever a voter strictly prefers the combined move over the status quo. From now on we refer to SPNE in weakly-undominated strategies as equilibrium.

Definition 1. An equilibrium of the agenda-auction game is a SPNE where weakly dominated actions are eliminated at stage 3.

We denote the equilibrium actions and strategies by a $*$ in the superscript, e.g., $s_{0}^{*}=\left(A^{*}(),. d_{0}^{*}(.,).\right)$.
The agenda-auction game is in general discontinuous. The reason is that as the proposal of proposer $i$ changes slightly, this may affect the choice of agenda by the agenda setter, and hence the outcome and payoffs in a discontinuous fashion. In the next theorem we use the idea of Simon and Zame (1990) existence result for discontinuous games. Their insight is that if upon a player's indifference this player's randomization is specified as a part of the equilibrium then this convexifies the bestreply correspondence. In our case, this means that the agenda setter's randomization at the second stage and then voters' randomizations at the third stage, whenever any of these is indifferent, must be specified as a part of an equilibrium correspondence, and not as a part of the description of the game. We remark that in all our examples considered below, this resolution of indifference by the agenda setter turns out to be deterministic. ${ }^{13}$

Simon and Zame (1990) existence theorem directly applies to simultaneous-moves normal-form games. We apply their theorem to our extensive form using backward induction, and the fact that the decision at the third stage is binary, at every history. The proof of the existence theorem is in the appendix.

Theorem 1. Suppose the preferences of all players $i \in N$ are quasi-concave, and for each proposer $i \in N p$, his jurisdiction $M_{i}$ is a compact subset of $R^{K}$. Then there exists an equilibrium of the agenda-auction game.

[^5]The application of the Simon and Zame (1991) result requires a bit of work as the agenda auction has three stages. Namely, the proof demonstrates that at all points at which the agenda setter's preference over different subsets of proposals changes, hence potentially causing a discontinuity in other voters' payoffs, the subgame-perfect equilibrium correspondence can be convexified. This is done by the voters' and the agenda setter's choice being specified as a part of equilibrium, i.e., when an agent is indifferent, any randomization is allowed. ${ }^{14}$

## 4. DISTRIBUTIVE POLITICS

We consider first the classic case of distributive politics. A pie of size 1 must be shared between the voters, precisely as in the setting considered by BF. For example, there is a fixed amount of resources to be allocated between different voters, or a budget must be split between different constituencies represented by the legislators. In the dynamic model of BF a proposer, who is randomly selected from the voters, proposes a split of the pie, and the voters' continuation payoffs determine the share of the pie that each must be offered in order to be in favor of the proposal; in equilibrium, the game ends in one period. In the present example, the status-quo distribution of the pie plays an analogous role to these continuation payoffs in the dynamic model. Additionally, in the agenda auction the proposer is selected according to how appealing his proposal is to the agenda setter, that is, how much surplus a proposer can allocate to the agenda setter.

In this static version of the BF model the equilibrium bears a resemblance to the equilibrium in BF. A proposal of $i \in N p$ is to give a majority of legislators their status-quo shares of the pie, and allocate the remaining surplus to the agenda setter. Such proposal is then approved in the voting stage. For example, when $x_{s q}=0$, the agenda setter obtains all the surplus. In a richer version of this example, status quo may be different from 0 , and the budget set may also be asymmetric. A non-zero status quo represents a situation where the outside option to not passing the budget proposal on the table implies utilities other than 0 to the voters; different voters may obtain different shares of the budget under the status quo. The status quo need not belong to the budget set, but in most applications it seems reasonable to assume that it does.

An example displaying these features is a legislative session where a budget must be passed, and there is a default budget allocation in case of an impasse, e.g., the previous budget. An asymmetric budget set may describe a situation where there are restrictions on allocations to constituencies of some legislators. For example, there may be an upper or a lower bound on resources that can be allocated to a given constituency. Another example of an asymmetry is when some voters may be expropriated, so that the budget set might be negative on those dimensions. We assume that the budget set is convex, which is an assumption that allows for enough flexibility to account for the aforementioned possibilities.

What follows is a brief formal treatment of this example. The set of proposers is the set of voters, $N p \cup\{0\}=N$, and there are as many dimensions $K=n+1$, where $n+1=|N|$. Each voter only cares about the share of the pie that he obtains, so that $\omega_{i, i}=1, \omega_{i, j}=0, j \neq i, \forall i \in N$. In the simplest version, the jurisdictions of all proposers coincide and are equal to the budget set

[^6]$M_{i}=B$, where $B=\left\{x \mid \sum_{i \in K} x_{i} \leq 1\right\}, \forall i \in N$. Let $y_{i, i}=\alpha>1, y_{i, j}=0, j \neq i, \forall i \in N$, so that the preferences of each voter are non-satiated on the feasible set. The budget set $B$ is a convex set, $B \subset R^{K}$, lying below the line $\sum_{i=1}^{K} x_{i}=1$. There is one slot on the agenda, $T=1$, which is the single decision on how the budget should be allocated.

The characterization of equilibrium outcomes is intuitive: any equilibrium involves a cheapest majority getting their status-quo shares, no matter which proposer is the one to propose the split, and the agenda setter pocketing the rest of the surplus. For example, if the budget set allows for expropriation, then some voters may indeed be expropriated in equilibrium, depending on the tradeoff between how much different voters may be expropriated (which depends on the budget set), and how "costly" their votes are relative to others (which depends on their status-quo allocation).

Proposition 1. Consider an environment of distributive politics with a convex budget set $B$ and a status quo $x_{s q} \in R^{N}$. In any equilibrium of the agenda auction, the outcome is given by

$$
\begin{gathered}
\max _{x \in B \cup\left\{x_{s q}\right\}} u_{0}(x), \\
\text { s.t., } \exists \bar{N} \subset N,|\bar{N}| \geq \frac{n+1}{2}: u_{j}(x) \geq u_{j}\left(x_{s q}\right), \forall j \in \bar{N} .
\end{gathered}
$$

## 5. NON-REDISTRIBUTIVE POLITICS

In this section we consider more general political environments where voters have preferences over political outcomes. For example, the outcome space $R^{K}$ may represent the ideological dimensions of some more or less important political decisions, such as, socially conservative vs. liberal policies, free trade vs protectionism, questions of religious significance, and so on. Such political outcomes might not be directly translatable to monetary payoffs, or profits. In particular, when preferences on some dimension do not satisfy local non-satiation, e.g., when voters' ideal points are interior to some feasible set of policies, or when the feasible set of policies is potentially unbounded. Another difference is that different voters may have different marginal rates of substitution between different policy dimensions, and due to restricted jursidictions, no transfers of a numeraire might be possible. Environments of non-redistributive politics are more general in that they can represent both, examples with and without money.

The simplest example of a non-distributive environment is the commonly studied case when $K=1$ and $T=1$. It is quite standard to consider voters with single-peaked preferences, which is here equivalent to assuming strictly quasi-concave preferences. If there are at least 2 proposers with the same jurisdiction, which includes the ideal points of all the voters, then the equilibrium proposals are both the same, and influenced by the agenda setter's preferences: both proposers then propose the closest point to the agenda setter's ideal point, which is majority feasible, and such that both proposers weakly prefer it to the status quo. This illustrates the impact of the agenda setter's preferences in a single-dimensional example.

The single-dimensional example illustrates how the proposers in the agenda auction compete with each other in a manner similar to Bertrand competition. This is similar to the example of distributive politics. But is different in that here, the proposers undercut the distance to the agenda setter's ideal point, rather than compete in the monetary surplus that they could allocate to the agenda setter. What the single-dimentional example does not illustrate is how this policy undercutting works when there are more dimensions. Will the components of the agenda-setter's ideal point on $T$ dimensions form an equilibrium of the agenda auction whenever it is feasible? The
answer to this is affirmative under fairly general assumptions, see Proposition 2 below. Another simplification in the environments of distributive politics, or a single-dimensional policy space, is that in such examples the majority-win set imposes few restrictions on the proposals with which the proposers could "out-bid" each other. In contrast, when there is more than one relevant policy dimension the shape of the majority-win set interacts with the proposers' incentives to outbid each others proposal. Many complications of this section arise from that problem.

In order to keep the model tractable, we maintain several assumptions throughout the section. We fix the status quo at the origin, $x_{s q}=0 \in R^{K}$. We then simplify the notation and denote the set of weakly majority-preferred policies over $x_{s q}$ by $O_{m a j}$. We also assume that the number of proposers (committees) equals the dimensionality of the space, $n p=K$. Next, proposers' jurisdictions are orthogonal lines (so that their directions form an orthogonal base), $M_{i}=\{x \in$ $R^{K}\left|x_{j}=0 \forall j \neq i,\left|x_{i}\right| \leq \bar{m}, \bar{m}>0\right\}$. We identify $\times_{i \in N p} M_{i}$ with $R^{n p}$, and slightly abuse notation by writing $m \in \times_{i \in N p} M_{i} \subset R^{n p}$, with the interpretation that $m_{i}$ is the non-zero component of proposer $i$ 's proposal. ${ }^{15}$ Finally, we assume that $T<n p=K$. Thus, we assume that each proposer supplies proposals as a "monopolist" on the dimension of his jurisdiction. Note that all these assumptions are standard. Since we assumed that $T<n p$, the main investigation of this section will be how this monopolistic provision of proposals trades-off with the competition for the slots on the agenda.

To simplify our exposition, we assume that the agenda setter cares equally about the moves along all dimensions, and his ideal point $y_{0}$ is equally far away from $x_{s q}=0$ on all dimensinos, i.e., $y_{0, j}=y_{0, j^{\prime}}, \forall j, j^{\prime} \in K$. This assumption allows us to abstract from the intrinsic trade-offs that the agenda setter might have to make between different dimensions, and rather focus on the trade-offs arising from the differences in proposals he is facing. This assumption also has a heuristic motivation in that such an agenda setter might be more likely to arise due to his relative impartiality in the face of voters' heterogeneous preferences. ${ }^{16}$ Without further loss of generality we fix $y_{0}$ in the positive orthant. Note that this assumption still allows for the agenda setter's ideal point to lie closer to the ideal point or further away from it, i.e., he might be a moderate or an extremist relative to the proposers and the rest of the voters. We summarize all this in the following assumption A1.

A1. Agenda setter's preference is uniform over different dimension. The agenda setter puts equal weights on all dimensions in the policy space. We normalize these weights, so that, $\omega_{0, j}=1, \forall j \leq K$. Furthermore, $y_{0}=(1,1, \ldots, 1)$.

In this section, we do not explicitly model voters in $N \backslash(N p \cup\{0\})$. Instead, we treat $O_{\operatorname{maj}}$ as an exogenous parameter - to that effect, we are implicitly assuming that there is a set of voters $N$, such that the preferences of these voters result in such $O_{m a j}$. For the purpose of determining the equilibrium outcomes, the voters in $N \backslash(N p \cup\{0\})$ then only enter at the voting stage, and thus only determine the feasibility of a given proposal through their voting decisions. Hence, we are implicitly assuming that when indifferent, each voter votes for the proposed agenda against the status quo - we can consequently omit the voting decisions in the description of players' strategies. ${ }^{17}$

[^7]Strategic incentives are thus separated from the feasibility restrictions imposed by the majority-win set, which is treated as a black box. This is a much simpler way of treating such environments, and is somewhat analogous to general equilibrium - each individual voter is "small enough" that varying his preference does not affect the majority win set.

We assume that all proposers have linear-distance preferences, $u_{i}(x)=\sum_{k=1}^{K} \omega_{i, k}\left|x_{k}-y_{k}\right|$, $\forall i \in N .{ }^{18}$

A2. Proposers' ideal points. The proposers' preferences are such that $y_{i} \geq 0, \forall i \in N p \cup\{0\}$.
A2 concerns the location of the ideal policies of proposers and agenda setter. Thus, proposers and agenda setter agree about the direction in which the policy should move, on all dimensions of the policy space, relative to the status quo. An interpretation of this is that proposers (i.e., committees) and the agenda setter are controlled by some majority, which corresponds to the arrangement of the US legislature. If A2 did not hold, so that in an extreme case the proposers ideal points and the majority's preference lay in the diagonally opposite orthant from the ideal point of the agenda setter, then the resulting situation would be similar to the case of moderate agenda setter below. However, it would most often result in a deadlock situation, whereby the unique equilibrium outcome were the status quo. Namely, the proposers would have strong incentives to undercut each others proposals, but only until the point of proposing no movement from the status quo - as no proposer would have an incentive to move the policy further away from his ideal point. Hence, the status quo would effectively provide a bound on their bids, similarly to an outside option.

A3. Vested committees. Each proposer cares most intensely about the policy dimension in the direction of his jurisdiction, and his ideal point is furthest away on that dimension: $\omega_{i, i} \geq 1>\omega_{i, j}$, $\left|y_{i, i}\right| \geq\left|y_{i, j}\right|, \forall i, j \in N p, j \neq i$.

A3 concerns the preferences of the proposers. Each proposer's incentive to move the status quo is strongest on the dimension of his jurisdiction. On the one hand, the heuristic justification is that a proposer on a given dimension cares more about that dimension than other dimensions, as such voters would presumably have highest incentive to compete for such an assignment. ${ }^{19}$ On the other hand, when proposers care most about their jurisdictions, this assumption should in principle imply the most severe policy distortion, everything else equal. It thus represents the worst case for the agency problem as the proposals should then be most biased from some median of the legislature. If A3 did not hold, this might result in negative competition among proposers, as a proposer might have incentives to propose a less desirable proposal to the agenda setter in order for someone else's proposal to be selected instead. ${ }^{20}$

Assumptions A2 and A3 together imply that all proposed policy moves must lie in the set $\{x \geq 0\} \subset R^{K}$, and assure that for any proposer $i \in N p$, some nontrivial proposal is majority

[^8]feasible, as long as $O_{m a j}$ intersects to a sufficient extent with the positive orhtant. Then no proposer plays a trivial role in the sense that the only move he would be willing to propose, and which would be majority preferred to $x_{s q}$, were the zero move. We maintain the assumptions A1-A3 throughout the section.

Since we assumed that $T<K$, all possible moves from the status quo are along hyper-planes of dimension $T$. The feasibility of a given move is determined by the intersection of the majoritywin set $O_{\text {maj }}$ with such a hyper-plane. For a set $O \subset R^{K}$, and a $I \subset K,|I| \leq T$, we denote $O_{I}=O \cap\left\{x \mid x_{k}=0, \forall k \notin O\right\}$. Hence, $O_{\text {maj, } I}$ is the intersection of $O_{\text {maj }}$ with the hyper-plane defined by the dimensions in $I$ (or the jurisdictions of proposers in $I$ ). The shapes of $O_{m a j, I}$, for different sets $I \subset N p$, determine what deviations from a given set of proposals can deliver moves that are feasible, and are potentially preferred by the agenda setter. If no proposer can, or has a motive to deviate to a more favorable proposal for the agenda setter, the resulting outcome is an equilibrium.

The first proposition of this section states that when the components of the agenda setter's ideal point are feasible, such a policy is an equilibrium of the agenda auction under fairly general conditions. Note that the agenda setter's ideal point may be further from the status quo than the ideal points of some of the proposers, and closer to the status quo than the ideal points of some others, i.e., the agenda setter need not be either completely moderate or a complete outlier.
Proposition 2. Suppose that A1-A3 hold, and that each proposer $i \in N p$ prefers $y_{0, i}$ to the status quo. Suppose also that there exist $I^{\prime}$ and $I^{\prime \prime} \neq I^{\prime}$, such that, $\left|I^{\prime}\right|=\left|I^{\prime \prime}\right|=T, y_{0, I^{\prime}} \in O_{\text {maj, } I^{\prime}}, y_{0, I^{\prime}} \in$ $O_{\text {maj, } I^{\prime \prime}}$.
Then, $y_{0, I}$ is an equilibrium outcome of the agenda auction, for every $I, I \subset K$, such that, $|I|=T$, and $y_{0, I} \in O_{m a j, I}$.
Proof. Step 1. That each proposer $i \in N p$ prefers $y_{0, i}$ to the status quo, along with A1-A3 imply that at $y_{0, i}$, everything else equal, proposer $i$ prefers to move to $y_{0, i}$, and keeping the policy at 0 on some other dimension $j$, than moving to $y_{0, j}$, and keeping the policy at 0 on dimension $i$. We show this for the case when $y_{i}<y_{0}$, as the case when $y_{i} \geq y_{0}$ is intuitively more obvious, and its proof follows the same lines. That $i$ prefers $y_{0, i}$ to the status quo implies $y_{0, i} \leq 2 y_{i, i}$. By A1, we have $y_{0, i}=y_{0, j}$. By A3, we have $\omega_{i, i}>\omega_{i, j}$, and $y_{i, i} \geq y_{i, j}$, so that, $y_{0, j}>y_{i, j}$. Hence,

$$
-\omega_{i, i}\left|y_{i, i}-y_{0, i}\right|-\omega_{i, j} y_{i, j}-\left(-\omega_{i, j}\left|y_{i, j}-y_{0, i}\right|-\omega_{i, i} y_{i, i}\right)=-\omega_{i, i}\left(y_{0, i}-2 y_{i, i}\right)+\omega_{i, j}\left(y_{0, i}-2 y_{i, j}\right),
$$

and by the above, the last expression is positive. Since $i$ 's utility is additively separable accross the different dimensions, we have proven the claim.
Step 2. Now let $m^{*}=y_{0}$, and let $A^{*}\left(m^{*}\right)=I^{*}$ be the equilibrium agenda choice by player 0 , the agenda setter. Take an $i \in N p$. If $i \notin A *\left(m^{*}\right)$, then $i$ is indifferent between deviating or not. If $i \in A^{*}\left(m^{*}\right)$, then in a subgame in which proposer $i$ deviated to $m_{i}^{\prime}$, the agenda setter would choose $A^{*}\left(m_{-i}^{*}, m_{i}^{\prime}\right)=\tilde{I}$. Since by assumption there exists at least one other $\tilde{I}$ besides $I^{*}$, such that $y_{0, \tilde{I}} \in O_{\text {maj, }, \tilde{I}}$, such $\tilde{I}$ would be feasible. Moreover, by A1, the agenda setter would be indifferent between the outcome of chosing $\tilde{I}$ under $\left(m_{-i}^{*}, m_{i}^{\prime}\right)$ and the outcome of choosing $I^{*}$ under $m^{*}$. By Step 1, such a deviation would hence be unprofitable for $i$.

We now consider two separate cases. The first, in which the agenda setter is a moderate relative to the proposers, and the second, where he is an outlier relative to the proposers. Formally, the agenda setter is a moderate if, $y_{0} \leq y_{i}, \forall i \in N p$; the agenda setter is an outlier if, $y_{0} \geq y_{i}, \forall i \in N p$. We first address of a moderate agenda sette relative to the proposers. That is, when his ideal point is closer to the status quo than the proposers' ideal points.

### 5.1. A moderate agenda setter

We have already shown that the agenda setter's ideal point is an equilibrium outcome of the agenda auction under fairly weak assumptions. A natural question to ask is under what conditions it is the unique equilibrium outcome, i.e., when must proposals unravel all the way to the agenda setter's ideal point by way of the proposers' undercutting of each other's proposals. That happens when at every point of the majority win set there is at least one proposer $i$, who is excluded from the agenda setter's selection, given current proposals, and such that $i$ can undercut at least one of the selected proposals. We call this condition feasible competition. ${ }^{21}$

Feasible Competition. Take a point $m \in R^{K}$ and a $I \subset N p$, such that $m_{I} \in O_{m a j, I}$, and denote $\bar{m}_{I}=\max _{i \in I} m_{i}$ and $\underline{m}_{I}=\min _{i \in I} m_{i}$. Then $m$ satisfies feasible competition for $I$, if there exist $j \notin I, i \in I, \bar{m}_{j} \in\left[\underline{m}_{I}, \bar{m}_{I}\right]$, and $\underline{m}_{j}<\bar{m}_{j}$, such that,

$$
m_{I \backslash\{i\}}+\left(0,0, \ldots, m_{j}^{\prime}, 0, \ldots, 0\right) \in O_{m a j, I \backslash\{i\} \cup\{j\}}, \forall m_{j}^{\prime} \in\left[\underline{m}_{j}, \bar{m}_{j}\right]
$$

The point $m$ satisfies feasible competition if it satisfies feasible competition for every $I \subset N p$, such that $m_{I} \in O_{m a j, I}$.

In the next proposition we formally prove that feasible competition is a sufficient condition for the agenda setter's ideal point to be the unique equilibrium outcome of the agenda auction.

Proposition 3. Assume A1-A3 hold, and let $T<n p$. Suppose that $y_{0}<\left(y_{1,1}, y_{2,2}, \ldots, y_{n p, n p}\right)$, and $\left\{x \mid 0 \leq x \leq y_{0}\right\}_{I} \subset O_{m a j, I}, \forall I \subset N p,|I|=T$. Further suppose that $y_{0, I} \in \operatorname{interior}\left(O_{m a j, I}\right), \forall I \subset$ $N p,|I|=T$, and that each point in $O_{m a j} \cap R_{+}^{n p}$ satisfies feasible competition.
Then, the set of equilibrium outcomes of the agenda auction is given by $X_{a a}^{*}=\left\{y_{0, I}|I \subset N p,|I|=\right.$ $T\}$.

Proof. Let $x_{a a}^{*}$ be an equilibrium outcome of the agenda auction. First suppose that $x_{a a, i}^{*}<y_{0, i}$, for some $i \in A^{*}\left(m^{*}\right)$ - recall that $\left.A^{*}\left(m^{*}\right)\right)$ is the set of proposers selected by the agenda setter, when the vector of proposals is given by $m^{*}$. Then, proposer $i$ would prefer to propose $m_{i}^{\prime}=y_{0, i}$, which would also be preferable to the agenda setter. Hence, it is not possible in equilibrium that $m_{a a, i}^{*}<y_{0, i}$. Now suppose that $x_{a a, i}^{*}>y_{0, i}$, for some $i$, so that $i \in A^{*}$. Let $m^{*}$ be the corresponding proposals. By feasible competition, there exists a $j \notin A^{*}$, and a proposal $m_{j}^{\prime}$, such that $m^{\prime}=\left(m_{1}^{*}, m_{2}^{*}, \ldots, m_{j}^{\prime}, \ldots, m_{n p}^{*}\right) \in O_{m a j, A^{*} \backslash\{i\} \cup\{j\}}$, and such that $m^{\prime} \succ_{0} m^{*}, m^{\prime} \succ_{j} m^{*}$. This implies that $m^{*}$ could not have been an equilibrium vector of proposals, a contradiction.

Feasible competition provides a tight sufficient condition for the equilibrium outcome of the agenda auction to be pinned down by the speaker's ideal point. It is precisely the condition which assures that at every majority and agenda-feasible point of the policy space, which does not coincide with the speaker's ideal point, one of the proposers has a feasible proposal with which he can "undercut"

[^9]one of the existing proposals. Feasible competition guarantees that equilibrium outcomes of the agenda auction are essentially unique - they are all given by some components of the speaker's ideal point.

There are several environments that satisfy feasible competition. In the single-dimensional example briefly described at the beginning of the section, feasible competition is always satisfied. If $\mathrm{T}=1$, there is a simple and easily verifiable necessary and sufficient condition for feasible competition. Define $\bar{x}_{i}=\max _{x \in O_{\operatorname{maj},\{i\}}} x_{i}$, for each $i \in N$, let $\bar{x}=\max _{i \in N} \bar{x}_{i}$, and let $\bar{I}=\left\{i \in N ; \bar{x}_{i}=\bar{x}\right\}$. Note that by Lemma $1, O_{m a j,\{i\}}=\left[0, \bar{x}_{i}\right]$, so that feasible competition is then satisfied if and only if, $|\bar{I}| \geq 2$. In that case, there are more than two legislators facing the same feasibility restriction, so that for any proposal that is feasible, there exists another proposal by a different legislator, which is closer to the speaker's ideal point.

When $T \geq 2$, matters are more complicated - once there are agenda and majority-feasible policy moves on more than one dimension, a deviation by a single proposer might no longer lie in the majority-feasible set. Conditions guaranteeing feasible competition are then much more complicated to formulate in general, and feasible competition must simply be checked as a property of the majority win set. One interesting case where feasible competition holds in general is the following. Suppose the policy space describes possible investments in various projects, so that each dimension $k$ represents a different project $k$; an investment $\alpha_{k}$ in project $k$ yields a return $\gamma_{i, k} \alpha_{k}$ to legislator $i$, where $\gamma_{i, k} \in(-1,1)$, so that $\gamma_{i, k}$ may also be negative. Assume that there is a budgeting restriction, e.g., the budget is of a size 1 , and that the sum of investments should balance the budget, $\sum_{k \in K} \alpha_{k} \leq 1$. Assume that $\gamma_{i, k}>0$, for a majority of legislators, $N m \subset N$, where $N p \cup\{0\} \subset N m$, and that $0<y_{0, k}<1$, and $y_{k, j} \geq 1, \forall j \in N p, k \in K$ - that is, each proposer would like to finance their own jurisdiction to the maximum, while the speaker would like to split the budget between different projects at least to some extent. In this environment, for any majority-feasible proposal in the positive orthant, a proposal that is different on one dimension by undercutting the previous proposal is also majority feasible. We will return to a similar example in the next section.

Examples where feasible competition holds are therefore neither knife edge, nor general. This illustrates how the shape of the majority-win set affects multiplicity of equilibria in the agenda auction. Whenever the speaker's ideal point is majority feasible (given the size of the agenda), it is an equilibrium outcome of the agenda auction. When feasible competition holds, it is the only equilibrium outcome, and when it does not, there might be other equilibrium outcomes.

### 5.2. Agenda setter as an outlier

We finally study the case where the agenda setter is an outlier relative to the proposers. The agenda setter's ideal point is further away from the status quo than the ideal points of the proposers. While the status quo limits the size of the move a proposer might be willing to propose, externalities stemming from moves proposed on the other dimensions will generally induce proposals that are much closer to the agenda setter's ideal point than if there were no such externalities. In general, competition again induces proposals close to the agenda setter's ideal point, and the status quo provides only a mild check on the agenda setter's power over policy outcomes.

Here we assume that the majority-win set doesn't impose any restrictions, which is equivalent to there being no voting stage. Such an assumption could be suitable for organizations, which are not based on voting. In the previous sections, we have already illustrated the kind of additional voting feasibility restrictions that arise due to the majority-win set. Since the agenda setter's ideal
point is fixed at $y_{0}=(1,1, \ldots 1)$, and he is an outlier relative to the proposers, any constraints imposed by the majority-win set mean that some outcomes lying weakly below $y_{0}$ are not feasible. We assume that there are no such feasibility restrictions. This assumption also eliminates the kind of complications illustrated in the previous subsection.

A4. Non-Restrictive majority win set. Majority-win set satisfies, $[0,1]^{I} \subset O_{m a j, I}, \forall I \subset K$.
To make the narrative more tangible, imagine an example of procurement. A mayor must decide between $n p$ different projects, and there are enough administrative resources and personnel to oversee only $T<n p$ projects. Each project is proposed by one of $n p$ city commissioners, who each benefit to a larger degree from the project they propose, but also get some benefit from any of the other projects. The status quo on the dimension of each project is that no level of the project is implemented. To get some level of a project implemented, each commissioner must incur a per-unit cost, which is higher for his own project than for the other projects. For example, a commissioner needs to administer parts of his own project, while on the other projects he can participate in a minor role. We assume that the net benefit to a commissioner displays decreasing marginal returns, and we make that particularly simple by assuming that his return on each project is piece-wise linear and additive across projects. For commissioner $i$, the benefit outweighs the administration of project $j$ at a rate $\omega_{i, j}$ until the level $y_{i, j}$, where this is reversed, so that $\omega_{i, j}$ is the ideal point of proposer $i$ on dimension $j$. The agenda setter wants to implement each project at the maximal level of the project, set at 1 , and his rate of return on each project is 1 . This formalization fits our framework precisely.

The example here could also pertain to a description of a firm, where a supervisor decides which projects are going to be carried out by the employees who propose them, and different employees care differently about different projects. In such setting no assumptions on the majority-win set are necessary - the supervisor has all the power to decide, and there is no subsequent voting. With no voting constraints the projects will still be proposed only at the level driven by competition between proposers.

Under A3, the intensity of change of preferences is strongest along a proposer's jurisdiction. Thus, when bidding further away from his ideal point, there will be a distance from the status quo at which the proposer will prefer a move on a different dimension than his own. Intuitively, when there is a very large move in the positive direction along dimension $i$, such a move makes $i$ 's utility more negative than an equally large move along dimension $j$. On the other hand, proposer $i$ clearly prefers a move along the dimension $i$ as long as the size of the move is less than $y_{i, i}$. Hence, there will be a distance, denoted by $\beta_{i, j}$, such that $i$ is indifferent between the move of size $\beta_{i, j}$ along dimension $i$ or dimension $j$. This distance $\beta_{i, j}$ is given by the following equation,

$$
-\omega_{i, i}\left(\beta_{i, j}-y_{i, i}\right)-\omega_{i, j}\left(y_{i, j}-0\right)=-\omega_{i, i}\left(y_{i, i}-0\right)-\omega_{i, j}\left(\beta_{i, j}-y_{i, j}\right) .
$$

This implies that, for each $i, j \in N p, i \neq j$,

$$
\begin{equation*}
\beta_{i, j}=2 \frac{\omega_{i, i} y_{i, i}-\omega_{i, j} y_{i, j}}{\omega_{i, i}-\omega_{i, j}} . \tag{1}
\end{equation*}
$$

To illustrate this suppose that proposer $j$ were making a proposal $m_{j} \leq 1$, which would put $j$ onto the agenda. The agenda setter is indifferent between two equidistant moves on two different dimensions. Then, in order for proposer $i$ to be just selected onto the agenda instead of $j, i$ would have to make a proposal $\hat{m}_{i}=m_{j}$. Whenever $\hat{m}_{i} \leq \beta_{i, j}$, making such a proposal would result in an
increase in $i$ 's utility, everything else equal. Hence, $m_{j}=\beta_{i, j}$ is precisely the proposal of $j$ at which proposer $i$ would be indifferent between getting onto the agenda by proposing $m_{i}=\beta_{i, j}=m_{j}$ and proposer $j$ being selected onto the agenda with $m_{j}$. The matrix of quantities $\beta_{i, j}$ thus determines the pairwise competitive interaction between all pairs of proposers $i$ and $j$.

As long as the proposers' ideal points are not too close to the status quo, by Proposition 2 the agenda setter's ideal point can always be supported as an equilibrium outcome. In fact, whenever a weaker condition, $\beta_{i, j} \geq 1, \forall i, j \in N p$, is satisfied, it is quite simple to verify that the unique equilibrium outcome of the agenda auction under A1-A4 is the agenda setter's ideal point $y_{0}=(1,1, \ldots, 1)$. The more interesting case arises when the agenda setter is a radical outlier, relative to the proposers, i.e., $\beta_{i, j} \leq 1, \forall i, j \in N p$. As a comparison note that if proposer $i$ cares only about the dimension of his own jurisdiction, i.e., $\omega_{i, j}=0, \forall j \neq i$, then the largest move that $i$ is willing to propose is $2 y_{i, i}$, independently of the other proposals.

From equation (1), $\beta_{i, j}$ might take a higher value for two different reasons: either $\omega_{i, j}$ is relatively high, or $y_{i, j}$ is relatively low. For example, in the former case, if $y_{i, j}=\frac{1}{2} y_{i, i}$, then for $\omega_{i, j}=\frac{1}{2} \omega_{i, i}$, $\beta_{i, j}=3 y_{i, i}$, while if $\omega_{i, j}=\frac{2}{3} \omega_{i, i}, \beta_{i, j}=4 y_{i, i}$. Hence in this case, $\beta_{i, j}$ is relatively high because proposer $i$ cares relatively intensely about the moves along the dimension $j$. In the latter case, the intensity of preference is fixed, and an ideal point closer to the status quo implies that a largersize move would equalize proposer $i$ 's disutility, were this move to occur on dimension $i$ or $j$. For example, if $\omega_{i, j}=\frac{1}{2} \omega_{i, i}$, then if $y_{i, j}=y_{i, i}, \beta_{i, j}=2 y_{i, i}$, while if $y_{i, j}=\frac{1}{2} y_{i, i}$, then $\beta_{i, j}=3 y_{i, i}$. Regardless of how the pairwise cutoff value $\beta_{i, j}$ came about, it summarizes the strategic effect that proposals of $j$ have on proposals of $i$.

We first characterize the equilibria without making any further assumptions. In every equilibrium, there should not exist a proposer who is not selected and would be willing to outbid current proposals; additionally, none of the selected proposers should have an incentive to lower their bid in order for another proposer to be selected. Because the competitive effect of proposers on eachother is pairwise, many different outcomes can be supported in equilibrium. These outcomes may be at different distances from the status quo. Equilibria depend on the agenda setter's choice in equilibrium, as well as out of equilibrium. We illustrate this with an example following the proposition.

Proposition 4. Assume that A1-A4 hold, $\beta_{i, j} \leq 1, \forall i, j \in N p, i \neq j$, and let $T<n p$. Then $m^{*} \in M$ and $A^{*} \subset N p,\left|A^{*}\right|=T$, support an equilibrium of the agenda auction, if and only if, there is a level of proposed moves $\alpha \in R$, with the following properties:

1. $\exists j \in N p \backslash A^{*}$, such that, $m_{i}^{*}=m_{j}^{*}=\alpha, \forall i \in A^{*}$, and $m_{k}^{*} \leq \alpha, \forall k \in N p \backslash A^{*}$;
2. For each $i \in A^{*}, \exists j \in N p \backslash A^{*}$, such that, $m_{i}^{*} \leq \beta_{i, j}$;
3. For each $j \in N p \backslash A^{*}, \exists i \in A^{*}$, such that, $\alpha \geq \beta_{j, i}$.

For each $\alpha, m^{*}$, and $A^{*}$, satisfying these conditions, the equilibrium outcome is given by $(\alpha, \ldots, \alpha)_{A^{*}}$.
Proof. Condition (1) is necessary for any equilibrium in order for the selection $A^{*}$ to be optimal for the agenda setter and proposals optimal for proposers in $A^{*}$. Also observe that in terms of equilibrium outcomes, it is enough to limit the attention to proposals of the form $m^{*}=(\alpha, \ldots, \alpha)-$ any equilibrium outcome is supportable by proposals of this form.

To see that for proposals of the form $m^{*}=(\alpha, \ldots, \alpha)$, conditions (2) and (3) guarantee an equilibrium, consider possible deviations. First, given proposals $m^{*}$, the agenda setter is indifferent between selecting any subset of proposers. Next, consider a subgame in which one of the proposers
$j \in N p \backslash A^{*}$ chose to deviate to a proposal $\hat{m}_{j}>\alpha$. Let the corresponding selection by the agenda setter be $\hat{A}=A^{*} \backslash\{i\} \cup\{j\}$, where $i \in A^{*}$, such that $\beta_{j, i} \leq \alpha-$ such an $i$ exists by condition (3). Such a selection is optimal for the agenda setter, and it makes the deviation by $j$ unprofitable, since $\hat{m}_{j}>\alpha \geq \beta_{j, i}$. Second, if a proposer $i \in A^{*}$ deviated to $\hat{m}_{i}<\alpha$, then let $\hat{A}=A^{*} \backslash\{i\} \cup\{j\}$, where $j$ is such that $\alpha \leq \beta_{i, j}$. Such a $j$ exists by (2). Again, such a selection is optimal for the agenda setter, and it makes the deviation by $i$ unprofitable. It is clear that a deviation by a proposer $i \in A^{*}$ to a proposal $\hat{m}_{i}>\alpha$ is utility diminishing for $i$, and a deviation by a proposer $j$ to a proposal $\hat{m}_{j}<\alpha$ makes no difference.

For the converse, let $m^{*}=(\alpha, \ldots, \alpha)$ and $A^{*}$ constitute an equilibrium, and first assume that (2) doesn't hold. Then there is an $i \in A^{*}$, such that $m_{i}>\beta_{i, j}, \forall j \in N p \backslash A^{*}$. Hence, if $i$ deviates to a $\hat{m}_{i}<\alpha$, by (1), some $j \in N p \backslash A^{*}$ is selected onto the agenda instead of $i$, so that such a deviation is profitable for $i$. Finally, if (3) is violated, then there exists a $j \in N p \backslash A^{*}$, such that $\beta_{j, i}>\alpha, \forall i \in A^{*}$. Hence, $j$ has a profitable deviation to a proposal $\hat{m}_{j}>\alpha$, regardless of how the agenda setter consequently selects the set of proposals (which must necessarily include $j$ ).

Returning to the above example of city politics, suppose that $n p=4$, and the parameters were such that the cutoffs of the 4 commissioners were given by the following table,

| $\beta_{i, j}$ | 1 | 2 | 3 | 4 |
| ---: | :---: | :---: | :---: | :---: |
| 1 |  | 0.7 | 0.9 | 0.9 |
| 2 | 0.9 |  | 0.8 | 0.8 |
| 3 | 0.8 | 0.9 |  | 0.7 |
| 4 | 0.7 | 0.8 | 0.7 |  |

Then, if $T=2$, for any $\alpha \in[0.7,0.9]$, proposals $m^{*}=(\alpha, \alpha, \alpha, \alpha)$ constitute equilibrium proposals. When $\alpha \in[0.7,0.8)$, one selection by the agenda setter that supports an equilibrium is $A^{*}=\{2,3\}$, when $\alpha \in[0.8,0.9), A^{*}=\{1,3\}$, supports an equilibrium, and when $\alpha=0.9, A^{*}=\{1,2\}$ supports an equilibrium. This illustrates the reasons why equilibrium in general won't be unique - the cutoff proposals are contingent on proposers, and the equilibrium is supported by a selection that the agenda setter would have chosen had a particular proposer deviated.

To show what affects the structure of equilibria, we further trim the example of this section by assuming that each $\beta_{i, j}$ can only take two possible values, $\beta_{i, j} \in\left\{\beta_{L}, \beta_{H}\right\}, \forall i, j \in N p$, where $\beta_{L}<\beta_{H} \leq 1$. The level of competition between the proposers is then affected by $T$, by the proportion of $\beta_{H}$ 's, and also by how dispersed $\beta_{H}$ 's are between the pairs of proposers. Heuristically, dispersion here describes how much the highly competitive dimensions vary across the proposers. If each proposer $i$ has a given number of dimensions $j$ with $\beta_{i, j}=\beta_{H}$, then the more such highly competitive dimensions vary across proposers $i$, the more competition there is for the slots on the agenda. The reason is that when there is more dispersion, then for any proposer $i$ proposing less than $\beta_{H}$, it is more likely that there might exist another proposer willing to out-bid such a proposal by $i$. We formally illustrate this intuition by studying three cases. First, where there is the least amount of dispersion; second, where there is some amount of dispersion; and third, where there is a maximal amount of dispersion.

A5.1. Non-dispersed competitive dimensions. Let $\beta_{L}<\beta_{H} \leq 1$. The jurisdictions are split in two nonempty sets $K_{H}, K_{L} \subset K, K_{H} \cap K_{L}=\emptyset$. For each proposer $i$, and $j \neq i, \beta_{i, j}=\beta_{H}$ if $j \in K_{H}$, and $\beta_{i, j}=\beta_{L}$ if $j \in K_{L}$.

A situation where competitive dimension are not dispersed according to A5.1 arises when all proposers care more intensely about the dimensions $K_{H}$. Condition A5.1 is satisfied when there is agreement between the proposers regarding the importance of different policy dimensions: the dimensions $K_{H}$ are the more important ones.

Proposition 5. Assume that A1-A4, A5.1 hold, and that $T<n p$. Then the equilibria of the agenda auction are given as follows.

1. Proposals $m_{i}^{*}=\beta_{H}$, and any selection $A^{*} \subset N p$ by the agenda setter, s.t., $\left|A^{*}\right|=T$ and $\left|K_{H} \backslash A^{*}\right| \geq 1$; the outcome is then $x_{i}^{*}=\beta_{H}$, for $i \in A^{*}$, and $x_{i}^{*}=0$, for $i \in N p \backslash A^{*}$.
2. If additionally, $T \leq K_{L}$, then there are also equilibria given by proposals $m_{i}^{*}=\alpha, \alpha \in$ $\left[\beta_{L}, \beta_{H}\right)$, and any selection $A^{*} \subset K_{L},\left|A^{*}\right|=T$; the outcome is then $x_{i}^{*}=\alpha$, for $i \in A^{*}$, and $x_{i}^{*}=0$, for $i \in N p \backslash A^{*}$.

Because of scarcity of slots, there is some competition between proposers. How vigorously proposers compete for these slots in any equilibrium depends on the off-equilibrium selection of proposals by the agenda setter. For example, suppose $T=1$, that proposals are given by $m_{i}=\beta_{H}$, $i \in N p$, and that in an equilibrium, the agenda setter selects a proposer $j \in K_{L}$, i.e., a proposer with the jurisdiction among the less competitive dimensions $K_{L}$. Should this proposer $j$ lower her proposal to $m_{j}^{\prime}<\beta_{H}$, the agenda setter would have to select some proposer $j^{\prime} \in K_{H}$ instead of $j$ in order that such a deviation not be beneficial for $j$. For an example of the equilibrium where the move is of size $\beta_{L}$, suppose proposals are given by $m_{i}=\beta_{L}, i \in N p$. If the agenda setter selected a proposal by $j \in K_{H}$, then any of the proposers would have an incentive to increase her proposal and get onto the agenda instead of $j$; but if the agenda setter selected a proposer $j \in K_{L}$, none of the proposers would have any incentive to increase her proposal.

A5.2. Some dispersion in Competitive dimensions: proposer blocks. Let $\beta_{L}<\beta_{H} \leq 1$. The proposers are split in two subsets $N p_{1}, N p_{2}, N p_{1}, N p_{2} \subset N p$, and $N p_{1} \cap N p_{2}=\emptyset$. For each proposer $i$, and $j \neq i, \beta_{i, j}=\beta_{H}$ if $i, j \in N p_{1}$, or $i, j \in N p_{2}$, and $\beta_{i, j}=\beta_{L}$ if $i \in N p_{1}, j \in N p_{2}$, or $i \in N p_{2}, j \in N p_{1}$.

A situation described in A5.2. arises when there are two groups of proposers, such that the proposers in each group agree that the jurisdictions of that group are the more important ones. There is some agreement in the preferences of the proposers within each block, but the two blocks of proposers have differing preferences.

Proposition 6. Assume that A1-A4, A5.2 hold, and that $T<n p$. Then the equilibria of the agenda auction are given as follows.

1. Proposals $m_{i}^{*}=\beta_{H}$, and any selection $A^{*} \subset N p$ by the agenda setter, s.t., $\left|A^{*}\right|=T$; the outcome is then $x_{i}^{*}=\beta_{H}$, for $i \in A^{*}$, and $x_{i}^{*}=0$, for $i \in N p \backslash A^{*}$.
2. If $T \in\left\{\left|N p_{1}\right|,\left|N p_{2}\right|\right\}$, then there are also equilibria given by proposals $m_{i}^{*}=\alpha, \alpha \in\left[\beta_{L}, \beta_{H}\right)$, and a selection $A^{*}$, s.t. $A^{*} \in\left\{N p_{1}, N p_{2}\right\}$; the outcome is then $x_{i}^{*}=\alpha$, for $i \in A^{*}$, and $x_{i}^{*}=0$, for $i \in N p \backslash A^{*}$.

A5.3. Dispersed cutoffs. Let $\beta_{L}<\beta_{H} \leq 1$, let $k_{H}$ be the number of high-cutoff dimensions of each proposer. For each proposer $i$, and $j \neq i, \beta_{i, j}=\beta_{H}$ if $j \in\left\{i+1, \ldots, i+k_{H}\right\}$, (when $i+\ell>K$, the corresponding dimension is $(i+\ell) \mathrm{MOD} K=i+\ell-K)$, and $\beta_{i, j}=\beta_{L}$, for any other $j$.

This last case describes one situation in which the high cutoff dimensions are allocated between the proposers in a manner that is as dispersed as possible, given the number of high-cutoff dimensions of each proposer. For example, when each proposer cares intensely about only one dimension other than his own, i.e., $k_{H}=1$, proposer 1 cares intensely about dimension 2, proposer 2 about dimension 3, and so on. This is then a case of minimal agreement between the proposers as to which dimensions are important. When $k_{H}>1$ there will necessarily have to be some overlap between such personally relevant dimensions across the proposers, and this overlap is minimized under a configuration such as the one in A5.3. More precisely, given a $k_{H}$, then for any group of $k_{H}+1$ proposers there will not exist a single dimension such that they would all agree that this dimension is a more relevant one. In this case only the outcomes that are closest to the agenda setter's ideal point can be supported in equilibrium.

Proposition 7. Assume that A1-A4, A5.3 hold, and that $T<n p$. Then the equilibria of the agenda auction are described by proposals $m_{i}^{*}=\beta_{H}, i \in N p$, and any selection $A^{*}$ by the agenda setter, s.t., $\left|A^{*}\right|=T$; the outcome is then $x_{i}^{*}=\beta_{H}$, for $i \in A^{*}$, and $x_{i}^{*}=0$, for $i \in N p \backslash A^{*}$.

The above three propositions give comparative statics on the number of high-cutoff dimensions per proposer relative to the number of slots on the agenda $T$. These results illustrate the importance of configuration of the high-cutoff dimensions. As the dispersion of these high-cutoff dimensions increases, the conditions for equilibria that are closer to the agenda setter's ideal point become less restrictive. The conditions for equilibria that are closer to the status quo (or the proposers' ideal points) become more restrictive. In the extreme case of the highest possible dispersion, only equilibria that are closest to the agenda setter's ideal point remain. The proofs of these propositions follow from Proposition 4.

## 6. CONTENT-NEUTRAL SCHEDULING AND WELFARE COMPARISONS.

A different legislative institution from the agenda auction is what we call content-neutral scheduling. Content-neutral scheduling is a simpler institution where rationing of the slots is independent of proposals' content. Most of the voting and political-economy literature abstracts from the details of the agenda-setting process by assuming such a content-neutral scheduling procedure. For example, BF (and the large ensuing literature on legislative bargaining) assume that rationing is probabilistic and the probability of a proposer being selected is independent of the substance of the proposal. What is qualitatively different under the content-neutral scheduling is that the proposers have no incentive to craft their proposals in any particular way, for the sake of getting these proposal on the agenda. In this section we compare our findings to this content-neutral benchmark, where the agenda setter has no power.

In the content-neutral scheduling game, stages 1 and 3 are the same as in the agenda-auction game. Stage 2 is eliminated, and instead, there is a random move by Nature prior to stage 1, whereby Nature selects $T$ proposers from $N p$ with equal probabilities - each proposer is picked with probability $\min \{T / n p, 1\}$; any other way of probabilistic rationing could work equally well, as long as these probabilities are indpendent of the proposals. In this content-neutral scheduling game, proposers have no incentive to provide proposals favorable to the agenda setter.

As in the agenda-auction game, we study the SPNE of the content-neutral scheduling game, after the elimination of weakly-dominated actions at the voting stage; we refer to that as an equilibrium. In the content-neutral scheduling game the outcome is generally stochastic since the selection of proposals is stochastic. However, conditional on the chosen set of proposers $I$ (in
the subgame after the Nature has chosen $T$ proposers), the outcome is deterministic in any purestrategy SPNE. Since the actual outcome depends on the jurisdictions of the chosen proposals, we make our comparisons conditional on the chosen set of proposers. Under the content-neutral scheduling, payoffs are continuous in players' strategies, so that the existence of equilibrium follows easily by standard arguments.

We frame this comparison between the agenda auction and the content-neutral scheduling as a welfare comparison of the equilibrium outcomes of the two insitutions. We compare these outcomes to some socially optimal policy. When voters have different ideal points, a policy which would be simultaneously optimal for all legislators generally does not exist; some policies are prefered by some voters, while other policies are prefered by other voters. We thus define some socially most desirable policy, which can be interpreted as the optimal policy of some representative, or median voter in the legislature. ${ }^{22}$ For any other policy, we define the linear distance to this socially optimal policy as the welfare loss. We denote the ideal policy by $\tilde{y} \in R^{K}$; as described above $\tilde{y}$ is in general a function of the legislators' ideal points. The welfare loss associated with a policy $x \in R^{K}$ is then given by,

$$
W(x, \tilde{y})=-\sum_{k=1}^{K}\left|x_{k}-\tilde{y}_{k}\right| .
$$

In the environment of purely redistributive politics, the comparison between the agenda auction and the content-neutral scheduling is particularly simple. Suppose Nature selected proposer $i$ to divide the pie. Then $i$ would want to obtain as much surplus as possible, while compensating the cheapest majority with their status-quo shares in order to secure their votes - this similar to the BF model, where a majority must be compensated with their continuation payoffs, were the game to continue beyond the first period. Thus, an equilibrium of the agenda auction is precisely an equilibrium of the content-neutral scheduling, where the agenda setter is the one who ends up being selected by Nature. We summarize this in the following proposition and corollary. Recall that $B$ is the convex set of feasible proposals of divisions of the pie.

Proposition 8. Consider an environment of distributive politics with a convex budget set $B$ and a status quo $x_{s q} \in R^{N}$. In any equilibrium of the content-neutral scheduling, where proposer $i$ is selected, the outcome is given by $x^{\{i\}, *}$,

$$
\begin{gathered}
x_{I S}^{\{i\}, *} \in \arg \max _{x \in B \cup\left\{x_{s q}\right\}} u_{i}(x), \\
\text { s.t., } \exists \bar{N} \subset N,|\bar{N}| \geq \frac{n+1}{2}: u_{j}(x) \geq u_{j}\left(x_{s q}\right), \forall j \in \bar{N} .
\end{gathered}
$$

Corollary 2. In the setting of purely distributive politics, the power to select among different proposals is equivalent to being the sole monopolist proposer.

To evaluate policy distortion under each of the two institutions, consider again the example of the environment in BF , where $B=\left\{x \mid x \geq 0, \sum_{k \in N} x_{k} \leq 1\right\}$ and let $\tilde{y}$ be the egalitarian policy, $\tilde{y}=\left(\frac{1}{n+1}, \ldots \frac{1}{n+1}\right)$. If $0 \in \arg \max _{i \in N} x_{s q, i}$, then the equilibrium policy will be most distorted under the agenda auction, relative to the average distortion under the content-neutral scheduling, and least distorted when $0 \in \arg \min _{i \in N} x_{s q, i}$. To see this, note that $\sum_{i \in N} x_{s q, i}=1$, so that as $x_{s q, 0}$

[^10]gets larger, $\frac{1}{n+2} \sum_{i>0} x_{s q, i}$ becomes smaller. In the first case then, the cheapest majority becomes cheaper and since the agenda setter has a maximal share of the pie to begin with, he would not belong to the cheapest majority under the content-neutral scheduling, unless he were the proposer. Similarly in the latter case. Thus, if the agenda setter is the one with the largest status-quo share of the pie, then, relative to the outcome of the independent scheduling, the agenda auction will on average lead to more distortion. Conversely, to obtain a most egalitarian distribution, the agenda setter should be one of the legislators with the smallest status-quo share.

In the environment of non-distributive politics a similar intuition applies as in the simpler case of sharing the pie. Under the agenda auction the proposers compete with each other in providing proposals that are as favorable as possible to the agenda setter, while under the content-neutral scheduling they have no reason to do so. Under the agenda auction the proposers compete with each other on different dimensions. Each proposer tries to undercut the distance of other proposals to the agenda setter's ideal point as long as such undercutting is feasible under the majority-win set. This then quite naturally leads to the equilibrium outcomes under the agenda auction being closer to the agenda setter's ideal point than the equilibrium outcomes under the content-neutral scheduling. More precisely, starting from an equilibrium set of proposals under the content-neutral scheduling, an equilibrium under the agenda auction is obtained through such an "undercutting procedure" where the undercutting proposals always remain inside the majority-win set; the procedure stops either because any such undercutting proposal would fall out of the majority-win set, or because the agenda setter's ideal point has been reached.

Proposition 9. Assume A1-A3 hold. Let $y_{i} \geq y_{0}$, for all $i \in N p$, and $O_{m a j, I} \cap R_{I,+}^{K} \neq \emptyset, \forall I \subset K$. Let $x^{*}$ be an equilibrium outcome under the independent scheduling, such that $x^{*} \geq y_{0}$. Then, either $x^{*}$ is also an equilibrium outcome under the agenda auction, or there exists an equilibrium outcome $x_{a a}^{*}$ under the agenda auction, such that $u_{0}\left(x_{a a}^{*}\right)>u_{0}\left(x^{*}\right)$.
Conversely, let $x_{a a}^{*}$ be an equilibrium outcome under the agenda auction, such that $x_{a a, i}^{*} \leq y_{i, i}$, for all $i \in N p$. Then either $x_{a a}^{*}$ is an equilibrium outcome under the independent scheduling, or there exists an equilibrium outcome $x^{*}$ under the independent scheduling, such that $u_{0}\left(x_{a a}^{*}\right)>u_{0}\left(x^{*}\right)$.

When the majority-win set is sufficiently large relative to the locations of the proposers' ideal points, then the unique equilibrium proposals under the content-neutral scheduling are comprised of each proposer proposing the component of their ideal point on the dimension of her jurisdiction. Denote this vector of proposals by $\bar{y}=\left(y_{1,1}, y_{2,2}, \ldots, y_{K, K}\right)$. Why would these then have to be the unique vector of proposals under content-neutral scheduling? The reason is that even if due to mis-coordination the proposals were at a boundary of the majority win set, when this boundary is sufficiently far from the status quo, each proposer would strictly prefer the status quo on his dimension, or any other dimension, over such a policy on the boundary of the majority-win set. Hence, the unique equilibrium proposals under the content-neutral scheduling are then given by $\bar{y}$. Consequently, as long as the socially optimal policy $\tilde{y}$ is closer to the agenda setter's ideal point $y_{0}$ than to $\bar{y}$, the social welfare will be higher under the agenda auction than under the content-neutral scheduling. We thus have the following corollary to Proposition $9 .{ }^{23}$

Corollary 3. Assume A1-A3, that $y_{0, k} \in\left[\tilde{y}_{k}, y_{k, k}\right], y_{0, k} \geq 0$, and $\times_{k \in I}\left[0,2 y_{k, k}\right] \subset O_{m a j, I}$, for any

[^11]$I \subset K,|I|=T$. Then, $W\left(x^{*}\right)<W\left(x_{a a}^{*}\right)$, for any equilibrium outcome $x^{*}$ of the content-neutral scheduling.

A direct policy implication of this analysis is that if a small or no policy change is more socially desirable and the agenda setter is a moderate, an institutio akin to the agenda auction may be a better institution. Of course, if a radical reform were needed, content-neutral scheduling might be more effective, which one could demonstrate by applying an argument similar to the above.

Finally, we turn to our analysis of the agenda auction when the agenda setter has radical preferences. Equilibria under the content-neutral scheduling are very simple under the assumptions A1-A5. As the majority-win set imposes no restriction, for each selected set of proposals, each proposer proposes the move equal to the component of their ideal point in their jurisdiction. Since these are generally much closer to the status quo than $\beta_{i, j}$, the equilibria under the agenda auction are much more extreme than under the independent scheduling when the agenda setter is a radical. However, as demonstrated in Proposition 4, even with no restriction from the majority win set, the outcomes of the agenda auction are moderated by the outside option provided by the status quo, and are generally not as extreme as the agenda setter's ideal point. Thus, when the agenda setter is a radical and the proposers are not, the outcomes are somewhat moderated under the agenda auction, but are more moderate under the independent scheduling.

In summary, under quite general conditions, the equilibrium outcomes under the agenda auction are somewhat more moderate than those of a content-neutral scheduling procedure. Taking an alternative perspective, under a given configuration of preferences of proposers and the agenda setter, whether one or the other institution in equilibrium results in an outcome closer to $\tilde{y}$ depends largely on the location of the socially optimal policy $\tilde{y}$. If moderate moves are socially preferred, then the agenda auction seems to be generally preferrable. Of course, if radical reform were needed it might still be easier to appoint a radical agenda setter than count on the fact that a large fraction of proposers had radical preferences. Under the assumption that the selection of the agenda setter is responsive to policy needs, the agenda auction may have relatively appealing normative properties. It may provide a reasonable amount of flexibility in the case when radical move from the status quo is socially optimal, at least as compared to the independent scheduling. At the same time, the agenda auction provides some check on the policy outcomes, when either proposers or the agenda setter are radical relative to the social optimum. This may be a part of the reason why many organizations are organized in a similar way

## 7. DISCUSSION

We have described a static extensive-form game with complete information, where proposers and agenda setter are given exogenously. This static framework is a relatively simple and abstract way to consider the agenda-setting problem, and the voting problem in general, when there is a restriction on the number of items that can be considered. The introduction of proposers, their jurisdictions, and the agenda setter puts more structure on the voting problem than the most structure-free environment in which a number of equivalent voters take a political decision. This additional structure guarantees the existence of stable voting outcomes as equilibria of the extensive-form game. On the opposite side, we impose the least possible structure on the agenda setting process, and in this way completely abstract from the protocol by which items are presented to the agenda setter and voted upon. ${ }^{24}$ This has allowed us to focus on competition between proposers in terms of

[^12]content of their proposals alone, given their own preferences, the preferences of the agenda setter, and the feasibility of different policy moves given by subsequent voting.

Even in the context of this static game, at least two immediate questions arise. First, how are different agents assigned their roles, either as proposers or the agenda setter? In the context of our analysis this question is of course crucially important as it will have a great impact on the outcomes of the ensuing game. However, we don't think that we are capable of providing a satisfactory general treatment here, as there seem to be in practice a variety of ways in which such roles are assigned. For example, they may be chosen by seniority of tenure in the institution, they may be assigned randomly, by virtue of the stake they hold, or they may be assigned through some voting procedure. For example, the chairpersons of legislative committees are appointed according to their seniority, affiliation to the party in control of the legislature, and influence they wield within the party. In the case of a board of a publicly traded company, only the shareholders with a sufficient stake in the company will typically be on the board, thus having opportunities to make proposals. In the case of local politics, the commissioners would either be appointed directly by the mayor himself, or would be voted into office by the entire voting body. There is a similar variety of ways of determining the agenda setter. In reality, all of these also have a component, which is perhaps best modeled as random. All these processes have very different implications on the policy positions of the agenda setter and the proposers. However, in any given application, one can likely describe this selection procedure with some degree of accuracy, and then analyze the resulting outcomes.

Another natural extension of our model in the context of a static game would be to relax the assumption of complete information. In many practical applications participants will have some degree of private information, which will be of strategic or normative importance. For instance, in the example of distributing a pie one could imagine introducing private information on reservation shares, i.e., minimal shares that different legislators might be willing to accept. This could either be a legislator's private observation of his share under the status quo, or a private parameter additional to the description of the status quo. Either way, such incomplete information would be very difficult to deal with in a dynamic framework, such as that of BF; but it would seem feasible in our simplified version of the BF model.

In the example of non-distributive politics, the simplest way of introducing incomplete information seems to be when parameter $\lambda$, describing how far the agenda setter's preference is from the status quo, is private information to the agenda setter. It also seems quite plausible to assume some uncertainty or asymmetric information on the majority win set, although this might perhaps be somewhat difficult to model in an elegant way. In that setting, maintaining majority win set as a separate parameter from the preferences of the proposers and the agenda setter might facilitate analyzing such a case. The set of questions one might be able to explore by extending our model to incomplete information seems to be rich and interesting.

Finally, an important question that we have not dealt with is the question of dynamics. In a dynamic setting, there are many periods, where status quo in the current period may be connected to the outcome of the previous one, and the agents consider discounted payoffs that they receive in each period. An aspect relevant to such a setting would be to consider either the proposers, or the agenda setter, or both, as short lived. Such assumptions could correspond to term limits, or to turnover due to ellections. In the present paper, the results that have some relevance to such dynamic considerations are those pertaining to the equilibrium outcomes relative to the status quo, in particular in the section on distributive politics. For example, in the context of sharing a pie,
model.
if the agenda setter persists over several periods, then he will be able to extract all the surplus eventually, while if the agenda setter changes, e.g., with the ruling party, the policy will be less distorted. Formulating fully dynamic models in our framework seems feasible, but is beyond the scope of the present paper.

## 8. APPENDIX

## Proof of Theorem 1.

Proof. We show that the payoff correspondence to the proposers at the first stage is upper hemicontinuous (uhc) and convex valued. First fix a vector of proposals $m \in \times_{i \in N p} M_{i}$ and fix an agenda $A \in \mathcal{A}_{T}$. Define the voting correspondence of each voter $i$ by $\bar{d}_{i}\left(x_{A}(m)\right)=\{$ "aye" $\}$ if $u_{i}\left(x_{A}(m)\right)>u_{i}\left(x_{s q}\right), \bar{d}_{i}\left(x_{A}(m)\right)=\{" n a y "\}$ if $u_{i}\left(x_{A}(m)\right)<u_{i}\left(x_{s q}\right)$, and $\bar{d}_{i}\left(x_{A}(m)\right)=\{\lambda " a y e "+$ $(1-\lambda)$ "nay" | $\lambda \in[0,1]\}$, where $\lambda$ "aye" $+(1-\lambda)$ "nay" is interpreted as a probability of $\lambda$ of voting the bill up, that is, in favor of the proposed policy over the $x_{s q}$. It is immediate that this correspondence is convex-valued and uhc. Recall the $q$-majority voting correspondence, $C_{q}$ : whenever one or more voters are indifferent and their mixing could change the result of the vote, its image is a set of possible probabilities over "aye" and "nay", and hence the outcome correspondence at the last stage is at those points given by the set of all possible probabilities over $x_{A}$ and $x_{s q}$. It is immediate that $C_{q}$ is also uhc.

Take the agenda setter's choice problem at stage 2 , fixing the proposals $m \in \times_{i \in N p} M_{i}$. Define the agenda setter's choice $\bar{A}_{m}$ as follows. Take a selection $\mu$ from the correspondence $C_{q}$ in order for the agenda setter to evaluate his choices and define the resulting choice as $\bar{A}_{m, \mu}$ (when he is indifferent take again the set of all possible randomizations over different agendas among which the agenda setter is indifferent). Let $\bar{A}_{m}=\cup_{\mu \in C_{q}(.)} \bar{A}_{m, \mu}$.

Finally, define $\bar{A}(m)=\bar{A}_{m}$, for each $m \in \times_{i \in N p} M_{i}$,

$$
\bar{A}: \times_{i \in N p} M_{i} \rightarrow \mathcal{A}_{T}
$$

From the above construction, $\bar{A}(m)$ is convex valued and uhc. Hence the payoff correspondence to the proposers in the first stage is convex valued and uhc. The proof follows from Theorem 1 in Simon and Zame (1990).

It is worth recalling the following observation from McKelvey and Wendell (1976), stating that $O_{m a j}$ is star-shaped. A direct implication is that $O_{m a j, I}$ is star-shaped as well, for any $I \subset N p$.

Lemma 1. If $N$ is finite, i.e., $O_{m a j}$ is the majority-win set arising from the preferences of a finite number of voters, then, for every $I \subset K$, the majority win set $O_{m a j, I}$ satisfies the following properties.

1. For each $x \in O_{m a j, I}, \lambda x+(1-\lambda) 0 \in O_{m a j, I}, \forall \lambda \in(0,1)$.
2. If additionally, voters have linear-distance preferences, $O_{m a j, I}$ is a finite union of convex sets, and under the weighted linear distance preferences, its boundary is a hyper-polygon.

Proof. The first part is not too difficult to prove, and is proven in McKelvey and Wendell (1976) for general quasi-concave preferences, and any $q$-majority rule, $q \geq \frac{1}{2}$. The second part is that each $I, O_{m a j, I}$ can be written as a finite union of convex sets. Each of these convex sets is obtained as
follows. Take some majority of voters, $N_{\operatorname{maj}} \subset N$, and on each subset of dimensions $I \subset K$, denote
 intersection of the sets of points preferred to 0 by each voter $i \in N_{m a j}$, on the subset of dimensions $I$. Therefore, $O_{\text {maj, } I ; N_{m a j}}$ is convex, and for each $I, O_{m a j, I}=\cup_{N_{m a j} \subset N} O_{m a j, I ; N_{m a j}}$. Since there are finitely many different majority selections $N_{m a j}$, the claim follows. It is also evident from this argument that the boundary of $O_{m a j, I}$ is piece-wise linear under the linear-distance preferences.

## Proof of Proposition 9.

Proof. Take an equilibrium $x^{*}$ under the content-neutral scheduling, and let $m^{*}$ be the vector of proposals and $I,|I|=T$, the set of proposers, s.t., $x^{*}=x_{I S}^{*, I}\left(m^{*}\right)$. If there is no proposer $j \in N p \backslash I$ who could deviate to a different proposal, $m_{j}^{\prime}$, such that there existed another proposer $i \in I$ over whose proposal $m_{j}$ were preferred by the agenda setter, then $x^{*}$ is an equilibrium of the agenda auction. We now construct an equilibrium outcome $x_{a a}^{*}$ of the agenda auction where such a deviating proposer exists, and by way of construction, we show that this equilibrium is closer to the agenda setter's ideal point. In order to do so, we will construct a sequence of proposals, arising as deviations from the original vector of proposals $m^{*}$. We will construct this sequence such that the distance of proposals to the agenda setter's ideal point is monotone decreasing, and hence increasing in the agenda setter's utility. We will then show that this sequence is finite, and that it's final element is an equilibrium proposal.

Let $\underline{\omega}=\min _{i \in N p} \omega_{i, i}$ and $\bar{\omega}=\max _{i \in N p, j \in N p, j \neq i} \omega_{i, j}$. Note that by A2, $\frac{\bar{\omega}}{\omega}<1$. Let $\Lambda^{I}$ be a finite index set, for each $I \subset N p,|I|=T$, and let $\mathcal{C}^{I}=\left\{C_{\lambda}^{I} \mid C_{\lambda}^{I} \subset O_{m a j, I}, \lambda \in \bar{\Lambda}^{I}\right\}$ be a finite collection of closed, convex sets, such that,

- $\cup \mathcal{C}^{I}=O_{m a j, I}$,
- $\operatorname{int}\left(C_{\lambda}^{I}\right) \cap \operatorname{int}\left(C_{\lambda^{\prime}}^{I}\right)=\emptyset, \lambda, \lambda^{\prime} \in \Lambda^{I}, \lambda \neq \lambda^{\prime}$,
- and $\operatorname{diam}\left(C_{\lambda}^{I}\right)<1-\frac{\omega}{\bar{\omega}}, \forall \lambda \in \Lambda^{I}, \forall I$,
where $\operatorname{diam}\left(C_{\lambda}^{I}\right)$ is the maximal absolute-value distance between any two points in $C_{\lambda}^{I}$. Note that such set of closed and convex sets exists by point 2 in the above lemma.

Next, we inductively construct the sequence $\left\{m^{\ell}\right\}$ of vectors of proposals and selected agendas $\left\{A^{\ell}\right\}$. Recall that $x_{a a}\left(m^{\ell}, A^{\ell}\right)$ is the outcome of the agenda auction when voters vote truthfully over a vector of proposals $m^{\ell}$ and selected agenda $A^{\ell}$.

So let $m^{1}=m^{*}, A^{1}=I$, and suppose that for a given $\ell \geq 1, \exists j \in N p \backslash A^{\ell}, i \in A^{\ell}$, and an $m_{j} \in M_{j}$, s.t.,

$$
\left.u_{0}\left(x_{a a}\left(m_{-j}^{\ell}, m_{j}\right), A^{\ell} \backslash\{i\} \cup\{j\}\right)\right)>u_{0}\left(x_{a a}\left(m^{\ell}, A^{\ell}\right)\right),
$$

and

$$
\left.u_{j}\left(x_{a a}\left(m_{-j}^{\ell}, m_{j}\right), A^{\ell} \backslash\{i\} \cup\{j\}\right)\right)>u_{j}\left(x_{a a}\left(m^{\ell}, A^{\ell}\right)\right) .
$$

Hence, $j$ is a proposer who can make a proposal that is preferred by the agenda setter and by himself to the ones that are being selected from $m^{\ell}$ - if such proposers $j$ and $i$ don't exist, then the vector $m^{\ell}$ along with the set $A^{\ell}$ constitute an equilibrium of the agenda auction, and the sequence terminates at such $\ell$-th element. Denote by $\lambda^{\ell}$ the index, s.t., $\left(m_{-j}^{\ell}, m_{j}\right)_{A^{\ell} \backslash\{i\} \cup\{j\}} \in C_{\lambda^{\ell}}^{A^{\ell} \backslash\{i\} \cup\{j\}}$. Thus, $\lambda^{\ell}$ is the index of the set in $\mathcal{C}^{A^{\ell} \backslash\{i\} \cup\{j\}}$ to which the deviating proposal belongs. The idea is
now to find the furthest proposal in this set $C_{\lambda^{\ell}}^{A^{\ell} \backslash\{i\} \cup\{j\}}$, such that the proposer $j$ would still have incentives to deviate. So let

$$
\tilde{m}_{j} \in \arg \max _{m_{j}, \text { s.t., }\left(m_{-j}^{\ell}, m_{j}\right)_{A^{\ell} \backslash\{i\} \cup\{j\}} \in C_{\lambda^{\ell}}^{A^{\ell} \backslash\{i\} \cup\{j\}}} u_{0}\left(x_{a a}\left(m^{\ell}, A^{\ell}\right)\right),
$$

subject to the constraint,

$$
\left.u_{j}\left(x_{a a}\left(m_{-j}^{\ell}, m_{j}\right), A^{\ell} \backslash\{i\} \cup\{j\}\right)\right) \geq u_{j}\left(x_{a a}\left(m^{\ell}, A^{\ell}\right)\right) .
$$

Finally, let $m^{\ell+1}=\left(m_{-j}^{\ell}, \tilde{m}_{j}\right)$, and let $A^{\ell+1}=A^{\ell} \backslash\{i\} \cup\{j\}$.
It is clear that for this sequence, $u_{0}\left(x_{a a}\left(m^{\ell}, A^{\ell}\right)\right)<u_{0}\left(x_{a a}\left(m^{\ell+1}, A^{\ell+1}\right)\right)$ for all $\ell \geq 1$. Now we show that for $\ell<\ell^{\prime}<\ell^{\prime \prime}$, if $A^{\ell}=A^{\ell^{\prime}}=A^{\ell^{\prime \prime}}$, then it is impossible that $\lambda^{\ell}=\lambda^{\ell^{\prime}}=\lambda^{\ell^{\prime \prime}}$. We prove that if $\lambda^{\ell}=\lambda^{\ell^{\prime}}$, then it must be that $m^{\ell^{\prime}}$ is on the boundary of $C_{\lambda^{\ell}}^{A^{\ell}}$. It follows, that $\lambda^{\ell^{\prime \prime}} \neq \lambda^{\ell^{\prime}}$. We show that for the case when $\ell^{\prime}=\ell+2$ - whenever the difference between $\ell$ and $\ell^{\prime}$ is greater, the statement is true a-fortiori.

When $\ell^{\prime}=\ell+2$, there are $j \in N p \backslash A^{\ell}$ and $i \in A^{\ell}$, such that $A^{\ell+1}=A^{\ell} \backslash\{i\} \cup\{j\}$, and $A^{\ell+2}=A^{\ell}$, and $m^{\ell+1}=\left(m_{-j}^{\ell}, \tilde{m}_{j}\right)$, where $\tilde{m}_{j}$ is constructed as above. By construction, the agenda setter prefers $m^{\ell+1}$ to $m^{\ell}$, implying that,

$$
m_{j}^{\ell+1}=\tilde{m}_{j}<m_{i}^{\ell} .
$$

By construction, proposer $i$ weakly prefers $m^{\ell^{\prime}}=m^{\ell+2}$ to $m^{\ell+1}$, and noticing that the only proposals that are different between the outcomes of $m^{\ell^{\prime}}$ and $m^{\ell+1}$ are their $i$-th and $j$-th components, this implies,

$$
-\omega_{i, i}\left(y_{i, i}-m_{i}^{\ell+2}\right)-\omega_{i, j}\left(y_{i, j}-0\right) \geq-\omega_{i, i}\left(y_{i, i}-0\right)-\omega_{i, j}\left(y_{i, j}-m_{j}^{\ell+1}\right) .
$$

We can reduce this expression to $\omega_{i, i} m_{i}^{\ell+2} \geq \omega_{i, j} m_{j}^{\ell+1}$, and again by the agenda setter's preference, we have $m_{i}^{\ell+2}<m_{j}^{\ell+1}$. Now let $\hat{m}_{i}$ be such that $\omega_{i, i} \hat{m}_{i}=\omega_{i, j} m_{j}^{\ell+1}$, so that $\hat{m}_{i}=\frac{\omega_{i, j}}{\omega_{i, i}} m_{j}^{\ell+1}$. Hence, $\hat{m}_{i}$ is the marginal proposal, such that $i$ is just indifferent between proposing $\hat{m}_{i}$ over $m_{j}^{\ell}$. Note that $\hat{m}_{i}$ need not be majority feasible, and $m_{i}^{\ell+2} \geq \hat{m}_{i}$. Since $m_{j}^{\ell+1}<m_{i}^{\ell}$, we have that

$$
m_{i}^{\ell}-\hat{m}_{i}>m_{j}^{\ell+1}\left(\frac{\omega_{i, j}}{\omega_{i, i}}-1\right) \geq\left(\frac{\omega}{\bar{\omega}}-1\right) .
$$

Hence, given that $m_{i}^{\ell} \in C_{\lambda^{\ell}}^{A^{\ell}}$ and $\operatorname{diam}\left(C_{\lambda^{\ell}}^{A^{\ell}}\right)<\frac{\omega}{\bar{\omega}}-1$, one of the following two options must be true. Either a proposal that is closer to the agenda setter's ideal point than any proposal in $C_{\lambda^{\ell}}^{A^{\ell}}$ is majority feasible and still preferred to $\hat{m}_{i}$ by $i$, in which case $\lambda^{\ell^{\prime}} \neq \lambda^{\ell}$; Or, $m^{\ell^{\prime}}$ is on the boundary of $C_{\lambda^{\ell}}^{A^{\ell}}$, which is closest to the agenda setter's ideal point. In the latter case, $\lambda^{\ell^{\prime}}=\lambda^{\ell}$. But then, $m^{\ell^{\prime \prime}}$ can no longer lie in $C_{\lambda^{\ell}}^{A^{\ell}}$, so that $\lambda^{\ell^{\prime \prime}} \neq \lambda^{\ell}$.

Hence, for each $C_{\lambda}^{I},|I|=T, \lambda \in \Lambda^{I}$, at most two vectors from the sequence $m^{\ell}$ lie in the set $C_{\lambda}^{I}$. Since each $\Lambda^{I}$ is finite, the set $\cup_{I,|I|=T} \Lambda^{I}$ is also finite, so that the sequence $m^{\ell}$ can have at most finitely many distinct elements, and the last element must be an equilibrium of the agenda auction. Since $u_{0}\left(x_{a a}\left(m^{\ell}, A^{\ell}\right)\right)>u_{0}\left(x_{a a}\left(m^{\ell^{\prime}}, A^{\ell^{\prime}}\right)\right), \forall \ell>\ell^{\prime}$, the constructed equilibrium of the agenda auction is closer to the agenda setter's ideal point than the equilibrium outcome of the original equilibrium of the content-neutral scheduling.

The proof of the converse statement is as follows. Take an equilibrium outcome of the agenda auction, $x_{a a}^{*}=x_{a a}\left(m^{*}, A^{*}\right)$, such that $x_{a a}^{*} \leq y_{i, i}$, and construct the equilibrium of the contentneutral scheduling $x^{*, I}$, with $I=A^{*}$. If the proposal of each proposer $i \in A^{*}$ is optimal, given what other proposers from the set $A^{*}$ are proposing, then $x_{a a}^{*}=x^{*, I}$. Otherwise, take proposers in $I$ in any order, and again inductively define the sequence of proposals $m^{\ell}$, but this time keeping the set of proposers constant. Let $m^{1}=m^{*}$. Given an $\ell$, and $i \in I$, let $\tilde{m}_{i}^{\ell}$ be given by,

$$
\tilde{m}_{i}^{\ell}=\arg \underset{\left(m_{-i}^{\ell, I}, m_{i}\right) \in O_{m a j}, I}{ } u_{i}\left(x_{I S}^{I}\left(m_{-i}^{\ell}, m_{i}\right),\right.
$$

and define $m^{\ell+1}=\left(m_{-i}^{\ell}, \tilde{m}_{i}^{\ell}\right)$. Since $x_{a a}^{*} \leq y_{i, i}$, at every step the deviating proposal by proposer $i$ must be increasing, and by the second part of the previous lemma, the procedure must stop in a finite number of steps, when for each $i \in I$ either $m_{i}^{\ell}=y_{i, i}$, or $m_{i}^{\ell, I}$ is on the boundary of $O_{m a j, I}$.

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[^0]:    ${ }^{1}$ For example, it is not possible that a legislature could meet in a plenary session for more than 24 hours a day and often there are constitutional limits on the number of days a given legislature can be in session.

[^1]:    ${ }^{2}$ In fact, the Rules Committee also gets to choose the rules for the vote, which gives it even more power.
    ${ }^{3}$ That final policy is consumed also by the voters who are not in favor of a given policy move is an important difference between voting as compared to market outcomes. See especially Demsetz (1981).
    ${ }^{4}$ Gilligan and Krehbiel (1990) also generate agency problems with legislative committees but because of informational asymmetries; see also Krehbiel (1992). For why and how such asymmetries may arise, see also the recent survey of the literature on information acquisition by committees by Gehring et al. (2005).

[^2]:    ${ }^{5}$ Stigler (1971) illustrates such agency problems in the case of regulatory agencies captured by the industries they are supposed to regulate. Our analysis suggests that in such cases agenda scheduling powers may figure importantly.
    ${ }^{6}$ Recent work by Levy and Razin (2010) is also related. They analyze a dynamic model of an all-pay contest of agenda formation where the probability to get a proposal on the agenda is increasing in an agent's payment, and there is a probability that the session ends before any proposal is implemented.

[^3]:    ${ }^{7}$ For example, in an Euclidean policy space one could make jurisdictions be discrete subsets of the policy space.
    ${ }^{8}$ This is clearly a simplification in that proposers are single voters, whereas in a committee, the preferences of members of the committee must be somehow aggregated; Loosely speaking, a proposer can be thought of as a representative member of a committe.

[^4]:    ${ }^{9}$ It is not necessary to interpret this constraint as a time constraint. Such constraint could also describe the limited admistrative resources that can be devoted to implementing moves on different policy dimension. Or it might even be imposed on the institution, perhaps stemming from the interests of some agent, whom might be interal or external to the legislature.
    ${ }^{10}$ An interpretation of this assumption is that no member of the legislature has a gate-keeping power. Our opinion is that this is the simplest modeling possibility.
    ${ }^{11}$ Why should the final policy be given by the sum of proposals on the agenda? One could consider other functional forms describing how the final policy is obtained from the proposals on the agenda. The sum of all proposals is the simplest, and it captures well the dependence of final policy on different proposals on the agenda along potentially different jursidictions. One could also imagine the sum of all proposals on the agenda as describing the reduced form of a process by which the final outcome could be achieved if each move was implemented separately.

[^5]:    ${ }^{12}$ We assume that all players here are sufficiently sophisticated for SPNE to be an appropriate equilibrium notion.
    ${ }^{13}$ For a simpler example with a similar intuition in terms of the equilibrium existence, consider a Bertrand game between two firms with different costs $c_{1}<c_{2}$. Interpret the demand side as a single consumer with a demand curve. Then an equilibrium will exist if whenever indifferent, the consumer may resolve this indifference in any way; clearly, assuming that the consumer randomizes with equal probabilities whenever indifferent will not work. In equilibrium, he will buy from the first firm for sure, at a price $c_{2}$.

[^6]:    ${ }^{14}$ A special case of our existence theorem is easier to prove, and we note that special case here, as it suffices for all our examples. If players' preferences are strictly quasi concave, rather than just quasi concave, and $M_{i}$ is convex for each $i \in N p$, then the subset of $\times_{i \in N p} M_{i}$ on which the subgame-perfect correspondence is single-valued is dense in $\times_{i \in N p} M_{i}$. It is convex-valued on the whole strategy space so the result then follows from the discussion in Simon and Zame (1991). The formal proof of this claim then follows along the lines of our existence theorem, and is just slightly simpler. Observe however that in many examples of political decisions, preferences might display some indifference, so that it is desirable to prove the existence result for the more general case of quasi concavity.

[^7]:    ${ }^{15}$ Given an $m \in M, m=\left(m_{1}, m_{2}, \ldots, m_{n p}\right)$, and $m_{j}^{\prime} \in M_{j}$, we denote by $\left(m_{-j}, m_{j}\right)=$ $\left(m_{1}, \ldots, m_{j-1}, m_{j}^{\prime}, m_{j+1}, \ldots m_{n p}\right)$ the vector of proposals when proposer $j$ deviates from $m_{j}$ to $m_{j}^{\prime}$.
    ${ }^{16}$ One could also think of the agenda setter as a leader of a particular majority group of voters. Then an agenda setter holding a view over different dimensions roughly corresponding to the average of his supporters might also be better able to sustain his power base in the long run.
    ${ }^{17}$ Under this assumption, the proof of Proposition 9 below provides a constructive proof for the existence of

[^8]:    equilibrium in the agenda auction, when the agenda setter's ideal point does not belong to the majority-win set.
    ${ }^{18}$ This assumption is not necessary for the voters in $N \backslash(N p \cup\{0\})$. It is also not necessary that $N$ is finite - what is needed for the purpose of this section is that Lemma 1 hold (see the Appendix).
    ${ }^{19}$ Apart from the self-interest motive, in reality, some committees have greater power and prestige than others, and appointments are influenced by seniority. Note that we do not assume that a proposer must be the most radical member of the legislature on a given dimension.
    ${ }^{20}$ It seems unlikely that proposers would seek assignments on jurisdictions that they cared little about, unless of course this were a part of a collusive scheme in which as a commitment device, a large number of proposers sought such appointments. But this would require prescient coordination among a large number of legislators from the majority, and it seems a much simpler scheme that they should select a different agenda setter instead.

[^9]:    ${ }^{21}$ For an intuitive comparison, one may again consider the standard Bertrand model with equal marginal costs, say $\underline{c}$. There, all prices between a given price above $\underline{c}$, and any other higher price, are feasible. If the current lowest price in the market is some $p>\underline{c}$, any producer may undercut this price, and thus carry the whole market. In contrast, in a political setting, the shape of $O_{m a j}$ is such that it is by no means guaranteed. The Bertrand case is essentially analogous to a rectangle-shaped $O_{m a j}$; even convexity of $O_{m a j}$ is neither necessary or sufficient, as the projections of $O_{m a j}$ on different subsets $I$ might have very different shapes and pose different restrictions. It is precisely the existence of some proposer who can undercut current "best" proposals, i.e., feasible competition, which guarantees that proposals will unravel, from any initial vector of proposals.

[^10]:    ${ }^{22}$ In a multi-dimensional policy space, the median voter may be a different one on each dimension. In that case we define the ideal policy as the vector comprised of the median ideal policies on each of the dimensions. In a purely redistributive environment, we define the ideal policy as the egalitarian policy.

[^11]:    ${ }^{23} \mathrm{We}$ comment that a large majority-win set is a simplifying assumption in that it automatically eliminates equilibrium outcomes which arise due to the irregular shape of the majority-win set. Such equilibria might either have relatively un-intuitive interpretation, or they may naturally arise if the restrictions of the majority-win set are binding. In either case, it seems intuitive that a similar welfare comparison to that in the corollary would still hold, but it would be much more complicated and less transparent.

[^12]:    ${ }^{24}$ The protocol itself may be relevant to what proposals make it on the agenda. However, when proposers are rational in the sense of having perfect foresight, the resulting outcome will be one of the equilibrium outcomes of our

