Liquidity Coinsurance, Moral Hazard and Financial Contagion*

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Abstract

We study the propagation of financial crises between regions characterized by moral hazard problems. The source of the problem is that banks are protected by limited liability and may engage in excessive risk taking. The regions are affected by negatively correlated liquidity shocks, so that liquidity coinsurance is Pareto improving. The moral hazard problem can be solved if banks are sufficiently capitalized. Under autarky, a limited investment is needed to achieve optimality, so that a limited amount of capital is sufficient to prevent risk-taking. With interbank deposits the optimal investment increases, and capital becomes insufficient to prevent excessive risk-taking. Thus bankruptcy occurs with positive probability and the crises spread to other regions via the financial linkages. Opening the financial markets is nevertheless Pareto improving; consumers benefit from liquidity coinsurance, although they pay the cost of excessive risk-taking. Finally, we show that in this framework a completely connected deposit structure is more conducive to financial crises than an incompletely connected structure.

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1 Introduction

It is sometimes claimed that the opening of financial markets may increase the instability of financial systems. In this paper we show that this claim may be correct, but that it does not necessarily imply that the creation of financial linkages across countries should be restrained.

The main idea is the following. Consider a two-region economy where, in each region, the banking sector has access to long-term investment opportunities and consumers give their assets to the banks in order to exploit such opportunities. The two regions have negatively correlated liquidity needs so that there are gains from trade from pooling the financial resources, for example through an interbank deposit market.

Banks can choose between a safe long-term asset and a riskier asset yielding a lower expected return (we will call this the 'gambling asset'). Investing in such risky assets may become attractive if the banks are protected by limited liability and are undercapitalized, since in that case the bank is gambling with depositors' money. The sub-optimal investment in the excessively risky asset may be prevented if the banks are sufficiently capitalized.

Suppose now that, under autarky, depositors optimally choose a low level of long-term investment. Then, under autarky, banks have enough capital and the moral hazard problem does not appear. Suppose next that, when financial linkages are established, depositors want to substantially increase the long-term investment; this will be the case if long–term investment becomes more attractive when liquidity coinsurance is present. At this point the depositors face a trade off. If they allow the banks to increase substantially the long–term investment then they will be undercapitalized, so that they will gamble with depositors' money. As an alternative they may restrict the amount of long–term investment (making therefore sure that banks remain sufficiently capitalized), thus giving up a substantial part of the gains from the creation of financial linkages. Provided that the gambling asset is not too bad, the depositor will prefer to have undercapitalized banks. This leads to a situation in which bankruptcy occurs with positive probability (when the 'gambling asset' has low returns), and bankruptcy in one country spreads to other countries.

Nevertheless, depositors are better off when financial linkages are established. The two regions can achieve a Pareto-superior allocation by exchanging deposits in the interbank market, thus providing liquidity coinsurance. This has to be traded off against the costs of greater exposure to financial crises. Notice that financial links will be established only when the benefits are greater than the costs, that is when the possibility of financial crises is limited. As a consequence, crises and financial contagion are rare events.

Furthermore, it turns out that the probability of contagion is greater the larger is the number of interbank deposit's cross-holdings. Thus, contrary to previous models, we find that a market organization in which each region is financially linked only to another region is less conducive to contagion than a market structure in which each region is financially linked to all other regions.

Various papers have analyzed contagion in the presence of financial links among banks. In particular, banks are connected to each other through interbank deposit markets that are desirable ex-ante, but during a crisis the failure of one institution can have direct negative payoff effects on the institutions to which it is linked (see Rochet and Tirole [14], Allen and Gale, [2], Aghion, Bolton and Dewatripoint [1], Freixas, Parigi and Rochet [10]). A common feature of these models is the reliance on some exogenous unexpected shock that causes a financial crisis to spill over into other financial institutions. Other explanations for financial contagion have looked at liquidity constraints (Kodres and Pritsker [13]), wealth constraints (Kyle and Xiong [12]), the incentive structure of financial intermediaries (Schinasi and Smith [15]), information asymmetry among investors (Kodres and Pritsker [13], Chen [6], Calvo [4], Calvo and Mendoza [5]). These are not mutually exclusive approaches, but they all involve a certain inability for the agents to correctly anticipate future events.

Moreover, recent empirical papers suggest that the inter-bank linkage channel may not be so important in spreading contagion as the theoretical literature has assumed so far. Sheldon and Maurer [16] for Switzerland, Furfine [11] for the US, Upper and Worms [17] for Germany, and Wells [18] for the UK estimate the matrix of bilateral exposure among banks, and then simulate the extent of contagion following a single bank failure. They find little potential for failures resulting from interbank linkages. However, these works assume a fixed structure of interbank claims, and therefore fail to capture all the ramifications of a bank failure. Cifuentes, Ferrucci and Shin [7] have in fact shown that when prices are allowed to change endogenously, the impact of an initial shock may be increased considerably.

At any rate, this empirical literature tries to measure the extent of contagion for a given exogenous shock on the solvency of one bank, an issue that we do not directly address in this paper. Instead, our paper models contagion in interbank deposit market as an endogenous phenomenon. Banks establish link and accept the risk of contagion only when the risk is not too big. The main implication is that contagion is a rare phenomenon, since otherwise the banks avoid establishing financial linkages.

The rest of the paper is organized as follows. Section 2 sets up the model and characterizes optimal risk-sharing. Section 3 presents the decentralized solution when the regions are in autarky. In this framework we study the role of bank capital, and its relation with moral hazard and aggregate uncertainty. Section 4 analyzes the decentralized environment when the regions are allowed to interact, with and without the moral hazard problem, and shows the conditions under which the establishment of financial linkages leads to increased instability. Section 5 contains the conclusions, and an appendix contains the proofs.

2 The Model

There are three dates ($t = 0, 1, 2$) and a single good, which serves as numeraire. There are three types of assets. A liquid asset (the *short asset*) that takes one unit of the good at date t and converts it into one unit of the good at date $t + 1$. An illiquid asset (the *safe asset*) that takes one unit of the good at date 0 and transforms it into $R > 1$ units of the good at date 2. Finally, in order to model moral hazard, a second illiquid asset is considered (the *gambling asset*) that takes one unit of the good at $t = 0$ and transforms it either into λR units ($\lambda > 1$) with probability η, or 0 units with probability $1 - \eta$ at date 2. We assume $\eta\lambda < 1$, so that risk-averse and risk-neutral agents strictly prefer the safe asset to the gambling asset. While the short asset and the safe asset are always available, the opportunity of investing in the gambling asset only appears with probability p.

We will assume that when the return on the gambling asset is λRx, only the portion Rx of the return is observable and can be used to pay the depositors. The fraction $(\lambda - 1)\, Rx$ is not observable and can be appropriated by the bank owners; for example, this may be on-the-job perks or simply extra money which is illegally diverted to other accounts.

An equivalent formulation is that a dollar invested in the gambling asset yields R with probability η and 0 with probability $1 - \eta$, but it also produces unobservable private benefits for an amount of B for the owners of the bank whenever it is successful. Setting $B = \lambda - 1$ makes the 'private benefit' model analytically equivalent to the 'unobservable extra return' model. Thus, our model can represent various situations that may cause a moral hazard problem.

Banks are protected by limited liability, so that when the return on the gambling asset is 0, the depositors obtain zero at time 2. These assumptions

	A	**B**
S_1	ω_H	ω_L
S_2	ω_L	ω_H

Table 1: Regional liquidity shocks

imply that no contract can be made contingent on the realization λR of the gambling asset.

There are two regions, labeled A and B. Each region contains a continuum of ex-ante identical consumers–depositors with an endowment of one unit of consumption good at date 0. Consumers have Diamond-Dybvig [9] preferences, that is,

$$U(c_1, c_2) = \begin{cases} u(c_1) \text{ with probability } \omega^i \\ u(c_2) \text{ with probability } (1 - \omega^i), \end{cases}$$

where the utility function $u(\cdot)$ is defined over non-negative levels of consumption, is strictly increasing, strictly concave, twice continuously differentiable, and satisfies Inada conditions. The probability ω^i represents the fraction of early consumers in region i, and it can take values ω_H and ω_L, with $\omega_H > \omega_L$. There are two equally likely states, S_1 and S_2. The realization of the liquidity preference shocks is state-dependent, and is given in Table 1.

Ex-ante, each region has the same probability of having a high liquidity preference. All the uncertainty related to liquidity is resolved at $t = 1$, when the state of nature is revealed and each consumer learns whether she is an early or late consumer. Consumer's type is private information. Notice that if we consider the two regions as a single economy there is no aggregate uncertainty, since the proportion of early consumers is $\gamma \equiv \frac{1}{2}\omega_L + \frac{1}{2}\omega_H$ in both states of the world.

Finally, in order to introduce bank capital we follow Allen and Gale [3] and consider a second class of agents (called investors) with risk neutral preferences. At each period t they are endowed with e_t units of the consumption good, and we assume $(e_0, e_1, e_2) = (e, 0, 0)$. They either consume or buy shares of the banks. If they become bank's shareholders, they are entitled to get dividends at $t = 1$ and $t = 2$. We denote by d_t the dividends paid to investors at time t, and assume the following utility function:

$$u(d_0, d_1, d_2) = Rd_0 + d_1 + d_2,$$

with $d_t \geq 0$ for $t = 0, 1, 2$. Since the investors can obtain a utility of Re by consuming immediately their endowment, they have to be rewarded at least

R for each unit of consumption they give up today. If they buy the shares of the banks for an amount e_0 then $d_0 = e - e_0$, and they get dividends d_1 and d_2 in the following periods. Overall, their utility is $R(e - e_0) + d_1 + d_2$. Investors buy bank's capital if the utility of doing so is higher than the utility of immediate consumption, that is $R(e - e_0) + d_1 + d_2 \geq Re$. The participation constraint of the investors can be written as

$$d_1 + d_2 \geq Re_0.$$

Notice that we assume that investors are risk neutral but their consumption is restricted to be positive. If negative consumption for investors were possible then full insurance could be achieved. The non–negativity of consumption for the investors, coupled with the assumption of zero endowments at $t = 1$ and $t = 2$ implies that the only way to share risk with depositors is through investment in bank capital remunerated by state contingent dividends. The consequence is that, when aggregate uncertainty is present, the optimal risk-sharing contract allows for different levels of consumption in different states.

Remark 1. We have chosen the simplest structure delivering the result that, when moral hazard is present, rational depositors may decide to accept the risk of having the owners taking gambles with their money in exchange of liquidity insurance. In general moral hazard can be completely eliminated if it is possible to capitalize sufficiently the banks. Therefore, the crucial ingredient for achieving the result is that there must be some cost, for depositors, in capitalizing the banks.

Given the assumption that the opportunity cost of capital for investors is R (equal to the return that banks can guarantee on the safe long–term asset), if the investors had an unlimited amount of capital it would always be possible to achieve the first best for depositors and avoid any moral hazard problem. This is the reason why we assume that amount of capital is limited. Essentially, limiting the existing capital is equivalent to say that the cost of capitalizing banks becomes infinite after a certain point.

Alternatively, we could assume that the supply of capital is unlimited *but* the opportunity cost of capital for investors is $R^* > R$. In this case there is a trade–off between capitalizing the banks and obtaining liquidity insurance. The premium $R^* - R$ acts as price for insurance that the depositors have to pay. In general the amount of insurance that the depositors want to buy is finite, so that the amount of bank capital will be finite and determined in equilibrium. Opening the financial markets will change the insurance opportunities for depositors, and therefore the optimal amount of bank capital. In such a framework we can obtain the same qualitative results as the ones obtained assuming that capital is limited. We will come back to the issue after proving our main result in section 4.

In this economy the Pareto efficient allocation can be characterized as the solution to the problem of a planner maximizing the expected utility of the consumers. The planner overcomes the problem of asymmetric liquidity needs of the two regions by pooling resources. Let y, x and z be the per capita amounts invested in the short, safe, and gambling assets, respectively. Since the gambling asset is dominated by the safe asset, optimality requires $z = 0$. The planner's problem is

$$\max_{\{x,y,c_1,c_2\}} \gamma u(c_1) + (1-\gamma) u(c_2)$$

subject to the feasibility constraints:

$$x + y \leq 1; \qquad \gamma c_1 \leq y; \qquad (1-\gamma)c_2 \leq Rx;$$

$$x \geq 0; \qquad y \geq 0; \qquad c_1 \geq 0; \qquad c_2 \geq 0.$$

It is obvious that optimality requires that the feasibility constraints are satisfied with equality, so we can write the problem as

$$\max_{y\in[0,1]} \gamma u\left(\frac{y}{\gamma}\right) + (1-\gamma) u\left(\frac{1-y}{1-\gamma}R\right). \tag{1}$$

Since u is strictly concave and satisfies the Inada conditions, the solution to problem 1 is unique and interior. The optimal value $y^* \in (0,1)$ is obtained from the first order condition

$$u'\left(\frac{y^*}{\gamma}\right) = Ru'\left(\frac{1-y^*}{1-\gamma}\right), \tag{2}$$

and once y^* has been determined by equation 2 we can use the feasibility constraints to determine the other variables, that is

$$c_1^* = \frac{y^*}{\gamma}, \qquad c_2^* = \frac{(1-y^*)}{1-\gamma}R, \qquad x^* = 1 - y^*. \tag{3}$$

Notice that (2) and (3) imply $u'(c_1^*) = Ru'(c_2^*)$, which in turn implies $u'(c_1^*) > u'(c_2^*)$ and $c_2^* > c_1^*$. Thus, the first-best allocation automatically satisfies the incentive constraint $c_2 \geq c_1$, that is late consumers have no incentive to behave as early consumers. We will denote the first-best allocation as $\delta^* \equiv (y^*, x^*, c_1^*, c_2^*)$, and U^* the expected utility achieved under the first best allocation.

We remark here that in the first best allocation, the capital owned by risk-neutral investors does not play a role. In fact, the allocation of risk-neutral investors' capital is indeterminate. They can give their money to the banks (as bank capital) for investment in the safe asset or they can consume their capital at time 0. This result is obtained because, when we analyze the first best allocation, we effectively rule out both aggregate uncertainty and moral hazard. We will see that the amount of bank capital plays an important role when either aggregate uncertainty or moral hazard are present.

3 Decentralized Economies in Autarky

The first best can be achieved only if the two regions pool their resources, so that aggregate uncertainty is eliminated. We now want to study the allocations that can be attained by a region in autarky, when there is aggregate uncertainty and (possibly) moral hazard. The structure we consider is the following:

- Banks can offer fully contingent contracts, specifying the fraction of each dollar of deposit to be invested in the short and safe assets respectively and the amount that the depositor can withdraw at each time t contingent on the realization of ω^i. A contract is therefore an array
$$\delta = \left\{x, y, c_1^L, c_1^H, c_2^L, c_2^H\right\},$$
where c_t^s is the amount that a depositor can withdraw at time t if the value of the liquidity shock is ω_s, with $s = L, H$.

- The fraction x invested in the illiquid asset can be misused by the bank owners and invested in the gambling asset; when this happens, the bank will pay c_t^s if the realization is λR, and 0 otherwise.

In our model the moral hazard problem cannot be solved through contracts, since outside parties cannot observe the investment choice of the bank or the extra return that it produces. On the other hand, limited liability prevents punishment when the return on the long-term investment turns out to be zero. Therefore, the only way to provide incentives to the bank to choose the safe asset is to require that the owners put enough of their capital in the bank. We now analyze the form of the optimal contract in autarky, with and without moral hazard.

3.1 Bank Capital and Aggregate Uncertainty

Call c_t^s and d_t^s the consumption of depositors and the dividend paid to investors at time t, with $t = 1, 2$, in state ω_s, with $s = L, H$. Notice that we allow for the possibility to roll over deposits from $t = 1$ to $t = 2$.

The allocation in autarky with aggregate uncertainty is given by the solution of the following problem:

$$\max_{x,y,e_0,\{c_t^s,d_t^s\}_{t=1,2}^{s=L,H}} \frac{1}{2}\left[\omega_H u\left(c_1^H\right) + (1-\omega_H) u\left(c_2^H\right)\right] + \frac{1}{2}\left[\omega_L u\left(c_1^L\right) + (1-\omega_L) u\left(c_2^L\right)\right]$$

subject to

$$\omega_s c_1^s + d_1^s \leq y; \qquad s = L, H$$

$$(1-\omega_s) c_2^s + d_2^s \leq Rx + \left(y - \omega_s c_1^s - d_1^s\right); \qquad s = L, H$$

$$\frac{1}{2}\left(d_1^H + d_2^H\right) + \frac{1}{2}\left(d_1^L + d_2^L\right) \geq Re_0;$$

$$y + x \leq 1 + e_0; \quad e_0 \geq 0; \quad e_0 \leq e; \quad d_t^s \geq 0; \quad c_t^s \geq 0; \quad s = L, H \quad t = 1, 2.$$

The first set of constraints says that the resources used at $t = 1$ to pay off depositors and investors have to be less than the amount invested in the short asset in every state of the world. The second set of constraints looks at the second period. In this case the resources available are given by the return on the investment in the safe asset Rx plus the resources rolled over from period 1, if any. The third constraint is the investors' participation constraint, and finally we have non-negativity and feasibility constraints. Let

$$\overline{\delta}(e) = \left\{\overline{y}(e), \overline{x}(e), \{\overline{c}_t^s(e)\}_{t=1,2}^{s=L,H}\right\} \tag{4}$$

be the optimal allocation offered to consumers under autarky when the amount of capital available is e. We have the following result.

Proposition 1 *There is a level of capital e^a such that, for each $e \geq e^a$ the optimal allocation $\overline{\delta}$ is the same and satisfies*

$$\overline{c}_1^H < \overline{c}_1^L \leq \overline{c}_2^L = \overline{c}_2^H.$$

For values of $e < e^a$ the expected utility of the consumers is strictly increasing in e, and it is constant for $e \geq e^a$.

The intuition for the result is as follows. First, dividends are paid only at date 2, since this way the capital can be invested in the (more profitable) safe asset rather than in the short asset. Second, the risk neutral investor only cares about the expected value of dividends. Thus, provided the non-negativity constraint for dividends is not violated, dividends can be made state-dependent in order to achieve identical consumption across states at period 2. If there is enough capital we don't have to worry about the non-negativity constraints for dividends in the second period, so that equality of consumption across states can be achieved when enough capital is present.

Notice that we have ruled out negative dividends and we have assumed $e_1 = 0$, so that no further injections of capital are possible at date 1. This implies that consumption in period 1 cannot be smoothed out. The presence of liquidity shocks that cannot be smoothed out implies that consumption in the first period must be lower when the liquidity shock is high. In particular, when the liquidity shock is high the consumers are entirely paid out the value of the short asset, i.e. $c_1^H = \frac{y}{\omega_H}$. When the shock is low, part of the short asset is consumed immediately and part is rolled over to the next period.

The allocation $\overline{\delta}(e)$ obviously gives a lower expected utility than the first-best allocation δ^*. We will call $\overline{U}(e)$ the expected utility achievable under the contract $\overline{\delta}(e)$. Also, define $\delta^a = \left\{y^a, x^a, \{c_t^{s,a}\}_{t=1,2}^{s=L,H}\right\}$ the contract offered when the capital is $e \geq e^a$. Thus δ^a is the optimal contract when the region is under autarky, there is no shortage of bank capital and there is no moral hazard.

3.2 Bank Capital and Moral Hazard

The previous analysis assumed that the bank was willing to invest money earmarked for long-term investment in the safe asset. Intuitively, this should be the case when the amount of bank capital is large with respect to the amount invested in the long-term asset, since bank's owners will be more reluctant to gamble with their own money. In particular, we want to answer the following question: If the consumers want a fraction $\overline{x}$ of deposits to be invested in the safe long asset, what is the minimum amount of bank capital needed to make sure that the bank will actually prefer the safe long asset to the gambling asset?

When the bank capital is e and a fraction x of deposits is earmarked for long term investment, the amount of money available for long term investment is $x + e$. The bank can split this amount between the safe asset and the gambling asset. Let $(\widehat{x}, \widehat{z})$ be the amounts invested in the safe asset and the gambling asset respectively. Essentially, we want to find the minimum amount of capital e such that the optimal choice of the bank is $(\widehat{x}, \widehat{z}) = (x+e, 0)$.

If the bank invests the whole amount $x+e$ in the safe asset, the return will be $R(x+e)$ and the profit at state $s = L, H$ is

$$d_2^s = R(x+e) + (y - \omega_s c_1^s) - (1-\omega_s) c_2^s.$$

On the other hand, if the bank puts the money in the gambling asset the profit in state s depends on the realization of the gambling asset, and it is therefore the random variable

$$\widetilde{d}_2^s = \begin{cases} \lambda R(x+e) + (y - \omega_s c_1^s) - (1-\omega_s) c_2^s & \text{with prob. } \eta \\ \\ 0 & \text{with prob. } (1-\eta). \end{cases}$$

The values of d_2^s have to satisfy the participation constraint for risk neutral investors, and competition among investors will imply that the constraint is satisfied with equality. Therefore

$$\frac{1}{2} d_2^L + \frac{1}{2} d_2^H = Re.$$

On the other hand, we have

$$\begin{aligned} E\left[\frac{1}{2}\widetilde{d}_2^L + \frac{1}{2}\widetilde{d}_2^H\right] &= \eta\left[(\lambda - 1) R(x+e) + \frac{1}{2} d_2^L + \frac{1}{2} d_2^H\right] \\ &= \eta\left[\lambda R(x+e) - Rx\right] \end{aligned}$$

Therefore, the bank will choose the safe asset if

$$Re \geq \eta\left[\lambda R(x+e) - Rx\right].$$

Define

$$\xi \equiv \frac{\eta(\lambda - 1)}{1 - \eta\lambda}. \tag{5}$$

We have proved the following result.

Proposition 2 *If the deposit contract offers a level of long-term investment x then the bank will invest in the safe asset only if the bank capital is $e \geq \xi x$.*

The value ξ is the lowest value of the ratio e/x such that the bank does not have incentives to select the gambling asset. If a contract includes a value of e and x such that $e < \xi x$ then it becomes common knowledge that the bank will invest the money in the gambling asset whenever it is available. Therefore, for a given level e of available capital, the investors have the choice between a contract with an investment x such that $e < \xi x$ in which the bank will gamble or a contract with an investment x such that $e \geq \xi x$, so that the bank will choose the safe asset.

For each given value e, define the problem

$$\max_{x,y,\{c_t^s,d_t^s\}_{t=1,2}^{s=L,H}} \frac{1}{2}\left[\omega_H u\left(c_1^H\right)+\left(1-\omega_H\right)u\left(c_2^H\right)\right]+\frac{1}{2}\left[\omega_L u\left(c_1^L\right)+\left(1-\omega_L\right)u\left(c_2^L\right)\right] \tag{6}$$

subject to

$$\xi x \leq e;$$

$$\omega_s c_1^s + d_1^s \leq y; \qquad s=L,H$$

$$\left(1-\omega_s\right)c_2^s + d_2^s \leq Rx + \left(y-\omega_s c_1^s - d_1^s\right); \qquad s=L,H$$

$$\frac{1}{2}\left(d_1^H+d_2^H\right)+\frac{1}{2}\left(d_1^L+d_2^L\right) \geq Re;$$

$$y+x \leq 1+e; \quad x\geq 0; \quad y \geq 0; \quad d_t^s \geq 0; \quad c_t^s \geq 0; \quad s=L,H; \quad t=1,2.$$

Program (6) maximizes the expected utility of the consumer subject to the constraint that the bank is willing to put the money earmarked for the illiquid investment into the safe asset rather than the gambling asset. Let's call $\delta^{ng}(e)$ the solution and $U^{ng}(e)$ the expected utility attained solving program (6). The function $U^{ng}(e)$ is continuous in e.

Furthermore, let x^a be the value of the long term investment in the contract solving the optimization problem without moral hazard as defined in (4) when e^a is the available capital (i.e., the capital that allows consumption smoothing in the second period, as described in Proposition 1). Then $U^{ng}(e)$ is strictly increasing up to $\max\{e^a, \xi x^a\}$. In fact, if $e < e^a$ then the expected utility must be strictly increasing since more capital implies that more risk sharing is possible, and if $e < \xi x^a$ the expected utility is increasing because more capital relaxes the moral hazard constraint.

Consider now the highest utility which can be achieved when the banks are allowed to gamble. This can be obtained solving the problem

$$\max_{x,y,\{c_t^s,d_t^s\}_{t=1,2}^{s=L,H}} \frac{1}{2}\left[\omega_H u\left(c_1^H\right)+\left(1-\omega_H\right)\left(\left(1-p+p\eta\right)u\left(c_2^H\right)+p\left(1-\eta\right)u\left(0\right)\right)\right]$$

$$+\frac{1}{2}\left[\omega_L u\left(c_1^L\right)+\left(1-\omega_L\right)\left(\left(1-p+p\eta\right)u\left(c_2^L\right)+p\left(1-\eta\right)u\left(0\right)\right)\right] \tag{7}$$

subject to

$$\xi x \geq e;$$

$$\omega_s c_1^s + d_1^s \leq y; \qquad s = L, H$$

$$(1-\omega_s)\, c_2^s + d_2^s \leq Rx + (y - \omega_s c_1^s - d_1^s)\,; \qquad s = L, H$$

$$\frac{1}{2}\left(d_1^H + d_2^H\right) + \frac{1}{2}\left(d_1^L + d_2^L\right) \geq Re;$$

$$y + x \leq 1 + e; \quad x \geq 0; \quad y \geq 0; \quad d_t^s \geq 0; \quad c_t^s \geq 0; \quad s = L, H; \quad t = 1, 2.$$

Notice that in this case the consumption offered at time 2 for every state of the world will be stochastic, of the form

$$\widetilde{c}_2^s = \begin{cases} \overline{c}_2^s & \text{with prob. } (1-p) + p\eta \\ 0 & \text{with prob. } p(1-\eta). \end{cases}$$

where $\overline{c}_2^s$ is the solution to program (7). Also notice that the constraints define a non-empty feasible set only if $e \leq \xi x$, that is only for low levels of capital. We will call $U^g(e)$ the expected utility obtained solving program (7).

4 Liquidity Coinsurance and Moral Hazard

Absent moral hazard problems the first-best allocation can be attained using an interbank market of deposits (Allen and Gale [2]). Since the two regions have negatively correlated liquidity needs, banks belonging to the two regions find it useful to exchange deposits between themselves. When a region turns out to have high liquidity needs then it liquidates the deposits held in the other region, and it gives them back when the other region needs them.

The first-best allocation can be attained by a decentralized banking system using interbank deposits as follows:

- each bank offers the contract $\delta^* = (y^*, x^*, c_1^*, c_2^*)$ to the consumers *and* the banks of the other region;
- each bank deposits $(\omega_H - \gamma)$ cents in a bank belonging to another region for each dollar deposited by consumers (and receives a deposit of $(\omega_H - \gamma)$ from a bank of the other region).

Under this arrangement, banks in the region hit by the high liquidity shock (i.e. $\omega^i = \omega_H$) withdraw their deposits from the bank in the other region at time 1, and at time 2 the funds move in the opposite direction. The interbank deposits are used as coinsurance instrument against the liquidity shock[1]. With perfect competition in the banking sector (and absent moral hazard), the equilibrium outcome will be that banks offer the contract yielding the first best allocation, thus maximizing consumers' expected utility.

Also observe that, if there is no moral hazard and the interbank deposit market is active, the level of bank capital does not play any role. All the capital is invested in the safe asset and paid back to investors, without affecting the first-best allocation for consumers.

Proposition 3 *If there is no moral hazard problem and the two representative banks exchange an amount $(\omega_H - \gamma)$ of deposits at $t = 0$ then the first best allocation δ^* can be implemented by a decentralized banking system offering standard deposit contracts.*

The interaction between the two regions eliminates aggregate uncertainty and is able to implement the first-best allocation. This makes the presence of bank capital not necessary, and its level would be indeterminate in both regions. We now study what allocations can be achieved when moral hazard is present.

4.1 Moral Hazard and Contagion

We now discuss what happens when we allow for both liquidity coinsurance and moral hazard. In general the optimal contract offered to depositors in the two regions will take into account both the possibility of liquidity coinsurance and the risk that, when banks are not sufficiently capitalized, the banks' owners may decide to invest in the gambling asset. Moral hazard can be prevented when a sufficient amount of capital is available, and in that case it will be possible to implement the full–insurance allocation discussed in Proposition 3.

In fact, the conditions under which banks in region A are willing to invest in the safe asset are exactly the same as before, i.e. an investment x in the safe asset can be supported only if $e \geq \xi x$. Remember that the optimal contract can specify how deposits (both from depositors of the own region

[1] Since the liquidity shocks in the two regions are perfectly negatively correlated, the insurance is perfect. Interbank deposits still play a role in smoothing out liquidity shocks as long as the shocks are not perfectly positively correlated across regions.

and from banks from other regions) should be invested. The moral hazard problem only appears when the contract requires to invest in the long–term asset. Thus, interbank deposits that are invested in the short–term asset do not create a moral hazard problem.

The highest possible utility that can be achieved is the one given by the first best allocation $\delta^* = (y^*, x^*, c_1^*, c_2^*)$. This allocation can be achieved through liquidity coinsurance between the two regions provided that banks have no incentive to invest in the gambling asset, i.e. $e \geq \xi x^*$ in both regions. We can therefore state the following result.

Proposition 4 *If $e \geq \xi x^*$ then the first best is attainable.*

When there is abundant capital moral hazard is not a problem and financial links between banks of the two regions do not increase the risk of bankruptcy or contagion in any region. Things are different when capital is scarce. In this case we can prove that there are always values of the parameters such that, under the optimal contract, the depositors prefer the 'gambling' contract. In other words, since the possibility of coinsurance makes the investment in the long-term asset very attractive, the depositors accept to take the risk that the assets may sometimes be misused (see Proposition 5). In fact, since the long–term investment is more attractive when liquidity coinsurance is possible than under autarky we will show (Proposition 7) that, under certain conditions on the parameters, the optimal contracts for depositors will prevent moral hazard under autarky but not when financial markets are opened.

To better understand the issue, suppose that the banks in both regions offer a contract $\overline{\delta} = (\overline{y}, \overline{x}, \overline{c}_1, \overline{c}_2)$ to the depositors of their region, and allow banks of other regions to make interbank deposits. In order to achieve coinsurance, each bank will deposit an amount $(\omega_H - \gamma)\,\overline{c}_1$ into a bank of the other region, and the investments in the short–term and long–term assets will be $\overline{y} = \gamma \overline{c}_1$ and $\overline{x} = (1 - \gamma)\,\overline{c}_2$.

Suppose, to fix ideas, that banks in region A invests the amount $\overline{x}$ in the gambling asset, rather than in the safe asset, while the banks in region B invest in the safe asset. Then the following will happen:

- With probability $(1 - p) + p\eta$ either the gambling asset does not appear or it appears and the gamble is successful. In the both cases the depositors of both regions receive the first best allocation and the banks in region B make zero profits. The bank in region A makes zero profits when the gambling asset does not appear and strictly positive profits otherwise.

- With probability $p(1-\eta)$ the gambling asset appears and the gamble fails. This, however, only becomes known in period 2. Early depositors get their first best allocation in both regions. Furthermore, in the second period bank A is bankrupt and
  - If $\omega^A = \omega_L$ then banks in region A lend money to banks in region B for liquidity insurance purposes at $t = 1$, and banks in region B give the money back at $t = 2$. Since banks in region A have gambled and get zero from the investment, the money returned by banks in region B is the only one available to pay depositors. Thus, late consumers of region A receive $\frac{\omega_H - \gamma}{1-\omega_L} c_2^*$ while the late consumers in region B receive the first best allocation. The banks in region B breaks even, and are unaffected by the bankruptcy in region A.
  - If $\omega^A = \omega_H$ then banks in region A borrow money from banks in region B at $t = 1$. However, at $t = 2$ banks in region A will be unable to give back the money, so that bankruptcy will spread also to region B. Late consumers in region B receive $\frac{1-\gamma}{1-\omega_L} c_2^*$, and the banks in region B goes bankrupt.

When it is understood that bank capital is insufficient to prevent moral hazard, the contract offered by the banks will maximize the consumers expected utility taking into account both the opportunities for coinsurance and the probability of bank failure in each region. The exact program for the determination of the optimal contract in spelled out in the appendix, as part of the proof of the next proposition. The main point however is that the expected utility generated by the optimal contract is a decreasing function of p, the probability that the gambling asset will appear, and in fact it will converge to the first–best utility as p goes to zero. This leads to the following result.

Proposition 5 *For each value $e < \xi x^*$ there is a value $p^e > 0$ such that if $p < p^e$ the depositors in the two regions prefer to let the banks to invest in the gambling asset.*

The intuition for the result can be grasped in the following way. Suppose that the banks are undercapitalized but still offer the contract $\delta^* = (y^*, x^*, c_1^*, c_2^*)$ and exchange interbank deposits for an amount $(\omega_H - \gamma) c_1^*$. Since the contract is not necessarily the optimal one, it puts a lower bound on the expected utility for depositors. In fact, when the gambling asset does not appear or it appears but does not fail, the depositors receive an expected utility of U^*. Thus, a lower bound on the expected utility that the depositors can obtain when they allow the banks to gamble is $(1 - p + p\eta)\, U^*$. As p goes to zero this expression converges to U^*, and it is therefore strictly higher than the expected utility that can be obtained when the investment in the long–term asset is limited in order to prevent moral hazard.

Proposition 5 implies that, provided bank capital is less than ξx^*, the depositors prefer to bear the burden of financial instability rather than restricting long-term investment. This, of course, provided that the burden of financial instability is limited, that is p is low. Thus, if financial instability is accepted as a consequence of the opening of financial markets, it must be the case that instability is a rare event.

In order to complete the argument and establish a link between the opening of financial markets and financial instability, we have to show that there are values of the parameters for which depositors prefer to prevent investment in the gambling asset under autarky, but allow it when financial markets open. This happens if the opening of the markets, by bringing new opportunities for coinsurance, increases substantially the utility of long-term investment. As a consequence, depositors will want to increase the long-term investment beyond the level $\frac{e}{\xi}$, thus accepting that banks will gamble. On the other hand, under autarky the desired level of long-term investment is smaller, so the depositors prefer to invest less than $\frac{e}{\xi}$ and avoid gambling.

We first establish conditions under which the long-term investment is higher when the regions exchange deposits than in autarky.

Proposition 6 *If R is sufficiently close to 1 and $e > e^a$then $x^a < x^*$.*

The simplest way to grasp the intuition for Proposition 6 is to consider the case $R = 1$. When capital is abundant, under autarky the optimal allocation will allow for an investment y^a in the short asset which is entirely consumed when the liquidity shock is high, while deposits are partially rolled over when the liquidity shock is low[2]. If $R = 1$ then it is optimal to consume the same amount in each state of the world and period, i.e. $c_t^s = 1$. This requires setting $y^a = \omega_H$, so that $c_1^H = \frac{y^a}{\omega_H} = 1$. This automatically implies $c_2^H = \frac{1-y^a}{1-\omega_H} = 1$. When the liquidity shock is low, individual consumption is $c_1^L = 1$, and aggregate consumption is $\omega_L c_1^L = \omega_L$. The second-period

[2]When $R = 1$ the optimal policy is not unique. Any investment $y \geq \omega_H$ will sustain the optimal consumption. However, for $R > 1$ the optimal policy is unique, and the policy we describe is the limit as R goes to one of the optimal policy.

consumption is $c_2^L = \frac{1-\omega_H+(\omega_H-\omega_L)}{1-\omega_L} = 1$, obtained by rolling over an amount $y^a - \omega_L = \omega_H - \omega_L$ to the second period.

The first best policy when coinsurance is allowed also sets $c_t^s = 1$ in each period and state, but now the investment in the short asset necessary to achieve this allocation is $y^* = \gamma < \omega_H = y^a$. Thus, for $R = 1$ we have $y^* < y^a$, and consequently $x^* > x^a$. Under autarky, we need a higher level of investment in the short asset to guarantee enough consumption in the ω_H state. When R is slightly above 1, the same intuition will apply. The difference is now that consumption in period 1 becomes more costly, since the return on the short asset is inferior to the return on the safe asset, and in particular it becomes costly to sustain consumption when the liquidity shock is high. Thus, the optimal allocation requires $c_1^H < c_1^L$. We have therefore two forces moving in opposite directions. On one hand, autarky requires investing more in the short asset in order to ensure enough consumption in state ω_H. On the other hand, for $R > 1$ optimality requires to curb c_1^H and therefore the investment in the short asset. When $R = 1$ only the first effect is present, so we have unambiguously $y^a > y^*$, but more in general when R is sufficiently close to 1 the first effect will dominate over the second.

When $x^a < x^*$ and $e^a = \xi x^a$, so that the optimal banking contract prevents moral hazard under autarky, then the opening of the markets leads to better coinsurance of liquidity needs *and* a positive probability of bankruptcy whenever p is sufficiently close to zero.

Proposition 7 *Suppose $x^a < x^*$, $e \in [\max\{\xi x^a, e^a\}, \xi x^*)$ and $p < p^e$. Then the two regions invest in the safe asset under autarky and in the gambling asset when interbank deposits are possible. Under the optimal allocation there is a strictly positive probability of bankruptcy and contagion.*

When $e \geq e^a$ each region selects the allocation δ^a, and $e \geq \xi x^a$ implies that banks prefer to invest in the safe asset rather than in the gambling asset. Thus, under autarky there is no bankruptcy.

When financial linkages are established, coinsurance against liquidity shocks becomes possible. The condition $x^a < x^*$ implies that, absent moral hazard problems, the depositors in the two countries would like to increase the investment in the long-term asset. In other words, the possibility of coinsurance makes long–term investment more valuable. However, since $e < \xi x^*$ the capital available is not sufficient to prevent investment in the gambling asset by the banks. The depositors have therefore to choose between curbing the long-term investment to $\frac{e}{\xi}$ or increase it and accept that firms will gamble whenever possible. When the probability that the gambling asset will appear is sufficiently small, the second alternative is more attractive.

It is interesting to analyze exactly what is the probability of bankruptcy and contagion when the situation is the one described in Proposition 7. Under the optimal contract each bank will invest an amount y^g in the short asset and a quantity $x^g = 1 - y^g$ in the long-term asset. Furthermore, each bank deposits an amount $(\omega_H - \gamma) c_1^g$ in the other region.

A region goes bankrupt in two cases. First, and obviously, when the gambling asset appears and the gamble fails, an event having probability $(1 - \eta) p$. Second, when the bank has enough money from the long-term investment (either because the gambling asset did not appear or because the gamble was successful) but in the second period the *other* region is unable to pay the interbank deposits. This is the case of contagion, since the inability to pay by the bank in region A is only due to the bankruptcy of the banks in region B. In general, a bank in region A is owed money from the bank of the region B when the liquidity shock of region A was high, i.e. $\omega^A = \omega_H$. This has probability $\frac{1}{2}$. On the other hand, the probability that the bank in region B is bankrupt is $(1 - \eta) p$. Thus, contagion from region B to region A occurs with probability $\frac{1}{2} p (1 - \eta) [1 - p (1 - \eta)]$, where the last term is the probability that bank A is solvent.

In our model contagion is a rare phenomenon due to the existence of the interbank deposit markets. It is rare because only if the probability of bankruptcy is low it is optimal to create financial linkages and invest in the long-term asset. The basic idea is that the possibility of coinsurance obtained by creating financial linkages increases the optimal amount of long–term investment. When bank capital is low, depositors optimally accept to let the banks gamble with the long-term investment, since it would be too costly to curb long–term investment at a level that prevents moral hazard.

Remark 2. A result similar to the one in Proposition 7 holds when the supply of capital is unlimited and the opportunity cost of capital for investors is $R^* > R$. As previously pointed out, the premium $R^* - R$ acts as price for insurance that the depositors have to pay. Putting more capital into a bank is costly for depositors, since they have to pay the premium $R^* - R$, but helps them in smoothing consumption across states and periods. Thus there will be a trade–off between costs and benefits, and the amount of capital will be determined in equilibrium as part of the optimal contract. Let e^a be the amount of bank capital determined under autarky and x^a the amount of long term investment, and suppose that e^a is large enough to prevent moral hazard problems. Next, suppose that coinsurance between regions becomes available. Since coinsurance between regions acts as a substitute for the insurance provided by bank capital and it costs less, the optimal contract

	A	**B**	**C**	**D**
S_1	ω_H	ω_L	ω_H	ω_L
S_2	ω_L	ω_H	ω_L	ω_H

Table 2: Regional liquidity shocks with multiple regions

will now require a lower level of bank capital and a higher level of long term investment, i.e. $e^* < e^a$ and $x^* > x^a$ (in fact, absent moral hazard problems the depositors can achieve full insurance with no bank capital). At the lower level of bank capital the moral hazard problems will appear. Thus the depositors have to choose between keeping the level of bank capital high (paying the cost $R^* - R$ per unit) and avoid moral hazard problems, or keeping bank capital low and accept the risk of instability. When p is sufficiently low they will prefer the second alternative. Thus, the qualitative prediction is the same as the one in Proposition 7. The model could be further generalized allowing for variable cost of capital. In this case we would have an increasing inverse supply function $R^*(\cdot)$, with $R^*(e)$ being the return needed to attract e units of bank capital.

4.2 Multiple Regions

So far we have assumed a two region economy. In such economy contagion occurs when the bank with the high liquidity shock in the first period faces a bank with a low return on the gambling asset.

Allowing for multiple regions does not change this basic transmission channel of contagion. However, with multiple regions we can analyze what structure of interbank deposit market is more resistant to contagion.

The current consensus in the literature seems to be that the more connected are the interbank deposit markets the better it is for the resilience of the banking system. The interbank deposit market turns out to be more vulnerable to contagion when the claim structure is less connected (Allen and Gale, [2]; Freixas, Parigi, and Rochet, [10]). This is not necessarily true in our model. In fact, in our model the conclusion is just the opposite: A more connected interbank deposit market increases the number of regions affected.

Assume there are 4 regions (called A, B, C, and D). There are two equally likely state of nature S_1 and S_2 and the realization of the liquidity preference shocks in each region is state dependent, as reported in Table 2. Again, notice that when all regions are pooled there is no aggregate uncertainty.

Figure 1: Different structures of the interbank deposit market.

As in Allen and Gale [2] we consider two interbank deposit market structures. In the *completely connected* structure each region receives deposits from and makes deposits to all other regions. In the *incompletely connected* structure, each region has relations only with the 'neighbors' (see Figure 1).

Let k^{in} be the amount of deposit that a bank put in the 'next' bank when interbank deposit market is incompletely connected. In order to equalize consumption across regions at $t = 1$, banks need to deposit (and receive) an amount $k^{in} = (\omega_H - \gamma)\, c_1^{in}$, where c_1^{in} is the consumption level promised to early consumers by the optimal contract.

Consider now the fully connected interbank deposit market. If c_1^{co} is the consumption promised in the first period and each bank deposits an equal amount in all other regions, then the amount of deposit made in each bank must be $k^{co} = \frac{1}{2}(\omega_H - \gamma)\, c_1^{co}$. This way, each bank receives a total of $\frac{3}{2}(\omega_H - \gamma)\, c_1^{co}$ from other banks. When the liquidity shock is high, the other region with a high liquidity shock withdraws $\frac{1}{2}(\omega_H - \gamma)\, c_1^{co}$, and the remaining amount can be used to pay depositors.

In our model contagion occurs when an otherwise solvent bank is unable to retrieve its deposits from another bank. Suppose for example that the state is S_1, so that region B and D have a low liquidity shock at period 1. Suppose further that the only region in which the gambling asset appears is A, and the gamble fails.

In the incompletely connected structure, when the state is S_1 region A withdraws deposits from region B and region C withdraws deposits from region D. Thus, after the first period the only deposits remaining are by B into C and by D into A. In the second period, D is supposed to receive back deposits from A, but it is unable to do so because A fails. Thus, financial crisis spreads to D (the same mechanism as in the two region economy). However, the contagious failures stop there. Region C has no linkage with D, and it can pay back the deposit to region B. In turn, B will be able to retrieve its deposits and it will pay its depositors.

Suppose now that the interbank deposit market is fully connected. In this case, at state S_1 regions A and C will withdraw all their deposits from other regions. After this, the interbank deposits remaining will be by region B and D, each holding a claim of $\frac{1}{2}(\omega_H - \gamma)\, c_1^{co}$ in all other regions. The *net* structure of claims (i.e. after eliminating the deposits that B has in D and D has in B) is that A owes $\frac{1}{2}(\omega_H - \gamma)\, c_1^{co}$ each to B and D, and the same is true for region C. Thus, if A fails then the failure will spread to both B and D; only region C is unaffected.

Summing up, under a fully connected interbank deposit market region A's bankruptcy spills over to regions D and B, while under an incomplete structure the only region affected by contagion is D. Thus, the number of regions affected by the contagion is higher under the connected structure. It should be noted however that in the connected case the amount of financial distress experienced by the banks affected by the contagion will typically be lower. When A fails and the market is connected the amount lost by D and B is $\frac{1}{2}(\omega_H - \gamma)\, c_1^{co}$; when the market is not connected only D is affected but its loss is $(\omega_H - \gamma)\, c_1^{in}$, an amount that will typically be higher[3]

It is worth exploring the reason for the difference between our paper and Allen and Gale [2]. To start with, the distribution of liquidity shocks in different in Allen and Gale [2]. Besides states S_1 and S_2 they allow for a 'zero probability' state S_3 in which aggregate liquidity needs at $t = 1$ exceed γ. In this unexpected state the banks are forced to liquidate the long–term asset at $t = 1$, which is assumed to be inefficient. Furthermore, early liquidation of the long–term asset may induce a bank run, since late consumers prefer to withdraw at $t = 1$ because they fear that at $t = 2$ they will be unable to obtain the promised return, thus forcing other banks to liquidate early and spreading the financial crisis.

The crucial difference with our paper is not that there is an additional state, but that contracts cannot be written (as in our model) contingent on the realization of the liquidity shock. If we were to introduce in our model a new state S_3 with high aggregate liquidity needs at $t = 1$, the optimal contract would simply prescribe a lower level of consumption at $t = 1$ in

[3]The values c_1^{co} and c_1^{in} are determined solving optimal programs of the type described in the proof of Proposition 5. When p is small, the two quantities will be very close.

that state. Either no early liquidation would be necessary, or the optimal contract would limit early liquidation to the amount that does not induce bank runs (i.e. the optimal contact would take into account the incentive compatibility constraint saying that in each state late consumers should be willing to wait). Thus, no contagion would occur.

Because contracts can be contingent on aggregate liquidity shocks, in our model it is irrelevant whether we allow or not for early liquidation of long–term asset, which is instead the crucial transmission channel in Allen and Gale [2]. In this paper we have assumed that aggregate liquidity needs at $t = 1$ are constant and equal to 1, but as we have just discussed there would be no difficulty introducing additional states with varying aggregate liquidity needs. Again, the crucial point is that the optimal contract will allow for state–contingent consumption at $t = 1$ (typically lower when liquidity needs are higher), and any early liquidation would be contracted out ex ante respecting incentive compatibility constraints.

In our model the only non–contractible variable, and the only source of contagion, is the return on the gambling asset. Since this realizes only at $t = 2$, no contagion at $t = 1$ occurs. Financial crises spread directly, when a failing bank is unable to pay debts to other banks. Thus, more contacts among banks increase the probability of contagion.

There is almost no empirical work on the relation between the structure of the interbank deposit market and the probability of contagion. As far as we know, the paper by Degryse and Nguyen (2004) is the only exception. They find that, in the Belgium banking system, a change from a completely connected structure (where all banks have symmetric links) towards an incomplete structure (where money centers are symmetrically linked to some banks, which are themselves not linked together) have decreased the risk and impact of contagion. This finding appears to be in accordance with our theoretical results. However, it is clear that more evidence is needed to shed light on this issue.

5 Conclusion

In this paper we show that financial contagion may arise as a phenomenon in the due course of the working of a market economy, without relying on unexpected contingencies or exogenous shocks. We consider an economy with two regions characterized by negatively correlated liquidity needs. In the presence of aggregate uncertainty and absent agency problems, the two regions can achieve the first-best allocation by pooling their assets by means of an interbank deposit market, thus creating financial links between the two regions.

The insurance provided by the interbank deposit market has to be traded off against the costs of possible imprudent investments made by banks. The opening of financial markets, while leading to an increase of the expected social welfare, may also increase financial instability, which is rationally taken into account by forward looking agents. From a positive point of view, the model predicts the quite robust empirical finding that financial contagion is rarely transmitted through interbank deposit markets.

Appendix

Proof of Proposition 1. We start observing that we can restrict attention, without loss of generality, to policies paying no dividends at time 1. Suppose that an optimal policy requires $d_1^s > 0$ for some s. Consider a new policy in which all variables are unchanged except that $\widehat{d}_1^s = 0$ and $\widehat{d}_2^s = d_1^s + d_2^s$. This policy is feasible and yields the same expected utility for all agents. Furthermore, we can restrict attention to policies in which the whole capital e is invested. If only $e_0 < e$ is invested then we can increase the capital to e and the dividends at time 2 by $R\,(e - e_0)$ in each state of the world, leaving all other variables unchanged. The policy yields the same utility and satisfies all the constraints.

Any optimal policy must be such that $x = 1 + e - y$, with $y \in [0, 1]$ (it must be the case that $y \leq 1$, because otherwise the participation constraint for the risk–neutral investors would be impossible to satisfy) and the resource constraint of the second period has to hold with equality at each state of the world. We can therefore write

$$d_2^s = R\,(1 + e - y) + (y - \omega_s c_1^s) - (1 - \omega_s)\, c_2^s, \qquad s = L, H.$$

Thus, the participation constraint for the risk-neutral investors can be written as

$$R\,(1 - y) + y \geq \frac{1}{2}\left(\omega_H c_1^H + (1 - \omega_H)\, c_2^H\right) + \frac{1}{2}\left(\omega_L c_1^L + (1 - \omega_L)\, c_2^L\,(\omega_L)\right).$$

If e is sufficiently large, then the positivity constraints on d_2^s will not bind.

We can therefore analyze the simpler problem:

$$\max_{y, \{c_t^s\}_{t=1,2}^{s=L,H}} \quad \omega_H u\left(c_1^H\right) + (1 - \omega_H)\, u\left(c_2^H\right) + \omega_L u\left(c_1^L\right) + (1 - \omega_L)\, u\left(c_2^L\right)$$

subject to:

$$\omega_s c_1^s \leq y; \qquad s = L, H$$

$$R\,(1 - y) + y \geq \frac{1}{2}\left(\omega_H c_1^H + (1 - \omega_H)\, c_2^H\right) + \frac{1}{2}\left(\omega_L c_1^L + (1 - \omega_L)\, c_2^L\right);$$

$$y \leq 1; \qquad y \geq 0; \qquad c_t^s \geq 0; \qquad s = H, L; \qquad t = 1, 2.$$

Since u satisfies the Inada condition the optimal y will be interior, and $c_t^s > 0$ for each t, s. Also, optimality requires $\omega_H c_1^H = y$. The Lagrangian can therefore be written as

$$\begin{aligned} L &= \omega_H u\left(\frac{y}{\omega_H}\right) + (1-\omega_H)\, u\left(c_2^H\right) + \omega_L u\left(c_1^L\right) + (1-\omega_L)\, u\left(c_2^L\right) - \mu\left(c_1^L - \frac{y}{\omega_L}\right) \\ &\quad -\lambda\left(\frac{1}{2}(1-\omega_H)\, c_2^H + \frac{1}{2}\left(\omega_L c_1^L + (1-\omega_L)\, c_2^L\right) - R(1-y) - \frac{1}{2}y\right) \end{aligned}$$

The first order conditions are:

$$y: \quad u'\left(\frac{y}{\omega_H}\right) + \frac{\mu}{\omega_L} - \lambda\left(R - \frac{1}{2}\right) = 0; \tag{8}$$

$$c_2^H: \quad u'\left(c_2^H\right) - \lambda\frac{1}{2} = 0; \tag{9}$$

$$c_1^L: \quad u'\left(c_1^L\right) - \frac{\mu}{\omega_L} - \lambda\frac{1}{2} = 0; \tag{10}$$

$$c_2^L: \quad u'\left(c_2^L\right) - \lambda\frac{1}{2} = 0. \tag{11}$$

Conditions (9) e (11) imply $c_2^L = c_2^H$; conditions (10) e (11) imply $c_1^L \leq c_2^L$. If $\mu > 0$ then $c_1^L = \frac{y}{\omega_L} > c_1^H$. If $\mu = 0$ then $c_1^L = c_2^L = c_2^H$ and

$$u'\left(\frac{y}{\omega_H}\right) = \lambda\left(R - \frac{1}{2}\right) > \lambda\frac{1}{2}u'\left(c_1^L\right),$$

which again implies $c_1^H < c_1^L$.

Consider now the general problem, with an arbitrary value of e.

$$\max_{y,\{c_t^s, d_t^s\}_{t=1,2}^{s=L,H}} \quad \omega_H u\left(c_1^H\right) + (1-\omega_H)\, u\left(c_2^H\right) + \omega_L u\left(c_1^L\right) + (1-\omega_L)\, u\left(c_2^L\right)$$

subject to:

$$\begin{gathered} \omega_s c_1^s \leq y; \qquad\qquad s = L, H \\ R(1-y) + y \geq \frac{1}{2}\left(\omega_H c_1^H + (1-\omega_H)\, c_2^H\right) + \frac{1}{2}\left(\omega_L c_1^L + (1-\omega_L)\, c_2^L\right); \\ R(1+e-y) + \left(y - \omega_L c_1^L\right) - (1-\omega_L)\, c_2^L \geq 0; \\ R(1+e-y) + \left(y - \omega_L c_1^H\right) - (1-\omega_L)\, c_2^H \geq 0. \end{gathered}$$

The Lagrangian can therefore be written as

$$L = \omega_H u\left(\frac{y}{\omega_H}\right) + (1-\omega_H)\, u\left(c_2^H\right) + \omega_L u\left(c_1^L\right) + (1-\omega_L)\, u\left(c_2^L\right) - \mu\left(c_1^L - \frac{y}{\omega_L}\right)$$

$$-\lambda\left[\frac{1}{2}\left(1-\omega_H\right)c_2^H+\frac{1}{2}\left(\omega_L c_1^L+\left(1-\omega_L\right)c_2^L\right)-R\left(1-y\right)-\frac{1}{2}y\right]$$
$$-\phi_L\left[\left(1-\omega_L\right)c_2^L-R\left(1+e\right)+\left(R-1\right)y+\omega_L c_1^L\right]$$
$$-\phi_H\left[\left(1-\omega_H\right)c_2^H-R\left(1+e\right)+\left(R-1\right)y+\omega_H c_1^H\right].$$

The first order conditions are:

$$y: \quad u'\left(\frac{y}{\omega_H}\right)+\frac{\mu}{\omega_L}-\lambda\left(R-\frac{1}{2}\right)-\phi_L\left(R-1\right)-\phi_H\left(R-1\right)=0;$$

$$c_2^H: \quad u'\left(c_2^H\right)-\lambda\frac{1}{2}-\phi_H=0;$$

$$c_1^L: \quad u'\left(c_1^L\right)-\frac{\mu}{\omega_L}-\lambda\frac{1}{2}-\phi_L=0;$$

$$c_2^L: \quad u'\left(c_2^L\right)-\lambda\frac{1}{2}-\phi_L=0.$$

If either ϕ_H or ϕ_L are strictly positive that by increasing e we strictly increase utility. ■

Proof of Proposition 3. In a competitive equilibrium the deposit contract offered by the representative banks maximizes the ex ante expected utility of the consumers. All we need to show is that the constraints faced by the representative banks, with the help of the interbank deposit market, are the same as the constraints faced by the social planner both in $t=1$ and $t=2$.

The region with high liquidity shock has the following budget constraints in $t=1$ and $t=2$:

$$\omega_H c_1 \leq y+(\omega_H-\gamma)c_1,$$

and

$$(1-\omega_H)c_2+(\omega_H-\gamma)c_2 \leq Rx.$$

Both constraints are the same of the social planner problem. The region with low liquidity shock has the following budget constraints in $t=1$ and $t=2$:

$$\omega_L c_1+(\omega_H-\gamma)c_1 \leq y,$$

and

$$(1-\omega_L)c_2 \leq Rx+(\omega_H-\gamma)c_2.$$

Since $\omega_H-\gamma=\gamma-\omega_L$, also this region offer first-best allocation since the constraints are the same of the social planner. ■

Proof of Proposition 5. Let U^* be the utility achieved under the first best contract $\delta^* = (y^*, x^*, c_1^*, c_2^*)$. If $e < \xi x^*$ then the first best allocation is not attainable, because whenever the banks offer δ^* they will invest in the gambling asset. As in the autarky case, banks can either offer a contract with a limited long–term investment x that avoids the moral hazard problem (i.e. such that $\xi x \leq e$) or a contract with $\xi x > e$ so that it becomes common knowledge that banks will gamble.

Notice that now an optimal contract will also specify the amount k of interbank deposits, i.e. each bank promises to deposit k and to receive k from a bank in the other region.

Conditional on avoiding gambling, the contract that maximizes the depositors utility is obtained solving the problem

$$\max_{x,y,k,\{c_t^s,d_t^s\}_{t=1,2}^{s=L,H}} \frac{1}{2}\left[\omega_H u\left(c_1^H\right) + (1-\omega_H)\, u\left(c_2^H\right)\right] + \frac{1}{2}\left[\omega_L u\left(c_1^L\right) + (1-\omega_L)\, u\left(c_2^L\right)\right] \tag{12}$$

subject to

$$\xi x \leq e;$$
$$\omega_H c_1^H + d_1^H \leq y + k;$$
$$\omega_L c_1^L + d_1^L \leq y - k;$$
$$(1-\omega_H)\, c_2^H + d_2^H \leq Rx + \left(y + k - \omega_H c_1^H - d_1^H\right) - k;$$
$$(1-\omega_L)\, c_2^L + d_2^L \leq Rx + \left(y - k - \omega_L c_1^L - d_1^L\right) + k;$$
$$\frac{1}{2}\left(d_1^H + d_2^H\right) + \frac{1}{2}\left(d_1^L + d_2^L\right) \geq Re;$$
$$y + x \leq 1 + e; \quad x \geq 0; \quad y \geq 0; \quad d_t^s \geq 0; \quad c_t^s \geq 0; \quad s = L, H; \quad t = 1, 2.$$

In fact, it is clear that in this case interbank deposits will perfectly insure against liquidity shocks, so that the problem can be alternatively written as

$$\max_{x,y,\{c_t,d_t\}_{t=1,2}} \gamma u\left(c_1\right) + (1-\gamma)\, u\left(c_2\right) \tag{13}$$

subject to

$$\xi x \leq e;$$
$$\gamma c_1 + d_1 \leq y$$
$$(1-\gamma)\, c_2 + d_2 \leq Rx + \left(y - \gamma c_1 - d_1\right)$$
$$d_1 + d_2 \geq Re;$$

$$y + x \leq 1 + e; \quad x \geq 0; \quad y \geq 0; \quad d_1 \geq 0; \quad c_t \geq 0; \quad t = 1, 2.$$

Let $U^{ng}(e)$ be the expected utility when moral hazard is prevented, that is the value of the objective function at the optimal point of program (13). We notice that the value $U^{ng}(e)$ does not depend on p, and since $e < \xi x^*$ we have $U^{ng}(e) < U^*$. Define

$$\Delta = U^* - U^{ng}(e).$$

We now discuss the optimal contract when banks are expected to invest in the gambling asset. Define

$$q = 1 - p + p\eta,$$

the probability that either the gambling asset does not appear or that it appears and the gamble is successful. Again, let k be the amount of interbank deposits that the banks exchange at time 0. Now it is understood that the amount will be withdrawn at time $t = 1$ when the region is hit by the high liquidity shock, and it will be returned at $t = 2$ only if the gambling succeeds.

When gambling is allowed, contracts can be written only contingent on the realization of the liquidity shock but not on the return on the gambling asset.[4] We claim that the optimal contract is obtained solving the following program

$$\max_{x,y,k,\{c_t^s,d_t^s\}_{t=1,2}^{s=L,H},c_2^C,c_2^F} \frac{1}{2}\left[\omega_H u\left(c_1^H\right) + (1-\omega_H)\left(qu\left(c_2^H\right) + (1-q)\,u(0)\right)\right] +$$

$$\frac{1}{2}\left[\omega_L u\left(c_1^L\right)\right] + \frac{(1-\omega_L)\,q}{2}\left[qu\left(c_2^L\right) + (1-q)\,u\left(c_2^C\right)\right]$$

$$+\frac{(1-\omega_L)(1-q)}{2}\left[qu\left(c_2^F\right) + (1-q)\,u(0)\right] \tag{14}$$

subject to

$$\xi x \geq e; \tag{15}$$

$$\omega_H c_1^H + d_1^H \leq y + k; \tag{16}$$

[4] This assumption is not essential. Allowing for contracts contingent on the return on the gambling asset would yield a higher expected utility for the optimal contract, and it would actually make it easier to have a higher expected utility when investment in the gambling asset is allowed than in the case in which long–term investment is limited in order to prevent moral hazard.

$$(1-\omega_H)\,c_2^H + d_2^H \leq Rx + \left(y + k - \omega_H c_1^H - d_1^H\right) - k; \tag{17}$$

$$\omega_L c_1^L + d_1^L \leq y - k; \tag{18}$$

$$(1-\omega_L)\,c_2^L + d_2^L \leq Rx + \left(y - k - \omega_L c_1^L - d_1^L\right) + k; \tag{19}$$

$$(1-\omega_L)\,c_2^C \leq Rx + \left(y - k - \omega_L c_1^L - d_1^L\right); \tag{20}$$

$$(1-\omega_L)\,c_2^F \leq \left(y - k - \omega_L c_1^L - d_1^L\right) + k; \tag{21}$$

$$\frac{1}{2}\left(d_1^H + q d_2^H\right) + \frac{1}{2}\left(d_1^L + q^2 d_2^L\right) + p\eta\left(\lambda - 1\right)Rx \geq Re; \tag{22}$$

$$y + x \leq 1 + e; \quad x \geq 0; \quad y \geq 0; \quad d_t^s \geq 0; \quad c_t^s \geq 0;; \quad s = L, H; \quad t = 1, 2.$$

$$k \geq 0; \quad c_2^C \geq 0; \quad c_2^F \geq 0 \tag{23}$$

We now explain in detail the objective function and the constraints. With

probability $\frac{1}{2}$ the region will have a high liquidity shock ω_H. In this case the bank will withdraw the interbank deposit k from the other region and it will pay the established amount c_1^H. The payments will have to satisfy the resource constraint (16). In the second period the bank has to pay k back to the other region and c_2^H to the depositor. With probability q it will be able to do that, and the payment will have to satisfy the resource constraint (17). With probability $1-q$ the gambling asset will fail, and the bank will have no money left to pay the depositors or the other banks. This explains the first row of the objective function.

With probability $\frac{1}{2}$ the firm will have a low liquidity shock at $t = 1$. In this case it pays the promised amount c_1^L to the depositors, and allows the firm in the other region to withdraw funds for k. The payments will have to satisfy the resource constraint (18). In the second period, with probability q the bank will have Rx from the gambling investment. If banks in the other region are also solvent, which happens with probability q, then they will pay back the deposit k. In this case the payments will have to satisfy the resource constraint (19). However, if the bank in the other region fails, which happens with probability $1-q$, the bank will be unable to retrieve the interbank deposits k, and the resource constraint will be given by (20). In this case the bank is affected by contagion and the depositors obtain a lower amount c_2^C (we have also assumed that no dividends are paid, which will always be true at the optimum). This explains the second row of the objective function.

Finally, the third row deals with the case in which the liquidity shock is low at $t = 1$ and the gambling asset fails, which happens with probability

$1-q$. If banks in the other region do not fail, then they will pay back their deposit, so that there will be some money left for late consumers. The resource constraint will be given by (21). If the other region also fails, the depositors get 0.

The participation constraint for investors (22) can be explained as follows. With probability $\frac{1}{2}$ the region will have a high liquidity shock. In that case the investors will receive d_1^H in the first period and d_2^H in the second period provided that the gambling asset does not fail, which happens with probability q. With probability $\frac{1}{2}$ the region will have a low liquidity shock, so that banks in the other region will withdraw k. In the first period investors receive d_1^L, but the dividend in the second period will be paid only if both the gambling asset in the region *and* the gambling asset in the other region do not fail. This happens with probability q^2. At last, whenever the gambling asset appears and yield λR, which happens with probability $p\eta$, the investors will be able to retain an amount $(\lambda-1)\,Rx$.

We can now complete the proof. Call $U^g(p)$ the utility achieved under liquidity coinsurance when the banks are allowed to gamble, i.e. the value of the objective function at the optimal point of program 14. The function $U^g(p)$ is continuous and decreasing in p, and $\lim_{p\downarrow 0} U^g(p) = U^*$. But this implies that, for p low enough $U^g(p) > U^* - \Delta = U^{ng}(e)$, so that gambling is preferred. ■

Proof of Proposition 6. The first-best value y^* is determined by the equation

$$u'\left(\frac{y}{\gamma}\right) = Ru'\left(\frac{R\,(1-y)}{1-\gamma}\right). \tag{24}$$

Consider now the determination of y^a, the short term investment under autarky when there is enough capital to make sure that consumption is constant in the second period. Look first at the case $c_1^L < \frac{y}{\omega_L}$ (some deposits are rolled to the second period when the liquidity shock is ω_L) so that $\mu = 0$ in the first order conditions (8) and (10). Since optimality requires $c_1^H = \frac{y}{\omega_H}$, the value y^a is determined by the equation

$$u'\left(\frac{y}{\omega_H}\right) = (R + (R-1))\,u'\left(\frac{R\,(1-y)}{1-\omega_H}\right). \tag{25}$$

The LHS of (24) is lower than the LHS of (25). If $R=1$ then the RHS of (24) is higher than the RHS of (25). It follows $y^a > y^*$ (in fact, when $R=1$ it is easy to see that the solution is $y^a = \omega_H > \gamma = y^*$). Since the solutions $y^a(R)$ and $y^*(R)$ are continuous in R, the inequality still holds for R sufficiently close to 1.

Consider now the case $\mu > 0$, so that $c_1^L = \frac{y}{\omega_L}$. In this case the solution implies $c_1^H < c_1^L < c_2^L = c_2^H$ and the first order conditions imply

$$\frac{1}{2}u'\left(c_1^H\right) + \frac{1}{2}u'\left(c_1^L\right) = Ru'\left(c_2\right). \tag{26}$$

Thus, this solution can never arise when $R = 1$, since the LHS of the equation would be strictly higher than the RHS. ■

References

[1] Aghion, P., P. Bolton and M. Dewatripont (2000), 'Contagion Bank Failures in a Free Banking System', *European Economic Review,* **44**: 713-718.

[2] Allen, F. and D. Gale (2000), 'Financial Contagion', *Journal of Political Economy*, **108**: 1-33.

[3] Allen, F. and D. Gale (2005), 'Systemic Risk and Regulation', mimeo, NYU, http://www.econ.nyu.edu/user/galed/papers/paper05-02-12.pdf

[4] Calvo, G. (1999), 'Contagion in Emerging Markets: When Wall Street is a Carrier', Unpublished paper, University of Maryland.

[5] Calvo, G. and E. Mendoza (1999), 'Rational Contagion and the Globalization of Securities Markets', *NBER Working Paper*, n. 7153, forthcoming *Journal of International Economics.*

[6] Chen, Y. (1999), 'Banking Panics: The Role of the First-Come, First-Served Rule and Information Externalities', *Journal of Political Economy*, **107**: 946-968.

[7] Cifuentes R., G. Ferrucci and H. S. Shin (2005), 'Liquidity Risk and Contagion', *Journal of European Economic Association*, **3**: 556-566.

[8] Degryse, H. and G. Nguyen (2004), 'Interbank Exposures: An Empirical Examination of Systemic Risk in the Belgian Banking System', Manuscript, University of Leuven.

[9] Diamond, D. and P. Dybvig (1983), 'Bunk Runs, Deposit Insurance and Liquidity', *Journal of Political Economy*, **91**: 401-419.

[10] Freixas, X., B. Parigi and J.C. Rochet (2000), 'Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank', *Journal of Money, Credit and Banking*, **32**: 611-638.

[11] Furfine, C. H. (2003), 'Interbank Exposures: Quantifying the Risk of Contagion', *Journal of Money, Credit and Banking*, **35**: 111-128.

[12] Kyle, A. S. and W. Xiong (2001), 'Contagion as a Wealth Effect', *Journal of Finance,* **56**: 1401-1440.

[13] Kodres, L. and M. Pritsker (2002), 'A Rational Expectation Model of Financial Contagion', *Journal of Finance*, **57**: 769-799.

[14] Rochet, J.C. and J. Tirole (1996), 'Interbank Lending and Systemic Risk', *Journal of Money, Credit and Banking*, **28**: 733-762.

[15] Schinasi, G. and T. Smith (1999), 'Portfolio Diversification, Leverage and Financial Contagion', *IMF Working Paper*, n. 136.

[16] Sheldon, G. and M. Maurer (1998), 'Interbank Lending and Systemic Risk: An Empirical Analysis of Switzerland', *Swiss Journal of Economics and Statistics*, **134**: 685-704.

[17] Upper, C. and A. Worms (2002), 'Estimating Bilateral Exposure in the German Interbank Market: Is There a Danger of Contagion?', Deutsche Bundesbank, Discussion Paper, n. 09/02.

[18] Wells, S. (2002), 'UK Interbank Exposures: Systemic Risk Implications', *Bank of England Financial Stability Review*, December.