

Welfare effects of housing price appreciation in the economy with binding credit constraints

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Abstract

This paper analyzes the effects of recent housing price appreciation on aggregate welfare. It generalizes previously available results by considering credit constraints together with endogeneity of housing prices. First, housing price appreciation implies improvement in aggregate welfare in a model with exogenous housing price and credit constraints. Then, housing price is endogenized by modeling the supply side of the housing market. In this model, housing price appreciation is caused by supply and demand shocks. The supply shock originates from a change in building permit cost. The demand shifts are generated by changes in household income and interest rates. Both credit-constrained and unconstrained versions of this model are considered. Finally, the combination of observed demand and supply shocks is used to quantify aggregate welfare effects on the US housing market from 1995 to 2004. The results demonstrate that demand shocks dominated during that period and the aggregate welfare improved as a result of housing price appreciation.

KEY WORDS : housing price appreciation, aggregate welfare, binding credit constraints, endogenous housing price, demand and supply side shocks

JEL CLASSIFICATION: R2, R20, R21, R31

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1 Introduction

In the second half of 1990s first half of 2000s a considerable increase in housing prices was observed in majority of developed countries. Particularly in the United States housing prices have risen at a rate exceeding growth rate of income and other asset prices during the last decade (Bajari et al (2005), Li and Yao(2004)). Between 1986 and 1994, the increase in housing prices was 22.1% as opposed to 41.9% for the period from 1996 to 2004, using the constant-quality housing price index published by the US Census Bureau (see Figure 1).

¹ This has stimulated research on the effects of housing price appreciation, particularly its link with monetary policy, its role in the business cycle and most importantly, its effects on consumption and consumer welfare (see for example Iacoviello and Minetti(2003), Iacoviello(2004), Li and Yao(2004), Campbell and Cocco(2005), Bajari et al(2005)).

Some papers have studied the effects of the increase in housing prices on the consumption and welfare of separate groups such as young renters, young homeowners and old homeowners. For example, Campbell and Cocco (2005) use UK micro-level data on real non-durable consumption growth and real housing price growth together with a life-cycle model to demonstrate a positive effect of an increase in the growth rate of housing prices on the growth rate of consumption. This effect is especially strong and significant for old homeowners and, still quite significant but smaller in magnitude, for young homeowners. Li and Yao (2004) also employ a life-cycle model of housing tenure choice to explore the effects of housing price shocks on household consumption and welfare. They find that for the homeowners less than

¹Similar observation can be made using other housing price measures e.g. the average purchasing price of housing from the Federal Housing Finance Board. It increased by 28.4% in the period 1986-1994 and by 68.9% from 1996 to 2004.

40 years old a permanent increase in housing price implies welfare losses while in case of older homeowners it implies an increase in their real non-durable consumption as well as welfare.

Bajari et al (2005) study the aggregate effects of housing price changes on consumer welfare. They develop a new approach to measuring the changes in consumer welfare due to changes in the prices of owner-occupied housing. This approach defines welfare adjustment as the transfer in the form of income required to keep expected discounted utility constant, given the change in housing prices. The authors claim that this measure is more accurate than the user cost employed in earlier studies. The reason is that the user cost (defined as the marginal rate of substitution between housing and non-durable consumption) is entirely static while the welfare adjustment involves dynamics. In addition, user costs fail to take into account the role of housing as an investment good. Using their measure of welfare adjustment, the authors show that there is no change in aggregate welfare due to an increase in the price of the existing stock of housing. This result is based on a simple market clearing condition, which implies that the losses of buyers are exactly compensated by the gains of sellers. This conclusion holds for both a deterministic version of the model where current state conveys no information about future states, as well as for a stochastic one, where the state follows a first order Markov chain.

Bajari et al (2005) abstract from rental markets and binding credit or borrowing constraints. However, for households subject to binding credit constraints, housing appreciation implies two kinds of effects: i) an increase in lifetime housing costs because of the necessity to buy a larger house in the future; ii) a benefit due to a relaxation of credit constraints (because of increased housing equity) and thus the opportunity for better consumption smoothing.

Thus, by abstracting from credit constraints, Bajari et al (2005) ignore the additional effects, which housing price appreciation has on credit-constrained households. Empirically, one can evaluate the importance of credit constraints from the fact that over 65% of owner occupied housing in US is mortgage-financed (according to American Housing Survey). Also, credit constraints are binding in the US economy since the maximum allowed loan-to-value ratio (LTV) for conventional mortgages in second half of 1990's beginning of 2000's was equal to 80%² (see Tsakaronis and Zhu (2004)) and average actual LTV for years 1995-2004 fluctuated between 75.1 and 79.9% (according to Monthly Interest Survey of Federal Housing Finance Board). From the modeling perspective, Ortalo-Magne and Rady(2005) identify a crucial role of capital gains and losses experienced by credit-constrained individuals in explaining housing market fluctuations.

In the first part of this paper, the aggregate welfare effects of housing price appreciation are studied in a model analogous to Bajari et al (2005) but with households subject to binding credit constraints. Two major forms of credit constraint have been used in the previous literature. One of the most widely used models of credit constraints is that of Kiyotaki and Moore (1997). The authors study how credit constraints interact with aggregate economic activity over the business cycle. In this model, borrowing is restricted so that the

²Maximum LTV in this context refers only to conventional (prime) single family mortgages. During the last decade rapidly growing sub-prime lending market has appeared in the US. Sub-prime mortgages usually have higher LTVs than conventional ones, since they are given to households unable to meet the usual down payment requirements. Sub-prime loans are not considered here. Although their share in total mortgages constantly grew between 1995-2004, the proportion of sub-prime mortgages didn't exceed 20 % of the total mortgages by 2004 (according to the US Federal Reserve Bank).

repayment of a loan in the next period does not exceed the next period's value of the asset serving as collateral. Similar borrowing constraints is used in Iacoviello and Minneti(2003), Iacoviello (2004), etc. A more efficient form of credit constraint, called a margin clause, is considered in Mendoza and Durdu (2004). They employ collateral constraints under which the borrowing of a small open economy cannot exceed a fraction of the current market value of the economy's equity holdings. This type of contract is more effective and is widely used in international capital markets by investment banks and other lenders as a mechanism to manage default risk. In contrast to the Kiyotaki-Moore constraint, the custody of collateral assets is transferred at the time of entering into a credit contract (in Kiyotaki Moore model it is transferred only in the next period, which is why it limits borrowing to the value of the asset in that period). Moreover, there is more flexibility and less risk for lenders since they can automatically make up shortfalls in the value of the collateral asset by liquidating it as soon as the price changes so that the value of the collateral is exactly equal to the debt.

Presented results show that in an economy with binding credit constraints housing price appreciation implies an overall improvement in aggregate welfare. In a model with the Kiyotaki-Moore type constraint, this result holds only with the additional assumption that housing prices follow a random walk. In the model with a margin clause this result is observed even in the simplest deterministic version. This is due to the fact that the margin clause constraint is immediately affected by the housing price appreciation as the current price enters this constraint. However, if Kiyotaki-Moore constraint is used, the next period's price enters the constraint and it is not necessarily affected by the change in current price.

In Bajari et al (2005), the housing prices are exogenous. In contrast, I allow housing price to be determined by the equilibrium in the housing market and to change due to

supply-side and demand-side shocks. Modeling of the supply side shock follows primarily Glaeser and Gyourko (2005). They show that the increase in housing prices since the 70's mainly reflects an increasing difficulty of obtaining regulatory approval for building houses. This can be explained by changing judicial tastes, decreasing ability to bribe regulators, and stricter formal procedures. Similarly, in my model an endogenous supply shock is generated by an increase in building permit costs. Besides analyzing the consequences of housing price appreciation driven by supply-side shocks, the theoretical model is used to explore the consequences of housing price appreciation driven by demand-side shocks. Inspection of the US data allows one to identify changes in income and interest rates as the most important demand-side shocks observed during 1995-2004. The effects of demand and supply-side shocks are analyzed for both credit constrained and unconstrained versions of the model.

The results of the endogenous price model demonstrate that the final welfare effect of housing price appreciation depends on its source. Housing price appreciation driven by negative supply-side shocks such as increase in building permit cost leads to welfare loss, while housing price appreciation driven by positive demand-side shocks such as increase in income or decrease in the interest rates implies a welfare gain. Comparison of welfare adjustments in a constrained and unconstrained model resulting from change in the building permit costs reveals that the relationship between them depends on the relative weight of housing in the utility function (under Cobb-Douglas form of preferences). Finally, the credit-constrained and unconstrained models are calibrated using a combination of actual demand and supply shocks in the US housing market in 1995-2004. The result demonstrates that housing price appreciation leads to an improvement in aggregate welfare.

The rest of the paper is organized as follows. Section 2 describes and solves the proposed

model with households facing credit constraints and interprets the results. Section 3 builds and solves the model with endogenous housing price in both credit-constrained and unconstrained versions in which the changes in the housing price are driven by supply side shocks. Section 4 interprets and compares the results of credit constrained and unconstrained models. Section 5 analyzes the welfare implications of housing price appreciation driven by demand side shocks. Section 6 determines the change in aggregate welfare due to housing price appreciation driven jointly by the supply side and demand side shocks. Section 7 concludes the paper.

2 Model with exogenous housing price and credit constraints

2.1 Model definition and solution

Consider an economy subject to credit constraints in which there are two goods: a composite consumption good c and housing h with a relative price q which is deterministic and exogenous as in the benchmark model . Also, there are risk-free assets in the form of bonds b . Households choose how many bonds to carry into next period b_{t+1} (b_{t+1} can be either positive or negative. In the latter case households are borrowers), how much housing stock to carry into next period h_{t+1} , and how much to consume now c_t . A household's investment into housing is denoted by x_t , and the investment in the risk-free asset (saving) is denoted by s_t . Households have real income y_t . The interest rate paid for borrowing or received for investment in bonds is exogenous and given by i_t . Adjustment of housing stock implies

transaction costs which enter into the budget constraint as a separate expenditure which is denoted by $f1\{x_t \neq 0\}$. Here $f1$ is an operator which takes fixed positive value when steady state investment into housing is not equal to zero and value zero when investment into housing is equal to zero. In this version of the model, the depreciation of housing and new construction is abstracted from and it is assumed that there is a fixed stock of housing traded between the agents.

Households are credit-constrained in the sense that they can borrow only up to a certain amount to finance their housing investment. Under margin clause constraint (Mendoza and Durdu(2004)) households can borrow only up to some fraction of their current wealth. In the present model, a household's current wealth consists of the current value of its housing stock which can be used as a collateral. Thus credit constraint takes the form $b_{t+1} \geq -mq_t h_{t+1}$ i.e. households can borrow only up to fraction $m < 1$ of the total value of their existing housing stock.

The problem of the household can be formulated in the following way:

$$V(h_t, b_t, q_t, y_t) = \max_{\{c_t, h_{t+1}, b_{t+1}\}} \{u(c_t, h_t) + \beta V(h_{t+1}, b_{t+1}, q_{t+1}, y_{t+1})\}, \quad (1)$$

s.t

$$c_t + q_t x_t + s_t + f1\{x_t \neq 0\} = y_t + i_t b_t, \quad (2)$$

$$b_{t+1} - b_t = s_t - \pi b_t, \quad (3)$$

$$h_{t+1} - h_t = x_t \quad (4)$$

$$b_{t+1} \geq -mq_t h_{t+1}. \quad (5)$$

Besides the credit constraint discussed above the optimization includes three additional constraints. One is the usual budget constraint. The second constraint says that real savings

(investment into bonds) should be equal to the difference between bondholding for the next period and the current bondholding net of inflation π . The third says that each period's investment in housing should be equal to the difference between the next period's housing stock and the current housing stock.

One can substitute (3) and (4) into (2) to simplify the maximization and obtain the following constraints:

$$c_t = y_t + i_t b_t - q_t(h_{t+1} - h_t) - (b_{t+1} - (1 - \pi)b_t) - f1\{x_t \neq 0\}, \quad (6)$$

$$b_{t+1} \geq -mq_t h_{t+1}. \quad (7)$$

The maximization of (1) subject to (6) and (7) gives the following F.O.C. and Envelope conditions:

$$\frac{\partial u(c_t, h_t)}{\partial c_t} = \lambda_t, \quad (8)$$

$$-q_t \lambda_t + \beta \frac{\partial V(h_{t+1}, b_{t+1}, q_{t+1}, y_{t+1})}{\partial h_{t+1}} + v_t m q_t = 0, \quad (9)$$

$$-\lambda_t + v_t + \beta \frac{\partial V(h_{t+1}, b_{t+1}, q_{t+1}, y_{t+1})}{\partial b_{t+1}} = 0, \quad (10)$$

$$\frac{\partial V(h_t, b_t, q_t, y_t)}{\partial h_t} = \frac{\partial u(c_t, h_t)}{\partial h_t} + \lambda_t q_t, \quad (11)$$

$$\frac{\partial V(h_t, b_t, q_t, y_t)}{\partial b_t} = \lambda_t (i_t + 1 - \pi). \quad (12)$$

where v is the Lagrange multiplier for the credit constraint and λ is the Lagrange multiplier for the budget constraint. From equation (10) it is possible to determine under what condition the multiplier of credit constraint v_t is positive meaning that constraint is binding. Substituting equation (8) and equation (12) into condition (10) and rearranging yields the following formula for v_t :

$$v_t = \frac{\partial u(c_t, h_t)}{\partial c_t} - \beta \frac{\partial u(c_{t+1}, h_{t+1})}{\partial c_{t+1}} (i_{t+1} + 1 - \pi).$$

This last equation implies that constraint is binding if the following holds :

$$\frac{\frac{\partial u(c_t, h_t)}{\partial c_t}}{\beta \frac{\partial u(c_{t+1}, h_{t+1})}{\partial c_t}} > i_{t+1} + 1 - \pi. \quad (13)$$

Thus credit constraint is binding if intertemporal marginal rate of substitution between consumption today and consumption tomorrow is higher than gross real interest rate.

Let's derive the user cost from this model and see how the existence of a binding credit constraint changes results of the benchmark model. User cost was defined in Dougherty and Vanrder(1982) as the marginal rate of substitution between housing consumption and other consumption. It is essentially the measure of the value of compensation which is necessary to force homeowners to give up one unit of housing. Combining the above equations, shifting the resulting equation one period back and expressing from it the marginal rate of substitution between housing consumption and composite good consumption yields:

$$\frac{\frac{\partial u(c_t, h_t)}{\partial h_t}}{\frac{\partial u(c_t, h_t)}{\partial c_t}} = mq_{t-1} \left(i_t - \pi - \frac{\Delta q_t}{mq_{t-1}} + \frac{m-1}{m} \right) + q_{t-1}(1-m) \frac{\frac{\partial u(c_{t-1}, h_{t-1})}{\partial c_{t-1}}}{\beta \frac{\partial u(c_t, h_t)}{\partial c_t}}. \quad (14)$$

From (13) it can be concluded that the imposition of credit constraint has an ambiguous effect on user cost. Since $m < 1$, the first term in this expression is unambiguously lower than the analogous term in Bajari et al's paper. However, user cost in this model includes an additional term which is positive and easy to quantify. This term can be interpreted as the inverse of the return on housing investment. Investing into one more unit of housing at time $t-1$ requires a reduction in consumption by q_{t-1} , and each unit of reduction in consumption

presupposes a loss equal to the marginal utility of consumption. On the other hand, in period t additional consumption is obtained, since the increased housing stock implies the possibility of higher borrowing in period t and therefore higher consumption. Thus, the expression in the denominator of the second term can be viewed as the benefit from investing into housing. The fraction m is also present in this term since borrowing increases only by the fraction m for each unit of increase in the housing stock. The user cost in the economy subject to credit constraints can be either higher or lower than in the benchmark paper depending on parameters.

Now the dynamic welfare adjustment first defined in Bajari et al(2005) should be derived for an economy subject to credit constraints. In this paper analysis is focused on the case with binding credit constraint (the condition that guarantees that it is binding is given above) and it is used with equality³. Let's define the welfare adjustment as compensation in the form of income necessary to keep a household's life-time utility unchanged or in other words to keep the value function constant given change in housing prices. This change in income is converted into utility terms by multiplying it by the marginal utility of wealth

³The analysis in this paper is concentrated on the period from 1995 to 2003 . For this period the assumption of credit constraint remaining constantly binding can be justified by the large increase in mortgage refinancing activity in the US. In particular the refinancing index, which is published by Mortgage Bankers Association of America and changes in which represent percent changes in mortgage refinancing applications compared with the previous month, increased from 1.5 in 1995 to around 10 in 2003. One of the crucial reasons behind this increase was the desire of the consumers to extract housing equity built -up as a result of housing price appreciation. This refinancing makes non-binding credit constraint binding again.

which is equal to the Lagrange multiplier of the budget constraint.

The change in the value function due to a change in housing price can be defined as :

$$\Delta V_i = \frac{\partial V(h_t, b_t, q_t, y_t)}{\partial q_t} \Delta q. \quad (15)$$

After the household is compensated for the change in lifetime utility due to change in housing prices, the final change in value function is given by:

$$\Delta V_T = \frac{\partial V(h_t, b_t, q_t, y_t)}{\partial q_t} \Delta q + \frac{\partial V(h_t, b_t, q_t, y_t)}{\partial y_t} \Delta y.$$

where ΔV_T stands for total change in value function. From this equation Δy is derived such that change in the value function equals zero. Based on Bajari et al (2005), an envelope theorem and the first order approximation is applied. Taking derivatives yields:

$$\begin{aligned} \frac{\partial V(h_t, b_t, q_t, y_t)}{\partial q_t} &= \frac{\partial u(c_t, h_t)}{\partial c_t} \frac{\partial c_t}{\partial q_t} = \\ &= \frac{\partial u(c_t, h_t)}{\partial c_t} (-x_t) + \left(\frac{\partial u(c_t, h_t)}{\partial c_t} m h_{t+1} - \beta \frac{\partial u(c_{t+1}, h_{t+1})}{\partial c_{t+1}} (1 - \pi + i_{t+1}) m h_{t+1} \right) \\ &\quad \frac{\partial V(h_t, b_t, q_t, y_t)}{\partial y_t} = \frac{\partial u(c_t, h_t)}{\partial c_t}. \end{aligned} \quad (16)$$

Thus in this economy, the effect of a price change on value function consists of two effects, a direct one and an indirect one. When housing price appreciates, there is a decrease in consumption due to more expensive investment into housing. This is the direct effect reflected in the first term in equatuon (16). On the other hand, due to the increase in price, the housing equity increases and borrowing constraint relaxes. This allows households to increase borrowing and, consequently, current consumption. This benefit net of the cost of repaying the additional borrowing in the next period is presented in parentheses in equation (16). This is the indirect effect.

Equating ΔV_T to zero and expressing Δy from the resulting equation yields the following formula for the individual welfare adjustment in this model:

$$\begin{aligned} \Delta y_t &= x_t \Delta q_t - mh_{t+1} \Delta q_t + \frac{\beta \frac{\partial u(c_{t+1}, h_{t+1})}{\partial c_{t+1}}}{\frac{\partial u(c_t, h_t)}{\partial c_t}} (1 - \pi + i_{t+1}) mh_{t+1} \Delta q_t = \\ &= x_t \Delta q_t - \left(mh_{t+1} \Delta q_t - \frac{\frac{(1 - \pi + i_{t+1})}{\partial u(c_t, h_t)}}{\frac{\partial c_t}{\beta \frac{\partial u(c_{t+1}, h_{t+1})}{\partial c_{t+1}}}} mh_{t+1} \Delta q_t \right). \end{aligned}$$

Taking into account equation (13) it can be seen that under the binding credit constraints the term in parenthesis in the last equation is positive. Using the utility function of the form $u(c, h) = \frac{(c^{1-\omega} h^\omega)^{1-\gamma}}{1-\gamma}$ based on Li and Yao (2004), the welfare adjustment can be presented in the following form :

$$\Delta y_t = x_t \Delta q_t - \left(mh_{t+1} \Delta q_t - \beta \frac{(1 - \pi + i_{t+1})(1 + \mu)^{\omega - \omega\gamma}}{(1 + \sigma)^{\omega - \omega\gamma + \gamma}} mh_{t+1} \Delta q_t \right).$$

where μ stands for housing stock growth rate and σ stands for composite good consumption growth rate.

Let me also discuss the result in case of using Kiyotaki-Moore constraint. This constraint limits the borrowing so that gross repayment next period does not exceed a fraction of next period's expected monetary value of the collateral asset. In terms of the present model it has the form $(1 + i_{t+1})b_{t+1} \geq -mE_t q_{t+1} h_{t+1}$. The crucial difference between margin clause and this constraint is that the next period's price rather than this period's price enters into the credit constraint. If the housing price next period is not affected by the

change in current price, the credit constraint will not be relaxed and consequently change in aggregate welfare will still be zero as in Bajari et al (2005). However several empirical papers have demonstrated that housing prices follow either random walk or AR(1) with high persistence. Using AR(1) assumption and applying the same procedure to the model with a Kiyotaki-Moore constraint, the following formula for the individual welfare adjustment can be derived:

$$\Delta y_t = x_t \Delta q_t - \frac{\rho m h_{t+1} \Delta q_t}{1 + i_{t+1}} + \frac{(1 - \pi + i_{t+1}) \rho m h_{t+1} \Delta q_t}{\frac{\partial u(c_t, h_t)}{\partial c_t} (1 + i_{t+1})} \frac{\frac{\partial c_t}{\partial u(c_{t+1}, h_{t+1})}}{\beta \frac{\partial c_{t+1}}{\partial u(c_{t+1}, h_{t+1})}} \quad (17)$$

Here the positive effect on consumption due to relaxation of credit constraint is discounted by the gross interest rate since it can be realized only next period.

2.2 Interpretation and quantification of the welfare adjustment

This section interprets and quantifies the final result. For convenience, here I restate the formula for individual welfare adjustment:

$$\Delta y_{j,t} = x_{j,t} \Delta q_t - \left(m h_{j,t+1} \Delta q_t - \beta \frac{(1 - \pi + i_{t+1})(1 + \mu)^{\omega - \omega \gamma}}{(1 + \sigma)^{\omega - \omega \gamma + \gamma}} m h_{j,t+1} \Delta q_t \right) \text{ for household } j. \quad (18)$$

Comparing the result in (18) to that of Bajari et al (2005), two crucial differences can be noted. First, as it was shown above, the term in parenthesis in equation (18) is positive which implies that for all households in the model economy the potential welfare loss is lower (welfare gain is higher) than in the benchmark paper since there is an additional beneficial effect of housing price appreciation on consumption. This effect comes in the form of relaxation of credit constraints which gives a better opportunity to smooth consumption.

Second, homeowners do get a certain benefit from housing price appreciation even without participating in housing transactions (when $x_{j,t} = 0$), which is quite consistent with reality. For instance, older homeowners can leave larger bequests or invest more in retirement accounts even without selling their house. Younger homeowners can shift their investment to risky assets or increase consumption.

The aggregate welfare adjustment is equal to the sum of individual adjustments defined by (17). Using the assumption of investment only into existing housing stock and summing up, the first term of the expression vanishes ($\sum x_{j,t} = 0$)⁴, yielding the following expression for the aggregate welfare adjustment:

$$\begin{aligned} W_t &= -\sum_j \left(mh_{j,t+1}\Delta q_t - \beta \frac{(1 - \pi + i_{t+1})(1 + \mu)^{\omega - \omega\gamma}}{(1 + \sigma)^{\omega - \omega\gamma + \gamma}} mh_{j,t+1}\Delta q_t \right) = \\ &= -(m\sum_j h_{j,t+1}\Delta q_t - \beta \frac{(1 - \pi + i_{t+1})(1 + \mu)^{\omega - \omega\gamma}}{(1 + \sigma)^{\omega - \omega\gamma + \gamma}} m\sum_j h_{j,t+1}\Delta q_t). \end{aligned} \quad (19)$$

Since it was shown that the term in parenthesis in equation (19) is positive, the total sum in equation (19) is negative. Thus, the aggregate welfare adjustment in this economy with exogenous housing prices and credit-constrained households is negative, implying that in aggregate less income is necessary to keep lifetime utility constant. That is, housing

⁴The term in parenthesis in equation (18) does not sum up to zero since all the households are subject to binding credit constraints implying that all the households are net borrowers. Here it is assumed that funds for borrowing are supplied from external sources. This appears to be a reasonable assumption for US economy. In the discussed period in the US economy, borrowing was largely financed by means of increasing external debt. In particular, from 1995 to 2003 total US external debt rose from 26 trillion dollars to 45 trillion dollars (Federal Reserve, US Treasury data). While the US government debt remained almost unchanged during this period, the US financial sector external debt increased by around 70% and business external debt increased by around 30% (Federal Reserve, US Treasury data).

price appreciation in an economy subject to binding credit constraints actually implies an improvement in aggregate welfare. Everybody in the economy who possesses any housing stock is made better off due to the relaxation of binding credit constraints. The finding is consistent with the observation that in certain years characterized by housing price appreciation developed countries experienced consumption growth or even a consumption boom (Campbell and Cocco(2004)).

It is possible to quantify the result in (19) and compare it to the result of Bajari et al (2005). The composite good consumption growth rate is approximated by the non-durable consumption growth rate taken from Bureau of Economic Analysis NIPA tables. The residential housing stock growth rate is taken from the Bureau of Economic Analysis Fixed Asset Tables. Inflation and risk-free interest rate are taken from IMF International Financial Statistics. Also, it is assumed that $m = 0.8$ (look in the introduction), $\gamma = 2$ (based on Li and Yao (2004)), $\omega = 0.2$ (based on Li and Yao (2004)), and $\beta = 0.98$.

The term $\sum_j h_{j,t+1} \Delta q_t$ can be interpreted as the change in the market value of the total housing stock, or in other words the change in the aggregate nominal housing wealth. The data on aggregate nominal housing wealth in the US can be obtained from several studies (such as Case, Quigley and Shiller (2001), Nothaft (2004), etc). However, when using it to quantify the result of this model, it is important to take into account three observations. Firstly, the model does not have the explicit choice of renting the house. Consequently, only the change in the value of owner-occupied housing stock should be considered. Secondly, the effect of relaxing borrowing constraints reflected in (18) should in reality be experienced only by credit-constrained households who take a mortgage when purchasing the house. Finally, due to considering the case of binding credit constraints, this result is true for

the households having mortgages with a maximum LTV (or close to it). Based on these considerations, the yearly change in the nominal housing wealth in US is multiplied by the share of owner-occupied housing in the total housing stock, by the share of mortgage-financed owner-occupied housing in the total owner occupied housing stock and also by the share of mortgages with LTV 70-80% (the average LTV in this group is 79%) in the total number of mortgages (see the data appendix for the data sources used to calculate these shares). The resulting numbers are then divided by the total number of households in the US economy (taken from Current Population Report of US Department of Commerce) to obtain per household change in aggregate welfare (in 2003 dollars) in the model with credit-constrained households. The results are displayed in Figure 2. The figure displays the absolute value of welfare change in (19) so the numbers are positive.

The obtained results contrast sharply with those of Bajari et al (2005), who found no effects of housing price appreciation on aggregate welfare in case of investing into existing housing stock. It turns out that when accounting for binding credit constraints, the housing price appreciation which occurred in the US between 1995 and 2003 improved aggregate welfare on average by 740 dollars per household a year or around 6700 dollars per household in total.

3 Model with endogenous housing prices: Supply side shocks

3.1 Households

The basic assumptions about the household sector in this model are analogous to the assumptions in section 2. The crucial difference is that the housing price is determined endogenously. To be more realistic, this version also takes into account physical depreciation of housing and assume that it occurs with constant rate δ .

The household problem in the economy with endogenous housing price and credit constraints can be formulated as follows:

$$V(h_t, b_t, y_t) = \max_{\{c_t, h_{t+1}, b_{t+1}\}} \{u(c_t, h_t) + \beta V(h_{t+1}, b_{t+1}, y_{t+1})\},$$

s.t

$$c_t + q_t x_{d,t} + s_t + f1\{x_t \neq 0\} = y_t + i_t b_t, \quad (20)$$

$$b_{t+1} - b_t = s_t - \pi b_t, \quad (21)$$

$$h_{t+1} - h_t = x_{d,t} - \delta h_t, \quad (22)$$

$$b_{t+1} \geq -mq_t h_{t+1}. \quad (23)$$

where subscript d denotes a variable belonging to the demand side of the housing market.

The Euler equations for this model are given by:

$$v_t = \frac{\partial u(c_t, h_t)}{\partial c_t} - \beta \frac{\partial u(c_{t+1}, h_{t+1})}{\partial c_{t+1}} (i_{t+1} + 1 - \pi), \quad (24)$$

$$\begin{aligned}
q_t \frac{\partial u(c_t, h_t)}{\partial c_t} &= \beta \frac{\partial u(c_{t+1}, h_{t+1})}{\partial h_{t+1}} + \beta \frac{\partial u(c_{t+1}, h_{t+1})}{\partial c_{t+1}} q_{t+1} (1 - \delta) + \\
&+ m q_t \left(\frac{\partial u(c_t, h_t)}{\partial c_t} - \beta \frac{\partial u(c_{t+1}, h_{t+1})}{\partial c_{t+1}} (i_{t+1} + 1 - \pi) \right).
\end{aligned} \tag{25}$$

In the unconstrained version of the endogenous price model, households are not subject to a credit constraint. Therefore, it is absent from their optimization problem. The rest of the problem is the same. Euler equations for this model are given by:

$$\frac{\partial u(c_t, h_t)}{\partial c_t} = \beta \frac{\partial u(c_{t+1}, h_{t+1})}{\partial c_{t+1}} (i_{t+1} + 1 - \pi), \tag{26}$$

$$q_t \frac{\partial u(c_t, h_t)}{\partial c_t} = \beta \frac{\partial u(c_{t+1}, h_{t+1})}{\partial h_{t+1}} + \beta \frac{\partial u(c_{t+1}, h_{t+1})}{\partial c_{t+1}} q_{t+1} (1 - \delta). \tag{27}$$

3.2 Construction firms

Supply side of the market is identical for both credit constrained and unconstrained versions of the model economy. In modeling the production of new housing I rely primarily on Amin and Capozza(1993). Let's assume that there is a perfectly competitive sector of construction firms that supply units to the housing market. The representative firm acts to maximize its profits taking the housing price as given. It has a production function given by $X_{s,t} = G(K_t, L_t) = K_t^\alpha L_t^{1-\alpha}$ where K_t is the amount of capital used, L_t is the amount land used and $\alpha < 1$. It is assumed that firms face constant returns to scale technology which implies a linear cost function with constant marginal cost, denoted by d . Output per unit of land is given by $x_{s,t} = g(k_t) = \frac{X_{s,t}}{L_t} = \left(\frac{K_t}{L_t} \right)^\alpha = (k_t)^\alpha$. Under these assumptions, the total cost of production is given by dk . Construction firms need to obtain a permit from the zoning authority, a process that involves costs. The cost of each permit is given by n , which includes both cash expenditures needed to obtain the building permit as well as the cost

of time necessary to obtain the building permit (in monetary terms). In real US economy regulation cost can vary either according to the value of the building project or according to the square footage of the constructed housing unit. Both the demand as well as the supply side of the model economy are calibrated in terms of average housing unit, which will be defined later. Consequently, the dollar value of the building permit cost is set according to the square footage of this typical unit. Under such calibration, one building permit is necessary to build one unit of output, that is, one average housing unit. This assumption is further justified by the fact that the entire US Census Bureau data on building permits is reported in terms of new privately owned housing units authorized in permit-issuing places, rather than in terms of number of obtained building permits per se.

With these assumptions, the maximization problem of a construction firm is given by:

$$\begin{aligned} \max_{k_t} \Pi_t &= q_t x_{s,t} - dk_t - nx_{s,t}, \\ \text{s.t. } x_{s,t} &= (k_t)^\alpha. \end{aligned}$$

From the maximization, one can get the optimal amount of input used by construction firm, which is:

$$k_t = \left(\frac{\alpha q_t - \alpha n}{d} \right)^{(1/(1-\alpha))}. \quad (28)$$

This gives the optimal amount of capital to land ratio chosen by the representative firm. Substituting back into the production function yields the amount of housing produced per unit of land:

$$x_{s,t} = g(k_t) = \left(\frac{\alpha q_t - \alpha n}{d} \right)^{(1/(1-\alpha))}. \quad (29)$$

Moreover, since in equilibrium all the firms act in the same way, multiplication of (29)

by the aggregate stock of land gives the aggregate supply of new housing produced.

3.3 Definition of equilibrium

Let's define the aggregate supply of land as \bar{L} . It is reasonable to assume that the supply of land is fixed in the short run. However, this doesn't imply that supply of new housing is fixed as well. It can increase if more housing is produced per unit of land. Let's assume that there is an exogenous output of composite consumption good, given by Y_t . The supply side of the consumption good market is not modeled explicitly, since the analysis is focused on the housing market. Also, the model with credit constraints is analyzed in the situation where credit constraint is binding. This implies that all households are net borrowers, with the amount of borrowing determined endogenously depending on the amount of housing consumption chosen. The equilibrium in credit market is not modeled here since the analysis is not focused on the behavior of the interest rate. It is assumed instead that there is an exogenously given supply of borrowing funds B_t , which is coming from abroad (evidence on US external debt presented in footnote 4). Finally it is assumed that there are J households and I firms in the economy.

The equilibrium consists of prices $\{q_t\}_{t=0}^{\infty}$, interest rates i_t , allocations $\{c_t, h_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ by households and the profit maximizing input demand of firms k_t , such that:

1) given prices, households solve their optimization problem (conditions (24)-(25) for the credit constrained economy and (26)-(27) for unconstrained economy) and firms maximize their profits (condition (28));

2) Markets clear, i.e.

$$\text{i) } \Sigma_j x_{jd,t} = g(k_t) \bar{L} \quad (\text{housing market}),$$

$$\text{ii) } \Sigma_j c_{j,t} = Y_t \quad (\text{goods market}),$$

$$\text{iii) } \Sigma_j b_{j,t+1} = B_t \quad (\text{for credit-constrained economy, bond market}),$$

$$b_{j,t+1} = 0 \quad (\text{for unconstrained economy, bond market}).$$

The last condition comes from the fact that in a standard unconstrained representative agent asset pricing model in equilibrium lending should compensate borrowing.

3.4 Characterization of the welfare adjustment

In this section the formula for welfare adjustment due to an endogenous housing price appreciation for an economy in a steady state is derived. The full derivation of steady state for both credit-constrained and unconstrained versions of the model is given in the appendix. Based on Li and Yao(2004) modified Cobb-Douglas utility function of the following form is used:⁵

$$u(c, h) = \frac{(c^{1-\omega} h^\omega)^{1-\gamma}}{1-\gamma}.$$

Suppose that the economy is in a steady state when building permit costs reflected in n increase. It is evident from (28) that this shifts down the profit-maximizing level of input and reduces the profit-maximizing output of the competitive firms per unit of land used. Consequently, the aggregate supply of new residential housing decreases and housing price appreciates (the expression for the response of housing price to the change in building permit costs is derived in the appendix). Similar to Section 2, the welfare adjustment is defined as

⁵Results under more general utility function, separable in housing consumption and composite good consumption, are available upon request.

the change in income necessary to keep lifetime utility constant when n changes. The change in value function resulting from the change in n is given by:

$$\Delta V = \frac{\partial V(h^{ss}, b^{ss}, y^{ss})}{\partial n} \Delta n + \frac{\partial V(h^{ss}, b^{ss}, y^{ss})}{\partial y} \Delta y.$$

where superscript ss denotes steady state values.

Using utility form defined above, calculating the corresponding derivatives, substituting them to the last equation, equating ΔV to zero and expressing Δy from the resulting equation yields the following formulas for the welfare adjustments:

$$\Delta y_t = \Delta n \omega \frac{\alpha}{q^{ss} - n(1 - \alpha)} \left(\frac{B(y^{ss} - f1\{x^{ss} \neq 0\})}{\beta(1 - \omega)D} \right), \quad \text{for the model with credit constraints} \quad (30)$$

$$\Delta y_t = \Delta n \omega \frac{\alpha}{q^{ss} - n(1 - \alpha)} (i^{ss} + \delta - \pi) \left(\frac{y^{ss} - f1\{x^{ss} \neq 0\}}{A} \right). \quad \text{for the unconstrained model} \quad (31)$$

where A,B and D are constants defined in the appendix.

4 Results of the endogenous price models driven by supply-side shocks: Interpretation and comparison

In this section the welfare adjustments in the models with endogenous housing prices driven by supply-side shocks are signed and compared.

The result in an economy with an endogenous housing price but without credit constraints is given by (32), where $A = (1 - \omega)i^{ss} + \omega\pi + \delta - \pi$ and $\alpha < 1$. The details of calibrating parameters π, i^{ss} and δ as well as the parameter values and the sources of calibration are

given in data appendix. Using the assumed values and setting $\omega = 0.56$ (justification for this is given later in the section) gives $A = 0.0338$, which implies that 5-th term in the product in (32) is positive. Also the 4-th term is positive. The 3-rd term is positive since it reflects the effect of change in regulation costs on the housing prices, which must be strictly positive. Change in n is positive by assumption. Consequently the individual welfare adjustment in this model is positive. Thus, in an economy with endogenous housing prices where households are not credit-constrained, the housing price appreciation driven by negative supply side shock leads to a welfare loss.⁶

In a model with both credit constraints and endogenous housing prices, the welfare adjustment is given by:

$$\Delta y_t = \Delta n \omega \frac{\alpha}{q^{ss} - n(1 - \alpha)} \left(\frac{B(y^{ss} - f1\{x^{ss} \neq 0\})}{\beta(1 - \omega)D} \right), \quad (32)$$

where $B = 1 - \beta(1 - \delta) - m(1 - \beta(i^{ss} + 1 - \pi))$ and $D = \frac{1 - \omega}{\omega\beta}B - m\pi + i^{ss}m + \delta$.

Looking at (39) (in the appendix) which defines the steady state housing stock in the credit-constrained economy it is easy to see that $D > 0$ is necessary for having positive steady state housing stock. Also, (38) (in the appendix) tells that positive consumption in the steady state requires $B > 0$ if $\omega < 1$ (since it is an exponent of housing in Cobb-Douglas utility function). Consequently, in this economy the welfare adjustment is positive. Thus, when endogenous housing price appreciation is driven by negative supply shocks and preferences are of Cobb-Douglas form, agents experience a welfare loss both with and without credit constraints.

⁶According to my definition positive Δy means welfare loss since people need more income to keep them indifferent between old and new prices.

One can compare the last two formulas for welfare adjustments to establish whether credit constraints alleviate or exacerbate the welfare loss from a negative supply shock. For simplicity let's abstract from fixed transaction costs; that is let's assume that $f1\{x^{ss} \neq 0\} = 0$. Also, to make a fair comparison let's ignore the possible difference between income of credit-constrained and unconstrained households and assume the same income for both economies.⁷ Examining (32) and (33), it is evident that for comparing those two results one should compare the terms $\frac{i^{ss} + \delta - \pi}{A}$ and $\frac{B}{\beta(1 - \omega)D}$. For the credit-constrained economy $i^{ss} = 0,057$, the level of the average effective interest rate on mortgages in US in 2004 (obtained from Monthly Interest Rate Survey of Federal Housing Finance Board). Also, it is important to recall that here an economy with binding credit constraints is considered. In this case the Lagrange multiplier of the credit constraint is positive, that is $v^{ss} > 0$. This fact creates differences in discount rates between credit-constrained and unconstrained households. Mathematically, the discount rate for the economy with binding credit constraints is given by:

$$\beta' = \frac{1 - \frac{v^{ss}}{\frac{\partial u(c^{ss}, h^{ss})}{\partial c}}}{i^{ss} + 1 - \pi},$$

while the discount rate for the economy without credit constraints is given by:

$$\beta' = \frac{1}{i^{ss} + 1 - \pi}.$$

Looking at the last two expressions and taking into account that $v_{ss} > 0$ and that the interest rate is higher in the economy with binding credit constraints, it is evident that the

⁷In case of accounting for potential differences in the incomes of credit constrained and unconstrained households, as I did in earlier drafts of the paper, the results of comparison are practically the same as in this draft.

discount factor in this economy should be lower than the discount factor in the unconstrained economy. Thus, for the economy with binding credit constraints I set $\beta = 0.96$, which is lower than the conventional 0.98-0.99. Finally $m = 0,8$ based on Tsakaronis and Zhu (2004) . Using all these values sensitivity analysis is performed by computing both terms mentioned above for values of preference parameter ω ranging from 0.1 to 0.9 where ω is the exponent of housing in Cobb-Douglas utility function. The results are presented in Table 1.

The table demonstrates that the welfare adjustment caused by a housing price appreciation due to an increase in regulation costs is lower in a credit-constrained economy than in an unconstrained economy for all $\omega \leq 0.5$ but is higher in the credit-constrained economy than in the unconstrained economy for all $\omega \geq 0.6$. Thus, the relationship between the welfare changes in credit-constrained and unconstrained models depends on the relative weight of housing in the agent's utility function. Since credit-constrained households intuitively have a lower housing stock than unconstrained ones, the marginal utility of housing for them is higher. Consequently, when housing consumption has a relatively high weight in the utility function, credit-constrained households loose more from a decrease in their steady state housing stock which has higher marginal utility for them, than unconstrained households.

It is possible to calculate ω using shares of housing and non-durable consumption in average annual expenditures in the US economy. According to the Consumer Expenditures Survey published by Bureau of Labor Statistics the share of housing in the expenditures in 2004 was equal to 32.1% and the share of non-durable consumption (aggregated from separate components given in the Consumption Expenditure Survey) was equal to 49%. On the other hand in my model the dollar value of one period expenditures on composite good (non-durable consumption) is given by c^{ss} (since the price of consumption is normalized at

1) and the dollar value of one period expenditures on housing is given by $\delta q^{ss} h^{ss}$ (since during one period households consume value of the depreciated housing stock). Looking at the steady state allocations in the appendix it is easy to see that in both credit-constrained and unconstrained versions of the economy the ratio $\frac{c^{ss}}{\delta q^{ss} h^{ss}}$ is a function of ω only and the other already calibrated parameters. On the other hand mathematically it is true that:

$$\frac{c^{ss}}{\delta q^{ss} h^{ss}} = \frac{\frac{c^{ss}}{Expenditures}}{\frac{\delta q^{ss} h^{ss}}{Expenditures}} = \frac{0.49}{0.321}.$$

Thus, ω can be calculated from this equation. For defining the plausible range of values for ω at first all the households in the actual economy are treated as unconstrained and ω is calculated from the above equation using steady state allocations of the unconstrained model . Then all the households are treated as credit-constrained and ω is calculated using allocations from the credit-constrained model.

The unconstrained model gives:

$$\frac{c^{ss}}{\delta q^{ss} h^{ss}} = \frac{(1 - \omega)(i^{ss} + \delta - \pi)}{\omega \delta} = \frac{0.49}{0.321} \quad \text{from which } \omega = 0.56.$$

The constrained model gives

$$\frac{c^{ss}}{\delta q^{ss} h^{ss}} = \frac{B(1 - \omega)}{\omega \beta \delta} = \frac{0.49}{0.321} \quad \text{from which } \omega = 0.64.$$

Since there are both types of households in the actual economy, the true value of ω should be between 0.56 and 0.64. In case of $\omega = 0.56$ the adjustment in constrained model is only marginally higher than that in the unconstrained economy since $\frac{i^{ss} + \delta - \pi}{A} = 1,33380$ and $\frac{B}{\beta(1 - \omega)D} = 1.37297$, while in case of $\omega = 0.64$, credit-constrained households clearly loose more from negative supply shock since $\frac{i^{ss} + \delta - \pi}{A} = 1.672835$ and $\frac{B}{\beta(1 - \omega)D} = 2.00179$.

5 Model with endogenous housing price : Demand-side shocks

5.1 Shifts in income as the reason of housing price appreciation

In general, changes in income constitute the most natural demand-side shock in any market including the housing market. Consequently, when searching for demand-side shocks affecting housing prices I first look at the dynamics of income in the US during the years of housing price appreciation. Annual figures for median household income in the US, obtained from the Current Population Survey of US Census Bureau are presented in Figure 3 together with constant-quality housing price index displayed previously in Figure 1.

The graph clearly shows that years of substantial housing price appreciation were characterized by a considerable upward shift in the median household income which, after staying nearly constant in the first half of the 90's, began to grow rapidly in the second half. Calculating the growth rate of income from US Census Bureau data indicates that in 1988-1994 median household income increased by only 17.7% while in 1995-2001 it grew by 24.5%. Empirical evidence would thus suggest that changes in income were an important demand-side driver of housing price appreciation in the last decade.

Let's denote by Δy_{new} the new change in income that is the welfare adjustment and by Δy_{old} the initial change in income that is the shock. The welfare adjustment is derived from the following equation:

$$\Delta V = \frac{\partial V(h^{ss}, b^{ss}, y^{ss})}{\partial c} \frac{\partial c^{ss}}{\partial y} \Delta y_{old} + \frac{\partial V(h^{ss}, b^{ss}, y^{ss})}{\partial h} \left(\frac{\partial h^{ss}}{\partial y} + \frac{\partial h^{ss}}{\partial q} \frac{\partial q^{ss}}{\partial y} \right) \Delta y_{old} + \frac{\partial V}{\partial y} \Delta y_{new}.$$

Equating ΔV to 0, using the steady state derived in the appendix, and expressing Δy_{new} from the resulting equation yields the following formulas for the welfare adjustments:

$$\Delta y_{new} = -\frac{B(1-\omega)}{\omega\beta D}\Delta y_{old} - \frac{B}{\beta} \left(\frac{1}{D} - \frac{y^{ss} - f1\{x^{ss} \neq 0\}}{Dq^{ss}} \frac{\partial q}{\partial y} \right) \Delta y_{old} \quad \text{for credit-constrained model,} \quad (33)$$

where

$$\frac{\partial q}{\partial y} = \frac{J\delta d^{(\alpha/(1-\alpha))}}{q^{ss}\bar{L}D\frac{\alpha^2}{1-\alpha}(\alpha q^{ss} - \alpha n)\frac{\alpha}{1-\alpha}^{-1} + \bar{L}D(\alpha q^{ss} - \alpha n)\frac{\alpha}{1-\alpha}} > 0.$$

and for unconstrained model:

$$\Delta y_{new} = -\frac{(1-\omega)(i^{ss} + \delta - \pi)}{A}\Delta y_{old} - (i^{ss} + \delta - \pi) \left(\frac{\omega}{A} - \frac{y^{ss} - f1\{x^{ss} \neq 0\}}{Aq^{ss}} \frac{\partial q}{\partial y} \right) \Delta y_{old}, \quad (34)$$

where

$$\frac{\partial q}{\partial y} = \frac{J\delta\omega d^{(\alpha/(1-\alpha))}}{Aq^{ss}\bar{L}\frac{\alpha^2}{1-\alpha}(\alpha q^{ss} - \alpha n)\frac{\alpha}{1-\alpha}^{-1} + A\bar{L}(\alpha q^{ss} - \alpha n)\frac{\alpha}{1-\alpha}} > 0.$$

The equation reflecting the response of housing price to changes in income was obtained as in previous cases by applying an implicit function theorem to the housing market clearing condition derived in the appendix. The second terms in the welfare adjustments given above are the final changes in housing stock due to interaction of income and substitution effects.

At this moment the sign of the last two results is ambiguous since the second term in both expressions is not necessarily negative. Intuitively it should be negative since in the case of housing the income effect usually dominates the substitution effect. These results are quantified in Section 6.

5.2 Changes in the interest rates as the reason of housing price appreciation

A decrease in mortgage interest rates and nominal interest rates on bonds generates an increase in the housing demand for both credit-constrained and unconstrained households. For the credit-constrained households who are net borrowers, a decrease in the mortgage rate implies lower current payments for their mortgages. This increases their disposable income, which in turn means that they can increase housing consumption and/or consumption of the composite good. For the unconstrained households housing and bonds can be viewed as the alternative investment opportunities or assets. Consequently, a decline in the interest rates on bonds makes housing a more attractive investment relative to bonds and the investment is shifted towards housing, thus further raising housing demand.

At this point, one should ask what happened to the nominal interest rates on bonds and mortgage interest rates in the real economy in 1990's. The evolution of the average effective interest rates on mortgages and long term government bond yields in the US from 1986 to 2003 is summarized in Figure 4. The figure clearly demonstrate a downward trend in the interest rates in 1995-2003. It thus appears quite important to study the welfare implications of housing price appreciation driven by a decrease in interest rates.

The welfare adjustment, defined as in the previous section, is derived from the following equation:

$$\Delta V = \frac{\partial V(h^{ss}, b^{ss}, y^{ss})}{\partial c} \frac{\partial c^{ss}}{\partial i} \Delta i + \frac{\partial V(h^{ss}, b^{ss}, y^{ss})}{\partial h} \left(\frac{\partial h^{ss}}{\partial i} + \frac{\partial h^{ss}}{\partial q} \frac{\partial q^{ss}}{\partial i} \right) \Delta i + \frac{\partial V}{\partial y} \Delta y = 0.$$

In the model with credit constraints welfare adjustment is given by:

$$\Delta y = (y^{ss} - f1\{x^{ss} \neq 0\}) \left(\frac{1-\omega}{\omega\beta}(1-\beta)(m-m^2) + \frac{Bm}{\beta\omega D^2} - \frac{Bm}{\beta\omega D} \left(\frac{(q^{ss}-n)(1-\alpha)}{D(q^{ss}-n(1-\alpha))} \right) \right) \Delta i. \quad (35)$$

In the unconstrained model it is given by:

$$\Delta y = (y^{ss} - f1\{x^{ss} \neq 0\}) \left(\begin{array}{c} -\frac{\omega(1-\omega)\delta}{A^2} + \frac{(i^{ss} + \delta - \pi)\omega(1-\omega)}{A^2} - \\ -\frac{\omega(i^{ss} + \delta - \pi)}{A} \left(\frac{(1-\omega)(q^{ss}-n)(1-\alpha)}{A(q^{ss}-n(1-\alpha))} \right) \end{array} \right) \Delta i. \quad (36)$$

6 US economy in 1995-2004: Actual aggregate welfare adjustment

In the previous sections, welfare adjustments in the model economy were derived for different supply and demand side shocks. In this section the aggregate welfare adjustment resulting from housing price appreciation driven by the combination of shocks observed in US housing market from 1995 to 2004 is computed.

According to the US Census Bureau in 2004 1,532,000 single-family housing units with an average area of 2,349 square feet per unit and 310,000 units in buildings with two units or more with an average area of 1,173 square feet per unit were built. Thus, in total 3,962,298,000 square feet of housing were built in the US in 2004. Dividing the total number of square feet produced by the total number of housing units produced yields that the area of an average housing unit was 2,151 square feet. Building permit cost is calculated according to the Craftman's National Construction Estimator taking into account square footage of the housing unit and US average construction cost per square foot and as a result is set to $n = 3016$. Using the report of the National Association of Realtors on the land use, which

says that in 2000 (the most recent available estimate) 658,000 acres of land were used for residential construction I set $\bar{L} = 658,000$. Finally, with this information it is possible to calculate the amount of output per unit of land in the real economy, which is equal to 5,428.41 square feet or 2.79 housing units.

With this information in hand the constant marginal cost d can be calculated. Using (29), which defines the output per unit of land, and solving it for d yields:

$$d = \left(\frac{(\alpha q^{ss} - \alpha n)^{(\alpha/(1-\alpha))}}{x_{s,t}} \right)^{((1-\alpha)/\alpha)}.$$

According to the Monthly Interest Rate Survey of Federal Housing Finance Board average purchase price of housing in the US in 2004 was 262,000 dollars. Also based on the National Association of Realtors' data on capital income and land income shares in the housing construction industry I set $\alpha = 0.4$. Finally, according to the calculation above, $x_{s,t} = 2.79$. Substituting all parameters into the last equation gives $d = 20586$.

At this point it is necessary to specify the structure of the population or, in other words, number of credit-constrained and unconstrained households. One can evaluate the degree of being credit-constrained by the current wealth or the accumulated wealth of the household. Even better indicator from this point of view can be the net worth of the household, that is, the value of the household's assets net of liabilities. The 2004 Survey of Consumer Finance by the Federal Reserve System reports the average net worth of American households according to the age of the household head (Table 2). Based on this data it is straightforward that households headed by individuals of the lowest two age groups are the most likely to be constrained. However, households headed by individuals of the age 35-44 have considerably higher net worth than do younger households. Moreover, according to the US Census Bureau,

households headed by individuals aged 35-44 have the second highest median income in the US economy. Consequently, in my research two different parametrizations are considered. Under the first one households headed by individuals of the aged 35-44 are assumed to be credit-constrained; under the second one they are considered unconstrained. Using Current Population Report of US Department of Commerce, I set $J_c = 44784339$ and $J_{uc} = 62888650$ for the first case and $J_c = 21737795$ and $J_{uc} = 85935104$ for the second case, where J_c and J_{uc} is the number of credit-constrained and unconstrained households respectively.

Now let's calculate an implied cumulative welfare adjustment for the actual US economy. According to constant-quality housing price index of the US Census Bureau, housing prices increased by 43.7% between 1995 and 2004. Also, median household income in the US increased by 30.1% between 1995 and 2004. Finally, the interest rate on long-term government bonds declined from 6.58 to 4.2 % (by 36.2%) during this period while the effective interest rate on mortgages declined from 8 to 5.7% (by 28.7%). The only unobservable is the change in the building permit cost or the supply-side shock. The idea is to calculate the elasticity of housing prices with respect to income and interest rates in both a constrained and an unconstrained economy and then to compute the total response of housing prices on demand side shocks. The supply-side shock or change in building permit costs can be computed so as to match the residual change in prices in the US economy.

To compute the response of housing prices to changes in demand side factors the following formulas are used:

$$\varepsilon_{qy,c} = \frac{J\delta d^{\alpha/(1-\alpha)}}{q^{ss} \bar{L} D \frac{\alpha^2}{1-\alpha} (\alpha q^{ss} - \alpha n) \frac{\alpha}{1-\alpha}^{-1} + \bar{L} D (\alpha q^{ss} - \alpha n) \frac{\alpha}{1-\alpha} \frac{y^{ss}}{q^{ss}}},$$

$$\varepsilon_{qy,uc} = \frac{J\delta\omega d^{(\alpha/(1-\alpha))}}{Aq^{ss}\bar{L}\frac{\alpha^2}{1-\alpha}(\alpha q^{ss} - \alpha n)\frac{\alpha}{1-\alpha}^{-1} + A\bar{L}(\alpha q^{ss} - \alpha n)\frac{\alpha}{1-\alpha}} \frac{y^{ss}}{q^{ss}},$$

$$\varepsilon_{qi,c} = \frac{m(1-\alpha)i^{ss}(q^{ss} - n)}{\omega D(q^{ss} - n(1-\alpha))},$$

$$\varepsilon_{qi,uc} = \frac{(1-\omega)(1-\alpha)i^{ss}(q^{ss} - n)}{A(q^{ss} - n(1-\alpha))}.$$

where $\varepsilon_{qy,c}$ is the elasticity of housing price with respect to income in the constrained economy, $\varepsilon_{qy,uc}$ is the elasticity of housing price with respect to income in the unconstrained economy, $\varepsilon_{qi,c}$ is the elasticity of price with respect to interest rate in the constrained economy and $\varepsilon_{qi,uc}$ is the elasticity of price with respect to interest rate in the unconstrained economy. These elasticities are computed for each of the variants of parametrization mentioned above and for each of the values of ω calculated in Section 5. The results are displayed in Table 3.

From the table it is evident that if assuming that households headed by individuals in the age group 35-44 are not constrained, the model-implied elasticities with respect to income changes are quite high in the unconstrained economy. Given the elasticities with respect to the other shocks, under such calibration the model-implied change in housing price due to actual changes in income and interest rates overshoots the actual quality-adjusted change in housing prices. Thus, the case with $J_c = 44784339$ and $J_{uc} = 62888650$ is used in what follows. Also, $\omega = 0.64$ is used. In this case the housing prices change in total by 38.7% due to a change in demand-side factors. Since between 1995 and 2004 housing prices changed by 43.7%, the change in housing price due to supply shock should have been equal to 5%. Now let's use the elasticity of housing prices with respect to regulation cost which is given by the following formula:

$$\varepsilon_{qs} = \frac{\alpha n}{q^{ss} - n(1 - \alpha)}.$$

Calculating this formula yields that $\varepsilon_{qs} = 0.085$. This implies that the building permit cost should have increased by 58.8% to match the actual change in housing price. Since the new building permit cost is equal to 3016 dollars, the old one would have been 1899 dollars, which implies the change of building permit cost of 1117 dollars.

Now the changes of variables in units rather than in percents are used to calculate the dollar value of the welfare adjustment resulting from housing price appreciation driven by all factors jointly. Thus $\Delta n = 1117$, $\Delta y_{old} = 10258$, $\Delta i_c = -2.3$ and $\Delta i_{uc} = -2.38$. Based on Global Property Guide I set transaction costs $f1\{x^{ss} \neq 0\}$ to the 9.07 % of housing price.

Using all of the above information each of the welfare adjustments derived previously is calculated for both credit-constrained as well as unconstrained versions of the model. Results are summarized in Table 4. According to prior expectations housing price appreciation driven by negative supply shock (building permit costs) results in welfare loss(positive Δy) while housing price appreciation driven by positive demand shock(income and interest rates) results in welfare improvement(negative Δy).

Given these results it is easy to calculate the cumulative aggregate welfare change in the actual US economy in 1995-2004. To make my result more informative the final cumulative welfare adjustment per household is expressed in terms of mean income in the US in 2004.

Under such measurement the total aggregate welfare adjustment is given by:

$$\Delta Y_{aggregate} = \frac{J_c}{J_c + J_{uc}} \frac{(\Delta y_{s,c} + \Delta y_{y,c} + \Delta y_{i,c})}{y_{mean}} + \frac{J_{uc}}{J_c + J_{uc}} \frac{(\Delta y_{s,uc} + \Delta y_{y,uc} + \Delta y_{i,uc})}{y_{mean}} = -0,318.$$

In this formula $\Delta y_{s,c}$ is welfare adjustment in the constrained economy due to housing price appreciation caused by supply shock, $\Delta y_{s,uc}$ is welfare adjustment in the unconstrained

economy due to housing price appreciation caused by to supply shock in the unconstrained economy, $\Delta y_{y,c}$ is welfare adjustment in the constrained economy due to housing price appreciation caused by income shock , $\Delta y_{y,uc}$ is welfare adjustment in the unconstrained economy due to housing price appreciation caused by income shock, $\Delta y_{i,c}$ is welfare adjustment in the constrained economy due to housing price appreciation caused by interest rate shock , $\Delta y_{i,uc}$ is welfare adjustment in the unconstrained economy due to housing price appreciation caused by interest rate shock in the unconstrained economy. Since the sign of the adjustment is negative the result implies the improvement in aggregate welfare. Thus, the housing price appreciation which took place in the US economy between 1995 and 2004 and which was driven by an observed combination of demand and supply side shocks improved the aggregate welfare per household by around 32% of mean household income in 2004 per household.

7 Summary

This paper explores the aggregate welfare effects of housing price appreciation in a general model with binding credit constraints and endogenous housing prices. First, the model with exogenous housing prices but with households subject to binding credit constraints is considered. It is demonstrated that in an economy with binding credit constraints housing price appreciation leads to an improvement in aggregate welfare. The result is due to the fact that credit-constrained model takes into account the welfare improving effect of the housing price appreciation, which implies relaxation of binding credit constraints. This effect is ignored in the previous models where households are assumed to be unconstrained.

A model with endogenous housing price, in which housing price appreciation is driven by supply and demand side shocks, is analyzed for both credit-constrained and unconstrained households. The supply side shocks are driven by the increases in building permit cost. Changes in income and interest rates are the demand side drivers. The relationship between welfare adjustments in the two modeling alternatives depends on the relative weight housing in the agent's utility function. The theoretical models are calibrated to calculate the actual welfare adjustment resulting from the combination of all considered shocks in the US housing market in 1995-2004. It is shown that the housing price appreciation from 1995 to 2004 led to per household improvement in the aggregate welfare by an amount equivalent to approximately 40% of mean household income in 2004.

Appendix

1. Derivation of steady state in the endogenous housing price model with credit constraints.

The steady state in the model with binding credit constraints should satisfy the following conditions:

$$h_{t+1} = h_t = h^{ss},$$

$$c_{t+1} = c_t = c^{ss},$$

$$b_{t+1} = b_t = b^{ss},$$

$$b^{ss} = -mq^{ss}h^{ss},$$

$$s^{ss} = b^{ss} - (1 - \pi)b^{ss} = \pi b^{ss},$$

$$x^{ss} = h^{ss} - (1 - \delta)h^{ss} = \delta h^{ss}.$$

Using the last 3 conditions budget constraint in the steady state can be rewritten as:

$$c^{ss} = y^{ss} - i^{ss}mq^{ss}h^{ss} - f1\{x^{ss} \neq 0\} - q^{ss}\delta h^{ss} + \pi mq^{ss}h^{ss}.$$

Rewriting (24) and (25) in the steady state and rearranging yields:

$$v_t = \frac{\partial u(c^{ss}, h^{ss})}{\partial c} - \beta \frac{\partial u(c^{ss}, h^{ss})}{\partial c} (i^{ss} + 1 - \pi),$$

$$q^{ss} = \beta \frac{\partial h}{\partial u(c^{ss}, h^{ss})} + \beta q^{ss}(1 - \delta) + mq^{ss}(1 - \beta(i^{ss} + 1 - \pi)).$$

Using in the last equation the utility form defined in Section 3.4 and rearranging the resulting equation yields.

$$c^{ss} = B \frac{1 - \omega}{\omega \beta} h^{ss} q^{ss}, \quad (37)$$

$$\text{where } B = 1 - \beta(1 - \delta) - m(1 - \beta(i^{ss} + 1 - \pi)).$$

Substituting (38) into the steady state budget constraint and rearranging the steady state level of housing stock is obtained:

$$h^{ss} = \frac{y^{ss} - f1\{x^{ss} \neq 0\}}{Dq^{ss}}, \quad (38)$$

$$\text{where } D = B \frac{1 - \omega}{\omega \beta} - mi^{ss} + m\pi + \delta.$$

Steady state level of consumption can be obtained by substituting (39) back to (38) :

$$c^{ss} = B \frac{(1 - \omega)(y^{ss} - f1\{x^{ss} \neq 0\})}{\omega \beta D}. \quad (39)$$

All the other endogenous variables can be now determined from various conditions. The results are given by the following

$$x^{ss} = \delta \frac{y^{ss} - f1\{x^{ss} \neq 0\}}{Dq^{ss}},$$

$$b^{ss} = -m \frac{y^{ss} - f1\{x^{ss} \neq 0\}}{D},$$

$$v^{ss} = (1 - \omega) \left(\frac{y^{ss} - f1\{x \neq 0\}}{D} B \frac{1-\omega}{\omega\beta} \right)^{-\gamma} \left(\frac{\omega\beta}{q^{ss} B(1-\omega)} \right)^{w(1-\gamma)} (1 - \beta(i^{ss} + 1 - \pi)).$$

Finally q^{ss} can be determined endogenously from market clearing condition by equating demand and supply:

$$J\delta \frac{(y^{ss} - f1\{x^{ss} \neq 0\})}{Dq^{ss}} = \left(\frac{\alpha q^{ss} - \alpha n}{d} \right)^{(\alpha/(1-\alpha))} \bar{L}. \quad (40)$$

The response of housing price to different shocks should be determined from the market clearing condition. It is not possible to explicitly solve (41) for housing price. But (41) represents an implicit function of q^{ss} in terms of model parameters only. For determining the response of housing price to different shocks implicit function theorem is applied to (41).

Let me demonstrate it here for supply side shock. Rearranging (40) and assuming that D is not equal to 0 yields:

$$d^{(\alpha/(1-\alpha))} J\delta (y^{ss} - f1\{x^{ss} \neq 0\}) - (Dq^{ss} \bar{L} (\alpha q^{ss} - \alpha n)^{(\alpha/(1-\alpha))}) = 0.$$

This is an implicit function which defines how the equilibrium price depends on building permit costs (parameter n). To determine how the equilibrium price changes in response to an increase in building permit cost the implicit function theorem is applied to this equation, yielding:

$$\frac{dq}{dn} = - \frac{\frac{\partial (d^{(\alpha/(1-\alpha))} J\delta (y^{ss} - f1\{x^{ss} \neq 0\}) - (Dq^{ss} \bar{L} (\alpha q^{ss} - \alpha n)^{(\alpha/(1-\alpha))}))}{\partial n}}{\frac{\partial (d^{(\alpha/(1-\alpha))} J\delta (y^{ss} - f1\{x^{ss} \neq 0\}) - (Dq^{ss} \bar{L} (\alpha q^{ss} - \alpha n)^{(\alpha/(1-\alpha))}))}{\partial q}} = \frac{\alpha q^{ss}}{q^{ss} - n(1 - \alpha)} > 0. \quad (41)$$

This is positive, since numerator is positive and denominator should be positive (cost of building permit multiplied by a number strictly less than one cannot exceed housing price).

The formulas for the responses of housing price to changes in income and interest rates are derived in a similar way from the market clearing condition.

2. Derivation of steady state in the unconstrained model.

Applying the same procedure as above one can derive the following conditions describing steady state:

$$h^{ss} = \frac{\omega(y^{ss} - f1\{x^{ss} \neq 0\})}{Aq^{ss}}, \quad (42)$$

where $A = (1 - \omega)i^{ss} + \omega\pi + \delta - \pi$,

$$c^{ss} = \frac{(1 - \omega)(i^{ss} + \delta - \pi)(y^{ss} - f1\{x^{ss} \neq 0\})}{A}, \quad (43)$$

$$x^{ss} = \delta \frac{\omega(y^{ss} - f1\{x^{ss} \neq 0\})}{Aq^{ss}}. \quad (44)$$

The market clearing condition is given by:

$$J\delta \frac{\omega(y^{ss} - f1\{x^{ss} \neq 0\})}{Aq^{ss}} = \left(\frac{\alpha q^{ss} - \alpha n}{d}\right)^{(\alpha/(1-\alpha))} \bar{L}.$$

The response of housing price in the unconstrained model to different shocks is derived in a similar way as previously from this market clearing condition.

Data appendix

When quantifying the result in (18) I use the American Housing Survey of the US Census Bureau, which reports the total number of housing units, the total number of the owner occupied housing units and the total number of mortgage-financed owner occupied housing units in US. Given this information one can calculate the share of owner occupied housing stock in the total housing stock and share of mortgage-financed owner occupied housing stock in the total owner occupied housing stock. The data from Monthly Interest Rate Survey of

Federal Housing Finance Board, which reports the proportions of mortgages with different LTV in the total number of mortgages, is also used in computation of (18).

When calibrating endogenous price model, the IMF International Financial Statistics is used to set the values for inflation rate and nominal interest rate in the unconstrained economy. The nominal interest rate is approximated by long-term (10 years) government bond yield and is set $i = 0.042$. Inflation is set $\pi = 0.02$. Depreciation rate δ is calibrated from several studies. Earlier studies such as Margolis(1982) and Malpezi and Ozane(1987) have estimated gross depreciation rate of 2% for the housing stock in the USA. Also, in the end of 1980s and beginning of 1990s the Congress raised depreciation period for housing in the US to 27.5 years, which implies a yearly depreciation rate of around 3.5%. Based on this range of estimates I set $\delta = 0.025$.

In this research the constant-quality housing price index published by US Census Bureau is used as the main housing price measure (Figure 1). The data of US Census Bureau on median household income (Figure 3) and new residential construction is also used. The data of Federal Housing Finance Board on average LTV as well as average effective interest rate on mortgages (Figure 4) is employed. Finally, the dynamics of long term bond yields (Figure 4) is taken from IMF International Financial Statistics and amount of land used in residential construction during a year is taken from the report of National Association of Realtors.

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Figure 1: Dynamics of constant-quality housing prices in US

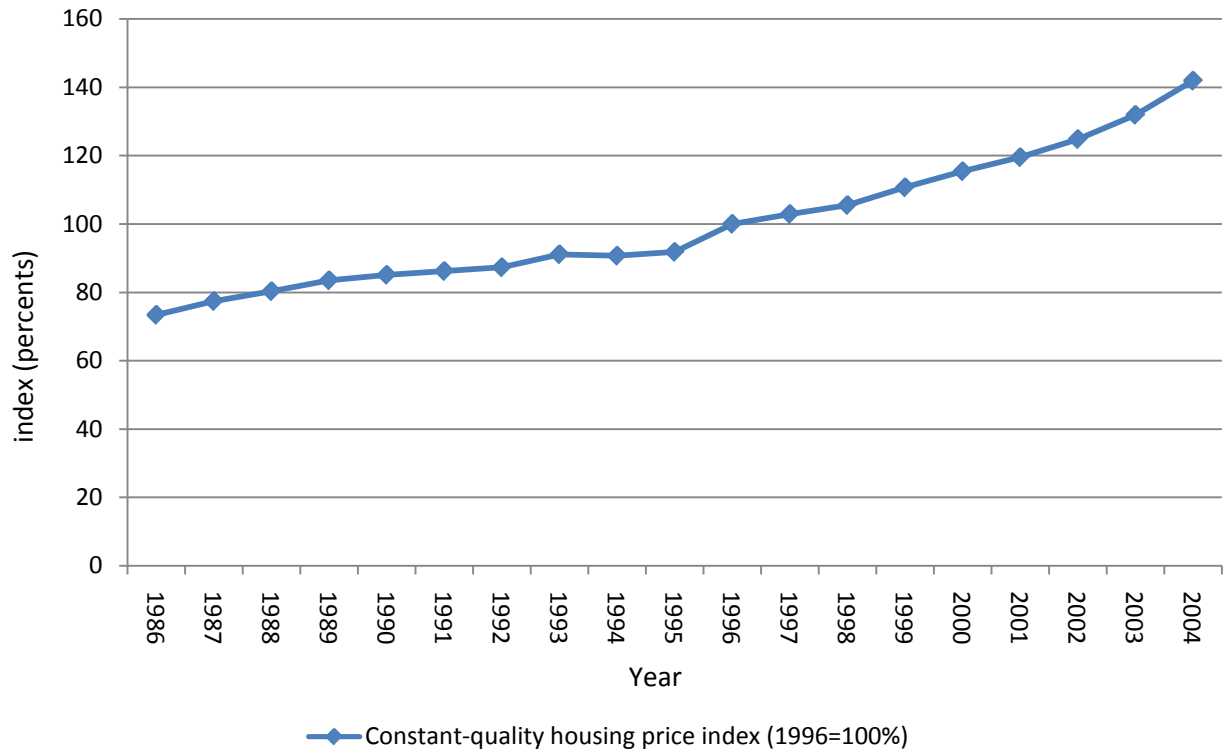


Figure 2: Welfare adjustment in the economy with exogenous housing price and binding credit constraints

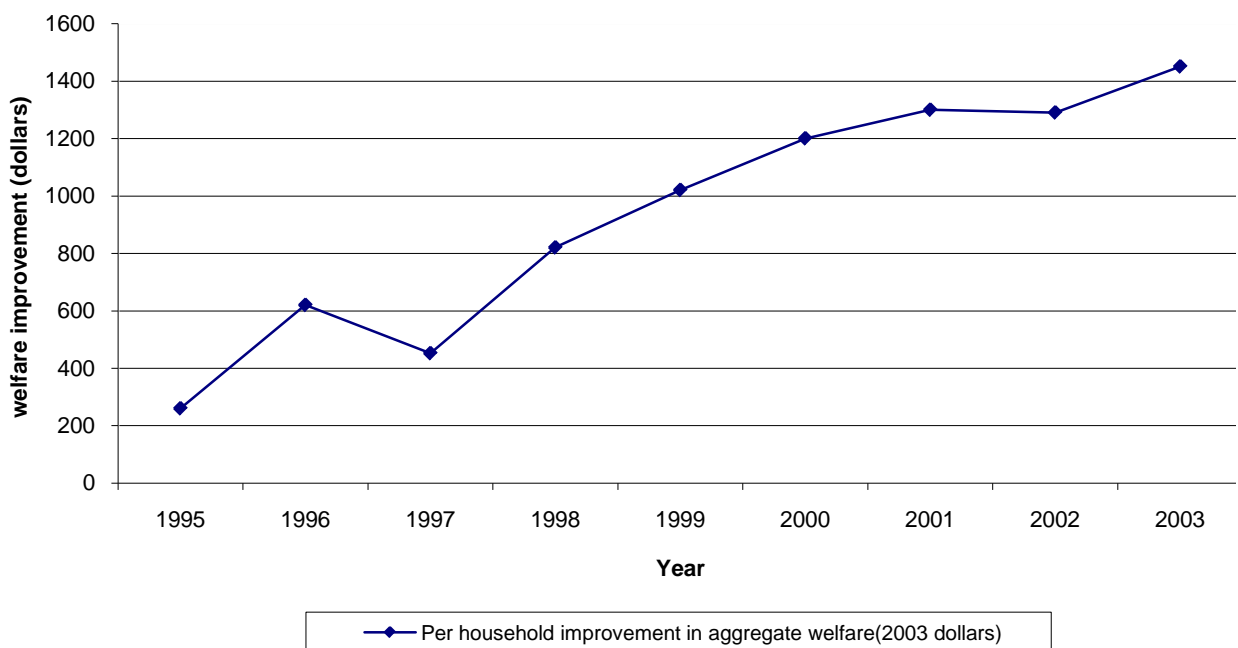


Figure 3: Joint dynamics of household's income and housing prices in US

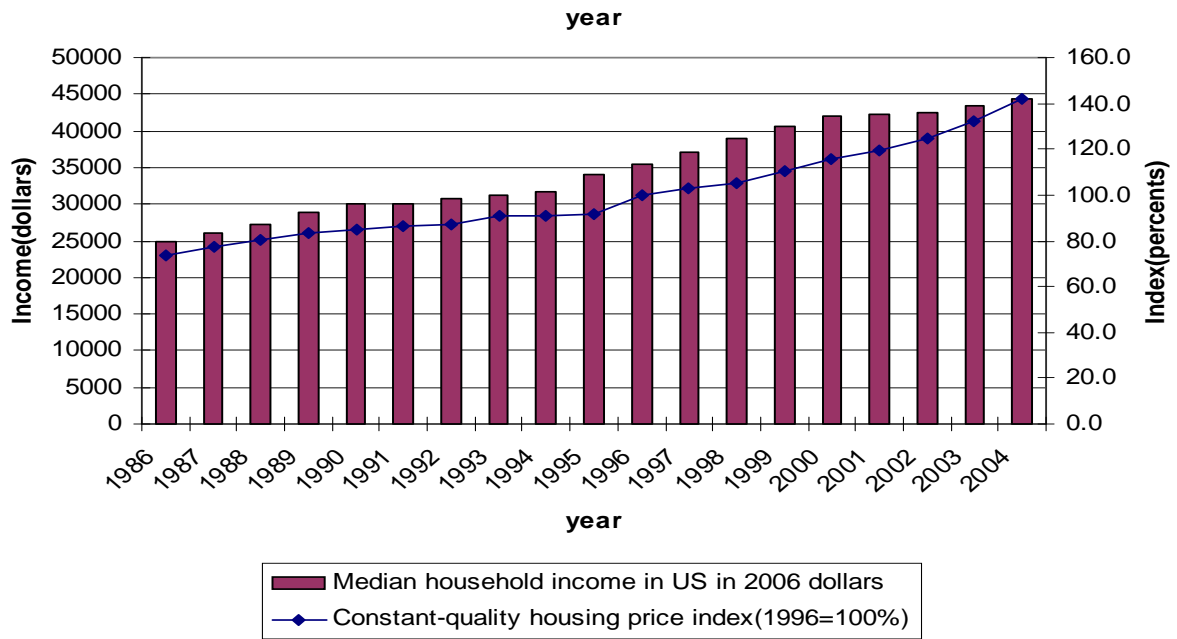


Figure 4: Evolution of effective interest rate on mortgages and long-term government bond yields in US in 1986-2004

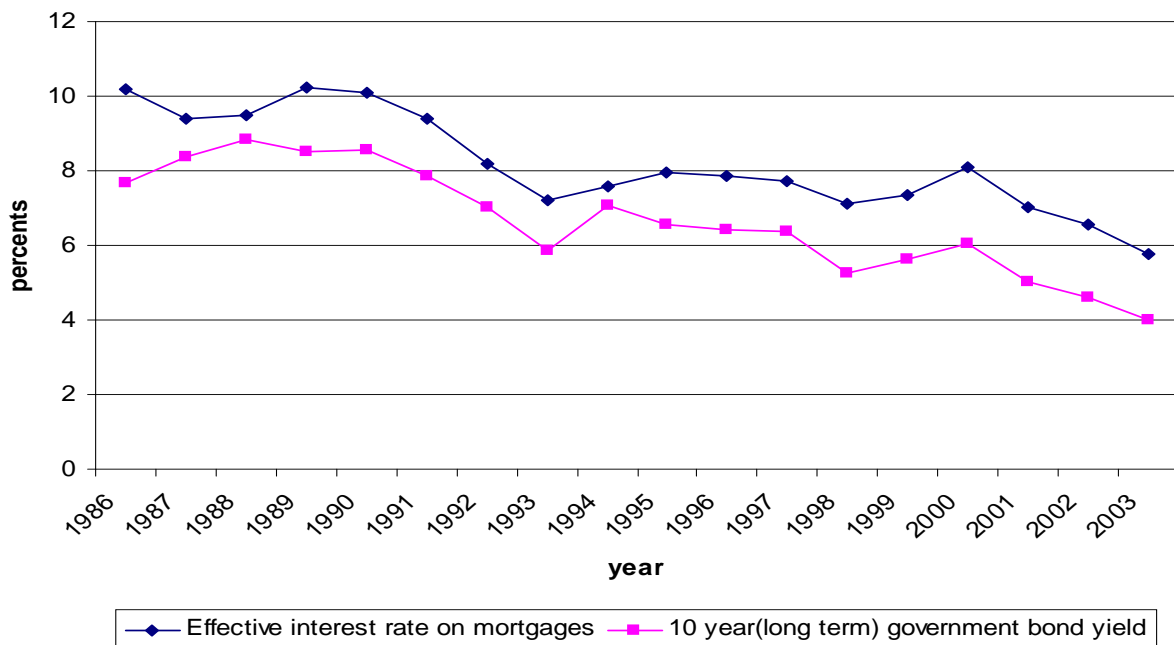


Table 1: Comparison of welfare adjustmetns in the credit constrained and unconstrained models(supply-side shocks)

	Unconstrained	Constrained
ω	$\frac{i^{ss} + \delta - \pi}{A}$	$\frac{B}{\beta(1 - \omega)\bar{D}}$
0.1	1.046781	0.121252
0.2	1.098154	0.274385
0.3	1.154829	0.473092
0.4	1.217672	0.740199
0.5	1.287749	1.116798
0.6	1.366385	1.685037
0.7	1.455248	2.636675
0.8	1.556474	4.546914
0.9	1.672835	8.291815

Table 2: Net worth of US households by age of the head

Age of household head	Mean net worth in constant 2004 dollars
Under 35	7 3500
35-44	299200
45-54	542700
55-64	843800
65-74	690900
75 and over	528100

Table 3: Model implied elasticities of the housing price with respect to each of the shocks under different parametrizations

Elasticities	Parametrization 1	Parametrization 2	Parametrization 3	Parametrization 4
$\varepsilon_{qy,uc}$	0.389	0.472	0.526	0.678
$\varepsilon_{qy,c}$	0.264	0.382	0.188	0.281
$\varepsilon_{qi,uc}$	0.235	0.264	0.235	0.264
$\varepsilon_{qi,c}$	0.357	0.38	0.357	0.38

where:

$\varepsilon_{qy,c}$ - the elasticity of price with respect to income in the constrained economy

$\varepsilon_{qy,uc}$ -the elasticity of price with respect to income in the unconstrained economy

$\varepsilon_{qi,c}$ - the elasticity of price with respect to interest rate in the constrained economy

$\varepsilon_{qi,uc}$ -the elasticity of price with respect to interest rate in the unconstrained economy.

J_c - number of credit-constrained households

J_{uc} - number of unconstrained households

ω - exponent on housing in the Cobb-Douglas utility function

Parametrization 1 - $J_c = 44784339$, $J_{uc} = 62888650$, $\omega = 0.56$

Parametrization 2 - $J_c = 44784339$, $J_{uc} = 62888650$, $\omega = 0.64$

Parametrization 3 - $J_c = 21737795$, $J_{uc} = 85935104$, $\omega = 0.56$

Parametrization 4 - $J_c = 44784339$, $J_{uc} = 62888650$, $\omega = 0.64$

Table 4: Welfare adjustments with respect to each of the shocks (in 2004 dollars)

Adjustment	$c(\text{constrained})$	$uc(\text{unconstrained})$
Δy_s	296	237
Δy_y	-8936	-10770
Δy_i	-32735	-1746