Stock market expectations and portfolio choice of American households.
Work in progress.

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Abstract

Using survey data on expectations and the composition of household savings, this paper aims at explaining the stockholding puzzle. The puzzle is in the fact that, despite the high historical returns and relatively low risk of stock-market based assets, many American households own no such assets, and many of those who do own a little. We develop a measurement model that is consistent with observed noise in survey measurement of expectations. The model is based on a theory of survey response and it allows us to separate noise from heterogeneity that is relevant in investment decisions. We estimate relevant heterogeneity and relate that to household investment behavior, with the help of a simple portfolio choice model. Our results confirm the validity of survey measures of expectations in predicting real behavior after measurement error is properly accounted for. A causal interpretation of the results suggest that heterogeneity in expectations is a major predictor of stockholding, and low average expectations, high uncertainty, and large heterogeneity in expectations explain much of the puzzle. We show important systematic variation in expectations, both in terms of levels and uncertainty. Our results are also informative about how people answer subjective probability questions in general. The results are preliminary and incomplete.

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1 Introduction[1]

Despite the superior historical performance of stocks over alternative bonds or bank accounts, American households hold surprisingly little stocks and stock-market based assets on average, and many hold none (see, for example, Campbell, 2006). This phenomenon is sometimes called the "stockholder puzzle" or the "stock market participation puzzle" (Mankiw and Zeldes, 1991, Haliassos and Bertaut, 1995). A related puzzle is the "equity premium puzzle", the fact that returns on stock-market based assets are too high: they cannot be rationalized by choices of a representative consumer with "sensible" risk preferences (Mehra and Prescott, 1985). While some authors suggest that there may be no puzzle - only that people are more risk-averse than economists think (Barsky et. al., 1997) -, most economists hold that the observed phenomena are puzzles indeed (Kocherlakota, 1996). The stockholding puzzle was reinforced and complemented by Poterba, Rauh, Venti, and Wise (2006), who showed the optimality of majority-stock portfolios in a realistic simulation-based analysis.

This paper shows that expectations over the performance of the stock market go a long way in explaining the puzzle, or to put it differently, the puzzle is mostly in people's expectations. In order to answer our main questions, we analyze answers to survey questions about stock market expectations, together with information about household savings, in the Health and Retirement Study (HRS). We separate relevant heterogeneity from survey noise with the help of a survey response model, which acknowledges that answering survey questions is a fundamentally different situation than making an investment decision. We estimate relevant heterogeneity in the location (mean) and uncertainty (variance) of one-year returns, and relate those to household portfolio choices. The relationship of expectations and household portfolio choice is kept simple. The substantive contribution of our analysis is in showing that heterogeneity of expectations together with discrepancy from historical moments go a long way in explaining observed portfolio behavior even in the most stylized setting.

Our methodological contribution is in developing a model of survey response and, based on that, a measurement model that is consistent with observed noise features. Our results validate the use of survey measures of expectations formulated as probability questions, in the sense that once survey noise is properly accounted for, these measures provide substantial and potentially unbiased information about expectations relevant for economic behavior.

The results show that people's actual expectations are consistent with portfolio choice; they are heterogenous; and, on average, they are more pessimistic and more uncertain than what historical returns would imply. We also find that households' investment behavior is broadly in line with their expectations, assuming "sensible" levels of risk aversion. We identify important systematic variation in stock-market expectations. Women, African Americans, and Hispanics have lower expectations, and women's expectations are also substantially more uncertain. More educated people have higher expectations (i.e. closer to the historical

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mean). Optimism about the weather, macroeconomic performance, and one's own survival probabilities are positively correlated with stock-market optimism, and symptoms of clinical depression are negatively correlated. Uncertainty over other events, proxied by the propensity to give 50-50 answers to other probability questions, is positively correlated with uncertainty about stock returns.

The formulation of the stockholding puzzle, and correspondingly, our approach to household investment decisions, is that of demand analysis, as is usual in household finance research (Campbell, 2006). While our analysis does not address equilibrium issues, it has important implications for equilibrium prices and turnover as well. Our results about high uncertainty on average and substantial heterogeneity in uncertainty support Weitzman's (2007) argument for the indefinite nature of the posterior variance of stock-market returns, and its potential role in explaining the equity premium. Moreover, the substantial heterogeneity of expectations is in line with the basic argument of many disagreement models (Hong and Stein, 2007) that postulate that heterogeneity in beliefs may be essential for trade.

Investigating survey measures of subjective probabilities is a relatively new line of research (see Manski, 2004). Within this literature, stock market expectations were analyzed by Dominitz and Manski (2006) and Winter et. al. (2006). Our approach differs from theirs in two ways. First, we connect expectations to investment behavior in a structural way. Second, we directly address the noise of measured subjective probabilities (in a spirit close to Hill, Perry and Willis, 2006). These two contributions together enable us to provide validation of survey measures of expectations as important and, if properly treated, potentially unbiased measures of relevant heterogeneity.

The remainder of the paper is structured the following way. Section 2 describes the main measurement problem and the data. It shows evidence for both signal and noise in the measurement. The descriptive analysis suggests that some noise features may be specific to the survey while others may be related to heterogeneity relevant for investment behavior as well. In section 3, we borrow a simple portfolio choice model and look at how heterogeneity in expectations affects portfolio choice. More importantly, we also develop a survey response model in order to relate measured probabilities to parameters of underlying relevant heterogeneity. Section 4 contains the baseline measurement model, and Section 5 shows the results. Section 6 contains an extension of the baseline model in applying the modal response hypothesis of Hill, Perry and Willis (2006). Section 7 concludes. The results are preliminary and incomplete.

2 Descriptive analysis

2.1 The measurement problem

The measurement problem is how to obtain statistics of people's expectations about the future performance of the stock market. In particular, we need to measure statistics that are sufficient for making the portfolio choice decision. In order to keep things simple and focus on the first two moments of returns expectations, we assume that people believe that yearly returns are i.i.d. and normally distributed. These assumptions are close to what we see in historical data. They imply that the mean and the standard deviation are sufficient statistics for the returns distribution. Moreover, as we shall see, it is exactly those statistics that are needed in simple portfolio choice theory (see later for more details).

The historical density of nominal yearly returns on the S&P 500 is depicted by the histogram in Figure 1.[2] The distribution is very close to normal and is i.i.d.; mean is 0.09, and standard deviation is 0.15. A person whose stock market expectations are based on the historical distribution, could be visualized by the density in Figure 1. Although not necessary for the analysis, it may sometimes help to assume that people represent those expectations by a mental image of the density function. The measurement problem is to extract the person's subjective expectations, e.g. the density function or sufficient statistics of it.

Although the mean and the standard deviation are sufficient statistics under the maintained assumption of i.i.d. normality, they are not straightforward to elicit in surveys. While it certainly makes sense to ask about expected returns, the same is not true for standard deviation: most people don't know what a standard deviation is, let alone have the ability to estimate it. Asking for specific probabilities is a more promising alternative.

The survey we use asks two such probabilities: the probability that returns will be positive, and the probability that they will be larger than 10 per cent. If, based on the density in their mind, respondents can calculate the appropriate probabilities, two probabilities exactly identify the distribution under the normality assumption. This is seen more easily from a cumulative distribution function, as the one in Figure 2. It shows the normal c.d.f. with the historical $\mu = 0.09$ and $\sigma = 0.15$.[3] The corresponding probability of positive returns (p_0) is 73 per cent, and the probability of returns at least 10% (p_{10}) is 47 per cent.

Identifying the mean and standard deviation from two probabilities is relatively straightforward, see later. Intuitively, the level of the probabilities (e.g. the average of the two) is informative about the expected value: the higher the probabilities, the higher the expectations. At the same time, the difference between the probabilities is informative about the spread of the distribution: the higher the difference the smaller the standard deviation (the steeper the c.d.f., or alternatively, the more probability mass is concentrated on the same support segment of the p.d.f.).

2.2 Stock market expectations in HRS 2002

We use data from the Health and Retirement Study (HRS). For an early review of the survey see Juster and Suzman (1995). HRS has had a number of probability questions from 1992

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[2]The relevant real, after tax return implicitly depends on both expectations of nominal returns and inflationary expectations. In addition, there are important tax considerations. In this paper, we ignore both inflation and taxes.

[3]Poterba et al. show a very similar distribution from the 1921 to 2001 time series, with the same mean but somewhat higher standard deviation (0.2).

on. It added questions on stock price expectations in the 2002 wave (see Manski, 2004, for a review of survey-based probability questions). HRS is representative of the 50 years old and older American population, and their households. Besides subjective probabilities, HRS collects data on the amount and structure of savings, including 401(k) accounts, a rich set of demographic variables, and measures of cognitive functioning.

In order to focus on households that are still in the wealth accumulation phase of the life cycle, we restricted our sample to the younger part of the survey. We kept people who were parts of the original HRS and War Babies study cohorts and were 55 to 65 years old in 2002.

Note that, while expectations are defined at the individual level, saving behavior is at the household level. HRS respondents are either couples or individuals without spouses (expectation of other possible members of the households are not elicited). In order to focus on our main question, and because of these data limitations, we simplify the problem of how households make financial decisions by picking one member per household. The one person we pick is the designated "financial respondent" of the household, the person who is the most knowledgeable about the savings and assets of the household, and who is therefore selected by the household to answer the asset questions. We also drop households with missing savings information in either 2002 or 2004 (less than 2 per cent of households). The sample for our main analysis consists of 3715 persons.

Households are asked whether they have investments in stocks or mutual funds. If "yes," we call these people "direct stockholders". HRS also asks about retirement accounts and their composition (the latter in a very simplified way). Persons who are not direct stockholders, but are in households with some stocks or mutual fund investments in retirement accounts are called "indirect stockholders." Stockholding is low (see Table 1). 56 per cent of the households with financial respondents between 55 and 65 years of age were neither direct nor indirect stockholders in 2002. This is the puzzle to explain. Also note that stockholding status is relatively stable.[4]

The main questionnaire of HRS 2002 contained two questions about the respondents' expectations of future performance of the U.S. stock market. One (p_0) asked what the respondent thought the probability is that the market will go up at all, and another one (p_{10}) about the probability that it will go up by at least 10 per cent. The questions themselves were phrased the following way.

We are interested in how well you think the economy will do in the next year.

p_0 question: By next year at this time, what is the percent chance that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more than they are today?

p_{10} question: By next year at this time, what is the chance they will have grown by 10 percent or more?

[4] There is probably some survey error in measures of stockholding but we do not focus on that error. The main reason is that in our models, stockholding will always be a left-hand side variable, and thus classical noise will have no or little effect on our estimates. At the same time, noise in stockholding measures would decrease the fit of our models.

Half of the respondents were asked the two probability questions in a different order: first the p_{10}, then the p_0 question. Respondents were randomly assigned to the two sequences. The purpose of the different ordering was to explore the potential effects of anchoring. The distributions of the two answers are slightly different (answer to the question that is asked first tends to have somewhat lower mean and smaller standard deviation), but the differences are very small and many times statistically insignificant. In what follows, we shall ignore the ordering of the questions.[5]

Of the 3715 respondents of the sample, 3049 (82%) answered both the p_0 and the p_{10} question. Of the 18 per cent whose answers are missing, the vast majority come from "I don't know" answers as opposed to refusals, and most people who said "I don't know" to one of the questions said the same to the other.[6] It seems, therefore, that the missing answers reflect genuine ignorance. Missing answers to other probability questions in the HRS are a lot less frequent, but they vary somewhat with the "difficulty" of the question (1 per cent for whether tomorrow will be a sunny day, and 4 per cent for whether the respondent's income will keep up with inflation or whether one would live to be a given age). It seems that, for many respondents, the stock market questions were just too difficult to answer. We shall further explore the nature of the missing answers later, when we analyze the noise component in the responses.

Summary statistics of the answers are in Table 2. The first thing to notice is the very low mean answers in 2002: a 49 per cent chance of positive returns (p_0) , and 39 per cent chance of at least 10% returns (p_{10}) . Recall that historically, these probabilities were 73 per cent and 47 per cent, respectively. The mean answers are, therefore, 24 and 8 percentage points lower, respectively. There is also a substantial amount of heterogeneity on the responses: standard deviation (i.e. typical cross-respondent difference) is close to 30 percentage points.

Answers in 2004 were somewhat more optimistic, less diverse, and there were fewer missing answers, as well. Table 2A shows that the 2002 versus 2004 comparison yields the same conclusion in each subsample (more valid answers, more optimistic answers, less diversity in the answers). Table 2A also shows that spouses of the financial respondents in our main sample are less optimistic and are more likely to give missing answers. Older HRS respondents are even less optimistic, more diverse, and are substantially more likely not to answer the stock market expectation questions.

2.3 Evidence for signal

Survey responses to subjective probability questions contain a significant amount of information in general (see, for example, Hurd and McGarry, 1995; Hurd and Smith, 2002; Van

[5] For example, the Kolmogorov-Smirnov tests rejects the null hypothesis of equal distributions for each pair of questions, while the Kruskal-Wallis test indicates the same distribution for the market up question regardless of question ordering.

[6] If one gave a non-valid answer to the first question, the second one was skipped. But of those who gave a 0 to 100 per cent probability answer to the first question, very few said "I don't know" on the second question. The rate is the same for those who got the market up question first and those who got he market up 10% question first.

der Klaauw and Wolpin, 2002; Dominitz and Manski, 2004, Finkelstein and McGarry, 2006). In earlier research on subjective probabilities, (Kézdi and Willis, 2003) found that stockholding is related to general optimism (the propensity to attach higher probabilities to more favorable events) and negatively related to the fraction of 50-50 or other focal answers to probability questions in general.

Table 3 shows mean responses to the core stock market expectation questions in HRS 2002 and 2004, by stock ownership. In both 2002 and 2004, stockholders (both direct and indirect) have significantly higher expectations than non-holders. In fact, in each survey year, stockholders attribute a higher likelihood of gains than losses, whereas non-holders attribute a smaller likelihood. In 2002, higher expectations show up for the p_{10} question as well, with similar relative differences. As we noted earlier, the distance between the two probability answers is a measure of the dispersion of the distribution: a larger distance is related to a smaller dispersion (more probability mass is concentrated on the same support segment). Table 3 shows that in 2002, on average, the distance is larger for stockholders (both direct and indirect).

These results suggest that reported probabilities are meaningful predictors of investment behavior. Stockholders' expectations exhibit higher mean and lower variance.

2.4 Evidence for noise

In this section we describe evidence of noise in the probability answers. Our main focus is on focal answers, rounding in general, violations of the law of probability, and test-retest noise, measured by the propensity to give a different answer to same question if asked twice. We also revisit the problem of missing probability answers. (Recall that nonresponse is 18 per cent and it most likely represents genuine ignorance.) We explore the extent of the noise features and also their potential information content. In particular, we investigate whether each phenomenon is related to stockholding, or other covariates. Our goal is to separate noise components that are specific to the survey situation from the potential information they may contain about heterogeneity relevant for investment decisions.

Figure 3 shows the histogram (empirical density) of each probability answer. The pictures are typical for survey probability answers; see Manski (2003) for examples. Table 4 shows the distribution of answers around focal and other round values. Virtually all answers are at some round numbers, including 0 and 100 per cent. Note that, strictly speaking, 0 and 100 per cent are not valid probabilities if the support is the real line and there is positive mass everywhere, as the normal assumption would imply. Of course, rounding may produce these focal answers given underlying probabilities consistent with normality.

Focal values at 50 per cent account for an especially large part of all answers. In the U.S., the expression "fifty-fifty" may be used as a synonym for "I don't know." Both models of survey response that we explore in this paper will exhibit the feature that extreme uncertainty leads to an answer at 50 per cent.

Quite a few respondents give answers that apparently violate the laws of probability. 14% of the respondents give a larger probability answer to the p_{10} question than the p_0 question, whereas the former set of events is a proper subset of the latter. Yet another 44% give the same answer to the two questions, implying a zero density between the no change and 10% increase. Under the normality assumption, this would imply an infinite variance. Table 5 shows the distribution of answers that imply positive, zero, or negative probability mass, by answers give to the p_0 question. Somewhat surprisingly, there is no clear relationship between the propensity to give focal or other round answer and the propensity to give zero mass or negative mass answers.

As we indicated before, HRS 2002 asked other stock market expectation questions from about 5 per cent of its respondents in an experimental module added to the core questionnaire. The module questions included the same questions as the core questionnaire. This gives us a unique opportunity to analyze differences in answers give to identical question within the same survey (typically 20-30 minutes apart from each other). Such differences are called test-retest differences in the literature of survey statistics and experimental psychology.

Table 6 shows that the overall distribution of the module answers ($p_{0'}$ and $p_{10'}$) are very similar to those in the core survey (p_0 and p_{10}, respectively).[7] In what follows, we are going to treat the core and module answers as drawn from the same distribution.

While the distributions are close, there are significant differences in the individual answers. Of the 292 respondents in the sample who answered all four questions, only 77 (26 per cent) gave the same answer to p_0 and $p_{0'}$. The correlation of the two market up answers is pretty low ($Corr\,(p_0, p_{0'}) = 0.48, \;\; Corr\,(p_{10}, p_{10'}) = 0.35$). Figure 4 shows the densities themselves, together with a fitted normal density for each. The differences are very close to being normal, with slightly more mass at 0 (no difference at all). Skewness-kurtosis tests cannot reject the normality of either distribution.

Table 7 shows how stock ownership is related to the different kinds of survey noise. Table 8 repeats the same in regressions, with the propensity to produce each noise feature on the left-hand side and broad stockholder status on the right-hand side, together with basic demographics, cognitive score, and wealth. The results show that item nonresponse is strongly negatively related to stockholding (86 of such respondents are non-holders, as opposed to 50 per cent of the rest of the sample). The propensity to give seemingly certain answers (0 or 100 per cent) is not related to anything except for some weak correlations to cognition and veteran status. The propensity to give a 50 per cent answer is not related to stockholding either but it is more strongly related to education, cognition, and demographics. Rounding to other numbers is weakly related to stockholding but barely anything else. The propensity to answer $p_0 = p_{10}$ and $p_0 < p_{10}$ are weakly negatively related to stockholding, education, and demographics. The absolute difference of core and module answers, a measure of test-retest noise, is not related to anything significant except that African Americans seem to give less noisy answers.

The results imply that nonresponse is strongly related to relevant heterogeneity in stock

[7] The mean and the entire distribution of p_0 and $p_{0'}$ are the same (Kolmogorov-Smirnov test p-value is 0.88). The same is less obvious for p_{10} and $p_{10'}$: module answers have a lower mean, and the distributions do not pass the Kolmogorov-Smirnov test (p=0.02). But even these distributions are not all that far from each other.

market expectations in a way that is consistent with genuine ignorance, Test-retest noise is not far from being purely random, which supports a classical measurement error interpretation. Rounding, the propensity to give focal answers, and apparent violations of the laws of probability may or may not be related to relevant heterogeneity.

3 Behavioral models

In order to facilitate measurement, we present two economic models of this section. The first model is that of investment behavior: it describes demand for stocks given expectations. The second model is that of survey response: it describes how people answer survey questions given their expectations. In the following section, we build our measurement models on these behavioral models in order to infer expectations from survey responses, and contrast stockholding predicted by those expectations to stockholding observed in the data.

3.1 Investment behavior

The investment behavioral model is the simple and very intuitive model of Merton (1969). Besides its simplicity it is logically consistent with our maintained assumption of i.i.d. normal returns.

Consider an individual who saves for retirement. For simplicity, assume that at time 0 she has wealth W_0 to invest and she wants to maximize the expected utility of W_T, her wealth when she retires at some predetermined time T. Assume that the only thing she cares about is her wealth at retirement (W_T), and that she has a conventional constant relative risk aversion (CRRA) utility function over W_T with parameter of relative risk aversion $1/\alpha$ (α denoting the coefficient of risk tolerance):

$$\max_{s_t} E_t \frac{W_T^{1-1/\alpha}}{1-1/\alpha} \tag{1}$$

She can choose between investing in two assets: a risk-free asset with known rate of return r ("bank account") and one risky asset ("stocks") with uncertain return. The instantaneous rate of return of the risky asset, denoted by dS/S, is assumed to follow a Brownian motion. The investment decision consists of choosing an optimal fraction of wealth invested into the risky asset for each time t between 0 and T, which we denote by s_t^*.

The equation of motion for the instantaneous return to the risky asset is given by

$$\frac{dS}{S} = \mu\ dt + \sigma\ dz \tag{2}$$

where dz is the increment to a standard Wiener process. This is a continuous time version of a random walk with drift, where the drift is μ and the variance is σ^2 (both normalized to the unity time-interval). Throughout the analysis we assume that the investor knows the random walk nature of the process and that its parameters are constant. We also assume that she also knows the parameters themselves, an assumption we will qualify later. A result of this equation of motion is that returns over the unit interval are distributed normally, with mean μ and variance σ^2.

With fraction s_t of wealth W_t invested into the risky asset at each time t, wealth follows a geometric Brownian motion given by

$$\frac{dW}{W} = (r + s_t(\mu - r))\, dt + s_t \sigma dz \tag{3}$$

where r is the known instantaneous rate of return on the risk-free asset. Subject to this budget constraint, the investor's problem is to maximize the expected utility of wealth at retirement, W_T, according to the expected utility function (1).

The well-known solution to this problem is a constant fraction of wealth invested into the risky asset

$$s_t^* = s^* = \alpha \frac{\mu - r}{\sigma^2}. \tag{4}$$

The optimal share invested into stocks is increasing in its mean return, decreasing in the return of the risk-free asset, and decreasing in the variance and the degree of risk aversion. $(\mu - r)/\sigma^2$ is also known as the Sharpe ratio. The Merton model's implication is that the optimal share of the risky asset is proportional to the Sharpe ratio, and the proportionality coefficient is the parameter of risk tolerance, the inverse of risk aversion.

The simple and elegant result in (4) comes at the cost of being at odds with a number of empirical regularities, especially if parameters of the returns process are substituted with those estimated from historical data. First of all, no matter how risk averse the investor is, s^* is always positive (historically, returns on stocks have been significantly higher than the rate on the risk-free asset, i.e. $\mu > r$). Yet many people hold no stocks at all – this is the "stockholder puzzle" (Haliassos and Bertaut, 1995). A related problem is that, given again historical moments of stock market returns, the aggregate amount in stocks should be higher than observed if people had "sensible" risk preferences – this is the "equity premium puzzle" (Mehra and Prescott, 1985; Kocherlakota, 1996.). In addition, observed portfolio structure of individual savings changes quite a lot, which is clearly at odds with the constant optimal fraction. Lastly, (4) obeys the "Tobin separation theorem" (Tobin, 1958), which suggests that the composition of the optimal portfolio should be independent of the optimal level of wealth. In the real world the two are strongly correlated: wealthier people tend to hold a significantly larger fraction of their savings in risky assets.

But, as a model of demand for the risky asset, there is nothing in this model that requires μ and σ^2 to take any specific value, such as their historical estimates. Heterogeneous beliefs in the parameters are also consistent with the model. Those who believe $\mu_i < r$ will hold a zero fraction of their savings in stocks (assuming no short sales). Also, people who believe the variance is sufficiently high will hold a tiny fraction which, in the presence of some transaction costs, may again lead to holding zero fraction in stocks (especially if very risk-averse).

Of course, this model is not naturalistic: numerous extensions have been considered in the literature, in which the simple investment rule of constant shares and continuous rebalancing results disappear. Taken as a positive model of household demand, an obvious problem with the model is that most people do not understand the concepts of Brownian motion or even standard deviations. Under the maintained hypothesis of normal returns, however, these parameters have very intuitive meanings associated with risk and return. The behavioral model is thus a parsimonious way of relating higher (lower) expectations or lower (higher) uncertainty to the desire to have a larger (smaller) share of stocks in savings. Moreover, Poterba, Rauh, Venti and Wise (2005) show that the constant share investment rule performs surprisingly well against alternative, more sophisticated ones.

3.2 Subjective beliefs about stock returns

Taken as a model of demand for stocks, all parameters may be heterogeneous in the population so that

$$s_i^* = \alpha_i \frac{\mu_i - r_i}{\sigma_i^2}$$

In this paper, we focus on heterogeneity in expectations: μ_i and σ_i. We always keep r_i constant, but in some of our estimates we make use of heterogeneity in α_i estimated by Kimball, Sahm and Shapiro (2007).

Nothing in the theory suggests that people's expectations should coincide with historical moments, at least not until equilibrium consequences are addressed. The main and very simple idea of our analysis is that beliefs may answer the stockholding puzzle with "sensible" risk preferences. Many people may hold no stocks because they do not expect them to bring higher returns than bank accounts ($\mu_i < r$), or they think stocks are prohibitively risky (σ_i^2 very large). The idea that prohibitively high subjective variance can explain low observed stockholding is emphasized by Weitzman (2007) as well.

In this subsection we introduce some notation to describe heterogeneity of beliefs. Let R_i denote annual returns on the stock market as perceived by individual i. If beliefs of individual i are identical to the historical record, the distribution is like the histogram in Figure 1. We maintain the assumption that individual i believes that yearly returns are $i.i.d.$ normally distributed random variables. Let $R_{i(t+1)}$ denote the return for next year $(t+1)$ as viewed by individual i at time t. We model this by writing

$$\begin{aligned} R_{i(t+1)} &= \mu_{it} + \eta_{it} \\ \eta_{it}|\mu_{it} &\sim N\left(0, \sigma_{it}^2\right) \end{aligned} \tag{5}$$

where μ_{it} is individual i's subjective expected value at time t, and η_{it} is the way she perceives, at time t, possible deviations from the expected value. She has subjective moments $E\left(R_{i(t+1)}\right) = \mu_{it}$ and $V\left(R_{i(t+1)}|\mu_{it}\right) = \sigma_{it}^2$: at time t, individual i perceives next year's returns as a random variable with mean μ_{it} and standard deviation σ_{it}. This is an atheoretical way of representing individual i's expectations, one that puts no restrictions on either μ or σ.

In what follows, we'll refer to R as the *fundamental subjective returns* (or *fundamental expectations about the returns*). Heterogeneity in R we label as *relevant heterogeneity.*

3.3 Survey response: noisy expectations

When the individual is approached by an interviewer and confronted with a question about her fundamental expectations, we assume that her answer is based on a possibly different (but hopefully related) object $\tilde{R}_{i(t+1)j}$ (where j denotes the particular question on the survey).

The idea here is that of survey noise. We assume that questions on the same survey are related to the same t. Sometimes we'll refer to $\tilde{R}$ as *noisy subjective returns* (or *noisy expectations*). Heterogeneity in $\tilde{R}$ we sometimes label as *measured heterogeneity.* The goal of the paper is to establish and estimate the relationship between measured heterogeneity and relevant heterogeneity. I.e. the relationship between (statistics of) $\tilde{R}_{i(t+1)j}$ and (statistics of) $R_{i(t+1)}$.

The survey tries measure expectations that would be relevant in an investment situation. At the same time, the survey situation is very different from an investment situation. There is considerably less time allowed, and there are practically no incentives to get the answers right. We assume that when confronted with probability question j on the survey, individual i retrieves a noisy version of the fundamental random variable:

$$\tilde{R}_{i(t+1)j} = \mu_{it} + \eta_{it} + v_{itj}, \tag{6}$$

where

$$v_{itj} | \mu_{it}, \eta_{it} \sim N\left(0, \sigma_v^2\right)$$

This noise is classical measurement error in the sense that it is independent of everything. It is also additive to μ but it won't be additive to the measured subjective probability variables.

Recall the stock market expectation questions of HRS come in the form of probabilities. There are two probabilities asked: the probability that the market will go up (p_0), and the probability that the market will go up by more than ten per cent (p_{10}). Some people in our sample answered the same pair of questions once again, in an experimental module, .

We assume that answers to each question j is based on a possibly different noisy expectation, $\tilde{R}_{i(t+1)j}$, $j = 0, 10, 0', 10'$. Core and module answers are obviously different measures of the fundamental object.

At the same time, it would make sense for adjacent answers to be affected by the same noise (the same draw v). However, the evidence of negative probability mass answers indicates that some people respond as if they forgot their previous answer. We assume that it is purely due to lack of attention on the survey, rather than the inability to think in terms of probabilities. Technically, we assume that for each question j, there is a new draw of survey noise v_{itj}, such that $Corr\left(v_0, v_{0'}\right) = Corr\left(v_{10}, v_{10'}\right) = 0$, but $Corr\left(v_0, v_{10}\right) = Corr\left(v_{0'}, v_{10'}\right) = \rho_v$.

3.4 Survey response: From noisy expectations to probability answers

In the survey people are asked to answer probability questions. Throughout this paper, we assume that whatever way people form that answer, it is based on the noisy expectations $\tilde{R}$ introduced above. A helpful story may be that people first form a mental image of the density function of $\tilde{R}$ in their head, and then they try to calculate the probability in question.

When looking at answers given within a survey, we assume that relevant expectations do not change. I.e. within a survey $\mu_{it} = \mu_i$ and $\sigma_{it} = \sigma_i$. To simplify notation, we omit the t subscript from now on.

The benchmark answer to each probability question j is the proper integral, which we shall call the precise probability and denoted by p_j^* :

$$p_{ij}^* = \Pr\left(\tilde{R}_{ij} > \tau_j | \mu_i, v_{ij}\right) = \Pr\left(\frac{\eta_i}{\sigma_i} > \frac{\tau_j - \mu_i - v_{ij}}{\sigma_i}\right) = \Phi\left(\frac{\mu_i + v_{ij} - \tau_j}{\sigma_i}\right) \tag{7}$$

so that $p_{i0}^* = \Phi\left(\frac{\mu_i + v_{i0}}{\sigma_i}\right)$ and $p_{i10}^* = \Phi\left(\frac{\mu_i + v_{i10} - 0.1}{\sigma_i}\right)$. The precise probability p_{ij}^* maps the individual mean (μ_i), the individual variance (σ_i) and the noise draw at the given probability question (v_{ij}) to a proper probability (τ_j is fixed by the probability question). The higher the individual mean or the noise draw, the higher the precise probability.

An important feature of the precise probability is that a mean-preserving spread in fundamental uncertainty (σ_i) pushes it towards 0.5. Mechanically, this is because given μ_i and v_{ij}, an increase in σ_i moves the index towards zero, for any value of τ_j. One consequence of this phenomenon is that in a population that is heterogenous in μ_i and has high enough uncertainty σ_i, the average precise probability would be biased towards 0.5 relative to the precise probability based on the average μ.

Another consequence is the fifty-fifty probability answer at the extreme. As fundamental uncertainty approaches infinity $(\sigma_i \to \infty)$, the index approaches zero, making the precise probability 0.5 $\left(p_{ij}^* \to 0.5\right)$. This is very much in line with the casual interpretation of a "fifty-fifty" answer reflecting ignorance. Infinitely large uncertainty can be interpreted as ignorance (returns can be anything with approximately equal likelihood), for example as an uninformative prior that was not sharpened by learning. Therefore, a person who is completely ignorant about the stock market would answer fifty per cent for both p_0 and p_{10} if she were to give the precise probability for an answer.

But assuming that people give exactly the precise probability as an answer is problematic for two reasons. First, in he spirit of our survey response models, it would not be rational for a respondent to put the effort necessary for the calculation. Calculating probabilities is a difficult task, no matter what density one has in mind. Second, while high uncertainty may explain the large fraction of 50-50 answers, the prevalence of 0, 100, and other rounded answers is incompatible with answers reflecting the precise probability itself.

A more realistic (indeed, more rational) model would assume that respondents make a guess of what that probability could be. We can model that "guessing" process in alternative ways. In this benchmark model, we stay agnostic about that process. Instead, we simply assume that an answer within a pre-specified interval can correspond to any precise probability within that interval. The admissible intervals are exogenously given and are the same for everyone. Formally, if the reported probability (p_{ij}) is in a pre-specified interval or 'bin' $\left[\underline{b}, \bar{b}\right]$, then the precise probability $\left(p_{ij}^{*}\right)$ implied by the parameters of the (noisy) density is also in this interval. In formulae:

$$p_{ij} \in \left[\underline{b}, \bar{b}\right] \Leftrightarrow p_{ij}^{*} \in \left[\underline{b}, \bar{b}\right] \Leftrightarrow \underline{b} \leq \Phi\left(\frac{\mu_i + v_{ij} - \tau_j}{\sigma_i}\right) < \bar{b} \tag{8}$$

When we implement the model, the bins will be defined (in percentage terms) as $[0, 5)$, $[5, 15)$, $[15, 25)$,..., $[95, 100]$. This way the bins allow for rounding to the nearest ten, and treat all other numbers not round (including 25 and 75 per cent).

For this model, rounding to 0 or 100 is no different from rounding to, say, 10 or 90. Rounding to 50 has a special role, not because of rounding itself (that is, again, assumed to be governed by the same mechanism), but because increasing fundamental uncertainty pushes precise probabilities $\left(p_{ij}^{*}\right)$ towards 0.5, that is inside the $[45, 55)$ interval. For a probability questions characterized by τ_j, and for a given mean μ_i and noise draw v_{ij}, there is always a large enough fundamental uncertainty σ_i that leads to an answer in the $[45, 55)$ interval.

This model is admittedly atheoretical: it is more of a statistical model, not a economic model of survey response behavior. The length and location of the intervals are given from outside, and the model is silent about why some people round while others don't. In its atheoretical way, however, the model is compatible with the difficulty of calculating the integral, and it allows people to pick the round number they feel closest to where the precise probability should be. It also allows for people to report an erroneous not-round probability as long as it belongs to the same interval as the precise probability. Since no information about rounding is used, this model is consistent with the fact that giving round answers is only weakly related to stockholding and demographics. And, last but not least, this is probably the simplest model that can deliver those results and is therefore a good benchmark.

3.5 Survey nonresponse

18 per cent of our sample of financial respondents did not answer the stock market probability questions but answered "don't know" instead. Correlations with observables (including stockholding) support the hypothesis that most of these "don't know" answers reflect genuine ignorance about the stock market. A straightforward way to model complete ignorance in our framework is to assume that people give missing answers if their relevant uncertainty is prohibitively high, i.e.

$$\sigma_i \to \infty$$

Such expectations automatically result in $s^{*} \to 0$.

In the remainder of the analysis we ignore people with missing stock market probability answers. One reason is technical: infinitely large variances are impossible to deal with. On the other hand, omission if these answers from the analysis does not decrease the validity of our results. We can think of the problem as sample selection on a right-hand side variable (σ_i), and if we want to extrapolate all results to the entire population, we simply have to add 18 per cent of $\sigma_i \to \infty$, $s_i = 0$ to the expectations and stockholding heterogeneity

4 Estimation

This is an empirical paper, with several objectives. In logical order, the first objective is methodological. We would like to show that answers to the probability questions considered here can be used to extract a lot of useful information about relevant heterogeneity in stock-market expectations. We test the validity of the extracted information against observed stockholding. One of the ways we carry out this test is similar in spirit to the Mehra and Prescott exercise: we estimate a single coefficient of risk tolerance (to be interpreted as a population average) and check whether it is in the "sensible" range of risk tolerance. We also check how much stockholding predicted by expectation and risk tolerance lines up with observed stockholding.

Once we established validity of our estimates of relevant expectations, we can turn to our more substantive questions. One objective is to estimate moments of the distribution of stock-market expectations in the population represented by our sample. In our notation, this means estimating mean and variance of μ_i and σ_i and their covariance. These moments help understanding to what extent mean and heterogeneity in expectations can explain mean level and heterogeneity in stockholding (established already when we validated the survey measures). Besides unconditional moments, we use our estimates to look at how stock-market expectations vary with observable individual characteristics (conditional means of μ_i and σ_i). For most of these individual characteristics we do not claim causal effects towards expectations.

In order for our results to shed light on the stockholding puzzle, we would like to interpret our results as expectations causing stockholding. On the other hand, serious endogeneity problems are likely to emerge both from omitted variables and simultaneity. In that sense, our results are reduced-form: they may show that observed stockholding is consistent with what observed expectations would imply but they cannot prove that stockholding is low (heterogeneous) because of low mean and high uncertainty of (heterogeneity in) stock-market expectations. Note however, that the validation of survey measures of stock-market expectations by observed stockholding does not require causality. Whether expectations cause stockholding, stockholding causes expectations, or both are caused by some third variable, the validation exercise can show that survey answers to probability questions do measure relevant heterogeneity in expectations (after, of course, survey noise is properly accounted for).

The objectives listed above can be achieved by estimating a structural model of stockholding and answers to probability questions.

4.1 Setup

We estimate a joint model of observed stockholding and observed answers to the probability questions asked in the core survey. We relate these observable variables to relevant heterogeneity (μ_i, σ_i, and possibly α_i) and survey noise (v_{ij}). The structure of the estimated models is therefore the following:

$$\begin{aligned} s_i &= h\left(\mu_i, \sigma_i, \alpha_i; u_{si}\right) \\ \left(p_{0i}, p_{10i}\right) &= k\left(\mu_i, \sigma_i, ; v_{0i}, v_{10i}\right) \end{aligned}$$

Observed stockholding (s_i) is assumed to depend on expectation parameters μ_i and σ_i and preference parameter risk tolerance α_i, as well as unobservables u_{si}. Survey answers to probability questions (p_{0i} and p_{10i}) are assumed to depend on expectation parameters $\mu_i$and σ_i, and question-specific survey noise v_{0i} and v_{10i}. The functional forms of h and k are outlined in this subsection.

The stockholding equation has the form of a two-way corner solution model,

$$s_i = \left\{ \begin{array}{lll} 0 & if & s_i^* < 0 \\ s_i^* & if & 0 \leq s_i^* \leq 1 \\ 1 & if & s_i^* > 1 \end{array} \right\} \tag{9}$$

Uncensored optimal share of stocks (s_i^*) is related to expectations and risk tolerance following the very simple relationship postulated by the portfolio choice model outlined above. We consider two versions. In the first and simpler version, risk tolerance is a single parameter to estimate: $\alpha_i = \alpha$. Its interpretation here is the average risk tolerance in the population represented by the sample. In the second version, we make use of estimated risk tolerance for our respondents by Kimball, Sahm and Shapiro (2007), a_i, and we estimate a regression parameter on it: $\alpha_i = \beta_\alpha a_i$. The interpretation of β_α is how much a_i, the Kimball-Sahm-Shapiro measure, captures risk tolerance in our model. $\beta_\alpha = 1$ would indicate that it does so perfectly; a deviation from 1 would indicate an imperfect measure.

$$s_i^* = \alpha \frac{(\mu_i - r)}{\sigma_i^2} + u_{si} \tag{10}$$

$$s_i^* = \beta_\alpha a_i \frac{(\mu_i - r)}{\sigma_i^2} + u_{si} \tag{11}$$

We estimated each of these equations both with and without an intercept.

Probability answers are related to relevant heterogeneity and noise as formulated above, in (7) and (8):

$$\begin{aligned} p_{i0} &\in \left[\underline{b}, \bar{b}\right] \Leftrightarrow \underline{b} \leq \Phi\left(\frac{\mu_i + v_{i0}}{\sigma_i}\right) < \bar{b} \\ p_{i10} &\in \left[\underline{b}, \bar{b}\right] \Leftrightarrow \underline{b} \leq \Phi\left(\frac{\mu_i + v_{i10} - 0.1}{\sigma_i}\right) < \bar{b} \end{aligned}$$

The bins $[\underline{b}, \bar{b}]$ are defined (in percentage terms) as $[0,5)$, $[5,15)$, $[15,25)$,..., $[95,100]$.

Because of survey noise and interval responses, (μ_i, σ_i) are not identified for each respondent separately. Instead, it is their conditional expectations that are identified. These conditional expectations we model in the following way:

$$\mu_i = \beta'_\mu x_{\mu i} + u_{\mu i} \tag{12}$$

$$\log(\sigma_i) = \beta'_\sigma x_{\sigma i} + u_{\sigma i} \tag{13}$$

4.2 Right-hand side variables

Right-hand side variables for expected value of returns ($x_{\mu i}$) and of uncertainty of returns ($x_{\sigma i}$) include standard demographics (gender, age, race, education) and expectations-specific instruments. The role of standard demographics is substantive: we would like to know how stock expectations vary across main demographic groups. The expectations-specific instruments are different in the two equations. The equation for expected value of stock market returns, or, in other words, stock market optimism, includes three instruments. The first one is a measure of optimism on other selected probability questions, labeled simply as "optimism". This is based on previous work of ours (Kézdi and Willis, 2003) and is used by Hill, Perry and Willis (2006) as well. There the measure optimism is defined as the principle component of all probability answers (except, of course, the question analyzed as a left-hand side variable). Here we use a narrower and simpler version of that index. It is defined as the average of the probability answers to weather, income, and survival expectations questions given by the individual in all of the surveys from year 1992 to 2002 (normalized to be in the $[0,1]$ range). The idea behind using this instrument is the effect optimism as a personal trait on stock market optimism.

The second instrument in the stock market optimism equation is the score created from psychological depression measures of HRS We call in "cheerfulness". It is a standardized score and is the inverse of depression, therefore more positive answers mean less depression. The third variable that appears in the equation of μ only is the level of the stock market before the interview (the closing S&P500 on the day of the last month before the interview). As HRS 2002 took place throughout much of the year, and the stock market had a roller-coaster drive during that year, we hoped to be able to see whether expectations track past events on the stock market.

The instrument used for imprecision in stock market expectations is The fraction of 50 per cent answers to all probability questions given by the individual in all of the surveys from year 1992 to 2002 (normalized to be in the $[0,1]$ range). The idea behind using the instrument is the effect uncertainty in general on uncertainty about stock market returns. This instrument is very similar to the one used in Hill, Perry and Willis (2006): they included the fraction 0 and 100 per cent answers as well, for reasons to be cleared later.

Besides learning about the relationship of these variables and heterogeneity in stock market expectations, the main role of the instruments is help identification.

4.3 Identification

The first and perhaps most important identification problem is separation of survey noise from relevant heterogeneity. The mechanical representation of the problem is that in the mapping from expectations to probability answers, the role of μ and v is interchangeable. Intuitively, the presence of survey noise makes separation of relevant heterogeneity from irrelevant noise because inter-personal difference in probability answers can be a result of both. In principle, we have three sources of identification. One is the fact that survey noise does not enter the stockholding decision, by definition. Joint estimation of stockholding with probability answers therefore helps identification.

Another source is the presence of instruments in the equation of μ and of $\log(\sigma)$ excluded from the equation of $\log(\sigma)$ and μ, respectively. The first set of instruments affect μ and thus μ/σ but not v/σ by assumption, while the second set affects σ and thus both μ/σ and v/σ. These help separating variance in μ/σ and v/σ and in turn, together with distributional assumptions, they help separating variance of μ, σ and v.

The third possible source of separating noise from relevant heterogeneity is the fact that for a subset of respondents, we observe answers to the same probability questions in the experimental module ($p_{0'}$ and $p_{10'}$) besides the core questionnaire answers (p_0 and p_{10}). By assumption (and supported by evidence), module answers are based on the same relevant expectations parameters μ and σ but different (and, most importantly, independent) noise. We could therefore incorporate the other two probability answers into the estimation model. But estimating the model with two additional probability answers for a small subset of respondents would greatly complicate matters. We can, however, estimate the most important moments of the noise distribution separately, using only the four probability answers. These estimates can be then used as calibrated values in the main estimation. Details of the identification and estimation of moments of the noise distribution are summarized in Appendix A.

A second important identification problem is the issue of causality: whether it is expectations causing stockholding or stockholding causing expectations.

There are two kinds of exclusion restrictions in our model. First, stockholding is assumed to be determined solely by expectations and risk tolerance (and unobservables). The effect of any observable characteristic is assumed to operate through these variables. Second, the expectations-specific instruments are each included in only one of the two expectations equations. It turns out that the second type of exclusion restrictions are not needed for identification. It also turns out, however, that the inclusion of each instrument in the other equation yields a statistically insignificant coefficient.

4.4 Maximum Simulated Likelihood

Unobserved stochastic components are assumed to be distributed normally with covariance matrix Ω :

$$\begin{pmatrix} u_{si} \\ u_{\mu i} \\ u_{\sigma i} \\ v_{i0} \\ v_{i10} \end{pmatrix} \sim N(0,\Omega) \ , \ \Omega = \begin{bmatrix} \sigma_{us}^2 & \rho_{s\sigma}\sigma_{us}\sigma_{u\sigma} & \rho_{s\mu}\sigma_{us}\sigma_{u\mu} & 0 & 0 \\ \rho_{s\sigma}\sigma_{us}\sigma_{u\sigma} & \sigma_{u\sigma}^2 & \rho_{\mu\sigma}\sigma_{u\sigma}\sigma_{u\mu} & 0 & 0 \\ \rho_{s\mu}\sigma_{us}\sigma_{u\mu} & \rho_{\mu\sigma}\sigma_{u\sigma}\sigma_{u\mu} & \sigma_{u\mu}^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_v^2 & \rho_v\sigma_v^2 \\ 0 & 0 & 0 & \rho_v\sigma_v^2 & \sigma_v^2 \end{bmatrix}$$

All parameters are estimated, except for σ_v^2 and ρ, which are calibrated in our benchmark estimates.

Estimation is complicated, because of the presence of survey noise adds two more unobservable component to the three-equation system, and because many of the relationships are nonlinear. Some of the error components need to be integrated out, and that integration has to be done numerically. The estimation therefore follows a Maximum Simulated Likelihood procedure. Technical details of the estimation procedure are to be found in Appendix B (not ready yet).

5 Main results

To be completed...

6 Extension: The Modal Response Model

6.1 How people answer survey questions of probabilities

In addition to our baseline results, we explore an alternative model of survey response to probability questions. The model is more complicated but also more theoretically grounded in modeling how people answer probability questions on surveys, and whether and how their fundamental uncertainty is related to the way they form those answers. This is an application of the "Modal Response Hypothesis" developed by Lillard and Willis (2001) and Hill, Perry and Willis (2006).

We have to put more structure on how we model individuals's perception of stock market returns. We assume that the individual's perceived stochastic deviation from the (perceived) mean has two sources: "genuine" i.i.d. risk (ε_{t+1}) and individual uncertainty about stock returns (δ_{it}) so that:

$$\begin{aligned} R_{i(t+1)} &= \mu_{it} + \eta_{it} = \mu_{it} + \delta_{it} + \varepsilon_{t+1} \\ (\delta_{it}, \varepsilon_{t+1}) &\sim N\left(\mathbf{0}, \mathbf{\Sigma}_\eta\right) \end{aligned} \tag{14}$$

where Σ_η is a diagonal matrix $\langle\sigma^2_{\delta it},\sigma^2_\varepsilon\rangle$.[8] As a result, $\sigma^2_{it} = V\left[\delta_{it}+\varepsilon_{t+1}\right] = \sigma^2_{\delta it}+\sigma^2_\varepsilon$: subjective returns variance is historical variance plus individual uncertainty. Note that with this decomposition we ruled out over-confidence: everybody thinks that the variance is at least as large as the historical variance.

Similarly to the model before, people are assumed to make a guess of what the precise probability could be. But here that guessing is modeled in an explicit way. The precise probability is a random variable, with a probability distribution, and people select a value of that random variable. The modal response hypothesis assumes that they pick the mode of the distribution of the random variable. One can visualize this process by assuming that people see a density of possible probability answers, and then they pick the mode of that density.

This model uses the decomposition of η into uncertainty (δ) and risk (ε), as defined in equation (14). Let q denote the probability that (noisy) returns are above the threshold asked in question j, conditional on person-specific uncertainty component δ and survey noise component ν :

$$q_{ij} = \Pr\left(\tilde{R}_{ij} > \tau_j|\mu_i,\delta_i,v_{ij}\right) = \int_{-\infty}^{\tau_j}\left(\mu_i+v_{ij}+\delta_i+\varepsilon\right)dF\left(\varepsilon\right) \tag{15}$$

q can be thought of as the answer to the survey question without individual-specific uncertainty (only risk). People who have fundamental uncertainty (δ) see not a single q but a distribution of q.The c.d.f. is denoted by G, the p.d.f. by g. They have the following form[9]:

$$\begin{aligned} G_i\left(q_{ij}\right) &\equiv \Pr\left[\Pr\left(\tilde{R}_{ij} > \tau_j|\mu_i,\delta_i,v_{ij}\right) < q_{ij}|\mu_i,v_{ij}\right] && (16)\\ &= \int_{-\infty}^{q_{ij}}\left[\int_{-\infty}^{\tau_j}\left(\mu_i+v_{ij}+\delta_i+\varepsilon\right)dF\left(\varepsilon\right)\right]dF\left(\delta_i\right) \\ g_i\left(q_{ij}\right) &\equiv \frac{\partial G_i\left(q_{ij}\right)}{\partial q_{ij}} = \frac{\sigma_\varepsilon}{\sigma_{\delta i}}\frac{\phi\left[\frac{\tau_j-\mu_i-v_{ij}+\sigma_\varepsilon\Phi^{-1}(p_j)}{\sigma_{\delta i}}\right]}{\phi\left(\Phi^{-1}\left(p_j\right)\right)} && (17)\end{aligned}$$

For each threshold τ and for given σ_ε, the shape of the density function g is determined by $(\mu+v)$ and σ_δ. Figures 6a and 6b show those shapes for $\tau=0$ and $\tau=0.1$, respectively.

When confronted with the question about what the probability is, people give an answer based on density g. One possible assumption is that they compute the expected value. But that would be very difficult. Moreover, the observed phenomenon of many focal answers is inconsistent with that. Instead, the modal response hypothesis of Hill, Perry and Willis (2006) assumes that people are likely to take shortcuts, and when confronted with the survey question, they pick the mode from the density g. In a sense, the mode is the most likely

[8] One way to rationalize individual uncertainty is in a Bayesian framework, used for example by Brennan (1998). In that framework, individuals know the returns variance conditional on mean returns, but they are uncertain about the mean. If uncertainty follows a normal distribution, overall variance would be a sum of the "true" risk variance and the variance induced by individual uncertainty about the mean.

[9] Details of derivation are available upon request

"true" q, and it is easy to locate if one has a mental image of the density g as in Figures 6a and 6b.

If the density is concave, the mode satisfies the first-order condition

$$p_{ij}^{foc} \equiv \Phi\left[(\mu_i + v_{ij} - \tau_j)\frac{\sigma_\varepsilon}{\sigma_\varepsilon^2 - \sigma_{\delta i}^2}\right] \tag{18}$$

If subjective uncertainty is smaller than historical market risk, $(\sigma_{\delta i}^2 < \sigma_\varepsilon^2)$, p^{foc} is a maximum, and it is therefore the mode. If subjective uncertainty is larger than historical market risk $(\sigma_{\delta i}^2 > \sigma_\varepsilon^2)$, p^{foc} is a minimum, and there are two modes, one at $p = 0$, another one at $p = 100\%$.

Note that answers are necessarily rounded to integers, and therefore a $p^* > 99.5\%$ would result in an answer of 100%, and similarly, $p^* < 0.5\%$ would result in an answer of 0%.

Following Willis et. al., we assume that if uncertainty is high enough, and/or μ is far enough from 0 or 100%, the actual answer is not 0 or 100% but 50%. when distinguish three cases here, depending on whether p^* is close or far from 0 and 100%.

The modal response hypothesis implies the following probability answers:

$$\begin{aligned} p_{ij}^{mode} &= p_{ij}^{foc} \quad if \quad \sigma_{\delta i}^2 < \sigma_\varepsilon^2 \quad \& \quad 0.005 \le p_{ij}^{foc} < 0.995 \\ &= 0 \quad if \quad \left[\sigma_{\delta i}^2 < \sigma_\varepsilon^2 \quad \& \quad p_{ij}^{foc} < 0.005\right] \quad OR \quad \left[\sigma_{\delta i}^2 \ge \sigma_\varepsilon^2 \quad \& \quad p_{ij}^{foc} \ge 0.995\right] \\ &= 1 \quad if \quad \left[\sigma_{\delta i}^2 < \sigma_\varepsilon^2 \quad \& \quad p_{ij}^{foc} \ge 0.995\right] \quad OR \quad \left[\sigma_{\delta i}^2 \ge \sigma_\varepsilon^2 \quad \& \quad p_{ij}^{foc} < 0.005\right] \\ &= 0.5 \quad if \quad \sigma_{\delta i}^2 \ge \sigma_\varepsilon^2 \quad \& \quad 0.005 \le p_{ij}^{foc} < 0.995 \end{aligned} \tag{19}$$

Observe a contrast to precise probability p^* as defined in (7). There, an increase in σ_i leads pushes the answer towards 0.5. But here, it does something different. Increased fundamental uncertainty is modeled as an increase in σ_{δ_i}, which would lead to an increase in σ_i. As long as $\sigma_\delta < \sigma_\varepsilon$, An increase in σ_δ pushes p^{foc} towards 0 if the uncertainty-free probability would be less than 0.5 $(\mu_i + v_{ij} - \tau_j < 0)$, and it pushes p^{foc} towards 1 if the uncertainty-free probability would be larger than 0.5. When uncertainty gets larger $(\sigma_{\delta i} > \sigma_\varepsilon)$ but not too large so that p^{foc} is close to 1 or 0, the previous phenomenon prevails, leading to a focal answer of 0 or 1 (if the uncertainty-free probability would be respectively less or greater than 0.5). However, when uncertainty is very large, the model assumes that people say fifty-fifty. In this modal response model, therefore, increased uncertainty first pushes answers towards 0 or 1, and a further increase pushes them to 0.5.

6.2 Estimation

Work in progress...

6.3 Results

Work in progress...

7 Conclusions

To be completed...

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Appendices

A Estimating variance and correlation of survey noise

$$
\begin{aligned}
R_{i(t+1)} &= \mu_{it} + \eta_{it} \\
\tilde{R}_{i(t+1)j} &= \mu_{it} + \eta_{it} + v_{itj} \\
v_{itj} &\perp \perp \mu_{it}, \sigma_{it}
\end{aligned}
$$

The noise components are assumed to be normally distributed

$$
\begin{aligned}
\begin{bmatrix} v_{i0} \\ v_{i10} \end{bmatrix} &\sim iidN(0, \Sigma) \quad \Sigma = \begin{bmatrix} \sigma_v^2 & (1-\rho)\sigma_v^2 \\ (1-\rho)\sigma_v^2 & \sigma_v^2 \end{bmatrix} \\
E[v_{ij}v_{ij'}] &= 0
\end{aligned}
$$

The goal of the exercise is to estimate moments of the noise distribution so we can calibrate those in the estimation. We are interested in :

$$\sigma_v^2 \text{ and } \rho$$

Use probability answers only, and assume that they are based on the precise probability

$$p_{ij}^* = \Pr\left(\tilde{R}_{ij} > \tau_j | \mu_i, v_{ij}\right) = \Pr\left(\frac{\eta_i}{\sigma_i} > \frac{\tau_j - \mu_i - v_{ij}}{\sigma_i}\right) = \Phi\left(\frac{\mu_i + v_{ij} - \tau_j}{\sigma_i}\right)$$

We assume that actual responses are equal to the precise probability:

$$p_{ij} = p_{ij}^*$$

We justify this by the fact that we are not going to use the individual answers themselves but their sample averages (to be more precise the sample average of various functions of the answers).

We are going to make use linearity of noise in the inverse of the probability answers:

$$\frac{\mu_i + v_{ij} - \tau_j}{\sigma_i} = \Phi(p_{ij})$$

A.1 Moment condition 1

Compare (moments of) core and module answers to the p_0 probability question and take expectation of the squares:

$$
\begin{aligned}
\Phi^{-1}(p_{i0}) - \Phi^{-1}(p_{i0'}) &= \frac{v_{i0} - v_{i0'}}{\sigma_i} \\
E\left[\left\{\Phi^{-1}(p_{i0}) - \Phi^{-1}(p_{i0'})\right\}^2\right] &= E\left[\left(\frac{v_{i0} - v_{i0'}}{\sigma_i}\right)^2\right] = E\left[(v_{i0} - v_{i0'})^2\right] E\left[\frac{1}{\sigma_i^2}\right]
\end{aligned}
$$

so that

$$E\left[\left\{\Phi^{-1}\left(p_{i0}\right)-\Phi^{-1}\left(p_{i0'}\right)\right\}^{2}\right]=2\sigma_{v}^{2}E\left[\frac{1}{\sigma_{i}^{2}}\right] \tag{20}$$

The same can be derived using p_{10} and $p_{10'}$, because there again $\Phi^{-1}\left(p_{i10}\right)-\Phi^{-1}\left(p_{i10'}\right)=\frac{v_{i10}-v_{i10'}}{\sigma_i}$, so that

$$E\left[\left\{\Phi^{-1}\left(p_{i10}\right)-\Phi^{-1}\left(p_{i10'}\right)\right\}^{2}\right]=2\sigma_{v}^{2}E\left[\frac{1}{\sigma_{i}^{2}}\right] \tag{21}$$

A.2 Moment condition 2

Compare adjacent core answers and take expectation of the squares:

$$\begin{aligned}
\Phi^{-1}\left(p_{i0}\right)-\Phi^{-1}\left(p_{i10}\right) &= \frac{v_{i0}-v_{i10}+0.1}{\sigma_i} \\
E\left[\left\{\Phi^{-1}\left(p_{i0}\right)-\Phi^{-1}\left(p_{i10}\right)\right\}^{2}\right] &= E\left[\frac{\left(v_{i0}-v_{i10}+0.1\right)^{2}}{\sigma_i^2}\right] \\
&= E\left[v_{i0}^2+v_{i10}^2+0.01-2v_{i0}v_{i10}+0.2v_{i0}-0.2v_{i10}\right]E\left[\frac{1}{\sigma_i^2}\right]
\end{aligned}$$

so that

$$E\left[\left\{\Phi^{-1}\left(p_{i0}\right)-\Phi^{-1}\left(p_{i10}\right)\right\}^{2}\right]=\left[2\left(1-\rho\right)\sigma_{v}^{2}+0.01\right]E\left[\frac{1}{\sigma_{i}^{2}}\right] \tag{22}$$

Again, we can do this for the other pair of answers, which are, in this case, the module answers, with the result of

$$E\left[\left\{\Phi^{-1}\left(p_{i0'}\right)-\Phi^{-1}\left(p_{i10'}\right)\right\}^{2}\right]=\left[2\left(1-\rho\right)\sigma_{v}^{2}+0.01\right]E\left[\frac{1}{\sigma_{i}^{2}}\right] \tag{23}$$

A.3 Moment condition 3

Compare adjacent core answers and take expectation of the squares:

$$\begin{aligned}
\left\{\Phi^{-1}\left(p_{i0}\right)-\Phi^{-1}\left(p_{i10'}\right)\right\} &= \frac{v_{i0}-v_{i10'}+0.1}{\sigma_i} \\
E\left[\left\{\Phi^{-1}\left(p_{i0}\right)-\Phi^{-1}\left(p_{i10'}\right)\right\}^{2}\right] &= E\left[\frac{\left(v_{i0}-v_{i10'}+0.1\right)^{2}}{\sigma_i^2}\right] \\
&= E\left[v_{i0}^2+v_{i10'}^2+0.01-2v_{i0}v_{i10'}+0.1v_{i0}-0.1v_{i10'}\right]E\left[\frac{1}{\sigma_i^2}\right]
\end{aligned}$$

so that

$$E\left[\left\{\Phi^{-1}\left(p_{i0}\right)-\Phi^{-1}\left(p_{i10'}\right)\right\}^{2}\right]=\left[2\sigma_{v}^{2}+0.01\right]E\left[\frac{1}{\sigma_{i}^{2}}\right] \tag{24}$$

A.4 Moment condition 4

Look at the probability to giving negative mass answers

$$\begin{aligned}
\Pr\left[p_{i0}<p_{i10}\right] &= \Pr\left[\Phi\left(\frac{\mu_i+v_{i0}}{\sigma_i}\right)<\Phi\left(\frac{\mu_i+v_{i10}-0.1}{\sigma_i}\right)\right]=\Pr\left[\frac{\mu_i+v_{i0}}{\sigma_i}<\frac{\mu_i+v_{i10}-0.1}{\sigma_i}\right]\\
&= \Pr\left[v_{i0}<v_{i10}-0.1\right]=\Pr\left[v_{i0}-v_{i10}<-0.1\right]=\Pr\left[v_{i0}-v_{i10}<-0.1\right]\\
&= \Pr\left[\frac{v_{i0}-v_{i10}}{\sqrt{2\left(1-\rho\right)\sigma_v^2}}<\frac{-0.1}{\sqrt{2\left(1-\rho\right)\sigma_v^2}}\right]=\Phi\left[\frac{-0.1}{\sqrt{2\left(1-\rho\right)\sigma_v^2}}\right]
\end{aligned}$$

so that

$$\Phi^{-1}\left[\Pr\left(p_{i0}<p_{i10}\right)\right]=\frac{-0.1}{\sqrt{2\left(1-\rho\right)\sigma_v^2}}$$

and therefore

$$\left\{\Phi^{-1}\left[\Pr\left(p_{i0}<p_{i10}\right)\right]\right\}^{2}=\frac{0.01}{2\left(1-\rho\right)\sigma_v^2} \tag{25}$$

And, similarly for module responses:

$$\left\{\Phi^{-1}\left[\Pr\left(p_{i0'}<p_{i10'}\right)\right]\right\}^{2}=\frac{0.01}{2\left(1-\rho\right)\sigma_v^2} \tag{26}$$

A.5 Rearranging the moment conditions

We have four types of conditions:

$$\begin{aligned}
2\sigma_v^2 E\left[\frac{1}{\sigma_i^2}\right] &= A\\
\left[2\left(1-\rho\right)\sigma_v^2+0.01\right]E\left[\frac{1}{\sigma_i^2}\right] &= B\\
\left[2\sigma_v^2+0.01\right]E\left[\frac{1}{\sigma_i^2}\right] &= C\\
\frac{0.01}{2\left(1-\rho\right)\sigma_v^2} &= D
\end{aligned}$$

We can rearrange these in order to get rid of the here ancillary moment, $E\left[1/\sigma_i^2\right]$:

$$\begin{aligned} \frac{2(1-\rho)\sigma_v^2 + 0.01}{2\sigma_v^2} &= \frac{B}{A} \\ \frac{2\sigma_v^2 + 0.01}{2\sigma_v^2} &= \frac{C}{A} \\ \frac{0.01}{2(1-\rho)\sigma_v^2} &= D \end{aligned}$$

From the third one, we get

$$\sigma_v^2 = \frac{0.005}{C/A - 1}$$

This way we can express ρ in two ways. First, using the first and second moment conditions, $(1-\rho)\sigma_v^2 + 0.005 = \frac{B}{A}\sigma_v^2$, thus $(1-\rho) + 0.005/\sigma_v^2 = \frac{B}{A}$ and so

$$\rho = 1 - \frac{B}{A} + 0.005/\sigma_v^2$$

Or alternatively from the fourth moment condition can be rewritten as $(1-\rho)\sigma_v^2 = 0.005/D$ and thus

$$\rho = 1 - \frac{0.005}{D\sigma_v^2}$$

A.6 Estimation of ρ and σ_v^2 by Minimum Distance

A crude approximation to optimal GMM as of now.

First we took averages of two versions of A, B, C, and D. These averages were simple means for A and C and weighted (by square root of sample size) averages for B and D. Then we estimated σ_v^2 by the formula above, and ρ in the two alternative ways.

There are two issues. The first one arises with moment conditions 1 through 3. The problem there is that $\Phi^{-1}(p)$ is not defined $p = 0$ or $p = 1$. Ad-hoc solution: replace then with $p = 0 + \varepsilon$ and $p = 1 - \varepsilon$, respectively. Various values for ε are considered for robustness checks (like 0.01 or 0.005 to 0.000001). The second problem arises with moment condition 4. While $\Pr\left[p_{i0} < p_{i10}\right] = \Pr\left[p_{i0} \leq p_{i10}\right]$ under normally distributed R if the p_i are the precise probabilities indeed, in practice we have quite a few cases with $p_{i0} = p_{i10}$. Whether we count them as $p_{i0} < p_{i10}$ or $p_{i0} > p_{i10}$ (or a fraction here, the other fraction there) has a large effect on the ρ estimate using moment condition 4.

The results are the following. For $\varepsilon = 0.01$, $\sigma_v^2 = 0.14$. As $\varepsilon \to 0$, we have $\sigma_v^2 \to 0.26$. As for ρ identified from the first three moment conditions only, for $\varepsilon = 0.01$, $\rho = 0.60$. As $\varepsilon \to 0$, $\rho \to 0.67$. When the fourth moment condition is used for estimating ρ, the results are very sensitive to how we count all the $p_{i0} = p_{i10}$ responses as $p_{i0} > p_{i10}$, we have $\rho = -0.8$ to -0.5

(depending on ε), results that are clearly counterintuitive. When we count all the $p_{i0} = p_{i10}$ responses as $p_{i0} < p_{i10}$, we have $\rho = 0.55$ to 0.80 (as ε is decreased from 0.01 towards 0). These latter results are very much in line with the other ρ estimate. When we count one half of the equal answers as greater, the other half as smaller (a middle-of-the-road approach), we get $\rho = 0.3$ to 0.45 (depending on ε).

Overall, taking uncertainty in the second ρ estimates into account, we can conclude in the following results:

$$\begin{aligned} \sigma_v^2 &\approx 0.14 \ \ to \ \ 0.26 \\ \rho &\approx 0.50 \ \ to \ \ 0.70 \end{aligned}$$

B Maximum Simulated Likelihood estimation

To be completed...

Tables

Table 1. Stockholding of households: distribution in per cent. HRS 2002 and 2004. Financial decision-makers between age 55 and 65.

	2002	2004
Direct stockholder	31	30
Indirect stockholder	13	12
Non stockholder	56	58
All	100	100

Table 2. Descriptive statistics of the probability answers in 2002 and 2004, by subsample.

	Main sample[a]			Spouses[b]			Older fin. resp.[c]			Older spouses[d]		
	p0	p10	$p0_{2004}$	p0	p10	$p0_{2004}$	p0	p10	$p0_{2004}$	p0	p10	$p0_{2004}$
Mean	49	39	52	46	38	49	39	31	43	39	32	46
Sd	30	28	26	28	27	26	31	28	28	29	27	27
Fraction missing[e]	0.18	0.18	0.13	0.21	0.21	0.14	0.33	0.33	0.31	0.28	0.28	0.24

Notes
[a] 55-65 years old, financial respondent of the household, HRS and War Babies cohort
[b] non-financial respondent member of the households in the main sample
[c] financial respondent, 66 years old or more, Children of the Depression and AHEAD cohorts
[d] non-financial respondent, 66 years old or more, Children of the Depression and AHEAD cohorts
[e] in 2002, missing if either p0 or p10 is missing (92 per cent of missing p0 or p10 answers are missing jointly)

Table 3. Stock market expectations by stockholding status (non-missing stockmarket expectations answers).

	HRS 2002			HRS 2004
	p0	p10	p0 – p10	p0
Direct stockholder	57	45	12	59
Indirect stockholder	54	44	10	56
Non stockholder	41	34	7	46
All	49	39	9	52

Table 4. Percentage of focal, round and other answers

	HRS 2002		HRS 2004
	p_0	p_{10}	p_0
Focal at 0	7	11	5
Focal at 50	26	24	31
Focal at 100	8	4	5
Other round answer*	53	54	53
Other answer	6	7	6
All	100	100	100
N	3049	3049	3246

*Rounded to nearest ten or 25 or 75

Table 5. Zero or negative probability mass between answers, by answer to the market up question

p_0	p0 > p10 (positive mass)	p0 = p10 (zero mass)	p0 < p10 (negative mass)	All
Focal at 0	0	86	14	100
Focal at 50	37	51	12	100
Focal at 100	63	37	0	100
Other round answer*	49	35	15	100
Other answer	44	32	24	100
All	43	43	14	100

Table 6. Core versus module answers to same questions (non-missing answers, n=185)

	p_0				p_{10}			
	Core	Module	Difference	Absolute Diff.	Core	Module	Difference	Absolute Diff.
Mean	53	51	1	22	42	36	7	24
Standard Dev.	31	30	31	21	30	28	32	22

Table 7. Stockholding and noise features.

	Percentage direct stockholder	Percentage indirect stockholder	Percentage not stockholder	All
p_0 focal at 0	19	8	73	100
p_0 focal at 50	34	15	51	100
p_0 focal at 100	52	19	32	100
p_0 other round answer*	37	15	48	100
p_0 other answer	34	11	55	100
Nonmissing p_0 and p_{10} answer	36	14	50	100
Missing p_0 or p_{10} answer	10	5	86	100
$p_0 > p_{10}$	41	15	44	100
$p_0 = p_{10}$ (zero mass)	33	13	54	100
$p_0 < p_{10}$ (negative mass)	29	14	57	100

Table 8. Predictors of the propensity to giving noisy answers
(linear probability models)

	Missing p_0		p_0 focal at 0 or 100		p_0 focal at 50		p_0 not round		$p_0 = p_{10}$		$p_0 < p_{10}$		$\lvert p_0 = p_{0\_module} \rvert$	
Stockholder	-0.215		-0.015		-0.021		0.003		-0.063		-0.039		-0.008	
	[18.95]**		[1.14]		[1.33]		[0.36]		[3.54]**		[3.17]**		[0.25]	
Education		-0.012		-0.001		-0.007		0.001		-0.007		-0.006		-0.005
		[4.32]**		[0.25]		[2.00]*		[0.35]		[1.94]		[2.08]*		[0.60]
Word recall		-0.006		-0.002		-0.011		0.003		-0.001		-0.004		-0.004
		[2.48]*		[0.72]		[3.56]**		[1.91]		[0.16]		[1.46]		[0.77]
counting back 7		-0.039		-0.017		0.021		-0.003		-0.005		0.002		0.021
		[6.66]**		[2.70]**		[2.80]**		[0.86]		[0.58]		[0.31]		[1.27]
female		0.059		-0.017		0.084		-0.02		0.076		0.029		0.070
		[4.11]**		[1.08]		[4.19]**		[2.04]*		[3.38]**		[1.84]		[1.82]
black		0.039		-0.004		-0.035		0.007		0.050		0.012		-0.114
		[1.96]*		[0.22]		[1.44]		[0.58]		[1.83]		[0.60]		[2.77]**
hispanic		0.170		-0.015		-0.047		-0.041		-0.085		0.061		0.001
		[5.70]**		[0.51]		[1.33]		[4.12]**		[2.10]*		[1.81]		[0.02]
log(wealth) (or 0)		-0.011		-0.003		0.004		-0.001		-0.006		-0.001		-0.006
		[5.66]**		[1.45]		[1.51]		[0.83]		[2.07]*		[0.48]		[1.30]
Born in U.S.		-0.070		-0.066		-0.003		-0.001		-0.045		0.026		-0.003
		[2.63]**		[2.22]*		[0.10]		[0.09]		[1.17]		[0.94]		[0.06]
Served in military		0.006		0.056		-0.003		-0.018		0.030		0.015		0.034
		[0.41]		[2.94]**		[0.12]		[1.67]		[1.19]		[0.92]		[0.79]
Constant	0.275	0.651	0.157	0.338	0.273	0.339	0.051	0.05	0.456	0.602	0.156	0.210	0.230	0.295
	[28.31]**	[14.29]**	[17.01]**	[6.81]**	[24.14]**	[5.88]**	[10.67]**	[2.28]*	[36.06]**	[9.14]**	[16.93]**	[4.43]**	[10.38]**	[2.30]*
Observations	3744	3772	3097	3130	3097	3130	3744	3772	3097	3130	3089	3122	193	194
R-squared	0.08	0.17	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.02	0.00	0.01	0.00	0.05

Robust t statistics in brackets
* significant at 5%; ** significant at 1%

Table 9. Summary statistics of right-hand side variables in the estimated structural model

Variable	Mean	Standard dev.	Obs.
Female	0.499	0.500	2986
Black	0.124	0.330	2986
Hispanic	0.053	0.224	2986
Education	13.4	2.6	2986
Optimism	0.520	0.153	2986
Cheerfulnes	0.000	0.996	2986
S&P 500	1026	99	2986
Fraction other 50 answers	0.165	0.085	2986
Risk tolerance	0.199	0.124	2986

Table 10. Coefficient estimates of the structural models

	(1)	(2)	(3)	(4)
Equation for mu				
Female	-0.047	-0.048	-0.042	
	(0.008)**	(0.008)**	(0.007)**	
Black	-0.097	-0.095	-0.098	
	(0.013)**	(0.013)**	(0.014)**	
Hispanic	-0.055	-0.053	-0.059	
	(0.015)**	(0.018)**	(0.014)**	
Education	0.011	0.011	0.010	
	(0.001)**	(0.001)**	(0.002)**	
Optimism	0.375	0.375	0.358	
	(0.034)**	(0.034)**	(0.037)**	
Cheerfulnes	0.011	0.011	0.011	
	(0.004)**	(0.004)**	(0.005)**	
S&P 500	0.0001	0.0001	0.000	
	(0.000)**	(0.000)**	(0.000)**	
Constant	-0.374	-0.395	-0.351	
	(0.047)**	(0.042)**	(0.049)**	
Equation for log(sigma)				
Female	0.203	0.212	0.198	
	(0.041)**	(0.041)**	(0.041)**	
Black	0.065	0.029	0.063	
	(0.062)	(0.062)	(0.063)	
Hispanic	-0.066	-0.104	-0.064	
	(0.094)	(0.094)	(0.094)	
Education	0.007	0.014	0.010	
	(0.008)	(0.008)	(0.008)	
Fraction other 50 answers	2.504	2.550	2.478	
	(0.245)**	(0.248)**	(0.247)**	
Constant	-1.736	-1.835	-1.764	
	(0.129)	(0.131)	(0.130)	
Equation for s				
log(alpha)	-1.359	-1.428		
	(0.113)**	(0.112)**		
log(risk tolerance)			0.779	
			(0.072)**	
Constant		0.215		
		(0.034)**		
Log likelihood	-15,323	-15,305	-15,327	
Observations	2986	2986	2986	2986

Table 11. Moments of relevant heterogeneity and goodness of fit

	(1)	(2)	(3)	(4)
E(μ)	0.014	0.002	0.013	
StdDev(μ)	0.129	0.129	0.128	
E(σ)	0.488	0.493	0.491	
StdDev(σ)	0.540	0.552	0.546	
E(α)	0.257	0.240	0.277	
Percent correctly predicted stockholder				
Overall	69	68	69	
if stockholder	62	66	62	
if not stockholder	75	70	75	

Figures

Figure 1. Histogram of yearly returns on the S&P 500 (August to August), between 1950 and 2005. Normal density with appropriate mean and standard deviation superimposed.

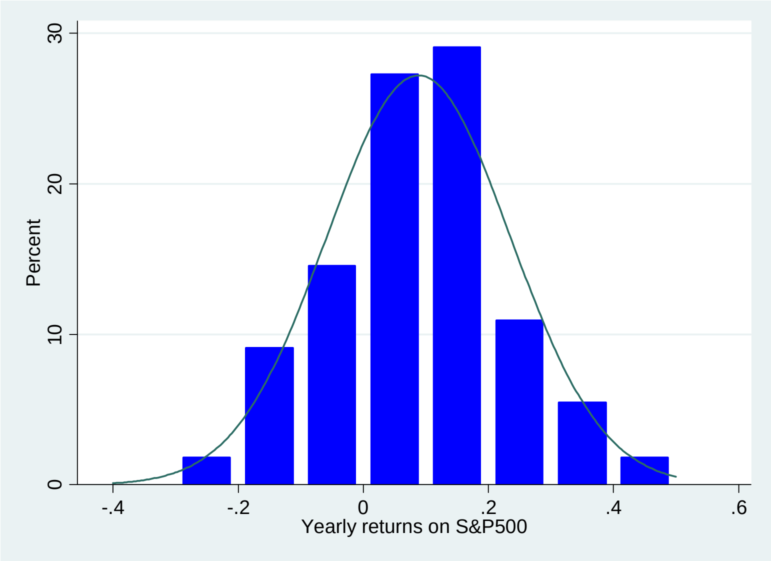


Figure 2.

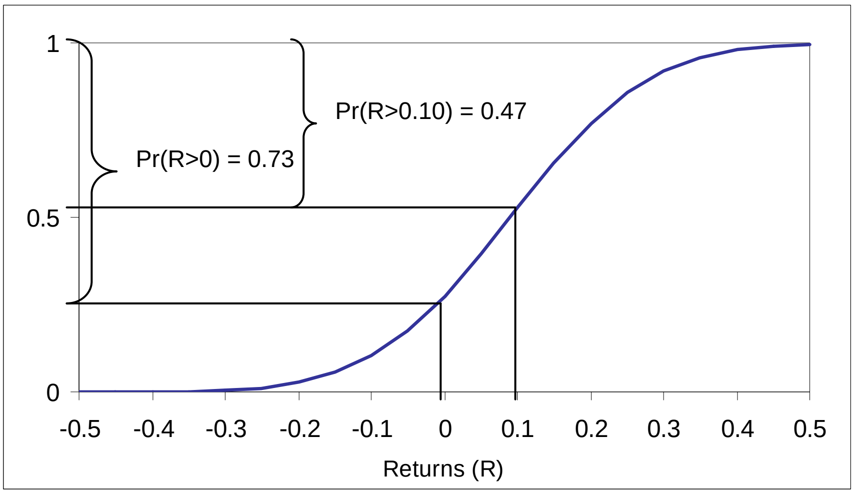
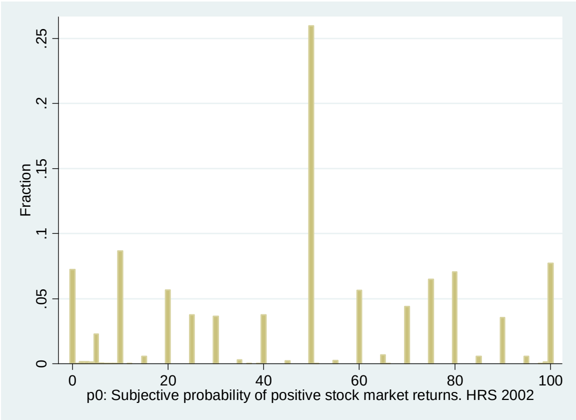
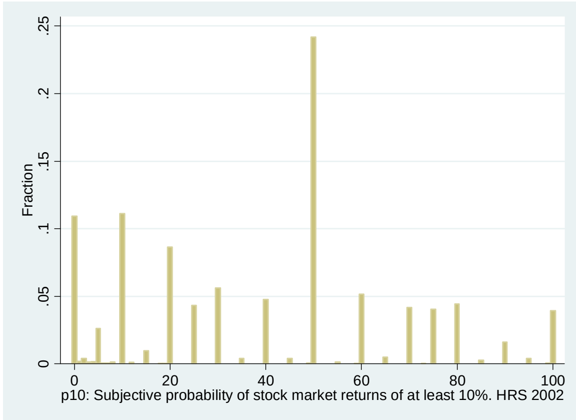


Figure 3.

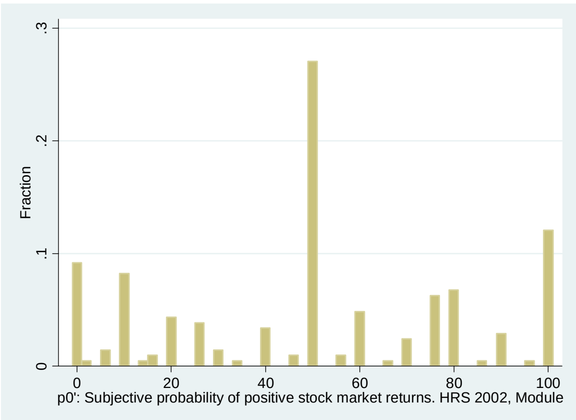


p0

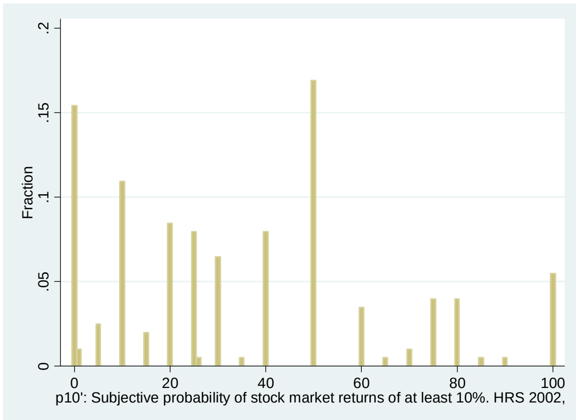


p10

Figure 3A. Answers from the experimental module, HRS 2002



p0'



p10'

Figure 5. Distribution of the difference between core survey and experimental module answers to the same probability question. Market up and market up 10% questions separately.

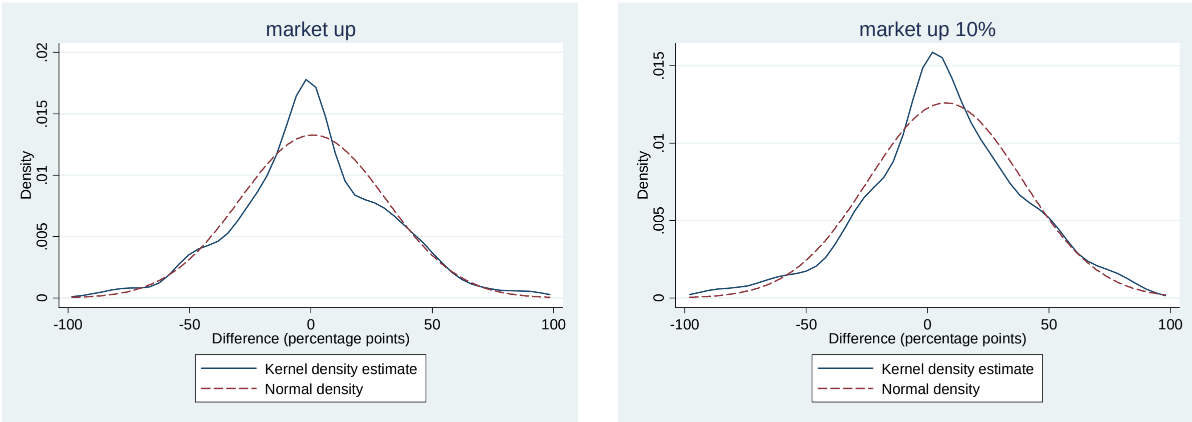


Figure 6a: $g(q_{ij})$ as in (9). τ=0 (probability of market up), σ_ε=0.15 (historical sd)

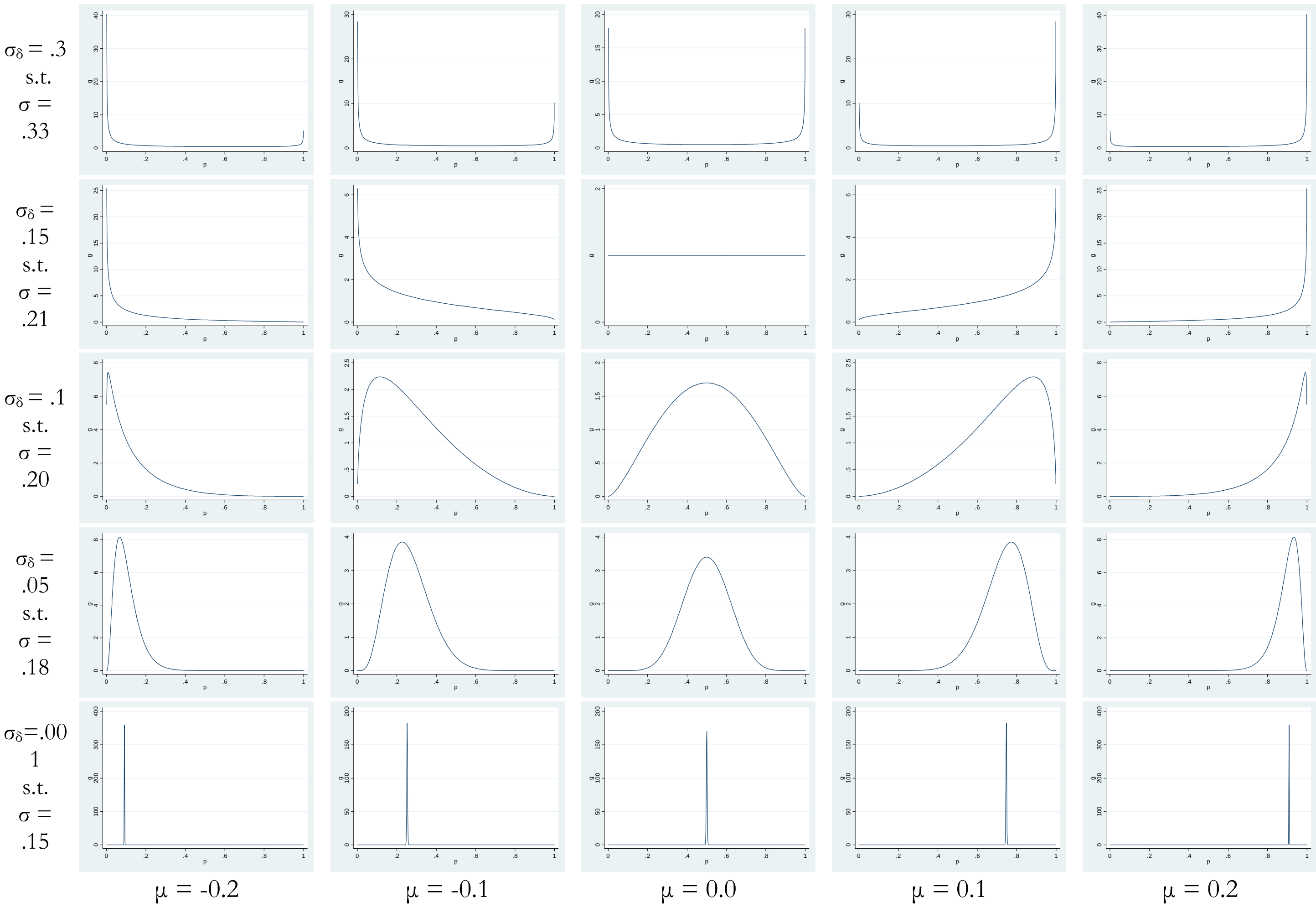


Figure 6B: $g(q_{ij})$ as in (9). τ=0.1 (probability of market up at least 10%), σ_ε=0.15 (historical sd)