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Branka Matyska

Dissertation

Prague, March 2021

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In dedication to past generations of researchers who laid the foundation for this work, to the current generation that inspired it, and to future generations that will improve it

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Abstract

This thesis uses economic theory and empirical estimation to evaluate the effects of macroprudential and fiscal policies. Chapter 1 assesses the efficiency of macroprudential capital requirements in the form of four market risk measures. The chapter generates a novel prediction that prudential instruments based on salience and the overweighting of tail market losses are beneficial for policymakers aiming to reduce the likelihood of a financial crisis. The results suggest that overweighting worst- and best-case outcomes can prevent fire sales, while overweighting intermediate losses leads to welfare improvements for the financial system after an uncertainty shock. This chapter illuminates how adverse liquidity and uncertainty shocks elicit policy responses, and how they affect bank risk attitudes and the time and the cross-sectional dimensions of systemic risk. Chapter 2 studies macroeconomic implications of Value at Risk financial regulation and derives optimal deposit insurance. The main finding is that optimal deposit insurance is risk-sensitive when banks are subject to risk-based capital requirements. Chapter 3 studies the impact of a fiscal stimulus package on firm dynamics and the US labor market. It shows that corporate income tax cuts increase job creation through delayed firm entry, and a reduction in job losses through lower firm exit rates. Wages of newly hired workers rise significantly, while aggregate wages exhibit a persistent rise in the wake of the policy change.

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Introduction

This thesis develops theoretical models and empirical analyses that evaluate the effectiveness of macroprudential and fiscal policies. The financial crisis that began in 2008 illustrates that market losses experienced by financial institutions can adversely affect output and market stability. This highlights the importance of proper regulatory risk management. The first two chapters focus on better evaluation of market risk, and more profound understanding of implications of risk-based capital requirements on systemic risk, welfare and optimal deposit insurance. Several questions arise : What are the implications of the Basel capital regulation for systemic risk, fire sales, and welfare? If regulators adopt spectral risk measures as capital requirements, how does this macroprudential regulation affect systemic risk and welfare? How should the optimal deposit insurance be implemented if banks are subject to risk-based capital requirements? Can corporate income tax cuts boost wages, help create new jobs, and prevent job losses? Brief answers follow.

Chapter 1 evaluates the effectiveness of macroprudential capital requirements in the form of market risk measures for alleviating systemic risk, fire sales, and welfare losses during crisis resolution. We develop a general equilibrium, heterogeneous agent model with financial institutions that are subject to risk-based capital requirement constraint and compare the benchmark Value at Risk to three spectral risk measures. The key idea of alternative regulation is probability weighting, so that regulators overweight or underweight outcomes relative to their objective probabilities. Within the context of our model, prudential instruments based solely on overweighting of tail market losses are preferable for policymakers aiming to reduce the likelihood of systemic crises. In the steady-state, the financial sector exhibits a twofold pattern: the financial sector is risk-averse or risk-seeking in market losses. Focusing on both downside and upside risks increases households' welfare but leads to risk-seeking preferences of banks and exacerbates the systemic risk. The results suggest that overweighting worst and best-case outcomes can prevent fire sales, while overweighting intermediate losses leads to welfare improvements for the financial sector after an uncertainty shock.

Chapter 2 has two aims. From a positive perspective, it aims to analyze the macroeconomic implications of capital requirements implemented as a Value at Risk (VaR) regulatory risk measure. From a normative perspective, the objective is to derive optimal deposit insurance. The chapter presents a two-period simple macro model with two agents : unconstrained households and constrained banks that optimize under risk-based capital requirements. We document the procyclicality of the balance sheet of the financial institution due to a risk-based Value at Risk constraint designed to limit the probability of market losses to a fixed acceptable threshold. When banks are subject to capital regulation, optimal deposit insurance is not fixed (risk-insensitive), but instead changes with market conditions. The insurance level depends on the riskiness and return of the bank's asset side of the balance sheet. In effect, risk-based capital requirements —such as Value at Risk —require risk-sensitive deposit insurance. When the government acts as a deposit insurance provider, capital ratios and interest rates are higher and more procyclical than they are without insurance.

Chapter 3 is an exploration of the effects of a fiscal stimulus package stimulus on firm dynamics and the labor market in the United States. We estimate and model the impact of corporate income tax cuts on employment through firms entry and exit. We first identify the effect of a corporate income tax cut on the net business and job creation in US data, using a narrative approach. We find a significant positive, though delayed, impact on job creation through firm entry and an immediate reduction in job losses through lower firm exit rates. Wages of new hires rise significantly, and aggregate wages exhibit a persistent rise in the wake of the policy change. We also find that incumbent firms respond strongly to investment tax credit incentives. Secondly, we lay out a dynamic stochastic general equilibrium model with endogenous firm entry and exit that is able to capture some of the patterns observed in the data. For comparison, we also study the dynamics in response to a tax cut in a model with homogeneous firms and a constant exit rate. We show that the workhorse general equilibrium business cycle model with entry, exit, and homogeneous firms is consistent with several patterns observed in the data. We show that output, entry, and exit rise as dividends are taxed less: firm churn and business dynamism increase. In a model with homogeneous firms, aggregate wage increases in response to tax cuts, consistent with our empirical findings, while in a model with heterogeneous firms, aggregate wages instead decline on impact.

Chapter 1

Salience, Systemic Risk and Spectral Risk Measures as Capital Requirements

"Everyone knows: Financial markets are risky. But in the careful study of that concept, risk, lies knowledge of our world and hope for quantitative control over it."
— Benoit B. Mandelbrot, *The (Mis)behavior of Markets(2004)*

1.1 Introduction

The severity and longevity of the 2008 financial crisis prompted policymakers and economists to search for effective macroprudential regulations. The extensive policy and academic debate highlights the lack of strong a consensus regarding macroprudential tools and financial regulation and supervision objectives. As emphasized by by Borio (2003), the primary aim is to limit widespread financial instability, and the ultimate goal is to minimize macroeconomic costs associated with financial instability. To date, macroprudential policy has focused on countercyclical capital requirements, loan to value ratio, and systemic surcharges to ensure financial stability.[1] However, the design of market risk measures has been predominantly neglected from the new prudential framework, although bank solvency crucially depends on their ability to withstand market losses. In the aftermath

[1] See, for example, Elliott (2011), Kahou and Lehar (2017), and Galati and Moessner (2013) for overviews of macroprudential tools.

of the crisis, the Financial Crisis Inquiry Commission (FCIC) concluded that the failure of policymakers to adequately measure the risk for asset-backed securities of the largest financial firms was among the prime causes of the crisis.

This paper fills the important gap in the design of regulatory market risk measure by answering three questions. First, how effective are spectral risk measures in reducing the likelihood of financial crises, and improving social welfare? Second, could spectral risk measures prevent fire sales caused by adverse financial shocks? Third, who might benefit or lose from adverse shocks, savers or the financial sector? The paper evaluates the effectiveness of alternative financial regulations in achieving macroprudential stability and efficiency goals.

We start by developing a heterogeneous agent stochastic general equilibrium model with a binding capital requirement constraint and endogenous systemic risk measured by the probability of the financial sector being undercapitalized. We juxtapose Value at Risk from the Basel framework and three spectral risk measures as risk-based capital requirements. The prominent feature of the spectral risk measures of Acerbi (2002) is that they relate the market risk measure to the decision maker's subjective probabilities. From a regulatory viewpoint, spectral risk measures are a promising generalization of Expected Shortfall as a market risk measure on Banking Supervision (2011). We first analyze the steady-state equilibrium in the presence and absence of macroprudential policy. Then, we investigate the ex-post role of four risk measures in crisis management following a sudden increase in borrowing costs, a decline in bank equity capital, and an uncertainty shock. Three shocks proxy for an exogenous drop in asset prices, comparable to the downfall of the housing market during the 2008 crisis. The critical questions are: what are the implications of macroprudential policy for systemic risk, endogenous risk, fire sales, and welfare?

The most important and novel feature of our model is its formulation of market risk measures consistent with the psychology of attention in Bordalo, Gennaioli, and Shleifer (2013b) and Tversky and Kahneman (1992). Specifically, decision-makers over or underweight outcomes relative to their objective probabilities because their ability to comprehend and evaluate probabilities is limited, and over/underweighting creates probability distortions. We devise macroprudential regulation such that the associated probability weighting function is convex, has an inverse S-shape, or is S-shaped.[2] With

[2]We include these three cases because, in most experimental settings, the literature has identified an inverse S-shaped proneness to probability distortions (Gonzalez and Wu 1999; Abdellaoui 2000; Bruhin,

the convex weighting function, regulators are pessimistic and overweight bank exposure to tail market losses using Wang (2000)'s distortions. The inverse S-shaped probability weighting function overweights small and underweights large probabilities. This implies the mixture of regulatory pessimism and optimism since worst and best-case outcomes are overweighted, while intermediate are underweighted. With an S-shaped weighting function, regulators underweight extreme outcomes and overweight intermediate ones. In this framework, regulators focus neither on favorable or unfavorable scenarios, but pay attention to average losses that arise in normal times. We denote three market risk measures Wang, Kahneman-Tversky (KT), and anti-KT.

The results show that financial crises are more likely when banks are unregulated than in the equilibrium attained by VaR capital requirements. Comparing four regulatory regimes, we find that focusing solely on tail market losses can limit the probability of a financial crisis and endogenous risk. Therefore, VaR and Wang regulations fulfill the primary macroprudential objective of mitigating widespread financial instability. However, focusing on upside risks by overweighting intermediate or best-case market scenarios achieves higher output per unit of bank equity. Specifically, KT and anti-KT fulfill the ultimate macroprudential goal of minimizing macroeconomic costs related to instability. In this respect, results contribute to the literature that reports on systemic risk-return tradeoff of capital requirements, in which lower crisis probability comes at the cost of lower output (e.g., Adrian and Boyarchenko (2018)).

Our results also provide evidence on the redistributive effects of financial regulation (e.g., Korinek and Kreamer (2014)). In equilibrium, KT and anti-KT capital requirements redistribute wealth from the financial sector to the rest of the economy. Bank welfare is lower under KT and anti-KT than under VaR and Wang, while household welfare is higher. At the same time, under KT and anti-KT, additional equity hurts bankers and benefits households. The welfare transfer is possible because capital requirements based on probability weighting play a twofold role: leverage limit and altering risk-sharing incentives.

Turning to crisis experiments, if aggregate bank equity declines or borrowing becomes more costly, the main result delivers the volatility paradox of Brunnermeier and Sannikov (2014), in that lower crisis probability is associated with higher price volatility and vice

Fehr-Duda, and Epper 2010), but also concave, convex, or S-shaped weighting function (Goeree, Holt, and Palfrey 2003; Goeree, Holt, and Palfrey 2002; Van de Kuilen and Wakker 2011; Qiu and Steiger 2011). In addition, Epper and Fehr-Duda (2017) find support for the coexistence of under and overweighting of tail outcomes and a context-dependent probability weighting function.

versa. In particular, borrowing frictions destabilize prices but lower crisis probability. Conversely, if aggregate bank equity becomes scarce, this leads to a rise in crisis probability and a decline in endogenous risk. All four macroprudential policies can manage either the likelihood of a crisis or financial panics when banks face adverse funding conditions. Still, a systemic-endogenous risk trade-off is reduced when regulators measure market risk using KT and anti-KT. When an economy faces an uncertainty shock, results suggest that substantial fire sales made from the banking sector to households under the VaR and Wang regulation, and that output and welfare will decline both for households and the banking sector. Meanwhile, anti-KT generates welfare improvements for banks. The advantage of the three risk measures is that regulators can mitigate the likelihood of a crisis despite the fire sales. The results further show that KT policy increases the probability of financial crises but successfully prevents fire sales and leads to a rise in output.

Finally, our results on bank risk attitudes present mixed evidence on predictions of prospect and salience theory of Tversky and Kahneman (1992) and Bordalo, Gennaioli, and Shleifer (2013b). When evaluating market losses, a twofold pattern emerges under VaR and Wang: the financial sector is risk-averse or risk-seeking in market losses. While we find the twofold preference pattern, prospect theory cannot explain bank's risk-taking patterns. Banks in our model are subject to capital requirements based on probability weighting, suggesting that both institutional and behavioral factors play essential roles in determining economic outcomes.

Given our findings, regulators could implement two macroprudential policy interventions in practice. The Tinbergen principle highlights the necessity of at least one independent policy instrument for each policy objective. What our results suggest is that VaR and Wang could target lessening crisis probability and endogenous risk, while KT and anti-KT can target preventing negative welfare and output spillovers. In practice, capital buffers can be designed to balance the ex-ante prevention of systemic risk and ex-post crisis management. In our framework, this objective translates into weighting downside and upside market risk measures according to regulators' preferences for systemic risk reduction or output and welfare loss. Second, regulators may enforce VaR or Wang policies during stable times to reduce the likelihood of a crisis while adjusting their choice of a risk measure when financial markets are disrupted.

This paper closely relates to the new wave of research on macroprudential policy tools in dynamic general equilibrium models (Angelini, Neri, and Panetta 2011; Angeloni and

Faia 2013; Adrian and Boyarchenko 2018; Bianchi et al. 2011; Bianchi and Mendoza 2018; Benes and Kumhof 2015; Benigno et al. 2013; Goodhart et al. 2012; Martinez-Miera and Suarez 2012).[3] Bianchi and Mendoza (2018) argue that a macroprudential debt and dividend tax can reduce the incidence and magnitude of financial crises and increase social welfare compared to the competitive equilibrium. Benes and Kumhof (2015) report that welfare gains can be derived from a macroprudential countercyclical capital buffer. This paper's distinguishing feature is its focus on systemic risk and the welfare implications of risk-based capital requirements. In this regard, the paper closest to ours is Adrian and Boyarchenko (2018), which shows that lower risk-based capital requirements simultaneously increase consumption growth and crisis probability. In their model, tighter liquidity requirements are more effective than tighter capital requirements because the likelihood of a crisis declines without impairing consumption growth. Unlike Adrian and Boyarchenko (2018), we also focus on spectral risk measures as capital requirements.

Our modeling approach builds on intermediary asset pricing literature as described by Brunnermeier and Sannikov (2014), He and Krishnamurthy (2013), He and Krishnamurthy (2019), Adrian and Boyarchenko (2018), and Korinek and Kreamer (2014). Our model is a simplified version of Brunnermeier and Sannikov (2014). In this literature, financial institutions are not a veil, in that asset prices and systemic risk depend on the intermediary capital. This literature introduces binding financial constraints to generate nonlinear price dynamics. For example, in He and Krishnamurthy (2013) banks face a constraint on outside equity financing. Adrian and Boyarchenko (2018) introduce liquidity requirements in addition to risk based capital requirements. The VaR and Wang risk measures produce countercyclical bank leverage as in Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013), while KT and anti-KT give rise to the procyclical leverage featured in Adrian and Boyarchenko (2018).

This paper complements the literature on risk measures, including Value at Risk and spectral risk measures (Acerbi 2002; Krokhmal, Palmquist, and Uryasev 2002; Szegö 2002; Cotter and Dowd 2006; Dowd and Blake 2006; Dowd, Cotter, and Sorwar 2008; Adam, Houkari, and Laurent 2008; Brandtner 2013; Adrian and Brunnermeier 2011). Most of the papers apply risk measures to portfolio optimization, but abstract from their implementation in the general equilibrium setting. A notable exception is Adrian and Brunnermeier (2011), who propose Co-VaR as the systemic risk measure - the financial system Value at

[3] Galati and Moessner (2013) and Kahou and Lehar (2017) provide comprehensive literature reviews of macroprudential policies.

Risk conditional on the institution being distressed. Instead of systemic risk measures, we propose alternative market risk measures. Among viable applications, the literature has suggested using spectral risk measures to devise optimal portfolio (Adam, Houkari, and Laurent 2008), or to calibrate margin requirements (Cotter and Dowd 2006). We thereby seek to assess the strengths and limitations of spectral risk measures and stimulate their further research in prudential regulation.

Finally, this paper is connected to literature that applies the probability weighting of prospect theory, notably in finance and insurance, where attitudes towards risk play a pivotal role.[4] For example, De Giorgi and Legg (2012) show that probability weighting leads to an increase in the required equity premium. Barberis and Huang (2008) analyze asset price implications of prospect theory and show that probability weighting leads to overpricing of securities with a positively skewed return distribution. As argued by Barberis (2013), the finance literature has used the pricing of skewness predicted by probability distortions to explain the low average returns on distressed stocks (Eraker and Ready 2015), the low average returns on stocks initially offered publicly, and insufficient diversification of household portfolios. Nonlinear probability weighting can explain behavior observed in insurance markets. For example, overweighting small probabilities creates a demand for property insurance policies, as reported by Sydnor (2010), and for automobile insurance (Barseghyan et al. 2013). Nonlinear distortions of probabilities can explain the phenomena mentioned above; investors overweight the tails of the distribution of potential gains or losses they are considering. The economics areas in which prospect theory has not been employed extensively include macroeconomics and financial regulation. In essence, this paper argues that probability weighting can offer useful insights into these areas.

Section 1.2 describes spectral risk measures and probability weighting as a key method to quantify expected losses. Sections 1.3 and 1.4 describe the equilibrium model with a macroprudential VaR and analyze the effectiveness of this policy from the systemic risk perspective after liquidity and uncertainty shocks. Section 1.5 presents an alternative regulation in the form of three spectral risk measures and studies their advantages and disadvantages. Section 1.6 concludes.

[4] Barberis (2013) and O'Donoghue and Somerville (2018) summarize applications of prospect theory and probability weighting.

1.2 Spectral risk measures and probability weighting

In this section, we briefly define spectral risk measures, which we use in section 1.5 to measure market risk and devise regulatory capital requirements. The key idea of alternative regulation is probability overweighting, where policymakers overweight losses that are salient to them.

1.2.1 Spectral risk measures

While the paper's primary goal is to investigate the role of spectral risk measures in systemic risk and welfare domains, we begin with simpler questions. How do regulators and investors measure market losses?

When computing risk measures, the starting point is the gain-loss probability distribution of bank assets. From a regulatory point of view, the purpose of capital requirements is to hold enough capital to absorb expected losses in the future. Policymakers have adopted Value at Risk and Expected Shortfall as market risk measures in Basel III and IV. Both measures calculate required regulatory capital based on the downside risk potential. VaR answers the question: what value of a given portfolio is at risk? In essence, it represents the maximum expected loss with a certain confidence level. In mathematical terms, VaR is the quantile of the probability loss distribution.[5] From the shareholders' perspective, the VaR quantile is a meaningful risk measure, because the default event itself is of primary concern and the size of a shortfall is secondary. On the other hand, Expected Shortfall measures average losses exceeding the VaR limit, which is the average expected size of a shortfall.

Nonetheless, both VaR and Expected Shortfall are special cases of spectral risk measures introduced by Acerbi (2002). Spectral risk measures are defined as the weighted average of quantiles of a loss probability distribution

$$M_g(X) = \int_0^1 g(p)F^{-1}(p)dp \tag{1.1}$$

where $F^{-1}(p)$ is a quantile function of a random variable X which measures market losses and $g(p)$ satisfies

1. $g(p) \geq 0$ (positivity),

[5] Quantile at level p is a an inverse of cumulative distribution function of a random variable X, that is $F^{-1}(p) = \inf \{x \quad : F(x) \equiv Prob\,[X \leq x] \geq p\}$.

2. $\int_0^1 g(p)dp = 1$ (sub-additivity), and

3. $g'(p) \geq 0$ (monotonicity).

The weighting function $g(p)$ is called the risk spectrum and reflects the regulatory degree of risk aversion or seeking.[6] It is related to the probability weighting function or decision weights in Tversky and Kahneman (1992) such that $G'(p) = g(p)$ holds. The first condition requires that the weights are weakly positive, while the second assumes that weights sum to one. The second property reflects diversification benefits and requires total portfolio losses to be lower than or equal to the sum of individual losses when assets are combined into a portfolio. The third property reflects risk aversion and requires that the weights attached to larger losses are no less than the weights attached to smaller losses. For VaR_α, the monotonicity condition does not hold, as it overweights the loss at the fixed confidence level α and underweights larger and smaller losses.

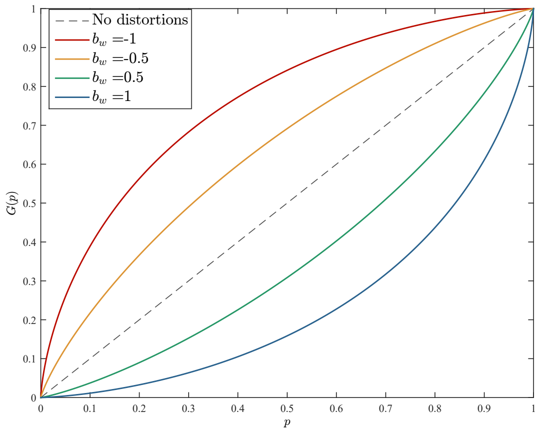
1.2.2 Probability weighting

Ultimately, the main challenge is how to represent the risk attitudes of regulators.[7] The key idea is to formulate risk attitudes consistent with the psychology of attention in Bordalo, Gennaioli, and Shleifer (2013b) and Tversky and Kahneman (1992), where investors evaluate lotteries by overweighting the most salient states. The probability weighting function constitutes "the local thinking" and captures the strength of investors' attention to salient market outcomes. For example, suppose regulators measure market losses using VaR. In that case, their behavior is consistent with extreme local thinking; regulators focus on a single rare event.

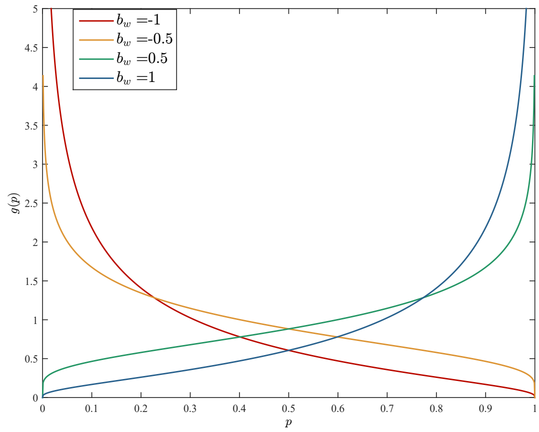
Figures 1.1a and 1.1b illustrate several probability weighting functions. The horizontal axis shows the objective probability of market loss, while the vertical axis shows its subjective probability or decision weight. In this respect, the 45-degree line corresponds to linear probability weighting, and deviations from that line represent underweighting or overweighting of the objective probabilities.

[6] In the case of VaR_α, the risk spectrum is a Dirac delta function $g(p) = \delta_{1-\alpha}(p)$, and equals zero everywhere except at $1-\alpha$ and has an integral over the interval $[0, 1]$ equal to one. With Expected Shortfall (ES_α), the risk spectrum is a step function, $g(p) = \frac{1}{1-\alpha}$ for $p \in [0, 1-\alpha]$ and $g(p) = 0$ for $p \in (1-\alpha, 1]$

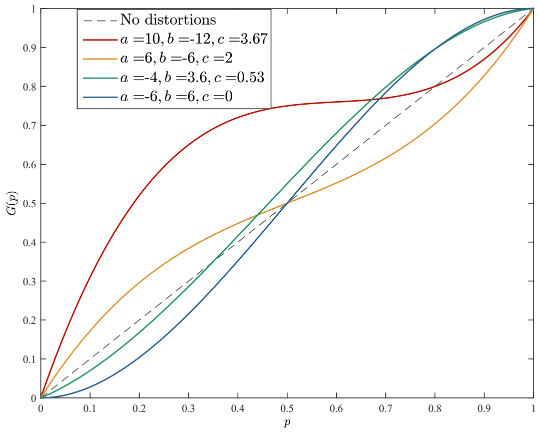
[7] The literature on spectral risk measures has predominantly used the utility function as guidance to construct the risk spectrum. Some choices of risk spectrum can be found in Dowd, Cotter, and Sorwar (2008) and Guegan and Hassani (2015).



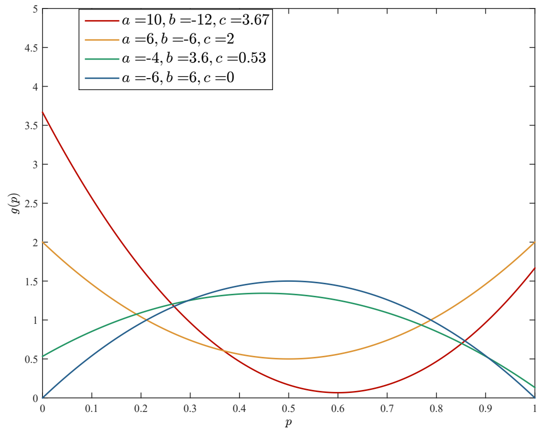
(a) Probability weighting function Wang



(b) Risk spectrum Wang



(c) Probability weighting function KT



(d) Risk spectrum KT

Figure 1.1: (Left) Probability weighting function of Wang's ($G(p) = \Phi(\Phi^{-1}(p) - b_w)$) and Kahneman-Tversky's risk measure ($G(p) = \frac{a}{3}p^3 + \frac{b}{2}p^2 + cp$). (Right) Risk spectrum of Wang's ($g(p) = \mathrm{e}^{-\frac{b_w^2}{2} + b_w\Phi^{-1}(p)}$) and Kahneman-Tversky's risk measure ($g(p) = ap^2 + bp + c$). No distortion corresponds to the objective probability ($G(p) = p$).

We devise macroprudential regulation in three ways. The associated probability weighting function is convex, has an inverse S-shape or is S-shaped. With the convex weighting function, regulators are solely concerned with banks' exposure to tail market losses and insure against it. They overweight the risk in tails according to Wang (2000). In technical terms, the probability weighting function for the Wang risk measure is equal to

$$G_W(p) = \Phi(\Phi^{-1}(p) - b_w), \tag{1.2}$$

and the risk spectrum is such that $g(p) = G'(p)$ is equal to[8]

$$g_W(p) = \mathrm{e}^{-\frac{b_w^2}{2} + b_w \Phi^{-1}(p)}, \tag{1.3}$$

where b_w is a parameter to be chosen. For negative values of b_w, the weighting function is convex and regulators overweight tail market losses, while positive values imply underweighting and a concave weighting function, as shown in Figure 1.1a.

Second, we construct decision weights in the spirit of Tversky and Kahneman (1992) by which regulators overweight small probabilities and underweight high probabilities. The weighting function is inversely S-shaped, and regulators disproportionately overweight the worst-case and best-case outcomes. We name this spectral risk measure the Kahneman-Tversky risk measure (KT). The risk spectrum and the probability weighting are[9]

$$g_{KT}(p) = ap^2 + bp + c \tag{1.4}$$

and

$$G_{KT}(p) = \frac{a}{3}p^3 + \frac{b}{2}p^2 + cp \tag{1.5}$$

for positive a, where a, b and c are parameters to be chosen.

In decision making under risk, the prospect theory of Tversky and Kahneman (1992) describes how people transform values and probabilities. Decision-makers derive utility from gains and losses from the reference point. They exhibit loss aversion, risk aversion for gains, and risk-seeking for losses. These systematic deviations from the Expected Utility are captured by a value function that is convex and steeper for losses and concave for gains. Apart from value transformations, decision-makers treat probabilities nonlinearly. *The possibility effect* reflects the tendency to overweight small probabilities, while higher probabilities or highly likely events are underweighted (*certainty effect*). Two effects together produce inverse S-shaped decision weights. This shape conveys a psychological mechanism underlying probability distortions, namely *diminishing sensitivity*: the decision-maker is less sensitive to changes in probability as they move away from two

[8] It is straightforward to prove this, where as before $\Phi(\cdot)$ is the cdf and $\phi(\cdot)$ the pdf of the standard normal distribution. $G'(p) = \frac{\partial \Phi(\Phi^{-1}(p) - b_w)}{\partial p} = \phi(\Phi^{-1}(p) - b_w)\frac{\partial(\Phi^{-1}(p) - b_w)}{\partial p} = \frac{\phi(\Phi^{-1}(p) - b_w)}{\phi(\Phi^{-1}(p))} = \mathrm{e}^{-\frac{b_w^2}{2} + b_w \Phi^{-1}(p)}$. The second equality follows from the chain rule of derivatives, the third from the derivative of inverse function and the fourth from the definition of $\phi(\cdot)$ and canceling common terms.

[9] In Tversky and Kahneman (1992), decision weights are equal to $G(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}}$. Because it is impossible to obtain an analytical expression for the spectral risk measures with the original decision weights, we use the third-order polynomial approximation.

reference points: 0 and 1. The first reference point defines outcomes which will certainly *not* happen, while the second determines events that will certainly happen. The risk spectrum in Figure 1.1d when a is positive conveys this intuition clearly with two peaks at the ends of the interval; distortions are more pronounced at the ends than in the middle of the distribution (red and orange line).

Figure 1.2: S&P 500 return and four market risk measures

Third, we consider the S-shaped weighting function, by which regulators underweight small and overweight high probabilities (Figure 1.1c, blue and green line). In this framework, regulators focus neither on favorable or unfavorable scenarios, but pay attention to average losses that arise in "normal" times. We call the risk measure associated with the S-shaped probability weighting function the *anti-KT* risk measure for simplicity and intuitive appeal. When a is negative, the risk spectrum peaks at the interior point in Figure 1.1d. In contrast to KT risk measure, regulators' attention is drawn to the intermediate reference point rather than to the extreme points.

Therefore, what constitutes the most salient outcomes for regulators changes across different regimes. The critical implication is that different probability weighting functions lead to quantitatively distinct market risk assessments. As an illustration, Figure 1.2 depicts risk assessments for the S&P500 daily return using the four risk measures described: VaR ($\alpha = 0.05$), Wang ($b = -1.3$), KT ($a = 10, b = -12, c = 3.37$), and anti-KT ($a = -4, b = 3.6, c = 0.53$). As anticipated, regulatory losses are the highest and most volatile using the VaR risk measure. For all four risk measures, the maximum value is reached during the 2008 market downturn and the beginning of the 2008/2009 financial crisis.

In section 1.3 and 1.5, we use these three spectral risk measures and VaR to set capital requirements in a heterogeneous agent model.

1.3 Model

The model in Chapter 1 is a simplified version of Brunnermeier and Sannikov (2014), with the capital requirement constraint imposed on the financial sector and capital as a single factor of production. There are two types of agents, unconstrained households and constrained banks. Heterogeneity in productivity and impatience and aggregate risk are the two minimum assumptions needed to generate fire sales, systemic risk, and borrowing in equilibrium. The constraint limits the level of borrowing depending on the amount of asset-side balance sheet risk, measured as a macroprudential VaR or three spectral risk measures in section 1.5.

We first derive the steady-state equilibrium with optimal consumption and investing choices of two agents and endogenous systemic risk. We summarize equilibrium equations in subsection 1.3.3 and contrast equilibrium dynamics when banks are regulated or not in section 1.4. Then, in subsection 1.4.2 and 1.4.3, we assess the efficiency of VaR in three crisis experiments: a permanent increase in borrowing costs, a decline in bank equity, and uncertainty shock. Finally, we assess the efficiency of three spectral risk measures in the steady-state in subsection 1.5.3 and after adverse shocks in subsection 1.5.4.

1.3.1 Preferences and production

There is a continuum of infinitely-lived households and intermediaries with preferences represented by the utility function

$$E\left[\int_0^\infty e^{-rt}\log \underline{c}_t dt\right], \tag{1.6}$$

$$E\left[\int_0^\infty e^{-\rho t}\log c_t dt\right]. \tag{1.7}$$

where $\underline{c}_t$ and c_t are households' and bankers' consumption in the current period. We do not assume that only intermediaries can directly hold productive capital. Both agents can produce a final good from the capital using linear production technology

$$y_t = ak_t, \tag{1.8}$$

$$\underline{y}_t = \underline{a}\ \underline{k}_t, \tag{1.9}$$

with bankers being more productive ($a > \underline{a}$) and impatient ($\rho > r$) than households. Capital supply is exogenous and evolves over time according to a geometric Brownian motion

$$\frac{dk_t}{k_t} = \sigma dW_t, \tag{1.10}$$

where W_t is a standard Brownian motion, and σ (the percentage volatility of capital) is a constant. The term σdW_t denotes *capital quality* shock and captures temporary random changes in expectations about the future productivity of capital. It is a simple way to introduce exogenous variations in the value of capital. As in Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2019) σdW_t is the only shock in the economy.[10] The price of capital p_t is endogenous in equilibrium and evolves as

$$\frac{dp_t}{p_t} = \mu_t^p dt + \sigma^p dW_t, \tag{1.11}$$

where μ^p is the expected price growth and σ^p price volatility. Because the financial and non-financial sectors maximize their utility, μ^p and σ^p arise endogenously. Macroprudential regulation assumes that aggregate risk is endogenous and dependent on market participants' collective behavior. In contrast, the microprudential perspective treats asset price fluctuations as exogenous, given by the market, so the aggregate risk is independent

[10] In both Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2019), the evolution of capital has a drift component. In particular, physical capital evolves as $dk_t = (\Phi(i_t) - \delta)dt + \sigma k_t dW_t$, where i_t denotes investment at time t which is subject to the adjustment cost function $\Phi(\cdot)$, and capital depreciates by δdt, where δ is a constant depreciation rate. Brunnermeier and Sannikov (2014) also consider idiosyncratic jump risk, and assume that capital managed by expert i evolves according to $dk_t = (\Phi(i_t) - \delta)dt + \sigma dW_t + k_t dJ_t^i$, where the term dJ_t^i is a zero-mean Poisson process with intensity λ and jump distribution $F(y)$, $y \in [-1, 0]$ (if $y = -1$, the entire capital of expert i is destroyed). We abstract from the drift and jump components in the evolution of capital. Adding investments would provide an additional channel by which changes in asset prices affect output. It would further imply different asset price levels when households hold all the capital in the economy. On the other hand, the jump component introduces default risk and makes bank debt risky by allowing interest rates to depend on default risk. With Value at Risk, bank debt is risky, in that the equilibrium interest rate depends on volatility. Nonetheless, we abstract from both extensions in order to focus on systemic risk and asset pricing implications of different market risk measures.

of the bank and household portfolios and consumption decisions. Therefore, we will see how the exogenous capital risk translates into endogenous price risk and macroprudential capital requirements.

1.3.2 Banks

There is a continuum of banks in the economy. Each bank raises funds from households by issuing debt and invests in physical capital. Meanwhile, capital requirements limit the level of external debt financing by forcing banks to hold enough net worth to absorb expected future losses. Therefore, banks play a dual role in the economy. First, banks foster economic growth because they have access to more productive technology compared to households. Second, since regulators impose risk-based capital requirements on banks, banks provide the risk-bearing capacity to the rest of the economy.

In principle, banks finance capital purchases by issuing debt

$$b_t = p_t k_t - n_t,$$

where n_t denotes bank's net worth, bank equity capital or wealth. At each period, the bank chooses how much to consume and borrow, so net worth evolves as

$$dn_t = ak_t d_t + d(p_t k_t) - r_t(p_t k_t - n_t)dt - c_t dt. \tag{1.12}$$

The first two terms are income from production and capital gains or losses, which reflect changes in the market value of the risky asset. The second two terms are debt repayment and consumption. Net worth is endogenous because it depends on consumption and borrowing decisions and the endogenous evolution of asset prices. In financial friction literature, and papers such as Kiyotaki and Moore (1997), Brunnermeier and Sannikov (2014), and He and Krishnamurthy (2013), bank equity plays a key role in pricing physical capital and investments, and in predicting financial crises.

Using Ito's product rule, market gains and losses evolve as

$$\frac{d(p_t k_t)}{p_t k_t} = (\mu_t^p + \sigma\sigma_t^p)dt + (\sigma + \sigma_t^p)dW_t. \tag{1.13}$$

The novel assumption of our model relates to the capital requirement constraint imposed on banks. Prudential capital requirements in the form of Value at Risk or three spectral risk measures enter the bank's optimization problem. To compute these measures, we define a loss process as a market loss on the bank balance sheet's asset-side.

Let $X_t \equiv p_t k_t$ denote the market value of capital at time t, so

$$dX_t = X_t(\mu_t^p + \sigma\sigma_t^p)dt + X_t(\sigma + \sigma_t^p)dW_t$$

follows a geometric Brownian motion. Solving for this stochastic differential equation, we obtain the market valuation of capital at time t

$$\begin{aligned} X_t &= X_0 + \int_0^t X_s(\mu_s^p + \sigma\sigma_s^p)ds + \int_0^t X_s(\sigma + \sigma_s^p)dW_s \\ &= X_0 \exp\left(\int_0^t (\mu_s^p + \sigma\sigma_s^p - \frac{1}{2}(\sigma + \sigma_s^p)^2)ds + \int_0^t (\sigma + \sigma_s^p)dW_s\right). \end{aligned}$$

The independence of increments property of a geometric Brownian motion gives the future market value of capital at time $t + \tau$

$$X_{t+\tau} = X_t \exp\left(\int_t^{t+\tau} (\mu_s^p + \sigma\sigma_s^p - \frac{1}{2}(\sigma + \sigma_s^p)^2)ds + \int_t^{t+\tau} (\sigma + \sigma_s^p)dW_s\right).$$

$X_{t+\tau}$ assumes that the capital exposure between time t and $t+\tau$ are kept unchanged. We define the balance sheet loss between periods t and $t+\tau$ as

$$Loss(t, t+\tau) \equiv X_t - X_{t+\tau}. \tag{1.14}$$

By defining losses in such a way, we assume that bank assets are marked-to-market. Therefore, marked-to-market gains and losses between two successive periods are captured by the change in the market value of capital between two periods, $X_t - X_{t+\tau}$. Since market losses are stochastic, when quantifying capital requirements, VaR_α computes the maximum loss over the horizon τ, which can be exceeded only with a small fixed probability α if the current portfolio were kept unchanged.

$$VaR_\alpha^{t,t+\tau} = \inf\{L \geq 0 : P(X_t - X_{t+\tau} \geq L|\mathcal{F}_t) \leq \alpha\}. \tag{1.15}$$

In other words, VaR_α is the $1-\alpha$ quantile of a market loss distribution

$$P(Loss(t, t+\tau) \leq VaR_\alpha^{t,t+\tau}) = 1 - \alpha.^{11}$$

[11] For example, in case α= 0.05, $VaR_\alpha^{t,t+\tau}$ implies that there is a 95% probability that the market loss

Importantly, we make the risk computations consistent with regulatory practice by assuming that the current portfolio composition is kept unchanged, and current market conditions will prevail over the horizon τ. By doing so, we *condition on information available at time* t, and project it to future periods when assessing expected losses. As a result, if *past* portfolio holdings and market conditions are relevant for risk assessment, it would imply a different market risk measure, namely backward-looking $VaR_{\alpha}^{t,t-\tau}$.

Theorem 1. *We have*

$$VaR_{\alpha}^{t,t+\tau} = p_t k_t (1 - \mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau + \Phi^{-1}(\alpha)(\sigma+\sigma_t^p)\sqrt{\tau})})$$

where $\Phi^{-1}(\cdot)$ *is the inverse of the cumulative distribution function of the standard normal distribution.*

PROOF. See Appendix 1.A. □

Capital requirements in the form of spectral risk measures are defined as a weighted average of VaR_p quantiles

$$SRM_t = \int_0^1 g(p) VaR(p)^{t,t+\tau} dp.$$

For different choices of the probability weighting function $G(p)$ and the risk spectrum $g(p) = G'(p)$ from section 1.2, the Wang, KT and anti-KT weighting functions, we obtain different assessment of regulatory market losses and capital requirements. We define by M_t the regulatory loss per unit of capital

$$M_t = \int_0^1 g(p)(1 - \mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau + \Phi^{-1}(p)(\sigma+\sigma_t^p)\sqrt{\tau})}) dp. \tag{1.16}$$

We assume that banks are constrained and must hold enough equity to absorb regulatory losses calibrated as spectral risk measure

$$p_t k_t M_t \leq n_t. \tag{1.17}$$

Referring to (1.16) and (1.17), we see that as the price volatility σ_t^p or price growth μ_t^p vary with market conditions, or regulators use a different risk spectrum $g(p)$, the bank can hold more or less units of capital k_t for the equity level n_t. Moreover, the capital

will not exceed the VaR threshold and a 5 % probability of experiencing a market loss larger than VaR.

requirement constraint (1.17) affects bank capital structure or bank debt-equity ratio. To see this, if we define leverage by $\frac{p_t k_t}{n_t}$, the constraint (1.17) puts a bound on the leverage banks can take depending on current market conditions. Therefore, (1.17) can be interpreted as the state-varying borrowing constraint. In principle, the amount of external debt financing depends on how regulators measure expected losses.

As in Brunnermeier and Sannikov (2014), we assume banks maximize their utility function defined by (1.7). In summary, banks choose capital k_t and consumption c_t to maximize utility (1.7) subject to the evolution of their net worth (1.12) and the regulatory capital requirement constraint (1.17). The optimization problem combines a standard expected utility consumption-portfolio model with a behavioral one that applies probability weighting to how regulators evaluate risk. This approach is consistent with that proposed by Barberis, Mukherjee, and Wang (2016), who argue that agents derive utility both from wealth levels and realized gains-losses, and as such, formulation in which agents' decisions are determined solely by prospect theory, should be avoided.

There is a unit mass of identical risk-averse households. Households finance their consumption purchases by holding bank debt and investing in physical capital. Unlike the financial sector, households are unconstrained in capital choice and face only endogenous evolution of their net worth

$$d\underline{n}_t = (r_t \underline{n}_t + \underline{a}\underline{k}_t)dt + p_t \underline{k}_t(\mu_t^p + \sigma\sigma_t^p - r_t)dt - \underline{c}_t dt + p_t \underline{k}_t(\sigma + \sigma_t^p)dW_t. \tag{1.18}$$

Households' net worth appreciates by earning interest rate r_t on bank debt and equity premium on capital, and depreciates through consumption. Analogous to banks' optimization problem, households maximize their expected discounted lifetime utility or value function. In particular, households choose consumption $\underline{c}_t$ and capital demand $\underline{k}_t$ in order to maximize value function subject to net worth evolution

$$\begin{aligned} V(n_0) = \max_{\underline{c}_t, \underline{k}_t} \quad & E\left[\int_0^\infty e^{-rt} \log \underline{c}_t dt\right] \\ s.t. \qquad & d\underline{n}_t = (r_t \underline{n}_t + \underline{a}\underline{k}_t)dt + p_t \underline{k}_t(\mu_t^p + \sigma\sigma_t^p - r_t)dt - \underline{c}_t dt + p_t \underline{k}_t(\sigma + \sigma_t^p)dW_t. \end{aligned} \tag{1.19}$$

Although households are unconstrained in capital demand, they experience market gains and losses if they hold some capital. While banks protect against market losses with regulatory requirements, households insure against downside market risk by providing risky debt financing to banks.

1.3.3 Equilibrium

Definition 1. Given the initial endowment of capital $(k_0, \underline{k}_0)$, an equilibrium is a collection of allocations $(k_t, \underline{k}_t, c_t, \underline{c}_t, n_t, \underline{n}_t,)$ and a price process p_t such that

(i) bank's maximization problem is solved,

(ii) household's maximization problem is solved,

(iii) markets for output and capital clear.

1.3.4 Household Euler and asset pricing equations

We first solve the household optimization problem by applying the dynamic programming approach. The households *Hamilton-Jakobi-Bellman* equation is

$$rV(\underline{n}_t) = \max_{\underline{c}_t, \underline{k}_t} \quad \log \underline{c}_t + V'(\underline{n}_t)[a\underline{k}_t + r_t\underline{n}_t + p_t\underline{k}_t(\mu_t^p + \sigma\sigma_t^p - r_t) - \underline{c}_t] + \frac{1}{2}V''(n)(\sigma + \sigma_t^p)^2 p_t^2 \underline{k}_t^2 \tag{1.20}$$

where $V(\underline{n}_t)$ denotes the household value function. The mathematical derivation of the HJB equation results from Ito's lemma. The intuition comes from the fact that Brownian motion has enough volatility even in small intervals, contributing to the drift whenever $V(\cdot)$ is convex or concave. In its economic interpretation, the right-hand side terms denominate instantaneous utility, gains or losses from the drift, and gains or losses from the volatility of net worth, while the left-hand side term represents an instantaneous value function.

The first-order condition for consumption implies the Euler equation; the optimal level of consumption is such that the marginal utility of consumption equals the marginal utility of wealth

$$\frac{1}{\underline{c}_t} = V'(\underline{n}_t). \tag{1.21}$$

The first-order condition for capital gives the asset pricing equation if households hold capital

$$\frac{\underline{a}}{p_t} + \mu_t^p + \sigma\sigma_t^p = r_t + \frac{-V''(\underline{n}_t)(\sigma + \sigma_t^p)^2 p_t \underline{k}_t}{V'(\underline{n}_t)}. \tag{1.22}$$

It conveys that the expected return on capital is equal to the interest rate plus the risk premium. In other words, the equity premium equals the risk premium. Substituting first-order conditions into the HJB equation gives the second-order linear differential equation, with a solution provided by the following proposition.

Theorem 2. *Households' optimal consumption and capital rules are linear in wealth and the value function is given by*

$$\underline{c}_t(\underline{n}_t) = r\underline{n}_t$$

$$\underline{k}_t(\underline{n}_t) = \frac{\frac{a}{p_t} + \mu_t^p + \sigma\sigma_t^p - r_t}{p_t(\sigma + \sigma_t^p)^2}\underline{n}_t$$

$$V(\underline{n}_t) = \frac{1}{r}\log(r\underline{n}_t) + \frac{1}{r^2}\left(r_t - r + \frac{(\frac{a}{p_t} + \mu_t^p + \sigma\sigma_t^p - r_t)^2}{2(\sigma + \sigma_t^p)^2} \right). \tag{1.23}$$

PROOF. See the Appendix 1.A. □

1.3.5 Bank Euler and asset pricing equations

We now solve for the bank maximization problem. Banks' *Hamilton-Jakobi-Bellman* equation is

$$\rho V(n_t) = \max_{c_t,k_t} \quad \log c_t + V'(n_t)[ak_t + r_t n_t + p_t k_t(\mu_t^p + \sigma\sigma_t^p - r_t) - c_t] + \frac{1}{2}V''(n)(\sigma + \sigma_t^p)^2 p_t^2 k_t + \xi\left[n_t - p_t k_t M\right]. \tag{1.24}$$

The optimal policies for consumption and capital demand and a value function are computed from two optimality conditions and the Lagrange multiplier on the capital requirement constraint.

$$\frac{1}{c_t} = V'(n_t) \tag{1.25}$$

$$\frac{a}{p_t} + \mu_t^p + \sigma\sigma_t^p = r_t + \frac{-V''(n_t)(\sigma + \sigma_t^p)^2 p_t k_t + \xi p_t M}{V'(n_t)} \tag{1.26}$$

$$\xi(n_t - p_t k_t M) = 0. \tag{1.27}$$

The banks' Euler equation (1.25) is analogous to that of the households. The Lagrange multiplier on ξ captures the tightness of the capital requirement constraint. Because households are unconstrained, the equity premium they earn on capital equals the risk premium. Referring to the asset pricing equation (1.26), banks receive additional compensation, $\xi p_t M_t$, which we denote *salience loss premium.* Positive salience loss premium implies that banks are risk-averse in losses and demand extra payment for being exposed to regulatory market losses. Conversely, when this premium is negative, banks are risk-seeking in market losses.[12]

[12] In the asset pricing literature, Garleanu and Pedersen (2011) and Fostel and Geanakoplos (2008)

We summarize the optimal consumption and capital choices of banks and value function in the following proposition.

Theorem 3. *Optimal consumption and capital rules and the value function of banks are given by*

$$c(n_t) = \rho n_t$$

$$k_t(n_t) = \frac{n_t}{p_t M_t}$$

$$V(n_t) = \frac{1}{\rho}\log(\rho n_t) + \frac{1}{\rho^2}\left(r_t - \rho + \frac{1}{M_t}(\frac{a}{p_t} + \mu_t^p + \sigma\sigma_t^p - r_t) - \frac{(\sigma+\sigma_t^p)^2}{2M_t^2}\right) \tag{1.28}$$

Proof. See the Appendix 1.A. □

From Proposition 2 and 3, the welfare of both agents is the sum of utility of current consumption and discounted future wealth. For households, future wealth consists of interest rate earnings and equity premium on physical capital minus risk premium adjustments. Banks accumulate future wealth through leveraged equity premium and liquidate through debt repayments and risk premium adjustment.

1.3.6 State variable evolution and Markov equilibrium

We solve for the stationary Markov equilibrium with the state variable defined by banks' wealth relative to the market value of the risky asset in the economy $\eta_t \equiv \frac{N_t}{p_t K_t}$, as in Brunnermeier and Sannikov (2014). K_t denotes aggregate capital supply in the economy, while N_t is the banks' aggregate net worth. We can summarize the Markov equilibrium in the state variable η_t, where all variables are functions of the current value of η_t. Law motion of η_t is summarized by the following proposition.

Theorem 4. *Banks' wealth share η_t evolves as*

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^\eta dW_t, \tag{1.29}$$

introduce similar compensation. Garleanu and Pedersen (2011) call this compensation the margin premium in returns; the more difficult it is to fund (i.e., the higher the margin or haircut), the higher the required yield will be. Similarly, Fostel and Geanakoplos (2008) call this excess premium the collateral value of the asset; the easier it is to use the asset as collateral, the higher the price and lower the required premium.

with the drift $\mu_t^\eta = \dfrac{1}{M_t}\dfrac{a}{p_t} - \rho + (\dfrac{1}{M_t} - 1)(\mu_t^p + \sigma\sigma_t^p - r_t - (\sigma + \sigma_t^p)^2)$ *and volatility* $\sigma_t^\eta = (\dfrac{1}{M_t} - 1)(\sigma + \sigma_t^p)$.

PROOF. See Appendix 1.A. □

The three market-clearing conditions are as follows. We denote by $\psi_t \equiv \dfrac{k_t}{K_t} = \dfrac{\eta_t}{M(\eta_t)}$ the banks' share of physical capital. The first condition states that aggregate capital demand in the economy is the sum of bank or household capital demand and equals the exogenous capital supply. Because short-selling of capital is not allowed, $\psi_t = \min(1, \dfrac{\eta_t}{M(\eta_t)})$. If short selling were allowed, ψ_t could be greater than one. Second, aggregate wealth is equal to the market value of the aggregate capital. Third, since there are no real investments in the economy, aggregate output equals aggregate consumption.

- *Market clearing for capital*

$$k_t + \underline{k}_t = K_t, \quad i.e. \quad \psi_t + (1 - \psi_t) = 1$$

- *Aggregate wealth*

$$N_t + \underline{N}_t = p_t(k_t + \underline{k}_t)$$

$$\eta_t + (1 - \eta_t) = 1$$

- *Market clearing condition for output*

$$\rho N_t + r\underline{N}_t = ak_t + \underline{ak}_t \quad i.e.$$

$$p_t(\rho\eta_t + r(1 - \eta_t)) = a\psi_t + \underline{a}(1 - \psi_t)$$

In sum, we obtain the system of ordinary differential and algebraic equations with the endogenous state variable $\eta_t \in [0, 1]$ and boundary conditions at $\eta_t = 0$.

1. Risk-based capital requirement

$$M_t = 1 - \mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma + \sigma_t^p)^2)\tau + \Phi^{-1}(\alpha)(\sigma + \sigma_t^p)\sqrt{\tau}} \tag{1.30}$$

2. Marked-to-market balance sheet

$$\sigma_t^\eta = (\frac{1}{M_t} - 1)(\sigma + \sigma_t^p) \tag{1.31}$$

3. Asset market feedback

$$\sigma_t^p = \sigma_t^\eta \eta_t \frac{p'(\eta_t)}{p(\eta_t)} \tag{1.32}$$

4. Marked-to-market balance sheet

$$\mu_t^\eta = \frac{1}{M_t}\frac{a}{p(\eta_t)} - \rho + (\frac{1}{M_t} - 1)(\mu_t^p + \sigma\sigma_t^p - r_t) + (1 - \frac{1}{M_t})(\sigma + \sigma_t^p)^2 \tag{1.33}$$

5. Asset market feedback

$$\mu_t^p = \frac{p'(\eta_t)}{p(\eta_t)}\mu_t^\eta \eta_t + \frac{1}{2}\frac{p''(\eta_t)}{p(\eta_t)}(\sigma_t^\eta \eta_t)^2 \tag{1.34}$$

6. Market clearing for output

$$p(\eta_t)\left(\rho\eta_t + r(1-\eta_t)\right) = a\psi(\eta_t) + \underline{a}(1-\psi(\eta_t)) \tag{1.35}$$

7. Households' equity premium

$$\frac{\underline{a}}{p(\eta_t)} + \mu_t^p + \sigma\sigma_t^p - r_t = \frac{1-\psi(\eta_t)}{1-\eta_t}(\sigma + \sigma_t^p)^2 \tag{1.36}$$

8. Banks' equity premium

$$\frac{a}{p(\eta_t)} + \mu_t^p + \sigma\sigma_t^p - r_t = \frac{1}{M}(\sigma + \sigma_t^p)^2 + \xi_t p(\eta_t) M_t \tag{1.37}$$

9. Stationary probability distribution (Kolmogorov forward equation)

$$0 = -\mu_t^\eta(\eta_t)\eta_t f(\eta_t) + \frac{1}{2}\frac{\partial}{\partial\eta}((\sigma_t^\eta(\eta_t)\eta_t)^2 f(\eta_t)). \tag{1.38}$$

The boundary conditions at $\eta = 0$ are

$$p(0) = \frac{\underline{a}}{r}, \quad \sigma^p(0) = 0, \quad \mu^p(0) = 0. \tag{1.39}$$

The solution to the Kolmogorov forward equation (1.38) provides the steady-state or stationary probability distribution of the state variable η_t. We are interested in localizing the maximum value of the stationary distribution. This value is significant because it conveys whether the economy is prone to systemic risk. For instance, if the probability that banks are distressed and have zero equity close to zero, $f(0) \approx 0$, systemic risk is negligible. The stationary wealth share distribution is given by

$$f(\eta) = C \frac{\mathrm{e}^{2\int_0^\eta \frac{\mu^\eta(\eta')}{\sigma^\eta(\eta')^2\eta'} d\eta'}}{\sigma^\eta(\eta)^2\eta^2}, \tag{1.40}$$

where C is the normalizing constant. The precise proposition and its proof are stated in Proposition 5 in Appendix 1.A.

The critical limitation of our model is related to stationary distribution. By abstracting from transition dynamics, we infer how prices and systemic risk behave in the long run. The absence of time derivative in the Kolmogorov equation (1.40) conveys the abstraction from transition dynamics. Even if macroprudential regulation may be relevant in the long term, its impact might be more pronounced in the short-run. Transition dynamics are significant in their own right because they account for the practical aspects of regulation. For instance, if the financial sector undertakes a macroprudential regulatory reform such as the change of the measure of market risk in Basel IV to reduce systemic risk, how long would it take until favorable results become evident, and what factors accelerate or delay the transition? We leave this extension for future endeavors.

1.4 Steady state with macroprudential VaR

In the benchmark model we solve the system for the following set of parameters. The productivity and preference parameters are similar to hose of Brunnermeier and Sannikov (2014) and Adrian and Boyarchenko (2018), while for Value at Risk we set a 30-day time horizon and a confidence level of $1 - \alpha = 95\%$.

$$a = 0.055, \quad \underline{a} = 0.04, \quad r = 0.04, \quad \rho = 0.05, \quad \sigma = 0.1, \quad \tau = 30, \quad \alpha = 0.05.$$

Figure 1.3 depicts the optimal values of output, asset price, systemic and endogenous risk, welfare, and various risk compensations as a function of the banks' wealth relative to the market value of the risky asset in the economy $\eta_t = \frac{N_t}{p_t K_t}$. η_t can be interpreted as the inside capital of the financial sector, where low values of η_t imply a scarcity of bank equity. Within our equilibrium specification, we consider two regulatory regimes. In the first regime, the financial sector is unregulated and does not insure against market losses (red line in Figure 1.3). This case corresponds to $\xi = 0$, and the simplified version of Brunnermeier and Sannikov (2014)'s model. In the second regime, capital requirements are measured by Value at Risk (blue line).

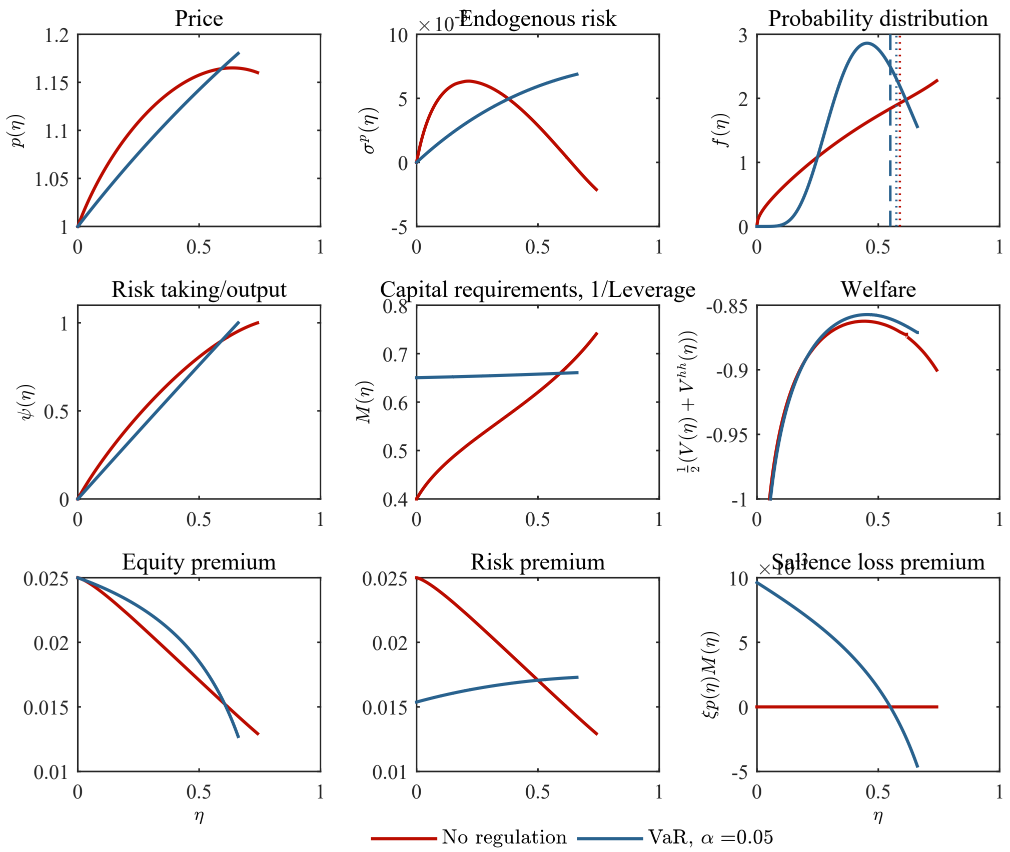


Figure 1.3: Equilibrium with macroprudential VaR
NOTES: Policy functions of the banks' wealth share η under the Value at Risk at a confidence level of $1-\alpha = 0.95$. No regulation corresponds to $\xi = 0$.

The prominent feature of our model, clearly illustrated in the top right panel in Figure 1.3, is the contrast in the steady-state distributions with and without regulation. The steady-state distribution measures the probability of each state η. The tightness of the capital requirement constraint ξ and equity growth μ^η endogenously determine distinct regions. The blue dashed line indicates the equity threshold ($\eta^\xi = 0.55$) below which capital requirement constraint binds and $\xi > 0$. Above η^ξ banks are unconstrained, while the left-hand side represents the constrained states. The blue dashed line is the steady-state level of equity $\eta^* = 0.57$ in the regulatory regime at which banks stop accumulating equity and $\mu^\eta = 0$. η^* is such that the marginal value of saving and the marginal value of consumption of an extra unit of net worth are equal. Banks will grow additional equity below η^*, and vice versa, deplete existing equity if η is higher than η^*. When banks are unregulated, they stop accumulating equity at level $\eta^*_{nr} = 0.59$, depicted by the red dashed line.

Probability distribution helps to explain the effects of macroprudential VaR on systemic risk. When banks are regulated, the distribution reaches the maximum in the middle region and the probability that η is zero close to zero ($f(0) \approx 0$). Therefore, a systemic risk, defined as periods in which bank equity is zero, is rare in the model. However, most of the weight is part of the state space where the capital constraint binds. The constrained region can be considered times of economic distress. Without regulation, the key finding is that bank equity has a higher probability of ending up at extreme equity levels. In other words, the financial sector is more likely to be undercapitalized (η close to 0) or overcapitalized ($\eta > \eta^*$). This result is consistent with Brunnermeier and Sannikov (2014), where the probability distribution is bimodal and peaks at the lowest and highest equity levels. Altogether, VaR seems to mitigate systemic risk.

The reason for lower systemic risk is that banks' precautionary savings are higher under the VaR regulation than when banks optimally self-insure without constraints. This self-insurance motive is illustrated by the equity premium in the bottom left panel. In both regimes, self-insurance is highest when bank equity is scarce, and η is low. The critical insight is that regulation generates two motives for precautionary savings, against both market losses and income uncertainty. The risk premium in the bottom middle panel captures precautionary incentives for future income uncertainty. On the other hand, the salience loss premium in the bottom right panel reflects self-insurance against market losses. In the absence of regulation, banks do not insure against the risk of financial crises because the salience loss premium is zero. With regulation, banks insure about 50 % more against income volatility than against future market losses. In particular, the maximum value of the risk premium is in the range of 1.5-1.8 %, while the salience loss premium at its maximum is 1%.

The salience loss premium reflects differences between regulator and bank assessments of downside market risk and required equity to absorb losses. At the cutoff η^ξ, banks behave as if future market losses are certain and equal to VaR losses. If the constraint is tight, the benefits of higher equity include positive salience loss premium term $\xi p_t M_t$. Banks are risk-averse in market losses and demand a positive salience loss premium to bear the resolution of uncertainty in downside market risk. Conversely, if the constraint is loose, the extra unit of equity is costly, and banks become risk-seeking in market losses and hope to avoid them. Therefore, VaR regulation elicits a twofold preference pattern; risk aversion and risk-seeking in market losses. The argument above explains why bank equity is valued differently depending on whether banks are regulated or not. In essence, the absence of regulation implies risk-neutrality in market losses even though banks are risk-averse in the traditional sense of wanting to smooth income and, thereby, consumption fluctuations.

Since both households and banks are forward-looking, it follows that differences in equity valuation across two regimes lead to differences in price valuations. The top left panel in Figure 1.3 compares price dynamics with and without regulation. As one might expect, the price of the risky asset rises with bank equity. This occurs because banks are more productive and impatient than households and want to borrow in equilibrium. Without productivity and discount rate differences, the price would be constant when bank equity varies. The risky asset price is lower than the competitive equilibrium price when the VaR constraint is tight and higher when it is loose. This is visible as two price functions intersect close to η^{ξ}. Therefore, asset prices reflect banks' liquidity valuations in addition to fundamentals.

A similar pattern is observed for bank risk-taking in the middle left panel in Figure 1.3. As the two graphs suggest, regulated banks take fewer risks if they are constrained, and more if they are unconstrained than unregulated banks. This asymmetric behavior of prices and risk-taking is a consequence of the asymmetric tightness of the VaR constraint. As a result, banks in the unregulated economy take "excessive" risks, compared with the regulated equilibrium that internalizes market losses. This explanation is consistent with the agency channel of risk-taking described in Freixas, Laeven, and Peydró (2015). According to the agency channel, banks take excessive risk because lenders and financial institutions share losses. Here, losses are shared between two sectors, since both households and banks fail to self-insure against market losses.

Referring to the price and risk-taking dynamics in both regimes in Figure 1.3, if bank equity declines, banks can no longer engage in productive opportunities and sell risky capital to households at a lower price. In other words, both equilibria exhibit fire sales. In general, fire sales arise when borrowers liquidate assets after an adverse balance sheet shock. Simultaneously, marginal buyers (households) value capital less than natural buyers (banks), so banks only find buyers for their risky assets at fire-sale prices. There is a question regarding how overweighting of market losses amplifies fire sales. In the 2008 downfall, financial institutions struggled to determine their exposure to potential losses on mortgage-backed securities, which caused fire sales of MBS and destabilized financial markets, as reported by Mizen (2008). Without regulation, the model can replicate the observed increase in price volatility associated with the period of financial distress. In particular, the top middle panel depicts endogenous risk, which is zero near η^*_{nr}, and rises abruptly when bank equity falls below η^*_{nr}. Fire sales produce a volatility spiral, defined in Brunnermeier and Sannikov (2014), in which selling capital depresses prices and makes them more volatile. Although we do not solve an optimal macroprudential policy that implements socially optimal allocations, VaR offsets the amplification of fire sales. Specifically, as the ψ function declines in the left middle panel in Figure 1.3 (blue line) going from high to low equity, price volatility σ^p in the upper middle declines (blue line). The critical implication for regulators is that by overweighting market losses, risky asset prices can decline without destabilizing financial markets. This implication is relevant because panics accompanying fire sales can trigger costly government interventions, such as the Federal Reserve's Large-Scale Asset Purchases (LSAPs).

In our model, precautionary motives are critical determinants of aggregate welfare. The right panel in the second row in Figure 1.3 shows that regulation can achieve higher welfare than a deregulated economy when bank equity is close to η^ξ. For both agents, welfare is a sum of the utility of current consumption levels and discounted consumption growth, minus consumption volatility, as seen from equations (1.23) and (1.28). A higher equity premium and leverage contribute to higher consumption growth, while a higher risk premium implies higher consumption volatility. The middle panel in the second row indicates that the leverage ratio, $\frac{1}{M_t}$, is higher with VaR regulation in the unconstrained region. Therefore, welfare improvements spurred by VaR are driven by the higher leveraged equity premium when equity is higher than η^ξ and by lower risk premium when equity is lower than η^ξ. Since macroprudential VaR can reduce crisis probability without reducing welfare, there is no welfare - systemic risk tradeoff. Similarly, Bianchi et al. (2011) show that an optimal *macroprudential debt-dividend tax* can reduce the incidence and magnitude of financial crises and increase social welfare compared to the competitive equilibrium without regulation. In this respect, macroprudential VaR may act as an implicit debt-dividend tax.

To summarize, compared to an economy with unregulated banks, VaR capital requirements can limit the likelihood of a financial crisis and increase aggregate welfare. Also, regulation can prevent amplification of fire sales, so selling capital depresses prices but does not increase endogenous price risk. In the following sections, we investigate how an economy with macroprudential VaR responds to adverse shocks such as an increase in borrowing costs, a decline in bank equity, or an increase in uncertainty. Three shocks proxy for an exogenous drop in asset prices, comparable to the 2008 housing market downfall. The next sections explore the implications of macroprudential VaR on systemic and endogenous risks, fire sales, and welfare.

1.4.1 Crisis management under macroprudential VaR

Economists and policymakers have suggested two primary channels through which the 2008 crisis constricted economic activity. One was the breakdown of housing prices, which discouraged household spending. The second one was fragility of the financial system, including its dependence on short-term funding, which resulted in widespread panic. In most chronicles of the financial crisis by policymakers, fire sales are described as the amplifier that helped transform a real estate crisis into a systemic crisis that threatened to cause the financial system to collapse. The conventional narrative is that the drop in housing prices caused rapidly spreading panic in the financial markets as investors assessed the extent of potential losses on mortgage-backed securities, which led to fire sales of these securities and insolvency of major financial firms. Clearly, fire sales, price volatility, and systemic risk interact, which has implications for financial regulation and policymakers' responses to future crises.

In the following subsections, we assess the effectiveness of macroprudential VaR in crisis management. To a certain extent, three crisis experiments aim to capture the 2008 downfall of the housing market. We explore whether regulators can prevent fire sales and systemic and endogenous risks from materializing by performing comparative static analysis to three model parameters.

First, we consider shocks to external financing conditions of the financial sector, in a sense that permanent change in discount rates increases funding costs. When external financing shock hits banks, borrowing becomes more expensive. We model the interest rate jump as an increase in households' impatience rate of r. Second, we examine the response of our economy to a decline in bank equity. To do so, we increase banks' impatience rate of ρ so that they consume a larger fraction of wealth. The tightening of borrowing costs and bank equity shortages disrupt the funding conditions of the financial sector. Third, uncertainty has received substantial attention as an essential factor in shaping the severity and duration of the 2008 crisis. For instance, Stock and Watson (2012) suggest that financial and uncertainty shocks were principal contributors to output declines during the great recession, while Bloom et al. (2018) propose that uncertainty shocks are new shocks that drive business cycles. We model uncertainty shocks as a permanent increase in exogenous risk σ, which captures the effect of higher shock volatility.

1.4.2 Funding shocks

Figure 1.4 plots the baseline VaR equilibrium ($r = 0.04$ and $\rho = 0.05$) plus an increase in borrowing costs (higher household impatience rate $r = 0.045$) and a decrease in bank capital (higher bank impatience rate $\rho = 0.55$). In both cases, the impatience of one agent rises by approximately 10 %. The top left panel shows that borrowing costs depress prices more when equity is low than when it is high; prices drop from 1 to about 0.9 at $\eta = 0$ (solid orange line). This is because households value capital less precisely in states when they hold all physical capital, and when bank equity is low. When bank equity deteriorates, prices are affected more when equity is high than when it is low. At maximum value, prices drop from 1.18 to 1.1 because banks now value capital less in states in which they hold all capital at $\psi = 1$ (dashed orange line).

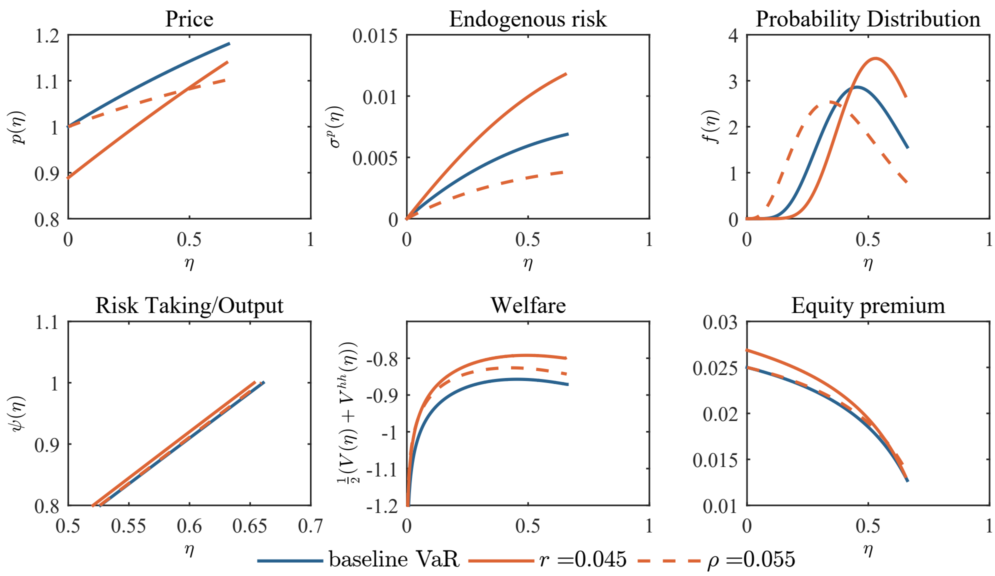


Figure 1.4: Response of macroprudential VaR to an increase in borrowing costs ($r = 0.045$) and a decline in bank equity capital ($\rho = 0.55$).

Simultaneously, the share of risky assets the banks hold remains almost unchanged, as the bottom left panel suggests. Since $\psi_t = \frac{\eta_t}{M_t}$, changes in the bank's risky asset holdings are determined by regulatory capital requirements. By definition, VaR loss is a function of total risk in the economy, exogenous risk σ, and endogenous price risk σ^p. Banks adjust risky asset holdings only slightly because risk-based capital requirements are not directly affected by discount rates. This argument and our findings demonstrate that disruptions in bank funding conditions are not sufficiently strong factors to produce substantial fire sales. With responsive prices and less responsive risk-taking, a rise in agents' impatience directly translates into welfare gains. Referring to the second-row middle panel, both funding shocks enhance aggregate welfare. These welfare improvements are attributable to boosts in the current consumption rate and consumption growth due to a rise in the equity premium, which is depicted in the bottom right panel.

The key result from the two funding shock experiments is the volatility paradox defined in Brunnermeier and Sannikov (2014), in the sense that systemic risk correlates negatively with endogenous risk. To see this clearly, we depict systemic risk in the top right and endogenous risk in the top middle panel in Figure 1.4 across two funding shocks. Tighter borrowing costs trigger a rise in endogenous risk and a decline in systemic risk (solid orange line). In fact, price volatility almost doubles, rising from 0.6 % to 1.2 % at the maximum point. Plots of declines in ban equity provide further evidence of the volatility paradox, now in the opposite direction. Specifically, if bank equity becomes scarce, this leads to higher systemic risk, but price volatility drops from 0.6% to 0.4% (dashed orange line). When funding shocks hit, therefore, macroprudential VaR can manage either crisis likelihood or financial panics. This trade-off is similar to those examined in the literature on systemic risk and intermediary asset pricing. For example, in Adrian and Boyarchenko (2018), tightening of capital and liquidity requirements reduces the probability of a crisis but increases price volatility. In contrast, in Brunnermeier and Sannikov (2014), if households and banks are unconstrained, more severe borrowing frictions lead to lower endogenous risk and lower crisis probability.

1.4.3 Uncertainty shock

Figures 1.5 summarizes results in the face of an uncertainty shock ($\sigma = 0.15$). In response to heightened uncertainty, regulators substantially raise capital requirements from 0.65 to about 0.83, prompting financial institutions to deleverage (bottom right panel). The vigorous regulatory response triggers substantial fire sales; prices drop (top right panel), and more capital is allocated to households (bottom right panel).

Compared to funding shocks, the volatility paradox disappears; the financial sys-

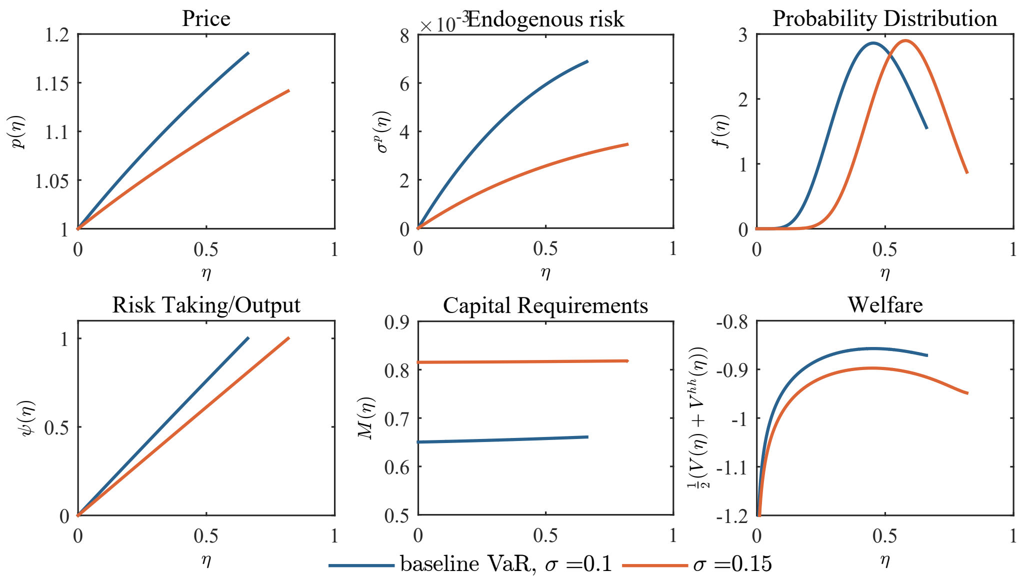


Figure 1.5: Uncertainty shock with macroprudential VaR

tem becomes capitalized enough to absorb the fall in asset prices without falling into panic and insolvency as endogenous and systemic risk are dampened (top middle and top right panel). Banks build precautionary savings to self-insure against the tail losses; the salience loss premium $\xi p_t M_t$ rises with higher capital requirements, which explains how crisis probability is reduced.

However, higher capital ratios reduce aggregate welfare and impede growth prospects by reducing the financial sector's risk-taking. The key to the welfare decline shown in the bottom right panel is the lower leverage of banks and higher consumption volatility due to exogenous risk increase. When uncertainty rises, our results imply a tradeoff between lower systemic and endogenous risks on one side and output and welfare contractions on the other. The welfare-systemic risk tradeoff also arises in Adrian and Boyarchenko (2018), where banks are subject to capital and liquidity requirements. Similarly, economic contraction due to heightened uncertainty is previously reported by Bloom et al. (2018), who show that uncertainty shocks leads to significant reductions of output and investments in a stochastic general equilibrium model with heterogeneous firms. Our results suggest that macroprudential VaR is incapable of preventing economic contractions caused by uncertainty shocks.

Overall, the main conclusion we derive from three crisis experiments is that macroprudential VaR effectively controls systemic and endogenous risks in volatile market environments shocks but fails to combat these risks when funding conditions fluctuate. The opposite holds for fire sales; regulation can prevent substantial fire sales when funding shocks hit the financial sector while generating fire sales in periods in which uncertainty is high.

1.5 Spectral risk measures as macroprudential capital requirements

As shown in the previous section, VaR may not be the most effective market risk measure when an economy faces uncertainty shock. Growth prospects become subdued, and aggregate welfare drops. Similarly, funding shocks produce a volatility paradox under VaR regulation. This section considers whether regulators can design a risk measure to reduce the likelihood of future financial crises and alleviate the crisis's economic costs? It explores whether such a risk measure can eliminate the volatility paradox, and discusses which agents benefit from new regulation. The theory of spectral risk measures may offer a possible solution. Knowing that VaR overweights the downside market risk of predetermined loss probability, we instead focus on spectral risk measures with the probability weighting function previously defined in section 1.2. Therefore, this section aims to construct a general framework for risk measures that simultaneously analyze upside and downside risks. As before, we assess the efficiency of the proposed regulation in the steady-state and after adverse shocks.

1.5.1 Specification of alternative regulations

In this subsection, we analytically compute spectral risk measures for Wang's and KT and anti-KT probability weighting functions from section 1.2. The Wang risk measure is

$$
\begin{aligned}
M_W &= \int_0^1 g_W(p)F^{-1}(p)dp \\
&= \int_0^1 \mathrm{e}^{-b_w^2+b_w\Phi^{-1}(p)}(1-\mathrm{e}^{(\mu_t^p+\sigma\sigma_t^p-\frac{1}{2}(\sigma+\sigma_t^p)^2)\tau+\Phi^{-1}(p)(\sigma+\sigma_t^p)\sqrt{\tau}})dp \\
&= 1-\mathrm{e}^{(\mu^p+\sigma\sigma^p)\tau+b_w(\sigma+\sigma^p)\sqrt{\tau}},
\end{aligned}
$$

where b_w is the parameter of the Wang probability weighting function given by equation (1.2).

Analytically, KT and anti-KR risk measures are computed as

$$
\begin{aligned}
M_{KT} &= \int_0^1 g_K T(p) F^{-1}(p) dp \\
&= \int_0^1 (ap^2 + bp^2 + c)(1 - \mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau + \Phi^{-1}(p)(\sigma+\sigma_t^p)\sqrt{\tau}}) dp \\
&= 1 - \mathrm{e}^{(\mu^p + \sigma\sigma^p)\tau} \left(c + (b+a)\Phi(\frac{(\sigma+\sigma^p)\sqrt{\tau}}{\sqrt{2}}) - 2aT(\frac{(\sigma+\sigma^p)\sqrt{\tau}}{\sqrt{2}}, \frac{1}{\sqrt{3}}) \right)
\end{aligned}
$$

, where $T(\cdot,\cdot)$ is Owen's T function, and a, b and c are parameters of the KT probability weighting function given by equation (1.5). The proof for M_{KT} is presented in Appendix 1.A.

1.5.2 Policy comparison

The severity and longevity of the 2008 financial crisis prompted policymakers to search for more effective macroprudential regulation tools and objectives. As emphasized by Borio (2003), the primary aim is to limit widespread financial instability, and the ultimate goal is to minimize macroeconomic costs associated with financial instability. In this section, we juxtapose a VaR risk measure and three spectral risk measures for fulfilling macroprudential goals. First, we consider the implications of three measures on systemic and endogenous risks, and the second, how these measures affect fire sales, output, and welfare. The first question aims to assess the effectiveness of prudential tools in fulfilling the "stability" objective, while the second question targets the "efficiency" objective that minimizes instability costs.

Table 1.1: Parameter values

Parameter	Wang	KT	anti-KT
b_w	-1.3		
a		10	-4
b		-12	3.6
c		3.66	0.53

As mentioned in section 1.2, the three risk measures are such that the associated probability weighting functions are convex (Wang), inversely S-shaped (KT), or S-shaped (anti-KT). With the convex weighting function, regulators are pessimistic and overweight bank exposure to tail market losses. The inverse S-shaped implies the mixture of regulatory pessimism and optimism, as worst-case and best-case outcomes are overweighted while intermediate are underweighted. With an S-shaped weighting function, regulators underweight extreme outcomes and overweight the intermediate ones. Table 1.1 summarizes parameter values used to compute equilibrium outcomes for three risk measures. Therefore, the parameter are set to $b_w = -1.3$ for the Wang risk measure, $a = 10, b = -12$, and $c = 3.66$ for the KT risk measure, and $a = -4, b = 3.6$, and $c = 0.53$ for the anti-KT risk measure.[13]

1.5.3 Policy comparison in the steady-state

Several implications emerge from the steady-state policy comparison in Figure 1.6 for regulator ability to fulfill the two prudential objectives. Among the crucial ones, we find that regulators achieve the macroprudential stability objective by focusing solely on tail market losses. In particular, the top right and top middle panels imply that financial instability in terms of systemic and endogenous risks is lowest under the VaR and Wang regulations.

High bank capital buffers are essential for limiting the probability of systemic crisis and financial panics. The bottom left panel shows that when the worst-case scenario is salient for regulators, as it is with the VaR and Wang risk measure, capital ratios are at approximately 50% and 65%. If regulators instead assess market risk by overweighting the worst-case and best-case (KT) or intermediate outcomes (anti-KT), the required capital ratios decline to about 15 %. Moreover, banks become risk-seeking in market losses as illustrated by the negative salience loss premium in the bottom right panel. Banks no longer fear market losses but hope to avoid them and fail to internalize the potential systemic costs of their risk-taking. As both regulators and banks shift from pessimism to optimism, banks hold less capital to absorb market losses and fail to self-insure. Such

[13]When choosing parameters, two restrictions are important. First, parameter a summarizes the strength of overweighting or underweighting, meaning higher a equals more overweighting and vice versa. Second, the important limitation when choosing KT parameters is that the risk spectrum should have the minimum value at the interior point around 1/2 to preserve an inverse S-shape of the probability weighting function. Similarly, for the anti-KT measure, the maximum value is attained at the interior point. When solving for equilibrium, we choose a and the point p^* where the risk spectrum achieves minimum or maximum value, while other two parameters are $b = -2p^*a$, $c = 1 - \frac{a}{3} - \frac{b}{2}$, which come from restrictions on the probability weighting function $G(1) = 1$, and the minimum or maximum of the quadratic risk spectrum $p^* = -\frac{b}{2a}$. By choosing p^*($p^* = 0.45$ for anti-KT, and $p^* = 0.6$ for KT) we directly control which market losses are most overweighted or underweighted. The results are robust for p^* in the range 0.55-0.65 for the KT risk measure, and p^* in the range 0.4-0.5 for the anti-KT risk measure.

risk attitudes increase the incidence of large losses that lead to higher probability of a crisis, as shown by the steady-state distribution that shifts to the left.

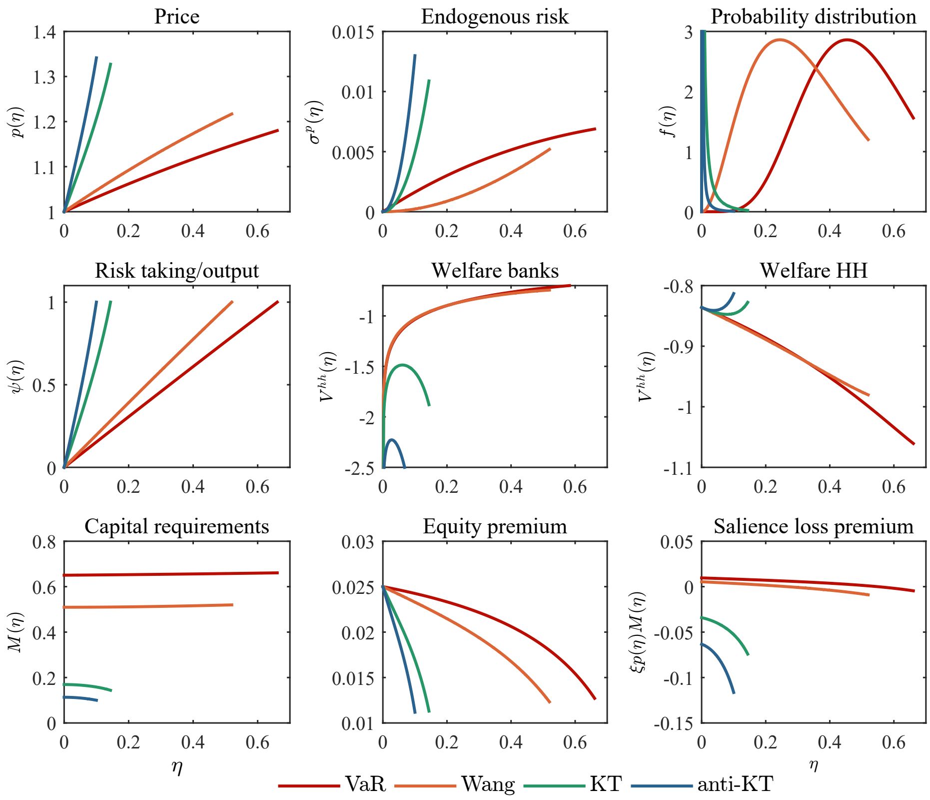


Figure 1.6: Steady-state policy comparison between VaR, Wang, KT and anti-KT

The top middle panel in Figure 1.6 depicts endogenous risk. Endogenous risk is financial risk created because banks and households do not internalize how the asset price responds to their collective portfolio decisions. Because of the capital requirement constraint, the endogenous risk arises in response to regulatory market losses. We find that the anti-KT risk measure produces the highest endogenous risk, while the endogenous risk is the lowest under the Wang regulation when regulators overweight the worst-possible loss. Related to these findings, Gennaioli, Shleifer, and Vishny (2012) argue that as investors recognize and disproportionately overweight tail losses, agents react less strongly to the news. According to Gennaioli, Shleifer, and Vishny (2012), when investors fail to account for improbable risks when trading new securities, financial instability rises sharply. Because market participants cannot imagine worst-case outcomes during quiet periods, they perceive new securities as being safer but end up bearing neglected tail risks. Eventually, investors start incorporating disregarded tail risks, which triggers a flight to safe traditional assets and spurs agents to overreact.

The second crucial result is that regulators partly achieve the macroprudential efficiency objective by focusing on upside risks, that is, on best-case or intermediate outcomes. With KT and anti-KT, output per unit of bank equity is higher than under VaR and Wang regulations (middle left panel). The intuition for this finding is straightforward. Higher capital requirements impose a constraint on bank risk-taking, leading to lower output since banks are more productive. Apart from the higher output, households are better off if capital requirements incorporate upside risks. This is because capital requirements redistribute wealth between agents.

To see redistributive effects, the second-row middle and right panel in Figure 1.6 depict the welfare of two agents. With VaR and Wang, households' welfare declines in η, while banks' welfare rises in η. A rise in bank equity hurts households, while banks absorb the benefits of additional equity. However, the KT and anti-KT graphs illustrate that capital requirements may redistribute wealth from the financial sector to the rest of the economy. Redistribution manifests in lower bank welfare and higher households' welfare under KT and anti-KT than under VaR and Wang. Second, with KT and anti-KT, higher bank equity hurts bankers and benefits households.

Most literature on financial regulation focuses on the systemic risk implications of prudential instruments and disregards redistributive effects. Welfare redistribution is achieved because capital requirements have a dual role; they affect bank leverage and risk-sharing between two agents. Looser regulation amplifies the leverage channel, while probability weighting alters risk-sharing incentives; agents may share the asset's downside risk and the upside risk, depending on which probabilities are underweighted or overweighted. Four risk measures fall into different spectrums. With VaR and Wang, banks do not share downside or upside market risks with households. With KT, banks do not share the downside risk but share upside (intermediate) risks. Finally, with anti-KT, banks share downside risk and upside (best-case) risks.

With KT and anti-KT, the leverage effects benefit households by increasing savings, and they also profit from the improved risk-sharing of upside risks. For bankers, more risk-sharing means lower equity premium and consumption growth (blue and green lines in the bottom middle panel), while higher leverage means higher consumption growth. The former effect dominates, and banks experience negative consequences when upside risks are shared. Related to this finding, Korinek and Kreamer (2014) shows that financial deregulation benefits banks due to increased financial risk-taking and because they earn greater expected returns. Deregulation hurts workers because higher risk-taking generates more frequent credit crunches. Our result suggests that policymakers can calibrate capital requirements that redistribute wealth from banks to households. Lower capital requirements can achieve such redistribution if the upside potential of risky bank investments is shared among two sectors.

Taking the two main results together, we find a stability-efficiency tradeoff of capital requirements based on probability weighting. Precisely, overweighting tail losses achieves the stability objective, while including upside risks better fulfills the efficiency objective. A similar tradeoff is found by Adrian and Boyarchenko (2018), where tighter capital requirements reduce the probability of a crisis but lead to lower welfare in terms of consumption growth. In sum, if the likelihood of a systemic crisis is the primary reason for regulating the banking industry, ex-ante systemic risk is best addressed when regulators focus only on tail losses. Nonetheless, we suggest that focusing solely on downside risk has limited benefits. A way forward in the prudential framework may be to include upside risks, particularly when regulators are concerned about stifling economic growth and welfare redistribution.

1.5.4 Policy comparison after adverse shocks

Apart from steady-state policy evaluation, crisis management is crucial when analyzing the prudential framework's strengths and weaknesses. Scenario analyses usually draw from stressful historical events such as the collapse of housing prices in 2008. We focus on sensitivity analyses such as uncertainty and funding shocks, which aim to replicate declines in asset prices, to identify four opportunities and limitations of four prudential frameworks.

Turning to funding shocks, we have shown that macroprudential VaR gives rise to a volatility paradox in that it can manage either the likelihood of a crisis or financial panics. The relevant question is, can probability weighting avoid this tradeoff? Figure 1.7 plots endogenous risk and the probability distribution of the state variable η under VaR and anti-KT.[14] Three cases are depicted, the baseline (gray line, $r = 0.04, \rho = 0.05$), an increase in borrowing costs (solid line, $r = 0.045, \rho = 0.05$), and a decline in bank equity (dashed line, $r = 0.04, \rho = 0.055$). If the probability distribution moves to the left or right from the baseline distribution, this indicates a rise and decline in systemic

[14] For brevity, we omit the Wang and KT regulation because Wang gives almost identical results as VaR, while the KT results are similar to those of anti-KT.

risk. As with VaR, tighter borrowing costs trigger endogenous risk rise and a decline in systemic risk (solid blue lines). Similarly, a decline in bank equity leads to higher systemic and lower endogenous risk (blue dashed lines). In the event of an adverse funding shock, the volatility paradox is present under alternative regulations.

The key implication of comparing overweighting the downside (VaR) and overweighting intermediate outcomes (anti-KT) is excess sensitivity of VaR and excess smoothness of anti-KT. In the latter case, systemic and endogenous risks appear to adjust insufficiently to discount rate shocks. A rise of approximately 10 % in household impatience rates produces a 70 % rise (from 0.7 to 1.2 p.p.) in endogenous risk under VaR, while under anti-KT, the increase is about 5% (from 1.3 to 1.36 p.p.) Meanwhile, if bank impatience rises by 10%, endogenous risk declines by 45 % under VaR (from 0.7 to 0.4 p.p.) and by 4 % under anti-KT (from 1.3 to 1.25 p.p.). A simplified intuition is that capital requirements are functions of exogenous risk σ and endogenous price risk σ^p, and losses around the median (anti-KT) are less sensitive than tail losses (VaR) to changes in the standard deviation of the market loss distribution. Accordingly, less responsive capital requirements lead to smaller adjustments in risky asset holdings and, therefore, less fluctuation in endogenous and systemic risk.

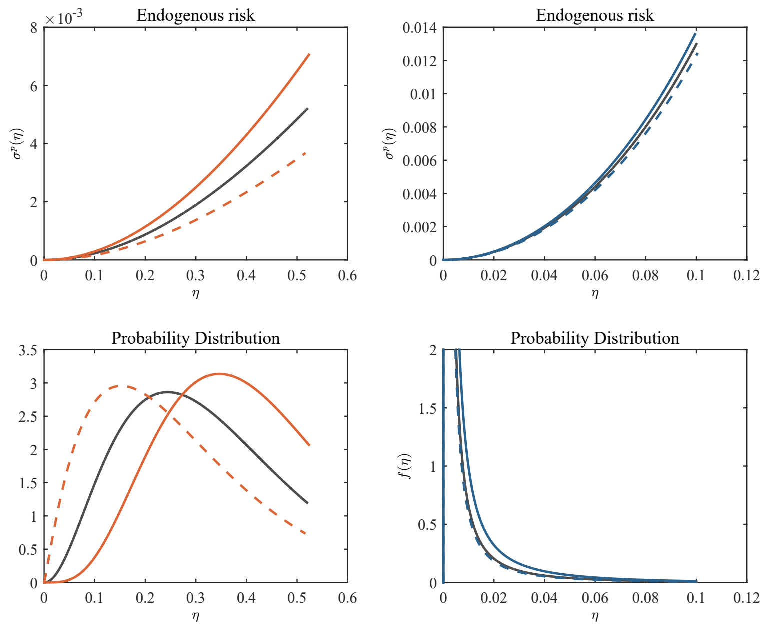


Figure 1.7: Response of endogenous and systemic risks to an increase in borrowing costs ($r = 0.045$) and a decline in bank equity ($\rho = 0.055$) under VaR (first column) and anti-KT regulations (second column).

Our final results in this section compare the efficacy of four risk measures when uncertainty shock arises. In section 1.4.3, we have shown that the main transmission channel of an uncertainty shock with VaR is a sharp reduction in risky asset holdings because capital ratios rise to reflect a more volatile market environment, causing fire sales and a decline in aggregate output. In particular, a tradeoff emerges between lower systemic and endogenous risks on one side and output and welfare contractions on the other. The VaR regulation achieves the macroprudential stability objective but fails to realize the efficiency goal. In light of these results, we examine whether spectral risk measures can achieve both the efficiency and stability goals.

Figure 1.8 shows the effects of an increase in exogenous risk σ from 10 to 15 p.p. on the probability of financial crises (bottom panel) and percentage changes in output, prices, and welfare (upper panel). As the upper panel suggests, under the Wang regulation, crisis dynamics are about the same as under VaR. By focusing solely on downside risk, two risk measures can reduce crisis probability and endogenous risk, but neither prevent fire sales or welfare losses of two agents. In both regulatory regimes, the maximum output decline peaks at about 20 %, while prices drop by about 5 % at peak.

The key insight of the uncertainty crisis experiment is that KT and anti-KT perform better with respect to macroprudential efficiency goals. First, the KT measure not only prevents fire sales, but the uncertainty shock stipulates fire *buys* and economic expansion, because banks are willing to buy capital at higher prices. While prices appreciate by a maximum of 10%, banks boost their risky asset purchases by up to 40 %. Banks are willing to increase risky asset holdings because regulators reduce capital requirements in response to heightened uncertainty, as illustrated in the top right subplot. In a run-up to the 2008 financial crisis, selling mortgage-backed securities depressed their price significantly because both buyers and sellers struggled to evaluate losses associated with these assets. Our results suggest that simultaneously overweighting the worst-case and best-case market outcomes may prevent fire sales. However, preventing fire sales does not necessarily mean that banks are better off. In fact, fire buys leave banks vulnerable to crises as systemic and endogenous risks rise sharply, and households can potentially benefit from fire sales.

Second, compared to VaR and Wang, KT redistributes welfare from banks to households, while anti-KT redistributes wealth from households to banks. For both agents, VaR and Wang yield up to 6% welfare losses. Household welfare gains under KT policy is up to 5 %, while banks experience welfare losses that reduce their welfare almost

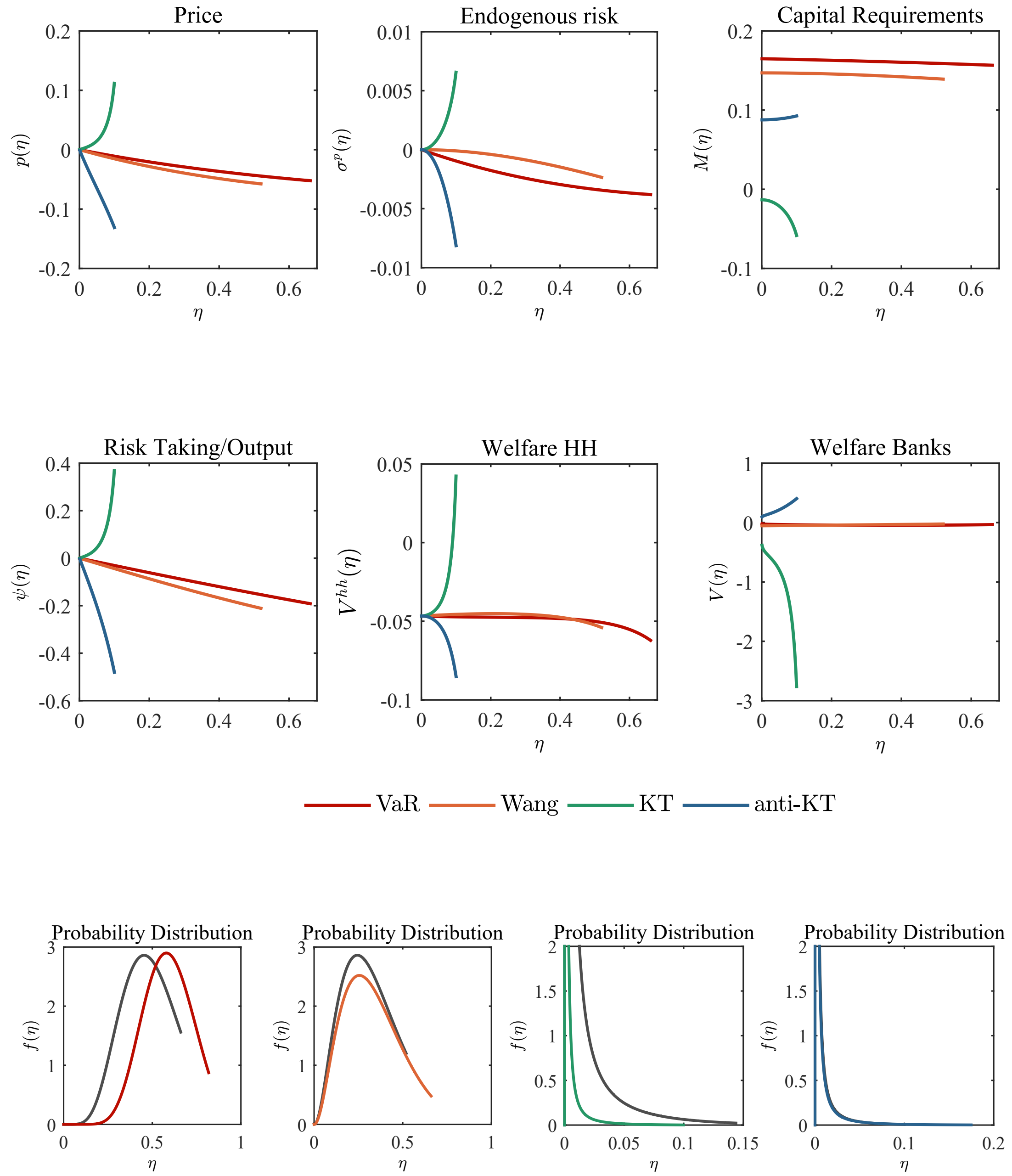


Figure 1.8: (Upper panel) Response of price, risk taking, endogenous risk and welfare to an increase in uncertainty from $\sigma = 0.1$ to $\sigma = 0.15$ under VaR, Wang, KT and anti-KT regulations, percentage change. (Bottom panel) Steady-state probability distribution, baseline (gray line) and after uncertainty shock (colored line).

threefold. Notice that, since the KT policy involves higher prices and risk-taking, there are welfare gains for banks in terms of current consumption. Still, these are outweighed by a sharp increase in future consumption volatility due to a rise in the risk premium. Meanwhile, higher prices benefit households at higher levels of bank equity. Under anti-KT, household welfare losses peak at 9 %, while banks' welfare increases in the range of 10 to 40 %. Bank welfare gains arise from reduced fluctuations in future consumption. Therefore, the main channel by which KT and anti-KT redistribute wealth compared to downside risk measures is by amplifying or dampening variations in future income and, accordingly, future consumption.

Ideally, policymakers strive for one prudential tool to mitigate various financial vulnerabilities and negative spillovers to the real economy after adverse shocks. The proposed regulation based on probability weighting approximates this ideal, one which regulators can conveniently achieve by switching between market risk measures. We have shown that overweighting tail losses are beneficial for policymakers aiming to reduce the likelihood of systemic crises. The inclusion of upside risk is valuable because it can redistribute wealth between two sectors and prevent fire sales. Given our findings, regulators could implement two policy interventions in practice. First, capital buffers can be designed to balance the ex-ante prevention of systemic risk and ex-post crisis management. Regulators can achieve this goal is by weighting downside and upside market risk measures according to their preferences for macroprudential stability or efficiency. Alternatively, regulators may enforce VaR or Wang policies during peaceful times to reduce likelihood of a crisis while adjusting their choice of risk measure when financial markets become disrupted.

1.5.5 Connection to salience and prospect theory

We compare our predictions to those of the prospect and salience theory of Tversky and Kahneman (1992) and Bordalo, Gennaioli, and Shleifer (2013a). In decision making under risk, prospect and salience theories examine why agents sometimes prefer to take the risk but sometimes avoid risk. Both approaches yield a fourfold choice pattern in risk preferences: the decision-maker is risk-seeking in losses of high probability and risk-averse in losses of low probability. And vice versa, agents are risk-seeking in low probability gains and risk-averse in high probability gains.

Three of our findings are relevant for prospect theory. First, under the VaR and Wang regulations, a twofold pattern emerges: the financial sector is risk-averse or risk-seeking in market losses. Second, KT and anti-KT regulations elicit risk-seeking preferences.

The positive salience loss premium in Figure 1.6 indicates risk aversion, while the negative reflects risk-seeking attitudes. Third, we find that banks take more risks if capital requirements rise and less risks when capital requirements decline.

We can reconcile the first prediction with salience theory, in which investors are risk-seeking when the asset's upside is salient and risk-averse when its downside is salient. By definition, an asset's salient payoff is defined as one most different from the average market payoff in a given state of the world. The upside is salient when potential market gains are higher than losses, while the downside is salient when losses are higher. The idea is that the preference shift occurs because there is a shift in salience from market losses to gains. The intuition for the twofold pattern is straightforward; banks think about expected losses measured by regulators, focus on the upside when market losses are lower than VaR or Wang, and focus on the downside when market losses are higher. In doing so, they overweight losses that drive their attention. For example, when the risky asset upside is salient, banks give themselves a small chance of avoiding market losses above VaR or Wang measures. Conversely, a salient downside triggers fear of extreme market losses.

Prospect theory distinguishes two drivers of risk attitudes: the curvature of the value function (convex for losses and concave for gains) and the probability weighting function (the possibility and the certainty effect). The value function captures an observation that agents evaluate financial outcomes as gains and losses from the reference point and are more sensitive to losses. Subjective probabilities reflect the tendency of the individual to pay comparatively more attention to less probable outcomes. While the value function and the certainty effect favor risk-seeking for losses, the possibility effect favors risk aversion for losses. In prospect theory, individuals tend to shift from avoiding risk to seeking risk in losses when the possibility effect reflects into the certainty effect; when a highly unlikely loss becomes highly likely. Because with VaR_α loss probability is fixed at level α, the change in bank attitude towards risk is inconsistent with prospect theory.

We can reconcile the second prediction with both theories. The explanation consistent with salience theory is that KT and anti-KT regulations elicit risk-seeking preferences because both measures draw attention to the risky asset's upside potential. The certainty effect of prospect theory explains risk-seeking. The certainty effect is present because losses become more likely with KT as steady-state distribution shifts to the left in comparison to VaR and Wang.

Lastly, we investigate how is bank risk-taking behavior affected by prior gains and

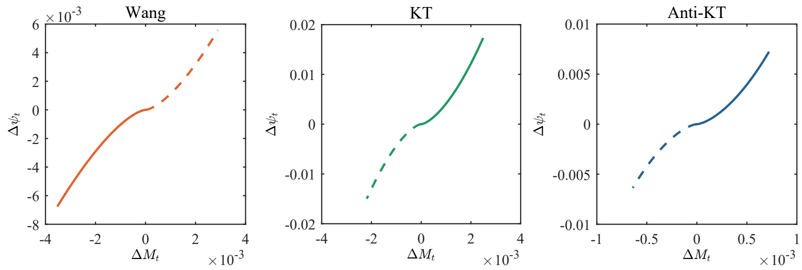


Figure 1.9: Changes in risk taking as a function of changes in capital requirements when borrowing costs increase ($r = 0.045$, solid lines), and bank equity declines ($\rho = 0.055$, dashed lines).

losses. Figure 1.9 depicts the relationship between changes in risk-taking (y-axis), $\Delta\psi = \psi_{after} - \psi_{before}$, and changes in capital requirements $\Delta M = M_{before} - M_{after}$ (x-axis) after two funding shocks. Gains on the x-axis represent a reduction in regulatory losses and loosening of capital requirements. In contrast, losses mean tightening of capital requirements.

Figure 1.9 implies that under three types of regulation, Wang, KT, and anti-KT, banks engage in more risk-taking after prior gains ($\Delta\psi > 0$ if $\Delta M > 0$), but become risk-averse after prior losses ($\Delta\psi < 0$ if $\Delta M < 0$). After a reduction in regulatory capital ratios, banks purchase risky assets, and vice versa, sell the assets when capital requirements rise. Initial gains and losses affect their subsequent choices in systematic ways - the risk attitudes shift around zero gain or loss (no change in capital requirements), resembling the reflection effect of prospect theory in Tversky and Kahneman (1992). The reflection effect asserts that decision-makers exhibit opposite risk attitudes depending on whether the outcomes are framed as possible gains or losses.

However, prospect theory, in which investors evaluate outcomes in terms of gains and losses relative to the reference point, predicts a disposition effect. Intuitively, the disposition effect captures investors' reluctance to realize losses and readiness to realize gains. Shefrin and Statman (1985) document the tendency of mutual fund investors to hold stocks which decline in price longer ("losers") and to sell too soon stocks which experience increases in price ("winners"). The disposition effect implies that prior gains generate risk aversion while prior losses nudge risk-seeking behavior. As suggested by Shefrin and Statman (1985), the S-shape value function (concavity for gains, and convexity for losses) explains such risk attitudes. Investors evaluate the gambling choice in the risk-averse part of the value function if the stock appreciates, using the purchase price as a reference point. As a consequence, price appreciation provides incentives to sell risky assets. If instead the stock price declines, the value function's risk-seeking region prompts investors to hold the asset.

Therefore, our prediction of banks' willingness to take fewer risks with prior losses, while prior gains increase their willingness to take risks, is inconsistent with prospect theory. One reason is that we abstract from the S-shape value function. The second reason is that the capital requirements we impose separate the bank's risk attitudes in choice and pricing. Risk attitude in choice reveals the bank's willingness to invest in risky assets and depends on measured regulatory losses. In technical terms, risk attitude in choice is equal to $\psi = \frac{\eta}{M}$. Instead, risk attitude in pricing conveys the banks' perception of losses and governs the salience loss premium, $\xi p_t M_t$. Third, prior capital requirements, M_{before}, serve as a reference point for assets sales or purchases, not prior price p_{before}. These three features combined explain the attitudes towards risk observed in Figure 1.9.[15]

To summarize, our findings on bank attitudes toward risk present mixed evidence on predictions of prospect and salience theory. While we find a twofold preference pattern of prospect and salience theory, prospect theory cannot explain bank risk-taking patterns. Banks in our model are subject to capital requirements based on probability weighting, suggesting that both institutional and behavioral factors play essential roles in determining economic outcomes.

1.6 Conclusion

The role of macroprudential policy instruments to mitigate the probability and the severity of systemic crises received substantial attention after the 2008 financial crisis. In this paper, we investigate the systemic risk and welfare implications of macroprudential regulation. To do so, we incorporate capital requirements in the form of four market risk measures into a continuous-time heterogeneous agent model. The key idea of the proposed spectral risk measures is probability weighting, in that regulators overweight market losses that are salient to them.

This paper's central finding is that, if limiting the likelihood of a crisis is the primary reason for regulating the banking industry, systemic and endogenous risks are best addressed when regulators overweight only tail losses. In turn, focusing on both the

[15]The derivative of $\Delta\psi$ with respect to ΔM equals $\frac{d\Delta\psi}{d\Delta M} = \frac{d\left(\frac{\eta}{M_{after}} - \frac{\eta}{M_{before}}\right)}{d(M_{before} - M_{after})} = \frac{\eta}{M_{before}M_{after}} > 0$.

downside and upside of the market is beneficial for policymakers aiming to prevent welfare and output costs of tighter capital requirements. When managing adverse internal or external funding shocks, four measures can either reduce crisis likelihood or amplification. In the face of a uncertainty shock, overweighting both the worst-case and best-case scenario prevents fire sales and output declines, while overweighting intermediate outcomes generates welfare improvements for banks. Given our findings, we suggest that VaR and Wang could target crisis probability and endogenous risk, while KT and anti-KT can target welfare and fire sales.

There are several model extensions and applications that we leave for future analysis. First, we would like to develop a more comprehensive theoretical model that includes loss aversion and a convex-concave value function of prospect theory. Loss aversion and convexity in the gains domain reduce decision-makers' willingness to take the risk and may imply richer equilibrium dynamics. Another fruitful direction would be to investigate transition dynamics from the benchmark Value at Risk capital requirements to alternative spectral risk measures. An additional benefit of this extension is that it would deliver a time-varying crisis probability. Finally, the empirical estimation of the probability weighting function from recapitalized banks during the recent crisis is a promising future direction.

1.A Omitted proofs

In this appendix, we provide proofs of the propositions stated in Section 1.3.

Proof of Proposition 1. Recall that we defined market losses as $X_t - X_{t+\tau}$ and $VaR_\alpha^{t,t+\tau}$ is the quantile with confidence level $1-\alpha$ of market losses. In other words, $VaR_\alpha^{t,t+\tau}$ is defined as

$$VaR_\alpha^{t,t+\tau} = \inf\{L \geq 0 : P(X_t - X_{t+\tau} \geq L|\mathcal{F}_t) \leq \alpha\} = (Q_{t,t+\tau}^\alpha)^-,$$

where

$$Q_{t,t+\tau}^\alpha = \sup\{L \in \mathbb{R} : P(X_{t+\tau} - X_t \leq L|\mathcal{F}_t) \leq \alpha\}$$

is the quantile of the projected market gains over the horizon of length τ and $x^- = \max\{0, -x\}$. Then we have

$$\begin{aligned}
&P(X_{t+\tau} - X_t \leq L|\mathcal{F}_t)\\
&= P\left(X_t \exp\left(\int_t^{t+\tau} (\mu_s^p + \sigma\sigma_s^p - \frac{1}{2}(\sigma + \sigma_s^p)^2)ds + \int_t^{t+\tau} (\sigma + \sigma_s^p)dW_s\right) - X_t \leq L \quad |\mathcal{F}_t\right)\\
&= P\left(\exp\left((\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma + \sigma_t^p)^2)\tau + (\sigma + \sigma_t^p)(W_{t+\tau} - W_t)\right) \leq 1 + \frac{L}{X_t} \quad |\mathcal{F}_t\right)\\
&= P\left((\sigma + \sigma_t^p)(W_{t+\tau} - W_t) \leq \log(1 + \frac{L}{X_t}) - (\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma + \sigma_t^p)^2)\tau \quad |\mathcal{F}_t\right)\\
&= \Phi\left(\frac{\log(1 + \frac{L}{X_t}) - (\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma + \sigma_t^p)^2)\tau}{(\sigma + \sigma_t^p)\sqrt{\tau}}\right)
\end{aligned}$$

where the last equality follows from the fact that the random variable $(\sigma+\sigma_t^p)(W_{t+\tau}-W_t)$ is conditionally normally distributed with zero mean and variance $(\sigma + \sigma_t^p)^2\tau$, and $\Phi(\cdot)$ is the cumulative distribution of the standard normal distribution. Therefore, we have

$$\begin{aligned}
&P(X_{t+\tau} - X_t \leq L|\mathcal{F}_t) \leq \alpha\\
&\Phi\left(\frac{\log(1 + \frac{L}{X_t}) - (\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma + \sigma_t^p)^2)\tau}{(\sigma + \sigma_t^p)\sqrt{\tau}}\right) \leq \alpha
\end{aligned}$$

$$L \leq X_t \left(\exp \left((\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma + \sigma_t^p)^2)\tau + \Phi^{-1}(\alpha)(\sigma + \sigma_t^p)\sqrt{\tau} \right) - 1 \right),$$

which implies

$$Q_{t,t+\tau}^{\alpha} = X_t \left(\exp \left((\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma + \sigma_t^p)^2)\tau + \Phi^{-1}(\alpha)(\sigma + \sigma_t^p)\sqrt{\tau} \right) - 1 \right).$$

Finally, we obtain the expression stated in the proposition

$$VaR_{\alpha}^{t,t+\tau} = p_t k_t \left(1 - \exp((\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma + \sigma_t^p)^2)\tau + \Phi^{-1}(\alpha)(\sigma + \sigma_t^p)\sqrt{\tau}) \right).$$

$\square$

Proof of Proposition 2. The proof for the optimal policy functions of household is given using the matching drifts method, while the guess-and-verify method is used to derive the proof for the value function. For the ease of exposition we omit the time subscript.

Step 1

We define by $\lambda = V'(\underline{n})$ the marginal utility of net worth or the stochastic discount factor of households, which follows the Brownian motion

$$d\lambda = \mu^{\lambda}\lambda dt + \sigma^{\lambda}\lambda dW_t.$$

By Ito's lemma we have

$$\mu^{\lambda}\lambda = V"(\underline{n})[\underline{ak} + r_t\underline{n} + p\underline{k}(\mu^p + \sigma\sigma^p - r_t) - c] + \frac{1}{2}V'''(\underline{n})(\sigma + \sigma^p)^2 p^2 \underline{k}^2$$

$$\sigma^{\lambda}\lambda = V''(\underline{n})(\sigma + \sigma^p)p\underline{k}.$$

Step 2

We obtain the envelope condition after substituting for household FOCs from the main text

$$(r - r_t)V'(\underline{n}) = V''(\underline{n})[\underline{ak} + r_t\underline{n} + p\underline{k}(\mu^p + \sigma\sigma^p - r_t) - c] + \frac{1}{2}V'''(\underline{n})(\sigma + \sigma^p)^2 p^2 \underline{k}^2$$

which implies

$$(r - r_t)\lambda = \mu^{\lambda}\lambda$$

$$\frac{d\lambda}{\lambda} = (r - r_t)dt + \sigma^\lambda dW_t.$$

<u>*Step 3*</u>

From the first order condition for consumption and log utility, we have that consumption is equal to $c = 1/\lambda$. Again using Ito's lemma for consumption as a function of a stochastic discount factor, we obtain expressions for consumption drift and volatility

$$\mu^c c = -\frac{1}{\lambda^2}\mu^\lambda + \frac{1}{2}\frac{2}{\lambda^3}(\sigma^\lambda)^2\lambda^2, \quad \sigma^c c = -\frac{1}{\lambda^2}\sigma^\lambda\lambda$$

$$\mu^c = -\mu^\lambda + (\sigma^\lambda)^2, \quad \sigma^c = -\sigma^\lambda$$

$$\mu^c = r_t - r + (\sigma^c)^2. \tag{1.41}$$

Note that we can rewrite the households' asset pricing equation as

$$-\sigma^\lambda = \frac{\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t}{\sigma + \sigma^p}. \tag{1.42}$$

<u>*Step 4*</u>

Ito's lemma gives expressions for consumption drift and volatility as a function of net worth

$$\mu^c c = c'(\underline{n})[a\underline{k} + r_t\underline{n} + p\underline{k}(\mu^p + \sigma\sigma^p - r_t) - c] + \frac{1}{2}c''(\underline{n})(\underline{n})(\sigma + \sigma^p)^2 p^2 \underline{k}^2, \quad \sigma^c c = c'(\underline{n})(\sigma + \sigma^p)p\underline{k}. \tag{1.43}$$

Using $\sigma^c = -\sigma^\lambda$ and substituting for σ^c in the asset pricing equation, we obtain

$$\underline{k}(\underline{n}) = \frac{\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t}{(\sigma + \sigma^p)^2}\frac{c(\underline{n})}{c'(\underline{n})p} \tag{1.44}$$

The function $c(\underline{n})$ can be found by equating drifts in (1.41) and (1.43) and using (1.42) and substituting for k from (1.44)

$$r_t - r + \left(\frac{\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t}{\sigma + \sigma^p}\right)^2 =$$

$$= \frac{c'(\underline{n})[\underline{ak} + r_t\underline{n} + p\underline{k}(\mu^p + \sigma\sigma^p - r_t) - c(\underline{n})] + \frac{1}{2}c''(\underline{n})(\underline{n})(\sigma + \sigma^p)^2 p^2 \underline{k}^2}{c(\underline{n})}$$

which yields the optimal policy rule for consumption $c(\underline{n}) = rn$. Then, the capital rule $k(\underline{n})$ is easily obtained by substituting for $c(n) = rn$ and $c'(n) = r$ in (1.44).

Step 5

Plugging back two policy rules into the HJB equation from the main text we obtain the new HJB equation

$$rV(\underline{n}) = \log(r\underline{n}) + V'(\underline{n})\underline{n}\left[\left(\frac{\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t}{\sigma + \sigma^p}\right)^2 + r_t - r\right] + \frac{1}{2}V''(\underline{n})\underline{n}^2\left(\frac{\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t}{\sigma + \sigma^p}\right)^2.$$

Finally, we guess and verify the value function form to be $V(\underline{n}) = B\log\underline{n} + D$. This functional form implies that $V'(\underline{n}) = B\frac{1}{n}$ and $V''(\underline{n}) = -B\frac{1}{n^2}$. Plugging this back into the new HJB, we find the coefficient to be equal to

$B = \frac{1}{r}$, and $D = \log\frac{1}{r} + \frac{1}{r^2}(r_t - r + \frac{(\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t)^2}{2(\sigma + \sigma^p)^2})$, which concludes the proof. □

Proof of Proposition 3. We solve for banks' optimal consumption and capital rules and the value function by using the same methods and steps as in the household's case.

Step 1

Let $\lambda = V'(n)$ represent banks' stochastic discount factor and let it follow Brownian motion

$$d\lambda = \mu^\lambda \lambda dt + \sigma^\lambda \lambda dW_t$$

By Ito's lemma we have

$$\mu^\lambda\lambda = V\text{"}(n)[ak + r_t n + pk(\mu^p + \sigma\sigma^p - r_t) - c] + \frac{1}{2}V'''(n)(\sigma + \sigma^p)^2 p^2 k^2$$

$$\sigma^\lambda\lambda = V''(n)(\sigma + \sigma^p)pk$$

Step 2

The envelope condition of banks is

$$\rho V'(n) = (\log c - V'(n)c)' + \left(V'(n)\left[a\frac{n}{pM} + r_t n + \frac{n}{M}(\mu^p + \sigma\sigma^p - r_t)\right]\right)'$$
$$+ \frac{1}{2}\left[V''(n)(\sigma + \sigma^p)^2\frac{n^2}{M^2}\right]' + \xi'(n)\left[n - pkM\right] + \xi(n)\left[n - pMk(n)\right]'$$

$$= \left(\frac{1}{c}c'(n) - V'(n)c(n) - V''(n)c\right) + V''(n)\left[a\frac{n}{pM} + r_t n + \frac{n}{M}(\mu^p + \sigma\sigma^p - r_t)\right]$$
$$+V'(n)\left[r_t + \frac{1}{M}(a + \mu^p + \sigma\sigma^p - r_t)\right] + V''(n)\left[(\sigma + \sigma^p)^2\frac{n}{M^2}\right]$$
$$+\frac{1}{2}\left[V'''(n)(\sigma + \sigma^p)^2\frac{n^2}{M^2}\right].$$

The equality follows from the fact that the constraint is binding and when substituting for $k = \frac{n}{pM}$. Using the first order condition for consumption and expression for drift and volatility of the stochastic discount function we obtain the rewritten envelope condition

$$\rho - \frac{1}{M}(\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t) - r_t = \mu^\lambda + (\sigma + \sigma^p)\frac{\sigma^\lambda}{M}.$$

Therefore, the stochastic discount factor of intermediaries evolves as

$$\frac{d\lambda}{\lambda} = \left(\rho - r_t - \frac{1}{M}(\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t) - (\sigma + \sigma^p)\frac{\sigma^\lambda}{M}\right) dt + \sigma^\lambda dW_t$$

giving us the expression for the drift of the SDF

$$\mu^\lambda = \rho - r_t - \frac{1}{M}(\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t) - (\sigma + \sigma^p)\frac{\sigma^\lambda}{M}. \tag{1.45}$$

We can also rewrite the first order condition for capital presented in the main text; that is the bank's asset pricing equation

$$\lambda\left(a + p(\mu^p + \sigma\sigma^p - r_t)\right) + (\sigma + \sigma^p)\sigma^\lambda\lambda p - \xi pM = 0. \tag{1.46}$$

Then we calculate the bank's stochastic discount factor, which differs from households' exactly in the third term

$$\mu^\lambda = \rho - r_t - \frac{\xi}{\lambda}.$$

If banks were unconstrained, ξ would be equal to zero and we would have the unconstrained financial sector without regulators imposing the capital requirement constraint, as in Brunnermeier and Sannikov (2014). Here, ξ could be interpreted as a marginal cost of the financial regulation in terms of a unit of net worth.

Step 3

The first order condition with respect to consumption is the same as in the household's case, $c = \frac{1}{\lambda}$. Using Ito's lemma we obtain the same expressions for consumption growth and volatility as in the household's case

$$\mu^c = -\mu^\lambda + (\sigma^\lambda)^2, \quad \sigma^c = -\sigma^\lambda. \tag{1.47}$$

Step 4

We also know by Ito's lemma $c(n)$

$$\mu^c c = c'(n)[ak + r_t n + pk(\mu^p + \sigma\sigma^p - r_t) - c] + \frac{1}{2}c''(n)(\sigma + \sigma^p)^2 p^2 k^2, \tag{1.48}$$

$$\sigma^c c = c'(n)(\sigma + \sigma^p)pk = c'(n)(\sigma + \sigma^p)\frac{n}{M}. \tag{1.49}$$

Performing the same steps as in the households case, by matching consumption drifts using (1.48),(1.49),(1.47),(1.46), and (1.45) we obtain

$$r_t - \rho + \frac{1}{M}(\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t) + \frac{(\sigma + \sigma^p)}{M}\left(-\frac{c'(n)(\sigma + \sigma^p)\frac{n}{M}}{c(n)}\right) + \left(-\frac{c'(n)(\sigma + \sigma^p)\frac{n}{M}}{c(n)}\right)^2$$
$$= \frac{c'(n)[\frac{n}{M}(\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t) + r_t n - c(n)] + \frac{1}{2}c''(n)(\sigma + \sigma^p)^2\frac{n^2}{M^2}}{c(n)}.$$

Guessing a linear consumption rule $c(n) = An + F$ and substituting it in matching drifts we obtain $c(n) = \rho n$.

Step 5

Plugging back policy rules into the bank's HJB equation, the HJB equation becomes

$$\rho V(n) = \log(\rho n) + V'(n)n\left[\frac{1}{M}(\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t) + r_t - \rho\right] + \frac{1}{2}V''(n)\frac{n^2}{M^2}(\sigma + \sigma^p)^2.$$

We guess and verify the value function form $V(n) = B\log n + D$. Plugging back into the HJB we get $B = \frac{1}{\rho}$ and $D = \log\frac{1}{\rho} + \frac{1}{\rho^2}\left(r_t - \rho + \frac{1}{M}(\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t) - \frac{1}{2}\frac{(\sigma+\sigma^p)^2}{M^2}\right)$, which concludes the proof.

□

Proof of Proposition 4. Recall that banks' net worth evolves as

$$\frac{dn_t}{n_t} = r_t dt + \frac{p_t k_t}{n_t}(\frac{a}{p_t} + \mu_t^p + \sigma\sigma_t^p - r_t)d_t - \frac{c_t}{n_t}d_t + \frac{p_t k_t}{n_t}(\sigma + \sigma_t^p)dW_t.$$

We know from Proposition 2 that banks' optimal capital and consumption policy functions are $k_t = \dfrac{n_t}{p_t M_t}$ and $c_t = \rho n_t$, respectively, which gives us

$$\frac{dn_t}{n_t} = \left(r_t + \frac{1}{M_t}(\frac{a}{p_t} + \mu_t^p + \sigma\sigma_t^p - r_t) - \rho \right) d_t + \frac{1}{M_t}(\sigma + \sigma_t^p)dW_t.$$

Further, total capital in the economy evolves as

$$\frac{d(p_t K_t)}{p_t K_t} = (\mu_t^p + \sigma\sigma_t^p)d_t + (\sigma + \sigma_t^p)dW_t.$$

Using Ito's formula for law motion of ratio of two geometric Brownian motions [16] where $\eta_t = \dfrac{n_t}{p_t K_t}$, we obtain

$$\begin{aligned}\frac{d\eta_t}{\eta_t} &= \left(r_t + \frac{1}{M_t}(\frac{a}{p_t} + \mu_t^p + \sigma\sigma_t^p - r_t) - \rho - \mu_t^p - \sigma\sigma_t^p + (\sigma + \sigma_t^p)^2 - \frac{1}{M_t}(\sigma + \sigma_t^p)^2 \right) dt \\ &+ (\frac{1}{M_t} - 1)(\sigma + \sigma_t^p)dW_t \\ &= \left(\frac{1}{M_t}\frac{a}{p_t} - \rho + (\frac{1}{M_t} - 1)(\mu_t^p + \sigma\sigma_t^p - r_t - (\sigma + \sigma^p)^2) \right) d_t + (\frac{1}{M_t} - 1)(\sigma + \sigma_t^p)dW_t.\end{aligned}$$

□

Theorem 5. *The stationary wealth distribution $f(\eta_t)$ satisfies the Kolmogorov forward equation*

$$0 = -\frac{\partial}{\partial\eta}(\mu_t^\eta(\eta)\eta_t f(\eta_t)) + \frac{1}{2}\frac{\partial^2}{\partial\eta^2}((\sigma_t^\eta(\eta_t)\eta_t)^2 f(\eta_t)) \tag{1.50}$$

on a closed interval. We assume that a detailed balance condition holds, meaning that no probability can "escape" from the interval $[0, \eta^*]$. [17] *In particular, we assume that η_t is*

[16] If we have two GBMs $\dfrac{dX_t}{X_t} = \mu_t^x d_t + \sigma_t^x dW_t$ and $\dfrac{dY_t}{Y_t} = \mu_t^y d_t + \sigma_t^y dW_t$, then we have $\dfrac{dX_t/Y_t}{X_t/Y_t} = (\mu_t^x - \mu_t^y + (\sigma_t^y)^2 - \sigma_t^x\sigma_t^y)d_t + (\sigma_t^x - \sigma_t^y)dW_t$.

[17] This is a sufficient condition for the stationary wealth share distribution to exist. A detailed balance condition or reversibility property of the Markov chain is a sufficient but not a necessary condition in order to have a stationary distribution. Other types of boundary condition are absorbing, $f(0) = 0$, implying that some probability mass can leave the domain $[0, \eta^*]$. An intuitive interpretation can be explained as follows. Suppose we have a two-state Markov chain with two states of the world, a good state in which banks have enough capital and a bad state when bank capital is scarce $\{g, b\}$. Let π_g and π_b denote the probability of being in a good or bad state respectively (mass of banks that are sufficiently or insufficiently capitalized) and T_{gb} and T_{bg} transition probabilities from a good to a bad state and vice versa from a bad to a good state. Reversibility reads $\pi_g T_{gb} = \pi_b T_{bg}$, the mass of banks moving from sufficient to insufficient capital is equal to the mass of banks moving from insufficient to sufficient capital, probability inflow to a good state is equal to probability outflow from a good state, and both states are visited in equilibrium. Absorbing conditions would imply that in equilibrium we end up in a good or a bad state, i.e. the probability distribution is degenerate. Since in our case η is a continuous

equipped with reflecting boundary conditions, one at $\eta = 0$ *and the endogenous reflecting boundary* $\eta = \eta^*$*. Then the stationary distribution is equal to* $f(\eta) = C\frac{e^{2\int_0^\eta \frac{\mu^\eta(\eta')}{\sigma^\eta(\eta')^2\eta'}d\eta'}}{\sigma^\eta(\eta)^2\eta^2}$.

PROOF OF PROPOSITION 5. The stationary Kolmogorov forward equation (1.50) can be rewritten as

$$\frac{dJ}{d\eta} = 0,$$

where $J(\eta) = -\mu_t^\eta(\eta)\eta + \frac{1}{2}\frac{\partial}{\partial\eta}((\sigma_t^\eta(\eta)\eta)^2 f(\eta))$ denotes the probability flux or the probability current associated with the equation (1.50). Since the derivative of the probability current is equal to zero for all $\eta \in [0, \eta^*]$, this means that the current is constant at the same interval, that is

$$J(\eta) = const.$$

From the reflecting boundary conditions we have $J(0) = J(\eta^*) = 0$, and from the integration of the Kolmogorov forward equation we conclude that the probability current must be constant and equal to 0 because $J(0) = 0$. Consequently, we have $J(\eta) = 0$ for $\eta \in [0, \eta^*]$. Hence, the stationary Kolmogorov equation becomes

$$0 = -\mu_t^\eta(\eta)\eta + \frac{1}{2}\frac{\partial}{\partial\eta}((\sigma_t^\eta(\eta)\eta)^2 f(\eta)) \tag{1.51}$$

Integrating the equation (1.51), we obtain the closed form solution in the main text, where C is the normalization constant.[18] □

PROOF OF M_{KT}. We need to evaluate the following integral

$$\begin{aligned} M_{KT} &= \int_0^1 (ap^2 + bp^2 + c)(1 - e^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau + \Phi^{-1}(p)(\sigma+\sigma_t^p)\sqrt{\tau}})dp \\ &= \underbrace{\int_0^1 (ap^2 + bp^2 + c)dp}_{=1} - \int_0^1 (ap^2 + bp^2 + c)e^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau + \Phi^{-1}(p)(\sigma+\sigma_t^p)\sqrt{\tau}} dp \end{aligned}$$

state variable, detailed balance implies no discontinuities in probability and its first derivative.

[18]If the boundary conditions are absorbing then the diffusion process η_t would eventually be absorbed by the boundary points 0 and η^*. Consequently, the stationary distribution is 0 (degenerate). If the boundary conditions were periodic, this would imply $C_1 = -\mu_t^\eta(\eta)\eta + \frac{1}{2}\frac{\partial}{\partial\eta}((\sigma_t^\eta(\eta)\eta)^2 f(\eta))$ where the constant C_1 is determined from the periodic boundary conditions $f(0) = f(\eta^*), J(0) = J(\eta^*)$. Periodic boundary conditions are used to model small-system processes that are part of a large system that exhibits fixed periodicity. For instance, this would be the case if the periodicity of systemic crises is predetermined, and the financial sector constitutes a small part of an economy. These conditions could be possibly used in a large scale continuous time DSGE models.

$$= 1 - (I + II + III)$$

which can be separated into the sum of three integrals. We first introduce the change of variables $\Phi^{-1}(p) = x$, $p = \Phi(x)$, $dp = \phi(x)dx$ in order to solve these integrals. The third integral is equal to

$$III = c\mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau} \int_{-\infty}^{\infty} \mathrm{e}^{(\sigma+\sigma_t^p)\sqrt{\tau}x} \phi(x)dx$$
$$= c\mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2\tau + \frac{1}{2}(\sigma+\sigma_t^p)^2\tau} \Phi(x - (\sigma + \sigma^p)\sqrt{\tau})\Big|_{-\infty}^{+\infty}$$
$$= c\mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p)\tau}$$

, where we have used $\int \mathrm{e}^{nx}\phi(x)dx = \mathrm{e}^{\frac{n^2}{2}}\Phi(x-n)$[19] The second integral is equal to

$$II = b\mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau} \int_{-\infty}^{\infty} \mathrm{e}^{(\sigma+\sigma_t^p)\sqrt{\tau}} \phi(x)\Phi(x)dx.$$

Introducing the integration by parts $u = \Phi(x)$ $du = \phi(x)dx$, and $dv = \mathrm{e}^{(\sigma+\sigma_t^p)\sqrt{\tau}x}\phi(x)dx$, $v = \mathrm{e}^{(\sigma+\sigma_t^p)^2\tau/2}\Phi(x - (\sigma + \sigma^p)\sqrt{\tau})$, we have

$$II = b\mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau + \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau} \left(\Phi(x)\Phi(x - (\sigma + \sigma^p)\tau)\Big|_{-\infty}^{+\infty} - \int_{-\infty}^{\infty} \phi(x)\Phi(x - (\sigma + \sigma^p)\sqrt{\tau})dx\right)$$
$$= b\mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p)\tau}(1 - 1 + \Phi(\frac{(\sigma + \sigma^p)\sqrt{\tau}}{\sqrt{2}})) = \mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p)\tau}\Phi(\frac{(\sigma + \sigma^p)\sqrt{\tau}}{\sqrt{2}}) \qquad ,$$

where we have used

$$\int_{-\infty}^{+\infty} \Phi(x) \cdot \phi(m + sx)dx = \int_{-\infty}^{+\infty} \Phi(\frac{x-m}{s}) \cdot \phi(x)\frac{dx}{s}$$
$$= \frac{1}{s}\left(1 - \Phi(\frac{m}{\sqrt{1+s^2}})\right).$$

Analogously, using the integration by parts $u = \Phi^2(x)$ $du = 2\phi(x)\Phi(x)dx$, and $dv = \mathrm{e}^{(\sigma+\sigma_t^p)\sqrt{\tau}x}\phi(x)dx$, $v = \mathrm{e}^{(\sigma+\sigma_t^p)^2\tau/2}\Phi(x - (\sigma + \sigma^p)\sqrt{\tau})$ and using

$$\int_{-\infty}^{+\infty} \Phi(x)^2 \cdot \phi(m + sx)dx = \int_{-\infty}^{+\infty} \Phi^2(\frac{x-m}{s}) \cdot \phi(x)\frac{dx}{s}$$
$$= \frac{1}{s}\left(1 - \Phi(\frac{m}{\sqrt{1+s^2}})\right) - \frac{2}{s}T\left(\frac{m}{\sqrt{1+s^2}}, \frac{s}{\sqrt{2+s^2}}\right),$$

[19]See wikipedia for a list of the integrals of Gaussian functions.

we find the third integral to be equal to

$$III = a\mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p)\tau}\left(\Phi(\frac{(\sigma+\sigma^p)\sqrt{\tau}}{\sqrt{2}}) - T(\frac{(\sigma+\sigma^p)\sqrt{\tau}}{\sqrt{2}}, \frac{1}{\sqrt{3}}\right),$$

which concludes the proof. $\square$

Chapter 2

Macroeconomics with Financial Sector Risk Constraints

2.1 Introduction

What are the macroeconomic implications of market risk measures in the Basel accords? At its core, the 2008 financial crisis emphasized that the widespread failures and losses of financial institutions and risk management policies can have severe macroeconomic consequences. According to Kashyap et al. (2008) and Mizen (2008), losses on mortgage-backed securities of investment banks propagated through the interbank lending market, causing a credit crunch and spillovers to the broader economy. Estimates suggest a remarkable 25% of output was lost to the 2008 financial crisis (Laeven and Valencia 2010). As indicated by Duffie (2019), most relevant authorities agree that the then existing regulation and supervision allowed large intermediaries to have insufficient capital and liquidity compared to the risk they held on their balance sheet. That said, the importance of analyzing the implications of risk management policies on the banking sector and broader economy has never been greater.

In this paper, we endeavor to bridge the gap between economic micro-founded theory and the actual regulations by utilizing a statistical measure of market risk. The questions we address are: What are the implications of the Basel financial regulation on leverage and interest rates? Should regulators provide deposit insurance and protect lenders? How should the optimal deposit insurance be implemented from a welfare perspective if banks are subject to VaR capital requirements? Answering the last question is crucial since most emerging economies have no explicit deposit insurance scheme, and our analysis could provide useful guidance in that direction. Therefore, the overreaching goals of the paper are twofold. From a positive perspective, one goal is to analyze the macroeconomic implications of capital requirements implemented as a Value at Risk risk measure. From a normative perspective, the goal is to develop the optimal deposit insurance from the welfare point of view, maximizing the representative households' expected utility and accounting for financial regulation.

To answer the questions we identified earlier, we consider the two-period version of Gertler and Kiyotaki (2010)'s model of government credit policies in the recent financial crisis. However, the nature of financial frictions differ. More specifically, in Gertler and Kiyotaki (2010) bankers exhibit a moral hazard behavior and can divert a fraction of funds, which potentially constrains the intermediaries' ability to obtain funds from depositors. In our model, we focus on ex-ante risk-based capital requirements implemented as Value at Risk (henceforth VaR). Regulators can perform two roles in our economy. First, they impose the minimum capital requirement constraint on banks, which is always binding. Second, regulators can act as deposit insurance providers, setting insurance fees, and repaying depositors in the case of bank insolvency.

The fundamental purpose of capital requirements is to quantify the downside market risk arising from banks holding risky assets on their balance sheets. When seen in isolation, capital requirements are designed to limit the probability of large market losses up to a small acceptable threshold. Before the Great Recession, the Basel accord used VaR to require financial institutions to meet capital requirements to cover the market risk they incur due to their daily portfolio adjustments. From the early 90s, VaR has become the standard measure of market risk which answers the question of what the maximum market loss is within a specified confidence interval. VaR was developed to provide senior management with a single number that can incorporate information about portfolio risk. VaR may penalize diversification, and it neglects tail losses that may hide behind the threshold. In response to these shortcomings, the post-crisis measure of risk, Expected Shortfall, measures average losses above the VaR threshold.

Among the papers close to this one in terms of the statistical tail-behavior of financial institutions' asset returns in equilibrium are Adrian and Brunnermeier (2011), who propose CoVaR as an optimal systemic risk measure, and Acharya (2009), who recommends the systemic expected shortfall. However, our emphasis is on market risk measures rather than systemic risk measures. Unlike our policy-oriented objective, most risk management literature is concerned with assessing the limitations of VaR from investors' perspectives. For example, extensive research recommends Expected Shortfall, distortion, and spectral risk measures to set the optimal portfolio instead of VaR(Artzner et al. 1999; Wang 2000; Acerbi and Tasche 2002; Acerbi 2002). A notable exception is Basak and Shapiro (2001), who also consider the expected utility maximization with a VaR constraint in an equilibrium setting. Similarly to Basak and Shapiro (2001), we investigate the effect of VaR constraint on risk-taking incentives of financial institutions or risk managers. Unlike their model, we focus on the implications of VaR on deposit insurance instead of stock market volatility.

Our paper is complementary to papers focused on capital regulation and its incorporation into macroeconomic models. Capital requirements have gained popularity in the aftermath of the crisis, as the severity of the crisis opened the debate on the efficiency of prudential instruments (Freixas, Laeven, and Peydró 2015). Most of the literature on prudential policy and policymakers has focused on countercyclical capital requirements. Currently, Basel III uses the credit-to-GDP gap as an early warning indicator of financial vulnerability and for setting countercyclical capital buffers (Committee et al. 2010). From a positive perspective, the benefits of countercyclical requirements have been analyzed more recently in Drehmann et al. (2010), Repullo and Suarez (2012), and Repullo and Suarez (2012). For instance, Drehmann and Gambacorta (2012) find that countercyclical buffers can limit credit procyclicality. In contrast, Repullo and Saurina Salas (2011) argue against the proposal of using the credit-to-GDP gap and recommend GDP growth as a guide for setting capital requirements instead. Whether capital requirements are procyclical or countercyclical is determined endogenously in our model.

We depart in several ways from recent contributions that study prudential policies from a positive and normative perspective. As an important example, Collard et al. (2017) jointly derive optimal monetary and prudential policies, setting the interest rate and capital requirements. Distinct from Collard et al. (2017), we focus on optimal deposit insurance while the equilibrium interest rate is determined by deposit supply and demand instead of monetary policy enforcing the interest rate policy rule. Repullo and Suarez (2012) study the implications of capital requirements embedded in Basel I, II, and III on credit supply and welfare. Their analysis of the impact of Basel II capital requirements is analogous to our analysis of the implication of VaR capital regulation. Unlike our model, Repullo and Suarez (2012) only consider the case with fixed deposit insurance and allow voluntary capital buffers. The second assumption means that banks may choose to hold capital above the minimum regulatory requirements to in order to anticipate future funding difficulties and raise equity capital in the future. In this way, Repullo and Suarez (2012) also capture the banks' precautionary motives independent of the regulators' precautionary motives. Moreover, Repullo and Suarez (2012) abstract from demand-side feedback effects. Our model is simple enough to incorporate depositors' demand and insurance at the expense of omitting the richer firm-bank relationship dynamics.[1]

This paper contributes to the literature on capital regulation in two ways. First, we show that VaR is an important driver of procyclical leverage in the banking sector. Specifically, banks increase leverage when favorable market conditions prevail, when the expected return is high, and volatility is low. Vice versa, banks decrease leverage when markets are characterized by high volatility and low return. In this way, the VaR measure with a fixed confidence level generates procyclical leverage of the financial sector, which is demonstrated empirically by Adrian and Shin (2010). Moreover, procyclicality of capital requirements translates into procyclical interest rates, leading to high values of the interest rate in good times and low values in bad times.

Second, we find that when banks are subject to capital regulation, optimal deposit insurance changes with market conditions. The insurance level depends on the riskiness and return of the bank's asset side of the balance sheet. In effect, we find that risk-based capital requirements, such as VaR, lead to optimal risk-sensitive deposit insurance. When monetary authority or regulators act as a deposit insurance provider, capital ratios and the interest rate are higher and more procyclical.

The remainder of the chapter is organized as follows. Section 2.2 presents the benchmark macro model without financial regulation. Section 2.3 gives a quick overview of risk measures incorporated in preceding and current financial regulations, specifically VaR and Expected Shortfall. Section 2.4 presents a macro model with bank risk measurement under the VaR constraint. Section 2.5 derives optimal deposit insurance when banks are subject to VaR capital requirements. Section 2.6 concludes. Mathematical proofs are provided in Appendix 2.A.1.

[1] The agency problem between banks and firms in Repullo and Suarez (2012) is a relevant factor for bank capital structure and regulation.

2.2 Macro model without financial regulation

In this subsection, we briefly present a simplified two-period version of Gertler and Kiyotaki (2010). Then, we introduce a market risk and VaR capital requirement constraint to manage risk and examine how regulation affects the equilibrium interest rate and leverage.

The models in Chapters 1 and 2 are similar but not identical. Both models have two agents, unconstrained households (savers) and banks (borrowers) with imposed risk-based capital requirement constraints. In Chapter 1, the model is a simplified version of Brunnermeier and Sannikov (2014)'s model, while in Chapter 2, we use a two-period version of Gertler and Kiyotaki (2010)'s model. Both models feature endogenous leverage, bank risk taking and the interest rate. Chapter 1 focuses on studying the implications of risk-based capital requirements on welfare, crisis probability, prices, and price volatility, all of which arise endogenously. The model in this chapter abstracts from asset prices and price volatility, while the default probability is exogenously given. The goal is to derive optimal deposit insurance when banks are subject to VaR capital requirements.

2.2.1 Household

The representative household consists of two types of members: bankers and savers. Agents maximize their utility or profit, and optimal consumption and saving choices are derived. There are two periods in the model. In the first period, savers are endowed with y units of the consumption good, which they allocate between deposits and consumption. The first-period budget constraint is equal to

$$c + d \leq y, \tag{2.1}$$

where d is deposit level household supply to the bank, and c is household consumption in the first period. In the second period, households consume their income, which is equal to the sum of gross return on deposits they invested in the first period, and the profits bankers bring to the household. The second-period budget constraint is

$$C \leq R^d d + \pi, \tag{2.2}$$

where R^d and π are the interest rate on deposits and the profit brought by bankers, respectively. The household chooses d, taking R^d and π as given, in order to maximize its lifetime utility

$$u(c) + \beta u(C) \tag{2.3}$$

subject to (2.1) and (2.2). The first order condition with respect to d gives the standard Euler intertemporal substitution equation

$$u'(c) = \beta R^d u'(C),$$

The Euler equation identifies that a higher interest rate R^d implies a higher willingness of a household to substitute towards tomorrow's consumption. Since the utility function is increasing in consumption, the second-period budget constraint is binding. Substituting for deposits d from the second- into a first-period budget constraint, we obtain the intertemporal budget constraint

$$c + \frac{C}{R^d} \leq y + \frac{\pi}{R^d}. \tag{2.4}$$

The left-hand side of (2.4) is the present discounted value of household consumption, while the right-hand side denotes the present discounted value of a household's income. Let the household preferences be given by CRRA utility with relative risk aversion γ

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

Then, the Euler optimality condition becomes

$$c^{-\gamma} = \beta R^d C^{-\gamma}. \tag{2.5}$$

Substituting for C from (2.5) into (2.4) we obtain optimal consumption

$$c = \frac{y + \frac{\pi}{R^d}}{\frac{(\beta R^d)^{\frac{1}{\gamma}}}{R^d} + 1}.$$

We can see that the households choose first-period consumption as a fraction of income, which is a decreasing function of the interest rate paid on deposits for $\gamma \leq 1$. Substituting

c into (2.4), we obtain household's deposit supply

$$d = y - c = y - \frac{y + \dfrac{\pi}{R^d}}{\dfrac{(\beta R^d)^{\frac{1}{\gamma}}}{R^d} + 1}.$$

2.2.2 Firms

There is a continuum of competitive firms in the economy. Unlike savers and bankers, firms have no endowment and need to issue security s in the first period to finance the purchase of goods in order to produce capital, with one-to-one technology

$$s = k. \tag{2.6}$$

In the second period, firms produce goods from capital using a linear production function

$$f(k) = R^k k, \tag{2.7}$$

where R^k is a gross return on capital which is certain and equal to μ. Notice that we implicitly assume that the firm production function has constant returns to scale. Since firms are competitive, they earn no profit, so the return on a security is equal to that on capital.

2.2.3 Banks

Banks play two roles in the benchmark model: they give credit to firms and provide liquidity in the form of deposits to households. In the model with regulation, banks also offer insurance against market risk. In the first period, bankers are endowed with N units of equity. We assume that they cannot issue additional equity but can only combine deposits with existing equity to finance asset purchases from firms

$$s = N + d^b. \tag{2.8}$$

At this point, we distinguish between a household's supply of deposits d and a bank's demand for deposits d^b. In equilibrium, due to the market clearing condition, the two will be the same. As seen from (2.8), in the first period, the bank's portfolio consists of riskless security on the asset side and equity and deposits on the liability side. In the second period, banks earn a profit

$$\pi = R^k s - R^d d^b. \tag{2.9}$$

Bankers are risk neutral and aim to maximize profit by choosing s and taking R^d as given.

2.2.4 Equilibrium without regulation

Benchmark equilibrium : The equilibrium consists of values of c, C, R^d, d, d^b, and R^d such that

1. the household's maximization problem is solved,
2. the bank's maximization problem is solved,
3. the market for deposits clears, $d = d^b$,
4. the market for securities clears, i.e. (2.6) holds.

Here, we consider only interior equilibrium, in which $c, C, d, d^b > 0$. For interior equilibrium to exist we must have that

$$R^d = R^k \equiv \mu. \tag{2.10}$$

This can easily be seen from the bank's problem. If $R^d < \mu$, then the bank would want to borrow an infinite amount, $d^b = \infty$, which exceeds the household endowment in the first period. Similarly, if $R^d > \mu$, banks would set deposits equal to zero, which violates the interior equilibrium requirement. To solve for the equilibrium value of deposits, we impose $d = d^b$ and substitute profits in (2.9) into the household's supply of deposit (2.2.1)

$$d = y - c = y - \frac{y + \frac{\pi}{R^d}}{\frac{(\beta R^d)^{\frac{1}{\gamma}}}{R^d} + 1} = y - \frac{y + \frac{\mu s - R^d d}{R^d}}{\frac{(\beta R^d)^{\frac{1}{\gamma}}}{R^d} + 1}$$

$$= y - \frac{y + \frac{\mu(N+d) - R^d d}{R^d}}{\frac{(\beta R^d)^{\frac{1}{\gamma}}}{R^d} + 1} = \frac{y(\beta R^d)^{\frac{1}{\gamma}} - \mu N - d(\mu - R^d)}{(\beta R^d)^{\frac{1}{\gamma}} + R^d}.$$

Solving for d we get

$$d = \left(1 + \frac{\mu - R^d}{(\beta R^d)^{\frac{1}{\gamma}} + R^d}\right)^{-1} \frac{y(\beta R^d)^{\frac{1}{\gamma}} - \mu N}{(\beta R^d)^{\frac{1}{\gamma}} + R^d} = \frac{y(\beta R^d)^{\frac{1}{\gamma}} - \mu N}{(\beta R^d)^{\frac{1}{\gamma}} + \mu}. \tag{2.11}$$

Substituting (2.11) into the first-period budget constraint (2.1) and the second-period constraint in (2.2), we obtain the household's consumption in the first period

$$c = y - d = y - \frac{y(\beta R^d)^{\frac{1}{\gamma}} - \mu N}{(\beta R^d)^{\frac{1}{\gamma}} + \mu} = \frac{\mu(y + N)}{(\beta R^d)^{\frac{1}{\gamma}} + \mu}, \tag{2.12}$$

and consumption in the second period

$$C = R^d d + \pi = R^d d + R^k s - R^d d = R^k(N + d) = \mu \frac{(\beta R^d)^{\frac{1}{\gamma}}(y + N)}{(\beta R^d)^{\frac{1}{\gamma}} + \mu}. \tag{2.13}$$

The benchmark equilibrium is defined by equations (2.10)-(2.13). In the following section, we will see how optimal choices are modified in the presence of a VaR constraint.

2.3 VaR and Expected Shortfall as risk measures

Recent accords of Basel II and III have adopted different measures of market risk. These are designed to quantify the portfolio risk of an uncertain financial position based on its downside risk potential. A popular risk measure, VaR is based on a quantile concept. VaR quantifies the worst market losses that can be expected with a small probability. From the shareholders' or managements' perspective, VaR at the company level is a meaningful measure of risk since the default event itself is of primary concern, and the size of a shortfall is only secondary. On the other hand, Expected Shortfall measures average losses exceeding the VaR limit, which is the average shortfall.

We can calculate both measures as follows. Let X be a random variable that represents profit-loss distribution or market gains or losses of a portfolio, and let confidence level $\alpha \in (0, 1)$. VaR is the largest loss that can occur with the confidence level no smaller than α

$$VaR_\alpha(X) = \min\{x| \quad F_X(x) \geq \alpha\} = q_\alpha(X),$$

where $F_X(x)$ is the cumulative distribution function of X. In simple mathematical terms, VaR is α-quantile, $q_\alpha(X)$. Note that $VaR(X)$ gives the size of losses that can occur with a probability no greater than $1 - \alpha$. On the other hand, Expected Shortfall with a confidence level α quantifies losses exceeding the $\alpha-$quantile. Therefore, $ES_\alpha(X)$ is defined by Artzner et al. (1999) as

$$ES_\alpha(X) = \frac{1}{\alpha}\int_0^\alpha VaR_p(X)dp = E[X|X \leq VaR_\alpha(X)].$$

VaR has often been criticized for violating subadditivity. This property reflects the notion that individual risks typically diversify (or, at least, do not increase) when investors combine risky positions into a portfolio. The VaR of a portfolio can be larger than the sum of VaRs of individual portfolio positions when violating subadditivity. In response to this deficiency, Artzner et al. (1999) proposes *coherent* risk measures, and four axioms they should satisfy. VaR is not coherent since it may discourage diversification, while Expected Shortfall is a coherent risk measure for any confidence level.[2] In the following section, we incorporate the above definitions of two market risk measures in the equilibrium model with market risk and financial regulation.

2.4 Macro model with VaR regulation

How does VaR affect the equilibrium interest rate and leverage? This is the central question we strive to answer in this section. Thus, we introduce financial regulation that changes the banks' and households' optimization problems. The return on capital investment is now uncertain, and banks use a VaR constraint to manage market risk. One can think of the risky security as granting direct loans to private firm borrowers, but there is a default risk that the borrower will fail to honor his loan obligations. We derive a constrained equilibrium and study the implications of financial regulation on the interest rate and leverage.

2.4.1 Banks

The risky security is traded in the first period in anticipation of its realized return in the second period. Let us make a simplifying assumption that return on capital, R^k, is uniformly distributed over the interval

$$[\mu - \sigma, \mu + \sigma],$$

[2]See Acerbi and Tasche (2002) for the proof.

where both μ and σ are known by the household and the bank. The mean and the variance of R^k are $E(R^k) = \mu, Var(R^k) = \frac{\sigma^2}{3}$. For a given confidence level α, VaR of the risky asset R^k is such that

$$Prob(R^k \leq VaR_{1-\alpha}) = 1 - \alpha \tag{2.14}$$

holds. This gives us the expression for the VaR quantile

$$VaR_{1-\alpha} = F^{-1}(1 - \alpha)$$

where $F^{-1}(\cdot)$ is the inverse of the cumulative distribution function of the return on capital R^k. With a uniformly distributed market return, $VaR_{1-\alpha}$ can easily be computed from

$$\int_{\mu-\sigma}^{VaR_{1-\alpha}} \frac{1}{2\sigma} dR^k = 1 - \alpha$$

$$VaR_{1-\alpha} = \mu + (1 - 2\alpha)\sigma$$

and graphically represented in Figure 2.1.

Now let us consider how the VaR capital requirements enter the bank's maximization

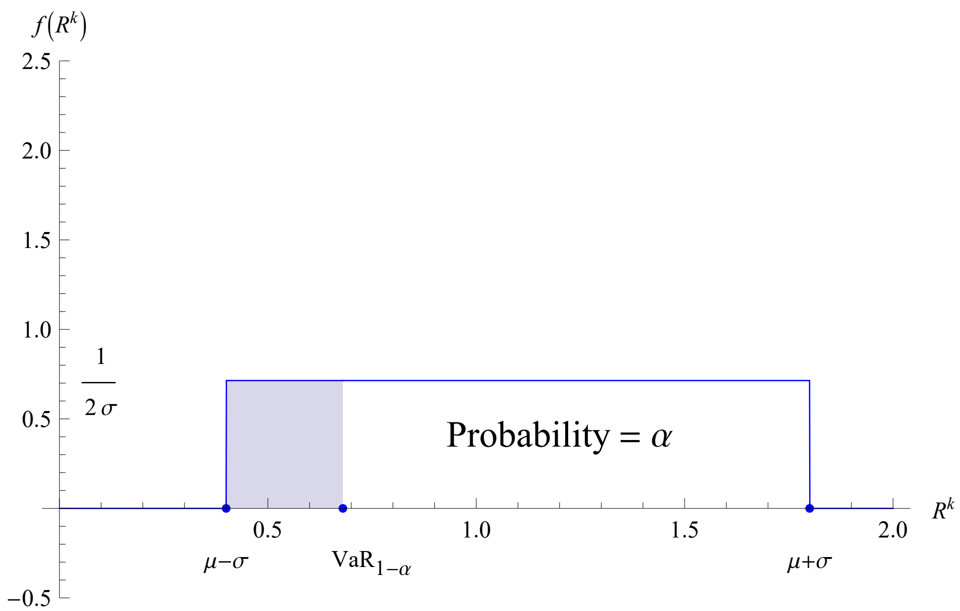


Figure 2.1: VaR of uniformly distributed return on capital

problem. Uncertainty about a project outcome injects risk into the bank's balance sheet. Banks maximize the expected profit subject to the constraint that the insolvency probability in the second period is kept at an acceptable level $1-\alpha$. Since the bank becomes insolvent if equity falls below zero, the constraint reads

$$Prob(R^k s - R^d(s-N) \leq 0) \leq 1-\alpha, \quad \text{or equivalently}$$

$$Prob(R^k \leq R^d(1-\frac{N}{s})) \leq 1-\alpha. \tag{2.15}$$

By choosing the size of s, banks operate at a probability of default of at most $1-\alpha$. Comparing the right-hand side terms in (2.14) and (2.15) inside the probability brackets we have that the default occurs if the realized return is below the $VaR_{1-\alpha}$ threshold. Therefore, the VaR constraint becomes

$$1-\frac{N}{s} \leq \frac{VaR_{1-\alpha}}{R^d}, \tag{2.16}$$

or after rearrangement

$$s - s\frac{VaR_{1-\alpha}}{R^d} \leq N. \tag{2.17}$$

Under VaR capital requirements, current equity must be sufficient to absorb the worst possible future loss. The VaR constraint forces banks to keep an equity-to-loans ratio of at least $1 - VaR_{1-\alpha}/R^d$.

In other words, the bank fails when $R^k s < R^d d$, in which case the bank does not pay $R^d d^b$. The reason is that if the bank defaults in the second period and does not meet its promised obligations to depositors, households will demand a higher payment in the non-default state. In the default state, which occurs with probability $1-\alpha$, the bank would pay $(R^k - D)$ to households per unit of capital, where D is the default cost. Therefore, expected total payment would be $(1-\alpha)\big(E(R^k|R^k < VaR_{1-\alpha}) - D\big)\, s$. We assume that default costs are exactly equal to the expected return on capital below the VaR threshold, so the bank's expected default costs offset profit. If the realized return is above the threshold, the bank pays $R^d d^b$ to the household and earns a profit of $sR^k - R^d d^b$.

In sum, the bank maximizes expected profit subject to the constraint

$$\begin{aligned} \max_s \quad & \alpha E(R^k|R^k \geq VaR_{1-\alpha})s - \alpha R^d(s-N) \\ \text{s.t.} \quad & s - sVaR_{1-\alpha}/R^d \leq N. \end{aligned} \tag{2.18}$$

The Lagrangian representation of the bank's problem is

$$\max_{s} \quad \alpha E(R^k|R^k \geq VaR_{1-\alpha})s - \alpha R^d(s-N) + \lambda(sVaR_{1-\alpha}/R^d + N - s).$$

The first order conditions are:

$$\begin{aligned}
[s]: &\quad \alpha(\mu + (1-\alpha)\sigma) - \alpha R^d + \lambda VaR_{1-\alpha}/R^d - \lambda = 0 \\
[\lambda]: &\quad \lambda(sVaR_{1-\alpha}/R^d + N - s) = 0,
\end{aligned}$$

where λ is the Lagrange multiplier associated with the constraint, and we have used a conditional expectation of return above the VaR threshold, where $E(R^k|R^k \geq VaR_{1-\alpha}) = \int_{\mu+(1-2\alpha)\sigma}^{\mu+\sigma} \frac{1}{2\sigma} R^k dR^k/(1-(1-\alpha)) = (\mu + (1-\alpha)\sigma)$. If the constraint is not binding, this implies that $\lambda = 0$. Substituting $\lambda = 0$ into $[s]$: equation, we obtain $R^d = \mu+(1-\alpha)\sigma > \mu$. But this would mean that the bank would earn no profit on deposits, and would be as well off as purchasing only amount N of securities. Since we are interested in cases when deposit demand is positive, the constraint is binding when $\lambda > 0$. As a result, profit maximization leads banks to choose the largest value of risky security holdings allowed by the VaR constraint

$$s = \frac{N}{1 - \dfrac{VaR_{1-\alpha}}{R^d}}. \tag{2.19}$$

The demand for deposits is given by

$$d^b = s - N = N\frac{VaR_{1-\alpha}}{R^d - VaR_{1-\alpha}}. \tag{2.20}$$

2.4.2 Household

Imposing the VaR constraint on bankers, households know that banks operate on the probability of default and that they obtain the return on deposits only in the non-default state. As before, bank default occurs when $R^k s < R^d d$. In the case of default, with probability $1-\alpha$, the household receives $\left(E(R^k - D|R^k < VaR_{1-\alpha})\right)s$, plus a deposit insurance payout $I\left(R^d - \left(E(R^k - D|R^k < VaR_{1-\alpha})\right)\right)s$, where I is the insurance compensation paid to households per unit of deposits. This section assumes that banks are not protected by deposit insurance ($I = 0$), which we relax in section 2.5.1. Since costly default exactly offsets the expected return below the VaR threshold, the households maximization problem is

$$\begin{aligned}
\max_{c,C^{ND}} \quad & u(c) + \beta\alpha E(u(C^{ND})|\text{not default}) \\
\text{s.t.} \quad & c + d = y \\
\text{s.t.} \quad & C^{ND} = R^d d + \pi(R^k).
\end{aligned} \tag{2.21}$$

We can rewrite the optimization problem in terms of deposits as

$$\max_d \quad \frac{(y-d)^{1-\gamma}}{1-\gamma} + \beta \int_{\mu+(1-2\alpha)\sigma}^{\mu+\sigma} \frac{(R^d d + \pi)^{1-\gamma}}{1-\gamma} \frac{1}{2\sigma} dR^k.$$

Differentiating with respect to d we get the first-order condition

$$-(y-d)^{-\gamma} + \frac{\beta}{2\sigma} R^d \int_{\mu+(1-2\alpha)\sigma}^{\mu+\sigma} (R^d d + \pi)^{-\gamma} dR^k = 0.$$

Since in equilibrium profit transfered to the household is $R^d d + \pi = R^k(N+d)$, we have

$$(y-d)^{-\gamma} = \frac{\beta}{2\sigma} R^d (N+d)^{-\gamma} \left(\frac{(\mu+\sigma)^{1-\gamma}}{1-\gamma} - \frac{(\mu+(1-2\alpha)\sigma)^{1-\gamma}}{1-\gamma} \right).$$

Denoting by $B = \frac{(\mu+\sigma)^{1-\gamma}}{1-\gamma} - \frac{(\mu+(1-2\alpha)\sigma)^{1-\gamma}}{1-\gamma}$, the household's deposit supply is

$$d = \frac{\left(\frac{BR^d\beta}{2\sigma}\right)^{\frac{1}{\gamma}} y - N}{1 + \left(\frac{BR^d\beta}{2\sigma}\right)^{\frac{1}{\gamma}}}, \tag{2.22}$$

first-period consumption

$$c = y - d = \frac{y+N}{1 + \left(\frac{BR^d\beta}{2\sigma}\right)^{\frac{1}{\gamma}}}, \tag{2.23}$$

and the total purchase of securities

$$s = N + d = \frac{(y+N)\left(\frac{BR^d\beta}{2\sigma}\right)^{\frac{1}{\gamma}}}{1 + \left(\frac{BR^d\beta}{2\sigma}\right)^{\frac{1}{\gamma}}}. \tag{2.24}$$

Equilibrium with VaR : The equilibrium consists of values of c, C, R^d, d, d^b, and R^d such that

1. the household's maximization problem is solved,
2. the bank's maximization problem is solved,
3. the market for deposits clears, $d = d^b$,
4. the market for securities clears, i.e. (2.8) holds.

Equilibrium can be summarized by equations (2.22)-(2.24) and (2.20). When banks are constrained or unconstrained, the first-period consumption is a fraction of the bank and the saver's total endowment. The financial regulation affects this fraction through B. We can think of B as per unit expected marginal utility of non-defaulted risky security. The higher the confidence level of α is, the higher is the utility.

2.4.3 Equilibrium with VaR regulation

In this section, we discuss the theoretical and numerical properties and implications of equilibrium with the VaR constraint. Table 2.1 describes our baseline parameters of the model. The discount rate β is set to 0.95, and we normalize the non-banking sector's size by y to 1. For the parameters pertaining to the banking sector, we follow Repullo and Suarez (2012). According to Repullo and Suarez (2012), an average *Total interest income* of all US commercial banks was 5.74% of *Earning assets*, *Total interest expense* was 2.32% of *Total liabilities*, and *Service charges on deposit accounts* were 0.55% in the pre-crisis years 2004-2007. This implies the intermediation margin of 3.97% on deposit-funded activities.

Therefore, we set μ to 6.29%, while α and σ and σ are set such that the equilibrium interest rate is 2.32%, an average intermediation margin is 3.97% and the loss given default parameter(LGD) is 0.45. The loss given default determines the loss of the loans of projects that fail, which is set according to the Basel II Internal Ratings-Based (IRB) foundation approach for unsecured corporate exposures. This calibration leads to a system of equations in N, α, and σ, which has a unique solution with positive parameter values. Solving for these parameters yields the steady-state default probability of 5.83% and the steady-state capital requirement of 40.1% of total assets.

Table 2.1: Baseline parameter values

μ	σ	α	β	y	N
1.0629	0.5094	0.9417	0.95	1	0.2435

Equilibrium characteristics are summarized in the following propositions, and their proofs are presented in Appendix 2.A.1.

Proposition 1. *The interest rate on deposits is increasing in μ and decreasing in σ.*

Proposition 2. *Leverage of the banking sector is procyclical.*

Proposition 3. *Leverage of the banking sector is decreasing in volatility.*

How does the interest rate vary with fundamentals, volatility, bank equity or default probability? If we assume log-preferences, the equilibrium interest rate is obtained by equating deposit demand (2.22) and supply (2.20)

$$R^d = \frac{2\sigma N + \beta B(\mu + (1-2\alpha)\sigma)(y+N)}{\beta B y}, \tag{2.25}$$

where $B = \log(\mu+\sigma) - \log(\mu + (1-2\alpha)\sigma)$. Without regulation the interest rate increases in a one-to-one fashion with respect to return on capital since $R^d = \mu$. With a VaR constraint the interest rate depends on μ nonlinearly. The interest rate is high when fundamentals are strong. This procyclicality is easily seen from

$$\frac{dR^d}{d\mu} = \frac{\dfrac{4\alpha\sigma^2 N}{(\mu+\sigma)(\mu+(1-2\alpha)\sigma)} + \beta(N+y)B^2}{\beta y B^2} > 0,$$

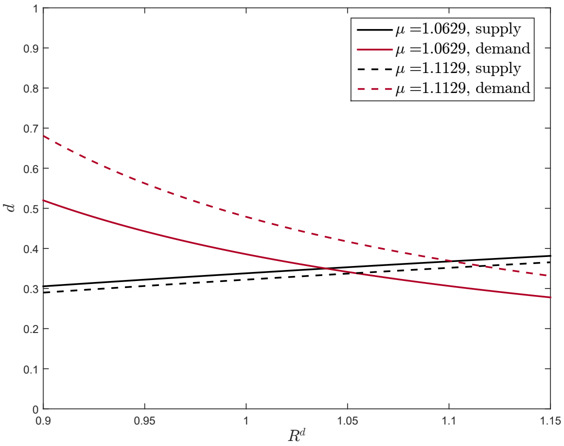
and Figure 2.2a and Figure 2.2b when we increase expected return and volatility by 5 p.p.

When expected return rises, banks are eager to capture their share of the pie by offering more loans to firms and increasing deposit demand. Although households reduce deposit supply, the resulting borrowing costs and leverage rise because of higher demand elasticity. Leverage is defined as the ratio of total assets to equity

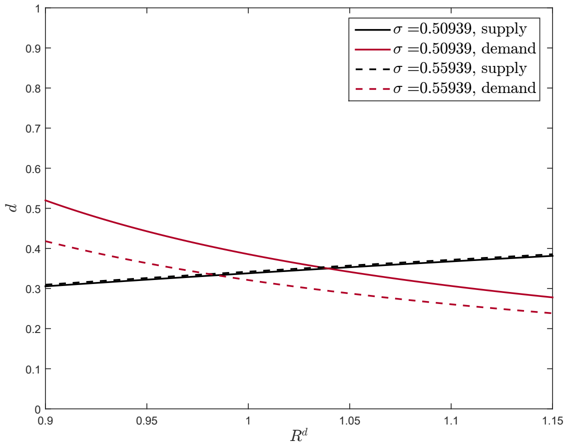
$$L = \frac{s}{N} = \frac{R^d}{R^d - VaR_{1-\alpha}}. \tag{2.26}$$

From Appendix 2.A.1 we know that the sign of changes in bank leverage with respect to changes in the expected return on capital $\frac{dL}{d\mu}$ depends on the sign of the following

Figure 2.2: Equilibrium interest rate with VaR



(a) increase in expected return



(b) increase in volatility

expression

$$R^d - VaR_{1-\alpha}\frac{dR^d}{d\mu}.$$

Substituting for R^d and $\frac{dR^d}{d\mu}$ and rearranging, the sign of $\frac{dL}{d\mu}$ is positive and proportional to

$$\frac{2\sigma N((\mu+\sigma)B - 2\alpha\sigma)}{\beta B^2(\mu+\sigma)y} > 0.$$

Under VaR regulation leverage rises as the expected return on capital increases. In contrast, in Gertler and Kiyotaki (2010), leverage is countercyclical. The empirical importance of VaR as a driver of procyclicality of leverage has been emphasized by Adrian and Shin (2010).

The opposite behavior of banks and households is observed when volatility rises. Banks curtail deposit demand, while the household's deposit supply is almost unchanged or slightly higher. From Appendix 2.A.1, the derivative of leverage with respect to volatility, $\frac{dL}{d\sigma}$ depends on the sign of

$$(1-2\alpha)R^d - VaR_{1-\alpha}\frac{dR^d}{d\sigma} = \frac{-2\mu N((\mu+\sigma)B - 2\alpha\sigma)}{\beta B^2(\mu+\sigma)y} < 0,$$

which is negative. To summarize, under VaR regulation, banks accumulate risk in the form of higher leverage in periods of sustained growth and in peaceful times. Under calibration in Table 2.1, leverage rises by 0.537 % in response to a 1% increase in expected return, and declines by 1.12 % when volatility rises by 1% when households have log preferences. Our results highlight procyclical risk-taking incentives of financial institutions that are subject to the VaR constraint. Related to this finding, Basak and Shapiro (2001) show that risky asset holdings exhibit peculiar behavior when investors maximize utility and quantify market risk using VaR. Investors can have both procyclical and countercyclical risk incentives. Three regions arise endogenously: good, medium, and bad states of the economy, depending on the state price density (Sharpe ratio $\frac{\mu-R^d}{\sigma}$). In good (low price density) and bad (high price density) states, investors' behavior is procyclical. In the middle region, it is countercyclical. In Basak and Shapiro (2001), the interest rate is exogenous (constant), while market volatility arises endogenously. In contrast, our model features exogenous volatility and endogenous interest rate. These two differences might explain the arising of countercyclical risk incentives in Basak and Shapiro (2001).

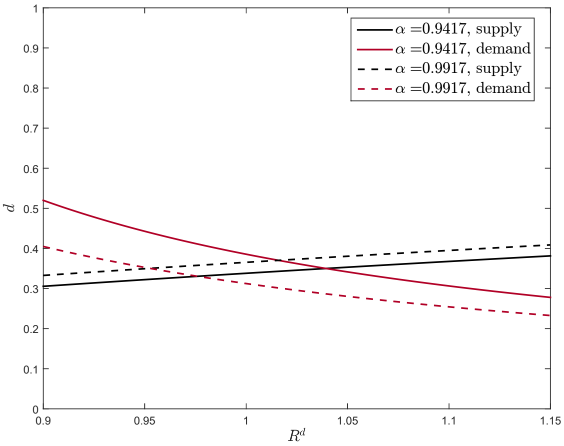
Another interesting property of the equilibrium interest rate is that it is increasing in the relative wealth of bankers

$$\frac{dR^d}{d(\frac{N}{y})} = \frac{2\sigma + \beta B(\mu + (1-2\alpha)\sigma)}{\beta B} > 0.$$

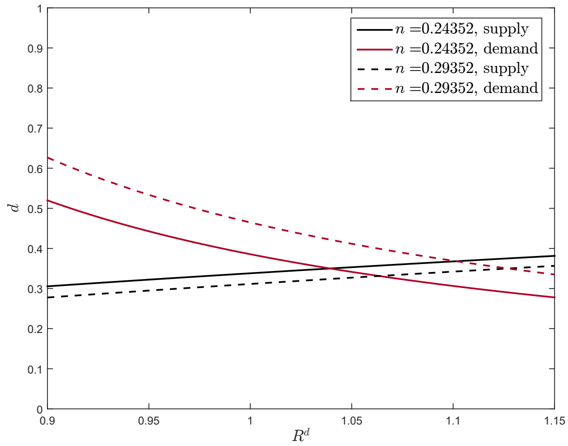
Figure 2.3b captures this finding when equity increases by 5 p.p. Holding everything else fixed, the more " skin in the game" the banker has, the higher is the cost of borrowing. The prediction of the external financing costs rising with the strength of the borrower's balance sheet is opposite to the financial accelerator prediction in Bernanke, Gertler, and Gilchrist (1999). In Gertler and Kiyotaki (2010), the financial accelerator emerges in equilibrium when bankers have incentives to divert funds. With VaR, higher equity conveys banks' incentives to engage in more risk taking, which induces households to reduce deposit supply and borrowing costs rise accordingly.

Would safer banks have lower leverage and borrowing costs? In the aftermath of the 2008 crisis, Gennaioli and Shleifer (2018) note that the two most important factors contributing to the crisis were flawed financial sector regulation and supervision and underestimated downside tail risks. In particular, they ascribe inaccurate beliefs about downside risk as the main contributor and suggest that investors and regulators assigned unjustifiably low probabilities to disastrous outcomes in the housing market. Figure 2.3a summarizes the effect of the default probability. If regulators allow for smaller default probability and impose a higher confidence level, such a policy would reduce banks' demand for risky security and increase households' supply, because banks are more resilient. As a result of more elastic demand adjustments, banks would become less lever-

Figure 2.3: Equilibrium interest rate with VaR



(a) increase in confidence level



(b) increase in bank capital

aged. Moreover, when banks operate at a lower probability of default, their borrowing costs decline, as illustrated in Figure 2.3c. Intuitively, safer banks have lower risk-taking incentives, which produces downward adjustments in borrowing rates. This finding complements the loan pricing implications of Basel regulation in Repullo and Suarez (2004), who show that banks charge a loan rate that is increasing in the probability of default when banks are subject to risk-based capital requirements. In our model, the interest rate is endogenous, while the loan rate is exogenous. In Repullo and Suarez (2004), loan rates are endogenous while bank intermediation costs are fixed. Interestingly, both bank borrowing and lending rates seem to reflect the riskiness of the bank balance sheet. Overall, we find that changes in fundamentals, uncertainty, and insolvency probability have opposite effects on households' and bankers' choices of deposit.

2.5 Deposit insurance

As we have shown in the previous subsection, VaR can amplify the risk-taking channel because it generates procyclical leverage. Moreover, so far we have assumed that capital requirements are the only tool available to manage borrowing and lending incentives and that households are not protected by deposit insurance if the bank defaults. Thus, we ask the question: what is the welfare-maximizing deposit insurance if banks are subject to VaR capital requirements? To answer this question, we allow for deposit insurance in subsection 2.5.1. Specifically, we find optimal deposit insurance when, as before, the VaR confidence level is fixed or predetermined by regulators. Then, we compare the interest rates and capital requirements in two regimes: with and without deposit insurance. In Diamond and Dybvig (1983) type models, deposit insurance is introduced as a government policy that prevents bank runs. However, we abstract from the "panic view" that prescribes financial crises to agents' tendency to withdraw deposits when everybody else does the same. Instead, in our model, bank crises arise due to the deterioration of market conditions and low realization of risky bank assets. The purpose of optimal deposit insurance is to provide consumption to the risk-averse depositors in the state in which banks default, comparable to government bail-outs.

2.5.1 Optimal deposit insurance

In this subsection, we derive the optimal deposit insurance policy if the government (monetary or prudential authority) acts as a deposit insurance provider. As in previous subsections, regulators implement $VaR_{1-\alpha}$ capital requirements. In this regime, households pay the insurance fee in the first period, and they are compensated only in the case when bank's return falls below the $VaR_{1-\alpha}$ threshold. Such transfers, that are conditional on a low return performance, proxy for unconventional monetary policies such as equity injection. Related to our goal of finding the optimal deposit insurance, Gertler and Kiyotaki (2010) evaluate the effectiveness of government ex-ante equity injections in alleviating financial distress. Unlike their model, we may interpret deposit insurance as ex-post equity injections. Without equity injections or insurance, bankers absorb losses from variations in fundamentals or volatility. With deposit protection or equity injection, however, losses are shared with households. As we will see, unlike equity injections, the exact amount of insurance coverage depends on how regulators measure risk.

We therefore solve two optimization problems. First, the social planner chooses the deposit insurance, which we denote by M. Second, the households' optimization problem is modified when deposits are insured, while banks are still subject to a binding VaR constraint as in previous subsections. The social planner solves the following problem

$$\max_{M} \quad \log(y-d-dM)+\frac{\beta}{2\sigma}\int_{\mu+(1-2\alpha)\sigma}^{\mu+\sigma}\log(R^k(N+d))dR^k+\frac{\beta}{2\sigma}\int_{\mu-\sigma}^{\mu+(1-2\alpha)\sigma}\log(\frac{dM}{1-\alpha})dR^k. \tag{2.27}$$

The first term represents households' first-period utility after paying the insurance fee. The second term is the expected utility in the case when the return on a risky asset is higher than the $VaR_{1-\alpha}$ threshold, and the third term is the expected utility of insurance coverage when bankers earn a return lower than this threshold, i.e. when the bank defaults. The optimization problem can be simplified to

$$\begin{aligned}\max_{M} \quad & \log(y-d-dM)+\beta(1-\alpha)\log(\frac{dM}{1-\alpha})+\beta\alpha\log(N+d) \\ +\beta & \frac{-2\alpha\sigma+(\mu+\sigma)\log(\mu+\sigma)-(\mu+(1-2\alpha)\sigma)\log(\mu+(1-2\alpha)\sigma)}{2\sigma}\end{aligned}$$

The first order condition reads

$$[M]: \quad -d(y-d-dM)^{-1}+\beta(1-\alpha)d(dM)^{-1}=0.$$

We further solve the households' optimization problem. In the first period, an insurance fee T is levied on households, and $\frac{T}{1-\alpha}$ is paid in the second period if the return falls below the $VaR_{1-\alpha}$ threshold. Therefore, the households solve the following maximization problem

$$\max_{d} \quad \log(y-d-T)+\frac{\beta}{2\sigma}\int_{\mu+(1-2\alpha)\sigma}^{\mu+\sigma}\log(R^d d+\pi)dR^k+\frac{\beta}{2\sigma}\int_{\mu-\sigma}^{\mu+(1-2\alpha)\sigma}\log(\frac{T}{1-\alpha})dR^k.$$

Since households cannot choose the deposit insurance fee, the last term does not affect supply of deposits. The first order condition gives

$$[d]: \quad -(y-d-T)^{-1}+\frac{\beta}{2\sigma}R^d\int_{\mu+(1-2\alpha)\sigma}^{\mu+\sigma}(R^d d+\pi)^{-1}=0.$$

In equilibrium, insurance coverage is financed by insurance fees,i.e., $T=dM$. The budget constraint of the government holds because $dM=(1-\alpha)\dfrac{dM}{1-\alpha}$. As before, the resource constraint implies $R^d d+\pi=R^k s$. The deposit supply is equal to

$$d=\frac{\frac{\beta}{2\sigma}BR^d y-N}{1+\frac{\beta}{2\sigma}BR^d(1+M)},$$

where, as before, $B=\log(\mu+\sigma)-\log(\mu+(1-2\alpha)\sigma)$. Because banks still adhere to VaR prudential regulation, deposit demand is still constrained by $VaR_{1-\alpha}$

$$d^b=N\frac{\mu+(1-2\alpha)\sigma}{R^d-(\mu+(1-2\alpha)\sigma)}.$$

From the market clearing condition for deposits, $d = d^b$, we obtain the equilibrium interest rate

$$R^d = \frac{\frac{\beta}{2\sigma}B(\mu + (1-2\alpha)\sigma)(y+N) + \frac{\beta}{2\sigma}B(\mu + (1-2\alpha)\sigma)MN + N}{\frac{\beta}{2\sigma}By},$$

and the equilibrium level of deposits

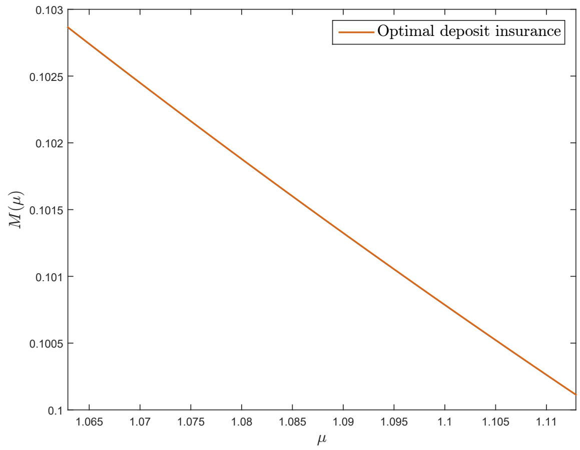
$$d = \frac{\frac{\beta}{2\sigma}B(\mu + (1-2\alpha)\sigma)y}{1 + \frac{\beta}{2\sigma}B(\mu + (1-2\alpha)\sigma)(1+M)}.$$

Substituting the equilibrium amount of deposits into the social planner's first order condition and solving for M, we obtain

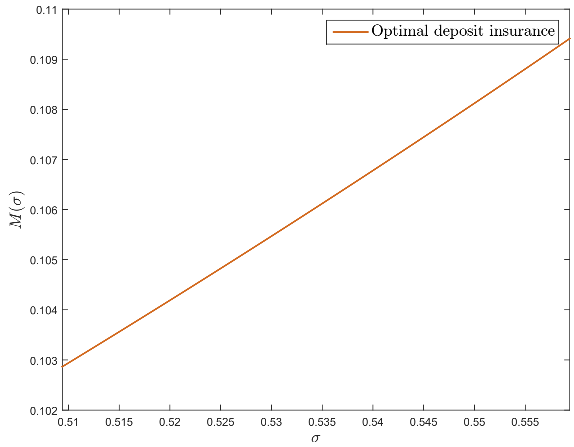
$$M = \frac{2\sigma(1-\alpha)}{B(\mu + (1-2\alpha)\sigma)}. \tag{2.28}$$

Our extension of the model with deposit insurance, while simplified, provides several useful insights. For example, we find that the optimal deposit insurance is inversely proportional to the VaR threshold and the expected marginal benefit of the non-defaulted asset, B. As seen from (2.28), variations in fundamentals, uncertainty, and default probability affect the optimal deposit insurance. Figure 2.4a shows that optimal deposit insurance decreases in response to a shock that boosts the risky asset return. Similarly, Figure 2.4b demonstrates that deposit insurance is an increasing function of market volatility. We find the comparative statics experiment with respect to α useful in the context of normative analysis of deposit insurance premiums. From a normative standpoint, Acharya et al. (2010) emphasize that the deposit insurance premium charged to banks should increase with banks' default risk. In our model, $1-\alpha$ proxies for default risk because below $VaR_{1-\alpha}$ threshold, banks do not pay out deposits to households, but households instead receive deposit insurance compensation. Although households insure against bank failure, we find that the deposit insurance fee increases with $1-\alpha$. Figure 2.4c confirms this prediction since as we move from right to left and decrease α (increase $1-\alpha$), the deposit insurance fee rises. All three figures imply that if banks are subject to capital regulation, optimal deposit insurance is not fixed but rather risk-sensitive as it changes with market conditions. Therefore, the insurance level is intrinsically tied to the riskiness and return of the bank's asset side of the balance sheet. In other words,

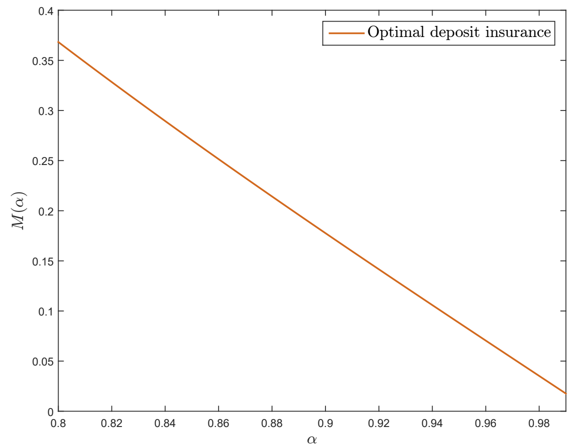
Figure 2.4: Optimal deposit insurance as a function of market return, volatility



(a) increase in expected return



(b) increase in volatility



(c) increase in α

risk-based capital requirements, such as VaR, require risk-sensitive deposit insurance.

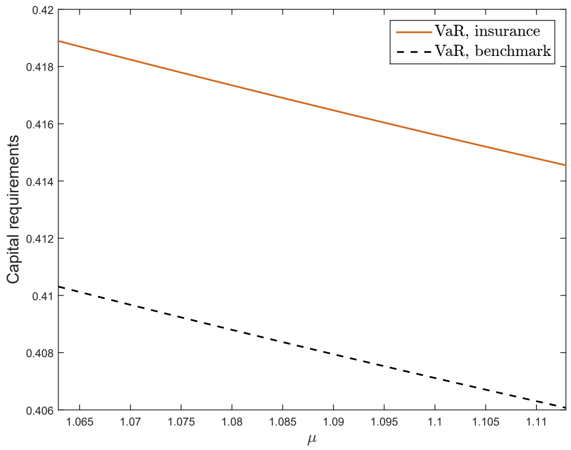
Allen, Carletti, and Leonello (2011) argue that the existing risk-insensitive deposit insurance scheme is inadequate because it assumes that financial crises and financial instability are panic-based events. In the panic view of Diamond and Dybvig (1983), bank runs or bank failures are multiple equilibrium phenomena and emerge as a consequence of coordination failure because individual agents find it optimal to withdraw deposits if they expect others to also withdraw their money early. Deposit insurance prevents bank runs by acting as an equilibrium selection device, and providing it is always costless and optimal. Further, Allen, Carletti, and Leonello (2011) emphasize that if financial crises are linked to deterioration in asset values, optimal deposit insurance might be risk-sensitive, but providing deposit insurance can be costly. In our model, the government compensates households if the risky asset return falls below the $VaR_{1-\alpha}$ threshold, so deposit insurance is intrinsically tied to asset deterioration by construction. In this case, as we have shown, the optimal deposit insurance is risk-sensitive.

2.5.2 Deposit insurance : capital requirements and interest rates

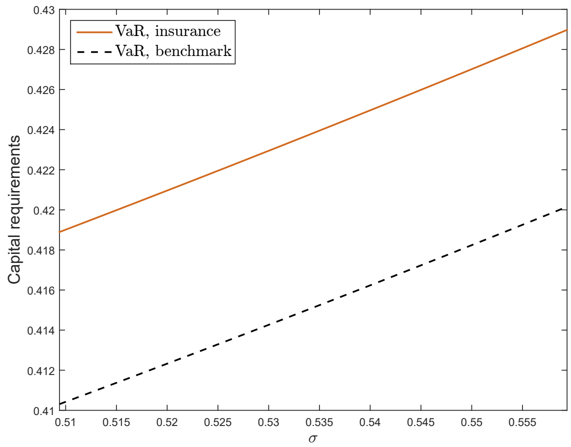
In this subsection, we compare capital requirements and interest rate under two regimes: benchmark VaR with fixed α, which captures pre-crisis Basel II regulation, and VaR with optimal deposit insurance with fixed α. Figure 2.5a and 2.5b show that the social planner chooses the higher level of capital requirements when deposit insurance is chosen optimally. As a result, banks invest less in a risky asset if the government protects lenders. This finding sheds light on the literature investigating whether the introduction of deposit insurance exacerbates bank the risk-taking channel. For example, DeLong and Saunders (2011) find evidence of risk-shifting incentives of publicly traded banks and trust after introducing fixed-rate deposit insurance in 1933. A few decades earlier, Keeley (1990) argued that fixed-rate deposit insurance poses a moral hazard for excessive risk taking. Our results show that when the deposit insurance is set optimally and depends on market conditions, the moral hazard problem does not arise. Crucially, households' deposit compensation changes with fundamentals, volatility, and default probability, unlike fixed-rate insurance, which is invariable to market conditions. From the perspective of market discipline, depositors seem to be more aware and concerned with banks' riskiness when deposit insurance is available.

Moreover, we find that capital ratios remain procyclical in both regimes. Procyclicality means that regulators impose lower capital ratios on banks in favorable market environments when the expected return is high and uncertainty is low, as shown in Figures 2.5a and 2.5b. To further investigate the effect of deposit insurance on the cyclicality of capital ratios, we plot the difference in capital ratios between two regimes $CR_{DI} - CR$ in Figure 2.5c and 2.5d. Figure 2.5c demonstrates that deposit insurance leads to more procyclical capital requirements when the expected return varies. This is evident in decreasing $CR_{DI} - CR$, which implies a lower slope of the orange graph than the black graph in Figure 2.5a. When fundamentals drive credit availability, deposit insurance amplifies economic contractions and expansions compared to no insurance. The same conclusion holds for volatility. When the credit cycle is driven by higher market volatility, optimal

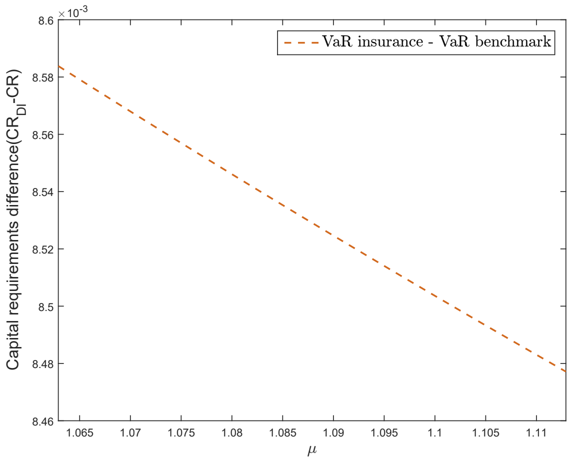
Figure 2.5: VaR capital ratios in two regimes



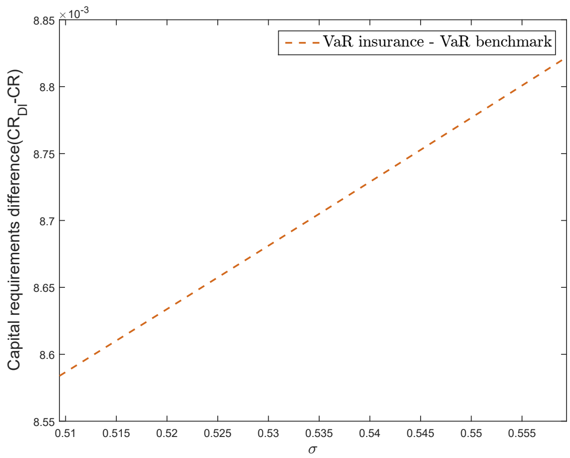
(a) increase in expected return



(b) increase in volatility



(c) increase in expected return



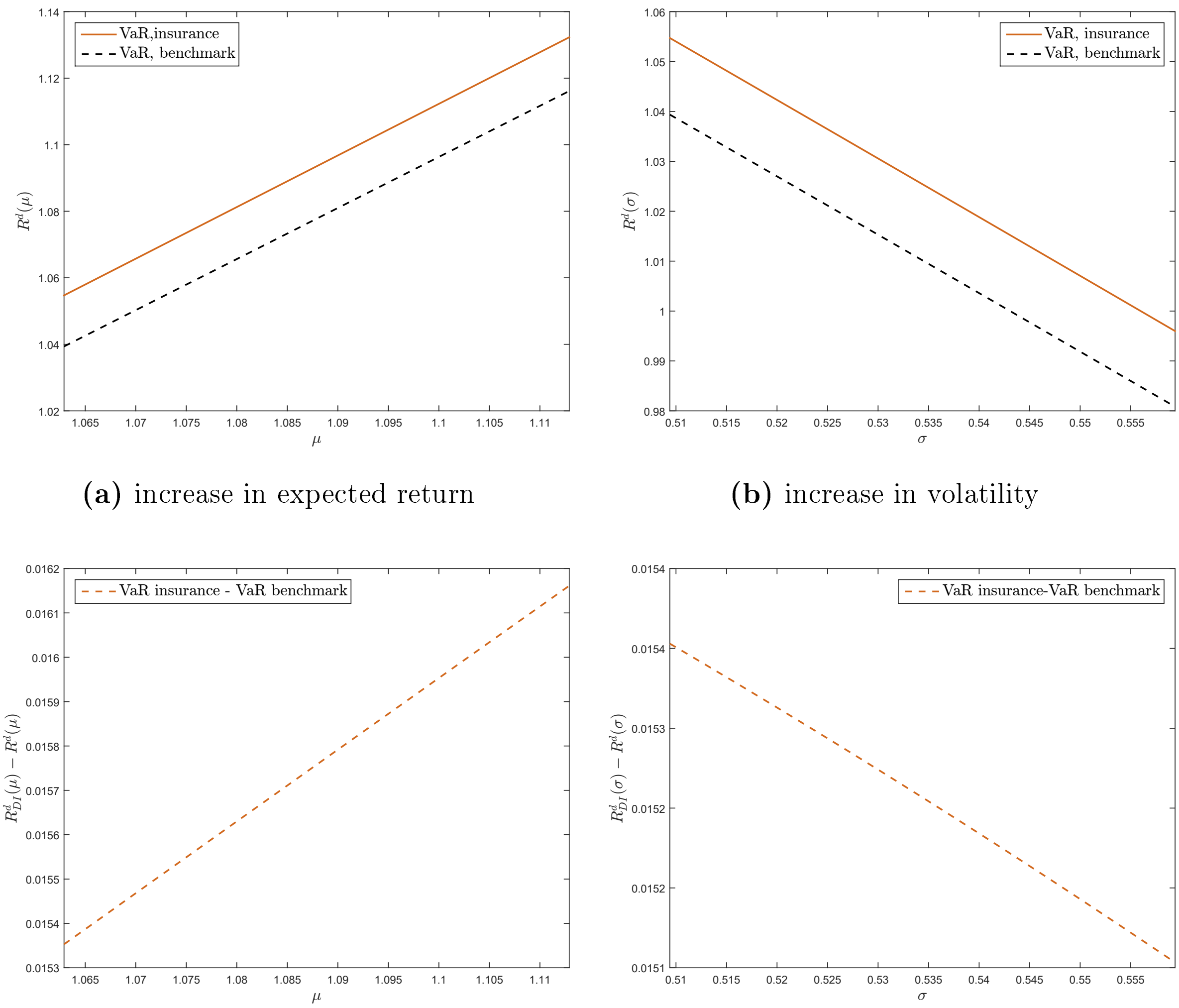
(d) increase in volatility

deposit insurance exacerbate credit fluctuations because $CR_{DI} - CR$ is increasing in Figure 2.5d.

Similar results apply for the interest rate. Figures 2.6a and 2.6b show that deposit insurance leads to higher borrowing costs. Compared to the benchmark VaR, deposit insurance seems to internalize the inefficiency of over-investments by increasing borrowing costs. This leads to tighter funding conditions for banks with a social planner, higher capital ratios, and fewer investments. By raising capital ratios, the social planner taxes debt to reduce risky asset investments. Meanwhile, the interest rate remains procyclical and increasing in μ and decreasing in σ. As we did for capital ratios, we plot the difference in the interest rate between two regimes $R^d_{DI} - R^d$ in Figure 2.6c and 2.6d. Both figures imply that the interest rate becomes more procyclical or sensitive to variations in expected return and volatility compared to the benchmark VaR.

Our result that optimal deposit insurance implies higher and more procyclical cap-

Figure 2.6: Equilibrium interest rate in two regimes



(a) increase in expected return

(b) increase in volatility

(c) increase in expected return

(d) increase in volatility

ital ratios sheds light on prudential policy discussions. While regulators and academics agree that capital requirements should be higher than the pre-crisis levels in the Basel II regulation, the debate regarding their cyclicality is unresolved. Predominately, the macroprudential perspective advocates reducing the procyclicality of bank capital ratios and leverage.[3] In their view, credit procyclicality is often seen as the leading indicator of

[3]As reported by Repullo and Suarez (2012), in the 2008 financial crisis the G20 Washington Summit called for policy recommendations that mitigate procyclicality of bank capital, leverage, and executive compensations.

the financial institutions' fragility, systemic risk, and credit crunch (Freixas, Laeven, and Peydró 2015). Related to our result, Repullo and Suarez (2012) show that if depositors are insured and there are high social costs of bank failure, optimal capital requirements are higher and less procyclical than implied by Basel II. Although we compute optimal deposit insurance instead of capital requirements, paying optimal insurance on rare events directly alters banks' incentives to invest in the risky asset and capital ratios. The difference in optimal procyclicality may be related to the effects of demand versus supply. Our model incorporates depositors' demand (endogenous interest rate) at the expense of omitting the richer firm-bank relationship dynamics. However, Repullo and Suarez (2012) abstract from the demand side, while the agency problem between banks and firms is a relevant factor for bank capital regulation because it determines default probabilities on loans.

To summarize the main implications of the two regimes, we find that the social planners' and competitive equilibria differ in two key respects: the level and procyclicality of risky capital investments and borrowing costs. When deposit insurance is available, optimal VaR leads to lower capital ratios, more investments, and cheaper borrowing than in the benchmark VaR. Conversely, when prudential or monetary authority provides optimal deposit insurance, this increases borrowing costs, tightens capital buffers, and reduces investments.

Before concluding, let us outline the crucial limitations of our model. The most important is that we abstract from endogenous systemic risk since $1-\alpha$ proxies for the exogenous systemic risk(default probability is fixed to $1-\alpha$ because of $VaR_{1-\alpha}$ capital requirements). In reality, when realized market losses are higher than the $VaR_{1-\alpha}$ threshold and erode bank equity, banks can adjust to capital regulation either by raising new equity or selling the risky assets. If the individual bank chooses to liquidate the risky asset, it can cause its price to plummet and lead to a fire sales spiral, which may cause the default of other banks. In this case, the endogenous default probability is higher than $1-\alpha$. If, instead, banks can raise equity when market losses are higher than the $VaR_{1-\alpha}$ threshold, the endogenous default probability would be lower than $1-\alpha$. In other words, in our model, market risk and credit risk are both equal to $1-\alpha$, while by endogenizing systemic risk (credit risk of the financial system), credit and market risk may not necessarily coincide. Conditional on the availability of deposit insurance and given the default probability, we have offered risk-sensitive optimal deposit insurance. Accounting for the endogeneity of systemic risk and testing for the robustness of optimal deposit insurance is essential before regulators implement such policies.

2.6 Conclusion

Risk-based capital requirements are a crucial component of prudential policy design that has received limited attention in the literature. In the first part of the paper, we consider a simple macro model with the financial sector subject to VaR capital requirements. We show that VaR is an important driver of procyclical leverage in the banking sector. Specifically, banks increase leverage when favorable market conditions prevail, when the expected return is high, and volatility is low. Vice versa, banks decrease leverage when markets are characterized by high volatility and low return.

In the second part of the paper, we compute the optimal deposit insurance by maximizing welfare conditional on VaR capital regulation. When banks are subject to capital regulation, optimal deposit insurance is not fixed (risk-insensitive) but instead changes with market conditions. The insurance level depends on the riskiness and return of the bank's asset side of the balance sheet. In effect, risk-based capital requirements, such as VaR, require risk-sensitive deposit insurance. When monetary authority or regulators act as a deposit insurance provider, capital ratios and the interest rate are higher and more procyclical than those without insurance.

Some extensions of the simple model are worthy of pursuit in the future. First, since capital regulation affects the financial institutions' risk-taking channel, the interaction of conventional monetary and prudential policy can be analyzed. Extending the model with the risk-taking channel of monetary policy could be a promising direction. Second, our model abstracts from systemic risk, a prevailing rationale for regulating the banking sector in the first place. Finding an optimal market risk measure that maximizes welfare while minimizing systemic risk may be a way forward in prudential policy design.

2.A Appendix

2.A.1 Omitted proofs

In this appendix, we present proofs omitted from the main text.

Proof of Proposition 1. Let us define H as residual demand using deposit supply(2.20), demand (2.22), and market clearing condition $d = d^b$

$$H \equiv N\frac{VaR_{1-\alpha}}{R^d - VaR_{1-\alpha}} - \frac{\left(\frac{BR^d\beta}{2\sigma}\right)^{\frac{1}{\gamma}} y - N}{1 + \left(\frac{BR^d\beta}{2\sigma}\right)^{\frac{1}{\gamma}}} = 0.$$

For simplicity, we will prove the proposition for log-preferences. To prove that R^d is increasing in μ, we use the implicit function theorem for $H(R^d, \mu)$

$$\frac{dR^d}{d\mu} = -\frac{\frac{\partial H}{\partial \mu}}{\frac{\partial H}{\partial R^d}}.$$

$$\frac{\partial H}{\partial \mu} = N\frac{VaR_\mu(1-\alpha)(R^d - VaR_{1-\alpha}) + VaR_\mu(1-\alpha)VaR_{1-\alpha}}{(R^d - VaR_{1-\alpha})^2}$$

$$+\frac{2\beta\sigma R^d y}{(\mu+\sigma)(\mu+(1-2\alpha)\sigma)(2\sigma+\beta BR^d)} + \frac{2\beta\sigma R^d(2\sigma N - \beta BR^d y)}{(\mu+\sigma)(\mu+(1-2\alpha)\sigma)(2\sigma+\beta BR^d)^2}$$

$$= \frac{R^d NVaR_\mu(1-\alpha)}{(R^d - VaR_{1-\alpha})^2} + \frac{4\alpha\beta\sigma^2 R^d(N+y)}{(\mu+\sigma)(\mu+(1-2\alpha)\sigma)(2\sigma+\beta BR^d)^2}.$$

Similarly,

$$\frac{\partial H}{\partial R^d} = -\frac{N(VaR_{1-\alpha})}{(R^d - VaR_{1-\alpha})^2} - \frac{\beta By}{\beta BR^d + 2\sigma} + \frac{\beta B(\beta BR^d y - 2\sigma N)}{(\beta BR^d + 2\sigma)^2}$$

$$= \frac{-NVaR_{1-\alpha}}{(R^d - VaR_{1-\alpha})^2} - \frac{2\sigma\beta B(N+y)}{(\beta BR^d + 2\sigma)^2} < 0.$$

In order to prove that $\frac{dR^d}{d\mu} > 0$, we need the sign of $VaR_\mu(1-\alpha)$, which denotes for the derivative of the VaR with confidence α with respect to μ. From the main text, we know that $VaR_{1-\alpha} = \mu + (1-2\alpha)\sigma$. Therefore, $VaR_\mu(1-\alpha) = 1$ and $\dfrac{\partial H}{\partial R^d} > 0$, and altogether implies $\dfrac{dR^d}{d\mu} > 0$. □

Proof of Proposition 2. Leverage of the banking sector is defined by (2.26). Then we have

$$\frac{dL}{d\mu} = \frac{\frac{dR^d}{d\mu}(R^d - VaR_{1-\alpha}) - R^d\left(\frac{dR^d}{d\mu} - \frac{dVaR_{1-\alpha}}{d\mu}\right)}{(R^d - VaR_{1-\alpha})^2}$$

$$= \frac{R^d - VaR_{1-\alpha}\frac{dR^d}{d\mu}}{(R^d - VaR_{1-\alpha})^2}.$$

From the proof of Proposition 1, we can calculate $\dfrac{dR^d}{d\mu}$, which yields

$$\frac{dR^d}{d\mu} = \frac{\frac{4\alpha\sigma^2 N}{(\mu+\sigma)(\mu+(1-2\alpha)\sigma)} + \beta(N+y)B^2}{\beta y B^2}.$$

We obtain

$$R^d - VaR_{1-\alpha}\frac{dR^d}{d\mu} = \frac{2\sigma N + \beta B(\mu + (1-2\alpha)\sigma)(y+N)}{\beta B y}$$
$$-(\mu + (1-2\alpha)\sigma)\frac{\frac{4\alpha\sigma^2 N}{(\mu+\sigma)(\mu+(1-2\alpha)\sigma)} + \beta(N+y)B^2}{\beta y B^2}$$
$$= \frac{2\sigma N((\mu+\sigma)B - 2\alpha\sigma)}{\beta B^2(\mu+\sigma)y} > 0,$$

which implies that leverage is an increasing function of the expected return μ. □

Proof of Proposition 3. Analogously to the proof of Proposition 2, using the expression for leverage defined by (2.26), we have

$$\frac{dL}{d\sigma} = \frac{\frac{dR^d}{d\sigma}(R^d - VaR_{1-\alpha}) - R^d\left(\frac{dR^d}{d\sigma} - \frac{dVaR_{1-\alpha}}{d\sigma}\right)}{(R^d - VaR_{1-\alpha})^2}$$

$$= \frac{R^d VaR_\sigma(1-\alpha) - VaR_{1-\alpha}\frac{dR^d}{d\sigma}}{(R^d - VaR_{1-\alpha})^2},$$

where $VaR_\sigma(1-\alpha)$ denotes the derivative VaR with respect to σ. This gives $VaR_\sigma(1-$

$\alpha) = \dfrac{d(\mu + (1-2\alpha)\sigma)}{d\sigma} = 1 - 2\alpha$. What is left to calculate is $\dfrac{dR^d}{d\sigma}$ which is equal to

$$\frac{dR^d}{d\sigma} = \frac{\left(\dfrac{-2\alpha\mu(2N\sigma + \beta(\mu + (1-2\alpha)\sigma)(N+y)B)}{(\mu+\sigma)(\mu + (1-2\alpha)\sigma)} + (2N + \dfrac{2\alpha\beta\mu(N+y)}{\mu+\sigma} + (1-2\alpha)(N+y)B)B\right)}{\beta y B^2}.$$

Finally, we obtain

$$(1-2\alpha)R^d - VaR_{1-\alpha}\frac{dR^d}{d\sigma} = \frac{-2\mu N((\mu+\sigma)B - 2\alpha\sigma)}{\beta B^2(\mu+\sigma)y} < 0.$$

This concludes the proof that leverage is a decreasing function of the volatility σ. $\square$

Chapter 3

The Employment Effects of Corporate Tax Shocks: New Evidence and Some Theory

Jointly with

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3.1 Introduction

The US Administration's 2017 tax reform reduced the rate for companies from 35% to 21%. Many expect that reducing the tax burden on firms will spur job growth by boosting economic activity. Whether such expectations are accurate is central to the design of fiscal policy, yet the subject has received surprisingly scant attention in the academic literature. Moreover, in an environment where very low interest rates constrain monetary policy, it is all the more important to understand the transmission of such measures and to gauge their effectiveness. Therefore, in this paper, we seek to estimate and model the impact of a fiscal stimulus package, in the form of corporate tax cuts, on employment through firm entry and exit.

A substantial amount of job creation and destruction is associated with firm turnover. Davis and Haltiwanger (1990) attribute 25% of US annual job destruction to firm exit and 20% of annual job creation to firm entry, while Spletzer (2000) reports roughly 20%

for these two measures. We investigate the extent to which job gains occur through new openings in response to tax incentives, and ask whether this margin is relevant, or whether established firms actually matter more for job creation. Similarly, to the extent that corporate tax reductions save jobs, we investigate whether existing firms shed fewer jobs, or if jobs are saved mainly through a reduction in closings.

The paper proceeds in three stages. First, it provides empirical evidence on the transmission of corporate income taxes to macroeconomic aggregates and to the labor market. In particular, we estimate the effect of temporary corporate income tax shocks on firm entry and exit dynamics, job flows and aggregate and newly-hired wages. This section employs structural vector autoregression (VAR) analysis to identify corporate income tax shocks in aggregate US data. To identify corporate income tax surprises we use the external instrument estimation strategy developed by Mertens and Ravn (2013). This method exploits the attractive features of both structural vector autoregressions and the narrative approach.

Secondly, an additional regression analysis on US state-level data is conducted as a robustness exercise. In this stage, we use variations in state-level corporate income taxes across US states to identify the effects of a fiscal stimulus on output, the labor market, wages and firm dynamics. The econometric approach is similar to that employed by Nakamura and Steinsson (2014) to identify the government spending multiplier and to Suárez Serrato and Zidar (2016), who identify the effects of business tax cuts on local economic activity.

Finally, we use this empirical evidence to develop a dynamic stochastic general equilibrium (DSGE) model as a laboratory to study the effects of tax shocks on both business and job creation. The DSGE model is then used to evaluate the permanent effects of a cut in the corporate tax rate. The fey features of our model are imperfect competition between firms and endogenous entry (which implies time variation in the number of producers), heterogeneous productivity levels across firms (which endogenizes the exit rate), and distortionary corporate income taxation. Our benchmark model features endogenous firm entry modeled as in Bilbiie, Ghironi, and Melitz (2012), i.e. potential entrants pay a sunk cost in terms of effective labor units. Moreover, to capture endogenous firm exit, we introduce heterogeneity in productivity across firms, as in Ghironi and Melitz (2005), which results in a time-varying proportion of low-productivity firms that exit each period.

The debate on the size (and even the sign) of fiscal policy effects revolves to a large extent around output multipliers. The ability of fiscal policy to stimulate net job creation, which is arguably more important for a policymaker, is an under-researched topic. Previous literature has focused mainly on estimating output and unemployment multipliers of tax changes in government spending (e.g. Monacelli, Perotti, and Trigari (2010)). For example, Lewis and Winkler (2017) study the effects of government spending expansions on net business formation. A notable exception is Monacelli, Perotti, and Trigari (2010), who examine how changes in different tax rates affect the labor market. They find larger effects for business taxes than for personal income taxes, but do not investigate the effects of taxes on job flows.

In the literature on endogenous firm entry, job gains and losses are typically not analyzed, the implicit assumption being that in many macroeconomic models, a firm and a worker are equivalent concepts.[1] Recent advances in jointly modeling job flows and firm dynamics have been made by Colciago and Rossi (2015), who show that the extensive margin of job creation arising from firm entry amplifies the response of labor market variables to technology shocks.

On an aggregate level, we find a significant positive, though delayed, impact on job creation through firm entry and an immediate reduction in job losses through lower firm exit rates. This suggests that the exit margin is relatively more important for establishment turnover than the entry margin in response to tax shocks. This finding contrasts with that of acyclical *product* exit[2], which has been used in the endogenous-entry literature as an argument to view exit as exogenous, similarly to capital depreciation in the real business cycle literature.[3] Wages of new hires rise significantly, while aggregate wages exhibit a persistent rise in the wake of the policy change. Accordingly, our results also suggest a higher degree of stickiness in the wages of existing firm-worker matches relative to those of newly-formed matches. On the state level, our results suggest that corporate income tax changes may be effective in incentivizing new firms to enter the market and reducing firm turnover, and also in creating jobs and boosting wages of new hires in the short-run.

In the third part of the paper, we lay out a dynamic stochastic general equilibrium model with endogenous entry of homogeneous firms and exit that is able to capture some of the patterns observed in the data. For comparison, we also study the dynamics in response to a tax cut in a model with heterogeneous firms and a constant exit rate.

[1]See, for instance, Bilbiie, Ghironi, and Melitz (2012), Etro and Colciago (2010), Lewis and Poilly (2012), and Lewis and Stevens (2015).

[2]See, for example, Bernard, Redding, and Schott (2010).

[3]Most notably, see Bilbiie, Ghironi, and Melitz (2012).

We show that the popular general equilibrium business cycle model with entry, exit, and homogeneous firms is consistent with several patterns observed in the data. We show that output, entry, and exit rise as dividends are taxed less; firm churn and business dynamism increase. In a model with heterogeneous firms the aggregate wage declines in response to tax cut in contrast to our empirical findings, while in a model with homogeneous firms aggregate wages instead rise on impact.

The remainder of the paper is structured as follows. Section 3.2 details our empirical analysis comprising aggregate as well as state-level econometric evidence for the US economy. Section 3.3 develops a model that is meant to capture our main empirical findings. Finally, Section 3.4 concludes.

3.2 Empirical evidence

We provide empirical evidence on the transmission mechanism of corporate tax shocks to firm dynamics and the labor market. The first subsection employs structural vector autoregression (VAR) analysis to identify corporate income tax shocks in aggregate US data, while the second subsection estimates reduced-form effects using panel regressions estimated on US state-level data.

3.2.1 Aggregate US evidence

Our first econometric approach estimates structural VARs on a mixture of macroeconomic, financial, labor market and fiscal policy variables for the aggregate US economy.

VAR Specification

In our *baseline specification*, we include a fixed set of five core variables, specifically: (1) the average corporate income tax rate, our policy variable, (2) corporate profits, (3) real output, (4) employment, and (5) the excess bond (external finance) premium developed by Gilchrist and Zakrajšek (2012) to capture financial frictions that affect firms' borrowing costs. We then estimate a number of *augmented VAR specifications* by appending, in turn, one additional variable to the vector of baseline variables. We do this for three sets of additional variables.

First, we add establishment entry and exit as measures of expansions and contractions in the economy's productive capacity along the extensive margin. The corresponding impulse responses could provide a first indication of whether significant job flows can be expected at the extensive margin. Second, we analyze employment changes in more detail by estimating, separately, the responses to corporate tax cuts of job creation by establishment births and job destruction by establishment deaths. Third, we use both aggregate wages and wages of newly hired workers, since the latter variable is more sensitive to aggregate labor market conditions, as shown by Haefke, Sonntag, and van Rens (2013). We explore how these wage measures respond to tax cuts, the idea being that wage increases, especially those of newly hired workers, might stand in the way of new job creation.

Method

To identify corporate income tax surprises, we use the external instrument estimation strategy developed by Mertens and Ravn (2013). The method exploits the attractive features of both structural vector autoregressions and the narrative approach. Identification is achieved by imposing the restrictions that narrative measures of exogenous tax changes correlate with the structural tax shock but are orthogonal to other structural shocks. There are no timing restrictions. The procedure has three stages. In the first stage, we estimate a reduced-form VAR by ordinary least squares. The second stage consists in regressing the VAR residuals of the policy indicator on the nonpolicy indicator by using narratives as instruments (two-stage least squares). In the third stage, we impose the covariance restrictions and compute impulse responses. We elaborate on the econometric framework as follows.

Consider a standard structural vector autoregression model. Let Y_t be a vector of n economic variables, including a constant term observed at time t, p the number of lags, A a nonsingular $n \times n$ matrix, B_i an $n \times n$ coefficient matrix with $i = 1, 2, \ldots p$, and ε_t an $n \times 1$ vector of uncorrelated structural shocks with zero mean and unit variance,

$$AY_t = B_1 Y_{t-1} + B_2 Y_{t-2} + \cdots + B_p Y_{t-p} + \varepsilon_t. \tag{3.1}$$

Pre-multiplying both sides of equation (3.1) by the inverse of A, we obtain the reduced form specification

$$Y_t = C_1 Y_{t-1} + C_2 Y_{t-2} + \cdots + C_p Y_{t-p} + u_t, \tag{3.2}$$

where $C_i = A^{-1} B_i$ and the reduced-form residuals are linear transformations of the structural shocks, $u_t = D\varepsilon_t$, with $D = A^{-1}$. Since the variance-covariance matrix of the reduced form residuals is symmetric $\Sigma_{uu} = DD'$, it provides $\frac{n(n+1)}{2}$ identifying restrictions. In order to compute impulse response functions implied by the reduced-form specification (3.2), the recursiveness assumption, which imposes that D is lower triangular, is predominantly used in the policy literature (see, for example, Blanchard and Perotti (2002)).

Mertens and Ravn's identification strategy differs from the preceding one in the following way. Denote by y_t^τ the column of the fiscal policy instrument variable, which in our specification is the average corporate income tax rate. Let ε_t^τ be the corresponding structural shock, $\varepsilon_t^{-\tau}$ the structural shocks to the non-policy variables and D^τ the associated column of matrix D. Similarly, we denote by u_t^τ the reduced-form residuals from the equation for the fiscal policy instrument, and by $u_t^{-\tau}$ the reduced-form residuals for all the other macro, labor or financial variables. Since our interest lies in identifying impulse responses to corporate tax shocks and not to other shocks, we only need to identify the elements of the associated column τ of matrix D.[4]

Covariance restrictions are obtained from additional assumptions imposed on an appropriate instrument for the policy shocks. Let Z_t be an instrumental variable for the structural shocks ε_t^τ. Here, the narratively identified measures of exogenous shocks to average tax rates from Romer and Romer (2010) are used as an instrument Z_t. Suitable instrumental variables satisfy two conditions, a strong instrument assumption and an exclusion restriction,

$$E[Z_t \varepsilon_t^\tau] = \Phi, \tag{3.3}$$

$$E[Z_t \varepsilon_t^{-\tau}] = 0, \tag{3.4}$$

where Φ is a matrix to be estimated. In our specification, since we have only one instrument for the structural shocks of the average corporate income tax, Φ is a scalar. Condition (3.3) states that the instrument Z_t needs to be sufficiently correlated with the underlying corporate tax shock. Condition (3.4) states that the instrument must not be correlated with the other structural shocks.

The procedure to obtain impulse response functions following a unit increase in the structural shock to the tax instrument is as follows. First, we estimate the reduced-form VAR in (3.2) to obtain the residuals to the policy and non-policy variables, u_t^τ and $u_t^{-\tau}$,

[4] In case we are interested in impulse responses to other shocks, the proposed identifying restrictions do not suffice and additional zero or sign restrictions need to be imposed.

respectively. Second, we regress the VAR residuals of the policy variable, u_t^τ, on the instrument Z_t to obtain the fitted values $\hat{u}_t^\tau$ and the covariance matrix Σ_{Zu^τ}. Third, we regress the residuals of the non-policy variables $u_t^{-\tau}$ on the fitted values $\hat{u}_t^\tau$ and obtain the covariance matrix $\Sigma_{Zu^{-\tau}}$. Lastly, we impose the identifying restrictions to obtain the matrix column D^τ and compute impulse responses. We can partition the matrix $D = \begin{bmatrix} D^{\tau,\tau} & D^{\tau,-\tau} \\ D^{-\tau,\tau} & D^{-\tau,-\tau} \end{bmatrix}$, which simplifies the identifying restrictions to be expressed as

$$D^{-\tau,\tau} = \Sigma_{Zu^\tau}^{-1} \Sigma_{Zu^{-\tau}} D^{\tau,\tau}. \tag{3.5}$$

For more details, see Mertens and Ravn (2013).

Data

Table 3.1 summarizes the data sources and transformations pertaining to the variables in our VAR. Data are quarterly and in logarithms; the sample period is 1979q1-2006q1. Job creation by openings and job destruction by closings are available at a quarterly frequency from the BLS's Business Employment Dynamics database, starting in 1992.[5] For the earlier period, we used yearly data available from US Census Bureau's Business Dynamics Statistics starting in 1976, and interpolated the missing quarterly values between 1976 and 1992 using the method developed by Chow and Lin (1971).[6] As for the related series, we used New Business Incorporations and Failures, respectively. The former were reported at a monthly frequency in the BEA's Survey of Current Business between 1948m1 and 1993m12.[7] The latter are taken from the Economic Report of the President (various issues), where the 1984 discontinuity was corrected in accordance with Naples and Arifaj (1997).

Results

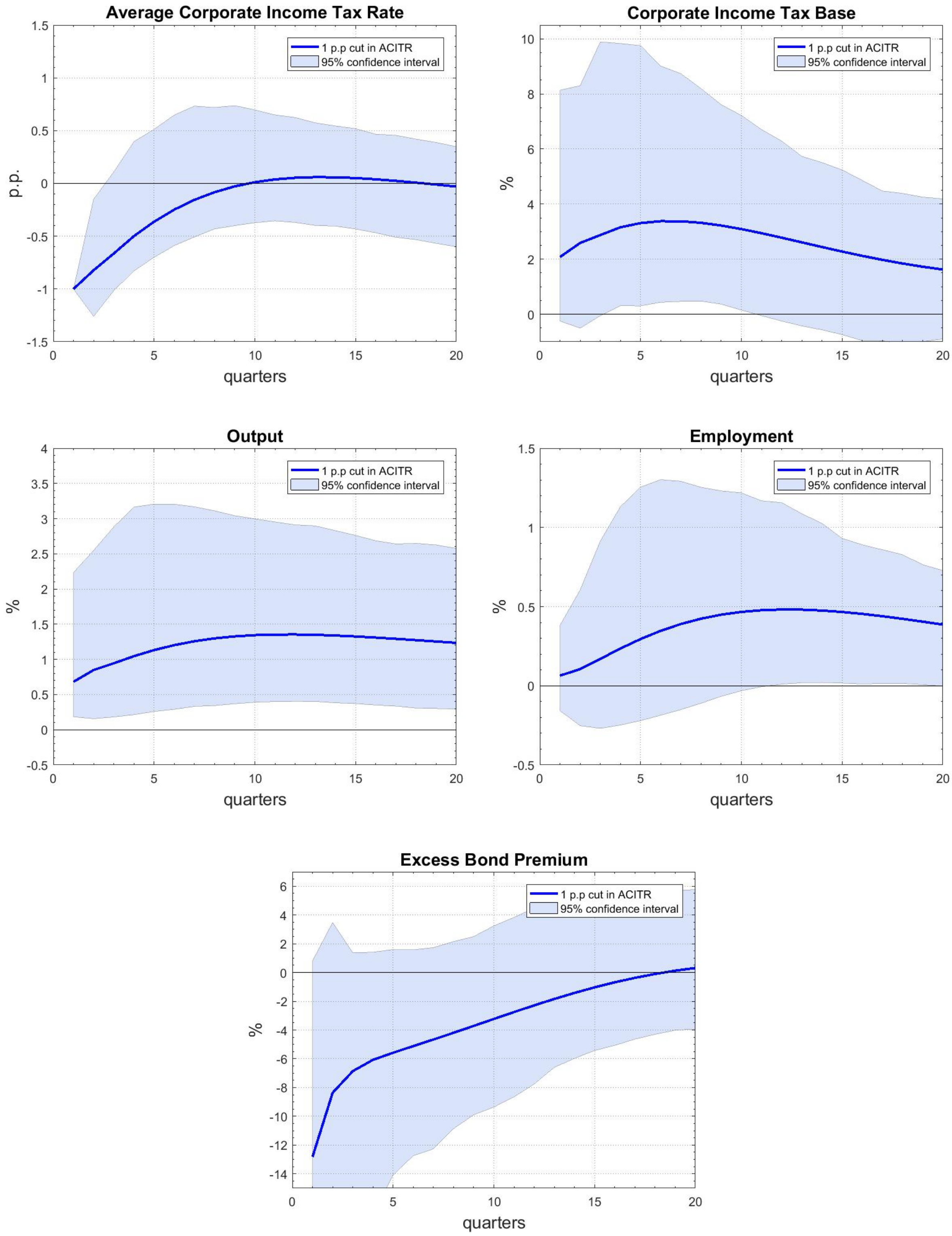
Figures 3.1 and 3.2 present the impulse responses to a policy shock given by a 1 p.p. reduction in the average corporate income tax rate. The solid black line represents the point estimate, while the gray shaded areas are the 95 % bootstrap confidence intervals.

[5]The BLS's Business Employment Dynamics database is available at: www.bls.gov/bdm/home.htm.

[6]The Business Dynamics Statistics can be downloaded from: https://www.census.gov/ces/dataproducts/bds/.

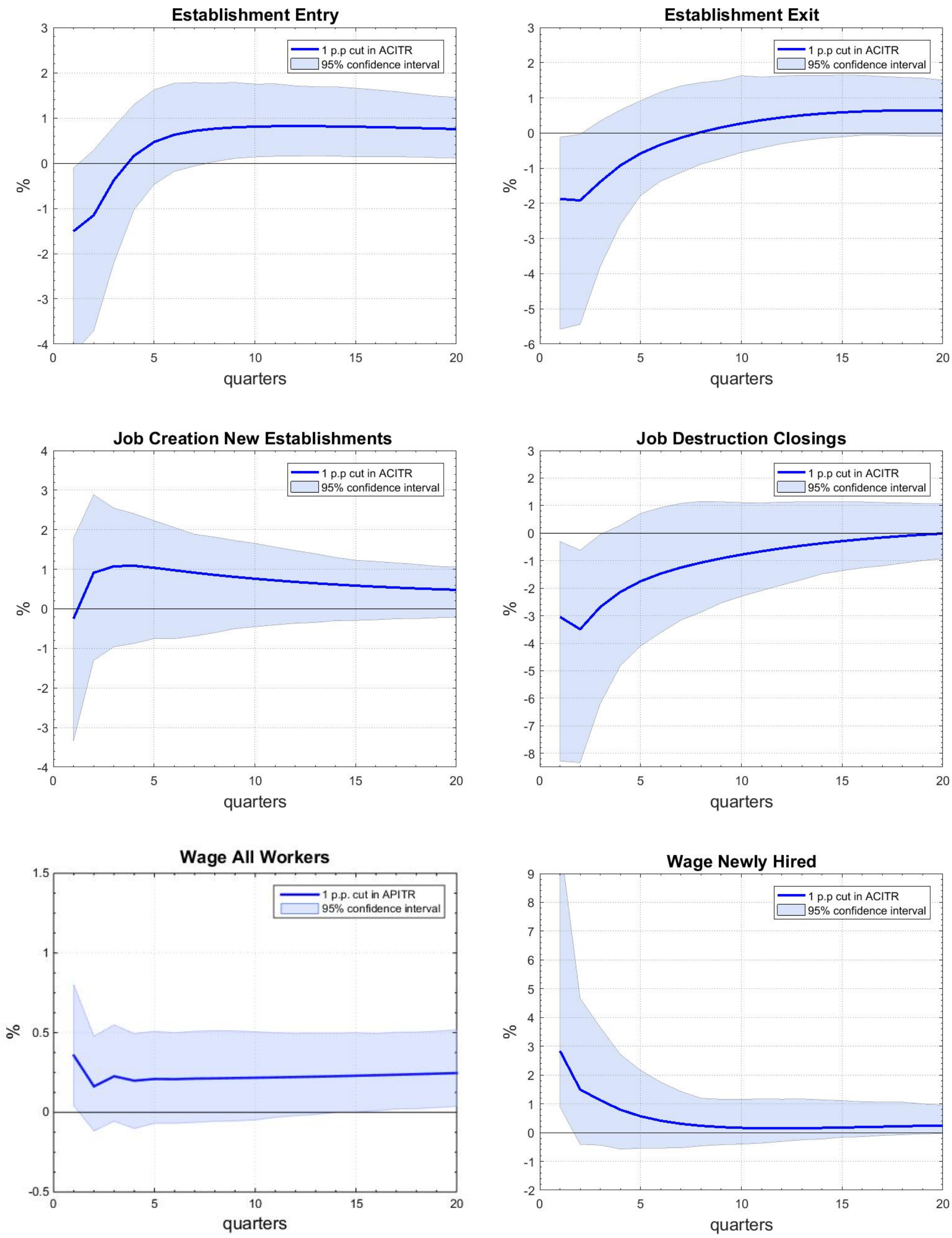
[7]Monthly data on new incorporations from 1948m1 until 1994m12 are available on page C-29 of this file: http://www.bea.gov/scb/pdf/NATIONAL/BUSCYCLE/1994/1194cpgs.pdf.

Figure 3.1: Impulse Responses: Baseline VAR Model



Notes: Figures show impulse responses to a 1 p.p. cut in the average corporate income tax rate (ACITR).

Figure 3.2: Impulse Responses: Augmented VAR Model



Note: Figures show impulse responses to a 1 p.p. cut in the average corporate income tax rate (ACITR).

Table 3.1: Aggregate US Data

Variable	Source	Transformation
Core Variables		
Average corporate income tax	Mertens and Ravn (2013)	None
Corporate profits	Mertens and Ravn (2013)	None
Real GDP	Mertens and Ravn (2013)	None
Employment	Mertens and Ravn (2013)	None
Excess bond premium	Gilchrist and Zakrajšek (2012)	None
Additional Variables		
Establishment entry	BLS BDM, Census BDS	Chow and Lin (1971)
Establishment exit	BLS BDM, Census BDS	Chow and Lin (1971)
Job creation entry	BLS BDM, Census BDS	Chow and Lin (1971)
Job destruction exit	BLS BDM, Census BDS	Chow and Lin (1971)
Wage all workers	Haefke, Sonntag, and van Rens (2013)	None
Wage newly hired	Haefke, Sonntag, and van Rens (2013)	None

Notes: BLS = Bureau of Labor Statistics, BDM = Business Employment Dynamics. BEA = Bureau of Economic Analysis, SCB = Survey of Current Business. Census BDS = US Census Bureau's Business Dynamics Statistics.

Core Variables Regarding the core variables shown in Figure 3.1, we find that 1 p.p cut in corporate income taxes raises output, profits and employment. The time profile of the response, however, differs across the core variables: output rises on impact and does so persistently, while firm profits increase with a lag.[8] Employment is the most sluggish variable of the three, taking three years to record a significant increase.

The tax cut appears to lower the external finance premium; however, the 95% confidence interval is rather wide and contains the zero line. According to Gilchrist and Zakrajšek (2012), the excess bond premium is a component of corporate bond credit spreads that is not directly attributable to expected default risk related to firm characteristics. Intuitively, credit spreads may anticipate future economic activity because they incorporate investors' expectations about future cash flows, which affect the business sector's profits, and in turn hiring decisions today. Our results suggest that a reduction in the corporate tax rate may reduce credit spreads through an increase in expected future profits, which decreases the risk of default. The resulting drop in credit costs in turn alleviates financial constraints on established firms, thereby possibly helping to prevent

[8]Note that in Mertens and Ravn (2013) identification strategy, all variables are allowed to respond instantaneously, which would not be the case under a Cholesky decomposition where output and other real variables are be predetermined in the current period.

firm exit and job destruction.

Additional Variables Expectations of higher future profits should, in theory, induce forward-looking firms to enter the market and create jobs. Establishment entry indeed rises in response to a tax cut, but only after some time. In contrast, the initial impact response is negative. We find a significant immediate drop in establishment exit and job destruction by deaths in response to a corporate tax cut.

This suggests that the exit margin is relatively more important for *establishment* turnover than the entry margin in response to tax shocks. Note how this finding contrasts with that of acyclical *product* exit – see Bernard, Redding, and Schott (2010) – that has been used in the endogenous-entry literature as an argument to view exit as exogenous, similarly to capital depreciation in the real business cycle literature.[9] Using plant-level data from 1972 until 1997, Lee and Mukoyama (2015) report that the entry margin is more volatile and displays greater selection effects.

While it is remarkable that establishment entry and exit show a qualitatively similar response pattern, a possibly related finding is recorded in Davis and Haltiwanger (2001), who note that the 1970s oil price hikes increased both job destruction and creation in the two years after the shock. The same paper also reports that 'oil and monetary shocks generate much greater short-run responses in job destruction than job creation in almost every sector', which underscores the relative importance of the exit margin for labor flows.

Our results thus suggest that in the short run, a reduction in taxes acts to save jobs - by reducing establishment exit - rather than helping to create new ones. Establishment births and the associated job creation rise only after a substantial delay. Figure 3.2 provides some suggestive evidence of what might drive this delay. Entry could be inhibited due to the increase in the wages of newly hired workers, which drives up entry costs if the latter involves wage payments. The initial decrease in the number of new firms entering the market coincides with the positive response of newly hired wages. This may suggest that entrants face entry costs in terms of marginal wages rather than fixed output costs, as in Jaimovich and Floetotto (2008), or aggregate wages, in Ghironi and Melitz (2005). An intriguing explanation for the initial decline in entry is given by Neira and Singhania (2018), who suggest that higher wages raise the opportunity cost to would-be entrepreneurs of starting a business. This story is consistent with the secular decline in the US startup rate that went hand-in-hand with a fall in the effective corporate tax rate

[9]Most notably, see Bilbiie, Ghironi, and Melitz (2012).

since 1978.

The two subplots in the bottom row of Figure 3.2 show an immediate strong positive response of wages of newly hired workers, whereas the response of average hourly wage to a tax reduction is smaller but persistent. The wage of new hires, unlike the aggregate wage, is volatile and responds more than one-to-one to changes in labor productivity in the first four quarters. These responses motivate the question of what kind of models of wage setting and labor market institutions are consistent with the observed wage response patterns.

In a frictionless labor market, workers can be replaced costlessly, so that each worker is marginal; differences in the wages of newly hired workers and incumbent workers cannot be an equilibrium outcome, implying the same behavior of two wage measures (Barro 1977). With search frictions in the labor market, hiring is a forward-looking decision. The number of newly created jobs is found by equalizing the cost of opening a vacancy with the expected net present value of profits that the firm will make once the vacancy has been filled. The latter in turn depends on the productivity and the wage of the marginal worker over the contracting period.

The increase in newly hired wages observed in Figure 3.1 implies that hiring a marginal worker becomes more expensive, which might explain the initial drop in establishment entry. The cyclical behavior of newly hired wages may be very different from that of the aggregate wage. Under certain bargaining arrangements, workers' bargaining power is pro-cyclical, consistent with the response of newly hired wages reported above. Haefke, Sonntag, and van Rens (2013) find that aggregate wages grow almost independently of aggregate productivity, while wages at the start of an employment relationship react strongly to changes in productivity. Though we consider an exogenous tax reduction rather than a productivity shock, our results also suggest a higher degree of stickiness in the wages of existing worker-firm matches relative to the wages of newly-formed matches.[10]

[10] Following Haefke, Sonntag, and van Rens (2013), we also estimate aggregate wage response using hourly compensation in the private nonfarm business sector from the BLS productivity and cost program. Wages from Haefke, Sonntag, and van Rens (2013) are the CPS averages for all employed workers between 25 and 60 years old in the private nonfarm business sector, excluding supervisory workers and correcting for composition bias and sampling error. Figure 3.4 in the Appendix displays the wage response. In this case, the aggregate wage initially drops and subsequently increases. The estimated wage response qualitatively exhibits similar behavior to the firm exit. One explanation for such a response is that with long-term wage contracting and a larger share of ongoing matches than new matches, a tax reduction induces firms with lower productivity and lower wages to stay in the market, when they would otherwise exit the market.

Robustness

We investigate the robustness of our results by considering two alternative VAR specifications. First, we consider a VAR in first differences of all observable variables. Second, we augment our baseline specification to control for the responses of government spending and labor income to corporate tax surprises, since these omitted variables can lead to misspecification. We find that the short-run and medium-run effects of corporate tax shock are robust to these alternative specifications.

3.2.2 US state-level evidence

In this section, we use variations in state-level corporate income taxes across US states to identify the effects of a fiscal stimulus on output, the labor market, wages and firm dynamics. The econometric approach is similar to that employed by Nakamura and Steinsson (2014) to identify the government spending multiplier and to Suárez Serrato and Zidar (2016), who identify the effects of business tax cuts on local economic activity.

Regression model

In the main empirical specification we follow Nakamura and Steinsson (2014) and employ difference-in-difference panel data framework

$$Y_{it} - Y_{it-1} = \beta(\tau^{CI}_{it} - \tau^{CI}_{it-1}) + \beta_x(X_{it} - X_{it-1}) + \alpha_i + \gamma_t + \varepsilon_{it}, \tag{3.6}$$

where Y_{it} is the logarithm of the dependent variable in state i in year t, and thus $Y_{it} - Y_{it-1}$ measures approximately the percentage growth of the dependent variable in state i over one year; τ^{CI}_{it} is the state-level corporate income tax rate in state i in year t, α_i and γ_t represent state and year fixed effects, and X_{it} is a vector of controls. By including state fixed effects, we allow for state-specific time trends in the dependent variable and account for unobserved time-invariant heterogeneity across states. The inclusion of time fixed effects allows us to control for aggregate shocks and policies, such as changes in federal taxes and aggregate monetary policy. The main aim is to estimate the coefficient β in (3.6) for real GDP and labor market variables in Table 3.2, namely the labor market multiplier.

The controls X_{it} we consider in our baseline specification are the variables that affect the corporate tax base, including investment tax credit rate and the research and development (R&D) tax credit rate, loss carryback rule, and loss carry-forward rule. In this specification, we also include per-capita government spending in X_{it}. To the extent that the decrease in corporate taxes needs to be financed locally, states may have to tighten other fiscal policies when cutting corporate taxes. Such a policy tightening may counteract the intended effect of tax reductions.

One potential caveat of estimating the effect of state corporate tax in (3.6) is that corporate tax is potentially endogenous to the state's business cycle, in which case coefficients would be biased. Therefore, in our second specification, we estimate equation (3.6) using an instrumental variables approach similar to Nakamura and Steinsson (2014). The idea is to instrument for state corporate tax using average corporate tax interacted with a state dummy. This instrument captures the differential sensitivity of corporate taxes across states to the national level of corporate tax. The identifying assumption is that the United States does not embark on tax reforms because states that have the highest corporate taxes are facing weaker labor market conditions relative to other states. In the first stage, we regress changes in state corporate taxes on changes in average tax and fixed effects, allowing for different sensitivities across states. The one-year corporate tax change in (3.6) is then computed from the fitted values of the first stage regression.

In our third specification, we apply identification by heteroskedasticity introduced by Lewbel (2012). This method identifies structural parameters in models with endogenous regressors where traditional instrumental variables are either weak or not readily available. To see how this method can be applied to estimate the effect of corporate tax on the labor market, suppose that $\tau^{CI} = X_1$ is an endogenous regressor, Y is an endogenous variable and X represents exogenous regressors:

$$Y = \beta_x X + \beta X_1 + \varepsilon_1 \tag{3.7}$$

$$X_1 = \gamma_x X + \gamma_y Y + \varepsilon_2 \tag{3.8}$$

As before, we are interested in estimating β in (3.7). Structural parameters may be identified given some heteroskedasticity. The identification comes from restricting correlations of $\varepsilon\varepsilon'$ with X, by assuming that $Cov(X, \varepsilon_j) \neq 0$. For this identification, estimators take the form of the generalized method of moments with higher moments restrictions or modified two-stage least squares. Since these estimates can be less reliable in comparison to those coming from standard exclusion restrictions, they can be used when instruments are not available, or together with traditional instruments to increase efficiency. We opt for the second option, and include lags of changes in corporate tax rate $\Delta\tau_{t-1}^{CI}, \Delta\tau_{t-2}^{CI}$ as instruments. The appropriate lag structure is chosen such that the p-value for the Hansen test of over-identification (Hansen J statistics) and the p-value for the instrument exogeneity test (C statistics) are such that we do not reject null hypotheses. We also limit estimation to more parsimonious models with a lag structure of less than three years, because of the duration of the election cycle and the precision of the estimates.

Data

Table 3.2 contains the data sources and variable transformations related to the state-level regressions. Data are yearly and cover the 1980-2006 period for wages of newly hired and incumbent workers, and the 1992-2010 period for all other variables.

Table 3.2: US State Level Data

Variable	Source	Transformation
Core Variables		
Corporate income tax	Suárez Serrato and Zidar (2016)	None
Investment tax credit	Suárez Serrato and Zidar (2016)	None
R&D tax credit	Suárez Serrato and Zidar (2016)	None
Real GDP	BEA	Deflated by US CPI
CPI	BLS	2010=1
Excess bond premium	Gilchrist and Zakrajšek (2012)	None
Labor Market Variables		
Establishment entry	BLS BDM	None
Establishment exit	BLS BDM	None
Job creation entry	BLS BDM	None
Job creation expansions	BLS BDM	None
Job destruction exit	BLS BDM	None
Job destruction contractions	BLS BDM	None
Real wage per worker	BEA	Deflated by US CPI
Wage all workers	Suárez Serrato and Zidar (2016)	Haefke, Sonntag, and van Rens (2013)
Wage stay workers	Suárez Serrato and Zidar (2016)	Haefke, Sonntag, and van Rens (2013)
Wage newly hired	Suárez Serrato and Zidar (2016)	Haefke, Sonntag, and van Rens (2013)

Notes: BLS = Bureau of Labor Statistics, BDM = Business Employment Dynamics, BEA = Bureau of Economic Analysis.

Results

Table 3.3 summarizes the effect of corporate tax changes on economic activity over two years. For brevity, we only report the estimated coefficients on state corporate taxes with standard errors and statistical significance. More elaborate tables with coefficients on the investment and R&D tax credits and government spending can be found in Appendix 3.A.3.

Table 3.3: EFFECTS OF CORPORATE TAX INCREASE ON LOCAL ECONOMIC ACTIVITY AFTER ONE YEAR

(1) FE	(2) IV	(3) het-IV	(1) FE	(2) IV	(3) het-IV
	Output			*Employment*	
-0.21	-0.27	-0.35**	-0.04	0.05	-0.21
(0.173)	(0.392)	(0.156)	(0.060)	(0.088)	(0.162)
	Establishment Entry			*Establishment Exit*	
-0.25	-4.12***	-1.48*	0.74	2.49***	-0.54
(0.871)	(0.999)	(0.832)	(0.511)	(0.945)	(0.489)
	Job Creation Births			*Job Creation Expansions*	
0.82	-6.86***	-0.12	0.07	-0.73	-0.80***
(1.029)	(2.047)	(0.833)	(0.378)	(0.681)	(0.236)
	Job Destruction Exit			*Job Destruction Contractions*	
1.84*	1.85	1.47*	0.21	-0.78	-0.23
(1.059)	(2.798)	(0.769)	(0.233)	(0.694)	(0.276)
	Real Wage per Worker			*Hourly Wage All Workers*	
-0.11	-0.33	-0.36***	0.01	0.01	-0.01
(0.125)	(0.316)	(0.077)	(0.014)	(0.013)	(0.015)
	Hourly Wage Newly Hired			*Hourly Wage Stay Workers*	
-0.86	-1.47	-2.74*	0.00	0.01	-0.01
(3.402)	(3.029)	(1.464)	(0.015)	(0.014)	(0.015)

Notes: In Table 3.3, columns (1) to (3) show the effect of an increase in corporate tax while controlling for the change in state investment tax credit, R&D tax credit,loss carry back rule and loss carry forward rule, and government spending. In column (2), we use average state corporate tax interacted with state dummy variables as an instrument for the state corporate tax in the two stage regression. We also estimate coefficients by identification through heteroskedasticity(Lewbel 2012) in column (3). Standard errors are clustered by state and statistical significance is indicated by p-values as follows: $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

To summarize the findings on entrants, we confirm the finding that entry of new firms on the market and job creation by those firms reacts negatively and significantly to a rise in corporate income taxes both in the short and long-term. In particular, column (3) in Table 3.3 shows that a 1 p.p. increase in the corporate income tax rate induces a significant 1.48 % decrease in the establishment entry growth. The Cragg-Donald statistic in Table 3.8 is 24.41, which exceeds the critical value (21.39) at 5% from Stock and Yogo (2005), implying that any bias from using the lags of change in corporate tax as instruments is less than 5% of the bias from an OLS regression. Relatedly, Suárez Serrato and Zidar (2016) find that a 1 p.p. cut in business taxes causes roughly a 4 p.p. increase in the establishment growth rate over ten years. We also find a 6.86 % decrease in jobs created by new firms after one year in column (2). However, one potential concern, given a 4.12 % decrease in establishment growth in column (2) and a 6.86 % decline in jobs created is that these coefficients may be biased. Since the first stage F-statistics is small (3.973), the average state corporate tax interacted is a weak instrument and estimates could be biased. Therefore, coefficients may be overestimated and we capture effects due to reallocation and establishment mobility. Increasing corporate taxes in one state might induce firms to open a new establishment in a neighboring state. This would increase establishment entry in the latter state in the absence of local state tax changes. Moreover, since in both cases the p-value of Wu-Hausmann is close to zero, we reject the hypothesis of an exogenous instrument in Tables 3.10 and 3.8 and estimates may be inconsistent. In this case, identification through heteroskedasticity points to a better estimate, as we do not reject the hypothesis of exogenous instruments.[11]

Similarly, as we find a significant effect of corporate taxes on the entry margin, we find a significant immediate increase in establishment exit of 2.49 % in response to a corporate tax cut after one year.[12] This finding suggests that both entry and exit margin are important for establishment turnover in response to tax shocks. Further result is a 1.47% significant change in the number of jobs destroyed by exiting firms.

Turning to incumbent firms, corporate income taxes significantly affect job creation. 1 .p.p. increase in the corporate income tax rate reduces the number of jobs created by expansions by 0.8 %.[13] Intuitively, since a tax increase reduces the net present value of future profits, this leads to a contraction of the workforce through a decrease in the hiring rate. Existing firms are reluctant to hire new workers, but when adjusting to a higher tax environment they seem willing to keep existing workers. We find no significant effect of income tax on firms' firing decisions.

Consistent with our VAR analysis in the previous section, we observe a higher degree

[11]See the p-value of Hansen J and C statistics in Table 3.10 and 3.8.

[12]Identification by average state corporate taxes produces consistent estimates, as the p-value of Wu-Hausmann is high in Table 3.9.

[13]See column (3) in Table 3.3.

of stickiness in the wages of all workers relative to the wages of newly-formed matches. We find that wages of newly hired workers decrease by 2.74%, while the aggregate wage declines only by 0.36%. On the other hand, the hourly wage per worker seems not to significantly respond to corporate tax, while we observe a persistent small increase in the VAR estimation. Note, however, that the aggregate wage is *per worker* compensation, while incumbent and newly hired wages are a measure of *per hour* compensation, which limits direct comparability.[14]

In our final result, output declines on impact, as the estimated coefficients for the output regression is -0.35 while the employment rate seems not to be significantly affected by corporate taxes. Overall, our results suggest that corporate income tax changes may be effective in incentivising new firms to enter the market and reducing firm turnover, but also in creating jobs and boosting wages of new hires in the short-run.

3.3 Model

In this section, we lay out a dynamic stochastic general equilibrium model that is able to capture some of the patterns observed in the data. Our benchmark model features endogenous firm entry modelled as in Bilbiie, Ghironi, and Melitz (2012), i.e. potential entrants pay a sunk cost in terms of effective labor units. Moreover, to be able to capture endogenous firm exit, we introduce heterogeneity in productivity across firms, as in Ghironi and Melitz (2005), which results in a time-varying proportion of low-productivity firms that exit each period. Firms operate under monopolistic competition and incur fixed overhead labor costs in addition to variable labor costs. Labor markets are perfectly competitive so that all firms pay workers the same wage. We abstract from capital.

We outline the model, show its dynamics in response to a tax cut, and discuss which model features are necessary to capture the main characteristics of the VAR responses.

3.3.1 Benchmark model

In any period there exists a mass of N_t firms and a distribution $\mu(z)$ of productivity levels over a subset of (z^*, ∞), where z^* is the lower bound cutoff level. Due to fixed costs of production, firms with low productivity will never produce. Given the productivity draw,

[14]Two measures of aggregate wages used in the VAR estimation come from Haefke, Sonntag, and van Rens (2013) and from the BLS Productivity and Cost Account; however, the latter measure is not available at the state level.

which remains fixed over the firm's lifetime, a firm will produce only if the discounted value of its future profits is positive. This will be the case as long as its productivity is above a given threshold that we denote as z_t^*. Since the discounted value of future profits is equal to the value of a firm at any given point in time, it follows that $z_t^* = \inf \{z : v^* (z) > 0\}$,where v denotes the firm's value. The size of the production sector, i.e. the number of producers, is endogenously determined. It fluctuates over time with the profitability of the market, inducing changes in z_t^*.

Firms, Technology and Price Setting

There is a continuum of monopolistically competitive firms indexed by their productivity level z. Firms produce differentiated goods according to the following production function

$$y_t^c(z) = Z_t z l_t^c, \tag{3.9}$$

where l_t^c is the quantity of variable labor input, Z_t is an aggregate technology shock and z is an idiosyncratic productivity level. Taking the real wage w_t as given, firms maximize profits subject to demand $y_t(z) = (p_t(z)/P_t)^{-\theta} Y_t^c$, where Y_t^c is the total demand for goods and $\theta > 1$ is the elasticity of substitution between goods varieties. This results in an optimal relative price, defined as $\rho(z) \equiv p_t(z)/P_t$, given by

$$\rho_t(z) = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t z}, \tag{3.10}$$

Real profits can be written as $d_t(z) = \frac{1}{\theta} rr_t(z) - \frac{w_t f^c}{Z_t}$, where $rr_t(z) \equiv \rho_t(z) y_t(z)$ are real revenues and f^c is a fixed labor cost of production. Given demand for good z, we can write real revenues as

$$rr_t(z) = \rho_t(z)^{1-\theta} Y_t^c. \tag{3.11}$$

Equation (3.11) implies that the ratio between real revenues of two firms with different productivity levels $\widetilde{z}$ and z^* is

$$\frac{rr_t(\widetilde{z})}{rr_t(z^*)} = \left(\frac{\widetilde{z}}{z^*} \right)^{\theta - 1}, \tag{3.12}$$

that is, it is a function of productivity only, as long as the wage rate is common across firms.

Firm Value, Entry and Exit The value of a firm with productivity z is given by

$$v_t(z) = (1-\delta_t)E_t \sum_{s=1}^{\infty} Q_{t,t+s} d_{t+s}(z), \tag{3.13}$$

where δ_t is the time-varying probability of exiting the market, and $Q_{t,t+s} \equiv \beta \frac{u'(c_{t+s})}{u'(c_t)}$ is the stochastic discount factor of the household that owns the firms (see below).

There is an unbounded set of potential entrants. In order to enter the market firms must pay a sunk cost in terms of labor given by $\frac{w_t f_t^e}{Z_t}$. Firms draw their productivity from a distribution with probability density function $g(z)$ after they have entered the market. Upon entry, a firm with a low productivity draw can decide to exit without producing. Prior to entry, firms do not know their productivity; thus the entry condition is determined considering the profit of the firm with average productivity. The entry condition reads

$$\frac{w_t f_t^e}{Z_t} = v_t(\widetilde{z}), \tag{3.14}$$

where $v_t(\widetilde{z})$ is the value of the firm with average productivity. After paying the entry costs, firms draw their productivity level from $g(z)$. Exit of new entrants and incumbent firms takes place at the end of the period. The cutoff productivity level is determined by the condition that the marginal firm must have a value of zero, $v_t(z^*) = 0$, Together with firm value (3.13), this implies that the profits of a firm characterized by the threshold productivity level, $d_t(z^*)$, equals zero. Any firm which draws a productivity below z^* will leave the market. The equilibrium distribution of productivity levels $\mu(z)$ is the conditional distribution of $g(z)$ over the range of productivity levels above the threshold $[z^*, \infty)$,

$$\mu(z) = \begin{cases} \frac{g(z)}{1-G(z^*)} & z \geq z^* \\ 0 & \text{otherwise} \end{cases},$$

where $G(z)$ is the cumulative distribution function, such that $1 - G(z^*)$ is the ex-ante probability of successful entry.

Price Index and Aggregate Profits In equilibrium, the aggregate price index is

$$P_t = \left[\frac{1}{1-G(z^*)} \int_{z^*}^{\infty} p_t(z)^{1-\theta} N_t g(z) dz \right]^{\frac{1}{1-\theta}}. \tag{3.15}$$

With the average productivity level given by

$$\widetilde{z}=\left[\frac{1}{1-G\left(z^{*}\right)}\int_{z^{*}}^{\infty}z^{\theta-1}g\left(z\right)dz\right]^{\frac{1}{\theta-1}}, \tag{3.16}$$

one can show that the price index simplifies to

$$P_{t}=N_{t}^{\frac{1}{1-\theta}}p_{t}(\widetilde{z}), \tag{3.17}$$

where $p_t(\widetilde{z})=\frac{\theta}{\theta-1}\frac{w_t}{Z_t\widetilde{z}}$ is the price set by the firm with productivity $\widetilde{z}$. Denoting by $\rho_t(\widetilde{z})$ the real price of the average-productivity firm, we obtain $1=N_t^{\frac{1}{1-\theta}}\rho_t(\widetilde{z})$. It can be shown that aggregate profits are the product of the number of firms and average firm profits, $D_t=N_t d_t(\widetilde{z})$. The key requirement to apply this aggregation procedure is that the cost of factors and price markups are not firm-specific. Also, aggregate labor used for production equals firm-level labor input multiplied by the number of firms, $L_t^c=N_t l_t^c(\widetilde{z})$.

Firm Dynamics The dynamics of the number of firms is given by

$$N_{t+1}=(1-\delta_t)(N_t+N_t^e). \tag{3.18}$$

Individual productivity levels are drawn from a Pareto distribution with scale parameter $\kappa>0$ and location $z_{\min}>0$.[15] The exit rate is determined from the cumulative distribution function $G(z)$,

$$\delta_t=1-\left(\frac{z_{\min}}{z_t^*}\right)^{\kappa}, \tag{3.19}$$

which is the probability of $z<z_t^*$. By definition, the number of exiting firms N_t^x is the exit rate multiplied by the total number of firms, $N_t^x=\delta_t N_t$. We see from (3.19) that, for a given scale parameter of the Pareto distribution κ, the firm exit rate is positively related to the threshold productivity level. When z_t^* rises, more firms fall below this cutoff level, thereby raising the proportion of firms that are forced to leave the market.

[15] Bena, Garlappi, and Grüning (2016) formulate an alternative specification of aggregate productivity, which is a combination of heterogeneous "incremental" innovation by incumbents and "radical" innovation by entrants. They show that such an innovation process generates endogenous firm creation and destruction, time-varying economic growth and countercyclical economic uncertainty. However, Bena, Garlappi, and Grüning (2016) assume that labor is supplied inelastically and therefore abstract from employment, job creation, and job destruction dynamics, which are the main labor market variables of interest in our model. With the same narrative VAR methodology as in Section 3.2.1, a promising direction for future endeavors would be to investigate the effects of corporate income tax cuts on the rates of radical and incremental innovations.

Households Household maximize expected lifetime utility,

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \chi \frac{L_t^{1+1/\phi}}{1+1/\phi} \right), \tag{3.20}$$

subject to the budget constraint

$$v_t(\widetilde{z}) \left(N_t + N_t^e \right) x_{t+1} + C_t = [(1-\tau_t^d) d_t(\widetilde{z}) + v_t(\widetilde{z})] N_t x_t + w_t L_t + T_t, \tag{3.21}$$

where $v_t(\widetilde{z})$ and $d_t(\widetilde{z})$ represent post-entry averages, i.e. profits and value in case of successful entry, τ_t^d is a dividend tax and T_t are lump-sum transfers from the government. The first order conditions for consumption C_t, labor L_t and shares x_{t+1} are summarized by a labor supply equation and an optimality condition for share holdings,

$$\chi L_t^{\frac{1}{\phi}} = \frac{w_t}{C_t}, \tag{3.22}$$

$$v_t(\widetilde{z}) = (1-\delta_t) \beta E_t \left\{ \frac{C_t}{C_{t+1}} [(1-\tau_{t+1}^d) d_{t+1}(\widetilde{z}) + v_{t+1}(\widetilde{z})] \right\}. \tag{3.23}$$

Market Clearing A single entrant requires f_t^e effective labor units or $l_t^e = \frac{f_t^e}{Z_t}$ standard labor units to enter. Aggregate labor demand arising from entry is therefore $L_t^e = \frac{N_t^e f_t^e}{Z_t}$. Then the aggregate labor market clearing condition is

$$L_t = N_t l_t^c(\widetilde{z}) + \frac{N_t^e f_t^e}{Z_t}. \tag{3.24}$$

Aggregate output is used only for consumption, $C_t = Y_t^c$. Imposing asset market clearing, $x_t = x_{t+1} = 1$, in the household budget constraint (3.21), we obtain the aggregate accounting relation,

$$v_t(\widetilde{z}) N_t^e + C_t = d_t(\widetilde{z}) N_t + w_t L_t. \tag{3.25}$$

A summary of the benchmark model's equilibrium conditions is provided in Table 3.4. We now turn to the transmission of tax cuts implied by the model and how this is shaped by the different model features and parameter values. For comparison, we also study the dynamics in response to a tax cut in a model with homogeneous firms and exogenous exit, similarly to Bilbiie, Ghironi, and Melitz (2012), see Table 3.5. In the model with symmetric firms, we no longer distinguish between the average and the marginal firm. Therefore, the equations relating to the marginal firm, i.e. the first three equations in

Table 3.4: EQUILIBRIUM CONDITIONS: BENCHMARK MODEL

Marginal firm's revenue	$rr_t^* = \widetilde{rr}_t(\widetilde{z}_t/z_t^*)^{1-\theta}$
Marginal firm's profits	$d_t^* = \frac{1}{\theta}rr_t^* - w_t f^c/Z_t$
Exit condition	$d_t^* = 0$
Exit rate	$\delta_t = 1 - (z_{\min}/z_t^*)^\kappa$
Average firm's productivity	$\widetilde{z}_t = (\frac{\kappa}{\kappa-(\theta-1)})^{1/(\theta-1)} z_t^*$
Average firm's revenue	$\widetilde{rr}_t = \widetilde{\rho}_t^{\,1-\theta} C_t$
Average firm's profits	$\widetilde{d}_t = \frac{1}{\theta}\widetilde{rr}_t - w_t f^c/Z_t$
Average firm's value	$\widetilde{v}_t = (1-\delta_t)\beta E_t\{\frac{C_t}{C_{t+1}}[(1-\tau_{t+1}^d)\widetilde{d}_{t+1} + \widetilde{v}_{t+1}]\}$
Entry condition	$\widetilde{v}_t = w_t f_t^e/Z_t$
Firm dynamics	$N_{t+1} = (1-\delta_t)(N_t + N_t^e)$
Price setting	$\widetilde{\rho}_t = \frac{\theta}{\theta-1} w_t/(Z_t\widetilde{z}_t)$
Price index	$1 = N_t^{1/(1-\theta)}\widetilde{\rho}_t$
Labor supply	$\chi L_t^{1/\phi} = w_t/C_t$
Resource constraint	$\widetilde{v}_t N_t^e + C_t = \widetilde{d}_t N_t + w_t L_t$

Table 3.4, drop out. Also, the exit rate is now a constant equal to δ, such that the fourth equation in Table 3.4 no longer applies. The equations pertaining to the average firm remain and we remove the tilde from the following variables: firm value v_t, profits d_t, revenues rr_t, and the relative price ρ_t.

Table 3.5: EQUILIBRIUM CONDITIONS: MODEL WITH SYMMETRIC FIRMS

Firm revenue	$rr_t = \rho_t^{1-\theta} C_t$
Firm profits	$d_t = \frac{1}{\theta}rr_t - w_t f^c/Z_t$
Firm value	$v_t = (1-\delta)\beta E_t\{\frac{C_t}{C_{t+1}}[(1-\tau_{t+1}^d)d_{t+1} + v_{t+1}]\}$
Entry condition	$v_t = w_t f_t^e/Z_t$
Firm dynamics	$N_{t+1} = (1-\delta)(N_t + N_t^e)$
Price setting	$\rho_t = \frac{\theta}{\theta-1} w_t/Z_t$
Price index	$1 = N_t^{1/(1-\theta)}\rho_t$
Labor supply	$\chi L_t^{1/\phi} = w_t/C_t$
Resource constraint	$v_t N_t^e + C_t = d_t N_t + w_t L_t$

3.3.2 Calibration

Steady state productivity is normalized to unity, $Z = 1$. We also set steady state labor L to one, and find the value of χ needed to support this normalization. In calibrating the model, we opt for parameter values that are commonly used in the business cycle literature. The discount rate β is set to 0.99, consistent with a 4% real interest rate in a quarterly model, which implies that the gross quarterly real interest rate is $R = 1.01$. The Frisch elasticity of labor supply, ϕ, is set to unity, such that we are effectively working with a quadratic labor disutility function. The elasticity of substitution across goods varieties θ is set to 3.8 as in Ghironi and Melitz (2005) (henceforth GM), implying a steady state price net markup of 36%. The Pareto distribution is calibrated as in GM (2005), where the location parameter z_{min} is set to unity and the scale parameter is set to $\kappa = 3.4$. From (3.19), we see that a greater value of κ leads to a higher firm exit rate, given a particular threshold productivity level z_t^*. The calibration choices for κ and θ ultimately drive the size of selection effects implied by the model. Consider the equation which determines the ratio of the average firm's and the marginal firm's productivity level, $\widetilde{z}_t/z_t^*$. Under our parameterization, this ratio equals $(\frac{\kappa}{\kappa-(\theta-1)})^{1/(\theta-1)} = 1.8580$. When the value of the average firm rises, it must be that the value of the marginal firm rises, too, pushing up the firm exit rate δ_t. The steady state exit rate is calibrated to equal 10% annually, i.e. $\delta = 0.025$. Following Ghironi and Melitz (2005), we normalize the entry cost f_e to one. The steady state dividend tax rate is set to 30%, such that $\tau^d = 0.3$. The tax rate τ_t^d is modelled as an autoregressive process with persistence parameter equal to 0.9. [16]

The recursive computation of the model's steady state is provide in Table 3.18 in the appendix.

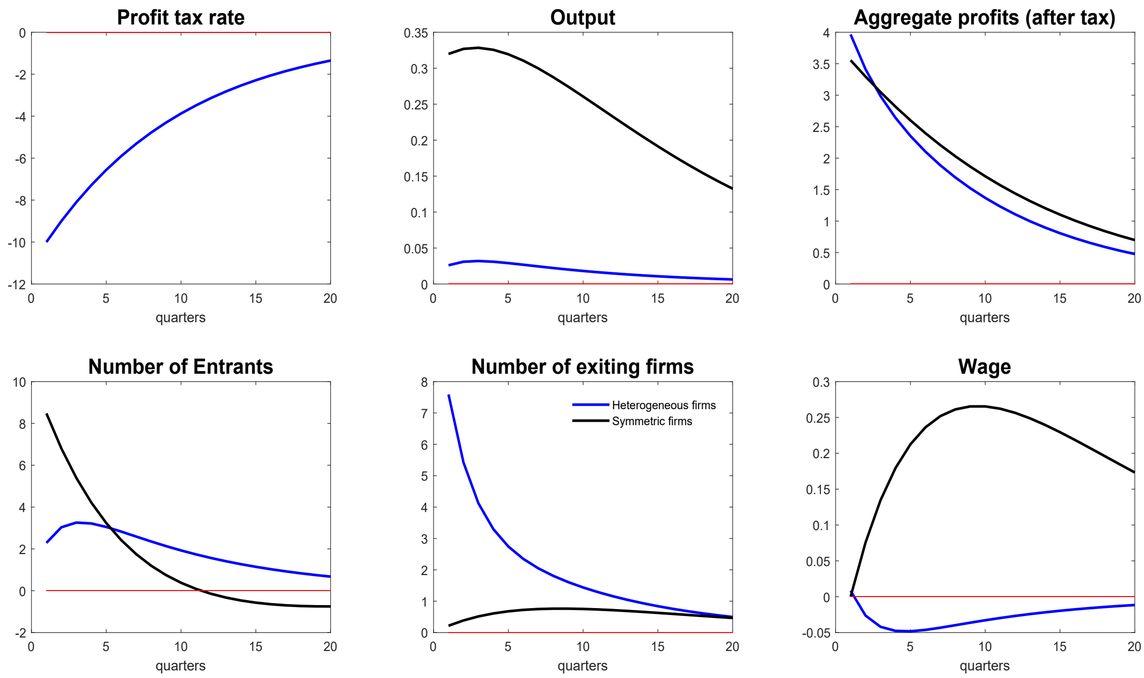
3.3.3 Model dynamics in response to a tax cut

This section presents the model-implied dynamics to a 10% cut in the dividend tax rate, in the benchmark model with heterogeneous firms and endogenous exit, and in the alternative model with homogeneous firms and a constant exit rate.

Firm dynamics. Figure 3.3 shows that entry and exit both rise as dividends are taxed less. Firm churn and 'business dynamism' increase; these model predictions are similar

[16] We model the dividend tax rate as an AR(1) process, which follows the tax rate's calibration to the impulse response function of the average corporate income tax rate in Figure 3.1. Since we focus on short-run labor and firm dynamics, we assume taxes are non-distortionary and financed by government lump-sum transfers. In doing so, we abstract from future distortionary tax pressure that could be relevant in the long run. For example, Croce, Nguyen, and Schmid (2012) show that when the government runs a zero-deficit labor tax policy, higher government expenditures directly translate into a higher tax, which leads to a decline in labor supply. When government aims to minimize labor fluctuations by introducing a counter-cyclical labor tax rate, the higher the sensitivity of labor taxes is to government debt, the higher are the long-lasting adverse fluctuations in labor.

Figure 3.3: Impulse Responses: Business Cycle Model



Notes: The figure shows the model-implied deviations from the steady state, in percentages, of selected variables in response to a 10% cut in the dividend tax rate τ_t^d.

to Sedláček and Sterk (2018). The explanation for the positive entry response is that the tax cut makes it more attractive to invest in new firms as the present discounted value of the stream of future after-tax profits rises. The firm exit rate, δ_t, rises. This is a direct consequence of the rise in the threshold productivity level z_t^*. As explained above, the ratio between the productivity levels of the average firm and marginal firm remains constant, such that a rise in $\widetilde{z}_t$ necessarily implies a rise in z_t^*. In our calibration, the average firm's productivity is always 1.858 times the marginal firm's productivity; likewise, the average firm's revenue is always $\frac{\kappa}{\kappa-(\theta-1)} = 5.67$ times the marginal firm's revenue. The rise in the exit is stronger than the rise in entry, such that the overall number of firms N_t falls.

Consumption, labor supply, and the wage rate. Recall that in this model, buying shares - i.e., investing in new firms - is the only way the households can transfer resources across time. When households decide to save more (in new firms) and labor supply is not very elastic, they consequently consume less today. The tax cut is financed with lump-sum taxes levied on households. The rise in taxes leads to a negative wealth effect, which shifts out the labor supply schedule and puts downward pressure on the wage rate. On the other hand, the rise in entry leads to more labor demand, which is a force that drives the wage up. A priori, it is not apparent how the wage responds to the tax cut. Here, the negative wealth effect dominates, and the wage declines when firms are heterogeneous. By contrast, labor demand prevails over the wealth effect when firms are homogeneous, and workers' compensation rises. Through the entry condition, the value of the average firm, $\widetilde{v}_t$, follows the dynamics of the wage.

Output and profits. Despite the crowding-out effect on consumption, aggregate output rises. This is because of the investment boom in new entrants, the increase in N_t^e. Despite the decline in the number of producers, aggregate after-tax profits respond positively, since the after-tax profits of the average firm, $(1 - \tau_t^d)\widetilde{d}_t$, increase.

3.4 Conclusion

This paper explores the effects of a corporate income tax stimulus on firm dynamics and the labor market in the United States. Positive effects on establishment entry are observed with a substantial delay, while immediate benefits are reaped in terms of lower firm exit and the associated job destruction. Our results show an initial drop in establishment entry, which may be related to the documented increase in the wages of newly hired workers. Wages of new hires rise significantly and return fast to the steady-state, while aggregate wages exhibit a persistent rise in the wake of the policy change. The divergent response patterns of the two wage measures warrant further investigation. We also find that incumbent firms respond strongly to investment tax credit incentives. An interesting direction for future research involves the decomposition of corporate tax into dividend tax and capital gains tax to study the implications of heterogeneity in tax reforms.

3.A Appendix

3.A.1 Augmented VAR with corporate taxes: additional variables

To test for the robustness of the aggregate wage response, we also estimate augmented VAR using hourly compensation in the private nonfarm business sector from the BLS productivity and cost program, rather than the average wage from Haefke, Sonntag, and van Rens (2013). In this case, the aggregate wage initially drops and subsequently increases. The estimated wage response qualitatively exhibits similar behavior to the firm exit. One explanation for such a response is that with long-term wage contracting and a larger share of ongoing matches than new matches, a tax reduction induces firms with lower productivity and lower wages to stay in the market that would otherwise exit the market. The difference in aggregate and average wage response could be due to correction for sampling and composition bias in Haefke, Sonntag, and van Rens (2013).

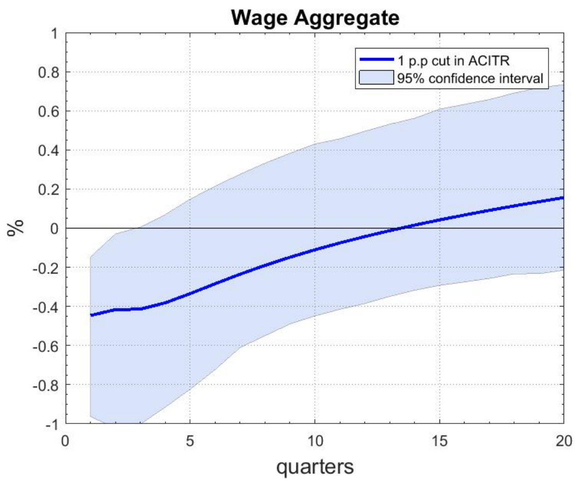


Figure 3.4: Impulse Responses: Augmented VAR Model with Corporate Taxes

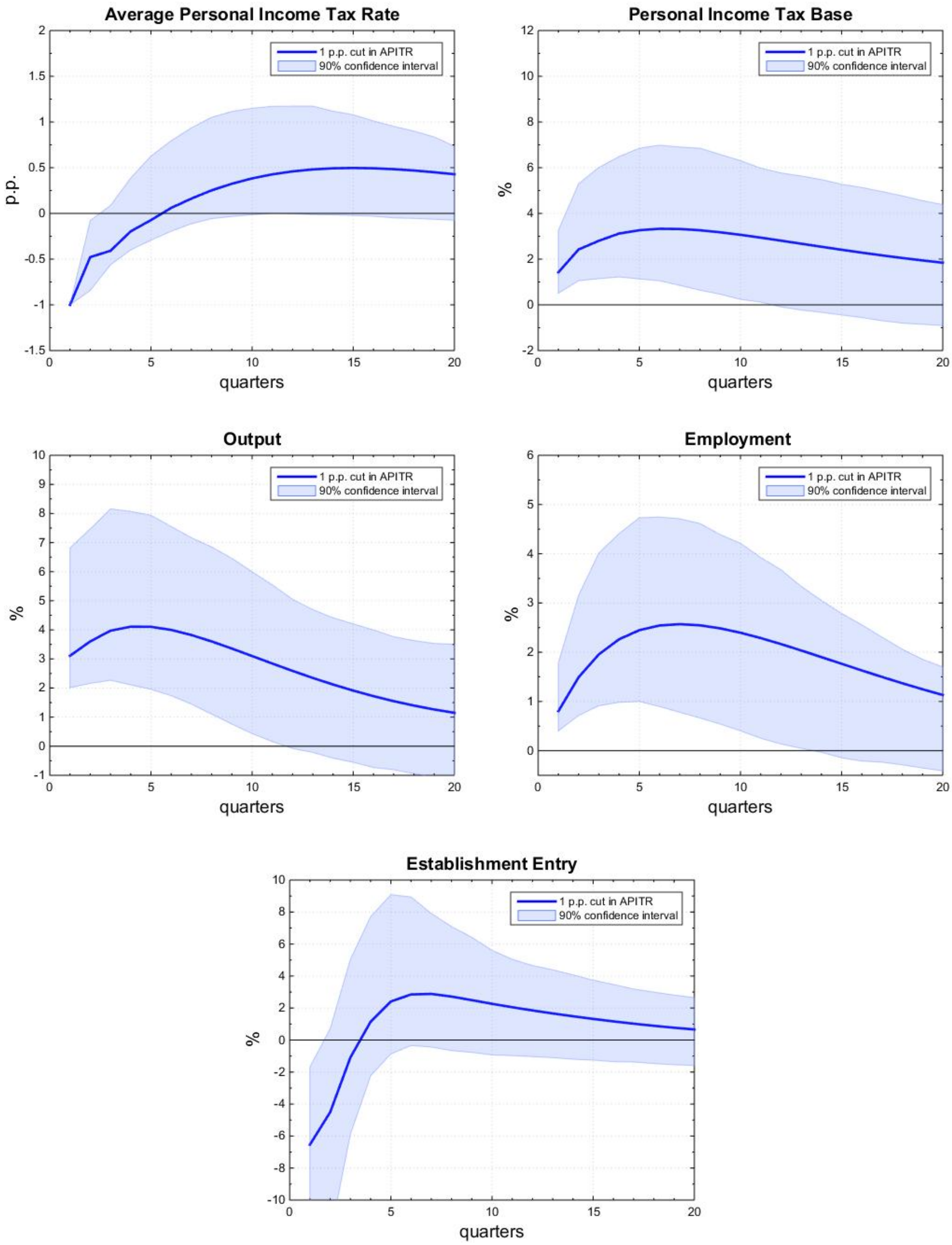
3.A.2 VAR with personal taxes

In order to test for the robustness of the qualitative response of establishment entry, we perform an analogous estimation as in our baseline, and an augmented VAR specification with the private income tax rate instead of the corporate tax rate. In particular, our five VAR variables include the private income tax rate, the private income tax base, employment, real output, and establishment entry. The estimation strategy is identical to the preceding one except that we use a different external instrument as a proxy for exogenous private income tax changes. All additional data is from Mertens and Ravn (2013). To the extent that differentiation between the private and the corporate tax rate should be irrelevant for entrants (albeit relevant for existing firms in the market) since their profits are de facto nonexistent before entry, we obtain a comparatively similar response of establishment entry. In both cases, the evidence suggests a short-term decrease and medium-term increase in entry rate in response to a tax cut.

3.A.3 State-level regressions

In this appendix, we report full results tables of the effect of corporate income tax increases on different endogenous variables, controlling for government per-capita spending. In Tables 3.6 to 3.17, all three columns show the effect of corporate tax shocks while controlling for state investment tax credit and R&D tax credit, loss carryback rule, loss carry forward rule and per capita government spending. The first column presents the results of the difference-in-difference estimation. In the second column, we report estimates that employs standard identification using average state corporate tax interacted with the state dummy as an instrumental variable. To asses the appropriateness of this instrument, we carry out tests of over-identification and orthogonality assumptions as well as the strength of the instruments. As the first test, we examine the F-statistics of the first-stage regression of our endogenous variable on the instruments. To asses the validity of our instrument, we report the p-values of Wu-Hausmann statistics and the p-value of Sargan statistics. In column (3), we use identification by heteroskedasticity introduced in Lewbel (2012), with a lags structure chosen such that p-value for the Hansen test of over-identification (Hansen J statistics) and p-value for instrument exogeneity test (C statistics) are such that we do not reject null hypotheses. With this approach, for job destruction by exiting firms, the appropriate lag is a one-year tax change. We choose two-year tax changes for wages of all workers, newly hired and stay workers, while for the remaining dependent variables, the first three-year lags are chosen. To test for the weakness of the instrument, we compare a Cragg-Donald statistic to critical values for instrument weakness developed by Stock and Yogo (2005). All regressions include state fixed effects and time fixed effects; standard errors are clustered by state and reported in brackets.

Figure 3.5: Impulse Responses: Augmented VAR Model with Personal Taxes



Notes: Figures show impulse responses to a 1 p.p cut in the average private income tax rate (APITR).

Table 3.6: Output

	(1) FE	(2) IV	(3) het-IV
Corporate Tax	-0.21	-0.27	-0.35**
	(0.173)	(0.392)	(0.156)
Investment Tax Credit	0.09**	0.09***	0.05***
	(0.037)	(0.035)	(0.019)
R&D Tax Credit	0.01	0.01	0.02
	(0.028)	(0.026)	(0.025)
Government Spending	0.44***	0.44***	0.44***
	(0.031)	(0.029)	(0.018)
Observations	768	768	672
R-squared	0.767	0.766	0.730
1st stage F-stat		3.973	
Prob>Wu-Hausmann		0.886	
Prob>Sargan		0.465	
Cragg-Donald statistic			24.41
Prob>Hansen J			0.934
Prob>C stat			0.830

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.7: Employment Rate

	(1) FE	(2) IV	(3) het-IV
Corporate Tax	-0.04	0.05	-0.21
	(0.060)	(0.088)	(0.162)
Investment Tax Credit	0.10***	0.10***	0.03***
	(0.028)	(0.027)	(0.013)
R&D Tax Credit	-0.01	-0.01	0.00
	(0.014)	(0.013)	(0.019)
Government Spending	0.04**	0.04***	0.05***
	(0.016)	(0.015)	(0.011)
Observations	768	768	672
R-squared	0.825	0.825	0.768
1st stage F-stat		3.973	
Prob>Wu-Hausmann		0.742	
Prob>Sargan		1	
Cragg-Donald statistic			24.41
Prob>Hansen J			0.669
Prob>C stat			0.661

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.8: Establishment Entry

	(1) FE	(2) IV	(3) het-IV
Corporate Tax	-0.25	-4.12***	-1.48*
	(0.871)	(0.999)	(0.832)
Investment Tax Credit	0.14	0.13	0.05
	(0.148)	(0.141)	(0.089)
R&D Tax Credit	-0.01	0.02	0.17
	(0.147)	(0.143)	(0.138)
Government Spending	-0.03	-0.02	-0.02
	(0.071)	(0.072)	(0.057)
Observations	768	768	672
R-squared	0.231	0.216	0.225
1st stage F-stat		3.973	
Prob>Wu-Hausmann		0.0487	
Prob>Sargan		0.469	
Cragg-Donald statistic			24.41
Prob>Hansen J			0.372
Prob>C stat			0.545

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.9: Establishment Exit

	(1) FE	(2) IV	(3) het-IV
Corporate Tax	0.74	2.49***	-0.54
	(0.511)	(0.945)	(0.489)
Investment Tax Credit	0.17	0.17**	0.07
	(0.101)	(0.086)	(0.071)
R&D Tax Credit	-0.01	-0.03	0.03
	(0.111)	(0.106)	(0.124)
Government Spending	-0.01	-0.01	0.03
	(0.066)	(0.063)	(0.058)
Observations	768	768	672
R-squared	0.407	0.404	0.412
1st stage F-stat		3.973	
Prob>Wu-Hausmann		0.299	
Prob>Sargan		0.523	
Cragg-Donald statistic			24.41
Prob>Hansen J			0.120
Prob>C stat			0.173

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.10: Job Creation Entry

	(1) FE	(2) IV	(3) het-IV
Corporate Tax	0.82	-6.86***	-0.12
	(1.029)	(2.047)	(0.833)
Investment Tax Credit	0.08	0.07	0.10
	(0.136)	(0.115)	(0.127)
R&D Tax Credit	-0.13	-0.06	-0.04
	(0.393)	(0.377)	(0.335)
Government Spending	0.11	0.13	0.10
	(0.108)	(0.102)	(0.082)
Observations	768	768	672
R-squared	0.180	0.148	0.165
1st stage F-stat		3.973	
Prob>Wu-Hausmann		0.00553	
Prob>Sargan		0.343	
Cragg-Donald statistic			24.41
Prob>Hansen J			0.402
Prob>C stat			0.235

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.11: Job Creation Expansions

	(1) FE	(2) IV	(3) het-IV
Corporate Tax	0.07	-0.73	-0.80***
	(0.378)	(0.681)	(0.236)
Investment Tax Credit	0.14***	0.14***	0.06*
	(0.050)	(0.045)	(0.035)
R&D Tax Credit	0.12	0.12	0.08
	(0.078)	(0.077)	(0.069)
Government Spending	0.12***	0.12***	0.14***
	(0.041)	(0.038)	(0.031)
Observations	768	768	672
R-squared	0.667	0.666	0.680
1st stage F-stat		3.973	
Prob>Wu-Hausmann		0.327	
Prob>Sargan		0.473	
Cragg-Donald statistic			24.41
Prob>Hansen J			0.375
Prob>C stat			0.180

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.12: JOB DESTRUCTION EXIT

	(1) FE	(2) IV	(3) het-IV
Corporate Tax	1.84*	1.85	1.47*
	(1.059)	(2.798)	(0.769)
Investment Tax Credit	0.07	0.07	0.08
	(0.126)	(0.121)	(0.121)
R&D Tax Credit	0.16	0.16	0.43
	(0.513)	(0.485)	(0.444)
Government Spending	0.04	0.04	0.12
	(0.103)	(0.098)	(0.097)
Observations	768	768	720
R-squared	0.322	0.322	0.317
1st stage F-stat		3.973	
Prob>Wu-Hausmann		0.997	
Prob>Sargan		0.738	
Cragg-Donald statistic			44.33
Prob>Hansen J			0.541
Prob>C stat			0.276

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.13: JOB DESTRUCTION CONTRACTIONS

	(1) FE	(2) IV	(3) het-IV
Corporate Tax	0.21	-0.78	-0.23
	(0.233)	(0.694)	(0.276)
Investment Tax Credit	0.11	0.10	0.06
	(0.075)	(0.072)	(0.041)
R&D Tax Credit	-0.04	-0.03	-0.06
	(0.066)	(0.060)	(0.054)
Government Spending	-0.03	-0.03	-0.03
	(0.048)	(0.046)	(0.040)
Observations	768	768	672
R-squared	0.736	0.734	0.732
1st stage F-stat		3.973	
Prob>Wu-Hausmann		0.279	
Prob>Sargan		0.403	
Cragg-Donald statistic			24.41
Prob>Hansen J			0.738
Prob>C stat			0.171

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.14: Real Wage per Worker, BEA

	(1) FE	(2) IV	(3) het-IV
Corporate Tax	-0.11	-0.33	-0.36***
	(0.125)	(0.316)	(0.077)
Investment Tax Credit	0.05	0.05*	0.01
	(0.030)	(0.029)	(0.012)
R&D Tax Credit	0.00	0.00	0.03*
	(0.020)	(0.019)	(0.019)
Government Spending	0.05***	0.05***	0.04***
	(0.014)	(0.013)	(0.010)
Observations	768	768	672
R-squared	0.535	0.534	0.497
1st stage F-stat		3.973	
Prob>Wu-Hausmann		0.409	
Prob>Sargan		0.189	
Cragg-Donald statistic			24.41
Prob>Hansen J			0.689
Prob>C stat			0.916

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.15: Hourly Wage All Workers, Zidar

	(1) FE	(2) IV	(3) het-IV
Corporate Tax	0.01	0.01	-0.01
	(0.014)	(0.013)	(0.015)
Investment Tax Credit	-0.01	-0.01	0.01
	(0.029)	(0.027)	(0.016)
R&D Tax Credit	-0.03	-0.03	-0.02
	(0.061)	(0.058)	(0.045)
Government Spending	0.04	0.04*	0.04**
	(0.022)	(0.021)	(0.019)
Observations	1,152	1,152	1,104
R-squared	0.234	0.234	0.222
1st stage F-stat		690.2	
Prob>Wu-Hausmann		0.557	
Prob>Sargan		0.971	
Cragg-Donald statistic			4971
Prob>Hansen J			0.817
Prob>C stat			0.131

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.16: Hourly Wage Stay Workers

	(1) FE	(2) IV	(3) het-IV
Corporate Tax	0.00	0.01	-0.01
	(0.015)	(0.014)	(0.015)
Investment Tax Credit	-0.01	-0.01	0.01
	(0.031)	(0.029)	(0.016)
R&D Tax Credit	0.01	0.01	0.04
	(0.058)	(0.056)	(0.045)
Government Spending	0.03*	0.03*	0.04**
	(0.019)	(0.018)	(0.017)
Observations	1,152	1,152	1,104
R-squared	0.206	0.206	0.189
1st stage F-stat		690.2	
Prob>Wu-Hausmann		0.459	
Prob>Sargan		0.571	
Cragg-Donald statistic			4971
Prob>Hansen J			0.648
Prob>C stat			0.225

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.17: Hourly Wage Newly Hired

	(1) FE	(2) IV	(3) het-IV
Corporate Tax	-0.86	-1.47	-2.74*
	(3.402)	(3.029)	(1.464)
Investment Tax Credit	0.00	0.02	-0.03
	(0.344)	(0.353)	(0.217)
R&D Tax Credit	0.52*	0.52**	0.47***
	(0.287)	(0.266)	(0.171)
Government Spending	0.03	0.03	-0.13
	(0.293)	(0.271)	(0.231)
Observations	580	580	545
R-squared	0.121	0.121	0.074
1st stage F-stat		3.790	
Prob>Wu-Hausmann		0.895	
Prob>Sargan		0.0192	
Cragg-Donald statistic			1218
Prob>Hansen J			0.676
Prob>C stat			0.424

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.18: Recursive Steady State Computation

Average firm's productivity	$\widetilde{z} = \left(\frac{\kappa}{\kappa-(\theta-1)}\right)^{\frac{1}{\theta-1}} z^*$
Marginal firm's productivity	$z^* = \frac{z_{\min}}{(1-\delta)^{\frac{1}{\kappa}}}$
Marginal firm's profits	$d^* = 0$
Fixed production cost	$f^c = f^e\big[\left(\frac{\widetilde{z}}{z^*}\right)^{\theta-1} - 1\big]^{-1}\frac{1-\beta(1-\delta)}{\beta(1-\delta)\left(1-\tau^d\right)}$
Marginal firm's output	$y^* = \frac{z^*}{(\mu-1)}f^c$
Average firm's output	$\widetilde{y} = y^*\left(\frac{\widetilde{z}}{z^*}\right)^{\theta}$
Average firm's labor input	$\widetilde{l}^c = \frac{\widetilde{y}}{Z\widetilde{z}}$
Number of firms	$N = \frac{L}{\widetilde{l}^c+\frac{f^c}{Z}+\frac{\delta}{1-\delta}f^e}$
Number of entrants	$N^e = \frac{\delta}{1-\delta}N$
Consumption	$C = Z\widetilde{z}\widetilde{l}^c N^{\frac{\theta}{\theta-1}}$
Wage	$w = \frac{C}{\frac{1-\beta}{1-\beta(1-\delta)}\left[\left(\frac{\widetilde{z}}{z^*}\right)^{\theta-1}-1\right]\frac{wf^c}{Z}N+L}$
Average firm's value	$\widetilde{v} = \frac{wf^e}{Z}$
Average firm's profits	$\widetilde{d} = \frac{1-\beta(1-\delta)}{\beta(1-\delta)}\widetilde{v}$
Average firm's revenue	$\widetilde{rr} = \left(\mu\frac{w}{Z\widetilde{z}}\right)^{1-\theta}C$
Marginal firm's revenue	$rr^* = \left(\frac{\widetilde{z}}{z^*}\right)^{1-\theta}\widetilde{rr}$
Weight on labor in utility	$\chi = \frac{w}{CL^{\frac{1}{\phi}}}$

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