Risk taking by entrepreneurs

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Abstract

Entrepreneurs bear substantial risk, but empirical evidence shows no sign of a positive premium. The paper develops a theory of endogenous entrepreneurial risk taking that explains why self-financed entrepreneurs may find it optimal to invest into risky projects offering no risk premium. The model has also a number of implications for firm dynamics supported by strong empirical evidence, such as positive correlation between survival, size, and age of businesses.

1 Introduction

Entrepreneurs bear substantial risk. According to recent estimates[1], to compensate for the extra risk entrepreneurial returns (return to private equity) should exceed public equity by at least 10 percent. Yet the evidence shows no signs of a positive premium.[2] A number of hypotheses have been offered to explain this puzzle, all of them based on the idea that entrepreneurs have a different set of preferences (e.g. risk tolerance or overoptimism.) This paper provides an alternative theory of endogenous entrepreneurial risk-taking that does not rely on individual heterogeneity.

The key ingredients in our theory are borrowing constraints, the existence of an outside opportunity and endogenous risk choice. A self-financed entrepreneur chooses every period how much to invest in a project, which is chosen from a set of alternatives. All available projects offer the same expected return but a different variance. After returns are realized, the entrepreneur decides whether to exit and take the outside opportunity (e.g. become a worker) or to stay in business.

The possibility of exit creates a nonconcavity in the entrepreneurs' continuation value: for values of wealth below a certain threshold, the outside opportunity gives higher utility; for higher wealth levels, entrepreneurial activity is preferred. Risky projects provide lotteries over future wealth that eliminate this nonconcavity and are particularly valuable to entrepreneurs with wealth levels close to this threshold. As the level of wealth increases, entrepreneurs invest in less risky projects.

It is the relatively poor entrepreneurs that decide to take more risk. At the same time, due to self-financing, they invest less in their projects than richer entrepreneurs. Correspondingly, the model implies that survival rates of the business are negatively correlated with business size. Moreover, if agents enter entrepreneurship with relatively low wealth levels (as occurs in a case with endogenous entry that we study), our model also implies that young businesses exhibit lower survival rates. Moreover, conditional on survival small (younger) firms grow faster than larger (older) ones. All these

[1]These calculations assume standard levels of risk aversion (CRRA=2). See Heaton and Lucas (2000).

[2]Moskowitz and Vissing-Jorgensen (2002) estimate the return to entrepreneurial investment using data from SCF (Survey of Consumer Finances) and FFA/NIPA(the Flow of Funds Accounts and National Income and Product Accounts) and report that the average return to all private equity is similar to that of the public market equity index.

implications are supported by strong empirical evidence from the literature on firm dynamics (see, e.g. Evans 1987, Dunne, Roberts and Samuelson 1989 and Davis and Haltiwanger 1992).

In order to stress the role of risk taking, our model allows entrepreneurs to choose completely safe projects with the same expected return. All exit in our model occurs precisely because low wealth entrepreneurs purposively choose risk. If risky projects were not available, no exit would occur.

As mentioned above, three features are key to our model: the existence of an outside opportunity, financial constraints and the endogenous choice of risk. Many papers consider some of these features separately, but as far as we know ours is the first that considers all of them together. Discrete occupational choices appear in several papers, following Lucas (1978). Borrowing constraints have been considered in several recent papers (Gomes 2001, Albuquerque and Hopenhayn 2002, Clementi and Hopenhayn 2002) and is consistent with the empirical evidence presented in Evans and Jovanovic (1989), Gertler and Gilchrist (1994) Fazzari, Hubbard and Petersen (1988) and others. The use of lotteries to convexify discrete choice sets was introduced in the macro literature by Rogerson (1988).

A number of papers address the question of which agents decide to become entrepreneurs. All these models rely on some source of heterogeneity. The classical work in this field is a general equilibrium model by Kihlstrom and Laffont (1979), where it is assumed that agents differ in their degrees of risk aversion. Obviously the least risk averse agents are selected into entrepreneurship, which is assumed to be a risky activity. In a recent paper, Cressy (1999) points out that different degrees of risk aversion can be the result of differences in wealth. In particular, if preferences exhibit decreasing absolute risk aversion (DARA), wealthier agents become entrepreneurs. The same happens in the occupational choice model described in the paper, but due to the presence of borrowing constraints.

The empirical regularities on firm dynamics have been explained in models by Jovanovic (1982) and Hopenhayn (1992) and Ericson and Pakes (1995). These models rely on exogenous shocks to firms' productivities and selection. In Jovanovic the source is learning about (ex-ante) heterogeneity in entrepreneurial skills. In Hopenhayn survival rates and the dynamics of returns are determined by an exogenous stochastic process of firms' productivity shocks and the distribution of entrants. In Ericson and Pakes the shocks affect the outcome of investments made by firms.

In contrast to the studies listed above, we do not assume any heterogeneity in risk aversion (as in Kihlstrom and Laffont), or in the returns to entrepreneurial activity (as in Jovanovic). In our setup risk taking is a voluntary decision of agents and not an ex ante feature of the available technology (as in Kihlstrom and Laffont, and Cressy). In contrast to Hopenhayn (1992), we endogenize the stochastic process that drives firm dynamics.

The paper is organized as follows. Section 2 describes the basic model of entrepreneurial risk choice. In this section the outside opportunity is described by a function of wealth with some general properties. This section gives the core results of the paper. Section 3 gives a detailed occupational choice model that endogenizes the outside value function. There is entry and exit from employment to entrepreneurship. We explore conditions under which risk taking occurs in equilibrium and provide benchmark computations to assess its value.

2 The Model

2.1 The Environment

The entrepreneur is an infinitely lived risk averse agent with time separable utility $u(c)$ and discount factor β. Assume $u(c)$ is concave, strictly increasing and satisfies standard Inada conditions. The entrepreneur starts a period with accumulated wealth w. At the beginning of each period he first decides whether to continue in business or to quit and get an outside value $R(w)$, which is an increasing and concave function of his wealth. Entrepreneurs are self-financed and while in business face the following set of investment opportunities.

There is a set of available projects with random return $\widetilde{A}k$, where k is the amount invested. Entrepreneurs must choose one of these projects and the investment level $k \leq w$. All projects offer the same expected return $E\widetilde{A} = A$, but different levels of risk. We assume the expected return $A > 1/\beta$. The distribution of a project's rates of return is concentrated in two points, $x \leq y$. (As shown later, this assumption is without loss of generality.) If the low return x is realized with probability $1 - p$, the average return is $A = (1 - p)x + py$, and the high return y may be expressed as

$$y = x + \frac{A - x}{p} \geq A. \tag{1}$$

Thus, we will identify every available project by the value of the lower return x and the probability of the higher return p. Denote by $\Omega_2(A)$ the set of available projects [3],

$$\Omega_2(A) = \{(x,p)|x \in [0,A], p \in [0,1]\}.$$

If $x = A$ or $p = 1$ the project is safe, delivering the return A for sure; for all other values of x and p the project is risky. The existence of riskless projects that are not dominated in expected return is obviously an extreme assumption. It is convenient for technical reasons and it helps to emphasize the point that risk taking is not necessarily associated with higher returns.

Intuitively, risk taking in this set up occurs due to the presence of the outside opportunity. Imagine that risky projects are not available. In this case the value of an active entrepreneur with current wealth w is defined by the standard dynamic problem[4]

$$V_l(w) = \max_k \{u(w-k) + \beta V_l(Ak)\}. \tag{2}$$

If $R(w)$ and $V_l(w)$ have at least one intersection, the value of the entrepreneur with the option to quit is a non-concave function $\max\{R(w), V_l(w)\}$. This nonconcavity suggests that a lottery on wealth levels could be welfare improving. As will be seen, in the absence of such lottery, an entrepreneur may find it beneficial to invest in a risky project.

If risk taking is possible, an entrepreneur with current wealth w that decides to stay in business, picks a project $(x,p) \in \Omega_2(A)$ and the amount of wealth $k \in [0,w]$ invested into this project. Given that the entrepreneur has no access to financing, consumption will equal $w-k$. By the beginning of the following period the return of the project is realized, giving the entrepreneur wealth yk in case of success or xk in case of failure. At this stage the entrepreneur must decide again whether to continue in business or to quit and take the outside value.

[3]Subindex 2 corresponds to the number of mass points of the payoffs'distribution

[4]Note that the the return in (2) is unbounded (due to $A\beta > 1$), so we must assume that the agents' utility function $u(c)$ is such that the solution to (2) exists. This is true for a general class of the utility functions, including CRRA.

Letting $V(w)$ denote the value for an entrepreneur with wealth w at the begging of the period (exit stage), the value $V_E(w)$ at the investment stage is given by:

$$
\begin{aligned}
&V_E(w) = \max_{k,x,p}\{u(w-k) + \beta[pV(yk) + (1-p)V(xk)]\}, \\
&s.t. \qquad y = x + \frac{A-x}{p},
\end{aligned} \tag{3}
$$

In turn, the agent's initial value and exit decision are given by:

$$
V(w) = \max\{V_E(w), R(w)\}. \tag{4}
$$

We will call (3)-(4) the *optimal risk choice problem* (ORCP). Its solution gives the entrepreneur's exit decision, consumption path and project risk choice. The latter is the main focus of our work. An entrepreneur who chooses $p < 1$ invests into a risky project. The risk of business failure is larger for smaller values of p. As we show below, risk taking decreases with the level of wealth while total investment increases. Using the scale of the project (i.e. total investment) as a measure of business size, the model implies that smaller firms take more risk and face higher failure rates.

2.2 The Solution

This section characterizes the solution to the entrepreneurial choice problem. We divide the problem in three steps: 1) project risk choice; 2) consumption/investment decision and 3) exit decision. A sketch of the main features of the solution is given here. More details and proofs are provided in the appendix.

2.2.1 Project risk choice

Let k denote the total investment in the project. The expected return is then Ak, independently of the level of risk chosen. Figure 1 illustrates this decision problem. If the end-of-period wealth is below w_E, the entrepreneur will quit and take the outside option; if it is above he will stay in business. The continuation value $V(Ak)$ is thus given by the envelope of the two concave[5] functions, $R(w)$ and $V_E(w)$. As a consequence of the option to exit, this value is not a concave function in end-of-period wealth.

[5]The outside value $R(w)$ is concave by assumption. Lemma 1 establishes the concavity of $V_E(w)$.

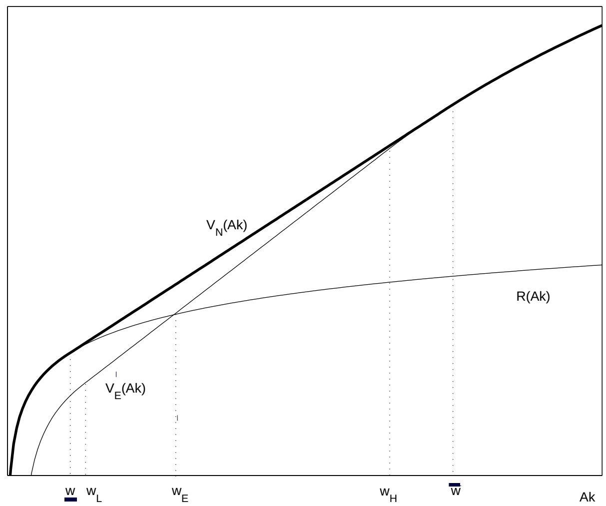

Figure 1: End-of-period expected value $V_N(Ak)$ of entrepreneur

The choice of project risk is used to randomize end-of-period wealth on the two points $\underline{w}$ and $\overline{w}$ depicted in Figure 1, giving an expected value that corresponds to the concave envelope of the two value functions considered.[6] Let $V_N(Ak)$ denote this function:

$$V_N(Ak) = \begin{cases} R(Ak) & \text{for } Ak \leq \underline{w}, \\ R(\underline{w}) + (Ak - \underline{w}) / (\overline{w} - \underline{w}) (V(\overline{w}) - R(\underline{w})) & \text{for } \underline{w} < Ak < \overline{w}, \\ V(Ak) & \text{for } Ak \geq \overline{w}. \end{cases}$$

As shown in the figure, depending on the level of investment k, we may distinguish three cases: If $Ak \leq \underline{w}$, it is optimal not to randomize and exit in the following period. In case $Ak \geq \bar{w}$, it is also optimal to invest in the safe project. Finally, if $\underline{w} < Ak < \bar{w}$, it is optimal to randomize between the two endpoints.

More formally, this choice is implied by the first order conditions for the

[6]The figure assumes that $R(w)$ and $V_E(w)$ have a unique intersection point. This obviously depends on the outside value function. In section 3 we derive this outside value function from a model of entrepreneurial choice and show that the single crossing property holds.

dynamic problem of the entrepreneur (3):

$$\begin{aligned}(x):&\qquad V'(yk)=V'(xk),\\(p):&\qquad V'(yk)=\frac{V(yk)-V(xk)}{yk-xk}.\end{aligned}\tag{5}$$

These two equations say that the possible project's payoffs must coincide with the tangent points $\underline{w}$ and $\overline{w}$. Thus the optimal randomization is accomplished by choosing the project with $x=\underline{w}/k$, $y=\overline{w}/k$ and $p=(Ak-\underline{w})/(\overline{w}-\underline{w})$. Note that the probability of the high payoff (”success”) increases linearly with the scale of the project k.

2.2.2 Consumption/Investment choice

Letting w denote the wealth of the entrepreneur and since projects are self-financed, the level of consumption $c=w-k$. The consumption/investment decision is the solution to the following problem:

$$V_E(w)=\max_k u(w-k)+\beta V_N(Ak).\tag{6}$$

As shown before, V_N is the concave envelope of $V(w)$ and $R(w)$ .The following lemma states that V_N and thus V_E are concave functions.

Lemma 1 *The functions $V_N(w)$ and $V_E(w)$ are concave.*

We proceed to characterize the consumption/savings decision. The first order conditions for problem (6) are given by:

$$u'(w-k)=\beta AV_N'(Ak),$$

where

$$V_N'(Ak)=\begin{cases}R'(Ak) & \text{for } Ak\le\underline{w}\\ R'(\underline{w})=V'(\bar{w}) & \text{for } \underline{w}<Ak<\bar{w}\\ V'(Ak) & \text{for } Ak\ge\bar{w}.\end{cases}$$

The above first order conditions imply that consumption is constant at a level c^* given by $u'(c^*)=\beta AR'(\underline{w})$ when optimal investment Ak falls in the risk taking region, $\underline{w}<Ak<\bar{w}$. This corresponds to initial wealth levels w such that $w_L<w<w_H$, where $w_L=\underline{w}/A-c^*$ and $w_H=\bar{w}/A-c^*$. In

this region, investment $k = w - c^*$ increases linearly with the agent's wealth and the probability of a successful realization increases. Outside this region, there is no risk taking and consumption and investment increase with wealth.

The above conditions also imply that once the wealth of the entrepreneur surpasses the threshold w_H, it grows continuously, remaining above $\bar{w}$ forever after. From that point on, there is no more risk taking. This is a special feature of the model explained by the existence of riskless projects and the absence of risk premia. In a more realistic setup, firms could recur in the set $w_L < w < w_H$ after a series of bad shocks.

A sharper characterization of the value of the entrepreneur $V_E(w)$ follows from the above comments. This value coincides with the value of a risk-free entrepreneur $V_l(w)$ for $w \geq w_H$; is linear in the intermediate range $w_L < w < w_H$; coincides with the value of the entrepreneur that invests into a safe project and quits in the following period for $w \leq w_L$. Note that if risky projects were not available, active entrepreneurs would face two options - either to stay or exit- at the beginning of the following period. Risk taking increases the entrepreneur's utility by eliminating this nonconcavity in the continuation value.

2.2.3 The optimal exit decision

The entrepreneur exits when $R(w) > V_E(w)$. A sufficient condition for this region to be nonempty is that $(1-\beta) R(0) > u(0)$. This condition is satisfied when the outside option includes some other source of income. When $R(w)$ crosses $V_E(w)$ at a unique point w_E -as in the example considered in section 3, this becomes the threshold for exit.

Suppose exit is given by a threshold policy with cutoff value w_E. Three situations may arise: *(i)* $w_E \leq w_L$; *(ii)* $w_L < w_E < w_H$ and *(iii)* $w_H < w_E$. For the last case, risk-taking would not be observed since entrepreneurs would exit once they are in the risk-taking region. In the other two cases risk-taking is observed. In case *(ii)*, the entrepreneur invests in a risky project, exits if it fails and stays forever if it succeeds. There is an upper bound on the probability of failure given by $(1 - p(w_E)) < 1$. In contrast, in case *(i)* there is no upper bound on project failure.[7]

[7]The example given in section 3 suggests that while case *(i)* is atypical, the other two cases may occur.

2.2.4 Characterization of the solution

The following Proposition summarizes the results derived in this section.

PROPOSITION 1 *Suppose the entrepreneur selects projects from the class $\Omega_2(A)$ with an expected return $A > 1/\beta$. Suppose the outside value of the entrepreneur $R(w)$ is concave. If $R(w)$ and $V_E(w)$ have a unique intersection point w_E, then: there exist wealth levels $w_L < w_H$ such that:*

(i) Entrepreneurs exit if $w \leq w_E$ and stay if $w > w_E$;

(ii) Letting $w_* = \max\{w_L, w_E\}$ and $w^* = \max\{w_H, w_E\}$:

  (a) entrepreneurs invest in risky projects and stay in business forever if $w \geq w^*$;

  (b) invest in risky projects if $w \in (w_*, w^*)$ and stay in business the following period with probability $p(w) = (Ak(w) - \underline{w})/(\overline{w} - \underline{w})$;

  (c) invest in safe projects if $w \leq w_*$ and exit in the following period.

(iii) If an entrepreneur chooses a risky project (i.e. $w \in (w_*, w^*)$), the probability of survival $p(w)$ and the level of investment $k(w)$ are increasing in w, while consumption $c(w)$ is constant.

The previous Proposition has some immediate implications for firm dynamics. In the following, we measure a firm's size by the level of its investment k.

CORROLLARY 1 *(i) Survival probability increases with firm size (ii) Conditional on survival, smaller firms have higher growth rates.*

The above results assume a single crossing of the functions $R(w)$ and $V_E(w)$. In case of multiple crossings, there will be more than one region of risk-taking. Within each of these regions, total investment will increase and the risk of failure decrease with wealth.

2.3 Extending the Class of Projects

In the above analysis we assume that the only projects available to entrepreneurs have returns concentrated in two points. In this section we show that this restriction is without loss of generality..

Let $\Omega(A) = \{\lambda | \int d\lambda = 1 \text{ and } \int z d\lambda(z) = A\}$. This is the set of all probability distributions of returns with mean A. Obviously, the class $\Omega_2(A)$ considered earlier is a subset of $\Omega(A)$. Thus, if we assume the entrepreneur chooses a project from $\Omega(A)$, all projects $(x, p) \in \Omega_2(A)$ are still available to him.

The following Proposition gives our main result in this section.

PROPOSITION 2 *Suppose the outside value of the entrepreneur $R(w)$ satisfies the assumptions of Proposition 1. Let the entrepreneur choose any project from $\Omega(A)$, where $\beta A > 1$. Then the distribution of returns of the project chosen is concentrated in two points, so the entrepreneurial decision is identical to the one described in Proposition 1.*

The proof of Proposition 2 is very intuitive. The decision problem (3) of the active entrepreneur is now given by:

$$\begin{aligned} V_E &= \max_{k,\lambda} \{u(w-k) + \beta \int V(zk) d\lambda(z)\}, \\ \text{s.t.:} & \qquad \int d\lambda = 1 \text{ and } \int z d\lambda(z) = A. \end{aligned} \tag{7}$$

Together with the exit decision (4) it forms a well defined dynamic programming problem which has a unique solution.

If $V_E(w)$ coincides with the value function (3) found in the previous section, the value of the entrepreneur $V(w)$ is a piecewise concave function over the intervals $(0, w_E)$ and $(w_E, +\infty)$ (this follows from Lemma 1). For any given distribution of returns λ, Let x_λ and y_λ be the expected returns on the intervals $(0, w_E)$ and $(w_E, +\infty)$, respectively. Let $p_\lambda = \lambda(w_E, +\infty)$, the probability of the upper set of returns. Consider an alternative project that pays either x_λ (with probability $1 - p_\lambda$) and y_λ (with probability p_λ). Given that the value function is concave in the two regions considered, the expected return of this project is at least as high as the original one.

3 An Example: Occupational Choice Model

In Section 2 no interpretation was provided for the outside value. In this Section we endogenize $R(w)$ in a simple occupational choice model, study conditions under which risk taking will and will not occur, and provide some simulation results.

The decision problem of the entrepreneur is defined by (3) and (4) of the previous section. An entrepreneur becomes a worker if he exits from business. Workers receive wage $\phi > 0$ every period and save in a risk free bond to smooth consumption over time. The rate of return to the risk-free bond is r. We assume that $1/\beta \leq 1 + r < A$. This assumption, combined with self-financing, implies that only relatively wealthy agents are willing to become entrepreneurs.

At the beginning of every period a worker gets randomly "hit with an idea" that allows him to become an entrepreneur. The probability of this event is $0 \leq q \leq 1$. If the worker chooses to become an entrepreneur he pays a fixed cost $\eta \geq 0$ and receives no wage ϕ. If the worker decides not to become an entrepreneur, his situation becomes identical to that of a worker who was not faced with this opportunity. Let $R_s(w)$ denote the value of the worker conditional on not becoming an entrepreneur in the current period. Prior to the realization of this shock, the value to the worker $R(w)$ is given by

$$R(w) = (1 - q)R_s(w) + q \max\{V_E(w - \eta), R_s(w)\}. \tag{8}$$

If the worker does not enter entrepreneurship, he must decide how much to save in the risk free bond, so the value $R_s(w)$ is given by:

$$R_s(w) = \max_a \{u(w + \phi - a) + \beta R((1 + r)a\}. \tag{9}$$

The above two equations, together with (3) and (4) fully characterize the behavior of the agents in this discrete occupational choice model.

The worker decides to become an entrepreneur only if: (i) he gets an opportunity; and (ii) his current wealth level is such that $V_E(w - \eta) \geq R_s(w)$. Denote by w_N the lowest wealth level at which workers are willing to enter entrepreneurship, $V_E(w_N - \eta) = R_s(w_N)$, which determines the entry threshold rule for the workers. The uniqueness of w_N is established in Lemma 2

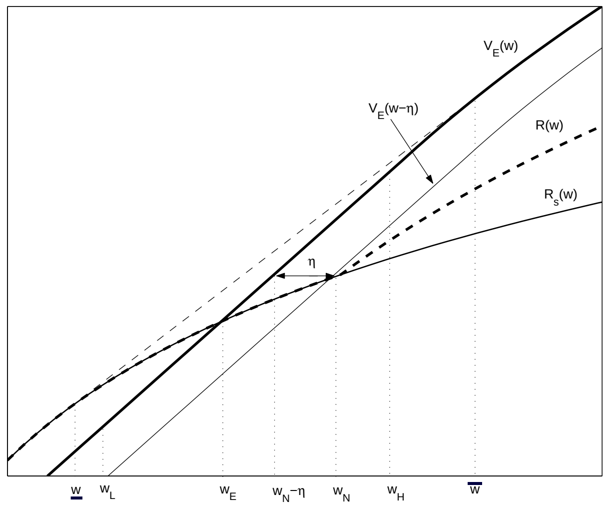

Figure 2: Value functions' allocation in the occupational choice model.

below. The entrepreneurial exit rule and investment decision were described in Proposition 1. In order to apply Proposition 1 to this occupational choice model we need to verify the uniqueness of the entrepreneurs' exit wealth level w_E. This is also done in Lemma 2:

Lemma 2 *There exist $w_N \geq w_E > 0$ such that (i) $V_E(w) > R(w)$ if and only if $w > w_E$; (ii) $V_E(w - \eta) > R_s(w)$ if and only if $w > w_N$.*

Figure 2 depicts the value functions previously defined. The intersection of $R_s(w)$ and $V_E(w - \eta)$ (solid thin lines) determines the entry wealth level w_N. For $w > w_N$, $V_E(w - \eta) > R_s(w)$, so every worker chooses entrepreneurship whenever this option is available to him. Since this occurs with probability q, $R(w)$ is a linear combination of $V_E(w - \eta)$ and $R_s(w)$ for $w \geq w_N$. If $w \leq w_N$, the worker does not enter entrepreneurship, independently of the realized opportunity, so $R(w) = R_s(w)$ in this region.

Apart from single crossing, Proposition 1 also assumes concavity of the outside value $R(w)$. However, it is easy to verify that the value function defined by (8) is not concave to the left of w_N.[8] In order to apply Proposition 1 to describe the entrepreneurs' behavior, Lemma 3 shows that the

[8]Jumps in the worker's consumption appear since the worker's wealth increases over time, and his continuation value, $R(w)$, has at least one kink.

entrepreneurs' project choice is unchanged if we replace the outside value $R(w)$ with its concave envelope $\widehat{R}(w)$. Note that the exit rule is determined by the intersection of $V_E(w)$ with $R(w)$, not with its concave envelope. As before, we denote by $\underline{w}$ and $\overline{w}$ the tangent points of $R(w)$ and $V_E(w)$ with their common tangent line; and by w_L the wealth level at which entrepreneur's expected payoff is equal to $\underline{w}$.

Lemma 3 *If the inequality $1/\beta \leq 1 + r < A$ holds then (i) $w_E > w_L$; (ii) entrepreneurial optimal behavior is unchanged if, instead of $R(w)$, its concave envelope $\widehat{R}(w)$ is used in (4).*

The results of Lemma 3 are based on the following intuition. Obviously, the common tangent line to $V_E(w)$ and $R(w)$ coincides with the common tangent line to $V_E(w)$ and $\widehat{R}(w)$, and so do the correspondent tangent points $\underline{w}$ and $\overline{w}$. Thus the project choice of entrepreneurs randomizing between $\underline{w}$ and $\overline{w}$ is not affected if $\widehat{R}(w)$ is used instead of $R(w)$. Potentially, other randomization regions may occur below w_L if $R(w)$ is not concave to the left of $\underline{w}$. But (i) of Lemma 3 together with the uniqueness of w_E established in Lemma 2 above imply that $V_E(w) < R(w)$ for all $w < w_L$, and thus no entrepreneur randomizing over the lower values of $R(w)$ chooses to stay in business. Therefore, the observed project choice of active entrepreneurs is unchanged if $R(w)$ is replaced with its concave envelope $\widehat{R}(w)$.

Lemmas 2 and 3 imply that all the assumptions of Proposition 1 hold and thus it can be used to characterize the behavior of entrepreneurs. The main properties of the agents' decision are summarized below:

PROPOSITION 3 *If entry, exit and investment choice is defined by (3), (4), (8), and (9), then there exist $0 < \underline{w} < w_E < w_N$ and $0 < w_H < \overline{w}$ such that*

(i) workers with wealth levels $w > w_N$ enter into entrepreneurship with probability q;

(ii) entrepreneurs exit from business if $w \leq w_E$ and stay otherwise;

(iii) entrepreneurs with wealth levels $w_E \leq w \leq \max\{w_E, w_H\}$ invest into risky projects, survival rates $p(w)$ of their businesses are bounded away from zero and increase with w, investment $k(w)$ also increases, while consumption $c(w)$ stays constant;

(iv) entrepreneurs with wealth levels $w > \max\{w_E, w_H\}$ invest into fully safe projects and stay in business forever; their investment $k(w)$ and consumption $c(w)$ increase in w.

From Proposition 3 it follows that if an entrepreneur enters with wealth levels $w < w_H$, he invests in a risky project, obtaining either $\underline{w}$ or $\overline{w}$ at the beginning of the following period, depending on the realization of the project's return. If the low return is realized, the entrepreneur exits in the next period with wealth $\underline{w} < w_E$, otherwise he invests into a fully safe project from next period on. The probability $p(w)$ of the high return determines the survival probability of the establishment. Those entrepreneurs who enter with higher levels of wealth choose higher $p(w)$ and thus are more likely to stay in business.

Risk taking not necessarily occurs in this environment. In particular, if the wealth level net of entry cost $(w_N - \eta)$ with which the entrepreneur starts business exceeds the upper bound of the randomization region (w_H), risky investments will never be chosen. In the environment described above this happens if $q = 1$, i.e., if there is no uncertainty about being able to enter entrepreneurship.

PROPOSITION 4 *There exist $\overline{q} < 1$ such that risk taking occurs if and only if $q \leq \overline{q}$.*

The result in the above Proposition is driven by the agents' desire to smooth consumption over time. In the absence of uncertainty, the worker correctly foresees his continuation value $\max\{R(w), V_E(w)\}$ and thus chooses a savings policy such that the downward jump in consumption at the moment he enters entrepreneurship is small. Correspondingly, the kink in the value function at the point of entry is so small that randomizations are not beneficial.

In contrast, in the presence of an uninsurable shock to entrepreneurial opportunities, the continuation value and optimal savings policy prior to the shock realization change after the resolution of this uncertainty. If the ex-post desired wealth increases compared to the ex-ante desired level , current consumption would obviously go down. The possibility of risk taking allows an entrepreneur to decrease the size of this downward jump in consumption. In particular, as a consequence of the outside opportunity, the entrepreneur consumes more than the safe investment policy suggests - actually, as much as the entrepreneur with wealth level w_H does - and the rest of his wealth is used to invest in risky projects. In the following period, independent of the project's payoff, he consumes $\overline{c} > c^*$ such that $u'(\overline{c}) = R'(\underline{w}) = V_E'(\overline{w})$. And only the future path of consumption will depend on the realized return of the risky project. Finally, only entrepreneurs with relatively low wealth levels use this consumption smoothing mechanism - because it is only for them that the outside opportunity provides the necessary insurance in case of project failure.

The following numerical example illustrates the implications of the preceding theoretical analysis. We use the following parameter values: $\beta = 0.98$, r is equal to the inverse of the rate of time preference $1/\beta$, the entry cost $\eta = 0$, and the expected entrepreneurial return A is 10 percent higher than the return to the risk-free bond. In this example we choose a logarithmic utility function $u(c) = \ln(c)$ and later consider how the behavior of entrepreneurs changes as the coefficient of relative risk aversion increases. Results from the simulations are presented in Table 1 below.

Table 1: Risk taking and welfare gains, $u(c) = \ln(c)$

	$p(w)$	Max. ΔW (%)	Av. ΔW (%)
$q = 0.1$	0.41	7.1	3.1
$q = 0.2$	0.71	1.8	0.7
$q = 0.3$	0.86	0.5	0.2
$q = 0.4$	0.93	0.1	0.05
$q = 0.55$	1	-	-

The first column of Table 1 reports the lowest survival rate that is observed in the economy. If the opportunity to enter entrepreneurship is a rare event ($q = 0.1$), workers faced with this opportunity at relatively low wealth levels decide to enter and invest into very risky projects, exiting with probability $1 - p(w) = 0.59$ in the following period. The welfare gain that these agents obtain due to the availability of risky projects is fairly high: they

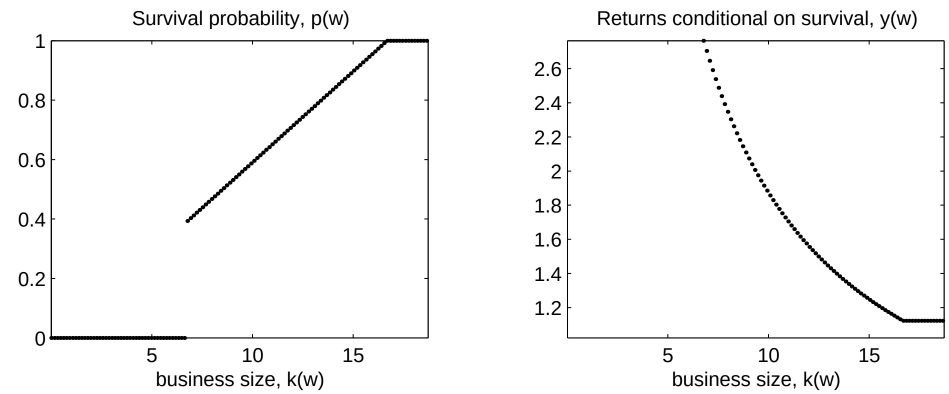

Figure 3: Project's survival and return, $u(c) = \ln(c)$, $q = 0.1$, $\eta = 0$.

would lose 7.1 percent of their life-time consumption if the risky investment were not available. Those agents who enter entrepreneurship with higher wealth levels take less risk and, correspondingly, benefit less from risky investments. The average welfare gain in the region of risk taking, conditional on entering, is equal to 3.1 percent of life time consumption. Workers' welfare also increases due to the opportunity of risk taking in the future. If $q = 1$, the expected life time consumption of the worker who starts with $w = 0$ rises by 1.3 percent when investments into risky projects are possible.

Figure 3 illustrates the relationship between the size of the projects $k(w)$, their survival rates $p(w)$, and returns $y(w)$ to investment (conditional on survival) observed in the economy. As was summarized in Corollary 1, larger establishments are more likely to survive, but experience lower rates of returns. Note that in this economy exit from entrepreneurship occurs only due to the presence of risk taking: if the risky projects are not available, the homogeneity of expected project's returns together with condition $1/\beta \leq 1 + r < A$ implies that all entrepreneurs continue operating their businesses once the entry decision has been made.

With the introduction of an entry cost, there are two counteracting effects: (a) workers enter into entrepreneurship at higher wealth levels and, consequently, choose higher survival probabilities; (b) the value of the worker decreases and becomes flatter, thereby aggravating the nonconcavity of the continuation value of the entrepreneur. Correspondingly, the potential gain from risk taking increases. For example, if $\eta = \phi$ (entry cost is equal to the wage level) and $q = 0.1$, the lowest observed probability of business survival is 0.61, while the gain in welfare for the most risky entrepreneurs is 9.3 percent of life-time consumption.

Going back to Table 1, we see that less risk taking occurs as the probability of the entrepreneurial opportunity increases. This happens because as q increases, workers make better predictions about their future optimal investment level, and thus when the entrepreneurial opportunity arrives it leads to smaller jumps in consumption, decreasing entrepreneur's incentives for risk taking. According to Proposition 4, there exists a maximum $\overline{q}$ such that no one makes risky investment if entry into entrepreneurship is possible with probability higher than $\overline{q}$. In the simulated example $\overline{q} = 0.52$.

Table 2: Sensitivity to the risk aversion coefficient, $q = 0.1$, $u(c) = \frac{c^{\sigma}-1}{1-\sigma}$

	$p(w)$	Max. ΔW (%)	Av. ΔW (%)
$\sigma = 1$	0.41	7.1	3.1
$\sigma = 2$	0.67	2.7	1.2
$\sigma = 3$	0.73	1.7	0.9
$\sigma = 4$	0.78	0.9	0.5

Finally, as agents become more risk averse, entrepreneurs take less risk and obtain lower welfare gain from it. Table 2 compares the range of risk taking and the associated welfare gains for different values of the coefficient of relative risk aversion σ. For example, if $\sigma = 2$, the minimal $p(w)$ increases to 0.67 and the largest welfare gain from making risky investments decreases to 2.7 percent of life-time consumption. However, even for large values of the coefficient of relative risk aversion ($\sigma = 4$), entrepreneurs that get an opportunity to enter at low wealth levels make risky investments (obtaining a welfare gain of 0.9% from the availability of risky projects) and about a quarter of them (22 percent) exit in the following period.

The type of uncertainty introduced in the model is not the only one which leads to risk taking. Any idiosyncratic shock that, in the absence of risk taking, would make the agent save more than planned in advance, may lead to risk choice. As an example, similar results were obtained in an environment where entrepreneurs are subject to shocks in the level of expected returns ($A = \{A_L, A_H\}$), even if these shocks are not persistent and for both values $A_i > 1 + r$. In particular, if at the time of entry an entrepreneur faces the high return, he will try to use the current good shock to spread the increase in consumption into future periods. In the absence of risky projects, this would require a sizable increase in investment and a corresponding fall in current consumption. The opportunity of risk taking allows the entrepreneur to spread the currently beneficial situation over future periods through a risky investment, obtaining a high return in case the project is successful. Again, only entrepreneurs with relatively low wealth levels are able to use this mechanism, because only for them the outside option serves as insurance against project failure.

4 Final Remarks

Entrepreneurship is risky, but there appears to be no premium to private equity. Any theory addressing this puzzle must rely, directly or indirectly, on a positive -or at least neutral- attitude towards risk. Earlier papers in this area assume directly that entrepreneurs have a lower degree of risk aversion. In our paper, the indirect utility function of the entrepreneur has a convex region, where riskiness is desired. However, this nonconvexity is created by the existence of an outside opportunity so it does not rely on assumptions about preferences for risk.

As a theory of risk taking, our model has specific implications. The combination of the outside option and financing constraints imply a desire for risk at low wealth levels, close to the exit threshold. As a consequence, risk taking decreases with the level of wealth, giving rise to the positive correlation between size (measured by investment) and survival found in the data. This is an implication of our theory that would be hard to derive just from the heterogeneity of preferences. As an example, Cressy (2000) justifies risk-taking by entrepreneurs assuming that higher wealth makes agents less risk averse. A consequence of this assumption is that larger firms should take more risk and thus exhibit more variable growth, which is counter to the data.

Entrepreneurs in our model are self-financed. This is obviously an extreme form of borrowing constraint. A recent paper by Clementi and Hopenhayn (2002), derive borrowing constraints as part of an optimal lending contract in the presence of moral hazard. The nonconvexity due to an outside (liquidation) option is also present in their model and there is a region where randomization is optimal.

We have chosen to keep our model stylized in order to get sharper results. As a downside, our model has some special unrealistic features. Most notably, risk-taking occurs only once; if the outcome is favorable, the entrepreneur takes no further risk and stays in business forever. These results follow from the possibility of choosing projects with arbitrary risk levels (including a fully safe one) and equal returns. Risk taking could last for more than one period if the variance of returns was bounded above. On the other hand, a lower bound on project risk or a return/risk trade-off, generates the possibility of future exit by firms that are currently outside the randomization region.

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