Multiple Shareholders and Control Contests*

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Abstract

We consider the allocation of corporate control in a shareholders' meeting in a company with large shareholders and a sea of atomistic shareholders. To attract the vote of small shareholders, the large shareholders submit competing proposals to limit benefit taking, knowing that the turnout and the vote of the small shareholders will depend on the relative merit of the proposals. Multiple shareholders are thus viewed as a device to reduce private benefits through competition for control. The optimal ownership will give the more efficient large shareholders enough shares to gain control, but allocate a relatively larger fraction of shares to the less efficient shareholder in order to reduce rents. We investigate when multiple blocks are likely to collude against minority shareholders, and when the ownership structure is immune to retrading.

Keywords: Controlling shareholders, private benefits of control, shareholder votes, multiple shareholders, shareholder coalitions, retrading-proofness.

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1 Introduction

In Berle and Means' (1932) classic theory of the modern corporation, companies are characterized by the dichotomy between employed managers and dispersed shareholders, but in what appears to be a contradiction, many of the largest publicly traded companies in the United States still have large shareholders with controlling share blocks. The concentration of ownership is even more salient in most other countries.[1] As record numbers of companies receive a public listing and large-scale privatization programs are undertaken, the understanding of ownership concentration and its effects attracts considerable attention far beyond a purely academic research interest. For example, ownership concentration is now frequently credited to affect firm performance and even the growth prospects of countries (e.g., Ang et.al.(2000), Claessens et.al. (2000)).

The theoretical literature on ownership structure has largely been confined to the analysis of a single large shareholder surrounded by a sea of atomistic shareholders.[2] Empirical literature on ownership structure, however, shows that multiple blockholders coexist in many companies. For example, in Germany, a quarter of large publicly listed companies have two or more large shareholders (Weigand (1999)), and so do 58 % of closely held corporations in the US (Gomes and Novaes (1999)).

The objective of this paper is to investigate the structure of corporate governance and the allocation of corporate control in the presence of multiple large shareholders. It provides a first attempt to model the full scope of strategic interactions between the various blockholders on the one hand, and between any of the blockholders and the small shareholders on the other hand. Even if there are just two blockholders - the case to which we stick in this paper - this is a game with three parties. The motivation to investigate this tripartite interaction including the possible voting coalitions is to get a handle on the following questions: How is corporate control allocated in a non-entrepreneurial company? Are small shareholders better off when facing multiple instead of a single dominant shareholder? When would we expect the large shareholders to collude in their efforts to gain control over the company?

Our model contributes also to the following two more methodological issues: Should we think of large shareholders as sharing power or as competing for control? And can we give a sharper meaning to the ubiquitous, but rather imprecise notion of a "controlling equity

[1] For a detailed account, see e.g. La Porta, Lopez-de-Silans and Shliefer (1999).

[2] Jensen and Meckling (1976), Shleifer and Vishny (1986), Bolton and von Thadden (1995), Maug (1995), Admati, Pfleiderer and Zechner (1993), Burkart, Gromb and Panunzi (1997).

stake"? How is it, as LaPorta, Lopez-de-Silanes and Shleifer (1999) put it, that in practice "10 per cent ownership of a U.S. firm (are considered) to be sufficient for control", and how can control be exercised with such a small stake, which would be tiny, say, in the context of parliamentary coalition formation?

In our model, the allocation for control is decided by a vote in the shareholders' meeting, which is interpreted as the vote for the board composition. The vote is cast on a one share-one vote basis, but small shareholders may choose to stay away if the benefits of participation do not justify the cost of getting informed.

We then consider two large shareholders submitting competing platforms proposals to the vote. A proposal's merit is, on the one hand, dependent on the shareholder's ability to develop the company's strategy. The large shareholders differ in their ability to do so. On the other hand, they can pledge (and commit) to limit the private benefits of control that will be taken at the minority shareholders' expense. In order to assemble a voting majority including the necessary votes from small shareholders, the two large shareholders will engage in a vigorous bidding competition pledging to limit the private benefits of control. The competition limiting private benefits is viewed as the key mechanism how the presence of multiple blockholders protects minority shareholders and can create value.

The benefit that the controlling shareholder gets away with is the rent explained by *(i)* differences in the shareholder's ability to create value and *(ii)* the difference in block size explaining how many small votes she needs to attract to her proposal to get a voting majority. In principle, therefore, the efficient allocation would be to give the efficient shareholder just enough shares that she can manage to win the vote, but to allocate a relatively larger block to the less efficient shareholder so that she can overcome her ability handicap by an advantage in block size, and drive equilibrium rents towards zero. A first, and surprising, conclusion is that in companies with optimized ownership structures, we should expect the second largest blockholder, not the largest blockholder to be in possession of effective control powers.

But this ownership structure must be seen as a sort of first-best benchmark. It does not take into account two constraints that may severely limit the feasibility of the target ownership structure.

First, rather than compete for the votes of small shareholders, blockholders may collude and rally behind a common platform. One might expect that the large shareholders would always prefer to do so. For example, in many countries shareholders are allowed to write and enforce explicit and shareholder agreements tying their votes.[3] But since there is a

[3]Only Italy and France, to our knowledge, require that listed companies report such agreements.

three-way interaction, the sustainability of collusion may be hampered by the possibility that any of the shareholders in the collusive coalition could defect and enlist instead the small shareholders. We find that shareholder coalitions are surprisingly hard to sustain: No "exploitative" coalition (splitting the maximum of private benefits among its members) will form if the efficient shareholder owns the largest block, or if her ability edge is large enough.

Second, our first best ownership structure neglects the possibility that shares can be retraded. Multiple blocks lead to competition that is undesirable from the blockholder's view, and a simple alternative to forming a voting coalition is to merge the blocks through block sales. Obviously, an ownership structure can only be immune against block mergers if the merger implies a corresponding value-reducing effect. In our model, we suggest that the two blockholders supply complementary monitoring services. We show that an ownership structure with multiple blocks will be retrading-proof if the monitoring complementarity is large enough, and if the efficient shareholder has a sufficiently large block already to make the reduction in control benefits competition relatively unattractive. We investigate also the possibility of downstairs share purchases. Again, we find that none of the shareholders will try to increase her stake by buying on the floor if the lead of the efficient shareholder is sufficiently large.

While complementarity in monitoring services is our reference why multiple block may be robust to retrading, there are, in our opinion, a couple of other mechanisms leading to retrading-proofness: *(i)* Multiple blocks allow a better risk sharing between large shareholders;[4] *(ii)* The implicit commitment to protect minority shareholders is attractive for firms that expect to go back to the capital market (firms with high growth opportunities) in the future; *(iii)* A second blockholder allows for a contingent shift in control; *(iv)* Asymmetric information about the firm's prospects or about private benefits.

To summarize, immunization of an ownership structure against collusion and retrading will usually require that the controlling shareholder also be the owner of the largest block.this indicates that the feasible ownership structure may be distant from the first best ownership structure, i.e. from the ownership structure that maximizes firm value and that an initial seller would wish to establish. In practice, the typical presumption is that the largest blockholder is also the controlling shareholder, which may well indicate how important theee constraints are.

Our model leads to several original empirical implications. First, variables like the

[4] Risk aversion is known to be a major obstacle to the sustainability of large share blocks (Admati, Pfleiderer and Zechner (1993))

maximum benefits of a monopolistic shareholder or an exploitative coalition or the social cost of benefit-taking can be taken to capture differences between legal system and the degree of investor protection they offer. Our findings offer only limited support for the idea that legal protection and competing shareholders are substitutes: The weaker the legal protection, they stronger usually the gain that can be made by doing away with shareholder competition, via collusion or retrading.

Second, for the empirical study of shareholder monitoring and block transfer premia, our paper points out that shareholder heterogeneity (captured by abilities) is as important as block size. Only controlling blocks should trade at a premium, while minority blocks trade at a discount, and block size is not a valid proxy for control power.

Two other recent papers have recently explicitly addressed the issue of competing blocks. In Gomes and Novaes (1999), multiple shareholders are less prone to dilute minority shareholders by in poor projects with positive private benefits, but they tend to disagree more often and valuable projects and will disagree. Bennedsen and Wolfenzon (2000) consider only closely held corporations. The controlling coalition of shareholders will take private benefits at the expense of minority shareholders, but less so if the winning coalition internalizes the negative effects of benefit-taking.

Our model shares with both papers the focus on conflicts about private benefits of control, but it adds a number of new elements. For example, both Gomes and Novaes and Bennedsen and Wolfenzon seem to implicitly acknowledge the importance of the retrading issue, without addressing it. Also, Gomes and Novaes have no role for minority shareholders whereas Bennedsen and Wolfenzon do not consider a non-cooperative interaction of the large shareholders, while our paper does both. We view as the main innovations of our paper *(i)* the explicit competition for control among the blockholders *(ii)* an explicit role for dispersed shareholders; *(iii)* the analysis of collusion *(iv)* and the analysis of retrading-proofness.

Related themes of multiple large shareholders have also been touched in other work. Pagano and Roell (1998) suggest that multiple blocks commit the firm to protect minority investors. Among the papers arguing that multiple blockholders are unlikely to emerge, Zwiebel (1995) determines ownership structures in a general equilibrium model, and shows that investors would sort to one large shareholder per firm only precisely in order to eschew the sort of competition over benefits that we model. Winton (1993) emphasizes the free-rider problem in monitoring efforts among multiple large shareholders. Similarly, in Bolton and von Thadden's (1998) liquidity-control trade-off, multiple large shareholders would increase the liquidity costs without offering compensating advantages in monitoring.

The paper is organized as follows. In Section **??**, the model is laid out. Section **??**

contains the basic analysis of the bidding strategies and the voting outcome, and its consequences for the optimal ownership structure. Section **??** gives a simplification of the model that allows closed form characterizations. In Section **??**, collusion is analyzed, and retrading-proofness in Section **??**. Empirical implications are discussed in Section **??**. Section **??** concludes.

2 The Model

We consider a company with two large shareholders, $i = 1, 2$, and an ocean of small shareholders, s. The company could be privately held or publicly listed. The continuum of small shareholders is modelled as the interval $[0, 1]$, endowed with Lebesgue measure λ. Prior to the contest, the two shareholders are endowed with fractions α_1 and α_2 of the shares of the company, with $\alpha_1+\alpha_2 = \bar{\alpha}$. The remainder of the shares, $1-\bar{\alpha}$, are distributed uniformly among the small shareholders.

A shareholders' meeting is convened in order to allocate control power. The decision is reached by a vote of the shareholders present, where each share carries one vote. The allocation of control power should be understood as the vote for the composition of the board of directors. We assume that only the two large shareholders can propose candidates for the company's board, and there is uneven number of seats; as a result, the outcome of the shareholders' vote will be an unequivocal allocation of control power to one of the two large shareholders. For the small shareholders, participation in the shareholders' vote is costly.[5] We define a measurable function ξ from $[0, 1]$ to $\mathbf{R}^+$, describing the voting cost of the small shareholders. We let $F(\kappa)$ describe the commonly known distribution of voting costs among small shareholders, i.e. $F(\kappa) = \lambda(\{x, \xi(s) \leq \kappa\})$. We suppose on the other hand that the large shareholders do not incur any cost to participate in the meeting.[6]

The winning shareholder defines the company's strategy which is an essential determinant of firm value. The two large shareholders have different abilities to define the strategy. Let θ_i denotes the ability of the controlling shareholder. Without loss of generality, we suppose that shareholder 1 is more competent than shareholder 2, $\theta_1 \geq \theta_2$ and let $\Delta\theta = \theta_1 - \theta_2$

[5] This cost reflects as much the effort to get informed about the proposals as it reflects the cost of attending the meeting in person. Proxy voting, therefore, may mitigate, but not eliminate this cost.

[6] This assumption is made for convenience, but does not imply any substantive difference between small and large. To see this, assume there is finite number n of shareholders, and assume all shareholders (small and large) have costs drawn from the same distribution $F(\xi)$. Then let n go to infinity, and keep the average cost per share constant by adjusting the cost to $F(\xi)/n$. The limit of this sequence corresponds to our assumption.

denote the difference in abilities.

The two large shareholders enter the voting by offering two different plans B_1 and B_2. The plans describe measures a shareholder commits to when in control in order to limit her control power and to protect the interests of minority shareholders (large and small). The proposal B_1 (B_2) describes the amount of private benefits that shareholder 1 (shareholder 2) can maximally take when getting control over the firm. The proposals limiting the control benefits to B_1 and B_2 are binding commitments that will be enshrined in the company charter and cannot be revoked by the board.

Once the vote has taken place, and the winner of the control contest is chosen, the two large shareholders provide monitoring efforts e_1 and e_2.. We assume that they incur monitoring costs $c_1(e_1)$ and $c_2(e_2)$. Finally, the value of the firm is realized. The firm value is given by $V = \theta_i \; + v(e_1, e_2)$, that is it is determined by the controlling shareholder's ability and the two shareholders' effort.

We suppose that the value of the firm is increasing and concave in the efforts of the two shareholders. Furthermore, we assume that the *monitoring efforts of the two large shareholders are complementary* , i.e. $\frac{\partial^2 v}{\partial e_i \partial e_j} \geq 0$. Finally, we assume that the monitoring costs are increasing and convex, and satisfy the boundary conditions $\lim_{e_i \to 0} c_i'(e_i) = 0, \lim_{e_i \to \infty} c_i'(e_i) = \infty$.

When the controlling shareholder extracts private benefits B_i, this results in a linear loss of the value of the firm, given by γB_i where $\gamma > 1$. To guarantee that the controlling shareholder has an incentive to extract private benefits, we require that $(1 - \gamma\alpha_i) > 0$ for $i = 1, 2$.

We assume that $\alpha_1 < 1/2$ and $\alpha_2 < 1/2$ to exclude trivial outcomes of the contest. Initially, we take the ownership structure (α_1, α_2) as given, but later we will ask how an initial seller of the firm (a founder-entrepreneur, a venture capitalist, a company spinning off assets, or a government privatizing state enterprises) will seek to partition the equity blocks in order to maximize proceeds. Figure 1 illustrates the time line of the model.

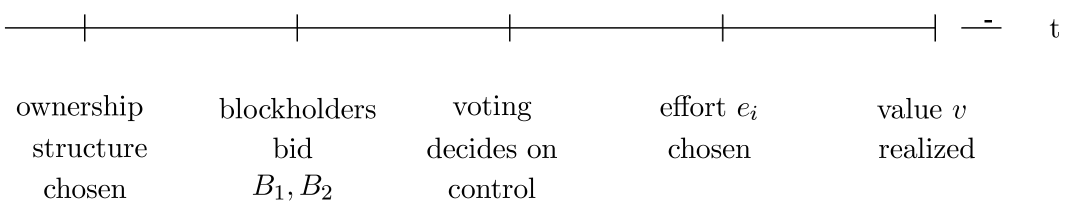

Figure 1: Time Line

3 Control Contests and Optimal Ownership Structure

3.1 Optimal Monitoring Effort Levels

We now turn to the analysis of the model, starting with the last stage of the interaction between the two shareholders. After the vote has taken place, the identity of the controlling shareholder, i, and the private benefits B_i have been determined, and the utilities of the two shareholders are given by

$$\begin{aligned} U_i &= \alpha_i(\theta_i \ + v(e_i, e_j)) + (1 - \gamma\alpha_i)B_i - c_i(e_i) \\ U_j &= \alpha_j(\theta_i, +v(e_i, e_j)) - \alpha_j B_i - c_j(e_j) \end{aligned}$$

The two shareholders select non-cooperatively their monitoring efforts e_i and e_j, as the solution to the first-order conditions

$$\begin{aligned} \alpha_i \frac{\partial v(e_i, e_j)}{\partial e_i} &= c_i'(e_i) \\ \alpha_j \frac{\partial v(e_i, e_j)}{\partial e_j} &= c_j'(e_j) \end{aligned}$$

Notice that, given the complementarity of efforts, the reaction functions are increasing. Furthermore, assuming that $(\frac{\partial^2 v}{\partial e_i \partial e_j} + \frac{\partial^2 v}{\partial e_i^2}) < c_i''(e_i)$ for $i = 1, 2$, the slope of the reaction functions is smaller than one, so that there exists a unique equilibrium (e_1^*, e_2^*). Using the technique developed in Dixit (1986), we obtain the following simple comparative statics on the equilibrium effort levels:

$$\frac{\partial e_i^*}{\partial \alpha_i} > \frac{\partial e_j^*}{\partial \alpha_i} > 0, \frac{\partial e_j^*}{\partial \alpha_j} > \frac{\partial e_i^*}{\partial \alpha_i} > 0.$$

Once the equilibrium choice of efforts has been made, we can define the value of the firm and the costs as

$$\begin{aligned} V(\alpha_1, \alpha_2) &= v(e_i^*(\alpha_1, \alpha_2), e_j^*(\alpha_1, \alpha_2)). \\ C_1(\alpha_1, \alpha_2) &= c_1(e_1^*(\alpha_1, \alpha_2)) \\ C_2(\alpha_1, \alpha_2) &= c_2(e_2^*(\alpha_1, \alpha_2)) \end{aligned}$$

Using these notations, the utilities of the two large shareholders and the small shareholders are given by

$$
\begin{aligned}
U_i &= \alpha_i(\theta_i + V(\alpha_1, \alpha_2)) + (1 - \gamma\alpha_i)B_i - C_i(\alpha_1, \alpha_2) \\
U_j &= \alpha_j(\theta_i + V(\alpha_1, \alpha_2)) - \gamma\alpha_j B_i - C_j(\alpha_1, \alpha_2) \\
U_s &= (1 - \bar{\alpha})(\theta_i + V(\alpha_1, \alpha_2)) - \gamma(1 - \bar{\alpha})B_i
\end{aligned}
$$

It is easy to see that the utilities of the second large shareholder and of the small shareholders is decreasing in B_i, while the utility of the large shareholder is increasing in B_i. Furthermore, the utility levels of all shareholders are increasing in the controlling shareholder'sss ability, so that, for an identical level of private benefits, $B_1 = B_2$, the small shareholders prefer to give control to shareholder 1.

We now consider the control contest between the two large shareholders. Let B_1 and B_2 be the private benefits proposed by the two shareholders. We establish the following Lemma, characterizing the preferences of the three types of shareholders.

Lemma 1 *Shareholder* 1 *always prefers her own plan,* B_1*. Shareholder* 2 *prefers her proposed plan* B_2 *if and only if* $(1 - \gamma\alpha_2)B_2 + \gamma\alpha_2 B_1. \geq \alpha_2\Delta\theta$*. The small shareholders prefer the plan* B_1 *if and only if* $\gamma(B_1 - B_2) \leq \Delta\theta$*. Whenever shareholder* 2 *prefers the plan* B_1*, the small shareholders also prefer* B_1*.*

Proof: See Appendix.

Figure 2 graphs the preferences of the second large shareholder and the small shareholders in the plane (B_1, B_2) and illustrates our argument, showing that small shareholders always prefer shareholder $1's$ plan when the second shareholder does.

Insert Figure 2

3.2 Voting Equilibria

We now use Lemma **??** to compute the equilibrium of the voting game. In the voting game, small shareholders simultaneously choose whether to participate in the meeting. As this simultaneous choice clearly results in coordination problems, we restrict our attention to *strong equilibria* of the voting game, i.e. equilibria such that no group of agents with positive measure has an incentive to deviate.[7]

We now define the utility of a small shareholder s in the voting game. We first observe that, when both shareholders prefer B_1, then the small shareholders also prefer B_1. In that case of unanimous agreement, it is a dominant strategy for any measurable subset of small shareholders not to vote, as their vote is costly and useless. Consider then a situation where shareholder 2 prefers her own plan to that of shareholder 1. Two cases may arise: either shareholder 2 has less shares than shareholder 1 ($\alpha_2 \leq \alpha_1$) or she has more shares ($\alpha_1 \leq \alpha_2$).. In both cases, if the preferences of the small shareholders agree with the preferences of the largest shareholder, it is a dominant strategy for any subset of small shareholders not to vote. We thus have two cases left to consider: *(i)* one where $\alpha_2 \leq \alpha_1$ and small shareholders prefer B_2 to B_1 and *(ii)* one where $\alpha_1 \leq \alpha_2$ and small shareholders prefer B_1 to B_2.

Case *(i)* Let μ denote the number of shares of small shareholders voting for B_2. The utility of a voting shareholder is $U_s = (1-\bar{\alpha})(\theta_2+V(\alpha_1,\alpha_2))-\gamma(1-\bar{\alpha})B_2-\xi(s)$ if $\mu+\alpha_2 \geq \alpha_1$ and $U_s = (1-\bar{\alpha})(\theta_1+V(\alpha_1,\alpha_2))-\gamma(1-\bar{\alpha})B_1-\xi(s)$ if $\mu+\alpha_2 < \alpha_1$.[8] If the small shareholder doesn't vote, she obtains the same utilities, without incurring the voting cost $\xi(s)$.

Case *(ii)* Similarly, let μ denote the number of shares of small shareholders voting for B_1. The utility of a voting shareholder is $U_s = (1-\bar{\alpha})(\theta_1+V(\alpha_1,\alpha_2))-\gamma(1-\bar{\alpha})B_1-\xi(s)$ if $\mu+\alpha_1 \geq \alpha_2$ and $U_s = (1-\bar{\alpha})(\theta_2+V(\alpha_1,\alpha_2))-\gamma(1-\bar{\alpha})B_2-\xi(s)$ if $\mu+\alpha_1 < \alpha_2$. If the small shareholder doesn't vote, she obtains the same utilities, without incurring the voting cost $\xi(s)$.

Proposition 1 *(i) There always is a unique strong equilibrium of the voting game.*

(ii) Suppose that $\alpha_1 \geq \alpha_2$ *If* $\gamma(B_1-B_2) < \Delta\theta + \frac{1}{1-\bar{\alpha}}F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}})$, *then no small shareholder participates in the vote, and shareholder* 1 *wins the control contest. If* $\gamma(B_1-B_2) \geq \Delta\theta + \frac{1}{1-\bar{\alpha}}F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}})$, *then a fraction* $\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}}$ *of the small shareholders vote, and shareholder* 2 *wins.*

(iii) Suppose that $\alpha_2 \geq \alpha_1$ *and* $(1-\gamma\alpha_2)B_2+\gamma\alpha_2 B_1 \geq \alpha_2\Delta\theta$. *If* $\Delta\theta - \frac{1}{1-\bar{\alpha}}F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) < \gamma(B_1-B_2)$, *then no small shareholder participates in the vote, and shareholder* 2 *wins. If*

[7] We emphasize that we adopt a purely non-cooperative our approach. The use of strong equilbiria is merely an equilibrium refinement in order to reduce the number of equilibria in the small shareholders' coordination problem, but does not indicate the use of cooperative game thery concepts.

[8] We suppose that in case of equality of the votes, ($\mu+\alpha_2 = \alpha_1$), the small shareholders and the second shareholder win the control contest. If $\alpha_1 = \alpha_2$ and $\mu = 0$, we assume that shareholder 1 wins the contest.

$\Delta\theta - \frac{1}{1-\bar{\alpha}}F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) \geq \gamma(B_1 - B_2)$, *then a fraction* $\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}$ *of the small shareholders vote, and shareholder* 1 *wins the control contest.*

(iv) If $\alpha_2 \geq \alpha_1$ *and* $(1-\gamma\alpha_2)B_2 + \gamma\alpha_2 B_1 < \alpha_2\Delta\theta$, *no small shareholder participates in the vote, and shareholder* 1 *wins the control contest.*

Proof: See Appendix.

Figures (3a), and (3b) (3c) illustrate the voting equilibria when $\alpha_1 \geq \alpha_2$ and $\alpha_2 \geq \alpha_1$. Notice that, when $\alpha_2 \geq \alpha_1$, two possible situations may arise, depending on the exact point where the two curves $(1-\gamma\alpha_2)B_2 + \gamma\alpha_2 B_1 = \alpha_2\Delta\theta$ and $\gamma(B_1 - B_2) = \Delta\theta - \frac{1}{1-\bar{\alpha}}F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}})$ intersect in the plane. We label those cases (b) and (c).

Insert Figures (3a), (3b) and (3c).

3.3 Control Contests

We now turn to the stage where the two large shareholders simultaneously choose the private benefits B_1 and B_2. The competition between the two large shareholders is reminiscent of a model of Bertrand competition between two firms with different marginal costs (see e.g. Shy (1996), p. 109). Hence, in order to solve for an equilibrium in pure strategies, we introduce a smallest money unit η. We say that money is *continuous* if $\eta = 0$ and *discrete* if $\eta > 0$. When money is discrete, the choices of private benefits are $B_1 = b_1\eta$ and $B_2 = b_2\eta$ for integer values b_1 and b_2.

Proposition 2 (i) *Suppose* $\alpha_1 \geq \alpha_2$ *and money is discrete. Then the Bertrand game where the two large shareholders choose private benefits* B_1 *and* B_2 *admits a unique equilibrium in pure strategies. As the smallest money unit* η *goes to zero, this equilibrium converges to* $(B_1^*, 0)$ *where* $B_1^* = \frac{1}{\gamma(1-\bar{\alpha})}F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}}) + \frac{1}{\gamma}\Delta\theta$.

(ii) *Suppose* $\alpha_2 \geq \alpha_1$. *If* $\frac{1}{1-\bar{\alpha}}\ F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) \leq \frac{1}{1-\gamma\alpha_2}\Delta\theta$, *and money is continuous, the game admits a unique equilibrium in pure strategies given by* (B_1^*, B_2^*) *where* $B_1^* = \frac{1}{\gamma}\Delta\theta - \frac{(1-\gamma\alpha_2)}{\gamma(1-\bar{\alpha})}F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}})$, $B_2^* = \frac{\alpha_2}{1-\bar{\alpha}}F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}})$. *If* $\frac{1}{1-\bar{\alpha}}\ F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) > \frac{1}{1-\gamma\alpha_2}\Delta\theta$ *and money is discrete, for* $\eta < \frac{1}{1-\bar{\alpha}}\ F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) - \frac{1}{1-\gamma\alpha_2}\Delta\theta$, *the game admits a unique equilibrium in pure strategies. As the smallest money unit* η *goes to zero, this equilibrium converges to* $(0, B_2^*)$ *where* $B_2^* = \frac{1}{\gamma(1-\bar{\alpha})}F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) - \frac{1}{\gamma}\Delta\theta$.

Proof: See Appendix.

Proposition ?? characterizes, for different parameter configurations, the equilibrium values of the private benefits extracted by the controlling shareholder. Notice that, as long as $\frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) \leq \frac{1}{1-\gamma\alpha_2}\Delta\theta$, shareholder 1 wins the control contest and is able to extract positive benefits. Figure 4 graphs the values of private benefits extracted for different values of α_1 (taking $\alpha_2 = \bar{\alpha} - \alpha_1$, where $\bar{\alpha}$ is an exogenous parameter). When α_1 is small, firm 2 extracts positive private benefits. These benefits are decreasing until $\alpha_1 = \tilde{\alpha}_1$ where $\tilde{\alpha}_1$ is the unique solution (when it exists) to the equation: $\frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\bar{\alpha}-2\alpha_1}{1-\bar{\alpha}}) = \frac{1}{1-\gamma(\bar{\alpha}-\alpha_1)}\Delta\theta$.[9] For $\alpha_1 = \tilde{\alpha}_1$, the private benefits are equal to zero. As α_1 becomes larger than $\tilde{\alpha}_1$, shareholder 1 starts extracting private benefits, and these benefits are increasing in α_1.

Insert Figure 4

3.4 Optimal Ownership Structure

The computation of the optimal ownership structure of the firm involves a comparison between different effects. Suppose that the objective of the social planner is to maximize the total value of the firm for all shareholders:

$$W = V(\alpha_1, \alpha_2) - C_1(\alpha_1, \alpha_2) - C_2(\alpha_1, \alpha_2) + \theta_i - (\gamma - 1)B_i$$

where i denotes the identity of the controlling shareholder.

We can separate the total value of the firm into two terms: $V(\alpha_1, \alpha_2) - C_1(\alpha_1, \alpha_2) - C_2(\alpha_1, \alpha_2)$ represents the value obtained by the monitoring efforts of the two large shareholders, $\theta_i - (\gamma - 1)B_i$ represents the added value due to the skill of the controlling shareholder and the loss in value due to the extraction of private benefits. From Figure 4, it is easy to see that the second term is maximized for $\tilde{\alpha}_1$ and $\tilde{\alpha}_2 = \bar{\alpha} - \tilde{\alpha}_1$. The combination of shares which maximizes the first term, $V(\alpha_1, \alpha_2) - C_1(\alpha_1, \alpha_2) - C_2(\alpha_1, \alpha_2)$, depends on the specific description of the monitoring efforts, and can only be described implicitly as $(\bar{\alpha}_1, \bar{\alpha}_2)$, where $\bar{\alpha}_2 = \bar{\alpha} - \bar{\alpha}_1$ and $\bar{\alpha}_1$ is the solution to the problem: $\max_{0 \leq \alpha_1 \leq \bar{\alpha}} v(e_1^*(\alpha_1, \bar{\alpha} - \alpha_1), e_2^*(\alpha_1, \bar{\alpha} - \alpha_1)) - c_1(e_1^*(\alpha_1, \bar{\alpha} - \alpha_1)) - c_2(e_2^*(\alpha_1, \bar{\alpha} - \alpha_1))$.

[9] If $\frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\bar{\alpha}}{1-\bar{\alpha}}) > \frac{1}{1-\gamma\bar{\alpha}}\Delta\theta$, the equation has a unique interior solution, $\tilde{\alpha}_1 > 0$. If $\frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\bar{\alpha}}{1-\bar{\alpha}}) \leq \frac{1}{1-\gamma\bar{\alpha}}\Delta\theta$, the equation has no solution, and there is no value of α_1 for which firm 2 obtains control of the firm. The lowest value of private benefits is then given by $B_1^* = \frac{1}{\gamma}\Delta\theta - \frac{1-\gamma\bar{\alpha}}{\gamma(1-\bar{\alpha})}F^{-1}(\frac{\bar{\alpha}}{1-\bar{\alpha}})$ for $\alpha_1 = 0$.

While we cannot give a general analytical solution to the optimal distribution of shares among the two large shareholders, we obtain the following simple characterization Lemma, in the case where $\bar{\alpha}_1 > \bar{\alpha}_2$, i.e. where shareholder 1 is the more efficient monitor.

Proposition 3 *Suppose that the monitoring value function $V(\alpha_1, \bar{\alpha} - \alpha_1) - C_1(\alpha_1, \bar{\alpha} - \alpha_1) - C_2(\alpha_1, \bar{\alpha} - \alpha_1)$ is strictly concave in α_1 and that distribution of shares maximizing the monitoring value of the firm satisfies $\bar{\alpha}_1 > \bar{\alpha}_2$. Then, in the optimal ownership structure $(\alpha_1^*, \alpha_2^*), \tilde{\alpha}_1 < \alpha_1^* < \bar{\alpha}_1$ and $\tilde{\alpha}_2 > \alpha_2^* > \bar{\alpha}_2$.*

Proof: See Appendix.

Lemma **??** shows that whenever shareholder 1's monitoring efforts are more valuable than shareholder 2's, the optimal distribution of shares requires a decrease in the share of the controlling shareholder, and an increase in the share of the minority shareholder. This results is easily understood: in order to reduce the extraction of private benefits from the controlling shareholder, the social planner must reduce her share relative to the share of the second shareholder. At the optimal point, the increase in value due to the reduction of private benefits extracted by the controlling shareholder's share must be equal to the decrease in value due to the reduction of the effort of the more able shareholder.

For an explicit characterization of the optimal ownership structure, we need to further specify the model.

4 A Linear-Quadratic Model

In order to illustrate the different effects affecting the value of the firm, and to compute exactly the optimal ownership structure of the firm, we restrict our attention henceforth to a specific parametrization of the model, which we call the *linear-quadratic model.* In this model, we assume that firm values are additive in effort levels, $v(e_1, e_2) = e_1 + e_2$. Moreover, the effort cost functions are quadratic, $c_1(e_1) = \frac{1}{2}c_1 e_1^2$ and $c_2(e_2) = \frac{1}{2}c_2 e_2^2$, with $c_2(1 - \bar{\alpha}) < c_1 < c_2$. The optimal effort levels are thus given by $e_1^* = \frac{\alpha_1}{c_1}$ and $e_2^* = \frac{\alpha_2}{c_2}$. The values and costs are obtained as $V(\alpha_1, \alpha_2) = \frac{\alpha_1}{c_1} + \frac{\alpha_2}{c_2}, C_1(\alpha_1, \alpha_2) = \frac{1}{2}\frac{\alpha_1^2}{c_1}$ and $C_1(\alpha_1, \alpha_2) = \frac{1}{2}\frac{\alpha_2^2}{c_2}$.. The distribution of shares maximizing the monitoring value of the firm is thus given by $\bar{\alpha}_1 = \frac{c_2 - c_1 + c_1\bar{\alpha}}{c_1 + c_2}$ and $\bar{\alpha}_2 = \frac{c_1 - c_2 + c_2\bar{\alpha}}{c_1 + c_2}$. It is easy to check that, as $c_1 < c_2$, $\bar{\alpha}_1 > \bar{\alpha}_2$.

We will conduct all further analyses in the framework of the linear-quadratic model. We will sometimes also assume that voting costs ξ are distributed uniformly.

To compute the distribution of shares minimizing the private benefits, suppose that voting costs are uniformly distributed over $[0,1]$ and assume that $\bar{\alpha}(1-\gamma\bar{\alpha}) > \Delta\theta(1-\bar{\alpha})^2$.. We then obtain: $\tilde{\alpha}_1 = \frac{3\bar{\alpha}}{4} - \frac{1}{2\gamma} + \frac{\sqrt{(3\gamma\bar{\alpha}-2)^2+8\gamma(\bar{\alpha}(1-\gamma\bar{\alpha})-\Delta\theta(1-\bar{\alpha})^2)}}{4\gamma}$. It is easy to check that $0 < \tilde{\alpha}_1 < \frac{\bar{\alpha}}{2}$. To compute the optimal distribution of shares, we write the total value of the firm as

$$\begin{aligned} W &= \frac{\alpha_1}{c_1} + \frac{(\bar{\alpha}-\alpha_1)}{c_2} - \frac{\alpha_1^2}{2c_1} - \frac{(\bar{\alpha}-\alpha_1)^2}{2c_2} \\ &+ \begin{cases} \theta_1 - \frac{(\gamma-1)(2\alpha_1-\bar{\alpha})}{\gamma(1-\bar{\alpha})^2} - \frac{(\gamma-1)}{\gamma}\Delta\theta & \text{if } \alpha_1 \geq \frac{\bar{\alpha}}{2} \\ \\ \theta_1 + \frac{(\gamma-1)(1-\gamma(\bar{\alpha}-\alpha_1))(\bar{\alpha}-2\alpha_1)}{\gamma(1-\bar{\alpha})^2} - \frac{(\gamma-1)}{\gamma}\Delta\theta & \text{if } \tilde{\alpha}_1 \leq \alpha_1 \leq \frac{\bar{\alpha}}{2} \\ \\ \theta_2 - \frac{(\gamma-1)(\bar{\alpha}-2\alpha_1)}{\gamma(1-\bar{\alpha})^2} + \frac{(\gamma-1)}{\gamma}\Delta\theta & \text{if } \tilde{\alpha}_1 > \alpha_1 \end{cases} \end{aligned}$$

The total value of the firm is thus a piece-wise quadratic function in α_1. To compute the optimal value of α_1, notice first that for $\tilde{\alpha}_1 > \alpha_1$, $\frac{\partial W}{\partial \alpha_1} > 0$. Hence the optimal distribution of shares must satisfy $\alpha_1 \geq \tilde{\alpha}_1$. Next denote by α_1^* and α_1^{**} the maximum of the quadratic functions for $\tilde{\alpha}_1 \leq \alpha_1 \leq \frac{\bar{\alpha}}{2}$ and $\alpha_1 \geq \frac{\bar{\alpha}}{2}$, respectively. Straightforward computations show that

$$\begin{aligned} \alpha_1^* &= \frac{(c_2-c_1)\gamma(1-\bar{\alpha})^2 + \bar{\alpha}c_1\gamma(1-\bar{\alpha})^2 + 3c_1c_2\bar{\alpha}(\gamma-1)\gamma - 2c_1c_2(\gamma-1)}{\gamma(c_2(1-\bar{\alpha})^2 + c_1(1-\bar{\alpha})^2 - 4c_1c_2(\gamma-1))} \\ \alpha_1^{**} &= \frac{(c_2-c_1)\gamma(1-\bar{\alpha})^2 + \bar{\alpha}c_1\gamma(1-\bar{\alpha})^2 - 2c_1c_2(\gamma-1)}{\gamma(c_2(1-\bar{\alpha})^2 + c_1(1-\bar{\alpha})^2)} \end{aligned}$$

It is easy to check that $\alpha_1^{**} < \alpha_1^*$. The optimal ownership structure can then be characterized as follows.

If $\alpha_1^* < \tilde{\alpha}_1$, the optimal distribution of shares is $\tilde{\alpha}_1$; if $\tilde{\alpha}_1 \leq \alpha_1^* \leq \frac{\bar{\alpha}}{2}$, the optimal distribution of shares is α_1^* ; if $\alpha_1^{**} < \frac{\bar{\alpha}}{2} < \alpha_1^*$, the optimal distribution of shares is $\frac{\bar{\alpha}}{2}$; if $\frac{\bar{\alpha}}{2} \leq \alpha_1^{**}$, the optimal distribution of shares is α_1^{**}.

It follows that the schedule of the optimal ownership can be represented as the step function depicted in Figure 5. We emphasize in particular that our model explains equal distribution of the two blocks, $\alpha_1 = \alpha_2$, as a generic event,

Insert Figure 5

5 Collusion

In this Section, we analyze the large shareholders' incentives to offer a common plan to the small shareholders. As opposed to the case where one of the two shareholders buys the shares of the other, in a situation of collusion, both shareholders exert monitoring efforts, and fully exploit the complementarity of monitoring efforts. At first glance, it thus appears that the two shareholders will *always* collude, as collusion enables them to extract larger private benefits, without incurring any loss in the value generated by monitoring efforts. A closer look at formal models of collusion shows however that the incentives to collude depend on the exact way in which the two shareholders bargain over the private benefits.

Consider for example a model where the two shareholders first bargain over the division of the total benefits, $\bar{B}$, and upon failure of the bargaining process, offer two alternative plans, B_1 and B_2 to the vote of the small shareholders. As long as $\bar{B} > \max\{B_1, B_2\}$, the bargaining process should end up in an efficient outcome, where the two shareholders collude and agree on a division of the total private benefits $\bar{B}$. This result however depends on strong assumptions on the process of bargaining and collusion. First, to obtain this result, one needs to clearly differentiate between a phase of bargaining between the two large shareholders, and a voting phase where the two shareholders offer alternative plans to the small shareholders. Second, the model supposes that both shareholders recognize the failure of an agreement, and can then resort to separate, individual plans to offer to the small shareholders. Third, this result is based on the assumption that the bargaining rule is efficient.

By contrast to the previous model, where the bargaining and voting phases are clearly differentiated, we propose in this Section a simple model of collusion, where bargaining and voting are closely interrelated. We suppose that shareholder 1 (the efficient shareholder) first proposes a division of the total benefits to shareholder 2, $(B_1, \bar{B} - B_1)$. Shareholder 2 either accepts the proposal of shareholder 1 or proposes an alternative plan, B_2 If she proposes an alternative plan, a meeting is convened, and all shareholders may vote between the plans $(B_1, \bar{B} - B_1)$ and B_2. In this model, shareholder 2 can thus counter the offer of shareholder 1 by offering an alternative plan to the small shareholders, whereas shareholder 1 is *committed* to the collusive plan she proposes to the other shareholder.

In order to compute the equilibrium of the game of collusion, we first need to characterize the strong equilibria of the voting game, once the plans $(B_1, \bar{B} - B_1)$ and B_2 have been chosen. To this end, notice that the small shareholders prefer the plan B_2 if and only if $B_2 \leq \bar{B} - \frac{\Delta\theta}{\gamma}$. The second shareholder prefers her plan to the collusive plan if and only if

$B_2 \geq \bar{B} + \frac{\alpha_2 \Delta\theta}{1-\gamma\alpha_2} - \frac{B_1}{1-\gamma\alpha_2}$. Figure 5 illustrates the preferences of the small shareholders and shareholder 2 in the plane (B_1, B_2), assuming $\Delta\theta < \gamma\bar{B}$.

Insert Figure 6

In order to characterize the strong equilibria of the voting game, we disregard the region of private benefits (B_1, B_2) where shareholder 2 weakly prefers the collusive plan to her own plan, as in this region, it is a weakly dominated strategy for shareholder 2 to propose an alternative plan. Following the same steps as in the proof of Proposition **??**, we can easily compute the strong equilibria of the voting game as follows.

Proposition 4 *Consider the region of private benefits* (B_1, B_2) *where shareholder* 2 *strictly prefers her plan to the collusive plan. Then the collusive game has the following unique strong equilibrium of the voting game:*

(i) *If* $\alpha_1 \geq \alpha_2$*, as long as* $B_2 > \bar{B} - \frac{\Delta\theta}{\gamma} - \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}})$*, then no small shareholder participates in the vote, and shareholder* 1 *wins the control contest. If* $B_2 \leq \bar{B} - \frac{\Delta\theta}{\gamma} - \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}})$*, then a fraction* $\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}}$ *of small shareholders votes, and shareholder* 2 *wins.*

(ii) *If* $\alpha_1 \leq \alpha_2$*, as long as* $B_2 < \bar{B} - \frac{\Delta\theta}{\gamma} + \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}})$*, then no small shareholder participates in the vote, and shareholder* 2 *wins the contest. If* $B_2 \geq \bar{B} - \frac{\Delta\theta}{\gamma} + \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}})$*,then a fraction* $\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}$ *of small shareholders votes, and shareholder* 1 *wins.*

Proof: See Appendix.

Figures (7a) (7b) and (7c) graph the regions where shareholders 1 and 2 win when $\alpha_1 \geq \alpha_2$ and $\alpha_1 \leq \alpha_2$ under the assumption that $\bar{B} - \frac{\Delta\theta}{\gamma} - \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}}) > 0$. If $\alpha_1 \leq \alpha_2$, two cases may arise, depending on whether $\frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) \leq \Delta\theta$ or $\frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) \geq \Delta\theta$. We label those cases (7b) and (7c).

Insert Figures (7a), (7b) and (7c).

We now solve for the equilibrium plans proposed by the two shareholders. Consider first the reaction function of shareholder 2, after she observes the collusive plan $(B_1, \bar{B} - B_1)$ proposed by shareholder 1. The following Lemma characterizes the optimal response of shareholder 2.

Lemma 2 *Suppose* $\bar{B} - \frac{\Delta\theta}{\gamma} - \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_1 - \alpha_2}{1-\bar{\alpha}}) > 0$. *Let* $(B_1, \bar{B} - B_1)$ *be the collusive plan proposed by shareholder* 1.

(i) *If* $\alpha_1 \geq \alpha_2$, *shareholder* 2 *accepts any collusive plan such that* $B_1 \leq B_1^* = \frac{\Delta\theta}{\gamma} + \frac{(1-\gamma\alpha_2)}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}})$. *Shareholder* 2 *rejects any collusive plan* $B_1 > B_1^*$ *and proposes an alternative plan* $B_2 = \bar{B} - \frac{\Delta\theta}{\gamma} - \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}})$.

(ii) *If* $\alpha_2 \geq \alpha_1$, *then shareholder* 2 *accepts any collusive plan* $B_1 \leq B_1^*$. *Shareholder* 2 *rejects any collusive plan* $B_1 > B_1^*$, *and proposes an alternative plan* B_2. *,where*

$$B_1^* = \begin{cases} \frac{\Delta\theta}{\gamma} - \frac{(1-\gamma\alpha_2)}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) & \text{if } \frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) \leq \Delta\theta \\ \\ \alpha_2 \Delta\theta & \text{if } \frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) \geq \Delta\theta \end{cases}$$

and

$$B_2 = \begin{cases} \bar{B} - \frac{\Delta\theta}{\gamma} + \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) & \text{if } \frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) \leq \Delta\theta \\ \\ \bar{B} & \text{if } \frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) \geq \Delta\theta \end{cases}$$

Proof: See Appendix.

As shareholder 1 always prefers the collusive plan to the alternative plan proposed by shareholder 2, in the first stage of the game, she will offer to the second shareholder the *lowest* value of private benefits which make her accept the collusive plan. From Lemma **??**, we can easily compute these values, and compare them with the private benefits extracted from shareholder 1 in the absence of collusion.

Proposition 5 *Suppose* $\bar{B} - \frac{\Delta\theta}{\gamma} - \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}}) > 0$. *Consider the model of collusion where shareholder* 1 *commits to a collusive plan.*

(i) *If* $\alpha_1 \geq \alpha_2$, *the efficient shareholder has no incentive to propose a collusive plan.*

(ii) *If* $\alpha_2 \geq \alpha_1$ *and* $\frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) \leq \Delta\theta$, *the efficient shareholder is indifferent between the collusive plan and the competitive outcome.*

(iii) *If* $\alpha_2 \geq \alpha_1$ *and* $\frac{1}{1-\gamma\alpha_2}\Delta\theta \geq \frac{1}{1-\bar{\alpha}}F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) \geq \Delta\theta$, *the efficient shareholder has no incentive to propose a collusive plan.*

(iv) *If* $\alpha_2 \geq \alpha_1$ *and* $\frac{1}{1-\bar{\alpha}}F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) > \frac{1}{1-\gamma\alpha_2}\Delta\theta$, *the efficient shareholder strictly prefers to offer the collusive plan.*

Proof: See Appendix.

6 Retrading

We have so far assumed that the initially fixed ownership structure will not be altered. For publicly held companies, anonymous buying and selling shares on the stock exchange, as well as "upstairs" trading of share blocks, are means to alter the ownership structure. And for privately held corporations, privately negotiated share trades should be possible.

In this Section, we consider when a given ownership structure would be immune to such retrading opportunities. To see why this question is important, note that if retrades are possible and the seller's target ownership structure is not viable, then the prices that shareholders are initially willing to pay should reflect their expectations for the long-run ownership structure, and not the initial partition.

The motive why the large shareholders would retrade is fairly obvious: If shareholder 2 sells her stake (or parts of it) to shareholder 1, this should be an easy means to reduce the competition wooing small shareholders. Though destructive for the firm value as a whole, it can increase the gain of the coalition of the two large shareholders. We consider this possibility below as *"upstairs"* retrading.

Similarly, any of the two large shareholders may want to increase her position relative to the competitor by purchasing stocks anonymously on the exchange, or *"downstairs"*.

6.1 Upstairs

We continue to analyze the linear-quadratic specification of the model. Moreover, we assume that costs ξ are uniformly distributed over the unit interval and that both firms have identical costs, $c_1 = c_2 = c$. If the optimal ownership structure was initially chosen, the efficient shareholder 1 will win the control contest, and the benefit she obtains can be stated as

$$B_1^* = \frac{\alpha_1 - \alpha_2}{\gamma(1-\bar{\alpha})^2} + \frac{\Delta\theta}{\gamma}\,. \tag{1}$$

We assume that the two large shareholders can privately negotiate a deal to sell shares between them. They will only be able to do so if their joint surplus increases, and we invoke the Nash-Rubinstein bargaining outcome for the splitting of the joint surplus.[10]

We find the following conditions for an allocation (α_1, α_2) to be immune against any retrading attempt on the upstairs market:

Proposition 6 (Retrading) *An allocation (α_1, α_2) will be immune to any blocktrading arrangements if the following two conditions hold:*

$$\alpha_1 \leq \frac{\bar{\alpha}}{c} + \frac{c(1-\gamma\bar{\alpha})}{\gamma(1-\bar{\alpha})^2} \tag{2}$$

and

$$\frac{\bar{\alpha}}{2\gamma(1-\bar{\alpha})^2} \geq \bar{B} + \frac{\Delta\theta}{\gamma} \, . \tag{3}$$

Proof: See Appendix.

For an interpretation of the conditions in Proposition **??**, let us first focus on the relevant case: We need only consider the case where α_1 increases at the expense of α_2. This is the obvious direction for a block trade since it allows to mitigate the control competition and hence, to increase B_1 at the small shareholder'sss expense. Further, retrading does then not alter the efficient outcome of the control contest, with shareholder 1 taking control, since a loss of $\bar{\alpha}\Delta\theta$ would accrue to both large shareholders combined otherwise.

We consider then the fundamental trade-off surrounding shareholder 1's possible acquisition of shareholder 2's stake. If shareholder 1 suggests to buy shareholder 2's entire block, then the competition for control is eliminated, meaning that the control benefits can be raised to $\bar{B}$ from B_1. On the other hand, shareholder 2 ceases to contribute valuable monitoring services, and shareholder 1 has to provide all the monitoring herself, at a higher marginal cost, since effort costs are convex. Let ΔY denote the change in the combined effort costs and gross[11] firm value for both blockholders due to the block trade. Let $\Delta B = \bar{B} - B_1$ be the change in private benefits. Thus, the block transaction would only increase the joint value of both shareholders if

$$\Delta Y < (1-\gamma\bar{\alpha})\Delta B \, . \tag{4}$$

[10]The results do not depend on using any paticular bargaining hypothesis.

[11]That is, prior to benefit taking.

Condition (**??**) takes into account that the benefit increase is inefficient, and that the large shareholders assume a portion $\bar{\alpha}$ of the social value loss of $(\gamma-1)\Delta B$. After substitution for the difference terms ΔB and ΔY, this condition becomes

$$\bar{\alpha}\left(\frac{\alpha_1+\alpha_2}{c}+\theta_1\right)-\frac{\alpha_1^2}{c}-\frac{\alpha_2^2}{c}+(1-\gamma\bar{\alpha})B_1\leq\bar{\alpha}\left(\frac{\alpha_1+\alpha_2}{c}+\theta_1\right)-\frac{(\alpha_1+\alpha_2)^2}{c}+(1-\gamma\bar{\alpha})\bar{B}\,. \tag{5}$$

Inequality (**??**) states the joint surplus of independent blockholdings (with control competition) on the left-hand side and the value after a block acquisition (without competition) in the right hand side. Clearly, if there was no benefit from the presence of multiple shareholders, in our model represented by the monitoring cost advantage $\Delta Y = \frac{\alpha_1^2}{c}+\frac{\alpha_2^2}{c}-\frac{(\alpha_1+\alpha_2)^2}{c}<0$, then a block acquisition would always occur. As emphasized in the introduction, there are other conceivable benefits from multiple block holdings, like risk diversification, commitment against future expropriation etc., which could assume a similar role. Condition (**??**) is directly derived from (**??**). (**??**) highlights that a high degree of ownership concentration, captured by $\bar{\alpha}$, and a low level of maximal benefits $\bar{B}$ make such a retrade less likely to be lucrative. Both is intuitive: the retrade comes at the expense of the small shareholders; the increased benefit taking will reduce the firm value by $(\bar{B}-B_1)\,\gamma$, of which the small shareholders bear a fraction $(1-\bar{\alpha})$.. The smaller the wealth transfer $(1-\bar{\alpha})\,(\bar{B}-B_1)\,\gamma$, the less attractive is retrading.

But acquiring shareholder 2's entire block is not the only option for shareholder 1. We also need to consider the possibility of a *partial retrading* between the two large shareholders, that is shareholder 1only buying a fraction of shareholder 2's holdings. Therefore, consider the other extreme point for shareholder 1's acquisition policy, to buy only an incremental fraction of shareholder 2's holdings. Such an incremental trade would be worthwhile if the marginal loss due effort costs and gross value, $\frac{\partial Y}{\partial \alpha_1}$, does not exceed the large shareholder's gain from increased benefit taking, $(1-\gamma\bar{\alpha})\frac{\partial B_1}{\partial \alpha_1}$. The further we go away from the initial ownership structure, the less is the cost sharing optimized, hence the larger becomes the first effect; but also the closer we get to maximum benefit taking even with control competition, so the smaller the second one. In other words, the two shareholder get the largest possible gain from a marginal reshuffle at the original ownership structure, and this is lucrative along as (**??**) holds, assuming $\Delta \to 0$.

If this initial reshuffle is not attractive, then no other partial retrade needs to be considered. Clearly, (**??**) is more likely to be satisfied the larger is α_1: Intuitively, the larger is shareholder 1's stake relative to shareholder 2's, the more costly to reduce burden sharing, and the less is control competition constraining benefit taking. It can be verified that this condition never holds if the blocks are symmetrically distributed, $\alpha_1 = \alpha_2$.

Two conditions are needed in order to exclude block trades, as there is a discontinuity when shareholder 1 acquires the last share held by shareholder 2 (this eliminates the possibility that anyone could raise a competing bid for control). In a sense, this discontinuity is more of an artefact of the model than a realistic feature, and if this discontinuity is dropped, the conditions that the ownership structure is immune against retrading coincide and, importantly, they become *weaker.*

6.2 Downstairs

We consider next whether any of the large shareholders would instead want to buy or sell shares on the trading floor from small shareholders. (We focus on shareholder 1's attempt to increase her stake, for the same reasons as in the analysis of upstairs trades.) We invoke ideas which are standard in the literature on corporate ownership since the seminal paper by Grossman and Hart (1980).

We assume that trading only occurs if it is consistent with a *subgame perfect equilibrium*, where all shareholders (including all small shareholders) hold correct beliefs of shareholder 1's intentions and trades only when this is beneficial.[12] We also assume that the large shareholder's trading activity is *not anonymous*, in the sense that changes in the block size are instantly observable to informed market participants.. This could either be due to the microstructure of the market, or because of disclosure regulation. Moreover, we assume that retrading does not alter the distribution of ξ in the remaining free float. We find:

Proposition 7 *If trading is not anonymous, then large shareholders will never sell on the stock market.*

Shareholder 1 will not buy on the market if

$$\alpha_1 \geq \frac{1+\gamma\bar{\alpha}}{\gamma} - \Delta\theta(1-\bar{\alpha})^2 \tag{6}$$

Shareholder 2 will not buy on the market if

$$\alpha_1 \geq \frac{2\bar{\alpha} - \Delta\theta(1-\bar{\alpha})^2}{3} \tag{7}$$

[12] This,assumption notably excludes that there are market participants forced to trade for liquidity rasons even at below-value prices, as is the case in many market microstructure models. Subgame perfection gives updating of beliefs to the ex post value, as there is complete information. An extension to asymmetric information (and refinement to Perfect Bayesian equilibrium) is possible, and gives the same result as long as we assume non-anonymity.

Proof: See Appendix.

This result basically expands on an argument developed e.g. in Burkart, Gromb and Panunzi (1997) for the case of a single large shareholder, that can be traced back to Grossman and Hart's (1980) well-known free-rider problem. The argument is as follows. Suppose there are no changes in control benefits, and suppose shareholder 1 wants to buy on the market, to increase her stake to α_1', say. Since dispersed shareholders immediately update their belief about the final firm value from $V(\alpha_1, \alpha_2)$ to $V(\alpha_1', \alpha_2)$. So shareholder 1 needs to spend $(\alpha_1' - \alpha_1)V(\alpha_1', \alpha_2)$ on the purchase. The purchase is only worth considering if

$$\alpha_1 V(\alpha_1, \alpha_2) - c_1(\alpha_1) < \alpha_1' V(\alpha_1', \alpha_2) - c_1(\alpha_1') - (\alpha_1' - \alpha_1)[V(\alpha_1', \alpha_2) = \alpha_1 V(\alpha_1', \alpha_2) - c_1(\alpha_1')$$

But this inequality will never hold, since when holding α_1, the shareholder's effort maximization means that the expression $\alpha_1 V(\alpha_1', \alpha_2) - c_1(\alpha_1')$ is maximized for $\alpha_1' = \alpha_1$. To this argument of Burkart, Gromb and Panunzi (1997), we need to add the fact that stock purchases will alter the equilibrium benefits B_1. Thus, unlike the earlier results with a single large shareholder, there will be instances where stock purchases on the market occur. Importantly, however, these are likely to be *marginal* purchases: Even when given the opportunity to buy unilaterally, the large shareholders will not absorb the entire free float since at some point, the marginal impact on benefits will level off. If (realistically) both shareholders buy simultaneously, there will be for the same reason a Nash equilibrium with a positive free float.

But the stable free float may be smaller than the seller's desired free float.

7 Empirical Implications

Some of the empirical implications that arise from our paper can be summarized as follows.

(i) Coexistence of Blocks, Ownership Structure, Collusion and Retrading-Proofness

The first group of empirical implications relates to the likelihood to observe multiple blocks of shares, their relative weight and the allocation of control in a cross-sectional sample. When formulating these hypotheses, we take into account that retrading and collusion are likely determinants of the stable, that is observable, ownership structures.

- The largest shareholder is more likely to be in control if the cost advantage in monitoring costs is large or if the legal protection of investor rights is effective (γ is large).

- Multiple blocks are more likely to be sustainable if shareholders are heterogenous, for example include one corporate shareholder and one other shareholder (capturing complementarities in monitoring).
- Multiple blocks are more likely to be sustainable if *(i)* the largest block is large compared to other blocks (since this is likely to guarantee retrading-proofness) or if *(ii)* if the efficient shareholder (proxied by industry distance, for example) holds the largest block or if the ability difference is small (both conditions make it more likely that shareholders can collude)

Finally, our model also allows inferences in the opposite direction: if the existence of multiple blocks is observed and blockholders seem to be fairly similar, this should indicate that equilibrium private benefits are small. If shareholders are proxied to be more heterogeneous, equilibrium private benefits should be more substantial.

(ii) Legal System

Our model can be easily interpreted as capturing variations across different legal systems. As in Shleifer and Wolfenzon (2000) and Burkart and Panunzi (2000), we would argue that a system with a poor state of investor protection is a system where the costs to transform company resources into private resources are small. As a consequence, the amount of private benefits that insiders will choose to extract when maximizing their utility is large, compared to legal systems affording a better protection of minority shareholders. Thus, a small parameter γ and also a large value of $\bar{B}$ and can be understood as variables capturing a legal system with a low state of investor protection. Then we get predictions like:

- In legal system with poor investor protection, multiple blocks will only be sustainable if either the largest block is very big or if there is collusion.

(iii) Block Transfers and Block Premia

Finally, our model allows for some empirical predictions relative to the growing empirical literature on block transfers, see e.g. Barclay and Holderness (1991), Crespi-Cladera and Renneboog (2000) and Nicodano and Sembenelli (2000).

- Controlling blocks should trade at a premium, minority blocks at a discount..
- Block premia should be large if the sale is a block merger, or if the prevalence of collusion is likely.

- Positive block premia coincide with negative stock price reactions if the sale is to a less efficient shareholder or the sale is a block merger. Positive block premia coincide with positive stock price reactions if the sale is to a more efficient shareholder.

8 Conclusion

In this paper, we investigate when a seller of a corporation, like a venture capitalist bringing a company public or a government undertaking a privatization program, would find it desirable to make room not just for one, but for several large blockholders. Blockholder are viewed to compete away private benefits in an effort to secure the vote of minority shareholders. If private benefits are value-destroying, this device to protect minority shareholders is in the interest of the seller, too.

The seller would typically like to reserve the largest block for a shareholder who is not taking over control because she as a ability handicap. However, an ownership structure that allocates only the second largest block to the controlling shareholder is very likely not retrading-proof. Moreover, the more the size handicap of the efficient shareholder, the more is she likely to organize collusion among the large shareholders, thus effectively circumventing the very reason why a second shareholder has been put in place.

Our theory emphasizes that empirical research on ownership structure effects on performance and on block transfers is incomplete if it does not try to proxy for difference in perceived shareholder ability.

This is only a preliminary attempt to model corporate governance by explicitly referring to voting games played in shareholder meetings. We have included only one reason why multiple share blocks may be mutually sold out in block mergers, but our model can easily be accommodated other mechanisms as well, thus giving richer testable hypotheses on when we would expect multiple blocks to survive.

Another extension would be to incorporate an explicit liquidity premium in the stock price in order to capture the motives why the seller would like to include a free float of $1 - \bar{\alpha} > 1/2$.

Appendix

Proof of Lemma ??: The inequalities determining the optimal behavior of shareholder 2 and of the small shareholders are clear. To prove that shareholder 1 always prefers her plan, note that

$$\begin{aligned}\alpha_1(\theta_1+V(\alpha_1,\alpha_2))+(1-\gamma\alpha_1)B_1-C_1(\alpha_1,\alpha_2) &> \alpha_1(\theta_1+V(\alpha_1,\alpha_2))-C_1(\alpha_1,\alpha_2)\\ &> \alpha_1(\theta_2+V(\alpha_1,\alpha_2))-C_1(\alpha_1,\alpha_2)\\ &> \alpha_1(\theta_2+V(\alpha_1,\alpha_2))-C_1(\alpha_1,\alpha_2)-\gamma\alpha_1 B_2.\end{aligned}$$

Finally, to prove that small shareholders always prefer B_1 when shareholder 2 does, notice that the equation $(1-\gamma\alpha_2)B_2+\gamma\alpha_2 B_1=\alpha_2\Delta\theta$ determines a strictly decreasing line in the plane (B_1,B_2). On the other hand, the equation $\gamma(B_1-B_2)=\Delta\theta$ determines a strictly increasing line in the same plane. For $B_2=0$, the two lines intersect at the point $B_1=\frac{1}{\gamma}\Delta\theta$. Hence, the two lines do not intersect in the positive orthant. We conclude that whenever shareholder 2 prefers the plan of shareholder 1, so do the small shareholders. **QED.**

Proof of Proposition ??: Suppose $\alpha_1\geq\alpha_2$ and $\gamma(B_1-B_2)-\Delta\theta<F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}})$. We show that the only strong equilibrium is the equilibrium where nobody votes. If a measure $\varepsilon<\alpha_1-\alpha_2$ vote, their votes do not change the outcome of the control contest, and hence it is a dominant strategy for them not to vote. If a measure $\varepsilon\geq\alpha_1-\alpha_2$ of the small shareholders vote, as $\gamma(B_1-B_2)-\Delta\theta<F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}})$, there must exist a positive measure δ of small shareholders for whom $\xi(s)>\gamma(B_1-B_2)-\Delta\theta$, and who prefer not to vote. Suppose now that $\gamma(B_1-B_2)-\Delta\theta\geq F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}})$. We show that there is a strong equilibrium where a fraction $\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}}$ of small shareholders with voting cost $\xi(s)<\gamma(B_1-B_2)-\Delta\theta$ participates in the vote. Small shareholders who do not vote have no incentive to deviate since they already obtain their preferred outcome, and do not incur the voting cost. If now a measure ε of voting small shareholders chooses to deviate, the outcome of the vote will be shareholder 1's plan, and as $\xi(s)<\gamma(B_1-B_2)-\Delta\theta$ for all voting small shareholders, this will induce a lower payoff . It is clear that there cannot be a strong equilibrium where a larger fraction of small shareholders chooses to vote, since the non-pivotal voters have an incentive to deviate. There does not exist a strong equilibrium where no small shareholder votes either, since a positive measure of shareholders has an incentive to vote, in order to ensure that shareholder 2 wins the control contest.

Similar arguments show that, when $\alpha_2\geq\alpha_{1.}$and $(1-\gamma\alpha_2)B_2+\gamma\alpha_2 B_1.\geq\alpha_2\Delta\theta$, the prescribed strategies form a strong equilibrium of the voting game. Finally, if $\alpha_2\geq\alpha_{1.}$and

$(1-\gamma\alpha_2)B_2 + \gamma\alpha_2 B_1. < \alpha_2\Delta\theta$, by Lemma **??**, all shareholders unanimously prefer shareholder 1's plan, and the small shareholders do not vote. QED.**QED.**

Proof of Proposition ??: Suppose $\alpha_1 \geq \alpha_2$. From Proposition **??**, we can divide the plane (B_1, B_2) into two regions, region A, where shareholder 1 wins the control contest, and region B where shareholder 2 wins the control contest. We first claim that there cannot be an equilibrium where (B_1, B_2) belong to region B. To see this note that, whenever $B_1 = 0$, shareholder 1 wins the control contest, and by Lemma **??**, she prefers to win the control contest, irrespective of the values of B_1 and B_2. Hence, for any point (B_1, B_2) in region B, shareholder 1 has a profitable deviation, $B_1 = 0$. Now consider a point (B_1, B_2) in region A, with $B_1 \geq B_1^*$. As $B_1 \geq B_1^*$ and (B_1, B_2) belongs to region A, we must have $B_2 > 0$. By choosing $B_2 = 0$, player 2 wins the control contest. Furthermore, by the last part of Lemma **??**, in the region where $B_1 \geq B_1^*$, shareholder 2 prefers to win the control contest, irrespective of the values of B_1 and B_2. Hence, shareholder 2 has a profitable deviation by choosing $B_2 = 0$. Next, consider a point (B_1, B_2) in region A, with $B_2 > 0$ and $B_1 < B_1^*$. As shareholder 1's utility is increasing in B_1 in region A, B_1 must be the maximal private benefit that player 1 can extract while winning the control contest. In other words, we must have

$$(b_1 + 1)\eta \geq b_2\eta + \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_1 - \alpha_2}{1-\bar{\alpha}}) + \frac{1}{\gamma}\Delta\theta \; . \tag{8}$$

However, as $B_1 < B_1^*$, we have

$$b_1\eta < \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_1 - \alpha_2}{1-\bar{\alpha}}) + \frac{1}{\gamma}\Delta\theta \, . \tag{9}$$

Clearly, the two inequalities **??** and **??** are inconsistent for any value $b_2 \geq 1$. Hence, the only possible equilibrium candidate is given by $B_2 = 0$ and $B_1 = \max\{b_1\eta | b_1\eta < \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}}) + \frac{1}{\gamma}\Delta\theta\}$. These strategies form an equilibrium, as shareholder 1 has no incentive to deviate, and shareholder 2 always loses the control contest and is indifferent among all possible values of B_2. As the discrete money unit converges to zero, the equilibrium converges to $(B_1^*, 0)$.

Suppose now that $\alpha_2 \geq \alpha_1$. Consider first the case where $\frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) \leq \frac{1}{1-\gamma\alpha_2}\Delta\theta$ (case of figure b2), and suppose that money is continuous Consider a point (B_1, B_2) in the region where shareholder 2 wins the contest. As shareholder 1 always wins the contest by offering $B_1 = 0$, she has a profitable deviation by choosing $B_1 = 0$. Consider now a point (B_1, B_2) in the region where shareholder 1 wins with $B_1 > B_1^*$. As $B_1 > B_1^*$, there exists a value B_2 such that $\frac{\alpha_2\Delta\theta}{1-\gamma\alpha_2} - \frac{\gamma\alpha_2 B_1}{1-\gamma\alpha_2} < B_2 < \frac{1}{\gamma(1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) - \frac{\Delta\theta}{\gamma} + B_1$. Hence, shareholder can win the contest by proposing B_2 when shareholder 1 proposes B_1. As

$B_1 > B_1^*$, shareholder 2 strictly prefers to win the contest, and this deviation is profitable. Now note that, for any value $B_1 \leq B_1^*$, shareholder 1 wins the control contest. Hence any offer $B_1 < B_1^*$ is dominated for shareholder 1 by the offer B_1^*. To finish the proof, note that (B_1^*, B_2^*) does indeed for an equilibrium, as shareholder 2 has no incentive to deviate (he loses the contest for any value of B_2 when shareholder 1 proposes B_1^*), and shareholder 1 has no incentive to deviate, as any value $B_1 > B_1^*$ would make shareholder 2 win the contest.

Finally suppose that $\alpha_2 \geq \alpha_1$ and $\frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) > \frac{1}{1-\gamma\alpha_2}\Delta\theta$ (case of figure 3b) and let money be discrete. This case turns out to be very similar to the case $\alpha_1 \geq \alpha_2$. We can again, from Proposition **??**, divide the plane (B_1, B_2) into two regions, region A, where shareholder 1 wins the control contest, and region B where shareholder 2 wins the control contest. Let the smallest money unit η be small enough so that $\eta < \frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) - \frac{1}{1-\gamma\alpha_2}\Delta\theta$. Then, for any possible point (B_1, B_2) in region A there exists a deviation for shareholder 2 which makes her win the control contest, and such that $(1-\gamma\alpha_2)B_2 + \gamma\alpha_2 B_1 > \alpha_2\Delta\theta$, i.e. shareholder 2 strictly prefers to win the contest. Hence, shareholder 2 has a profitable deviation and (B_1, B_2) cannot be an equilibrium. For any point (B_1, B_2) in region B with $B_2 \geq B_2^*$, shareholder 1 has a profitable deviation, by offering $B_1 = 0$ and winning the contest. Finally, by an argument similar to the argument of the case $\alpha_1 \geq \alpha_2$, a point (B_1, B_2) in region B with $B_2 < B_2^*$ and $B_1 > 0$ cannot be an equilibrium. Hence the only possible equilibrium is given by $B_1 = 0$ and $B_2 = \max\{b_2\eta | b_2\eta < \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) - \frac{1}{\gamma}\Delta\theta\}$ and it is easy to see that these strategies indeed form a equilibrium, which converges to $(0, B_2^*)$ as the smallest money unit η goes to zero. **QED.**

Proof of Proposition ??: Consider the derivative of the total value at any point where $\alpha_1 \geq \bar{\alpha}_1$:

$$\frac{\partial W}{\partial \alpha_1} = \frac{\partial V(\alpha_1, \bar{\alpha}-\alpha_1) - C_1(\alpha_1, \bar{\alpha}-\alpha_1) - C_2(\alpha_1, \bar{\alpha}-\alpha_1)}{\partial \alpha_1} - (\gamma - 1)\frac{\partial B_1}{\partial \alpha_1} < 0.$$

At any point where $\alpha_1 \leq \tilde{\alpha}_1$,

$$\frac{\partial W}{\partial \alpha_1} = \frac{\partial V(\alpha_1, \bar{\alpha}-\alpha_1) - C_1(\alpha_1, \bar{\alpha}-\alpha_1) - C_2(\alpha_1, \bar{\alpha}-\alpha_1)}{\partial \alpha_1} - (\gamma - 1)\frac{\partial B_2}{\partial \alpha_1} > 0\,.$$

QED.

Proof of Proposition ??: The proof is omitted, since it uses the same arguments as the proof of Proposition **?? QED.**

Proof of Lemma ??: Consider first the case $\alpha_1 \geq \alpha_2$. Let B_1^* be the intersection between the two lines: $B_2 = \bar{B} - \frac{\Delta\theta}{\gamma} - \frac{1}{\gamma(1-\bar{\alpha})}F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}})$ and $B_2 = \bar{B} + \frac{\alpha_2\Delta\theta}{1-\gamma\alpha_2} - \frac{B_1}{1-\gamma\alpha_2}$, i.e. $B_1^* =$

$\frac{\Delta\theta}{\gamma} + \frac{(1-\gamma\alpha_2)}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}})$. If $B_1 \leq B_1^*$, for any alternative plan B_2 that shareholder 2 prefers to the collusive plan, shareholder 1 wins the contest between B_2 and the collusive plan. Hence, accepting the collusive plan is a weakly dominant strategy for the second shareholder. If $B_1 > B_1^*$, shareholder 2 can win the contest by proposing $B_2 \leq \bar{B} - \frac{\Delta\theta}{\gamma} - \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}})$. As $B_1 > B_1^*$, shareholder 2 strictly prefers to win the contest for $B_2 = \bar{B} - \frac{\Delta\theta}{\gamma} - \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}})$, and her optimal response is to choose the highest private benefits that she can extract, $B_2 = \bar{B} - \frac{\Delta\theta}{\gamma} - \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}})$. A similar line of reasoning enables us to compute the optimal response when $\alpha_2 \geq \alpha_1$ and $\frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) \leq \Delta\theta$. If $\alpha_2 \geq \alpha_1$ and $\frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) \geq \Delta\theta$, shareholder 2 wins the contest by proposing any value B_2. If $B_1 \leq B_1^* = \alpha_2\Delta\theta$, shareholder 2 prefers the collusive plan, and it is a weakly dominant strategy to accept the plan ; if $B_1 > B_1^*$, shareholder 2 strictly prefers to propose the alternative plan $B_2 = \bar{B}$. **QED.**

Proof of Proposition ??: The proof involves a direct comparison between the private benefits obtained without collusion, characterized in Proposition**??** and the private benefits obtained under collusion given by Lemma **??**. If $\alpha_1 \geq \alpha_2, \frac{1}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}}) + \frac{1}{\gamma}\Delta\theta > \frac{\Delta\theta}{\gamma} + \frac{(1-\gamma\alpha_2)}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_1-\alpha_2}{1-\bar{\alpha}})$. If $\alpha_2 \geq \alpha_1$ and $\frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) \leq \Delta\theta$, the two values are equal. If $\alpha_2 \geq \alpha_1$ and $\frac{1}{1-\gamma\alpha_2}\Delta\theta \geq \frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) \geq \Delta\theta, \alpha_2\Delta\theta < \frac{\Delta\theta}{\gamma} - \frac{(1-\gamma\alpha_2)}{\gamma(1-\bar{\alpha})} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}})$. Finally, if $\alpha_2 \geq \alpha_1$ and $\frac{1}{1-\bar{\alpha}} F^{-1}(\frac{\alpha_2-\alpha_1}{1-\bar{\alpha}}) > \frac{1}{1-\gamma\alpha_2}\Delta\theta$, shareholder 1 loses the contest in the competitive case, and strictly prefers the collusive outcome. **QED.**

Proof of Proposition ??:

The condition for no partial share trade to be feasible is that for all $\alpha_1^{'} \in (\alpha_1, \bar{\alpha}]$

$$\int_{\alpha_1}^{\alpha_1^{'}} \frac{\partial Y}{\partial \alpha} d\alpha + (1-\gamma\bar{\alpha}) \int_{\alpha_1}^{\alpha_1^{'}} \frac{\partial B_1}{\partial \alpha} d\alpha \leq 0 \,, \tag{10}$$

where $Y = \bar{\alpha}\,(\theta_1 + e_1 + e_2) - c(\alpha_1) - c(\alpha_2)$ and $\alpha_1^{'} \geq \alpha_1$ denotes any target ownership structure for shareholder 1 different from the initial one. Then

$$\frac{\partial Y}{\partial \alpha_1^{'}} = \frac{d}{d\alpha_1^{'}} \left(-\frac{\alpha_1^2}{2c} - \frac{\alpha_2^2}{2c} \right) = -\frac{2\alpha_1^{'} - \bar{\alpha}}{c} \tag{11}$$

and

$$\frac{\partial B_1}{\partial \alpha_1^{'}} = \frac{2}{\gamma(1-\bar{\alpha})^2} \,. \tag{12}$$

After imposing the local condition $\alpha_1^{'} = \alpha_1$ in condition (**??**) and evaluating (**??**) and (**??**) for $\alpha_1^{'} = \alpha_1$, we get (**??**).

Next, note that $\frac{\partial Y}{\partial \alpha_1'}$ is decreasing in α_1' whereas $\frac{\partial B_1}{\partial \alpha_1'}$ is constant. Thus, we need to check only the two boundary conditions of (**??**), where either $\alpha_1' = \alpha_1$ or $\alpha_1' = \bar{\alpha}$. If (**??**) holds for these cases, then it must also be satisfied for all intermediate α'.

Substituting $\alpha_2 = \bar{\alpha} - \alpha_1$ and (**??**) in (**??**) gives after simplification

$$\frac{\alpha_1(\bar{\alpha} - \alpha_1)}{c} + \frac{(2a_1 - \bar{\alpha})(1 - \gamma\bar{\alpha})}{\gamma(1 - \bar{\alpha})^2} \geq (1 - \gamma\bar{\alpha}) \left(\bar{B} - \frac{\Delta\theta}{\gamma} \right)$$

which is identical to (**??**). **QED.**

Proof of Proposition ??: Consider first whether shareholder 1 has a strategy that would alter her stake to $\alpha_1' \neq \alpha_1$.

In any SPE, all players hold correct beliefs about equilibrium strategies, so all small shareholders update their belief about the final firm value from $V(\alpha_1, \alpha_2)$ to $V(\alpha_1', \alpha_2)$. So shareholder 1 needs to expend (at least) $(\alpha_1' - \alpha_1)V(\alpha_1', \alpha_2)$ when buying shares ($\alpha_1' > \alpha_1$), or will get (at most) $(\alpha_1' - \alpha_1)V(\alpha_1', \alpha_2)$ when selling shares ($\alpha_1' < \alpha_1$). After the change in block size, shareholder 1 will exert effort so as to maximize

$$\alpha_1' V(\alpha_1', \alpha_2) - c_1(\alpha_1')$$

Moreover, if the shareholder were to sell shares, then her utility from control benefits will strictly decrease, inequality (**??**) holds a fortiori. Since this argument does not depend on shareholder 1 assuming control, the same argument holds for shareholder 2.

Thus, the change in block size will not increase shareholder 1's utility if

$$\begin{aligned} & \alpha_1 V(\alpha_1, \alpha_2) - c_1(\alpha_1) + (1 - \gamma\alpha_1) B_1(\alpha_1, \alpha_2) \\ > \; & \alpha_1' V(\alpha_1', \alpha_2) - c_1(\alpha_1') + (1 - \gamma\alpha_1') B_1(\alpha_1', \alpha_2) - (\alpha_1' - \alpha_1)[V(\alpha_1', \alpha_2) - \gamma B_1(\alpha_1', \alpha_2) + \gamma B_1(\alpha_1, \alpha_2)] \\ = \; & \alpha_1 V(\alpha_1', \alpha_2) - c_1(\alpha_1') + (1 - \gamma\alpha_1) B_1(\alpha_1', \alpha_2) - \gamma(\alpha_1' - \alpha_1) B_1(\alpha_1, \alpha_2) \end{aligned} \tag{13}$$

Since $\alpha_1 V(\alpha_1', \alpha_2) - c_1(\alpha_1')$ is strictly concave, and since

$$\frac{\partial}{\partial \alpha_1'} (1 - \gamma\alpha_1') B_1(\alpha_1', \alpha_2) = \frac{1 + \gamma(\bar{\alpha} - \alpha_1')}{\gamma(1 - \bar{\alpha})^2} - \Delta\theta \tag{14}$$

which is strictly decreasing in α_1', the difference between left-hand side and right-hand side in (**??**) is strictly increasing as shareholder 1 increases her stake $\alpha_1' \geq \alpha_1$. Thus, the *critical* condition for shareholder 1's trading on the floor is where $\alpha_1' = \alpha_1$. This is identical to the local condition of (**??**) at $\alpha_1' = \alpha_1$, or

$$\alpha_1 \frac{\partial V(\alpha_1, \alpha_2)}{\partial \alpha_1} - \frac{\partial c_1(\alpha_1)}{\partial \alpha_1} + \frac{\partial}{\partial \alpha_1} (1 - \gamma\alpha_1) B_1(\alpha_1, \alpha_2)$$

Next, $\alpha_1 \frac{\partial V(\alpha_1,\alpha_2)}{\partial \alpha_1} - \frac{\partial c_1(\alpha_1)}{\partial \alpha_1} = 0$ by the implicit function theorem. Thus, a necessary and sufficient condition is that

$$\frac{\partial}{\partial \alpha_1} (1-\gamma\alpha_1)B_1(\alpha_1,\alpha_2) = \frac{1+\gamma(\bar{\alpha}-\alpha_1)}{\gamma(1-\bar{\alpha})^2} - \Delta\theta < 0 ,$$

which is identical to (**??**) Similarly, a necessary and sufficient condition for shareholder 2 not buying on the market is

$$\frac{\partial}{\partial \alpha_2} - \gamma\alpha_2 B_1(\alpha_1,\alpha_2) = \frac{2\gamma\alpha_2 - \gamma\alpha_1}{\gamma(1-\bar{\alpha})^2} - \Delta\theta < 0 ,$$

which is identical to (**??**). **QED.**

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