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Monetary and Fiscal Policy in Small Open Economies

Aliya Algozhina

Dissertation

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To my aunt Tatyana Makatova. An intelligent, radiant, spiritually rich, and gorgeous lady, who died from cancer in April 2013.

Contents

Abstract

This dissertation studies monetary and fiscal policies jointly, with their respective policy rules in a small open economy. Policy interactions have attracted new research interest since the 2008 crisis due to a global increase in fiscal debt. The first chapter extends the standard New Keynesian model of a small open economy with the structural specifics relevant for emerging market countries: two instruments of monetary policy – interest rate and foreign exchange interventions, two instruments of fiscal policy – public consumption and public investment, two types of households – forward-looking and rule-of-thumb consumers, and foreign debt via collateral constraint. Imperfect capital mobility is assumed, as foreign borrowings are restricted and there is a positive steady state difference between the domestic and foreign interest rates, due to more impatient households in the domestic economy. Parameters are calibrated for Hungary and the model's simulation is compared between two cases: with and without a collateral constraint.

The results show that fiscal and monetary policy shocks transmit to the economy differently from the standard Mundell-Fleming model. A positive public investment shock can cause exchange rate depreciation and crowd out private capital, expanding output due to the accumulation of public capital. Public consumption, in contrast, alters the demand side of the economy, appreciating the exchange rate and stimulating foreign debt. A monetary policy shock as a sudden increase in the domestic interest rate affects the main variables consistent with the New Keynesian prediction of nominal rigidities, yet the dynamics of foreign debt depend on whether a collateral constraint is on or off the model. The collateral constraint makes the model volatile, due to its Lagrange multiplier entering the uncovered interest rate parity and affecting the exchange rate.

The second chapter expands the model further by concentrating on a subset of developing countries that export oil. Thus, the oil production sector, a Sovereign Wealth Fund (SWF), and a world oil price shock are additionally included. The two types of monetary policy rule, CPI targeting and product price targeting (PPT) according to Frankel and Catao (2011), are examined across exchange rate regimes and pro/counter/acyclical fiscal stance based on a loss measure. The loss measure is, according to De Paoli (2009), represented as a sum of variances in domestic price inflation, aggregate output, and real exchange rate that is minimized to find the optimal Taylor rule in a small open economy.

Based on calibration for Kazakhstan, the study reveals that the best policy combination is a countercyclical fiscal stance and managed exchange rate regime with the PPT monetary anchor. This allows the fiscal policy to countercyclically offset a volatile terms of trade shock, to which developing countries are often exposed. It also allows the exchange rate to be managed by the central bank's interventions, which seem beneficial in providing a stable exchange rate since the economy borrows from abroad, imports foreign goods, and depends on the world oil price. It also suggests the appropriate monetary policy to target product price inflation, which includes oil price inflation that is important for the oil sector's exports and delivers a better stabilization of exchange rate than the CPI anchor. However, if a flexible exchange rate regime is institutionally chosen, then the CPI targeting should be adopted, since it effectively stabilizes the domestic price inflation and aggregate output.

The third chapter in this dissertation finds an optimal public investment path for resource-rich low-income countries by modifying a perfect foresight general equilibrium model of Berg, Portillo, Yang, and Zanna (2013) in several respects: The policy rule for public capital is introduced. Public capital accumulation involves the effective public investment with its absorptive capacity constraint costs captured by a single parameter. External saving is an additional fiscal instrument which clears the government budget. There is a variable share of resource revenues to accumulate the SWF, and the natural resource sector has its real FDI shock. Based on calibration for African countries, the study finds that the front-loaded public investment path is optimal given an initial one-period resource windfall, absorptive capacity constraints in the economy, and capital scarcity. This result also holds under less productive public capital, while a scenario of no resource windfall produces the welfare loss due to a steady increase in consumption tax to finance public investment.

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Aliya Algozhina

Introduction

Macroeconomic models have been extensively transformed from their real business cycle theory, which explains economic fluctuations by a technology shock, to the New Keynesian framework, which includes monopolistic competition and nominal rigidities to highlight a non-neutrality of the monetary sector. The latter dynamic stochastic general equilibrium (DSGE) models have provided a foundation for monetary policy analysis not only among academic researchers, but also practitioners in various institutions, including central banks. Since 1993, monetary policy has been conveniently specified by its Taylor rule, which interprets the reaction function of a central bank in a simplified manner.

Fiscal policy, in contrast, does not have a universally accepted rule, apart from a standard government budget constraint, and may vary depending on models' specifics and country experiences. The fiscal role is, therefore, often downplayed and relatively limited in the field of monetary economics, which basically supports the independence of central banks. The traditional quantity theory of price level has been recently challenged by the fiscal theory of price level, and thus concepts of monetary dominance versus fiscal dominance have emerged. The global crisis of 2008, when many countries accumulated their fiscal debt, has brought attention to monetary and fiscal policy interactions and whether a central bank is capable alone of achieving its price stability goal.

This dissertation studies monetary and fiscal policies jointly, with their respective policy rules in a single DSGE framework for emerging market economies. The first paper extends the standard New Keynesian model of a small open economy with the structural specifics relevant for emerging market countries: two instruments of monetary policy – interest rate and foreign exchange interventions, two instruments of fiscal policy – public consumption and public investment, two types of households – forward-looking and rule-of-thumb consumers, and foreign debt via collateral constraint. The second paper expands the model further by concentrating on a subset of developing countries that export oil. Thus, the oil production sector, Sovereign Wealth Fund (SWF)[1], and world oil price shock have been additionally included. The two types of monetary policy rule, CPI targeting and product price targeting (PPT) according to Frankel and Catao (2011), are examined across exchange rate regimes and pro/counter/acyclical fiscal stance. The third paper abstracts from monetary policy and stochastic framework, finding an optimal public investment path for resource-rich low-income countries in a perfect foresight general equilibrium model.

[1]Since 2000, the number and assets of SWFs have rapidly increased because of commodity prices and Asian current account surpluses (Klitzing, Lin, Lund & Nordin, 2010). Countries tend to have small financial systems to absorb huge capital inflows and therefore set up SWFs separately from their government and central bank to prepare for the future when surpluses may cease. The commodity-exporting countries thereby try to cushion the effects of volatile world prices on the economy and budget, still having a positive fiscal debt to keep their limited securities market functional, including open market operations by the central bank.

1 Monetary and Fiscal Policy Interactions in an Emerging Open Economy: A Non-Ricardian DSGE Approach

[2]

1.1 Introduction

For over two decades remarkable progress has been made in macroeconomic modeling by synthesizing the New Keynesian theory and the real business cycle theory. As a result, in recent years macroeconomic linkages have been intensively modeled using a DSGE approach, which primarily highlights the influential role of monetary policy (Christiano, Eichenbaum & Evans, 2005; Smets & Wouters, 2007). Central banks worldwide develop their core DSGE models to frame their policy decisions, discuss clearly the sources of fluctuations, and perform counterfactual policy experiments. Although there are still challenges (Tovar, 2009), the DSGE models inject an increased discipline to judgement, thinking, and communication about monetary policy. Apart from advanced economies, those models are estimated for emerging markets as well (Castillo, Montoro & Tuesta, 2013; Silveira, 2008; Andrle, Hledik, Kamenik & Vlcek, 2009; Jakab & Vilagi, 2008; Zeman & Senaj, 2009; Iordanov & Vassilev, 2008; Lee, 2012). However, fiscal policy in this framework is usually passive; thus, it is either ignored or specified simply by a balanced government budget with a limited role for fiscal debt. In other words, Ricardian equivalence holds, due to forward-looking consumers, and monetary dominance is assumed, resulting in a weak fiscal side in the models.

Yet, the post-crisis situation shows that active fiscal policy has been implemented

[2] The earlier version of this work was issued as the *CERGE-EI Working Paper 2012, 476,* 1-28.

globally, causing high fiscal debt across countries. In addition, the developed world has reached its zero lower bound of interest rates, when expansionary fiscal policy can be quite effective in terms of stimulating economic activity (Christiano, Eichenbaum & Rebelo, 2011; Eggertsson, 2011; Woodford, 2011) and may interact with the influential monetary policy that should be captured jointly in a model. Even earlier, Benigno and Woodford (2003) pointed out the problem of modeling these two policies in isolation, which appeared to be more inter-related than expected, based on their analysis of optimal monetary and fiscal policy within a single framework.

The consequences of one policy decision to another occur because, on the one hand, the interest rate set by the monetary policy affects a burden of fiscal debt, which may appropriately adjust in response to the interest rate change, while on the other hand, fiscal stimulus changes output, which may in turn adjust a tradeoff between inflation and output facing the monetary policy. Moreover, according to the fiscal theory of price level (Leeper, 1991; Sims, 1994; Cochrane, 2011), a persistent fiscal deficit without tax or spending adjustments inevitably causes inflation, thus constraining monetary policy to achieve its goal of price stability. Both policies, therefore, should take into account the consequences of their decisions on the targets of the other policy in order to be consistent and endogenously effective in a macroeconomic outcome.

Currently, there are two streams of literature on monetary and fiscal policy interactions in a DSGE framework. The first one deals with the optimal policy rules, assuming that either tax or government spending is the only fiscal instrument modeled jointly with the Taylor-type monetary policy rule (Schmitt-Grohe & Uribe, 2007; Leith & Wren-Lewis, 2013; Chadha & Nolan, 2007). The second stream focuses on the fiscal multiplier defined as the ratio of a change in output to an exogenous change in the fiscal instrument (Woodford, 2011; Davig & Leeper, 2011; Cogan, Cwik, Taylor & Wieland, 2010; Christiano, Eichenbaum & Rebelo, 2011; Eggertsson, 2011), apart from the various econometric estimations, which generally suffer from endogeneity, proper identification of fiscal shocks without any mix with automatic stabilizers, and ignorance of fiscal debt dynamics. Both of these streams of DSGE models, though, do not impose any heterogeneity of households, assuming instead a representative agent who optimizes his future consumption path by appropriate savings. This might result in a relatively low fiscal multiplier, because once there is a fiscal expansion, active monetary policy tightens and a high interest rate encourages households to save rather than to consume; thus, consumption declines.

Realizing this problem in assessing fiscal stimulus, researchers have suggested incorporating two types of households: savers or traditional Ricardian households who are also known as the standard optimizers having savings in assets, and spenders or non-Ricardian households who do not have access to financial markets and simply consume their disposable income each period (Mankiw, 2000). The latter type is sometimes referred to as the rule-of-thumb or liquidity-constrained households[3] in the literature. Gali, Lopez-Salido, and Valles (2007) have extended, therefore, the standard New Keynesian model by incorporating these two types of households which allowed them to demonstrate that government spending has an effect on consumption consistent with the evidence, due to interaction between the behavior of the rule-of-thumb consumers and sticky prices. A global integrated monetary and fiscal policy model constructed at the IMF (Kumhof, Laxton, Muir & Mursula, 2010) also distinguishes these two types of households and estimates that multipliers of two-year stimulus range from 0.2 to 2.2 depending on the fiscal instrument, the extent of monetary accommodation, and the presence of a financial accelerator mechanism (Freedman, Kumhof, Laxton, Muir & Mursula, 2009).

However, the above models are applicable to the developed world and do not take into account three structural specifics relevant for an emerging open economy. First, an emerging economy conducts its hybrid monetary policy using at least two instruments: the interest rate in accordance with the standard Taylor rule to target inflation and foreign exchange interventions to manage the exchange rate (Ghosh, Ostry & Chamon, 2016). Second, fiscal policy can be active trying to stimulate the economy through an increase of public consumption and/or public investment[4] and not through cutting taxes, which are

[3]The liquidity-constrained households are relevant to assume especially in the emerging market countries, which often have their underdeveloped financial system domestically.

[4]Public consumption and public investment should be treated as the two separate fiscal policy instruments, because the former stimulates demand, while the latter can affect supply by accumulating public capital in a production function. According to Aschauer (1989), there is a positive relationship between

relatively inflexible to change and distortionary. Third, emerging economies often have a private sector heavily indebted to the foreign world due to their underdeveloped domestic financial market to finance investments. Thus, they are vulnerable to an external shock of sudden stops, which is exactly the case they faced due to the global financial crisis of 2008. Moreover, sudden stops seem to be related to collateral constraint (Mendoza, 2010; Mendoza, 2006; Chari, Kehoe & McGrattan, 2005; Kiyotaki & Moore, 1997) rather than to a financial accelerator mechanism à la Bernanke, Gertler, and Gilchrist (1999). This is because a sudden shrinkage of foreign funds supply can abruptly cause an economic downturn in developing economies, which used to constantly have capital inflows earlier.

The aim of this paper, therefore, is to build a DSGE model for an emerging open economy capturing these three structural specifics with a collateral constraint on foreign borrowings. Based on the constructed model, the focus is to understand how multiple instruments of monetary and fiscal policy interact to jointly affect the economy under two cases: with and without a collateral constraint. Imperfect capital mobility is assumed, as foreign borrowings are restricted and there is a positive steady state difference between the domestic and foreign interest rates, due to more impatient households in the domestic economy. According to the impossible trinity, therefore, an independent monetary policy in terms of inflation targeting and a managed exchange rate regime are feasible under imperfect capital mobility. In addition to monetary and fiscal policy shocks (interest rate, public investment, and public consumption), the impulse-response functions to a foreign demand shock are also examined, which increase the foreign interest rate.

The model takes stock of existing relevant studies, combining them and modifying their approaches to fit the specifics of an emerging market economy. In particular, the New Keynesian framework of a small open economy laid out in Gali (2015) is extended by two types of households, similar to Gali, Lopez-Salido, and Valles (2007), and introducing a foreign exchange interventions rule, like in Benes, Berg, Portillo, and Vavra (2015). The collateral constraint is borrowed from Faia and Iliopulos (2011), yet it is pinned on

public investment and the growth rate of labor productivity, while public consumption is negatively related to the growth of output per hour in the G-7 countries.

private physical capital, as opposed to durable goods. The Cobb-Douglass production function has three inputs: labor, private capital, and public capital, which is accumulated by public investment (Traum & Yang, 2015).

The calibration of the model is based on Hungary as the first economy among all emerging markets severely hit by the global financial crisis already felt in mid-October 2008. International organizations were called on for support using their emergency financing arrangements. In 2009, Hungarian real GDP fell by 6.7 percent, the euro-forint exchange rate depreciated by 12 percent, unemployment increased to 9.8 percent, positive net exports were 10 times higher than in 2008 due to the collapse in imports and capital outflows, and foreign exchange reserves of the central bank dropped significantly, especially in the second quarter of 2009. The main vulnerability of Hungary originated from its high public and private sector debt: fiscal debt amounted to 66 percent of GDP, while external debt reached 97 percent of GDP at the end of 2007 (IMF, 2008).

In section 1.2, the model is outlined with its two types of households, standard optimizers and rule-of-thumb households, firms acting in a monopolistically competitive market, two monetary policy rules, and respective fiscal policy rules. Section 1.3 describes the calibrated values for parameters, the list of which is provided in Appendix 1.6.1. Section 1.4 examines the impulse-response functions and volatilities with a collateral constraint and without it, including sensitivity to a higher population fraction of rule-of-thumb households than in the baseline simulation. Section 1.5 concludes.

1.2 Model

The model has several frictions: an incomplete asset market, capital adjustment costs, collateral constraint, and the Calvo price setting. The crucial underlying assumption is that the foreign world is a saver, while the domestic economy is a borrower; thus, the foreign discount factor is higher than the domestic discount factor, as the domestic households might be relatively impatient compared to the foreign world. This assumption implies in turn that the interest rate of an emerging economy is always higher than the foreign interest rate, which is consistent with the evidence, reflecting a possible default risk premium and/or expectations about future exchange rate depreciation.

Since there are two types of households, only forward-looking consumers borrow from abroad and have a collateral constraint on physical capital. They also hold the domestic government bonds, own firms, rent capital to those firms, decide about investment, and receive transfers from the central bank. The firms monopolistically set prices on their intermediate goods à la Calvo (1983) and their profits are transferred to the forward-looking households. Labor market is assumed to be competitive, without unions or high households' bargaining power over wages in the emerging market setting.

The Taylor rule includes lagged interest rate, CPI inflation, and output, but there is also a rule for the foreign exchange reserves responding to the exchange rate and its change (Benes et al., 2015). Public consumption and public investment respond to fiscal debt and output, capturing a procyclical fiscal policy.

The foreign world is exogenously captured by its Phillips curve, AR(1) process for output, and the Taylor rule for its interest rate. All foreign variables are denoted by an asterisk.

1.2.1 Households

The economy is populated by a continuum of households on the interval [0,1], where the fraction μ is rule-of-thumb households. They do not have access to financial markets and consume all of their disposable income each period. In other words, they act myopically without any effect of a future policy on their economic decisions. The other $(1-\mu)$ fraction of households are forward-looking households who hold government bonds, invest in private capital, rent the capital to firms, borrow from abroad, and receive profits from the monopolistic firms and transfers from the central bank. The labor market is competitive, wages are the same across all households, and both types of households work the same number of hours. The superscript S indicates a variable associated with savers (forward-looking consumers) and N with non-savers (rule-of-thumb households).

The forward-looking household maximizes its utility (Schmitt-Grohe & Uribe, 2003):

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{[C_t^S - \phi^{-1} N_t^{\phi}]^{1-\sigma} - 1}{1-\sigma}, \qquad \phi > 1,\ \sigma > 1 \tag{1}$$

subject to the following budget constraint:

$$C_t^S + I_t^S + b_t + R_{t-1}^* \frac{e_t}{e_{t-1}} \frac{b_{t-1}^*}{\pi_t} + T_t^S = W_t N_t + R_t^k K_{t-1}^S + R_{t-1} \frac{b_{t-1}}{\pi_t} + b_t^* + \Pi_t + CB_t, \tag{2}$$

where $b_t = \frac{B_t}{P_t}$ is the real purchases of government bonds, e_t is a nominal exchange rate (the units of domestic currency per unit of a foreign currency), $b_t^* = e_t \frac{B_t^*}{P_t}$ is the real foreign borrowings denominated in composite consumption goods, R_{t-1} and R_{t-1}^* are the nominal gross domestic and foreign interest rates, T_t^S is the real lump-sum taxes, W_t is a real wage, R_t^k is the real rental cost of private capital, $\pi_t = \frac{P_t}{P_{t-1}}$ is inflation, Π_t is the real profits transferred from the monopolistic firms[5], and CB_t is the central bank's transfers in a form of real foreign exchange reserves (see equation 23).

The law of motion for private capital is specified according to Gali, Lopez-Salido, and Valles (2007):

$$K_t^S = (1-\delta) K_{t-1}^S + \kappa \left(\frac{I_t^S}{K_{t-1}^S} \right) K_{t-1}^S, \qquad \kappa' > 0,\ \kappa'' \leq 0,\ \kappa'(\delta) = 1,\ \kappa(\delta) = \delta \tag{3}$$

The collateral constraint relates gross foreign liabilities to a future value of capital (durable goods in Faia & Iliopulos, 2011) and always binds, assuming that foreign debt is permanently high in this economy[6]:

$$R_t^* b_t^* = E_t \{ \Omega \frac{Q_{t+1} \pi_{t+1}}{e_{t+1}/e_t} K_t^S \}, \tag{4}$$

where Q_t is a real shadow value of capital (Tobin's Q) and Ω is an upper bound of leverage ratio.

[5] $\Pi_t = Y_t(p_{h,t} - MC_t)$, where Y_t is final output, $p_{h,t}$ is the relative domestic price of goods to composite consumption, and MC_t is the marginal costs of monopolistic firms to composite consumption.

[6] Occasionally binding collateral constraint is ruled out because it requires global solution methods, which may be infeasible to apply in this complex model.

The problem of forward-looking household is, therefore, to maximize utility (1) with respect to consumption C_t^S, investment I_t^S, capital K_t^S, government bonds b_t, foreign borrowings b_t^*, and labor N_t subject to the budget constraint (2), capital accumulation (3), and collateral constraint (4). The first-order conditions of this problem are below, where λ_t, λ_t^k, and $\lambda_t\lambda_t^c$ are the Lagrange multipliers to the constraints (2), (3), (4), respectively.

$$\frac{1}{\left[C_t^S - \frac{N_t^\phi}{\phi}\right]^\sigma} = \lambda_t \tag{5}$$

$$Q_t = \frac{1}{\kappa'\left(\frac{I_t^S}{K_{t-1}^S}\right)}, \quad \text{where } Q_t = \frac{\lambda_t^k}{\lambda_t} \tag{6}$$

$$Q_t = E_t\left\{\beta\frac{\lambda_{t+1}}{\lambda_t}\left[R_{t+1}^k + Q_{t+1}\left(1-\delta+\kappa(\frac{I_{t+1}^S}{K_t^S}) - \kappa'(\frac{I_{t+1}^S}{K_t^S})\frac{I_{t+1}^S}{K_t^S}\right)\right] + \lambda_t^c\Omega\frac{Q_{t+1}\pi_{t+1}}{e_{t+1}/e_t}\right\} \tag{7}$$

$$\frac{1}{R_t} = \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t\pi_{t+1}}\right\} \tag{8}$$

$$\frac{1}{R_t^*} = \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\frac{e_{t+1}}{e_t\pi_{t+1}}\right\} + \lambda_t^c \tag{9}$$

$$W_t = N_t^{\phi-1} \tag{10}$$

The rule-of-thumb household has the same preferences as the saver. It chooses only consumption C_t^N and labor N_t and its budget constraint is simply this:

$$C_t^N + T_t^N = W_tN_t \tag{11}$$

The first-order conditions with respect to C_t^N and N_t are identical to the saver's solutions. Thus, the rule-of-thumb household faces the same labor supply condition (10).

Each $i \in \{S, N\}$ type of household has the CES consumption preferences over domestic and foreign goods with $\eta > 0$ as an elasticity of substitution between goods:

$$C_t(i) = \left[\gamma^{\frac{1}{\eta}}C_{H,t}^{\frac{\eta-1}{\eta}}(i) + (1-\gamma)^{\frac{1}{\eta}}C_{F,t}^{\frac{\eta-1}{\eta}}(i)\right]^{\frac{\eta}{\eta-1}},$$

where γ is a home-bias parameter, while $(1-\gamma)$ is a degree of openness. The standard consumption expenditures minimization by a household delivers the following CPI index:

$$P_t = \left[\gamma P_{h,t}^{1-\eta} + (1-\gamma) P_{f,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \quad \text{or} \quad 1 = \gamma p_{h,t}^{1-\eta} + (1-\gamma) RER_t^{1-\eta},$$

where $p_{h,t}$ is a relative price of domestic goods to composite consumption and $RER_t = \frac{e_t P_t^*}{P_t}$ is a relative price of foreign goods to composite consumption (real exchange rate).

The aggregate consumption is a sum of two households' consumption: $C_t = \mu C_t^N + (1-\mu) C_t^S$. Similar to private consumption, investment is the CES basket with the same home-bias parameter γ and CPI for simplicity.

1.2.2 Firms

Following the basic New Keynesian framework, there are monopolistically competitive firms producing differentiated intermediate goods, and a perfectly competitive firm producing a final domestic good. The final domestic producer has a constant returns technology:

$$Y_t = \left(\int_0^1 V_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $V_t(j)$ is the input amount of intermediate good j and $\varepsilon > 1$ is the elasticity of substitution between differentiated intermediate goods. It maximizes profit taking as given the domestic final good's price P_t^h and intermediate goods' prices $P_t^h(j)$ such that the optimal demand allocation is as follows:

$$V_t(j) = \left(\frac{P_t^h(j)}{P_t^h} \right)^{-\varepsilon} Y_t \tag{12}$$

Each intermediate goods firm maximizes its profit subject to three constraints. First, the identical Cobb-Douglass production function includes private capital, labor, and public capital:

$$Y_t(j) = K_{t-1}(j)^{\alpha} N_t(j)^{1-\alpha} K_{G,t-1}^{\psi}, \tag{13}$$

where the usage of public capital is common to all intermediate firms. The second constraint is the demand schedule each firm j faces (12). The third one is that some firms cannot adjust their prices due to price stickiness (Calvo, 1983). Each period, a fraction $(1-\theta)$ of firms adjusts their prices, while the respective fraction θ keeps their prices unchanged; thus, θ is an index of price stickiness and the domestic price index evolves as follows:

$$(P_t^h)^{1-\varepsilon} = \theta(P_{t-1}^h)^{1-\varepsilon} + (1-\theta)(P_t^{h*})^{1-\varepsilon}$$

Cost minimization, taking the wage and rental cost of capital denominated in domestic goods as given, provides the following real marginal costs common to all intermediate firms:

$$mc_t = \frac{w_t^{1-\alpha}(r_t^k)^\alpha}{K_{G,t-1}^\psi (1-\alpha)^{1-\alpha}\alpha^\alpha} \tag{14}$$

The price setting decision involves picking $P_t^{h*}$to maximize

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ D_{t,t+k} Y_{t+k}(j) \left(\frac{P_t^{h*}}{P_{t+k}^h} - mc_{t+k} \right) \right\},$$

where $D_{t,t+k} = \beta^k E_t(\frac{U_{C_{t+k}^S}}{U_{C_t^S}})$ is a stochastic discount factor coming from the forward-looking household's problem, subject to the demand constraint according to (12):

$$Y_{t+k}(j) = \left(\frac{P_t^{h*}}{P_{t+k}^h} \right)^{-\varepsilon} Y_{t+k}$$

The first-order condition of this price setting decision is as follows:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ D_{t,t+k} Y_{t+k}(j) \left(\frac{P_t^{h*}}{P_{t+k}^h} - \frac{\varepsilon}{\varepsilon - 1} mc_{t+k} \right) \right\} = 0, \tag{15}$$

where $\frac{\varepsilon}{\varepsilon-1}$ is a frictionless price markup.

The Phillips curve for a small open economy is derived according to Gali (2015) in Appendix 1.6.3. It differs from the standard closed-economy version due to a distinction between domestic price inflation and CPI inflation; thus, the economy's average real marginal costs include the real exchange rate in addition.

1.2.3 Fiscal policy

The government collects lump-sum taxes T_t and issues one-period bonds b_t to finance the government purchases, which include public consumption G_t^C and public investment G_t^I. The government budget constraint in real terms is as follows:

$$(1-\mu)b_t + T_t = p_t^g(G_t^C + G_t^I) + (1-\mu)R_{t-1}\frac{b_{t-1}}{\pi_t}, \tag{16}$$

where $T_t = (1-\mu)T_t^S + \mu T_t^N$ and p_t^g is a relative price of government purchases to composite consumption with its own home-bias parameter γ_2.

$$p_t^g = \left[\gamma_2 p_{h,t}^{1-\eta} + (1-\gamma_2)RER_t^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{17}$$

Public investment is productive so that the law of motion for public capital is given by:

$$K_{G,t} = (1-\delta^g)K_{G,t-1} + G_t^I \tag{18}$$

The two fiscal instruments, public investment and public consumption, have the following rules responding to fiscal debt and output in order to capture a business cycle (Traum & Yang, 2015):

$$\widehat{G_t^I} = \rho_{GI}\widehat{G_{t-1}^I} + (1-\rho_{GI})(\vartheta_{GI}\widehat{Y}_t - \gamma_{GI}\widehat{b}_{t-1}) + \epsilon_t^{GI} \tag{19}$$

$$\widehat{G_t^C} = \rho_{GC}\widehat{G_{t-1}^C} + (1-\rho_{GC})(\vartheta_{GC}\widehat{Y}_t - \gamma_{GC}\widehat{b}_{t-1}) + \epsilon_t^{GC} \tag{20}$$

Hats, hereafter, denote the deviation of variables from their steady state. Both types of public spending have exogenous shocks to be analyzed separately as the two distinct instruments of fiscal policy stimulus.

Since fiscal debt clears the government budget constraint, the lump-sum taxes require an additional equation, which includes fiscal debt and public spending similar to Gali, Lopez-Salido, and Valles (2007):

$$\widehat{T}_t = \varphi_b \widehat{b}_{t-1} + \varphi_I \widehat{G_t^I} + \varphi_C \widehat{G_t^C} \tag{21}$$

1.2.4 Monetary policy

The domestic interest rate is not equal to foreign interest rate and allows for an independent monetary policy under imperfect capital mobility. The nominal interest rate responds to its lagged value, CPI inflation, and output according to the Taylor rule below:

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + (1-\rho)\left[\phi_\pi \pi_t + \phi_y \widehat{Y}_t\right] + \epsilon_t, \tag{22}$$

where ρ is an interest rate smoothing parameter, ϕ_π and ϕ_y are the inflation and output responses, respectively. There is an exogenous interest rate shock to be examined along with fiscal shocks, which all follow AR(1) processes with a persistence parameter ρ_ϵ.

Every period, the central bank receives interest on its foreign exchange reserves and invests into a new stock of reserves. This flow of revenues is transferred to forward-looking households, savers:

$$CB_t = R^*_{t-1} \frac{e_t}{e_{t-1}} \frac{fxr^*_{t-1}}{\pi_t} - fxr^*_t, \tag{23}$$

where $fxr^*_t = e_t \frac{FXR^*_t}{P_t}$ is the real foreign exchange reserves in composite consumption goods.

A managed exchange rate regime is associated with the foreign exchange interventions as an additional monetary policy instrument. They represent the purchases/selling of foreign currency by a central bank, and accumulate the foreign exchange reserves according to their rule (Benes et al., 2015), responding to the exchange rate and its rate of depreciation[7].

$$\widehat{fxr^*_t} = \rho_{fxr} \widehat{fxr^*_{t-1}} + (1-\rho_{fxr})(\alpha_1 \widehat{e}_t + \alpha_2 \bigtriangleup \widehat{e}_t), \quad \alpha_1 < 0, \ \alpha_2 < 0 \tag{24}$$

[7]The higher $\widehat{e}_t$, the more the nominal exchange rate depreciates.

This rule shows that the more the exchange rate depreciates/appreciates, the more the foreign exchange reserves fall/accumulate, implying the selling/purchases of foreign currency by a central bank, respectively.

1.2.5 Market clearing conditions

For simplicity, an elasticity of substitution between domestic and foreign goods is assumed to approach one ($\eta \to 1$). Thus, the domestic goods market clearing condition is as follows:

$$p_t^h Y_t = \gamma \left[C_t + (1-\mu) I_t^S \right] + \gamma_2 p_t^g (G_t^C + G_t^I) + (1-\gamma) e_t y_t^*, \tag{25}$$

where the last term stands for the exports of domestic goods, which include the foreign demand/output $y_t^* = \frac{Y_t^*}{P_t}$, nominal exchange rate e_t, and openness parameter $(1-\gamma)$.

Labor, capital, and final goods markets clear according to the following conditions, respectively:

$$N_t = \int_0^1 N_t(j) dj, \qquad K_t = \int_0^1 K_t(j) dj = (1-\mu) K_t^S$$

$$Y_t = C_t + (1-\mu) I_t^S + p_t^g (G_t^C + G_t^I) + NX_t \tag{26}$$

The balance of payments equates its current account, net exports, with the financial account coming from the budget constraint of households-savers: foreign borrowings of households and foreign exchange reserves transferred to savers.

$$NX_t = (1-\mu) \left(R_{t-1}^* \frac{e_t}{e_{t-1}} \frac{b_{t-1}^*}{\pi_t} - b_t^* + fxr_t^* - R_{t-1}^* \frac{e_t}{e_{t-1}} \frac{fxr_{t-1}^*}{\pi_t} \right) \tag{27}$$

1.2.6 The foreign world

The foreign world is specified by the following three equations for its output, interest rate, and inflation, respectively:

$$\widehat{y}_t^* = \rho_{Y^*} \widehat{y}_{t-1}^* + \epsilon_t^{y^*} \tag{28}$$

$$\widehat{R}_t^* = \phi_\pi^* \pi_t^* + \phi_y^* \widehat{y}_t^* \tag{29}$$

$$\pi_t^* = \beta^* E_t \pi_{t+1}^* + \lambda^* \left(\sigma + \frac{\phi^* + \alpha^*}{1 - \alpha^*} \right) \widehat{y}_t^* \tag{30}$$

The Phillips curve (30) is in accordance with the New Keynesian model (Gali, 2015), assuming that the foreign world is a relatively large economy. A foreign demand shock ultimately increases the foreign interest rate through output directly and inflation, which is affected by output in the Phillips curve.

The model includes 24 endogenous variables constituting a system of 24 equations, where the variables are represented in log-deviation from their steady state: inflation π_t, aggregate consumption of households $\widehat{C}_t$, hours worked $\widehat{N}_t$, domestic interest rate $\widehat{R}_t$, net exports $\widehat{NX}_t$, foreign exchange reserves $\widehat{fxr_t^*}$, foreign debt $\widehat{b}_t^*$, investment $\widehat{I_t^S}$, private capital $\widehat{K_t^S}$, public capital $\widehat{K}_{G,t}$, nominal exchange rate $\widehat{e}_t$, fiscal debt $\widehat{b}_t$, public consumption $\widehat{G_t^C}$, public investment $\widehat{G_t^I}$, lump-sum taxes $\widehat{T}_t$, output $\widehat{Y}_t$, relative price of domestic goods to composite consumption $\widehat{p_t^h}$, relative price of government purchases to composite consumption $\widehat{p_t^g}$, domestic consumer prices $\widehat{P}_t$, foreign consumer prices $\widehat{P_t^*}$, foreign interest rate $\widehat{R_t^*}$, foreign inflation π_t^*, foreign output $\widehat{y}_t^*$, and the Lagrange multiplier to a collateral constraint $\widehat{\lambda_t^c}$. The system of log-linear equations consists of the Taylor rule (22), foreign exchange interventions (24), public investment, public consumption, and taxes rules (19, 20, and 21), three foreign world expressions (28, 29, and 30), and the other 16 equations presented in Appendix 1.6.4.

1.3 Calibration

All parameters can be divided into three sets: standard values borrowed from other studies because of the non-availability of respective Hungarian data, fixed values borrowed from the estimated Hungarian DSGE model (Jakab & Vilagi, 2008), and specifically calibrated parameters for this model. The list of parameters is provided in Appendix 1.6.1.

The first set includes depreciation rates for private and public capital $\delta = 0.025$, $\delta^g = 0.02$ (Traum & Yang, 2015), the elasticity of substitution between differentiated intermediate goods $\varepsilon = 9$, output response in the Taylor rule $\phi_y = \phi_y^* = 0.125$, persistence in the monetary policy shock $\rho_\epsilon = 0.5$ (Gali, 2015), the elasticity of investment with respect to Tobin's Q $z = 1$ (Gali, Lopez-Salido & Valles, 2007), the inverse of intertemporal elasticity of substitution for consumption $\sigma = 2$, the elasticity of wages with respect to hours worked $\phi = \phi^* = 1.45$ (Schmitt-Grohe & Uribe, 2003), the exchange rate change response in the interventions rule $\alpha_2 = -0.62$ (Gartner, 1987), while α_1 is fixed at -0.12. The foreign parameters are set to their standard values: discount factor $\beta^* = 0.99$, inflation response in the Taylor rule $\phi_\pi^* = 1.5$ (Gali, 2015), price stickiness $\theta^* = 0.75$ (Gali, Lopez-Salido & Valles, 2007), output elasticity to capital $\alpha^* = 0.32$, persistence in the foreign output $\rho_{Y^*} = 0.8$, and a ratio of foreign output to domestic output y_y^* is technically feasible at 5.

The second set consists of posterior estimates obtained by Jakab and Vilagi (2008) and their fixed parameters: the interest rate smoothing $\rho = 0.76$, inflation response in the Taylor rule $\phi_\pi = 1.37$, price stickiness $\theta = 0.9$, and the fraction of rule-of-thumb households $\mu = 0.25$.

The third largest set contains calibrated parameters using the averages of Hungarian data[8] for the steady state of variables derived in Appendix 1.6.2. In particular, the GDP ratios of private consumption, public consumption, public investment, net exports, external debt, fiscal debt, and net foreign assets of the central bank are $c_y = 0.66$, $g_y^C = 0.1$, $g_y^I = 0.04$, $nx_y = -0.0002$, $b_y^* = 1.16$, $b_y = 2.77$, and $fxr_y^* = 0.69$, respectively. The degree of openness is calculated as a ratio of imports to GDP, $1 - \gamma = 0.69$; thus, the home-bias parameter in consumption and investment γ is equal to 0.31, while it is assumed to be higher for government purchases $\gamma_2 = 0.9$ as its large share may go to the wages of public servants. The discount factor is set to 0.97 because the average T-bill rate is used as a proxy for the policy interest rate, which is 3 percent per quarter. The upper bound of leverage ratio Ω appears to be 0.14 using the collateral constraint at steady state. The elasticity of output with respect to private capital α is equal to 0.45,

[8] The data over 1995Q1-2011Q3 include real GDP, CPI-deflated private consumption, public consumption, fixed capital formation, exports, imports, T-bill rate, CPI, net foreign assets of the central bank, fiscal debt, and fiscal revenues, which are all from the IMF's International Financial Statistics. The euro-forint exchange rate and external debt in euro are from the webpage of the Hungarian Central Bank.

which is higher than its standard value because of the specific steady state rental cost of capital (32). The elasticity of output with respect to public capital ψ corresponds to 0.08 based on the equation (33). Fiscal parameters are calibrated based on the steady state expressions for public consumption, public investment, and lump-sum taxes in Appendix 1.6.2: $\vartheta_{GI} = 1.03$, $\gamma_{GI} = 0.38$, $\gamma_{GC} = 0.4$, and $\varphi_I = 0.3$.

Some parameters are obtained by running regressions according to the model's equations, using the seasonally adjusted log of real data. For example, the autoregressive coefficient ρ_{GC} in the public consumption equation (20) is equal to 0.4, while the output response of public consumption appears to be positive $\vartheta_{GC} = 1.18$, suggesting a procyclical fiscal policy. The lump-sum taxes' responses to fiscal debt $\varphi_b = 0.4$ and to public consumption $\varphi_C = 0.3$ are obtained by regressing the government revenues on fiscal debt and public consumption. The empirical counterpart of foreign exchange reserves equation (24) gives its only significant persistence parameter $\rho_{fxr} = 0.53$.

1.4 Results: simulation

The exogenous shocks take an increase of 25 basis points in the foreign output/demand and domestic interest rate, while fiscal shocks rise by 1 percent to examine the impulse-response functions in this section. This, in the absence of further changes caused by responses of other variables, would imply an increase of 1 percent in the annualized domestic interest rate and foreign demand. A shock to foreign demand translates into an increase of foreign interest rate by 74 basis points produced by this quarterly model, which does not offset the initial boost of foreign output specified as an AR(1) process, excluding feedback from the interest rate.

This section analyzes the baseline simulation with a collateral constraint and an alternative, standard version without it. The impulse-response functions to a foreign demand shock are examined first, which according to a variance decomposition of shocks, explains the largest share in variables. Second, the volatility of main variables is presented in comparison with a no collateral constraint version. As a sensitivity test, the fraction of rule-of-thumb households is increased to see differences in the impulse-response functions to a foreign demand shock. Finally, a monetary policy shock and fiscal shocks are examined under both cases: with and without a collateral constraint.

Impulse-response functions to a foreign demand shock

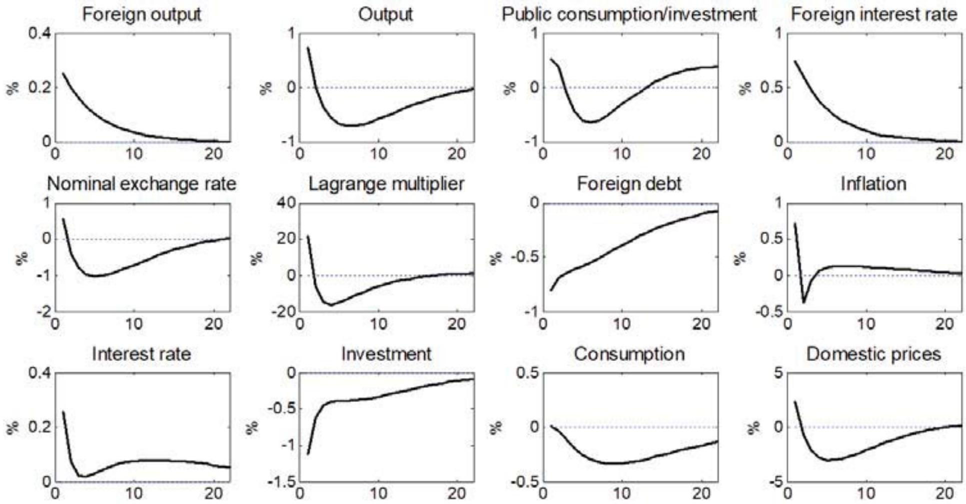


Figure 1. Collateral constraint case

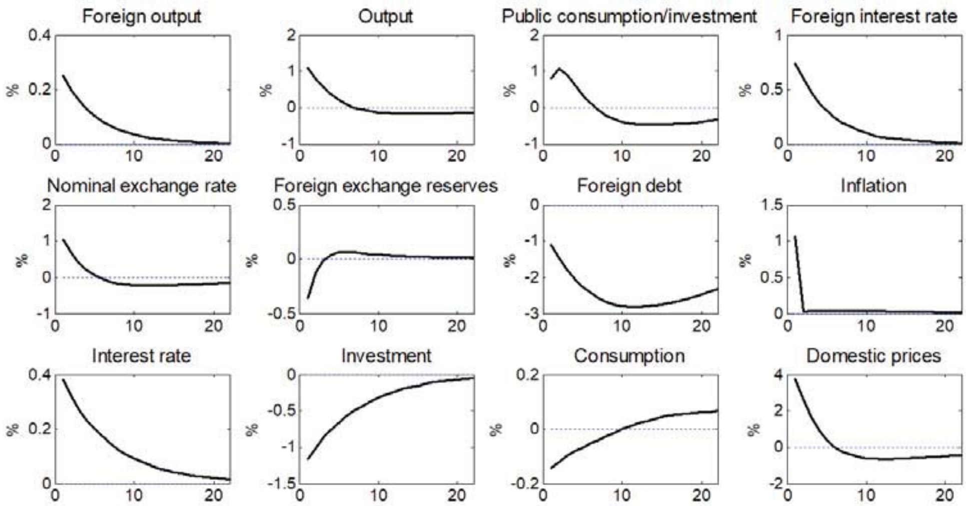


Figure 2. No collateral constraint

In Figure 1, a foreign output shock increases the foreign demand on domestic goods to be exported according to the market clearing condition (25), stimulating the domestic output, to which public spending procyclically responds. The shock also raises the foreign interest rate, which responds to increased inflation and output according to the foreign Taylor rule (29). High foreign interest rate depreciates the nominal exchange rate, as capital may flow out, and makes borrowings from abroad expensive, raising their shadow value or Lagrange multiplier to the collateral constraint. The higher Lagrange multiplier, associated also with the increased marginal costs of borrowing, discourages foreign debt. The exchange rate depreciation, meanwhile, brings CPI inflation due to a complete pass-through effect. The domestic interest rate positively responds to inflation and contributes to lower private investment accordingly. Note that a reversed hump-shaped pattern of the Lagrange multiplier translates into several variables: the exchange rate through a specific uncovered interest rate parity (UIP) condition (37), the relative prices of domestic goods and government purchases' prices due to their indices containing the exchange rate, and thereby output as well.

In contrast, if there is no collateral constraint that may be considered as a move towards perfect capital mobility[9], the impulse-response functions do not exhibit the reversed hump-shaped pattern for variables in Figure 2, but rather their decaying effects consistent with the shock are observed. However, consumption immediately falls, discouraged by the high interest rate and exchange rate depreciation, which makes imported goods expensive. These dynamics for consumption do not hold under the collateral constraint in Figure 1, where consumption shows no change in the first period because of its upward pressure stemming from the increased Lagrange multiplier that triggers a substitution effect from investment to consumption.

Table 1 shows that most variables are volatile if the model has a collateral constraint, indicating standard deviations of the variables in percentage increase from a no collateral constraint case. The collateral constraint as an additional equation reduces the volatilities

[9]By removing a collateral constraint, the model still has imperfect capital mobility, since domestic and foreign interest rates are different.

in investment, capital, inflation, thus interest rate, and foreign debt, on which net exports depend. Government bonds appear therefore as a volatile saving asset for households to flexibly adjust and compensate for restrictions on the foreign debt. Taxes are also volatile, which are positive for savers at a steady state, while negative for rule-of-thumb households, being essentially their transfers. Thus, fiscal variables seem to be affected by shocks the most, two of which are fiscal shocks, when foreign borrowings are restricted or, in other words, "capital controls" are imposed. The exchange rate fluctuations, in turn, induced by the Lagrange multiplier in UIP influence other variables: the relative prices of domestic goods and government purchases' prices due to their indices containing the exchange rate, foreign exchange reserves responding to the exchange rate, rental cost of capital which is rewritten in terms of hours worked in the equation (38), and thus output as well.

Table 1. **Volatilities with a collateral constraint relative to without it**
(in % increase)

$Var(\widehat{b_t})$	122.6	$Var(\widehat{N_t})$	31.8	$Var(\widehat{NX_t})$	-52
$Var(\widehat{T_t})$	90.9	$Var(\widehat{fxr_t^*})$	31.6	$Var(\widehat{R_t})$	-35.9
$Var(\widehat{e_t})$	51.6	$Var(\widehat{Y_t})$	27.2	$Var(\widehat{K_{G,t}})$	-33.2
$Var(\widehat{C_t})$	45.5	$Var(\widehat{G_t^C})$	3.3	$Var(\widehat{I_t^S})$	-13
$Var(\widehat{p_t^g})$	43	$Var(\widehat{G_t^I})$	2.9	$Var(\widehat{K_t^S})$	-12.8
$Var(\widehat{p_t^h})$	42.6	$Var(\widehat{b_t^*})$	-95.4	$Var(\pi_t)$	-10.9

Volatility represents a standard deviation calculated as a square root of the variance. Numbers in Table 1 indicate the percentage increase of volatilities produced by the model with a collateral constraint from the corresponding ones generated by the version without a collateral constraint.

As a sensitivity test, the transmission mechanism of a foreign demand shock is worth examining at the higher fraction of rule-of-thumb households than the baseline version, $\mu = 0.75$ versus $\mu = 0.25$. Appendix 1.6.5 illustrates that liquidity-constrained households, which cannot smooth their consumption due to no access to saving instruments, make the variables even more volatile, especially those related to prices: Lagrange multiplier, nominal exchange rate, relative prices, inflation, and foreign exchange reserves which respond to the exchange rate. Consumption increases because output has increased due to higher foreign demand on domestic goods, and therefore the income of rule-of-thumb households rises, which can only consume. For these non-savers, foreign borrowings do not matter, and thus their shadow value falls, exerting less pressure on the foreign currency and thereby appreciating the nominal exchange rate. Inflation, however, remains high because of the increased output and associated production costs. The magnitude of output response is smaller in Appendix 1.6.5 compared to Figure 1, because a majority of households is liquidity-constrained, not receiving income other than wages. Thus, a small fraction of savers, owners of monopolistic firms, has to meet the increased foreign demand on domestic goods.

In Figure 3, a positive interest rate shock appreciates the exchange rate, reducing inflation and thus making real fiscal debt higher. The increased interest rate discourages consumption and investment, ultimately contracting output. These dynamics are consistent with a standard prediction of the New Keynesian setting. This model, however, adds an insight about foreign debt. The monetary tightening leads to a higher shadow value of foreign borrowings, which implies that the marginal costs of those borrowings increase, causing a fall in the foreign debt. Under a no collateral constraint, Figure 4 illustrates that the interest rate does not change immediately, since there is no feedback from the Lagrange multiplier to UIP and low inflation, due to the exchange rate appreciation, does not require monetary tightening. As a result, the foreign debt also remains unchanged initially.

Impulse-response functions to a monetary policy shock

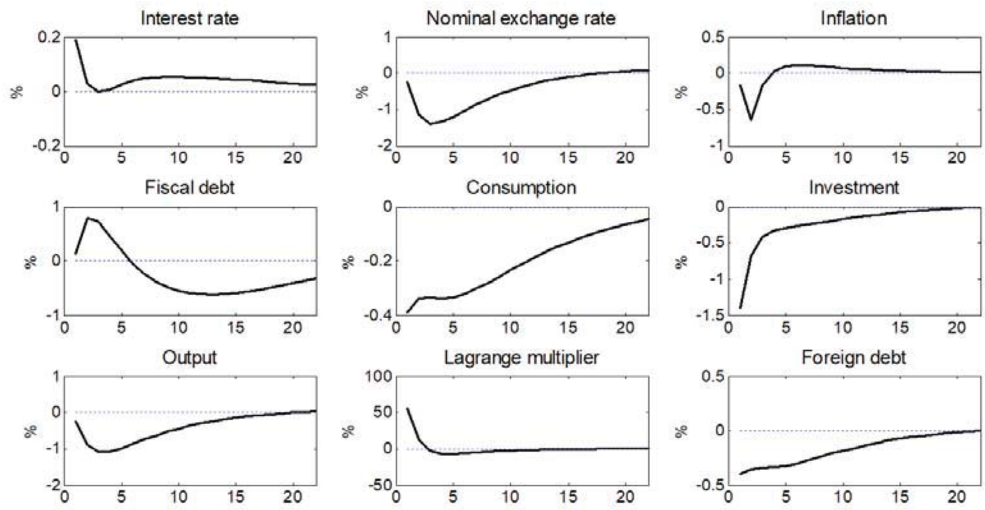


Figure 3. Collateral constraint case

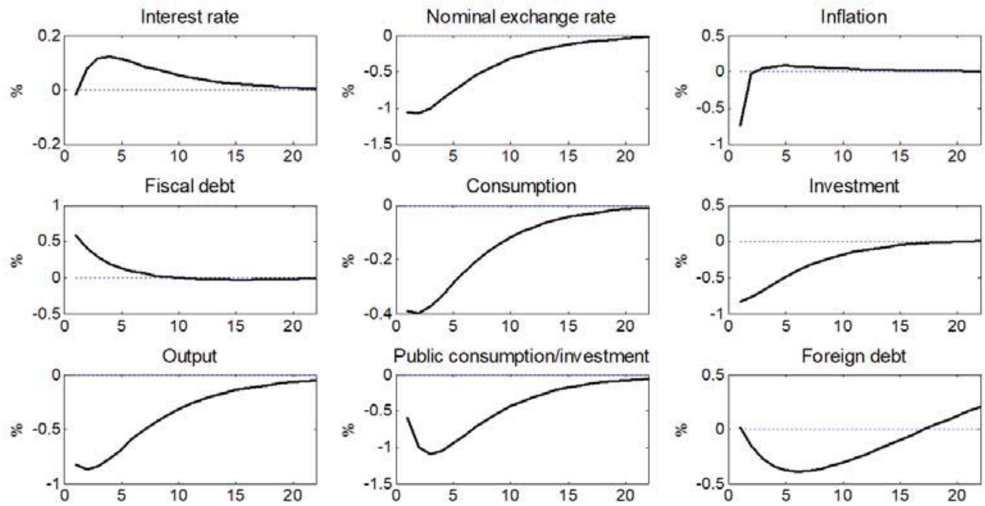


Figure 4. No collateral constraint

In Figures 5 and 6, fiscal shocks appreciate the exchange rate as the standard Mundell-Fleming model would predict, lowering inflation. Thus, the interest rate falls. However, the shocks affect foreign debt differently. An increase of public investment expands output through public capital accumulation as a productive input, encouraging more foreign borrowings for higher private consumption since there is no collateral constraint. In contrast, public consumption stimulates demand, not the supply side of the economy. Therefore, further foreign borrowings are not feasible within a given capacity of the total output.

Impulse-response functions to fiscal shocks, no collateral constraint

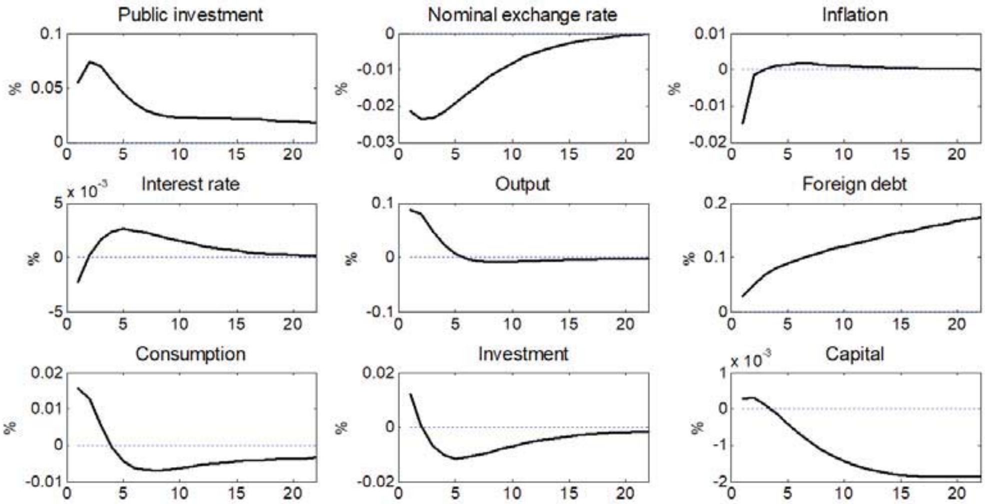


Figure 5. Public investment shock

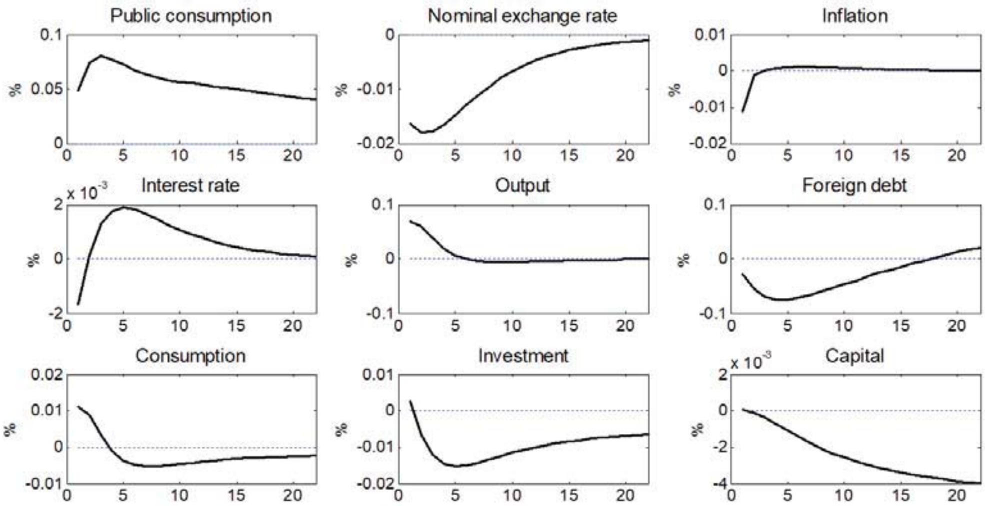


Figure 6. Public consumption shock

The collateral constraint changes a transmission mechanism of fiscal shocks. Due to accumulating public capital, the public investment shock (in Figure 7) crowds out private investment and thereby private capital. Thus, the borrowing limit shrinks, raising its shadow value. This higher Lagrange multiplier, associated with the increased marginal costs of borrowing, leads to a decline in the foreign debt which, together with the shadow value effects through UIP, depreciates the exchange rate: On the one hand, a fall in foreign borrowings means that capital outflows take place, depreciating the nominal exchange rate. On the other hand, UIP includes the Lagrange multiplier to collateral constraint (37), and thus an increase of the latter as a shadow value of foreign debt denominated in foreign currency puts higher demands on that currency, depreciating also the nominal exchange rate of the domestic currency. As a result, this exchange rate depreciation caused by public investment is not in line with the traditional Mundell-Fleming model which assumes perfect capital mobility, rather than a collateral constraint on foreign borrowings, and does not distinguish between the types of public spending.

Figure 8, in contrast, illustrates that public consumption does not crowd out private capital, leading to a fall in the Lagrange multiplier. Thus, the foreign debt initially rises to finance private investment, which is also stimulated by the low interest rate. A public consumption shock brings standard exchange rate appreciation, which contributes to low inflation through its pass-through effect. Thus, the interest rate decreases as a response to inflation.

Impulse-response functions to fiscal shocks, with a collateral constraint

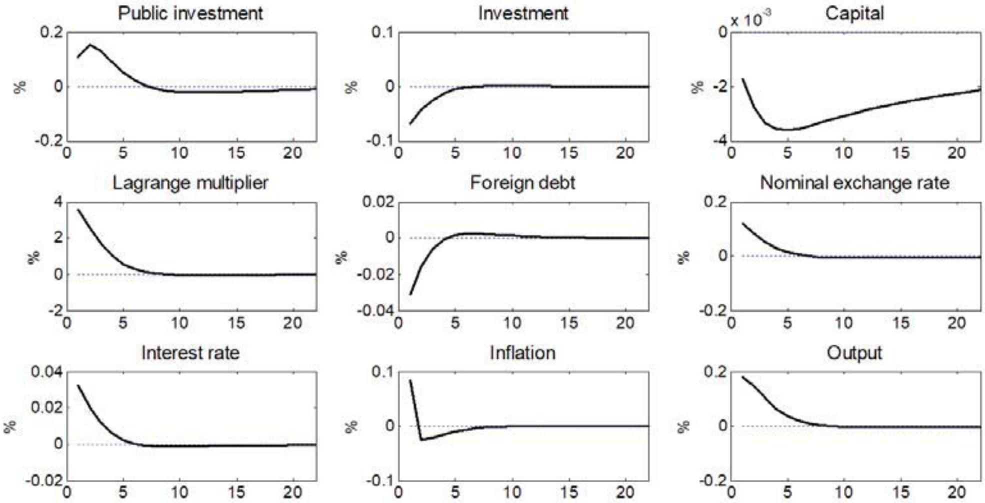


Figure 7. Public investment shock

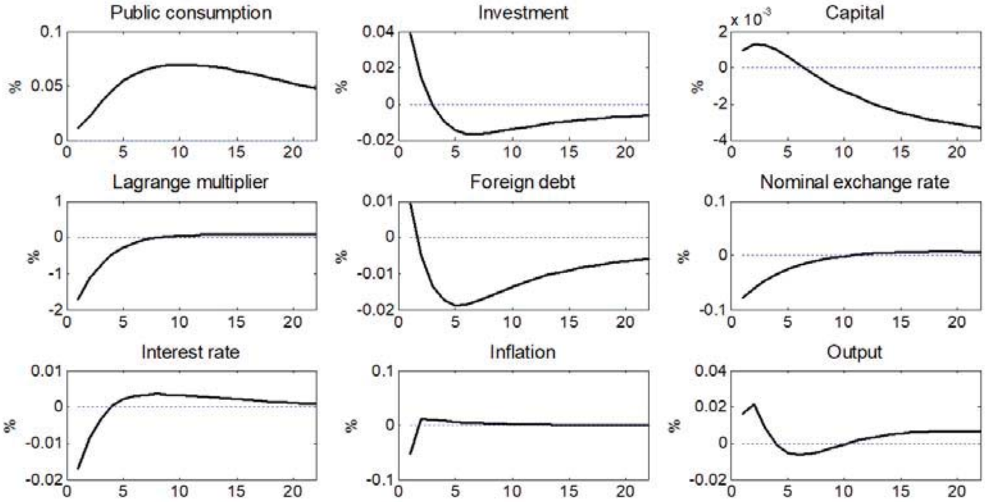


Figure 8. Public consumption shock

Overall, three main findings are worth highlighting. First, public investment and public consumption are the two distinct fiscal instruments which differently affect the nominal exchange rate, foreign debt, and private capital. Second, a collateral constraint alters the transmission mechanism of fiscal, monetary policy, and foreign demand shocks and makes the model volatile due to the Lagrange multiplier as a separate variable, which enters UIP. Third, a high fraction of liquidity-constrained households acts as an additional friction, producing small effects on output and destabilizing the price-related variables.

1.5 Conclusion

This paper develops a DSGE model for an emerging open economy to understand its monetary and fiscal policy interactions in the collaterally-constrained environment of indebtness to the foreign world. The model captures a set of structural specifics: two monetary instruments—interest rate and foreign exchange interventions; two fiscal instruments—public consumption and public investment; and the foreign debt of the private sector to finance investments. The constructed framework combines the New Keynesian model of a small open economy with two types of households, forward-looking optimizers and rule-of-thumb consumers, relaxing the assumption of Ricardian equivalence and integrating the exogenous foreign world. Two versions of the model are examined: with a collateral constraint on foreign borrowings and without it.

In terms of novel contributions, the paper achieves the following. First, the standard UIP is extended by including an additional term that is explicitly derived from the optimization problem of a forward-looking household. This term appears to be the Lagrange multiplier to a collateral constraint or a shadow value of foreign debt, whereas Benes et al. (2015) introduced it ad hoc as being foreign exchange reserves. This model also has the foreign exchange reserves, but not only in the interventions rule and in the budget constraint of households, like in Benes et al. (2015), but also in the balance of payments equation which is consistent with the real practice of statistics compilation at central banks. These two micro-founded equations related to a collateral constraint and the reserves of the central bank seem to be the right approach to follow in an emerging open economy framework, where foreign exchange intervention is an additional monetary policy instrument and foreign borrowings, denominated in foreign currency, need to be collateralized by physical capital.

Second, according to this study, fiscal and monetary policy shocks transmit to the economy, with its imperfect capital mobility, differently from the standard Mundell-Fleming model. A positive public investment shock can cause exchange rate depreciation and crowd out private capital, expanding output due to the accumulation of public capital, which is an input in the production function. Public consumption, in contrast, alters the demand side of the economy, appreciating the exchange rate and stimulating foreign debt. A monetary policy shock as a sudden increase in interest rate affects the main variables consistent with the New Keynesian prediction of nominal rigidities, yet the dynamics of foreign debt depend on whether a collateral constraint is on or off the model.

Third, the collateral constraint makes the model volatile, due to its Lagrange multiplier entering UIP and affecting the exchange rate. This result may explain why emerging markets need their hybrid monetary policy, a combination of inflation targeting and managed exchange rate regime; they are vulnerable to external shocks and prone to a sudden stop crisis, as they borrow from abroad. Finally, a high fraction of liquidity-constrained households also makes the variables volatile. In addition, the output response tends to be small, as those households only consume each period out of their wages, without any other income and savings, due to the lack of access to financial markets.

In conclusion, this paper suggests several policy implications for emerging market countries. It is important to distinguish public investment and public consumption as two separate fiscal policy instruments affecting the economy differently. These countries should avoid high external indebtness of the private sector by advancing their domestic financial market. Furthermore, they are advised to keep in mind that their monetary policy with its two instruments can act in a non-standard way because of the specific UIP condition, which includes an additional term that may affect the exchange rate due to a collateral constraint. The foreign exchange reserves also exist, which respond to the exchange rate dynamics and enter the balance of payments equation. Finally, reducing the number of liquidity-constrained households can be crucial through various poverty alleviation programs.

1.6 Appendix

1.6.1 Table of parameters

Parameter	Definition
$\beta = 0.97$	discount factor
$\gamma = 0.31$	home-bias in consumption and investment
$\gamma_2 = 0.9$	home-bias in government purchases
$\Omega = 0.14$	upper bound of leverage ratio
$\alpha = 0.45$	output elasticity to private capital
$\psi = 0.08$	output elasticity to public capital
$\delta = 0.025$	depreciation rate of private capital
$\delta^g = 0.02$	depreciation rate of public capital
$\phi = \phi^* = 1.45$	wage elasticity to hours worked, domestic and foreign
$\sigma = 2$	inverse of intertemporal elasticity of substitution for consumption
$\theta = 0.9$	index of price stickiness
$\varepsilon = 9$	elasticity of substitution between differentiated intermediate goods
$\mu = 0.25$	fraction of rule-of-thumb households
$\phi_\pi = 1.37$	inflation response in the Taylor rule
$\phi_y = \phi_y^* = 0.125$	output response in the Taylor rule, domestic and foreign
$\alpha_1 = -0.12$	exchange rate response in the interventions rule
$\alpha_2 = -0.62$	exchange rate change response in the interventions rule
$\gamma_{GC} = 0.4$	response of public consumption to fiscal debt
$\gamma_{GI} = 0.38$	response of public investment to fiscal debt
$\vartheta_{GI} = 1.03$	response of public investment to output
$\vartheta_{GC} = 1.18$	response of public consumption to output
$\varphi_b = 0.4$	response of lump-sum taxes to fiscal debt
$\varphi_C = 0.3$	response of lump-sum taxes to public consumption
$\varphi_I = 0.3$	response of lump-sum taxes to public investment
$\rho_{GC} = \rho_{GI} = 0.4$	persistence in public consumption and public investment
$\rho = 0.76$	interest rate smoothing in the Taylor rule
$\rho_{fxr} = 0.53$	persistence in foreign exchange reserves
$z = 1$	investment elasticity to Tobin's Q
$c_y = 0.66$	GDP ratio of private consumption
$g_y^C = 0.1$	GDP ratio of public consumption
$g_y^I = 0.04$	GDP ratio of public investment
$nx_y = -0.0002$	GDP ratio of net exports
$b_y^* = 1.16$	GDP ratio of external debt
$b_y = 2.77$	GDP ratio of fiscal debt
$fxr_y = 0.69$	GDP ratio of foreign exchange reserves
$y_y^* = 5$	ratio of foreign output to domestic output
$\beta^* = 0.99$	foreign discount factor
$\phi_\pi^* = 1.5$	foreign inflation response in the Taylor rule
$\theta^* = 0.75$	foreign price stickiness
$\alpha^* = 0.32$	foreign output elasticity to capital
$\rho_{Y^*} = 0.8$	persistence in foreign output
$\rho_\epsilon = 0.5$	persistence in fiscal and monetary policy shocks

1.6.2 Steady state

The endogenous variables at their steady state, denoted by bars, are shown in this appendix and they are identical for both versions of the model: with and without a collateral constraint. At steady state, there is no inflation, thus $\overline{\pi} = 1$ implying that flexible prices persist in the long run. The first-order condition with respect to government bonds b_t (8) gives that $\overline{R} = \frac{1}{\beta}$, while with respect to foreign debt b_t^* (9) suggests $\overline{\lambda^c} = \beta^* - \beta$ at steady state. Similarly, $\overline{R}^* = \frac{1}{\beta^*}$.

The public investment equation (19) can be represented as $\overline{G_I} = \frac{\overline{G_I}^{\rho_{GI}} \overline{Y}^{(1-\rho_{GI})\vartheta_{GI}}}{\overline{b}^{(1-\rho_{GI})\gamma_{GI}}}$, which allows expressing $\overline{G_I} = \frac{\overline{Y}^{\vartheta_{GI}}}{\overline{b}^{\gamma_{GI}}}$. Similarly, the steady state public consumption is as follows based on the equation (20) $\overline{G_C} = \frac{\overline{Y}^{\vartheta_{GC}}}{\overline{b}^{\gamma_{GC}}}$. The public capital accumulation equation (18) provides public capital $\overline{K_G} = \frac{\overline{G^I}}{\delta^g} = \frac{\overline{Y}^{\vartheta_{GI}}}{\delta^g \overline{b}^{\gamma_{GI}}}$, from which fiscal debt is expressed in terms of output and public capital $\overline{b} = \left(\frac{\overline{Y}^{\vartheta_{GI}}}{\overline{K_G}\delta^g}\right)^{1/\gamma_{GI}}$. The steady state lump-sum taxes can be found by plugging the previous three equations for fiscal debt, public consumption, and public investment into the tax equation (21) $\overline{T} = \frac{\overline{Y}^{\frac{\vartheta_{GI}\varphi_b - \vartheta_{GI}\varphi_C\gamma_{GC} + \vartheta_{GC}\varphi_C\gamma_{GI}}{\gamma_{GI}}}}{(\overline{K_G}\delta^g)^{\frac{\varphi_b - \gamma_{GC}\varphi_C - \gamma_{GI}\varphi_I}{\gamma_{GI}}}}$. Therefore, the government budget constraint (16) is represented in terms of output and public capital:

$$\frac{\overline{Y}^{\frac{\vartheta_{GI}\varphi_b - \vartheta_{GI}\varphi_C\gamma_{GC} + \vartheta_{GC}\varphi_C\gamma_{GI}}{\gamma_{GI}}}}{(\overline{K_G}\delta^g)^{\frac{\varphi_b - \gamma_{GC}\varphi_C - \gamma_{GI}\varphi_I}{\gamma_{GI}}}} = (1-\mu)(\overline{R}-1)\left(\frac{\overline{Y}^{\vartheta_{GI}}}{\overline{K_G}\delta^g}\right)^{1/\gamma_{GI}} + \overline{K_G}\delta^g + \overline{Y}^{\vartheta_{GC} - \frac{\vartheta_{GI}\gamma_{GC}}{\gamma_{GI}}} (\overline{K_G}\delta^g)^{\frac{\gamma_{GC}}{\gamma_{GI}}} \tag{31}$$

The first-order condition with respect to capital (7), given that $\overline{Q} = 1$, yields the following rental cost of capital, which under a no collateral constraint would be without the last term:

$$\overline{R}^k = \frac{1}{\beta} - (1-\delta) - \frac{\overline{\lambda^c}\Omega}{\beta} \tag{32}$$

The law of one price holds. There is also an assumption of symmetric steady state $(\frac{\overline{P_f}}{\overline{P_h}} = 1)$ and a unit elasticity of substitution between domestic and foreign goods ($\eta = 1$). Thus, the real/nominal exchange rate and relatives prices at steady state are equal to one.

At steady state, the firm equates its marginal costs (14) with the inverse of price frictionless mark-up $\frac{\varepsilon}{\varepsilon-1}$; thus, wage is found as:

$$\overline{W} = (1-\alpha)\left(\frac{\overline{K_G}^{\psi}\alpha^{\alpha}(\varepsilon-1)}{(\overline{R}^k)^{\alpha}\varepsilon}\right)^{\frac{1}{1-\alpha}}$$

The labor supply condition (10) gives $\overline{N} = \overline{W}^{\frac{1}{\phi-1}}$. Therefore, the production function

(13) suggests the following output expressed in terms of private capital and public capital:

$$\overline{Y} = [(1-\mu)\overline{K^S}]^{\alpha}\overline{K_G}^{\frac{\psi\phi}{\phi-1}} \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}(\varepsilon-1)}{(\overline{R}^k)^{\alpha}\varepsilon} \right)^{\frac{1}{\phi-1}} \tag{33}$$

This means that the government budget constraint (31) can be rewritten in terms of both types of capital constituting the first equation in a system. The second equation comes from the aggregate demand condition shown gradually below.

The law of motion for physical capital (3) suggests $\overline{I^S} = \delta\overline{K^S}$.

The domestic goods market clearing condition (25) provides private consumption:

$$\overline{C} = \frac{\overline{Y}}{\gamma} - \frac{\gamma_2}{\gamma}(\overline{G_C} + \overline{G_I}) - \frac{1-\gamma}{\gamma} - (1-\mu)\overline{I^S}$$

The budget constraint of rule-of-thumb households (11) gives their taxes $\overline{T^N} = \overline{WN} - \overline{C^N}$, where their consumption is assumed to be equal to the consumption of savers at steady state, thus to aggregate consumption as well, given that the latter sums up the consumption of both households: $\overline{C} = \mu\overline{C^N} + (1-\mu)\overline{C^S}$. The taxes of savers are equal to $\overline{T^S} = \frac{\overline{T} - \mu\overline{T^N}}{(1-\mu)}$ since $T_t = \mu T_t^N + (1-\mu)T_t^S$.

The steady state foreign exchange reserves, according to their rule (24), are equal to 1 given that $\overline{e} = 1$.

Foreign debt can be found from the budget constraint of savers (2) in terms of private and public capital:

$$\overline{b^*} = \frac{\overline{C} - \overline{N}^{\phi} + \overline{T}}{(1-\mu)(1-\overline{R^*})} - \frac{(\overline{R}^k - \delta)\overline{K^S}}{(1-\overline{R^*})} - \frac{\overline{b}(\overline{R}-1)}{(1-\overline{R^*})} - \frac{(1-\frac{\varepsilon-1}{\varepsilon})\overline{Y}}{(1-\overline{R^*})} + \overline{fxr^*}$$

The balance of payments equation (27) provides net exports:

$$\overline{NX} = (1-\mu)\left(\overline{R^*} - 1\right)\overline{b^*} + (1-\overline{R^*})\overline{fxr^*}$$

Therefore, the aggregate demand condition (26) can be utilized as a second equation in the system, to find private and public capital:

$$\overline{Y} = \overline{C} + (1-\mu)\delta\overline{K^S} + \overline{K_G}\delta^g + \overline{G_C} + \overline{NX} \tag{34}$$

Since private and public capital are found in the system of two equations (31 and 34), the other steady state variables can be extracted from the expressions above.

1.6.3 The Phillips curve

The Phillips curve for CPI inflation in a small open economy has been derived according to Gali (2015). The log-linearized optimal price setting condition (15) delivers the standard equation for domestic inflation π_t^h:

$$\pi_t^h = \beta E_t \pi_{t+1}^h + \lambda \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\widehat{MC}_t,$$

where $\widehat{MC_t}$ is the log deviation of the economy's average real marginal costs from their steady state and $\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}$.

CPI inflation includes the domestic inflation π_t^h and the terms of trade, which can be alternatively represented by the real exchange rate RER_t:

$$\pi_t = \pi_t^h + \frac{1-\gamma}{\gamma} \triangle \widehat{RER}_t, \tag{35}$$

where $\triangle \widehat{RER_t} = \triangle \widehat{e_t} + \pi_t^* - \pi_t$.

The Phillips curve therefore is as follows, taking into account the previous CPI inflation equation (35):

$$\pi_t = \beta E_t \pi_{t+1} + \frac{\lambda(1-\alpha)}{1-\alpha+\alpha\varepsilon}\widehat{MC}_t - \beta\frac{1-\gamma}{\gamma}E_t \triangle \widehat{RER}_{t+1} + \frac{1-\gamma}{\gamma} \triangle \widehat{RER}_t,$$

where $\widehat{MC}_t = \widehat{W}_t - (\widehat{Y}_t - \widehat{N}_t) + \frac{1-\gamma}{\gamma}\widehat{RER}_t$. Wages can be substituted with the log-linearized labor supply condition (10), while $\widehat{RER}_t = \widehat{e}_t + \widehat{P}_t^* - \widehat{P}_t$ resulting finally in the following Phillips curve:

$$\begin{aligned} \pi_t \;=\; & \beta E_t \pi_{t+1} + \frac{1-\gamma}{\gamma}\left(\lambda\frac{1-\alpha}{1-\alpha+\alpha\varepsilon} + \beta + 1\right)\widehat{e}_t + \lambda\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\left(\phi\widehat{N}_t - \widehat{Y}_t\right) \\ & + \frac{(1-\gamma)\lambda(1-\alpha)}{\gamma(1-\alpha+\alpha\varepsilon)}(\widehat{P}_t^* - \widehat{P}_t) + \frac{1-\gamma}{\gamma}(\pi_t^* - \pi_t) - \beta\frac{1-\gamma}{\gamma}E_t(\pi_{t+1}^* - \pi_{t+1}) - \\ & - \frac{1-\gamma}{\gamma}\widehat{e}_{t-1} - \beta\frac{1-\gamma}{\gamma}E_t\widehat{e}_{t+1} \end{aligned} \tag{36}$$

1.6.4 Log-linearized equations

The other 16 log-linearized equations of the model are listed in this appendix.

The Phillips curve of CPI inflation (36) is derived above.

The log-linearization of the first-order condition of a saver with respect to foreign debt

(9) provides the UIP condition[10]:

$$\widehat{R}_t = E_t\widehat{e}_{t+1} - \widehat{e}_t + \frac{\beta^*}{\overline{\beta}}\widehat{R}^*_t + \frac{\overline{\lambda^c}}{\beta}\widehat{\lambda}^c_t \tag{37}$$

The combination of log-linearized first-order conditions for capital (7) and investment (6) given that $\widehat{R^k_{t+1}} = \phi N_{t+1} - \widehat{K^S_t}$ delivers the following[11]:

$$\begin{aligned}\widehat{I^S_t} - \widehat{K^S_{t-1}} &= [\beta(1-\delta) + \Omega\overline{\lambda^c}](E_t\widehat{I^S_{t+1}} - \widehat{K^S_t}) + z[1-\beta(1-\delta) - \Omega\overline{\lambda^c}](\phi E_t\widehat{N}_{t+1} \\ &\quad -\widehat{K^S_t}) - z(1-\Omega\overline{\lambda^c})(\widehat{R}_t - E_t\pi_{t+1}) + z\Omega\overline{\lambda^c}(E_t\pi_{t+1} + \widehat{e}_t - E_t\widehat{e}_{t+1} + \widehat{\lambda^c_t})\end{aligned} \tag{38}$$

The collateral constraint (4) is log-linearized below, taking into account that $\widehat{Q_{t+1}} = \frac{\widehat{I^S_{t+1}} - \widehat{K^S_t}}{z}$ and $z = 1$:

$$\widehat{b}^*_t = E_t\pi_{t+1} - \widehat{R}^*_t + \widehat{e}_t - E_t\widehat{e}_{t+1} + \widehat{I^S_{t+1}} \tag{39}$$

The law of motion for private capital (3) is as follows:

$$\widehat{K^S_t} = (1-\delta)\widehat{K^S_{t-1}} + \delta\widehat{I^S_t} \tag{40}$$

Public capital (18) evolves according to:

$$\widehat{K}_{G,t} = (1-\delta^g)\widehat{K}_{G,t-1} + \delta^g\widehat{G^I_t} \tag{41}$$

The production function (13) implies:

$$\widehat{Y}_t = \alpha\widehat{K^S_{t-1}} + (1-\alpha)\widehat{N}_t + \psi\widehat{K}_{G,t-1} \tag{42}$$

The aggregate consumption equation is derived according to Gali, Lopez-Salido, and Valles (2007) by combining the Euler equation (8), budget constraint of the rule-of-thumb households (11), and the relationship $C_t = \mu C^N_t + (1-\mu)C^S_t$:

$$\widehat{C}_t = E_t\widehat{C}_{t+1} + \Theta_n(\widehat{N}_t - E_t\widehat{N}_{t+1}) - \Theta_i(\widehat{R}_t - E_t\pi_{t+1}) + \mu\overline{TC}^{-1}(\widehat{T_{t+1}} - \widehat{T}_t), \tag{43}$$

where $\Theta_n = \left[\mu\overline{N}^\phi\phi + (1-\mu)\overline{N}^\phi\right]\overline{C}^{-1}$ and $\Theta_i = (\sigma\overline{C})^{-1}(1-\mu)(\overline{C} - \phi^{-1}\overline{N}^\phi)$.

The government budget constraint (16) is represented in terms of fiscal debt:

$$\widehat{b}_t = \overline{R}(\widehat{b}_{t-1} - \pi_t) + \widehat{R}_{t-1} + \frac{g^I_y}{(1-\mu)b_y}\widehat{G^I_t} + \frac{g^C_y}{(1-\mu)b_y}\widehat{G^C_t} + \frac{g^C_y + g^I_y}{(1-\mu)b_y}\widehat{p}^g_t - \frac{\overline{T}}{\overline{b}(1-\mu)}\widehat{T}_t \tag{44}$$

[10]The model's version without a collateral constraint has its UIP as $\widehat{R}_t = E_t\widehat{e}_{t+1} - \widehat{e}_t + \widehat{R}^*_t$.

[11]This equation (38) under a no collateral constraint transforms into $\widehat{I^S_t} - \widehat{K^S_{t-1}} = \beta(1-\delta)(E_t\widehat{I^S_{t+1}} - \widehat{K^S_t}) + z[1-\beta(1-\delta)](\phi E_t\widehat{N}_{t+1} - \widehat{K^S_t}) - z(\widehat{R}_t - E_t\pi_{t+1})$.

The log-linearization of the balance of payments equation (27) results in:

$$\begin{aligned}\widehat{NX}_t &= \frac{\overline{R^*} b_y^*(1-\mu)}{nx_y}\widehat{b}_{t-1}^* + \frac{\overline{R^*}(1-\mu)(b_y^* - fxr_y)}{nx_y}(\widehat{e}_t - \widehat{e}_{t-1} - \pi_t) - \frac{b_y^*(1-\mu)}{nx_y}\widehat{b}_t^* \quad (45)\\ &-\frac{(1-\mu)\overline{R^*}fxr_y}{nx_y}\widehat{fxr_{t-1}^*} + \frac{(1-\mu)fxr_y}{nx_y}\widehat{fxr_t^*} + \frac{(1-\mu)(b_y^* - fxr_y)}{nx_y}\widehat{R}_{t-1}^*\end{aligned}$$

The log-linearized relative price of government purchases to composite consumption (17), assuming $\eta \to 1$, is this:

$$\widehat{p_t^g} = \gamma_2 \widehat{p_t^h} + (1-\gamma_2)(\widehat{e_t} + \widehat{P_t^*} - \widehat{P}_t), \quad (46)$$

where $\widehat{RER_t} = (\widehat{e_t} + \widehat{P_t^*} - \widehat{P}_t)$. Since prices are used in this equation and in the Philips curve (36), the domestic and foreign CPI inflations are as follows:

$$\pi_t = \widehat{P}_t - \widehat{P_{t-1}} \quad (47)$$

$$\pi_t^* = \widehat{P_t^*} - \widehat{P_{t-1}^*} \quad (48)$$

The domestic goods market clearing condition (25) can be rewritten as:

$$\widehat{Y}_t + \widehat{p_t^h} = \gamma c_y \widehat{C}_t + (1-\mu)(1-i_y)\gamma \widehat{I_t^S} + \gamma_2 g_y^C \widehat{G_t^C} + \gamma_2 g_y^I \widehat{G_t^I} + \gamma_2 (g_y^C + g_y^I)\widehat{p_t^g} + (1-\gamma) y_y^* (\widehat{e}_t + \widehat{y_t^*}), \quad (49)$$

where $(1-i_y) = 1 - c_y - g_y^C - g_y^I - nx_y$.

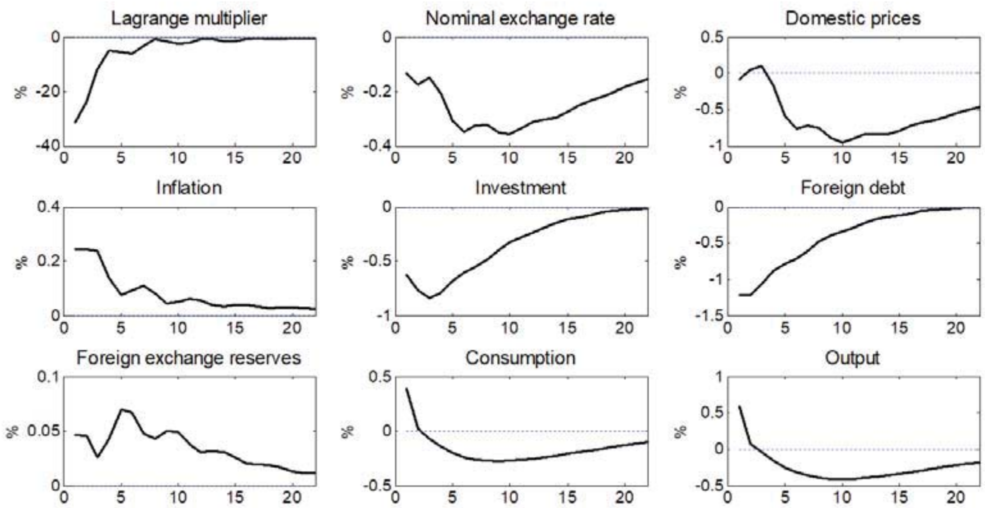
The real GDP (26) is represented in terms of investment:

$$\widehat{I_t^S} = \frac{1}{(1-i_y)(1-\mu)}\left[\widehat{Y}_t - c_y \widehat{C}_t - g_y^C \widehat{G_t^C} - g_y^I \widehat{G_t^I} - (g_y^C + g_y^I)\widehat{p_t^g} - nx_y \widehat{NX}_t\right] \quad (50)$$

The budget constraint of a saver (2) is log-linearized as well, by combining the aggregate relationships for consumption $C_t = \mu C_t^N + (1-\mu) C_t^S$ and taxes $T_t = \mu T_t^N + (1-\mu) T_t^S$, and the budget constraint of a rule-of-thumb household (11):

$$
\begin{aligned}
\widehat{C}_t \quad = \quad & [\frac{\phi\overline{N}^{\phi} + (1-\mu)\overline{K^S R^k}\phi}{\overline{C}} - \frac{(1-\mu)(\varepsilon-1)\phi}{c_y\varepsilon}]\widehat{N_t} + \frac{(1-\mu)b_y}{c_y}\widehat{R_{t-1}} - \frac{\overline{T}}{\overline{\overline{C}}}\widehat{T}_t \qquad (51) \\
& + \frac{(1-\mu)b_y\overline{R}}{c_y}\widehat{b_{t-1}} + \frac{1-\mu}{c_y}(b_y^*\widehat{b_t^*} - (1-i_y)\widehat{I_t^S} - b_y\widehat{b}_t) - \frac{(1-\mu)b_y^*\overline{R^*}}{c_y}\widehat{b_{t-1}^*} \\
& + \frac{(1-\mu)\overline{R^*}(b_y^* - fxr_y)}{c_y}\widehat{e_{t-1}} - (1-\mu)\frac{fxr_y}{c_y}\widehat{fxr_t^*} + [\frac{\overline{R^*}(1-\mu)(fxr_y - b_y^*)}{c_y} \\
& - \frac{(1-\mu)(\varepsilon-1)(1-\gamma)}{c_y\varepsilon\gamma}]\widehat{e}_t + \frac{(1-\mu)(fxr_y - b_y^*)}{c_y}\widehat{R_{t-1}^*} + \frac{(1-\mu)}{c_y}(\widehat{Y}_t + \widehat{p_t^h}) \\
& + \overline{R^*}(1-\mu)\frac{fxr_y}{c_y}\widehat{fxr_{t-1}^*} + \frac{(1-\mu)\overline{R^*}(b_y^* - fxr_y) - (1-\mu)b_y\overline{R}}{c_y}\pi_t
\end{aligned}
$$

1.6.5 Impulse-response functions to a foreign demand shock ($\mu = 0.75$)



Collateral constraint case

2 Monetary Policy Rule, Exchange Rate Regime, and Fiscal Policy Cyclicality in a Developing Oil Economy

[12]

2.1 Introduction

Most macroeconomic DSGE models are constructed for the developed world, incorporating its advanced market structure and relevant policy environment. Emerging market economies have their own unique features, which can modify the existing core frameworks in several respects. First, public investment should be considered separately from public consumption as a growth inducing instrument of fiscal policy (Berg, Portillo, Yang & Zanna, 2013), since it is usually associated with infrastructure and human capital, which developing countries often lack (Rioja, 2003; Sab & Smith, 2002). Second, monetary policy is typically a hybrid of inflation targeting and a managed exchange rate regime; thus, interest rate and foreign exchange interventions represent the two separate instruments of monetary policy (Ghosh et al., 2016). Third, in an underdeveloped domestic financial market, the investments of firms are often financed by foreign funds, so that physical capital and foreign debt can be linked through a collateral constraint (Faia & Iliopulos, 2011 linked durables goods with the foreign debt). Fourth, households are heterogeneous in their income and access to a financial market; a certain portion of the population may be liquidity constrained with only wages, without savings (Mankiw, 2000; Gali, Lopez-Salido & Valles, 2007). These four structural specifics are incorporated in the model of Chapter 1 calibrated for Hungary as the first emerging market economy to be severely hit by the

[12] The earlier version of this work was issued as the *CERGE-EI Working Paper 2016, 572,* 1-41.

global financial crisis of 2008.

This second chapter extends the Hungarian model for a subset of emerging open economies which export oil. The oil-exporting developing economies obviously differ from other emerging countries and need to be examined through their own DSGE framework. The particular features of an oil economy are as follows: The oil and non-oil production sectors should be specified separately. The economy is exposed to a volatile exogenous world oil price shock. SWF is established collecting the oil tax revenues, saving them abroad, and partly transferring to the government budget[13]. Finally, motivated by Frankel and Catao (2011), monetary policy can follow product price targeting (PPT) as an alternative to CPI; thus, these two anchors need to be compared in a general equilibrium framework jointly with fiscal policy based on some welfare measure, to determine which one is preferred.

Frankel and Catao (2011) argue that commodity exporting economies are better off targeting the output price index, which includes export commodities and excludes import products; such monetary policy is automatically countercyclical against the volatile terms of trade shock. The argument is that if the world oil price increases and there is PPT, then monetary policy tightens by raising its interest rate, thus causing exchange rate appreciation, which is the objective of offsetting the initial positive terms of trade shock. Conversely, an adverse terms of trade shock, such as a fall in oil price, can be mitigated by the exchange rate depreciation under PPT. The CPI inflation targeting, in contrast, does not respond to export prices, but to import prices. If there is an adverse terms of trade shock, such as an increase of import prices, CPI targeting brings exchange rate appreciation, further exacerbating the initial negative shock for producers of tradable goods, who use imports as their intermediate inputs. "Bottom line: a Product Price Targeter would appreciate in response to an increase in world prices of its commodity exports, not in response to an increase in world prices of its imports. CPI targeting gets this backwards." (Frankel & Catao, 2011, p. 4).

[13] The mechanism of SWF accumulation differs across countries, but since the model is calibrated for Kazakhstan, its experience is specifically captured.

The aim of this paper is, therefore, to construct a DSGE model for a developing oil economy capturing its structural specifics, as defined above, to examine the CPI/PPT monetary policy rule under a flexible/managed exchange rate regime combined with a pro/counter/acyclical fiscal policy. In order to assess whether an anchor of price stability should be the CPI or PPT, a welfare measure is adopted. According to De Paoli (2009), the welfare for a small open economy is represented as a loss function of three variables: variations in domestic price inflation, aggregate output, and real exchange rate. Based on this loss measure, inflation and output responses of the Taylor rule are optimized for managed and flexible exchange rate regimes, distinguished by the presence of foreign exchange interventions.

Fiscal policy cyclicality is associated with the oil output response of public spending. This is because the business cycle of an oil producing economy tends to correlate more with its oil sector's output rather than aggregate output; thus, commodity boom/bust is the cycle, to which fiscal policy responds. Since this model focuses on the oil price shock affecting the real oil output, the latter needs to be directly included in fiscal rules to ensure that fiscal policy transmits the shock into the economy. Acyclical fiscal policy assumes the zero oil output response of public spending and is taken as a benchmark to calculate loss in deviation from it; thus, the pro/countercyclical fiscal stance corresponds to the positive/negative oil output response, respectively. The impulse-response functions to a fall in world oil price shock, also referred to as adverse terms of trade shock, are analyzed to understand its transmission mechanism.

The calibration is based on Kazakhstan as a small open, oil exporting economy hit by the global financial crisis of 2008 due to high private sector's foreign debt. Since 2006, the IMF has included Kazakhstan in its "fuel exporters" group analyzed in the *World Economic Outlook*[14]. In 2000, Kazakhstan established its SWF managed by the National Bank on behalf of the Ministry of Finance. Oil tax revenues directly accumulate the SWF which is invested abroad, but regularly, there are ad hoc transfers from SWF to the

[14]The classification is made on the evidence that over five years the average share of fuel exports in total exports exceeds 40 percent.

government budget. Monetary policy is independently conducted by the National Bank pursuing a primary goal of price stability and intervening in the foreign exchange market to avoid speculative attacks.

In section 2.2, the model is outlined with its two types of households, standard optimizers and rule-of-thumb households, non-oil firms acting in a monopolistically competitive market, capital-intensive oil producer, two monetary policy rules for two instruments, and respective fiscal policy rules. Section 2.3 describes the model calibration. Section 2.4 examines the main results and sensitivity tests followed by the conclusion.

2.2 Model

The model has several frictions: an incomplete asset market, investment adjustment costs, collateral constraint, and the Calvo price setting. The underlying assumption is that the rest of the world is a saver, while the domestic economy is a borrower; thus, the foreign discount factor is higher than the domestic discount factor, as the domestic households might be relatively impatient compared to the rest of the world. This implies that the interest rate of an emerging economy is always higher than the foreign interest rate, which is consistent with the evidence (Reinhart & Reinhart, 2008). Therefore, imperfect capital mobility is assumed, since there is a positive difference between the domestic and foreign interest rates, and foreign borrowings are restricted. According to the impossible trinity, in turn, an independent monetary policy in terms of inflation targeting and a managed exchange rate regime are feasible under imperfect capital mobility.

The model uniquely specifies oil production as a capital-intensive sector with only capital input, for simplicity, which is accumulated by FDI that responds to the world oil price. The oil sector is owned by the government and foreigners who pay royalty taxes that accumulate the SWF. There are also transfers from SWF to the government budget. These specifics related to the oil sector capture the country case of Kazakhstan, contributing thereby to the limited literature on DSGE models for commodity-exporting, emerging market economies. The following subsections describe the model structure in detail.

2.2.1 Households

The economy is populated by a continuum of households on the interval [0,1], where the fraction μ is rule-of-thumb households. They do not have access to financial markets and consume all of their disposable income each period. The other $(1-\mu)$ fraction of households are forward-looking households who hold government bonds, borrow from abroad, invest in non-oil physical capital, rent the capital to non-oil firms, and receive profits from those monopolistic non-oil firms and transfers from the central bank. The labor market is competitive, wage is the same across all households, and both types of households work the same number of hours. The superscript S indicates a variable associated with savers (forward-looking households), while N is for non-savers (rule-of-thumb households).

The forward-looking household maximizes its utility (Schmitt-Grohe & Uribe, 2003):

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{[C_t^S - \phi^{-1} N_t^{\phi}]^{1-\sigma} - 1}{1-\sigma}, \qquad \phi > 1,\ \sigma > 1 \tag{52}$$

subject to the following budget constraint:

$$C_t^S + I_t + b_t + R_{t-1}^* \frac{RER_t}{RER_{t-1}} \frac{b_{t-1}^*}{\pi_t^*} + T_t^S = W_t N_t + R_t^{kno} K_{t-1}^{no} + R_{t-1} \frac{b_{t-1}}{\pi_t} + b_t^* + \Pi_t + CB_t, \tag{53}$$

where $b_t = \frac{B_t}{P_t}$ is the real purchase of government bonds, RER_t is a CPI-based real exchange rate (the price of a foreign goods basket in terms of the domestic goods basket), $b_t^* = RER_t \frac{B_t^*}{P_t^*}$ is the real foreign borrowings expressed in domestic goods (all foreign variables are denoted by an asterisk), R_{t-1} and R_{t-1}^* are the nominal gross domestic and foreign interest rates respectively, T_t^S is the real lump-sum taxes, W_t is a real wage, R_t^{kno} is the real rental cost of non-oil physical capital, $\pi_t = \frac{P_t}{P_{t-1}}$ is inflation, Π_t is the real profits of monopolistic non-oil firms[15], and CB_t is the central bank's transfers in a form

[15] $\Pi_t = Y_t^{no}(p_{h,t} - MC_t)$, where Y_t^{no} is non-oil output, $p_{h,t}$ is the relative domestic price of non-oil goods to composite consumption, and MC_t is the marginal costs of non-oil firms to composite consumption.

of real foreign exchange reserves (see equation 78).

The law of motion for non-oil capital is specified according to Berg et al. (2013), incorporating the investment adjustment costs:

$$K_t^{no} = (1-\delta)K_{t-1}^{no} + \left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t, \quad \text{where } \kappa > 0 \tag{54}$$

The collateral constraint relates gross foreign liabilities to a future value of capital (durable goods in Faia & Iliopulos, 2011) and always binds, assuming that foreign debt is permanently high in this economy[16]:

$$R_t^* b_t^* = E_t\{\Omega \frac{Q_{t+1}\pi_{t+1}^*}{RER_{t+1}/RER_t} K_t^{no}\}, \tag{55}$$

where Q_t is a shadow value of non-oil capital (Tobin's Q) and Ω is an upper bound of leverage ratio.

The problem of the saver is, therefore, to maximize the utility (52) with respect to consumption C_t^S, investment I_t, capital K_t^{no}, government bonds holdings b_t, foreign borrowings b_t^*, and hours worked N_t subject to the budget constraint (53), capital accumulation equation (54), and collateral constraint (55). The first-order conditions of this problem are in Appendix 2.6.3.

The rule-of-thumb household has the same preferences as the saver. It chooses only consumption and labor and its budget constraint is simply this:

$$C_t^N + T_t^N = W_t N_t \tag{56}$$

Each $i \in \{S, N\}$ type of household has the composite CES consumption preferences over domestic and foreign goods with $\eta > 0$ as an elasticity of substitution between goods:

$$C_t(i) = \left[\gamma^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}}(i) + (1-\gamma)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}(i)\right]^{\frac{\eta}{\eta-1}},$$

[16] Occasionally binding collateral constraint is ruled out because it requires global solution methods, which may be infeasible to apply in this complex model.

where γ is a home-bias parameter, while $(1-\gamma)$ is a degree of openness. The standard consumption expenditures minimization by a household delivers the following CPI index:

$$P_t^{1-\eta} = \gamma P_{h,t}^{1-\eta} + (1-\gamma) P_{f,t}^{1-\eta} \text{ or } 1 = \gamma p_{h,t}^{1-\eta} + (1-\gamma) RER_t^{1-\eta}, \tag{57}$$

where $p_{h,t}$ is a relative price of domestic goods to composite consumption and RER_t is also a relative price of foreign goods to composite consumption.

The aggregate consumption in turn is $C_t = \mu C_t^N + (1-\mu) C_t^S$. Similar to private consumption, investment is the CES basket with the same home-bias parameter γ and CPI for simplicity.

2.2.2 Non-oil and oil production sectors

Following the basic New Keynesian framework, there are monopolistically competitive non-oil firms producing differentiated intermediate goods, and a perfectly competitive non-oil firm producing a final domestic good. The final domestic non-oil producer has a constant returns technology:

$$Y_t^{no} = \left(\int_0^1 X_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $X_t(j)$ is the input amount of intermediate good j and $\varepsilon > 1$ is the elasticity of substitution between differentiated intermediate goods. It maximizes profit taking as given the domestic final good's price P_t^h and intermediate goods' prices $P_t^h(j)$ such that the optimal demand allocation is as follows:

$$X_t(j) = \left(\frac{P_t^h(j)}{P_t^h} \right)^{-\varepsilon} Y_t^{no} \tag{58}$$

Each intermediate goods non-oil firm has an identical Cobb-Douglass production function, which includes the non-oil private capital, labor, and public capital:

$$Y_t^{no}(j) = u^{no} K_{t-1}^{no}(j)^{\alpha} N_t(j)^{1-\alpha} K_{G,t-1}^{\psi}, \tag{59}$$

where the level of technology u^{no} is just constant (unity) and the usage of public capital is common to all firms.

Intermediate goods producers solve their problem in two stages. First, cost minimization subject to the production function (59) provides the following marginal costs common to all non-oil firms, taking the wage and rental cost of capital, denominated in domestic non-oil goods, as given:

$$mc_t = \frac{w_t^{1-\alpha} (r_t^{kno})^{\alpha}}{u^{no} K_{G,t-1}^{\psi} (1-\alpha)^{1-\alpha} \alpha^{\alpha}} \tag{60}$$

Second, intermediate non-oil producers choose the price P_t^{hop} to maximize their discounted real profits:

$$\sum_{m=0}^{\infty} \theta^m E_t \left\{ D_{t,t+m} Y_{t+m}^{no}(j) \left(\frac{P_t^{hop}}{P_{t+m}^h} - mc_{t+m} \right) \right\}, \tag{61}$$

where $D_{t,t+m} = \beta^m E_t(\frac{U_{C_{t+m}^S}}{U_{C_t^S}})$ is a stochastic discount factor coming from the forward-looking household's problem, subject to the demand constraint according to (58):

$$Y_{t+m}^{no}(j) = \left(\frac{P_t^{hop}}{P_{t+m}^h} \right)^{-\varepsilon} Y_{t+m}^{no}$$

A fraction $(1-\theta)$ of non-oil firms adjusts their prices each period, while the respective fraction θ keeps their prices unchanged; thus, θ is an index of price stickiness according to Calvo (1983). The domestic price index evolves as follows:

$$(P_t^h)^{1-\varepsilon} = \theta (P_{t-1}^h)^{1-\varepsilon} + (1-\theta)(P_t^{hop})^{1-\varepsilon}$$

The first-order condition of this price setting decision (61) is below:

$$\sum_{m=0}^{\infty} \theta^m E_t \left\{ D_{t,t+m} Y_{t+m}^{no}(j) \left(\frac{P_t^{hop}}{P_{t+m}^h} - \frac{\varepsilon}{\varepsilon - 1} mc_{t+m} \right) \right\} = 0, \tag{62}$$

where $\frac{\varepsilon}{\varepsilon-1}$ is a frictionless price markup.

The production function of an oil firm has only capital input, assuming that oil production is a capital-intensive sector, and to avoid any complications originating from the possible labor mobility between two sectors:

$$Y_t^o = (K_{t-1}^o)^{\alpha^o} \tag{63}$$

The oil capital is accumulated by FDI which responds to the world oil price:

$$K_t^o = (1-\delta)K_{t-1}^o + FDI_t^* \tag{64}$$

$$\widehat{FDI_t^*} = \rho_{FDI}\widehat{FDI_{t-1}^*} + (1-\rho_{FDI})\widehat{P_t^{o*}} \tag{65}$$

The world oil price follows the AR(1) process and has an exogenous shock referred to as the terms of trade shock:

$$\widehat{P_t^{o*}} = \rho_o\widehat{P_{t-1}^{o*}} + \epsilon_t^o \tag{66}$$

The oil firm receives its profits Π_t^{o*} net of royalties levied on production quantity at a rate τ^o:

$$\Pi_t^{o*} = (1-\tau^o)P_t^{o*}Y_t^o \tag{67}$$

The oil sector is owned by foreigners and the government. The dividend share of oil profits that the government receives is denoted by ι^{div}.

2.2.3 Fiscal policy

The government collects its lump-sum taxes T_t and transfers from the SWF, TR_t. It issues one-period bonds to finance the government purchases, which include public consumption G_t^C and public investment G_t^I. The government budget constraint in real terms is as follows[17]:

$$(1-\mu)b_t + T_t + \underbrace{\frac{R^*_{t-1}}{\pi^*_t}(1-\rho_{swf})SWF^*_{t-1}RER_t}_{TR_t} = p^g_t(G^C_t + G^I_t) + (1-\mu)R_{t-1}\frac{b_{t-1}}{\pi_t}, \quad (68)$$

where $T_t = (1-\mu)T^S_t + \mu T^N_t$ and p^g_t is a relative price of government purchases to composite consumption with its own home-bias parameter γ_2.

$$p^g_t = \left[\gamma_2 p^{1-\eta}_{h,t} + (1-\gamma_2)RER^{1-\eta}_t\right]^{\frac{1}{1-\eta}} \quad (69)$$

Public investment is productive so that the law of motion for public capital is given by:

$$K^G_t = (1-\delta^g)K^G_{t-1} + G^I_t \quad (70)$$

Oil tax revenues, denominated in foreign goods, consist of royalties and government share of the oil sector's profits

$$T^{o*}_t = \tau^o P^{o*}_t Y^o_t + \iota^{\text{div}}\Pi^{o*}_t \quad (71)$$

which go directly to the SWF, accumulated according to the equation below.

$$SWF^*_t = \rho_{swf}\frac{R^*_{t-1}}{\pi^*_t}SWF^*_{t-1} + T^{o*}_t, \quad (72)$$

where ρ_{swf} is a persistence in the SWF process after its interest is accrued, while $(1-\rho_{swf})$ fraction of interest income transfers to the government budget.

Two fiscal instruments, public investment and public consumption, have the following rules, with their oil output response (ϑ_{GI} and ϑ_{GC}) associated with fiscal cyclicality. The real oil output is affected by the world oil price shock through FDI, accumulating oil

[17] Alternatively, the government budget constraint can be represented as $(1-\mu)b_t + T_t + T^{o*}_t RER_t + \frac{R^*_{t-1}}{\pi^*_t}SWF^*_{t-1}RER_t = p^g_t(G^C_t + G^I_t) + (1-\mu)R_{t-1}\frac{b_{t-1}}{\pi_t} + SWF^*_t RER_t$. By plugging the SWF equation (72) into it, the government budget constraint boils down to (68).

capital; therefore, it is directly included to fiscal rules, so that fiscal policy transmits the shock into the economy, which has the cycle of oil sector's boom/bust. The SWF transfers, in contrast, are a stock variable representing rather an annuity value which is affected by the real exchange rate.

$$\widehat{G_t^I} = \rho_{GI}\widehat{G_{t-1}^I} + (1-\rho_{GI})[\vartheta_{GI}\widehat{Y_t^o} - \gamma_{GI}\widehat{b}_{t-1} + \gamma_{TR}^{GI}\widehat{TR}_t] \tag{73}$$

$$\widehat{G_t^C} = \rho_{GC}\widehat{G_{t-1}^C} + (1-\rho_{GC})[\vartheta_{GC}\widehat{Y_t^o} - \gamma_{GC}\widehat{b}_{t-1} + \gamma_{TR}^{GC}\widehat{TR}_t] \tag{74}$$

This specification of referring pro/counter/acyclical fiscal policy to positive/negative/zero values for ϑ_{GI} and ϑ_{GC}, respectively, is consistent with a notion of cyclically adjusted or structural fiscal balances, according to which a cyclical component, related to automatic stabilizers, should be removed mostly from taxes and public transfers, while public spending on wages, goods, and services is usually independent of the business cycle, thus not requiring any adjustment (Bornhorst, Dobrescu, Fedelino, Gottschalk & Nakata, 2011).

Since fiscal debt clears the government budget constraint, the lump-sum taxes need a separate equation, which includes fiscal debt, public spending, and SWF transfers specific to this model:

$$\widehat{T}_t = \varphi_b\widehat{b}_{t-1} + \varphi_I\widehat{G_t^I} + \varphi_C\widehat{G_t^C} - \varphi_{TR}\widehat{TR}_t \tag{75}$$

2.2.4 Monetary policy

As domestic interest rate is not equal to foreign interest rate, an independent monetary policy is feasible under imperfect capital mobility. The nominal interest rate responds to its lagged value, CPI inflation, and aggregate output according to the CPI targeting Taylor rule below:

$$\widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)\left[\phi_\pi\pi_t + \phi_y\widehat{Y}_t\right], \tag{76}$$

where ρ is an interest rate smoothing parameter, ϕ_π and ϕ_y are the inflation and output responses, respectively.

The PPT Taylor rule, in contrast, uses the product price inflation, which is a weighted average of oil price inflation in real terms $\pi_t^o = \triangle\ \widehat{P_t^{o*}} + \triangle\ \widehat{RER}_t$ and domestic price inflation $\pi_t^h = \pi_t - \frac{1-\gamma}{\gamma} \triangle\ \widehat{RER}_t$, according to Appendix 2.6.5**,** with weights corresponding to the GDP share of the oil s_o and non-oil $(1 - s_o)$ sectors, respectively.

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + (1-\rho) \left[\phi_\pi \left(s_o \pi_t^o + (1-s_o)\pi_t^h\right) + \phi_y \widehat{Y}_t\right] \tag{77}$$

After rearranging, the PPT rule boils down to:

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + (1-\rho) \left[\phi_\pi \left(s_o \triangle\ \widehat{P_t^{o*}} + (1-s_o)\pi_t + \frac{s_o - 1 + \gamma}{\gamma} \triangle\ \widehat{RER}_t\right) + \phi_y \widehat{Y}_t\right]$$

Every period, the central bank receives interest on its foreign exchange reserves and invests into a new stock of reserves. This flow of revenues is transferred to forward-looking households, savers:

$$CB_t = R_{t-1}^* \frac{RER_t}{RER_{t-1}} \frac{fxr_{t-1}^*}{\pi_t^*} - fxr_t^*, \tag{78}$$

where $fxr_t^* = RER_t \frac{FXR_t^*}{P_t^*}$ is the real foreign exchange reserves expressed in composite consumption goods.

A managed exchange rate regime is associated with the foreign exchange interventions as an additional monetary policy instrument. They represent the purchases/selling of foreign currency by a central bank, and accumulate the foreign exchange reserves according to their rule (Benes et al., 2015), responding to the exchange rate and its rate of depreciation[18].

$$\widehat{fxr_t^*} = \rho_{fxr} \widehat{fxr_{t-1}^*} + (1-\rho_{fxr})(\alpha_1 \widehat{RER}_t + \alpha_2 \triangle\ \widehat{RER}_t), \quad \alpha_1 < 0,\ \alpha_2 < 0 \tag{79}$$

This rule shows that the more the exchange rate depreciates/appreciates, the more the foreign exchange reserves fall/accumulate, implying the selling/purchases of foreign currency by a central bank, respectively.

A flexible exchange rate regime is associated with the zero values for α_1 and α_2 para-

[18] The higher $\widehat{RER}_t$, the more the real exchange rate depreciates.

meters in the foreign exchange interventions rule.

2.2.5 Market clearing conditions

For simplicity, an elasticity of substitution between domestic and foreign goods is assumed to approach one ($\eta \to 1$); thus, the domestic non-oil goods market clearing condition is as follows:

$$p_t^h Y_t^{no} = \gamma\left[C_t + (1-\mu)I_t\right] + \gamma_2 p_t^g (G_t^C + G_t^I) \tag{80}$$

The real GDP on its supply and demand sides is:

$$Y_t = p_t^h Y_t^{no} + Y_t^o P_t^{o*} RER_t = C_t + (1-\mu)I_t + p_t^g (G_t^C + G_t^I) + NX_t \tag{81}$$

The labor and capital markets clear according to their conditions:

$$N_t = \int_0^1 N_t(j)dj, \qquad (1-\mu)K_t^{no} = \int_0^1 K_t^{no}(j)dj$$

The balance of payments equates its current account with the financial account, combining the budget constraints of government and households-savers. The current account includes net exports, interest income of SWF assets (as those assets are saved abroad) minus the foreign share of the oil sector's profits, while the financial account represents the interest payments on foreign debt, a new foreign borrowing of households, foreign exchange reserves transferred to households, and FDI.

$$NX_t + \frac{R_{t-1}^*}{\pi_t^*}(1-\rho_{swf})SWF_{t-1}^* RER_t - (1-\iota^{\text{div}})RER_t \Pi_t^{o*} =$$
$$= (1-\mu)\left(R_{t-1}^* \frac{RER_t}{RER_{t-1}} \frac{b_{t-1}^*}{\pi_t^*} - b_t^*\right) + fxr_t^* - R_{t-1}^* \frac{RER_t}{RER_{t-1}} \frac{fxr_{t-1}^*}{\pi_t^*} - RER_t FDI_t^*$$

2.2.6 The rest of the world

The rest of the world is a large economy governed by three exogenous equations for its output, interest rate, and inflation, respectively:

$$\widehat{Y}_t^* = \rho_{Y*}\widehat{Y}_{t-1}^* + \epsilon_t^{Y^*} \tag{82}$$

$$\widehat{R_t^*} = \phi_\pi^* \pi_t^* + \phi_y^* \widehat{Y}_t^* \tag{83}$$

$$\pi_t^* = \beta^* E_t \pi_{t+1}^* + \lambda^* \left(\sigma + \frac{\phi^* + \alpha^*}{1 - \alpha^*} \right) \widehat{Y}_t^* \tag{84}$$

The equilibrium of this model consists of households' and firms' optimality conditions (92, 93, 94, 95, 100, and 109), capital accumulation equations (96, 97, and 98), SWF accumulation (99), outputs (101, 102, and 103), the government budget constraint (104), fiscal policy (73, 74, and 75), monetary policy (76 or 77 and 79), the balance of payments (108), FDI process (65), market clearing conditions (106 and 107), price equations (66, 91, and 105), and the rest of the world (82, 83, and 84).

2.3 Calibration

All parameters can be divided into three sets: standard values borrowed from other studies because of the non-availability of relevant data, estimates from time-series regressions according to the model's equations, and calibrated parameters based on a steady state of the model. The list of parameters is provided in Appendix 2.6.1, excluding the GDP ratios and parameters for the rest of the world which are described in this section.

The first set includes the depreciation rates for private and public capital $\delta = 0.025$, $\delta^g = 0.02$ (Traum & Yang, 2015), the elasticity of substitution between differentiated intermediate goods $\varepsilon = 9$ (Gali, 2015), price stickiness $\theta = 0.9$ (Jakab & Vilagi, 2008), the inverse of intertemporal elasticity of substitution for consumption $\sigma = 2$ (Schmitt-Grohe & Uribe, 2003), investment adjustment costs parameter $\kappa = 20$ (Berg et al., 2013), and the fiscal debt response of lump-sum taxes $\varphi_b = 0.4$ (Algozhina, 2012). The foreign parameters are set to their standard values: the elasticity of wages with respect to hours worked $\phi^* = 1.45$ (Schmitt-Grohe & Uribe, 2003), discount factor $\beta^* = 0.99$, inflation and output responses in the Taylor rule $\phi_\pi^* = 1.5$, $\phi_y^* = 0.125$ (Gali, 2015), price stickiness $\theta^* = 0.75$ (Gali, Lopez-Salido & Valles, 2007), output elasticity to capital $\alpha^* = 0.32$, and output persistence $\rho_{Y^*} = 0.8$.

The second set consists of significant OLS estimates according to the model's equations based on quarterly Kazakh data in real terms, described in Appendix 2.6.2. Since the data are trending and integrated of first order, a new method proposed by Hamilton (2017) has been used to extract a cyclical component: 8-quarters future or current value is regressed on a constant and current or 8-lags value. The residuals from this two-year projection would represent cyclical factors and describe a true-data generating process, as opposed to a cyclical component produced by the Hodrick-Prescott filter. These forecast errors from a univariate time series in log can be treated as a data-consistent analogous object to the stationary variables in the model, expressed in deviation from their steady state and matched with trending observed data. A cyclical component of each time series, retrieved in this way, is then regressed on a cyclical component of other variables according to the model's equations.

The estimates of the public consumption rule (74) are as follows, with t-statistics in parentheses, suggesting the only significant autoregressive coefficient $\rho_{GC} = 0.44$, which is also set for the persistence in public investment $\rho_{GI} = 0.44$.

$$\widehat{G_t^C} = \underset{(2.1)}{0.04} + \underset{(2.5)}{0.44}\widehat{G_{t-1}^C} + \underset{(1.58)}{0.14}\widehat{Y_t^o} - \underset{(-0.37)}{0.02}\widehat{b_{t-1}} - \underset{(-0.82)}{1.3 \cdot 10^{-6}}\widehat{TR_t}$$

R-sq. 0.37, Adj. R-sq. 0.29
DW 1.6, N 37 obs.

According to the lump-sum taxes equation (75), the regression of cyclical components of non-oil fiscal revenues on public consumption, lagged public debt, SWF transfers to the government budget[19], and public investment, which is proxied by the data of fiscal capital expenditures, produces the significant response to public consumption $\varphi_C = 0.95$.

$$\widehat{T_t} = \underset{(-0.89)}{-0.03} + \underset{(0.22)}{0.02}\widehat{b_{t-1}} + \underset{(0.68)}{0.08}\widehat{G_t^I} + \underset{(3.9)}{0.95}\widehat{G_t^C} - \underset{(-0.24)}{6.1 \cdot 10^{-7}}\widehat{TR_t}$$

R-sq. 0.34, Adj. R-sq. 0.26
DW 1.15, N 37 obs.

The interest rate smoothing ρ, according to the empirical CPI Taylor rule, appears to be 0.88.

$$\widehat{R_t} = \underset{(-0.37)}{-0.04} + \underset{(16.6)}{0.88}\widehat{R_{t-1}} + \underset{(0.6)}{0.04}\pi_t - \underset{(-0.94)}{1.2}\widehat{Y_t}$$

R-sq. 0.87, Adj. R-sq. 0.86
DW 1.6, N 49 obs.

The foreign exchange interventions rule (79) results in the following, suggesting the

[19]The SWF transfers to the government budget are zeros in 2006:Q3 and 2006:Q4. Thus, it is not possible to take a log of them, and therefore the coefficient in front of this variable is very low.

persistence parameter $\rho_{fxr} = 0.77$ and the exchange rate response $\alpha_1 = \frac{-0.33}{1-0.77} = -1.4$. The exchange rate change response α_2 is insignificant; therefore it is set to -0.1. These values of α_1 and α_2 apply to a managed exchange rate regime, whereas they are zeros if a flexible exchange rate regime is examined.

$\widehat{fxr_t^*} = 0.008 + 0.77\widehat{fxr_{t-1}^*} - 0.33\widehat{RER_t} + 0.29 \vartriangle \widehat{RER_t}$ R-sq. 0.76, Adj. R-sq. 0.75

(0.45) (8.78) (-3.2) (0.65) DW 2.1, N 61 obs.

An empirical counterpart of the world oil price equation (66) gives the persistence in the oil price process $\rho_o = 0.85$, while the standard deviation of residuals is 0.2.

$\widehat{P_t^{o*}} = 0.003 + 0.85\widehat{P_{t-1}^{o*}} + \epsilon_t^o$ R-sq. 0.72, Adj. R-sq. 0.717

(0.1) (12.76) (s.d. 0.2) DW 1.3, N 65 obs.

The FDI equation (65) produces the significant FDI persistence ρ_{FDI} of 0.3.

$\widehat{FDI_t^*} = 0.007 + 0.3\widehat{FDI_{t-1}^*} + 0.01\widehat{P_t^{o*}}$ R-sq. 0.09, Adj. R-sq. 0.03

(0.07) (1.7) (0.04) DW 1.92, N 33 obs.

The third set includes the parameters calibrated to a steady state of the model which corresponds to data averages[20]. The GDP ratios of private consumption, public consumption, net exports, FDI, foreign debt, fiscal debt, oil output, public investment, and foreign exchange reserves are as follows, respectively: $c_y = 0.61$, $g_y^C = 0.08$, $nx_y = 0.07$, $fdi_y = 0.09$, $b_y^* = 2.17$, $b_y = 0.5$, $s_o = 0.52$, $g_y^I = 0.07$, and $fxr_y = 0.52$. The degree of openness is calculated as a ratio of imports to GDP, $1 - \gamma = 0.32$; thus, the home-bias parameter in private consumption and investment γ is equal to 0.68, while it is assumed to be higher for public spending $\gamma_2 = 0.9$, as its large share may go to the wages of public servants. The domestic discount factor is around 0.978 because the average T-bill rate is used as a proxy for the policy interest rate, 2.3 percent per quarter[21]. The upper bound of leverage ratio Ω appears to be 0.54. The elasticity of output with respect to private capital α is equal to 0.3 as a share of capital income to GDP, while with respect to public capital it is $\psi = 0.16$ suggested by a steady state wage equation in Appendix 2.6.4. Using

[20]The steady state is natural and inefficient in Appendix 2.6.4, since it is at flexible prices and with monopolistic competition.

[21]The domestic interest rate matters for the government bonds in this model, as investments are financed by foreign funds rather than the domestic financial market.

data on wages, the elasticity of wages with respect to hours worked ϕ is 1.45 according to the labor supply condition (90), in which hours are obtained from the non-oil production function (59). The royalties rate levied on oil production quantity $\tau^o = 0.27$ is calculated as the SWF inflows share in oil output. The dividend share of oil profits that the government receives ι^{div} is set to 0.05, while the elasticity of oil output with respect to oil capital α^o is technically feasible at 0.7. The persistence in SWF process ρ_{swf} is equal to 0.747 to match the GDP ratio of SWF assets $swf_y = 0.65$.

There are three types of fiscal policy: procyclical, countercyclical, and acyclical. The acyclical fiscal policy is a benchmark to calculate welfare loss in deviation from it. It is associated with the zero oil output response of public consumption and public investment in their rules ($\vartheta_{GC} = 0$ and $\vartheta_{GI} = 0$). The procyclical fiscal policy corresponds to the positive oil output response of public spending ($\vartheta_{GC} = 0.4$ and $\vartheta_{GI} = 0.4$), while the countercyclical fiscal policy is simulated at their negative values ($\vartheta_{GC} = -0.4$ and $\vartheta_{GI} = -0.4$). Those are the two parameters which differ across fiscal cyclicality, while the rest hold the same. The fiscal debt responses of public consumption $\gamma_{GC} = 0.3$ and public investment $\gamma_{GI} = 0.3$ are assumed to be equal. The response of public consumption to SWF transfers γ_{TR}^{GC} is set to 0.2, fixing it slightly lower than $\gamma_{GC} = 0.3$, whereas public investment response to the SWF transfers γ_{TR}^{GI} is 0.1. The parameters of the lump-sum taxes equation (75) are as follows: public investment response $\varphi_I = 0.2$ and SWF transfers response $\varphi_{TR} = -0.3$. The latter is calculated according to taxes at a steady state under acyclical fiscal policy:

$$\varphi_{TR} = \frac{\varphi_b \ln \overline{b} + \varphi_I \ln \overline{G_I} + \varphi_C \ln \overline{G_C} - \ln \overline{T}}{\ln \overline{TR}}$$

2.4 Results

This section describes results in the following order. The welfare measure derived by De Paoli (2009) is explained, based on which a grid search of Taylor rule parameters is made. The model has been simulated with two shocks: the world oil price and foreign output shocks with their standard deviations of 0.2 for both. Given these optimal monetary policy parameters, the impulse-response functions to a negative world oil price shock, interpreted as the worsening of a terms of trade shock, are analyzed to understand its transmission mechanism. The welfare loss components are examined across fiscal policy cyclicality, exchange rate regimes, and monetary policy's price anchors at calibrated parameters of the foreign exchange interventions rule. Furthemore, these parameters of exchange rate policy are varied at the optimal Taylor rule to find their loss-minimizing values[22]. Finally, weights for loss components are assumed to be not equal, following the parametrization of De Paoli (2009), to compare with the baseline results.

In a small open economy with monopolistic competition and nominal rigidities, De Paoli (2009) has used a linear-quadratic approach to derive welfare as a second-order approximation of households' utility. The linear terms in this objective function have been eliminated by a second-order approximation of her model's equilibrium conditions in order to take into account the effect of second moments on the mean of the variables. As a result, the objective loss function becomes a purely quadratic expression of domestic price inflation, output gap, and real exchange rate.

In this model, such a loss measure is adopted as a sum of variances in domestic price inflation π_t^h, aggregate output $\widehat{Y}_t$, and real exchange rate $\widehat{RER_t}$. This loss function is minimized to find the Taylor rule: inflation ϕ_π and output responses ϕ_y across pro/counter/acyclical fiscal stance. Monetary policy can be hybrid, combining a managed exchange rate regime with a CPI/PPT anchor, or pure inflation targeting associated with the CPI/PPT under a flexible exchange rate regime. The foreign exchange interventions rule is set to its calibrated parameters ($\alpha_1 = -1.4$ and $\alpha_2 = -0.1$) for a managed exchange rate regime, while those parameters are zeros under a flexible exchange rate regime.

[22] The existing literature has not yet explored an optimal exchange rate policy combined with monetary policy in a DSGE framework. Meanwhile, Ilzetzki, Reinhart, and Rogoff (2017) find that less flexible exchange rate arrangements account for around 80 percent of all countries out of 194 over 1946-2016, or about one-half of world GDP.

Table 1. **Optimal monetary policy responses**

	Managed exchange rate		Flexible exchange rate	
Procyclical fiscal policy				
	CPI targeting	PPT	CPI targeting	PPT
ϕ_π	11	14	13	16
ϕ_y	2.35	1.1	2.85	1.35
Countercyclical fiscal policy				
	CPI targeting	PPT	CPI targeting	PPT
ϕ_π	11	14	12	15
ϕ_y	2.6	1.35	3.35	1.6
Acyclical fiscal policy				
	CPI targeting	PPT	CPI targeting	PPT
ϕ_π	11	14	13	15
ϕ_y	2.6	1.1	3.1	1.6

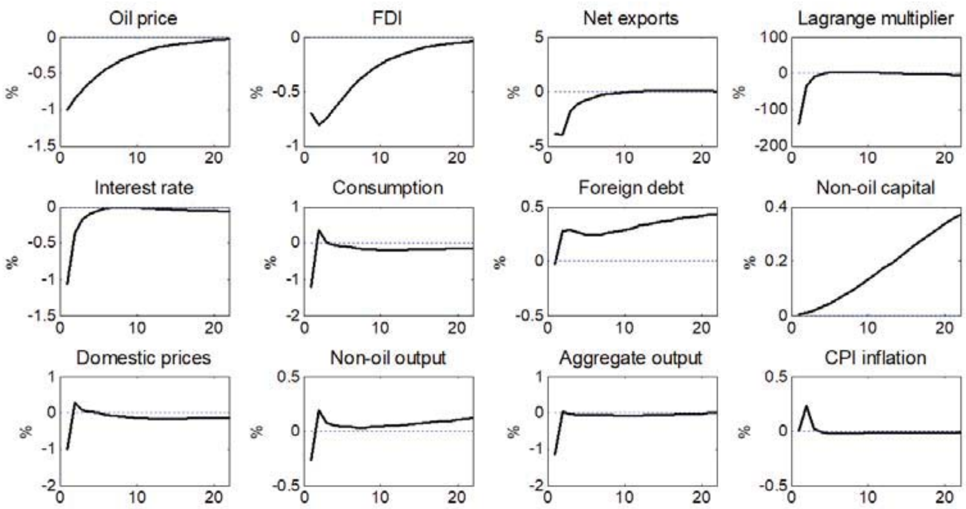
Table 1 shows the results of grid search made in a range 0-20 with a step of 1 for inflation response ϕ_π and 0.1-5.1 with a step of 0.25 for output response ϕ_y. The reason for high monetary policy parameters compared to their standard values, commonly found in the literature, is in the collateral constraint of this model. The Lagrange multiplier to the collateral constraint appears in the first-order conditions of a forward-looking household's problem in Appendix 2.6.3, the log-linearization of which results in two equations containing this shadow value of relaxing the borrowing constraint (94 and 100 in Appendix 2.6.6). One of this equations is essentially the UIP condition, which determines the real exchange rate that affects inflation, due to its complete pass-through effect, and aggregate output via oil sector or net exports. As a result, the model produces an additional volatility, which tries to be stabilized by a more aggressive monetary policy than it would have been in a standard framework without the collateral constraint[23].

[23] The first paper in this dissertation compares volatilities across two cases: with and without a collateral constraint. Faia and Iliopulos (2011), using the same collateral constraint for their durable goods, also found that consumption, output, and inflation become volatile in the open economy setting.

According to Table 1, the PPT inflation response is higher than the CPI targeting because the PPT Taylor rule includes oil price inflation, which needs to be properly stabilized in the presence of oil price shock. A flexible exchange rate regime, across CPI/PPT, has higher inflation responses than the price anchors produce under a managed exchange rate. This is because the former exchange rate regime has only one monetary policy instrument, the interest rate, which should still respond to a change in the real exchange rate as the terms of trade, due to consumption of foreign goods. A managed exchange rate regime, in contrast, has the foreign exchange interventions in addition to the interest rate that stabilize the exchange rate; therefore, the inflation response of interest rate does not need to be high.

As for optimal output response, Table 1 suggests that it is lower under a PPT rule because oil output can be stabilized by the PPT inflation response, which controls for oil price inflation. Thus, there is no need for a strong reaction to aggregate output, since its volatile oil output is taken care by the inflation response already. This, in contrast, does not hold under a CPI rule. A flexible exchange rate regime, meanwhile, causes the output response to be higher than a managed exchange rate does at respective CPI/PPT anchors. This is because the absence of foreign exchange interventions in the former case, which would otherwise better stabilize the exchange rate and thereby aggregate output, limits the ability of monetary policy to deal with output volatility, leaving this exclusively to the interest rate, which therefore has to respond strongly to output.

Figure 1. **Impulse-response functions to a negative world oil price shock of 1%: countercyclical fiscal policy combined with PPT monetary rule under a managed exchange rate regime**



Figures 1 and 2 show the impulse-response functions to a negative world oil price shock produced at the optimized monetary policy parameters according to Table 1. A flexible exchange rate regime, across all cases of policy combination, produces no changes in the foreign exchange reserves, otherwise the impulse-response functions stay the same. The figures, in text, display the dynamics of a countercyclical fiscal stance and managed exchange rate regime, while the rest can be found in Appendix 2.6.7. A sudden drop in oil price discourages the oil sector's FDI and decreases net exports. A fall in FDI implies that the economy is less indebted to the foreign world, whereas an increase of trade deficit needs to be financed externally. These two effects on the balance of payments front lead to a fall in the Lagrange multiplier to collateral constraint, since the shadow value of relaxing the borrowing constraint goes down, as this borrowing limit extends. The interest rate also decreases, because the UIP condition (100 in Appendix 2.6.6) includes the Lagrange multiplier, suggesting that a difference between domestic real interest rate and foreign real interest rate should be equal to a sum of expected change in the real exchange rate and this shadow value of relaxing the borrowing constraint. Therefore, the Lagrange multiplier positively affects the domestic interest rate, and its fall means that the marginal cost of borrowing declines, triggering a substitution effect from consumption, which immediately drops, to investment, as foreign debt and non-oil capital accumulate over time. Low consumption decreases the domestic prices and contributes to a fall in non-oil output, as hours worked are discouraged by low wages that are production costs to be covered by low domestic prices. Since non-oil output and oil price decline, the aggregate output falls as a result of combining two production sectors.

Figure 1, in contrast to Figure 2, shows a larger drop in the interest rate because a decrease in oil price shock affects the oil price inflation, to which the PPT Taylor rule responds in Figure 1. The Lagrange multiplier falls to a larger extent under PPT, influenced by the more decreased interest rate. As the interest rate goes down, inflation has an obvious spike in the second period. These spikes, due to the lower interest rate of PPT, are also observed in consumption, domestic prices, hours worked, and non-oil output as opposed to the dynamics of CPI rule in Figure 2.

Figure 2. **Impulse-response functions to a negative world oil price shock of 1%: countercyclical fiscal policy combined with CPI monetary rule under a managed exchange rate regime**

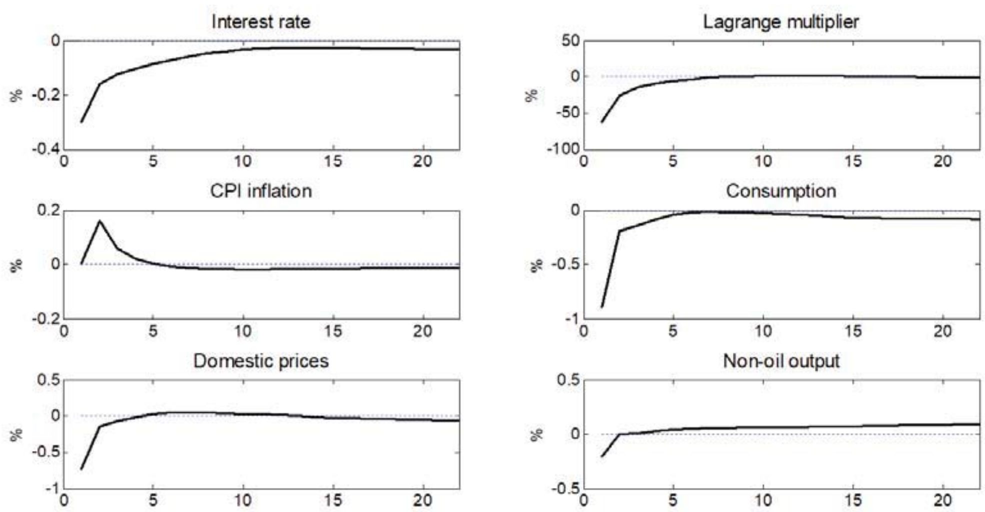


Table 2 summarizes the numerical results of loss measure L as a sum of variances in domestic price inflation, aggregate output, and real exchange rate. The results are produced at the optimal Taylor rule parameters and calibrated foreign exchange interventions rule ($\alpha_1 = -1.4$ and $\alpha_2 = -0.1$ in the equation 79). All entries are in percent deviation from a benchmark policy combination: acyclical fiscal stance and a CPI monetary anchor under a flexible exchange rate regime. Positive values mean the percentage increase in loss relative to the benchmark, while negative values indicate lower loss contributed by a respective entry.

Table 2. **Loss components** (in %)

	Procyclical fiscal policy			
	Managed exchange rate		Flexible exchange rate	
	CPI targeting	PPT	CPI targeting	PPT
L	-4.56	-5.29	1.55	2.13
$Var(\pi_t^h)$	-0.009	0.01	-0.006	0.014
$Var(\widehat{Y}_t)$	-3.67	-2.12	1.21	3.9
$Var(\widehat{RER_t})$	-0.88	-3.18	0.35	-1.78
	Countercyclical fiscal policy			
	Managed exchange rate		Flexible exchange rate	
	CPI targeting	PPT	CPI targeting	PPT
L	-6.59	-7.56	-0.66	-0.3
$Var(\pi_t^h)$	0.004	0.025	0.01	0.03
$Var(\widehat{Y}_t)$	-4.77	-3.61	-0.5	2.05
$Var(\widehat{RER_t})$	-1.83	-3.97	-0.17	-2.38
	Acyclical fiscal policy			
	Managed exchange rate		Flexible exchange rate	
	CPI targeting	PPT	CPI targeting	PPT
L	-5.98	-6.86	0	0.46
$Var(\pi_t^h)$	-0.004	0.019	0	0.022
$Var(\widehat{Y}_t)$	-4.8	-3.15	0	2.18
$Var(\widehat{RER_t})$	-1.18	-3.73	0	-1.74

Values are in the percent deviation of corresponding entry from the benchmark acyclical fiscal policy combined with a CPI targeting monetary rule and flexible exchange rate regime.

Table 2 reports that a countercyclical fiscal stance and PPT monetary rule under a managed exchange rate regime is the best policy combination, delivering lower variances in aggregate output and real exchange rate than the benchmark does. This is because fiscal stimulus dampens a negative terms of trade shock by countercyclically offsetting it and stabilizing the ultimate effects on the economy better than procyclical and acyclical fiscal stances. In fact, a procyclical fiscal policy is the worst, since it transmits an external terms of trade shock to the domestic economy, which becomes dependent on volatile foreign shocks rather than staying resilient to them. A managed exchange rate regime is strongly preferred across all fiscal cyclicalities because a stabilization of exchange rate matters for a small open economy with its collateral constraint. Such an economy imports foreign goods and borrows from abroad, so that fluctuations in the exchange rate exacerbate not only relative price of imports (terms of trade), but also foreign debt. Therefore, foreign exchange interventions, which stabilize the exchange rate, are beneficial and they better smooth the aggregate output volatility at the same time.

The PPT anchor helps to achieve a more stable exchange rate than the CPI target shows in Table 2. This is because the interest rate of PPT responds to a change in the real exchange rate more than the CPI target, due to expressing the oil price inflation in real terms. Yet, the CPI rule provides a better stabilization of aggregate output and domestic price inflation, as the impulse-response functions do not display spikes in consumption, domestic prices, and non-oil output unlike PPT. It is driven by the CPI Taylor rule, which has higher optimal output and inflation responses, since the latter is not adjusted by the GDP share of non-oil output like under PPT. Therefore, if there is a flexible exchange rate regime, the CPI targeting outperforms the PPT.

Overall, Table 2 supports Frankel and Catao (2011) who recommends the PPT monetary anchor for a commodity exporting economy, since the exchange rate depreciates more in response to a fall in the world oil price. Note that the exchange rate depreciation does not occur immediately and is delayed due to sticky prices. However, PPT performs well under a managed exchange rate regime by improving the exchange rate stabilization, at the cost of higher volatilities in aggregate output and domestic price inflation. The latter two variables are better stabilized by the CPI targeting which is preferred under a flexible exchange rate regime.

Table 3. **Loss components at optimal α_1 and α_2** (in %)

	Procyclical FP		Countercyclical FP		Acyclical FP	
	Managed exchange rate regime					
	CPI targeting	PPT	CPI targeting	PPT	CPI targeting	PPT
L	-24.93	-26.81	-26.74	-29.4	-26.37	-28.63
$Var(\pi_t^h)$	-0.02	-0.03	-0.028	-0.018	-0.024	-0.03
$Var(\widehat{Y}_t)$	-24.27	-25.8	-26.5	-27.9	-26.57	-26.74
$Var(\widehat{RER}_t)$	-0.64	-0.98	-0.21	-1.5	0.22	-1.86
α_1	-9	-4	-9	-4	-9	-4
α_2	-5	-6	-6	-6	-6	-6

Values are in the percent deviation of corresponding entry from the benchmark acyclical fiscal policy combined with a CPI targeting monetary rule and flexible exchange rate regime.

The results of managed exchange rate regime, discussed so far, apply to the calibrated foreign exchange interventions rule ($\alpha_1 = -1.4$ and $\alpha_2 = -0.1$ in the equation 79). However, this rule can be optimized given the monetary policy responses provided by Table 1. The grid search of parameters α_1 and α_2 is therefore made in a range between -20 and 0 with a step of 1. Table 3 summarizes the numerical results of loss components under a managed exchange rate regime at optimal values for α_1 and α_2. Since a flexible exchange rate regime is associated with zero α_1 and α_2, its respective columns would be the same as in Table 2, thus they are omitted in this case.

The results show that output stabilization is significantly improved by a more active exchange rate policy, in which interventions strongly respond to the exchange rate now. This is because a stable exchange rate is preferred for this open economy, which borrows from abroad to accumulate physical capital, imports foreign goods for consumption and investment, and exports oil that is dependent on the world oil price. It has been assumed, though, that weights for loss components are equal to one, implying that policymakers have equal stabilization goals over domestic price inflation, aggregate output, and real exchange rate.

Table 4. **Loss components with different weights at optimal α_1 and α_2** (in %)

	Procyclical fiscal policy			
	Managed exchange rate		Flexible exchange rate	
	CPI targeting	PPT	CPI targeting	PPT
L	-3.27	-4.13	0.84	-3.55
$0.54Var(\pi_t^h)$	-0.03	-0.05	-0.009	0.02
$0.03Var(\widehat{Y}_t)$	-1.84	-1.95	0.09	0.29
$0.86Var(\widehat{RER}_t)$	-1.4	-2.13	0.76	-3.86
	Countercyclical fiscal policy			
	Managed exchange rate		Flexible exchange rate	
	CPI targeting	PPT	CPI targeting	PPT
L	-2.5	-5.41	-0.4	-4.97
$0.54Var(\pi_t^h)$	-0.04	-0.02	0.01	0.05
$0.03Var(\widehat{Y}_t)$	-2	-2.13	-0.04	0.15
$0.86Var(\widehat{RER}_t)$	-0.46	-3.26	-0.37	-5.17
	Acyclical fiscal policy			
	Managed exchange rate		Flexible exchange rate	
	CPI targeting	PPT	CPI targeting	PPT
L	-1.56	-6.12	0	-3.59
$0.54Var(\pi_t^h)$	-0.03	-0.04	0	0.03
$0.03Var(\widehat{Y}_t)$	-2.01	-2.03	0	0.17
$0.86Var(\widehat{RER}_t)$	0.48	-4.05	0	-3.79

Values are in the percent deviation of corresponding entry from the benchmark acyclical fiscal policy combined with a CPI targeting monetary rule and flexible exchange rate regime.

As a sensitivity test, the weights for loss components can differ, following, for example, the parametrization of De Paoli (2009): 0.54, 0.03, and 0.86 for variances in the domestic price inflation, output, and real exchange rate, respectively. These weights are used to calculate entries in Table 4 at the same Taylor rule and foreign exchange interventions rule optimally found earlier. Ideally, the weights should be derived in terms of deep parameters based on a second-order approximation of households' utility, suggesting future research on finding a model-consistent loss function. At this stage, Table 4 is rather a preliminary exercise to assess properties of the model and ensure that the model has been constructed correctly. An analytical representation of welfare derivation may yet require a substantial simplification of the model, to go forward.

The results indicate that even a flexible exchange rate regime should be combined with PPT, since a weight on the real exchange rate is relatively high now. This supports the previous finding that a PPT monetary rule stabilizes the exchange rate better than a CPI anchor. A further analysis of Table 4 should be avoided though, since the variances are produced by the Taylor rule and foreign exchange interventions rule that are optimized based on the loss function with equal weights for its components.

2.5 Conclusion

This paper develops a DSGE model for an emerging oil economy to study the loss-minimizing monetary policy jointly with a pro/counter/acyclical fiscal stance. The study reveals that the best policy combination is a countercyclical fiscal stance and managed exchange rate regime with the PPT monetary anchor. This allows the fiscal policy to countercyclically offset a volatile terms of trade shock, to which developing countries are often exposed, and the exchange rate to be managed by the central bank's interventions, which seem beneficial in providing a stable exchange rate since the economy borrows from abroad, imports foreign goods, and depends on the world oil price. It also suggests the monetary policy to target product price inflation, which includes oil price inflation important for the oil sector's exports and delivers better stabilization of exchange rate than the CPI anchor. However, if a flexible exchange rate regime is institutionally chosen, then the CPI targeting should be adopted, since it stabilizes well the domestic price inflation and aggregate output. In conclusion, the adverse effects of a volatile terms of trade shock can be mitigated by an appropriate fiscal and monetary policy combination to smooth variances in the domestic price inflation, aggregate output, and real exchange rate in a small open economy.

2.6 Appendix

2.6.1 Table of parameters

Parameter	Definition
$\beta = 0.978$	discount factor
$\gamma = 0.68$	home-bias in consumption and investment
$\gamma_2 = 0.9$	home-bias in government purchases
$\Omega = 0.54$	upper bound of leverage ratio
$\mu = 0.5$	fraction of rule-of-thumb households
$\alpha = 0.3$	non-oil output elasticity to private capital
$\psi = 0.16$	non-oil output elasticity to public capital
$\alpha^o = 0.7$	oil output elasticity to private capital
$\phi = 1.45$	wage elasticity to hours worked
$\sigma = 2$	inverse of intertemporal elasticity of substitution for consumption
$\delta = 0.025$	depreciation rate of private capital (oil and non-oil)
$\delta^g = 0.02$	depreciation rate of public capital
$\theta = 0.9$	index of price stickiness
$\varepsilon = 9$	elasticity of substitution between differentiated intermediate goods
$\kappa = 20$	investment adjustment costs parameter
$\phi_\pi = 14$	inflation response in the Taylor rule
$\phi_y = 1.35$	output response in the Taylor rule
$\alpha_1 = -1.4$	exchange rate response in the interventions rule
$\alpha_2 = -0.1$	exchange rate change response in the interventions rule
$\tau^o = 0.27$	oil royalty rate
$\iota^{\text{div}} = 0.05$	dividend share of oil profits accrued to the government
$\gamma_{GC} = \gamma_{GI} = 0.3$	response of public consumption/investment to fiscal debt
$\vartheta_{GC} = \vartheta_{GI} = -0.4$	response of public consumption/investment to oil output
$\gamma_{TR}^{GC} = 0.2$	response of public consumption to SWF transfers
$\gamma_{TR}^{GI} = 0.1$	response of public investment to SWF transfers
$\varphi_b = 0.4$	response of lump-sum taxes to fiscal debt
$\varphi_{TR} = -0.3$	response of lump-sum taxes to SWF transfers
$\varphi_C = 0.95$	response of lump-sum taxes to public consumption
$\varphi_I = 0.2$	response of lump-sum taxes to public investment
$\rho_{GC} = \rho_{GI} = 0.44$	persistence in public consumption/investment
$\rho_{FDI} = 0.3$	persistence in the FDI process
$\rho_{swf} = 0.747$	persistence in the SWF process
$\rho = 0.88$	interest rate smoothing in the Taylor rule
$\rho_{fxr} = 0.77$	persistence in the foreign exchange reserves of a central bank
$\rho_o = 0.85$	persistence in the world oil price process
$\sigma_{\epsilon_t^o} = \sigma_{\epsilon_t^{Y*}} = 0.2$	standard deviation of the world oil price and foreign output shocks

2.6.2 Data description

Data are outlined here which are used in the OLS regressions, calculated GDP ratios, and other calibrated parameters. Most data are manually retrieved from the non-English websites of respective institutions, indicated in parentheses below. Some data are obtained based on a formal request to those institutions.

Real GDP is the GDP at constant prices of 1994 in mln tenge according to the National Accounts over 1994Q1-2012Q2 (Agency of Statistics).

Private consumption is the consumption expenditures of households at constant prices of 1994 in mln tenge according to the National Accounts over 1994Q1-2012Q2 (Agency of Statistics).

Public consumption is public consumption at constant prices of 1994 in mln tenge according to the National Accounts over 1994Q1-2012Q2 (Agency of Statistics).

Fixed capital formation is gross fixed capital formation at constant prices of 1994 in mln tenge according to the National Accounts over 1994Q1-2012Q2 (Agency of Statistics).

Imports are the imports of goods and services at constant prices of 1994 in mln tenge according to the National Accounts over 1994Q1-2012Q2 (Agency of Statistics).

Net exports are the net exports at constant prices of 1994 in mln tenge according to the National Accounts over 1994Q1-2012Q2 (Agency of Statistics).

Oil output is the mining industry's output in mln tenge according to the composition of total industrial output statistics over 1998Q3-2012Q2 (Agency of Statistics).

CPI is a quarterly consumer price index over 1994Q1-2012Q2 (National Bank).

Real public debt is the CPI-deflated public debt, including debt guaranteed by the state, in mln tenge over 1999Q4-2012Q2 (Ministry of Finance).

Real SWF transfers to the government budget represent the CPI-deflated oil revenues of government budget till 2007 and SWF transfers to the government budget since 2007 in mln tenge over 2001Q2-2012Q2 (Ministry of Finance).

Real fiscal capital expenditures are the CPI-deflated capital expenditures of government budget in mln tenge over 2000Q1-2012Q2 (Ministry of Finance).

Real non-oil fiscal revenues are the CPI-deflated difference between total fiscal revenues and SWF transfers to the government budget in mln tenge over 2000Q1-2012Q2 (Ministry of Finance).

T-bill rate is an effective annual return on medium-term Treasury bills in percent over 1998Q1-2012Q2 (Statistical Bulletin of the National Bank).

Producer price index is a quarterly producer price index over 1994Q1-2012Q2 (Agency of Statistics).

World oil price is a petroleum UK Brent price USD/barrel over 1993Q4-2012Q2 (International Financial Statistics of the IMF).

Real foreign exchange reserves are the CPI-deflated net foreign exchange reserves of the National Bank in mln tenge over 1994Q4-2012Q2 (National Bank).

Real exchange rate is a bilateral real exchange rate, tenge per 1 USD, over 1995Q1-2012Q2 (National Bank).

Real FDI is the US CPI-deflated foreign direct investment in mln USD according to the balance of payments statistics over 2002Q1-2012Q2 (National Bank). The US CPI index is retrieved from the International Financial Statistics of the IMF.

Foreign debt is an external debt of banks and other private entities in mln USD according to the balance of payments statistics over 1995Q1-2012Q2 (National Bank).

Wage is an average monthly wage of a hired employee in tenge over 1994Q1-2012Q4 (Agency of Statistics).

SWF inflows represent the oil revenues of government budget till 2007 and inflows into SWF since 2007 in mln tenge over 2001Q2-2012Q4 (Ministry of Finance).

SWF assets are the stock of SWF at the end of period in mln tenge over 2001Q2-2012Q4 (Ministry of Finance).

2.6.3 First-order conditions

The first-order conditions of the forward-looking household's problem are listed in this appendix, where λ_t, λ_t^k, and $\lambda_t \lambda_t^c$ are the Lagrange multipliers to the budget constraint (53), capital accumulation (54), and collateral constraint (55), respectively. In particular, equations (85), (86), (87), (88), (89), (90) below are the first-order conditions with respect to consumption, investment, non-oil capital, government bonds, foreign debt, and hours worked, respectively.

$$\frac{1}{\left[C_t^S - \frac{N_t^\phi}{\phi}\right]^\sigma} = \lambda_t \tag{85}$$

$$\frac{1}{Q_t} = 1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \kappa\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}} + \beta\kappa E_t\left\{\frac{Q_{t+1}\lambda_{t+1}}{Q_t\lambda_t}\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right\}, \tag{86}$$

where $Q_t = \frac{\lambda_t^k}{\lambda_t}$ is a shadow value of non-oil capital.

$$Q_t = E_t\left\{\beta\frac{\lambda_{t+1}}{\lambda_t}\left[R_{t+1}^{kno} + Q_{t+1}(1-\delta)\right] + \lambda_t^c\Omega\frac{Q_{t+1}\pi_{t+1}^*}{RER_{t+1}/RER_t}\right\} \tag{87}$$

$$\frac{1}{R_t} = \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t\pi_{t+1}}\right\} \tag{88}$$

$$\frac{1}{R_t^*} = \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\frac{RER_{t+1}}{RER_t\pi_{t+1}^*}\right\} + \lambda_t^c \tag{89}$$

$$W_t = N_t^{\phi-1} \tag{90}$$

The first-order conditions of a rule-of-thumb household with respect to consumption C_t^N and hours worked N_t are identical to the saver's solutions above. Thus, a non-saver faces the same labor supply condition (90).

2.6.4 Steady state

The model's steady state assumes zero inflation, thus it is at flexible prices. Variables at steady state are denoted by bars and presented in this appendix.

The first-order condition of a forward-looking household with respect to the government bonds (88) gives that $\overline{R} = \frac{1}{\beta}$, while with respect to foreign debt (89) suggests $\overline{\lambda^c} = \beta^* - \beta$ at steady state. Similarly, $\overline{R}^* = \frac{1}{\beta^*}$.

An oil producer equalizes the marginal product of capital to its price:

$$\alpha^o(1-\tau^o)(\overline{K^o})^{\alpha^o-1} = \overline{R^{ok}} = \frac{1}{\beta} - (1-\delta),$$

from which the steady state of oil capital can be found.

$$\overline{K^o} = \left[\frac{1/\beta - (1-\delta)}{\alpha^o(1-\tau^o)}\right]^{\frac{1}{\alpha^o-1}}$$

Since oil capital is known, the oil output, FDI, and SWF are obtained from their respective equations (63), (65), and (67, 71, 72):

$$\overline{Y^o} = (\overline{K^o})^{\alpha^o}, \quad \overline{FDI^*} = \delta\overline{K^o}, \quad \overline{SWF} = \frac{[\tau^o + \iota^{\text{div}}(1-\tau^o)]\overline{Y^o}}{1-\rho_{swf}/\beta^*}$$

The law of one price holds. There is also an assumption of symmetric steady state ($\frac{\overline{P_f}}{\overline{P_h}} = 1$) and a unit elasticity of substitution between domestic and foreign goods ($\eta = 1$). Thus, the real exchange rate and relatives prices at steady state are equal to one.

The SWF transfers to the government budget are as follows:

$$\overline{TR} = \overline{R^*}(1-\rho_{swf})\overline{SWFRER}$$

The public capital accumulation equation (70) gives public investment at steady state:

$$\overline{G_I} = \delta^g\overline{K_G}$$

Fiscal debt is represented in terms of public capital, using the public investment equation (73) and the expression above:

$$\bar{b} = \left(\frac{\overline{Y}_o^{\vartheta_{GI}}\,\overline{TR}^{\gamma_{TR}^{GI}}}{\delta^g\overline{K_G}}\right)^{\frac{1}{\gamma_{GI}}}$$

Public consumption is as follows based on its rule (74), in which fiscal debt can be plugged from the previous equation:

$$\overline{G_C} = \frac{\overline{Y_o}^{\vartheta_{GC}} \overline{TR}^{\gamma^{GC}_{TR}}}{\overline{b}^{\gamma_{GC}}}$$

The lump-sum taxes equation (75) suggests taxes at steady state:

$$\overline{T} = \frac{\overline{b}^{\varphi_b} \overline{G_I}^{\varphi_I} \overline{G_C}^{\varphi_C}}{\overline{TR}^{\varphi_{TR}}}$$

The government budget constraint (68) can be used to obtain public capital by substituting the taxes, SWF transfers, public consumption, public investment, and fiscal debt with their respective previous expressions:

$$\overline{T} + \overline{TR} = \overline{G_C} + \overline{G_I} + (1-\mu)(\overline{R}-1)\overline{b}$$

The first-order condition with respect to non-oil capital (87) yields the following rental cost of capital:

$$\overline{R^{kno}} = \frac{1}{\beta} - (1-\delta) - \frac{\overline{\lambda^c}\Omega}{\beta}$$

The price setting problem of a non-oil firm suggests that the marginal costs (60) equate with the inverse of price frictionless mark-up $\frac{\varepsilon}{\varepsilon-1}$ at steady state; thus, wages are:

$$\overline{W} = (1-\alpha)\left[\frac{\overline{K_G}^{\psi}\alpha^{\alpha}(\varepsilon-1)}{(\overline{R^{kno}})^{\alpha}\varepsilon}\right]^{\frac{1}{1-\alpha}}$$

The labor supply condition (90) gives $\overline{N} = \overline{W}^{\frac{1}{\phi-1}}$.

As aggregate output is a sum of non-oil and oil output $\overline{Y} = \overline{Y^{no}} + \overline{RERY^o} = \overline{N}^{1-\alpha}\overline{K_G}^{\psi}\overline{K^{no}}^{\alpha} + \overline{RERY^o}$, the non-oil capital is obtained in terms of aggregate output:

$$\overline{K^{no}} = \left(\frac{\overline{Y} - \overline{RERY^o}}{\overline{N}^{1-\alpha}\overline{K_G}^{\psi}}\right)^{\frac{1}{\alpha}}$$

The law of motion for capital (54) relates private investment with the non-oil capital: $\overline{I} = \delta\overline{K^{no}}$.

The collateral constraint (55) allows finding the foreign debt:

$$\overline{b^*} = \frac{\Omega\overline{K^{no}}}{\overline{R}^*}$$

The steady state foreign exchange reserves, according to their rule (79), are equal to 1 given that $\overline{RER} = 1$.

The balance of payments equation provides net exports:

$$\begin{aligned}\overline{NX} \;=\;& (1-\mu)\left(\overline{R^*}-1\right)\overline{b^*}+(1-\overline{R^*})\overline{fxr^*}-\overline{RERFDI^*}-\overline{R^*}(1-\rho_{swf})\overline{SWFRER}+\\ &+(1-\iota^{\text{div}})\overline{RER}(1-\tau^o)\overline{Y^o}\end{aligned}$$

The taxes of rule-of-thumb households are equal to:

$$\overline{T^N}=\frac{\overline{T}-(1-\mu)\overline{T^S}}{\mu}$$

given that $T_t=\mu T_t^N+(1-\mu)T_t^S$, while the taxes of savers can be derived from their budget constraint (53), assuming that both types of household have equal consumption at steady state:

$$\overline{T^S}=\mu\left[(\overline{R^{kno}}-\delta)\overline{K^{no}}+\overline{b}(\overline{R}-1)+\overline{b^*}(1-\overline{R^*})+(1-\frac{\varepsilon-1}{\varepsilon})\overline{Y^{no}}+(\overline{R^*}-1)\overline{fxr^*}\right]+\overline{T}$$

The budget constraint of a rule-of-thumb household (56) provides its consumption $\overline{C^N}=\overline{WN}-\overline{T^N}$, which is assumed to be equal to the saver's consumption, thus to aggregate consumption as well due to the sum of both households' consumption: $\overline{C}=\mu\overline{C^N}+(1-\mu)\overline{C^S}$.

The real GDP condition (81) can be utilized to derive the aggregate output, by plugging into variables expressed in terms of output according to their steady state equations above:

$$\overline{Y}=\overline{C}+(1-\mu)\overline{I}+\overline{G_C}+\overline{G_I}+\overline{NX}$$

2.6.5 The Phillips curve

The Phillips curve for CPI inflation in a small open economy has been derived according to Gali (2015).

The log-linearized optimal price setting condition (62) delivers a standard equation for domestic inflation π_t^h:

$$\pi_t^h=\beta E_t\pi_{t+1}^h+\lambda\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\widehat{MC}_t,$$

where $\widehat{MC_t}$ is the log deviation of the economy's average real marginal costs from their steady state and $\lambda=\frac{(1-\beta\theta)(1-\theta)}{\theta}$.

The CPI inflation includes the domestic inflation π_t^h and the terms of trade, which can be alternatively represented by the real exchange rate RER_t:

$$\pi_t=\pi_t^h+\frac{1-\gamma}{\gamma}\bigtriangleup\widehat{RER_t} \tag{91}$$

The Phillips curve then is as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \widehat{MC}_t + \frac{1-\gamma}{\gamma} \vartriangle \widehat{RER}_t - \beta \frac{1-\gamma}{\gamma} E_t \vartriangle \widehat{RER}_{t+1},$$

where $\widehat{MC}_t = \widehat{W}_t - (\widehat{Y_t^{no}} - \widehat{N}_t) + \frac{1-\gamma}{\gamma}\widehat{RER}_t$. Wages can be substituted with the log-linearized labor supply condition (90), so that the Phillips curve used in the model is:

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \frac{1-\gamma}{\gamma}\left(\lambda \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} + \beta + 1\right) \widehat{RER}_t - \frac{1-\gamma}{\gamma}\widehat{RER}_{t-1} \\ &\quad -\beta \frac{1-\gamma}{\gamma} E_t \widehat{RER}_{t+1} + \lambda \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\left(\phi \widehat{N}_t - \widehat{Y_t^{no}}\right) \end{aligned} \tag{92}$$

2.6.6 Log-linearized equations

The aggregate consumption equation is derived according to Gali, Lopez-Salido, and Valles (2007) by combining the Euler equation (88), budget constraint of the rule-of-thumb households (56), and the relationship $C_t = \mu C_t^N + (1-\mu) C_t^S$:

$$\widehat{C}_t = E_t \widehat{C}_{t+1} + \Theta_n(\widehat{N}_t - E_t \widehat{N}_{t+1}) - \Theta_i(\widehat{R}_t - E_t \pi_{t+1}) + \mu \overline{TC}^{-1}(\widehat{T_{t+1}} - \widehat{T}_t), \tag{93}$$

where $\Theta_n = \left[\mu \overline{N}^{\phi} \phi + (1-\mu)\overline{N}^{\phi}\right] \overline{C}^{-1}$ and $\Theta_i = (\sigma \overline{C})^{-1}(1-\mu)(\overline{C} - \phi^{-1}\overline{N}^{\phi})$.

The combination of the first-order condition with respect to non-oil capital (87) and investment (86) given that $\widehat{R_t^{kno}} = \phi \widehat{N}_t - \widehat{K_{t-1}^{no}}$ delivers the following:

$$\begin{aligned} \kappa(1+\beta)\widehat{I}_t &= \left(\beta(1-\delta) + \Omega\overline{\lambda^c}\right)\left[\kappa(1+\beta)E_t\widehat{I_{t+1}} - \kappa\beta E_t \widehat{I_{t+2}} - \kappa \widehat{I}_t\right] + \kappa\beta E_t \widehat{I_{t+1}} \\ &\quad +(1-\beta(1-\delta) - \Omega\overline{\lambda^c})E_t\left[\phi \widehat{N_{t+1}} - \widehat{K_t^{no}}\right] - (1-\Omega\overline{\lambda^c})\left(\widehat{R}_t - E_t\pi_{t+1}\right) \\ &\quad +\kappa \widehat{I_{t-1}} + \Omega\overline{\lambda^c}(E_t\pi^*_{t+1} + \widehat{RER}_t - E_t\widehat{RER}_{t+1} + \widehat{\lambda_t^c}) \end{aligned} \tag{94}$$

The collateral constraint (55) combined with the first-order condition with respect to investment (86) yields:

$$\widehat{b_t^*} = E_t\pi^*_{t+1} - \widehat{R_t^*} + \widehat{K_t^{no}} + \widehat{RER}_t - E_t\widehat{RER}_{t+1} + \kappa(1+\beta)E_t\widehat{I_{t+1}} - \kappa\beta E_t\widehat{I_{t+2}} - \kappa\widehat{I}_t \tag{95}$$

The law of motion for non-oil capital (54) is as follows:

$$\widehat{K_t^{no}} = (1-\delta)\widehat{K_{t-1}^{no}} + \delta\widehat{I}_t \tag{96}$$

Similarly, the public capital accumulation (70) in its log-linearized form is below:

$$\widehat{K_t^G} = (1-\delta^g)\widehat{K_{t-1}^G} + \delta^g\widehat{G_t^I} \tag{97}$$

The oil capital is accumulated by FDI according to its equation (64):

$$\widehat{K_t^o} = (1-\delta)\widehat{K_{t-1}^o} + \delta\widehat{FDI_t^*} \tag{98}$$

The combination of oil tax revenues equation (71), SWF accumulation (72), and the profits of oil producer (67) corresponds to:

$$\widehat{SWF}_t = \rho_{swf}\overline{R^*}(\widehat{SWF}_{t-1} - \pi_t^*) + \frac{[\tau^o + \iota^{\text{div}}(1-\tau^o)]s_o}{swf_y}(\widehat{Y_t^o} + \widehat{P_t^{o*}}) + \rho_{swf}\widehat{R_{t-1}^*} \tag{99}$$

The log-linearization of the first-order condition of a saver with respect to foreign debt (89) provides the following UIP condition:

$$\widehat{R}_t = E_t\pi_{t+1} + \frac{\beta^*}{\beta}\widehat{R}_t^* - E_t\pi_{t+1}^* + E_t\widehat{RER}_{t+1} - \widehat{RER}_t + \left(\frac{\beta^*}{\beta} - 1\right)\widehat{\lambda}_t^c \tag{100}$$

The non-oil and oil production functions (59 and 63) give respectively:

$$\widehat{Y_t^{no}} = \alpha\widehat{K_{t-1}^{no}} + (1-\alpha)\widehat{N}_t + \psi\widehat{K}_{G,t-1} \tag{101}$$

$$\widehat{Y_t^o} = \alpha^o\widehat{K_{t-1}^o} \tag{102}$$

The aggregate output is as follows:

$$\widehat{Y}_t = (1-s_o)\widehat{Y_t^{no}} + (1-s_o)\widehat{p_t^h} + s_o(\widehat{Y_t^o} + \widehat{RER}_t + \widehat{P_t^{o*}}) \tag{103}$$

The government budget constraint (68) in terms of fiscal debt results in:

$$\begin{aligned}\widehat{b}_t \quad = \quad & \overline{R}(\widehat{b}_{t-1} - \pi_t) + \widehat{R}_{t-1} + \frac{g_y^I}{(1-\mu)b_y}\widehat{G_t^I} + \frac{g_y^C}{(1-\mu)b_y}\widehat{G_t^C} + \frac{g_y^C + g_y^I}{(1-\mu)b_y}\widehat{p_t^g} \\ & -\frac{\overline{T}}{\overline{b}(1-\mu)}\widehat{T}_t - \frac{(1-\rho_{swf})swf_y}{b_y(1-\mu)}\widehat{R_{t-1}^*} - \frac{(1-\rho_{swf})\overline{R^*}swf_y}{b_y(1-\mu)}(\widehat{RER}_t + \widehat{SWF}_{t-1} - \pi_t^*)\end{aligned} \tag{104}$$

The log-linearized relative price of government purchases to composite consumption (69), assuming $\eta \to 1$, is this:

$$\widehat{p_t^g} = \gamma_2\widehat{p_t^h} + (1-\gamma_2)\widehat{RER}_t \tag{105}$$

The domestic goods market clearing condition (80) can be rewritten as:

$$\widehat{Y_t^{no}} + \widehat{p_t^h} = \frac{\gamma c_y}{(1-s_o)}\widehat{C}_t + \frac{(1-\mu)(1-i_y)\gamma}{(1-s_o)}\widehat{I}_t + \frac{\gamma_2 g_y^C}{(1-s_o)}\widehat{G_t^C} + \frac{\gamma_2 g_y^I}{(1-s_o)}\widehat{G_t^I} + \frac{\gamma_2(g_y^C + g_y^I)}{(1-s_o)}\widehat{p_t^g}, \tag{106}$$

where $(1-i_y) = 1 - c_y - g_y^C - g_y^I - nx_y$.

The real GDP (81) is represented in terms of investment:

$$\widehat{I}_t = \frac{1}{(1-i_y)(1-\mu)}\left[\widehat{Y}_t - c_y\widehat{C}_t - g_y^C\widehat{G_t^C} - g_y^I\widehat{G_t^I} - (g_y^C + g_y^I)\widehat{p_t^g} - nx_y\widehat{NX}_t\right] \tag{107}$$

The log-linearization of the balance of payments equation results in:

$$\begin{aligned}
\widehat{NX}_t \;=\; & \frac{\overline{R^*}b_y^*(1-\mu)}{nx_y}\widehat{b_{t-1}^*} + \frac{\overline{R}^*(1-\rho_{swf})swf_y - (1-\mu)\overline{R^*}b_y^* + \overline{R^*}fxr_y}{nx_y}\pi_t^* \qquad (108)\\
& +[\frac{\overline{R^*}b_y^*(1-\mu)}{nx_y} - \frac{\overline{R}^*(1-\rho_{swf})swf_y}{nx_y} - \frac{\overline{R^*}fxr_y}{nx_y} - \frac{fdi_y}{nx_y} + \\
& +\frac{(1-\iota^{\text{div}})(1-\tau^o)s_o}{nx_y}]\widehat{RER}_t + \frac{\overline{R^*}fxr_y - \overline{R^*}b_y^*(1-\mu)}{nx_y}\widehat{RER}_{t-1} + \\
& +\frac{(1-\iota^{\text{div}})(1-\tau^o)s_o}{nx_y}(\widehat{Y}_t^o + \widehat{P_t^{o*}}) - \frac{\overline{R}^*(1-\rho_{swf})swf_y}{nx_y}\widehat{SWF}_{t-1} + \\
& +\frac{(1-\mu)b_y^* - (1-\rho_{swf})swf_y - fxr_y}{nx_y}\widehat{R}_{t-1}^* - \frac{\overline{R^*}fxr_y}{nx_y}\widehat{fxr_{t-1}^*} + \\
& +\frac{fxr_y}{nx_y}\widehat{fxr_t^*} - \frac{b_y^*(1-\mu)}{nx_y}\widehat{b_t^*} - \frac{fdi_y}{nx_y}\widehat{FDI_t^*}
\end{aligned}$$

The budget constraint of a saver (53) is log-linearized as well, by combining the aggregate relationships for consumption $C_t = \mu C_t^N + (1-\mu)C_t^S$ and taxes $T_t = \mu T_t^N + (1-\mu)T_t^S$, and the budget constraint of a rule-of-thumb household (56):

$$\begin{aligned}
\widehat{C}_t \;=\; & [\frac{\phi\overline{N}^{\phi} + (1-\mu)\overline{K^{no}}\,\overline{R^{kno}}\phi}{\overline{C}} - \frac{(1-\mu)(1-s_o)(\varepsilon-1)\phi}{c_y\varepsilon}]\widehat{N}_t + \frac{(1-\mu)b_y}{c_y}\widehat{R_{t-1}} \qquad (109)\\
& -\frac{\overline{T}}{\overline{C}}\widehat{T}_t + \frac{(1-\mu)b_y\overline{R}}{c_y}(\widehat{b_{t-1}} - \pi_t) + \frac{1-\mu}{c_y}(b_y^*\widehat{b_t^*} - (1-i_y)\widehat{I}_t - b_y\widehat{b}_t) - \\
& -\frac{(1-\mu)b_y^*\overline{R^*}}{c_y}\widehat{b_{t-1}^*} + \frac{(1-\mu)b_y^*\overline{R^*} - \overline{R^*}fxr_y}{c_y}(\widehat{RER_{t-1}} + \pi_t^*) - \frac{fxr_y}{c_y}\widehat{fxr_t^*} + \\
& +[\frac{\overline{R^*}fxr_y - (1-\mu)b_y^*\overline{R^*}}{c_y} - \frac{(1-\mu)(1-s_o)(\varepsilon-1)(1-\gamma)}{c_y\varepsilon\gamma}]\widehat{RER_t} + \\
& +\frac{fxr_y - (1-\mu)b_y^*}{c_y}\widehat{R_{t-1}^*} + \frac{(1-\mu)(1-s_o)}{c_y}(\widehat{Y_t^{no}} + \widehat{p_t^h}) + \overline{R^*}\frac{fxr_y}{c_y}\widehat{fxr_{t-1}^*}
\end{aligned}$$

2.6.7 Impulse-response functions

Figure 1. Impulse-response functions to a negative world oil price shock of 1%: acyclical fiscal policy combined with PPT monetary rule under a managed exchange rate regime

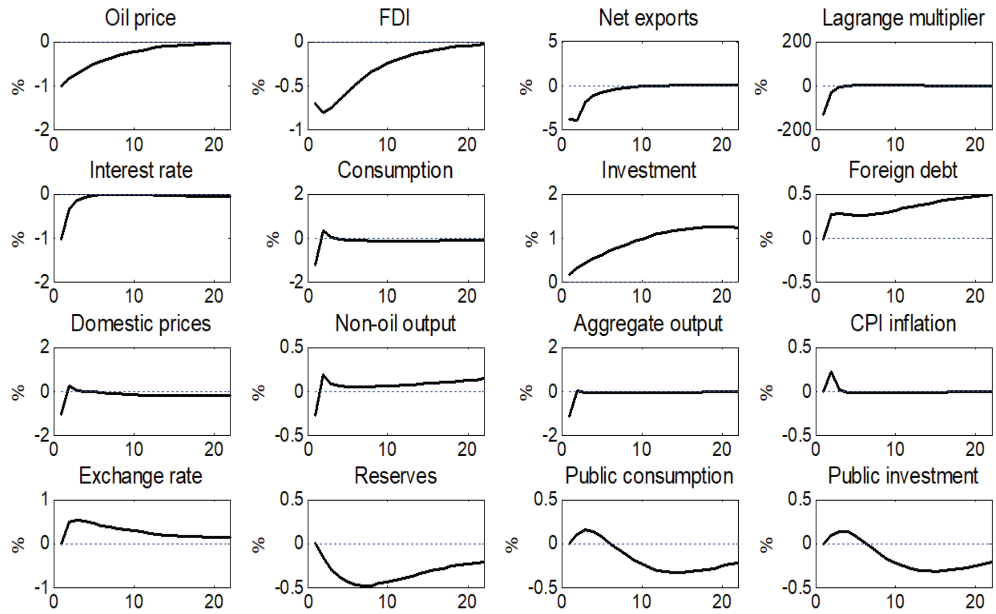


Figure 2. Impulse-response functions to a negative world oil price shock of 1%: acyclical fiscal policy combined with CPI monetary rule under a managed exchange rate regime

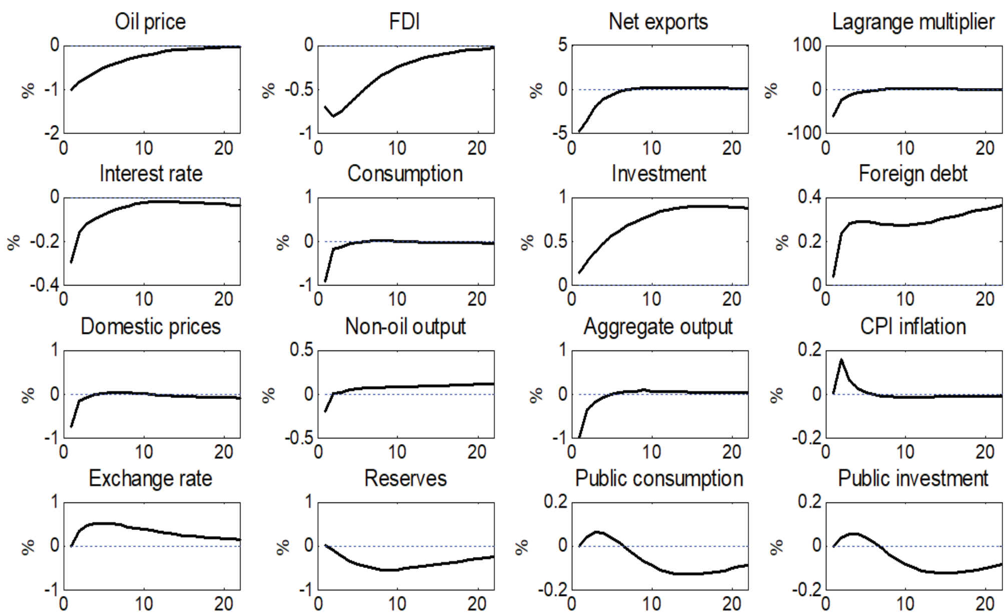


Figure 3. Impulse-response functions to a negative world oil price shock of 1%: pro-cyclical fiscal policy combined with PPT monetary rule under a managed exchange rate regime

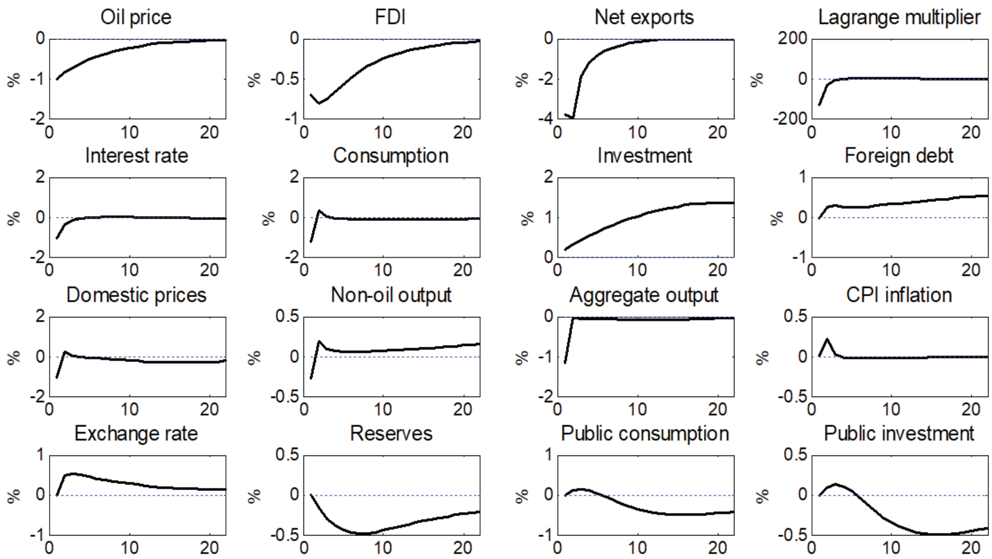
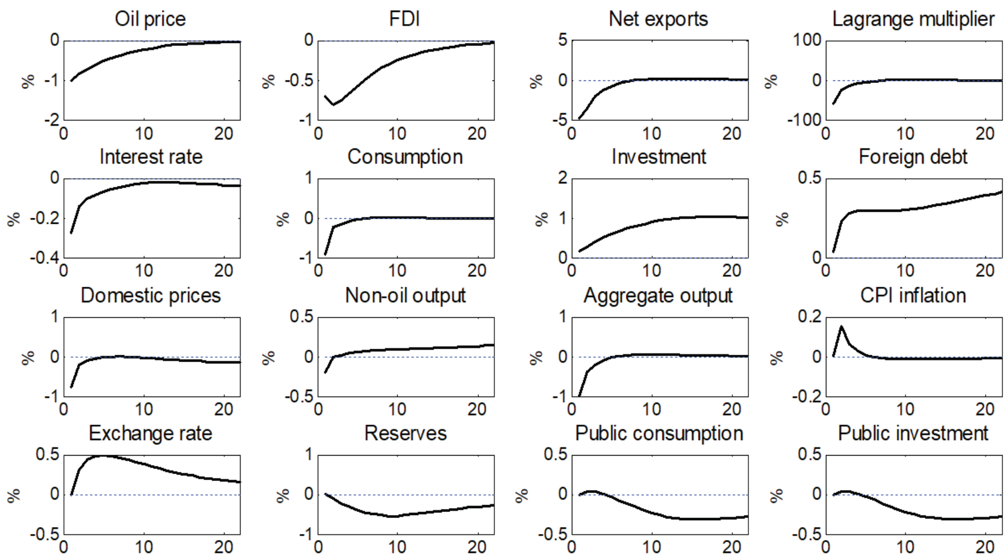


Figure 4. Impulse-response functions to a negative world oil price shock of 1%: pro-cyclical fiscal policy combined with CPI monetary rule under a managed exchange rate regime



A flexible exchange rate regime, across all figures of policy combination, produces no changes in the foreign exchange reserves, otherwise the impulse-response functions stay the same.

3 Optimal Public Investment in Resource-Rich Low-Income Countries

[24]

3.1 Introduction

Resource-rich low-income countries are often considered as the most vulnerable economies in the world for three basic reasons. First, they are exposed to volatile external shocks: commodity world price fluctuations, capital inflows/outflows, and geological discovery/depletion of natural resources. Second, they are prone to a natural resource curse due to weak institutions, high income inequality, inefficient governance, and Dutch Disease problems (Van der Ploeg, 2011). Third, their current generation is poor, living in an environment of capital scarcity, an underdeveloped financial system and high absorptive costs for investment to build home-grown capital. In such specific conditions, finding an optimal rule-based policy to manage resource windfalls with a sustainable development objective is crucial, but challenging.

Several studies have recently concluded that resource abundant developing economies are better off investing their resource windfalls domestically rather than saving them abroad in a Sovereign Wealth Fund for future generations (Berg, Portillo, Yang & Zanna, 2013; Van der Ploeg & Venables, 2011; Collier, Van der Ploeg, Spence & Venables, 2010). This is due to the lack of growth-inducing domestic capital such as infrastructure and human capital, which have higher social value and returns than foreign assets in those economies. The fact of poor, impatient, and credit-constrained current households, who need to consume now, suggests a policy focus of benefiting them, as opposed to saving for

[24]This project was assigned during my summer internship-2012 at the IMF's Research Department (Washington, DC).

future individuals, who may well be in a relatively wealthier position given a sustainable development path over time.

This paper utilizes the perfect foresight general equilibrium model of Berg et al. (2013), who find that the sustainable domestic investment of resource windfalls is preferable to saving them abroad. However, in drawing this conclusion, they arbitrarily compare a 26 percent increase in public investment to 40 percent at given adjustment speed. The objective of this study is to find an optimal policy rule for public capital, which is pinned down by two parameters – a new steady state level of increased public capital and its adjustment speed. These are optimized on a grid to maximize a household's utility. Since public capital is a stock variable, the associated optimal public investment path is obtained accordingly. Therefore, this paper extends Berg et al. (2013) in several respects: The policy rule for public capital is introduced. Absorptive capacity constraint costs in public investment are captured by a single parameter. External savings clear the government budget. There is a variable share of resource revenues to accumulate the SWF, and the natural resource sector has its FDI shock.

In section 3.2, the model is outlined representing households, producers of traded and non-traded goods, natural resource sector, and fiscal policy. Section 3.3 describes the calibration of parameters, the list of which is provided in Appendix 3.7.1. Section 3.4 discusses the findings of optimal policy for public capital and public investment at different absorptive capacity constraints. Sensitivity analysis to a reduced output elasticity of public capital is presented in section 3.5, including a scenario of no resource windfall shock. Section 3.6 concludes.

3.2 Model

The model is a small open, real economy with no external public or private debt, but with FDI in a natural resource sector. This "closed" assumption of a financial account captures the limited access of low-income countries to foreign funds and facilitates the study of an increase of public investment solely financed by a resource windfall rather than by external borrowing. The domestic public debt is fixed to avoid a drop in household's consumption due to increased savings in the government bonds than finance the scaling-up of public capital.

The model has a representative household, who consumes and pays consumption tax, supplies labor and pays fixed labor tax, owns firms of traded and non-traded goods, holds a constant amount of government bonds and receives fixed remittances from abroad and fixed transfers from the government budget. The producers of traded and non-traded goods are perfectly competitive, who differ in terms of their total factor productivity (TFP) and have public capital as an additional input in their Cobb-Douglass production function. The natural resource sector is assumed to be capital-intensive with its real FDI shock, thus there is no labor input in this sector to avoid the complications from possible labor mobility. Public investment is productive, effectively accumulating public capital and yet containing absorptive capacity constraints.

3.2.1 Households

A representative household maximizes its expected utility by choosing composite consumption C_t and labor L_t:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{\kappa}{1+\psi} L_t^{1+\psi} \right] \tag{110}$$

subject to the budget constraint:

$$(1+\tau_t^c)C_t + B = (1-\tau^l)w_t L_t + RB + \Omega_t^T + \Omega_t^N + s_t RM^* + Z, \tag{111}$$

where σ and ψ are the inverses of the elasticity of intertemporal substitution for consumption and labor, τ_t^c and τ^l are the consumption and labor tax rates, Ω_t^T and Ω_t^N are the real profits transferred from the producers of traded and non-traded goods, s_t is a CPI-based real exchange rate, RM^* is remittances in the units of foreign consumption (denoted by an asterisk), Z is the government transfers, B is the government bonds, and

R is the domestic real interest rate.

The composite CES consumption bundle C_t includes traded (C_t^T) and non-traded (C_t^N) goods:

$$C_t = \left[\varphi^{\frac{1}{\chi}}(C_t^N)^{\frac{\chi-1}{\chi}} + (1-\varphi)^{\frac{1}{\chi}}(C_t^T)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}, \tag{112}$$

where φ is a consumption home-bias parameter and χ is the intratemporal elasticity of substitution between traded and non-traded goods. Composite consumption is set as a numeraire for the economy, so that by the assumed law of one price for traded goods, s_t is also the relative price of traded goods to composite consumption, while p_t^N is the relative price of non-traded goods to composite consumption:

$$1 = \varphi(p_t^N)^{1-\chi} + (1-\varphi)s_t^{1-\chi} \tag{113}$$

Labor supply of a household consists of labor efforts made in the traded (L_t^T) and non-traded (L_t^N) sectors with ρ as an elasticity of substitution; thus, there is imperfect labor mobility between these two sectors:

$$L_t = \left[\delta^{-\frac{1}{\rho}}(L_t^N)^{\frac{1+\rho}{\rho}} + (1-\delta)^{-\frac{1}{\rho}}(L_t^T)^{\frac{1+\rho}{\rho}}\right]^{\frac{\rho}{1+\rho}} \tag{114}$$

A real wage index combines the real wage rates in each sector:

$$w_t = \left[\delta(w_t^N)^{1+\rho} + (1-\delta)(w_t^T)^{1+\rho}\right]^{\frac{1}{1+\rho}} \tag{115}$$

3.2.2 Producers of traded and non-traded goods

The Cobb-Douglass production function of sector $j \in \{T, N\}$ includes public capital K_{t-1}^G as an additional input with its output elasticity of α^G:

$$Y_t^j = z_t^j(K_{t-1}^j)^{1-\alpha^j}(L_t^j)^{\alpha^j}(K_{t-1}^G)^{\alpha^G} \tag{116}$$

The law of motion for private capital has quadratic investment adjustment costs with a relevant parameter $\kappa^j > 0$:

$$K_t^j = (1-\delta^j)K_{t-1}^j + \left[1 - \frac{\kappa^j}{2}\left(\frac{I_t^j}{I_{t-1}^j} - 1\right)^2\right] I_t^j \tag{117}$$

The traded and non-traded sectors are perfectly competitive and differ in terms of their TFP. There is a constant TFP parameter z^N for the non-traded sector and learning-by-doing externalities are in the TFP of the traded sector:

$$z_t^N = z^N, \qquad \ln z_t^T = \rho_{zT} \ln z_{t-1}^T + d \ln Y_{t-1}^T \tag{118}$$

A producer maximizes its net present-value profits, weighted by the marginal utility of household λ_t:

$$E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \underbrace{[p_t^j Y_t^j - w_t^j L_t^j - I_t^j]}_{=\Omega_t^j} \tag{119}$$

through choosing labor, capital, and investment subject to the capital accumulation equation (117).

3.2.3 Natural resource sector

The natural resource production is assumed to be a capital-intensive sector and has capital input:

$$Y_t^o = z_t^o (K_{t-1}^o)^{\alpha^o}, \tag{120}$$

which is accumulated by the FDI denominated in foreign consumption goods.

$$K_t^o = (1-\delta^o)K_{t-1}^o + FDI_t^* \tag{121}$$

The only shock in the model is a real FDI shock transmitting through the resource output:

$$\ln FDI_t^* = \rho_{FDI} \ln FDI_{t-1}^* + \varepsilon_t^{FDI} \tag{122}$$

The profits of the resource sector include royalties levied on production quantity at a rate τ^o:

$$\Omega_t^{o*} = (1-\tau^o)P_t^{o*}Y_t^o \tag{123}$$

The resource sector is owned by foreigners and government: the dividend share of resource profits that the government receives is denoted by ι^{div}. The resource revenues consist of royalties and dividends:

$$T_t^o = s_t[\tau^o P_t^{o*}Y_t^o + \iota^{\text{div}}\Omega_t^{o*}] \tag{124}$$

3.2.4 Fiscal policy

The government collects its resource revenues, non-resource revenues representing consumption and labor taxes, and interest income from the SWF. Fiscal expenditures include transfers, interest payments on bonds, and government purchases which are a sum of public consumption and public investment. Thus, the government budget constraint is as follows:

$$ES_t = T_t^o + \underbrace{\tau_t^c C_t + \tau^l w_t L_t}_{T_t^{NO}} + s_t r^* SWF_{t-1}^* - Z - (RB - B) - p_t^G \underbrace{(G_t^C + G_t^I)}_{G_t}, \tag{125}$$

where a residual variable ES_t indicates the external savings that accumulate SWF. The external savings themselves are a time-varying share of resource revenues ϕ_t.

$$SWF_t^* = \rho_{swf}SWF_{t-1}^* + \frac{ES_t}{s_t}, \quad ES_t = \phi_t T_t^o, \quad \phi_t = \phi\frac{T_t^o}{T^o} \tag{126}$$

The policy rule to be examined for the optimal increase of public capital $\frac{K_{nss}^G}{K^G}$ and adjustment speed γ is as follows:

$$K_t^G = (1-e^{-\gamma t})K_{nss}^G + e^{-\gamma t}K^G, \tag{127}$$

where K_{nss}^G is a new steady state public capital, while K^G is an initial steady state level of public capital.

Public capital accumulation involves the effective public investment $\epsilon\widetilde{G}_t^I$ with its absorptive capacity constraint costs pinned down by the parameter $b > 0$:

$$K_t^G = (1-\delta^g)K_{t-1}^G + \epsilon\widetilde{G}_t^I, \qquad \widetilde{G}_t^I = \left[1 - b\left(\frac{G_t^I}{G^I} - 1\right)^2\right] G_t^I \tag{128}$$

Similar to private consumption, government purchases are the CES bundle of traded and non-traded goods with a variable degree of home-bias ν_t:

$$G_t = \left[\nu_t^{\frac{1}{\chi}}(G_t^N)^{\frac{\chi-1}{\chi}} + (1-\nu_t)^{\frac{1}{\chi}}(G_t^T)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}} \tag{129}$$

This parameter is time-varying, according to Berg et al. (2013), to distinguish the home-bias of additional public spending (ν_g) from its steady state value (ν), since the analysis focuses on the allocation of additional public spending to public investment:

$$\nu_t = \nu + (\nu_g - \nu)\frac{p_t^G G_t - p^G G}{p_t^G G_t} \tag{130}$$

The relative price of government purchases to composite consumption is accordingly as follows:

$$p_t^G = \left[\nu_t(p_t^N)^{1-\chi} + (1-\nu_t)s_t^{1-\chi}\right]^{\frac{1}{1-\chi}} \tag{131}$$

3.2.5 Market clearing conditions

The market clearing condition for the non-traded sector requires that its supply is equal to demand:

$$Y_t^N = (p_t^N)^{-\chi}\underbrace{[\varphi(C_t + I_t^N + I_t^T) + \nu_t(p_t^G)^{\chi}G_t]}_{D_t^N} \tag{132}$$

The aggregate output consists of traded, non-traded, and resource sectors' output:

$$Y_t = s_t Y_t^T + p_t^N Y_t^N + s_t P_t^{o*} Y_t^o \tag{133}$$

The current account deficit includes the domestic absorption, output, remittances, and interest income of SWF:

$$CA_t^d = (C_t + \underbrace{I_t^T + I_t^N + I_t^o}_{I_t} + p_t^G G_t) - Y_t - s_t[RM^* + r^* SWF_{t-1}^*] \tag{134}$$

The balance of payments is specified by the following variables: current account deficit, FDI, foreign share of resource profits, and the difference of SWF assets.

$$CA_t^d = s_t[FDI_t^* - (1 - \iota^{\text{div}})\Omega_t^{o*} - (SWF_t^* - \rho_{swf} SWF_{t-1}^*)] \tag{135}$$

The equilibrium system of equations consists of solutions to the household's and firms' optimization problems, private and public capital accumulation equations, government budget constraint, fiscal policy, SWF accumulation, price equations, market clearing conditions, balance of payments equation, and FDI process. The dynamics of the model are driven by a large temporal FDI shock, so that resource output eventually reverts to its pre-windfall level. The equilibrium is solved non-linearly from the initial pre-windfall steady state to a new steady state of increased public capital.

3.3 Calibration

The model is calibrated on annual data for the CEMAC region (Central African Economic and Monetary Community), which includes Cameroon, Central African Republic, Chad, Congo, Equatorial Guinea, and Gabon. The FDI shock persistence is set to 0.8 with a standard deviation of 6.26 to double the resource output-to-GDP ratio over the next ten years. The domestic real interest rate is 10 percent, giving a discount factor of 0.91 associated with the presence of impatient households. SWF earns a real return of 2.7 percent, whereas public capital, due to its scarcity, has a higher net return of 9.12 percent at its annual depreciation rate of 10 percent and output elasticity of 0.1. As a sensitivity test, the return on public capital lower than the SWF's interest rate (1.47 percent) is also examined, by changing its output elasticity to 0.06. The tightness of absorptive capacity constraints b is varied across 0.1, 0.2, and 0.3 to observe the differences in optimal increase of public investment. The remaining parameters are consistent with the calibration of Berg et al. (2013) for the CEMAC region and listed in Appendix 3.7.1.

3.4 Results

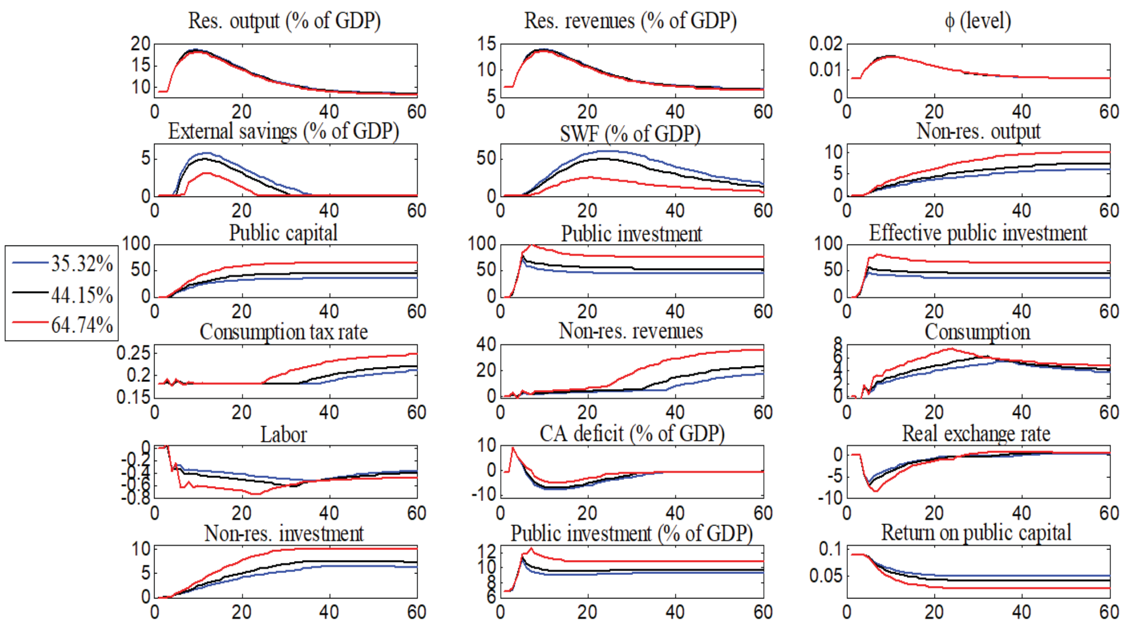
The optimal policy parameters to increase public capital specified by equation (127) are found in two steps. First, the search of welfare-maximizing public capital at a new steady state K_{nss}^{G} is implemented based on a discounted sum of household's utility. Second, given this optimal level of public capital, the utility-maximizing adjustment speed γ is found over a 100 year period. These two steps are repeated at each absorptive capacity constraint b, which characterizes the tightness of public investment costs in the economy. Technically, the non-linear model is solved in such a way that external savings eventually clear the government budget constraint, and public investment adjusts to avoid an initial hike in consumption tax rate.

Table 1. **Main results** ($\alpha^G = 0.1$)	$b = 0.1$	$b = 0.2$	$b = 0.3$
Effective public investment per \$1 invested	0.6613	0.6613	0.6604
Optimal increase of public capital	64.74%	44.15%	35.32%
Optimal adjustment speed	0.16	0.14	0.14
Optimal increase of public investment at new SS	74.4%	52.6%	43.4%
Overshooting magnitude of public investment	102%	79%	71.1%
Overshooting magnitude of effective public investment	81.1%	56.7%	45.2%
Consumption tax rate increase at new SS	39.4%	27.9%	23.2%
St dev of consumption growth over first 10 years	1.36	0.75	0.76
Welfare gain w.r.t. original pre-windfall steady state	3.2%	2.5%	2.1%

The main results are summarized in Table 1. A parameter $b = 0.3$ can be interpreted as \$0.6604 effective public investment accumulating public capital per \$1 invested. As absorptive capacity constraints become less tight (b declines), the effective public investment per \$1 invested increases and the optimal levels of public capital and public investment rise, which are highest at $b = 0.1$. The adjustment speed to reach a new increased level of public capital appears to be at its value which produces an overshooting public investment path that turns out to be optimal across all b. This suggests that front-loaded public investment is preferred thanks to a resource windfall in low-income countries with their capital scarcity and underdeveloped domestic financial market.

In order to finance public investment in the long run, the consumption tax rate has to rise, since the resource windfall is an initial one-period shock. In terms of welfare gain, which is measured as a percentage increase in consumption from the original pre-windfall steady state, the loose absorptive capacity constraint delivers the best outcome. Yet consumption is very volatile in the first several years given the higher magnitude of optimal overshooting public investment at $b = 0.1$.

Figure 1. **Optimal rates of public capital increase**



Y-axis is in percent deviation from the initial steady state in all figure hereafter unless denoted otherwise.

A temporary FDI shock hits the economy in Figure 1, with different absorptive capacity constraints and their respective optimal rates of public capital increase according to Table 1. In response to the shock, a resource output-to-GDP ratio doubles to 18 percent and resource revenues rise to 14 percent of GDP during the next ten years. The saving share of resource revenues ϕ_t, though quite small, yet generates a large increase of external savings in the SWF. A blue solid line depicts the lowest increase of public capital associated with the tight absorptive capacity constraint, and therefore an excess of resource windfall is saved more in the SWF rather than invested domestically. A black dashed line corresponds to the dynamics under $b = 0.2$ as a middle case. A red dotted line, associated with a loose absorptive capacity constraint and thus high accumulation of public capital, shows the welfare preferred case, as it delivers permanently higher consumption and permanently lower labor (higher leisure) than the other two lines.

Consumption tax rate as a part of non-resource revenues increases in the later period, since external savings eventually deplete to maintain public investment. The current account deficit initially rises due to a temporal FDI shock, but then declines as resource output and savings in the SWF expand. The magnitude of public capital increase affects the extent of exchange rate appreciation: the more the government invests, the more the exchange rate appreciates. Return on public capital, meanwhile, depends on its availability: capital scarcity generates its higher return and vice versa (a blue solid line versus a red dotted line in Figure 1).

Figure 2. **Different adjustment speed**

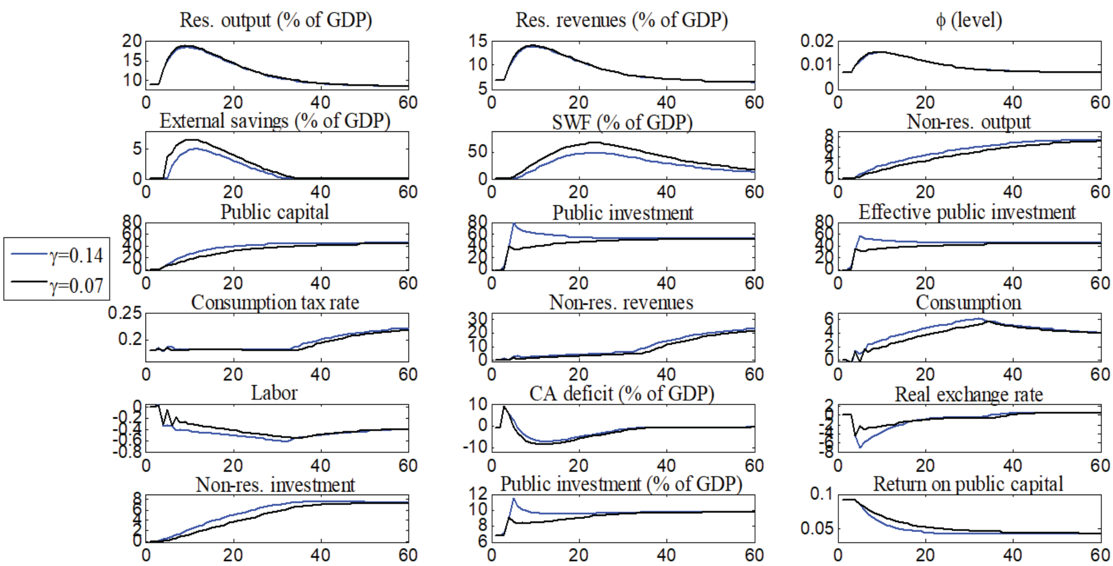


Figure 2 compares the impulse-response functions across two adjustment speeds at $b = 0.2$. A blue solid line illustrates the optimal overshooting public investment path ($\gamma = 0.14$), while a black dashed line shows no overshooting dynamics relative to a new steady state level ($\gamma = 0.07$). Consumption and labor under non-optimal public investment appear to be more volatile than they are under a preferable front-loaded policy. In terms of welfare gain, the optimal public investment policy is equivalent to a 2.5 percent increase in consumption from the original pre-windfall steady state, while the non-overshooting path at $\gamma = 0.07$ produces a welfare gain of 1.9 percent.

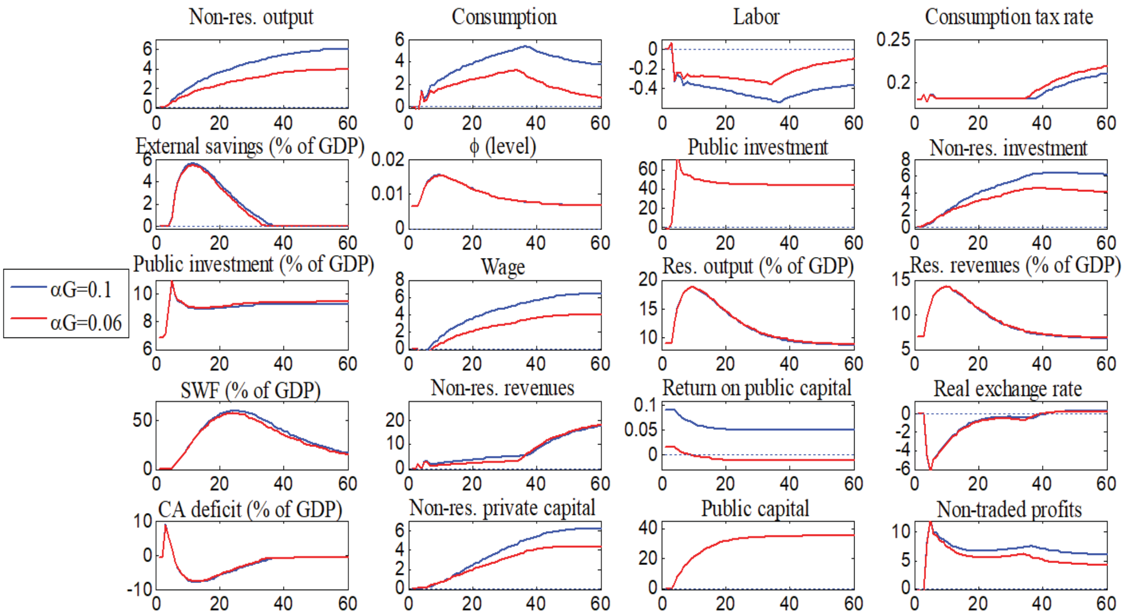
3.5 Sensitivity analysis

This section examines two cases. Public capital is less productive compared to the baseline model and, thereafter, there is no resource windfall in the first place. If public capital has its output elasticity of $\alpha^G = 0.06$, as opposed to baseline $\alpha^G = 0.1$, and its return is therefore lower than the SWF's interest rate by 1.23 percentage points, then the government should accumulate less capital, but at a faster adjustment speed (see Table 2). This is because, over time, the return on public capital decreases as it is expanded by public investment; thus, the overshooting public investment path is still preferred. However, relatively volatile consumption takes place at the tight absorptive capacity constraint $b = 0.3$ as opposed to $b = 0.1$ in Table 1. This is because public capital, being less productive, does not need to increase much, but should adjust to that level fast.

Table 2. **Sensitivity analysis:** $\alpha^G = 0.06$	$b = 0.1$	$b = 0.2$	$b = 0.3$
Effective public investment per $1 invested	0.695	0.694	0.694
Optimal increase of public capital	26.1%	20.36%	16.06%
Optimal adjustment speed	0.29	0.28	0.28
Optimal increase of public investment at new SS	27%	21.5%	17.1%
Overshooting magnitude of public investment	59.3%	45.7%	38.4%
Overshooting magnitude of effective public investment	53.7%	39.6%	32.3%
Consumption tax rate increase at new SS	16.3%	13%	10.3%
St dev of consumption growth over first 10 years	0.84	0.91	0.98
Welfare gain w.r.t. original pre-windfall steady state	1.1%	0.9%	0.8%

Figure 3 contrasts the dynamics of a public capital increase by 35.32 percent at $b = 0.3$, according to Table 1, versus its behavior under a reduced output elasticity, $\alpha^G = 0.06$. A solid blue line shows that relatively productive public capital generates higher non-resource output, consumption, and wages, since labor also becomes productive allowing households to have more leisure. However, consumption tax rate is higher in the long run under $\alpha^G = 0.06$, because more tax revenues are needed to finance the same level of public investment, as low output, due to less productive public capital, creates fewer fiscal revenues.

Figure 3. **Different output elasticity of public capital**



In order to compare the main results with a scenario of no initial resource windfall, a version of zero FDI shock is simulated within the same framework. Table 3 summarizes the results without a resource windfall across different absorptive capacity constraints. The percentage increase of public capital is the same as in the baseline version with FDI shock, but the adjustment speed is significantly lower, suggesting a gradual increase of public investment instead of its earlier overshooting path, due to the absence of an initial resource windfall.

Table 3. **No resource windfall:** $\alpha^G = 0.1$	$b = 0.1$	$b = 0.2$	$b = 0.3$
Effective public investment per \$1 invested	0.6882	0.6887	0.6888
Optimal increase of public capital	64.74%	44.15%	35.32%
Optimal adjustment speed	0.001	0.001	0.001
Optimal increase of public investment at new SS	41%	28.4%	23%
Consumption tax rate increase at new SS	20.7%	14.3%	11.6%
St dev of consumption growth over first 10 years	0.024	0.017	0.014
Welfare gain w.r.t. original steady state	-0.01%	-0.001%	-0.003%

Negative numbers for welfare gain mean that a scenario of no resource windfall is worse than the original steady state without any increase of public capital. This is due to the absence of external savings as an additional fiscal buffer to finance public investment initially, and the only instrument is consumption tax, which is distortionary for the welfare contributing consumption component.

Figure 4. **Dynamics under no resource windfall**

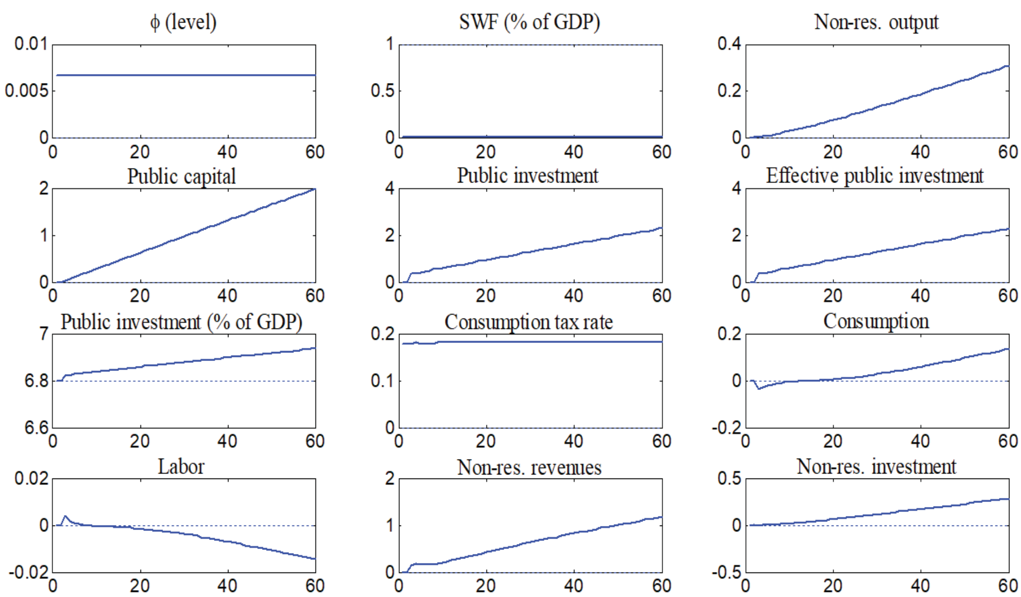


Figure 4 shows the dynamics of public capital increase by 35.32 percent at $b = 0.3$ with its adjustment speed of 0.001. The saving share of resource revenues ϕ_t is at its constant calibrated value, while the SWF is zero. The gradual scaling up of public investment is financed by a steady increase of consumption tax rate, which reduces consumption in the beginning and is thus not welfare improving.

Table 4. **No resource windfall:** $\alpha^G = 0.1$, $\beta = 0.98$	$b = 0.1$	$b = 0.2$	$b = 0.3$
Effective public investment per \$1 invested	0.6613	0.6613	0.6604
Optimal increase of public capital	64.74%	44.15%	35.32%
Optimal adjustment speed	0.019	0.024	0.024
Optimal increase of public investment at new SS	74.4%	52.6%	43.4%
Consumption tax rate increase at new SS	40.6%	28.75%	23.9%
St dev of consumption growth over first 10 years	0.44	0.38	0.31
Welfare gain w.r.t. original steady state	-0.51%	-0.42%	-0.35%

Table 4 is produced at a higher discount factor $\beta = 0.98$ than the baseline one $\beta = 0.91$, which corresponds to a real interest rate of 2 percent. A high discount factor implies that households are patient and value the future more today, than impatient households with their low discount factor. The former case, giving preferences for savings and investments, suggests a significantly high optimal increase of public investment in Table 4 relative to Table 3, which appears to be identical to the main results with a resource windfall in Table 1. The more households are patient, the more public investment is preferred, but since there is no resource windfall, consumption tax increases to finance the public investment; thus, welfare gain falls more. In other words, under no resource windfall, the economy is worse off than in its initial steady state without any public investment, suggesting that it matters how fiscal expenditures are financed. Commodity-rich economies can benefit and improve their welfare by investing returns from their natural resources domestically, using the front-loaded public investment policy.

3.6 Conclusion

This paper examines the optimal fiscal policy to accumulate public capital through investing resource revenues domestically rather than saving them abroad in the SWF of resource-rich low-income countries. The model is a modified version of Berg et al. (2013)

in several respects: The fiscal policy rule is expressed in terms of public capital as a stock variable and the public investment path is obtained from capital. The tightness of absorptive capacity constraints is captured by a single parameter b in the equation for effective public investment. External saving is a clearing fiscal instrument rather than distortionary consumption tax, and there is a variable share of resource revenues saved in the SWF as opposed to its fixed share.

This study finds the optimal level of public capital and its adjustment speed to that new increased steady state. The associated optimal public investment path is front-loaded regardless of absorptive capacity constraints and productivity of public capital. Less productive public capital suggests the lower magnitude of increase for capital and public investment, but should move at a faster adjustment speed to its new steady state level. The gradual non-overshooting increase of public investment causes consumption volatility and is not preferred under a no resource windfall either, since consumption tax becomes the only source for financing fiscal expenditures within this model. To conclude, resource-rich low-income countries can significantly gain from their commodity blessing by prudent public investment policy.

3.7 Appendix

3.7.1 Table of parameters

Parameter	Definition
$\beta = 0.909$	discount factor
$\sigma = 2$	inverse of intertemporal elasticity of substitution for consumption
$\psi = 10$	inverse of Frisch elasticity of labor supply
$\varphi = 0.5$	home-bias in private consumption
$\nu = 0.6$	home-bias in government purchases
$\nu_g = 0.5$	home-bias in government purchases above the initial steady state
$\chi = 0.44$	elasticity of substitution between T and N goods
$\rho = 1$	elasticity of substitution between two types of labor
$\alpha^T = 0.65$	labor income share in traded sector
$\alpha^N = 0.45$	labor income share in non-traded sector
$\alpha^G = 0.1$	output elasticity of public capital
$\alpha^o = 0.9$	resource capital income share
$d, \rho_{zT} = 0.1$	learning-by-doing externalities
$\kappa^T, \kappa^N = 25$	investment adjustment cost in T and N sectors
$\delta^T, \delta^N, \delta^g, \delta^o = 0.1$	depreciation rates for K^T, K^N, K^G, and K^o
$\epsilon = 0.7$	public investment efficiency
$\iota^{\text{div}} = 0.4$	share of resource dividends accrued to the government
$\tau^c = 0.18$	consumption tax rate
$\tau^l = 0.08$	labor tax rate
$\tau^o = 0.58$	resource royalty rate
$r^* = 0.027$	real interest rate of SWF
$b = 0.1, 0.2, 0.3$	tightness of absorptive capacity constraints
$\phi = 0.0065$	constant share of resource revenues in external savings
$\rho_{FDI} = 0.8$	persistence of FDI process
$\sigma_{FDI} = 6.26$	standard deviation of FDI shock
$\rho_{swf} = 0.956$	AR(1) coefficient in SWF process
$\frac{C}{GDP} = 57.2\%$	consumption in percent of GDP
$\frac{C^T}{GDP} = 28.6\%$	consumption of traded goods in percent of GDP
$\frac{I}{GDP} = 17\%$	investment in percent of GDP
$\frac{Y^o}{GDP} = 9\%$	resource output in percent of GDP
$\frac{G^C}{GDP} = 13.3\%$	public consumption in percent of GDP
$\frac{G^I}{GDP} = 6.8\%$	public investment in percent of GDP
$\frac{G^T}{GDP} = 8.04\%$	government purchases of traded goods in percent of GDP
$\frac{EX}{GDP} = 21.6\%$	exports in percent of GDP
$\frac{B}{GDP} = 11.6\%$	public debt in percent of GDP
$\frac{SWF^*}{GDP} = 1\%$	SWF in percent of GDP

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