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Essays on Allocations in Two-Sided Markets

Jan Šedek

Dissertation

Prague, May 27, 2021

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Abstract

Chapter 1 studies strategy-proofness in a congested market with asymmetric information and interdependences in players preferences. The market consists of players and depletable locations. Knowing about the asymmetries of information and interdependences in preferences, the players choose one of two locations. In case of congestion, the rejected players are costlessly allocated to the other location. We show that, under correlated preferences, asymmetric information causes strategy-proofness to fail. We further provide a characterization of strategy-proofness of the allocation mechanism. Finally, we provide several sufficient conditions for strategy-proofness including independence of preferences.

Chapter 2 studies information acquisition incentives and welfare in a congested market with independent preferences. The players first learn about their preferences over two locations, after which they choose a location. In case of congestion, the rejected players are costlessly allocated to the other location. First, we show that for independent preferences, the allocation game with information acquisition tends to exhibit complementarities in information acquisition. This results in equilibrium multiplicity. Second, we show that due to prevailing positive externalities the equilibrium, in which more agents learn, welfare-dominates the non-learning equilibrium. Finally, we show that abolishing uncertainty about allocation chances leads to a welfare improvement for the players. The results can be applied, for instance, to matching markets. In particular, the second welfare result can be applied to a school choice setting, in which randomization is often used to break ties. We argue that the welfare of students would be improved if tie-breaking occurred before information acquisition - a change which is technically possible to implement.

Chapter 3 studies a monopolistic platform's decision on how to allocate sellers to consumers in a two-sided market for depletable goods. We find that the platform recommends low quality sellers and thus facilitates sales of lower quality products in order to increase the size of the market. This is achieved by a mechanism in which the platform commits to a rule through which it diverts some demand from high quality products to low quality products in times of low demand in order to satisfy the participation constraint of low quality sellers and ensure they earn non-negative profits, and thereby enable them to stay on the market. The rule consequently increases the number of sellers on the market in times of high demand and the platform can extract additional profits through transaction fees.

In Czech:

Kapitola 1 se zaměřuje na strategy-proof mechanismy na trzích s asymetrickými informacemi a interdependentními preferencemi hráčů. Modelovaný trh se skládá z hráčů

a vyčerpatelných lokací. Hráči si zvolí jednu ze dvou lokací. Lokace však má pouze omezenou kapacitu. V případě, že se lokace přeplní, jsou odmítnutí hráči přesunuti na zbylou lokaci s volnou kapacitou. Tento článek ukazuje, že v případě korelovaných preferencí přestávají být mechanismy strategy-proof. Charakterizace strategy-proofness alokačních mechanismů je předložena. Na závěr, je předloženo několik postačujících podmínek na strategy-proofness, včetně nezávislosti preferencí.

Kapitola 2 se zaměřuje na incentivy učení a blahobyt na trzích s vyčerpatelnými statky. Abychom se vyhnuli absenci strategy-proofness předpokládáme, že preference hráčů jsou statisticky nezávislé. Hráči si po zjištění informací o dvou dostupných lokacích jednu z nich vyberou. V případě, že se lokace přeplní, jsou odmítnutí hráči přesunuti na zbylou lokaci s volnou kapacitou. Nejprve ukážeme, že v takové hře vykazují učící strategie hráčů strategickou komplementaritu. To má za následek nejednoznačnost řešení v podobě Nashových rovnováh. Ukážeme, že pozitivní externality mají za následek, že v rovnováhžných stavech, v kterých se učí více hráčů, dosahují hráči vyššího blahobytu. Na závěr ukážeme, že snížení nejistoty v šancích na přijetí hráče lokací, má za následek zvýšení blahobytu hráčů. Výsledek je možné přímo aplikovat na problém s volbou školy, v kterém se často využívá loterie za účelem alokace studentů se stejnou prioritou. Tvrdíme, že studentům by se dařilo lépe, kdyby se tyto loterie realizovaly v dostatečném předstihu před přihlašováním studentů na školy.

Kapitola 3 se zaměřuje na chování monopolistických platforem a jejich rozhodování jak k sobě alokovat kupující a prodejce na trzích s vyčerpatelnými statky. Ukážeme, že platformy mají incentiv párovat kupující k prodejcům s nízkou kvalitou za účelem aby tito prodejci na platformě zůstali aktivní. Platforma tak v důsledku uměle zvětšuje trh na kterém působí. Trh tak může pojmout více kupujících v období zvýšené poptávky. Platforma v důsledku realizuje dodatečné zisky v těchto obdobích.

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Jan

Introduction

The unifying theme of this dissertation is allocation mechanisms in markets which experience congestion. Allocation mechanisms are relevant in many economic contexts, in which the desired market outcome cannot be achieved through price mechanisms. Sometimes it is not feasible to implement prices efficiently, while in other times they are not even desirable. While examples of the former are ubiquitous, examples of the latter include markets for education and organ donation. In practice, even in the presence of prices, and perhaps in current pandemic times more than ever, we still sometimes face congestion or shortages. While shortages are mostly associated with undersupplies of demanded commodities, congestion is often associated with heterogeneous and indivisible goods. However, in current times, when we observe queues in front of supermarkets and shortages of protective masks, other sanitary products, and even certain food items, we are reminded more than ever that shortages are not only a problem of the past which can be ignored in the expectation that it can be solved solely via a price mechanism.

The idea behind the allocation mechanism described in chapters one and two is that it is uncertain whether a person will be allocated what he chose, as allocations depend on how many people make the same choice. The literature on market design provides many different allocation mechanisms which fit this description, such as the celebrated deferred acceptance mechanism (Gale and Shapley 1962). Nevertheless, the easiest way to motivate this setting is with a first-come-first-served allocation rule. The mechanism can then be applied to a person deciding which retail outlet to go to to acquire a certain good. Much like with limited places in schools, which are often allocated through a deferred acceptance mechanism, the success of the action depends on how many people choose the same retail outlet, in which the supply of the particular good is limited. More specifically, we assume that each person prefers one of two locations (schools or retail outlets)[1]. However, each location has a limited capacity to provide the good. If the person is not able to obtain the good in his chosen location, he still can obtain it in another location, although he prefers it from his chosen location or his chosen location sells it for a lower price.

Chapter one takes a deferred acceptance mechanism in a model in which players have uncertain preferences over the locations. Previous literature has shown that this mechanism is strategy-proof in a setting with perfect information and in a setting where information is independent across players. Strategy-proofness simply means that players choose the location they prefer in the case of perfect information, and the location they prefer in expectation in the case of uncertain preferences. In this chapter we allow the information to be interdependent across players and show under what conditions strategy-proofness is achieved. Finally, we provide a counter-example which shows that strategy-proofness fails for correlated preferences.

In contrast with chapter one, in which the players are endowed with information, in chapter two the players acquire information at a cost. We assume no failure of strategy-proofness by using a sufficient condition for strategy-proofness from chapter one. This allows us to study information acquisition incentives and welfare in congested markets. We first show that the mechanism exhibits complementarity in information acquisition - the more players learn, the higher the learning incentives are. This contrasts with previous literature, which suggests that in such a mechanism the result should be the opposite. Further, this causes the setting to exhibit multiple equilibria, one in which no one is learning, because the incentives to learn are not high enough, and another one in which most of the population is learning. We then move to the welfare results, which show that learning imposes a positive externality on other players, and consequently the non-learning equilibrium exhibits underlearning. Finally, we show that abolishing the allocation uncertainty in the mechanism is associated with a welfare improvement. This is driven by the fact that learning becomes more efficient as players have more information about the payoffs from learning. This can be implemented in practice, for example, by reserving the good in advance at the location for the player. We provide such an implementation rule for the case of the school choice setting.

Chapter three deviates from this setting and instead studies an allocation problem of a monopolistic platform. We motivate this setting with websites including Airbnb and booking.com, which have large market shares for short term rental of apartments.

[1] or goods from the preferred location

However, such platforms often also have power over the information they provide to consumers, hence they can heavily influence demand for the goods. While the sellers are free to decide whether to sell through the platform or not, the platform can influence their decision by promoting their product. The main result of the paper is that the platform shifts demand towards low quality sellers to induce them to stay on the platform in order to satisfy realizations of higher demand. The high quality sellers, who lose from this, stay on the platform regardless, as their expected profits remain positive. This gives the platform additional profits, because the platform's profit is proportional to the size of the market. However, this profit is offset by the loss of profit stemming from the fact that higher quality products sell at higher prices. Chapter three analyzes this trade-off and shows a solution to the platform's problem.

Chapter 1

Strategy-Proofness in Congested Markets with Asymmetric Information and Interdependent Preferences[1]

1.1 Introduction

Chances of allocation with a location (a job, a place in a school, a depletable good) often depend on how many other agents choose the same thing and who they are. In economic literature, allocation and matching models are built to determine the outcomes in these markets.

The most famous examples of matching markets are marriage markets, the college admission problem, the house allocation problem, school choice, and labor markets. While some of these markets are run in a decentralized way, with either side of the market approaching the other and forming matches, other markets, such as markets for school places, dorm rooms in colleges, and some labor markets are often run centrally through an allocation algorithm.

Whether the markets are run centrally or whether a decentralized mechanism determines the allocation, a strategy-proof mechanism is often desirable. Strategy-proofness means that the players who participate on the market can simply choose what they most prefer instead of forming strategies based on the expected choices of others.

However, most matching and allocation literature assumes complete information for the agents, an assumption which is convenient but hardly realistic. We build a finite player model with two locations to analyze the strategy-proofness property in a setting

[1]This project has received funding from Charles University, GAUK project No: 570218 and the European Research Council, under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 678081)

with information asymmetries and interdependences in preferences. We provide several sufficient conditions for strategy-proofness of the allocation mechanism and a characterization of strategy-proofness in terms of the location choice rules together with player preferences. Finally, we provide a counter-example with common values which establishes a failure of strategy-proofness in this setting.

1.2 Literature Review

This paper contributes to literature on matching initiated by Gale and Shapley (1962) who discovered the deferred acceptance (DA) mechanism, known for its properties of strategy-proofness and stability. It is well known that the strategy-proofness of the DA mechanism has limits in matching models (Dubins and Freedman 1981). It is also well known that strategy-proofness cannot be satisfied for both sides of the market (Roth 1990) for stable mechanisms. However, there are theoretical models suggesting that strategy-proofness on both sides is not necessary for large markets (Kojima and Pathak 2009). Moreover, there is a large number of applications in which it is reasonable to assume that one side of the market [2] behaves non-strategically, such as in Abdulkadiroğlu and Sönmez 2003, and Roth 1984.

Roth (1990) also establishes strategy-proofness for a DA mechanism under uncertainty about preferences, assuming independence. However, to our knowledge, there is limited literature on the topic of matching or allocation with asymmetric information or interdependences in preferences. One exception is Kloosterman and Troyan (2018), who analyze a school choice model with a DA mechanism under asymmetric information, and discuss the curse of acceptance which results from asymmetric information. Unlike Kloosterman and Troyan (2018), we analyze a model with a general class of mechanisms and possible indifferences in preferences. Second, Liu et al. (2014) build a matching model with asymmetric information, but focus on the stability of allocations. Finally, Che and Koh (2016) build a college admission model with preference uncertainties and focus on strategic behavior of colleges.

Several papers build a matching model with information acquisition (which can result in asymmetric information), but independent preferences. Bade (2010) does so for a house allocation problem. Chen and He (2015) do so for a school choice problem. Immorlica et al. (2018) build a search-like model for school choice.

[2] In this paper we refer to this "passive" side as locations.

Section 1.3 gives the setup of the model. Section 1.4 provides a definition of strategy-proofness with sufficiency and characterization theorems. Section 1.5 delivers the counter-example of the failure of strategy-proofness. Section 1.6 concludes.

1.3 Model Setup

There are $n \in \mathbb{N}$ players and 2 locations. Each player can be allocated to a single location, while location $j \in \{1, 2\}$ can be allocated up to capacity $q^j \in \mathbb{N}$ players. The combined capacities are assumed to be large enough to accommodate all players, $q^1 + q^2 \geq n$. An allocation is thus a pair (μ^1, μ^2), with $\mu^j \in M = \{0, 1\}^n$, where $\mu_i^j = 1$ whenever the ith player is allocated to the jth location.

Each player $i \in \{1, \ldots, n\}$ simultaneously chooses a location $a_i \in A \equiv \{1, 2\}$. We denote the aggregate action $a \equiv (a_1, \ldots, a_n) \in A^n$. Given a, each location chooses for each player i an action from $A_L \equiv \{\text{"Accept"}, \text{"Reject"}\}$. Each location j has a **stochastic choice rule** $\mathcal{D}^j : A^n \mapsto \Delta(A_L^n)$ which allows location j to *Accept* or *Reject* players stochastically, conditional on the aggregate action a. The stochastic choice rule implies that location j accepts player i with probability $D_i^j(a)$. The choice rule is assumed to satisfy the following two properties. First, the location cannot accept players who did not choose it: $\forall i, j, a : a_i \neq j \implies D_i^j(a) = 0$.

Second, the choice rule is **non-wasteful** if $\forall i, j, a$:

- $|\{i : a_i = j\}| \leq q^j, a_i = j \implies D_i^j(a) = 1$.
- $|\{i : a_i = j\}| > q^j, z \in \text{supp}(\mathcal{D}^j(a)) \implies |\{i : z_i = \text{Accept}\}| = q^j$,

that is, if fewer than q^j players choose a location, all are accepted, and if more than q^j players choose a location, exactly q^j of them are accepted.

In case a player i is accepted by location j he is allocated there and not to the other location $j' \neq j$, that is, the final allocation μ is such that $\mu_i^j = 1$ and $\mu_i^{j'} = 0$. In case a player is not accepted by location j he is automatically allocated to location $j' \neq j$, that is, the final allocation μ is such that $\mu_i^j = 0$ and $\mu_i^{j'} = 1$. Consequently the set of choice rules $\{\mathcal{D}^1, \mathcal{D}^2\}$ together with the aggregate action a determine the allocation μ.

When choosing his action, each player knows the number of other players n, the available actions A, the capacities q^1, q^2 and the choice rules $\mathcal{D}^1, \mathcal{D}^2$.

Each player chooses his action to maximize his expected utility. Allocating player i at location j gives player i utility u_i^j and consequently for a fixed action profile $a = (a_i, a_{-i})$

player i receives expected utility

$$U_i(j, a_{-i}) \equiv D_i^j(j, a_{-i})u_i^j + (1 - D_i^j(j, a_{-i}))u_i^{j'}$$

from choosing action j. A player i has a utility vector $\theta_i \equiv (u_i^1, u_i^2)$. All players have a prior belief F with support in $\mathbb{R}^{2n}$ over the profile of types $\theta \equiv (\theta_1, \ldots, \theta_n)$. The case of independent preferences is of particular interest: $F = \prod_i^n F_i$, where F_i is a marginal distribution over θ_i. However, prior F allows for interdependences. Additionally, the set of all players is partitioned into sets I (Informed) and U (Uninformed). The players $i \in I$ receive a private signal $\sigma_i = \theta_i$, revealing to player i the realization of his type. The players $i \in U$ receive an uninformative signal $\sigma_i = \varnothing$. Finally, we denote $\tilde{F}^i$ a posterior belief over θ, that player i holds after the signals are received, and $\tilde{F}_i^i$ the corresponding marginal distribution over own θ_i. We assume that players know the sets I and U, but they do not have knowledge of the content of the other players' signals. The **allocation mechanism** H is thus summarized by $(n, A, q^1, q^2, \mathcal{D}^1, \mathcal{D}^2, F, I, U)$.

Players $i \in I$ have a strategy $s_i : \mathbb{R}^2 \mapsto A$, which consists of actions after receiving signal σ_i and players $i \in U$ simply choose an action $s_i : A$. Fixing the sets I and U, one can write with a slight abuse of notation $a_{-i} = s_{-i}(\theta)$.[3] Given posterior beliefs $\tilde{F}^i$, the optimal action of player i: a_i^* then maximizes player i's expected utility:

$$\max_{a_i} E_{\tilde{F}^i}[U_i(a_i, s_{-i}(\theta))]. \tag{1.1}$$

A Bayesian Equilibrium is a strategy profile $s^* \equiv (s_1^*, \ldots, s_n^*)$ such that all players choose according to (1.1) given H.

1.4 Strategy-Proofness

The purpose of this section is to show the conditions under which the allocation mechanism H is strategy-proof. The property of strategy-proofness is desired from allocation mechanisms in many applications. There is a vast literature concerning the strategy-proofness of allocation mechanisms in settings with complete information, however, to the best of our knowledge, there is no generalization of the definition to a setting with asymmetric information. We define the strategy-proofness of mechanism H in the allo-

[3]The first abuse of the notation is that for $i \in U$, s_i does not depend on θ. Second, for $i \in I$ we should write $a_{-i} = s_{-i}(\theta_{-i})$. We drop the subscripts on θ to ease notation

cation game with asymmetric information as follows:

Definition 1. A mechanism H is **strategy-proof** if:

$$\forall i \in \{1, \dots, n\}, \sigma_i, s_{-i}, j \neq j' : E_{\tilde{F}^i}[u_i^j] > E_{\tilde{F}^i}[u_i^{j'}] \implies a_i^* = j,$$

The definition requires the players to have dominant strategies for all possible posterior beliefs $\tilde{F}^i$, which are implied by the prior F and the signal realizations σ_i. In the case of an informative signal, the realization of θ_i is revealed to player i and, consequently, for an informative signal σ_i the beliefs $\tilde{F}^i$ are degenerate. The expectation over player i's utility $E_{\tilde{F}^i}[u_i^j] > E_{\tilde{F}^i}[u_i^{j'}]$ is then simply the realized utility $u_i^j > u_i^{j'}$.

Furthermore, the setting with complete information is a special case of a setting with prior and private signals. This is achieved by choosing the prior F to be a degenerate prior. In such a case, we refer to **strategy-proofness under certainty**. This property is well established in the literature (Roth 1990). However, strategy-proofness is easily achieved also for the case when all players are perfectly informed about their own preferences, but remain uninformed about the preferences of others. In such a case, we refer to **strategy-proofness under certainty about a player's own preferences**. Such preferences are achieved in our model by choosing $U = \varnothing$ so that all players are Informed. The following proposition formalizes the notion.

Proposition 1. *Mechanism H is* ***strategy-proof under certainty about a player's own preferences****, for the special case of $U = \varnothing$.*

Proof. $U = \varnothing$ implies that $\forall i : \sigma_i = \theta_i = (u_i^1, u_i^2)$. Fix player i, s_{-i} and a profile of types θ, where θ_i is such that $u_i^1 > u_i^2$ and consequently $\tilde{F}_i^i$ is such that $u_i^1 > u_i^2$ with probability 1. Assuming the converse inequality $u_i^1 < u_i^2$ follows an analogical argument. According to (1.1), i chooses location 1 if:

$$\begin{aligned} &E_{\tilde{F}^i}[D_i^1(1, s_{-i}(\theta))]u_i^1 + E_{\tilde{F}^i}[1 - D_i^1(1, s_{-i}(\theta))]u_i^2 \\ &\qquad > E_{\tilde{F}^i}[D_i^2(2, s_{-i}(\theta))]u_i^2 + E_{\tilde{F}^i}[1 - D_i^2(2, s_{-i}(\theta))]u_i^1. \end{aligned}$$

Which together with $u_i^1 > u_i^2$ implies that location 1 is chosen if:

$$E_{\tilde{F}^i}[D_i^1(1, s_{-i}(\theta)) + D_i^2(2, s_{-i}(\theta))] > 1.$$

This inequality holds due to the following argument. First, observe that $D_i^j(a_i, s_{-i}(\theta)) \geq 0$

is a probability. Second, observe that at least one of the probabilities is equal to 1. This is due to the assumption $q^1 + q^2 \geq n$.

We prove the remaining part by contradiction: Fix any a_{-i} and suppose that both $D_i^1(1, a_{-i}) < 1$ and $D_i^2(2, a_{-i}) < 1$. Then, by non-wastefulness, both $|\{i : a_i = 1\}| > q^1$ and $|\{i : a_i = 2\}| > q^2$. Consequently, $|\{i : a_i = 1\}| + |\{i : a_i = 2\}| = n > q^1 + q^2$ which yields a contradiction. □

The proof of proposition 1 in fact shows that a player i has a dominant strategy if he is informed, $i \in I$. If all players are informed then strategy-proofness is achieved and proposition 1 arises. The following proposition gives the sufficient condition for **strategy-proofness** of mechanism H.

Proposition 2. *Mechanism H is* ***strategy-proof*** *for the case of independent preferences.*

$$\prod_i F_i = F$$

Proof. Fix i. First notice that for $i \in I$, $\sigma_i = \theta_i$, $\forall \theta_i : \tilde{F}^i_i(\theta_i) \in \{0, 1\}$ and consequently the result is established by the proof of proposition 1. For $i \in U$, $\tilde{F}^i = F$, fix s_{-i} and let $E_F[u_i^1] > E_F[u_i^2]$ without loss of generality. According to (1.1), $a_i = 1$ is chosen if:

$$E_F[D_i^1(1, s_{-i}(\theta))u_i^1 + (1 - D_i^1(1, s_{-i}(\theta)))u_i^2] \\ > E_F[D_i^2(2, s_{-i}(\theta))u_i^2 + (1 - D_i^2(2, s_{-i}(\theta)))u_i^1]. \quad (1.2)$$

Since θ_i are independent, $s_{-i}(\theta)$ is independent of θ_i and consequently one can decompose the expectation:

$$E_F[D_i^1(1, s_{-i}(\theta))]E_F[u_i^1] + E_F[1 - D_i^1(1, s_{-i}(\theta))]E_F[u_i^2] \\ > E_F[D_i^2(2, s_{-i}(\theta))]E_F[u_i^2] + E_F[1 - D_i^2(2, s_{-i}(\theta))]E_F[u_i^1].$$

This inequality holds following the same argument as in the proof of proposition 1, except that now we can use the assumption $E_F[u_i^1] > E_F[u_i^2]$, instead of $u_i^1 > u_i^2$. □

We will return to an example of a failure of strategy-proofness under the assumption of interdependent preferences in the next section. The next two results provide more conditions for achieving strategy-proofness.

Proposition 3. *(Sufficient condition) Mechanism H satisfies strategy-proofness if:*

$$\forall i \in \{1, \ldots, n\}, s_{-i}, j \neq j' : E_F[u_i^j] > E_F[u_i^{j'}] :$$

$$Cov_F[D_i^j(j, s_{-i}(\theta)) + D_i^{j'}(j', s_{-i}(\theta)), u_i^j - u_i^{j'}] \geq 0. \tag{1.3}$$

Proof. First notice that for any $i \in I$, $\sigma_i = \theta_i$, and thus $\forall \theta_i : \tilde{F}_i^i(\theta_i) \in \{0, 1\}$ and the result is established by the proof of proposition 1.

For $i \in U$, $\tilde{F}^i = F$, fix s_{-i} and let $E_F[u_i^1] > E_F[u_i^2]$ without loss of generality. In the case of interdependent preferences, (1.1) can be written as:

$$\begin{aligned} &E_F[D_i^1(1, s_{-i}(\theta))]E_F[u_i^1] + Cov_F[D_i^1(1, s_{-i}(\theta)), u_i^1] \\ &\quad + E_F[1 - D_i^1(1, s_{-i}(\theta))]E_F[u_i^2] + Cov_F[1 - D_i^1(1, s_{-i}(\theta)), u_i^2] \\ &\quad > E_F[D_i^2(2, s_{-i}(\theta))]E_F[u_i^2] + Cov_F[D_i^2(2, s_{-i}(\theta)), u_i^2] \\ &\quad + E_F[1 - D_i^2(2, s_{-i}(\theta))]E_F[u_i^1] \\ &\quad + Cov_F[1 - D_i^2(2, s_{-i}(\theta)), u_i^1]. \end{aligned} \tag{1.4}$$

which is, following the argument in the proof of proposition 2, satisfied if:

$$\begin{aligned} &Cov_F[D_i^1(1, s_{-i}(\theta)), u_i^1] + Cov_F[1 - D_i^1(1, s_{-i}(\theta)), u_i^2] \\ &\quad > Cov_F[D_i^2(2, s_{-i}(\theta)), u_i^2] + Cov_F[1 - D_i^2(2, s_{-i}(\theta)), u_i^1]. \end{aligned}$$

which, after some manipulation yields the result. □

One obvious case when the sufficient condition of proposition 3 is satisfied is when q^1, q^2 are large enough to guarantee that the mechanism H is never congested. In such a case, for both $j, \forall a$: $D_i^j(a) = 1$. The covariance term in proposition 3 is then trivially equal to 0. Indeed, with no congestion effect, there is no need to take other players' behavior into account. Proposition 3 is particularly practical due to the following corollary, which can be applied in the case of no asymmetric information $I = \varnothing$:

Corollary 1. *Player i chooses his most preferred alternative if the action of other players is constant across θ.*

Proof. Let $s_{-i}(\theta)$ be constant across θ, then for $j \in \{1, 2\} : D_i^j(j, s_{-i}(\theta))$ is constant across θ and consequently:

$$Cov_F[D_i^j(j, s_{-i}(\theta)) + D_i^{j'}(j', s_{-i}(\theta)), u_i^j - u_i^{j'}] = 0,$$

which together with proposition 3 yields the result. $\square$

Corollary 1 is particularly important because the action of other players is constant whenever the other players are known to be Uninformed. In a follow-up paper, we apply corollary 1 extensively to a game with learning. The next result provides an *if and only if* condition for strategy-proofness.

Proposition 4. *(Characterization) Mechanism H is* ***strategy-proof*** *if and only if:*

$$\forall i \in \{1, \ldots, n\}, s_{-i}, j \neq j' : E_F[u_i^j] > E_F[u_i^{j'}] :$$

$$E_F[(D_i^j(j, s_{-i}(\theta)) + D_i^{j'}(j', s_{-i}(\theta)))(u_i^j - u_i^{j'})] > E_F[u_i^j - u_i^{j'}]. \tag{1.5}$$

Proof. For any $i \in I$, $\sigma_i = \theta_i$, $\forall \theta_i : \tilde{F}_i^i(\theta_i) \in \{0, 1\}$ and the result is established by the proof of proposition 1.

For $i \in U$, $\tilde{F}^i = F$, fix s_{-i} and let $E_F[u_i^1] > E_F[u_i^2]$ without loss of generality. Then, inequality (1.4) follows from (1.1), written equivalently as:

$$\begin{aligned} &Cov_F[D_i^j(j, s_{-i}(\theta)) + D_i^{j'}(j', s_{-i}(\theta)), u_i^j - u_i^{j'}] \\ &\qquad > E_F[u_i^j - u_i^{j'}](1 - E_F[D_i^j(j, s_{-i}(\theta)) + D_i^{j'}(j', s_{-i}(\theta))]), \end{aligned}$$

or equivalently as

$$\begin{aligned} &E_F[(D_i^j(j, s_{-i}(\theta)) + D_i^{j'}(j', s_{-i}(\theta)))(u_i^j - u_i^{j'})] \\ &\qquad - E_F[D_i^j(j, s_{-i}(\theta)) + D_i^{j'}(j', s_{-i}(\theta))]E_F[u_i^j - u_i^{j'}] \\ &\qquad\qquad > E_F[u_i^j - u_i^{j'}](1 - E_F[D_i^j(j, s_{-i}(\theta)) + D_i^{j'}(j', s_{-i}(\theta))]), \end{aligned}$$

which yields the result. $\square$

1.5 Failure of Strategy-Proofness Under Common Values

The previous section gave a set of results demonstrating under what conditions strategy-proofness is satisfied. This section provides a counter-example in which strategy-proofness fails. For the purpose of illustration we focus on the case of common values, where all players are of the same type.

Example 1. Interdependencies in preferences can cause a failure of the **strategy-proofness** of mechanism H in a setting with asymmetric information.

Proof. Let $n = 2$ and $q^1 = q^2 = 1$. Let $\theta = (\hat{\theta}, \hat{\theta})$, where $\hat{\theta}$ is equal to $\theta^{1>2}$ with probability p and $\theta^{2>1}$ with probability $1-p$. When player i has type $\theta^{j>j'}$, he receives utility $\hat{u} > 0$ from location j and 0 from location j'. Consequently, we drop subscript i on u_i. That is for both $i \in \{1, 2\}$, the following table summarizes the prior F:

Table 1.1: Utilities and Prior

	$\theta^{1>2}$	$\theta^{2>1}$
$u_i^1 = u^1$	$\hat{u}$	0
$u_i^2 = u^2$	0	$\hat{u}$
$F(\theta)$	p	$1-p$

Let $p > 1/2$, so that location 1 is, ex-ante, better:

$$E[u^1] = p\hat{u} > (1-p)\hat{u} = E[u^2].$$

Let the choice rules $D_i^j(a)$ be according to the following table, which is common knowledge:

Table 1.2: Choice Rules

$D_i^j(a_i, a_{-i})$	$D_i^1((1,1))$	$D_i^2((2,2))$	$D_i^1((1,2))$	$D_i^2((2,1))$
$i = 1$	1	0	1	1
$i = 2$	0	1	1	1

First, observe that the last two columns of the table are a consequence of **non-wastefulness**, that is, whenever the players choose different locations, they are both allocated to their choice. Second, observe that we choose the choice rules in such a way that in case of congestion, $a = (1, 1)$ or $a = (2, 2)$, player 1 is accepted by location 1 and player 2 is accepted by location 2. This asymmetry, together with the assumption that location 1 is ex-ante better, plays a crucial role in the example. We will show that the **strategy-proofness** property fails for the *disadvantaged* player 2, who is rejected by the ex-ante better location 1 in case of $a = (1, 1)$.

Suppose that the first player is *informed*, $1 \in I$. That is, player 1 learns the realization of $\hat{\theta}$ and chooses a_1^* according to the realization. Further suppose that the *disadvantaged* player 2 is *uninformed*, $2 \in U$. **Strategy-proofness** requires player 2 to choose location 1, but we will show that this does not happen.

Proof of proposition 1 shows that, after receiving σ_1, player 1 chooses the location 1 for $\sigma_1 = \theta^{1>2}$, which happens with probability p and location 2 for signal $\sigma_1 = \theta^{2>1}$, which happens with probability $1 - p$. Knowing this, the expected utility of player 2, $U_2(a_2, a_1^*(\theta))$ is:

Table 1.3: Expected Utility of Player 2

$U_2(a_2, a_1^*(\theta))$	$p : a_1^*(\theta^{1>2}) = 1$	$1 - p : a_1^*(\theta^{2>1}) = 2$
$a_2 = 1$	0	0
$a_2 = 2$	0	$\hat{u}$

The top-left cell is 0, because with probability p, both players have type $\theta^{1>2}$ and both players chose location 1, but location 1 is allocated to player 1, leaving player 2 with location 2, which yields 0. The top-right cell is zero, because, while player 2 was allocated to his chosen location 1, thanks to the fact that player 1 chose location 2, both players share type $\theta^{2>1}$ and thus location 1 yields 0. The bottom left cell is 0, because, while player 2 received his chosen location 2, location 2 yields 0, because both players share type $\theta^{1>2}$. Finally, the bottom-right cell is $\hat{u}$, because both players share type $\theta^{2>1}$ and both players chose location 2, but location 2 accepts player 2 and yields $\hat{u}$.

Table 1.3 shows that the *disadvantaged* player 2 cannot benefit from his choice of location 1 when player 1 is *informed*. In fact, the only case in which he is allocated to location 1 is exactly when location 1 is worse than location 2. Knowing this, the *disadvantaged* player 2 chooses location 2 due to:

$$(1 - p)\hat{u} > 0.$$

To summarize, for the *informed advantaged* player 1 and the *uninformed disadvantaged* player 2, player 2 chooses location 2 despite the fact that his posterior beliefs are such that $E_{\tilde{F}^2}[u^1] > E_{\tilde{F}^2}[u^2]$, thus violating **strategy-proofness**. □

Example 1 illustrates that, in the case of interdependent preferences, **strategy-proofness** is complicated by knowing what others know. To see the intuition behind why **strategy-proofness** fails for interdependent preferences, suppose a player i does not know his θ_i. Perhaps player i has a low chance of being allocated to location j in case of congestion, but location j is ex-ante attractive. However, due to interdependence, θ_i is positively correlated with θ_{-i} and consequently location j is good for player i when it is good for the other players $-i$. Furthermore, player i might expect that other players $-i$ are better informed about their types. Finally, knowing that other players know their types, player i knows he will find location j congested exactly when he would like to be allocated there and when he has a low chance to be allocated there. Consequently, the choice of location j is not a good choice, since the only way to be allocated there is when the realized type of player i reveals that location j is bad. This prediction ultimately leads player i to choose a different location and to violate strategy-proofness.

The fact that *uninformed* player i learns that his type is such that location j is bad in realization from being allocated to location j is an example of the winners curse, which parallels the classic result from auctions with common values.

1.6 Conclusion

We have shown that strategy-proofness, an often-desired property of allocation mechanisms, fails in settings with asymmetric information and interdependent preferences. Furthermore, we have provided several conditions imposed on the mechanism together with preferences under which strategy-proofness is satisfied. The model can be applied to a variety of settings. Perhaps the most attractive application is a setting with learning, which naturally brings asymmetric information into the mechanism. Alternatively, a study of welfare effects and effects on uninformed (socially disadvantaged) players could shed light on new shortcomings of currently applied mechanisms.

Chapter 2

Information Acquisition Incentives and Welfare in Congested Markets[1]

2.1 Introduction

In many contexts, economic agents commit to choices which are difficult or costly to change - enrolling in a university or signing a work or lease contract. These types of choices deplete the goods and limit others making the same choice. Before entering the market, agents often first learn about what they would like to take away from it - a process which can be costly and lengthy. Consequently, during the search process, one does not know if he will be able to obtain the good he is currently learning about. A high school student in his final years is learning about which program at what university would be the best for him before he sends an application. A prospective tenant learns about locations which are best for him due to his commute, security, or desired amenities before he schedules a property viewing, etc. Often there is very limited time for learning after a final offer from a university or a real estate agent is made.

This paper builds a model to discuss learning incentives and welfare in settings with congestion. If the allocation mechanism is such that the player's (economic agent) choice of location (school, job, or a property) does not depend on other players' choices, the matching literature (Dubins and Freedman 1981) identifies the allocation mechanism as strategy-proof. We use the result in Sedek (2020) that asymmetric information stemming from learning has no influence on a player's choice of a location under the assumption of

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independent preferences. While the allocation mechanism is strategy-proof, and thus the location choice is non-strategic, the same cannot be said about the choice of information. This is because the choices of other players still influence the value of information.

We first discuss strategic effects in the allocation mechanism. We show that, despite the location choice exhibiting strategic substitutability, which is driven by congestion, the information acquisition tends to exhibit strategic complementarity. This surprising property is due to learning which causes players to switch from their default action. When players are uninformed, the widely chosen default action tends to deepen the congestion problem and thus discourages learning. As players are learning, congestion is alleviated, and information becomes more desirable. Thus, this model serves as an example of strategic complementarity in information acquisition arising naturally from a game with strategic substitutability in action. This stands in contrast to Hellwig and Veldkamp (2009), who find that information acquisition tends to inherit the strategic properties of the underlying game in a beauty contest setting.

Second, we discuss the implications of learning incentives on welfare. The complementarity in information acquisition translates into equilibrium multiplicity. We first show that, for stable equilibria, the equilibria with more learning are welfare-dominant over the other equilibria. We show that this is due to prevailing positive externalities from learning.

Third, welfare results provide a comparative static exercise of the following form. All equilibria in a game with ex-ante symmetric players are welfare-dominated by a unique equilibrium in a related game, where the players are ex-ante asymmetric, but perfectly informed about their allocation chances. We apply this result to a school choice setting.

In a school choice setting, students (players) often face uncertainties in admission chances due to tie-breakers, which are lotteries evaluated after the applications are submitted. We argue that eliminating this uncertainty by informing students about their tie-breakers before the students gather information would increase student welfare. This comes at the cost of sacrificing an opportunity for a natural experiment, arising from the lotteries as suggested by Abdulkadiroğlu et al. (2017).

2.2 Literature Review

To our knowledge, there is limited literature on the topic of matching or allocation of indivisible goods with information acquisition. There are a few recent papers on school choice with information acquisition. Chen and He (2017) find that, under strategy-proof mechanisms with independent student preferences, students have no incentive to learn about their cardinal preferences or the preferences of other students. Immorlica et al. (2018) study a school choice continuum economy with information acquisition where students have independent preferences and their priorities are strict. They find that information deadlocks, a situation in which some students are waiting for other students to reveal their preferences through the mechanism, can occur. We support this finding by showing that our finite setting tends to exhibit strategic complementarities in information acquisition. Bade (2010) studies the housing allocation problem with information acquisition and shows that no allocation mechanism can achieve the first best welfare outcome.

Our model serves as an example of a setting with strategic substitutability, but complementartity in information acquisition. Hellwig and Veldkamp (2009) build a beauty contest model with information acquisition to show that, in a beauty contest setting, this is not possible. Colombo, Femminis, and Pavan (2014) study the effects of information acquisition on welfare in a general strategic complementarity/substitutability setting.

Finally, several papers have built matching models with interdependencies in preferences. Among these is Sedek (2020), who analyses the same setting as this paper, but without learning and with interdependent preferences. Sedek (2020)[2] then dives into results about strategy-proofness. Kloosterman and Troyan (2018) analyze school choice with a Gale-Shapley mechanism under asymmetric information and discuss the curse of acceptance which results from asymmetric information. Other related papers with interdependent preferences include Che and Koh (2016), Che, Kim, and Kojima (2015), and Chakraborty, Citanna, and Ostrovsky (2010).

Our final welfare result contributes to the school choice literature originated by Abdulkadiroğlu and Sönmez (2003). The main theoretical papers in this strand of literature are: Abdulkadiroğlu, Pathak, and Roth 2009; Balinski and Sonmez 1998; Haeringer and Klijn 2009. There is a growing experimental and empirical literature on the same topic: Calsamiglia, Haeringer, and Klijn 2010; Chen and Sonmez 2006; De Haan et al. 2015; Calsamiglia and Güell 2014; Calsamglia, Fu, and Güell (2018).

In contrast to the main result of this paper, Chen (2018) argues that, under the widely used Boston mechanism in school choice setting, uncertainty in priorities is associated with higher expected utility ex-ante.

The rest of this paper is organized as follows. Section 2.3 formalizes the model. Section

[2]Included in this dissertation as chapter 1.

2.4 provides results on strategy-proofness. Section 2.5 shows the results on strategic effects in information acquisition and welfare results. Section 2.6 delivers the main result of the paper. Section 2.7 discusses the application of the model in a school choice setting. Section 2.8 concludes.

2.3 Model Setup

There are $n \in \mathbb{N}$ players and 2 locations. Each player can be allocated to a single location, while each location can be allocated to $q \in \mathbb{N}$ players up to capacity. The combined capacities are assumed to be large enough to allocate all players, $2q \geq n$. An allocation is thus a pair (μ^1, μ^2), with $\mu^j \in M = \{0,1\}^n$, where $\mu_i^j = 1$ whenever the ith player is allocated to the jth location.

The allocation game with information acquisition has two stages, first the information acquisition stage, and then the location choice stage. In the information acquisition stage, each player simultaneously chooses a binary action, whether to learn or not $\alpha_i \in A_0 \equiv \{\text{"Learn"}, \text{"Not Learn"}\}$. In the location choice stage, each player $i \in \{1, \ldots, n\}$ simultaneously chooses a location $a_i \in A \equiv \{1, 2\}$. We denote the aggregate action in the location choice stage $a \equiv (a_1, \ldots, a_n) \in A^n$. Given a, each location chooses for each player i an action from $A_L \equiv \{\text{"Accept"}, \text{"Reject"}\}$. Each location j has a **stochastic choice rule** $\mathcal{D}^j : A^n \mapsto \Delta(A_L^n)$ which allows location j to *Accept* or *Reject* players stochastically, conditional on the aggregate action a. The stochastic choice rule implies that location j accepts player i with probability $D_i^j(a)$. The choice rule is assumed to satisfy the following two properties. First, the location cannot accept players who did not choose it: $\forall i, j, a : a_i \neq j \implies D_i^j(a) = 0$.

Second, the choice rule is **non-wasteful** if $\forall i, j, a$:

- $|\{i : a_i = j\}| \leq q^j, a_i = j \implies D_i^j(a) = 1.$
- $|\{i : a_i = j\}| > q^j, z \in \text{supp}(\mathcal{D}^j(a)) \implies |\{i : z_i = \text{Accept}\}| = q^j,$

that is, if fewer than q^j players choose a location, all are accepted, and if more than q^j players choose a location, exactly q^j of them are accepted.

In case a player i is accepted by location j he is allocated there and not to the other location $j' \neq j$, that is, the final allocation μ is such that $\mu_i^j = 1$ and $\mu_i^{j'} = 0$. In case a player is rejected by location j he is automatically allocated to location $j' \neq j$, that is, the final allocation μ is such that $\mu_i^j = 0$ and $\mu_i^{j'} = 1$. Consequently, the set of choice rules $\{\mathcal{D}^1, \mathcal{D}^2\}$ together with the aggregate action a determine the allocation μ. The **allocation mechanism** H can be then summarized as $H \equiv (n, A, q^1, q^2, \mathcal{D}^1, \mathcal{D}^2)$.

When choosing actions, each player knows the number of other players n, the number of locations, the capacities q, the available actions A and A_0, the choice rules $\mathcal{D}^1, \mathcal{D}^2$, the dynamic structure of the game, and the allocation mechanism H. In the location stage of the game, players do not know the actions chosen by other players in the information acquisition stage.

Each player chooses his actions to maximize expected utility. Allocating player i at location j gives player i utility u_i^j and, consequently, for a fixed action profile $a = (a_i, a_{-i})$ player i receives

$$U_i(j, a_{-i}) \equiv D_i^j(j, a_{-i})u_i^j + (1 - D_i^j(j, a_{-i}))u_i^{j'}$$

from choosing location j. A type of player i is summarized by his utility vector $\theta_i \equiv (u_i^1, u_i^2)$. The profile of types $\theta \equiv (\theta_1, \dots, \theta_n)$ is distributed according to F with support in $\mathbb{R}^{2n}$. We assume each player to have an identical marginal distribution F_i over his type θ_i and we assume θ_i to be independent, that is $F = \prod_i^n F_i$. In case of $E_F[u_i^1]$ we drop the subscript i as the expectation is the same for all players. In case the player chooses $\alpha_i =$ "Learn" in the information acquisition stage, he learns the realization of θ_i, but has to pay cost $c \geq 0$. In case he chooses $\alpha_i =$ "Not Learn" he is left only with the knowledge of F, which is costless.

A player i chooses a pure strategy $s_i \equiv (\alpha_i, a_i, \tilde{a}_i(\theta_i)) \in S$ which consists of the learning decision $\alpha_i \in A_0$, the action $a_i \in A$ in the information set where he chose "Not Learn" and an action $\tilde{a}_i(\theta) : \mathbb{R}^2 \mapsto A$ for each information set which corresponds to a realization of θ_i after he chose "Learn". When player i chooses his actions, he holds beliefs $\phi_i \equiv (F, F, \tilde{F}_{\theta_i})$ about the profile of player types in each information set. Initial beliefs and beliefs after the action "Not Learn" are the prior F. The beliefs after action "Learn" $\tilde{F}_{\theta_i} \in \Delta\theta$ are, for each realization of θ_i, derived by Bayes rule from the realization of θ_i and prior F.

Observe that, for a fixed strategy profile $s \equiv (s_1, \dots, s_n)$, the aggregate action a is stochastic, as it depends on the realization of θ through learning. We define σ_i and $\boldsymbol{\sigma}$ to satisfy $a_i \equiv \sigma_i(s_i, \theta)$ and $a = \boldsymbol{\sigma}(s, \theta) = ((\sigma_1(s_1, \theta_1), \dots, (\sigma_n(s_n, \theta_n)))$, for a fixed strategy profile s and a fixed realization of θ. The best response then solves the following three problems.

First, in the location choice stage, the player who chose $\alpha_i =$ "Not Learn" chooses a_i to maximize his expected utility, given beliefs F

$$\max_{a_i} E_F[U_i(a_i, \boldsymbol{\sigma}_{-i}(s_{-i}, \theta_{-i}))]. \tag{2.1}$$

We denote the corresponding value function $\hat{V}_i : S^{n-1} \mapsto \mathbb{R}$. Second, the player who chose $\alpha_i =$ "Learn" and learned to be type θ_i chooses a by maximizing:

$$\max_{a_i} E_{\tilde{F}}[U_i(a_i, \boldsymbol{\sigma}_{-i}(s_{-i}, \theta_{-i}))]. \tag{2.2}$$

We denote the corresponding value function $\tilde{V}_i : S^{n-1} \times \mathbb{R}^2 \mapsto \mathbb{R}$. Further, we denote by $a^* \in A$ the player's optimal action in the location choice stage. In the information acquisition stage, player i chooses "Learn" if and only if his expected **value of information** exceeds the information cost:

$$VI_i(s_{-i}) \equiv E_F[\tilde{V}_i(s_{-i}, \theta_{-i})] - \hat{V}_i(s_{-i}) \geq c. \tag{2.3}$$

A Perfect Bayesian Equilibrium is a profile of a strategies and beliefs $((s_1^*, \phi_1^*), \ldots, (s_n^*, \phi_n^*))$, such that all players choose locations according to (2.1) and (2.2) and choose information according to (2.3).

2.4 Strategy-Proofness

We begin by showing that the allocation mechanism H exhibits a convenient property of **strategy-proofness**.

Definition 2. A mechanism H is **strategy-proof** if:

$$\forall i \in \{1, \ldots, n\}, s_{-i}, \theta_i, G \in \{F, \tilde{F}_{\theta_i}\}, j \neq j' : E_G[u_i^j] > E_G[u_i^{j'}] \implies a_i^* = j,$$

If **strategy-proofness** is satisfied, every player has a weakly dominant strategy in the location choice stage of the game. This is particularly due to the assumption of independent preferences. We restate the next result from Sedek (2020), which can be directly applied to our model:

Proposition 5. *Mechanism H is* ***strategy-proof*** *if players' preferences under a prior F are independent*

$$\prod_i F_i = F.$$

The result greatly simplifies the problem and allows us to work conveniently with the value of information. For a discussion of a model without learning, but with interdependent preferences, see Sedek (2020).

Proposition 5 shows that the behavior of players in the location choice stage of the game is determined by the prior F and thus it remains to solve for the information acquisition decisions. The following table summarizes the optimal actions of player i in the location choice stage, in case location 1 is ex-ante better, $E_F(u^1) > E_F(u^2)$.

Table 2.1: Actions in Location Choice Stage

a_i	$\theta_i^{u_i^1>u_i^2}$	$\theta_i^{u_i^2>u_i^1}$
] $\alpha_i =$ "Learn"	1	2
$\alpha_i =$ "Not Learn"	1	1

where $\theta_i^{u_i^j>u_i^{j'}}$ is an event in which the i'th players type is such that $u_i^j > u_i^{j'}$.

Unfortunately, in the general case, information acquisition decisions depend on which players are learning. This is because the value of choosing a location can depend on whether a certain other player chooses the same location. To study information acquisition incentives, we impose a specific choice rule in the next section. We assume ex-ante symmetry to show results for externalities and information acquisition incentives. We return to the ex-ante asymmetric setup in section 2.6.

2.5 Ex-ante Symmetric Players

In this section, we assume that players are ex-ante symmetric. Our setup allows for asymmetries only in the stochastic choice rules $\mathcal{D}^j$. Consequently, the assumption of ex-ante symmetric players is equivalent to imposing an assumption on the stochastic choice rules. Thus, we capture ex-ante symmetry by the following assumption.

Assumption 1. (ex-ante symmetry) Let $\bar{a}^j \equiv |\{\iota \in N : a_\iota = j\}|$ denote[3] the number of players who choose j in a, then the players are ex-ante symmetric in the allocation game

[3] Where $N \equiv \{i \in \mathbb{N}, i \leq n\}$ corresponds to the set of players.

with information acquisition if:

$$D_i^j(a) = D^j(a) = \min\{\frac{q}{\bar{a}^j}, 1\}.$$

The consequence of ex-ante symmetry is that, when player i is deciding what to do, who does what is irrelevant as long as the distribution of the actions in aggregate action a remains the same.

2.5.1 Strategic Effects

As the payoffs do not now depend on who chooses the same location, table 2.1 allows us to write payoffs in terms of the number of players who choose the same location.

Let $\bar{U}_i(a_i, \bar{a}_{-i}^j) \equiv U_i(a_i, a_{-i})$, where $\bar{a}_{-i}^j = |\{\iota \in N : a_\iota = j\}|$. $\bar{U}(a_i, \bar{a}_{-i}^j)$ is then the expected utility player i receives by choosing location a_i, when $\bar{a}_{-i}^j$ other players choose location j. This notation will be convenient in discussion of strategic effects. We next introduce a definition of **strategic substitutability in location choice**:

Definition 3. Mechanism H exhibits **strategic substitutability in location choice** if: $\forall i \in \{1, \ldots, n\}, \forall j \neq j', u_i^j \geq u_i^{j'}, \bar{a}_{-i}^j \leq n-2:$

$$\bar{U}_i(j, \bar{a}_{-i}^j) - \bar{U}_i(j', \bar{a}_{-i}^j) \geq \bar{U}_i(j, \bar{a}_{-i}^j + 1) - \bar{U}_i(j', \bar{a}_{-i}^j + 1). \tag{2.4}$$

that is, if the choice of a location (weakly) lowers the relative value of choosing that location in comparison with the other location for all other players. This differs from the usual definition of strategic subtitutability, which relates rather to the monotonicity of the best response function in the actions of other players. However, as the location choice stage has dominant strategies, the best response function is constant and thus the usual definition does not make sense in this context. Nonetheless, one can see that both sides of inequality 2.4 are a relative payoff of choosing j rather than j' and, thus, if this relative payoff decreases with the number of players who choose j, player i becomes "less prone" to choose location j. In a setting without dominant strategies, this could eventually lead to player i switching from location j to j' and in this sense, the definition captures the essence of strategic substitutability. We argue that the usual definition of strategic substitutability captures a consequence of a decrease in the relative payoff and thus, in a sense, it follows from this definition.

Proposition 6. *The ex-ante symmetric allocation game with information acquisition exhibits* ***strategic substitutability in location choice****.*

Proof. Fix i, j and $j' \neq j$. First, notice that $D^j(a) = \min\{q/\bar{a}^j, 1\}$ is weakly decreasing in $\bar{a}^j$ and

$$D^{j'} = \min\{\frac{q}{\bar{a}^{j'}}, 1\} = \min\{\frac{q}{n - \bar{a}^j}, 1\}$$

is weakly increasing in $\bar{a}^j$. Consequently, due to $u_i^j \geq u_i^{j'}$, $\bar{U}_i(j, \bar{a}_{-i}^j)$ is weakly decreasing in $\bar{a}_{-i}^j$ and $\bar{U}_i(j', \bar{a}_{-i}^j)$ is weakly increasing in $\bar{a}_{-i}^j$. Finally, inequality 2.4 can be written as:

$$\bar{U}_i(j, \bar{a}_{-i}^j + 1) - \bar{U}_i(j, \bar{a}_{-i}^j) \leq \bar{U}_i(j', \bar{a}_{-i}^j + 1) - \bar{U}_i(j', \bar{a}_{-i}^j),$$

the left hand side of which is non-positive, while the right hand side is non-negative due to the monotonicites established above. □

The intuition behind proposition 6 is that, as many players choose the same location, the chances of allocation to this location become (weakly) lower and the expected utility of choosing this location is thus (weakly) lower as well — the congestion effect.

We next define an analogical concept for the information acquisition stage of the game. To do this, recall that the players in the location choice stage of the game behave according to their dominant strategies, that is, according to table 2.1. Let $k \equiv |\{\iota \in N : \alpha_\iota = \text{"Learn"}\}|$ be the number of players who choose to learn. $\psi^j(k, \theta) : \mathbb{N} \times \mathbb{R}^n \mapsto \mathbb{N}$ is a function that gives $\bar{a}^j$, the realized number of players that choose location j, given that k players are learning. $\psi^j(k, \theta)$ together with $\alpha \equiv (\alpha_1, \ldots, \alpha_n)$ then completely determines $\bar{a}^j$. In case we are interested only in a_{-i} we use notation k_{-i} as the number of other players who are learning and $\psi_{-i}^j(k_{-i}, \theta)$ analogically. It is now possible and convenient to define the value of information as a function of the number of players learning. Let $\bar{VI}_i(k_{-i}) \equiv VI_i(s_{-i})$, where $\bar{a}_{-i}^j = \psi_{-i}^j(k_{-i}, \theta)$.

Definition 4. Mechanism H exhibits **strategic complementarity in information acquisition** if: $\forall i \in \{1, \ldots, n\}, k_{-i} \leq n - 2$:

$$\bar{VI}_i(k_{-i} + 1) \geq \bar{VI}_i(k_{-i}). \tag{2.5}$$

Definition 4 captures the idea that learning (weakly) increases the value of learning for other players. Plugging the value-of-information definition into definition 4 reveals the analogy between definition 4 and definition 3. Much like in definition 3, the **complementarity in information acquisition** simply means that the relative payoff of choosing the action "Learn" rather than "Not Learn" is increasing with the number of players learning. Unlike in definition 3, the information acquisition stage does not necessarily exhibit dominant strategies and thus we refer to the counterpart of definition 4, $\bar{VI}_i(k_{-i}+1) \leq \bar{VI}_i(k_{-i})$ as **strategic substitutability in information acquisition**.

In the next proposition, we derive the value of the information function. Characterization of strategic effects will follow. We denote with p the probability of $u_i^1 > u_i^2$, which is determined by F_i.

Proposition 7. *Suppose that prior F is such that, $E_F[u^j] > E_F[u^{j'}]$, then the value of information in the ex-ante symmetric allocation game with information acquisition is:*

$$VI_i(s_{-i}) = (1-p)\Big(E_F[D^{j'}(j', a_{-i})] + E_F[D^j(j, a_{-i})] \\ - 1\Big)\Big(E_F[u_i^{j'}|u_i^{j'} > u_i^j] - E_F[u_i^j|u_i^{j'} > u_i^j]\Big), \quad (2.6)$$

where $E_F[u_i^j|u_i^\gamma > u_i^{\gamma'}]$ is the conditional expected utility from being allocated to location j given that for player i location γ is better than location γ'

Proof. Fix i and s_{-i}. Let F be such that $E_F[u^1] > E_F[u^2]$ without loss of generality.[4] Due to proposition 5, the actions in the location choice stage are determined by table 2.1 and thus the uninformed player chooses location 1 and the value of being uninformed for player i is:

$$\hat{V}_i(s_{-i}) = E_F[D^1(1, a_{-i})]E_F[u_i^1] + E_F[1 - D^1(1, a_{-i})]E_F[u_i^2]$$

The value of being informed is:

$$\tilde{V}_i(\theta_i, s_{-i}) = p(E_F[D^1(1, a_{-i})]E_F[u_i^1|u_i^1 > u_i^2] + E_F[1 - D^1(1, a_{-i})]E_F[u_i^2|u_i^1 > u_i^2]) \\ + (1-p)(E_F[D^2(2, a_{-i})]E_F[u_i^2|u_i^2 > u_i^1] + E_F[1 - D^2(2, a_{-i})]E_F[u_i^1|u_i^2 > u_i^1]).$$

[4] Generality could be regained by assuming a particular behavior for the indifferent players.

Combing both expression yields, after some manipulation, the value of information:

$$VI_i(s_{-i}) = (1-p)\Big(E_F[D^2(2,a_{-i})] + E_F[D^1(1,a_{-i})] \\ -1\Big)\Big(E_F[u_i^2|u_i^2 > u_i^1] - E_F[u_i^1|u_i^2 > u_i^1]\Big). \quad (2.7)$$

□

Expression (2.6) has several natural properties. The first term in expression (2.6) shows that the value of information is proportional to the probability of the ex ante worse location being better in realization. This is because only in that state of the world is the location choice changed. Moreover, the second term shows that the higher the expected chances of being accepted by any location are (the lower the congestion), the higher the value of information is. Finally, the third term shows that the value of information is proportional to the expected difference between the options, conditional on the ex-ante worse option being better in realization.

Unfortunately, the strategic effects in information acquisition are not as simple as in the case of location choice. It turns out that the **strategic complementarity in information acquisition** depends both on the parameters of the model and on how many players are learning. Since learning naturally causes actions to be stochastic, we state the following proposition in terms of a limit when the number of players $n \to \infty$. This is a simplification, as it causes the fraction of players who choose each location to converge by the law of large numbers. Naturally, the $2q \geq n$ cannot be satisfied unless q increases with n. To be able to show the results in terms of the parameters of the model, we denote q^n the capacity for the n player model and $Q \equiv \lim_{n\to\infty} q^n/n$ the capacity in terms of a population fraction. The expected chances of admission $E_F[D^j(a)]$ then also converge, which allows us to present the next proposition. Finally let $\bar{k} \equiv k/n$, the fraction of players who are learning.

Proposition 8. *In the limit as $n \to \infty$, the strategic effects in information acquisition of the ex-ante symmetric allocation game with information acquisition depend on model parameters in the following way, for:*

- $1-p < Q$: *the game exhibits complementarity in information acquisition.*
- $1-p > Q$: *exists a threshold $\hat{k}$, such that for:*
  - $\bar{k} \leq \hat{k}$, *the game exhibits complementarity in information acquisition.*

– $\bar{k} \geq \hat{k}$, *the game exhibits substitutability in information acquisition*

Proof. Since learning makes aggregate location choice a a random variable, it could be the case that both $E_F[D^1(a)] < 1$ and $E_F[D^2(a)] < 1$ for finite n. However, in the limit as $n \to \infty$, the law of large numbers ensures that either $E_F[D^1(a)] \to 1$ or $E_F[D^2(a)] \to 1$ or both. This is because the fraction of players who choose location 1 converges to a number which is either smaller than Q or greater than $1 - Q$. In the former case, the majority of players choose the second location, which can only happen when a majority of players learn. In the latter case, the majority of players choose the first location, which can either be a consequence of low learning or large p.

Consequently, the expression (2.6) is always positive, because the last term is the gain from being allocated to the ex-ante worse location in the state where it is better in realization and thus is positive.

In case of $1 - p < Q$, we know that $E_F[D^2(a)] \to 1$ since for $D^2(a) < 1$, many players have to learn about their preferences and choose location 2. However, even if every player learns, the fraction of players who choose location 2 converges to $(1-p)$, which is strictly less than its capacity Q. In this case, the value of information becomes:

$$VI_i(a) = (1-p)E_F[D^1(a)][E_F[u^2|u^2 > u^1] - E_F[u^1|u^2 > u^1]].$$

Thus, to show complementarity in information acquisition, it remains to prove that $E_F[D^1(a)]$ is weakly increasing in k, the proof of which we leave to the appendix.

Finally, for $1 - p > Q$ let $\hat{k}$ be such that if $\bar{k} > \hat{k}$ then $D^2(a) < 1$. Such a $\hat{k}$ exists, because if every player learns, the law of large numbers ensures in the limit as $n \to \infty$, the fraction of players who learn and choose location 2 converges to $1-p > Q$. The value of information depending on $\bar{k}$ is:

$$VI_i(a) = \begin{cases} (1-p)D^1(a)[E_F[u^2|u^2 > u^1] - E_F[u^1|u^2 > u^1]] & ;\bar{k} < \hat{k} \\ (1-p)D^2(a)[E_F[u^2|u^2 > u^1] - E_F[u^1|u^2 > u^1]] & ;\bar{k} > \hat{k} \end{cases}.$$

Both cases above are possible. The case $\bar{k} < \hat{k}$ requires that $E_F[D^1(a)]$ is increasing in $\bar{k}$, proof of which appears in the appendix. For the case $\bar{k} > \hat{k}$ of substitutability, it remains to prove that $E_F[D^2(a)]$ is weakly decreasing in $\bar{k}$. The proof is analogical to $E_F[D^1(a)]$ being increasing in $\bar{k}$ and thus is left out. □

The strategic complementarity in information acquisition is driven by the following mechanism. An uninformed player always approaches the ex-ante better location. An informed player sometimes learns that the ex-ante worse location is in fact better for him. Consequently, information makes a player less likely to choose the ex-ante better location. As players are learning, the ex-ante better location becomes less congested and chances of allocation to it grow. An uninformed choice for the ex-ante better location becomes riskier as the chance matters less for the allocation. This in turn makes information more valuable.

However, as the second part of proposition 8 shows, when a sufficient number of players are learning, the situation can be reversed if learning often results in choosing of the ex-ante worse location ($1 - p > Q$). In such a case, the ex-ante worse location can become congested, which in turn decreases the value of information as more players are learning.

While the result is in the limit for simplicity, we only need the limit to have distribution of $\bar{a}^j$ converging above or below a certain threshold. The proof of proposition 8 suggests that even if that does not happen, the term $E_F[D^1(a) + D^2(a) - 1]$ should still remain positive and the part which does not cancel with the constant should in principle follow the same monotonicity as in the proof of proposition 8. However, due to the technical difficulty of proving this, we leave it unproven.

2.5.2 Solution

We now describe the solution of the game in the limit as $n \to \infty$. Since the allocation game with information acquisition for some values of parameters exhibits complementarity in information acquisition, the game exhibits multiple equilibria. The first row of figure 2.1 shows the value of information for player i as a function of $\bar{k}$. When $\bar{k}$ is such that the value of information $VI_i(s_{-i})$ is lower than the constant cost c, player i decides not to learn and consequently the aggregate learning $\bar{k}$ drops. When $VI_i(s_{-i})$ exceeds c, player i decides to learn and consequently $\bar{k}$ increases. This results in two stable equilibria, equilibrium $\bar{k} = 0$, henceforth a "non-learning" equilibrium and a "learning" equilibrium where $0 < \bar{k} \leq 1$. In addition to the stable equilibria, figure 2.1 shows an unstable equilibrium, which will be omitted from further discussion due to its instability. Let k^* denote the fraction of players learning in the learning equilibrium and let $E_F[u^1] > E_F[u^2]$, then for $q > 1 - p : k^* = 1$ and for $q < 1 - p : k^*$ solves the indifference condition[5]:

$$c = (1-p)E_F[D^2(\psi(k^*,\theta))]\big(E_F[u^2|u^2 > u^1] - E_F[u^1|u^2 > u^1]\big). \tag{2.8}$$

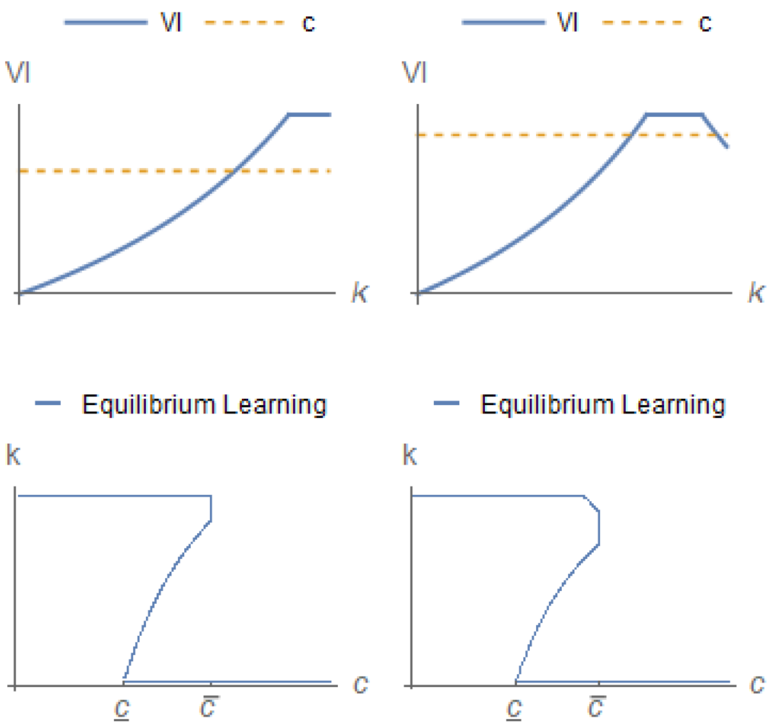

Figure 2.1: Multiple equilibria

The second row of figure 2.1 shows the equilibrium fraction of players learning in terms of information costs. Let $E_F[u^1] > E_F[u^2]$, then for low information costs $c < \underline{c} \equiv (1-p)\frac{q}{n}(E_F[u^2|u^2 > u^1] - E_F[u^1|u^2 > u^1])$, the only equilibrium is the learning equilibrium. For high information costs $c > \bar{c} \equiv (1-p)(E_F[u^2|u^2 > u^1] - E_F[u^1|u^2 > u^1])$, the only equilibrium is the non-learning equilibrium. For information costs between $(\underline{c}, \bar{c})$ the learning, the non-learning and the unstable equilibria emerge.

2.5.3 Externalities and Welfare

In case of multiple equilibria, the equilibria can be welfare ordered in such a way that the stable equilibrium, where some players learn (learning equilibrium) exhibits higher utilitarian welfare than the equilibrium where no players learn (non-learning equilibrium).

[5]In the limit, the condition holds with equality.

This is mostly due to the fact that positive externalities from learning seem to prevail. We first define what we mean by positive externalities from learning.

Definition 5. Let k be the number of players who are learning. An additional player imposes a positive externality by learning on player i if: $\forall a_i, \theta$

$$EXT_i(a_i, k) \equiv E_F[\bar{U}_i(a_i, \psi(k+1, \theta)) - \bar{U}_i(a_i, \psi(k, \theta))] \geq 0,$$

where $\psi \equiv (\psi^1, \psi^2)$[6].

We say that externality on player i is positive only if the impact on his expected utility is positive for all of his possible actions. The next proposition shows that this is the case as long as the ex-ante worse location is not congested.

Proposition 9. *Let $E_F[u^j] > E_F[u^{j'}]$. Let k be the number of players who are learning, then the learning of an additional player imposes a positive externality on other players, $\forall i, a_i, \theta : EXT_i(a_i, k) \geq 0$ if:*

$$E_F[D^{j'}(\psi^{j'}(k, \theta))] = 1.$$

Proof. Let $E_F[u^1] \geq E_F[u^2]$, $E_F[D^2(\psi^2(k, \theta))] = 1$ without loss of generality. The externality imposed on a player i can be written as:

$$\begin{aligned} EXT_i(a_i = j, k) = E_F[D^j(\psi^j(k+1, \theta))]E_F[u^j] \\ + E_F[1 - D^j(\psi^j(k+1, \theta))]E_F[u^{j'}] \\ - E_F[D^j(\psi^j(k, \theta))]E_F[u^j] + E_F[1 - D^j(\psi^j(k, \theta))]E_F[u^{j'}]. \end{aligned}$$

For $a_i = 1$, the externality is:

$$\begin{aligned} EXT_i(1, k) = (E_F[D^1(\psi^1(k+1, \theta))] \\ - E_F[D^1(\psi^1(k, \theta))])(E_F[u^1] - E_F[u^2]) \geq 0. \end{aligned}$$

The inequality follows from $E_F[D^1(\psi^1(k, \theta))]$ being weakly increasing in k, which is proven in the appendix. For $a_i = 2$:

$$EXT_i(2, k) = (E_F[D^2(\psi^2(k+1, \theta))] - E_F[D^2(\psi^2(k, \theta))])(E_F[u^2] - E_F[u^1]) = 0.$$

[6]Technically, either ψ^1 or ψ^2 carries sufficient information for $\bar{U}$, and we define ψ to avoid any confusion regarding what enters $D^1(\bar{a}^1)$ and $D^2(\bar{a}^2)$ within $\bar{U}$.

The result does not hold when $E_F[D^2(\psi^2(k,\theta))] < 1$ and $E_F[D^1(\psi^1(k,\theta))] = 1$, in this case trivially: $EXT_i(1,k) = 0$, but

$$EXT_i(2,k) = (E_F[D^2(\psi^2(k+1,\theta))] - E_F[D^2(\psi^2(k,\theta))])(E_F[u^2] - E_F[u^1]) \leq 0$$

since $D^2(\psi(k,\theta))$ is decreasing in k. □

Since a_i can take only values from $\{1,2\}$, the previous proposition shows that, in the allocation game with information acquisition, a player who is not learning benefits from other players learning. As the player who is learning chooses with probability $p : a_1$ and with probability $1-p : a_2$, the next corollary shows that a player who is learning benefits from other players learning in expectation.

Corollary 2. *Let $E_F[u^j] > E_F[u^{j'}]$, k be the number of players who are learning, then a player who is learning benefits from other players' learning in expectation as long as:*

$$E_F[D^{j'}(\psi^{j'}(k,\theta))] = 1.$$

Proof.

$$E_F[\tilde{V}_i(\theta, s_{-i})] = p\bar{U}_i(1, \psi(k,\theta)) + (1-p)\bar{U}_i(2, \psi(k,\theta)) \geq 0.$$

Consequently:

$$E_F[EXT_i(a_i,k)] = pEXT_i(a_1,k) + (1-p)EXT_i(a_2,k) \geq 0.$$

□

Corollary 2 suggests that, for some parameters of the model, welfare might not be increasing in k. This is because the externality can be negative for high k, when the ex-ante worse location becomes congested. The next result shows that even when the second location is influenced by congestion and welfare becomes decreasing in k, the welfare of the resulting equilibrium dominates the non-learning equilibrium. We again state the next result in the limit to make the fraction of players who choose each location converge.

Proposition 10. *Let s^* be an equilibrium with a fraction of players who are learning $k^* > 0$. Let s^N be an equilibrium such that $\forall \iota \in s^N : \alpha_\iota =$ "Not Learn". Then, as*

$n \to \infty$, s^* *dominates* s^N *in terms of players' utilitarian welfare:*

$$W^L \equiv \sum_{\iota=1}^{nk^*} E_F[\tilde{V}_\iota(\theta, s^*_{-\iota}) - c] + \sum_{\iota>nk^*}^{n} E_F[\hat{V}_\iota(s^*_{-\iota})] \geq \sum_{\iota=1}^{n} E_F[\hat{V}_\iota(s^N_{-\iota})] \equiv W^N.$$

Proof. Let $E_F[u^1] > E_F[u^2]$. The case of $1-p < Q$, leads with $n \to \infty$ to $E_F[D^2(\psi(k,\theta))] = 1$, the ex-ante worse location having excess capacity and consequently, by proposition 9, the setting exhibits positive externalities from learning. This in turn implies the result.

In the case of $1-p > Q$, for $n \to \infty$ only $k^* < 1$ players learn in equilibrium s^*, where k^* solves the indifference condition (2.8). In this equilibrium $E_F[D^2(\psi^2(k^*,\theta))] < 1$ and $E_F[D^1(\psi^1(k^*,\theta))] = 1$. The expected welfare of this equilibrium is:

$$\begin{aligned} W^L = nk^*(1-p)\big(&E_F[D^2(\psi^2(k^*,\theta))]E_F[u^2|u^2 > u^1] \\ &+ E_F[1 - D^2(\psi^2(k^*,\theta))]E_F[u^1|u^2 > u^1]\big) \\ &+ nk^*pE_F[u^1|u^1 > u^2] - cnk^* + n(1-k^*)E_F[u^1]. \end{aligned}$$

Using indifference condition (2.8), W^L simplifies to:

$$W^L = nE_F[u^1]. \tag{2.9}$$

The welfare of the non-learning equilibrium is:

$$W^N = nQE_F[u^1] + n(1-Q)E_F[u^2],$$

which is clearly inferior to (2.9) for all n. □

2.6 Main Result

The following proposition abandons the assumption of ex-ante symmetric players and shows comparative statics with respect to stochastic choice rules $\mathcal{D}^j$. Since $\mathcal{D}^j$ are complicated objects, and comparative statics are thus a challenging task, we will build an asymmetric choice rule with a clear interpretation and compare it to the ex-ante symmetric case.

Suppose that each location j has an (preference) ordering υ^j over the players. That is, location j prefers players with lower index in υ^j. Then suppose that ν^j is a cutoff player such that:

$$\sum_{\iota=1}^{\nu_j^*(a)} \mathbb{1}(a_\iota = j) \leq q.$$

Then our new stochastic choice rule $\tilde{\mathcal{D}}^j$ satisfies the following:

$$\tilde{D}_i^j(a) \equiv \begin{cases} 1 & i \leq \nu_j^*(a) \\ 0 & o.w. \end{cases}$$

Clearly, what we strive for by using this definition is to remove the stochasticity of the stochastic choice rule. The advantage of $\tilde{\mathcal{D}}$ over the ex-ante symmetric case is that $\tilde{\mathcal{D}}$ is deterministic as long as all players know the orderings v^j. We assume this. The next proposition shows that a move from the ex-ante symmetric case to $\tilde{\mathcal{D}}$ is associated with a welfare improvement in the limit, as the number of players goes to infinity.

Proposition 11. *As $n \to \infty$, a change from $D_i^j(a) = q/(\bar{a}^j)$ to $\tilde{D}_i^j(a)$ weakly increases the utilitarian welfare of players.*

Proof. Let $\underline{c} < c < \bar{c}$, otherwise the result holds trivially. Let $E_F[u^1] \geq E_F[u^2]$. We denote $\tilde{k}$ the number of players who learn in equilibrium with $\tilde{D}_j^i(a)$. We will decompose the situation into four cases:

First, let $p < Q$ and $1-p < Q$. Since $1-p < Q$, $\forall k : E_F[D_i^2(\psi^2(k,\theta))] = 1$. Player i who is ranked in v^1 before player Q observes that upon choosing location 1, he is allocated there with certainty $\forall k : \tilde{D}_{i<Q}^1(\psi^1(k,\theta)) = 1$. Consequently, the value of information for this player is $(1-p)(E_F[u^2|u^2 > u^1] - E_F[u^1|u^2 > u^1])$. Since $c < \bar{c}$, the value of information results in a decision to learn. Since all such players are learning, in the limit $n \to \infty$, $(1-p)$ fraction of them chooses location 2. This is expected by other players, and consequently players who are ranked in v^1 between Q and $Q + (1-p)Q$ know that their chance of being accepted by location 1 in equilibrium must be $\tilde{D}_{Q<i<Q+Q(1-g)}^1(\psi^1(\tilde{k},\theta)) = 1$. Iterating this argument yields $\nu = Q/p$, which is the index in v^1 of the final player who can be sure that he will be allocated to location 1 with certainty. Since $p < Q$ all players learn and consequently $\tilde{k} = n$. Proposition 10 then implies the welfare ordering.

Second, for $1-p < Q < p$. The equilibrium under $\tilde{D}_i^j(a)$ is characterized by a population fraction m^1, such that for all players $i \leq nm^1$ the probability of being allocated to location 1, $\tilde{D}_{i\leq nm_1}^1(\psi^1(\tilde{k},\theta))$ converges with $n \to \infty$ to 1 and for all $i > nm^1$ the probability converges to 0. In equilibrium, the value of information converges to

$$\bar{VI}_i(\psi(\tilde{k})) \rightarrow \begin{cases} (1-p)(E_F[u^2|u^2 > u^1] - E_F[u^1|u^2 > u^1]) & ; i \leq nm^1 \\ 0 & ; i > nm^1 \end{cases} \tag{2.10}$$

therefore all $i \leq nm^1$ learn and all $i > nm^1$ do not. Consequently the threshold $m^1 \rightarrow Q/p$. The average welfare of this equilibrium converges to:

$$\frac{W^{m^1}}{n} \rightarrow \frac{Q}{p}(pE_F[u^1|u^1 > u^2] + (1-p)E_F[u^2|u^2 > u^1]) + (1 - \frac{Q}{p})E_F[u^2] - \frac{Q}{p}c,$$

which is larger than: $Q/pE_F[u^1] + (1-Q/p)E_F[u^2]$, whereas the learning equilibrium from the model with uncertainty about locations preferences has average welfare:

$$\frac{q}{n}E_F[u^1] + \frac{n-q}{n}E_F[u^2] = QE_F[u^1] + (1-Q)E_F[u^2] < \frac{Q}{p}E_F[u^1] + (1 - \frac{Q}{p})E_F[u^2].$$

Third, for $p < Q < 1-p$ it can happen that the second location is congested if enough players are learning. In that case, the equilibrium under certainty about location preferences is characterized by a population fraction m^2, such that for all players $i \leq nm^2$ the probability of being accepted by location 2 converges to 1 and for all $i > nm^2$ the probability converges to 0. In the equilibrium, the value of information is the same as (2.10), except that now the threshold is nm^2. In this case, the equilibrium threshold converges to: $m^2 \rightarrow Q/(1-p)$. The average welfare of this equilibrium converges to:

$$\frac{W^{m^2}}{n} \rightarrow \frac{Q}{1-p}(pE_F[u^1|u^1 > u^2] + (1-p)E_F[u^2|u^2 > u^1]) + (1 - \frac{Q}{1-p})E_F[u^1] - \frac{Q}{1-p}c,$$

which is at least $E_F[u^1] > QE_F[u^1] + (1-Q)E_F[u^2]$.

Finally, the case $Q < p$ and $Q < 1-p$ is impossible since $Q \geq 1/2 \geq \min\{p, 1-p\}$. □

The result suggests that reducing the uncertainty around the chances of allocation increases the welfare of the players by causing more efficient investment into information acquisition. While an increase in welfare due to reduced uncertainty may seem obvious, it does not hold for interdependent preferences. We refer to the counterexample, which shows a lack of strategy-proofness in Sedek (2020) and claim that this example also fails the previous welfare result.

2.7 School Choice Application

Allocation of places in schools in many cities is administered via a centralized Gale-Shapley algorithm (Abdulkadiroğlu, Pathak, and Roth 2009). When they apply, students are given their priority scores for each school. The priority score is determined by characteristics such as proximity to the school or having a sibling enrolled in the school. Subsequently, students submit their preference lists to the city. Finally, the city runs the Gale-Shapley algorithm, which is a special case of our mechanism H. Due to the nature of priority scores, many students have the same priority score for a particular school. Ties frequently arise, which are often broken using a lottery. Consequently, students face admission uncertainty caused by the lotteries. By strategy-proofness, this should not influence student behavior in applications, but it does influence student behavior in information acquisition. We apply proposition 11 to argue that students would benefit if the indifferences were broken before learning.

In terms of our model, we conduct a comparative statics exercise in the spirit of proposition 11. With $D^j(a)$, all students are in the same indifference set for a school, and thus they face a lottery each time they apply to an oversubscribed school. With $\tilde{D}^j(a)$, the priorities are strict and all students know their relative ranking for the school before they apply. Consequently, in equilibrium, students know whether they will be accepted to a particular school if they apply. This knowledge facilitates the information acquisition decision for the students and consequently improves their welfare.

Conjecture 1. (of proposition 11) Under the assumption of independent preferences, students enjoy higher welfare when priorities are strict, compared to a situation in which all students have the same priority when participating in a Gale-Shapley school choice mechanism.

We argue that the result would work in the same direction if only a subset of students lack the information about their chances of admission.

Finally, we motivate conjecture 1 by showing how welfare improvement is attainable in practice. We propose the following implementation by adjusting the timing of the admission procedure. Suppose that priorities, as is often the case in practice, are natural numbers. First, draw a tie-breaking lottery number $U[0, 1]$ for each student-school pair.

Second, compute the final (strict) priority score for each student school pair as the sum of their original priority score and the tie-breaker. Finally, inform the students about their score for each school long before applications are sent.

Conjecture 2. (policy recommendation) Resolving the tie-breaking lotteries well in advance of application submissions may increase student welfare from participation in the Gale-Shapley school choice mechanism.

2.8 Conclusion

We study information acquisition incentives and welfare in a model with independent preferences and find that information acquisition exhibits strategic complementarity, even though the setting exhibits strategic substitutability in its underlying allocation mechanism. Multiple equilibria naturally arise from strategic complementarities. Prevailing positive externalities from information acquisition consequently lead to the learning equilibrium welfare dominating the non-learning equilibrium. One way to escape the non-learning equilibrium is to abolish uncertainties in allocation chances. The final welfare result shows that such a change indeed increases players welfare. We apply the result to a school choice setting and argue that large indifference sets in priorities harm students by introducing uncertainty into their information acquisition decisions. The priorities can be better designed to reflect this and consequently will improve student welfare.

2.A Appendix - Remainder of Proof of Proposition 8

It remains to prove that: $E_F[D^1(a)] = E_F[D^1(\psi^1(k,\theta))]$ is increasing in $\bar{k}$ (and k) for $E_F[u^1] > E_F[u^2]$:

Proof. Clearly, for a fixed n, $E_F[D^1(\psi^1(k,\theta))]$ is increasing in $\bar{k} = k/n$ if and only if it is increasing in k we will prove the latter, that is

$$\forall k \leq n-1 : E_F[D^1(\psi^1(k+1,\theta))] \geq E_F[D^1(\psi^1(k,\theta))].$$

Let $\hat{D}^1(a) \equiv q/(\bar{a}^1)$. Recall that $D^1(a) = \min\{\hat{D}^1(a), 1\}$. $\bar{a}^1$ is a random variable composed of player i, players who do not learn, and players who learn and, who find themselves to be type $\theta^{u^1>u^2}$, which happens with probability p. Due to independent preferences, $\bar{a}^1 = n - k + \gamma^k$, where γ^k follows $BIN(p,k)$. Since:

$$E_F[\min\{\hat{D}^1(\psi^1(k,\theta)),1\}] = P(\hat{D}^1(\psi^1(k,\theta)) \geq 1)1 + P(\hat{D}^1(\psi^1(k,\theta)) < 1)E(\hat{D}^1(\psi^1(k,\theta))|\hat{D}^1(\psi^1(k,\theta)) < 1),$$

which is weakly increasing in k when:

$$P(\hat{D}^1(\psi^1(k+1,\theta)) \geq 1)(E_F[\hat{D}^1(\psi^1(k+1,\theta))|\hat{D}^1(\psi^1(k+1,\theta)) < 1] - E_F[\hat{D}^1(\psi^1(k,\theta))|\hat{D}^1(\psi^1(k,\theta)) < 1]) + (P(\hat{D}^1(\psi^1(k,\theta)) \geq 1) - P(\hat{D}^1(\psi^1(k+1,\theta)) \geq 1)) (E_F[\hat{D}^1(\psi^1(k,\theta))|\hat{D}^1(\psi^1(k,\theta)) < 1] - 1) \geq 0.$$

Both summands are positive when the probability $P(\hat{D}^1(\psi^1(k,\theta)) \geq 1)$ and the expectation $E(\hat{D}^1(\psi^1(k,\theta))|\hat{D}^1(\psi(k,\theta)) < 1)$ are both increasing in k.

The former is equal to $P(\frac{q}{n-k+\gamma^k} \geq 1) = P(\gamma^k \leq q-n+k)$ and it is increasing when:

$$P(\gamma^{k+1} \leq q-n+k+1) > P(\gamma^k \leq q-n+k)$$

$$pP(\gamma^k \leq q-n+k) + pP(\gamma^k = q-n+k+1) + (1-p)P(\gamma^k \leq q-n+k) > P(\gamma^k \leq q-n+k),$$

which simplifies to: $pP(\gamma^k = q-n+k+1) > 0$ and thus $P(D^1(\psi^1(k,\theta)) < 1)$ is increasing in k.

The remaining step is to prove that $E_F[\hat{D}^1(\psi^1(k,\theta))|\hat{D}^1(\psi^1(k,\theta)) < 1]$ is weakly increasing in k. To prove it, we prove that $E_F[\hat{D}^1(\psi^1(k,\theta))]$ is weakly increasing in k.

$$E_F[\hat{D}^1(\psi^1(k+1,\theta))] = \sum_{\gamma=0}^{k+1} \frac{q}{n-(k+1)+\gamma}\binom{k+1}{\gamma}p^\gamma(1-p)^{k+1-\gamma}$$
$$= p\sum_{\gamma=0}^{k} \frac{q}{n-k+\gamma}\binom{k}{\gamma}p^\gamma(1-p)^{k-\gamma} + (1-p)\sum_{\gamma=0}^{k} \frac{q}{n-(k+1)+\gamma}\binom{k}{\gamma}p^\gamma(1-p)^{k-\gamma}$$
$$= pE_F[\hat{D}^1(\psi(k,\theta))] + (1-p)\sum_{\gamma=0}^{k} \frac{q}{n-(k+1)+\gamma}\binom{k}{\gamma}p^\gamma(1-p)^{k-\gamma}$$

and consequently: $E_F[\hat{D}^1(\psi(k+1,\theta))] \geq E_F[\hat{D}^1(\psi(k,\theta))]$ since:

$$\sum_{\gamma=0}^{k} \frac{q}{n-(k+1)+\gamma} \binom{k}{\gamma} g^{\gamma}(1-g)^{k-\gamma} \geq \sum_{\gamma=0}^{k} \frac{q}{n-k+\gamma} \binom{k}{\gamma} g^{\gamma}(1-g)^{k-\gamma}.$$

□

Chapter 3

Strategic Behavior of an Online Platform

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3.1 Introduction

Platform intermediaries in two-sided markets are of increasing interest, as we have seen a surge in their numbers in recent years. These platforms choose many different design features, such as the rating system they use, the order and the way in which they display search results, their search filtering options, and many other visual and design aspects and platform features. It is becoming increasingly obvious that all of these features affect the allocation outcomes and the choices of consumers.[1]

We focus on platforms such as booking.com and Airbnb, which have significant market shares, and where the product sold through the platform is depletable, aggregate demand for the product fluctuates, and the consumer buys one unit at a time. We show that in such a setting the platform has incentives to allocate some low quality sellers to consumers when there is a state of low demand, in order to attract sellers to the market and therefore to benefit from larger market size when there is a state of high demand.

This paper studies the platform's allocation decisions. In our model, the platform commits to a stochastic allocation rule and a transaction fee which is constant across sellers. The sellers subsequently set prices and decide whether or not they will participate on the market. After that, the demand is realized, the allocation is realized, and consumers decide whether or not they will buy the allocated products. We assume the platform charges the same transaction fee to all sellers, and therefore earns more revenue

[1] For example, Anderson 2012; Mellinas, María-Dolores, and García 2015; Ert and Fleischer 2016; Dinerstein et al. 2018, Martin and Norton 2009, and others.

from transactions made by higher quality sellers who charge higher prices than lower quality sellers. However, the platform also realizes higher profits if the volume of transactions is higher, that is, if more sellers participate on the market. The platform therefore chooses its allocation decision together with the transaction fee strategically, taking into account its influence on the participation decisions of the sellers. While the platform cannot influence the participation decisions of particular sellers by charging them different transaction fees (it cannot price discriminate among firms), it can influence their decision by manipulating the allocations. A low quality seller who faces a higher probability of selling his product is more likely to stay on the market, generating additional profits. On the contrary, a high quality seller who enjoys a positive profit can be allocated to a consumer with lower probability and still stay on the market.

Even though we completely disable price discrimination by the platform, this is not crucial for our results. In reality, it is likely that there will be some small degree of price discrimination (for example, several pricing options), but as long as the price discrimination is not perfect and the platform does not charge a different fee to every individual seller, our result and the mechanism that we describe in this paper remain relevant.

This paper is organized as follows: Section 3.2 summarizes the relevant literature, Section 3.3 describes our model, Section 3.4 derives an alternative formulation of the problem, Section 3.5 derives the optimal behavior of a profit-maximizing platform, and Section 3.6 concludes.

3.2 Literature Review

This paper studies the behavior of a platform intermediary that enables trade between consumers and sellers. It thus builds on the growing body of mostly theoretical literature on two-sided platforms (e.g., Rochet and Tirole 2003; Rochet and Tirole 2006; Armstrong 2006, Weyl 2010). Most of the literature studies pricing strategies of platforms in a setting in which the two sides of the market exert externalities upon each other, i.e. the benefit of joining the platform for one side of the market is increasing in the size of the other side. We diverge from both of these: we fix the consumer side and assume exogenous consumer affiliation, and we also fix the pricing strategy of our platform to a simple transaction fee imposed on the sellers, with no charges being levied on the customers, which is what we often observe with online platforms in reality. Instead, we focus on a new decision of the platform of how to allocate heterogeneous sellers to consumers when there is an excess of sellers. This direction is motivated by an abundance of empirical and experimental research showing that online platforms make a number of design decisions that all have a surprisingly significant influence on consumers' choices.

We follow Hagiu and Jullien (2014) by adding a new platform design decision and thus expand the study of platforms. Hagiu and Jullien focused on the ability and incentive of a platform to divert consumers to unsolicited content such as advertisements in order to increase profits. In Hagiu and Jullien (2014), the platform trades-off consumer traffic (market size) for higher revenues derived from advertising exposure. Similarly, we incorporate findings from the empirical and experimental literature on online platforms and model the fact that platforms have the ability to influence or even manipulate consumer product choices by choosing the design and rules of the platform, especially the order in which the platform lists search output. For example, Anderson (2012) finds that if a hotel's position on the list drops by one (e.g. being 3rd vs 2nd or 10th vs 9th), its likelihood of being selected drops by 11.5%. In an experimental setting with relatively short lists[2], Ert and Fleischer (2016) and Murphy, Hofacker, and Mizerski (2006) confirm that the top positions are most likely to be selected, and that likelihood decreases along with the position. However, they also find that the likelihood of being selected increases for the very last position on the list, which may be due to the shortness of their lists where the last position is more visible. Both of these findings are consistent with the well documented primacy and recency effects.

Another example of how platform design, and specifically the way products are displayed and ordered for consumers, affects outcomes is Dinerstein et al. (2018), who exploit and analyze a design change that eBay implemented in 2011. The change consisted of altering the algorithm that orders searches from a more relevance based and price independent algorithm to a design which put more emphasis on price. Dinerstein et al. (2018) show that the switch created more price competition and led to lower prices and fewer purchases being made.

In addition to search output ordering, the platform makes many other design choices such as the choice of the rating system. Mellinas, María-Dolores, and García (2015) and Mellinas, María-Dolores, and García (2016) point out a previously unnoticed characteristic, that the booking.com rating scale goes from 2.5 to 10 - it does not start at 0 or 1 as one might think, and it thus inflates scores. They study the rating system and

[2] Ert and Fleischer (2016) use a list of 10 hotels, and Murphy, Hofacker, and Mizerski (2006) perform two experiments, one with 6 links and the other with 7 links in a list.

its attributes and conclude that the system distorts the ratings so as to shift the hotels with the lowest scores upwards towards medium scores. In turn, Anderson (2012) finds that a 1% increase in a review score increases the occupancy of a listing by up to 0.54%. The rating system thus represents another channel through which the platform can affect allocations. Ert (2014) summarizes several other seemingly trivial design aspects of websites which affect consumer choices in non-trivial ways. For example, Martin and Norton (2009) show that organization of information in online settings can significantly influence consumer choices. In a lab experiment, they show that grouping different attributes into one category induces consumers to put less weight on them in their decision making, as opposed to listing the same attributes in separate categories. Further using online experiments, Mandel and Johnson (2002) show that priming by altering background pictures and colours affects consumer choices, and Weinmann, Schneider, and vom Brocke (2016) additionally review several principles of digital nudging.

Though the reality is, of course, more complex, we simplify the choice of the platform in our model to choosing the probabilities with which the sellers of a particular quality are matched with a consumer. The platform can thus choose whether to allocate a higher or lower quality seller to a consumer. This principle is very similar to Dinerstein et al. (2018) in which the platform chooses a visibility function, or the probability of a product being considered by consumers.

We find that a platform may want to systematically divert consumers towards lower quality products. The motivation for doing so is somewhat similar to that in Kremer, Mansour, and Perry (2014), who show that a platform or a principal might want to give some consumers inferior recommendations in order to induce them to explore new alternatives and reveal new information which can be useful for other consumers. Similarly, in Hagiu and Jullien (2014), the platform diverts consumers to unsolicited content. The logic is comparable to (Jullien 2011)'s divide and conquer strategy. In his paper, the platform may want to subsidize one side of the market in order to attract it to join the platform and increase the benefits of joining for the other side, from which the platform then collects revenue. Though all of these papers are very different and model different situations in different settings, they share the feature that the platform does something seemingly counter-intuitive in order to maximize its profits. In our model, the platform effectively subsidizes low quality sellers in a state of low demand by matching them with consumers in order to induce sellers to join the platform and thus recoup revenue from a larger market in a state of high demand.

3.3 The Model

3.3.1 Setup

There are three types of agents in a two-sided market. Sellers, buyers and a monopolistic platform which manages the market. The buyers and sellers meet through the platform and cannot interact without it.

Each seller is selling a single unit of a good of quality $q \in Q \equiv [0, \bar{q}]$, which summarizes his type. There is a continuum of measure $\bar{q}$ of sellers uniformly distributed on interval Q. Each buyer can buy a single unit of the good from a seller. There are two possible states of the world: the mass of buyers who come to the platform is $D < \bar{q}$ in state *Low* which occurs with probability g, and it is $\bar{q}$ in state *High* which occurs with probability $1-g$. In either state the platform allocates buyers to sellers. Consequently, in state *High* all sellers can receive a buyer, but in state *Low*, there are fewer buyers than sellers and some sellers cannot be allocated a buyer.

We assume that the platform cannot distinguish between buyers and the allocation decision can thus be described by $\mu^{State} : Q \mapsto \{0, 1\}$, where the seller q receives a buyer, whenever $\mu(q) = 1$. Consequently μ^{High} is trivially $\mu^{High}(q) = 1$ for all q [3] and $\mu^{Low} \equiv \mu$ (for simplicity of notation), is then chosen by the platform, subject to a constraint $\int_Q \mu = D$. We denote the set of allocation functions which satisfy this constraint with $M \equiv \{\mu : \int_Q \mu = D\}$.

The game has three stages. First, the platform sets a transaction fee $t \in [0, 1]$ and a stochastic allocation rule for the state *Low*, $h \in \Delta(M)$ [4], $A_P \equiv [0, 1] \times \Delta(M)$. Second, all sellers $q \in Q$ simultaneously decide whether to participate on the platform or not and set prices p_q, $A_S \equiv \{"Participate", "Not"\} \times \mathbb{R}$. Third, the buyers arrive and the state of the world is realized - the mass of buyers is revealed. Every buyer meets a single seller according to the realized allocation rule h [5], observes his allocated seller (q, p_q), and decides whether he buys from him or not $A_B \equiv \{"Buy", "Not"\}$.

Each buyer has private information on his appreciation for quality θ, which is dis-

[3] This is a consequence of the fact that, in this state there are enough buyers to be matched with all sellers, and every seller and every buyer wants to be matched.

[4] Choosing an allocation function μ can be thought of as choosing a pure strategy, this is equivalent to allowing for mixed strategies. A crucial assumption is that $h(q)$ cannot be conditional on the sellers' choice of price with $h(q, p_q)$. We return to this assumption later on.

[5] This is guaranteed, because the platform can anticipate sellers' participation decisions and giving a non-participating seller non-zero probability of meeting a buyer is wasteful.

tributed (independently of other buyers) on $[0,1]$ according to a cumulative distribution function F with associated density f. The buyers' utility function is $u \equiv \theta q - p_q$ if he purchases good q and 0 if he does not. A seller q's expected profit is

$$\pi_S(q, p_q, h(q), t) \equiv (1-t)(1-g+gh(q))P(\text{Sale of q}|\mu(q)=1)p_q$$

if he *Participates* and $c : Q \mapsto \mathbb{R}$ if he does *Not*. We assume the opportunity cost of the seller to be weakly increasing in quality $\frac{\partial c}{\partial q} \geq 0$. The platform's expected profit is:

$$\pi_P(h,t) \equiv t \int_0^{\bar{q}} (1-g+gh(q))P(\text{Sale of q}|\mu(q)=1)p_q \mathbb{1}_{IR(q,h,t)\geq 0} dq$$

where $IR(q, h(q), t) \geq 0$ is an individual rationality constraint[6] of seller q, which will be satisfied for sellers who *Participate*.

We assume a subgame perfect equilibrium and thus, by backwards induction, the buyer θ observes his allocated (q, p_q) and *Buys* iff:

$$\theta \geq \frac{p_q}{q} \tag{3.1}$$

After observing (h,t), knowing the buyers' strategy, the participating seller q maximizes his expected profit:

$$\max_{p_q} (1-t)(1-g+gh(q))(1-F(\frac{p_q}{q}))p_q, \tag{3.2}$$

We denote the solution for firm q's problem $p : Q \mapsto \mathbb{R}_+$[7]. The seller q *Participates* if the maximum of (3.2) is larger than $c(q)$ or equivalently if:

$$IR(q, h(q), t) \equiv \pi_S(q, p(q), h(q), t) - c(q) \geq 0.$$

Finally, the platform, knowing the sellers' strategy, chooses $h : Q \mapsto [0,1]$ and $t \in [0,1]$ to solve:

$$\max_{h,t} t \int_0^{\bar{q}} (1-g+gh(q))(1-F(\frac{p(q)}{q}))p(q) \mathbb{1}_{IR(q,h(q),t)\geq 0} \, dq \tag{3.3}$$

[6]To be defined properly after formulation of the sellers' problem.

[7]The assumptions sufficient for the existence and uniqueness of this solution are presented in section 3.3.2

$$s.t: \int_0^{\bar{q}} h(q)dq = D \tag{3.4}$$

$$IR(q,h(q),t) \geq 0 = \{q : h(q) \geq \frac{1}{g}\Big(\frac{c(q)}{(1-F(\frac{p(q)}{q}))(1-t)p(q)} - (1-g)\Big)\}$$

Equation (3.4) is a consequence of allocating measure D of buyers to sellers in realization. We discuss how this constraint follows from the general problem in Appendix A. The constraint is with equality, because it is strictly better to allocate all buyers. The solution to the platform's problem will be denoted as h^*, t^*.

3.3.2 Assumptions and Preliminary Analysis

This section introduces several assumptions and discusses some preliminary results. The assumptions we make are: h is not a function of p_q, f is continuous, a technical assumption $F(x) + xf(x)$ is strictly monotone in x and $c(q)$ is concave. Given these assumptions, it turns out that $p(q)$ is weakly increasing in q and the boundary of the individual rationality set $h_0 : Q \times \mathbb{R} \mapsto \mathbb{R}$, implicitly defined by $IR(q, h_0(q,t), t) = 0$, is decreasing in q. These consequences, while rather technical, are important for later results. This section provides a discussion of the assumptions above.

h is not allowed to be a function of p_q

We assume that h is a function of q, but not p_q. Allowing for $h(q, p_q)$ allows the platform to influence price p_q through $h(q, p_q)$. Effectively, the platform could choose a p_q and then force the seller to set the price to this level by giving it a probability to sell only if the seller chooses the same p_q. Nevertheless, we argue that, in our model, the platform has no incentive to do such a thing, as the seller-optimal price $p(q)$ is also optimal for the platform. This is driven by the fact that the seller finds himself in a monopolistic position with the buyer after they are matched and thus, the price he chooses extracts all possible surplus from the buyer and the platform cannot do any better. This would not be the case, for example, if two sellers were competing after being allocated to a single buyer. In this paper we focus on the pricing decision of the platform and thus we leave out a discussion of including competition among sellers into the sellers' pricing problem.

f is continuous and $F(x) + xf(x)$ is strictly monotone in x

We need the assumption of continuous f for the existence of the solution to the sellers' problem. The solution of seller q, $p(q)$ is given by the first order condition:

$$(1 - g + gh(q))(1 - t)(1 - F(\frac{p(q)}{q}) - \frac{p(q)}{q} f(\frac{p(q)}{q})) = 0$$

interior solution exists for a continuous f [8] and solves:

$$1 - F(\frac{p(q)}{q}) - \frac{p(q)}{q} f(\frac{p(q)}{q}) = 0$$

To guarantee uniqueness of the solution to the previous equation, we would further need $F(x) + xf(x)$ to be strictly monotone. This is guaranteed, for example, for uniform F.

$p(q)$ is increasing and linear in q

Proposition 12. *$p(q)$ is increasing and linear in q, given f is continuous and $F(x) + xf(x)$ is strictly monotone in x.*

Proof. Let a be the solution of:

$$1 - F(a) - af(a) = 0,$$

then $p(q) = aq$, showing linearity.

We show that $p(q)$ is increasing in q by contradiction: If $p(q)$ is not increasing in q, then because it is linear there is an $a \leq 0$ s.t:

$$1 - F(a) - af(a) = 0$$

but for $a \leq 0$, $F(a) = f(a) = 0$ as a consequence the first order condition cannot be satisfied. □

Thanks to Proposition 12, $p(q)$ can be written as $p(q) = aq$, where $a > 0$.

[8] Setting $p_q = 0$ and $p_q = q$ shows existence together with continuity of f.

$h_0(q,t)$ is decreasing in q for concave costs

Finally, we show that the boundary of the individual rationality set, $h_0(q,t)$ (the lowest h for which individual rationality is satisfied given q and t), is decreasing in q for concave costs.

Lemma 1. *If* $\frac{\partial^2 c(q)}{\partial^2 q} < 0$*, then* $\frac{\partial h_0(q,t)}{\partial q} < 0$

Proof. $h_0(q,t)$ is defined implicitly by:

$$(1-t)(1-g+gh_0(q,t))(1-F(\frac{p(q)}{q}))p(q)-c(q)=0$$

This yields after some manipulation:

$$h_0(q,t)=\frac{1}{g}\Big(\frac{c(q)}{(1-F(\frac{p(q)}{q}))(1-t)p(q)}-(1-g)\Big)$$

Then, using Proposition 12, the derivative is:

$$\frac{\partial h_0(q,t)}{\partial q}=\frac{1}{g(1-t)}\frac{\partial}{\partial q}[\frac{c(q)}{(1-F(\frac{p(q)}{q}))p(q)}]=\frac{1}{g(1-t)(1-F(\frac{p(q)}{q}))a}\frac{\partial}{\partial q}[\frac{c(q)}{q}]$$

Using the assumption $\frac{\partial^2 c(q)}{\partial^2 q} < 0$ implies that average costs $\frac{c(q)}{q}$ are decreasing in q which yields the result. □

3.4 Analysis of the Platform's Problem

The platform's problem is a functional analysis problem with a discontinuous solution and thus is difficult to tackle using formal tools. Throughout this section, we show that the problem can alternatively be formulated in terms of multivariate real analysis, by introducing the following result:

Theorem 1. $\forall t \in (0,1], \exists q_1 \le q_2 \le q_3 \in \mathbb{R}$ *s.t:*

$$h^*(q)=\begin{cases} h_0(q,t) & q_1 \le q \le q_2 \\ 1 & q \ge q_3 \\ 0 & o.w. \end{cases}$$

where $q_2 = \min\{q_0, q_3\}$, *with* $q_0 \equiv h_0^{-1}(0)$[9].

We prove the result in Appendix B. The remainder of this section discusses the intuition behind the shape of h^*. Panels of Figure 3.1 depict how function h^* looks for different parameters of the model. While panel A shows the situation where $q_2 = q_0$, panel B shows the situation with $q_2 = q_3$. In both cases, the sellers with $q_1 \leq q \leq q_2$ are allocated the exact probability $h_0(q)$ that makes them indifferent between *Participating* and *Not*, because their expected profit is 0. On the other hand, the sellers to the right of q_2 enjoy positive expected profits. The shape of h^* is driven by two opposing sources of profit for the platform. First, the platform benefits from all transactions, but it benefits more from higher q which is associated with higher $p(q)$. This effect results in an incentive to set high $h(q)$ for high q. Second, the platform benefits from an increase in the mass of the participating sellers. By satisfying the individual rationality condition for an additional seller and making that seller "*Participate*", the platform generates additional profits in state *High*, in which there are plenty of buyers. This effect incentivizes the platform to satisfy the individual rationality condition for as many sellers as possible. Finally, observe that while the profit from the first source is realized only in the bad state of the world, the profit from the second source is realized in both states of the world[10]. In an interior solution, the platform balances the two effects and equalizes the marginal profits from both sources, giving rise to the shape of h^* in Figure 3.1.

The idea behind the proof is roughly that the problem can be thought of as optimally allocating a mass of D between 0 and $\bar{q}$ such that the objective (3.3) is maximized. To the right of the h_0 curve, the same mass allocated on higher q yields higher returns. This is due to the monotonicity of the objective function, which is driven by Proposition 12, $p(q)$ function being increasing in q (Lemma 2 in Appendix B follows). Similar monotonicity applies to mass changes on the left side of the h_0 curve holding the mass of participating sellers constant (Corollary 3 in Appendix B follows). The mass shift which changes the shape of the h_0 curve has additional gains (losses) when it causes additional sellers to "*Participate*" ("*NotParticipate*") and consequently there is a trade-off between allocating the mass to satisfy the $IR(q)$ constraint for additional sellers, the extensive margin,

[9] Technically q_0 and h_0 should be functions of t, however, throughout this section we will keep t fixed to an arbitrary value in interval $(0, 1]$ and thus for the sake of notation simplicity we will act as if t is a parameter of the model, rather than a variable.

[10] With probability weights h_0 and 1 for states *Low* and *High* respectively.

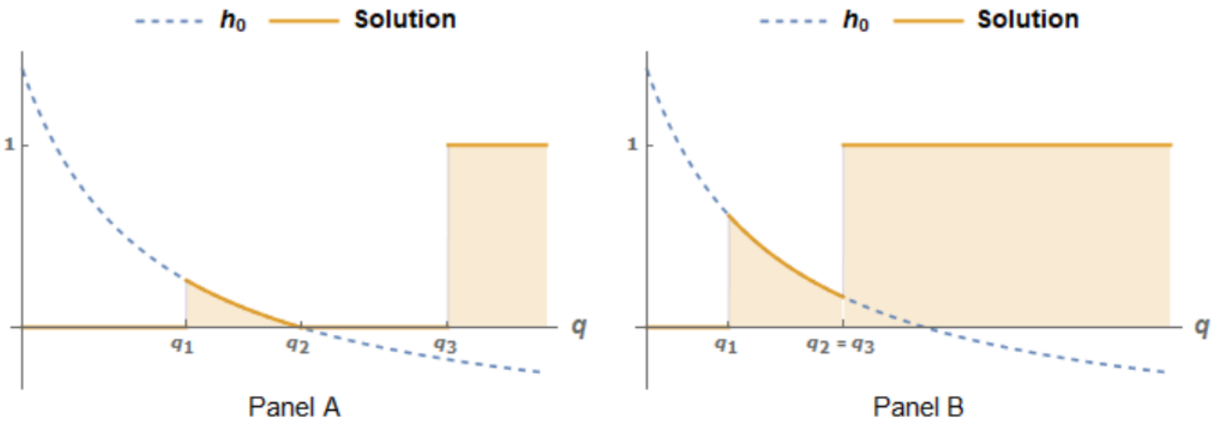

Figure 3.1: Shape of h^*.

and allocating it so as to gain additional profit from the sellers who are already present on the market, the intensive margin.

3.5 Solving the Platform's Problem

Theorem 1 allows us to write the platform's maximization problem equivalently as:

$$\max_{q_1,q_2,q_3,t} \pi(q_1,q_2,q_3,t)$$
$$s.t: \int_{q_1}^{q_2} h_0(q,t)dq + \bar{q} - q_3 = D \tag{3.5}$$
$$t \in [0,1]$$

where

$$\pi(q_1,q_2,q_3,t) \equiv t[g\int_{q_1}^{q_2}(1-F(\frac{p(q)}{q}))p(q)h_0(q,t)dq + g\int_{q_3}^{\bar{q}}(1-F(\frac{p(q)}{q}))p(q)dq + (1-g)\int_{q_1}^{\bar{q}}(1-F(\frac{p(q)}{q}))p(q)dq] \tag{3.6}$$

$$h_0(q,t) = \frac{1}{g}\Big(\frac{c(q)}{(1-F(\frac{p(q)}{q}))(1-t)p(q)} - (1-g)\Big)$$

$$q_2 = \min\{h_0^{-1}(0), q_3\}$$

This is a real analysis problem and thus the first order conditions can be derived from the related Lagrangean. The solution to the platform's problem $(q_1^*, q_2^*, q_3^*, t^*)$ then solves the following set of equations:

$$\frac{\partial}{\partial t} = \frac{\pi(q_1^*, q_2^*, q_3^*, t^*)}{t^*} + t^* g \int_{q_1^*}^{q_2^*} (1 - F(\frac{p(q)}{q}))p(q)\frac{\partial h_0(q,t)}{\partial t}dq - \lambda \int_{q_1^*}^{q_2^*} \frac{\partial h_0(q,t)}{\partial t}dq = 0 \quad (3.7)$$

$$\frac{\partial}{\partial q_1} = t^* g(1 - F(\frac{p(q_1^*)}{q_1^*}))(-p(q_1^*)h_0(q_1^*, t^*)) + t^*(1-g)(1-F(\frac{p(q_1^*)}{q_1^*}))(-p(q_1^*)) + \lambda h_0(q_1^*, t^*) = 0 \quad (3.8)$$

$$\frac{\partial}{\partial q_2} = t^* g(1 - F(\frac{p(q_2^*)}{q_2^*}))p(q_2^*)h_0(q_2^*, t^*) - \lambda h_0(q_2^*, t^*) = 0 \quad (3.9)$$

$$\frac{\partial}{\partial q_3} = t^* g(1 - F(\frac{p(q_3^*)}{q_3^*}))(-p(q_3^*)) + \lambda = 0 \quad (3.10)$$

$$\frac{\partial}{\partial \lambda} = D - \int_{q_1^*}^{q_2^*} h_0(q, t^*)dq - \bar{q} + q_3^* = 0 \quad (3.11)$$

Proposition 13. *Either $q_2^* = q_3^*$ or $q_2^* = q_0(t^*)$.*

Proof. Equation 3.9 implies either: $h_0(q_2^*, t^*) = 0$ or $t^* g p(q_2^*)(1 - F(\frac{p(q_2^*)}{q_2^*})) = \lambda$ For $h_0(q_2^*, t^*) = 0$, q_2^* solves:

$$h_0(q_2^*, t^*) = 0$$

and thus it solves the same equation as $q_0(t^*)$.

For $t^* g p(q_2^*)(1 - F(\frac{p(q_2^*)}{q_2^*})) = \lambda$: 3.10 implies $q_2^* = q_3^*$. □

Proposition 13 confirms the existence of two possible shapes of h^* as depicted in Figure 3.1. The case $q_2^* = q_0$ corresponds to panel a, while the case $q_2^* = q_3^*$ corresponds to panel b.

Proposition 14. *The trade-off between the extensive and intensive margins is captured by the following equation:*

$$g h_0(q_1^*, t^*)p(q_1^*) + (1-g)p(q_1^*) = g h_0(q_1^*, t^*)p(q_3^*) \quad (3.12)$$

Proof. combining equation 3.8 and 3.10 yields:

$$t^*g(1-F(\frac{p(q_1^*)}{q_1^*}))(-p(q_1^*)h_0(q_1^*)) + t^*(1-g)(1-F(\frac{p(q_1^*)}{q_1^*}))(-p(q_1^*)) \\ - t^*g(1-F(\frac{p(q_3^*)}{q_3^*}))(-p(q_3^*))h_0(q_1^*) = 0$$

The use of Proposition 12 yields:

$$t^*g(1-F(a))(-p(q_1^*))h_0(q_1^*))+t^*(1-g)(1-F(a))(-p(q_1^*))-t^*g(1-F(a))(-p(q_3^*))h_0(q_1^*) = 0$$

which after some manipulation yields the result. □

Proposition 14 confirms the previously described intuition behind Figure 3.1. For a small $\epsilon > 0$, the left hand side of equation 3.12 can be interpreted as the marginal benefit of allocating a rectangle of $h_0(q_1^*)\epsilon$ next to point q_1^*. In such a case, the platform benefits by extracting with probability g an additional profit of $h_0(q_1^*)p(q_1^*)$ and with probability $1-g$ profit of $p(q_1^*)$. The right hand side is then analogically the marginal benefit of allocating a rectangle $\epsilon h_0(q_1^*)$ to the point q_3^*. In this case, the additional profit is with probability g, $h_0(q_1^*)p(q_3^*)$ and with probability $1-g$ it is 0, because in state *High* the firms around q_3^* "*Participate*" with or without the additional mass.

We do not focus on t in this paper, but for completeness, the platform of course faces another trade-off when choosing the transaction fee. Since an increase in t shifts h_0 to the right, decreasing the set of sellers who decide to ȷ*Participate*ȷ, the platform can charge a lower transaction fee, attract more sellers, and generate a larger market, or it can charge higher fees, which discourage more sellers from participating and thus decrease the market size. The optimal t balances the two effects.

3.6 Conclusion

We show that online platforms have incentives to manipulate the demand that their sellers face by manipulating consumer choices. Specifically, in our model, the platform's recommendation strategy is nonlinear in the firm quality and redistributes some demand from high quality to low quality sellers. While the direction of the effect suggests that the demand manipulation is detrimental to welfare, this is not certain as the manipulation also enlarges the market. A full welfare analysis of this effect would be an interesting topic for further research. Other possible directions to explore include allowing more complicated pricing strategies of the platform, or studying competition among several platforms.

3.A Appendix - Constraint Derivation

Derivation of:

$$\int_0^{\bar{q}} h(q)dq = D. \tag{3.13}$$

This section shows how constraint 3.13 follows from a mixed strategy h supported by two pure strategies $h_i \in M, i \in 1, 2$. Generalization to finite pure strategies is then straightforward.

$$h_i = \begin{cases} 1; & q \in Q_i \\ 0; & o.w. \end{cases}$$

Let $h = \nu h_1 + (1-\nu)h_2$, where $\nu \in [0,1]$, then:

$$h = \begin{cases} 1; & q \in Q_1 \cap Q_2 \\ \nu; & q \in Q_1 \setminus Q_2 \\ 1-\nu; & q \in Q_2 \setminus Q_1 \\ 0; & o.w. \end{cases}$$

$$\int_Q hdq = \int_{Q_1 \setminus Q_2} hdq + \int_{Q_2 \setminus Q_1} hdq + \int_{Q_1 \cap Q_2} hdq$$

$$= \int_{Q_1 \setminus Q_2} \nu h_1 dq + \int_{Q_2 \setminus Q_1} (1-\nu)h_2 dq + \int_{Q_1 \cap Q_2} (\nu h_1 + (1-\nu)h_2)dq$$

$$= \nu \int_{Q_1 \setminus Q_2} h_1 dq + (1-\nu) \int_{Q_2 \setminus Q_1} h_2 dq + \nu \int_{Q_1 \cap Q_2} h_1 dq + (1-\nu) \int_{Q_1 \cap Q_2} h_2 dq$$

$$= \nu \int_{Q_1} h_1 dq + (1-\nu) \int_{Q_2} h_2 dq = \nu D + (1-\nu)D = D$$

3.B Appendix - Proof of Theorem 1

We first prove several lemmas; the proof of the theorem combines the lemmas at the end of the appendix.

Lemma 2. *Suppose h^* is a part of the solution to the platform's problem with $t^* \in (0,1]$ then if $\exists \alpha_1, \alpha_2 : \int_{\alpha_1}^{\alpha_2} h^*(q)dq = \alpha_2 - \alpha_1 \implies \int_{\alpha_1}^{\bar{q}} h^*(q)dq = \bar{q} - \alpha_1$*

Proof. Suppose not, fix any $t^* \in (0,1]$ and suppose $\exists \beta_1, \beta_2$ s.t. $\alpha_2 \leq \beta_1 < \beta_2$ and $\exists \delta > 0$ s.t. $\int_{\alpha_1}^{\alpha_2} 1 - h_0(q)dq \geq \delta$ and $\int_{\beta_1}^{\beta_2} h(q)dq = \beta_2 - \beta_1 - \delta$. Such δ exists iff. $h_0(\alpha_2) < 1$. [11] For $h_0(\alpha_2) < 1$, take:

$$h'(q) = \begin{cases} h_0 & \alpha_1 < q < \hat{q} \\ 1 & \beta_1 < q < \beta_2 \\ h & o.w. \end{cases}$$

where $\hat{q}$ solves: $\int_{\alpha_1}^{\hat{q}} 1 - h_0(q)dq = \delta$

then $\int_0^{\bar{q}} h^*(q)dq = \int_0^{\bar{q}} h'(q)dq$ and we will show that h' yields higher profit, which is a contradiction to the optimality of h^*. h' yields higher profit if:

$$\int_{\alpha_1}^{\hat{q}} (1 - g + g(h^*(q) - h'(q)))(1 - F(\frac{p(q)}{q}))p(q)dq < \int_{\beta_1}^{\beta_2} (1 - g + g(h'(q) - h^*(q)))(1 - F(\frac{p(q)}{q}))p(q)dq$$

This inequality holds because $(1 - F(\frac{p(q)}{q}))p(q)$ is increasing in q. One can take:

$$\begin{aligned} &\int_{\alpha_1}^{\hat{q}} (1 - g + g(h^*(q) - h'(q)))(1 - F(\frac{p(q)}{q}))p(q)dq \\ &< \int_{\alpha_1}^{\hat{q}} (1 - g + g(h^*(q) - h'(q)))(1 - F(\frac{p(\hat{q})}{\hat{q}}))p(\hat{q})dq \\ &< \int_{\beta_1}^{\beta_2} (1 - g + g(h'(q) - h^*(q)))(1 - F(\frac{p(\beta_1)}{\beta_1}))p(\beta_1)dq \\ &< \int_{\beta_1}^{\beta_2} (1 - g + g(h'(q) - h^*(q)))(1 - F(\frac{p(q)}{q}))p(q)dq \end{aligned} \tag{3.14}$$

The middle inequality then integrates to:

$$(1 - g + g\delta)(1 - F(\frac{p(\hat{q})}{\hat{q}}))p(\hat{q}) < (1 - g + g\delta)(1 - F(\frac{p(\beta_1)}{\beta_1}))p(\beta_1)$$

which holds, because $\hat{q} < \alpha_2 < \beta_1$. □

Let $q_3 \equiv \inf\{\alpha_1 : \int_{\alpha_1}^{\bar{q}} h(q)dq = \bar{q} - \alpha_1\}$

Lemma 3. *(Monotonicity under h_0) For all $\alpha_2 < \beta_1 < \beta_2 \leq q_0$, where $q_0 \equiv h_0^{-1}(0)$, let*

[11] In case of $h_0(\alpha_2) > 0$, one can take $h'(q) = 0$ for $\alpha_1 < q < \hat{q}$ and the rest of the proof is the same.

α_1 be such that:

$$\int_{\alpha_1}^{\alpha_2} h_0(q)dq = \int_{\beta_1}^{\beta_2} h_0(q)dq \equiv \delta$$

then:

$$\int_{\alpha_1}^{\alpha_2} (1-g+gh_0(q))(1-F(\frac{p(q)}{q}))p(q)dq < \int_{\beta_1}^{\beta_2} (1-g+gh_0(q))(1-F(\frac{p(q)}{q}))p(q)dq$$

Proof. The trick is the same as in the previous proof. Realizing that $(1-F(\frac{p(q)}{q}))p(q)$ is increasing in q,

$$\begin{aligned}\int_{\alpha_1}^{\alpha_2} &(1-g+gh_0(q))(1-F(\frac{p(q)}{q}))p(q)dq \\ &< \int_{\alpha_1}^{\alpha_2} (1-g+gh_0(q))(1-F(\frac{p(\alpha_2)}{\alpha_2}))p(\alpha_2)dq \\ &< \int_{\beta_1}^{\beta_2} (1-g+gh_0(q))(1-F(\frac{p(\beta_1)}{\beta_1}))p(\beta_1)dq \\ &< \int_{\beta_1}^{\beta_2} (1-g+gh_0(q))(1-F(\frac{p(q)}{q}))p(q)dq \quad (3.15)\end{aligned}$$

The middle inequality then simplifies to:

$$(1-g+g\epsilon)(1-F(\frac{p(\hat{q})}{\hat{q}}))p(\hat{q}) < (1-g+g\epsilon)(1-F(\frac{p(\beta_1)}{\beta_1}))p(\beta_1)$$

Which is true due to the fact that $(1-F(\frac{p(q)}{q}))p(q)$ is increasing in q. □

Lemma 3 says that the platform always prefers to assign mass towards higher q, when choosing among sellers for whom the individual rationality constraint is just satisfied.

Lemma 4. *If $\exists \alpha_1, \alpha_2 \leq q_3$ s.t: $\int_{\alpha_1}^{\alpha_2} h(q)dq > \int_{\alpha_1}^{\alpha_2} h_0(q)dq \geq 0$ then h is not a solution for the platform's problem.*

Proof. Let $\delta \equiv \int_{\alpha_1}^{\alpha_2} h(q) - h_0(q)dq$. For simplicity, we prove the case in which $q_3 - \delta \geq q_0 \geq \alpha_2$, the other cases are analogical, requiring special treatment to define h'. In this case:

$$h' = \begin{cases} 1 & q > q_3 - \delta \\ h_0 & \alpha_1 \leq q \leq \alpha_2 \\ h & o.w. \end{cases}$$

yields higher profit. $\square$

It follows from Lemma 4 that it is wasteful to allocate anything strictly between 1 and $h_0(q)$ mass to any seller.

Corollary 3. *(Of Lemma 3 and 4) Suppose h^* is a part of the solution of the platform's problem with $t^* \in (0,1]$ then if $\exists \alpha_1, \alpha_2 : \int_{\alpha_1}^{\alpha_2} h(q)dq = \int_{\alpha_1}^{\alpha_2} h_0(q)dq \implies \int_{\alpha_1}^{q_2} h(q)dq = \int_{\alpha_1}^{q_2} h_0(q)dq$, where $q_2 \equiv \min\{q_0, q_3\}$.*

Corollary 3 together with Lemma 5 describes the shape of the solution to the left of point q_2. Let $q_1 \equiv \inf\{\alpha_1 : \int_{\alpha_1}^{q_2} h(q)dq = \int_{\alpha_1}^{q_2} h_0(q)dq\}$

Lemma 5. *If $\exists \alpha_1, \alpha_2 \leq q_1$ s.t: $0 < \int_{\alpha_1}^{\alpha_2} h(q)dq < \int_{\alpha_1}^{\alpha_2} h_0(q)dq$ then h is not a solution for the platform's problem.*

Proof. Let $\delta \equiv \int_{\alpha_1}^{\alpha_2} h(q)dq$. Let β be such that $\int_{\beta}^{q_1} h_0(q) = \delta$. For simplicity, we prove the case in which $\alpha_2 \leq \beta$, the other case is analogical, requiring special treatment to define h'. In this case:

$$h' = \begin{cases} 0 & \alpha_1 \leq q \leq \alpha_2 \\ h_0(q) & \beta \leq q \leq q_1 \\ h & o.w. \end{cases}$$

yields higher profit. $\square$

Finally, we are ready to prove Theorem 1:

Proof. By Lemma 5, for $q \leq q_1 : h^*(q) = 0$ almost everywhere. By Corollary 3 for $q \in [q_1, q_2] : h^*(q) = h_0(q)$ almost everywhere. By Lemma 2 for all $q \geq q_3 : h^*(q) = 1$ almost everywhere. Finally by Lemma 4 for all $q \in [q_2, q_3] : h(q) = 0$ almost everywhere. This concludes the proof. $\square$

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